Year 10
Mathematics
Unit 17

(adapted for 2022 year 10)

Name: _____

Class: _____

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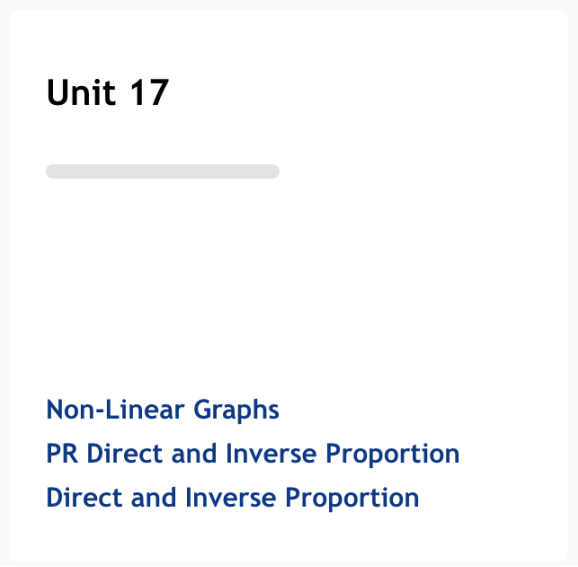
See unit 17 course on drfrostmaths.com

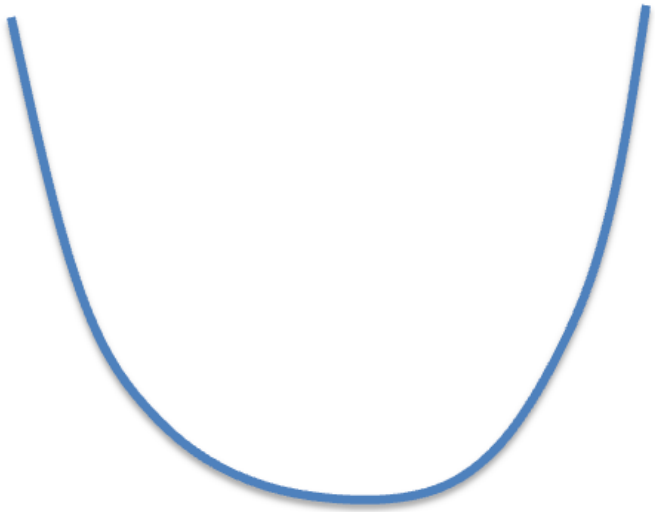


1 Non-Linear Graphs

Quadratic Graphs

$y = ax^2 + bx + c$

When $a > 0$



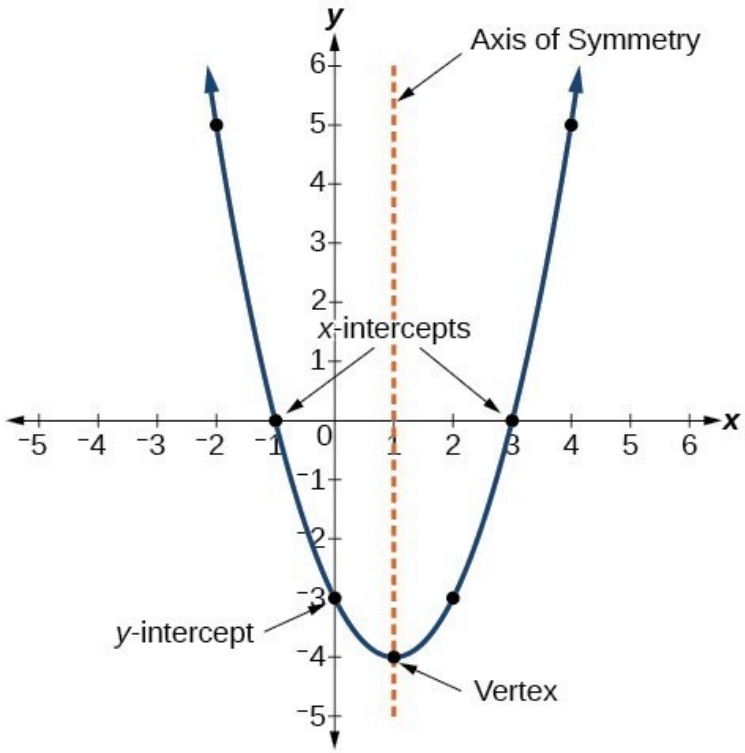
$y = ax^2 + bx + c$

When $a < 0$

The line for a quadratic equation is known as a **parabola**.

Interpreting Quadratic Graphs

- **y-intercept** – where the graph intercepts the y-axis
- **x-intercept** or **root** or **solution** – where the graph intercepts the x-axis
- **Turning point** or **vertex** or **minimum/maximum** – where the graph stops decreasing and starts increasing or vice-versa

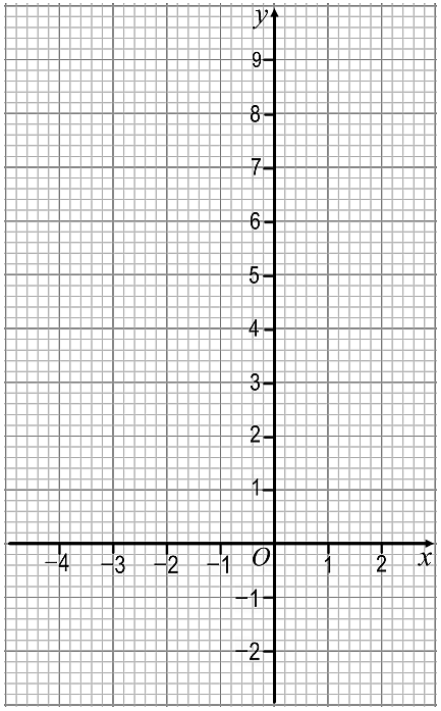


Worked Example

a) Complete the table and draw the graph of $y = x^2 + 2x$ for $x = -4$ to $x = 2$
b) Write down the equation of the line of symmetry of your graph
c) Use your graph to find:
i) the value of y when $x = 0.5$
ii) the values of x when $y = 6$

Here is a table of values for $y = x^2 + 2x$.

x	-4	-3	-2	-1	0	1	2
y	8		0	-1			8

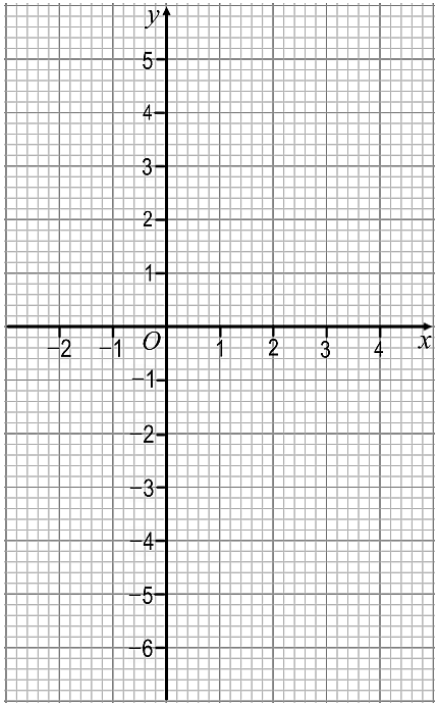


Worked Example

a) Complete the table and draw the graph of $y = x^2 - 2x - 4$ for $x = -2$ to $x = 4$
b) Write down the equation of the line of symmetry of your graph
c) Write down the values of x where the graph crosses the x-axis

Here is a table of values for $y = x^2 - 2x - 4$.

x	-2	-1	0	1	2	3	4
y		-1	-4			-1	



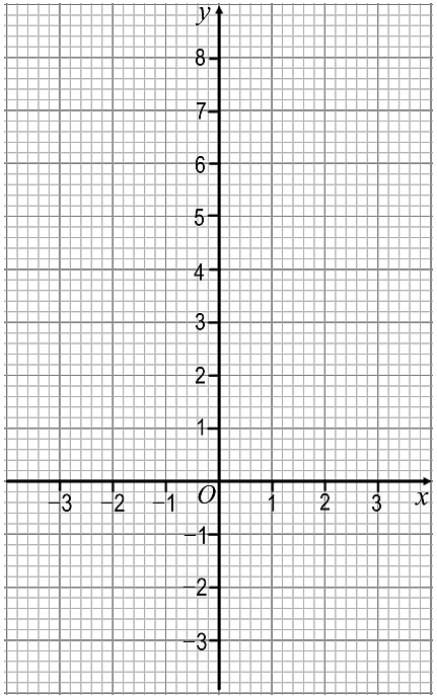
Fluency Practice

1. Here is a table of values for $y = x^2 - 2$.

x	−3	−2	−1	0	1	2	3
y	7		−1	−2			7

a) Complete the table of values.

b) On the grid, draw the graph of $y = x^2 - 2$ for $x = -3$ to $x = 3$.



c) Write down the equation of the line of symmetry of your graph.

d) Write down the coordinates of the minimum point.

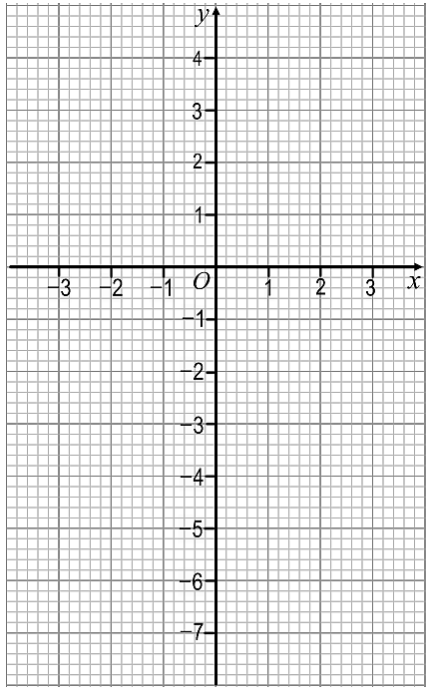
Fluency Practice

2. Here is the table of values for $y = 3 - x^2$.

x	−3	−2	−1	0	1	2	3
y	−6		2	3		−1	

a) Complete the table of values.

b) On the grid, draw the graph of $y = 3 - x^2$ for $x = -3$ to $x = 3$.



c) Write down the coordinates of the maximum point.

d) Write down the values of x where the graph crosses the x-axis.

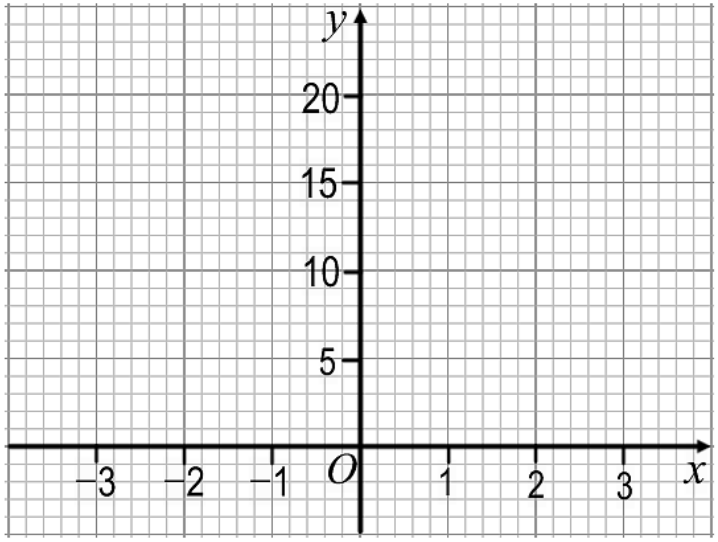
Fluency Practice

3. Here is a table of values for $y = 2x^2 + 1$.

x	−3	−2	−1	0	1	2	3
y		9		1	3	9	

a) Complete the table of values.

b) On the grid, draw the graph of $y = 2x^2 + 1$ for $x = -3$ to $x = 3$.



