



**Year 10  
Mathematics  
Unit 20**

Tasks



**Name:** \_\_\_\_\_

**Class:** \_\_\_\_\_

## Contents Page

- 1 [Graphical Simultaneous Equations](#)
- 2 [Linear Simultaneous Equations](#)
- 3 [Combinations and Permutations](#)
- 4 [Advanced Statistics](#)

See unit 20 course on [drfrostmaths.com](https://www.drfrostmaths.com)

### Unit 20

---

PR Graphical Simultaneous Equations

Graphical Simultaneous Equations

PR Linear Simultaneous Equations

Linear Simultaneous Equations

Combinations and Permutations

PR Advanced Statistics

Advanced Statistics

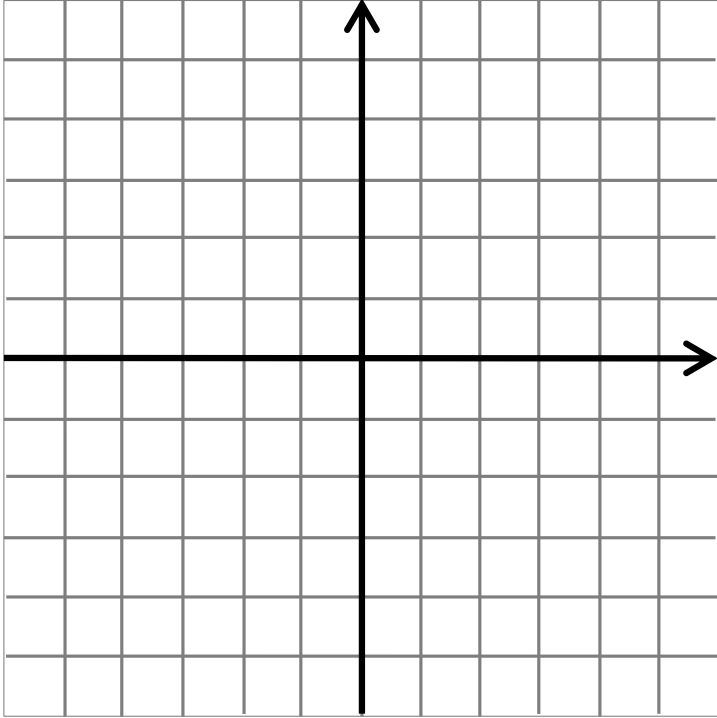
# 1 Graphical Simultaneous Equations

### Worked Example

Solve graphically:

$$y = 2x + 5$$

$$y = -2x - 1$$

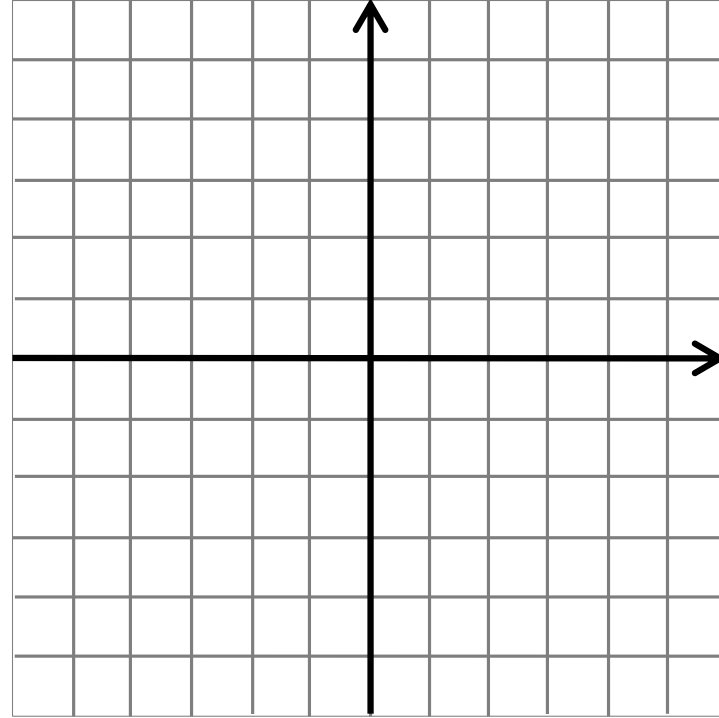


### Worked Example

Solve graphically:

$$y = 2x - 3$$

$$y = -2x + 5$$

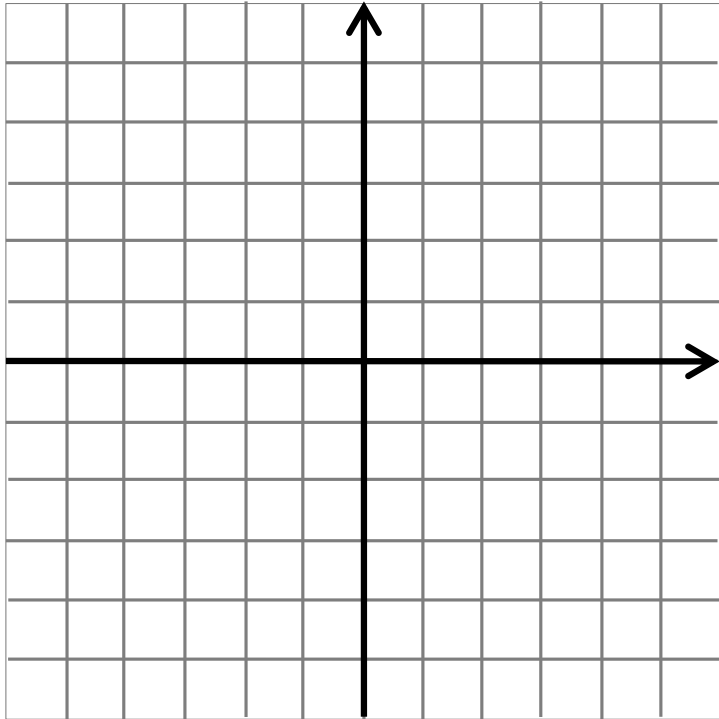


### Worked Example

Solve graphically:

$$2x - y = 8$$

$$x - y = 1$$

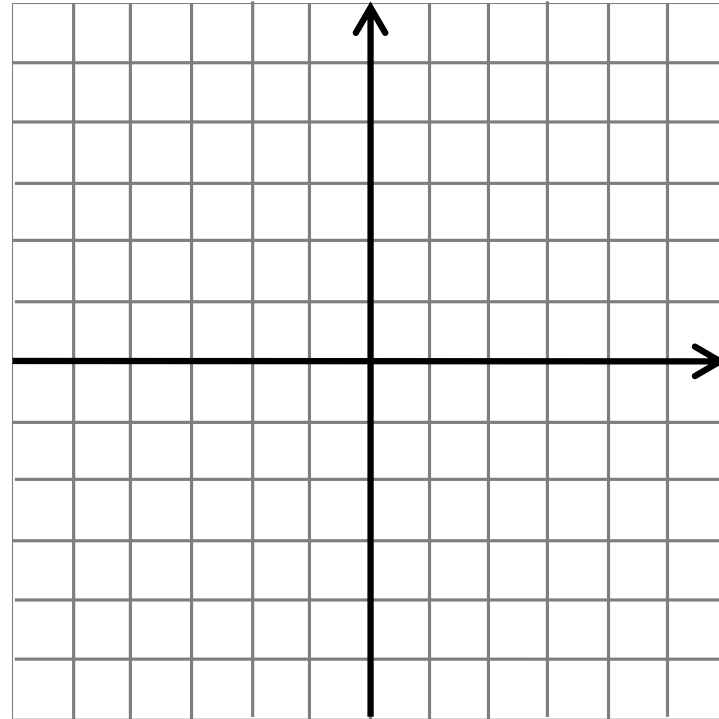


### Worked Example

Solve graphically:

$$-2x + y = 1$$

$$x + y = 10$$



## Solving Simultaneous Equations

### Graphically

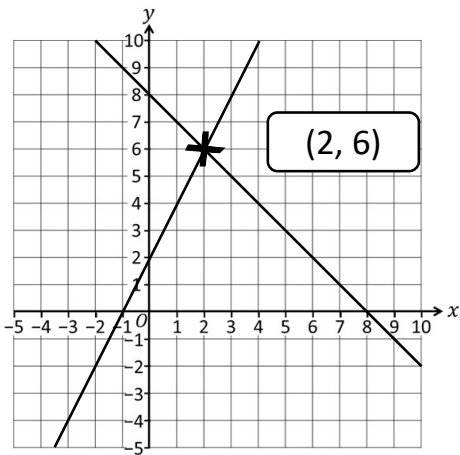
Plot each equation on the same grid.

You may want to rearrange the equations first.

The intersection shows the values  $(x, y)$  that satisfy both equations.

Check the solution by substituting values back into the equations.

$$y = 2x + 2 \quad y + x = 8$$

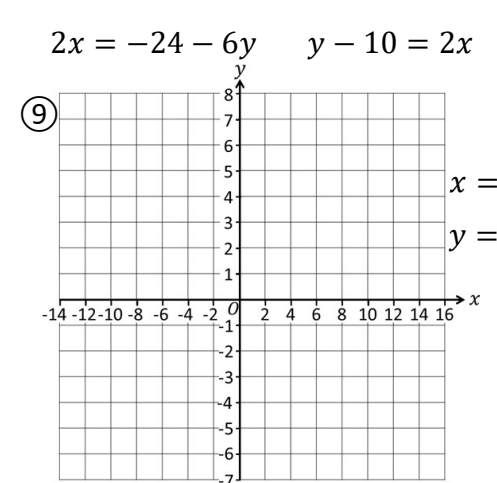
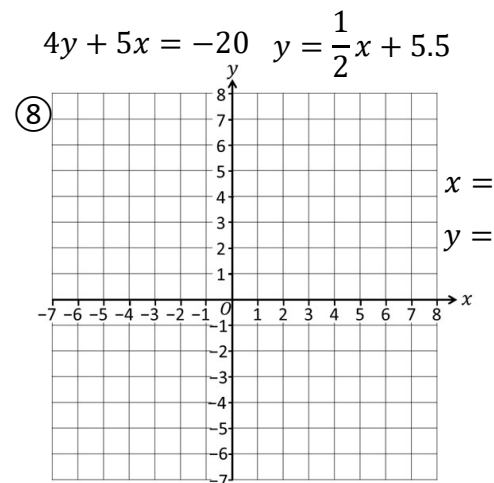
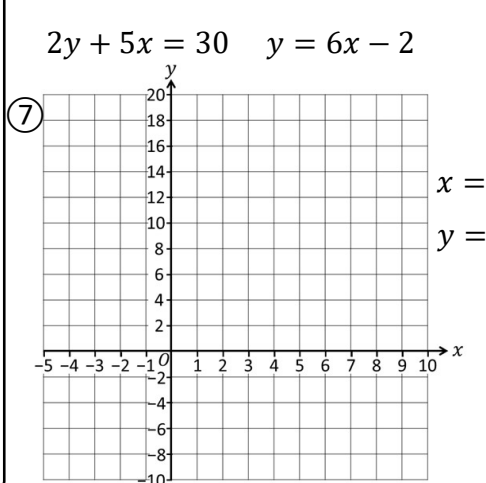
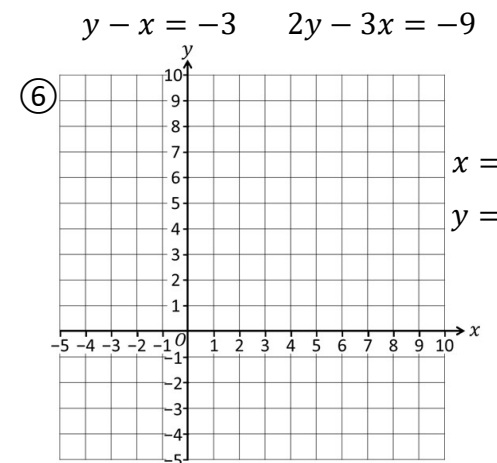
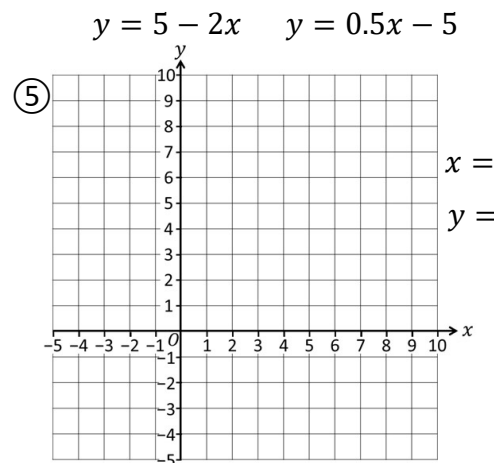
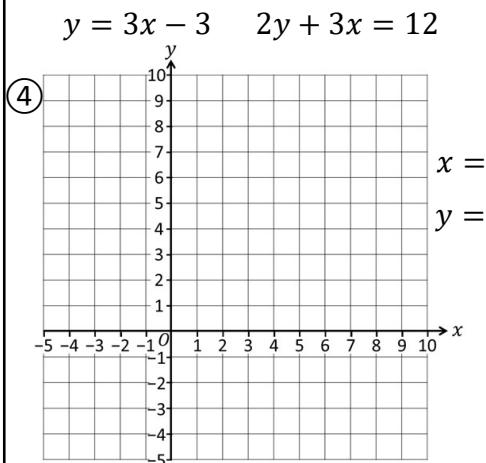
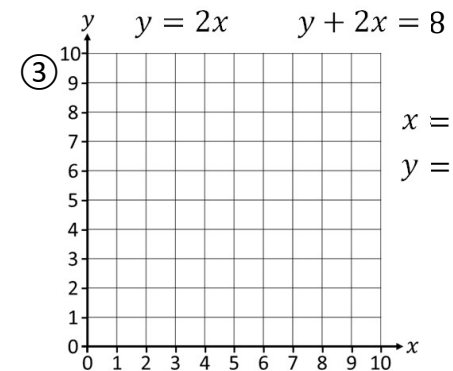
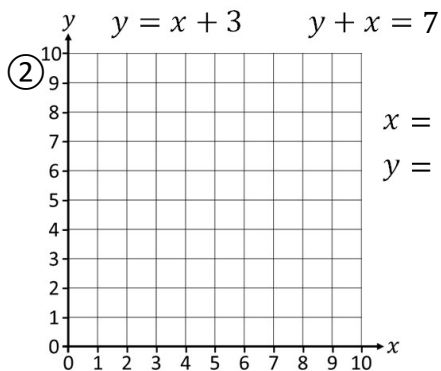
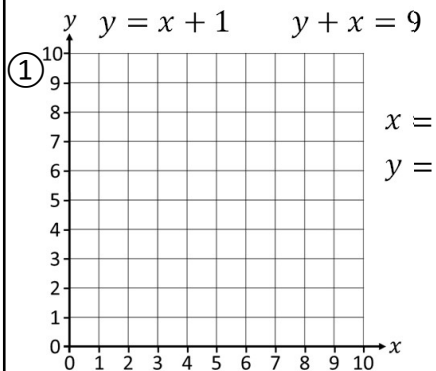


Solution:  $x = 2 \quad y = 6$

Substitute to check:

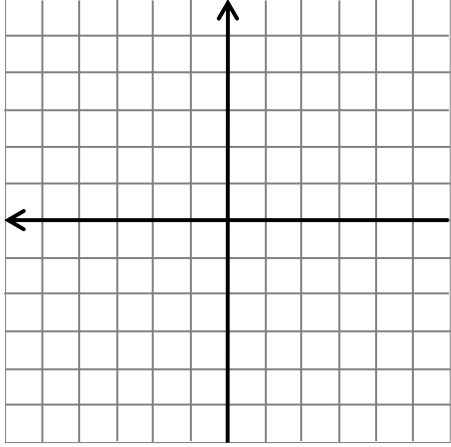
$$y = 2x + 2 \quad 6 = 2(2) + 2 \quad \checkmark$$

$$y + x = 8 \quad 6 + 2 = 8 \quad \checkmark$$



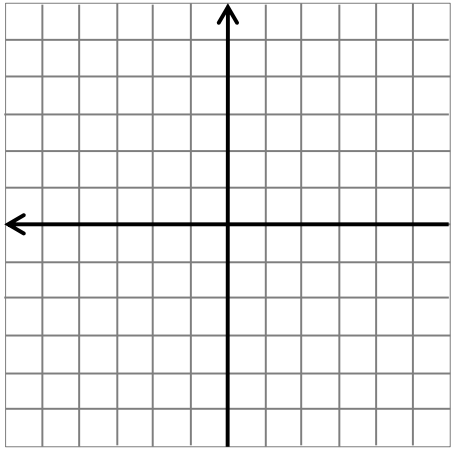
For each pair of equations draw the lines for each, the point of intersection represents the solution.

