Mathematics
Unit 19

Name: _____

Class: _____

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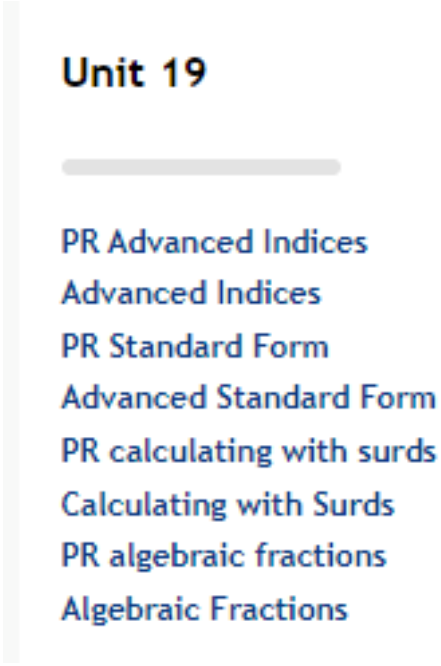
See unit 19 course on drfrostmaths.com



Indices Recap

Multiplication Law:
$y^a \times y^b = y^{a+b}$

Division Law:
$y^a \div y^b = y^{a-b}$

Power Law:
$(y^a)^b = y^{ab}$

Activity: can you come up with at least two INTERESTING examples and non-examples of each of the 3 rules

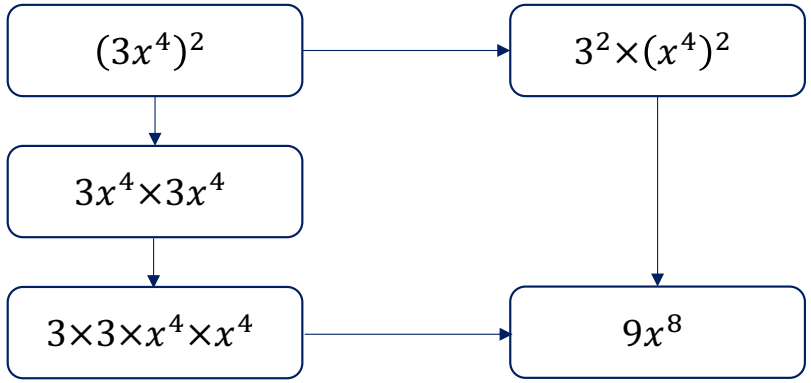
EXAMPLES	NON-EXAMPLES

Worked Example	Your Turn
Simplify: a) $y^{11} \times y^5$ b) $6y^3 \times 2y^5$ c) $y^5 \div y^2$ d) $8y^3 \div 2y$ e) $(y^3)^7$ f) $(3y^4)^2$	Simplify: a) $x^5 \times x^{-2}$ b) $7x^5 \times 8x^{-3}$ c) $y^5 \div y^4$ d) $15y^3 \div 3y$ e) $(y^7)^8$ f) $(5y^4)^3$

Worked Example	Your Turn
Simplify: a) $\frac{a^6 \times a^4}{a^2}$ b) $(4a^6b^3)^2$ c) $\frac{8a^5b^3}{4ab^7}$	Simplify: a) $\frac{a^6 \times a^{-4}}{a^2}$ b) $(2a^6b^3)^4$ c) $\frac{12a^2b^3}{4ab^7}$

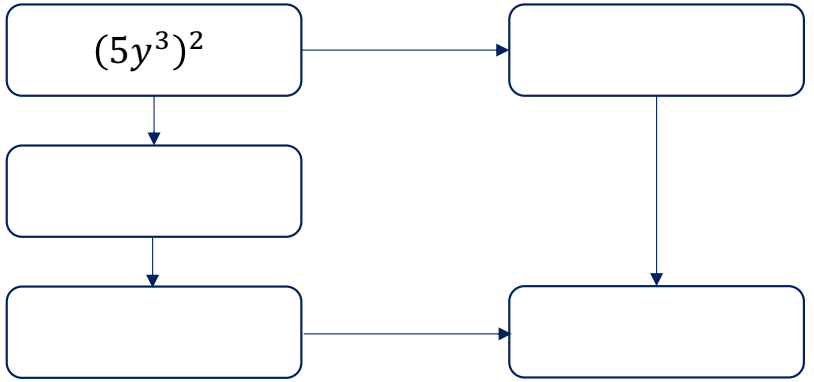
Task 1 – Raising a power to a power

Ex



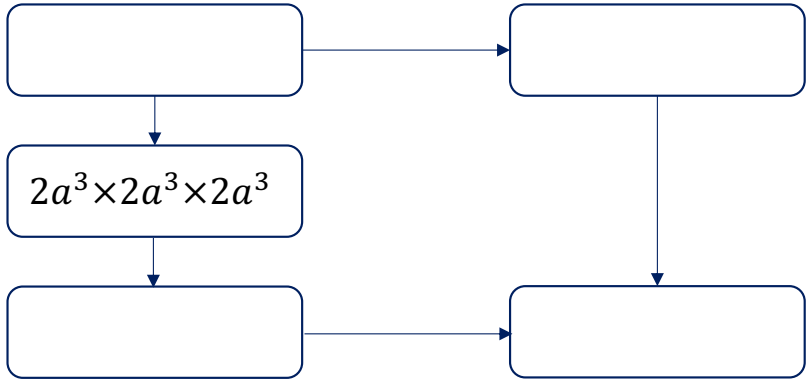
$(3x^4)^2 = 9x^8$

1



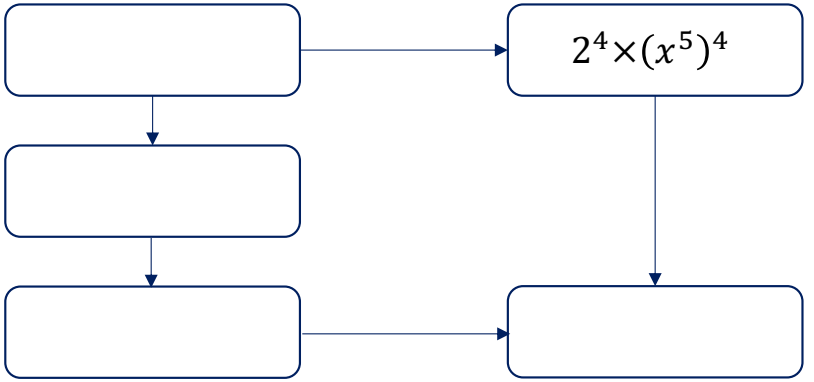
$(5y^3)^2 =$ ____

2



____ = ____

3



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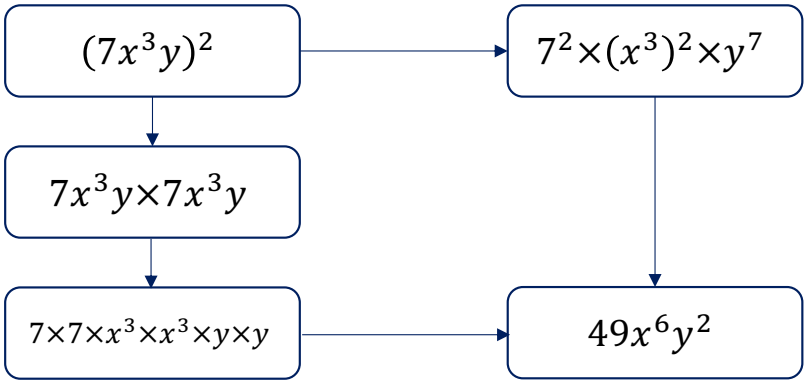
Extension – Can you create question/flowchart of your own?

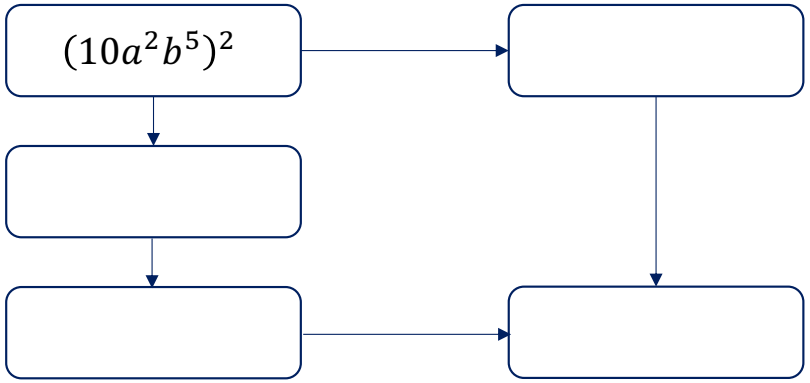
Task 2 – Raising a power to a power

Ex



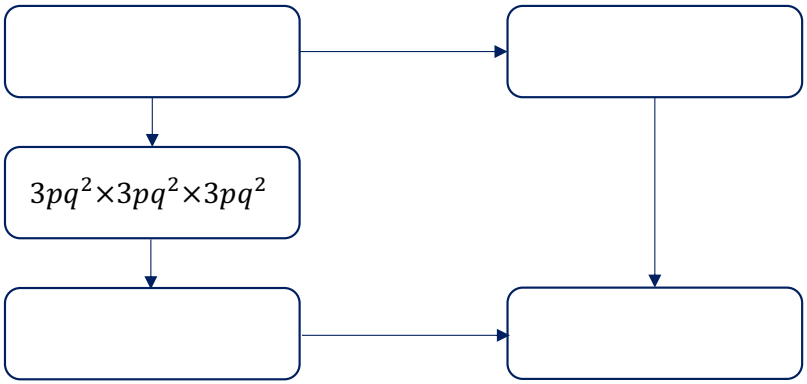
$(7x^3y)^2 = 49x^6y^2$

4



$(10a^2b^5)^2 =$ ____

5



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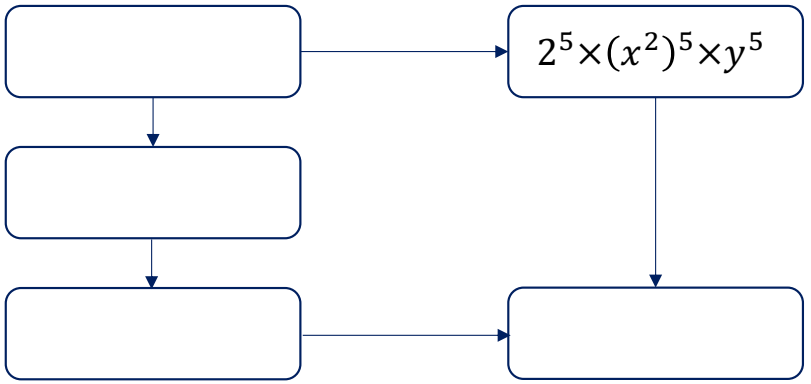
6