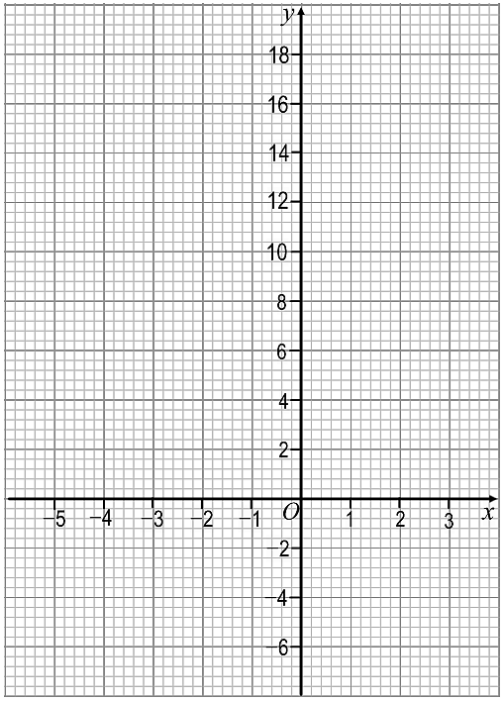
c) Use your graph to find:

i) the value of y when $x = -2.5$

ii) the two values of x when $y = 6$.

Fluency Practice

4. a) On the grid, draw the graph of $y = x^2 + 3x - 2$ for the values of x from −5 to 3.

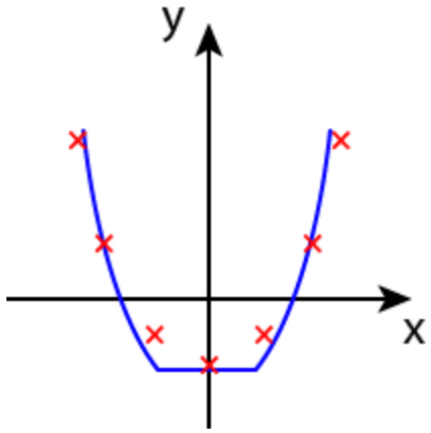


b) Use your graph to:

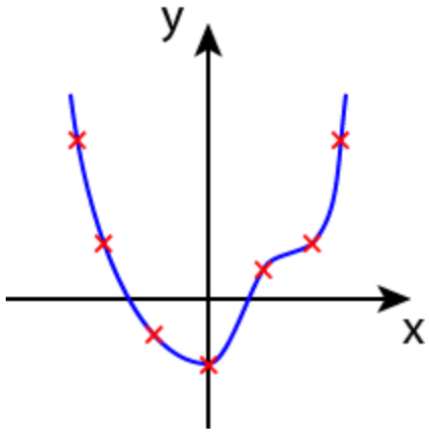
i) write down the values of x when the graph crosses the x-axis

ii) draw in and write down the equation of the line of symmetry.

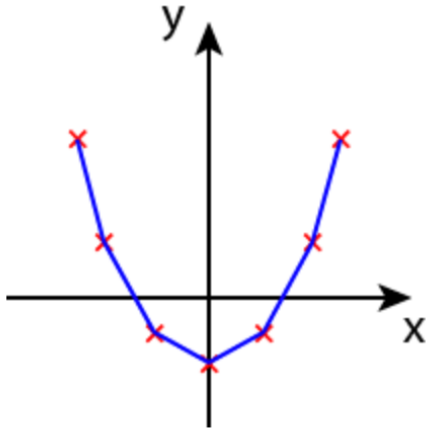
Common Mistakes



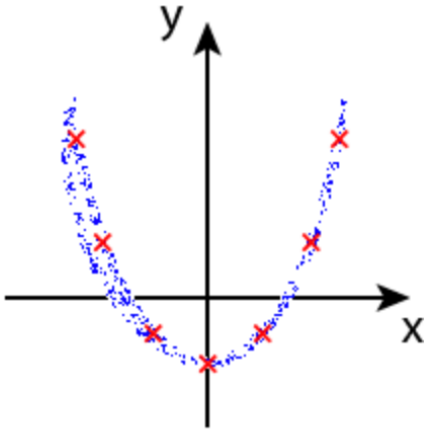
The parabola does not have a clearly defined minimum or maximum point.



A coordinate pair is either calculated or plotted incorrectly.



Drawing line segments between each coordinate pair suggest the relationship between them is linear which it is not.



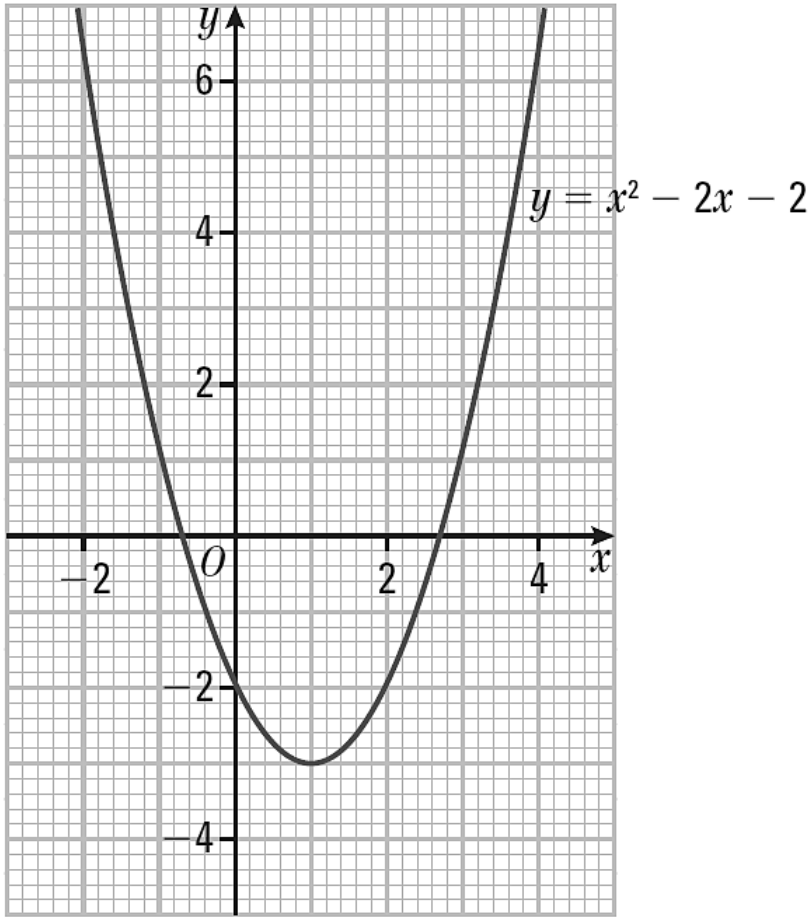
The graph is does not pass through each of the coordinate pairs to form a clear defined and smooth parabola.

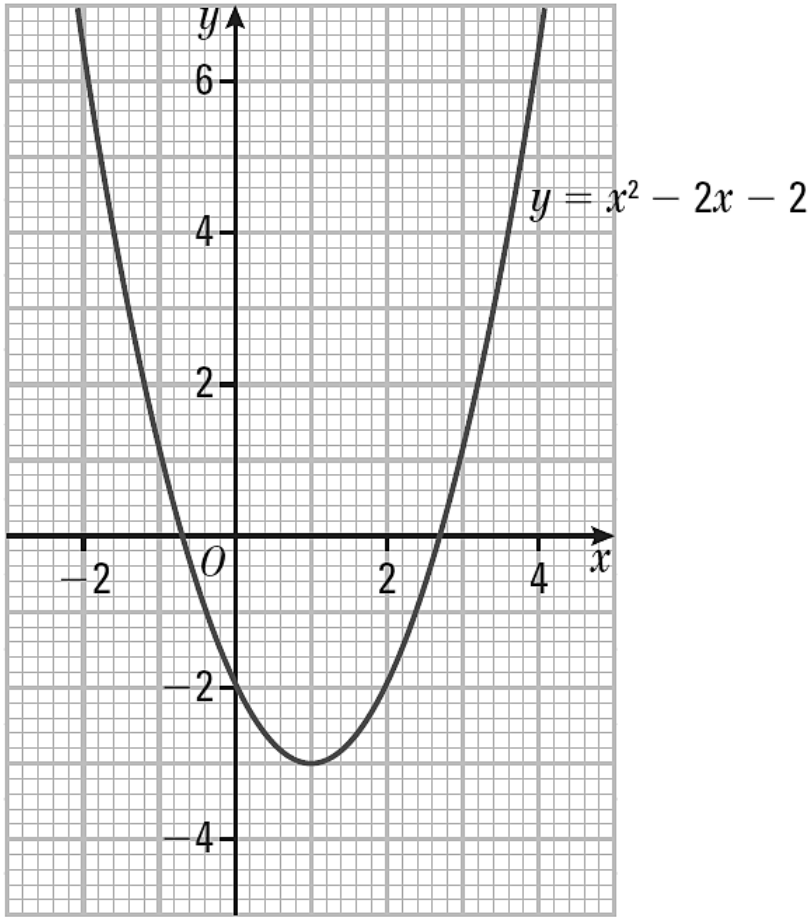
Worked Example

Use this graph to solve these equations:

a) $x^2 - 2x - 2 = 0$

b) $x^2 - 2x - 5 = 0$

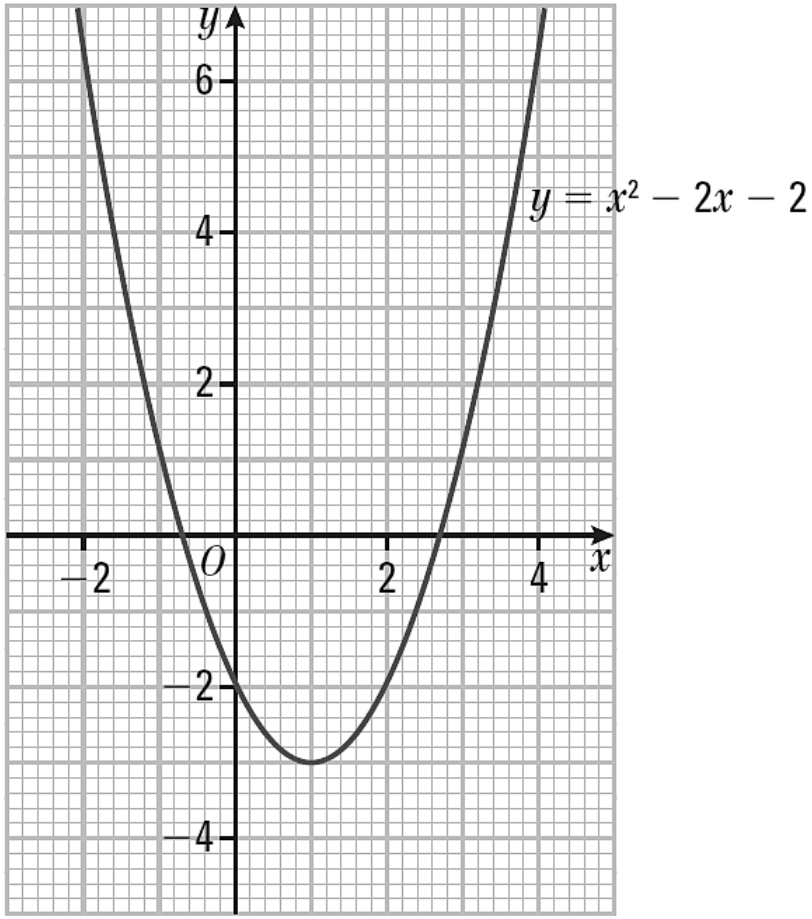




Worked Example

Use this graph to solve these equations:

c) $x^2 - 2x - 2 = x$

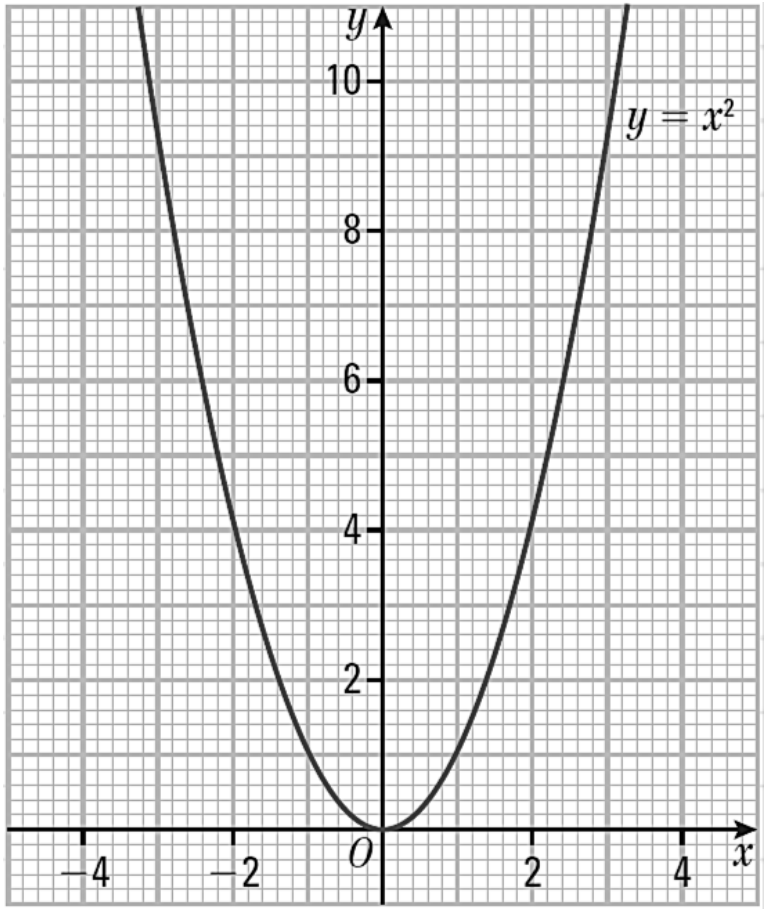


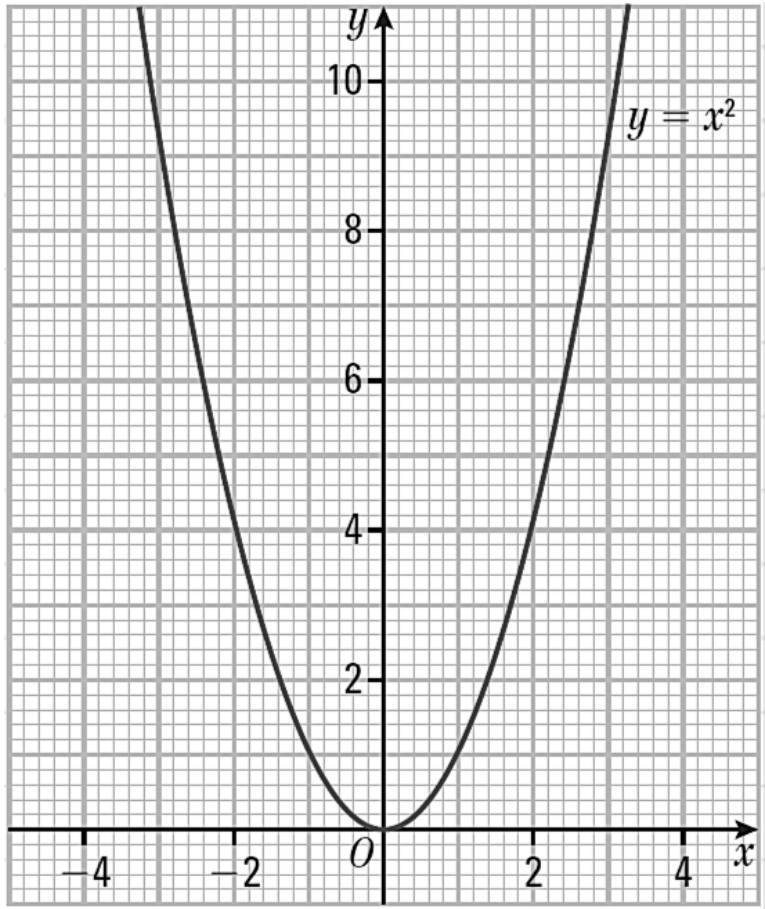
Worked Example

Use this graph to solve these equations:

a) $x^2 = 2x + 3$

b) $x^2 = x + 4$



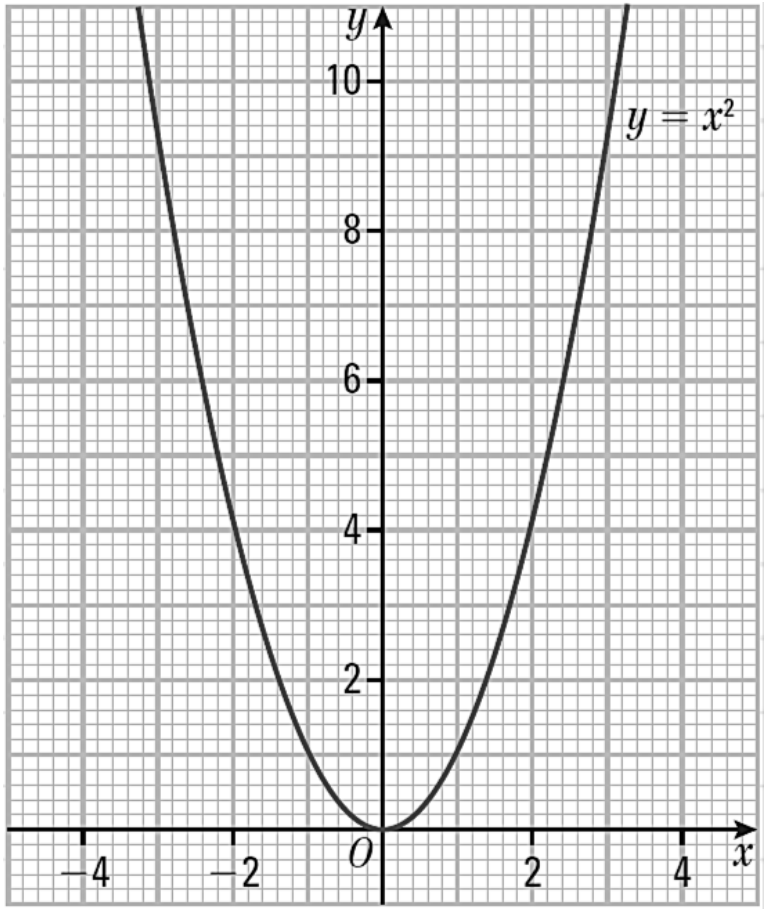


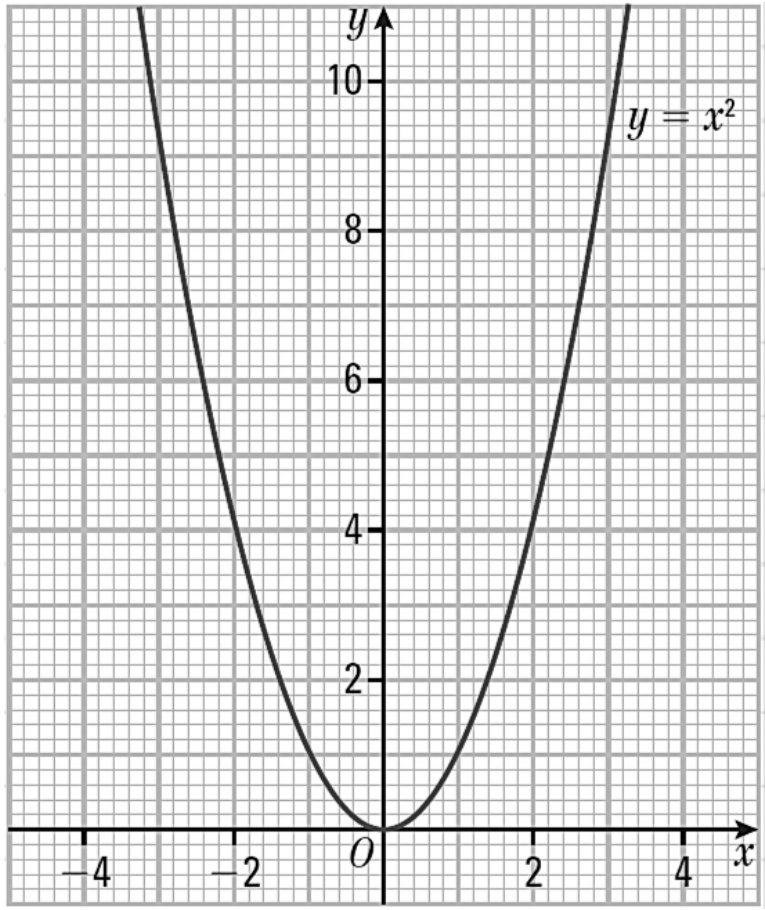
Worked Example

Use this graph to solve these equations:

c) $x^2 + x - 1 = 0$

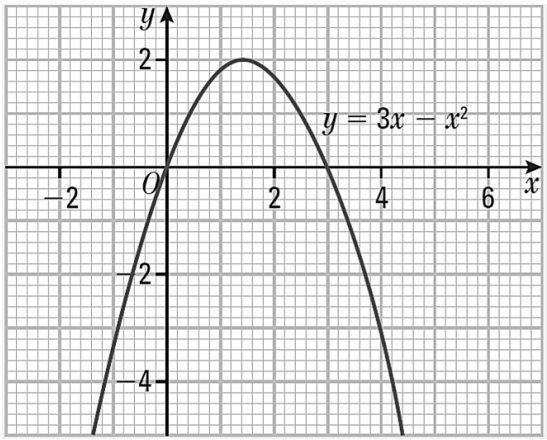
d) $x^2 - 2x - 1 = 0$





Fluency Practice

1. Use this graph to solve the equations.

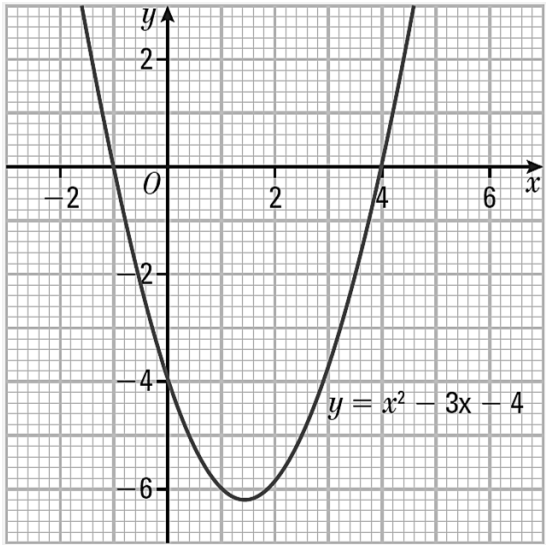


a) $3x - x^2 = 0$

b) $3x - x^2 = 1$

c) $3x - x^2 = -4$

2. Use this graph to solve the equations.



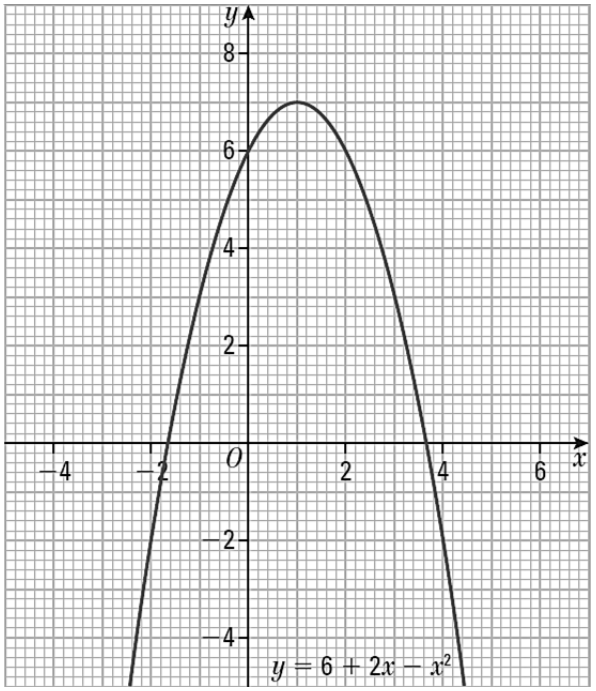
a) $x^2 - 3x - 4 = 0$

b) $x^2 - 3x - 4 = 2$

c) $x^2 - 3x - 4 = -5$

Fluency Practice

3. Use this graph to solve the equations.



a) $6 + 2x - x^2 = 0$

b) $4 + 2x - x^2 = 0$

c) $6 + 2x - x^2 = x$

d) $3 + 3x - x^2 = 0$

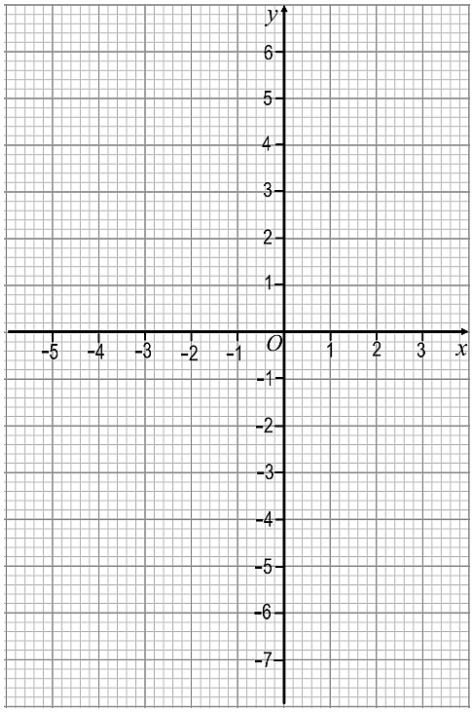
Fluency Practice

4. Here is a table of values for $y = x^2 + 3x - 4$.

x	-5	-4	-3	-2	-1	0	1	2
y	6	0		-6		-4		

a) Complete the table of values.

b) On the grid, draw the graph of $y = x^2 + 3x - 4$.

