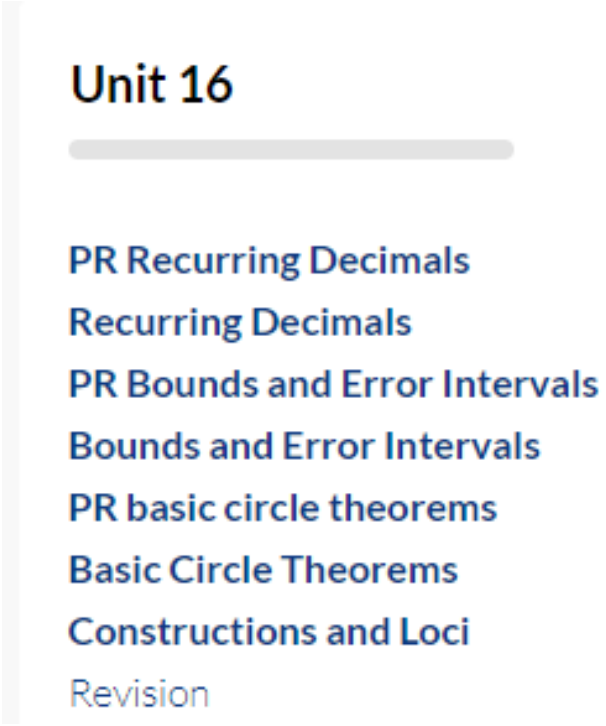
Mathematics
UNIT 16

Name: _____

Class: _____

Contents Page

Please see unit 16 course on drfrostmaths.com



PR – converting a fraction into a decimal

Recall from earlier – definition of a RATIONAL NUMBER

A rational number is any number that can be written as $\frac{a}{b}$ where a and b are integers.

Rational numbers can be located exactly on the number line.
These numbers are rational:

-3 $\quad -1\frac{1}{2}$ $\quad 0$ $\quad \frac{3}{4}$ $\quad 3$ $\quad 5\frac{1}{2}$ $\quad 6.4$

as they can be written in the form $\frac{a}{b}$

i.e. $\frac{-3}{1}$ $\quad \frac{-3}{2}$ $\quad \frac{0}{1}$ $\quad \frac{3}{4}$ $\quad \frac{3}{1}$ $\quad \frac{11}{2}$ $\quad \frac{64}{10}$

	Fraction	Factorised	The Law of Cancellation	Simplest Form	Factors of Denominator	Kind of Decimal
(i)	$\frac{8}{12}$					
(ii)	$\frac{3}{16}$					
(iii)	$\frac{9}{27}$					
(iv)	$\frac{12}{30}$					
(v)	$\frac{7}{32}$					
(vi)	$\frac{15}{21}$					
(vii)	$\frac{3}{10}$					
(viii)	$\frac{3}{18}$					
(ix)	$\frac{6}{33}$					
(x)	$\frac{3}{75}$					

Recurring Decimals to fractions

Examples: how do we write the following using 'dot' notation:

(a) 0.5555...

(b) 0.1111...

(c) 0.121212...

(d) 0.363636...

(e) 0.919191...

(f) 0.727272...

(g) 0.125125...

(h) 0.621621...

(i) 0.204204...

Recurring Decimals to Fraction – Algebraic Proof

How do we ELIMINATE the recurring part of a decimal.

Take for example $x = 0.\dot{1}\dot{7}$ (or 0.17171717171717….)

We can **scale** up x to $10x, 100x, 1000x$ ….

Which if these would have the same recurring part after the decimal point?

So how do we eliminate the recurring part?

What about $x = 0.0\dot{1}\dot{7}$ (or 0.017171717171717….)

ACTIVITY: go back a page and do the above for all the questions

Worked Example	Thinking	Your Turn
Express as a simplified fraction: $0.22222222\ldots$ $0.8888888\ldots$		Express as a simplified fraction: $0.777777777\ldots$

Worked Example	Thinking	Your Turn
Express as a simplified fraction: $0.4949494949\ldots$ $0.5454545454\ldots$		Express as a simplified fraction: $0.2727272727\ldots$

Worked Example	Thinking	Your Turn
Express as a simplified fraction: $0.365365365365\ldots$ $0.279279279279\ldots$		Express as a simplified fraction: $0.837837837837\ldots$

Worked Example	Thinking	Your Turn
Express as a simplified fraction: $1.4545454545\ldots$ $2.3737373737\ldots$		Express as a simplified fraction: $4.747474747474\ldots.$

Worked Example	Thinking	Your Turn
Express as a simplified fraction: $2.34545454545\ldots$ $3.78989898989\ldots$		Express as a simplified fraction: $7.57979797979\ldots$

Worked Example	Thinking	Your Turn
Express as a simplified fraction: $2.3456456456456\ldots$ $3.7654654654654\ldots$		Express as a simplified fraction: $7.530930930930\ldots$

Worked Example	Thinking	Your Turn
Write the fraction $0.1\dot{3}\dot{6} \times 0.\dot{5}$ as a fraction in its simples form		Write the fraction $0.6\dot{8}\dot{1} \times 0.\dot{1}$ as a fraction in its simples form

Exam-style question 1

a) Write $\frac{5}{14}$ as a decimal.

b) Write $0.0\dot{5}\dot{7}$ as a fraction.

Exam-style question 2

Show that $1.\dot{9} = 2$

Exam-style question 3

List the following in order of size, from smallest to largest:

$$\frac{34}{99},\ 0.34,\ \frac{1}{3},\ \frac{340}{999},\ 0.3$$

Challenge

Can you find the value of this infinite sum?

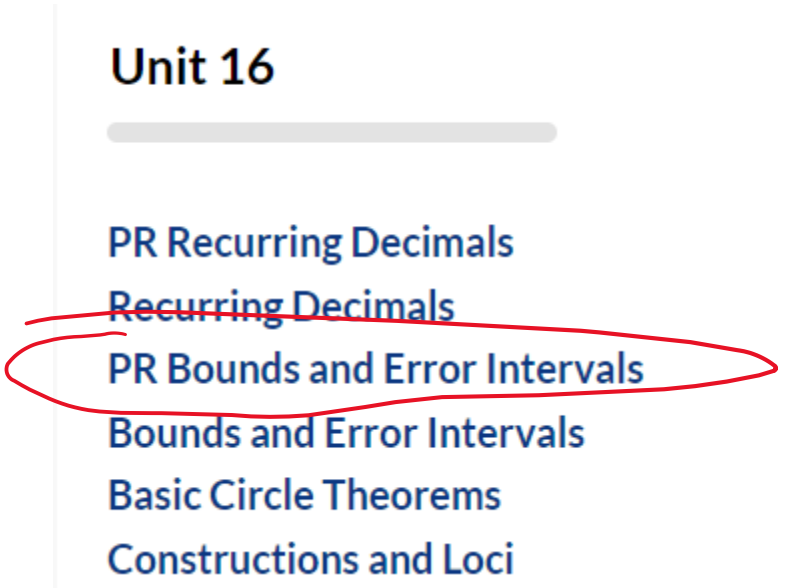
$$\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\frac{1}{10{,}000}+\ldots$$

Write your answer as a fraction.

Extra Notes

Pre Requisite Work

See Skills and Exam Qs to check below:



Bounds of Accuracy

BOUNDS OF ACCURACY

Introduction

When someone says that a distance is 50 metres, what do they mean? Measurements in real life can never be made with absolute accuracy – there is always a certain amount of error. So 50 metres could be accurate to the nearest metre, or to the nearest 10 metres, for example. Knowing within what interval the true distance lies can be very important in many applications of mathematics. When measurements are combined in a calculation, and each value has a certain amount of error, things can get complicated – and sometimes the result can be counterintuitive.

A number has been rounded to 30 to the nearest 10.

What could the number be?

What is the lowest and highest possible value it could be?

Upper and Lower Bounds

This smallest possible value is called the lower bound. The largest possible value is called the upper bound.

When a measure is expressed to a given unit, the maximum error is half of this unit.

For a value x, the error interval is:
least possible value $\leq x <$ greater possible value

Upper and Lower Bounds (1)

I DO	WE DO	YOU DO
60 has been rounded to the nearest 10. Use the number line to work out what the lowest and highest values could be. →	1200 has been rounded to the nearest 100. Use the number line to work out what the lowest and highest values could be. →	Use the number line to work out what the lowest and highest values could be in each of the following cases. 60 has been rounded to the nearest 10. → 2700 has been rounded to the nearest 100. →
0.4 has been rounded to the nearest 0.1. Use the number line to work out what the lowest and highest values could be. →	2.73 has been rounded to the nearest 0.01. Use the number line to work out what the lowest and highest values could be. →	3.7 has been rounded to the nearest 0.1. → 8.13 has been rounded to the nearest 0.01. →

Value	Rounded To	Lower Bound	Upper Bound	Error Interval
6000	Nearest 1000			
6000	Nearest 100			
600	Nearest 100			
600	Nearest 10			
6000	Nearest 10			
60	Nearest 10			
60	Nearest Whole			
6	Nearest Whole			
6000	Nearest Whole			
600	Nearest Whole			

Upper and Lower Bounds (2)

I DO	WE DO	YOU DO
For each of the following: a) Use the number line to work out what the lowest and highest values could be. b) Write your answer to part a) as an inequality (error interval)		
4.3 rounded to 1 dp.	10.36 rounded to 2 dp.	**ANSWER IN YOUR BOOKS** a) 4.8 rounded to 1 dp b) 5.8 rounded to 1 dp. c) 11.6 rounded to 1 dp. d) 11.61 rounded to 2 dp. e) 11.16 rounded to 2 dp. f) 14.16 rounded to 2 dp. g) 14.162 rounded to 3 dp. h) 14.160 rounded to 3 dp.

Extension: Determine the error intervals (as an inequality) for each of the I Do, We Do, You Do questions in ***Upper and Lower Bounds (1)***

Upper and Lower Bounds (3)

I DO	WE DO	YOU DO
For each of the following: a) Use the number line to work out what the lowest and highest values could be. b) Write your answer to part a) as an inequality (error interval)		
8 rounded to 1 sf.	8.2 rounded to 2 sf.	**ANSWER IN YOUR BOOKS** a) 6 rounded to 1 sf b) 6.4 rounded to 2 sf. c) 6.8 rounded to 2 sf. d) 3.4 rounded to 2 sf. e) 13.4 rounded to 3 sf. f) 1.34 rounded to 3 sf. g) 0.134 rounded to 3 sf. h) 0.1034 rounded to 4 sf.

Intelligent practice

Value	Rounded To	Place Value	+/−	Lower Bound	Upper Bound	Error Interval
400	1 Significant Figure					
400	3 Significant Figures					
400	2 Significant Figures					
40	2 Significant Figures					
4	2 Significant Figures					
4	1 Significant Figure					
0.4	1 Significant Figure					
0.3	1 Significant Figure					
30	2 Significant Figures					
0.03	1 Significant Figure					

Try these *Complete the following table:*

	Measurement	Largest possible error	Upper bound	Lower bound
Height of a tree	50 m to the nearest m			
Mid-day temperature	28°C to the nearest degree			
Weight of a letter	32 g to the nearest g			
Time to complete task	40 minutes to the nearest minute			
Length of caterpillar	3.4 cm to 1 decimal place			
Patient's temperature	38.6°C to 1 decimal place			
Weight of parcel	2.9 kg to 1 decimal place			
Time to reach 60 mph	6.2 seconds to 1 decimal place			
Length of shelf	2.75 m to 2 decimal places			
Weight of fish	1.64 kg to 2 decimal places			
Sprint time	10.27 seconds to 2 decimal places			
Height of a hill	480 m to the nearest 10 m			
Width of drive	560 cm to the nearest 10 cm			
Weight of cake	1200 g to the nearest 10 g			
Weight of cake	1200 g to the nearest 100 g			
Length of a runway	1900 m to 2 significant figures			
Length of a runway	1900 m to 3 significant figures			
Weight of an aircraft	170 000 kg to 2 significant figures			
Weight of an aircraft	170 000 kg to 3 significant figures			

Extension

Question 1: Declan is considering buying a sofa that is 207cm long.
The space that Declan wants to the sofa is 210cm to the nearest 10cm.
Should Declan buy the sofa?

Question 2: Mr Jones wants to buy a notebook for every student in year 11.
He knows there are 300 students in year 11 to the nearest 100.
What is the greatest possible number of notebooks that he would have to buy?

Question 3: The length of a field is 400m to the nearest 10m.
Rebecca says the lower bound is 350m and the upper bound is 450m.
Is she correct?

Question 4: The table shows the prices of posting large letters.
Gerard wants to post two large letters:

		1st Class
Size	Weight up to and including	Price
Large Letter	100g	95p
	250g	£1.26
	500g	£1.68
	750g	£2.42

- a large letter weighing 230g to the nearest 10 grams
- a large letter weighing 500g to the nearest 10 grams

(a) What is the smallest possible price for posting both large letters?

(b) What is the greatest possible price for posting both large letters?

Question 5: Below is a question that was posted online and the results.
Explain which answer you agree with.

A number has been rounded to 10, correct to 1 significant figure. What are the lower and upper bounds?

30% 5 and 15

55% 9.5 and 10.5

12% 9.5 and 15

3% 7.5 and 12.5

397 votes • Final results

Calculating with Upper and Lower Bounds (1)

I DO

Two numbers a and b have been rounded.

a is 300 to the nearest 100.
b is 80 to the nearest 10.

Complete the table below.

LB		UB
	a 300	
	b 80	

What would the LB and UB be for

$\boldsymbol{a + b}$

WE DO

Two numbers c and d have been rounded.

c is 6000 to the nearest 1000.
d is 50 to the nearest 10.

Complete the table below.

LB		UB
	c 6000	
	d 50	

What would the LB and UB be for

$\boldsymbol{c + d}$

YOU DO

ANSWER IN YOUR BOOKS

Four numbers e, f, g and h have been rounded.

e is 4000 to the nearest 1000.
f is 200 to the nearest 100.
g is 120 to the nearest 10.
h is 70 to the nearest 10.

Copy and complete the table below

LB		UB
	e	
	f	
	g	
	h	

Calculate the LB of:

$e + f \quad g + h \quad e + g \quad g + 2f$

Calculate the UB of:

$e + h \quad h + e \quad 2e + g \quad 3e + 4g$

Calculating with Upper and Lower Bounds (2)

I DO

Two numbers p and q have been rounded.

p is 320 to the nearest 10.
q is 7.4 to the nearest 0.1.

Complete the table below.

LB		UB
	p 320	
	q 7.4	

What would the LB and UB be for

$\boldsymbol{p - q}$

WE DO

Two numbers r and s have been rounded.

r is 700 to the nearest 100.
s is 3 to the nearest integer.

Complete the table below.

LB		UB
	r 700	
	s 3	

What would the LB and UB be for

$\boldsymbol{r - s}$

YOU DO

ANSWER IN YOUR BOOKS

Four numbers t, u, v and w have been rounded.

t is 8000 to the nearest 1000.
u is 70 to the nearest 10.
v is 8.3 to the nearest 0.1.
w is 2.78 to the nearest 0.01.

Copy and complete the table below

LB		UB
	t	
	u	
	v	
	w	

Calculate the LB of:

$t - u \quad u - t \quad 2t - v \quad v - w$

Calculate the UB of:

$t - u \quad v - w \quad 3t - w \quad w - 3t$

Calculating with Upper and Lower Bounds (3)

I DO

x is 6200 to the nearest 100.
y is 20 to the nearest 10.

Complete the table below.

LB		UB
	x 6200	
	y 20	

What would the LB and UB be for

$\boldsymbol{xy}$

$\boldsymbol{\frac{x}{y}}$

WE DO

x is 1200 to 2sf.
y is 0.75 to 2dp.

Complete the table below.

LB		UB
	x 1200	
	y 0.75	

What would the LB and UB be for

$\boldsymbol{xy^2}$

$\boldsymbol{\frac{x}{y}}$

YOU DO

x is 170 to the nearest 10.
y is 0.6 to 1dp.

Complete the table below.

LB		UB
	x 170	
	y 0.6	

What would the LB and UB be for

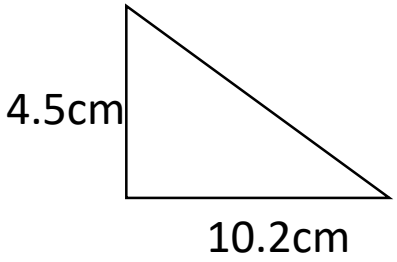
$\boldsymbol{xy}$

$\boldsymbol{\frac{x}{y}}$

Upper and Lower Bounds with Geometry

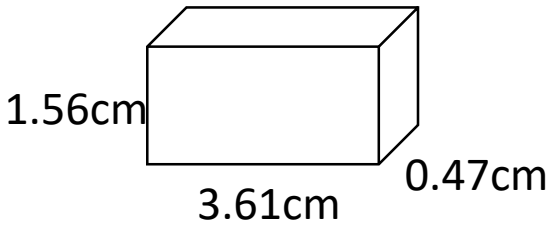
I DO

The height and width of the triangle below have been rounded to 1dp.
Work out the LB and UB for the **area** of the triangle.



WE DO

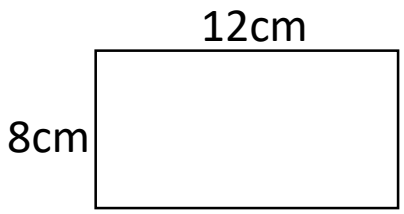
The dimensions of the cuboid below have been rounded to 2dp.
Work out the LB and UB for the **volume** of the cuboid.



Upper and Lower Bounds with Geometry

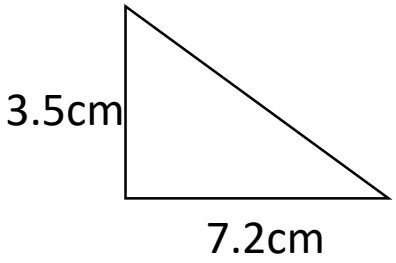
YOU DO 1

The height and width of the rectangle below have been rounded to the **nearest whole number**.
Work out the LB and UB for the **area** of the rectangle.



YOU DO 2

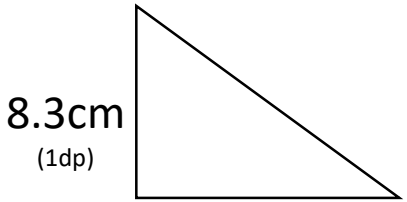
The height and width of the triangle below have been rounded to **1dp**.
Work out the LB and UB for the **area** of the triangle.



Upper and Lower Bounds with Geometry

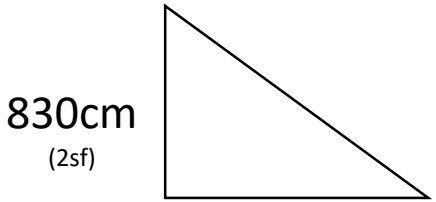
YOU DO 3

The height and width of the triangle below have been rounded as shown in brackets.
Work out the LB and UB for the **area** of the triangle.



YOU DO 4

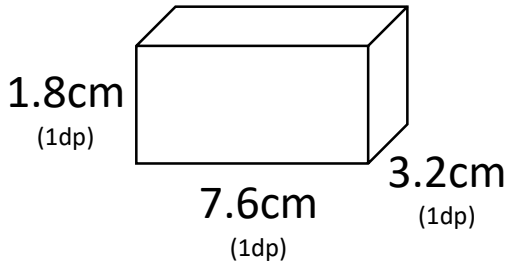
The height and width of the triangle below have been rounded as shown in brackets.
Work out the LB and UB for the **area** of the triangle.



Upper and Lower Bounds with Geometry

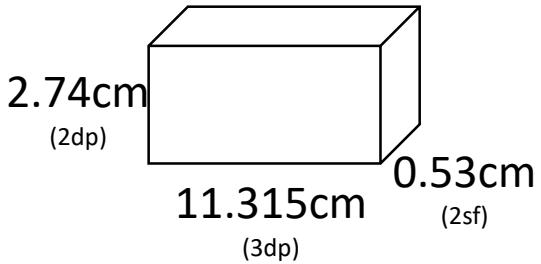
YOU DO 5

The dimensions of the cuboid below have been rounded as shown in brackets.
Work out the LB and UB for the **volume** of the cuboid.



