Year 10
Mathematics
Unit 18

Name: _____

Class: _____

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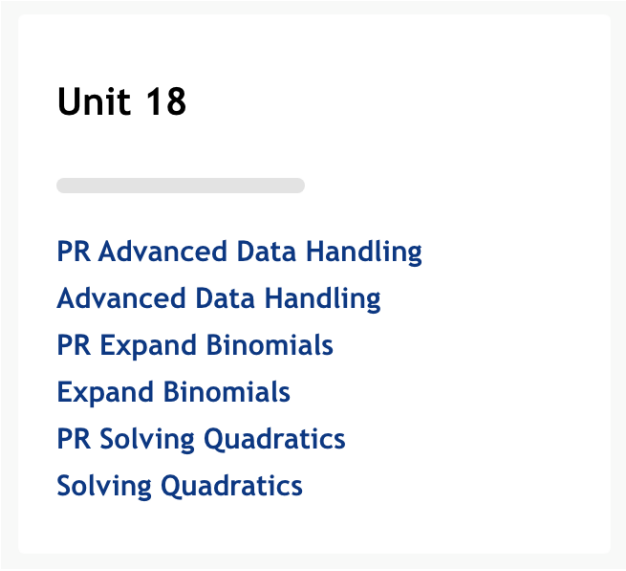

1 Advanced Data Handling

Ungrouped Frequency Tables

Worked Example

25 packets of sweets were opened. The numbers of sweets in the packets were:

11, 8, 9, 12, 10, 10, 9, 8, 9, 13, 9, 11, 10, 10, 12, 12,10,10, 10, 11,12, 8, 9, 8, 9

Construct a frequency table to show this data:

Number of sweets	Frequency

Mode of Ungrouped Data

Worked Example

Determine the modal score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Determine the modal score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Range of Ungrouped Data

Worked Example

Determine the range of the scores:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Determine the range of the scores:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Median of Ungrouped Data

Fluency Practice

Number of pieces of data:

Position of the median:

(a) 4, 10, 11, 12, 12, 15, 20

(b) 4, 10, 11, 12, 12, 15

(c) 10, 11, 12, 12, 15

(d) 10, 11, 12, 12

(e) 1, 3, 6, 8, 9, 12

Number of pieces of data:	Position of the median:
7	
11	
10	
41	
24	
	8
	3.5
	40
	21.5

Fluency Practice

Number of pets	Frequency	Which pieces of data are in this category?
0	3	1st 2nd 3rd
1	2	4th 5th
2	4	

Number of pets	Frequency	Which pieces of data are in this category?
0		1st 2nd
1		3rd
2		4th 5th 6th 7th 8th
3		9th 10th
4		11th 12th 13th

Number of pets	Frequency	Which pieces of data are in this category?
0	8	
1	9	
2	13	
3	12	
4	9	

Number of pets	Frequency	Which pieces of data are in this category?
0	5	
1	1	
2	3	

Number of pets	Frequency	Which pieces of data are in this category?
0		1st
1		2nd 3rd 4th
2		5th 6th 7th 8th
3		9th 10th
4		11th 12th

Number of pets	Frequency	Which pieces of data are in this category?
0		1st to 13th
1		14th to 29th
2		30th to 59th
3		60th to 80th
4		81st to 92nd

Number of pets	Frequency	Which pieces of data are in this category?
0	2	
1	1	
2	5	

Number of pets	Frequency	Which pieces of data are in this category?
0	21	1st to 21st
1	15	22nd to...
2	18	
3	25	
4	32	

Number of pets	Frequency	Which pieces of data are in this category?
0	1	
1	3	
2	3	

Number of pets	Frequency	Which pieces of data are in this category?
0	10	
1	12	
2	15	
3	20	
4	5	

Worked Example

Calculate the median score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Calculate the median score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Worked Example

Calculate the median score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	7

Your Turn

Calculate the median score:

Score	Frequency
0	9
1	6
2	2
3	4
4	4
5	8

Mean of Ungrouped Data

Worked Example

Calculate the mean score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Calculate the mean score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Worked Example

The table gives information about the numbers of badges gained by the girls in a Guide group.

a) Write down the mode.

b) Find the range.

c) Work out the median

d) Calculate the mean.

Number of badges	Frequency
0	2
1	8
2	4
3	3
4	5
5	3

Grouped Frequency Tables

Worked Example

80 people take part in a survey. Their ages are shown in the frequency table. How many respondents are in their thirties?