1.  $y = 3x - 1$   
 $y = 2x$



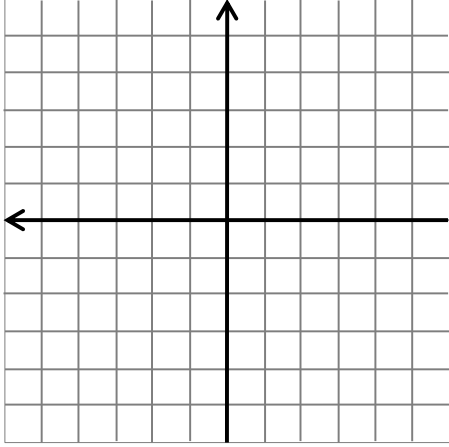
Point of intersection (\_\_\_\_, \_\_\_\_) so  $x =$  \_\_\_\_ &  $y =$  \_\_\_\_

2.  $y = 2x - 1$   
 $y = x$



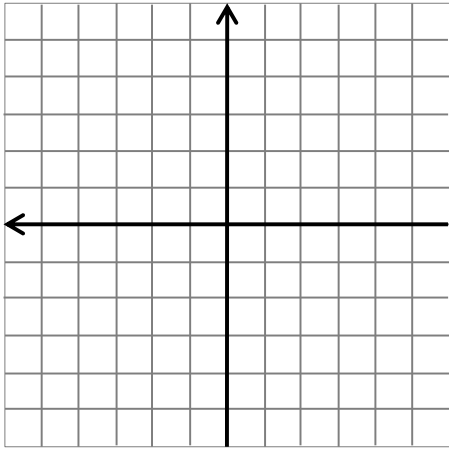
Point of intersection (\_\_\_\_, \_\_\_\_) so  $x =$  \_\_\_\_ &  $y =$  \_\_\_\_

3.  $y = 3x - 2$   
 $y = x - 2$



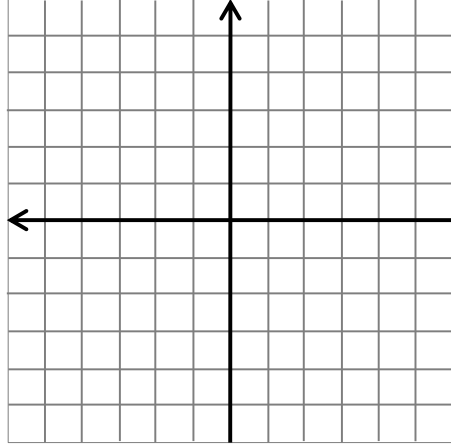
Point of intersection (\_\_\_\_, \_\_\_\_) so  $x =$  \_\_\_\_ &  $y =$  \_\_\_\_

4.  $y = 3 - 2x$   
 $y = x$



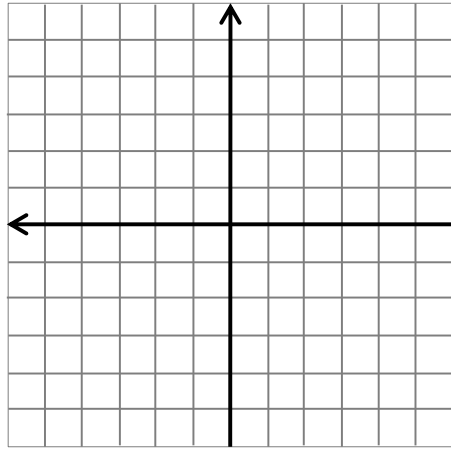
Point of intersection (\_\_\_\_, \_\_\_\_) so  $x =$  \_\_\_\_ &  $y =$  \_\_\_\_

5.  $x + y = 5$   
 $y = 2x - 1$



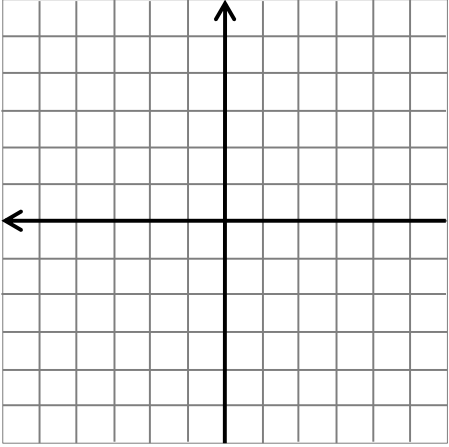
Point of intersection (\_\_\_\_, \_\_\_\_) so  $x =$  \_\_\_\_ &  $y =$  \_\_\_\_

6.  $2x + y = 6$   
 $x + y = -6$



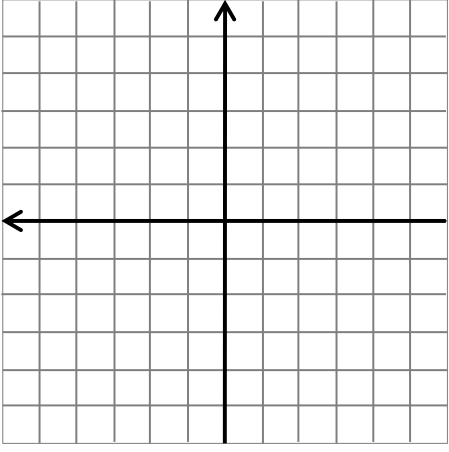
Point of intersection (\_\_\_\_, \_\_\_\_) so  $x =$  \_\_\_\_ &  $y =$  \_\_\_\_

7.  $x - y = 3$   
 $x + y = 5$



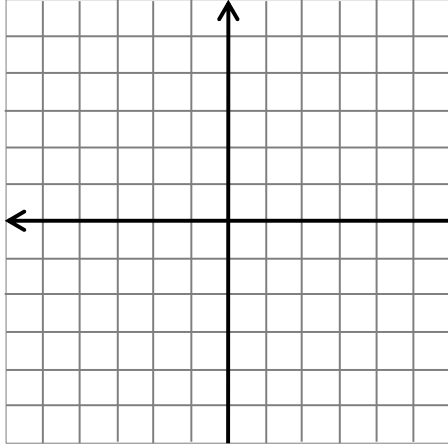
Point of intersection (\_\_\_\_, \_\_\_\_ ) so  $x =$  \_\_\_\_ &  $y =$  \_\_\_\_

8.  $x + y = -5$   
 $y = 4x$



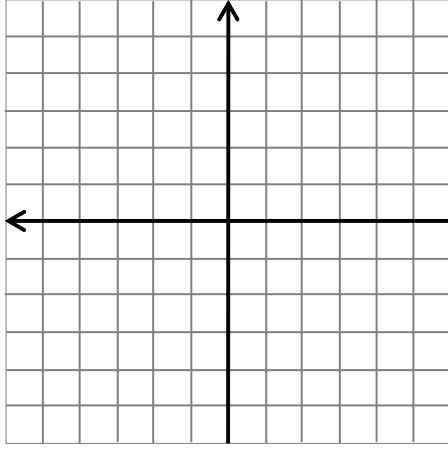
Point of intersection (\_\_\_\_, \_\_\_\_ ) so  $x =$  \_\_\_\_ &  $y =$  \_\_\_\_

9.  $y = 3x - 2$   
 $x + y = 2$



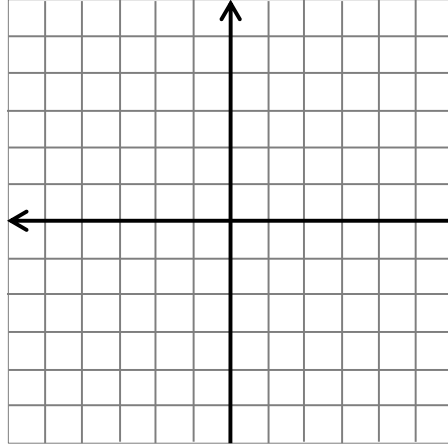
Point of intersection (\_\_\_\_, \_\_\_\_ ) so  $x =$  \_\_\_\_ &  $y =$  \_\_\_\_

10.  $2x - 3y = 6$   
 $x + 3y = 3$



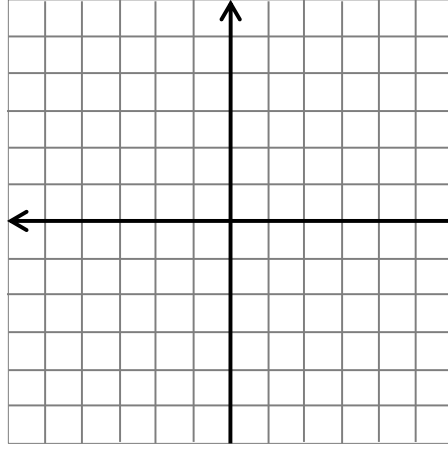
Point of intersection (\_\_\_\_, \_\_\_\_ ) so  $x =$  \_\_\_\_ &  $y =$  \_\_\_\_

11.  $x + 4y = 4$   
 $x - 2y = 4$



Point of intersection (\_\_\_\_, \_\_\_\_ ) so  $x =$  \_\_\_\_ &  $y =$  \_\_\_\_

12.  $x + 3y = 6$   
 $x + 2y = 5$



Point of intersection (\_\_\_\_, \_\_\_\_ ) so  $x =$  \_\_\_\_ &  $y =$  \_\_\_\_

Point of intersection (\_\_\_\_, \_\_\_\_ ) so  $x =$  \_\_\_\_ &  $y =$  \_\_\_\_

Point of intersection (\_\_\_\_, \_\_\_\_ ) so  $x =$  \_\_\_\_ &  $y =$  \_\_\_\_



## Extra Notes

## 2 Linear Simultaneous Equations

### Steps to solve simple linear simultaneous equations

- 1) Decide if the equations are in the correct form (ready to add or subtract).
- 2) Decide whether we need to manipulate the equations first.
- 3) Decide if we need to add or subtract.
- 4) Successfully add or subtract algebraic expressions, possible involving negative numbers.
- 5) Solve a linear equations, possibly involving negative numbers.
- 6) Substitute the solution into an algebraic equation.
- 7) Solve another linear equation.
- 8) Substitute the two solutions into one of the two algebraic equations to check the answer.

**Worked Example**

Solve:

$$4x + 3y = 23$$

$$2x + 3y = 19$$

**Your Turn**

Solve:

$$8x + 3y = 31$$

$$2x + 3y = 19$$

**Worked Example**

Solve:

$$4x - 3y = 23$$

$$2x + 3y = 19$$

**Your Turn**

Solve:

$$8x + 3y = 31$$

$$2x - 3y = 19$$

### Worked Example

Solve:

$$2x - 5y = 16$$

$$2x + 3y = 0$$

### Your Turn

Solve:

$$3x + 5y = 2$$

$$3x - 2y = -5$$

**Worked Example**

Solve:

$$2x + 3y = 11$$

$$3x + y = 13$$

**Your Turn**

Solve:

$$4x + 3y = 5$$

$$2x - 5y = 9$$

## Worked Example

Solve:

$$3x + 2y = 9$$

$$5x + 7y = 4$$

## Your Turn

Solve:

$$2x + 3y = 9$$

$$5x + 7y = 23$$

**Worked Example**

Solve:

$$5x = -4y + 22$$

$$3x = 7y - 15$$

**Your Turn**

Solve:

$$5x + 7y - 12 = 0$$

$$8x = -4y + 3$$



### Worked Example

Two numbers have a sum of 45 and a difference of 13. Find the numbers.

### Your Turn

Two numbers have a sum of 49 and a difference of 19. Find the numbers.

### Worked Example

When shopping in Jamaica, 5 coconuts and 14 bananas cost me \$8.70, and 8 coconuts and 9 bananas cost \$9.90. Find the cost of each coconut and banana.

### Your Turn

Four nectarines and three peaches cost \$2.90, and three nectarines and a peach cost \$1.90. Find the cost of each fruit.

### Worked Example

In my pocket I have only 5-cent and 10-cent coins. How many of each type do I have if I have 24 coins altogether and their total value is \$1.55?

### Your Turn

Martin collects 20-cent and 50-cent coins. He has 37 coins, and the total value of the coins is \$11.30. How many coins does Martin have of each type?

### Worked Example

Solve the following simultaneous equations:

$$2x + 5y = 25$$

$$y = 5x - 22$$

### Your Turn

Solve the following simultaneous equations:

$$5x + 4y = 30$$

$$y = 3x - 1$$

## Extra Notes

### 3 Combinations and Permutations

You have to pick a team of one boy and one girl. The boys are Adam, Bob and Charles. The girls are Diane and Elaine. List out all the possible choices of team. (You may use 'A' for Adam' and so on)

There were 3 boys and 2 girls to choose from, and we needed one of each. Without listing, we previously saw there were  $3 \times 2$  total possibilities.

**If there are  $m$  choices for one thing and  $n$  for the other, there are  $m \times n$  possibilities when we choose from both.**

### Worked Example

There are a choice of 9 starters and 5 mains on the menu of a restaurant.

Jake selects one starter and one main.

How many ways of choosing are there?

### Your Turn

There are a choice of 10 starters and 8 mains on the menu of a restaurant.

Anna selects one starter and one main.

How many ways of choosing are there?

## Worked Example – Repeated vs Non-Repeated

- a) A restaurant serves 9 dishes. Amir and Ben both choose a dish. How many different ways of choosing are there?
- b) A restaurant serves 9 dishes. Amir and Ben choose **DIFFERENT** dishes. How many different ways of choosing are there?



## Worked Example – Ordered vs Non-Ordered

- a) I want a football team of two people, **consisting of just a goalkeeper and a striker**. I have 5 people to choose from. How many possibilities are there?
- b) I want a football team of two people. I have 5 people to choose from. How many possible teams are there?

## Your Turn

In each case think carefully whether we divide by 2 (if the items in our selection are 'unordered') or not.

- a) In a race of 8 people, a Gold and a Silver medal are awarded. How many ways can the medals be awarded?
- b) In a class of 30 students I select 2 to give a detention to. How many ways can I do this?
- c) The Queen has a head male corgi that she wants to breed. She picks 2 amongst her 6 female corgis. How many selections can she make?
- d) A bag has 5 differently coloured counters. Alice picks one then Bob picks one. How many possible selections are there?

### Worked Example

- a) There are 21 boys and 17 girls in a basketball club. Ella selects one boy and one girl from the club. How many ways of choosing are there?
- b) There are 21 boys and 17 girls in a basketball club. Laura selects two boys from the club. How many ways of choosing are there?

### Your Turn

- a) There are 22 boys and 28 girls in a basketball club. Ella selects one boy and one girl from the club. How many ways of choosing are there?
- b) There are 22 boys and 28 girls in a basketball club. Laura selects two boys from the club. How many ways of choosing are there?

## Extra Notes

## 4 Advanced Statistics

# Histograms

Group	Frequency	Group width	Frequency density
$8 \leq p < 12$	20		
$8 \leq p < 12$	10		
$8 \leq p <$	10		1.25
$\leq p < 28$	10	20	
$25 \leq p <$		20	0.6
$\leq p < 65$	12		0.24
$20 \leq p <$	120	50	
$200 \leq p < 250$			1.6
$\leq p < 250$		20	4
$\leq p < 175$	20	80	
$\leq p < 175$	20		0.333 ...
$15 \leq p <$		60	0.3

## Worked Example

Plot a histogram:

Height, $x$ (cm)	Frequency
$140 < x \leq 155$	6
$155 < x \leq 175$	14
$175 < x \leq 185$	6
$185 < x \leq 190$	21





## Your Turn

Plot a histogram:

Price, $y$ (£)	Frequency
$0 < y \leq 10$	4
$10 < y \leq 20$	9
$20 < y \leq 25$	8
$25 < y \leq 35$	10
$35 < y \leq 50$	12



# histograms

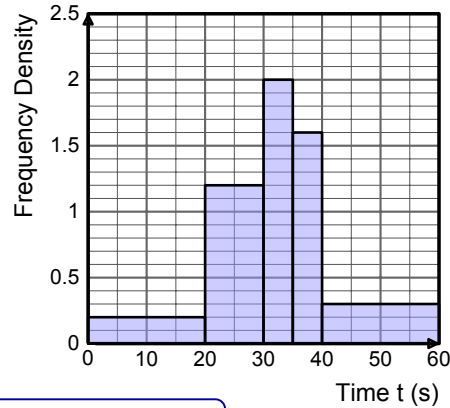
## learn by heart

Histograms are used for **continuous data**. The frequency is represented by the **area** of each bar, and the vertical scale is **frequency density**, calculated for each bar using the formula:

$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$

## example

Time, t (s)	Frequency	Class Width	Frequency Density
$0 < t \leq 20$	4	20	0.2
$20 < t \leq 30$	12	10	1.2
$30 < t \leq 35$	10	5	2
$35 < t \leq 40$	8	5	1.6
$40 < t \leq 60$	6	20	0.3



The class width is the range of times included in the row.

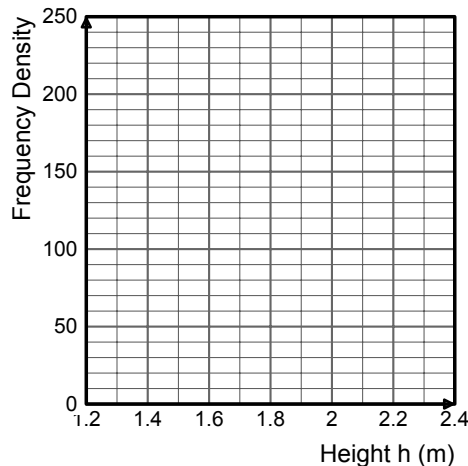
$$\text{Frequency Density} = \frac{\text{Frequency}}{\text{Class Width}}$$

## exercise

1. The data in the table shows the heights of a group of people.

Work out the frequency densities and complete the histogram.

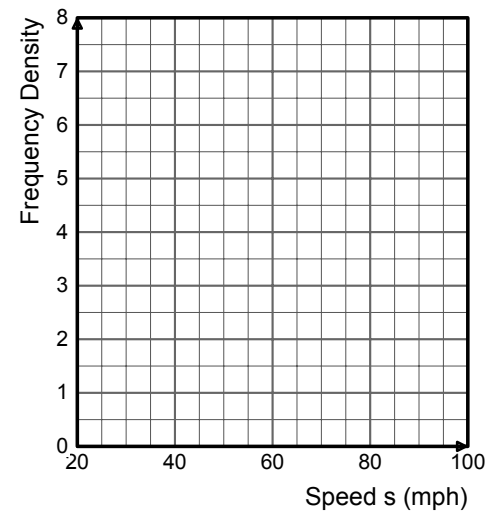
Height h (m)	Frequency	Frequency Density
$1.4 \leq h < 1.6$	12	
$1.6 \leq h < 1.7$	20	
$1.7 \leq h < 1.8$	17	
$1.8 \leq h < 2.1$	12	



2. The data in the table shows the speeds of vehicles passing a point on a motorway.

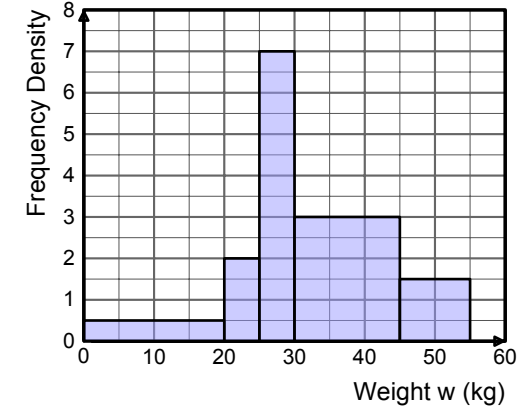
Complete the histogram.

Speed s (mph)	Frequency	Frequency Density
$30 < s \leq 55$	16	
$55 < s \leq 65$	23	
$65 < s \leq 70$	35	
$70 < s \leq 75$	32	
$75 < s \leq 95$	8	



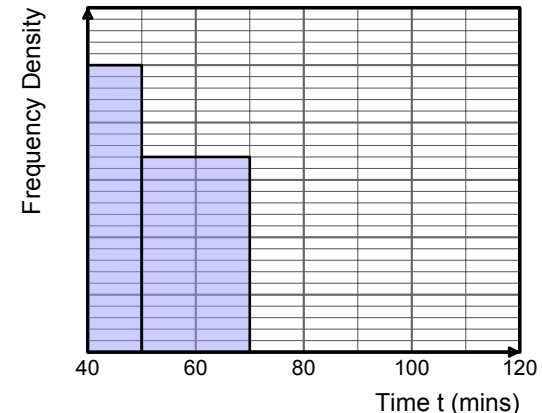
3. The histogram shows the weights of a number of children. Complete the frequency table.