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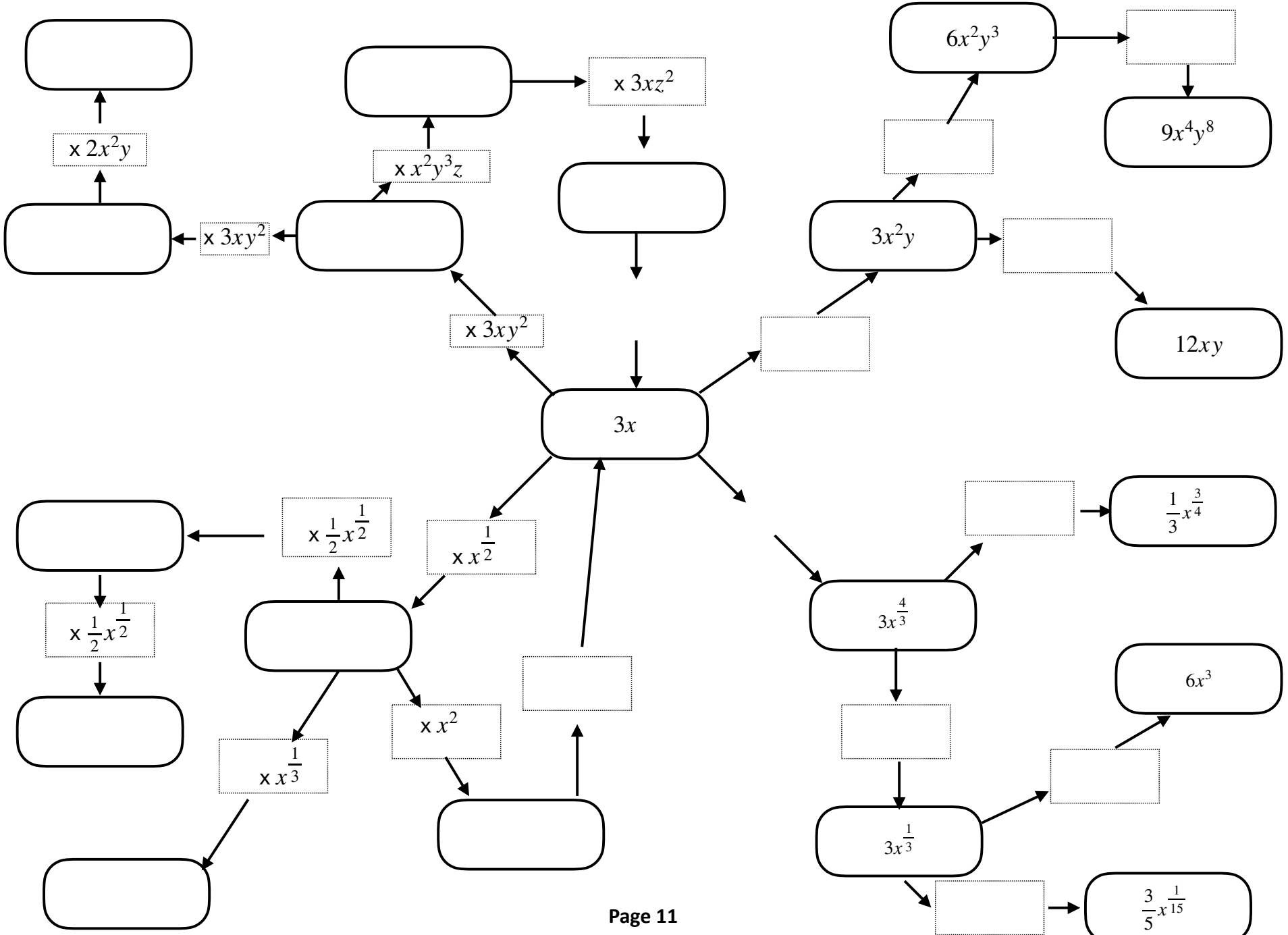
Extension – Can you create question/flowchart of your own?


x $2x^2y$
x $3xz^2$
x x^2y^3z
x $3xy^2$
$6x^2y^3$
$9x^4y^8$
$3x^2y$
$12xy$
x $3xy^2$
$3x$
x $\frac{1}{2}x^{\frac{1}{2}}$
x $x^{\frac{1}{2}}$
$\frac{1}{3}x^{\frac{3}{4}}$
x $\frac{1}{2}x^{\frac{1}{2}}$
$3x^{\frac{4}{3}}$
$6x^3$
x x^2
x $x^{\frac{1}{3}}$
$3x^{\frac{1}{3}}$
$\frac{3}{5}x^{\frac{1}{15}}$

The Power Zero

$2^4 = 16$ $2^3 = 8$ $2^2 = 4$ $2^1 = 2$ $2^0 = 1$

Any non-zero number divided by itself equals 1, i.e. $2 \div 2 = 1$

Using the exponent rule for division:

$$\frac{2^1}{2^1} = 2^{1-1} = 2^0 = 1$$

Worked Example	Your Turn
Simplify: a) $4x^0$ b) $x^4 \times x^0$ c) $\frac{x^9}{x^0}$ d) $x^0 \div x^{-2}$	Simplify: a) $8x^0$ b) $x^0 \times x^8$ c) $\frac{x^0}{x^{18}}$ d) $x^{-4} \div x^0$

Negative Indices

$2^4 = 16 \quad 2^3 = 8 \quad 2^2 = 4 \quad 2^1 = 2 \quad 2^0 = 1 \quad 2^{-1} = \frac{1}{2} \quad 2^{-2} = \frac{1}{4} \quad 2^{-3} = \frac{1}{8}$

$$\frac{2^3}{2^7} = \frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2 \times 2 \times 2 \times 2} = \frac{1}{2^4}$$

Using the exponent rule for division:

$$\frac{2^3}{2^7} = 2^{3-7} = 2^{-4}$$

Therefore

$$\frac{1}{2^4} = 2^{-4}$$

Worked Example	Your Turn
Evaluate: a) 3^{-2} b) -3^{-2} c) $(-3)^{-2}$	Evaluate: a) 5^{-3} b) -5^{-3} c) $(-5)^{-3}$

Fluency Practice

For the following terms,

a) Write with a positive exponent
b) Evaluate:

1.	26^{-1}	2.	2^{-1}	3.	10^{-2}	4.	2^{-2}
5.	-26^{-1}	6.	-2^{-1}	7.	-10^{-2}	8.	-2^{-2}
9.	$(-26)^{-1}$	10.	$(-2)^{-1}$	11.	$(-10)^{-2}$	12.	$(-2)^{-2}$
13.	2^{-5}	12.	-7^{-3}	13.	$(-8)^{-2}$	14.	$(-10)^{-5}$
15.	$(-4)^{-3}$	16.	9^{-4}	17.	-11^{-2}	18.	$(-3)^{-3}$
19.	-3^{-4}	20.	25^{-2}	21.	$(-2)^{-6}$	22.	15^{-2}

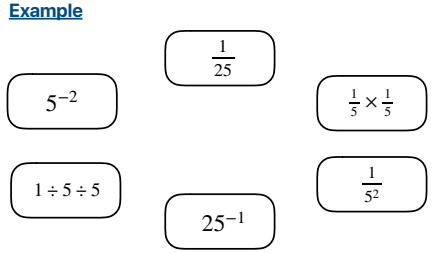
Decide if there are mistakes in the following and explain how to fix the answer:

a) $4^{-2} = -16$ b) $10^{-3} = \frac{1}{30}$

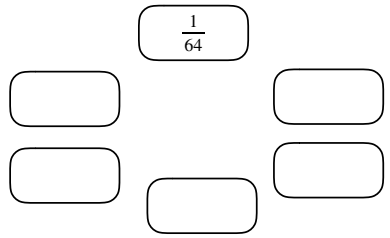
Extension

1 Complete each collection of equivalent expressions.

Example



(a)



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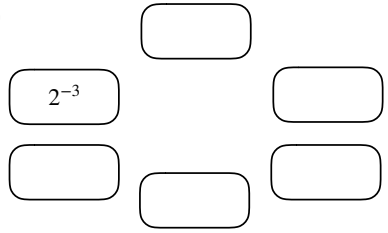

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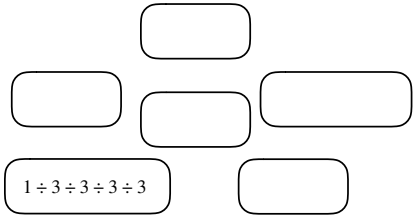
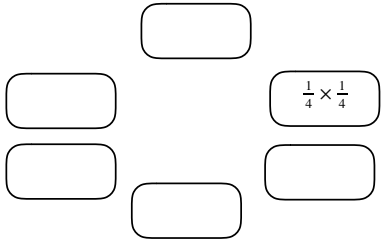


(d)



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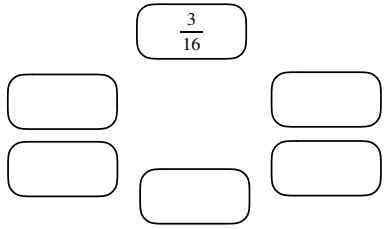

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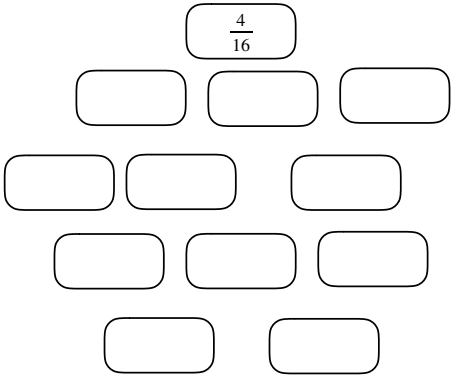

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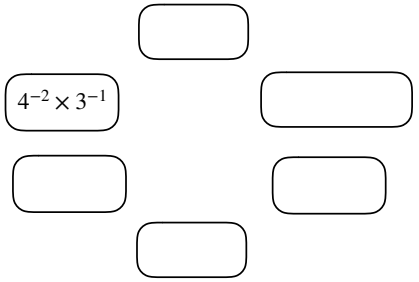


Worked Example	Your Turn
Write $\frac{1}{4^2}$ in index form	Write $\frac{1}{5^3}$ in index form

Worked Example	Your Turn
Simplify: a) $\left(\frac{3}{10}\right)^{-2}$ b) $\left(-\frac{3}{10}\right)^{-2}$	Simplify: a) $\left(\frac{2}{5}\right)^{-3}$ b) $\left(-\frac{2}{5}\right)^{-3}$

Worked Example	Your Turn
Rewrite the following with a positive index: a) x^{-3} b) $2x^{-3}$ c) $\frac{1}{2}x^{-3}$ d) $(2x)^{-3}$	Rewrite the following with a positive index: a) a^{-2} b) $4a^{-2}$ c) $\frac{1}{4}a^{-2}$ d) $(4a)^{-2}$

Worked Example	Your Turn
Rewrite the following with a negative index: a) $\frac{1}{x^5}$ b) $\frac{3}{x^5}$ c) $\frac{1}{3x^5}$	Rewrite the following with a negative index: a) $\frac{1}{d^{10}}$ b) $\frac{9}{d^{10}}$ c) $\frac{9}{18d^{10}}$

Expanding Single and Double Brackets

Q14 Which expression is equivalent to $4(2x - 7)$?