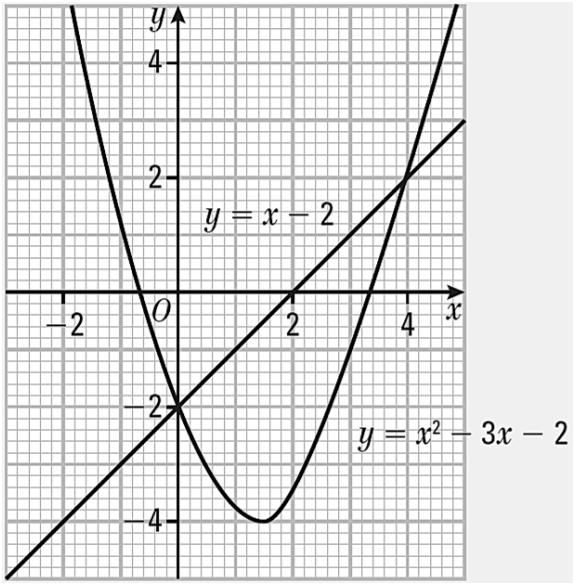


c) Use your graph to solve the equation $x^2 + 3x - 4 = 2$.

d) By drawing a suitable straight line on your graph, solve the equation $x^2 + 3x - 4 = x + 1$.

Fluency Practice

5. The graphs $y = x^2 - 3x - 2$ and $y = x - 2$ are shown below.



a) Show that the equation $x^2 - 3x - 2 = x - 2$ can be rewritten as $x^2 - 4x = 0$.

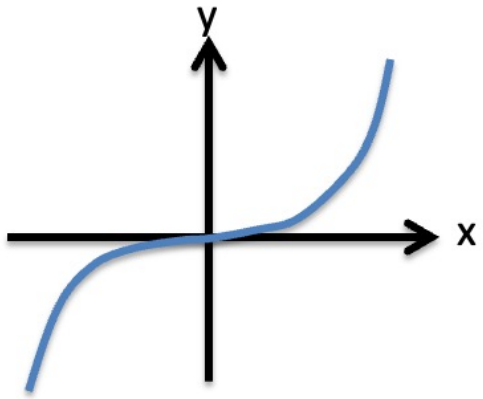
b) Solve the equation $x^2 - 4x = 0$.

c) The equation $x^2 - 2x - 4 = 0$ can be solved by drawing a suitable straight line on the graph.
Find the equation of this straight line and solve the equation $x^2 - 2x - 4 = 0$.

Cubic Graphs

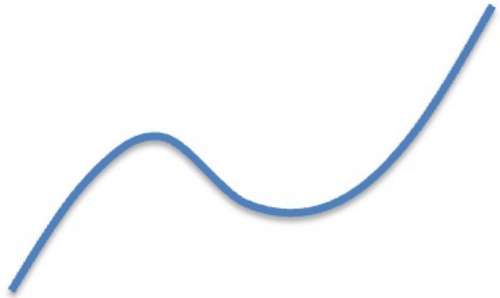
$y = ax^3$

When $a > 0$



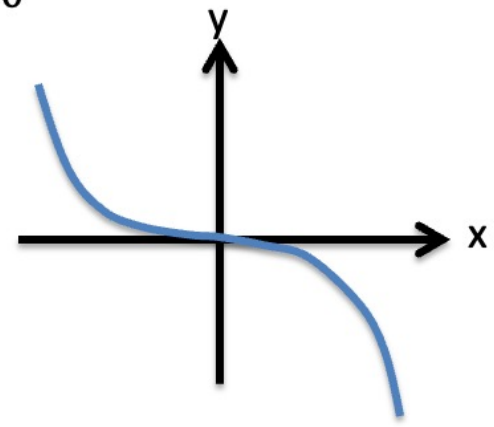
$y = ax^3 + bx^2 + cx + d$

When $a > 0$



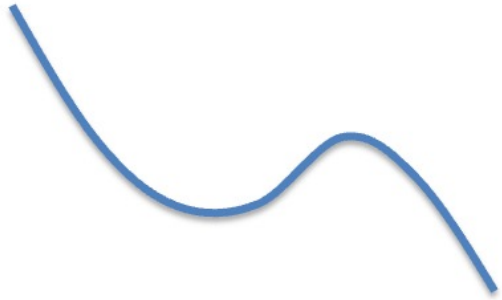
$y = ax^3$

When $a < 0$



$y = ax^3 + bx^2 + cx + d$

When $a < 0$



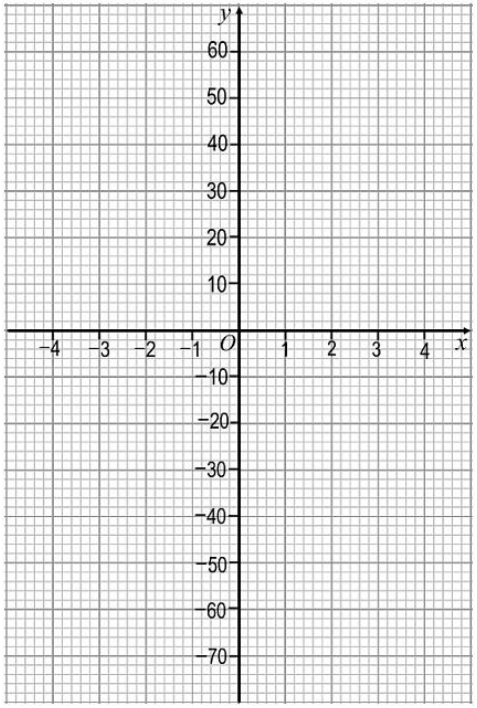
Worked Example

a) Complete the table and draw the graph of $y = x^3 - 4$ for $x = -4$ to $x = 4$

b) Use the graph to find the value of y when $x = 4$

Here is a table of values for $y = x^3 - 4$.

x	-4	-3	-2	-1	0	1	2	3	4
y									



Worked Example

a) Complete the table and draw the graph of $y = x^3 - 4x^2 + 5$ for $x = -2$ to $x = 5$

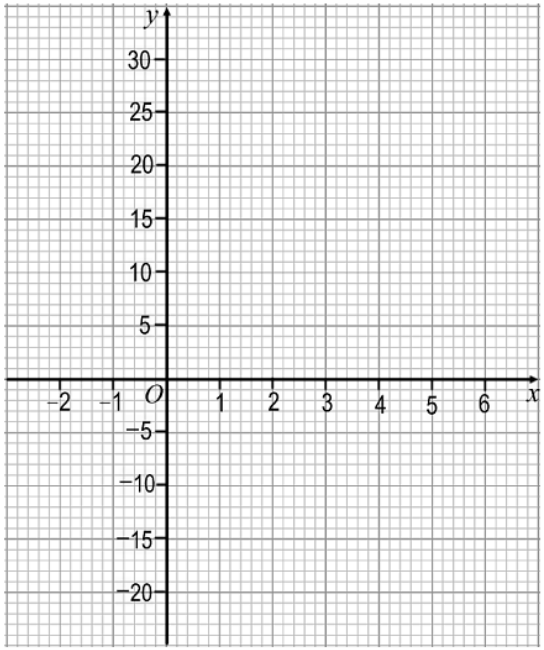
b) Use your graph to find the solutions to:

i) $x^3 - 4x^2 + 5 = 0$

ii) $x^3 - 4x^2 - x + 5 = 0$

Here is a table of values for $y = x^3 - 4x^2 + 5$.

x	-2	-1	0	1	2	3	4	5
y	-19		5			-4	5	



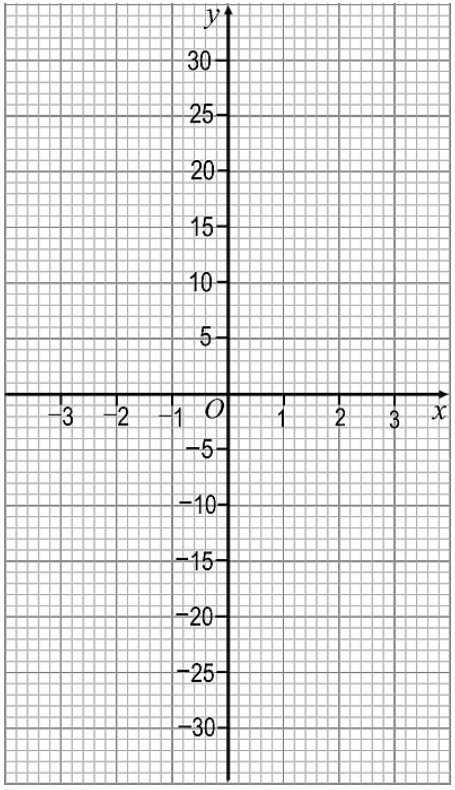
Fluency Practice

1. Here is a table of values for $y = x^3 + 1$.

x	−3	−2	−1	0	1	2	3
y							

a) Complete the table of values.

b) On the grid, draw the graph of $y = x^3 + 1$ for $-3 \leqslant x \leqslant 3$.



c) Use your graph to find the value of y when $x = 1.5$.

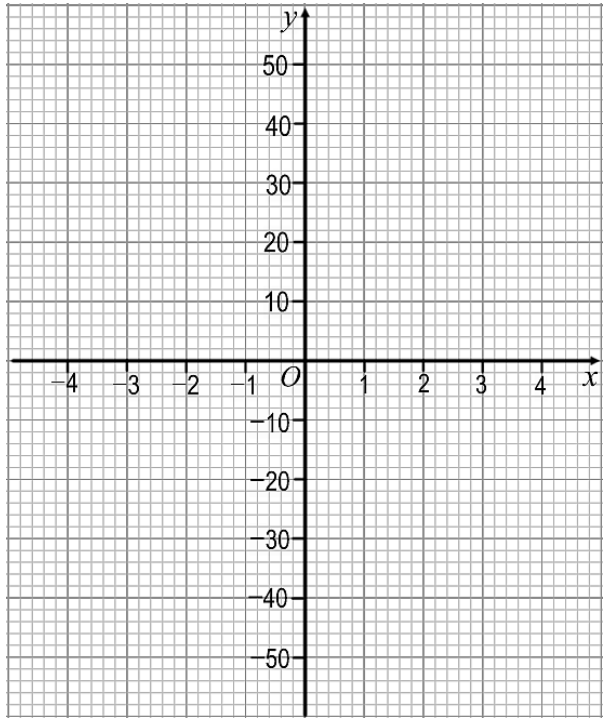
Fluency Practice

2. Here is the table of values for $y = x^3 - 5x$.

x	−4	−3	−2	−1	0	1	2	3	4
y		−12			0	−4		12	44

a) Complete the table of values.

b) On the grid, draw the graph of $y = x^3 - 5x$ for $-4 \leqslant x \leqslant 4$.



c) Use your graph to find the solutions to the equation $x^3 - 5x = 0$.