Weight w (kg)	Frequency
$0 < w \leq 20$	
$20 < w \leq 25$	
$25 < w \leq 30$	
$30 < w \leq 45$	
$45 < w \leq 55$	



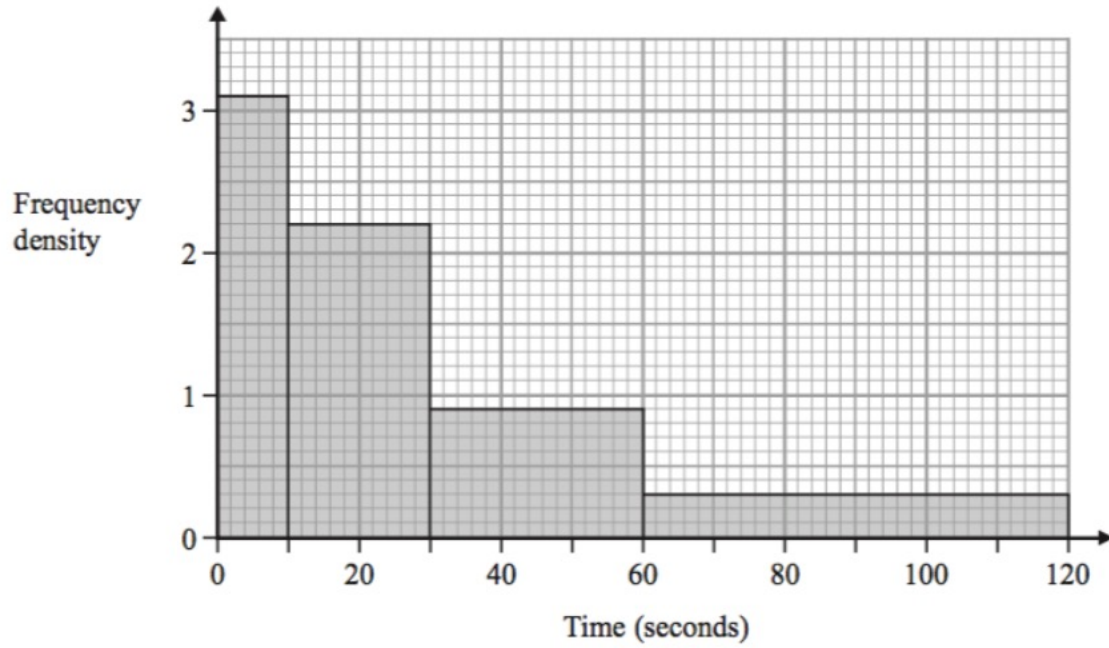
4. The table and histogram show the time taken by runners to complete a race. Complete the table and histogram.

Time t (mins)	Frequency
$40 < t \leq 50$	25
$50 < t \leq 70$	
$70 < t \leq 100$	81
$100 < t \leq 120$	12



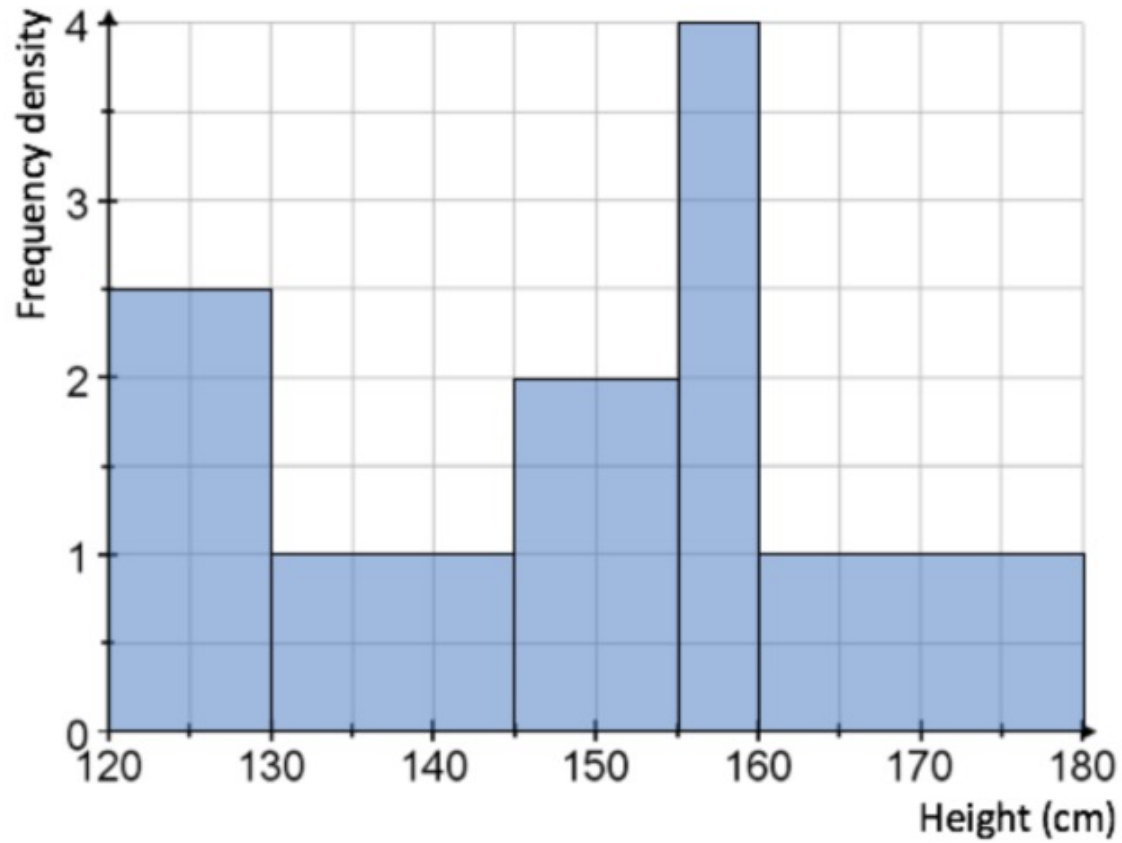
## Worked Example

Draw a frequency table from the histogram:



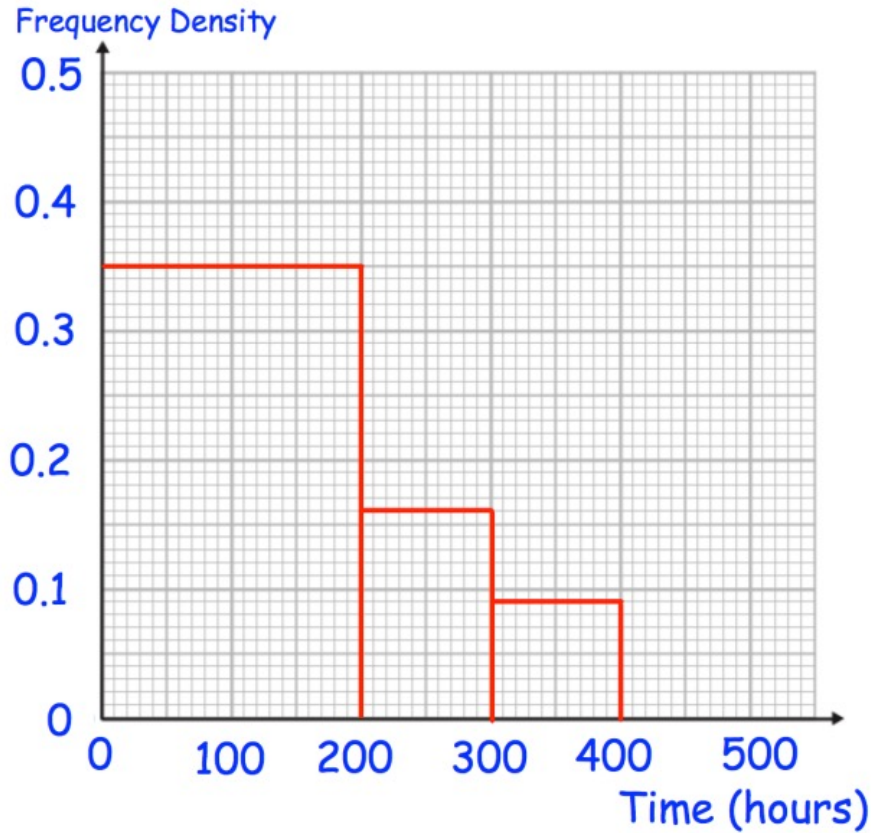
## Your Turn

Draw a frequency table from the histogram:



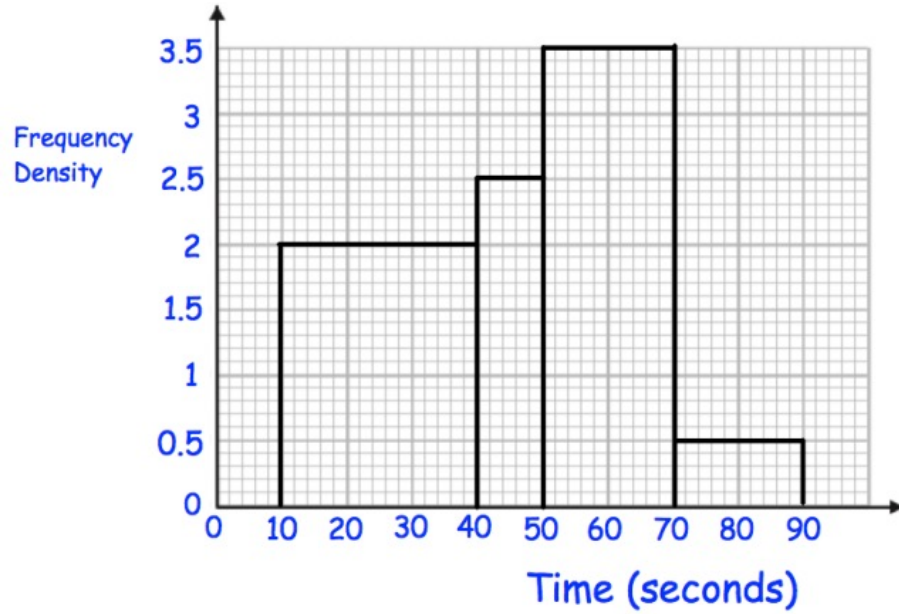
## Worked Example

- Estimate the number of pilots who have flown under 350 hours.
- Work out the percentage of pilots who have flown under 350 hours.



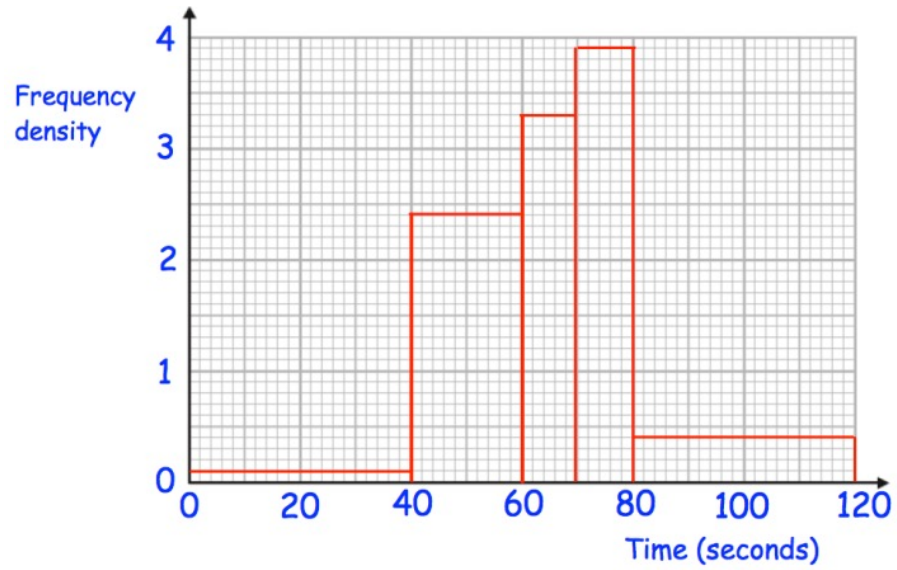
## Your Turn

- Estimate the number of students who took less than 60 seconds to complete the puzzle.
- Work out the percentage of students who took less than 60 seconds to complete the puzzle.



## Worked Example

Estimate the median time.



## Your Turn

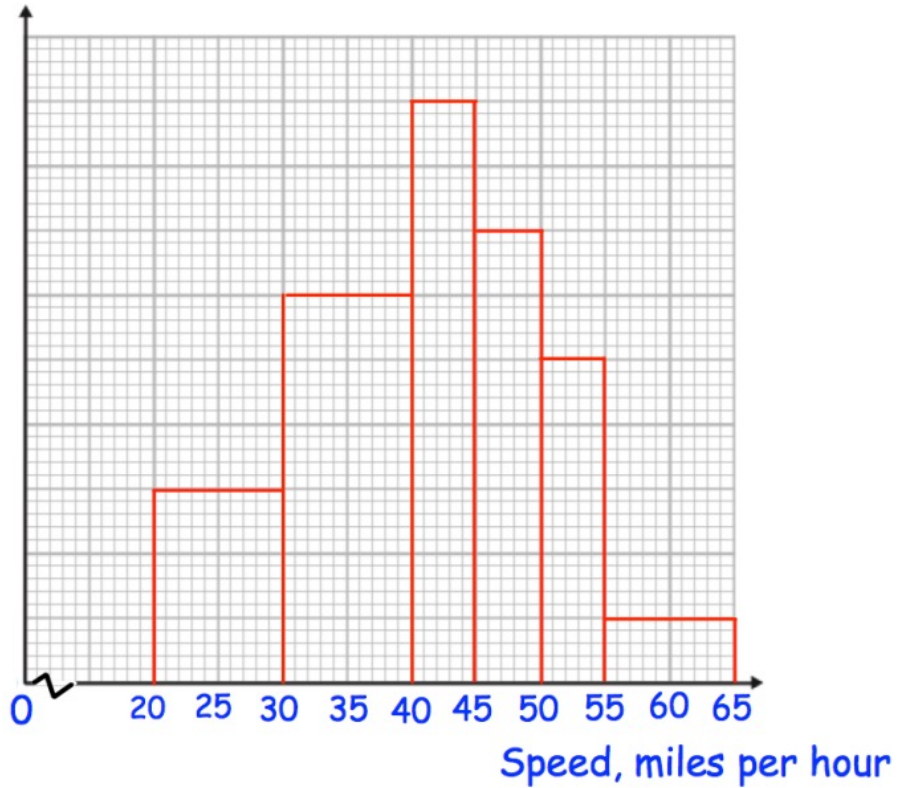
Estimate the median weight.





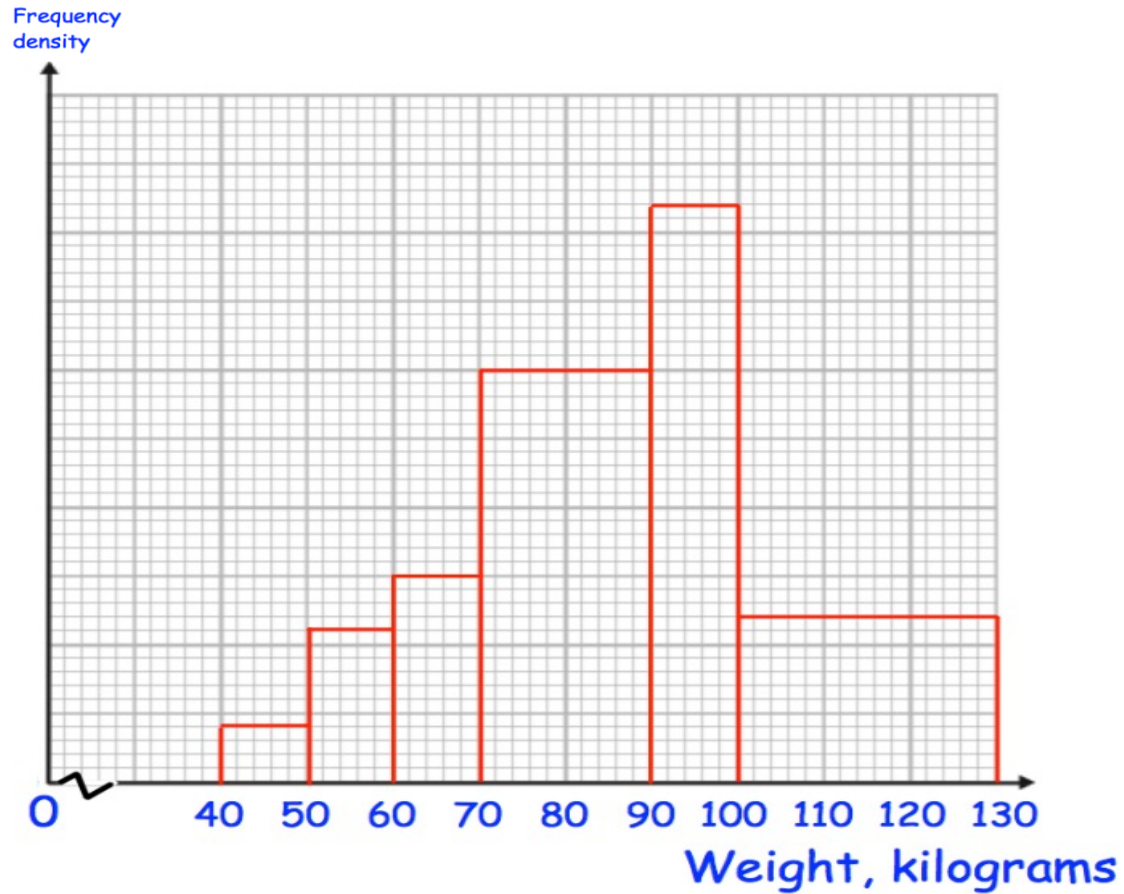
## Worked Example

There were 82 cars on the road. 14 cars were travelling over 50 mph. Estimate the number of cars that were travelling between 40 and 49 mph.



## Your Turn

There were 504 athletes measured. 45 athletes weigh under 60 kg. Estimate the number of athletes between 70 and 95 kg.



## Worked Example

Lenny collects the heights of 111 plants and records the data in the table below.

Height ( $y$ cm)	Frequency
$50 < y \leq 65$	15
$65 < y \leq 85$	28
$85 < y \leq 105$	26
$105 < y \leq 120$	15
$120 < y \leq 125$	27

A histogram was drawn and the class  $50 < y \leq 65$  was represented by a rectangle of width 6 cm and height 9 cm.

Calculate the width and the height of the rectangle representing the class  $85 < y \leq 105$ .

## Your Turn

Liam collects the heights of 98 plants and records the data in the table below.