A	$8x - 7$
B	$8x - 28$
C	$6x - 11$
D	$42x - 7$

Q26 Expand and simplify

$$a - 3(2a + 1)$$

A	$3 - 5a$
B	$-5a - 3$
C	$5a - 1$
D	$1 - 5a$

Expand $(x - 2)(x + 3)$

A $x^2 + x - 6$

C $x^2 + x + 6$

B $x^2 + 2x - 6$

D $x^2 + 5x - 6$

What is the correct expansion of $(x + 13)^2$?

A) $x^2 + 169$

B) $x^2 - 169$

C) $x^2 - 26x + 169$

D) $x^2 + 26x + 169$

Q9 Expand and simplify $(3a^2bc^3)^3$

A	$9a^5bc^6$
B	$27a^6b^3c^9$
C	$3a^5b^3c^6$
D	$9a^6b^3c^9$

What is the correct answer?

Ext: what error has been made in the other 3 options?

Mathematics Mastery

Fractional Indices

$$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$$

$$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$$

$$8^{\frac{3}{3}} = \left(8^{\frac{1}{3}}\right)^3 = \left(\sqrt[3]{8}\right)^3 = (2)^3 = 8$$

$$8^{\frac{4}{3}} = \left(8^{\frac{1}{3}}\right)^4 = \left(\sqrt[3]{8}\right)^4 = (2)^4 = 16$$

$$8^{\frac{5}{3}} = \left(8^{\frac{1}{3}}\right)^5 = \left(\sqrt[3]{8}\right)^5 = (2)^5 = 32$$

$$8^{\frac{m}{3}} = \left(8^{\frac{1}{3}}\right)^m = \left(\sqrt[3]{8}\right)^m = (2)^m$$

$$x^{\frac{1}{5}} = \sqrt[5]{x}$$

$$x^{\frac{2}{5}} = \left(x^{\frac{1}{5}}\right)^2 = (\sqrt[5]{x})^2$$

$$x^{\frac{3}{5}} = \left(x^{\frac{1}{5}}\right)^3 = (\sqrt[5]{x})^3$$

$$x^{\frac{4}{5}} = \left(x^{\frac{1}{5}}\right)^4 = (\sqrt[5]{x})^4$$

$$x^{\frac{m}{5}} = \left(x^{\frac{1}{5}}\right)^m = (\sqrt[5]{x})^m$$

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = (\sqrt[n]{x})^m$$

Worked Example	Your Turn
Simplify: a) $2a^3(3a^2 + 5a^{-4})$ b) $p^{\frac{1}{2}}\left(2p^{\frac{1}{2}} - p^{-\frac{3}{2}}\right)$ c) $x^2\left(x^{\frac{1}{3}} - x^{\frac{1}{4}}\right)$	Simplify: a) $3a^{-2}(4a^5 + 2a)$ b) $2p^{\frac{1}{3}}\left(3p^{\frac{2}{3}} - p^{-\frac{1}{3}}\right)$ c) $n^{\frac{3}{5}}\left(n^{\frac{1}{2}} + \frac{1}{n^{\frac{1}{2}}}\right)$

Worked Example	Your Turn
Simplify: $(2m^9 - m^{-2})(6m^{-3} + m^5)$	Simplify: $(7x^3 - x^{-4})(4x^{-2} + x^9)$

Fractional Indices

$x^{\frac{1}{2}} \times x^{\frac{1}{2}} = \left(x^{\frac{1}{2}}\right)^2 = x^1$ $x^{\frac{1}{2}}$ squared is x therefore the square root of x is $x^{\frac{1}{2}}$ i.e. $\sqrt{x}$

$x^{\frac{1}{3}} \times x^{\frac{1}{3}} \times x^{\frac{1}{3}} = \left(x^{\frac{1}{3}}\right)^3 = x^1$ $x^{\frac{1}{3}}$ cubed is x therefore the cubed root of x is $x^{\frac{1}{3}}$ i.e. $\sqrt[3]{x}$

$x^{\frac{1}{4}} \times x^{\frac{1}{4}} \times x^{\frac{1}{4}} \times x^{\frac{1}{4}} = \left(x^{\frac{1}{4}}\right)^4 = x^1$ The fourth power of $x^{\frac{1}{4}}$ is x therefore the fourth root of x is $x^{\frac{1}{4}}$ i.e. $\sqrt[4]{x}$

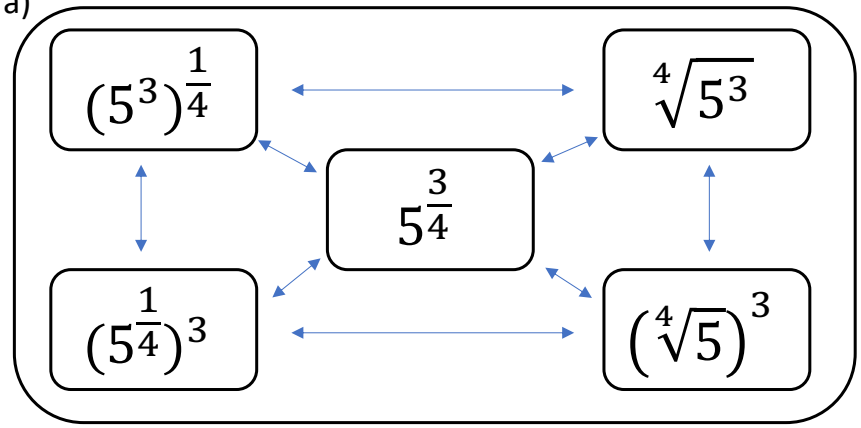
$x^{\frac{1}{n}} \times x^{\frac{1}{n}} \times x^{\frac{1}{n}} \times x^{\frac{1}{n}} \times \ldots = \left(x^{\frac{1}{n}}\right)^n = x^1$ The n^{th} power of $x^{\frac{1}{n}}$ is x therefore the n^{th} root of x is $x^{\frac{1}{n}}$ i.e. $\sqrt[n]{x}$

Worked Example	Your Turn
Evaluate: a) $64^{\frac{1}{2}}$ b) $64^{-\frac{1}{2}}$ c) $\left(\frac{81}{16}\right)^{\frac{1}{4}}$ d) $\left(\frac{81}{16}\right)^{-\frac{1}{4}}$	Evaluate: a) $64^{\frac{1}{3}}$ b) $64^{-\frac{1}{3}}$ c) $\left(\frac{81}{16}\right)^{\frac{1}{2}}$ d) $\left(\frac{81}{16}\right)^{-\frac{1}{2}}$

Fluency Practice

1) Complete the boxes, this first one has been done for you.

a)



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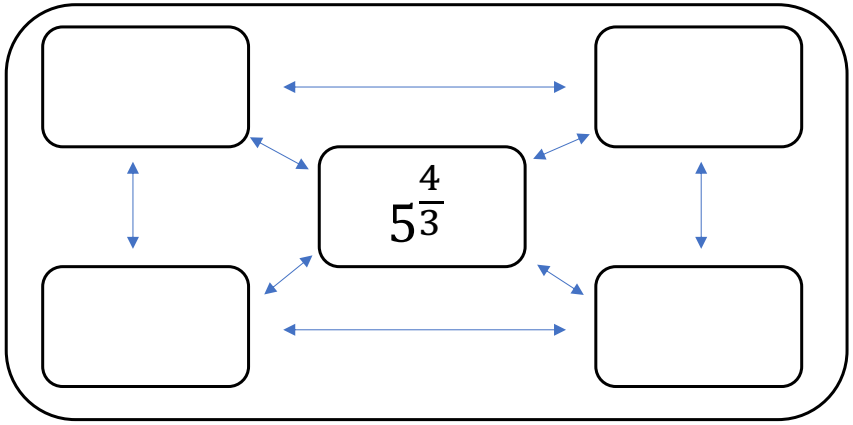

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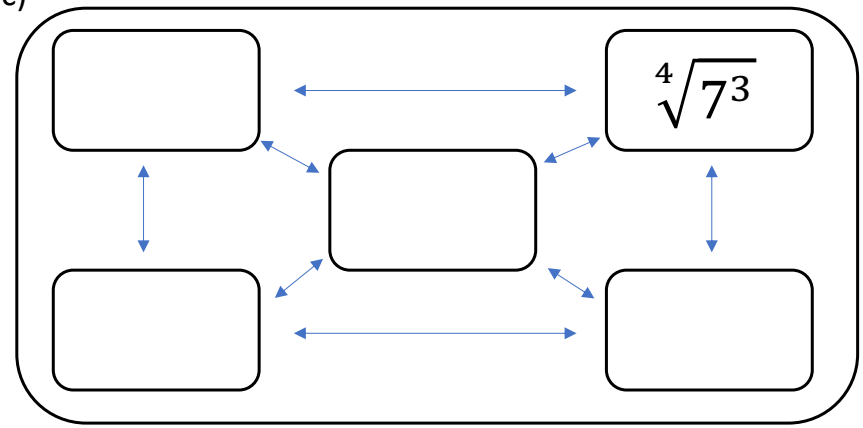

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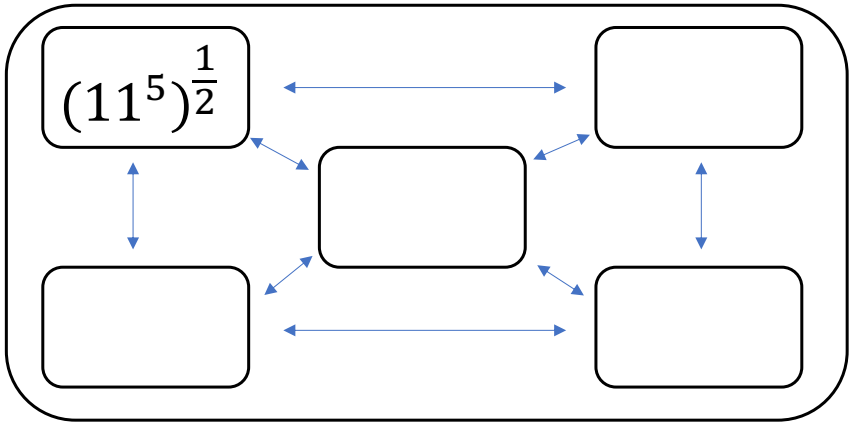
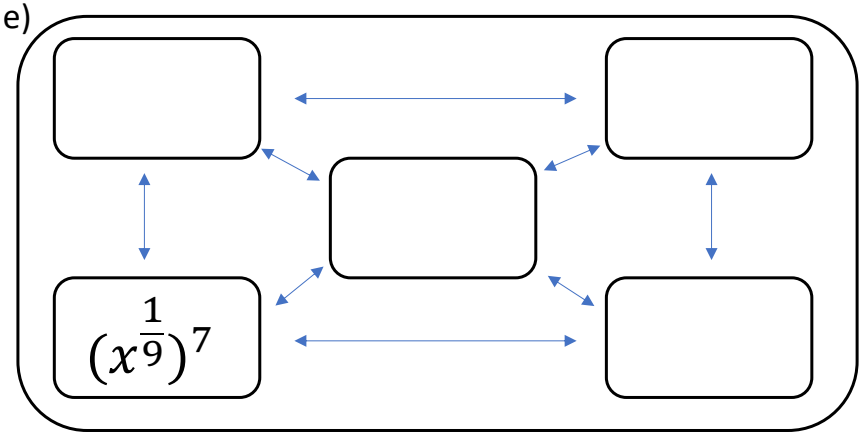