YOU DO 6

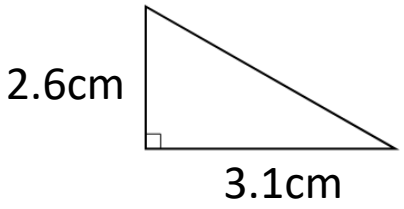
The dimensions of the cuboid below have been rounded as shown in brackets.
Work out the LB and UB for the **volume** of the cuboid.



Upper and Lower Bounds with Geometry – Pythagoras' Theorem

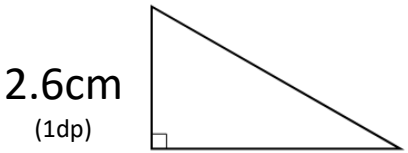
WE DO

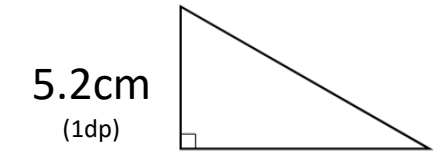
Use Pythagoras' Theorem to find the UB of the **hypotenuse** below.
All lengths are correct to **1dp**.

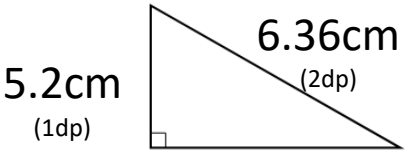


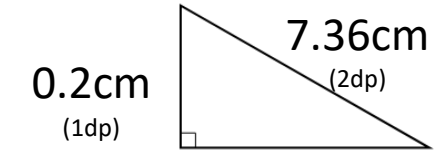
YOU DO

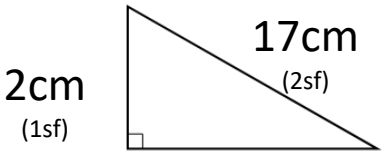
Use Pythagoras' Theorem to find the LB and UB of the missing sides below.
All lengths have been rounded as shown in brackets.

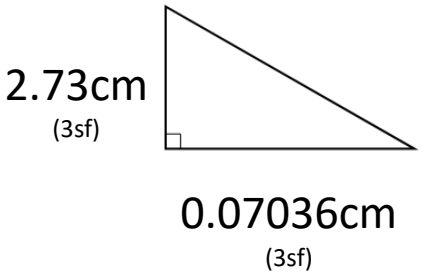












Upper and Lower Bounds Revision

(a)	(b)	(c)	(d)
Find the upper and lower bounds of 286 metres to the nearest metre.	Find the upper and lower bounds of 21 cm to the nearest cm.	Find the upper and lower bounds of 7.8 cm to 1 decimal place.	Find the upper and lower bounds of 5.24 kg to 2 decimal places.
(e)	**(f)**	**(g)**	**(h)**
Find the upper and lower bound of 80 cm to 1 significant figure.	Find the upper and lower bound of 5.6 kg to 2 significant figures.	A square has a side length of 4.1 cm to 1 decimal place. Find the lower bound of the perimeter of the square.	A rectangle measures 10 cm by 15 cm, both to the nearest cm. Find the upper bound of the area of the rectangle.
(i)	**(j)**	**(k)**	**(l)**
$a = b - c$ $c = 18$ correct to 2 significant figures. $b = 4.7$ correct to 1 decimal place. Find the upper and lower bounds of a.	$p = \frac{q}{r}$ $q = 20$ correct to 1 significant figure. $r = 6.3$ correct to 1 decimal place. Find the lower bound of p to 3 significant figures.	$c = \frac{d - e}{f}$ $d = 46, e = 8.5, f = 15,$ all correct to 2 significant figures. Find the upper bound of c to 2 decimal places.	$x = \frac{3a}{g - b}$ $a = 28, b = 12, g = 18,$ all correct to 2 significant figures. Find the lower bound of x to 3 significant figures.

Truncation

When we truncate a number, we find an estimate for the number without doing any rounding. To truncate a number, we miss off digits past a certain point in the number, filling in zeros if necessary to make the truncated number approximately the same size as the original number.

- To truncate a number to 1 decimal place, miss off all the digits after the first decimal place.
- To truncate a number to 2 decimal places, miss off all the digits after the second decimal place.
- To truncate a number to 3 significant figures, miss off all the digits after the first 3 significant figures (the first non-zero digit and the next two digits). Fill in any spaces with zeros to make the number approximately the same size as the original value.

Examples

Truncate 3.784 to 1 decimal place and 2 decimal places

- 3.784 truncated to 1 decimal place is 3.7
- 3.784 truncated to 2 decimal places is 3.78

Truncate $63{,}854$ and 0.04988 to 3 significant figures

- $63{,}854$ truncated to 3 significant figures is 63800
- 0.04988 truncated to 3 significant figures is 0.0498

Worked Example	Your Turn
A number, x, when truncated to 1 decimal place, is equal to 123.4. a) Write the upper bound. b) Write the lower bound. c) Write the error interval.	A number, x, when truncated to 1 decimal place, is equal to 567.8. a) Write the upper bound. b) Write the lower bound. c) Write the error interval.

Worked Example	Your Turn
A number, x, when truncated to 2 decimal places, is equal to 12.34. a) Write the upper bound. b) Write the lower bound. c) Write the error interval.	A number, x, when truncated to 2 decimal places, is equal to 56.78. a) Write the upper bound. b) Write the lower bound. c) Write the error interval.

Error Intervals

Video 377 on www.corbettmaths.com

Examples

Click here

Scan here

Workout

Question 1: The mass of a coin is 8 grams to the nearest gram.
Complete the error interval for the mass of the coin

..... g ≤ mass < ..... g

Question 2: The distance between two cities is 900km to the nearest 100km.
Complete the error interval for the distance

..... km ≤ distance < ..... km

Question 3: Frank rounds a number, *y*, to the nearest ten.
His result is 20
Write down the error interval for *y*

Question 4: Lily rounds a number, *y*, to the nearest whole number.
Her result is 5
Write down the error interval for *y*

Question 5: Freya rounds a number, *y*, to one decimal place.
Her result is 6.4
Write down the error interval for *y*

Question 6: Oscar rounds a number, *y*, to the nearest integer.
His result is 100
Write down the error interval for *y*

Question 7: A number, *n*, is rounded to 1 decimal place.
The result is 1.3
Using inequalities, write down the error interval for *n*.

Question 8: A number, *n*, is rounded to 2 decimal places.
The result is 6.27
Using inequalities, write down the error interval for *n*.

Question 9: Elliott weighs 56.2kg.
This mass, *m*, is to the nearest 100g.
Write the error interval due to rounding.

Error Intervals

Video 377 on www.corbettmaths.com

Question 10: A number, *x*, is 21 when rounded to 2 significant figures.
Write down the error interval.

Question 11: A number, *y*, is 15000 when rounded to 2 significant figures.
Write down the error interval.

Question 12: A number, *y*, is 680000 when rounded to 3 significant figures.
Write down the error interval.

Question 13: The length of a line, *l*, was given as 2.8cm, truncated to 1 decimal place.
Complete the error interval for *l*

..... *cm* ≤ *l* < ..... *cm*

Question 14: A number, *y*, is 0.37 when truncated to 2 decimal places.
Complete the error interval for y

..... ≤ *y* < .....

Question 15: A number, *n*, is truncated to 1 decimal place.
The result is 18.1
Using inequalities, write down the error interval for *n*.

Question 16: A number, *n*, is truncated to 3 decimal places.
The result is 4.066
Using inequalities, write down the error interval for *n*.

Apply

Question 1: The length of each side of a regular hexagon is 4.7cm to 1 decimal place.
Write the error interval for the perimeter, P

Question 2: Grace and George complete a crossword.
It takes Grace 9 minutes to complete the crossword to the nearest minute.
It takes George 11 minutes to complete the crossword to the nearest minute.

Show that the total time for both people to complete the crossword could be 20 minutes 50 seconds.

Question 3: A man jogs 200 metres to the nearest 10 metres.
It takes him 30 seconds to the nearest 10 seconds.

Work out the error interval for his speed, *s*.

Applying Limits of Accuracy

Video 184 on www.corbettmaths.com

Examples

Apply

Click here

Scan here

Question 1: India has pieces of ribbon, each measuring 7cm to the nearest centimetre.

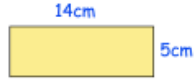
(a) find the least total length of 10 pieces of ribbon.
(b) find the greatest total length of 10 pieces of ribbon.

Question 2: Leon has four barrels, each with a mass of 30kg to the nearest 10 kg.

(a) find the the minimum possible mass of the four barrels.
(b) find the maximum possible mass of the four barrels.

Question 3: A rectangle has a length of 14cm and width of 5cm, both to nearest centimetre.

Find (a) the maximum possible area and
(b) the minimum possible area.



Question 4: The classes in a primary school have 20 students to the nearest 10.
There are 7 classes in the primary school.
Work out the greatest possible number of students that attend the school.

Question 5: Harry and Peter take part in a race.
It took Harry 30 seconds to the nearest 10 seconds to finish the race.
It took Peter 43 seconds to the nearest second to finish the race.
Work out the minimum possible difference between their finishing times.

Question 6: An average orange weighs 131g to the nearest gram.
A net contains 8 oranges.
The net weighs 10g to the nearest gram

What is the maximum possible weight of the net of oranges?

Question 7: Megan has 2 litres of fruit juice to the nearest litre.
She pours the fruit juice into glasses that hold 100ml to the nearest 10ml.
Work out the lowest possible number of glasses she can fill.

Applying Limits of Accuracy
Video 184 on www.corbettmaths.com

Question 8: A rectangular football pitch has a width of 72m, measured to the nearest metre.
The length of the pitch is 105m, measured to the nearest 5 metres.

Work out the lower bound for the perimeter of the pitch.

Question 9: The lengths of time taken for 4 people to complete a puzzle are listed below.
Each time is given to one decimal place.

20.8 seconds 35.1 seconds 19.7 seconds 41.3 seconds

(a) Work out the greatest possible range
(b) Work out the smallest possible mean.

Question 10: Mr Rodgers wants to keep 28 new maths textbooks on a shelf in his classroom.
Each book has a mass of 700g correct to 1 significant figure.
The shelf can hold up to 20kg to the nearest kilogram.
Can the shelf safely hold the textbooks?

Question 11: The base of a triangle is 30cm, correct to 2 significant figures.
The height of the triangle is 40cm, correct to 1 significant figure.
Calculate the upper bound for the area of the triangle

Question 12: Kelly drove a distance of 120 miles, to the nearest 10 miles, in a time of 2 hours, to the nearest hour.
Work out the difference between Kelly's greatest possible and lowest possible average speed.

Question 13: Rosie is buying strawberries, apples and grapes for a picnic.
She buys 4kg of strawberries and 3kg of grapes, both to the nearest kilogram.
Rosie buys 50 apples to the nearest 10 apples.
A kilogram of strawberries costs £1.20 to the nearest 10p
A kilogram of grapes costs £1.30 to the nearest 10p
An apple costs 20p each to the nearest 10p.
Work out the upper bound for the amount of money Rosie would have to pay

Question 14: A circle has an area of 600cm^2 to 2 significant figures.
Work out the lower bound of the radius.

Question 15: $w = aT$
Given a = 15 correct to 2 significant figures
and w = 700 correct to 2 significant figures
Calculate the upper bound for T

Exercise 1

1 If a time t was 27 seconds correct to the nearest second, determine:

(a) The lower bound. ?

(b) The upper bound. ?

(c) The error interval. ?

2 If a time t was 80 seconds correct to the nearest 10 seconds, determine the error interval of t. ?

3 *[Edexcel GCSE(9-1) Nov 2017 3F Q23b, Nov 2017 3H Q5b]* Jess rounds a number, x, to one decimal place. The result is 9.8.
Write down the error interval for x.

?

4 *[Edexcel GCSE(9-1) June 2017 3F Q23aii]*
Harley's house has a value of £160 000 correct to 2 significant figures.
Write down the greatest possible value of the house. ?

5 A weight measurement w is truncated to 24.35kg to 2 decimal places. What is the upper bound of w? ?

6 *[Edexcel GCSE(9-1) Mock Set 3 Autumn 2017 3H Q8]*
Kiera used her calculator to work out the value of a number x. She wrote down the first two digits of the answer on her calculator. She wrote down 7.3.
Write down the error interval for x.

?

7 A laser measures a distance and displays 3 metres to some given degree of accuracy. Find the lower and upper bound when the accuracy was:

To the nearest metre. ?

To the nearest cm. ?

To the nearest mm. ?

8 An Events Organiser planning a concert is told that a stadium has a capacity of 30,000, correct to 1 significant figure. The organiser wants to ensure that anyone he sells tickets to is guaranteed a seat. How many tickets can he sell? ?

☠ A cube is $10m^3$ correct to 1 significant figure. What is the maximum and minimum possible side length?

?

UPPER AND LOWER BOUNDS

EXAM-TYPE QUESTIONS

Ref: G145. 1R1

A1 Zoe weighs 62 kg, correct to the nearest kilogram. Write down the lower bound for Zoe's weight.	**A2** The length of line $AB = 8.3$ cm, correct to 2 significant figures. Write down the upper bound for the length of AB.	**A3** Anu weighs 83 kg, correct to the nearest half kilogram. Write down the upper bound for Anu's weight.	**A4** The length of line $CD = 27$ cm, correct to the nearest 0.5 cm Write down the lower bound for the length of CD.
B1 Correct to the nearest millimetre, the length of a side of a regular hexagon is 3.6 cm Calculate the upper bound for the perimeter of the hexagon.	**B2** The perimeter of a square is 24 cm, correct to the nearest half centimetre. Work out the lower bound for the length of a side.	**B3** Correct to 1 significant figure, the area of a rectangle is 80 cm^2. Correct to 2 significant figures, the length of the rectangle is 12 cm. Calculate the upper bound for the width.	**B4** Correct to 2 significant figures the area of a square is 230 cm^2. Calculate the lower bound for the perimeter of the square.
C1 $x = 1.8$ correct to 1 decimal place. Calculate the lower bound for the value of $4x + 1$	**C2** Correct to 1 significant figure, $a = 20$ and $b = 5$ Work out the upper bound of $5(a - b)$	**C3** $x = p(q - r)$ $p = 42$, $q = 24$ and $r = 14$ all correct to 2 significant figures. Work out the lower bound for the value of x.	**C4** Correct to 2 significant figures, $w = 58$, $x = 28$ and $y = 18$ Calculate the upper bound of $\frac{w}{x - y}$
D1 Jada has 100 litres of oil, correct to the nearest litre. The oil is poured into tins of volume 1.5 litres, correct to one decimal place. Calculate the upper bound for the number of tins that can be filled.	**D2** There are 300 sheets of paper in a pile, correct to the nearest 10 sheets. The height of the pile is 160 mm, correct to the nearest 10 mm. Calculate the upper bound for the thickness of one sheet.	**D3** The distance to school is 2.8 km, correct to the nearest 0.1 km. Sam walks at a speed of 5 km/h, correct to the nearest km/h. Calculate the upper bound for the time Sam takes to walk to school.	**D4** Correct to 2 decimal places, the volume of a solid cube is 42.88 cm^3 Calculate the lower bound for the surface area of the cube.

Giving answer to “suitable degree of accuracy”

$$m = \frac{\sqrt{s}}{t}$$

$s = 3.47$ correct to 2 decimal places. $t = 8.132$ correct to 3 decimal places. By considering bounds, work out the value of m to a suitable degree of accuracy. You must show all your working and give a reason for your final answer.

Test Your Understanding

$$q = \frac{r^2}{s}$$

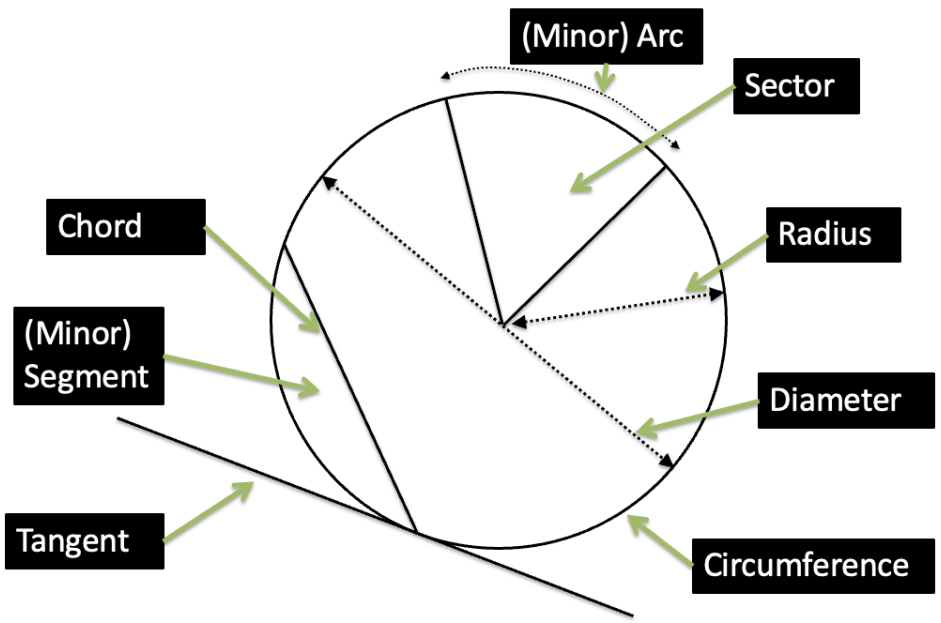
$r = 2.87$ correct to 2 decimal places. $s = 3.584$ correct to 3 decimal places. Work out the value of q to a suitable degree of accuracy, giving a reason for your answer.

Extra Notes

Circle Theorems

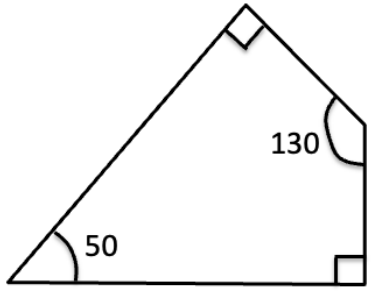
Circle Theorems

Parts of a Circle

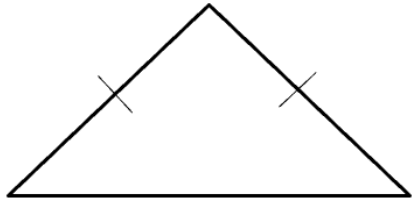


Non-Circle Theorems

These are not circle theorems, but are useful in questions involving circle theorems.



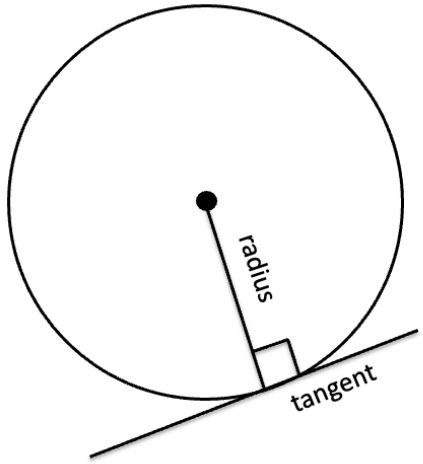
Angles in a quadrilateral add up to 360.



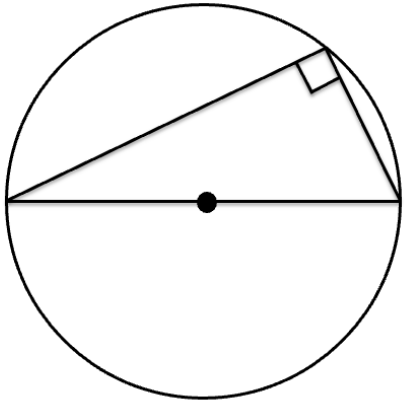
Base angles of an isosceles triangle are equal.

Circle Theorems

Circle Theorems Involving Right Angles



"Angle between radius and tangent is 90°".

