



Mathematics
Unit 19
Tasks



DO NOT WRITE INSIDE

Intelligent Practice

Simplify:

1) $y^{13} \times y^4$

2) $6y^{13} \times 5y^4$

3) $y^{13} \div y^4$

4) $40y^{13} \div 8y^4$

5) $(y^{13})^4$

6) $(3y^{13})^4$

7) $7y^4 \div y^2$

8) $7y^4 \times y^2$

9) $(7y^4)^2$

10) $2x^7 \times 5x^4$

11) $12y^5 \times 5x^4$

12) $12y^5 \div 6y^4$

13) $12y^5 \div 12y^{-4}$

14) $(12y^5)^2$

15) $(12y^{-3})^2$

16) $12y^{-3} \div 4y^2$

17) $12y^{-3} \div 4y^{-2}$

18) $12y^{-3} \times 4y^{-2}$

Intelligent Practice

Simplify:

$$1) \frac{a^3 \times a^5}{a^6}$$

$$2) \frac{a^6}{a^3 \times a^5}$$

$$3) \frac{x^6}{a^3 \times a^5}$$

$$4) \frac{12x^6}{2a^3 \times 3a^5}$$

$$5) \frac{12x^6}{2x^3 \times 3x^5}$$

$$6) 2x^3 \times 3x^5$$

$$7) 2x^3y^2 \times 3x^5y^2$$

$$8) 12x^6y^2 \times 3x^5y^2$$

$$9) 12x^6y^2 \div 3x^5y^2$$

$$10) \frac{12x^6y^2}{3x^5y^2}$$

$$11) \frac{12x^6y^8}{3x^5y^2}$$

Intelligent Practice

Simplify:

1) $3x^0$

2) $-10x^0$

3) $\frac{3}{10}x^0$

4) $0.9x^0$

5) $x^9 \times x^0$

6) $4x^9 \times x^0$

7) $5x^0 \times x^9$

8) $5x^0 \times$
 $4x^9$

9) $x^5 \div x^0$

10) $4x^5 \div x^0$

11) $x^5 \div 8x^0$

12) $4x^5 \div 8x^0$

13) $\frac{x^0}{x^{-6}}$

14) $\frac{14x^0}{x^{-6}}$

15) $\frac{x^0}{7x^{-6}}$

16) $\frac{14x^0}{7x^{-6}}$

EXTENSION:

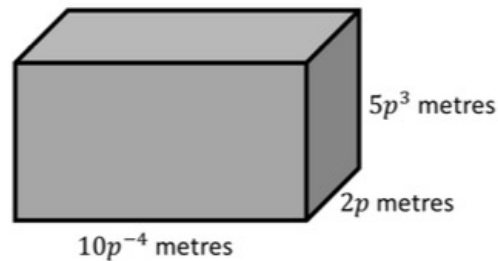
Task 1

1. If $2^3 \times 2^a = 2^0$, what is the value of a ?
2. If $4^{-3} \times 4^a = 1$, what is the value of a ?
3. If $x^f \times x^g = 1$ and $f < 0$ and $g > 0$, find possible values for f and g .

Task 2

A cuboid has dimensions as shown.

Show that the volume of the cuboid is 100 cubic metres.



Fluency Practice

1. Write in index form:

a) $\frac{1}{5^2}$

b) $\frac{1}{3^4}$

c) $\frac{1}{8^3}$

d) $\frac{1}{4^5}$

e) $\frac{1}{10^3}$

f) $\frac{1}{2^6}$

2. Write in the form 2^n :

a) $\frac{1}{2}$

b) $\frac{1}{4}$

c) $\frac{1}{8}$

d) $\frac{1}{32}$

e) $\frac{1}{64}$

f) $\frac{1}{256}$

3. Write in the form 5^n :

a) $\frac{1}{5}$

b) $\frac{1}{125}$

c) $\frac{1}{625}$

d) $\frac{1}{3125}$

e) $\frac{1}{78,125}$

f) $\frac{1}{390,625}$

4. Arrange in ascending order:

$$\frac{1}{50}$$

$$5^{-2}$$

$$\frac{3}{10}$$

$$2^{-3}$$

Fluency Practice

Simplify the following:

1. $\left(\frac{1}{5}\right)^{-1}$

2. $\left(\frac{2}{5}\right)^{-1}$

3. $\left(\frac{3}{5}\right)^{-1}$

4. $\left(\frac{4}{5}\right)^{-1}$

5. $\left(\frac{1}{4}\right)^{-1}$

6. $\left(\frac{3}{4}\right)^{-1}$

7. $\left(\frac{1}{3}\right)^{-2}$

8. $\left(\frac{2}{3}\right)^{-2}$

9. $\left(\frac{1}{5}\right)^{-2}$

10. $\left(\frac{2}{5}\right)^{-2}$

11. $\left(\frac{3}{5}\right)^{-2}$

12. $\left(\frac{4}{5}\right)^{-2}$

13. $\left(\frac{2}{3}\right)^{-3}$

14. $\left(\frac{4}{3}\right)^{-3}$

15. $\left(\frac{7}{8}\right)^{-2}$

16. $\left(-\frac{1}{10}\right)^{-4}$

17. $\left(-\frac{4}{9}\right)^{-3}$

18. $\left(-\frac{9}{10}\right)^{-2}$

19. $\left(-\frac{5}{3}\right)^{-3}$

20. $\left(-\frac{3}{2}\right)^{-4}$

21. $\left(-\frac{3}{10}\right)^{-3}$

22. $\left(-\frac{3}{2}\right)^{-3}$

23. $\left(-\frac{8}{5}\right)^{-3}$

24. $\left(-\frac{3}{8}\right)^{-2}$

25. $\left(-\frac{6}{5}\right)^{-2}$

Fluency Practice

Rewrite the following with positive indices:

1. x^{-6}

2. x^{-7}

3. a^{-8}

4. a^{-10}

5. p^{-11}

6. k^{-14}

7. $2s^{-3}$

8. $7z^{-3}$

9. $3d^{-4}$

10. $3d^{-7}$

11. $14x^{-1}$

12. $\frac{12}{3}x^{-2}$

13. $\frac{16x^{-5}}{4}$

14. $\frac{a^{-9}}{2}$

15. $\frac{e^{-5}}{6}$

16. $\frac{y^{-8}}{3}$

17. $\frac{g^{-7}}{10}$

18. $\frac{2x^{-4}}{3}$

19. $\frac{4f^{-5}}{7}$

20. $\frac{5x^{-8}}{9}$

21. $(3x)^{-3}$

22. $(-3x)^{-3}$

23. $(5x)^{-2}$

24. $(-5x)^{-2}$

25. $(-7a)^{-3}$

Fluency Practice

Task 1

Rewrite the following with negative indices:

1. $\frac{1}{x^4}$

2. $\frac{1}{x}$

3. $\frac{1}{d^{10}}$

4. $\frac{1}{b^{13}}$

5. $\frac{1}{y^x}$

6. $\frac{1}{m^n}$

7. $\frac{3}{x^{13}}$

8. $\frac{5}{x^9}$

9. $\frac{7}{y^{10}}$

10. $\frac{3}{x^{11}}$

11. $\frac{4}{x^3}$

12. $\frac{12}{3x^2}$

13. $\frac{a}{x^n}$

14. $\frac{1}{2x^4}$

15. $\frac{1}{3x^7}$

16. $\frac{1}{8a^5}$

17. $\frac{1}{7b^6}$

18. $\frac{3}{4x^5}$

19. $\frac{2}{5x^{12}}$

20. $\frac{1}{8a^5}$

Task 2

Match the equivalent pairs of expressions

$(3x)^{-1}$

$(3x)^{-2}$

$3x^{-1}$

x^3

x^{-3}

$3x^{-2}$

$\frac{1}{x^3}$

$\frac{1}{x^{-3}}$

$\frac{1}{9x^2}$

$\frac{3}{x}$

$\frac{1}{3x}$

$\frac{3}{x^2}$

Calculate $5^{-2} + 6^{-2}$

Find 2^{-3} of 40

Solve $2^{-2}x + 4 = 10$

Simplify $2^{-1}x + 3^{-2}y - x - 2^{-1}y$

Expand $4^{-2}(t + 6)$

Simplify $\sqrt{9^{-1}}$

*Find 2^{-1} of 6×10^{-4}

**A straight line is given as $x - 5y = 10$

Find the gradient in the form a^{-x} , where a and x are integers.

Task 1

Expand and fully simplify:

- $x(x^3 - 4)$
- $x(x^2 - x^{-2})$
- $2y^{-1}(3y^2 - 2y^3)$
- $2y^2(3y^4 + 5y^{-2})$
- $m(2m^{-1} - 4m^{-4})$
- $3c(4c^3 - 6c^{-4})$
- $4a^2(2a^{-1} + 3a^{-2})$
- $t^{-2}(3t^{-2} - t^2)$
- $4dz(3dz - d^{-\frac{1}{2}})$
- $a^{-2}(a + a^{-1})$
- $2w^5\left(\frac{1}{w} + 4w^{-2}\right)$
- $x^2\left(x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)$
- $x^{\frac{1}{2}}(x^4 + x^3)$
- $uz(3u + u^3)$
- $x^{\frac{1}{4}}(x^8 + x^6)$
- $3m^{\frac{3}{2}}\left(m^{\frac{3}{2}} + \frac{3}{m^{\frac{1}{2}}}\right)$
- $n^{\frac{1}{3}}\left(2n^{\frac{4}{3}} - \frac{1}{n^{\frac{2}{3}}}\right)$
- $b^{\frac{1}{3}}(b^2 + 2b^{-1})$
- $x^{-\frac{1}{3}}(x^{-2} + x^4)$
- $x^{\frac{1}{2}}\left(\frac{3}{x^4} - \frac{2}{x^{\frac{1}{5}}}\right)$
- $p^3(p^{-2} + p^3)$
- $x^{-3}(x^5 + x^2)$
- $5x^{\frac{1}{2}}\left(2x^{\frac{1}{2}} + 3x^{\frac{3}{2}}\right)$
- $3a^{-1}(4a^3 + 2a)$
- $2u^{-5}(u + 2u^5)$
- $3m^2(2m^2 + 7m^{-4})$
- $27. a^{\frac{1}{2}}\left(a^{\frac{1}{2}} + a^{-\frac{1}{2}}\right)$
- $p^{\frac{1}{3}}\left(p^{\frac{2}{3}} + p^{\frac{1}{3}}\right)$
- $e^{-\frac{2}{3}}\left(e^{\frac{7}{3}} - 2e^{\frac{2}{3}}\right)$
- $5n^4\left(n^{-2} + \frac{2}{n^3}\right)$
- $p^4\left(3p^{-4} - \frac{2}{p^3}\right)$
- $3a\left(a^{\frac{1}{2}} + 2a^{-2}\right)$
- $x^{\frac{1}{2}}(2x - 3)$
- $2p^{\frac{3}{4}}\left(p^{\frac{1}{4}} - p\right)$
- $x^{\frac{1}{5}}\left(2x^{\frac{9}{5}} + \frac{3}{x^{\frac{1}{5}}}\right)$
- $2a^{\frac{5}{3}}\left(\frac{1}{a^{\frac{2}{3}}} - 4a^{\frac{4}{3}}\right)$
- $x^{-4}\left(x^{\frac{1}{2}} - x^2\right)$
- $x^2\left(\frac{1}{x^2} + \frac{1}{x^3}\right)$
- $x^{-\frac{1}{2}}\left(\frac{1}{x^4} - \frac{3}{x^6}\right)$

Task 2

Lauren writes down the following statement:

$$p^{\frac{1}{3}}\left(p^{\frac{2}{3}} - p^{-\frac{1}{3}}\right) = p - 1$$

Is this statement true? Justify your answer.

Fluency Practice

Expand and fully simplify:

1. $(m^{-2} + m^3)(m^{-2} + m)$

2. $(x^{-3} + x^{-6})(x^7 + x^{-2})$

3. $(x + x^{-1})^2$

4. $(b^{\frac{1}{2}} + 1)^2$

5. $(3k^{\frac{1}{2}} - 2)^2$

6. $\left(x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{2}}}\right)\left(x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{2}}}\right)$

7. $\left(c^{\frac{1}{2}} + \frac{1}{c^{\frac{1}{2}}}\right)^2$

8. $\left(x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{4}}}\right)\left(x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{4}}}\right)$

9. $\left(x^{\frac{2}{3}} + y^{\frac{1}{2}}\right)\left(x^{\frac{1}{3}} - y^{\frac{1}{2}}\right)$

10. $\left(5c^{\frac{3}{4}} + 9c^{\frac{1}{2}}\right)\left(6c^{\frac{1}{3}} - 2c^{\frac{1}{4}}\right)$

Expand and simplify

a $x(x^2 - x^{-1})$

b $2x^3(x^{-1} + 3)$

c $x^{-1}(3x - x^3)$

d $4x^{-2}(3x^5 + 2x^3)$

e $\frac{1}{2}x^2(6x + 4x^{-1})$

f $3x^{\frac{1}{2}}(x^{-\frac{1}{2}} - x^{\frac{3}{2}})$

g $x^{-\frac{3}{2}}(5x^2 + x^{\frac{7}{2}})$

h $x^{\frac{1}{3}}(3x^{\frac{5}{3}} - x^{-\frac{4}{3}})$

i $(x^2 + 1)(x^4 - 3)$

j $(2x^5 + x)(x^4 + 3)$

k $(x^2 - 2x^{-1})(x - x^{-2})$

l $(x^2 - x^{\frac{3}{2}})(x - x^{\frac{1}{2}})$

Fluency Practice

Task 1

Rewrite the following using the radical sign:

1. $x^{\frac{1}{5}}$ 2. $x^{\frac{1}{6}}$ 3. $x^{\frac{1}{7}}$ 4. $x^{\frac{1}{8}}$ 5. $x^{\frac{1}{9}}$ 6. $x^{\frac{1}{10}}$ 7. $x^{\frac{1}{m}}$
8. $x^{-\frac{1}{2}}$ 9. $x^{-\frac{1}{3}}$ 10. $x^{-\frac{1}{4}}$ 11. $x^{-\frac{1}{5}}$ 12. $x^{-\frac{1}{6}}$ 13. $x^{-\frac{1}{7}}$ 14. $x^{-\frac{1}{a}}$

Task 2

Rewrite the following using the radical sign:

1. $4x^{\frac{1}{3}}$ 2. $4x^{-\frac{1}{3}}$ 3. $\frac{1}{4}x^{\frac{1}{3}}$ 4. $\frac{1}{4}x^{-\frac{1}{3}}$
5. $15y^{\frac{1}{10}}$ 6. $15y^{-\frac{1}{10}}$ 7. $\frac{1}{15}y^{\frac{1}{10}}$ 8. $\frac{1}{15}y^{-\frac{1}{10}}$
9. $-14x^{\frac{1}{8}}$ 10. $-14x^{-\frac{1}{8}}$ 11. $-\frac{1}{14}x^{\frac{1}{8}}$ 12. $-\frac{1}{14}x^{-\frac{1}{8}}$
13. $18c^{\frac{1}{2}}$ 14. $18c^{-\frac{1}{2}}$ 15. $\frac{1}{18}c^{\frac{1}{2}}$ 16. $\frac{1}{18}c^{-\frac{1}{2}}$
17. $-20k^{\frac{1}{9}}$ 18. $-20k^{-\frac{1}{9}}$ 19. $-\frac{k^{\frac{1}{9}}}{20}$ 20. $-\frac{k^{-\frac{1}{9}}}{20}$
21. $3c^{\frac{1}{7}}$ 22. $3c^{-\frac{1}{7}}$ 23. $\frac{c^{\frac{1}{7}}}{3}$ 24. $\frac{1}{3}c^{-\frac{1}{7}}$

Task 3

Rewrite the following using fractional exponents:

1. $\sqrt[3]{a}$ 2. $\sqrt[4]{k}$ 3. $\sqrt[5]{p}$ 4. $\sqrt[8]{x}$ 5. $\sqrt[3]{m}$
6. $\sqrt[7]{z}$ 7. $\sqrt[10]{w}$ 8. $\sqrt[4]{t}$ 9. $\sqrt[3]{a}$ 10. $\sqrt[4]{k}$
11. $\sqrt[10]{l}$ 12. $\sqrt[7]{n}$ 13. $\sqrt[13]{p}$ 14. $\sqrt[7]{m}$ 15. $\sqrt[3]{p}$

Which is bigger?

$$10^{\frac{1}{2}} \quad \text{or} \quad 20^{\frac{1}{3}}$$

$$20^{\frac{1}{2}} \quad \text{or} \quad 40^{\frac{1}{3}}$$

$$40^{\frac{1}{2}} \quad \text{or} \quad 80^{\frac{1}{3}}$$

$$5^{\frac{1}{2}} \quad \text{or} \quad 10^{\frac{1}{3}}$$

Fluency Practice

Task 1

Evaluate

- | | | | |
|---|--|--|---|
| 1. $25^{\frac{1}{2}}$ | 2. $25^{-\frac{1}{2}}$ | 3. $16^{\frac{1}{2}}$ | 4. $16^{-\frac{1}{2}}$ |
| 5. $9^{\frac{1}{2}}$ | 6. $9^{-\frac{1}{2}}$ | 7. $100^{\frac{1}{2}}$ | 8. $100^{-\frac{1}{2}}$ |
| 9. $36^{\frac{1}{2}}$ | 10. $36^{-\frac{1}{2}}$ | 11. $8^{\frac{1}{3}}$ | 12. $8^{-\frac{1}{3}}$ |
| 13. $125^{\frac{1}{3}}$ | 14. $125^{-\frac{1}{3}}$ | 15. $1000^{\frac{1}{3}}$ | 16. $1000^{-\frac{1}{3}}$ |
| 17. $16^{\frac{1}{4}}$ | 18. $16^{-\frac{1}{4}}$ | 19. $32^{\frac{1}{5}}$ | 20. $32^{-\frac{1}{5}}$ |
| 21. $216^{\frac{1}{3}}$ | 22. $216^{-\frac{1}{3}}$ | 23. $512^{\frac{1}{3}}$ | 24. $512^{-\frac{1}{3}}$ |
| 25. $\left(\frac{25}{49}\right)^{\frac{1}{2}}$ | 26. $\left(\frac{25}{49}\right)^{-\frac{1}{2}}$ | 27. $\left(\frac{36}{121}\right)^{\frac{1}{2}}$ | 28. $\left(\frac{36}{121}\right)^{-\frac{1}{2}}$ |
| 29. $\left(\frac{625}{1296}\right)^{\frac{1}{4}}$ | 30. $\left(\frac{625}{1296}\right)^{-\frac{1}{4}}$ | 31. $\left(\frac{216}{343}\right)^{\frac{1}{3}}$ | 32. $\left(\frac{216}{343}\right)^{-\frac{1}{3}}$ |
| 33. $\left(\frac{343}{1000}\right)^{\frac{1}{3}}$ | 34. $\left(\frac{343}{1000}\right)^{-\frac{1}{3}}$ | 35. $\left(\frac{125}{729}\right)^{\frac{1}{3}}$ | 36. $\left(\frac{125}{729}\right)^{-\frac{1}{3}}$ |

Task 2

- a) Given that $y = x^{\frac{1}{3}}$, find y when $x = 64$.
- b) Given that $y = x^{\frac{1}{2}}$, find y when $x = 16$.
- c) Given that $y = x^{\frac{1}{4}}$, find y when $x = 81$.
- d) Given that $y = x^{\frac{1}{3}}$, find y when $x = 125$.