Age range	Frequency
$20 \leq \text{age} < 30$	8
$30 \leq \text{age} < 40$	
$40 \leq \text{age} < 50$	12
$50 \leq \text{age} < 60$	16
$60 \leq \text{age} < 70$	11
$70 \leq \text{age} < 80$	10
$80 \leq \text{age} < 90$	9
	80

Modal Class of Grouped Data

Worked Example

Determine the modal class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 20$	3
$20 < x \leq 40$	2
$40 < x \leq 46$	6
$46 < x \leq 50$	7

Your Turn

Determine the modal class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	15
$10 < x \leq 20$	6
$20 < x \leq 40$	4
$40 < x \leq 46$	12
$46 < x \leq 50$	8

Range of Grouped Data

Worked Example

Determine the upper and lower bounds for the range:

Mass, x (kg)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 20$	3
$20 < x \leq 40$	2
$40 < x \leq 46$	6
$46 < x \leq 50$	7

Your Turn

Determine the upper and lower bounds for the range:

Mass, x (kg)	Frequency
$10 < x \leq 20$	5
$20 < x \leq 30$	3
$30 < x \leq 50$	2
$50 < x \leq 56$	6
$56 < x \leq 60$	7

Median Class of Grouped Data

Worked Example

Determine the median class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 20$	3
$20 < x \leq 40$	2
$40 < x \leq 46$	6
$46 < x \leq 50$	7

Your Turn

Determine the median class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	15
$10 < x \leq 20$	6
$20 < x \leq 40$	4
$40 < x \leq 46$	12
$46 < x \leq 50$	8

Median, UQ, LQ and IQR of Grouped Data

Worked Example

Jack collects the heights of 100 flowers and records the data in the table below.

Height (y cm)	Frequency
$40 < y \leq 50$	7
$50 < y \leq 60$	14
$60 < y \leq 70$	59
$70 < y \leq 80$	11
$80 < y \leq 90$	9

Use interpolation to estimate the median.
Give your answer correct to 1 decimal place.

Your Turn

James collects the heights of 80 flowers and records the data in the table below.

Height (x cm)	Frequency
$35 < x \leq 40$	4
$40 < x \leq 45$	9
$45 < x \leq 50$	26
$50 < x \leq 55$	13
$55 < x \leq 60$	8
$60 < x \leq 65$	20

Use interpolation to estimate the median.
Give your answer correct to 1 decimal place.

Worked Example

Jack collects the heights of 100 flowers and records the data in the table below.

Height (y cm)	Frequency
$40 < y \leq 50$	7
$50 < y \leq 60$	14
$60 < y \leq 70$	59
$70 < y \leq 80$	11
$80 < y \leq 90$	9

Use interpolation to estimate the interquartile range.
Give your answer correct to 1 decimal place.

Your Turn

James collects the heights of 80 flowers and records the data in the table below.

Height (x cm)	Frequency
$35 < x \leq 40$	4
$40 < x \leq 45$	9
$45 < x \leq 50$	26
$50 < x \leq 55$	13
$55 < x \leq 60$	8
$60 < x \leq 65$	20

Use interpolation to estimate the interquartile range.
Give your answer correct to 1 decimal place.

Midpoint of Two Numbers

Worked Example

Numbers	Midpoint
40 and 60	

Your Turn

Numbers	Midpoint
40 and 70	

Intelligent Practice

Numbers	Midpoint
1. 8 and 10	
2. 7 and 11	
3. 2 and 16	
4. 22 and 36	
5. 22 and 46	
6. 22 and 47	
7. 22 and 48	
8. 21 and 48	
9. 21 and 47	
10. 42 and 94	

Numbers	Midpoint
11. 142 and 194	
12. 14.2 and 19.4	
13. 7.1 and 9.7	
14. 7 and 9.6	
15. -9.6 and -7	
16. -9.9 and -7	
17. -9.9 and -6.9	
18. -6.9 and 9.9	
19. $-6\frac{3}{4}$ and $9\frac{3}{4}$	
20. $-6\frac{3}{5}$ and $9\frac{3}{4}$	

Estimated Mean of Grouped Data

Worked Example

Calculate an estimate for the mean:

Mass, x (kg)	Frequency
$0 < x \leq 8$	3
$8 < x \leq 16$	6
$16 < x \leq 24$	7
$24 < x \leq 32$	4

Your Turn

Calculate an estimate for the mean:

Mass, x (kg)	Frequency
$0 < x \leq 8$	3
$8 < x \leq 16$	0
$16 < x \leq 24$	7
$24 < x \leq 32$	4

Worked Example

Bob asked each of 40 friends how many minutes they took to get to work. The table shows some information about his results.

a) Write down the modal class.
b) Work out the upper and lower bounds for the range.
c) Work out the class in which the median lies.
d) Calculate an estimate for the median.
e) Calculate an estimate for the mean.