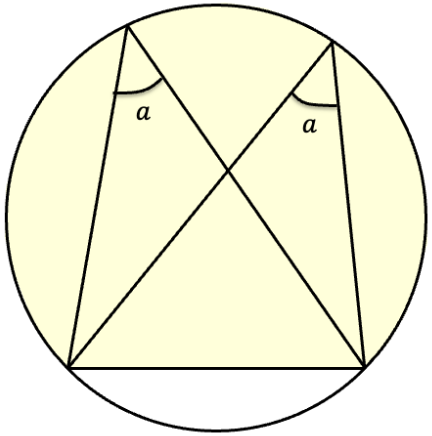


"Angle in semicircle is 90°."

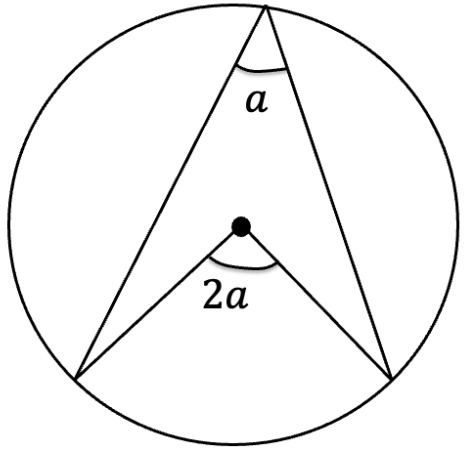
Note that the hypotenuse of the triangle MUST be the diameter.

Tip: Remember the wording in the black boxes, because you are often required to justify in words a particular angle in an exam.

Circle Theorems Involving Other Angles



"Angles in same segment are equal."

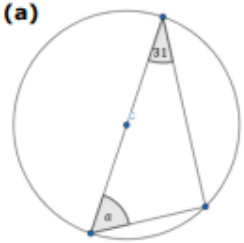


"Angle at centre is twice the angle at the circumference."

RIGHT-ANGLE IN A SEMI-CIRCLE PRACTICE GRID

Find the missing angles.

(a)



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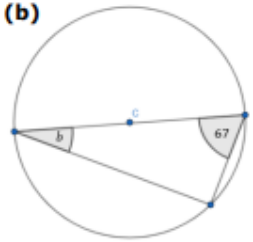

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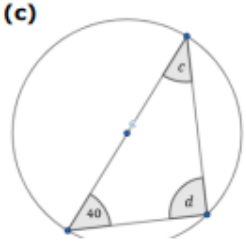

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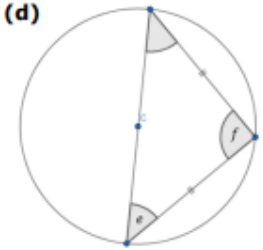

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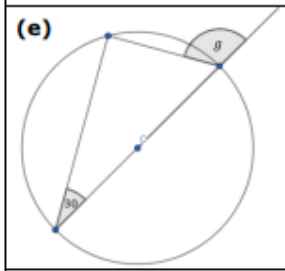

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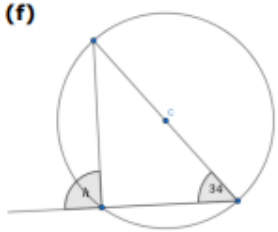

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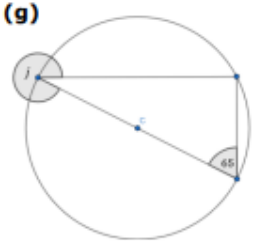

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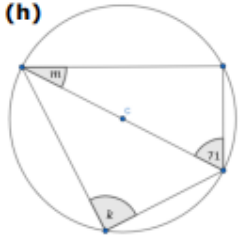

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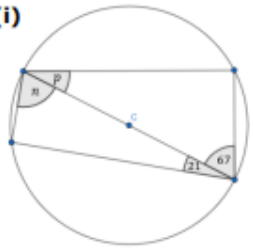

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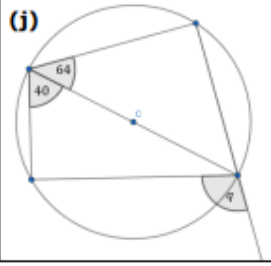
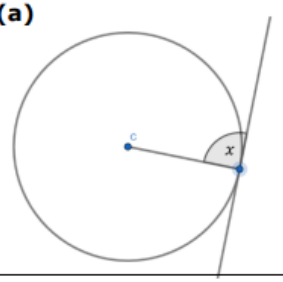


CIRCLE THEOREMS AND TANGENTS PRACTICE GRID

Find the missing angles.

(a)



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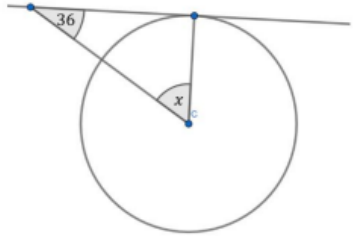

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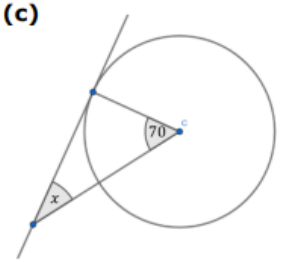

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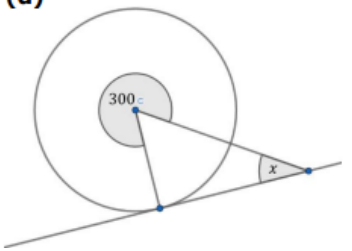
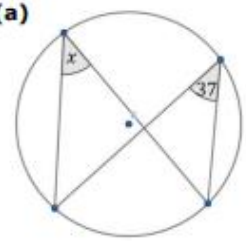


ANGLES FROM THE SAME SEGMENT PRACTICE GRID

Find the missing angles.

(a)



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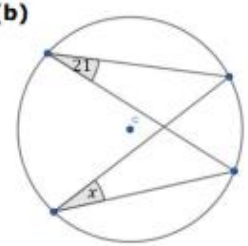

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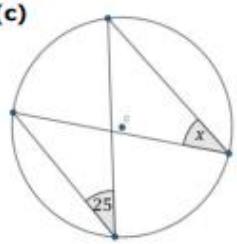

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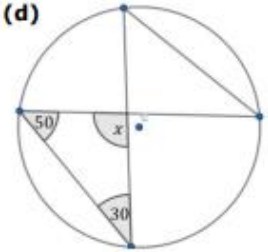

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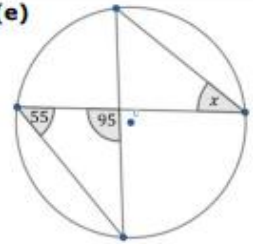

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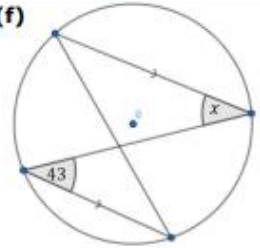

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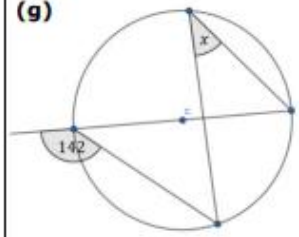

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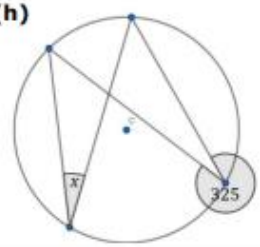

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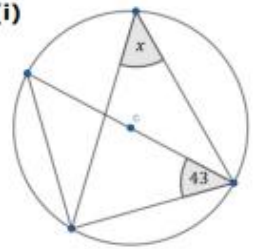

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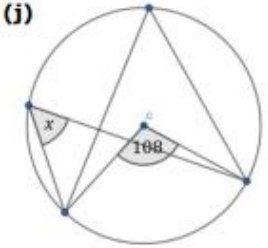
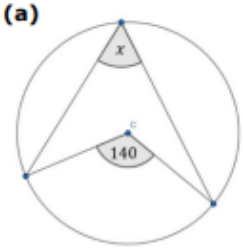


ANGLE AT THE CENTRE PRACTICE GRID

Find the missing angles.

(a)



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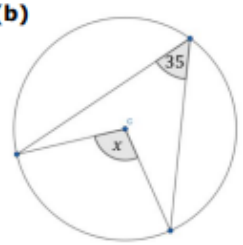

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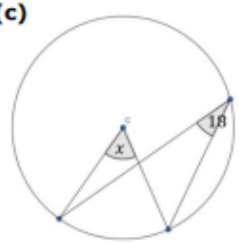

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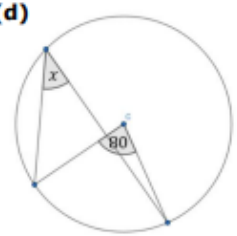

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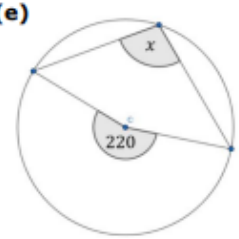

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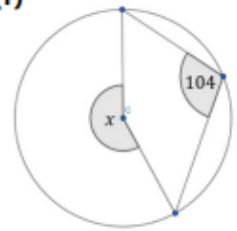

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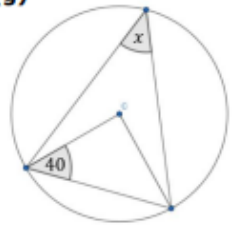

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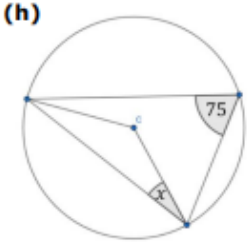

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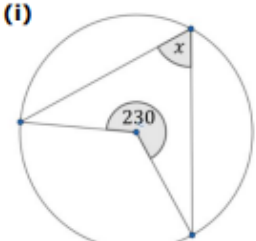

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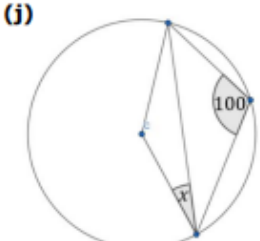
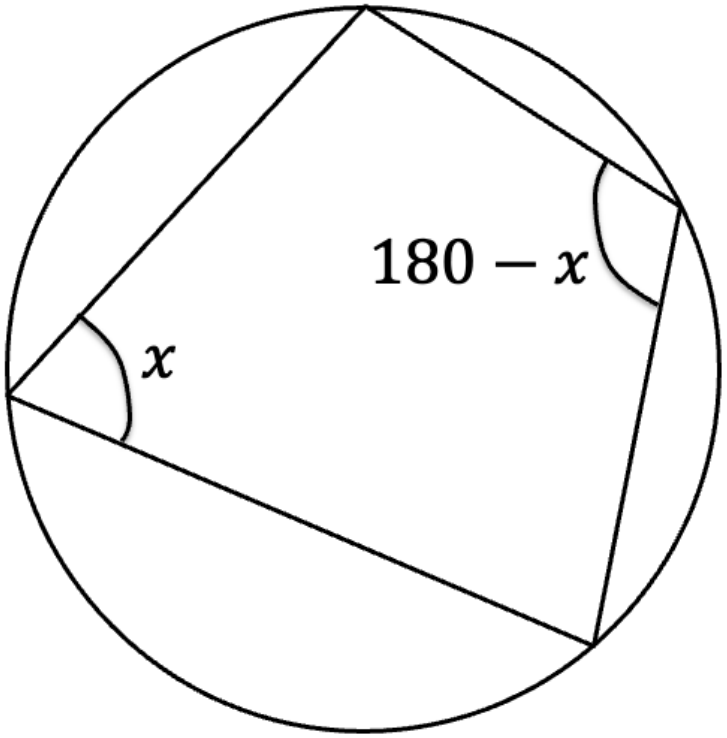


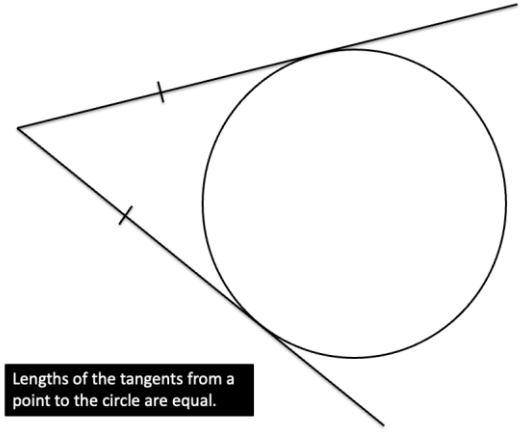
Circle Theorems

Circle Theorems Involving Other Angles

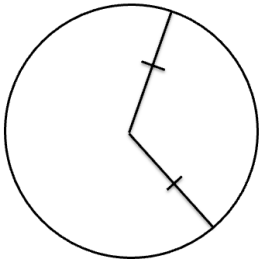


Opposite angles of **cyclic quadrilateral** add up to 180.

Circle Theorems Involving Lengths



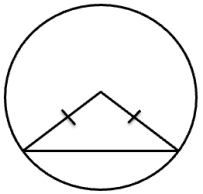
Lengths of the tangents from a point to the circle are equal.



Radius is of constant length

Tip: When you have multiple radii, put a mark on each of them to remind yourself they are the same length.

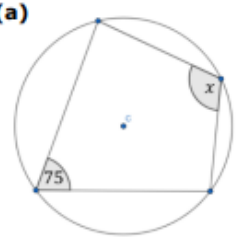
This result is that any triangle with one vertex at the centre, and the other two on the circumference, must be **isosceles**.



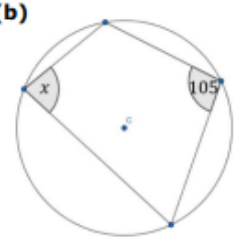
CYCLIC QUADRILATERALS PRACTICE GRID

Find the missing angles.

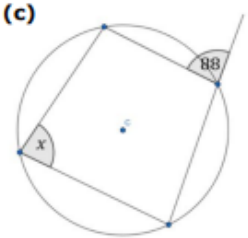
(a)



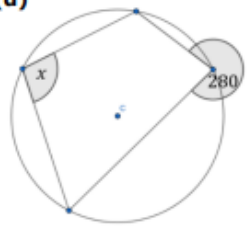
(b)



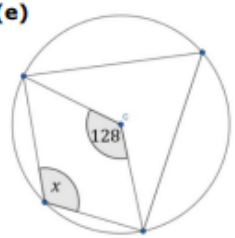
(c)



(d)



(e)



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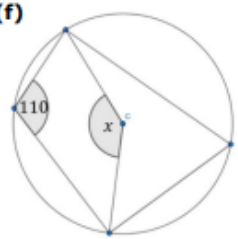
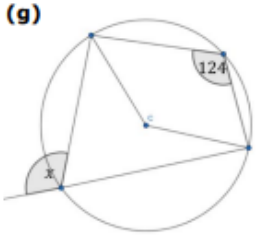
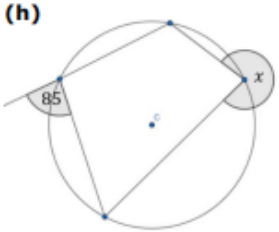


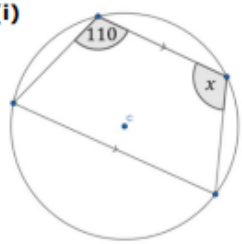
(g)



(h)



(i)



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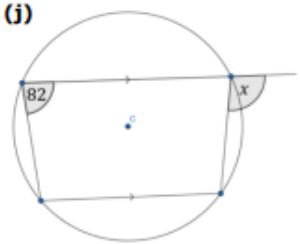

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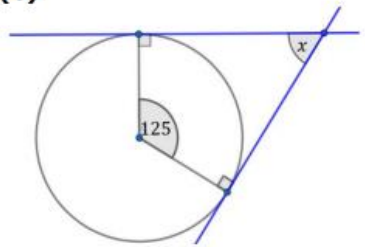

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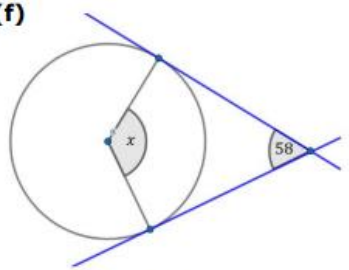

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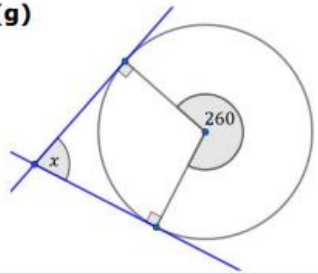

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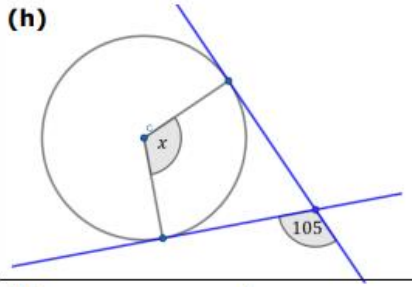

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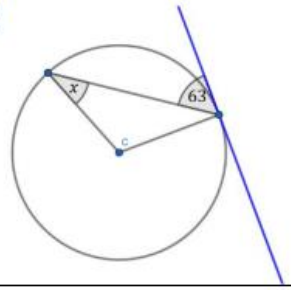

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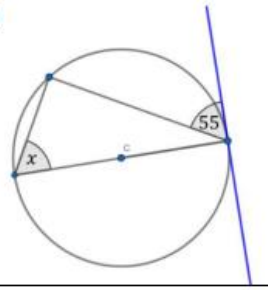
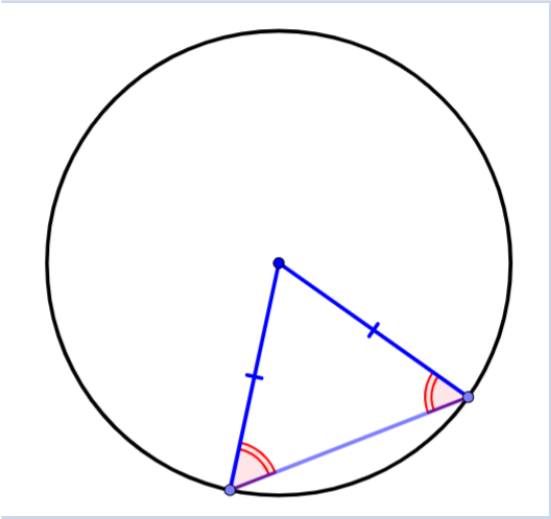
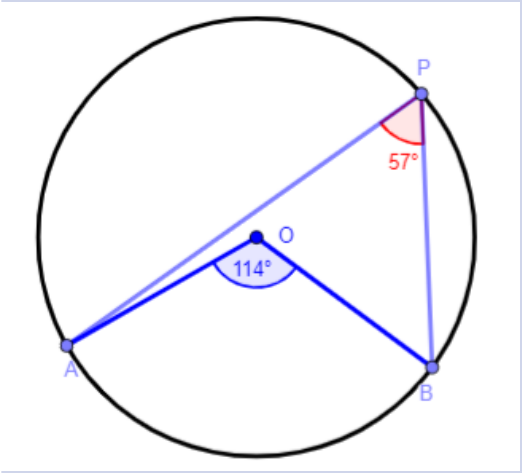
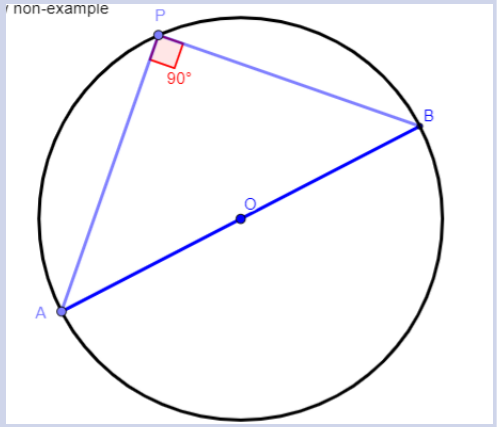