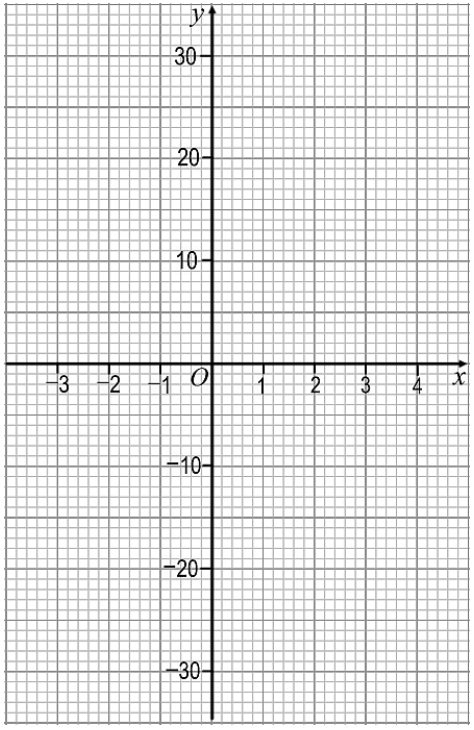
Fluency Practice

3. Here is a table of values for $y = 6x + x^2 - x^3$.

x	-3	-2	-1	0	1	2	3	4
y		0	-4			8	0	

a) Complete the table of values.

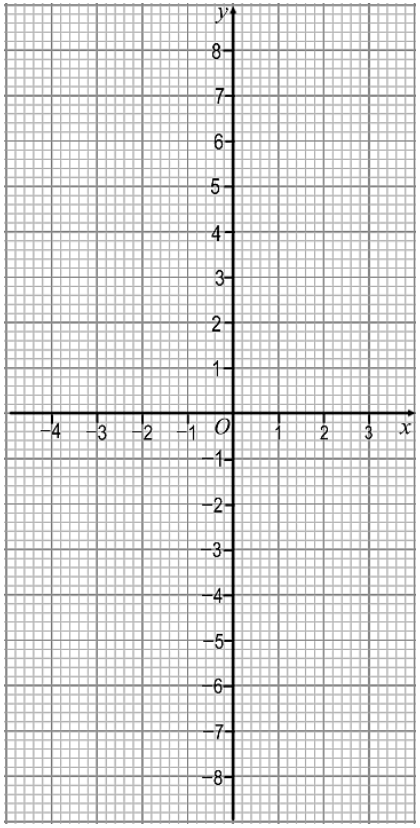
b) On the grid, draw the graph of $y = 6x + x^2 - x^3$ for $-3 \leqslant x \leqslant 4$.



c) By drawing a suitable line on your diagram, solve the equation $6x + x^2 - x^3 = x - 2$.

Fluency Practice

4. a) On the grid, draw the graph of $y = x^3 + x^2 - 4x - 2$ for the values of x from -3 to 2.



b) By drawing a suitable line on your diagram, solve the equation $x^3 + x^2 - 5x - 2 = 0$.

Reciprocal Graphs

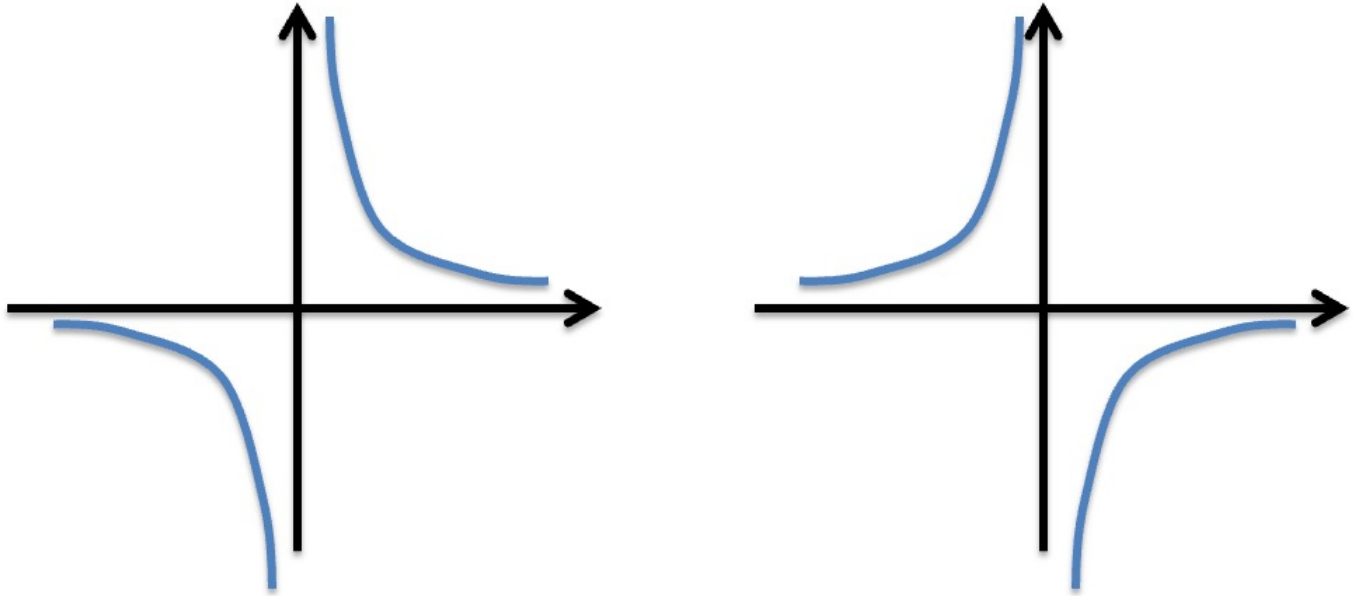
$$y = \frac{a}{x}$$

When $a > 0$

a is a constant while x is a variable, so we might have $y = \frac{3}{x}$

$$y = \frac{a}{x}$$

When $a < 0$



The lines $x = 0$ and $y = 0$ are called asymptotes.

An asymptote is a straight line which the curve approaches at infinity.

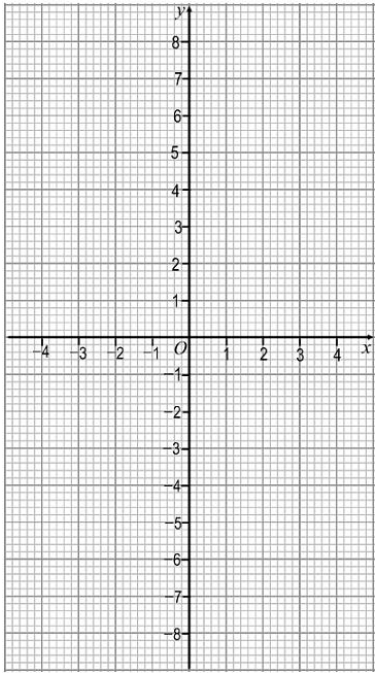
Worked Example

Complete the tables and draw the graph of $y = \frac{2}{x}$ for $x = -5$ to $x = 5$

Here is a table of values for $y = \frac{2}{x}$.

x	0.25	0.4	0.5	0.8	1	2	4	5
y								

x	−0.25	−0.4	−0.5	−0.8	−1	−2	−4	−5
y								



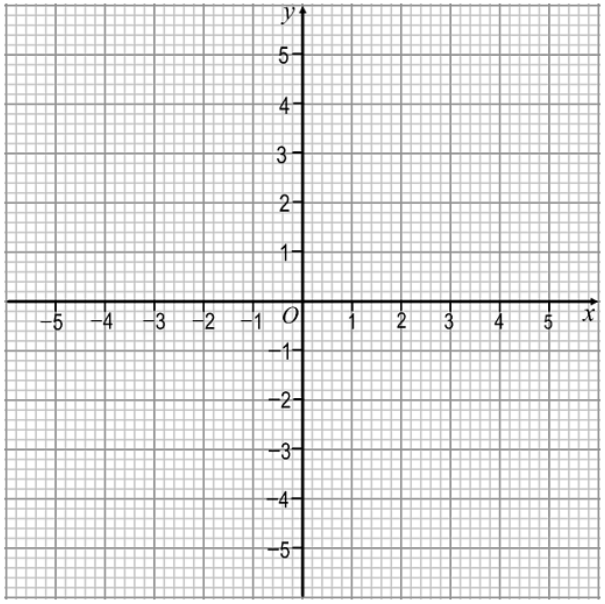
Worked Example

Complete the tables and draw the graph of $y = -\frac{1}{x}$ for $x = -5$ to $x = 5$

Here is a table of values for $y = -\frac{1}{x}$.

x	0.2	0.4	0.5	0.8	1	2	3	4	5
y									

x	−0.2	−0.4	−0.5	−0.8	−1	−2	−3	−4	−5
y									



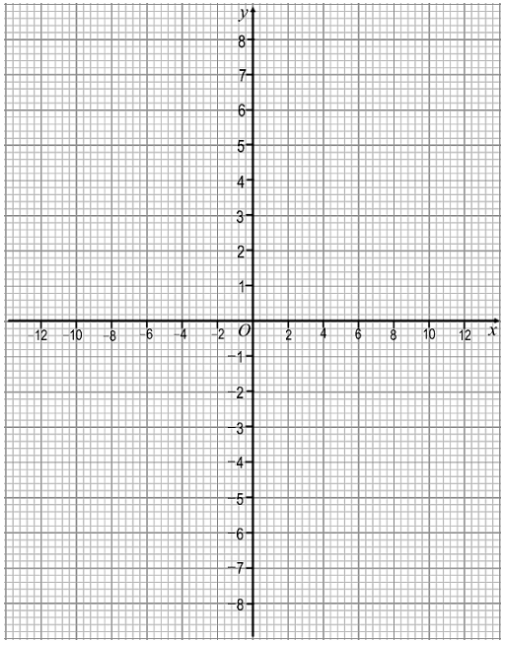
Worked Example

Complete the tables and draw the graph of $y = \frac{4}{x-1}$ for $x = -12$ to $x = 12$

Here is a table of values for $y = \frac{4}{x-1}$.

x	1.5	2	3	5	6	9	11
y							

x	0.5	0	-1	-3	-4	-7	-9
y							



Fluency Practice

1. Here are some table of values for $y = \frac{4}{x}$.

x	0.2	0.4	0.5	1	2	4	5	8	10
y		10		4	2		0.8		

x	-10	-8	-5	-4	-2	-1	-0.5	-0.4	-0.2
y									

a) Complete the table of values.

b) On your additional sheet, draw the graph of $y = \frac{4}{x}$ for $-10 \leqslant x \leqslant 10$.

c) Use your graph to find an estimate for the solutions of $\frac{4}{x} = 4 - x$.

2. On your additional sheet, draw the graph of $y = -\frac{3}{x}$ for $-10 \leqslant x \leqslant 10$.

Fluency Practice

3. a) Here are some table of values for $y = \frac{8}{x+2}$.

x	−12	−10	−7	−6	−4	−3	−1	0	2	3	6	8
y												

b) On your additional sheet, draw the graph of $y = \frac{8}{x+2}$ for $-12 \leqslant x \leqslant 12$.

c) For which values of x is $y = \frac{8}{x+2}$ not defined?