Time taken (m minutes)	Frequency
$0 < m \leq 10$	3
$10 < m \leq 20$	8
$20 < m \leq 30$	11
$30 < m \leq 40$	9
$40 < m \leq 50$	9

Extra Notes

2 Expand Binomials

Expanding Triple Brackets

Worked Example	Your Turn
Expand and simplify: $(x+2)(x-3)(x-4)$	Expand and simplify: $(x+4)(x-6)(x-8)$

Worked Example	Your Turn
Expand and simplify: $(5x + 2)(7x - 3)(x - 4)$	Expand and simplify: $(5x + 4)(7x - 6)(x - 8)$

Worked Example	Your Turn
Expand and simplify: $(5x + 2)^3$	Expand and simplify: $(7x - 6)^3$

Extra Notes

3 Solving Quadratics

Multiplication by Zero

Fluency Practice

Find the value of $(x-3)(x-7)$ if
a) $x = 8$ b) $x = 7$ c) $x = 3$

a) If $x = 8$ $(x-3)(x-7) = (8-3)(8-7)$
$= (5)(1)$
$= 5$

b) If $x = 7$ $(x-3)(x-7) = (4)(0)$
$= 0$

c) If $x = 3$ $(x-3)(x-7) = (0)(-4)$
$= 0$

1. Find the value of $(x-4)(x-2)$ if
a) $x = 6$ b) $x = 4$ c) $x = 2$

2. Find the value of $(x-5)(x-9)$ if
a) $x = 5$ b) $x = 10$ c) $x = 9$

3. Find the value of $(x-7)(x-1)$ if
a) $x = 1$ b) $x = 8$ c) $x = 7$

4. Find the value of $(x-4)(x-6)$ if
a) $x = 4$ b) $x = 6$ c) $x = 3$

5. Find the value of $(x-6)(x-7)$ if
a) $x = 2$ b) $x = 6$ c) $x = 9$

Find the value of $(x-2)(x+4)$ if
a) $x = 2$ b) $x = 4$ c) $x = -4$

a) If $x = 2$ $(x-2)(x+4) = (0)(6)$
$= 0$

b) If $x = 4$ $(x-2)(x+4) = (2)(8)$
$= 16$

c) If $x = -4$ $(x-2)(x+4) = (-6)(0)$
$= 0$

6. Find the value of $(x-3)(x+5)$ if
a) $x = 6$ b) $x = 3$ c) $x = -5$

7. Find the value of $(x-4)(x+6)$ if
a) $x = 0$ b) $x = -6$ c) $x = 4$

8. Find the value of $(x-7)(x+2)$ if
a) $x = -7$ b) $x = -2$ c) $x = 7$

9. Find the value of $(x+4)(x+5)$ if
a) $x = -4$ b) $x = -5$ c) $x = 0$

10. Find the value of $(x+7)(x+1)$ if
a) $x = -4$ b) $x = -1$ c) $x = -7$

The results of this exercise show that if the product of two factors is 0, then either one or both of these factors must be 0

In general we can say

if	$A \times B = 0$		
then either	$A = 0$	or/and	$B = 0$

Fluency Practice

In questions 1 to 12 find, if possible, the value or values of A. Note that if $A \times 0 = 0$ then A can have any value.

1. $A \times 6 = 0$

2. $A \times 7 = 0$

3. $A \times 4 = 0$

4. $A \times 0 = 0$

5. $3 \times A = 12$

6. $8 \times A = 8$

7. $A \times 10 = 0$

8. $A \times 9 = 18$

9. $A \times 20 = 0$

10. $A \times 3 = 21$

11. $0 \times A = 0$

12. $4 \times A = 0$

13. If $AB = 0$ find a) A if $B = 2$ b) B if $A = 10$

14. If $AB = 0$ find a) A if $B = 5$ b) B if $A = 5$

15. If $AB = 0$ find a) A if $B = 10$ b) B if $A = 3$

16. If $AB = 0$ find a) B if $A = 6$ b) A if $B = 0$

Find a and b if $a(b - 3) = 0$

Either $a = 0$ or/and $b - 3 = 0$

i.e., either $a = 0$ or/and $b = 3$

Find a and b if:

17. $a(b - 1) = 0$

18. $a(b - 5) = 0$

19. $a(b - 2) = 0$

20. $(a - 3)b = 0$

21. $(a - 9)b = 0$

22. $a(b - 4) = 0$

23. $a(b - 10) = 0$

24. $(a - 1)b = 0$

25. $(a - 7)b = 0$

26. $(a - 12)b = 0$

Quadratics Equations

Previously we have considered equations such as $x - 1 = 0$ and $3x + 2 = 0$. These are examples of *linear equations*. The first equation is true only for $x = 1$ and the second only for $x = -\frac{2}{3}$.