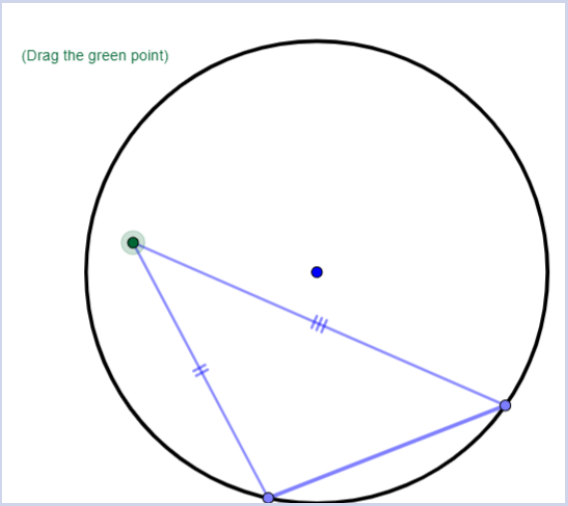
Examples

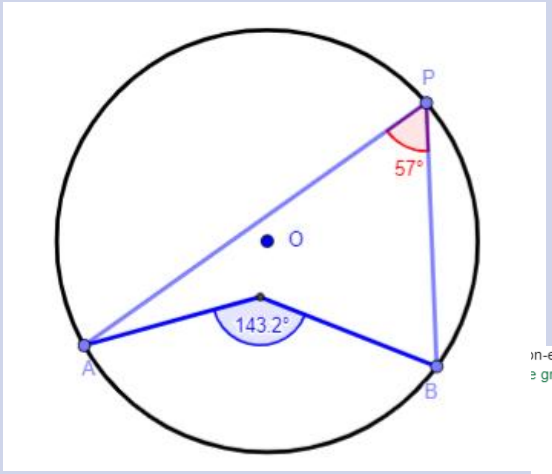


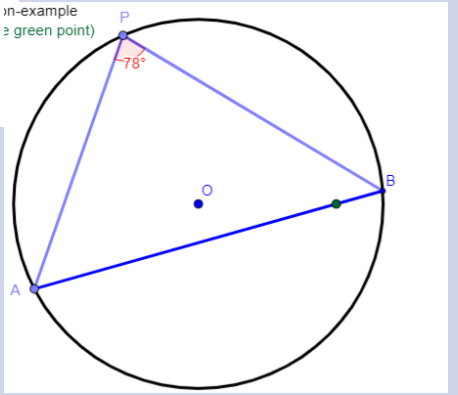




Non-Examples

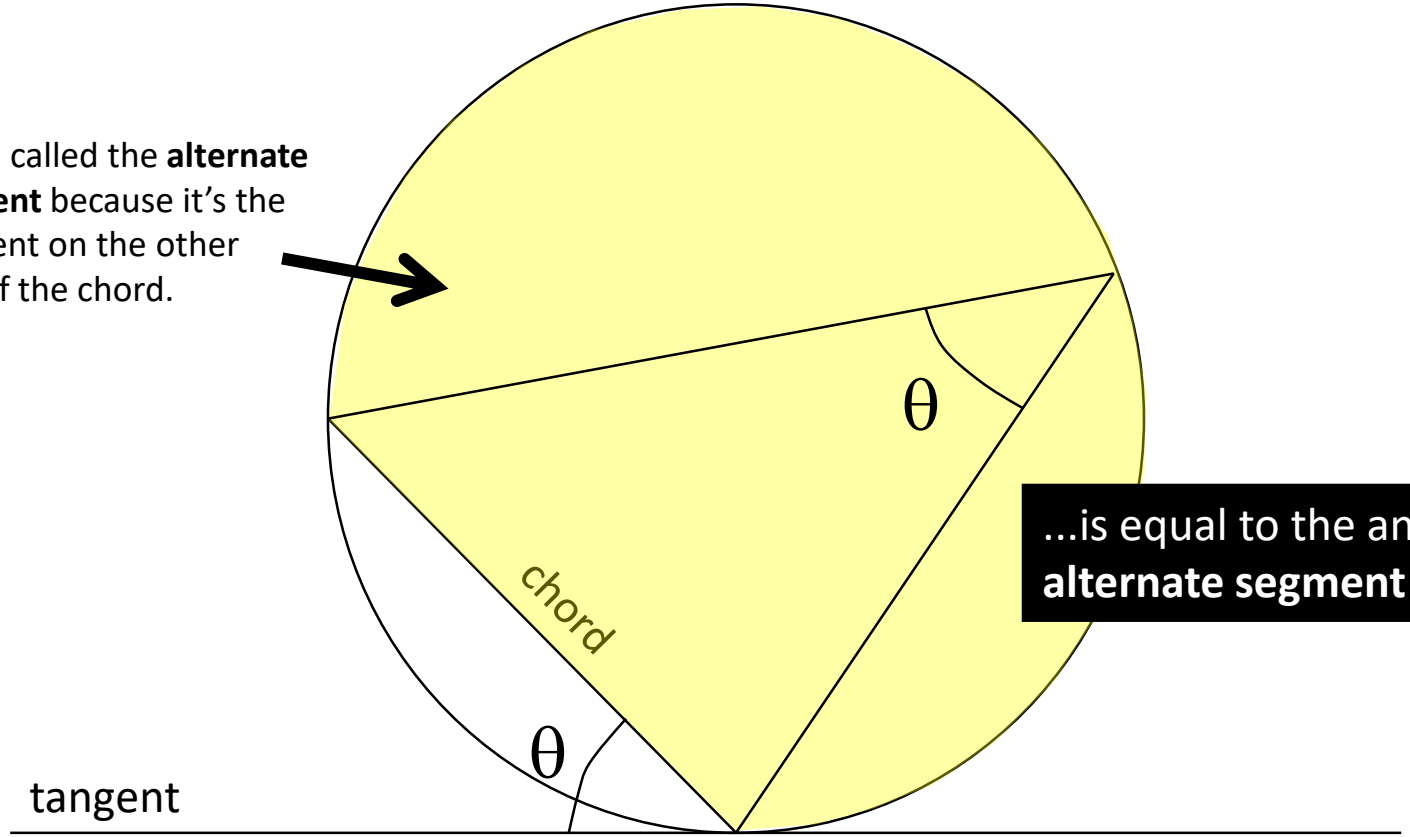




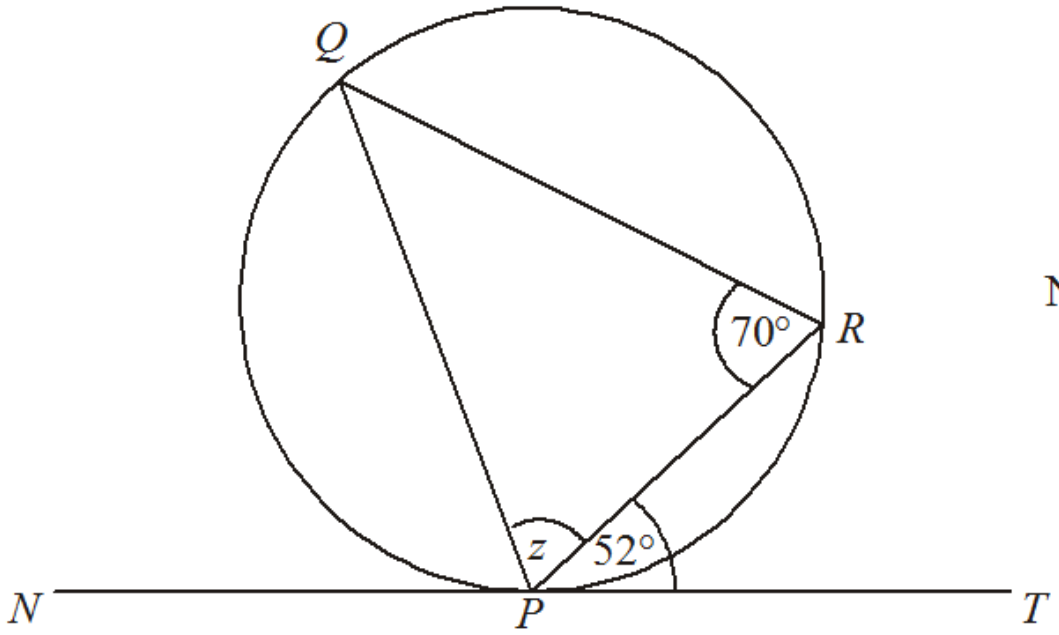


Alternate Segment Theorem

This one is probably the hardest to remember and a particular favourite in the Intermediate/Senior Maths Challenges.



Check Your Understanding



Not drawn accurately

?

Check Your Understanding

Source: IGCSE Jan 2014 (R)

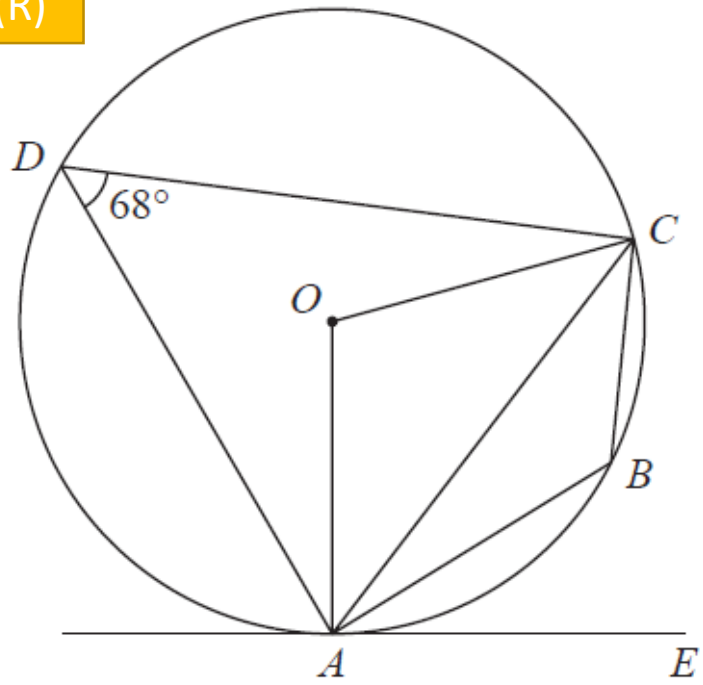


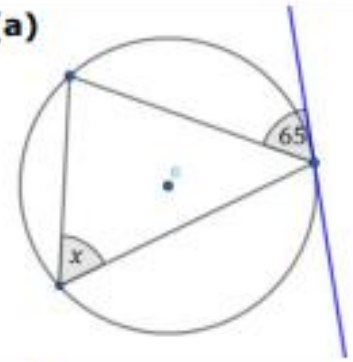
Diagram **NOT** accurately drawn

Angle ABC =	Angle AOC =	Angle CAE =
?	?	?
Give a reason:	**Give a reason:**	**Give a reason:**
?	?	?

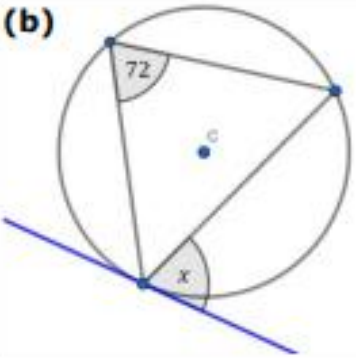
ALTERNATE SEGMENT THEOREM PRACTICE GRID

Find the missing angles.

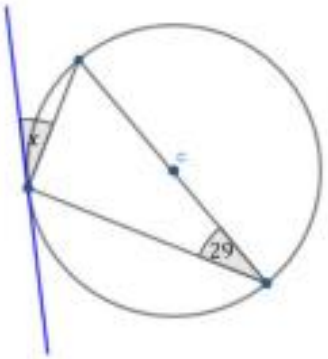
(a)



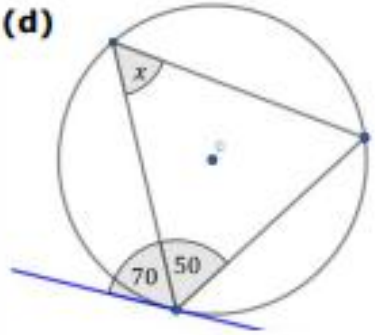
(b)



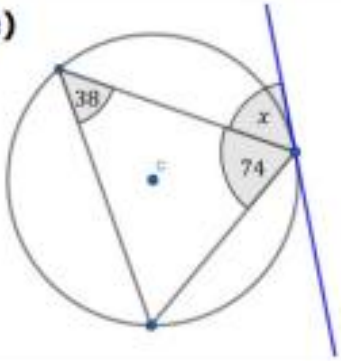
(c)



(d)



(e)



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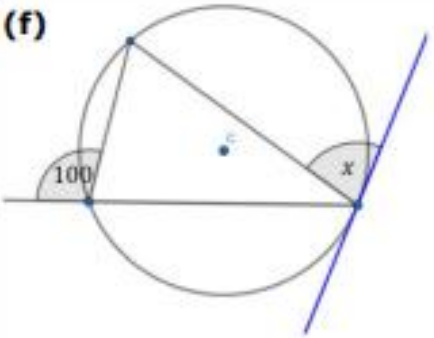
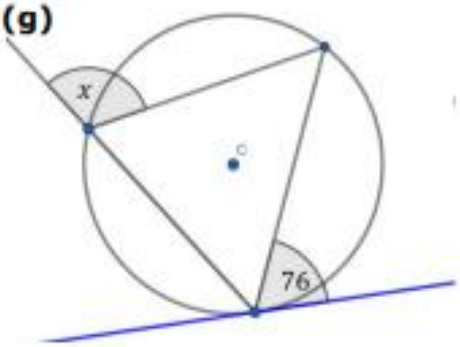
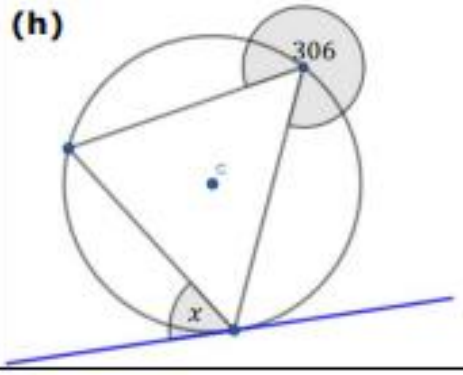


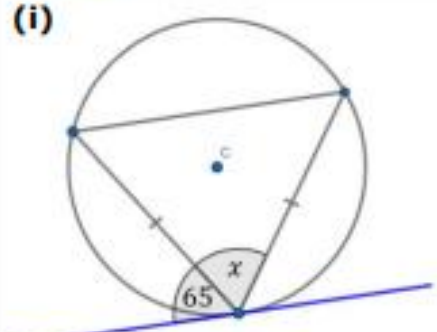
(g)



(h)



(i)



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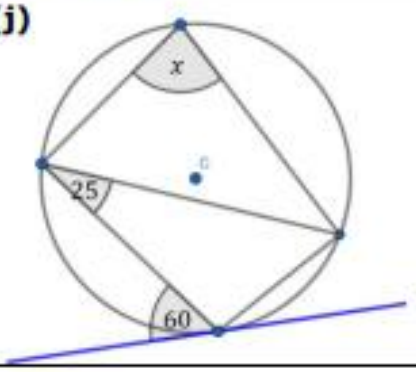
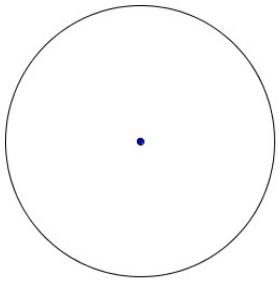
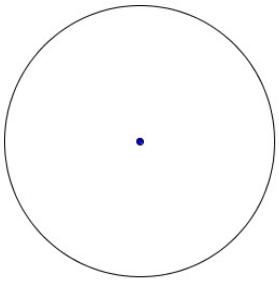


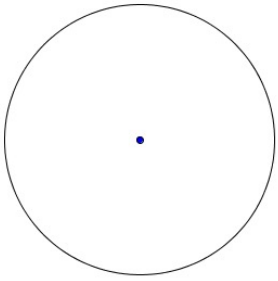
A tangent meets the radius at 90°



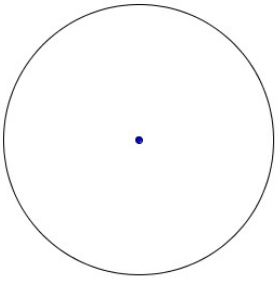
The angle in a semicircle is a right angle.



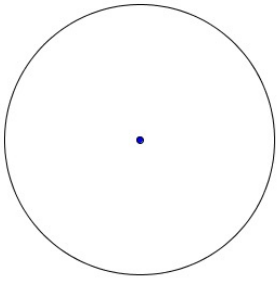
Any triangle with two points at the circumference and one in the centre is isosceles.



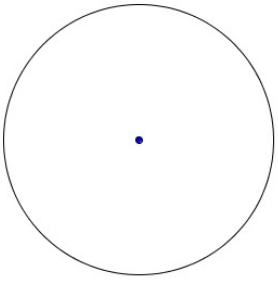
The angle subtended by an arc at the centre of a circle is twice the angle subtended at the circumference.



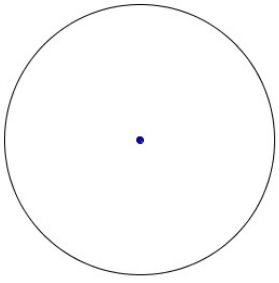
The sum of the opposite angles of a cyclic quadrilateral is 180.



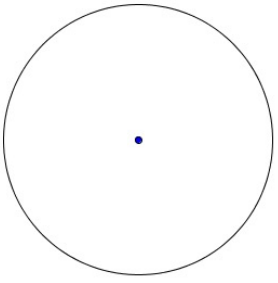
Angles in the same segment are equal



The angle between a chord and the tangent at the point of contact is equal to the angle in the alternate segment.



Any triangle created by two crossing tangents and the line joining their point of contact is isosceles.