Fluency Practice

e)



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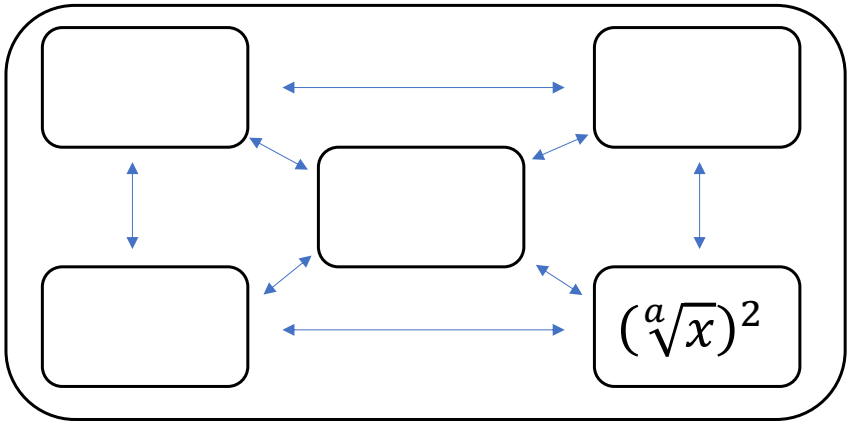
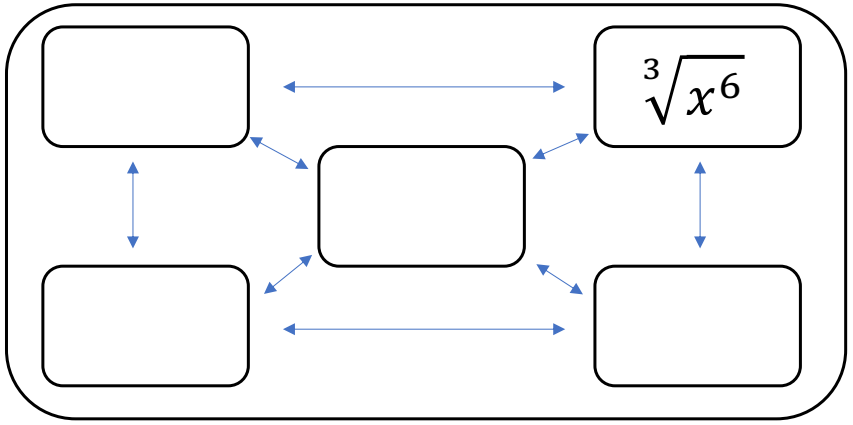


g)



2) Can you think of another way to write g?

3) Why is $(5^2)^{\frac{1}{3}} = (5^{\frac{1}{3}})^2$?

4) Which is more similar to $\left(\sqrt[3]{5}\right)^2$: $(5^2)^{\frac{1}{3}}$ *or* $(5^{\frac{1}{3}})^2$? Explain your answer?

5) Which is the most helpful representation when we want to find the value of $16^{\frac{3}{2}}$? Why?

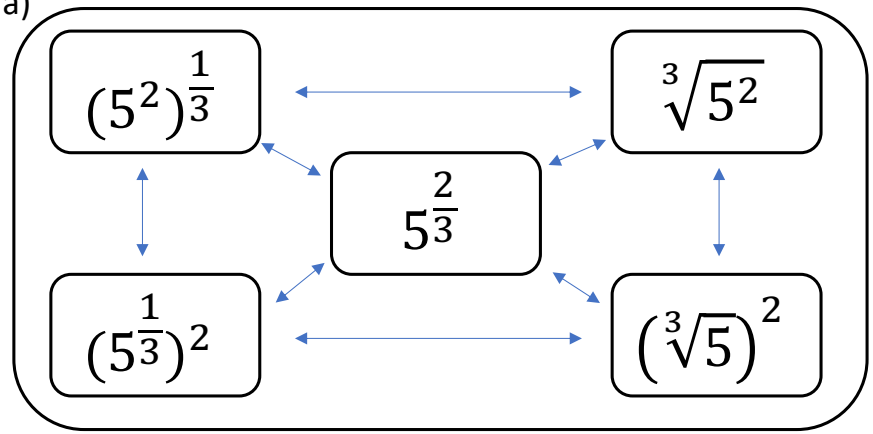
6) Which is the most helpful representation when we want to simplify $\left(\sqrt[5]{7}\right)^2 \times \sqrt[2]{7^3}$? Why?

7) How many different ways could you represent $x^{-\frac{5}{4}}$

Fluency Practice

1) Complete the boxes, this first one has been done for you.

a)



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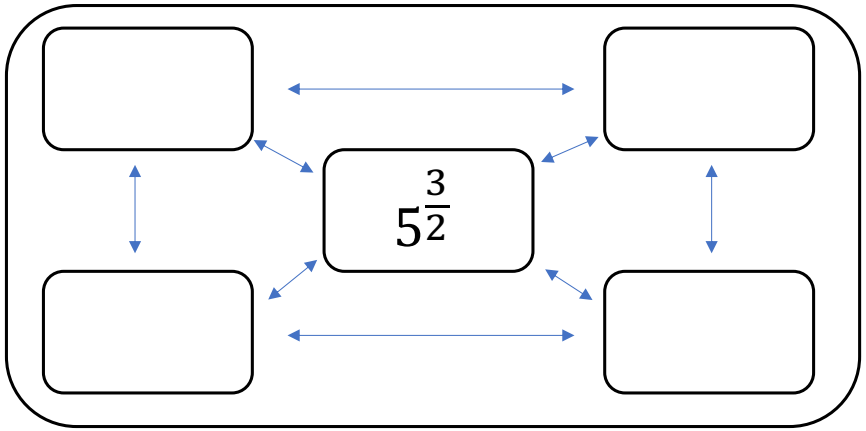
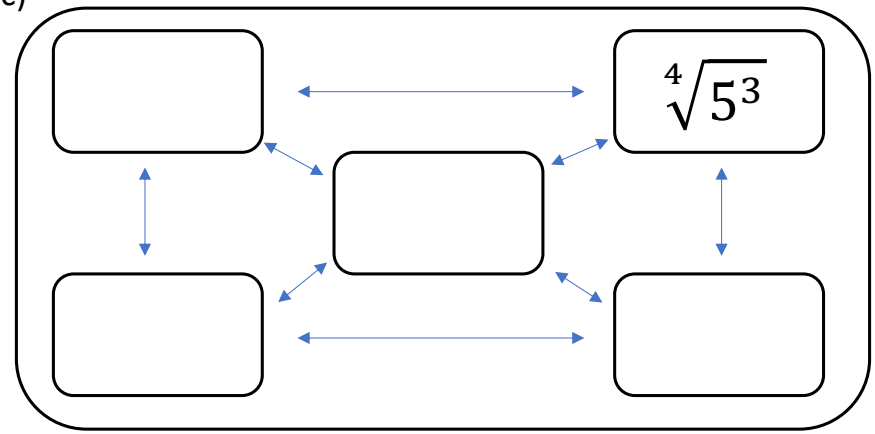


c)



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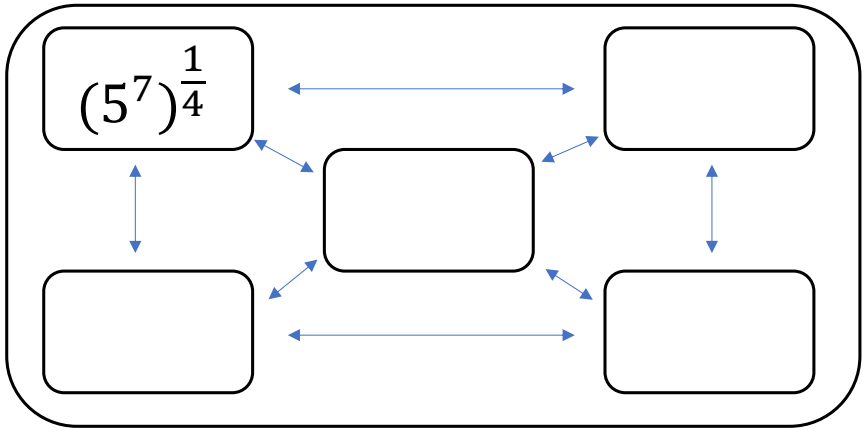
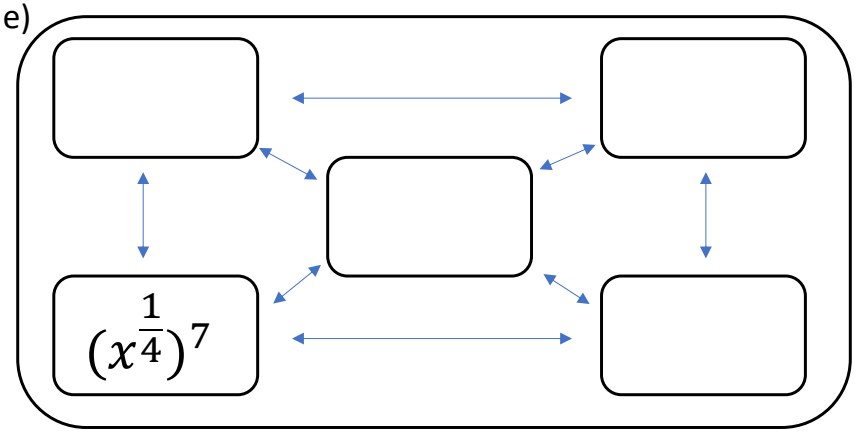


Fluency Practice

e)



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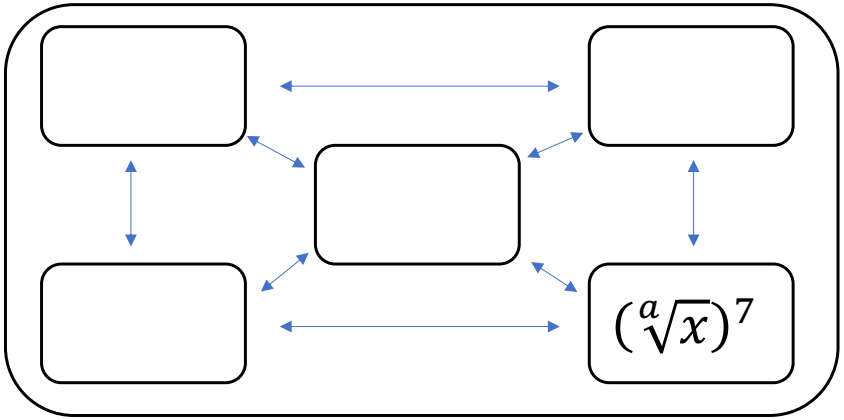
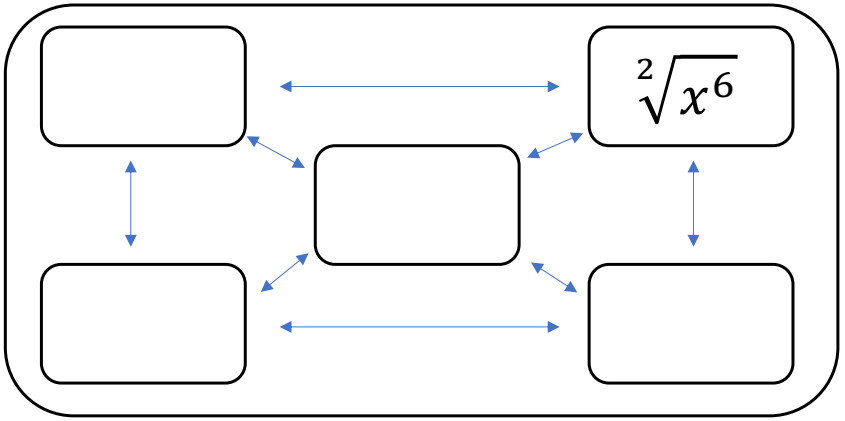


g)



2) Can you think of another way to write g?