Height ( $x$ cm)	Frequency
$0 < x \leq 20$	22
$20 < x \leq 30$	23
$30 < x \leq 45$	15
$45 < x \leq 55$	14
$55 < x \leq 60$	17
$60 < x \leq 80$	7

A histogram was drawn and the class  $45 < x \leq 55$  was represented by a rectangle of width 1.5 cm and height 3.5 cm.

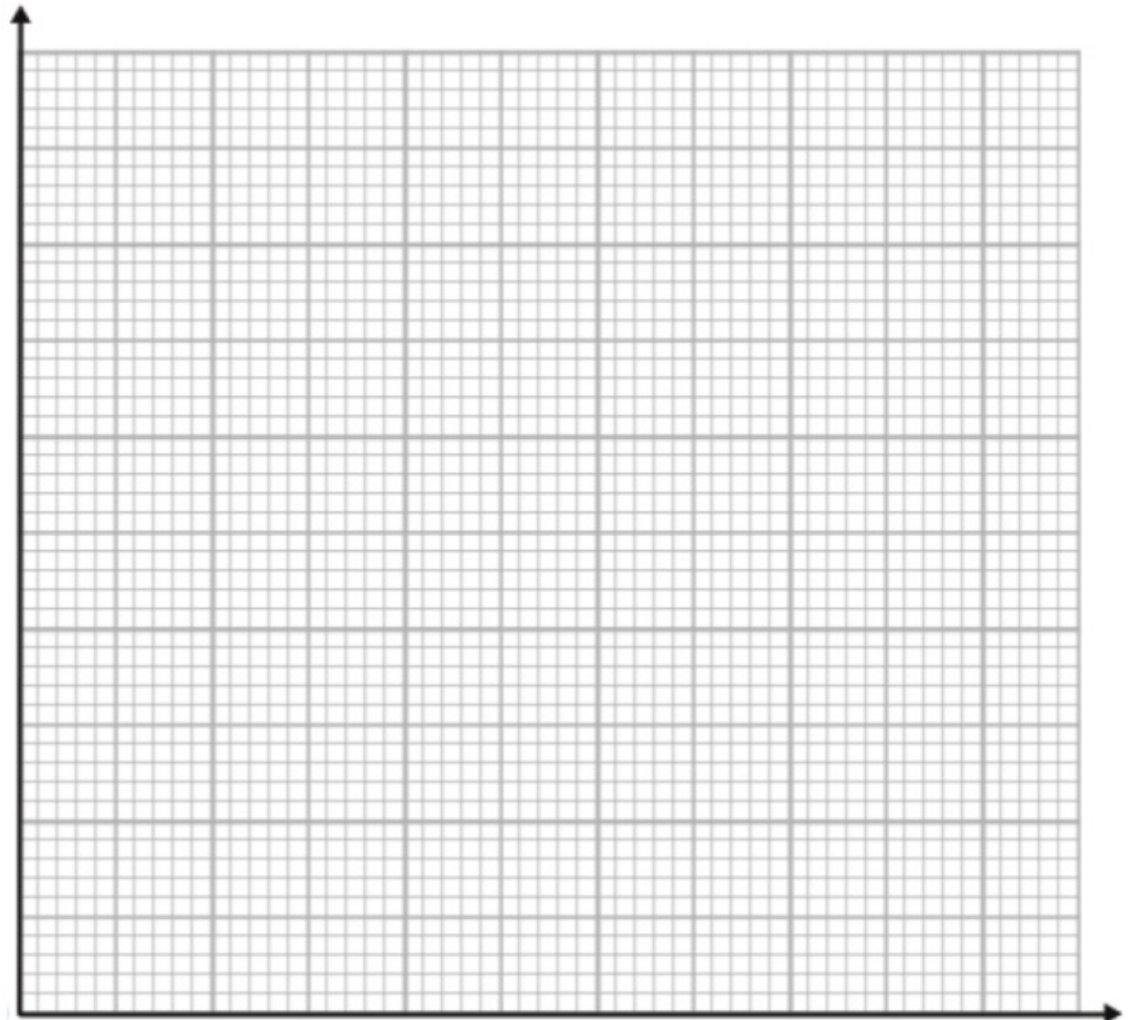
Calculate the width and the height of the rectangle representing the class  $30 < x \leq 45$ .

# Frequency Polygons

## Worked Example

Draw a frequency polygon for the information:

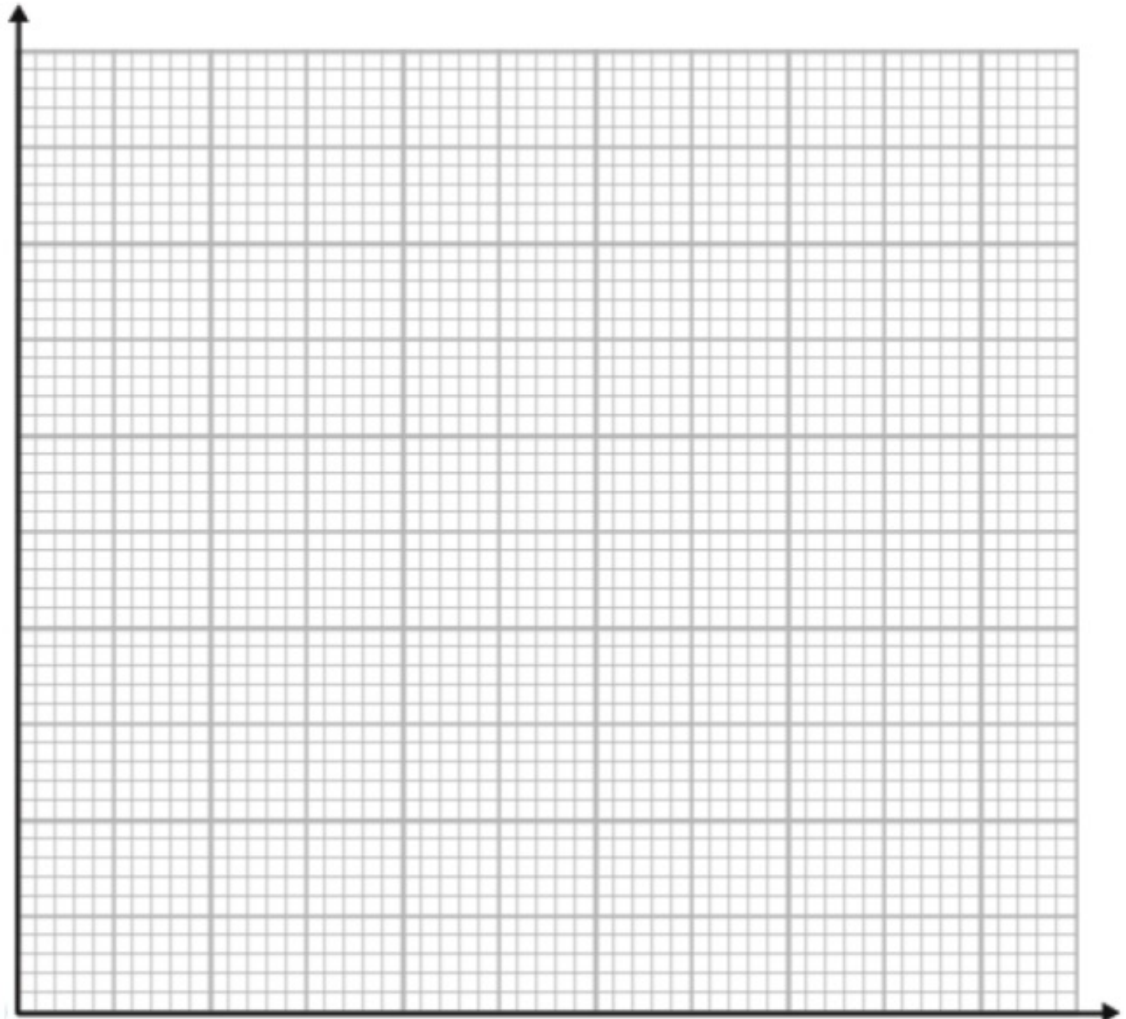
Lengths (cm)	Frequency
$0 < L \leq 0.5$	8
$0.5 < L \leq 1$	17
$1 < L \leq 1.5$	20
$1.5 < L \leq 2$	10
$2 < L \leq 2.5$	5



## Your Turn

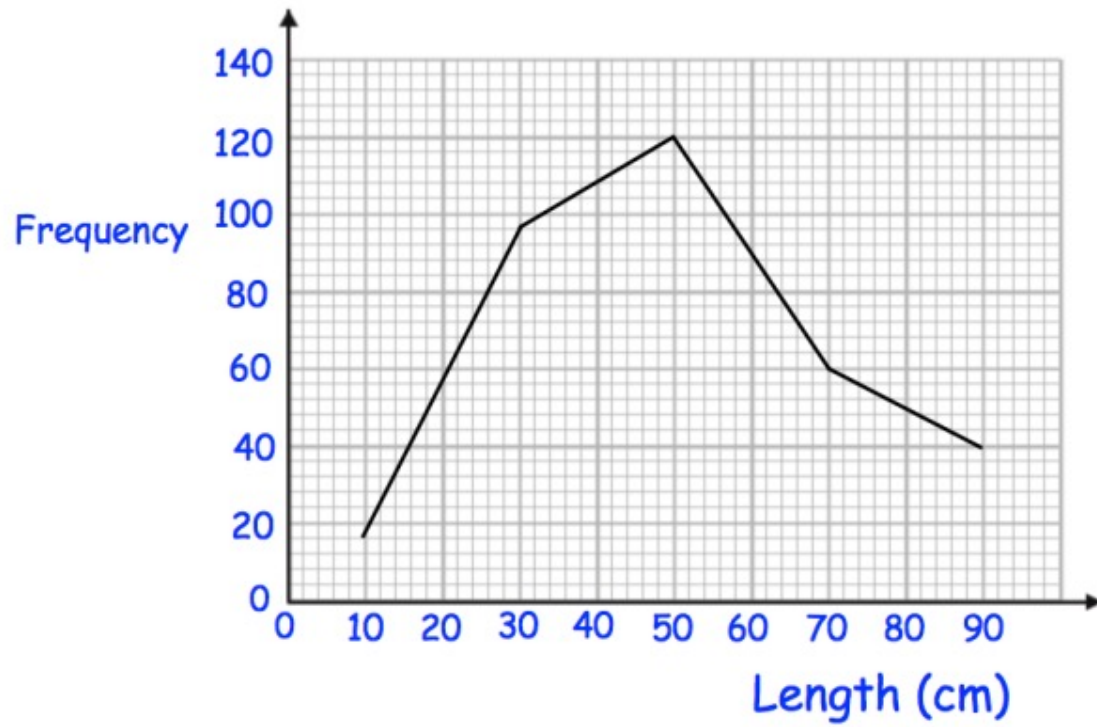
Draw a frequency polygon for the information:

Time (minutes)	Frequency
$0 < t \leq 10$	10
$10 < t \leq 20$	28
$20 < t \leq 30$	46
$30 < t \leq 40$	23
$40 < t \leq 50$	12



## Worked Example

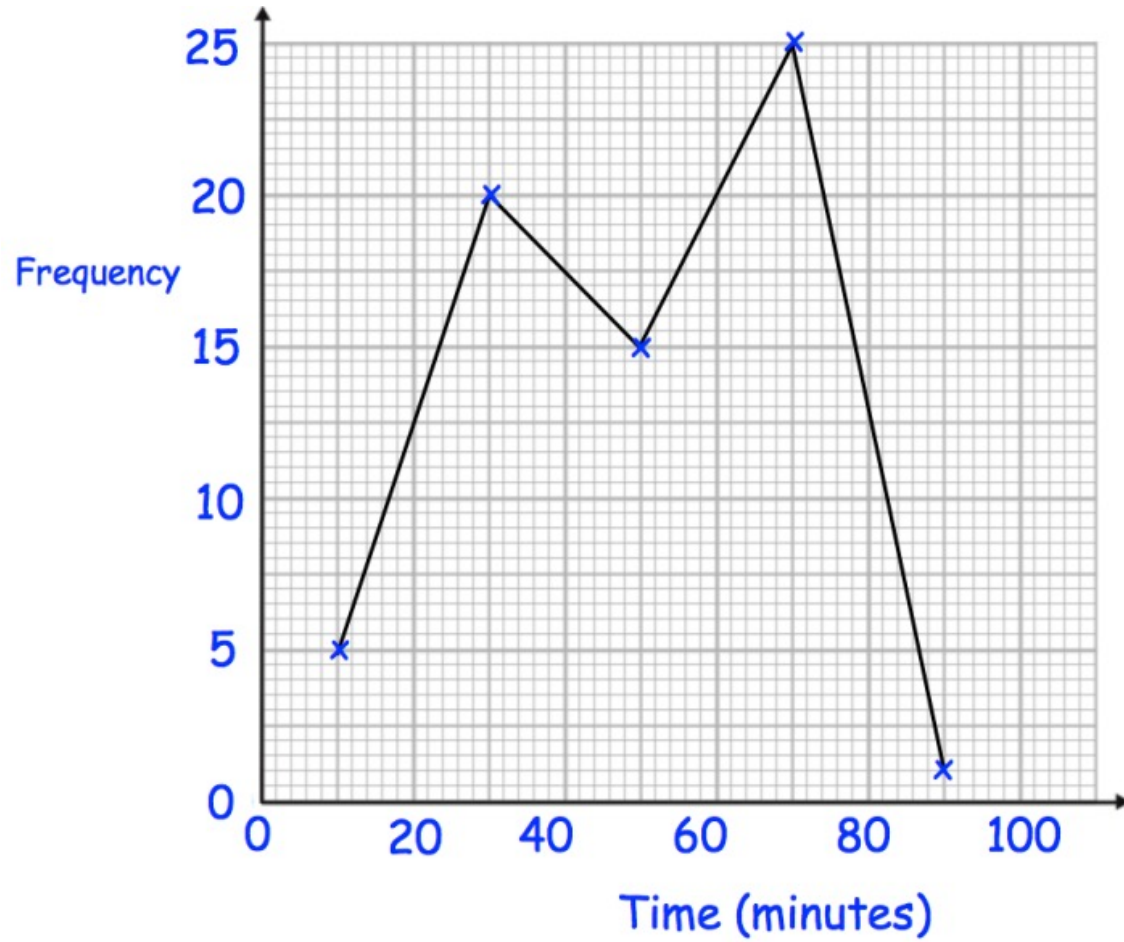
Estimate the mean length:





## Your Turn

Estimate the mean length:



## Cumulative Frequency Graphs

## Worked Example

James collects the running times of 90 athletes and records the data in the table below.

Time ( $y$ seconds)	Frequency
$40 < y \leq 45$	6
$45 < y \leq 50$	11
$50 < y \leq 55$	56
$55 < y \leq 60$	10
$60 < y \leq 65$	7

Complete the cumulative frequency table.

Time ( $y$  seconds)    Cumulative frequency

$40 < y \leq 45$

$40 < y \leq 50$

$40 < y \leq 55$

$40 < y \leq 60$

$40 < y \leq 65$

## Your Turn

John collects the running times of 50 athletes and records the data in the table below.

Time ( $y$ seconds)	Frequency
$15 < y \leq 20$	4
$20 < y \leq 25$	8
$25 < y \leq 30$	15
$30 < y \leq 35$	8
$35 < y \leq 40$	8
$40 < y \leq 45$	7

Complete the cumulative frequency table.

Time ( $y$  seconds)    Cumulative frequency

$15 < y \leq 20$

$15 < y \leq 25$

$15 < y \leq 30$

$15 < y \leq 35$

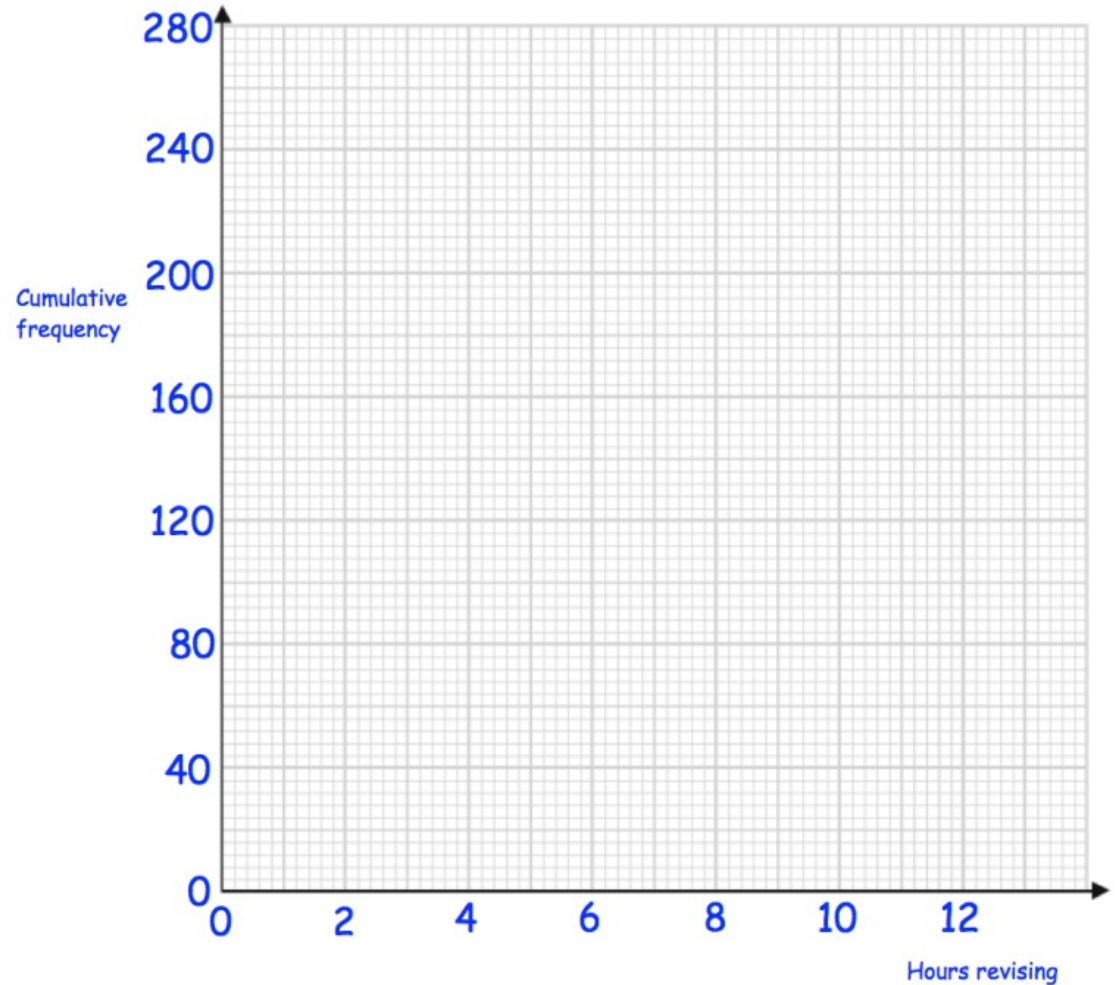
$15 < y \leq 40$

$15 < y \leq 45$

## Worked Example

Plot a cumulative frequency graph:

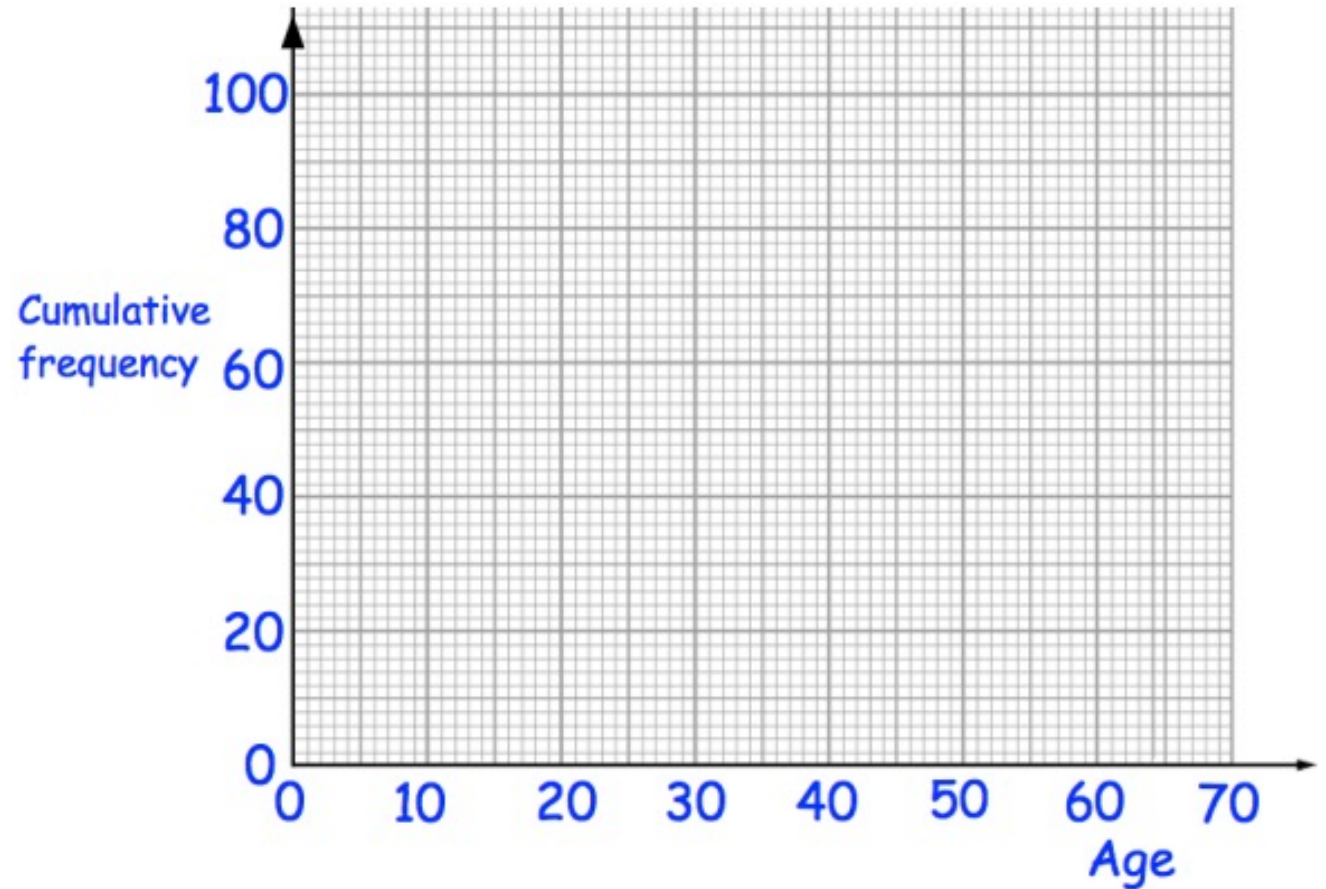
Number of hours (h)	Frequency
$0 < h \leq 2$	20
$2 < h \leq 4$	32
$4 < h \leq 6$	48
$6 < h \leq 8$	120
$8 < h \leq 10$	24
$10 < h \leq 12$	16



## Your Turn

Plot a cumulative frequency graph:

Age, $x$ years	Frequency
$20 < x \leq 30$	12
$30 < x \leq 40$	30
$40 < x \leq 50$	28
$50 < x \leq 60$	22
$60 < x \leq 70$	8

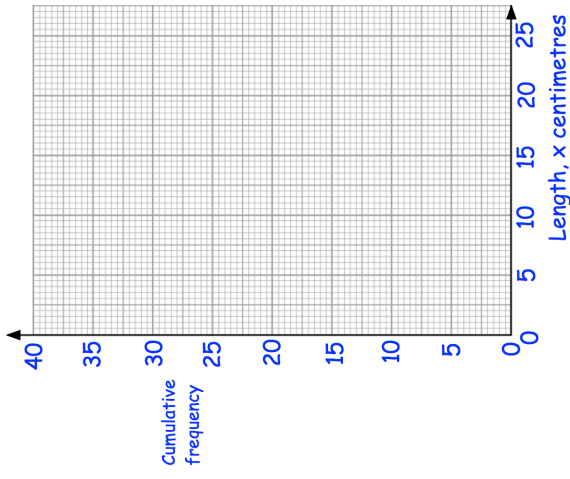


Question 1: The table shows information about the lengths of a type of fish caught in a lake

- (a) Complete the cumulative frequency table  
 (b) Draw a cumulative frequency graph for your table.

Length, x cm	Frequency
$0 < x \leq 5$	3
$5 < x \leq 10$	10
$10 < x \leq 15$	21
$15 < x \leq 20$	4
$20 < x \leq 25$	1

Length, x cm	Cumulative Frequency
$0 < x \leq 5$	
$0 < x \leq 10$	
$0 < x \leq 15$	
$0 < x \leq 20$	
$0 < x \leq 25$	

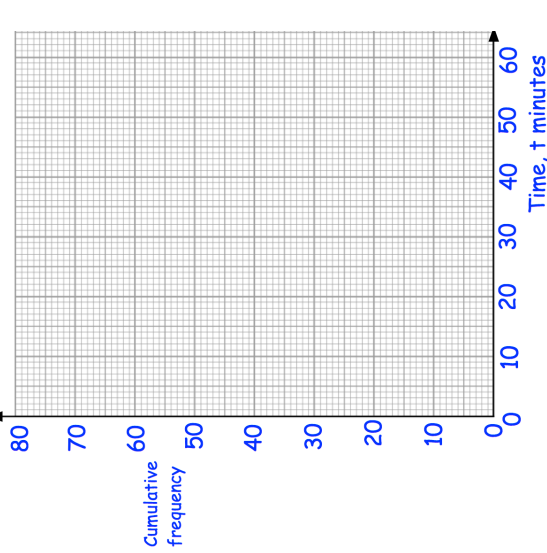


Question 2: The table shows information about the time taken to complete a puzzle

- (a) Complete the cumulative frequency table  
 (b) Draw a cumulative frequency graph for your table.

Time, t minutes	Frequency
$0 < t \leq 10$	3
$10 < t \leq 20$	11
$20 < t \leq 30$	15
$30 < t \leq 40$	27
$40 < t \leq 50$	16
$50 < t \leq 60$	8

Time, t minutes	Cumulative Frequency
$0 < t \leq 10$	
$0 < t \leq 20$	
$0 < t \leq 30$	
$0 < t \leq 40$	
$0 < t \leq 50$	
$0 < t \leq 60$	

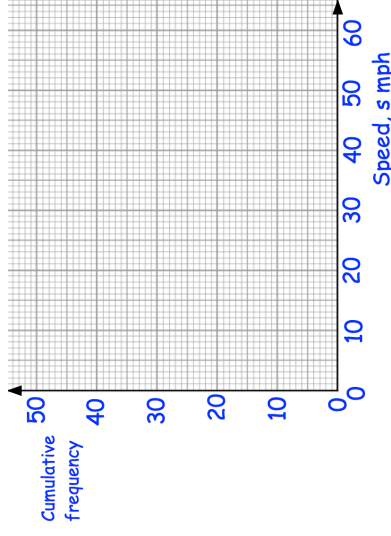


Question 3: The table shows information about the speed of vehicles on a road.

- (a) Complete the cumulative frequency table  
 (b) Draw a cumulative frequency graph for your table.

Speed, s mph	Frequency
$0 < s \leq 10$	2
$10 < s \leq 20$	4
$20 < s \leq 30$	14
$30 < s \leq 40$	21
$40 < s \leq 50$	9

Speed, s mph	Cumulative Frequency
$0 < s \leq 10$	
$0 < s \leq 20$	
$0 < s \leq 30$	
$0 < s \leq 40$	
$0 < s \leq 50$	

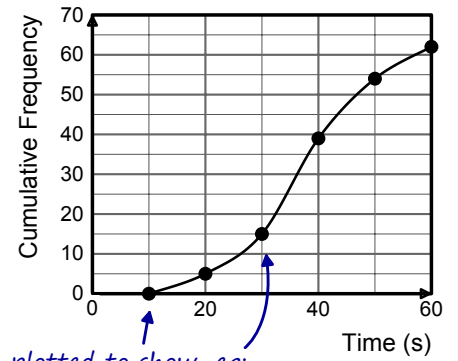


# cumulative frequency graphs

## example

Construct a cumulative frequency graph for this data set:

Time $t$ (s)	Frequency	Cumulative Frequency
$10 \leq t < 20$	5	5
$20 \leq t < 30$	10	15
$30 \leq t < 40$	24	39
$40 \leq t < 50$	15	54
$50 \leq t < 60$	8	62



*Points are plotted to show, eg:  
There are 0 values less than 10 seconds.  
There are 15 values less than 30 seconds.*

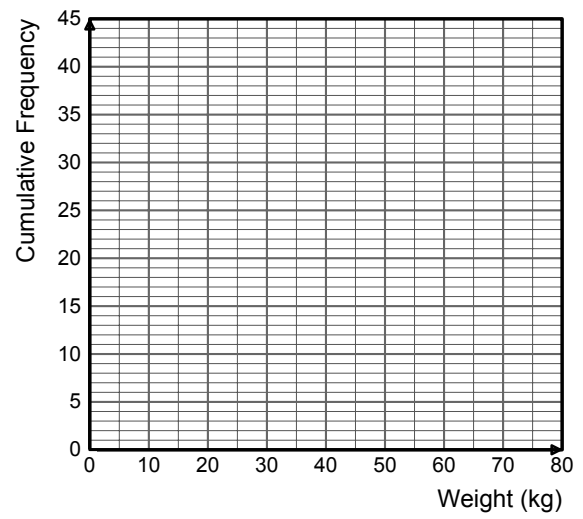
## exercise

- The frequency table shows the weights of 40 dogs.
  - Complete the cumulative frequency table and graph.

Weight $w$ (kg)	Frequency
$0 \leq w < 20$	6
$20 \leq w < 30$	9
$30 \leq w < 40$	14
$40 \leq w < 60$	7
$60 \leq w < 80$	4

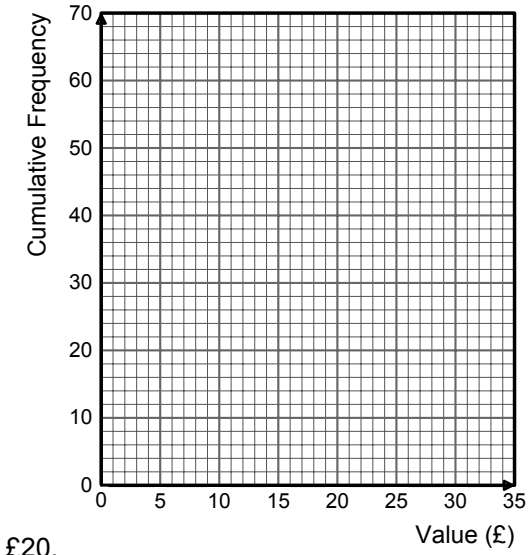
Weight $w$ (kg)	Cumulative Frequency
$w < 20$	6
$w < 30$	15
$w < 40$	
$w < 60$	
$w < 80$	

- Use your graph to estimate the number of dogs that weighed less than 35kg.



- The table shows the values of 60 collectable stamps.
  - Complete the table and cumulative frequency graph.

Value $v$ (£)	Freq.	C. Freq.
$0 \leq v < 5$	22	
$5 \leq v < 10$	17	
$10 \leq v < 15$	8	
$15 \leq v < 20$	5	
$20 \leq v < 25$	4	
$25 \leq v < 30$	4	

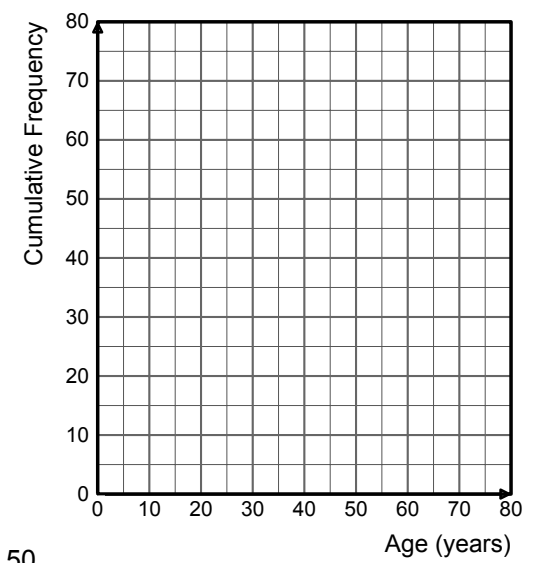


- Use your graph to estimate the number of stamps:

- valued at less than £8.
- valued at £8 or more.
- valued between £8 and £20.

- The table shows the ages of people watching a film at the cinema.
  - Complete the table and cumulative frequency graph.

Age $a$ (years)	Freq.	C. Freq.
$0 \leq a \leq 5$	6	
$5 < a \leq 10$	20	
$10 < a \leq 20$	8	
$20 < a \leq 30$	8	
$30 < a \leq 40$	17	
$40 < a \leq 60$	9	
$60 < a \leq 80$	6	



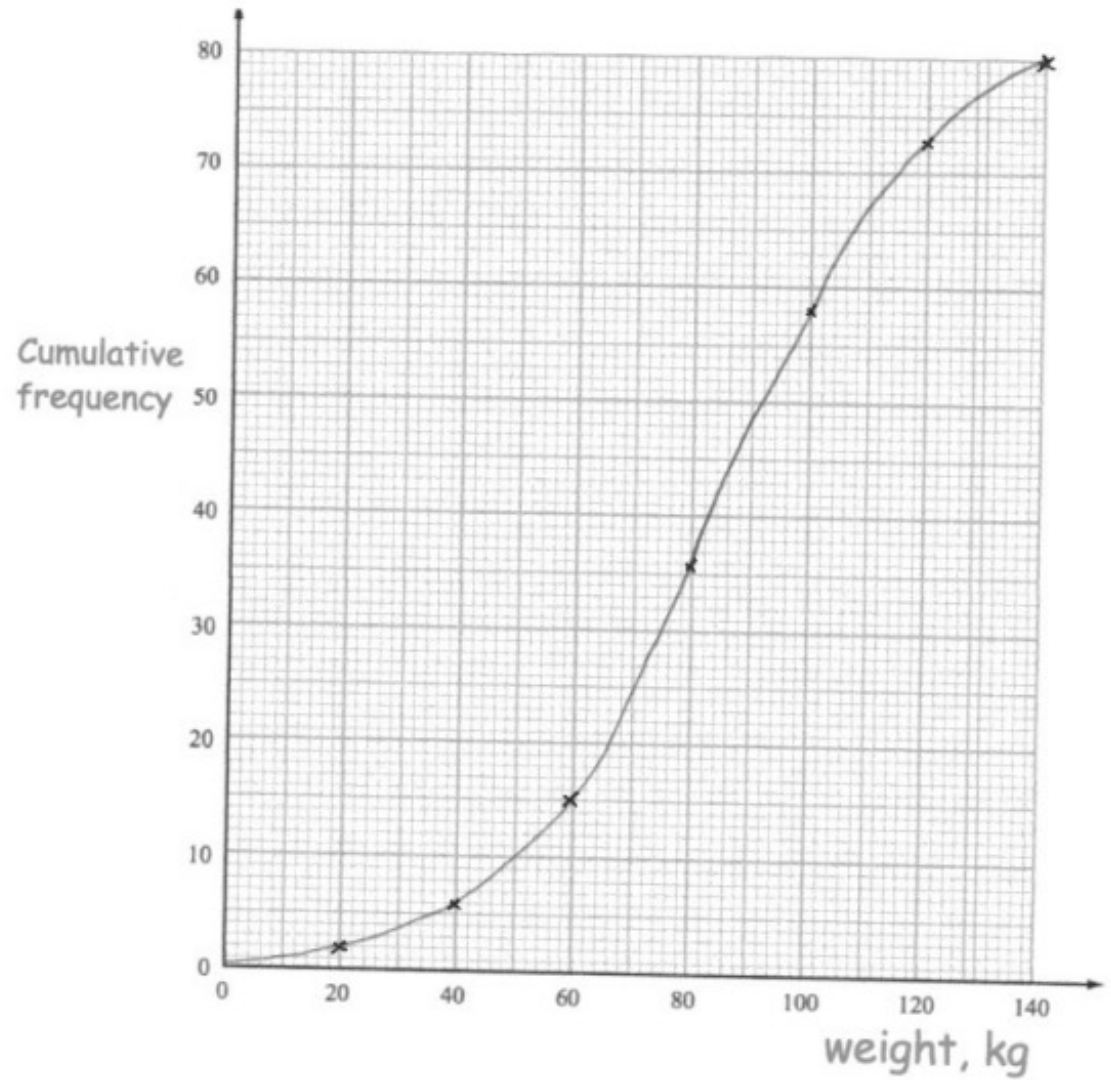
- Use your graph to estimate the number of people:

- aged 25 or less.
- aged 50 or less.
- aged between 25 and 50.
- aged more than 35.