Review

Corbett maths

Circle Theorems
Videos 64/65 on Corbettmaths

Examples

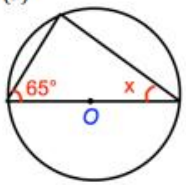
Workout

Click here

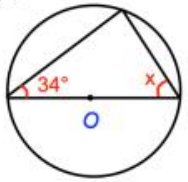
Scan here

Question 1: Find the missing angles labelled in each of these circles

(a)



(b)



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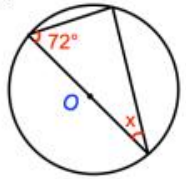

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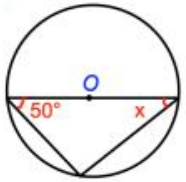

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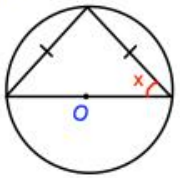

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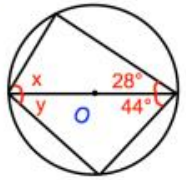

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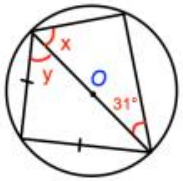

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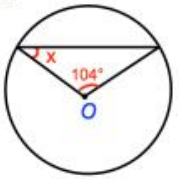
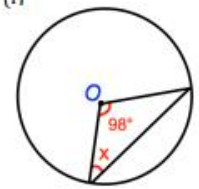


(i)



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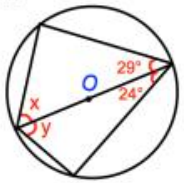
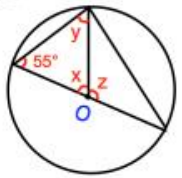


(k)



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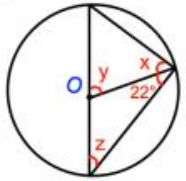

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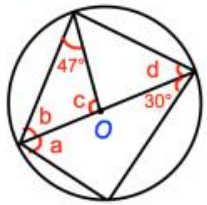
(m)



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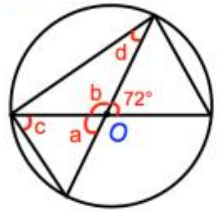

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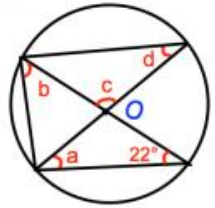

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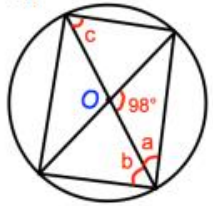

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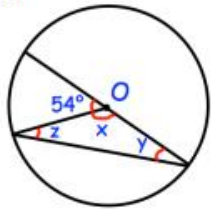

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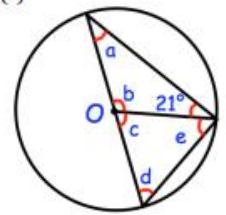

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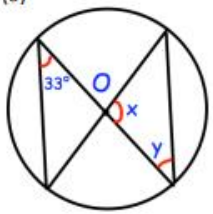

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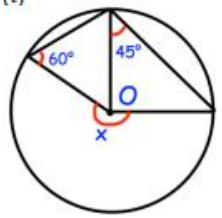

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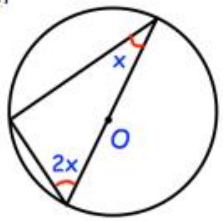

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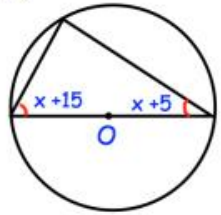

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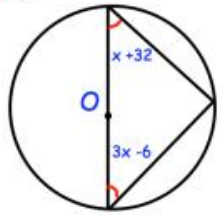

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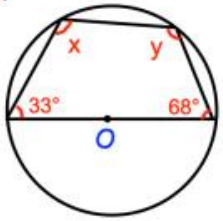

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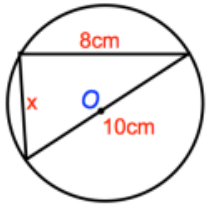
Question 2: Calculate the length of sides labelled in the circles below

(a)



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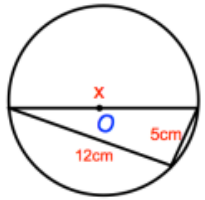

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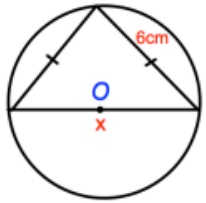
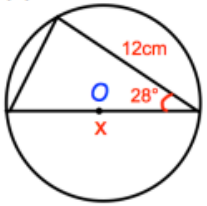


Question 3: Calculate the length of sides labelled in the circles below

(a)



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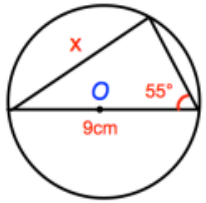

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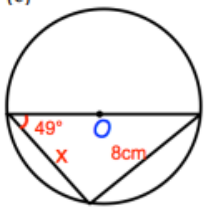
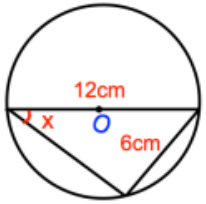


Question 4: Calculate the size of the missing angles

(a)



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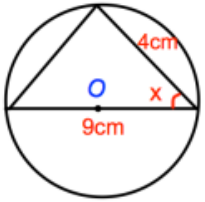

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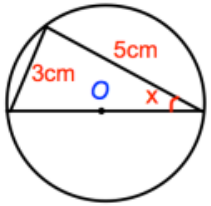
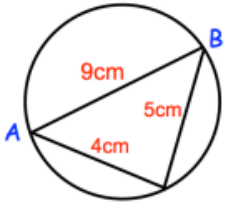


Question 5: State, with a reason, if AB is the diameter in each circle below.

(a)



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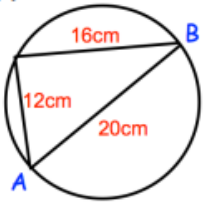

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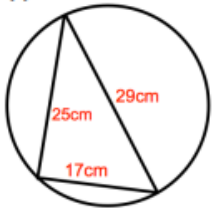

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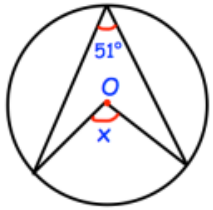
Question 6: Find the missing angles labelled in each of these circles

(a)



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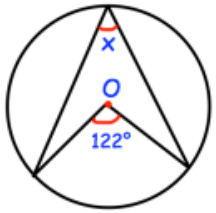

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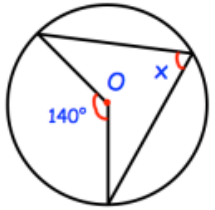

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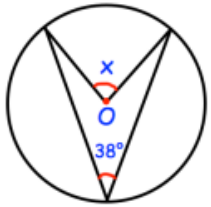

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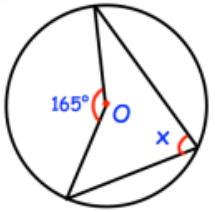

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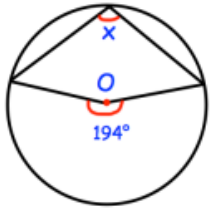

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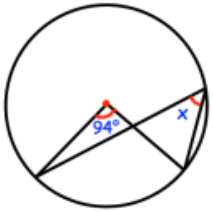

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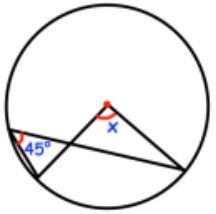

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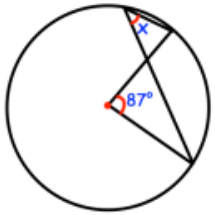

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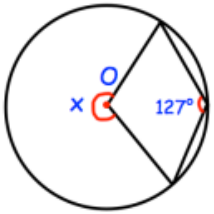

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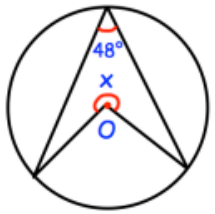

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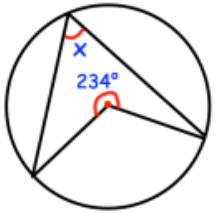

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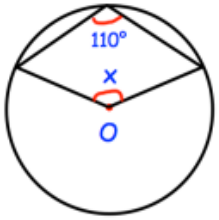
(m)



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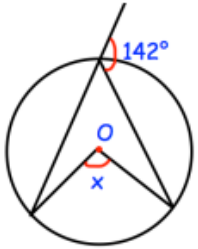

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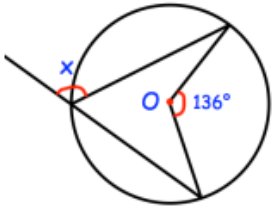

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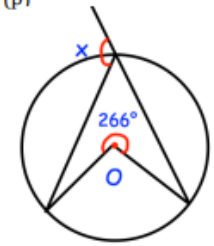

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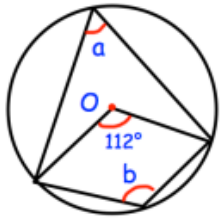

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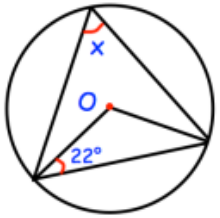

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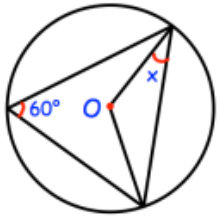

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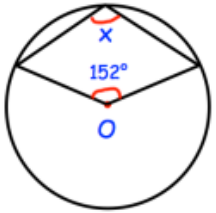

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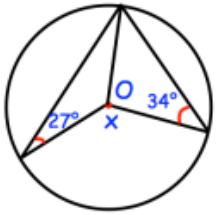

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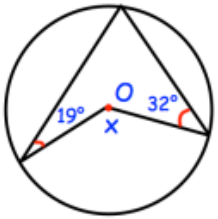

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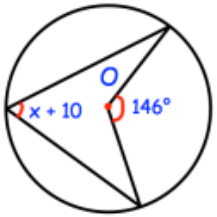

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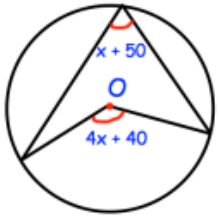

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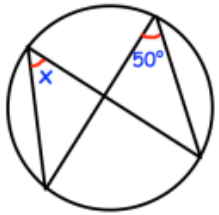
Question 7: Find the missing angles labelled in each of these circles

(a)



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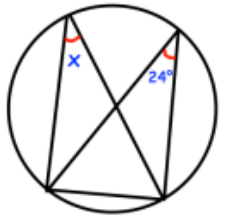

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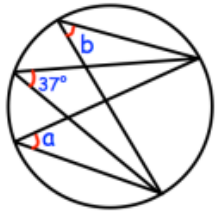

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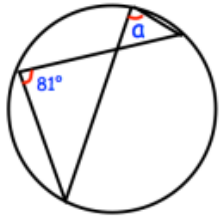

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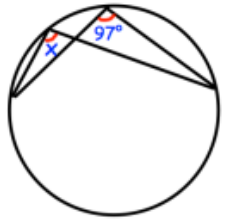

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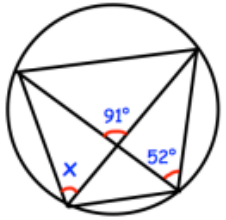

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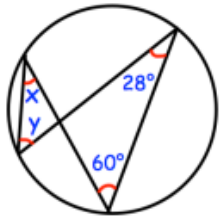

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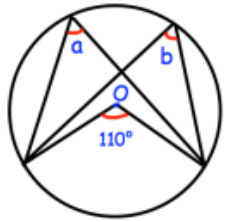
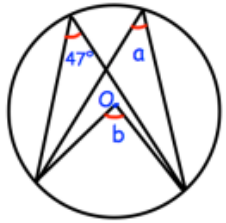


(i)



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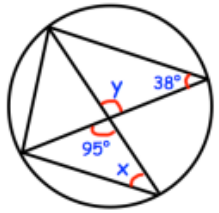

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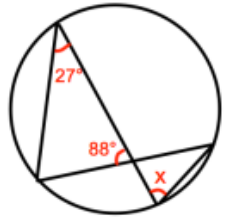

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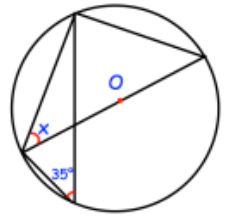

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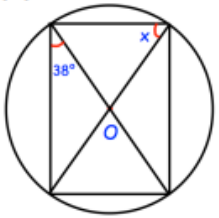
(m)



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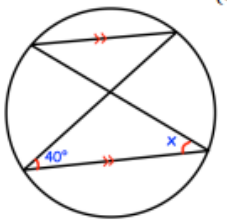

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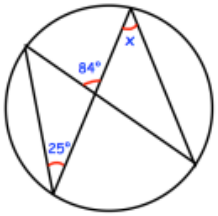

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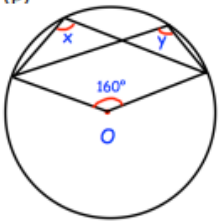

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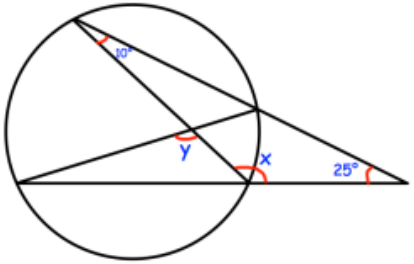
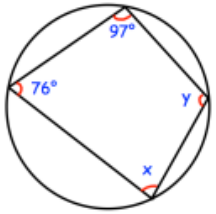
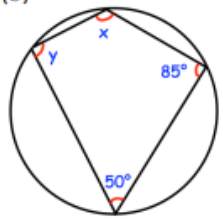


Question 8: Find the missing angles labelled in each of these circles

(a)



(b)



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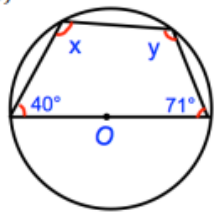

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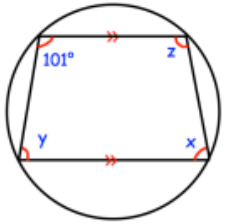

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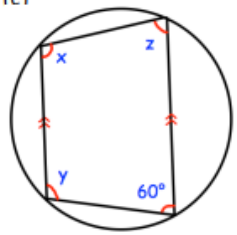

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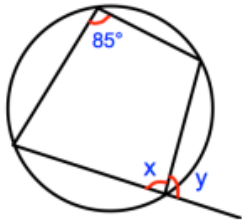

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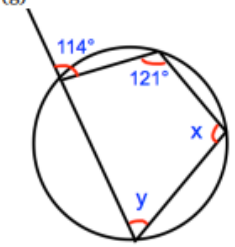
(g)



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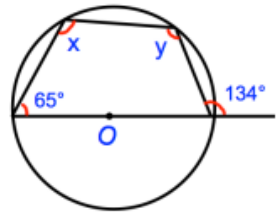

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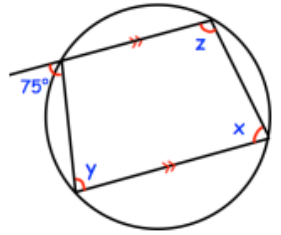

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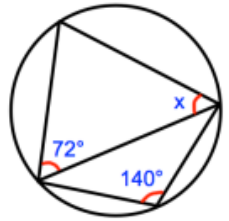

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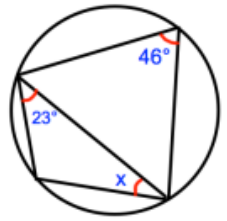

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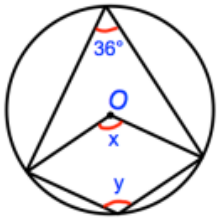

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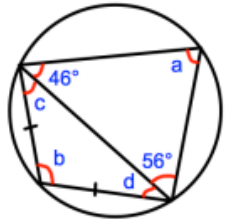

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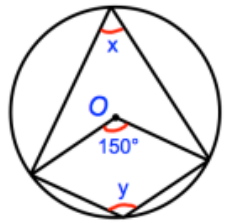

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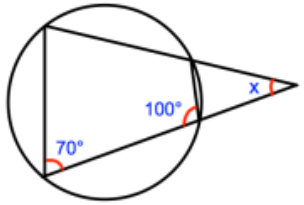

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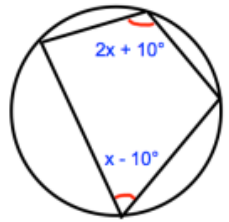

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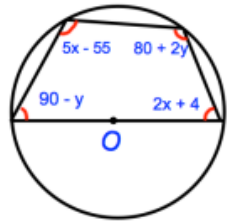

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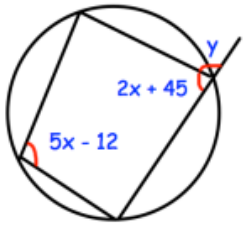
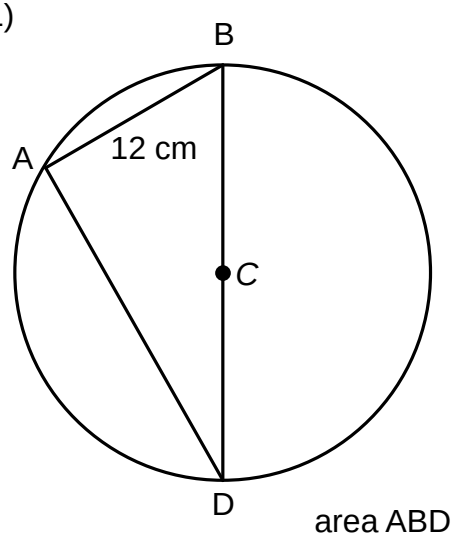


circle geometry and areas

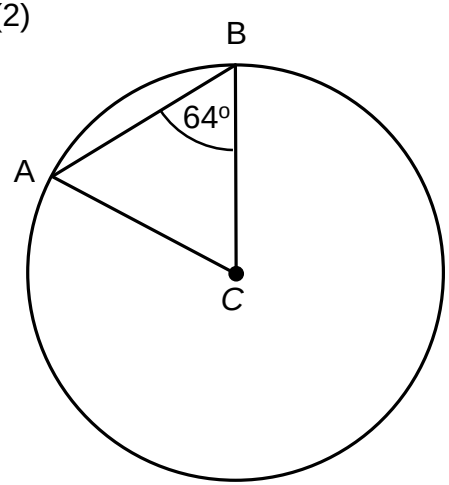
C is the centre of a circle, radius 10 cm

(1)



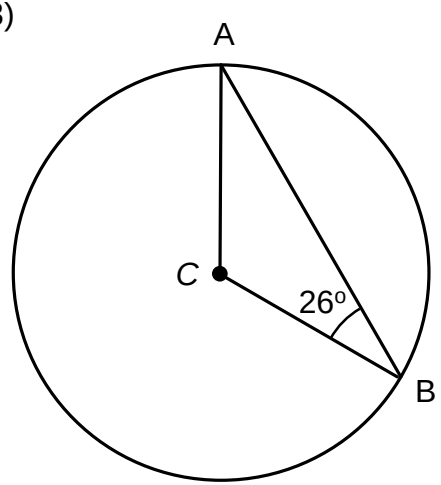
area ABD ?

(2)



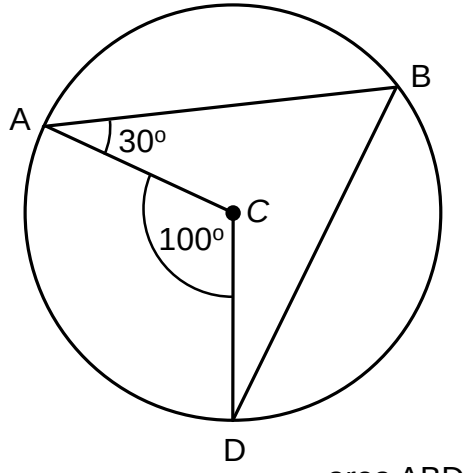
area AB*C* ?

(3)



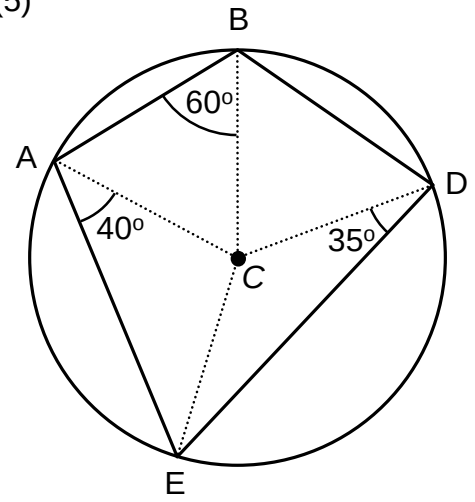
area AB*C* ?

(4)



area ABD*C* ?

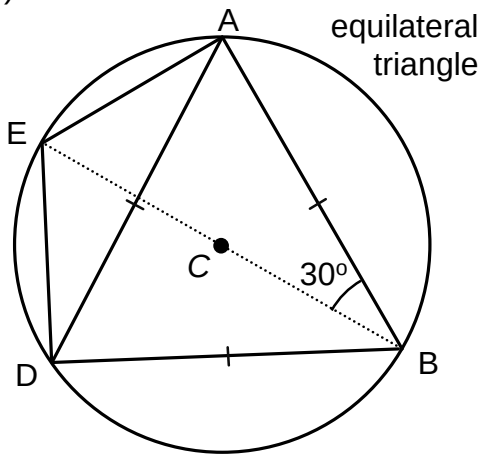
(5)



area ABDE ?