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5) Which is the most helpful representation when we want to find the value of $16^{\frac{3}{2}}$? Why?

6) Which is the most helpful representation when we want to simplify $\left(\sqrt[5]{7}\right)^2 \times \sqrt[2]{5^3}$? Why?

Worked Example	Your Turn
Evaluate: a) $25^{\frac{3}{2}}$ b) $25^{-\frac{3}{2}}$ c) $\left(\frac{36}{25}\right)^{\frac{3}{2}}$ d) $\left(\frac{36}{25}\right)^{-\frac{3}{2}}$	Evaluate: a) $81^{\frac{3}{4}}$ b) $81^{-\frac{3}{4}}$ c) $\left(\frac{81}{256}\right)^{\frac{3}{4}}$ d) $\left(\frac{81}{256}\right)^{-\frac{3}{4}}$

Review

$$y^a \times y^b = y^{a+b}$$

$$y^a \div y^b = y^{a-b}$$

$$(y^a)^b = y^{ab}$$

$$(yz)^a = y^a z^a$$

$$\left(\frac{y}{z}\right)^a = \frac{y^a}{z^a}$$

$$y^0 = 1$$

$$y^{-a} = \frac{1}{y^a}$$

$$y^{\frac{1}{b}} = \sqrt[b]{y}$$

$$y^{\frac{a}{b}} = (\sqrt[b]{y})^a$$

$$y^{-\frac{1}{b}} = \frac{1}{\sqrt[b]{y}}$$

$$y^{-\frac{a}{b}} = \frac{1}{(\sqrt[b]{y})^a}$$

Change of Base

What do you notice about all of the numbers: 1, 10, 100, 1000, …

They are all powers of 10.

What do you notice about all of the numbers: $2, 8, 4$,16….

They are all powers of 2.

We could replace the numbers with 2^1, 2^3 and 2^2 so that we have a consistent base.

Worked Example	Your Turn
a) Write 27 as a power of 3 b) Write 27^x as a power of 3 c) Write 8^{2x} as a power of 2	a) Write 8 as a power of 2 b) Write 8^x as a power of 2 c) Write 8^{3x} as a power of 2

Worked Example	Your Turn
Find the value of each of the following: a) $\sqrt{3^6 \times 16}$ b) $\sqrt[3]{3^6 \times 8}$ c) $\sqrt[4]{3^8 \times 16}$	Find the value of each of the following: a) $\sqrt{2^4 \times 9}$ b) $\sqrt[3]{64 \times 3^3}$ c) $\sqrt[4]{81 \times 256}$

Worked Example	Your Turn
Solve the equation: $3^x = \frac{1}{9}$	Solve the equation: $4^x = \frac{1}{64}$

Worked Example	Your Turn
Solve the equation: $\left(\frac{1}{3}\right)^{x} = 27$	Solve the equation: $\left(\frac{1}{4}\right)^{x} = 64$

Worked Example	Your Turn
Find the value of x that satisfies: a) $2^x \times 2^{x-3} = 32$ b) $2^{2x} \div 2^{x-3} = 32$	Find the value of x that satisfies: a) $3^x \times 3^{x-2} = 81$ b) $3^{3x} \div 3^{x-2} = 81$

Worked Example	Your Turn
Find the value of x that satisfies: $125^{\frac{1}{4}} \times 5^{2x+3} = 25^{\frac{2}{3}}$	Find the value of x that satisfies: $64^{\frac{1}{4}} \times 4^{3x+1} = 16^{\frac{2}{3}}$

Extra Notes

Calculating in Standard Form

Without using a calculator, work out the following, giving your answer in standard form.

(a) $(2 \times 10^5) + (3 \times 10^4)$

(b) $(6.2 \times 10^7) - (5 \times 10^6)$

(c) $(3 \times 10^{-2}) + (7 \times 10^{-1})$

(d) $(1.5 \times 10^{-4}) - (9 \times 10^{-5})$

(e) $(2 \times 10^5) \times (3 \times 10^4)$

(f) $(6 \times 10^8) \div (2 \times 10^4)$

(g) $(1.5 \times 10^{-4}) \times (3 \times 10^8)$

(h) $(4.4 \times 10^7) \div (1.1 \times 10^{-3})$

Using a calculator, work out the following, giving your answer in standard form.

(a) $(1.25 \times 10^5) + (3.4 \times 10^5)$

(b) $(2.7 \times 10^{-4}) - (1.28 \times 10^{-5})$

(c) $(3.87 \times 10^{-2}) \times (5.3 \times 10^4)$

(d) $\frac{4.1 \times 10^6}{1.73 \times 10^{-2}}$

(e) $(7.3 \times 10^{-2})^2$

(f) $\sqrt{(3.6 \times 10^{11})}$

(a) Given that $F = ma$, find F when $m = 1.2 \times 10^{-12}\ g$ and $a = 4.5 \times 10^9 m/s^2$.

(b) Denmark has a population of 5.36×10^6 and Jamaica has a population of 2.56×10^6. How many more people live in Denmark than in Jamaica?

(a) The mass of Saturn is 5.686×10^{26} tonnes and the mass of the Earth is 6.04×10^{21} tonnes. How many times heavier is Saturn than Earth?

(b) In 2009 the world population was 6.77×10^9. In 2019 it was 7.73×10^9. Calculate the percentage increase in population between 2009 and 2019.

Area and Perimeter with... Standard Form

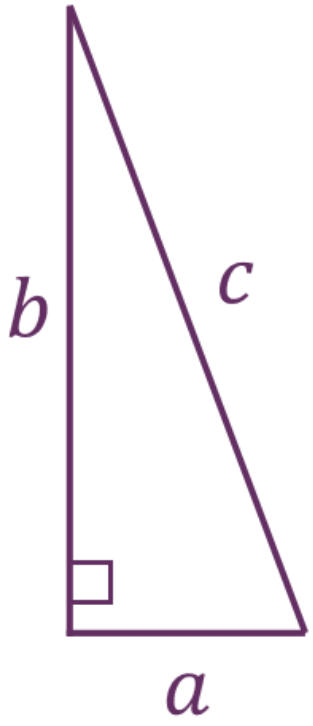
Fill in the gaps, giving all answers in standard form.

	w	h	Area	Perimeter
1)	3×10^6	4×10^6		
2)	9×10^5	1.2×10^6		
3)		3×10^4	2.4×10^8	
4)	3×10^5			6.6×10^6
5)		3×10^4	6×10^7	
6)			6×10^{12}	1×10^7

Area, Perimeter, and Pythagoras with... Standard Form

Fill in the gaps, giving all answers in standard form.

	a	b	c	Area	Perimeter
1)	3×10^6	4×10^6			
2)	5×10^9		1.3×10^{10}		
3)	8×10^4			6×10^9	
4)			2.5×10^{22}	8.4×10^{43}	5.6×10^{22}
5)		9.9×10^{-4}	1.01×10^{-3}		
6)				2.4×10^{11}	2.4×10^6



Calculate the following, giving **all** your answers in standard form.

1) a) Convert $0.4\ \text{mm}^2$ into km^2.
   b) Convert $8000\ \text{m}^2$ into km^2.
   c) How many tiles of area $0.4\ \text{mm}^2$ would it take to fill an area of $8000\ \text{m}^2$?

2) a) Calculate how many square centimetres there are in $2\ \text{km}^2$
   b) Calculate many square centimetres there are in $90\ 000\ \text{m}^2$.
   c) A farmer has farms measuring $2\ \text{km}^2$ and $90\ 000\ \text{m}^2$. Find the total area of her land in cm^2.

3) a) Convert $3 \times 10^{12}\ \text{mm}^2$ into square metres.
   b) Convert $4 \times 10^{-2}\ \text{km}^2$ into square metres.

   A small island has an area of $3 \times 10^{12}\ \text{mm}^2$. Each year, erosion reduces its area by $4 \times 10^{-2}\ \text{km}^2$.
   c) What will the area of the island be one year from now in square metres?
   d) How many years will it take for the island disappear entirely?

4) The Moon has a surface area of $1.44 \times 10^{7}\ \text{km}^2$. The sole of my shoe has an area of roughly $2.4 \times 10^{4}\ \text{mm}^2$.
   By converting both areas to m^2, approximate how many steps it would take to walk on the Moon's entire surface.

1) Assuming that each pair of numbers is the start of an arithmetic sequence, find:
(i) the next three terms, (ii) the nth term rule, (iii) the 200^{th} term.

2) Assuming that each pair of numbers is the start of a geometric sequence, find:
(i) the next three terms, (ii) the ratio between the first and third terms,
(iii) the ratio between the second and fifth terms.

a) $2 \times 10^3, 6 \times 10^3$

b) $2 \times 10^3, 2 \times 10^4$

c) $2 \times 10^3, 2.4 \times 10^3$

d) $2 \times 10^3, 3 \times 10^4$

e) $2 \times 10^3, 1.8 \times 10^4$

f) $2 \times 10^3, 1.8 \times 10^3$

g) $2 \times 10^3, 2 \times 10^5$

h) $2 \times 10^3, 2 \times 10^2$

i) $2 \times 10^{-2}, 6 \times 10^{-2}$

j) $2 \times 10^{-3}, 1.2 \times 10^{-2}$

Solving Linear Equations with... **Standard Form**

1) $x + 3 \times 10^6 = 5 \times 10^6$

2) $0.7x + 3.3 \times 10^6 = 5.4 \times 10^6$

3) $1.3x - 3.7 \times 10^{-3} = 5.4 \times 10^{-3}$

4) $(2.3 \times 10^3)x = 9.2 \times 10^{-5}$

5) $(6.1 \times 10^{11})x = 8 \times 10^6 - (3.5 \times 10^{11})x$

6) $3 \times 10^{-2} + 5x = 3x + 8 \times 10^{-2}$

7) $(3 \times 10^{-2})x + 5 = 3 + (8 \times 10^{-2})x$

8) $8x + 2.6 \times 10^8 = 12x + 1.2 \times 10^8$

9) $x + 3 \times 10^5 = 5 \times 10^6$

10) $0.7x - 1.1 \times 10^4 = 5.4 \times 10^6$

11) $1.3x + 5.3 \times 10^{-4} = 9 \times 10^{-7}$

12) $(9.2 \times 10^3)x = 2.3 \times 10^{-5}$

13) $(1.2 \times 10^{11})x = 8 \times 10^6 - (5 \times 10^9)x$

14) $3 \times 10^{-2} + 5x = 3x + 8 \times 10^{-3}$

15) $(2 \times 10^{-2})x - 7 = 11 + (8 \times 10^{-3})x$

16) $11x + 2.4 \times 10^8 = 1.2 \times 10^{12} - 13x$

12 **(a)** Work out.
Give your answers in standard form.