## Worked Example

Using the cumulative frequency graph, estimate the:

- Number of people who weight less than 40 kg
- Number of people who weigh more than 100 kg

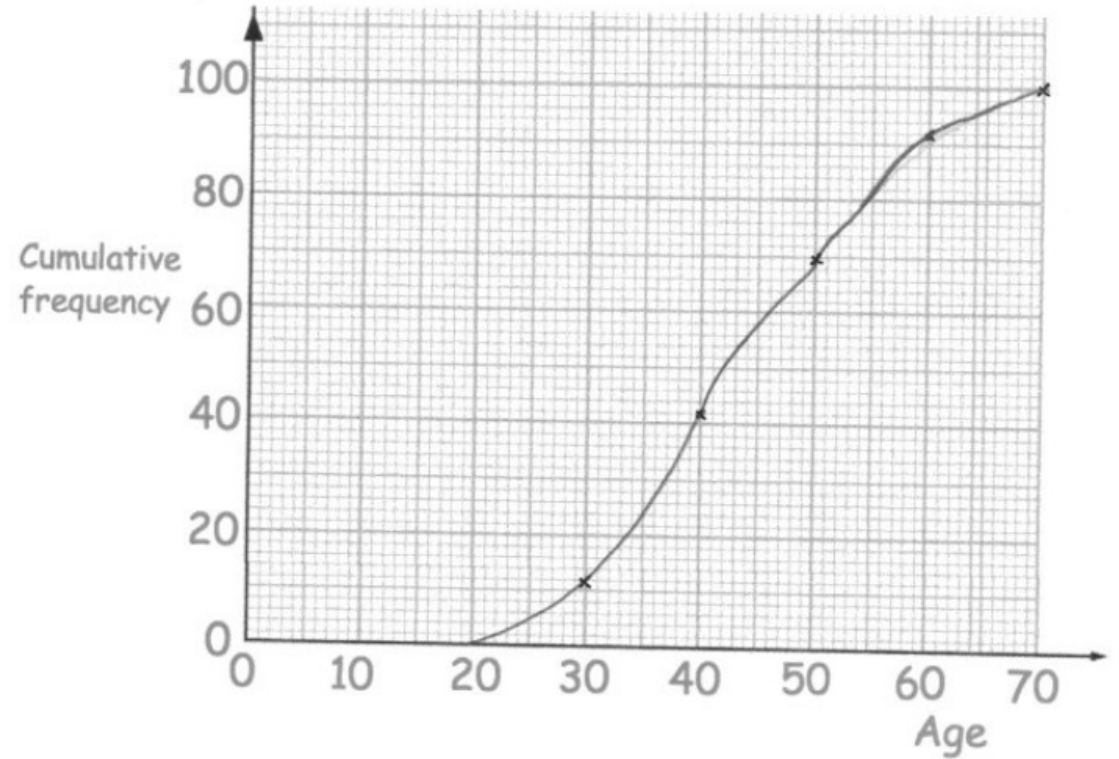




## Your Turn

Using the cumulative frequency graph, estimate the:

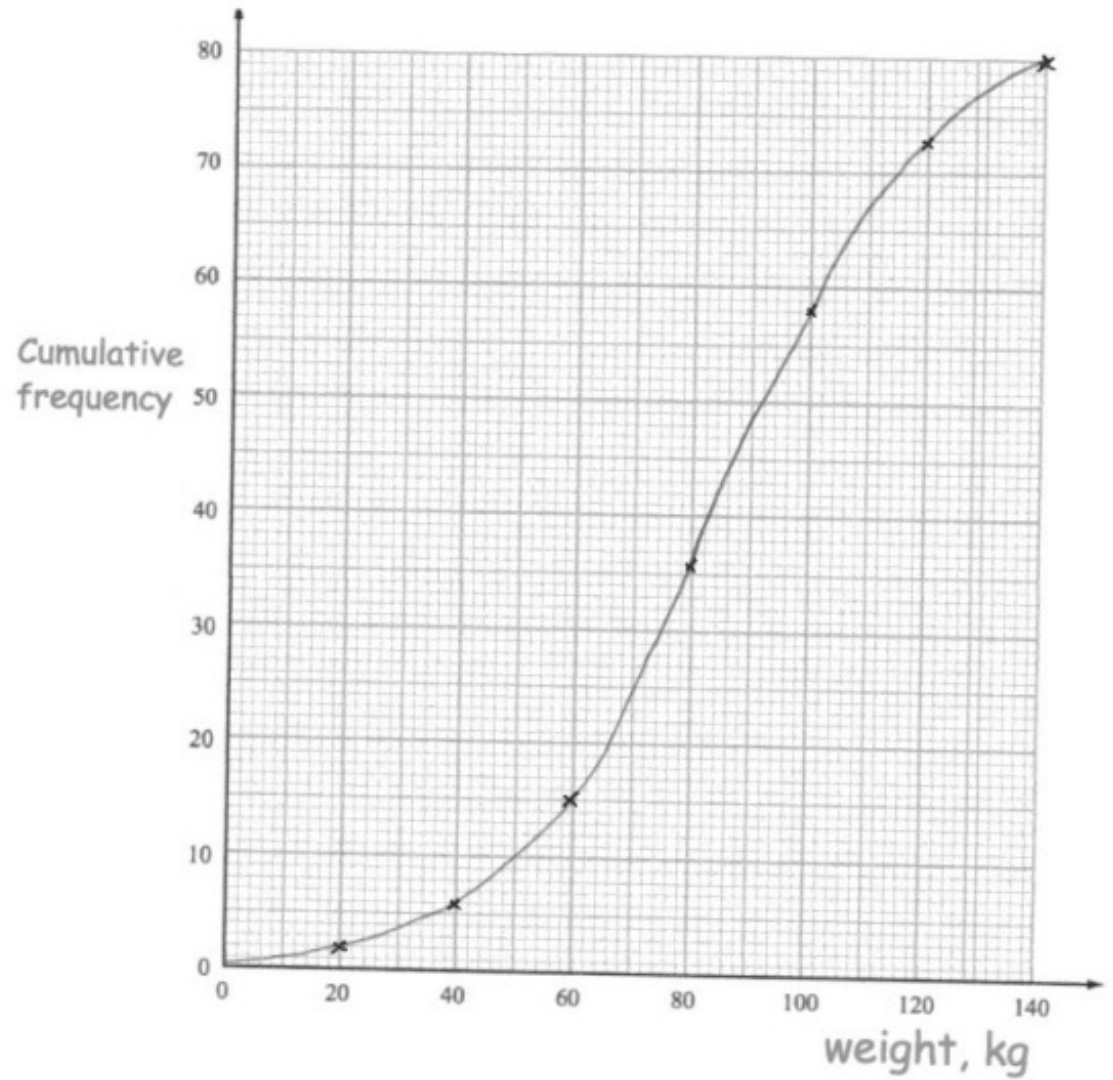
- a) Number of people younger than 35
- b) Number of people older than 62



## Worked Example

Using the cumulative frequency graph, estimate the:

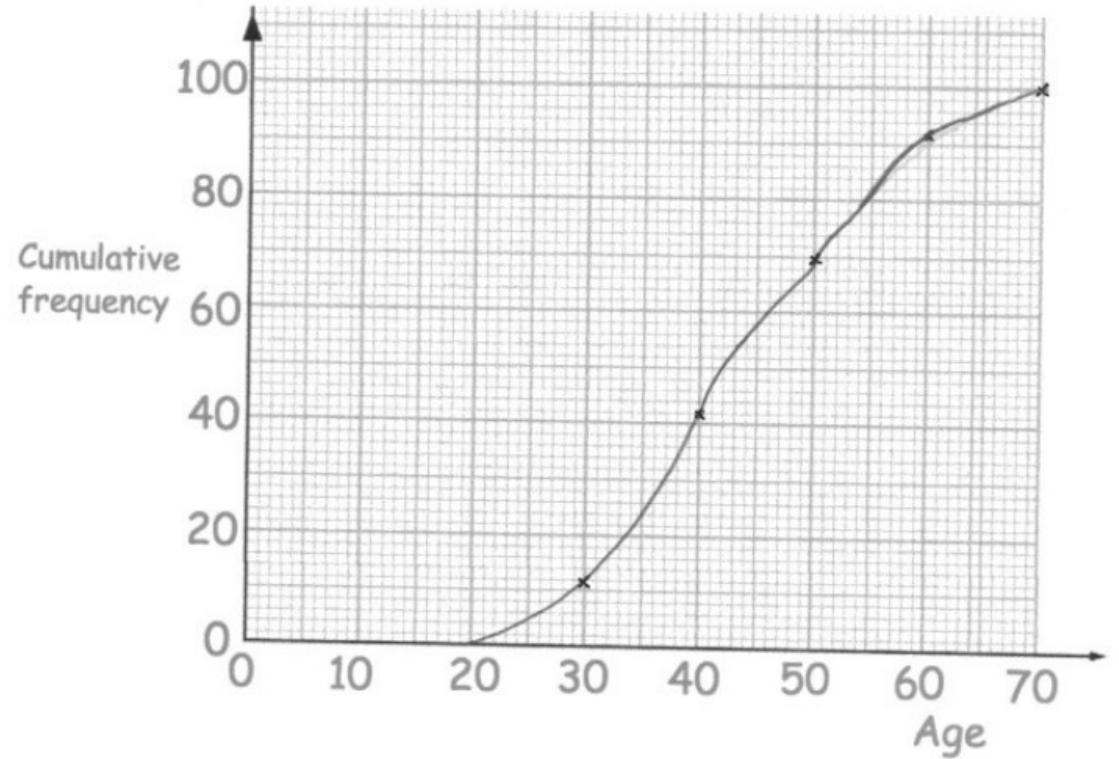
- Median weight
- Lower quartile weight
- Upper quartile weight
- Interquartile range of the weights



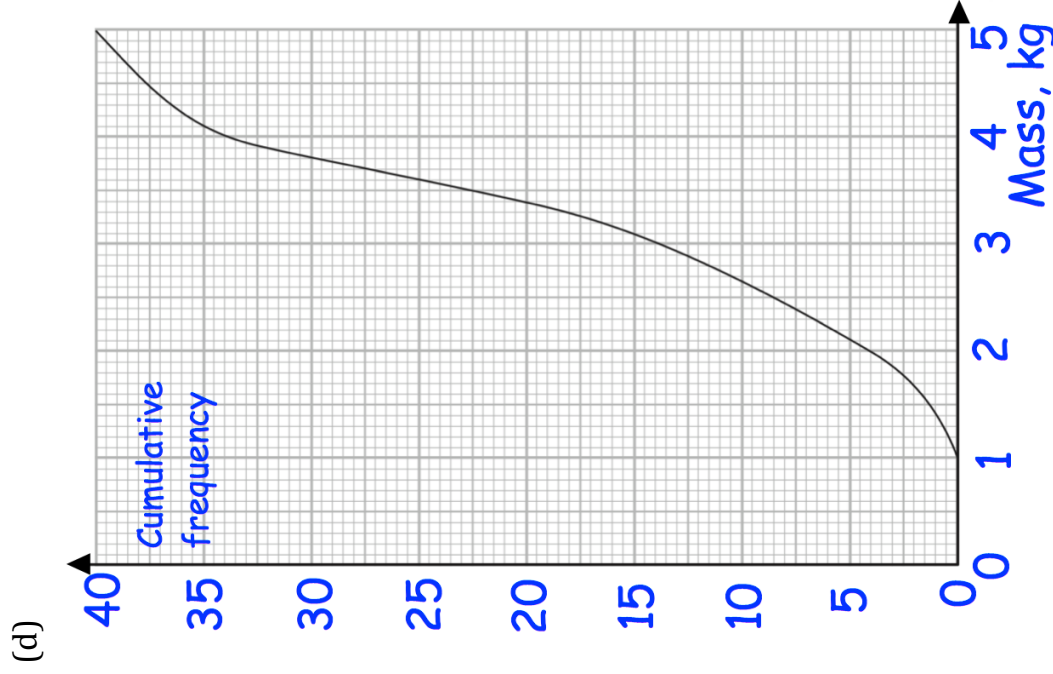
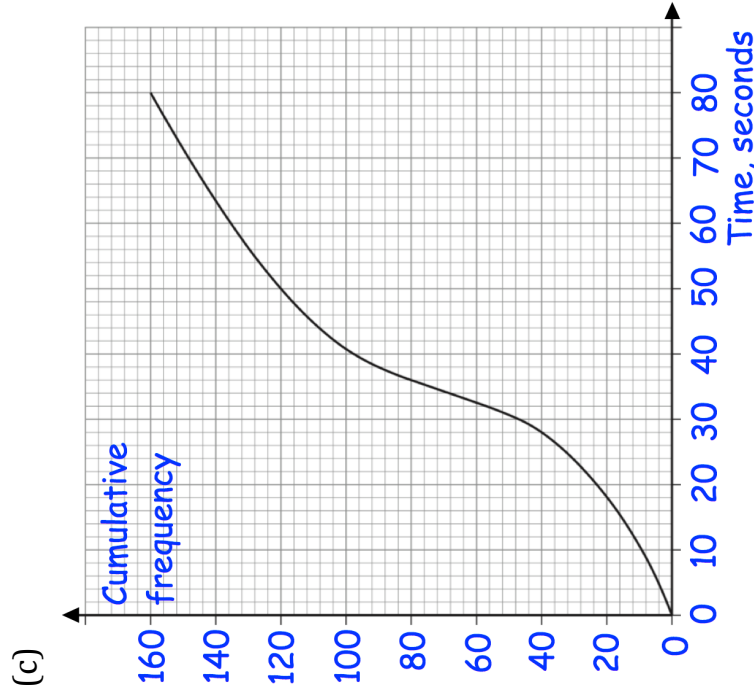
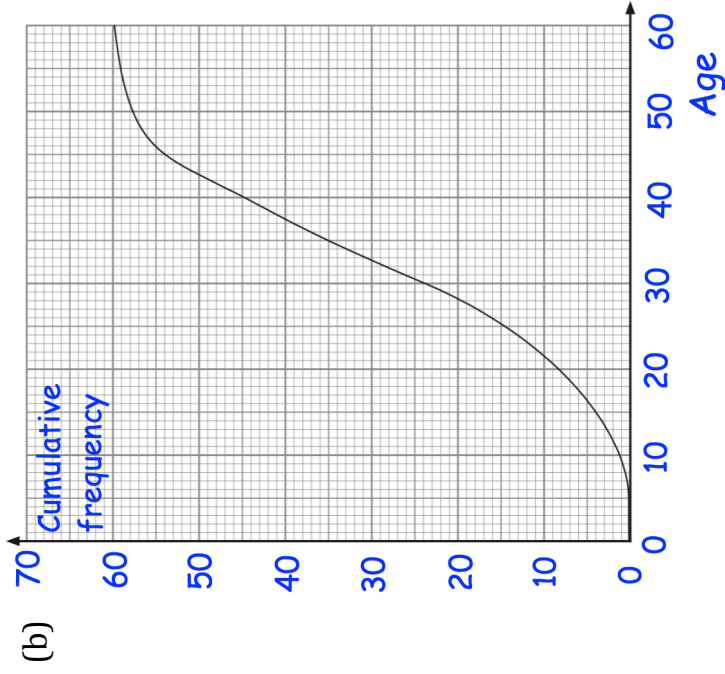
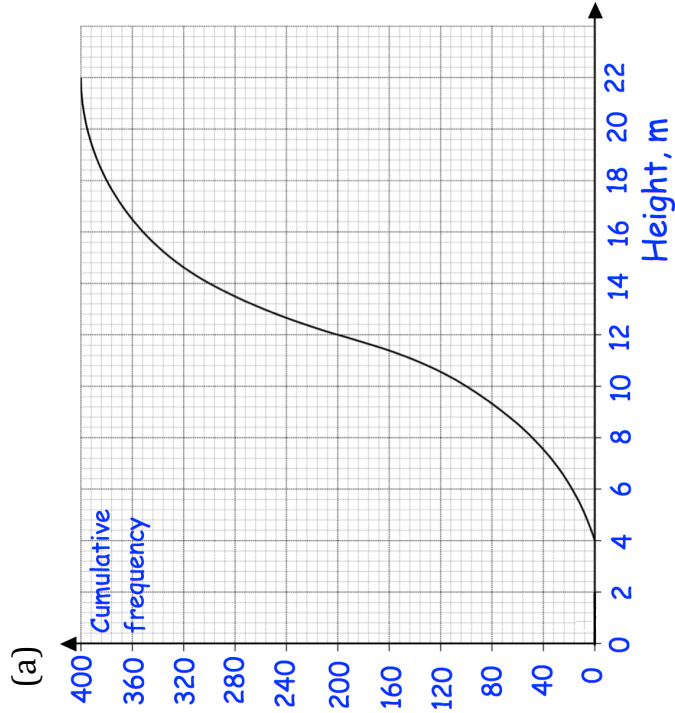
## Your Turn

Using the cumulative frequency graph, estimate the:

- Median age
- Lower quartile age
- Upper quartile age
- Interquartile range of the ages



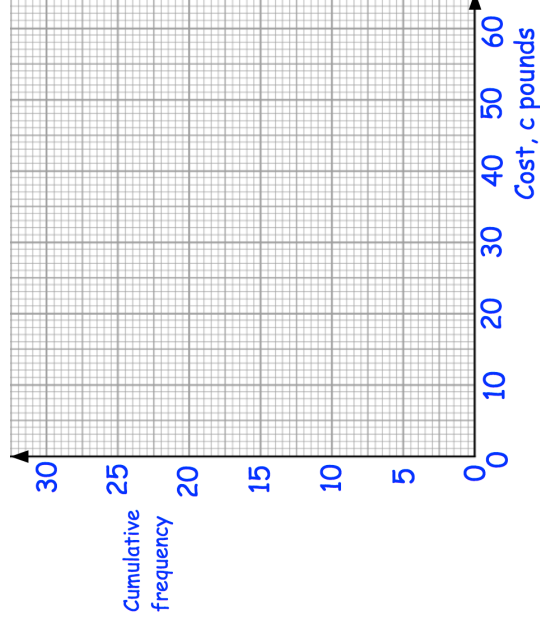
Question 4: Use each cumulative frequency graph to find an estimate for the median.