Task 3

For $a > 1$, put the following in order of size from smallest to largest:

$$a^0, a^2, a, a^{-2}, a^{\frac{1}{2}}$$

Task 4

Solve for a :

$$\frac{x \times x^4 \times x^a}{\sqrt{x}} = \sqrt{\frac{(x^a)^8}{\left(\frac{1}{x^4}\right)}}$$

Task 5

The statement $x > x^{\frac{1}{2}}$ is:

Always True

Sometimes True

Never True

Fluency Practice

Task 1

Write in index form:

- | | | | |
|------------------------|-------------------------|-------------------------------|---------------------------------|
| 1. $\sqrt{a^3}$ | 2. $5\sqrt{a^3}$ | 3. $\frac{1}{\sqrt{a^3}}$ | 4. $\frac{5}{\sqrt{a^3}}$ |
| 5. $\sqrt{w^5}$ | 6. $4\sqrt{w^5}$ | 7. $\frac{1}{\sqrt{w^5}}$ | 8. $\frac{4}{\sqrt{w^5}}$ |
| 9. $\sqrt[3]{x^2}$ | 10. $9\sqrt[3]{x^2}$ | 11. $\frac{1}{\sqrt[3]{x^2}}$ | 12. $\frac{9}{\sqrt[3]{x^2}}$ |
| 13. $\sqrt[3]{w^4}$ | 14. $-6\sqrt[3]{w^4}$ | 15. $\frac{1}{\sqrt[3]{w^4}}$ | 16. $-\frac{6}{\sqrt[3]{w^4}}$ |
| 17. $\sqrt[5]{m^2}$ | 18. $-2\sqrt[5]{m^2}$ | 19. $\frac{1}{\sqrt[5]{m^2}}$ | 20. $-\frac{2}{\sqrt[5]{m^2}}$ |
| 21. $\sqrt[9]{k^4}$ | 22. $-12\sqrt[9]{k^4}$ | 23. $\frac{1}{\sqrt[9]{k^4}}$ | 24. $-\frac{12}{\sqrt[9]{k^4}}$ |
| 25. $\sqrt[6]{x^5y^7}$ | 26. $\sqrt[4]{a^7b^3c}$ | 27. $\sqrt[7]{p^2q^3r^4}$ | 28. $\sqrt[7]{p^2q^3r^4}$ |

Task 2

Write with a radical sign in simplest form:

- | | | | | |
|---------------------------|---------------------------|----------------------------|---------------------------|----------------------------|
| 1. $x^{\frac{3}{4}}$ | 2. $m^{\frac{7}{5}}$ | 3. $p^{\frac{3}{2}}$ | 4. $y^{\frac{5}{8}}$ | 5. $c^{\frac{4}{3}}$ |
| 6. $4(x^{\frac{3}{4}})$ | 7. $9(m^{\frac{7}{5}})$ | 8. $16(p^{\frac{3}{2}})$ | 9. $5(y^{\frac{5}{8}})$ | 10. $10(c^{\frac{4}{3}})$ |
| 11. $x^{-\frac{3}{4}}$ | 12. $m^{-\frac{7}{5}}$ | 13. $p^{-\frac{3}{2}}$ | 14. $y^{-\frac{5}{8}}$ | 15. $c^{-\frac{4}{3}}$ |
| 16. $4(x^{-\frac{3}{4}})$ | 17. $9(m^{-\frac{7}{5}})$ | 18. $16(p^{-\frac{3}{2}})$ | 19. $5(y^{-\frac{5}{8}})$ | 20. $10(c^{-\frac{4}{3}})$ |

Task 3

Find equivalent terms from the following:

x^{-3}	x^3	x^2	\sqrt{x}	$\sqrt[3]{x}$	x^{-1}	$2x^{-1}$	$2\sqrt{x}$
$x^{\frac{3}{2}}$	$x^{\frac{1}{2}}$	$\sqrt{x^3}$	$x\sqrt{x}$	$x^{-\frac{1}{3}}$	x^{-2}	$\frac{1}{2x^2}$	$x^{\frac{2}{3}}$
$x^{-\frac{1}{2}}$	$2x^{-2}$	$\frac{\sqrt{x}}{x}$	x^4	x^{-4}	$\frac{1}{2\sqrt{x}}$	x^0	$x^{-\frac{3}{2}}$
$\frac{1}{2}x^{-\frac{1}{2}}$	$\frac{1}{2}x^{-2}$	$x^{\frac{1}{3}}$	x	$\frac{2}{\sqrt{x}}$	$\frac{1}{x}$	$\frac{1}{2x}$	$\sqrt[3]{x^2}$

Fluency Practice

Task 1

Evaluate:

1. $8^{\frac{4}{3}}$

2. $27^{\frac{5}{3}}$

3. $64^{\frac{5}{6}}$

4. $81^{\frac{3}{2}}$

5. $625^{\frac{3}{4}}$

6. $9^{\frac{3}{2}}$

7. $100^{\frac{3}{2}}$

8. $3125^{\frac{2}{5}}$

9. $8^{\frac{5}{3}}$

10. $4^{\frac{5}{2}}$

11. $81^{\frac{3}{4}}$

12. $125^{-\frac{2}{3}}$

13. $243^{-\frac{2}{5}}$

14. $32^{-\frac{2}{5}}$

15. $16^{-\frac{3}{4}}$

16. $4^{-\frac{5}{2}}$

17. $64^{-\frac{2}{3}}$

18. $8^{-\frac{4}{3}}$

19. $32^{-\frac{4}{5}}$

20. $125^{-\frac{2}{3}}$

21. $1000^{-\frac{2}{3}}$

22. $81^{-\frac{3}{4}}$

23. $625^{-\frac{3}{4}}$

24. $(-8)^{\frac{4}{3}}$

25. $(-27)^{\frac{5}{3}}$

26. $(-3125)^{\frac{2}{5}}$

27. $(-8)^{\frac{5}{3}}$

28. $(-125)^{-\frac{2}{3}}$

29. $(-243)^{-\frac{2}{5}}$

30. $(-32)^{-\frac{2}{5}}$

31. $(-64)^{-\frac{2}{3}}$

32. $(-8)^{-\frac{4}{3}}$

33. $(-32)^{-\frac{4}{5}}$

34. $(-125)^{-\frac{2}{3}}$

35. $(-1000)^{-\frac{2}{3}}$

36. $\left(\frac{4}{25}\right)^{\frac{3}{2}}$

37. $\left(\frac{4}{25}\right)^{-\frac{3}{2}}$

38. $\left(\frac{8}{125}\right)^{\frac{2}{3}}$

39. $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$

40. $\left(\frac{49}{100}\right)^{\frac{3}{2}}$

41. $\left(\frac{49}{100}\right)^{-\frac{3}{2}}$

42. $\left(\frac{8}{125}\right)^{\frac{2}{3}}$

43. $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$

44. $\left(\frac{16}{100}\right)^{\frac{5}{2}}$

45. $\left(\frac{16}{100}\right)^{-\frac{5}{2}}$

46. $\left(\frac{64}{1000}\right)^{\frac{4}{3}}$

47. $\left(\frac{64}{1000}\right)^{-\frac{4}{3}}$

48. $\left(\frac{16}{25}\right)^{\frac{3}{2}}$

49. $\left(\frac{16}{25}\right)^{-\frac{3}{2}}$

50. $\left(\frac{32}{243}\right)^{\frac{6}{5}}$

Fluency Practice

Task 2

a) Arrange the following in ascending order:

$$25^{\frac{1}{2}}, 8^{\frac{2}{3}}, 27^{\frac{1}{3}}$$

b) Which is the odd one out? Explain your answer.

$$64^{\frac{1}{2}}, 16^{\frac{3}{4}}, 9^{\frac{2}{3}}, 4^{\frac{3}{2}}$$

Task 3

Gina has completed her homework.

Can you spot any mistakes?

Question 1

Work out $9^{\frac{1}{2}}$

4.5

Question 2

Work out $27^{\frac{2}{3}}$

18

Task 4

a) Given that $y = x^{\frac{3}{5}}$, find y when $x = 32$.

b) Given that $y = x^{\frac{3}{2}}$, find y when $x = 49$.

c) Given that $y = x^{\frac{2}{3}}$, find y when $x = 1000$.

d) Given that $y = x^{\frac{3}{4}}$, find y when $x = 81$.

e) Given that $y = x^{\frac{5}{6}}$, find y when $x = 64$.

f) Given that $y = x^{\frac{3}{4}}$, find y when $x = 10,000$.

Task 5

Find equivalent pairs from the following:

$8^{\frac{2}{3}}$	$(-1)^{\frac{1}{3}}$	$9^{\frac{3}{2}}$	$16^{\frac{1}{4}}$	$\frac{2}{3}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{64}{125}$
$\left(\frac{1}{2}\right)^{-3}$	$\left(\frac{2}{5}\right)^{-2}$	$\left(\frac{1}{9}\right)^{-\frac{1}{2}}$	$36^{-\frac{1}{2}}$	$\frac{1}{3}$	4	1	$\frac{1}{2}$
3^{-2}	$\left(\frac{16}{25}\right)^{\frac{3}{2}}$	3^{-1}	$4^{-\frac{3}{2}}$	8	-1	$\frac{1}{9}$	3
$8^{-\frac{1}{3}}$	$(-1)^{-2}$	$\left(\frac{1}{5}\right)^{-1}$	$\left(\frac{4}{9}\right)^{\frac{1}{2}}$	5	$\frac{25}{4}$	27	2

Task 6

- $(4x)^{\frac{3}{4}}$
- $(9m)^{\frac{7}{5}}$
- $(16p)^{\frac{3}{2}}$
- $(5y)^{\frac{5}{8}}$
- $(10c)^{\frac{4}{3}}$
- $(4x)^{-\frac{3}{4}}$
- $(9m)^{-\frac{7}{5}}$
- $(16p)^{-\frac{3}{2}}$
- $(5y)^{-\frac{5}{8}}$
- $(10c)^{-\frac{4}{3}}$
- $\left(\frac{4x^6}{9y^4}\right)^{\frac{5}{2}}$
- $\left(\frac{243x^5}{32y^{20}}\right)^{\frac{3}{5}}$
- $\left(\frac{64x^3}{27y^6}\right)^{\frac{2}{3}}$

EXTENSION

2^{\square} \square^2 16 \square^2	\square^2	2^{\square} \square^2 16 \square^2	\square^2
2^{\square} \square^2 16 \square^2	\square^2	2^{\square} \square^2 8 \square^2	\square^2
3^{\square} \square^3 27 \square^3	\square^3	3^{\square} \square^3 27 \square^3	\square^3
3^{\square} \square^3 27 \square^3	\square^3	4^{\square} \square^2 4 \square^2	\square^2

2^{\square} \square^2 16 \square^2	\square^2	2^{\square} \square^2 16 \square^2	\square^2
2^{\square} \square^2 16 \square^2	\square^2	2^{\square} \square^2 8 \square^2	\square^2
3^{\square} \square^3 27 \square^3	\square^3	3^{\square} \square^3 27 \square^3	\square^3
3^{\square} \square^3 27 \square^3	\square^3	10^{\square} \square^2 10,000 \square^2	\square^2

1. Fill in the blanks

$$1. 100^{\frac{3}{2}} = (\sqrt{\square})^{\square} = 10^{\square} = ?$$

$$2. 16^{\frac{5}{4}} = (\sqrt{\square})^{\square} = \square^5 = ?$$

$$3. 64^{\frac{\square}{\square}} = (\sqrt[3]{64})^4 = \square^{\square} = ?$$

$$4. \square^{\frac{\square}{5}} = (\sqrt{32})^{\square} = \square^{\square} = 8$$

$$5. \square^{\frac{\square}{\square}} = (\sqrt[3]{\square})^{\square} = 3^5 = ?$$

$$6. \square^{\frac{3}{2}} = (\sqrt{\square})^{\square} = \square^{\square} = 216$$

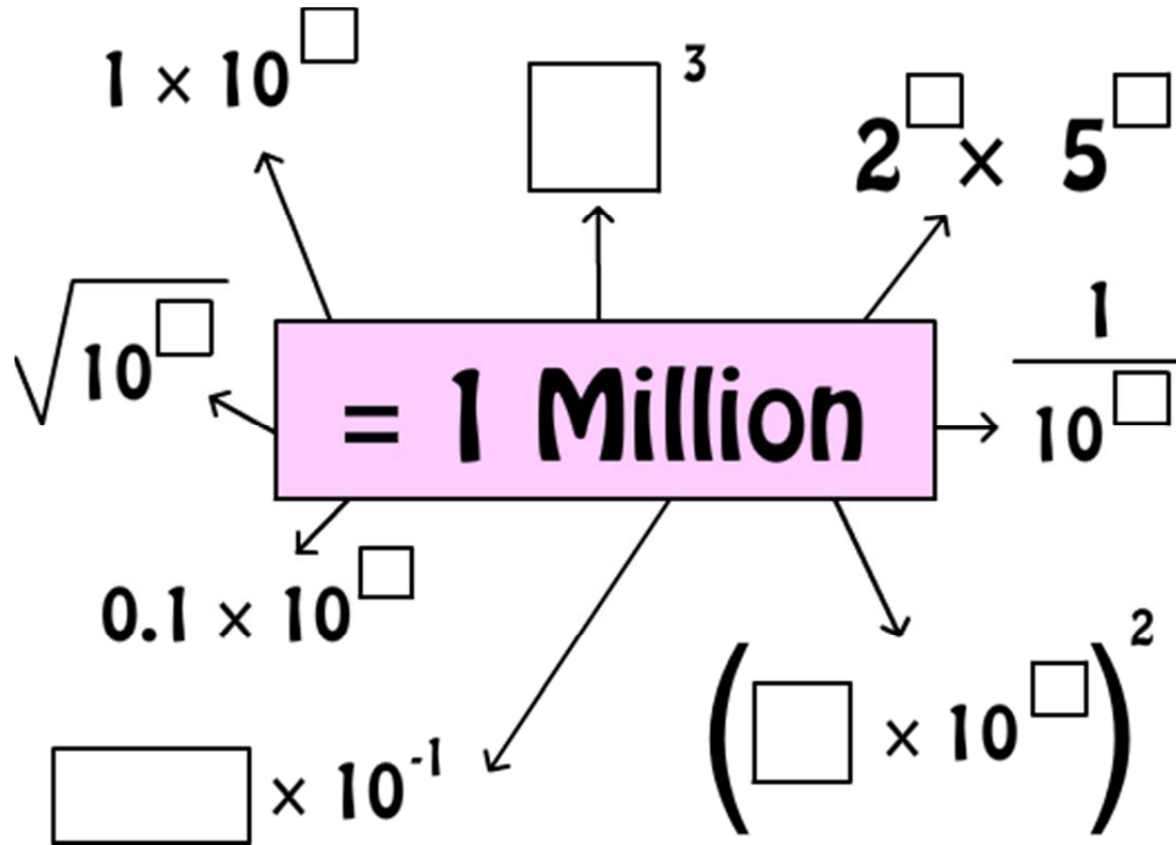


[Edexcel GCSE June 2003-6H Q17b]

Let $x = 2^p$, $y = 2^q$,

If $xy = 32$ and $2xy^2 = 32$, find the value of p and the value of q .

Fluency Practice



Match up each expression from the first column with its partner from the second column, and write the answers in the table below

First column		Second column	
1	\sqrt{x}	A	x^2
2	$\frac{1}{x}$	B	$\frac{1}{2}x^{-3}$
3	$\frac{1}{\sqrt{x}}$	C	$\frac{1}{2}x^{-1}$
4	$\sqrt[3]{x}$	D	x^{-4}
5	$\frac{1}{x^2}$	E	x^{-1}
6	$\frac{1}{2x^3}$	F	$\frac{1}{2}x$
7	$\frac{2}{x^3}$	G	x^4
8	$x\sqrt{x}$	H	x^2
9	$\frac{\sqrt{x}}{x^2}$	I	$2x$
10	$\frac{1}{x^{-2}}$	J	$x^{\frac{1}{3}}$
11	$\frac{1}{x^2} \times \frac{1}{x^3}$	K	$x^{\frac{-1}{2}}$
12	$(\sqrt[3]{x})^2$	L	$\frac{1}{2}x^{\frac{2}{3}}$
13	$\sqrt{x^7}$	M	x^{-2}
14	$\sqrt{\left(\frac{1}{x^8}\right)}$	N	$x^{\frac{3}{2}}$
15	$\frac{1}{\sqrt{x^{-8}}}$	O	$2x^{-1}$
16	$x \times \sqrt{x \times x^6}$	P	$x^{\frac{1}{2}}$
17	$\sqrt{\left(\frac{4}{x^2}\right)}$	Q	x^{-5}
18	$\sqrt{\left(\frac{1}{4x^2}\right)}$	R	$x^{\frac{2}{3}}$
19	$\sqrt{\left(\frac{x^2}{4}\right)}$	S	$2x^{-3}$
20	$\sqrt{4x^2}$	T	$x^{\frac{7}{2}}$

Table for Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Mental Mathemagician

$2^1 = 2$	$2^{11} = 2,048$	$2^{21} = 2,097,152$
$2^2 = 4$	$2^{12} = 4,096$	$2^{22} = 4,194,304$
$2^3 = 8$	$2^{13} = 8,192$	$2^{23} = 8,388,608$
$2^4 = 16$	$2^{14} = 16,384$	$2^{24} = 16,777,216$
$2^5 = 32$	$2^{15} = 32,768$	$2^{25} = 33,554,432$
$2^6 = 64$	$2^{16} = 65,536$	$2^{26} = 67,108,864$
$2^7 = 128$	$2^{17} = 131,072$	$2^{27} = 134,217,728$
$2^8 = 256$	$2^{18} = 262,144$	$2^{28} = 268,435,456$
$2^9 = 512$	$2^{19} = 524,288$	$2^{29} = 536,870,912$
$2^{10} = 1024$	$2^{20} = 1,048,576$	$2^{30} = 1,073,741,824$

Without using a calculator or paper, can you mentally compute these expressions?