4. a) Complete the table of values for $y = 3 - \frac{2}{x}$, $x \neq 0$.

x	−3	−2	−1	−0.5	−0.1	0.1	0.5	1	2	3
y										

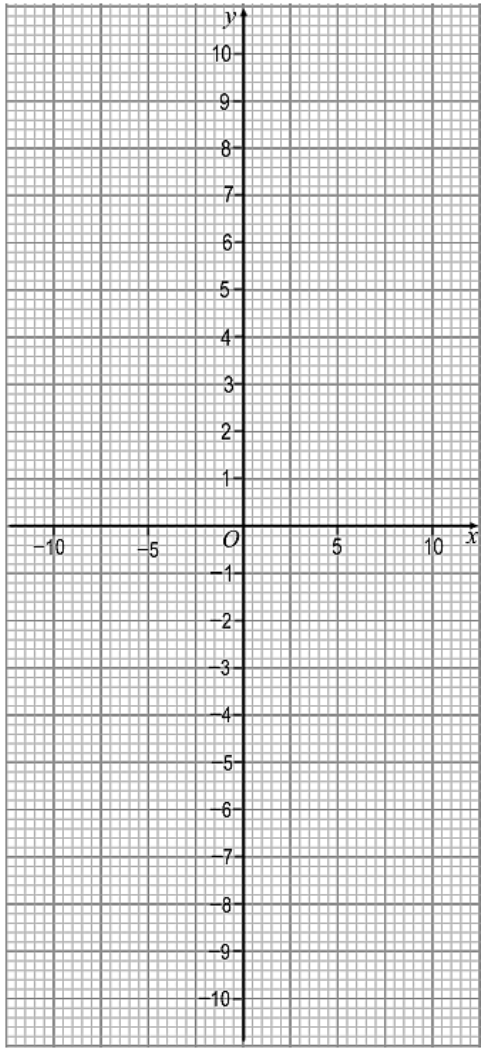
b) On your additional sheet, draw the graph of $y = 3 - \frac{2}{x}$ for $-3 \leqslant x \leqslant 3$.

c) This graph approaches two lines without touching them. These lines are called asymptotes. Write down the equation of each of these two lines.

Fluency Practice

Question 1

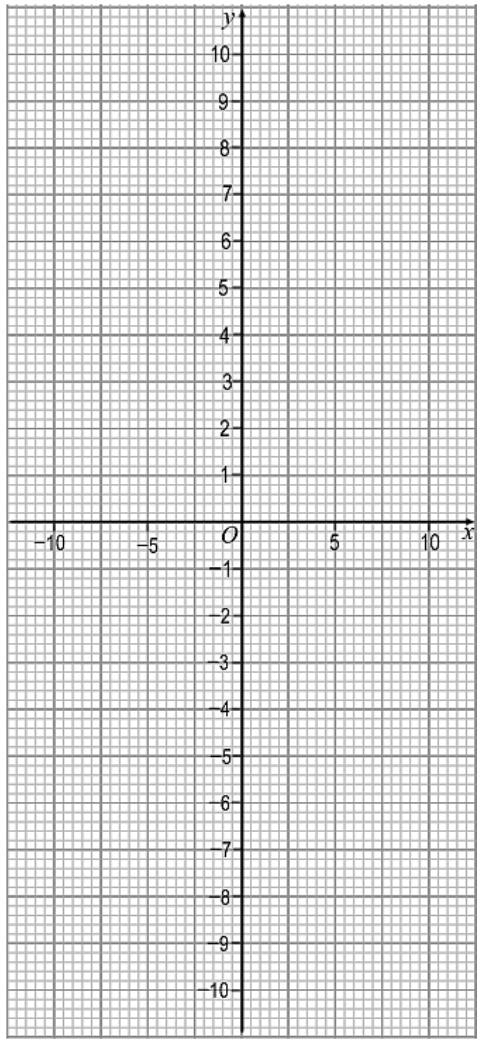
$y = \frac{4}{x}$



Fluency Practice

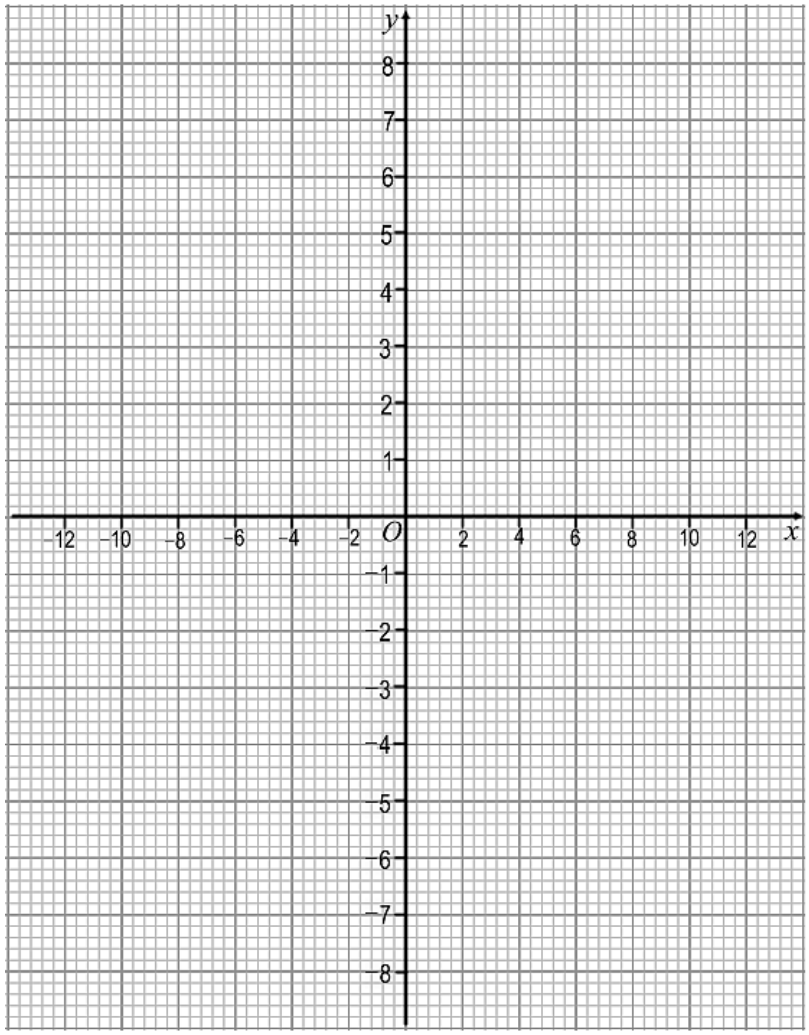
Question 2

$y = -\frac{3}{x}$



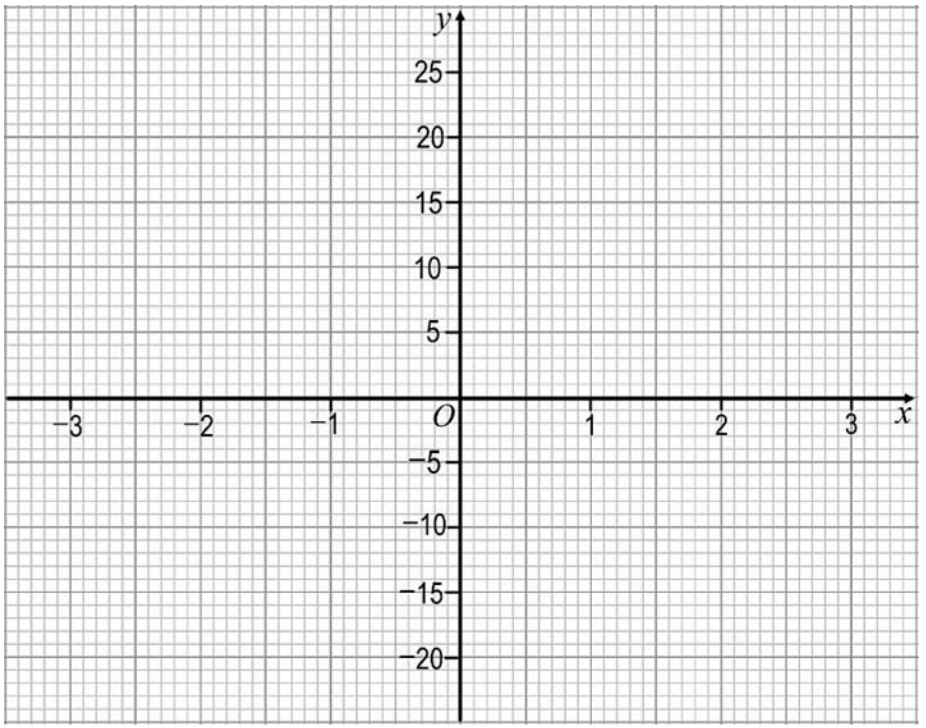
Fluency Practice

Question 3 $y = \frac{8}{x+2}$



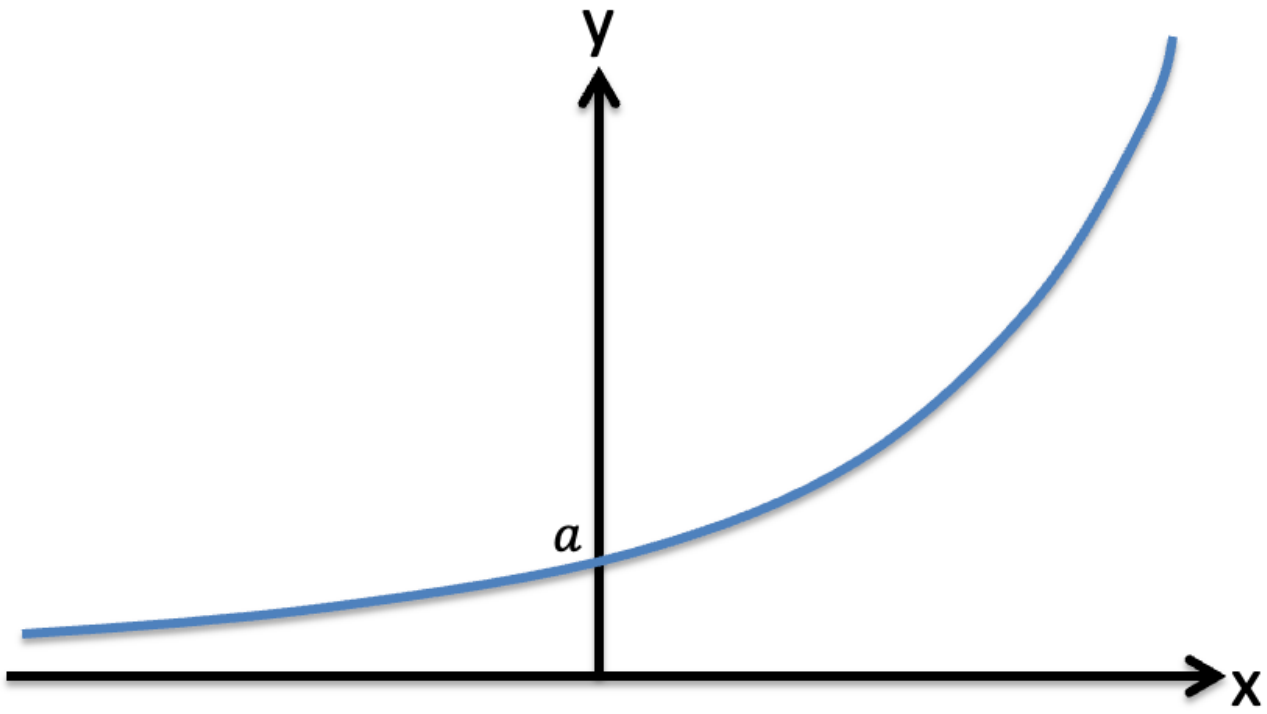
Fluency Practice

Question 4 $y = 3 - \frac{2}{x}$



Exponential Graphs

$$y = a \times b^x$$



The y-intercept is a because $a \times b^0 = a \times 1 = a$.