(i) $3 \times 10^4 + 2.7 \times 10^2$

(a)(i) ..... **[2]**

(ii) $5 \times 10^6 \times 7 \times 10^8$

(ii) ..... **[2]**

(b) Estimate.

$$\sqrt{\frac{0.621 \times 7.94}{0.334}}$$

(b) ..... **[2]**

14 **(a)** Write 543 000 in standard form.

(a) ..... **[1]**

(b) Write 6.3×10^{-2} as an ordinary number.

(b) ..... **[1]**

(c) Pierre is given this question.

> Work out.
> 61000 × 4000
> Give your answer in standard form.

Pierre's answer is 24.4×10^7.

Is Pierre correct?
Explain your answer.

.....

..... **[1]**

22 **(a)** Beth is given the following question.

> Work out
>
> $$4.1 \times 10^5 \times 3 \times 10^2.$$
>
> Give your answer in standard form.

This is Beth's answer to the question.

$$12.3 \times 10^7$$

Explain why Beth's answer is incorrect.

.....

..... **[1]**

(b) Show that

$$4.5 \times 10^2 + 7.3 \times 10^3 = 7.75 \times 10^3.$$

[2]

13 A company makes sweets.
The sweets are put into packets.

Here are some facts.

1.47×10^7 sweets are made every day	3.5×10^5 packets of sweets are produced every day

(a) Calculate the mean number of sweets in one packet.

(a) ..... **[2]**

A company makes sweets.
The sweets are put into packets.

Here are some facts.

1.47×10^7 sweets are made every day	3.5×10^5 packets of sweets are produced every day

(b) Sweets are made on 288 days each year.

Calculate the number of sweets made each year.
Give your answer in standard form.

(b) ..... **[3]**

(b) Work out.

N48

$5 \times 10^4 - 1.6 \times 10^3$

Give your answer in standard form.

(b) ..... **[3]**

13 **(a)** Write 0.003 16 in standard form.

N47

(a) ..... **[1]**

(b) Work out.

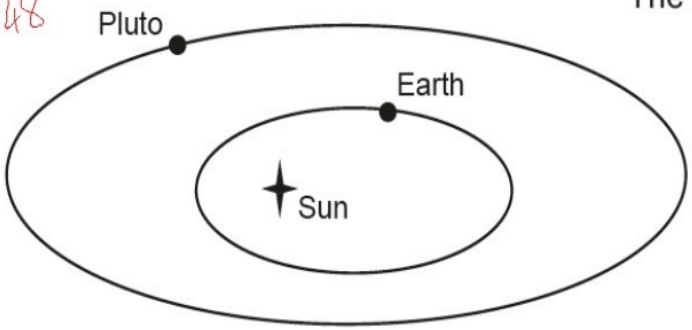
N48 $2 \times 10^2 \times 4 \times 10^5$

Give your answer in standard form.

(b) ..... **[2]**

22 Earth and Pluto go around the Sun.
Their distance to the Sun varies.

N47
N48



The table shows the closest distance that Earth and Pluto get to the Sun.

	Closest distance to the Sun (km)
Earth	1.47×10^8
Pluto	4.44×10^9

(a) Show that the closest distance of Pluto to the Sun is roughly 30 times the closest distance of Earth to the Sun. **[2]**

(b) Give a reason why we **cannot** use this information to say

The distance of Pluto to the Sun is always
30 times the distance of Earth to the Sun.

.....

..... **[1]**

Country	Population
England	5.35×10^7
Wales	3.07×10^6
Scotland	5.31×10^6
Northern Ireland	1.82×10^6

(c) The total population of the UK is predicted to reach 73.3 million in 2037.

Calculate the predicted percentage increase in the UK population from 2012 to 2037.
Give your answer correct to 2 significant figures.

(c) ..... % **[4]**

22 A newborn baby has an approximate mass of 3.5 kilograms.

N48 A human cell has an approximate mass of 2.7×10^{-11} grams.

Use these values to estimate the number of human cells in a newborn baby.
Give your answer in standard form, correct to 2 significant figures.

..... **[5]**

5 A company makes sweets.
The sweets are put into packets.

N48

Here are some facts.

1.47×10^7 sweets are made every day	3.5×10^5 packets of sweets are produced every day

(a) Calculate the mean number of sweets in one packet.

(a) ..... **[2]**

3 A newborn baby has an approximate mass of 3.5 kilograms.

N48 A human cell has an approximate mass of 2.7×10^{-11} grams.

Use these values to estimate the number of human cells in a newborn baby.
Give your answer in standard form, correct to 2 significant figures.

..... **[5]**

17 A grain of salt weighs 6.48×10^{-5} kg on average.
A packet contains 0.35 kg of salt.

N48

(a) Use this information to calculate the number of grains of salt in the packet.

(a) ..... **[2]**

(b) Explain why your answer to part **(a)** is unlikely to be the actual number of grains of salt in the packet.

.....

..... **[1]**

1 Work out $(2 \times 10^3) \times (4 \times 10^4)$, giving your answer in standard form.

N48

..... **[2]**

17 The table below shows the area, in square kilometres (km^2), of some countries.

Country	Area (km^2)
Australia	7.69×10^6
Latvia	6.46×10^4
Luxembourg	2.59×10^3
Russia	1.71×10^7
Singapore	7.24×10^2
Sweden	4.50×10^5

(a) Write the area of Sweden as an ordinary number.

N47

(a) ..... km^2 **[1]**

(b) Which of the above countries has the smallest area?

N47

(b) ..... **[1]**

(c) Alexis says

N48 The area of Australia is approximately three times larger than the area of Luxembourg.

Is she correct?
Show how you decide.

Alexis is ..... because .....

..... **[2]**

(d) Work out the total area of Russia and Australia.
Give your answer in standard form, correct to 2 significant figures.

N48

(d) ..... km^2 **[4]**

2 Use the formula $F = \frac{s}{\sqrt{tm}}$ to find the value of F when

N48
N50

$s = 5.8 \times 10^6$

$t = 4.1 \times 10^8$

$m = 3.7 \times 10^{-2}$.

Give your answer in standard form, correct to 2 significant figures.

..... **[4]**

17 The table below shows the number of barrels of oil produced per day by some countries.

Country	Barrels of oil produced per day
USA	1.17×10^7
China	3.98×10^6
UK	9.39×10^5
Cameroon	9.32×10^4
Japan	3.92×10^3

(a) Write the number of barrels of oil produced per day by Cameroon as an ordinary number.

N47

(a) ..... **[1]**

(b) How many more barrels of oil per day did China produce than the UK?
Give your answer in standard form, correct to 3 significant figures.

N48

(b) ..... **[4]**

(c) Jamal says the USA produced approximately three times more barrels of oil than Japan.

N48

Is he correct?
Show how you decide.

Jamal is ..... because .....

..... **[2]**

1 **(a)** Write these numbers in standard form.

N47 **(i)** 6500

(a)(i) ..... **[1]**

(ii) 0.0584

(ii) ..... **[1]**

(b) Work out $(4.2 \times 10^5) \times (1.8 \times 10^{-2})$, giving your answer in standard form.

N48

(b) ..... **[1]**

26 (a) Write 2.673×10^4 as an ordinary number.

.....

(1)

(b) Write 0.0704 in standard form.

.....

(1)

(c) Calculate $(4.515 \times 10^6) \div (3.01 \times 10^{-2})$
Give your answer in standard form.

.....

(2)

(Total for Question 26 is 4 marks)

4.62×10^8 tins of beans were sold last year.
These tins of beans cost a total of £300.3 million.

(c) Work out the average cost per tin of beans.

N48
R26

£.....

(2)

15 (a) Write 4.7×10^{-1} as an ordinary number.

.....

(1)

(b) Work out the value of $(2.4 \times 10^3) \times (9.5 \times 10^5)$
Give your answer in standard form.

21 Work out $\dfrac{0.06 \times 0.0003}{0.01}$

Give your answer in standard form.

.....

(Total for Question 21 is 3 marks)

25 Work out $(13.8 \times 10^{7}) \times (5.4 \times 10^{-12})$
Give your answer as an ordinary number.

.....

(Total for Question 25 is 2 marks)

(c) Work out $\dfrac{2.3 \times 10^{4} \times 6.7 \times 10^{3}}{5 \times 10^{-8}}$

N48

.....

(2)

18 Work out the value of $\dfrac{2.645 \times 10^{9}}{1.15 \times 10^{3}}$

Give your answer in standard form.

N47
N48

.....

(Total for Question 18 is 2 marks)

4.62×10^{8} tins of beans were sold last year.
These tins of beans cost a total of £300.3 million.

(c) Work out the average cost per tin of beans.

N48
R26

£.....

(2)

9 Find the value of $\dfrac{(6.67 \times 10^{-11}) \times (7.35 \times 10^{22})}{(1.74 \times 10^{6})^2}$

Give your answer correct to 1 decimal place.

N48
N50

.....

(Total for Question 9 is 2 marks)

8 (a) Write 7.97×10^{-6} as an ordinary number.

.....

(1)

(b) Work out the value of $(2.52 \times 10^{5}) \div (4 \times 10^{-3})$
Give your answer in standard form.

.....

(2)

(Total for Question 8 is 3 marks)

10 The table shows some information about eight planets.

Planet	Distance from Earth (km)	Mass (kg)
Earth	0	5.97×10^{24}
Jupiter	6.29×10^{8}	1.898×10^{27}
Mars	7.83×10^{7}	6.42×10^{23}
Mercury	9.17×10^{7}	3.302×10^{23}
Neptune	4.35×10^{9}	1.024×10^{26}
Saturn	1.28×10^{9}	5.68×10^{26}
Uranus	2.72×10^{9}	8.683×10^{25}
Venus	4.14×10^{7}	4.869×10^{24}

(a) Write down the name of the planet with the greatest mass.

.....

(1)

(b) Find the difference between the mass of Venus and the mass of Mercury.

Nishat says that Neptune is over a hundred times further away from Earth than Venus is.

(c) Is Nishat right?
You must show how you get your answer.

9 $T = \sqrt{\frac{w}{d^3}}$

N48
N50 $w = 5.6 \times 10^{-5}$
$d = 1.4 \times 10^{-4}$

(a) Work out the value of T.
Give your answer in standard form correct to 3 significant figures.