(6)

ABD is an equilateral triangle

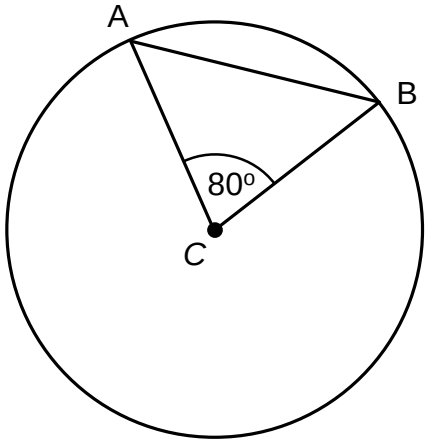


area ABDE ?

circle geometry and areas

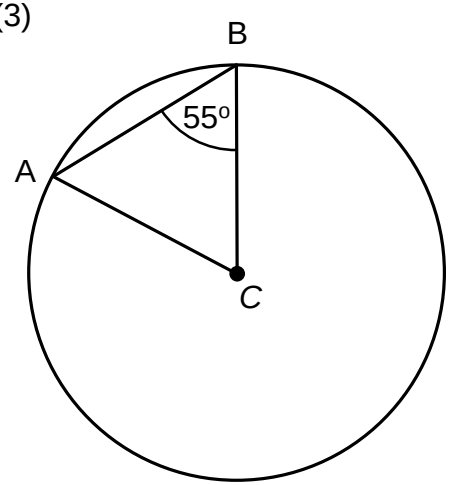
C is the centre of a circle, radius 10 cm

(1)



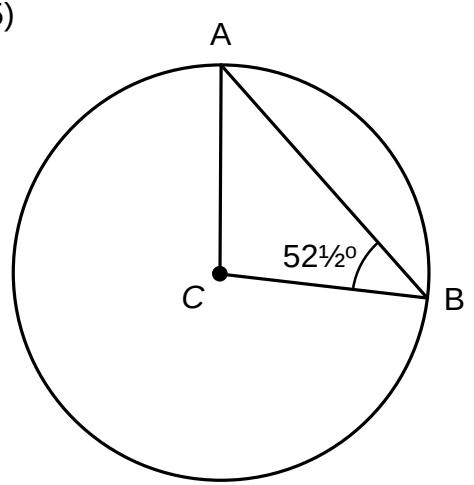
area AB*C* ?

(3)



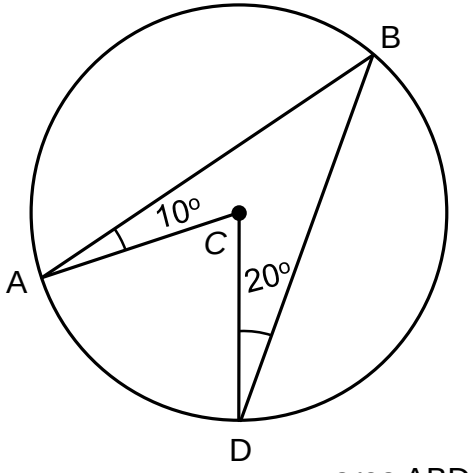
area ABC ?

(5)



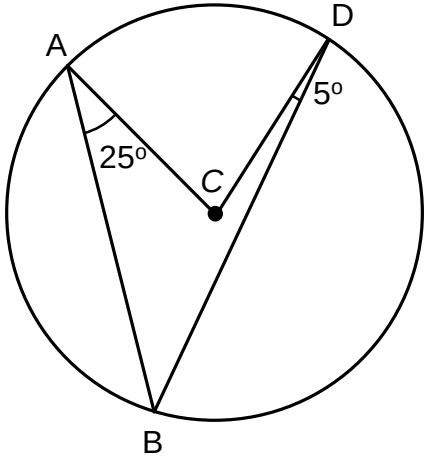
area ABC ?

(2)



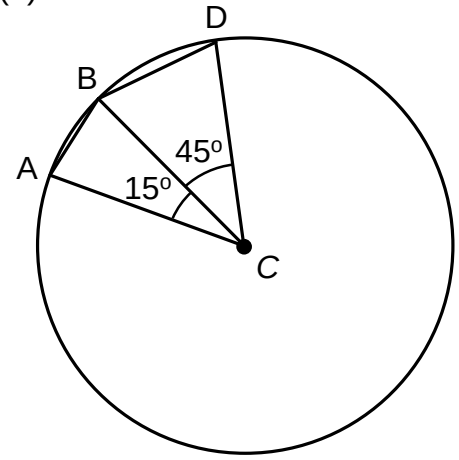
area ABD*C* ?

(4)



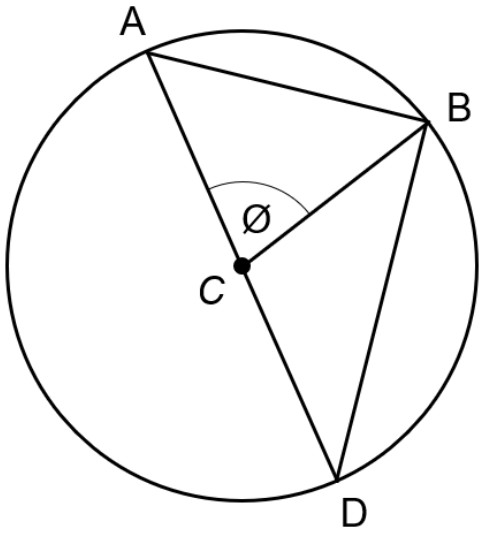
area ABD*C* ?

(6)



area ABD*C* ?

EXTENSION



explain why the area of triangle ABC = area of triangle BDC

explain why sin Ø = sin(180° – Ø)

check that:

$\sin(10^o) + \sin(50^o) = \sin(70^o)$

$\sin(15^o) + \sin(45^o) = \sin(75^o)$

$\sin(20^o) + \sin(40^o) = \sin(80^o)$

try to find other, similar results

EXAM STYLE QUESTIONS

Exam-style question 1

The diagram shows a circle with centre O.
P and Q are points on the circle.

Angle OPQ is 54°.

Work out the size of angle POQ.
You must give a reason for your answer.

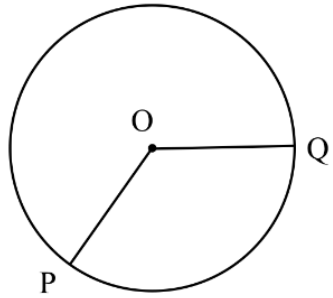


Diagram **NOT** accurately drawn

Exam-style question 2

The diagram shows a circle with centre O.

Work out the value of x.
You must give a reason for your answer.

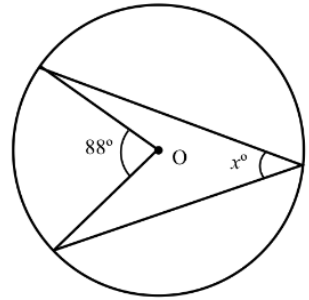


Diagram **NOT** accurately drawn

Exam-style question 3

The diagram shows a circle.

Work out the value of a.
You must give a reason for your answer.

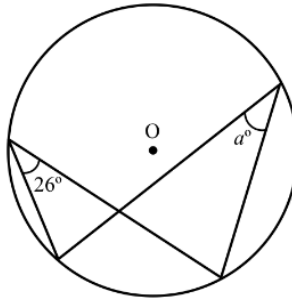


Diagram **NOT** accurately draw

Exam-style question 4

The diagram shows a circle with centre O.

Work out the value of y.
You must give a reason for your answer.

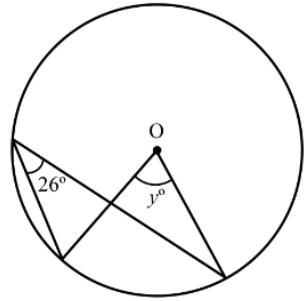


Diagram **NOT** accurately drawn

Exam-style question 5

The diagram shows a circle. ABCD is a quadrilateral whose vertices lie on the circumference of the circle.

Work out the size of angle BCD.
You must give a reason for your answer.

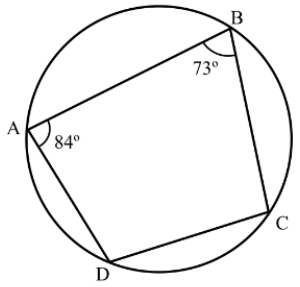


Diagram **NOT** accurately drawn

Exam-style question 6

The diagram shows a circle with centre O, a tangent to the circle at point X, a chord XY, and a further point on the circle labelled Z.

a) Write down the value of m.

b) Given that $\angle OXY = 36°$, work out the size of $\angle XZY$. You must give a reason for your answer.

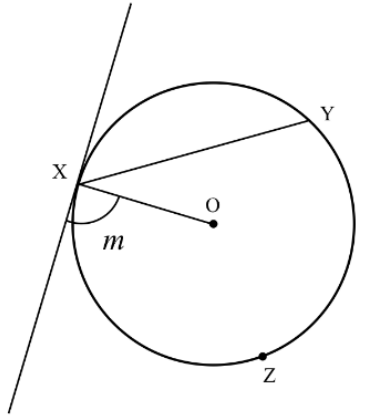


Diagram **NOT** accurately drawn

Exam-style question 7

The diagram shows a circle with centre O, a tangent to the circle at point X, a diameter XZ, chords XY and YZ, and a point P on the tangent.

a) Write down the size of $\angle XYZ$.

b) Without assuming the alternate segment theorem, prove that $\angle PXY = \angle XZY$

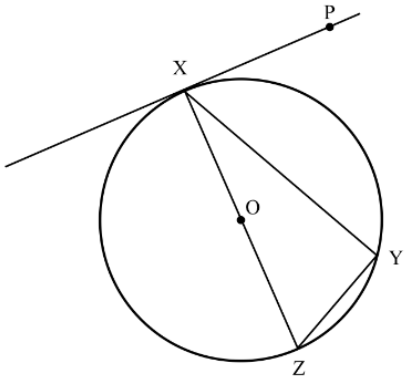


Diagram **NOT** accurately drawn

Exam-style question 8

The diagram shows a circle with centre O.

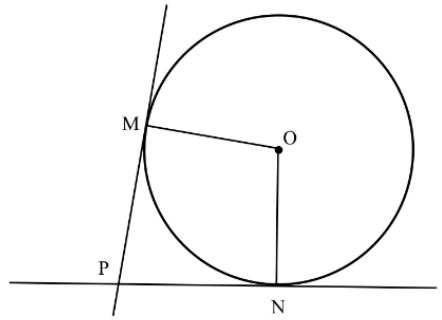
M and N are points on the circle.

PM and PN are tangents to the circle.

a) Given that $\angle MPN = 77°$, work out the size of $\angle MON$. You must give reasons for your answer.

b) Show that triangle OMP is congruent to triangle ONP.

Diagram **NOT** accurately drawn



Exam-style question 9

The diagram shows a circle with centre O. The points P, Q, R and S are on the circumference of the circle.

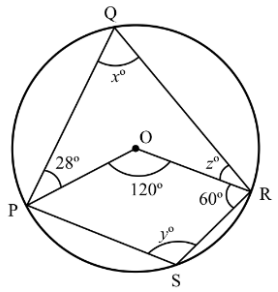
$\angle POR = 120^\circ$, $\angle OPQ = 28^\circ$, and $\angle ORS = 60^\circ$

a) Work out the value of x. You must give a reason for your answer.

b) Work out the value of y. You must give a reason for your answer.

c) Work out the value of z. You must give a reason for your answer.

Diagram **NOT** accurately drawn



Exam-style question 10

The diagram shows a circle with centre O, a tangent to the circle at point A, point X on the tangent, diameter AB, and a line segment BX.

$\angle AXB = 54^\circ$

BX intersects the circle at two points: B and C.

a) Label the point C on the diagram.

b) Work out the size of $\angle BAC$. Give reasons for your answer.

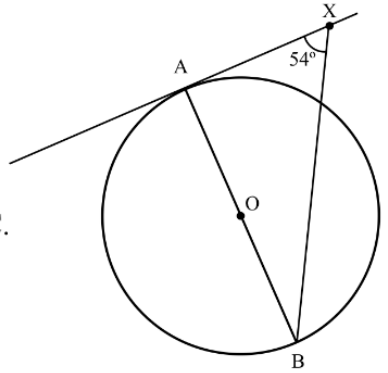


Diagram **NOT** accurately drawn

Challenge

The diagram shows a circle, a tangent to the circle at point A, a cyclic pentagon ABCDE, chords AD and BE, and points P and Q on the tangent.

$\angle PAB = 40^\circ$, $\angle ABE = 30^\circ$, and $\angle BED = 45^\circ$

Find the size of $\angle DAE$.

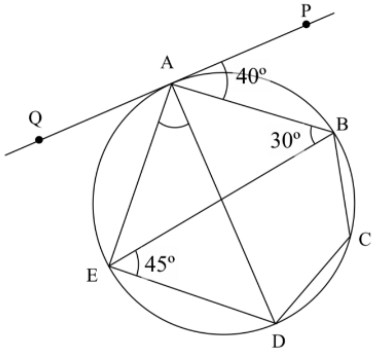


Diagram **NOT** accurately drawn

Compass constructions and Loci

To ‘construct’ something in the strictest sense means to draw it using only two things:

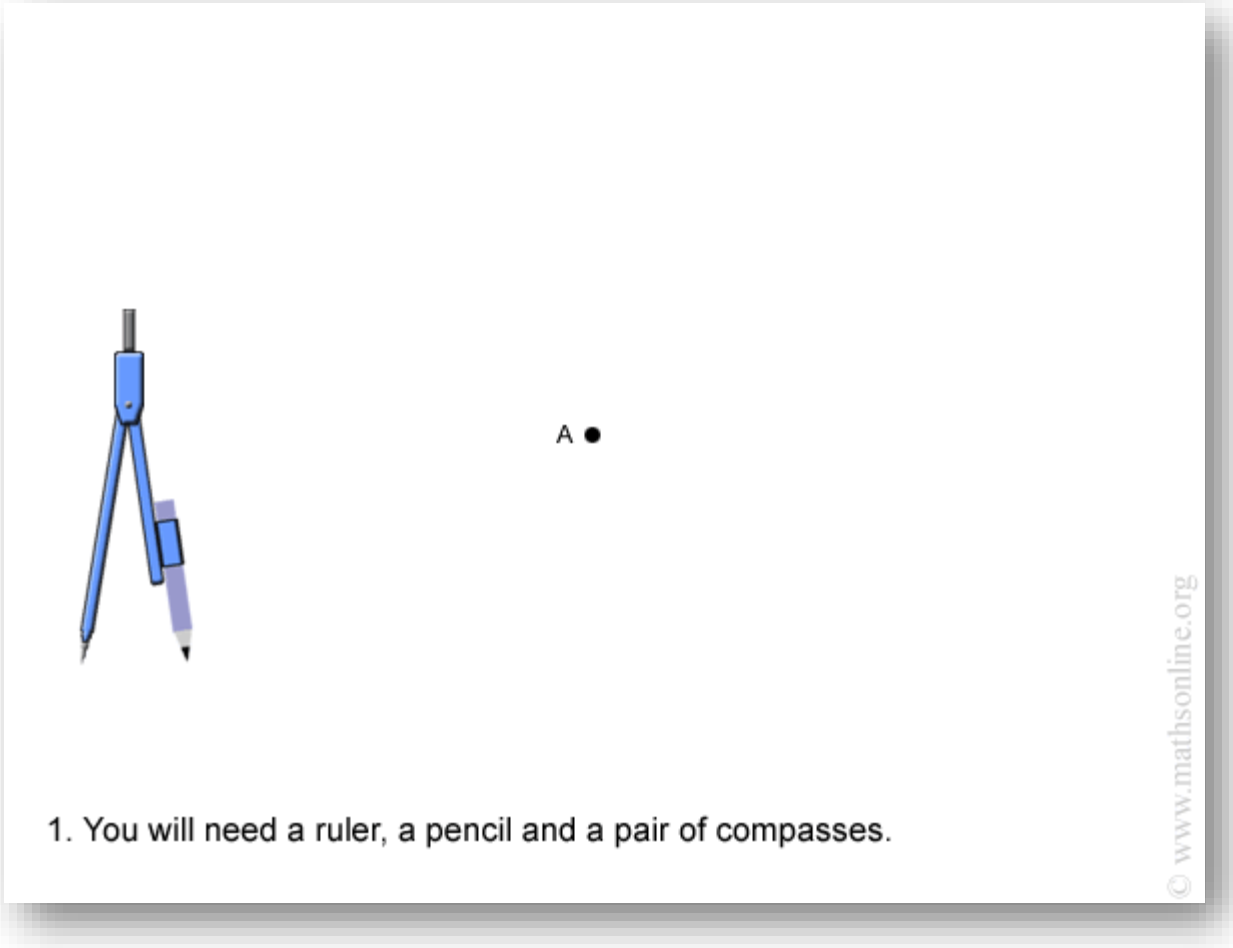
- Compass
- Straight Edge (Apart from where a length is specified, you are not allowed to measure lengths)

- **Bisect** means cut into two equal parts.

- **Equidistant** means equal distance from

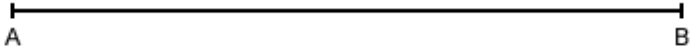
Loci of Points a fixed distance from a given point

Instructions:



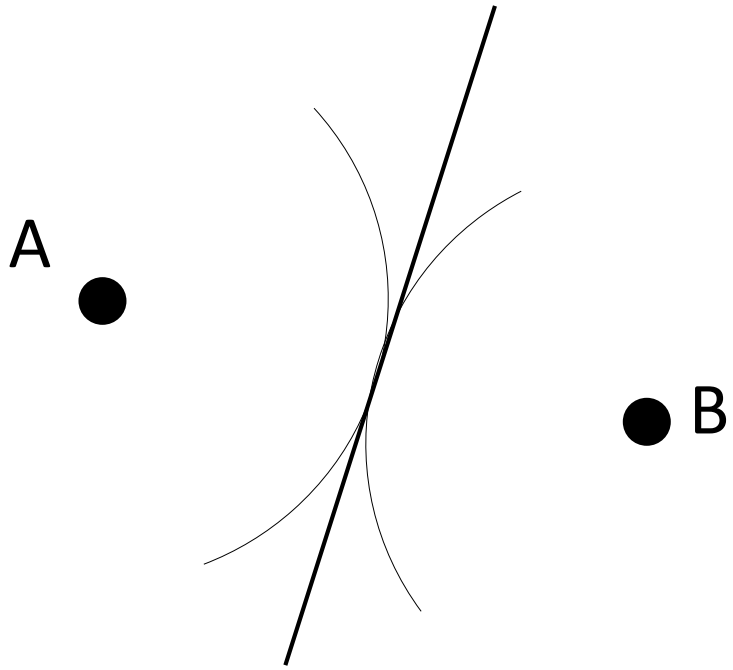
Perpendicular Bisector of a line

Instructions:

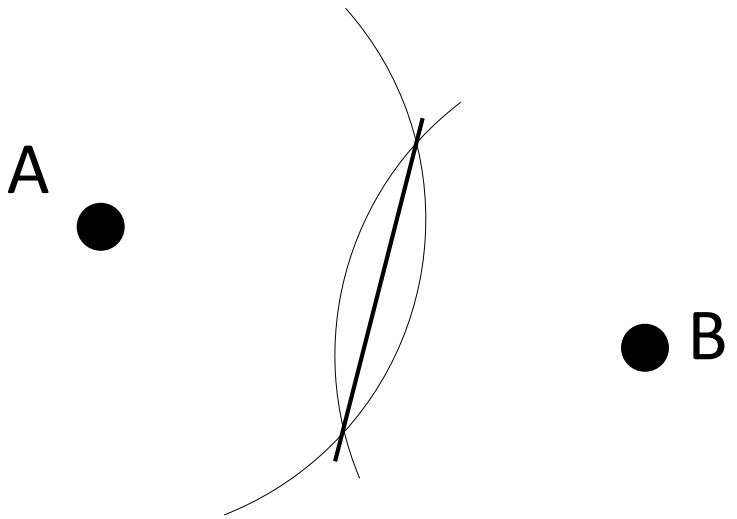


1. You will need a ruler, a pencil and a pair of compasses.

Common Losses of Exam Marks



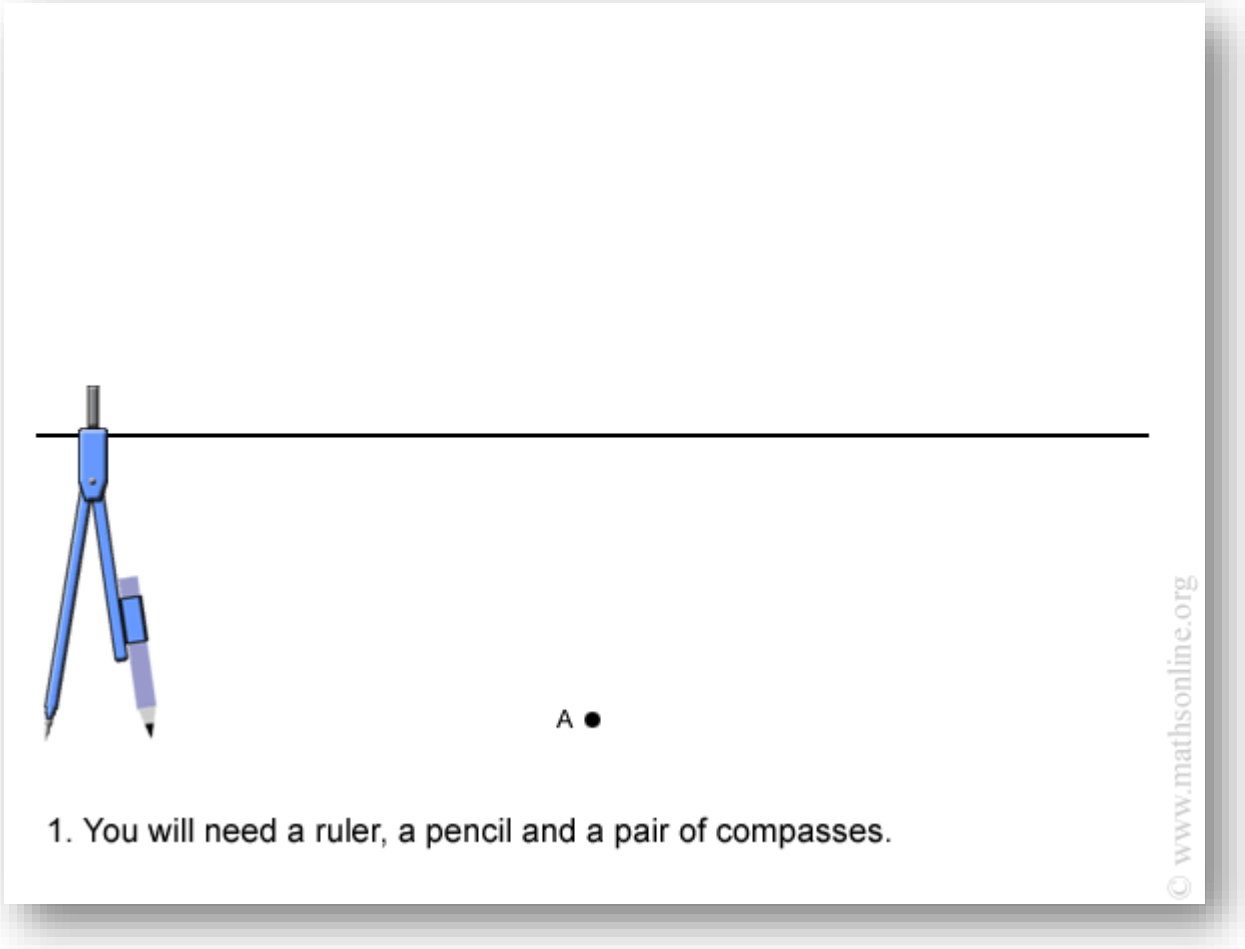
Le Problemo:
Arcs don't overlap enough, so points of intersection to draw line through is not clear.