If, however, we consider the equation

$$(x - 1)(x - 2) = 0$$

we find that it is true either when $x - 1 = 0$ or when $x - 2 = 0$, i.e. either when $x = 1$ or when $x = 2$

There are, therefore, two values of x that satisfy the equation $(x - 1)(x - 2) = 0$

Expanding the left-hand side gives

$$x^2 - 3x + 2 = 0$$

Equations like this, which contain an x^2 term, are called *quadratic equations*.

When we are given a quadratic equation we can often factorise the left-hand side into two linear factors,

e.g. $$x^2 - 5x + 4 = 0$$

gives $$(x - 4)(x - 1) = 0$$

It is this technique that concerns us in the present chapter.

Worked Example	Your Turn
What values of x satisfy the equation $x(x-9)=0$?	What values of x satisfy the equation $(x+6)x=0$?

Worked Example	Your Turn
What values of x satisfy the equation $(x - 9)(x + 5) = 0$?	What values of x satisfy the equation $(x + 6)(x - 5) = 0$?

Worked Example	Your Turn
Solve the equation $(2x - 3)(3x + 1) = 0$	Solve the equation $(3x + 2)(2x - 1) = 0$

Solving Quadratics Equations by Factorising

The previous two exercises suggest that if the left-hand side of a quadratic equation can be expressed as two linear factors, we can use these factors to solve the equation.

Worked Example	Your Turn
Solve the equation $x^2 + 2x - 8 = 0$	Solve the equation $x^2 + 2x - 15 = 0$

Worked Example	Your Turn
Solve the equation $x^2 - 49 = 0$	Solve the equation $x^2 - 64 = 0$

Worked Example	Your Turn
Solve the equation $3x^2 + 2x = 0$	Solve the equation $2x^2 - 3x = 0$

Worked Example	Your Turn
Solve the equation $x^2 - 4x + 4 = 0$	Solve the equation $x^2 + 14x + 49 = 0$

Worked Example	Your Turn
Solve the equation $5x^2 + 13x - 6 = 0$	Solve the equation $5x^2 + 7x - 6 = 0$

Worked Example	Your Turn
Solve the equation $4x^2 - 9 = 0$	Solve the equation $16x^2 - 81 = 0$

Worked Example	Your Turn
Solve the equation $x^2 - x = 12$	Solve the equation $x^2 = 2x + 3$

Worked Example	Your Turn
Solve the equation $12x^2 + 10x - 12 = 0$	Solve the equation $18x^2 - 15x - 18 = 0$

Worked Example	Your Turn
Solve the equation $x(x-2)=15$	Solve the equation $(x-3)(x+2)=6$

Worded Problems

Worked Example

I think of a positive number x, square it and then add three times the number I first thought of. If the answer is 54, form an equation in x and solve it to find the number I first thought of.

Worked Example

A rectangle is $4cm$ longer than it is wide. If it is xcm wide and has an area of $77cm^2$, form an equation in x and solve it to find the dimensions of the rectangle.

Worked Example

The sum of two numbers is 13 and the sum of their squares is 97. Find the numbers.

Solution of Quadratic Equation by Formula

If we apply the method of completing the square to the general quadratic equation $ax^2+bx+c=0$, where a, b and c are positive or negative numbers, we can establish a formula for solving the equation.

Consider the general equation $ax^2+bx+c=0$

Divide both sides by a $x^2+\frac{b}{a}x+\frac{c}{a}=0$

Subtract $\frac{c}{a}$ from each side $x^2+\frac{b}{a}x=-\frac{c}{a}$

Complete the square on the LHS and add the same quantity to the RHS. $x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=-\frac{c}{a}+\frac{b^2}{4a^2}$

Therefore $\left(x+\frac{b}{2a}\right)^2=\frac{-4ac+b^2}{4a^2}$

Take square roots of each side $x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$

Subtract $\frac{b}{2a}$ from each side $x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

i.e. $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

This is called the *formula* for solving quadratic equations. It gives values of x, or roots of the equation, for any given values of a, b and c (provided that b^2-4ac is not negative).