$T =$ .....

(2)

7 (a) Write the number 0.000 086 23 in standard form.

N47

.....

(1)

(b) Work out $\frac{3.2 \times 10^3 + 5.1 \times 10^{-2}}{4.3 \times 10^{-4}}$

N48
Give your answer in standard form, correct to 3 significant figures.

.....

(2)

(Total for Question 7 is 3 marks)

11 In May 2019, the distance between Earth and Mars was 3.9×10^7 km.

N48 In May 2019, a signal was sent from Earth to Mars.
Assuming that the signal sent from Earth to Mars travelled at a speed of 3×10^5 km per second,

(a) how long did the signal take to get to Mars?

..... seconds

(2)

The speed of the signal sent from Earth to Mars in May 2019 was actually less than 3×10^5 km per second.

(b) How will this affect your answer to part (a)?

.....

.....

.....

(1)

27 Work out $(3.42 \times 10^{-7}) \div (7.5 \times 10^{-6})$
Give your answer in standard form.

N48

.....

(Total for Question 27 is 2 marks)

10 A person's heart beats approximately 10^5 times each day.
A person lives for approximately 81 years.

(a) Work out an estimate for the number of times a person's heart beats in their lifetime.
N48 Give your answer in standard form correct to 2 significant figures.

.....

(2)

2×10^{12} red blood cells have a total mass of 90 grams.

(b) Work out the average mass of 1 red blood cell.
Give your answer in standard form.

N48

..... grams

(2)

(Total for Question 10 is 4 marks)

H/F

23 (a) Write 4.5×10^5 as an ordinary number.

N47

.....

(1)

(b) Write 0.007 in standard form.

N47

.....

(1)

(c) Work out $4.2 \times 10^3 + 5.3 \times 10^2$
Give your answer in standard form.

N48

.....

(2)

27 Work out $\frac{9.12 \times 10^{10}}{3.2 \times 10^{4}}$

N48

Give your answer in standard form.

[2 marks]

Answer _____

20 (a) Write 0.000 97 in standard form.

[1 mark]

N47

Answer _____

20 (b) Work out $\frac{3 \times 10^{5}}{4 \times 10^{3}}$

N48

Give your answer as an ordinary number.

[2 marks]

Answer _____

30 (a) Work out $\frac{2 \times 10^{14}}{8 \times 10^{9}}$

N48

Give your answer in standard form.

[2 marks]

Answer _____

H/F

27 (b) Work out $\frac{1.8 \times 10^{2}}{3 \times 10^{-1}}$

N48 Give your answer as an ordinary number.

[2 marks]

Answer _____

Extra Notes

Multiplying Surds

To simplify $\sqrt{a} \times \sqrt{b}$:

- Use the fact $\sqrt{a}\times\sqrt{b} = \sqrt{a\times b}$
- Simplify your answer.

EXT: can you prove why this is true using indices laws?

Worked Example	Your Turn
Simplify: a) $\sqrt{5} \times \sqrt{6}$ b) $\sqrt{3} \times \sqrt{6}$	Simplify: a) $\sqrt{5} \times \sqrt{7}$ b) $\sqrt{3} \times \sqrt{8}$

Worked Example	Your Turn
Simplify: a) $2\sqrt{5} \times \sqrt{6}$ b) $3\sqrt{3} \times 2\sqrt{6}$	Simplify: a) $2\sqrt{5} \times \sqrt{7}$ b) $3\sqrt{3} \times 2\sqrt{8}$

Multiplication Madness

Think of this like a normal multiplication table, just with terms missing everywhere. Fill in all blanks.

	$-\sqrt{5}$	$\sqrt{2}$			$-\sqrt{6}$			$2\sqrt{3}$	$-\sqrt{2}$			-2
3												
		$\sqrt{10}$				$3\sqrt{5}$		$2\sqrt{15}$				
				$16\sqrt{2}$								$8\sqrt{2}$
												4
$\sqrt{7}$	$-\sqrt{35}$						$2\sqrt{14}$					
									$2\sqrt{6}$			
		6				$9\sqrt{2}$						
					$-9\sqrt{2}$							
			$4\sqrt{6}$					$8\sqrt{3}$				
$-2\sqrt{5}$											-20	
$\sqrt{2}$			$2\sqrt{3}$		$-2\sqrt{3}$					$-\sqrt{6}$		
		4										

Surds: Multiplication Squares

Can you fill in the missing numbers in these multiplication squares?

x	$\sqrt{10}$	$\sqrt{6}$
$\sqrt{8}$	$\sqrt{}$	$2\sqrt{}$
$\sqrt{6}$	$2\sqrt{}$	

x	$\sqrt{6}$	$\sqrt{3}$
$\sqrt{12}$	$\_\_\sqrt{8}$	
$\sqrt{7}$	$\sqrt{3} \times \sqrt{}$	$\sqrt{}$

x	$3\sqrt{6}$	$2\sqrt{8}$
$\sqrt{6}$		$\_\_\sqrt{48}$
$\sqrt{3}$	$\_\_\sqrt{2}$	$4\sqrt{}$

x	$\sqrt{9}$	$2\sqrt{3}$
$2\sqrt{3}$	$\_\_\sqrt{}$	
$3\sqrt{3}$	$\_\_\sqrt{}$	

x	$\sqrt{}$	$\sqrt{}$
$\sqrt{6}$	$\sqrt{48}$	$\sqrt{30}$
	$\sqrt{72}$	$\sqrt{45}$

x		$2\sqrt{5}$
	$\sqrt{60}$	10
	$4\sqrt{6}$	$4\sqrt{}$

Challenge: Make up one of your own!

Dividing Surds

To simplify $\sqrt{a} \div \sqrt{b}$:

- Use the fact $\sqrt{a} \div \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
- Simplify your answer.

EXT: can you prove why this is true using indices laws?

Worked Example	Your Turn
Simplify: a) $\sqrt{60} \div \sqrt{2}$ b) $\sqrt{60} \div \sqrt{3}$	Simplify: a) $\sqrt{90} \div \sqrt{3}$ b) $\sqrt{90} \div \sqrt{2}$

Worked Example	Your Turn
Simplify: a) $2\sqrt{60} \div \sqrt{2}$ b) $12\sqrt{60} \div 2\sqrt{3}$	Simplify: a) $3\sqrt{90} \div \sqrt{3}$ b) $12\sqrt{90} \div 3\sqrt{2}$

Adding and subtracting Surds

To simplify $\sqrt{a} + \sqrt{b}$ or $\sqrt{a} - \sqrt{b}$:

- Simplify both surds if possible.
- Then add/subtract the surds by collecting like terms.

EXT: find a counter example for:

$$\sqrt{a} + \sqrt{b} = \sqrt{a+b} \quad and \quad \sqrt{a} - \sqrt{b} = \sqrt{a-b}$$

Worked Example	Your Turn
Simplify: a) $2\sqrt{5} + 5\sqrt{5}$ b) $2\sqrt{20} + 5\sqrt{5}$ c) $2\sqrt{20} + 5\sqrt{10}$	Simplify: a) $2\sqrt{6} + 5\sqrt{6}$ b) $2\sqrt{54} + 5\sqrt{6}$ c) $2\sqrt{20} + 5\sqrt{15}$

Worked Example	Your Turn
Simplify: $\frac{2\sqrt{20}+5\sqrt{5}}{\sqrt{5}}$	Simplify: $\frac{2\sqrt{54}-5\sqrt{6}}{\sqrt{6}}$

Worked Example	Your Turn
Expand and simplify: a) $2(4+\sqrt{3})$ b) $-\sqrt{3}(4+\sqrt{3})$ c) $\sqrt{12}(4+\sqrt{3})$	Expand and simplify: a) $-2(\sqrt{3}+4)$ b) $\sqrt{3}(\sqrt{3}+4)$ c) $\sqrt{27}(\sqrt{3}+4)$

Worked Example	Your Turn
Expand and simplify: a) $(2-\sqrt{3})(4+\sqrt{3})$ b) $(2-\sqrt{3})^2$	Expand and simplify: a) $(\sqrt{3}-2)(\sqrt{3}+4)$ b) $(\sqrt{3}-2)^2$

Worked Example	Your Turn
Expand and simplify: a) $\left(2-\sqrt{20}\right)\left(4+\sqrt{5}\right)$ b) $(2-2\sqrt{20})(4+5\sqrt{5})$	Expand and simplify: a) $\left(\sqrt{54}-2\right)\left(\sqrt{6}+4\right)$ b) $(2\sqrt{54}-2)(5\sqrt{6}+4)$

Rationalising Surds

To rationalise $\frac{1}{\sqrt{x}}$:

- Multiply the numerator and denominator by the surd in the denominator i.e., $\sqrt{x}$
- Simplify your answer.

To rationalise $\frac{1}{y+\sqrt{x}}$:

- Multiply the numerator and denominator by the conjugate of the denominator i.e., $y - \sqrt{x}$
- Simplify your answer by multiplying out the brackets.
- Simplify your answer by collecting like terms.

Worked Example	Your Turn
Rationalise: a) $\frac{3}{\sqrt{5}}$ b) $\frac{3}{2\sqrt{5}}$ c) $\frac{3+\sqrt{5}}{\sqrt{5}}$	Rationalise: a) $\frac{10}{\sqrt{5}}$ b) $\frac{3}{2\sqrt{6}}$ c) $\frac{10+\sqrt{5}}{\sqrt{5}}$

Identifying Conjugates

Recall the difference of two squares below:

Use this to define a conjugate:

Is $\sqrt{3}-1$ the conjugate of $\sqrt{3}+1$?

Is $-\sqrt{3}+1$ the conjugate of $\sqrt{3}+1$?

Is $-\sqrt{3}+1$ the conjugate of $1+\sqrt{3}$?

Is $1-\sqrt{3}$ the conjugate of $1+\sqrt{3}$?

Is $-1-\sqrt{3}$ the conjugate of $1-\sqrt{3}$?

Is $1+\sqrt{3}$ the conjugate of $1-\sqrt{3}$?

Is $1+\sqrt{5}$ the conjugate of $1-\sqrt{5}$?

Is $1-3\sqrt{5}$ the conjugate of $1+3\sqrt{5}$?

Is $3\sqrt{5}-1$ the conjugate of $1+3\sqrt{5}$?

Is $3\sqrt{5}-1$ the conjugate of $3\sqrt{5}+1$?

Is $-3\sqrt{5}-1$ the conjugate of $3\sqrt{5}+1$?

Is $-3\sqrt{5}-1$ the conjugate of $3\sqrt{5}-1$?