Question 5: For each table below (i) draw a cumulative frequency graph and (ii) use your graph to find an estimate of the median

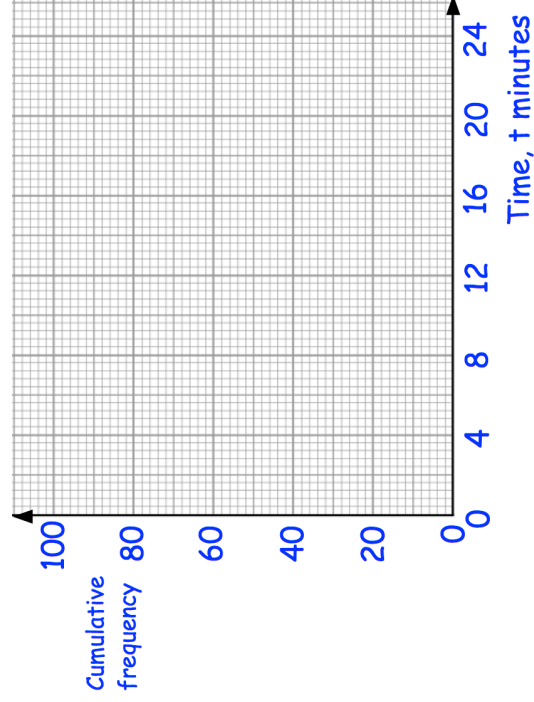
(a)

Cost, c pounds	Frequency
$0 < c \leq 10$	2
$10 < c \leq 20$	7
$20 < c \leq 30$	12
$30 < c \leq 40$	6
$40 < c \leq 50$	2
$50 < c \leq 60$	1



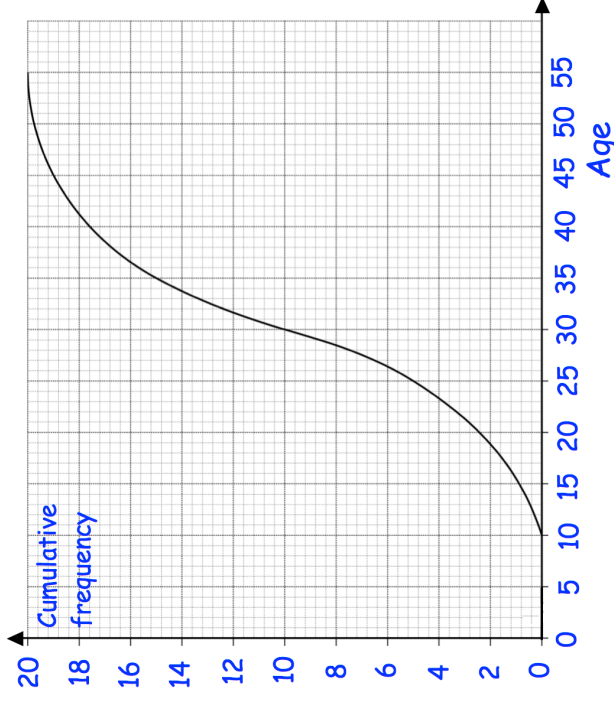
(b)

Time, t minutes	Frequency
$0 < t \leq 4$	5
$4 < t \leq 8$	11
$8 < t \leq 12$	19
$12 < t \leq 16$	25
$16 < t \leq 20$	31
$20 < t \leq 24$	9

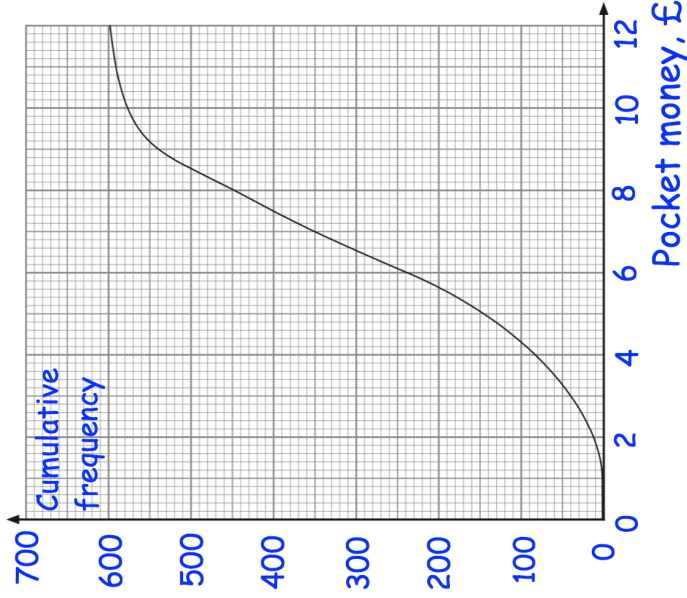


Question 6: Use each cumulative frequency graph to find the (i) lower quartile (ii) upper quartile (iii) interquartile range

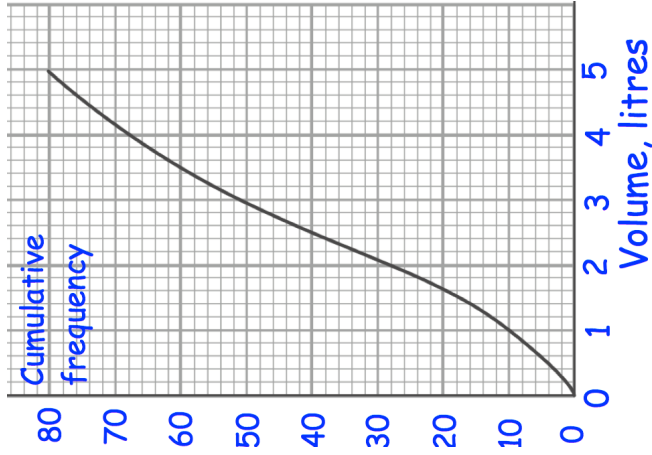
(a)



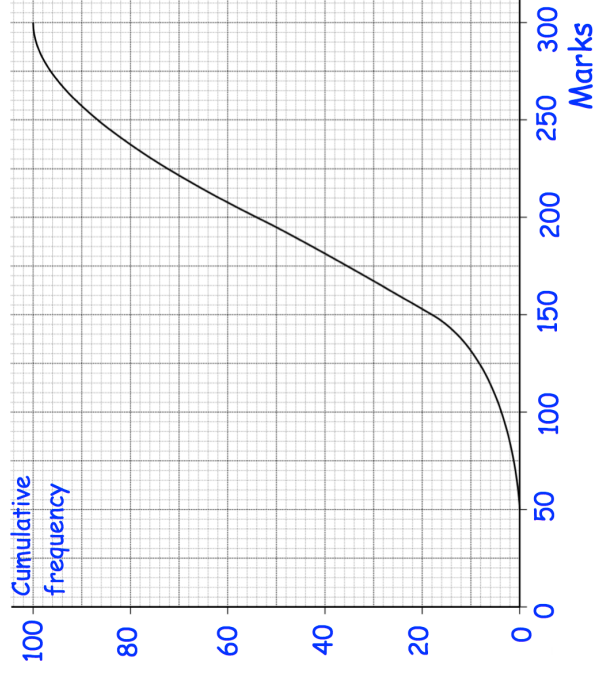
(b)



(c)

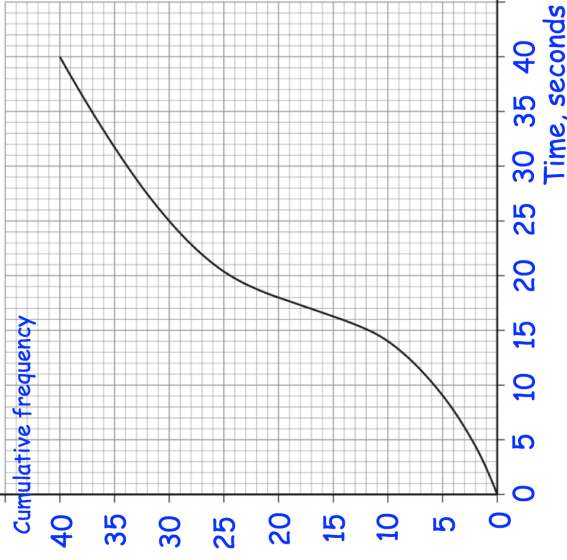


(d)



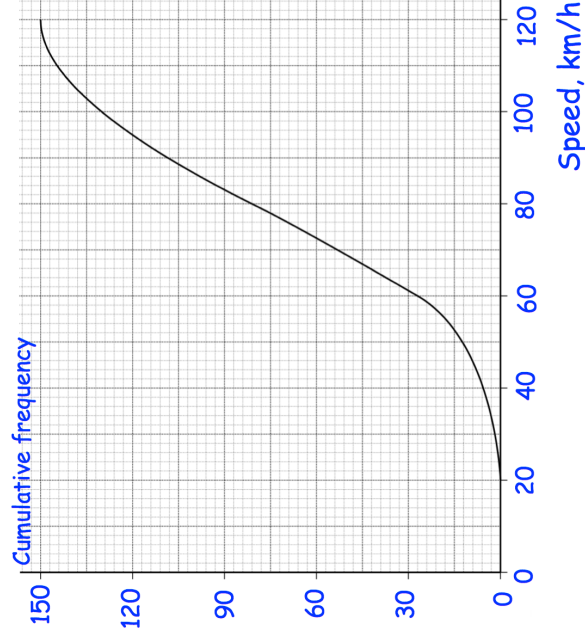
Question 7: The graph shows information about the time taken to solve a puzzle.

- (a) How many people took less than 30 seconds?
- (b) How many people took less than 10 seconds?
- (c) How many people took longer than 25 seconds?
- (d) How many people took longer than 35 seconds?
- (e) The fastest 10 people completed the puzzle in under how many seconds?
- (f) The slowest 2 people completed the puzzle in longer than how many seconds?



Question 8: The graph shows information about the speed of cars on a road.

- (a) How many cars travelled under 50km/h?
- (b) How many cars travelled over 110km/h?
- (c) 42 cars were exceeding the speed limit. What is the speed limit?
- (d) Mr Rodgers says 18% of the cars were travelling too slowly on this road. Below what speed does he feel is too slow?



# cumulative frequency graphs - medians & quartiles

## example

The cumulative frequency graph shows the heights of some plants. Find estimates for  $Q_1$ ,  $Q_2$  and  $Q_3$  and draw a box plot for the data.

*There are 80 pieces of data.  
The quartiles can be found at:*

$$Q_1 : \frac{1}{4} \times 80 = 20\text{th value}$$

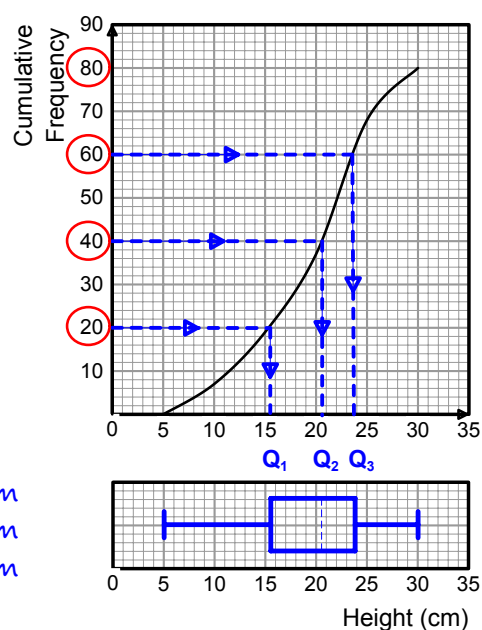
$$Q_2 : \frac{1}{2} \times 80 = 40\text{th value}$$

$$Q_3 : \frac{3}{4} \times 80 = 60\text{th value}$$

$$Q_1 = 16\text{cm}$$

$$Q_2 = 21\text{cm}$$

$$Q_3 = 24\text{cm}$$

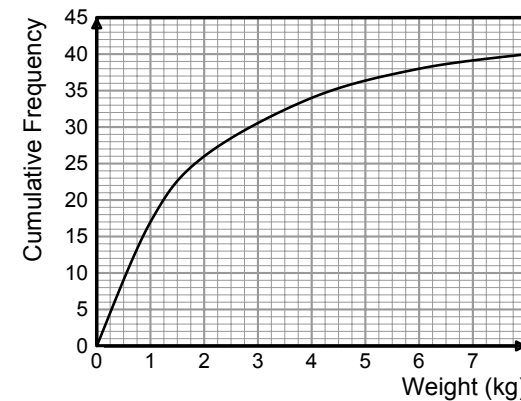


2. The cumulative frequency graph shows the weight of a number of parcels.

a) Find estimates for:

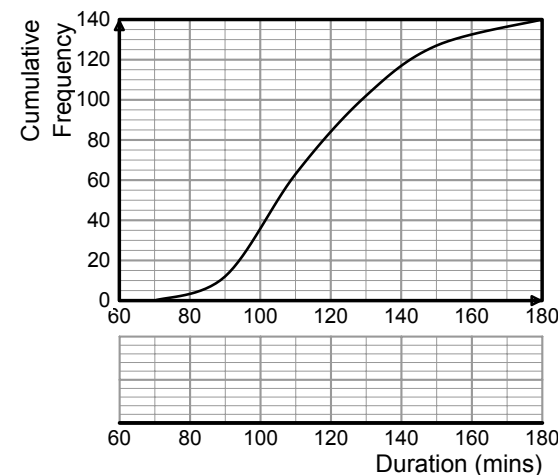
- the lower quartile
- the median
- the upper quartile

b) True or false? 80% of the parcels weighed less than 4kg.



3. The cumulative frequency graph shows the duration of a number of films, in minutes.

Find estimates for the median and quartiles and construct a boxplot in the space below the graph.

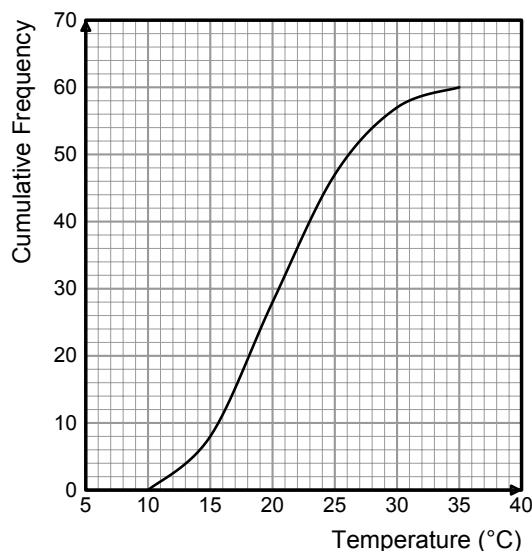


## exercise

1. The cumulative frequency graph shows the maximum temperature recorded on 60 days.

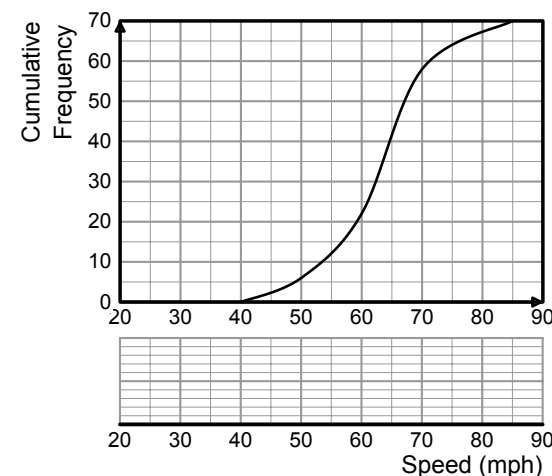
Find estimates for:

- the median
- $Q_1$
- $Q_3$
- the interquartile range
- the range



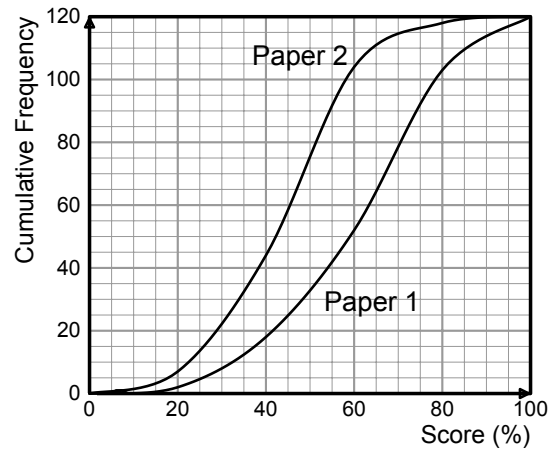
4. The cumulative frequency graph shows the speeds of vehicles on a motorway.

Find estimates for the median and quartiles and construct a boxplot in the space below the graph.



5. The cumulative frequency graph shows the scores of a year group for two exam papers.

- a) Which paper did the pupils find easier? Justify your answer.
- b) Certificates were given to pupils who achieved over 70% on a paper. Estimate the number of certificates given for each paper.

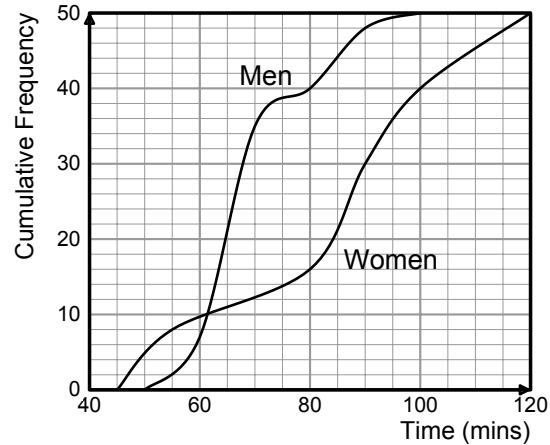


6. The cumulative frequency graph shows the times taken by 50 men and 50 women to complete a race.

- a) Complete the table:

	Men	Women
Median		
Range		
IQR		

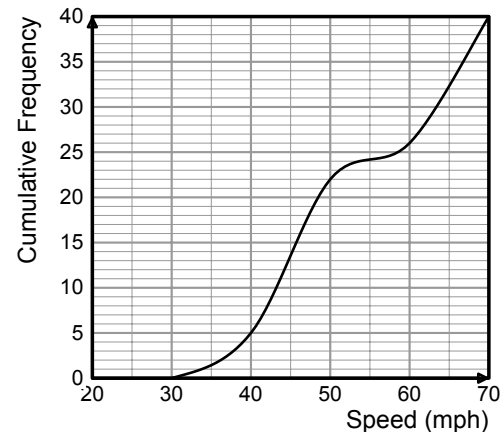
- b) Compare the two data sets.