Remember that a is the coefficient of x^2
b is the coefficient of x
c is the constant number term.

Since the two values of x are

$$-\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a} \quad \text{and} \quad -\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}$$

the sum of the two roots is always $\left(\frac{-b}{2a}\right)+\left(\frac{-b}{2a}\right)=-\frac{b}{a}$

This provides a useful check that your answers are correct.

Quadratic Formula

a general quadratic equation can always be written:

$$ax^2 + bx + c = 0$$

the solutions to a general quadratic equation are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a is the number in front of the x^2

b is the number in front of the x

c is the (constant) number

Worked Example	Your Turn
Write down the values of a, b and c in: a) $5x^2 + 2x - 3 = 0$ b) $x^2 + 2x - 3 = 0$ c) $x^2 + 2x = 4x - 3$	Write down the values of a, b and c in: a) $5x^2 - 2x + 3 = 0$ b) $x^2 - 2x + 3 = 0$ c) $x^2 - 2x = -4x + 3$

Intelligent Practice

Questions	a	b	c
$3x^2 + 5x + 1 = 0$			
$0 = 3x^2 + 5x + 1$			
$0 = 3x^2 + 5x + 2$			
$3x^2 + 4x + 2 = 0$			
$0 = 3x^2 + 4x - 2$			
$3x^2 - 4x + 2 = 0$			
$x^2 - 4x + 2 = 0$			
$x^2 + 2 - 4x = 0$			
$1 + 2x - 4x^2 = 0$			
$1 + 2x = 4x^2$			

Intelligent Practice

Questions	a	b	c
$2x = 4x^2 + 1$			
$1 = 4x^2 + 2$			
$4x^2 + 2x = 0$			
$4x^2 + 2 = 0$			
$2(2x^2 + 1) = 0$			
$-2(2x^2 + 1) = 0$			
$-2(2x^2 + 1) = 2x$			
$-2(2x^2 + 1) = 2x + 2$			
$-2(2x^2 + 1) = x^2 + 2x + 2$			
$-2(2x^2 + x + 1) = x^2 + 2x + 2$			

Discriminant

The expression $b^2 - 4ac$ in the quadratic formula is called the discriminant, because it can "discriminate" between the possible types of answer:

- When $b^2 - 4ac$ is positive, we get two real solutions
- When $b^2 - 4ac$ is zero, we get just one real solution (both answers are the same)
- When $b^2 - 4ac$ is negative, we get a pair of complex solutions

Worked Example	Your Turn
Given that $a = 5, b = 6, c = -7$ work out the value of $b^2 - 4ac$	Given that $a = -6, b = 7, c = 8$ work out the value of $b^2 - 4ac$

Solving Quadratic Equations by the Formula

Worked Example	Your Turn
Use the formula to solve the equation $x^2 - 9x - 2 = 0$ giving your answers correct to two decimal places.	Use the formula to solve the equation $x^2 - 2x - 9 = 0$ giving your answers correct to two decimal places.

Worked Example	Your Turn
Use the formula to solve the equation $3x^2 + 7x - 2 = 0$ giving your answers correct to two decimal places.	Use the formula to solve the equation $3x^2 - 9x + 2 = 0$ giving your answers correct to two decimal places.

Worked Example	Your Turn
Solve the equation $4x^2 = 7x + 1$ giving your answers correct to two decimal places.	Solve the equation $7x^2 = 4x + 1$ giving your answers correct to two decimal places.

Fill in the Gaps

Quadratic Equation	a, b and c	$b^2 - 4ac$	$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$	$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$	Solutions to 3sf
$x^2 + 5x + 1 = 0$	$a = 1, b = 5, c = 1$	$5^2 - 4 \times 1 \times 1 = 21$	$x = \frac{-5 + \sqrt{21}}{2}$	$x = \frac{-5 - \sqrt{21}}{2}$	
$2x^2 + 5x + 1 = 0$	$a = 2, b = 5, c = 1$	$5^2 - 4 \times 2 \times 1 = 17$			
$2x^2 - 5x + 1 = 0$	$a = 2, b = -5, c = 1$	$(-5)^2 - 4 \times 2 \times 1 = 17$	$x = \frac{5 + \sqrt{17}}{2}$		
$x^2 - 7x + 3 = 0$					
$2x^2 - 7x + 3 = 0$					
$5x^2 + x - 2 = 0$					
	$a = 3, b = 5, c = 2$				
			$x = \frac{-9 + \sqrt{89}}{4}$	$x = \frac{-9 - \sqrt{89}}{4}$	

Worded Problems

Extra Notes