Worked Example	Your Turn
Rationalise: a) $\frac{6}{4+\sqrt{3}}$ b) $\frac{6}{\sqrt{3}+5}$	Rationalise: a) $\frac{6}{4-\sqrt{3}}$ b) $\frac{6}{\sqrt{3}+4}$

Worked Example	Your Turn
Rationalise: a) $\frac{6}{4+2\sqrt{3}}$ b) $\frac{6}{2\sqrt{3}+5}$	Rationalise: a) $\frac{6}{4-2\sqrt{3}}$ b) $\frac{6}{2\sqrt{3}+4}$

Worked Example	Your Turn
Rationalise: $\frac{4}{\frac{1}{\sqrt{3}}+\sqrt{3}}$	Rationalise: $\frac{3}{\sqrt{2}+\frac{1}{\sqrt{2}}}$

Fluency Practice

For each question express the answer as a surd in the form $a\sqrt{x}$ where a is an integer

$$\frac{6}{\frac{1}{\sqrt{5}} + \sqrt{5}}$$

$$\frac{8}{\sqrt{3} - \frac{1}{\sqrt{3}}}$$

$$\frac{18}{\frac{2}{\sqrt{7}} + \sqrt{7}}$$

$$\frac{5\sqrt{2}}{\frac{3}{\sqrt{2}} + \sqrt{2}}$$

$$\frac{3\sqrt{5}}{\sqrt{5} - \frac{2}{\sqrt{5}}}$$

$$\frac{\frac{4}{\sqrt{3}} + 2\sqrt{3}}{\sqrt{3} + \frac{2}{\sqrt{3}}}$$

Question	Working		Answer
$\frac{5}{\sqrt{3}}$	$\times\frac{\sqrt{3}}{\sqrt{3}}$	$=\frac{5\sqrt{3}}{\sqrt{9}}$	$=\frac{5\sqrt{3}}{3}$
$\frac{\sqrt{3}}{\sqrt{7}}$	$\times\frac{\sqrt{7}}{\sqrt{7}}$		
$\frac{5\sqrt{5}}{\sqrt{6}}$			
$\frac{2+\sqrt{3}}{\sqrt{5}}$	$\times\frac{\sqrt{5}}{\sqrt{5}}$	$=\frac{\sqrt{5}(2+\sqrt{3})}{\sqrt{25}}$	$=\frac{2\sqrt{5}+\sqrt{15}}{5}$
$\frac{3-\sqrt{5}}{\sqrt{2}}$			
$\frac{1+\sqrt{2}}{2\sqrt{3}}$			
$\frac{\sqrt{2}-3\sqrt{5}}{5\sqrt{2}}$			

Question	Working		Answer
$\frac{3}{2+\sqrt{2}}$	$\times\frac{2-\sqrt{2}}{2-\sqrt{2}}$	$=\frac{3(2-\sqrt{2})}{4-\sqrt{4}}$	$=\frac{6-3\sqrt{2}}{2}$
$\frac{8}{4-\sqrt{3}}$			
$\frac{\sqrt{5}}{6+\sqrt{5}}$			
$\frac{3\sqrt{5}}{3-\sqrt{7}}$			
$\frac{7+\sqrt{2}}{3-\sqrt{2}}$	$\times\frac{3+\sqrt{2}}{3+\sqrt{2}}$	$=\frac{(7+\sqrt{2})(3+\sqrt{2})}{9-\sqrt{4}}$	$=\frac{23+10\sqrt{2}}{7}$
$\frac{1-\sqrt{8}}{5+\sqrt{2}}$			
$\frac{a+\sqrt{b}}{a\sqrt{b}}$			

Worked Example	Your Turn
Find in its simplest form $a : b$, given: $a = \sqrt{5} + \sqrt{c}$ $b = \sqrt{80} + \sqrt{d}$ c and d are positive integers $c : d = 1 : 16$	Find in its simplest form $a : b$, given: $a = \sqrt{7} + \sqrt{c}$ $b = \sqrt{63} + \sqrt{d}$ c and d are positive integers $c : d = 1 : 9$

Worked Example	Your Turn
Express b and c in terms of a: $\left(a+\sqrt{12}\right)^2 = b + c\sqrt{3}$	Express b and c in terms of a: $\left(a+\sqrt{8}\right)^2 = b + c\sqrt{2}$

Worked Example	Your Turn
Find the value of a and b: $\left(a-3\sqrt{5}\right)^2=b-42\sqrt{5}$	Find the value of a and b: $\left(a-2\sqrt{3}\right)^2=b-20\sqrt{3}$

Worked Example	Your Turn
What is the surd square root of $52 + 16\sqrt{3}$?	What is the surd square root of $55 + 30\sqrt{2}$?

Extra Notes

Simplifying Algebraic Fractions

Some of these fractions can be simplified, others cannot. Can you decide why each can or cannot be simplified?

$$\frac{2\times3^2}{5\times7} \qquad \frac{2\times3^2}{3\times7} \qquad \frac{ab}{a} \qquad \frac{ab+c}{a} \qquad \frac{ab+ac}{a}$$

Worked Example	Your Turn
Simplify: $\frac{6x}{10x^2}$	Simplify: $\frac{6x}{10x^3}$

Worked Example	Your Turn
Simplify: a) $\frac{5x+10}{x+2}$ b) $\frac{x^2+5x+6}{x+2}$	Simplify: a) $\frac{x+2}{5x+10}$ b) $\frac{x^2+5x+6}{2x+4}$

Multiplying algebraic fractions

Reminder, how do you:

- Multiply a fraction by a whole number
- Multiply a fraction by another fraction

Worked Example	Your Turn
$\frac{6x}{2y} \times \frac{4y}{5}$	$\frac{5a}{2b} \times \frac{5b}{30}$

Fill In The Blanks...

Multiplying Algebraic Fractions

Question	Write as a Single Fraction	Simplify Numerator and Denominator	Simplified Answer (where possible)
$\frac{x}{4} \times \frac{2x}{3}$	$\frac{x \times 2x}{4 \times 3}$	$\frac{2x^2}{12}$	$\frac{x^2}{6}$
$\frac{x}{6} \times \frac{4y}{5}$	$\frac{x \times 4y}{6 \times 5}$	$\frac{4xy}{30}$	
$\frac{2}{x} \times \frac{3xy}{5}$	$\frac{2 \times 3xy}{x \times 5}$		
$\frac{4x}{3y} \times \frac{2y}{x}$			
$\frac{2}{7x} \times \frac{3xy}{4}$			
$\frac{x^2}{8} \times \frac{4y}{x}$			
$\frac{2y}{x} \times \frac{9x^2y}{4}$			
$\frac{10y}{x^2} \times \frac{3xy^2}{5}$			
$\frac{4yz}{3} \times \frac{x^2}{6y^3}$			
$\frac{2x^3}{15yz} \times \frac{5x^2y^2}{z^3}$			
$\frac{\square}{5y^2} \times \frac{4x^2y}{\square}$	$\frac{\square \times 4x^2y}{5y^2 \times \square}$	$\frac{24x^3y}{15y^2}$	
$\frac{12x}{5yz} \times \frac{\square}{\square}$			$\frac{6x^3y}{25z}$

Worked Example 1

Simplify $\frac{2x^2+7x-15}{x^2-36} \times \frac{2x+12}{2x^3-3x^2}$ fully.

Reflective Process

- Factorise the numerators (if you can)
- Factorise the denominators (if you can)
- Replace the expressions with their factorised versions
- Cancel the common factors
- Rewrite without the expressions that you have crossed out

Worked Example 2

Simplify $\frac{3x^2+8x+5}{x^2-25} \times \frac{5x^2-25x}{3x^2+5x}$ fully.

Your Turn 1	Your Turn 2
Simplify $\frac{2x^2-17x+21}{x^2-49} \times \frac{5x^2+15x}{2x^2-3x}$ fully.	*[AQA IGCSE FM Practice paper set 1 P1 Q10]* Simplify fully $\frac{3x^2-x-14}{9x^2-4} \div \frac{x+2}{3x^2+2x}$

Adding and Subtracting Algebraic fractions

What must you do in order to add or subtract fractions?

Worked Example	Your Turn
$\frac{x}{5}+\frac{3x}{8}$	$\frac{5}{x}+\frac{8}{3x}$

Worked Example	Your Turn
Write the following expression as a single fraction in its simplest form: $\frac{8}{2y} + \frac{3}{3x^2y^2}$	Write the following expression as a single fraction in its simplest form: $\frac{5}{6b} + \frac{3}{4a^3b}$

Fill In The Blanks...

Adding and Subtracting Algebraic Fractions

Question	With a Common Denominator	Unsimplified Answer	Simplified Answer (where possible)
$\frac{x}{4}+\frac{7x}{20}$	$\frac{5x}{20}+\frac{7x}{20}$	$\frac{12x}{20}$	
$\frac{7x}{18}-\frac{2x}{9}$	$\frac{7x}{18}-\frac{4x}{18}$		
$\frac{2x}{3}+\frac{x}{4}$	$\frac{\square}{12}+\frac{\square}{12}$		
$\frac{17x}{30}+\frac{x}{10}$	$\frac{17x}{\square}+\frac{3x}{\square}$		
$\frac{x}{6}+\frac{11x}{24}$			
$\frac{3x}{4}-\frac{7x}{36}$			
$\frac{7}{2x}+\frac{3}{x}$	$\frac{\square}{2x}+\frac{\square}{2x}$		
$\frac{6}{5x}-\frac{9}{20x}$			
$\frac{5}{x}+\frac{2}{x^2}$	$\frac{\square}{x^2}+\frac{2}{x^2}$		
	$\frac{\square}{xy}-\frac{\square}{xy}$	$\frac{3-y}{xy}$	
	$\frac{\square}{4x^2}+\frac{\square}{4x^2}$	$\frac{7x+6}{4x^2}$	
$\frac{3}{10xy}-\frac{2}{x^2}$			

Worked Example	Your Turn
Write the following expression as a single fraction in its simplest form: $\frac{1}{x^2-1}+\frac{1}{x+1}$	Write the following expression as a single fraction in its simplest form: $\frac{1}{a^2-9}+\frac{1}{a-1}$

Worked Example	Your Turn
Write the following expression as a single fraction in its simplest form: $\frac{6}{x^2-4}-\frac{14}{x+2}$	Write the following expression as a single fraction in its simplest form: $\frac{4}{a^2-9}-\frac{5}{a-3}$

Solving equations with algebraic fractions

Key strategies

- Simplify to a single fraction using the LCM
- Multiply ***ALL*** terms by the LCM
- At every step, where possible, simplify and/or factorise

Worked Example	Your Turn
Solve $\frac{x+4}{2}+\frac{x+1}{5}=5$	Solve $\frac{x-4}{2}+\frac{x-1}{5}=2$

Worked Example	Your Turn
Solve $\frac{x+1}{3} - \frac{x-3}{5} = 1$	Solve $\frac{x+2}{3} - \frac{x-6}{5} = 2$

Worked Example	Your Turn
Solve $\frac{4}{x+6}+\frac{5}{x+8}=1$	Solve $\frac{4}{x+3}+\frac{5}{x+4}=2$

Worked Example	Your Turn
Solve $\frac{3}{x-6}+\frac{4}{x-9}=1$	Solve $\frac{3}{x-2}+\frac{4}{x-3}=3$

Worked Example

A coach is due to reach its destination 30 kilometres away at a certain time. Its start is delayed by 18 minutes, but by increasing the average speed by 5 km/h the driver arrives on time. How long did the journey actually take? What was the intended average speed?

Extra Notes