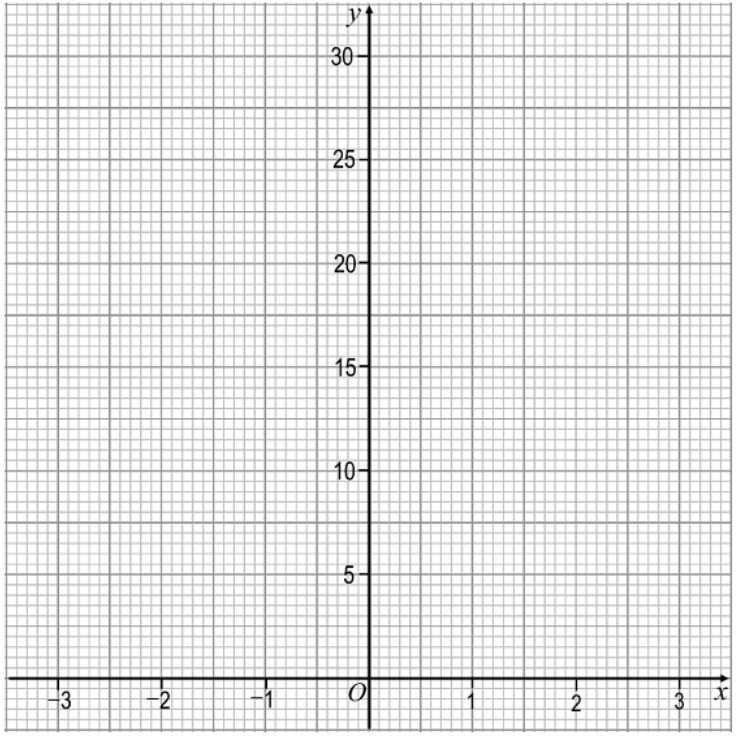
(unless $a = 0$, but let's not go there!)

Worked Example

a) Complete the tables and draw the graph of $y = 3^x$ for $x = -3$ to $x = 3$
b) Use your graph to estimate the solution to $3^x = 20$

Here is a table of values for $y = 3^x$.

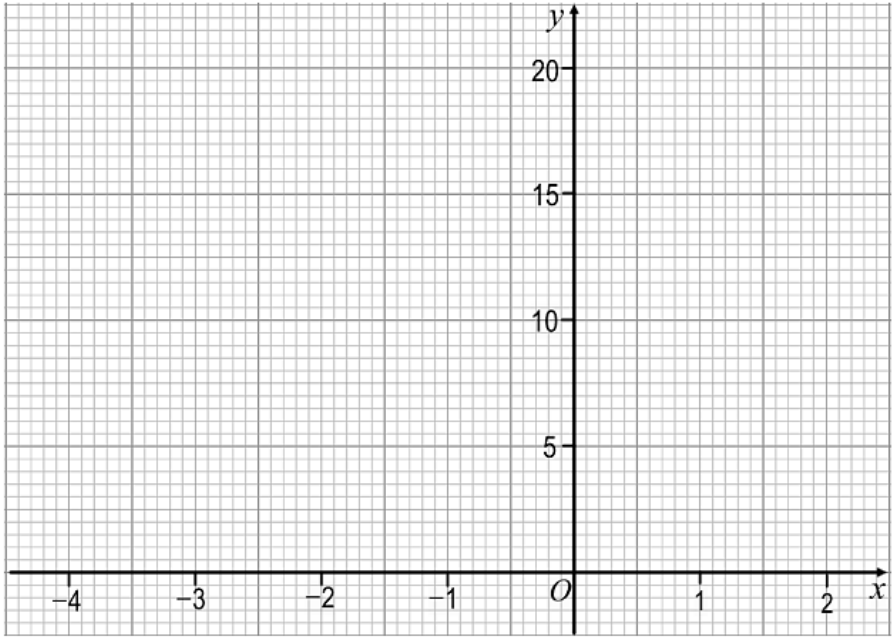
x	-3	-2	-1	0	1	2	3
y							



a) Complete the tables and draw the graph of $y = 2^{-x}$ for $x = -4$ to $x = 2$
b) Use your graph to estimate
i) the value of y when $x = 0.5$
ii) the solution to the equation $2^{-x} = 10$

Here is a table of values for $y = 2^{-x}$.

x	-4	-3	-2	-1	0	1	2
y							



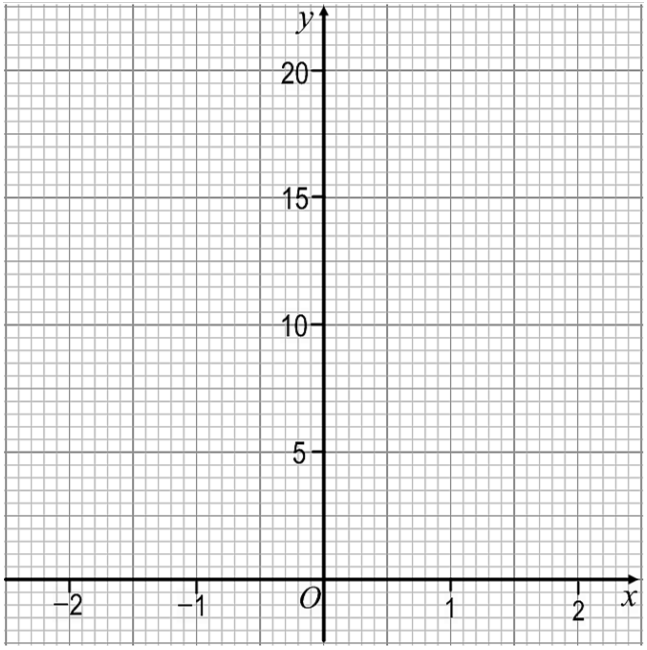
Fluency Practice

1. Here is a table of values for $y = 4^x$.

x	-2	-1	0	1	2
y					

a) Complete the table of values.

b) On the grid, draw the graph of $y = 4^x$ for $-2 \leqslant x \leqslant 2$.



c) Use your graph to find an estimate for:

i) the value of y when $x = 1.5$

ii) the value of x when $y = 11$

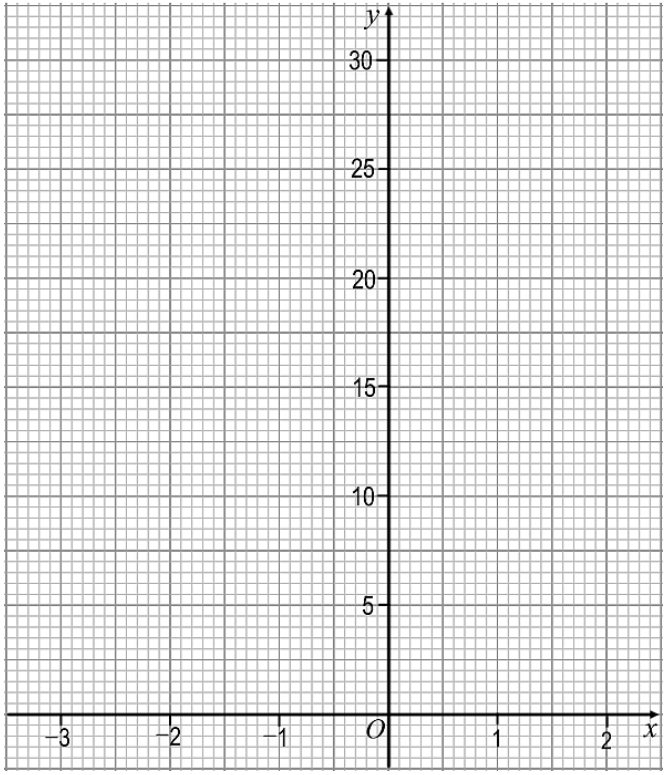
Fluency Practice

2. Here is the table of values for $y = 3^{-x}$.

x	-3	-2	-1	0	1	2
y						

a) Complete the table of values.

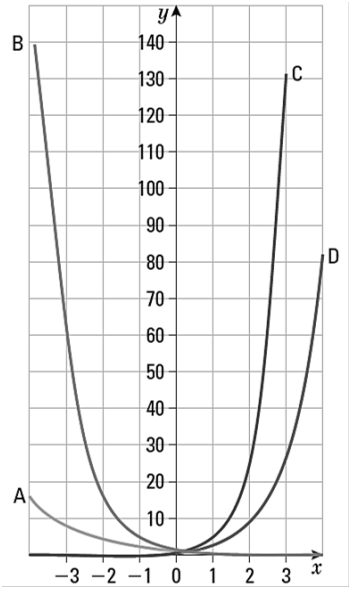
b) On the grid, draw the graph of $y = 3^{-x}$ for $-3 \leqslant x \leqslant 2$.



c) Use your graph to find the solution to the equation $3^{-x} = 7$.

Fluency Practice

3. The diagram shows the graphs of $y = 3^x$, $y = 2^{-x}$, $y = 5^x$ and $y = \left(\frac{1}{4}\right)^x$.



Match each graph to its equation.

4. The number of rabbits, n, in a particular population grows at a rate given by the equation $n = 5 \times 2^y$ where y is the number of years.

   a) How many rabbits were there initially (when $y = 0$)?

   b) How many rabbits are there after 6 years?

   c) How many years will it take for the rabbit population to exceed 5000?

Extra Notes

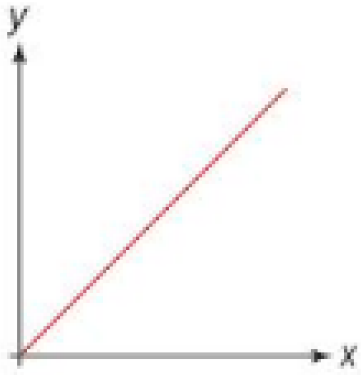
2 Direct and Inverse Proportion

Direct Proportion

y is directly proportional to x
y is proportional to x
y varies directly to x

$y \propto x$

$y = kx$
k is called the constant of proportionality



The graph of $y = kx$ is a straight line that passes through the origin.

Worked Example

y is directly proportional to x.

a) Find the constant of proportionality (k).

b) Find the missing value.

c) Write an equation connecting x and y.

x	1	2
y	5	

Your Turn

y is directly proportional to x.

a) Find the constant of proportionality (k).

b) Find the missing value.

c) Write an equation connecting x and y.

x		12
y	30	72

Worked Example	Your Turn
y is proportional to x. When $x = 2$, $y = 12$. a) Write an equation linking x and y and find the constant of proportionality (k). b) Use your answer to part a) to find the value of y when $x = 4$. c) Use your answer to part a) to find the value of x when $y = 72$.	y is proportional to x. When $x = 3$, $y = 24$. a) Write an equation linking x and y and find the constant of proportionality (k). b) Use your answer to part a) to find the value of y when $x = 9$. c) Use your answer to part a) to find the value of x when $y = 72$.

Worked Example	Your Turn
y is proportional to x. When $x = 2$, $y = 20$. a) Find y when $x = 5$. b) Find x when $y = 90$.	y is proportional to x. When $x = 4$, $y = 20$. a) Find y when $x = 5$. b) Find x when $y = 90$.

Worked Example	Your Turn
y is directly proportional to x. When $y = 20, x = 2$. a) Find y when $x = 5$. b) Find x when $y = 200$.	b is directly proportional to a. When $\mathrm{b} = 30, a = 5$. a) Find b when $\mathrm{a} = 2$. b) Find a when $b = 3000$.

Worked Example

y is directly proportional to x^2.

a) Work out the constant of proportionality (k).

b) Write an equation connecting y and x.

x	1	2	5
y	3	12	75

Your Turn

y is directly proportional to x^2.

a) Work out the constant of proportionality (k).

b) Write an equation connecting y and x.

x	1	3	7
y	5	45	245

Worked Example	Your Turn
y is proportional to x^2. When $y = 90$, $x = 3$. Work out the value of: a) y when $x = 5$. b) x when $y = 160$.	y is proportional to x^2. When $y = 18$, $x = 3$. Work out the value of: a) y when $x = 7$. b) x when $y = 72$.

Worked Example	Your Turn
y is directly proportional to the square of x. When $y = 36, x = 3$. a) Find y when $x = 5$. b) Find x when $y = 400$.	b is directly proportional to the square of a. When $\mathrm{b} = 12, a = 2$. a) Find b when $\mathrm{a} = 3$. b) Find a when $\mathrm{b} = 300$.

Worked Example	Your Turn
y is proportional to $\sqrt{x}$. When $y = 6$, $x = 9$. Work out the value of: a) y when $x = 16$. b) x when $y = 10$.	y is proportional to $\sqrt{x}$. When $y = 20$, $x = 16$. Work out the value of: a) y when $x = 36$. b) x when $y = 20$.