- 32×16
- $4 \times 64 \times 1024$
- $4 \times 16 \times 32 \times 64$
- $1024 \div 64$
- $1,048,576 \div 32,768$
- $1,073,741,824 \div 64$
- 128^2
- $(1,024)^3$
- $\sqrt{65,536}$
- $\sqrt[3]{134,217,728}$
- $\sqrt[5]{1,073,741,824}$
- $(1,048,576)^{1/2}$
- $(262,144)^{2/3}$
- $\frac{524,288}{(16,384)^{\frac{1}{2}}}$
- $\frac{(67,108,864)^{1/2}}{16^3(256)^{1/4}}$
- $\frac{1}{4} \cdot (65,536)$

EXTENSIONS

square and cube roots: give your answer as a fraction

(1) $\sqrt{0.36}$

(1) $\sqrt{2\frac{1}{4}}$

(2) $\sqrt{1\frac{24}{25}}$

(2) $\sqrt{1\frac{15}{49}}$

(3) $\sqrt{2\frac{14}{25}}$

(3) $\sqrt{1\frac{11}{25}}$

(4) $\sqrt{3.24}$

(4) $\sqrt[3]{1\frac{271}{729}}$

(5) $\sqrt{4.84}$

(5) $\frac{1}{2}\sqrt[3]{15.625}$

(6) $\sqrt{5\frac{19}{25}}$

(6) $2(2\frac{46}{49})^{-\frac{1}{2}}$

(7) $\sqrt[3]{1\frac{91}{125}}$

(7) $\sqrt{\frac{2}{7} \times 3\frac{3}{8} \times 1\frac{5}{16}}$

(8) $(1\frac{61}{64})^{-\frac{1}{3}}$

(8) $3(3.375)^{-\frac{1}{3}}$

no calculator

$$(2\frac{1}{4})^{-\frac{1}{2}} \times (3\frac{3}{8})^{\frac{1}{3}}$$

$$(1.25)^{-\frac{1}{3}} \times (10)^{\frac{1}{3}}$$

$$(1.5)^{-2} \times (16)^{-\frac{1}{2}} \times (81)^{\frac{3}{4}}$$

$$(48)^{\frac{1}{2}} \times (9)^{\frac{1}{6}} \times (3)^{-\frac{5}{6}}$$

$$(20)^{\frac{2}{3}} \div (100)^{\frac{1}{6}} \times (\frac{5}{2\sqrt{2}})^{\frac{2}{3}}$$

$$(12)^{\frac{1}{3}} \div (81)^{\frac{1}{6}} \times (162)^{\frac{1}{3}}$$

Fluency Practice

Write in terms of the given number:

- 1) 16 as a power of 2
- 2) 16 as a power of 4
- 3) 64 as a power of 4
- 4) 125 as a power of 5
- 5) 4^2 as a power of 2
- 6) 4^3 as a power of 2
- 7) 4^4 as a power of 2
- 8) 27^3 as a power of 3
- 9) 27^{3x} as a power of 3
- 10) 25^{3x} as a power of 5

Fluency Practice

Solve the following equations:

1) $3^x = 243$

2) $3^x = \frac{1}{243}$

3) $4^x = 64$

4) $4^x = \frac{1}{64}$

5) $5^x = 625$

6) $5^x = \frac{1}{625}$

Fluency Practice

Solve the following equations:

$$1) \left(\frac{1}{3}\right)^x = 81$$

$$2) \left(\frac{1}{3}\right)^x = \frac{1}{81}$$

$$3) \left(\frac{1}{3}\right)^x = \frac{1}{27}$$

$$4) \left(\frac{1}{3}\right)^x = 27$$

$$5) \left(\frac{1}{4}\right)^x = 64$$

$$6) \left(\frac{1}{4}\right)^x = 256$$

$$7) \left(\frac{1}{4}\right)^x = \frac{1}{256}$$

$$8) \left(\frac{5}{25}\right)^x = 625$$

Fluency Practice

Find the value of x that satisfies:

1) $2^{3x} \times 2^{x-3} = 32$

2) $2^{2x} \times 2^{x+3} = 64$

3) $3^{2x} \times 3^{x-6} = 81$

4) $3^{x+2} \times 3^{x-2} = 243$

5) $2^{4x} \div 2^{x-2} = 32$

6) $3^{6x} \div 3^{x-5} = 243$

Fluency Practice

Find the value of x that satisfies:

$$1) \quad 16^{\frac{1}{4}} \times 2^x = 8^{\frac{2}{3}}$$

$$2) \quad 81^{\frac{1}{4}} \times 3^x = 27^{\frac{2}{3}}$$

$$3) \quad 25^{\frac{1}{2}} \times 125^x = 5^{\frac{4}{2}}$$

$$4) \quad 27^{\frac{1}{4}} \times 3^x = 9^{\frac{2}{3}}$$

Solving Equations with Indices

1.

$$2^{2x+4} = (2^{x+1})^3$$

2.

$$3^{2x-1} = 81^{0.75}$$

3.

$$\frac{1}{3^{2x}} = 3^{2x} \times 3^{x+5}$$

4.

$$2^{6x-5} = 8^{x+6}$$

5.

$$\sqrt{6^{4x+3}} = \sqrt[3]{6^{7x-2}}$$

6.

$$3^{2x} \times \frac{1}{9^{4x-1}} = 27^{2(x+3)}$$

7.

$$\sqrt[4]{32^{1-x}} = \frac{1}{64^{3x+2}}$$

8.

$$\sqrt[x]{7^{2x-1}} = 2401^{x-0.5}$$

1. Find the value of ...

a. $144^{\frac{1}{2}}$

b. $625^{\frac{1}{4}}$

c. $27^{\frac{1}{3}}$

d. $\left(\frac{1}{16}\right)^{0.25}$

e. $0.01^{\frac{1}{2}}$

f. $\left(\frac{125}{64}\right)^{\frac{1}{3}}$

g. $\left(\frac{1}{64}\right)^{-\frac{1}{6}}$

h. $81^{-\frac{1}{4}}$

2. Find the value of ...

a. $16^{\frac{3}{2}}$

b. $1000^{\frac{2}{3}}$

c. $32^{\frac{3}{5}}$

d. $\left(\frac{1}{4}\right)^{\frac{5}{2}}$

e. $\left(\frac{1}{81}\right)^{\frac{3}{4}}$

f. $\left(\frac{27}{125}\right)^{\frac{2}{3}}$

g. $\left(\frac{3125}{243}\right)^{\frac{4}{5}}$

h. $625^{-\frac{3}{4}}$

3. Find the missing values

a. $16^{\frac{3}{\square}} = 64$

b. $\square^{\frac{2}{5}} = 25$

c. $\left(\frac{\square}{81}\right)^{\frac{3}{4}} = \frac{8}{\square}$

d. $\left(\frac{100}{\square}\right)^{\frac{\square}{2}} = \frac{1000}{27}$

e. $\left(\frac{\square}{125}\right)^{\frac{2}{\square}} = 25$

4. Write in the form 9^n

a. 81

b. 3

c. 27

d. 243

5. Write in the form 64^n

a. 8

b. 4

c. 16

d. 512

6. Find the value of x

a. $9^{\frac{x}{4}} = 3$

b. $16^{\frac{x+2}{4}} = 2$

c. $125^{\frac{2x-1}{6}} = 5$

d. $27^{\frac{5x-7}{6}} = 81$

e. $32^{\frac{2x-3}{10}} = 4$

f. $\left(\frac{4}{25}\right)^{5x-2} = \frac{2}{5}$

g. $2 \times 9^{\frac{3x-7}{4}} = 6$

h. $3 \times 4^{2x+8} = 24$

$$\text{a)} \quad 0.25^{2-x} = \frac{256}{2^{x+3}}$$

$$\text{b)} \quad \frac{27^{3x-2}}{243} = 81^{3x-7}$$

$$\text{c)} \quad \frac{1}{5} \times \left(\frac{1}{625} \right)^{1-x} = 25^{3x+1}$$

$$\text{d)} \quad 2^x \times 5^x = 0.1 \times (10^{x-1})^5$$

$$\text{e)} \quad 625^{-2x+1} \times \frac{1}{5} = 125^3$$

$$\text{f)} \quad \frac{81^{5-2x} \times 243^{x-2}}{9^{5x-1}} = \frac{1}{3}$$

$$\text{g)} \quad \frac{3^{-9x} \times 3^{-4x+2}}{3^8} = 3^{-5}$$

$$\text{h)} \quad \frac{(3^3)^{-3x} \times (3^{-2})^{2x-1}}{(3^4)^2} = 3^{-5}$$

$$\text{i)} \quad \frac{10^{\frac{1}{3}} \times 1,000}{10^x} = 0.01$$

$$\text{j)} \quad 2 \times 4^{6x} + 64 \times 64^{2x-1} = 39$$

$$\text{k)} \quad \frac{16^{-x} \times \left(\frac{1}{256} \right)^{-x-3}}{64^{2x}} = 2,048$$

$$\text{l)} \quad \frac{625^{1-x} \times 25^{x-1}}{3,125^x} = 5 \times \frac{1}{5^{6x}}$$

$$\text{m)} \quad \frac{5^{4-4x} \times 5^{2x-2}}{5^{5x}} = 5^1 \times 5^{-6x}$$

$$\text{n)} \quad 15^{2x} - 3 \times 225^x = -30$$

$$\text{o)} \quad \frac{16^{x-1} \times 2^{5x+2}}{32^{2-3x}} = 128$$

$$\text{p)} \quad \frac{32^{1-x} \times 16^{2-2x}}{128^{3x}} = 4,096$$

$$\text{q)} \quad 5^{2x} + 125 \times 25^x = 252$$

$$\text{r)} \quad 2^{x+3} \sqrt[3]{64} = 6^{-2x} \sqrt[3]{128^2}$$

$$\text{s)} \quad 2 \times 4^{4x} + 32 \times 16^{2x-1} = 1,024$$

$$\text{t)} \quad 4^{x+3} \times 8^{x-3} = 128$$

$$\text{u)} \quad \frac{32^{x-1} \cdot 512^{1-2x} \cdot (4^{-1})^{-3x} \cdot (0.125^2)^{x+2}}{256^{-x+3} \cdot 8^{5-5x}} = 4,096$$

$$\text{v)} \quad \frac{27^{-3x} \times \left(\frac{1}{9} \right)^{2x-1}}{81^2} = \frac{1}{243}$$

SSDD

$$y = 8^x$$

What is y when $x = -2$

$$y = 8^x$$

What is x when $y = 4$

$$y = 8^x$$

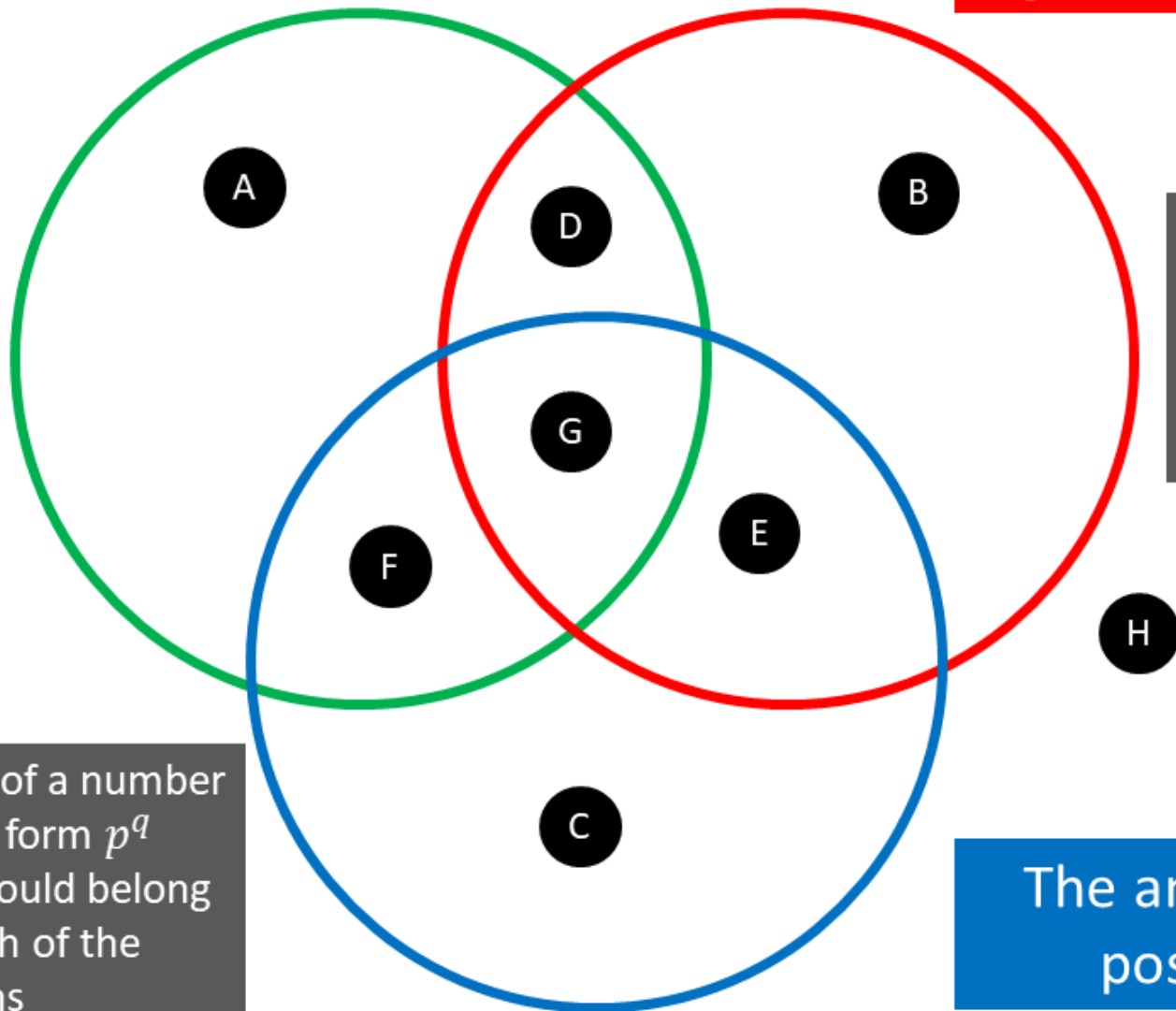
What is y^2 as a power of 2 in terms of x

$$y = 8^x$$

What is x when
 $y = 16^{(2x-3)}$

The power (q) is negative

The base number (p) is a fraction



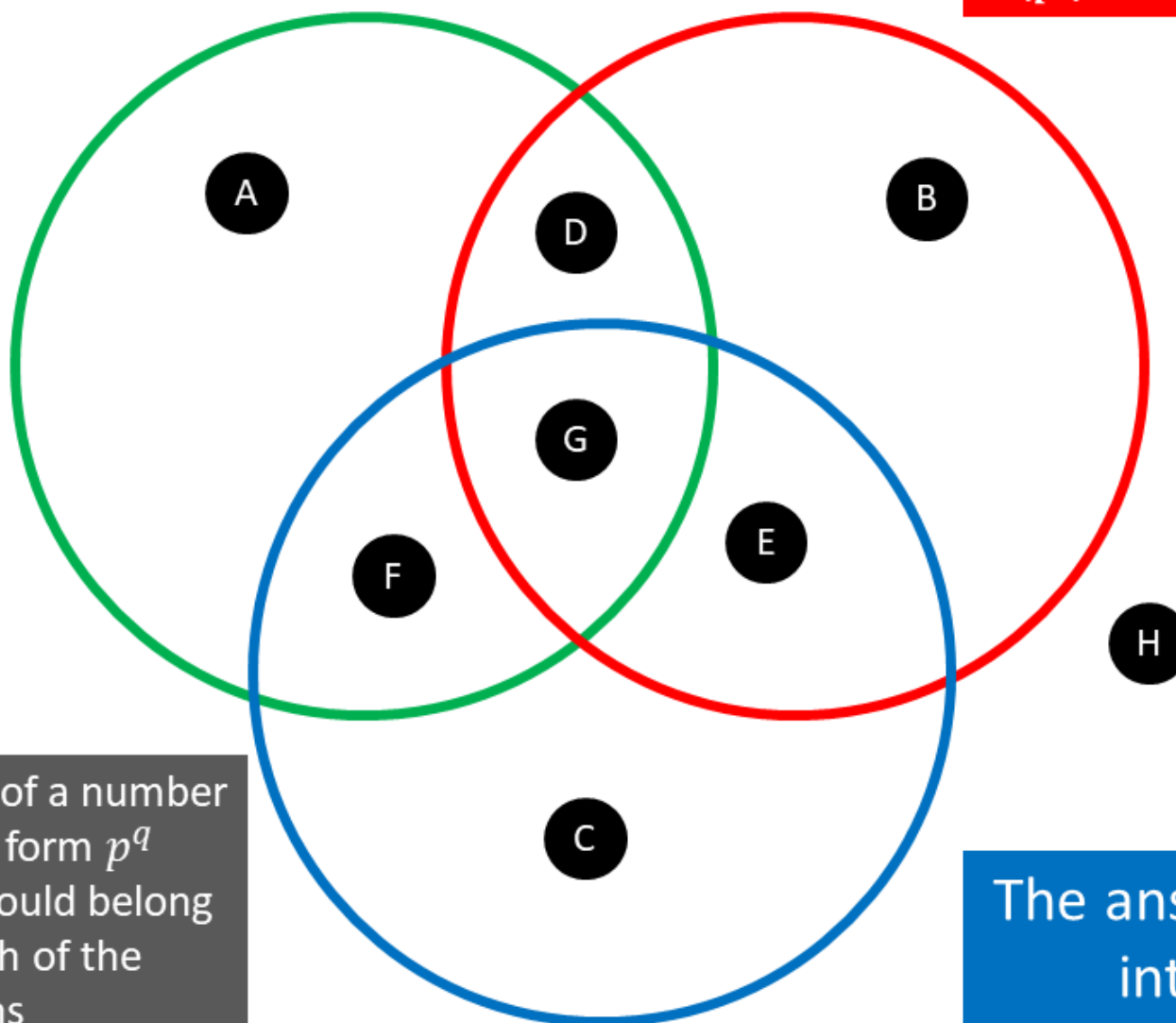
If you think a region is impossible to fill, convince me why!

Think of a number in the form p^q that could belong in each of the regions

The answer is positive

The power (q) is a fraction

The base number (p) is a fraction



If you think a region is impossible to fill, convince me why!

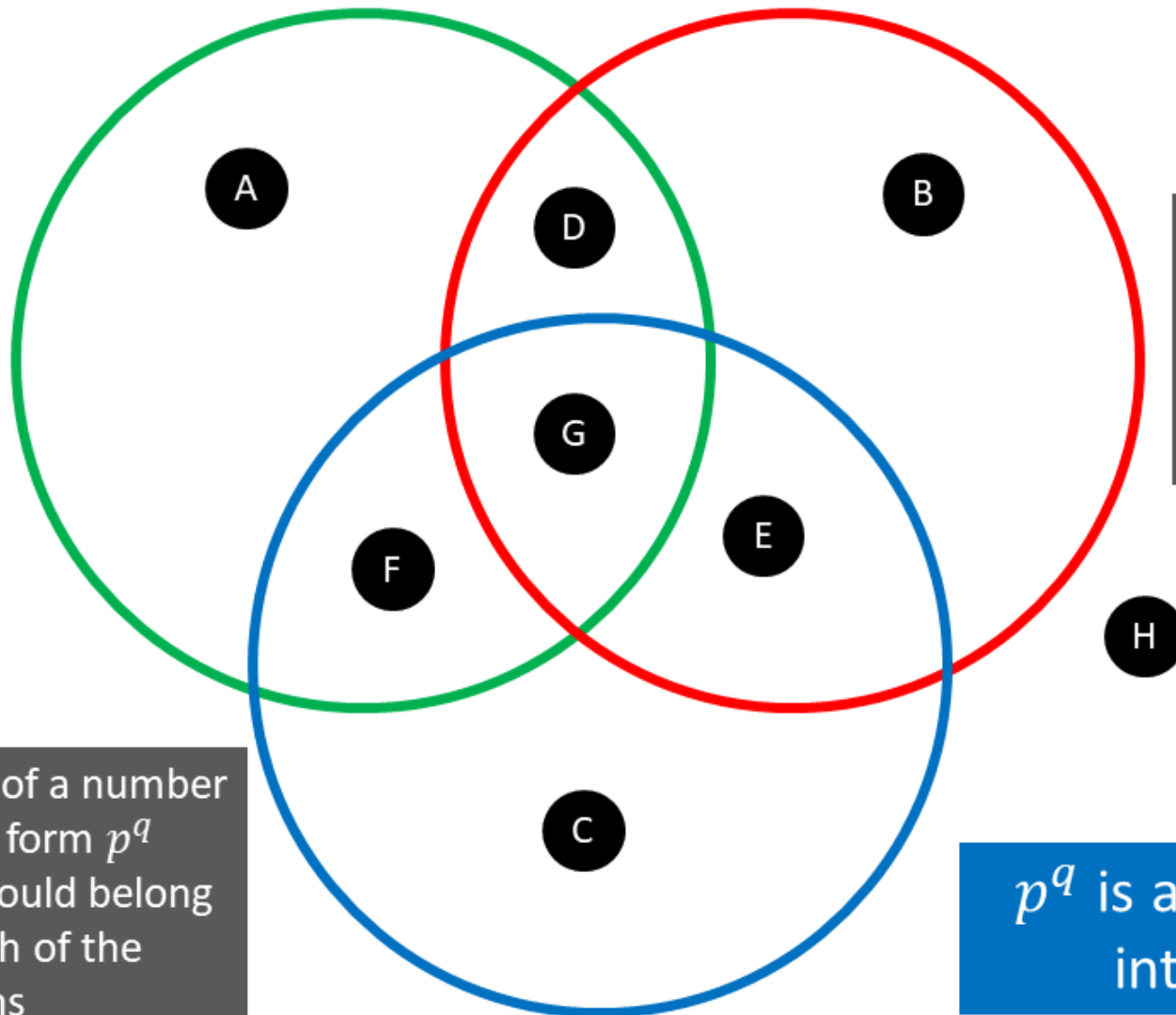
Think of a number in the form p^q that could belong in each of the regions

The answer is an integer

The power (q) is negative

@mrbartonmaths

The power (q) cannot be written as a unit fraction



If you think a region is impossible to fill, convince me why!