Le Problemo:
Locus is not long enough.
(Since it's actually infinitely long, we want to draw it sufficiently long to suggest it's infinite)

Drawing a Perpendicular to a Line through a Point

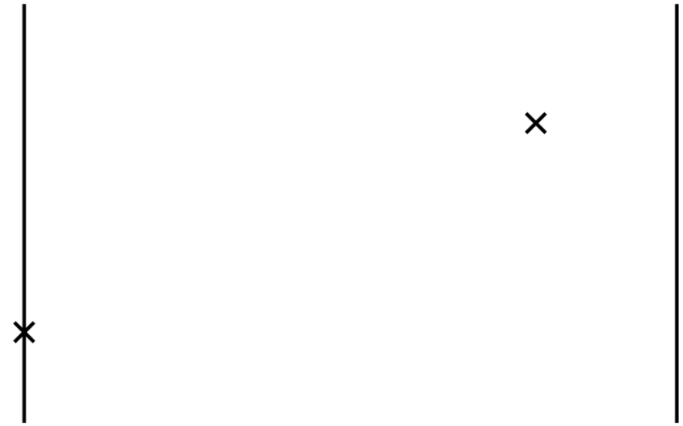
Instructions:



Exercises:

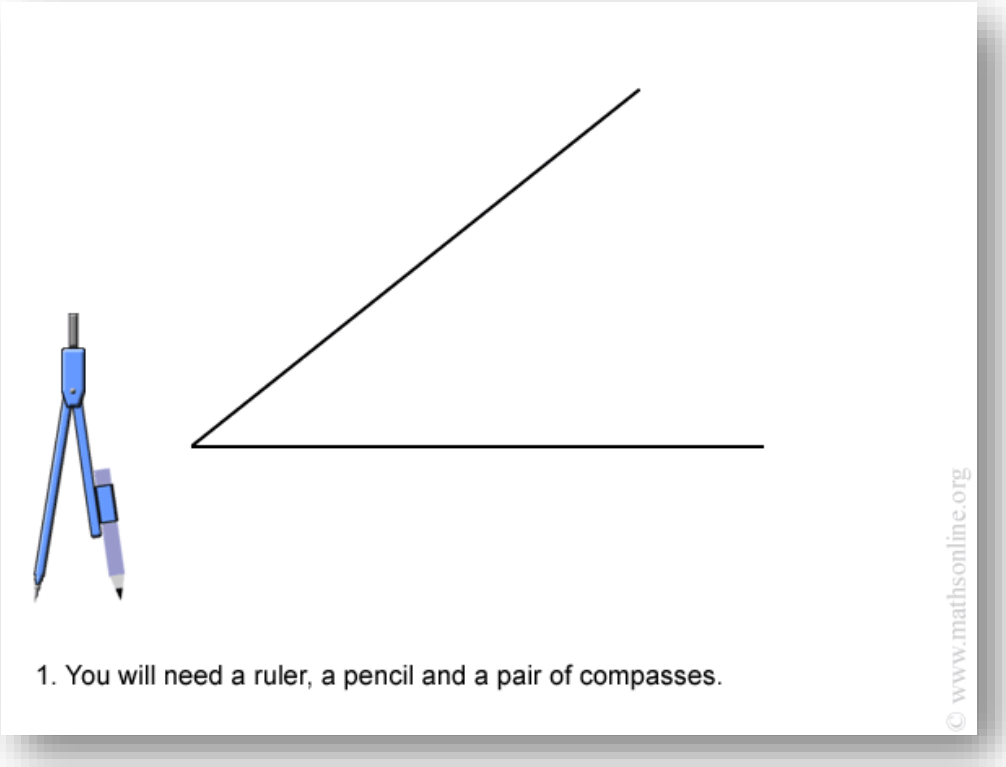
Draw a line and mark crosses on it to divide it into four sections of equal length. Use only a straight edge and compasses.

Roughly copy these diagrams. Then construct the perpendicular to each line that passes through the cross.



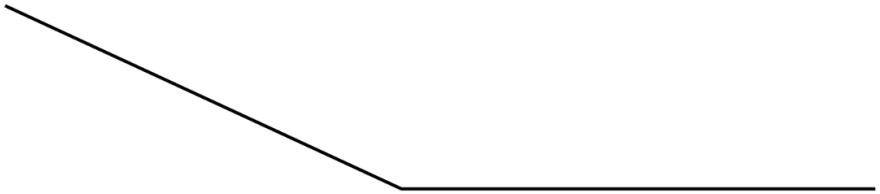
Bisecting an Angle

Instructions:

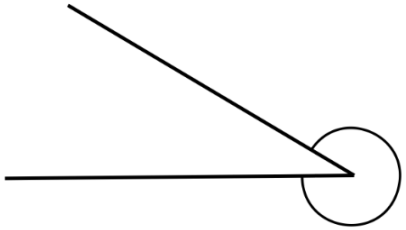


Exercises:

Use only a straight edge and compasses to divide this angle into four equal angles.



Fred is asked to bisect the reflex angle shown.

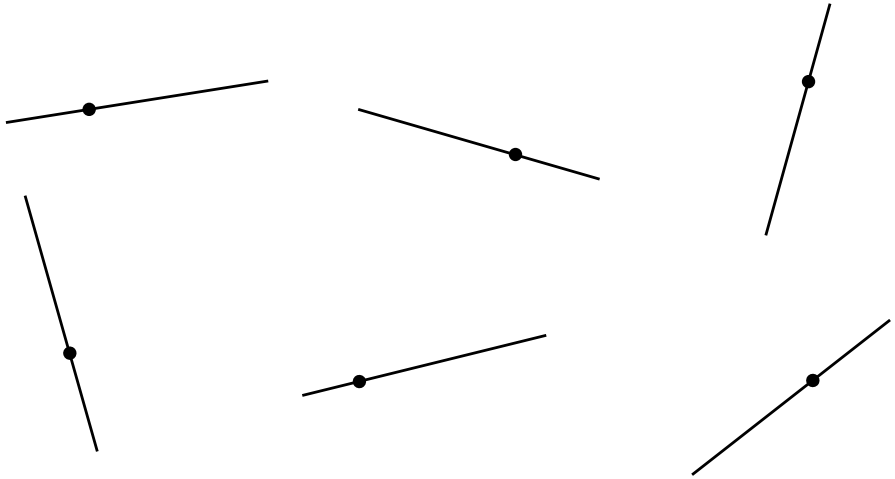


Fred says he can't do this because he only knows how to bisect angles that are less than 180^{o}.

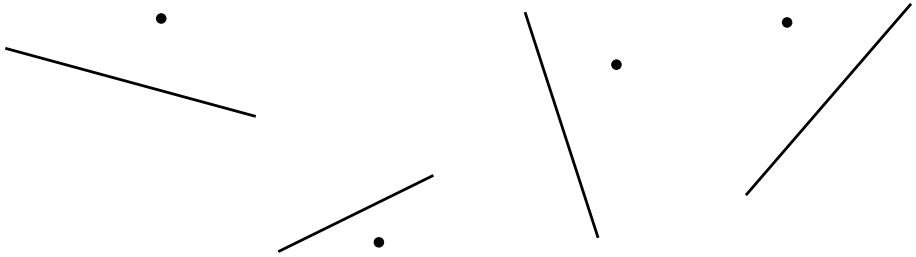
Explain why Fred does in fact have enough knowledge to bisect the reflex angle.

Fluency Practice

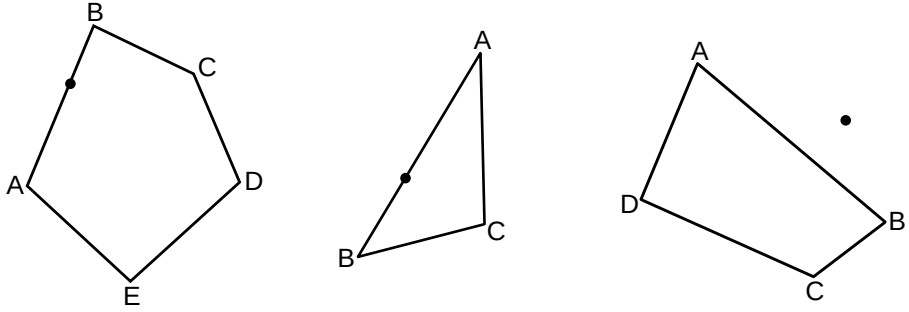
Q3 Construct the perpendicular bisector of the following lines through the given point.



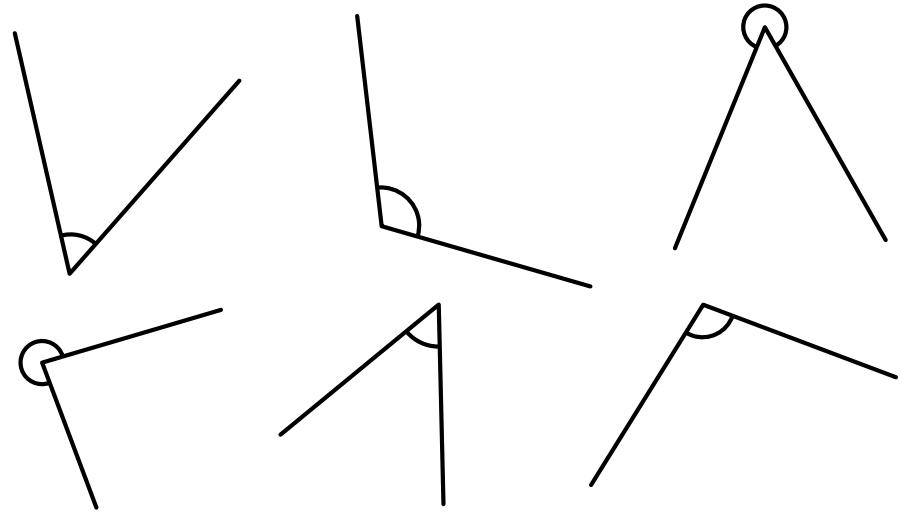
Q4 Construct the perpendicular bisector of the following lines through the given point.



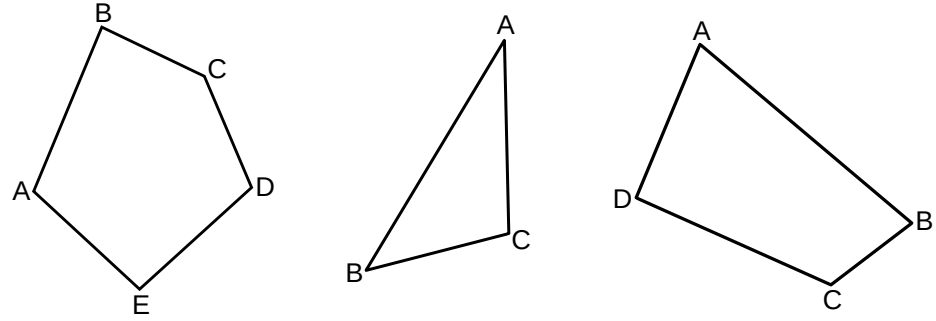
Q5 Construct the perpendicular bisector of the line **AB** through the given point.



Q6 Construct the **angle bisector** of the following angles.



Q7 Construct the angle bisector of ∠**ABC** for each of the following shapes..



Q8 Construct a 90º angle.

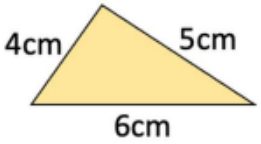
Q9 Construct a 45`º angle.

Constructing Triangles

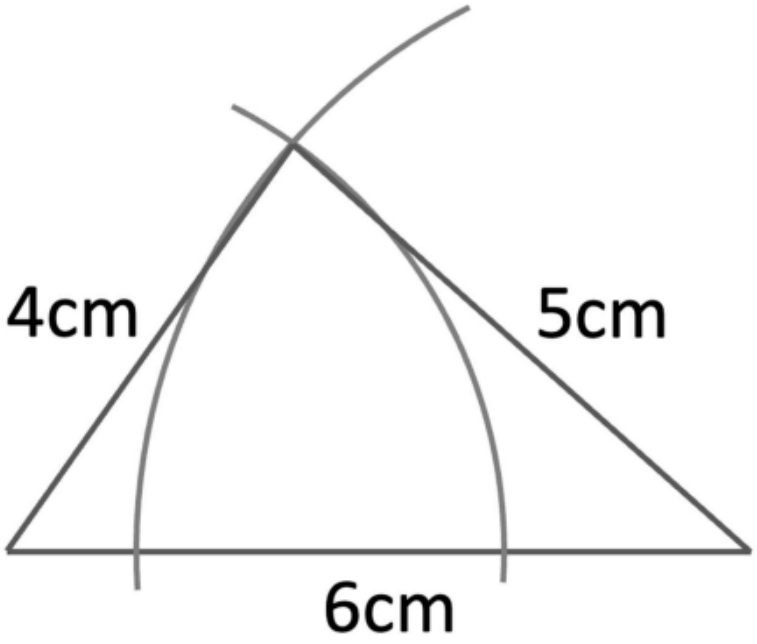
You can construct a unique triangle when you know:
Two sides and the angle between them **(SAS)**
Two angles and a side **(ASA)**
Three sides **(SSS)**

SSS

Using a ruler and compass only, construct the following SSS triangle accurately.



1) Draw a 6cm line with a ruler.
2) Draw two arcs with lengths 4cm and 5cm from each end of the line.
3) Join the ends of the line to the intersection.



Worked Example

Construct a triangle with:

- A side length of $10\ cm$
- A side length of $6\ cm$
- A side length of $8\ cm$

Your Turn

Construct a triangle with:

- A side length of $5\ cm$
- A side length of $3\ cm$
- A side length of $4\ cm$

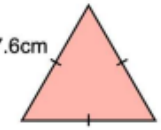
Fluency Practice

Q1 Use your compass and ruler only to construct the following **SSS** triangles.

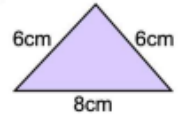
[a]

[b]

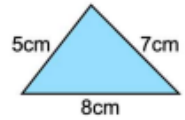
[c]



[d]

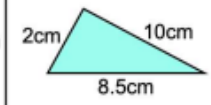


[e]

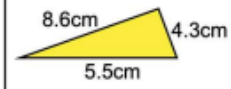


[f]

[g]

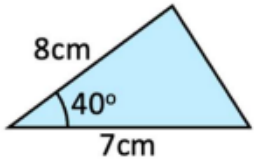


[h]

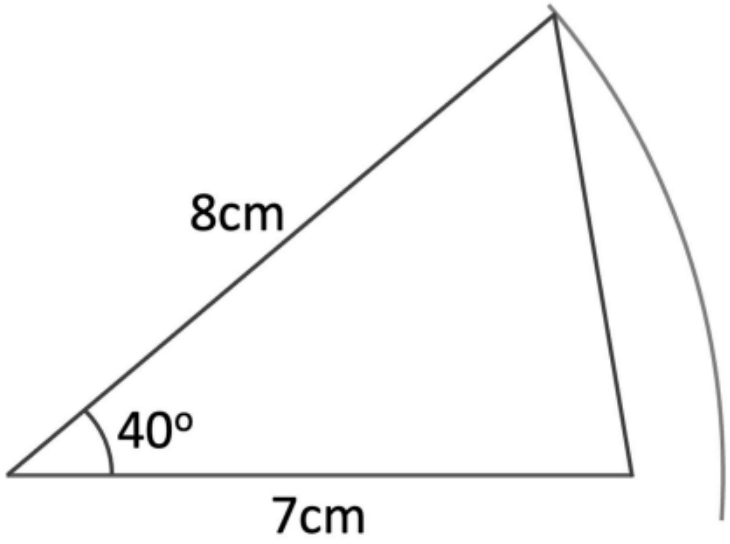


SAS

Using a ruler, compass and protractor, construct the following SAS triangle accurately.



1) Draw a 7cm line with a ruler.
2) Draw an arc with length 8cm.
3) Measure an angle of 40°.
4) Draw a line through the angle to the arc.
5) Join up the end of the lines.



Worked Example

Construct a triangle with:

- A side length of $10\ cm$
- An angle of $30°$
- A side length of $8\ cm$

Your Turn

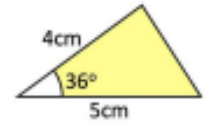
Construct a triangle with:

- A side length of $5\ cm$
- An angle of $30°$
- A side length of $4\ cm$

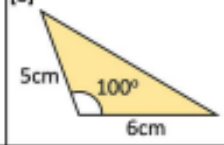
Fluency Practice

Q2 Use your compass and ruler only to construct the following triangles.