Worked Example	Your Turn
y is directly proportional to the square root of x. When $y = 36, x = 16$. a) Find y when $x = 25$. b) Find x when $y = 900$.	b is directly proportional to the square root of a. When $\mathrm{b} = 36, a = 144$. a) Find b when $\mathrm{a} = 49$. b) Find a when $\mathrm{b} = 243$.

Worked Example	Your Turn
a) y is directly proportional to $x + 2$. When $y = 20$, $x = 2$. Find y when $x = 5$. b) y is directly proportional to $x^2 + 4$. When $y = 52$, $x = 3$. Find y when $x = 5$.	a) y is directly proportional to $x + 2$. When $y = 12$, $x = 2$. Find y when $x = 8$. b) y is directly proportional to $2x^2$. When $y = 36$, $x = 3$. Find y when $x = 5$.

Worked Example	Your Turn
A is directly proportional to B^2. Find the percentage increase in A when B is increased by 10%.	A is directly proportional to B^2. Find the percentage increase in A when B is increased by 20%.

Inverse Proportion

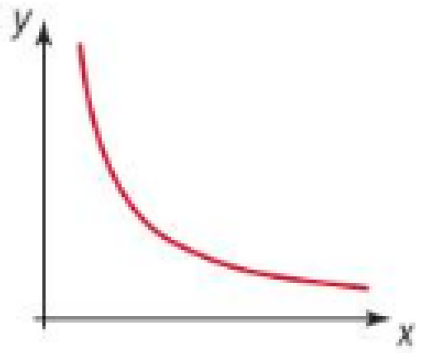
y is inversely proportional to x
y varies inversely or indirectly to x

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

k is called the constant of proportionality

The graph of $y = \frac{k}{x}$ is a reciprocal graph.



Worked Example

y is inversely proportional to x.

a) Work out the constant of proportionality (k).

b) Find the missing values in the table.

c) Write an equation connecting x and y.

x	1	2	
y	80		8

Your Turn

y is inversely proportional to x.

a) Work out the constant of proportionality (k).

b) Find the missing values in the table.

c) Write an equation connecting x and y.

x		9	12
y	120	40	

Worked Example	Your Turn
y is inversely proportional to x. When $x = 2$, $y = 50$. a) Work out the value of y when $x = 20$. b) Work out the value of x when $y = 12.5$.	y is inversely proportional to x. When $x = 5$, $y = 50$. a) Work out the value of y when $x = 10$. b) Work out the value of x when $y = 25$.

Worked Example	Your Turn
y is inversely proportional to x. When $y = 5, x = 2$. a) Find y when $x = 5$. b) Find x when $y = 0.5$.	b is inversely proportional to a. When $\mathrm{b} = 10, a = 3$. a) Find b when $\mathrm{a} = 5$. b) Find a when $\mathrm{b} = 0.25$.

Worked Example

y is inversely proportional to x^2.

a) What is the constant of proportionality (k).

b) Find the missing value.

c) Write an equation for y in terms of x.

d) Find the value of y when $x = 15$.

e) Find the value of x when $y = 62500$.

x	1	5	8
y	100	4	

Your Turn

y is inversely proportional to $\sqrt{x}$.

a) What is the constant of proportionality (k).

b) Find the missing values.

c) Write an equation for y in terms of x.

d) Find the value of y when $x = 36$.

e) Find the value of x when $y = 0.16$.

x	1	9		25
y	4	$\frac{4}{3}$	1	

Worked Example	Your Turn
y is inversely proportional to the cube of x. When $x = 2$, $y = 5$. a) Work out the value of y when $x = 4$. b) Work out the value of x (to 2dp) when $y = 8$.	y is inversely proportional to the square root of x. When $x = 4$, $y = 32$. a) Work out the value of y when $x = 100$. b) Work out the value of x when $y = 10$.

Worked Example	Your Turn
y is inversely proportional to the square of x. When $y = 6, x = 10$. a) Find y when $x = 5$. b) Find x when $y = 1.5$.	b is inversely proportional to the square of a. When $\mathrm{b} = 6, a = 5$. a) Find b when $\mathrm{a} = 10$. b) Find a when $\mathrm{b} = 6$.

Worked Example	Your Turn
y is inversely proportional to the square root of x. When $y = 4, x = 25$. a) Find y when $x = 4$. b) Find x when $y = 2.5$.	b is inversely proportional to the square root of a. When $\mathrm{b} = 4, a = 9$. a) Find b when $\mathrm{a} = 16$. b) Find a when $\mathrm{b} = 6$.

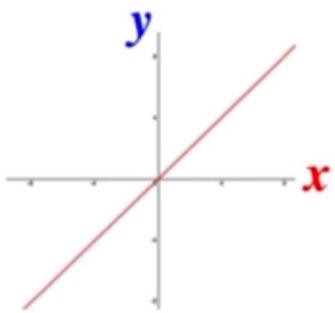
Worked Example	Your Turn
y is inversely proportional to $x + 3$. When $y = 52$, $x = 3$. Find y when $x = 5$.	y is inversely proportional to $2x + 1$. When $y = 30$, $x = 4$. Find y when $x = 7$.

Fill in the Gaps

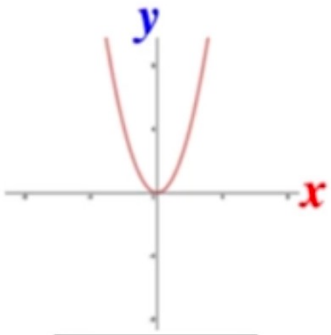
Type	**Statement**	k-Formula	k value x = 2 , y = 4	Final Formula
y is proportional to x	$y \propto x$	$y = k\,x$		
x is proportional to y				
y is inversely proportional to x	$y \propto \frac{1}{x}$	$y = \frac{k}{x}$		
x is inversely proportional to y				
y is proportional to the square of x				
x is proportional to the square of y				
x is proportional to $\sqrt{y}$				
Y is inversely proportional to $\sqrt{x}$				
Y is proportional to x^3				
x is proportional to 3 more than y				

Worked Example	Your Turn
A, B and C are three variables. A is directly proportional to C^2. A is also directly proportional to B^3. $C = 94$ when $B = 0.5$. Find B when $C = 730$.	D, E and F are three variables. D is directly proportional to F^2. D is also directly proportional to E^3. $F = 47$ when $\mathrm{E} = 0.25$. Find E when $\mathrm{F} = 365$.

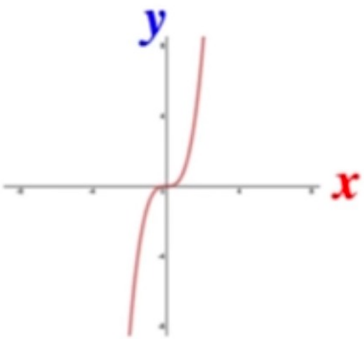
Graphs



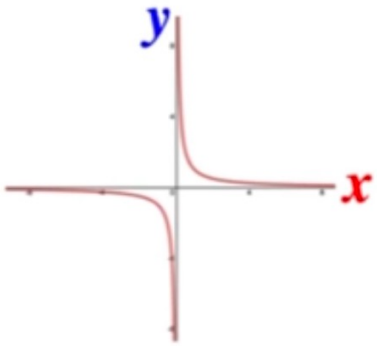
$y = kx$



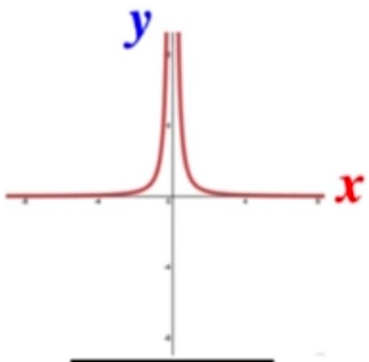
$y = kx^2$



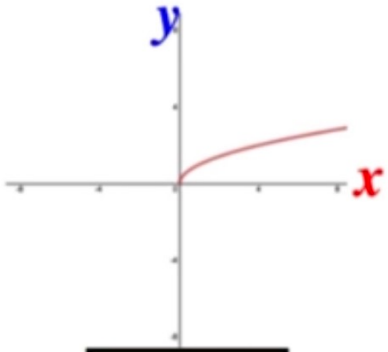
$y = kx^3$



$y = \frac{k}{x}$



$y = \frac{k}{x^2}$

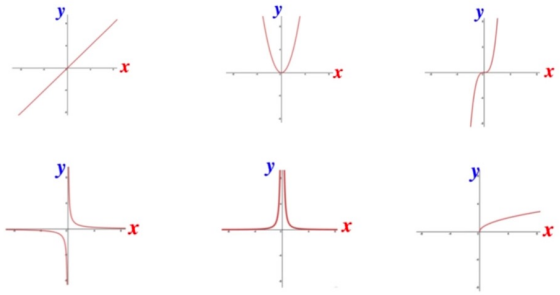


$y = k\sqrt{x}$

Fluency Practice

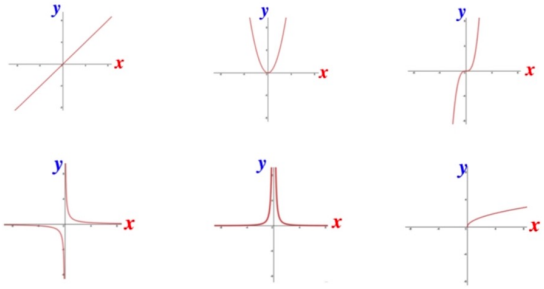
y is proportional to the square of x

Which of the following could be the graph demonstrating between y and x?



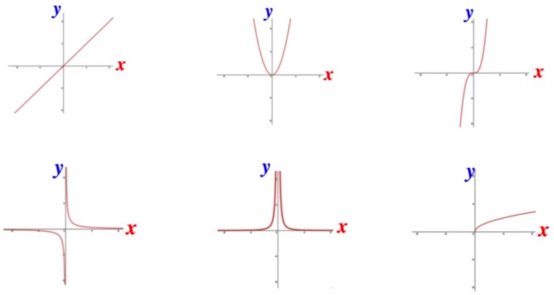
$y \propto x$

Which of the following could be the graph demonstrating between y and x?



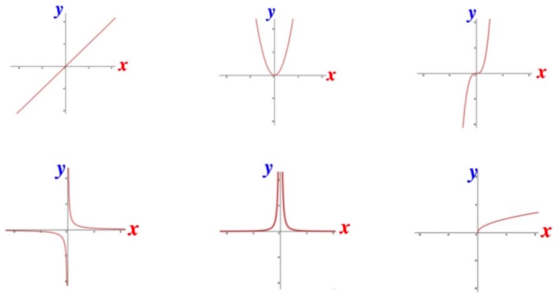
y is inversely proportional to x

Which of the following could be the graph demonstrating between y and x?



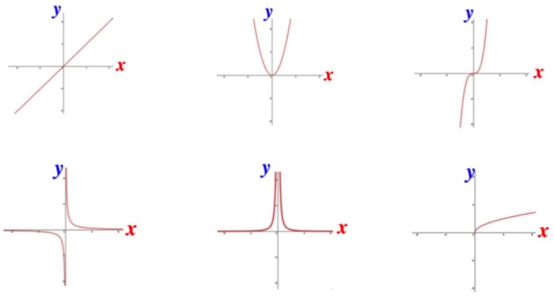
$y \propto x^3$

Which of the following could be the graph demonstrating between y and x?



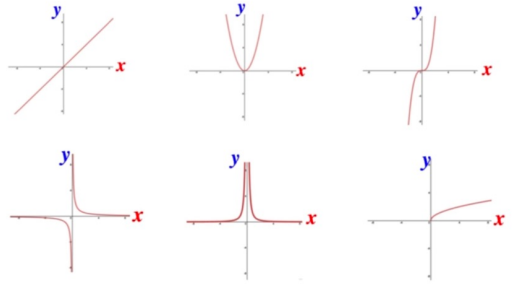
y is inversely proportional to the square of x

Which of the following could be the graph demonstrating between y and x?



$y \propto \sqrt{x}$

Which of the following could be the graph demonstrating between y and x?



Extra Notes