Think of a number in the form p^q that could belong in each of the regions

p^q is a positive integer

CHALLENGES

$$2^3 + 2^3 + 3^4 + 3^4 + 3^4 = ?$$

- $2^3 + 3^4$
 4^5
 $2^4 + 3^5$
 $2^6 + 3^{12}$

$$\sqrt{9^{16}x^2}$$

- 9^{4x}
 3^{4x}

- 9^{8x^2}
 3^{8x^2}

$$\frac{6^{2007} - 6^{2006}}{30} = ?$$

- 30
 0.2
 6^{2005}
 6^{2007}
 36

$$27^{-\frac{x}{3}} + 81^{\frac{1-x}{4}}$$

If the expression above can be stated in the form of $\frac{a}{b^x}$ for positive integers a and b , what is the value of $a + b$?

$$\begin{cases} ab = a^b \\ \frac{a}{b} = a^{3b} \end{cases}$$

a and b are real numbers, such that $a > 1$ and $b \neq 0$, satisfying the above system.

Find b^{-a} .

Task 2 Solving Exponential Equations			
$2^a = 32$	$4^b = 1$	$2^{2c} = \frac{1}{2}$	$5^{d-9} = \frac{1}{25}$
$10^{1-e} = 10000$	$4^{2f} + 1 = 65$	$7^{2g+1} = 7^{3g-2}$	$3^{2h-4} = 27$
$5^{3i-8} = 25^{2i}$	$3^{2j-5} = 27^j$	$3^2 \times 3^k = 27^k$	$2^{l-13} = \frac{1}{4}$
$9(3^m) = 27^{m-1}$	$8^{n-5} = \sqrt{8}$	$97^{p+5} = \sqrt{97} \times 97^{2p}$	$4^{q+4} = \frac{1}{64}$
$3^{r^2-3r} = 81$	$4^{2s^2+2s} = 8$	$3^t - 3^{2-t} = 8$	$(u+3)^{u+6} = 1$

-8	-4	0	4	$-\frac{3}{2}$	$\frac{5}{2}$
-7	-3	1	a 5	$-\frac{1}{2}$	$\frac{7}{2}$
-6	-2	2	7	$\frac{1}{2}$	$\frac{9}{2}$
-5	-1	3	11	$\frac{3}{2}$	$\frac{11}{2}$

Task 3 Find 6 solutions to each of the equations below. What would the next question be?		
$(n^2 - 5n + 5)^{(n^2-11n+30)} = 1$	$(n^2 - 7n + 11)^{(n^2-13n+42)} = 1$	

Task 1 Multiplying Powers

$x\sqrt{x}$	$(x - 2)$	$(x + 3)$	$(x + 5)$	$(x + 2)$
$(x - 3)$	$(x + 1)$	$(x - 1)$	x^2	\sqrt{x}
	$x^{-\frac{3}{2}}$	$\frac{1}{x}$	$(x^2 + 2x + 4)$	

Select expressions from those above that multiply to give the answers below.
 The first answer has been completed as an example.

$x^2 + 8x + 15$		$x^3 + 2x^2$		$x^2 - 9$	
$(x + 5)$	$(x + 3)$				
x^2		$x^{\frac{3}{2}} - 3x^{\frac{1}{2}}$		1	
\sqrt{x}		$\frac{4}{x} + x + 2$		$x^2 + 6x + 5$	
$x^3 + 2x^2 - 15x$		$x^3 + 2x^2 - 5x - 6$		$x^3 - 2x^2 - 5x + 6$	

Fluency Practice

Simplify:

1) $\sqrt{2} \times \sqrt{3}$

2) $\sqrt{4} \times \sqrt{3}$

3) $\sqrt{5} \times \sqrt{3}$

4) $\sqrt{6} \times \sqrt{3}$

5) $\sqrt{24} \times \sqrt{3}$

6) $\sqrt{12} \times \sqrt{3}$

7) $\sqrt{12} \times \sqrt{4}$

8) $\sqrt{11} \times \sqrt{4}$

9) $\sqrt{10} \times \sqrt{4}$

10) $\sqrt{10} \times \sqrt{8}$

11) $\sqrt{20} \times \sqrt{4}$

12) $\sqrt{20} \times \sqrt{5}$

13) $\sqrt{20} \times \sqrt{10}$

14) $\sqrt{20} \times \sqrt{20}$

15) $\sqrt{20} \times \sqrt{40}$

16) $\sqrt{20} \times \sqrt{40} \times \sqrt{2}$

17) $\sqrt{20} \times \sqrt{40} \times \sqrt{3}$

18) $(\sqrt{20} \times \sqrt{40} \times \sqrt{3})^2$

Fluency Practice

Simplify:

1) $2\sqrt{2} \times \sqrt{3}$

2) $2\sqrt{4} \times \sqrt{3}$

3) $\sqrt{5} \times 2\sqrt{3}$

4) $\sqrt{6} \times 2\sqrt{3}$

5) $\sqrt{24} \times 3\sqrt{3}$

6) $2\sqrt{12} \times 3\sqrt{3}$

7) $2\sqrt{12} \times 3\sqrt{4}$

8) $2\sqrt{11} \times 3\sqrt{4}$

9) $\sqrt{10} \times 2\sqrt{4}$

10) $\sqrt{10} \times 2\sqrt{8}$

11) $\sqrt{20} \times 2\sqrt{4}$

12) $5\sqrt{20} \times 3\sqrt{5}$

13) $5\sqrt{20} \times 3\sqrt{10}$

14) $5\sqrt{20} \times 3\sqrt{20}$

15) $2\sqrt{20} \times 3\sqrt{40}$

16) $2\sqrt{20} \times 2\sqrt{40} \times 2\sqrt{2}$

17) $2\sqrt{20} \times 2\sqrt{40} \times \sqrt{8}$

18) $2\sqrt{20} \times 2\sqrt{40} \times 2\sqrt{8}$



Surds.



Multiplying Surds.

A. Express these surds in the form $a\sqrt{b}$.

- 1). $\sqrt{8}$ 2). $\sqrt{27}$ 3). $\sqrt{20}$ 4). $\sqrt{32}$ 5). $\sqrt{80}$
6). $\sqrt{44}$ 7). $\sqrt{75}$ 8). $\sqrt{72}$ 9). $\sqrt{45}$ 10). $\sqrt{108}$
11). $\sqrt{28}$ 12). $\sqrt{125}$ 13). $\sqrt{245}$ 14). $\sqrt{192}$ 15). $\sqrt{405}$
16). $\sqrt{112}$ 17). $\sqrt{63}$ 18). $\sqrt{180}$ 19). $\sqrt{99}$ 20). $\sqrt{48}$

B. Express each of the following as the square root of a single number.

- 1). $3\sqrt{2}$ 2). $2\sqrt{5}$ 3). $6\sqrt{2}$ 4). $4\sqrt{5}$ 5). $3\sqrt{3}$
6). $2\sqrt{3}$ 7). $5\sqrt{5}$ 8). $7\sqrt{2}$ 9). $6\sqrt{3}$ 10). $5\sqrt{11}$
11). $9\sqrt{3}$ 12). $10\sqrt{7}$ 13). $8\sqrt{5}$ 14). $11\sqrt{3}$ 15). $15\sqrt{6}$

C. Work out the following. Leave the answer in surd form where appropriate.

- 1). $\sqrt{3} \times \sqrt{6}$ 2). $\sqrt{6} \times \sqrt{2}$ 3). $\sqrt{10} \times \sqrt{5}$ 4). $\sqrt{8} \times \sqrt{5}$ 5). $\sqrt{10} \times \sqrt{2}$
6). $\sqrt{3} \times \sqrt{3}$ 7). $\sqrt{2} \times \sqrt{8}$ 8). $\sqrt{14} \times \sqrt{2}$ 9). $\sqrt{2} \times \sqrt{9}$ 10). $\sqrt{5} \times \sqrt{15}$
11). $\sqrt{3} \times \sqrt{8}$ 12). $\sqrt{5} \times \sqrt{5}$ 13). $\sqrt{2} \times \sqrt{18}$ 14). $\sqrt{6} \times \sqrt{6}$ 15). $\sqrt{5} \times \sqrt{30}$

D. Surds in the form $a\sqrt{b}$ can be multiplied.

$$\text{E.g. } 3\sqrt{3} \times 4\sqrt{2} = 12\sqrt{6}$$

Work out the following. Simplify where possible.

- 1). $2\sqrt{7} \times 4\sqrt{3}$ 2). $2\sqrt{5} \times 3\sqrt{2}$ 3). $2\sqrt{3} \times 3\sqrt{3}$ 4). $5\sqrt{3} \times 7\sqrt{2}$
5). $4\sqrt{2} \times 5\sqrt{2}$ 6). $2\sqrt{3} \times 6\sqrt{5}$ 7). $2\sqrt{8} \times 5\sqrt{5}$ 8). $2\sqrt{3} \times 3\sqrt{8}$
9). $2\sqrt{2} \times 3\sqrt{8}$ 10). $2\sqrt{32} \times 3\sqrt{2}$ 11). $5\sqrt{6} \times 2\sqrt{3}$ 12). $2\sqrt{7} \times 3\sqrt{3}$
13). $5\sqrt{5} \times 4\sqrt{4}$ 14). $4\sqrt{24} \times 2\sqrt{3}$ 15). $5\sqrt{3} \times 4\sqrt{21}$ 16). $3\sqrt{5} \times 4\sqrt{24}$
17). $4\sqrt{8} \times 3\sqrt{5}$ 18). $3\sqrt{2} \times 8\sqrt{40}$ 19). $4\sqrt{18} \times 2\sqrt{5}$ 20). $2\sqrt{6} \times 5\sqrt{42}$

Fluency Practice

Simplify:

1) $\sqrt{6} \div \sqrt{3}$

2) $\sqrt{6} \div \sqrt{2}$

3) $\sqrt{12} \div \sqrt{2}$

4) $\sqrt{12} \div \sqrt{3}$

5) $\sqrt{15} \div \sqrt{3}$

6) $\sqrt{18} \div \sqrt{3}$

7) $\sqrt{72} \div \sqrt{3}$

8) $\sqrt{36} \div \sqrt{3}$

9) $\sqrt{36} \div \sqrt{4}$

10) $\sqrt{40} \div \sqrt{4}$

11) $\sqrt{40} \div \sqrt{40}$

12) $\sqrt{40} \div \sqrt{5}$

13) $\sqrt{100} \div \sqrt{5}$

14) $\sqrt{200} \div \sqrt{10}$

15) $\sqrt{400} \div \sqrt{20}$

16) $\sqrt{800} \div \sqrt{40}$

17) $\sqrt{800} \div \sqrt{40} \div \sqrt{2}$

18) $\sqrt{800} \div \sqrt{40} \div \sqrt{20}$

Fluency Practice

Simplify:

1) $2\sqrt{6} \div \sqrt{3}$

2) $4\sqrt{6} \div \sqrt{2}$

3) $4\sqrt{12} \div 2\sqrt{2}$

4) $4\sqrt{12} \div 2\sqrt{3}$

5) $6\sqrt{15} \div 3\sqrt{3}$

6) $6\sqrt{18} \div 3\sqrt{3}$

7) $24\sqrt{72} \div 3\sqrt{3}$

8) $12\sqrt{36} \div 3\sqrt{3}$

9) $12\sqrt{36} \div 4\sqrt{4}$

10) $20\sqrt{40} \div 4\sqrt{4}$

11) $20\sqrt{40} \div 20\sqrt{40}$

12) $20\sqrt{40} \div 5\sqrt{5}$

13) $10\sqrt{100} \div 5\sqrt{5}$

14) $20\sqrt{200} \div 10\sqrt{10}$

15) $40\sqrt{400} \div 20\sqrt{20}$

16) $80\sqrt{800} \div 40\sqrt{40}$

17) $80\sqrt{800} \div 40\sqrt{40} \div 2\sqrt{2}$

18) $80\sqrt{800} \div 40\sqrt{40} \div 2\sqrt{20}$

Dividing Surds.

A. Work out the following. Simplify where possible.

- 1). $\sqrt{10} \div \sqrt{2}$ 2). $\sqrt{20} \div \sqrt{5}$ 3). $\sqrt{150} \div \sqrt{3}$ 4). $\sqrt{90} \div \sqrt{2}$ 5). $\sqrt{72} \div \sqrt{6}$
6). $\sqrt{42} \div \sqrt{7}$ 7). $\sqrt{55} \div \sqrt{5}$ 8). $\sqrt{144} \div \sqrt{8}$ 9). $\sqrt{48} \div \sqrt{6}$ 10). $\sqrt{126} \div \sqrt{7}$
11). $\sqrt{80} \div \sqrt{5}$ 12). $\sqrt{588} \div \sqrt{3}$ 13). $\sqrt{320} \div \sqrt{10}$ 14). $\sqrt{1050} \div \sqrt{3}$ 15). $\sqrt{320} \div \sqrt{5}$

B. Surds in the form $a\sqrt{b}$ can be divided.

$$\text{E.g. } 8\sqrt{6} \div 2\sqrt{3} = 4\sqrt{2}$$

Work out the following. Simplify where possible.

- 1). $6\sqrt{15} \div 2\sqrt{3}$ 2). $14\sqrt{3} \div 7\sqrt{3}$ 3). $8\sqrt{6} \div 2\sqrt{3}$ 4). $20\sqrt{15} \div 4\sqrt{5}$
5). $10\sqrt{2} \div 2\sqrt{2}$ 6). $15\sqrt{7} \div 3\sqrt{7}$ 7). $10\sqrt{30} \div 5\sqrt{5}$ 8). $18\sqrt{32} \div 3\sqrt{8}$
9). $27\sqrt{24} \div 3\sqrt{8}$ 10). $24\sqrt{28} \div 3\sqrt{2}$ 11). $2\sqrt{27} \div 2\sqrt{3}$ 12). $21\sqrt{3} \div 3\sqrt{3}$
13). $32\sqrt{35} \div 4\sqrt{5}$ 14). $10\sqrt{24} \div 2\sqrt{3}$ 15). $8\sqrt{32} \div 4\sqrt{2}$ 16). $4\sqrt{48} \div 4\sqrt{3}$
17). $2\sqrt{10} \div 2\sqrt{2}$ 18). $12\sqrt{28} \div 3\sqrt{7}$ 19). $15\sqrt{30} \div 5\sqrt{5}$ 20). $30\sqrt{150} \div 5\sqrt{6}$

Fluency Practice

Simplify:

1) $2\sqrt{3} + 5\sqrt{3}$

2) $2\sqrt{3} + 5\sqrt{6}$

3) $2\sqrt{3} + 5\sqrt{12}$

4) $2\sqrt{3} + 5\sqrt{27}$

5) $2\sqrt{3} + 6\sqrt{27}$

6) $2\sqrt{30} + 6\sqrt{27}$

6) $2\sqrt{30} + 6\sqrt{27}$

7) $2\sqrt{30} + 6\sqrt{120}$

8) $6\sqrt{30} + 2\sqrt{120}$

9) $6\sqrt{60} + 2\sqrt{240}$

10) $6\sqrt{6} + 2\sqrt{24}$

11) $6\sqrt{6} + 2\sqrt{24} + 5\sqrt{36}$

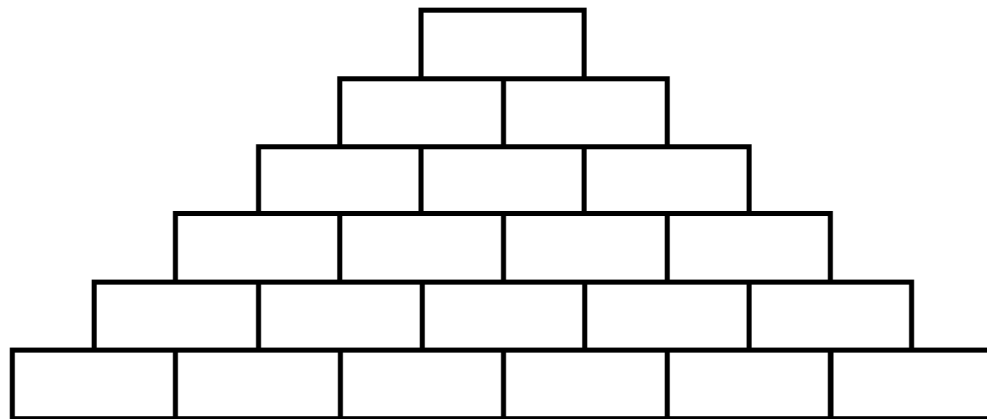
12) $6\sqrt{16} + 2\sqrt{24} + 5\sqrt{36}$

EXTENSION

Pyramid II

Cut out the bricks below. Place the bricks in a pyramid shape so that each brick is the sum of the two bricks beneath it.

$7\sqrt{2} + \sqrt{147}$	$\sqrt{2} - \sqrt{75}$	$\sqrt{3}$
$2\sqrt{2} - \sqrt{432}$	$-5\sqrt{3}$	$\sqrt{3} + \sqrt{8}$
$-9\sqrt{3}$	$\sqrt{18} + \sqrt{48}$	$-3\sqrt{3} + \sqrt{722}$
$2\sqrt{2}$	$-\sqrt{2} - \sqrt{48}$	$12\sqrt{2} + 8\sqrt{3}$
$5\sqrt{2} + \sqrt{3}$	$2(\sqrt{3} + \sqrt{2})$	$-\sqrt{2} + \sqrt{3}$
$\sqrt{98} - \sqrt{363}$	$2\sqrt{3}$	$\sqrt{2} + \sqrt{12}$
$\sqrt{2}$	$\sqrt{8} - \sqrt{27}$	$\sqrt{27} + 4\sqrt{2}$



Mixed Questions.

A. Work out the following. Simplify where possible.

- 1). $(\sqrt{2})^3$ 2). $(\sqrt{3})^3$ 3). $(\sqrt{2})^5$ 4). $(\sqrt{3})^4$ 5). $(\sqrt{5})^5$
6). $(3\sqrt{2})^2$ 7). $(2\sqrt{7})^2$ 8). $(2\sqrt{3})^3$ 9). $(2\sqrt{2})^3$ 10). $(2\sqrt{3})^2$
11). $(2\sqrt{5})^2$ 12). $(5\sqrt{3})^2$ 13). $(2\sqrt{5})^3$ 14). $(3\sqrt{6})^2$ 15). $(3\sqrt{5})^3$

B. Given that $\sqrt{2} = 1.41$, $\sqrt{3} = 1.73$ and $\sqrt{5} = 2.24$ find the values of each of the following:

- 1). $\sqrt{18}$ 2). $\sqrt{8}$ 3). $\sqrt{48}$ 4). $\sqrt{12}$ 5). $\sqrt{75}$
6). $\sqrt{20}$ 7). $\sqrt{32}$ 8). $\sqrt{27}$ 9). $\sqrt{50}$ 10). $\sqrt{45}$
11). $\sqrt{72}$ 12). $\sqrt{98}$ 13). $\sqrt{108}$ 14). $\sqrt{80}$ 15). $\sqrt{125}$
16). True or false ? To calculate the square roots of **all** the whole numbers from 1 to 100 you only need the square roots of all the prime numbers between 1 and 100.