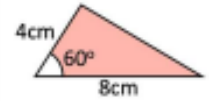
[a]



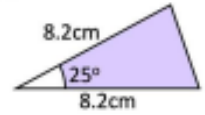
[b]



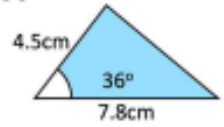
[c]



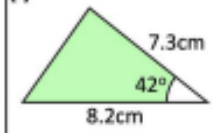
[d]



[e]

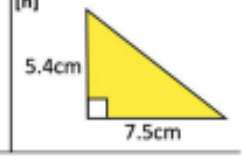


[f]



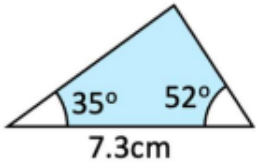
[g]

[h]

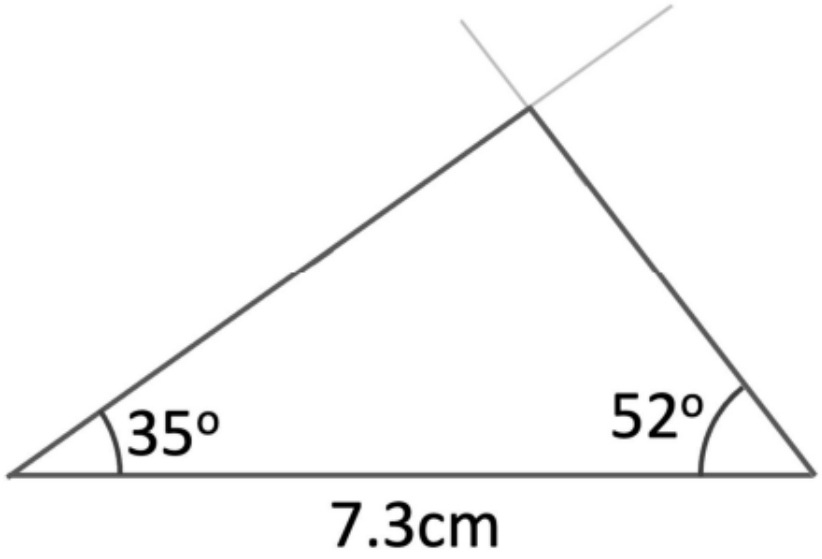


ASA

Using a ruler, compass and protractor, construct the following ASA triangle accurately.



1) Draw a 7.3cm line with a ruler.
2) Measure both angles.
3) Draw a feint line through each angle and label them.
4) Draw a solid line over each feint line up to the intersection.



Worked Example

Construct a triangle with:

- An angle of $30°$
- A side length of $10\ cm$
- An angle of $45°$

Your Turn

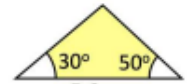
Construct a triangle with:

- An angle of $30°$
- A side length of $5\ cm$
- An angle of $60°$

Fluency Practice

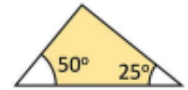
Q3 Use your compass and ruler only to construct the following triangles.

[a]



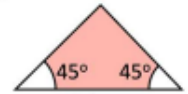
5.6cm

[b]



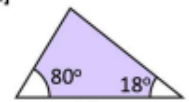
10cm

[c]



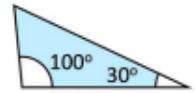
8.1cm

[d]



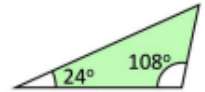
6.3cm

[e]



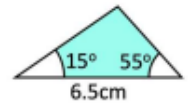
3.2cm

[f]



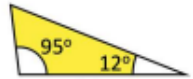
5.4cm

[g]



6.5cm

[h]

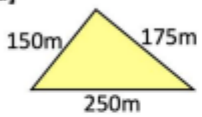


4.2cm

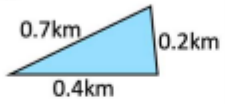
Extension

Q4 Use the scale of 1cm = 50m, construct the following triangles.

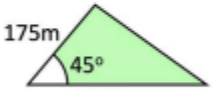
[a]



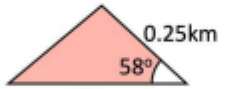
[b]



[c]



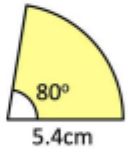
[d]

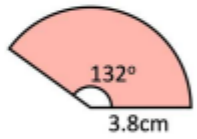


Q5 Accurately draw two different isosceles triangles with an angle of 40°.

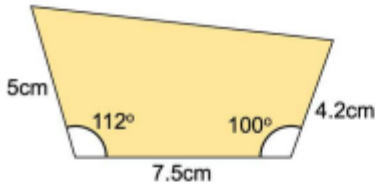
Q6 Construct an equilateral triangle with side length of 7cm. By measuring its height, work out its area.

Q7 Accurately draw the sectors below.

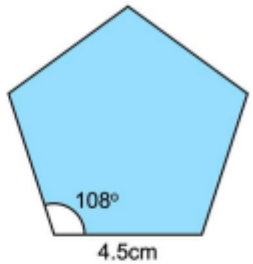




Q8 Accurately construct the following quadrilateral and find the length of the missing side.

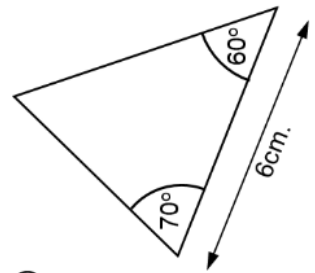


Q9 Use the information below to accurately construct a regular pentagon.

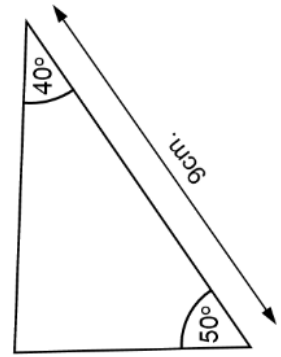


construct the following shapes accurately

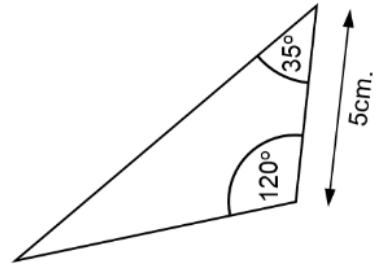
1)



2)



3)



4) an equilateral triangle with a side length of 6cm

5) a square with a side length of 5.5cm

6) an isosceles triangle with equal sides of length 7cm and two equal angles of size 70°

7) a rhombus with side length 5cm and one angle of 80°

8) a right angled triangle with one angle of 60° and the longest side being 9cm

9) a square with a diagonal of length 8cm

10) a parallelogram with sides of length 8cm and 4.5cm with an angle of 55° between them

Everything in the GCSE specification

- Construct triangles including an equilateral triangle
- Construct the perpendicular bisector of a given line
- Construct the perpendicular from a point to a line
- Construct the perpendicular from a point on a line
- Construct the bisector of a given angle
- Construct angles of 60º, 90º , 30º, 45º
- Construct a regular hexagon inside a circle
- Construct:
  -a region bounded by a circle and an intersecting line
  - a given distance from a point and a given distance from a line
  - equal distances from 2 points or 2 line segments
  - regions which may be defined by ‘nearer to’ or ‘greater than’

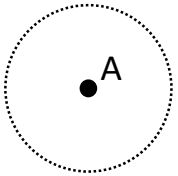
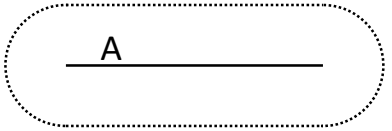
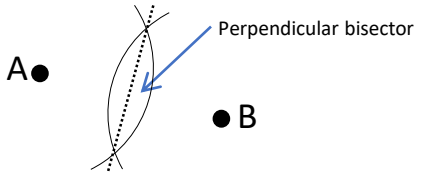
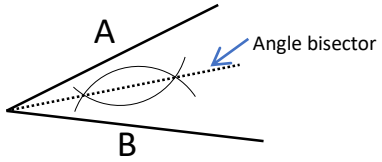
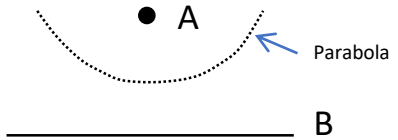
Extra Notes

Loci

Loci

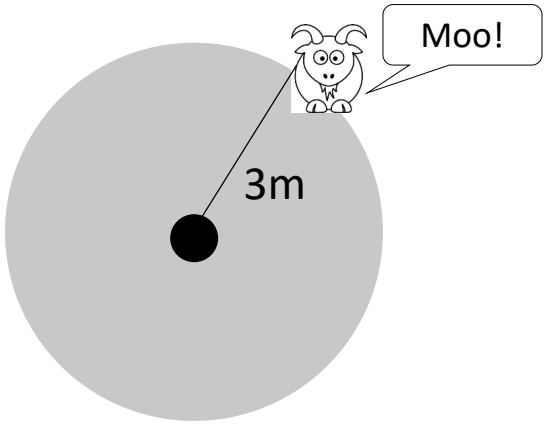
A **locus of points** is a set of points satisfying a certain condition.

We can use our constructions from last lesson to find the loci satisfying certain conditions…

Loci involving:			
Thing A	Thing B	Interpretation	Resulting Locus
Point	-	A given distance from point A	
Line	-	A given distance from line A	
Point	Point	Equidistant from 2 points or given distance from each point.	
Line	Line	Equidistant from 2 lines	
Point	Line	Equidistant from point A and line B	

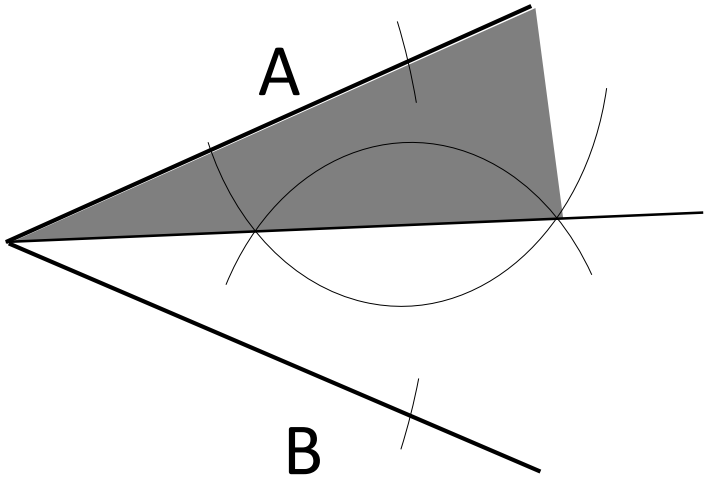
Regions satisfying descriptions

Loci can also be **regions** satisfying certain descriptions.



A goat is attached to a post, by a rope of length 3m. Shade the locus representing the points the goat can reach.

A goat is now attached to a metal bar, by a rope of length 3m. The rope is attached to the bar by a ring, which is allowed to move freely along the bar. Shade the locus representing the points the goat can reach.



Shade the region consisting of points which are closer to line A than to line B.

Common schoolboy error: Thinking the locus will be oval in shape.

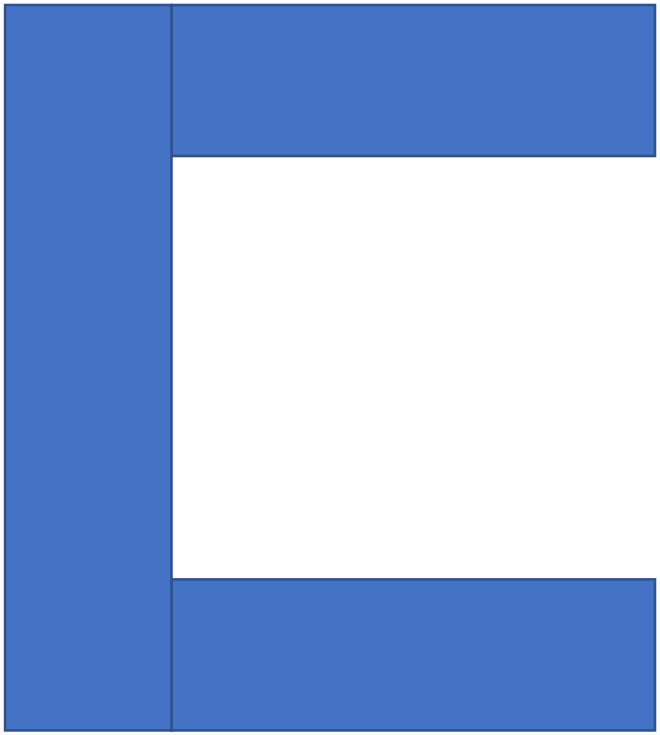
As always, you MUST show construction lines or you will be given no credit.

Examples

Q

I'm at most 2m away from the walls of a building. Mark this region with R.

Scale: 1m : 1cm

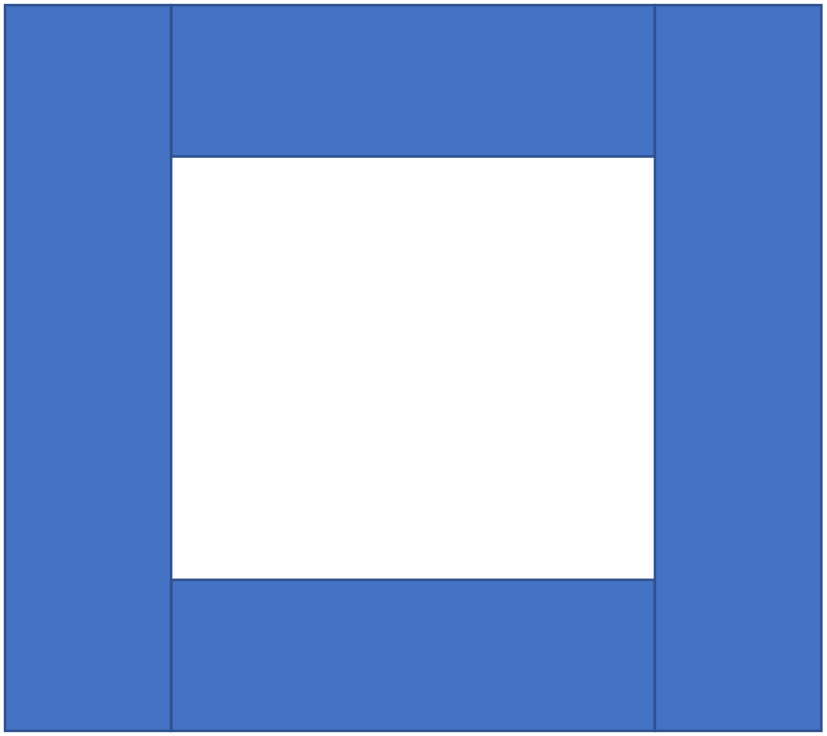


Examples

Q

I’m 2m away from the walls of a building.

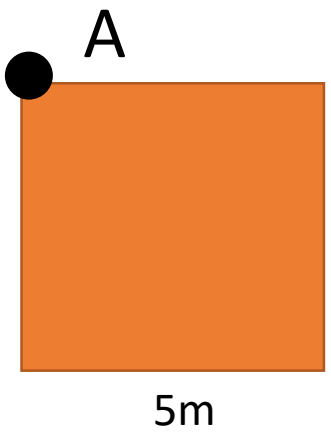
Scale: 1m : 1cm



Examples

Q

Scale: 1m : 1cm



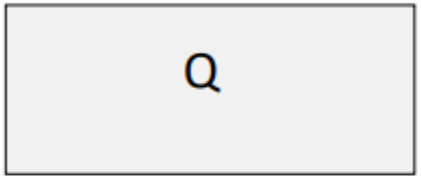
My goat is attached to a fixed point A on a square building, of 5m x 5m, by a piece of rope 10m in length. Both the goat and rope are fire resistant. What region can he reach?

Bonus question: What is the area of this region, is in terms of π?

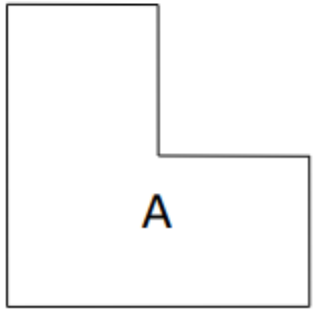
?

Loci Exercises

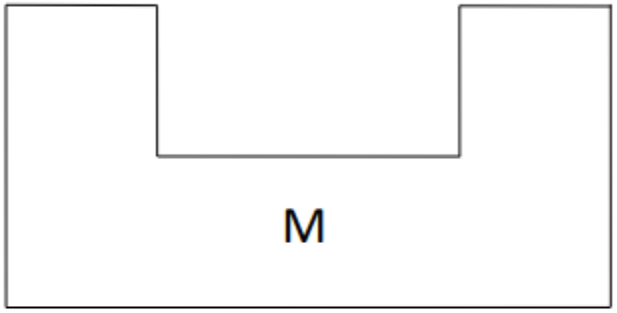
1. Mr Dumpleton is 2cm from shape Q. Shade the region he could be in.



2. Sketch the region in which you are at most 2cm from shape A.

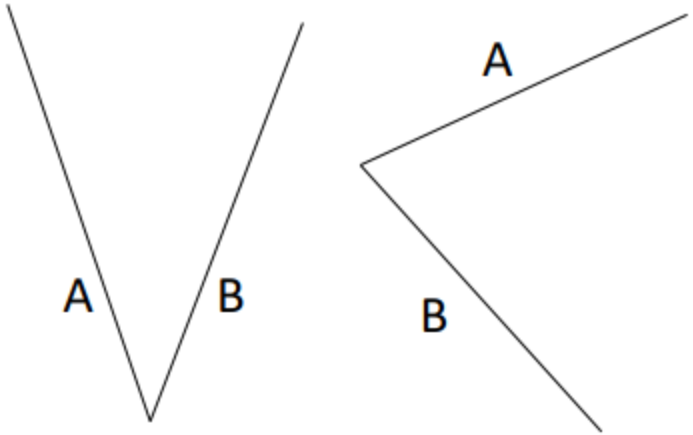


3. Draw the locus representing points which are 1cm from the edges of polygon M (this could include the inside).



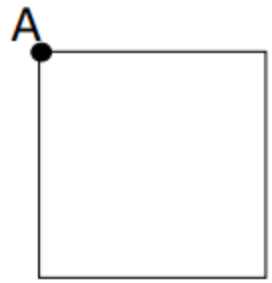
4. Sketch the region which is at most 5cm from A and 3cm from B.

5. Find the locus for which the points are equidistant from lines A and B.

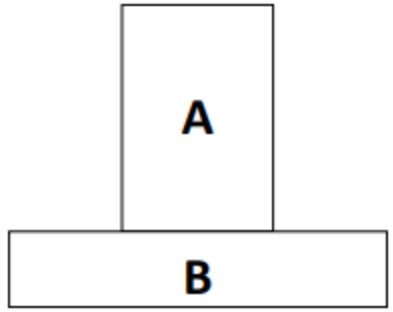


6. Draw the locus representing points which are equidistant from A and B.

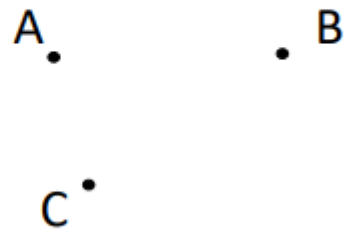
7. Mr Belemet is tied by a rope, of length 4cm to a fixed point A. Shade the region in which Mr Belemet can graze.



8. Sketch the region at most 3cm away from A and at most 2cm away from B.



9. Sketch the region where you are at most 2.5cm from A, at least 2cm from B, and at most 1.5cm from C.



10. Shade the region within rectangle ABCD which is:
    a. Closer to AB than to CD, and closer to BC than to AB.

b. Closer to AB than to CD, and at most 3cm away from A.

c. Closer to AB than to AD, less than 4cm away from A, and more than 1cm away from CD.

d. Closer to BC than to AD, more than 3cm away from B, and closer to AB than to BC.

11. ☠ For the following questions, calculate the area of the locus, in terms of the given variables (and π where appropriate). Assume that you could be inside or outside the shape unless otherwise specified.

e. x metres away from the edges of a square of length l.

_____

f. x metres away from the edges of a rectangle of sides w and h.

_____

g. x metres away from the edges of an equilateral triangle of side length y.

_____

h. Inside a square ABCD of side x metres, being at least x metres from A, and closer to BC than to CD.

_____

i. Being inside an equilateral triangle of side $2x$, and at least x away from each of the vertices.

_____

j. Being attached to one corner on the outside of $x \times x$ square building (which you can't go inside), by a rope of length $2x$.

_____

k. At most x metres away from an L-shaped building with two longer of longer sides $2x$ and four shorter sides of x metres.

_____

l. Being attached to one corner on the outside of $w \times h$ square building (which you can't go inside), by a rope of length x (where $x < w + h$). You may wish to distinguish between the cases when $x < w$ and/or $x < h$ and otherwise.

Extra Notes