C. Simplify

- 1). $\sqrt{8} + \sqrt{2}$ 2). $\sqrt{20} - \sqrt{5}$ 3). $\sqrt{3} + \sqrt{12}$ 4). $\sqrt{8} - \sqrt{2}$
5). $\sqrt{27} + \sqrt{12}$ 6). $\sqrt{125} - \sqrt{20}$ 7). $\sqrt{48} + \sqrt{75}$ 8). $\sqrt{18} + \sqrt{72}$
9). $\sqrt{75} - \sqrt{27}$ 10). $\sqrt{80} - \sqrt{20}$ 11). $\sqrt{108} - \sqrt{27}$ 12). $\sqrt{27} - \sqrt{12}$
13). $\sqrt{147} - \sqrt{108}$ 14). $\sqrt{48} - \sqrt{27}$ 15). $\sqrt{98} + \sqrt{8} + \sqrt{2}$ 16). $\sqrt{99} - \sqrt{44} - \sqrt{11}$
17). $3\sqrt{2} - \sqrt{18}$ 18). $\sqrt{175} - 4\sqrt{7}$ 19). $3\sqrt{8} + \sqrt{50}$ 20). $5\sqrt{5} + \sqrt{20}$
21). $2\sqrt{45} + 3\sqrt{20}$ 22). $3\sqrt{32} - 2\sqrt{18}$

Fluency Practice

Simplify:

$$1) \frac{2\sqrt{3}+5\sqrt{3}}{\sqrt{3}}$$

$$2) \frac{2\sqrt{3}+5\sqrt{12}}{\sqrt{3}}$$

$$3) \frac{2\sqrt{3}+5\sqrt{27}}{\sqrt{3}}$$

$$4) \frac{5\sqrt{27}-2\sqrt{3}}{\sqrt{3}}$$

$$5) \frac{10\sqrt{27}-4\sqrt{3}}{2\sqrt{3}}$$

$$6) \frac{10\sqrt{27}-30\sqrt{3}}{2\sqrt{3}}$$

$$7) \frac{10\sqrt{270}+30\sqrt{30}}{2\sqrt{30}}$$

$$8) \frac{10\sqrt{120}+30\sqrt{30}}{2\sqrt{30}}$$

$$9) \frac{10\sqrt{120}+30\sqrt{30}}{50\sqrt{30}}$$

Fluency Practice

Expand and simplify:

1) $3(4 + \sqrt{5})$

10) $\sqrt{5}(4 - \sqrt{5})$

2) $3(\sqrt{5} + 4)$

11) $2\sqrt{5}(4 - \sqrt{5})$

3) $6(\sqrt{5} + 4)$

12) $\sqrt{20}(4 - \sqrt{5})$

4) $6(\sqrt{5} - 4)$

13) $\sqrt{20}(\sqrt{4} - \sqrt{5})$

5) $6(4 - \sqrt{5})$

14) $\sqrt{20}(\sqrt{6} - \sqrt{5})$

6) $-6(4 - \sqrt{5})$

15) $\sqrt{20}(\sqrt{6} - \sqrt{10})$

7) $-6(-4 - \sqrt{5})$

16) $\sqrt{20}(\sqrt{12} - \sqrt{10})$

8) $6(4 + \sqrt{5})$

17) $\sqrt{10}(\sqrt{6} - \sqrt{5})$

9) $\sqrt{5}(4 + \sqrt{5})$

18) $\sqrt{10}(\sqrt{6} - \sqrt{5} + \sqrt{24})$

Fluency Practice

Expand and simplify:

1) $(\sqrt{5} + 2)(\sqrt{5} + 1)$

2) $(2 + \sqrt{5})(1 + \sqrt{5})$

3) $(2 - \sqrt{5})(1 + \sqrt{5})$

4) $(2 - \sqrt{5})(1 - \sqrt{5})$

5) $(2 + 2\sqrt{5})(1 + \sqrt{5})$

6) $(\sqrt{5} + 2)(\sqrt{5} - 1)$

7) $(\sqrt{7} + 2)(\sqrt{7} - 1)$

8) $(2\sqrt{7} + 2)(3\sqrt{7} - 1)$

9) $(2\sqrt{7} + 2)(3\sqrt{7} - 6)$

10) $(\sqrt{7} + 6)(\sqrt{7} - 6)$

11) $2(\sqrt{7} + 6)(\sqrt{7} - 6)$

12) $(\sqrt{7} - 6)(\sqrt{7} - 6)$

13) $(\sqrt{7} - 6)^2$

14) $(\sqrt{7} - 6)^3$

Fluency Practice

Expand and simplify:

1) $(2 + \sqrt{3})(4 + \sqrt{12})$

2) $(2 + \sqrt{3})(4 - \sqrt{12})$

3) $(2 - \sqrt{3})(4 + \sqrt{12})$

4) $(2 - \sqrt{3})(4 - \sqrt{12})$

5) $(\sqrt{3} - 2)(\sqrt{12} - 4)$

6) $(\sqrt{12} - 2)(\sqrt{3} - 4)$

7) $(\sqrt{27} - 2)(\sqrt{3} - 4)$

8) $(\sqrt{27} - 3)(\sqrt{3} - 5)$

9) $(\sqrt{54} - 3)(\sqrt{6} - 5)$

10) $(2\sqrt{54} - 3)(\sqrt{6} - 5)$

11) $(2\sqrt{54} - 3)(4\sqrt{6} - 5)$

12) $(2\sqrt{44} - 3)(4\sqrt{11} - 5)$

13) $(3\sqrt{44} - 2)(5\sqrt{11} - 4)$

14) $(3\sqrt{44} - 2)^2$

Fluency Practice

Rationalise:

$$1) \frac{5}{\sqrt{2}}$$

$$2) \frac{6}{\sqrt{2}}$$

$$3) \frac{6}{\sqrt{3}}$$

$$4) \frac{4}{\sqrt{3}}$$

$$5) \frac{4}{\sqrt{6}}$$

$$6) \frac{5}{\sqrt{6}}$$

$$7) \frac{5}{2\sqrt{6}}$$

$$8) \frac{5}{3\sqrt{6}}$$

$$9) \frac{5}{10\sqrt{6}}$$

$$10) \frac{5}{10\sqrt{5}}$$

$$11) \frac{5}{\sqrt{5}}$$

$$12) \frac{8}{\sqrt{8}}$$

$$13) \frac{10}{\sqrt{8}}$$

$$14) \frac{10}{\sqrt{12}}$$

$$15) \frac{10}{2\sqrt{3}}$$

$$16) \frac{10+\sqrt{3}}{2\sqrt{3}}$$

$$17) \frac{10-\sqrt{3}}{2\sqrt{3}}$$

$$18) \frac{12-\sqrt{3}}{2\sqrt{3}}$$

Fluency Practice

Rationalise:

1) $\frac{2}{\sqrt{2}+3}$

2) $\frac{2}{3+\sqrt{2}}$

3) $\frac{2}{3-\sqrt{2}}$

4) $\frac{7}{3-\sqrt{2}}$

5) $\frac{7}{\sqrt{2}-3}$

6) $\frac{7}{\sqrt{7}-3}$

7) 7) $\frac{\sqrt{7}}{\sqrt{7}-3}$

8) $\frac{\sqrt{5}}{\sqrt{5}-3}$

9) $\frac{\sqrt{5}}{5-\sqrt{3}}$

10) $\frac{2\sqrt{5}}{5-\sqrt{3}}$

11) $\frac{4\sqrt{5}}{5-\sqrt{3}}$

12) $\frac{4\sqrt{5}}{5-2\sqrt{3}}$

13) $\frac{4\sqrt{5}}{5-3\sqrt{3}}$

14) $\frac{4\sqrt{3}}{5-3\sqrt{3}}$

Rationalise means make the denominator a rational number.

Rationalise



- | | | | | | | | | | |
|------|-------------------------------|------|------------------------------|------|-------------------------------|------|-------------------------------|------|-------------------------------|
| 1). | $\frac{3}{\sqrt{3}}$ | 2). | $\frac{10}{\sqrt{5}}$ | 3). | $\frac{21}{\sqrt{7}}$ | 4). | $\frac{8}{\sqrt{2}}$ | 5). | $\frac{24}{\sqrt{6}}$ |
| 6). | $\frac{1}{\sqrt{3}}$ | 7). | $\frac{1}{\sqrt{2}}$ | 8). | $\frac{1}{\sqrt{5}}$ | 9). | $\frac{2}{\sqrt{3}}$ | 10). | $\frac{9}{\sqrt{15}}$ |
| 11). | $\frac{21}{\sqrt{6}}$ | 12). | $\frac{8}{\sqrt{18}}$ | 13). | $\frac{2}{\sqrt{5}}$ | 14). | $\frac{9}{\sqrt{6}}$ | 15). | $\frac{30}{\sqrt{75}}$ |
| 16). | $\frac{\sqrt{12}}{\sqrt{50}}$ | 17). | $\frac{\sqrt{12}}{\sqrt{3}}$ | 18). | $\frac{3\sqrt{2}}{\sqrt{10}}$ | 19). | $\frac{3\sqrt{7}}{\sqrt{21}}$ | 20). | $\frac{4\sqrt{5}}{\sqrt{20}}$ |

Eg. Rationalise $\frac{1}{\sqrt{7} - \sqrt{5}} = \frac{1 \times (\sqrt{7} + \sqrt{5})}{(\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5})} = \frac{\sqrt{7} + \sqrt{5}}{7 - 5} = \frac{\sqrt{7} + \sqrt{5}}{2}$

Eg. Rationalise $\frac{1}{\sqrt{2} + 1} = \frac{1 \times (\sqrt{2} - 1)}{(\sqrt{2} + 1)(\sqrt{2} - 1)} = \frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1$

- | | | | | | | | | | |
|------|---------------------------------|------|---------------------------------|------|---------------------------------|------|----------------------------------|------|----------------------------------|
| 21). | $\frac{1}{\sqrt{5} + \sqrt{2}}$ | 22). | $\frac{1}{\sqrt{3} - \sqrt{2}}$ | 23). | $\frac{4}{\sqrt{7} + \sqrt{5}}$ | 24). | $\frac{6}{\sqrt{13} - \sqrt{7}}$ | 25). | $\frac{4}{\sqrt{5} + \sqrt{3}}$ |
| 26). | $\frac{7}{\sqrt{3} + 2}$ | 27). | $\frac{4}{\sqrt{11} - 3}$ | 28). | $\frac{12}{\sqrt{7} + 3}$ | 29). | $\frac{6}{\sqrt{13} - 2}$ | 30). | $\frac{6}{\sqrt{24} - \sqrt{6}}$ |

Surds Revision

(a)	(b)	(c)	(d)
Write $\sqrt{108}$ in the form $k\sqrt{3}$	Write $\sqrt{45} + \sqrt{20}$ in the form $k\sqrt{5}$	Write $\sqrt{96} - \sqrt{24}$ in the form $k\sqrt{6}$	Expand $\sqrt{2}(5 + \sqrt{8})$
(e)	(f)	(g)	(h)
Expand and simplify $(7 + \sqrt{3})(4 - \sqrt{3})$	Expand and simplify $(5 + 2\sqrt{2})(6 - \sqrt{2})$	Expand and simplify $(5 + 3\sqrt{2})^2$	Rationalise the denominator and simplify fully $\frac{15}{\sqrt{18}}$
(i)	(j)	(k)	(l)
Rationalise the denominator and simplify fully $\frac{5 + 4\sqrt{3}}{\sqrt{3}}$	Express $\frac{\sqrt{3} + \sqrt{27}}{\sqrt{2}}$ as a single surd.	Rationalise the denominator and simplify fully $\frac{\sqrt{3} + 5}{2 - \sqrt{3}}$	$(4 + \sqrt{a})(7 - \sqrt{a}) = 23 + k\sqrt{a}$ Find the values of the positive integers a and k .

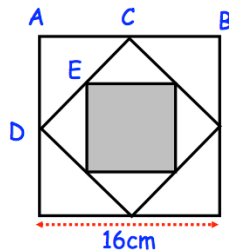
EXTENSION

Question 4: A shed has dimensions, in metres, of height $\sqrt{5}$, width $\sqrt{6}$ and length $\sqrt{10}$.
Find the volume of the shed.
Give your answer as a simplified surd.

Question 5: Mrs Jenkins is making decorations for a wedding.
She needs $18\sqrt{5}$ metres of ribbon in total.
Mrs Jenkins has 40 metres of ribbon.
Does she have enough ribbon?



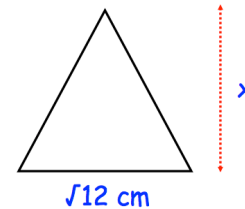
Question 6: The midpoints of the sides of a square of side 16cm are joined to form another square. This process is then repeated to create the shaded square.



Find the area of the shaded square.

Question 7: The area of a rectangle is $\sqrt{125} \text{ cm}^2$.
The length of the rectangle is $(2 + \sqrt{5}) \text{ cm}$.
Calculate the width of the rectangle.
Express your answer in the form $a + b\sqrt{5}$, where a and b are integers.

Question 8: The triangle below has an area of $2\sqrt{6} \text{ cm}^2$.
Find the height of the triangle, x .
Give your answer as a simplified surd.



Fluency Practice

Write each ratio in its simplest form.

$$\sqrt{20} : \sqrt{45}$$

$$\sqrt{63} : \sqrt{28} : \sqrt{175}$$

$$a = \sqrt{11} + \sqrt{c}$$

$$b = \sqrt{44} + \sqrt{d}$$

Given that $c:d$ is in the
ratio 1:4

Find the ratio $a:b$

$$a = \sqrt{3} + \sqrt{c}$$

$$b = \sqrt{48} + \sqrt{d}$$

Given that $c:d$ is in the
ratio 1:16

Find the ratio $a:b$

$$a = \sqrt{7} + \sqrt{c}$$

$$b = \sqrt{63} + \sqrt{d}$$

Given that $c:d$ is in the
ratio 1:9

Find the ratio $a:b$

$$a = \sqrt{12} + \sqrt{c}$$

$$b = \sqrt{192} + \sqrt{d}$$

Given that $c:d$ is in the
ratio 1:4

Find the ratio $a:b$

Fluency Practice

what are the surd square roots of these surds?

(1) $9 + 4\sqrt{5}$

(6) $36 + 16\sqrt{2}$

(11) $11 + 4\sqrt{6}$

(2) $25 + 4\sqrt{21}$

(7) $49 + 20\sqrt{6}$

(12) $8 - 2\sqrt{15}$

(3) $7 - 4\sqrt{3}$

(8) $29 - 12\sqrt{5}$

(13) $18 - 12\sqrt{2}$

(4) $8 + 2\sqrt{7}$

(9) $52 + 16\sqrt{3}$

(14) $55 + 30\sqrt{2}$

(5) $36 - 10\sqrt{11}$

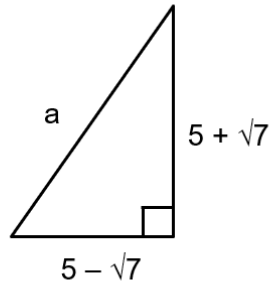
(10) $64 - 24\sqrt{7}$

(15) $42 - 24\sqrt{3}$

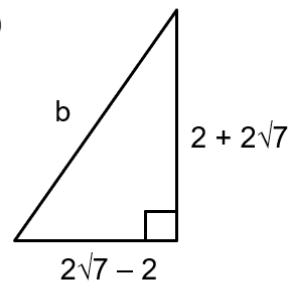
Purposeful Practice

find the missing length

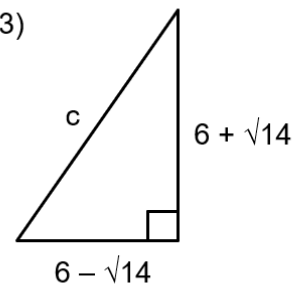
(1)



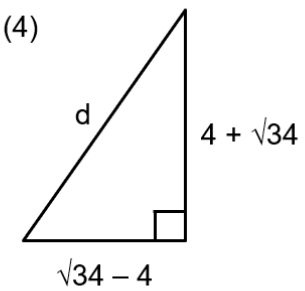
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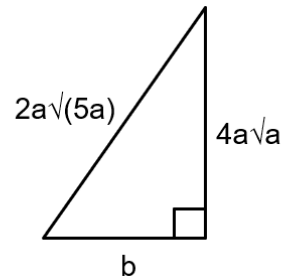
(3)



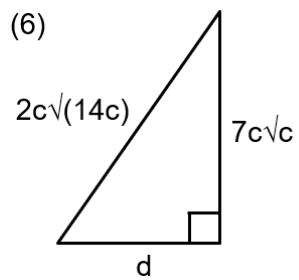
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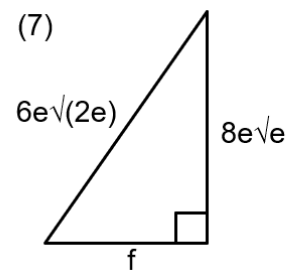
(5)



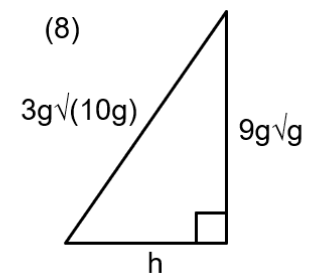
(6)



(7)



(8)



Purposeful Practice

For each set of numbers find: (i) the mean, (ii) the median, (iii) the range.

Leave your answers in their simplest form.

a) $5\sqrt{3}$ $4\sqrt{3}$ $6\sqrt{3}$

b) $\sqrt{3}$ $\sqrt{108}$ $\sqrt{12}$

c) $2\sqrt{12}$ $4\sqrt{12}$ $3\sqrt{27}$

d) $2\sqrt{27}$ $3\sqrt{12}$ $\sqrt{243}$

e) $2\sqrt{75}$ $\sqrt{48}$ $5\sqrt{12}$

f) $\sqrt{243}$ $5\sqrt{27}$ $3\sqrt{75}$

g) $\sqrt{20}$ $\sqrt{20}$ $\sqrt{80}$ $\sqrt{320}$

h) $\sqrt{28}$ $\sqrt{63}$ $3\sqrt{63}$ $4\sqrt{7}$

i) $-\sqrt{99}$ $2\sqrt{44}$ $-\sqrt{11}$ $4\sqrt{44}$

j) 0 $-2\sqrt{44}$ $-4\sqrt{44}$ $\sqrt{44}$

k) $\sqrt{2}$ $-\sqrt{18}$ $\sqrt{48}$ $\sqrt{8}$

l) $2\sqrt{150}$ $-2\sqrt{54}$ $2\sqrt{24}$ $-2\sqrt{96}$

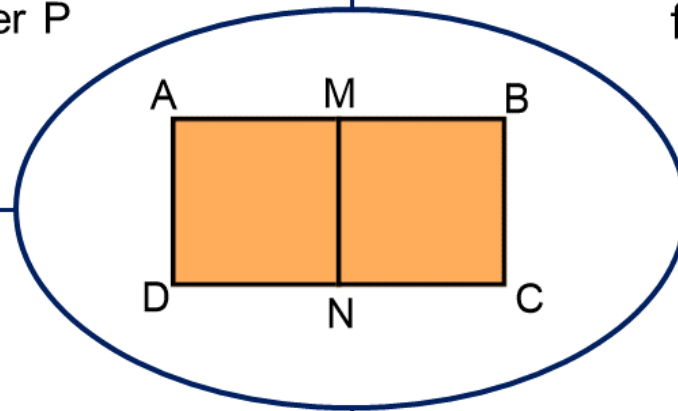
Fluency Practice

A rectangle made of two squares – KS4

AD = 11cm (2sf).

Give the upper and lower bounds for the perimeter P of ABCD.

If AD = 3cm, calculate the length of AC. Give your answer in the form $p\sqrt{q}$ where p & q are integers.



The area of AMND is 2m^2 .

Find the size of $\angle ABD$.
(calculator needed)

- What is the height of rectangle ABCD?
- Find the length of DC. Give your answer in the form \sqrt{r} where r is an integer.

Fluency Practice

Calculate the area of this shape:



$\sqrt{27}$ cm

$\sqrt{45}$ cm

Calculate the perimeter of this shape:



$\sqrt{27}$ cm

$\sqrt{45}$ cm

Calculate the length of the longest diagonal.



$\sqrt{27}$ cm

$\sqrt{45}$ cm

The length of the longest diagonal is $\sqrt{125}$ cm.

Calculate the length of the missing side.



? cm

$\sqrt{45}$ cm

Simplify 24:54

Simplify $\frac{4a}{15} \div \frac{2a^2}{3b}$

Simplifying

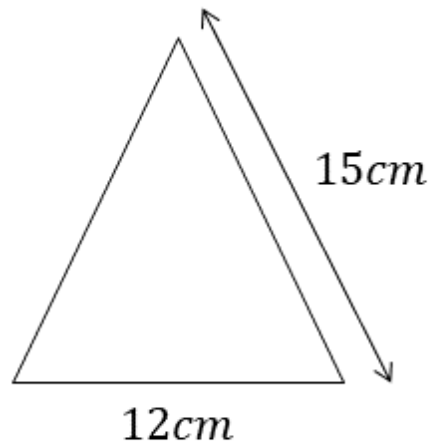
Rationalise the denominator

$$\frac{3\sqrt{2}}{6 + \sqrt{2}}$$

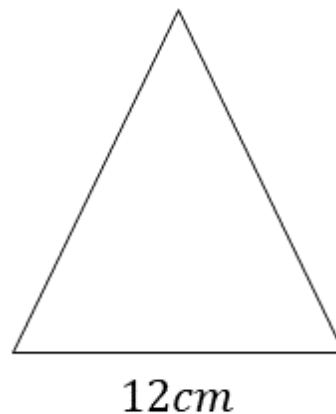
Solve for x

$$\frac{x}{(x+3)} - \frac{9}{x(x+3)} = 2$$

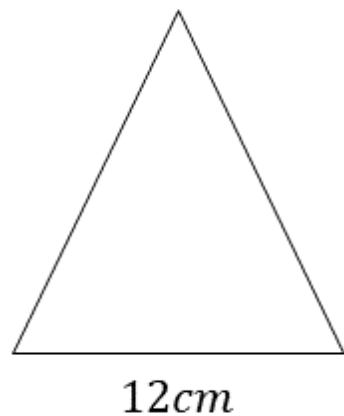
Calculate the area of the triangle?



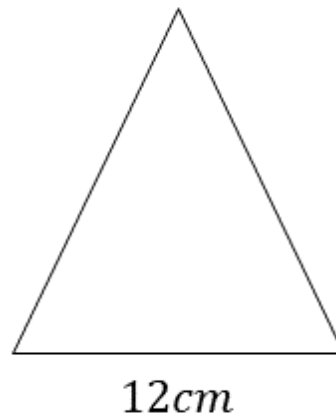
The area of the triangle is $18\sqrt{5}\text{ cm}^2$ calculate the height of the triangle



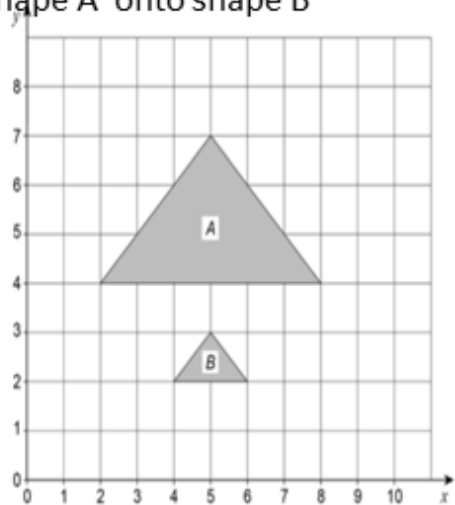
The area of this isosceles triangle is $30\sqrt{5}\text{ cm}^2$ calculate perimeter of the triangle



The perimeter of this isosceles triangle is 50cm calculate the area of the triangle.

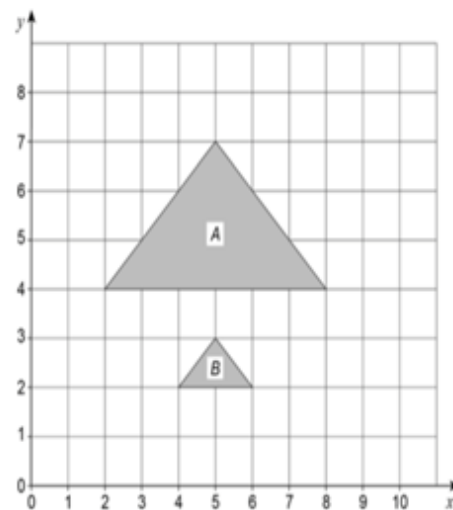


i) Describe fully the single transformation that maps shape A onto shape B



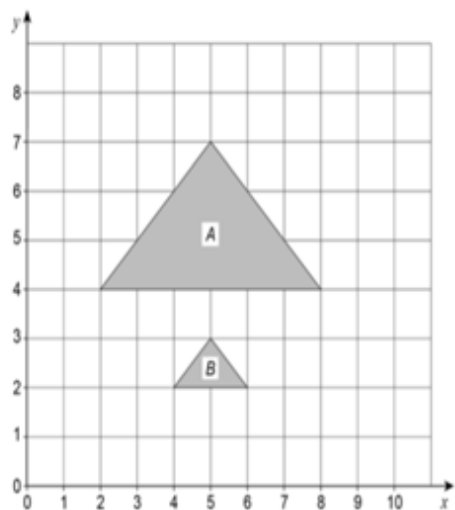
[3 marks]

ii) Find the area of triangle A



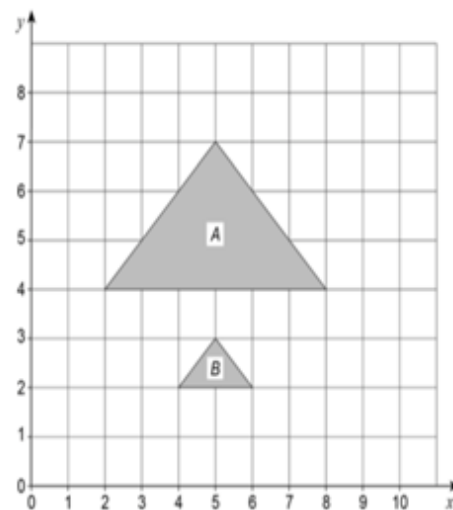
[3 marks]

iii) Find the perimeter of triangle B. Give your answer in the form $a + b\sqrt{c}$ where a b and c are integers



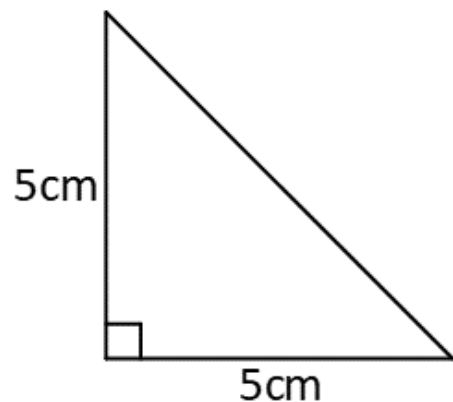
[3 marks]

iv) Triangle A is mapped onto Triangle B below. Triangle B is mapped onto a new triangle, Triangle C using the same scale factor. Work out the area of triangle C.

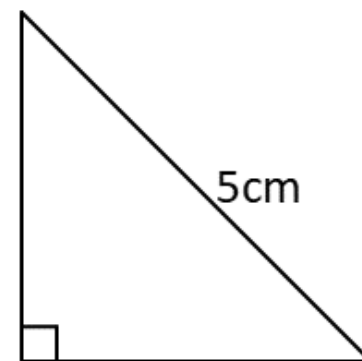


[3 marks]

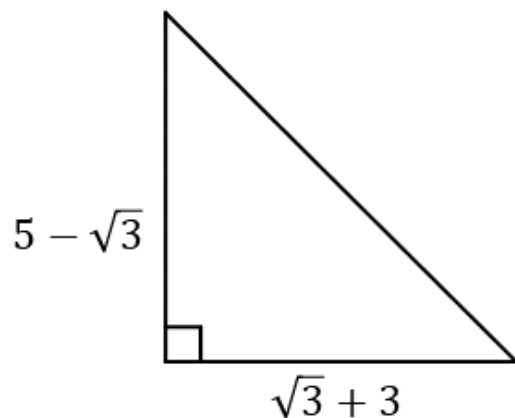
An isosceles right-angled triangle has a short side of length 5cm as shown. What is the area of the triangle?



An isosceles right-angled triangle has a hypotenuse of length 5cm as shown. What is the area of the triangle?

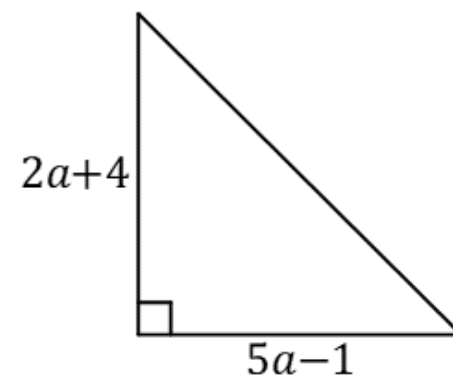


Triangle area



A right-angled triangle has perpendicular side lengths of $\sqrt{3} + 3$ and $5 - \sqrt{3}$ centimetres.

What is the exact area of the triangle?



A right-angled triangle with perpendicular side lengths of $2a + 4$ and $5a - 1$ centimetres has an area of 168cm^2 .

What is the length of the shortest side?

Find the area of a rectangle whose dimensions are $\sqrt{8}cm$ by $\sqrt{50}cm$.

$\sqrt{8}$ and $\sqrt{50}$ are 2 sides in a right-angled triangle. Find 2 possible answers for the length of the third side.

Two circles have radii $\sqrt{8}$ and $\sqrt{50}$ respectively. The ratio of their areas can be written in the form 1:n where n is a fraction. Find n.

A special ruler measures in roots. The width of a rectangle was found to be exactly $\sqrt{8}cm$ long. Another rectangle had a width of $\sqrt{50}cm$ long. What is the scale factor of enlargement between the two rectangles?

Write down an estimate for the number below.

$$\sqrt{40}$$

Write down the two consecutive integers that the number below falls between.

$$\sqrt{40}$$

Write the number below in the form $a\sqrt{b}$, where a and b are integers.

$$\sqrt{40}$$

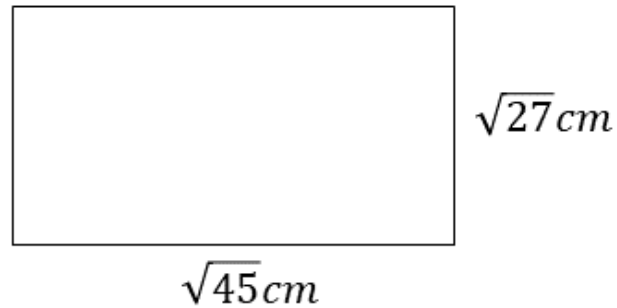
The number below is the hypotenuse of a right angled triangle.

The other two sides of the triangle are integer lengths.

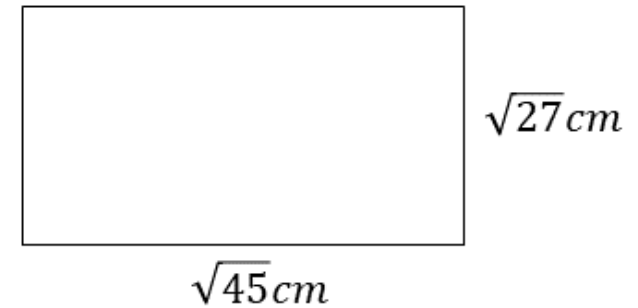
Write down the lengths of the other two sides.

$$\sqrt{40}$$

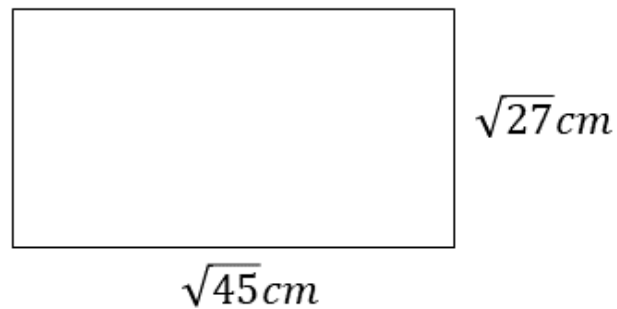
Calculate the area of this shape?



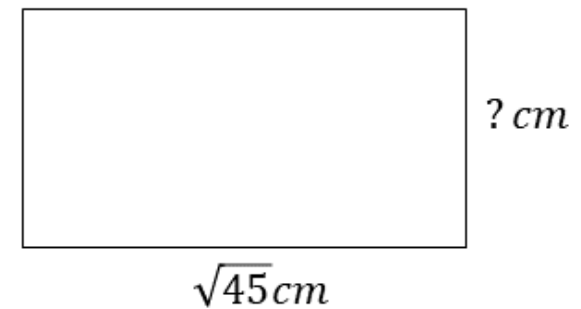
Calculate the perimeter of this shape?



Calculate the length of the longest diagonal.



The length of the longest diagonal is $\sqrt{72} cm$.
Calculate the length of the missing side.



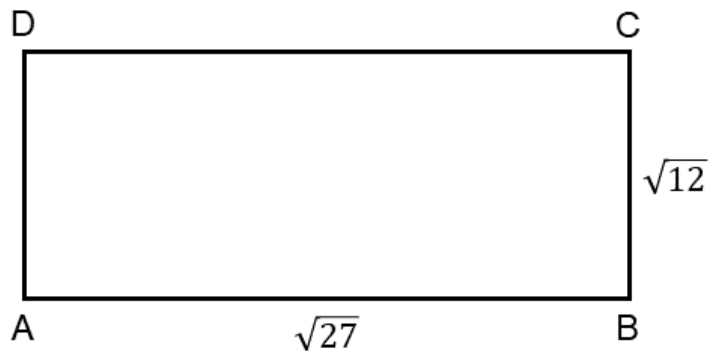
Calculate the perimeter of the rectangle.



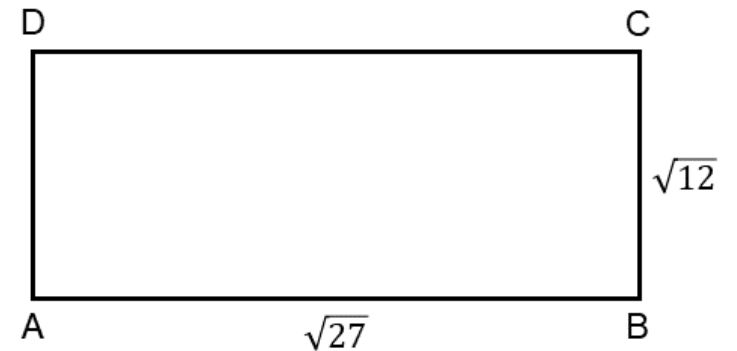
Calculate the area of the rectangle.

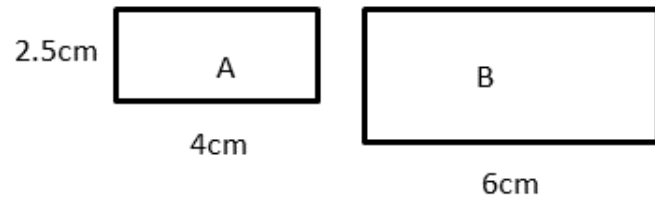


Calculate the length AC.

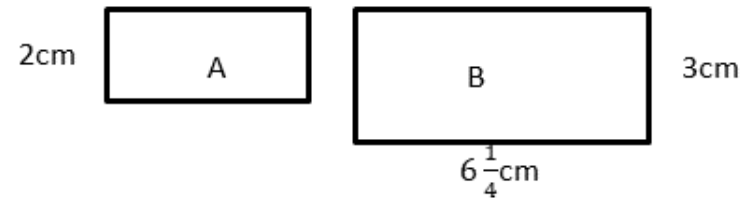


Write the ratio of AB:BC in its simplest form.

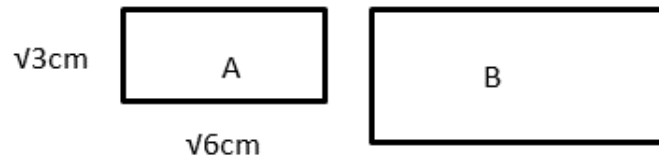




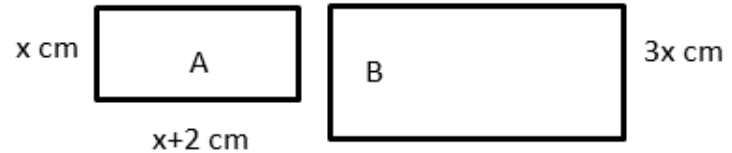
These rectangles are similar. Find the area of B.



These rectangles are similar. Find the length of rectangle A.



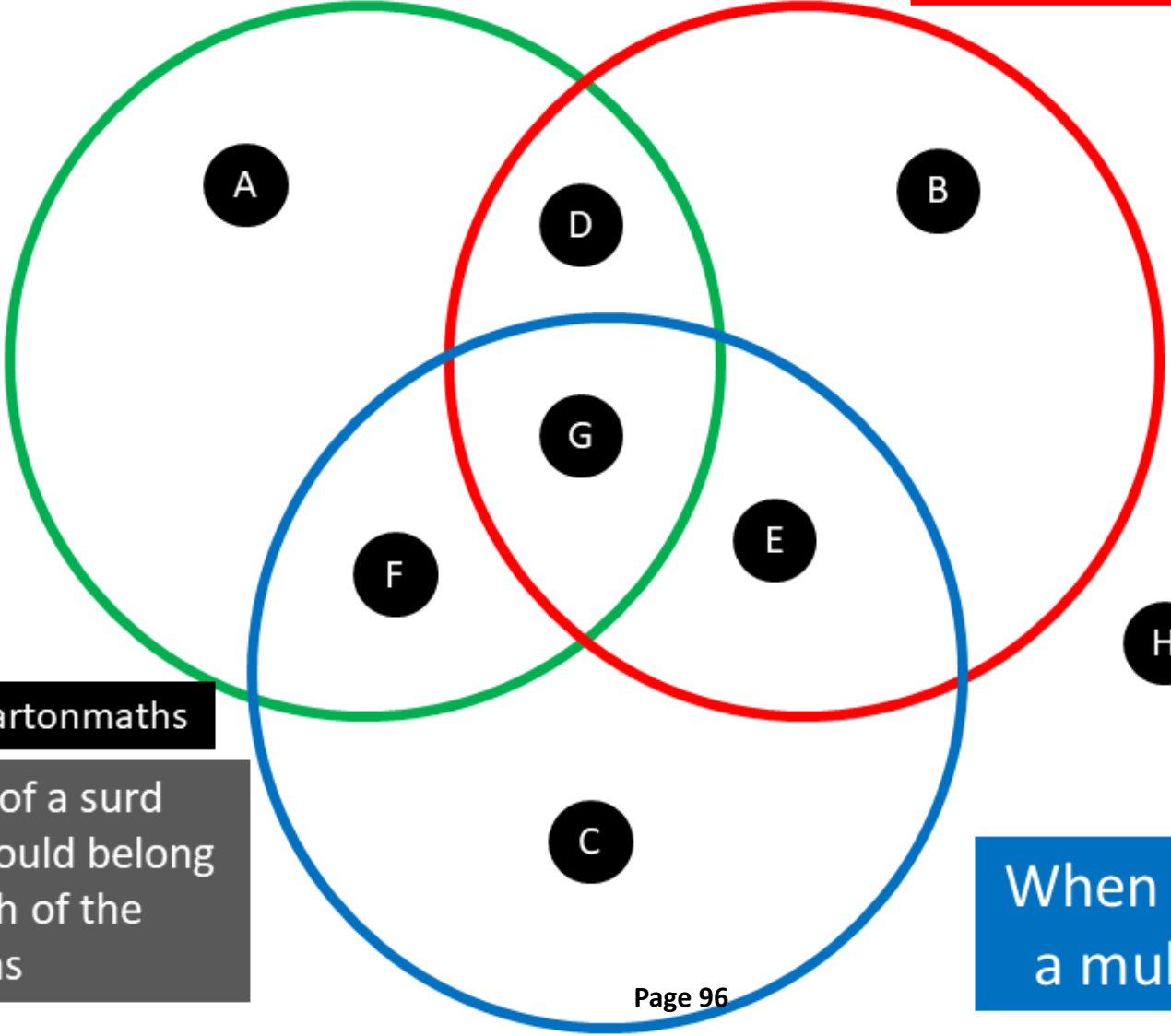
These rectangles are similar. The scale factor is 4. Find the area of B in its simplest form.



These rectangles are similar. A has an area of 35 cm^2 . Find the length of rectangle B.

Can only be simplified in one way

When squared is a factor of 36



If you think a region is impossible to fill, convince me why!

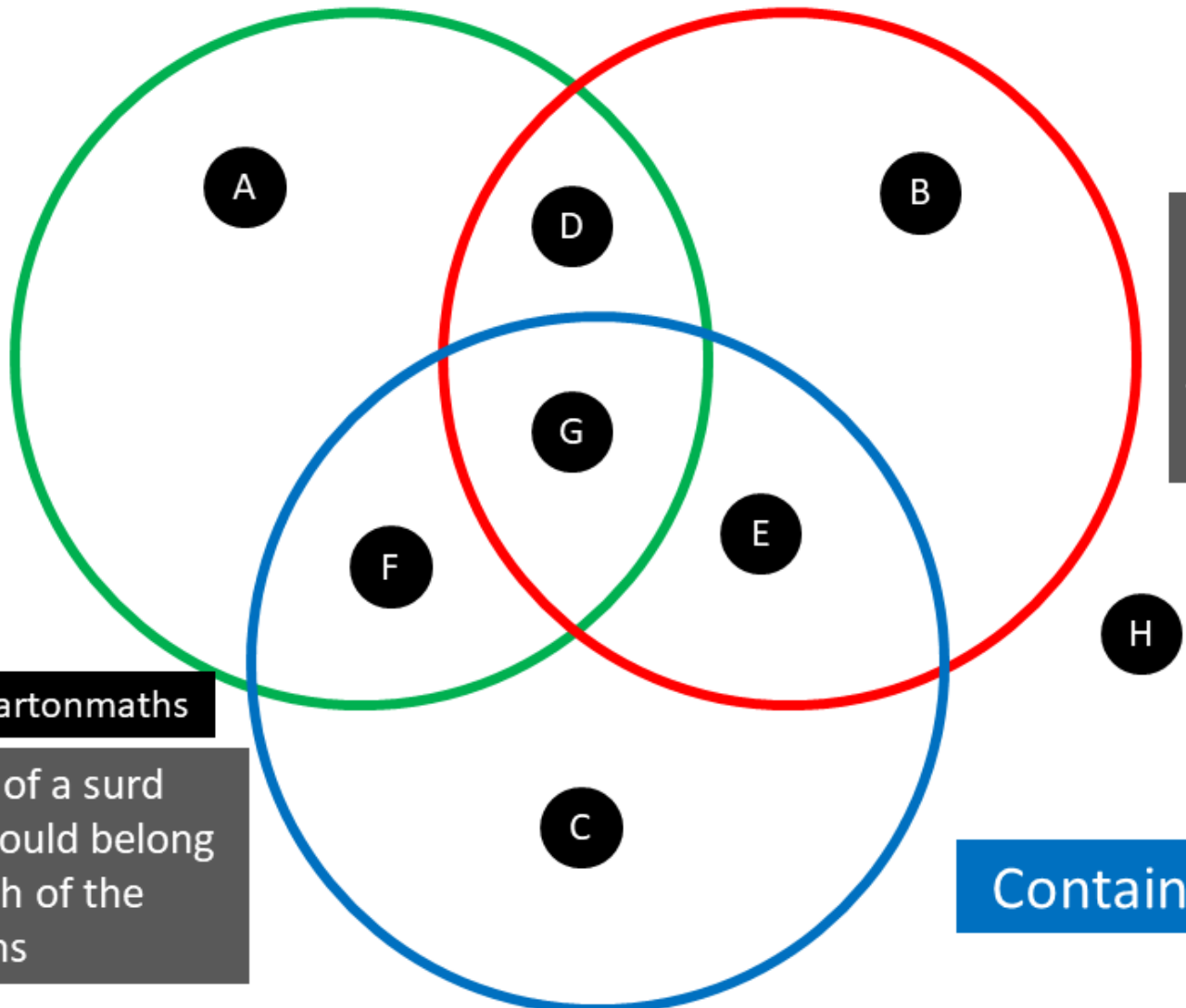
@mrbartonmaths

Think of a surd that could belong in each of the regions

When squared is a multiple of 3

Cannot be simplified

Contains the digit 4



If you think a region is impossible to fill, convince me why!

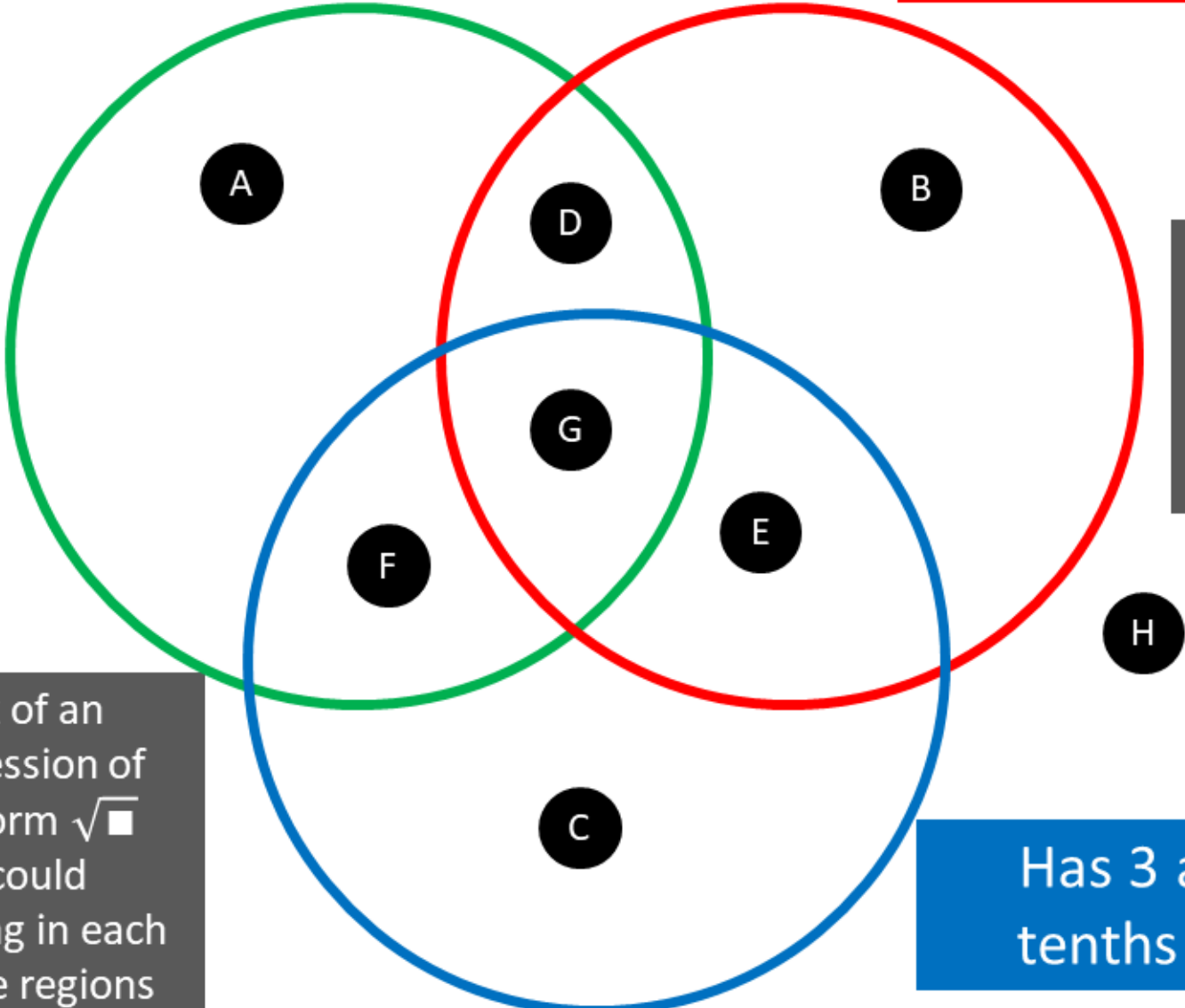
@mrbartonmaths

Think of a surd that could belong in each of the regions

Contains 3 digits

Less than 7

Decimal terminates



If you think a region is impossible to fill, convince me why!

Think of an expression of the form $\sqrt{\square}$ that could belong in each of the regions

Has 3 as its tenths digit

Challenge

Show clearly that the mean and median of $\sqrt{3}$, $\sqrt{12}$, $\sqrt{48}$ and $\sqrt{75}$ are equal.

Simplify

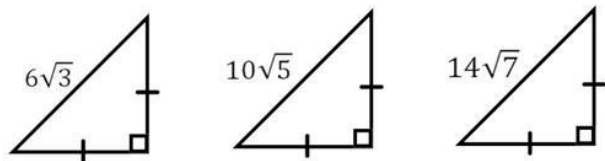
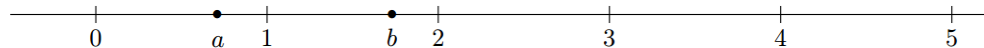
$$\sqrt{1 + \frac{1}{2}} \sqrt{1 + \frac{1}{3}} \sqrt{1 + \frac{1}{4}} \sqrt{1 + \frac{1}{5}} \sqrt{1 + \frac{1}{6}} \sqrt{1 + \frac{1}{7}}$$

Complete the multiplication table.

×		
	$2\sqrt{6}$	$\sqrt{6} - 2$
	$2\sqrt{6} + 6$	1

On the number line shown below, a is a number between 0 and 1, and b is a number between 1 and 2. Mark possible positions on this line for

$$\sqrt{a}, \sqrt{b}, a^2, b^2 \text{ and } \sqrt{\frac{b}{a}}$$



These triangles all share a rather wonderful property. Find the shorter sides and you'll see!

Can you find other triangles with this property?

Find the sum of

$$\frac{1}{\sqrt{1+\sqrt{2}}} + \frac{1}{\sqrt{2+\sqrt{3}}} + \text{and so on up to } + \frac{1}{\sqrt{99+\sqrt{100}}}$$

Fluency Practice

1) $\frac{3}{12}$

6) $\frac{12x}{6x}$

11) $\frac{10xy}{12xy^2}$

2) $\frac{3x}{12}$

7) $\frac{12xy}{6x}$

12) $\frac{10x^2y}{12xy^2}$

3) $\frac{3}{12x}$

8) $\frac{6x}{12xy}$

13) $\frac{10x^2y^2}{12xy^2}$

4) $\frac{12x}{3}$

9) $\frac{10x}{12xy}$

14) $\frac{10x^2y^3}{12xy^2}$

5) $\frac{12x}{6}$

10) $\frac{10x}{12xy^2}$

15) $\frac{100x^2y^4}{120xy^2}$

Fluency Practice

$$1) \frac{2(x+3)}{3(x+3)}$$

$$6) \frac{(x+3)(x+4)}{2x+8}$$

$$11) \frac{2x^2+14x+24}{3x^2-15x-108}$$

$$2) \frac{2(x+3)}{(x+3)}$$

$$7) \frac{x^2+7x+12}{2x+8}$$

$$12) \frac{2x^2+14x+24}{3x^2+4x-15}$$

$$3) \frac{(x+3)}{2(x+3)}$$

$$8) \frac{x^2+7x+12}{(x+4)(x-9)}$$

$$13) \frac{14x-24-2x^2}{3x^2+4x-15}$$

$$4) \frac{x+3}{2x+6}$$

$$9) \frac{x^2+7x+12}{x^2-5x-36}$$

$$14) \frac{14x-24-2x^2}{3x^2-4x-15}$$

$$5) \frac{(x+3)(x+4)}{2x+6}$$

$$10) \frac{2x^2+14x+24}{x^2-5x-36}$$

$$15) \frac{14x-24-2x^2}{4x+15-3x^2}$$

Fluency Practice

Question 3: Simplify the following algebraic fractions

- (a) $\frac{(x+6)(x+3)}{(x+3)}$ (b) $\frac{(x-1)(x+1)}{(x-1)}$ (c) $\frac{(x-3)}{(x-4)(x-3)}$
- (d) $\frac{(x+7)^2}{(x+7)}$ (e) $\frac{(x-3)(x+2)}{(x+2)(x+9)}$ (f) $\frac{(x+2)(x+4)^2}{(x+4)}$
- (g) $\frac{(x+1)(x+2)(x+3)}{(x+2)(x+3)(x+4)}$ (h) $\frac{x(x+3)^2}{x(x+1)(x+3)}$

Question 4: Simplify the following algebraic fractions

- (a) $\frac{x^2 + 5x + 4}{x^2 + 4x + 3}$ (b) $\frac{x^2 + 6x + 9}{x^2 - 2x - 15}$ (c) $\frac{x^2 - 2x}{x^2 + 2x - 8}$
- (d) $\frac{x^2 - 7x + 10}{x^2 + 3x - 10}$ (e) $\frac{x^2 + 8x + 15}{x^2 - x - 12}$ (f) $\frac{x^2 + 13x + 40}{x^2 + 14x + 48}$
- (g) $\frac{x^2 - 2x - 8}{x^2 + 6x - 40}$ (h) $\frac{x^2 + 10x + 24}{x^2 - 36}$ (i) $\frac{x^2 + 4x - 45}{x^2 + 10x + 9}$
- (j) $\frac{x^2 + 11x}{x^2 - 121}$ (k) $\frac{x^2 - 1}{x^2 + x}$ (l) $\frac{x^2 - 15x + 44}{x^2 - 16}$
- (m) $\frac{x^2 - x - 6}{x^2 - 2x - 3}$

Question 5: Simplify the following algebraic fractions

- (a) $\frac{3x^2 + 7x + 4}{x^2 + 3x + 2}$ (b) $\frac{x^2 - 2x - 8}{3x^2 + 7x + 2}$ (c) $\frac{5x^2 - 13x - 6}{x^2 - 9}$
- (d) $\frac{2x^2 + 3x - 2}{2x^2 - 15x + 7}$ (e) $\frac{9x^2 - 1}{3x^2 - 13x + 4}$ (f) $\frac{x^2 + 17x + 70}{5x^2 + 38x + 21}$
- (g) $\frac{3x^2 + 5x - 12}{12x^2 - 19x + 4}$ (h) $\frac{3x^2 + 11x + 6}{9x^2 + 21x + 10}$ (i) $\frac{4x^2 + x - 3}{4x^2 + 9x + 5}$
- (j) $\frac{9x^2 - 30x + 25}{6x^2 + 5x - 25}$ (k) $\frac{10x^2 - 23x + 12}{4x^2 + 4x - 15}$ (l) $\frac{20x^2 + 21x + 4}{16x^2 - 1}$

EXTENSION:

simplify

$$\frac{(a-b)^2 + 2b(a-b)}{a+b} + b$$

$$\frac{(a+b)^2 + b(a-b)}{a+3b}$$

EXTENSIONS

tough algebra questions

(1) simplify $(a + b)^2 - (a - b)^2$

(2) expand $(a - b)(a^2 + ab + b^2)$

(3) expand (i) $(n - 1)(n^2 + n + 1)$
 (ii) $(n - 1)(n^3 + n^2 + n + 1)$

(4) show that $(n + 1)(n + 2) + (n + 2)^2 - (n + 1)(n + 6)$ is a perfect square

(5) if $\frac{1}{2}(\frac{2}{3}(2a + b) + a - 3b) = \frac{1}{4}(a + 2b)$ show that $b = \frac{7}{8}a$

(6) simplify $\frac{a^2b + a}{b^2a + b}$

(7) simplify $\frac{k^2 - 7k + 10}{k^2 - 5k}$

(8) simplify $\frac{d^2 - 8d + 15}{2d^2 - 7d - 15}$

(9) if $a = t - \frac{1}{t}$ and $b = t + \frac{1}{t}$ show that $b^2 - a^2 = 4$

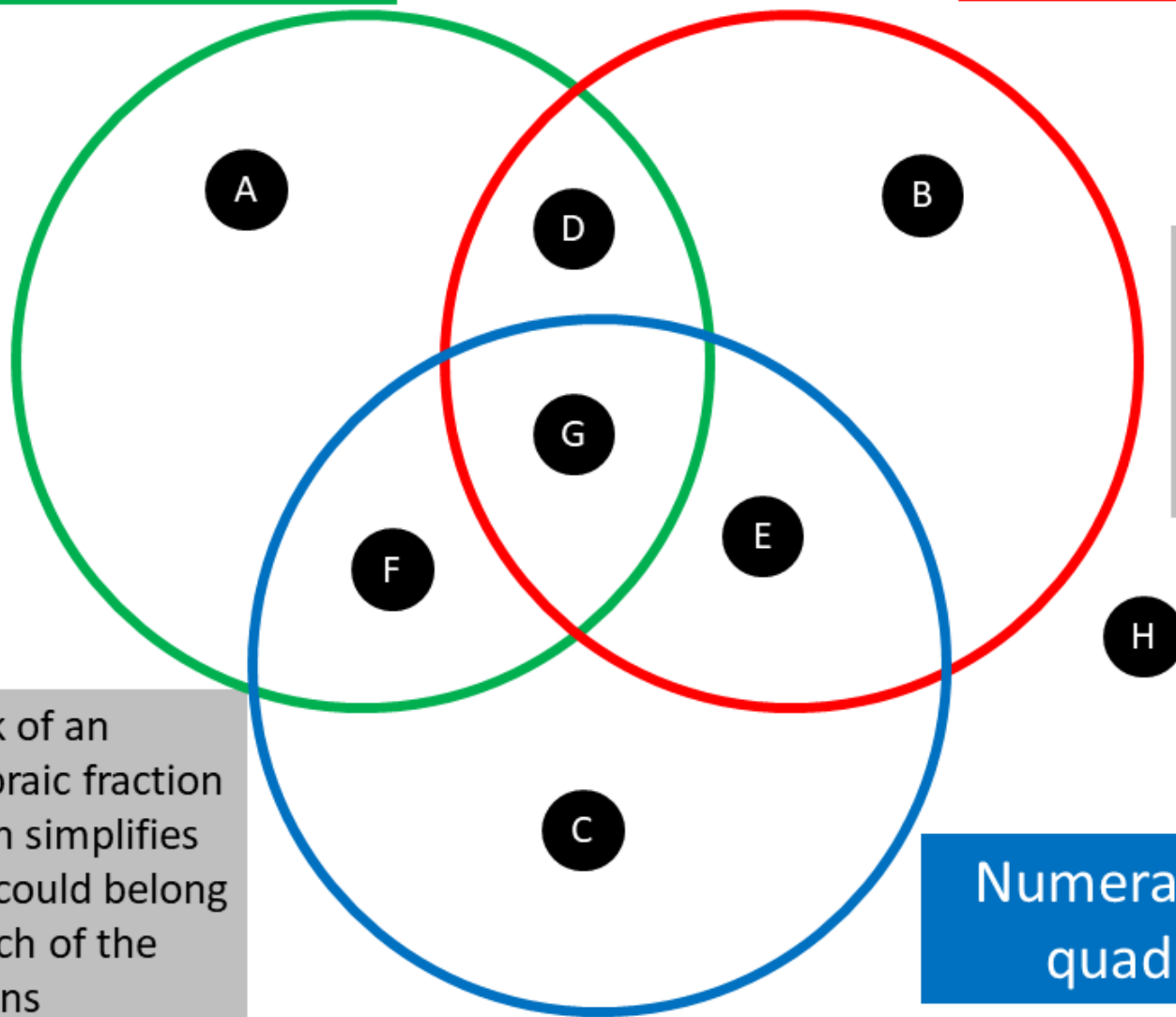
(10) if $a = \frac{b}{b + c}$ show that $\frac{a}{1 - a} = \frac{b}{c}$

Use the digits 0-9 (at most one time each) to fill in the boxes below so that the rational expression on the left simplifies to the rational expression on the right.

$$\frac{x^2 + \boxed{}x + \boxed{}\boxed{}}{x^2 - \boxed{}\boxed{}} = \frac{x + \boxed{}}{x - \boxed{}}$$

Denominator is
 $x^2 + 6x + 8$

Equivalent to $\frac{1}{x+4}$



If you think a region is impossible to fill, convince me why!

Think of an algebraic fraction which simplifies that could belong in each of the regions

Numerator is a quadratic

Fluency Practice

Question 1: Express the following as a single fraction.

(a) $\frac{2}{g} \times \frac{3}{h}$

(b) $\frac{3}{c} \times \frac{a}{4}$

(c) $\frac{w}{x} \times \frac{3}{a}$

(d) $\frac{3a}{7} \times \frac{2c}{9}$

(e) $\frac{a}{e} \times \frac{f}{b}$

(f) $\frac{e}{8} \times \frac{d}{8}$

(g) $\frac{x}{2} \times \frac{x}{5}$

(h) $\frac{7}{y} \times \frac{2}{y}$

(i) $\frac{3}{w} \times \frac{x}{4} \times \frac{y}{w}$

(j) $\frac{2x}{5} \times \frac{3x}{7}$

(k) $\frac{x}{y} \times \frac{x}{y}$

(l) $\frac{6a}{7c} \times \frac{5a}{c}$

Question 2: Express the following as a single **simplified** fraction.

(a) $\frac{2x}{y} \times \frac{y}{4}$

(b) $\frac{3a}{c} \times \frac{5}{6}$

(c) $\frac{4}{5a} \times \frac{5w}{8}$

(d) $\frac{3a}{7} \times \frac{2c}{9}$

(e) $\frac{10g}{w} \times \frac{w}{5}$

(f) $\frac{4x}{5y} \times \frac{3y}{8x}$

(g) $\frac{2y}{3} \times \frac{2y}{wy}$

(h) $\frac{6x}{5y} \times \frac{4x}{3y}$

(i) $\frac{x^2}{a} \times \frac{a^2}{x^2}$

(j) $\frac{ab}{c} \times \frac{c}{ae}$

(k) $\frac{6c}{w^2} \times \frac{15w^3}{2c^2}$

(l) $\frac{2a^4}{3b^3} \times \frac{6b^2}{5a}$

(m) $\frac{2a^3b}{3} \times \frac{6}{ab^2}$

(n) $\frac{x^4y^4}{z^2} \times \frac{z}{x^6y}$

(o) $\frac{14a^2bc^3}{9} \times \frac{6b^3}{21a^3c}$

BRONZE**SILVER****GOLD**

1. $\frac{3}{5} \times \frac{x}{y}$

2. $\frac{x}{y} \times \frac{3}{5}$

3. $\frac{x}{y} \times \frac{3}{6}$

4. $\frac{x}{y} \times \frac{3}{6x}$

5. $\frac{x}{xy} \times \frac{3}{6}$

6. $\frac{7x}{xy} \times \frac{3}{6}$

1. $\frac{7x}{xy} \times \frac{3}{6y}$

2. $\frac{7x}{xy} \div \frac{3}{6y}$

3. $\frac{7x}{xy} \div \frac{6y}{3}$

4. $\frac{7x}{xy} \times \frac{6y}{3}$

5. $\frac{7x}{xy} \times \frac{6y}{3} \times \frac{4y}{14}$

6. $\frac{7x}{xy} \div \frac{6y}{3} \times \frac{4y}{14}$

1. $\frac{7x}{xy} \div \frac{6y}{3} \div \frac{4y}{14}$

2. $\frac{7x^2}{xy} \div \frac{6y}{3} \div \frac{4y}{14}$

3. $\frac{7x^2}{xy} \div \left(\frac{6y}{3} \div \frac{4y}{14} \right)$

4. $\frac{7x^2}{xy} \div \left(\frac{6y}{3} \times \frac{4y}{14} \right)$

5. $\frac{7x^2}{xy} \div \left(\frac{6y}{9} \times \frac{4y}{14} \right)$

Fluency Practice

Simplify the following as far as possible:

- 1) $\frac{5}{x^2+x-2} \times \frac{x-1}{2}$
- 2) $\frac{9}{x^2-4x+3} \times \frac{x-3}{2}$
- 3) $\frac{7}{x-2} \div \frac{8}{x^2-x-2}$
- 4) $\frac{3}{x-4} \div \frac{8}{x^2+x+20}$

Simplify the following as far as possible:

- 1) $\frac{x^2-4x+3}{4x} \times \frac{3}{x^2-2x+1}$
- 2) $\frac{x^2+3x-4}{7x} \times \frac{3x}{x^2+7x+12}$
- 3) $\frac{x^2-2x-3}{2x} \div \frac{3}{x^2-7x+12}$
- 4) $\frac{x^2+4x+3}{3x} \div \frac{3}{x^2+3x+2}$

Simplify the following as far as possible:

- 1) $\frac{x^2-2x+1}{x^2-1} \times \frac{x^2+4x+3}{x^2+2x-3}$
- 2) $\frac{x^2+4x-5}{x^2-x-2} \times \frac{x^2-x-2}{x^2+6x+5}$
- 3) $\frac{x^2+4x-5}{x^2+x-2} \div \frac{x^2+10x+25}{x^2+8x+15}$
- 4) $\frac{x^2+x-2}{x^2-2x-8} \div \frac{x^2-4x+3}{x^2-6x+9}$

Fluency Practice

Question 3: Express the following as a single fraction. **Simplify** if possible.

(a) $\frac{x}{4} \times \frac{x-3}{2}$

(b) $\frac{x}{9} \times \frac{6}{x+7}$

(c) $\frac{x+1}{15} \times \frac{5}{x}$

(d) $\frac{1}{x+3} \times \frac{2}{x+1}$

(e) $\frac{3x+2}{3} \times \frac{x+1}{3}$

(f) $\frac{x+4}{x-4} \times \frac{x-2}{x+5}$

(g) $\frac{x+1}{x-7} \times \frac{x-5}{x+1}$

(h) $\frac{7}{2x+8} \times \frac{x+4}{14}$

(i) $\frac{4}{2x-1} \times \frac{6x-3}{x+7}$

(j) $\frac{x+8}{15} \times \frac{10}{x^3+8x^2}$

(k) $\frac{4}{x-2} \times \frac{x^2-2x}{8}$

(l) $\frac{x^2+5x+6}{4} \times \frac{2}{x+2}$

(m) $\frac{x^2+2x-8}{x^2+5x+6} \times \frac{x+2}{x+4}$

(n) $\frac{x^2+x-6}{x^2-25} \times \frac{x^2+10x+25}{x^2-4}$

(o) $\frac{3x^2+8x-3}{25} \times \frac{30}{6x^2+13x-5}$

BRONZE

$$1. \frac{3x}{2} + \frac{4x}{5}$$

$$2. \frac{x}{2} + \frac{4x}{5}$$

$$3. \frac{x}{2} + \frac{x}{5}$$

$$4. \frac{x}{2} - \frac{x}{5}$$

$$5. \frac{x}{5} - \frac{x}{2}$$

SILVER

$$1. \frac{5x}{5} - \frac{x}{2}$$

$$2. \frac{5}{x} + \frac{2}{x}$$

$$3. \frac{5}{x^2} + \frac{2}{x}$$

$$4. \frac{5}{2x^2} - \frac{2}{x}$$

$$5. \frac{6}{2x^2} - \frac{2}{x}$$

GOLD

$$1. \frac{6}{2xy} + \frac{2}{x}$$

$$2. \frac{6}{2xy} + \frac{2}{3x}$$

$$3. \frac{6}{2xy} - \frac{2}{3x^2}$$

$$4. \frac{6y}{2xy} + \frac{2}{3x^2}$$

$$5. \frac{y}{2xy} - \frac{2}{3x^3}$$

Question 1: Express the following as a single simplified fraction.

(a) $\frac{x}{3} + \frac{x}{5}$

(b) $\frac{c}{2} + \frac{c}{7}$

(c) $\frac{w}{3} + \frac{w}{9}$

(d) $\frac{x}{2} - \frac{x}{3}$

(e) $\frac{a}{5} - \frac{a}{9}$

(f) $\frac{m}{2} - \frac{m}{8}$

(g) $\frac{m}{3} + \frac{2m}{7}$

(h) $\frac{3x}{5} + \frac{x}{2}$

(i) $\frac{3c}{4} + \frac{5c}{9}$

(j) $\frac{m}{2} - \frac{2m}{5}$

(k) $\frac{3n}{4} - \frac{5n}{9}$

(l) $\frac{7h}{8} - \frac{5h}{12}$

Question 2: Express the following as a single simplified fraction.

(a) $\frac{2}{x^2} + \frac{5}{x}$

(b) $\frac{5}{6x} - \frac{1}{3x}$

(c) $\frac{2}{fg} - \frac{4}{9}$

(d) $\frac{6}{ac} + \frac{2}{3}$

(e) $\frac{9}{w} + \frac{wx}{4}$

(f) $\frac{d}{3} + \frac{2}{d^2}$

(g) $\frac{m^2}{6} - \frac{9}{4m}$

(h) $\frac{3}{4b^2} - \frac{1}{2b}$

(i) $\frac{ac}{5} + \frac{4}{c}$

(j) $\frac{x^3}{w} - \frac{2}{wx^2}$

(k) $\frac{2}{ab^2} - \frac{3}{b^3}$

(l) $\frac{xy}{5} - \frac{1}{x}$

Question 3: Express the following as a single simplified fraction.

(a) $\frac{x+4}{3} + \frac{x+1}{2}$

(b) $\frac{2x+1}{2} + \frac{x+3}{5}$

(c) $\frac{5x-2}{3} + \frac{2x+7}{4}$

(d) $\frac{3x+11}{2} + \frac{x-20}{3}$

(e) $\frac{5x-4}{2} + \frac{x+1}{3}$

(f) $\frac{x-4}{8} + \frac{2x-3}{2}$

(g) $\frac{7x+4}{2} - \frac{x+1}{3}$

(h) $\frac{9x+1}{5} - \frac{x+2}{2}$

(i) $\frac{3x+11}{2} - \frac{2x-3}{6}$

(j) $\frac{x-8}{3} - \frac{x-5}{7}$

(k) $\frac{4x-7}{10} - \frac{2x-9}{3}$

(l) $\frac{5-x}{6} - \frac{8-3x}{10}$

Question 4: Express the following as a single simplified fraction.

(a) $\frac{2}{x+5} + \frac{3}{x+1}$

(b) $\frac{2}{x+1} + \frac{1}{x+3}$

(c) $\frac{4}{x+5} - \frac{2}{x-1}$

(d) $\frac{x+1}{x-2} + \frac{x+3}{x+5}$

(e) $\frac{x+3}{2x+1} - \frac{x-2}{x-1}$

(f) $\frac{x}{x+7} + \frac{2x+5}{3x+1}$

(g) $\frac{3}{x+1} + \frac{x+7}{(x+1)(x+2)}$

(h) $\frac{1-x}{(x-7)(x+1)} - \frac{2}{x-7}$

Simplify the following as far as possible:

$$1) \frac{6}{2xy} + \frac{2}{x}$$

$$2) \frac{6}{2xy} + \frac{2}{3x}$$

$$3) \frac{6}{2xy} - \frac{2}{3x^2}$$

$$4) \frac{6y}{2xy} + \frac{2}{3x^2}$$

$$5) \frac{y}{2xy} - \frac{2}{3x^3}$$

Simplify the following as far as possible:

$$1) \frac{1}{x^2-4} - \frac{1}{x-2}$$

$$2) \frac{x}{x^2-4} - \frac{1}{x-2}$$

$$3) \frac{x}{x^2-x-2} - \frac{1}{x-2}$$

$$4) \frac{x-3}{x^2-x-2} - \frac{1}{x-2}$$

$$5) \frac{x-3}{x^2-x-2} - \frac{2x}{x-2}$$

Simplify the following as far as possible:

$$1) \frac{2x+1}{x+4} - \frac{x-5}{x-2}$$

$$2) \frac{11x+27}{2x^2+11x-6} - \frac{3}{x+6}$$

Challenge- Express the following as a fraction in its simplest form:

$$\frac{3}{x-1} + \frac{2}{x+1} + \frac{4}{x-3}$$

Fluency Practice

Q1

Solve the following equations.

$$[\text{a}] \frac{x+3}{4} + \frac{x+4}{5} = 2$$

$$[\text{e}] \frac{x+2}{2} + \frac{7x-7}{4} = 6$$

$$[\text{b}] \frac{x-1}{2} + \frac{x+3}{6} = 4$$

$$[\text{f}] \frac{x+5}{3} + \frac{2x+3}{4} = 2$$

$$[\text{c}] \frac{x-1}{4} - \frac{x+1}{6} = 1$$

$$[\text{g}] \frac{x+3}{6} + \frac{3x+8}{5} = 9$$

$$[\text{d}] \frac{x+2}{2} + \frac{x+3}{3} = 1$$

$$[\text{h}] \frac{x+2}{6} + \frac{7x+6}{2} = 6$$

Q2

Solve the following equations.

$$[\text{a}] \frac{3x+1}{2} + \frac{5x+7}{6} = 2$$

$$[\text{e}] \frac{8x-1}{5} + \frac{4x-5}{2} = 3$$

$$[\text{b}] \frac{6x-1}{3} - \frac{3x+1}{7} = 10$$

$$[\text{f}] \frac{3x-2}{6} + \frac{5x+6}{7} = 7$$

$$[\text{c}] \frac{2x+5}{2} - \frac{3x+1}{5} = 2$$

$$[\text{g}] \frac{9x+8}{6} + \frac{4x+3}{5} = 5$$

$$[\text{d}] \frac{2x+4}{7} + \frac{2x-3}{2} = 1$$

$$[\text{h}] \frac{2x+7}{6} + \frac{2x-8}{3} = 5$$

Fluency Practice

Question 5: Solve the following equations

(a) $\frac{x+5}{3} + \frac{x+1}{2} = 8$

(b) $\frac{x+6}{5} + \frac{x-3}{2} = 13$

(c) $\frac{2x-1}{4} + \frac{x-3}{5} = 3$

(d) $\frac{x+5}{10} - \frac{x+4}{2} = 1$

(e) $\frac{x-3}{4} - \frac{x-8}{3} = 4$

Fluency Practice

Q3

Solve the following equations.

$$[a] \frac{2}{x+6} + \frac{4}{x+8} = 2$$

$$[b] \frac{4}{x-5} + \frac{2}{x-8} = 3$$

$$[c] \frac{2}{x-9} + \frac{4}{x-7} = 2$$

$$[d] \frac{3}{x-1} + \frac{2}{x-3} = 1$$

$$[e] \frac{1}{2x-7} + \frac{6}{x+4} = 1$$

$$[f] \frac{1}{2x-5} + \frac{4}{x+2} = 1$$

$$[g] \frac{5}{2x-3} + \frac{2}{x-6} = 1$$

$$[h] \frac{2}{5x+2} + \frac{5}{x+4} = 1$$

Q4

Solve the following equations.

$$[a] \frac{4}{2x-1} + \frac{6}{4x-7} = 2$$

$$[b] \frac{5}{2x-6} + \frac{3}{2x+6} = 1$$

$$[c] \frac{6}{4x+1} + \frac{4}{2x+3} = 2$$

$$[d] \frac{2}{4x-9} + \frac{4}{2x-3} = 2$$

$$[e] \frac{6}{2x-3} + \frac{4}{3x-7} = 2$$

$$[f] \frac{3}{4x+7} + \frac{6}{2x+5} = 3$$

$$[g] \frac{3}{4x+9} + \frac{2}{2x+7} = 1$$

$$[h] \frac{3}{2x-1} + \frac{5}{2x-7} = 2$$

Question 6: Solve the following equations

$$(a) \frac{4}{x+1} + \frac{2}{x-2} = 3$$

$$(b) \frac{2}{x-3} + \frac{1}{x-4} = 2$$

$$(c) \frac{2}{x-5} - \frac{2}{x-4} = 1$$

$$(d) \frac{2}{x+1} - \frac{10}{x+4} = -1$$

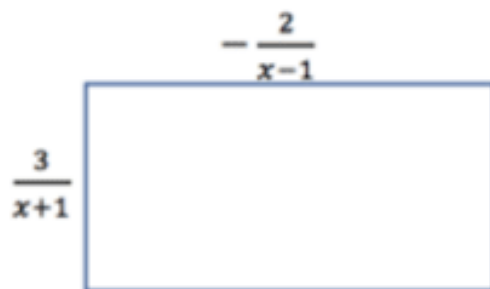
$$(e) \frac{3}{x-2} + \frac{3}{x+2} = 2$$

$$(f) \frac{11}{(x-1)(x+4)} + \frac{5}{x-1} = 1$$

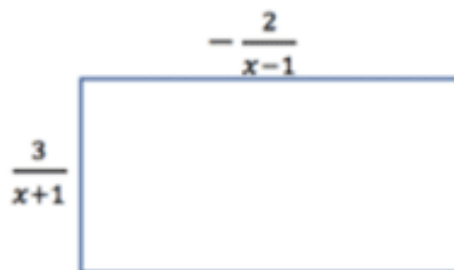
Fluency Practice

- 1.** When its average speed increases by 10 m.p.h. the time taken for a car to make a journey of 105 miles is reduced by 15 minutes. Find the original average speed of the car.
- 2.** Find the price of potatoes per kilogram if, when the price rises by 5 p per kg, I can buy 1 kg less for £2.10.
- 3.** Tickets are available for a concert at two prices, the dearer ticket being £3 more than the cheaper one. Find the price of each ticket if a youth group can buy ten more of the cheaper tickets than the dearer tickets for £180.
- 4.** In order to go on holiday Peter converts £300 into French francs. Had he bought three months earlier he would have received the same number of francs for £50 less, since the rate of exchange was two francs to the £ more than on the day he bought them. Find the present rate of exchange.
- 5.** From a piece of wire 42 cm long, a length $10x$ cm is cut off and bent into a rectangle whose length is one and a half times its width. The remainder is bent to form a square. If the combined area of the rectangle and square is 63 cm^2 find their dimensions.
- 6.** The members of a club hire a coach for the day at a cost of £210. Seven members withdraw which means that each member who makes the trip must pay an extra £1. How many members originally agreed to go?

- a) Find an expression for the **area** of the rectangle below:

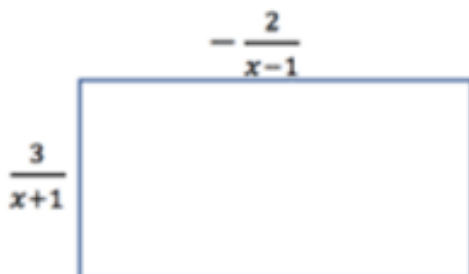


- b) Find a simplified expression for the **perimeter** of the rectangle below:

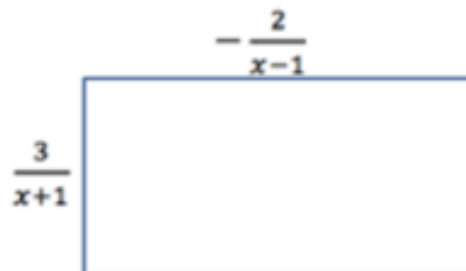


A Rectangle and 2 Fractions

- c) Find the value of x for which the length of the rectangle is the same as the width.



- d) Find the value of x for which the perimeter of the rectangle is 12.



Algebraic Fractions Revision

(a)	(b)	(c)	(d)
Simplify $\frac{x(x-5)}{x^2}$	Simplify $\frac{x^2+2x}{x^2+7x+10}$	Simplify $\frac{x^2-x-6}{x^2+6x+8}$	Simplify $\frac{3x^2-13x+4}{x^2-16}$
(e)	(f)	(g)	(h)
Simplify $\frac{x^2}{x-2} \times \frac{3x-6}{4x}$	Simplify $\frac{x(x-1)}{x^2+2x} \div \frac{x^2-1}{3x}$	Write as a single fraction $\frac{x}{3} + \frac{2x}{5}$	Write as a single fraction $\frac{5x}{6} - \frac{x}{4}$
(i)	(j)	(k)	(l)
Simplify fully $\frac{5}{x^2} + \frac{4}{3x}$	Simplify fully $\frac{3}{x+2} - \frac{2}{x-1}$	Write as a single fraction $\frac{2x-1}{5} + \frac{x+3}{4}$	Simplify fully $\frac{x}{2x+4} - \frac{3}{x^2+5x+6}$
(m)		(n)	
Solve $\frac{x}{3} + \frac{2x-1}{4} = 1$		Solve $\frac{2(x+1)}{5} = \frac{8+x}{6}$	
		(o)	
		Solve $\frac{5}{x+3} + \frac{3}{x-1} = 4$	