7. The cumulative frequency graph shows the speeds of some vehicles on a road.

One of the cars is to be chosen at random. Work out estimates for these probabilities:

- a)  $P(\text{speed} \leq 45)$
- b)  $P(\text{speed} \leq 65)$
- c)  $P(45 < \text{speed} \leq 65)$



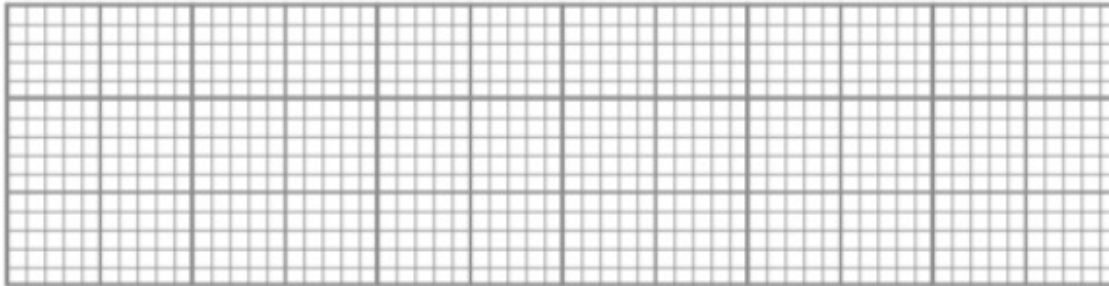


## Box Plots

## Worked Example

Draw a box plot to show this information:

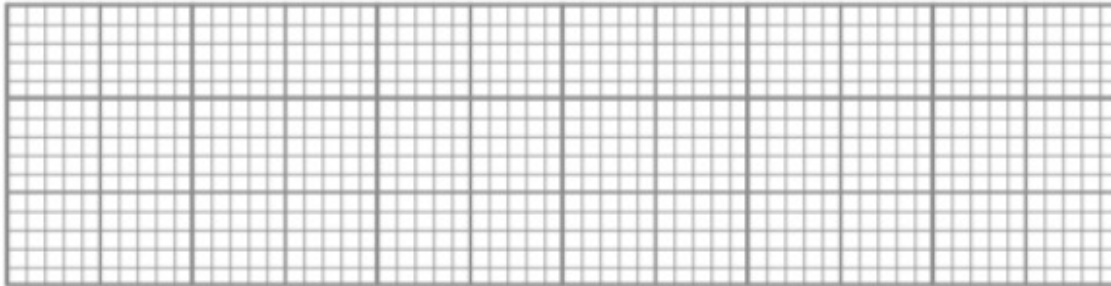
Lowest	51 kg
Lower Quartile	60 kg
Median	71 kg
Upper Quartile	74 kg
Highest	83 kg



## Your Turn

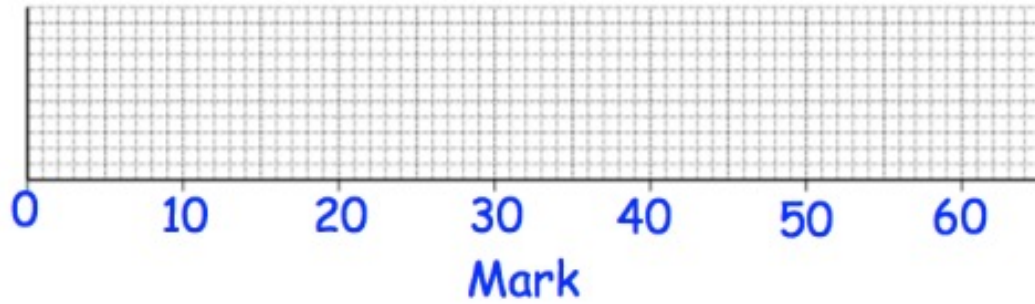
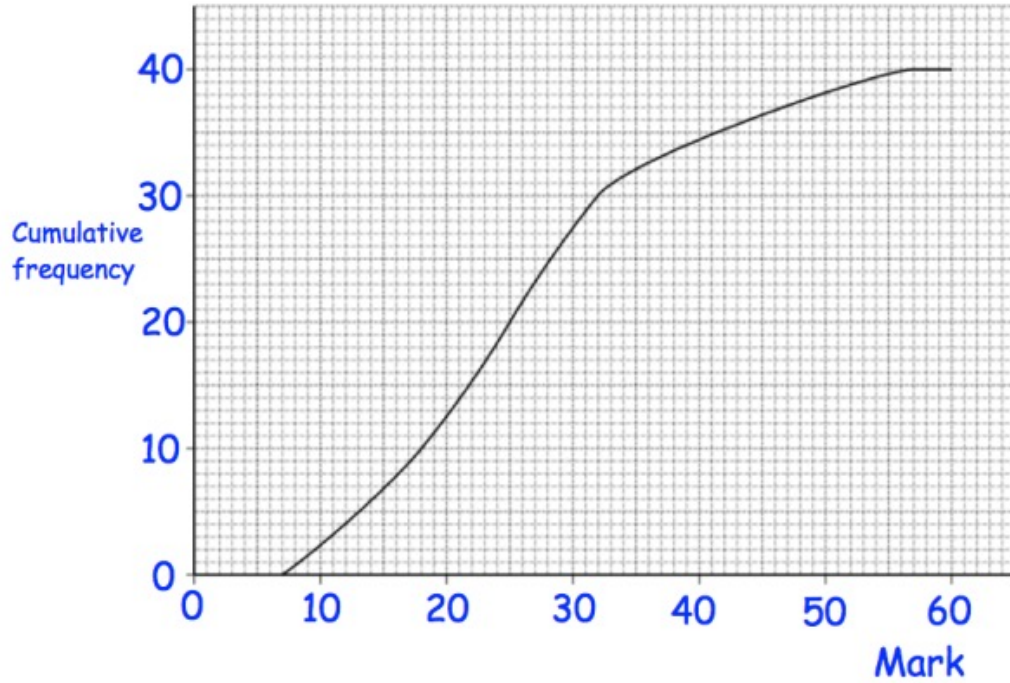
Draw a box plot to show this information:

Lowest	68kg
Lower Quartile	74kg
Median	82kg
Upper Quartile	88kg
Highest	100kg



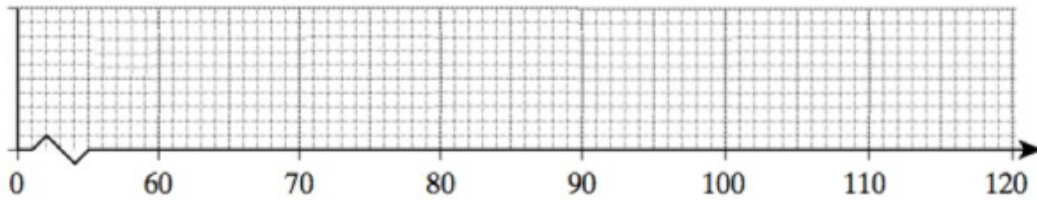
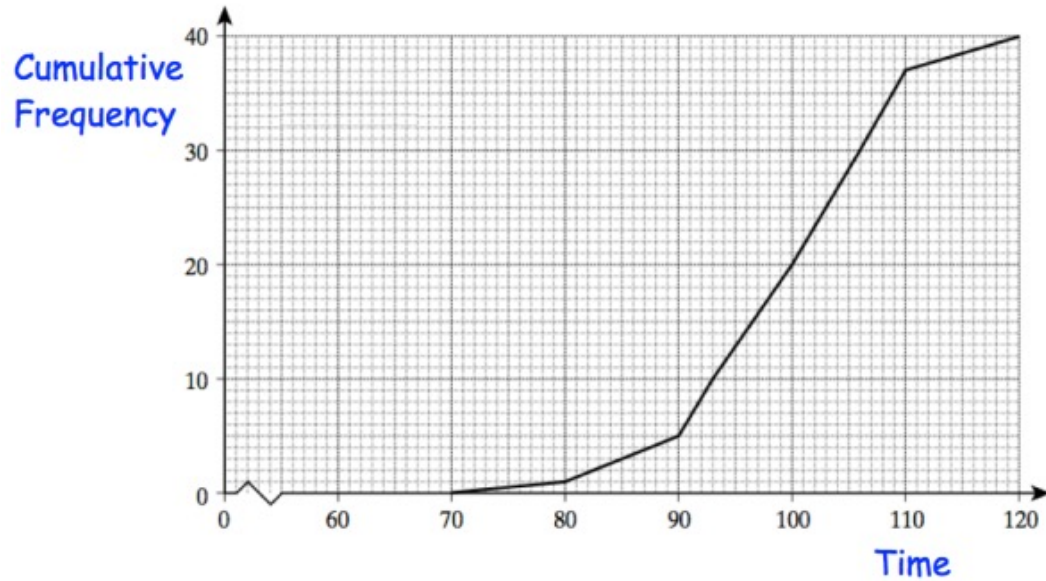
## Worked Example

Using the cumulative frequency graph, draw a box plot:



## Your Turn

Using the cumulative frequency graph, draw a box plot:



Question 1: Some students complete a quiz. The cumulative frequency graph shows their results

(a) How many students completed the quiz?

(b) Complete the frequency table below.

(c) What percentage of the students scored above 20 marks?

Marks	Frequency
$0 < m \leq 5$	
$5 < m \leq 10$	
$10 < m \leq 15$	
$15 < m \leq 20$	
$20 < m \leq 25$	



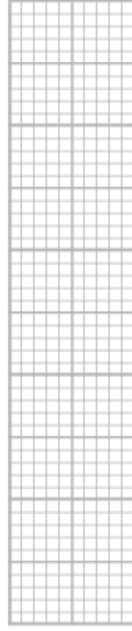
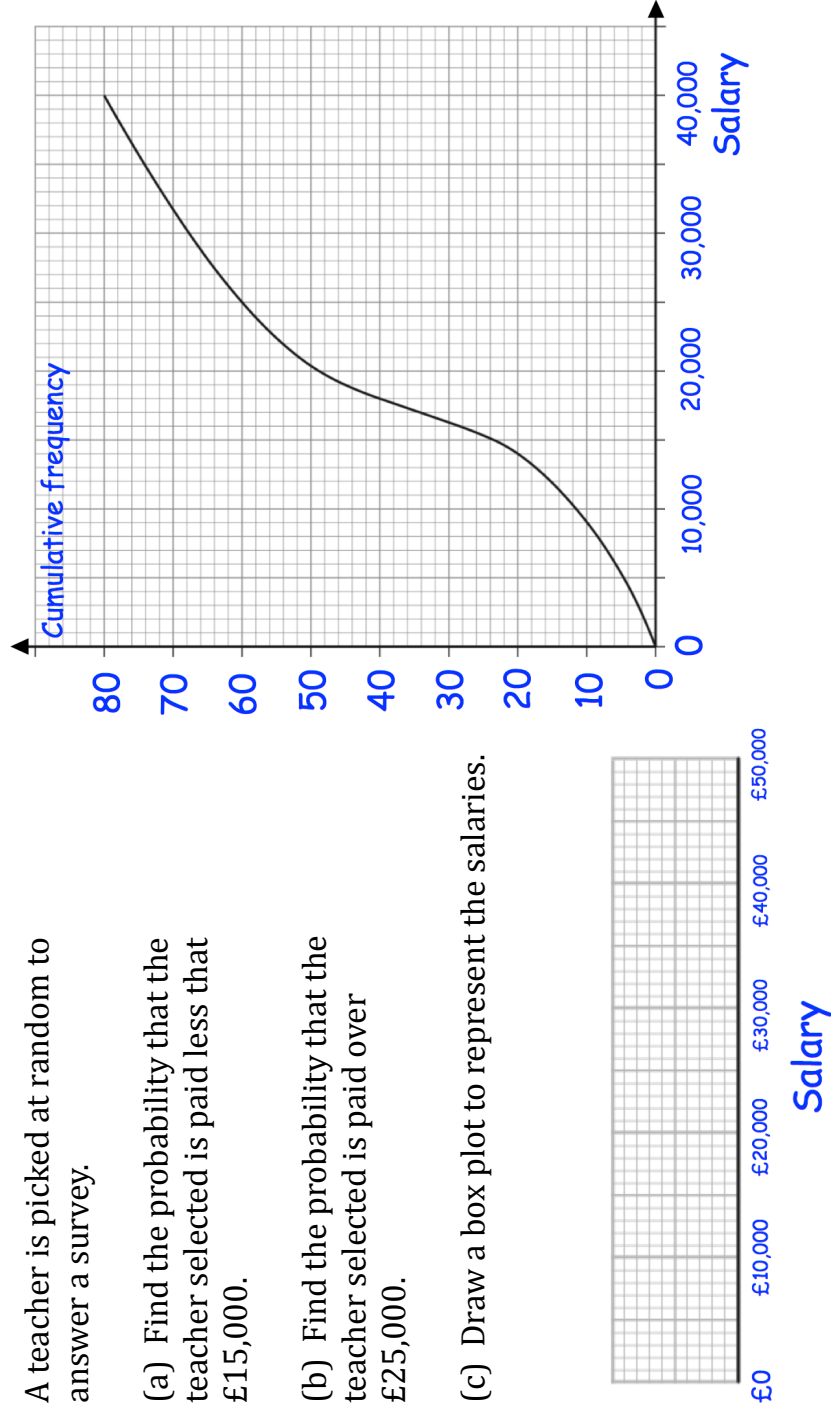
Question 2: The cumulative frequency graph below shows the salaries of 80 teachers. The lowest salary is £4,000 and the highest salary is £39,000.

A teacher is picked at random to answer a survey.

(a) Find the probability that the teacher selected is paid less than £15,000.

(b) Find the probability that the teacher selected is paid over £25,000.

(c) Draw a box plot to represent the salaries.

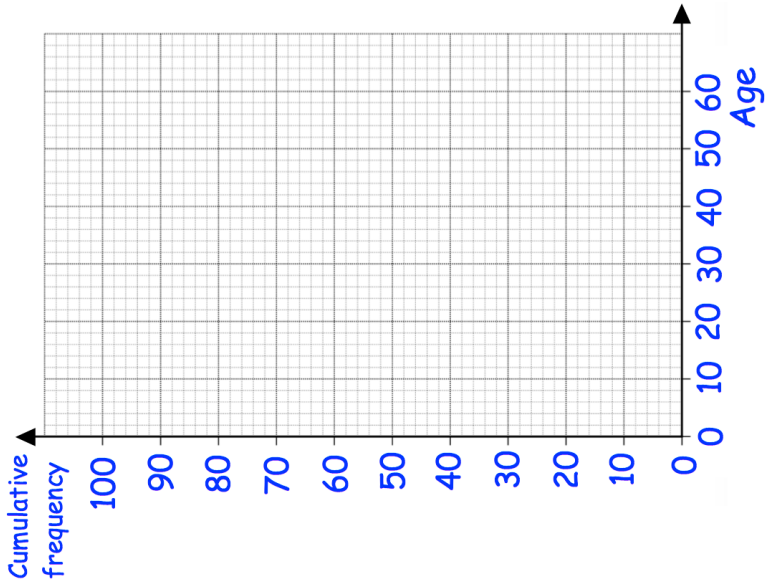


Question 3: The table shows information about the members of Abbeyville Cricket Club

Age	Frequency
$0 < A \leq 10$	2
$10 < A \leq 20$	5
$20 < A \leq 30$	19
$30 < A \leq 40$	38
$40 < A \leq 50$	25
$50 < A \leq 60$	11

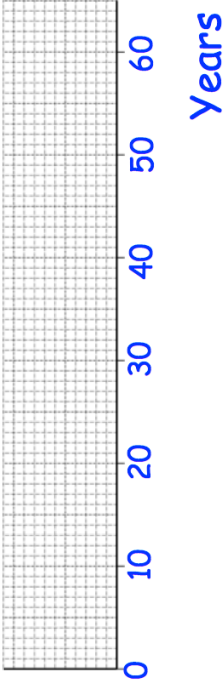
The youngest member is 9 and the oldest member is 58.

(a) Draw a cumulative frequency graph to represent this information.



(b) Draw a box plot to represent this information

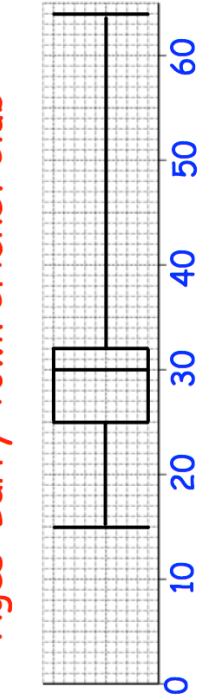
**Ages: Abbeyville Cricket Club**



(c) Work out the interquartile of the ages of the members of Abbeyville Cricket Club.

The box plot below shows information about Barry Town Cricket Club

**Ages: Barry Town Cricket Club**



(d) Write down the median age of the members of Barry Town Cricket Club

(e) Compare the distributions of the ages of the members of Abbeyville Cricket Club to the ages of the members of Barry Town Cricket Club.

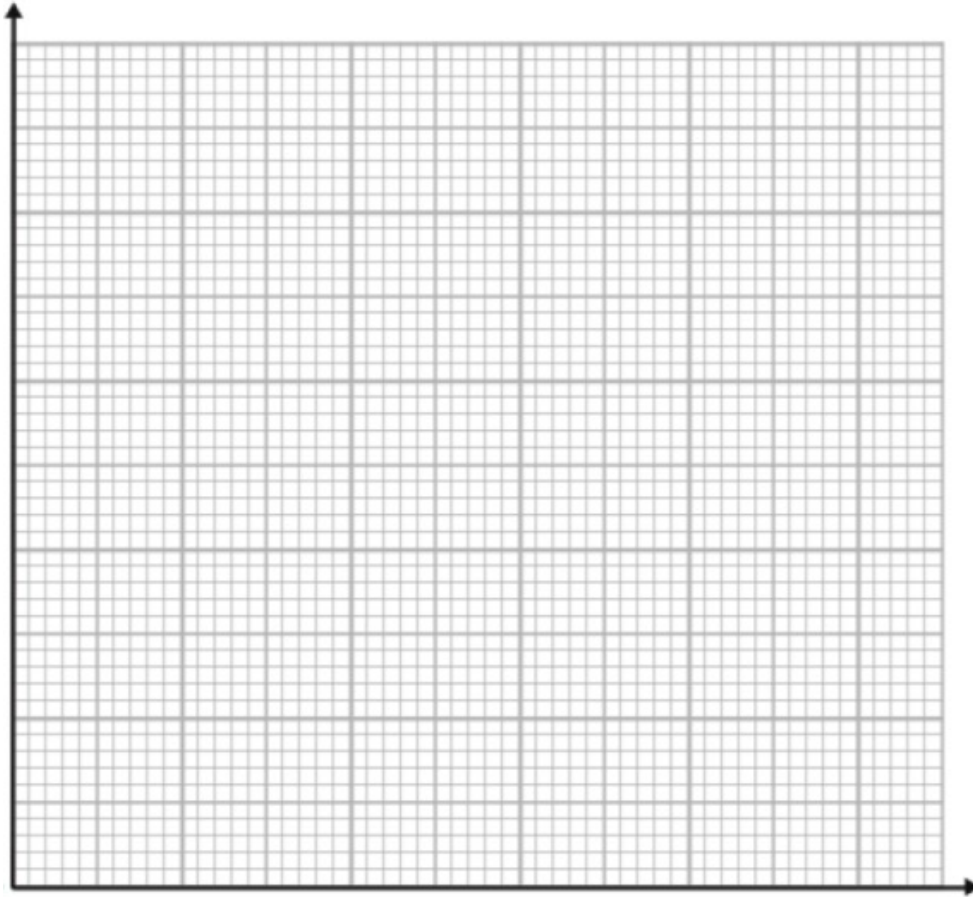
## Line Graphs



## Worked Example

The table shows the number of customers to a shop over several days. Draw a line graph for the data.

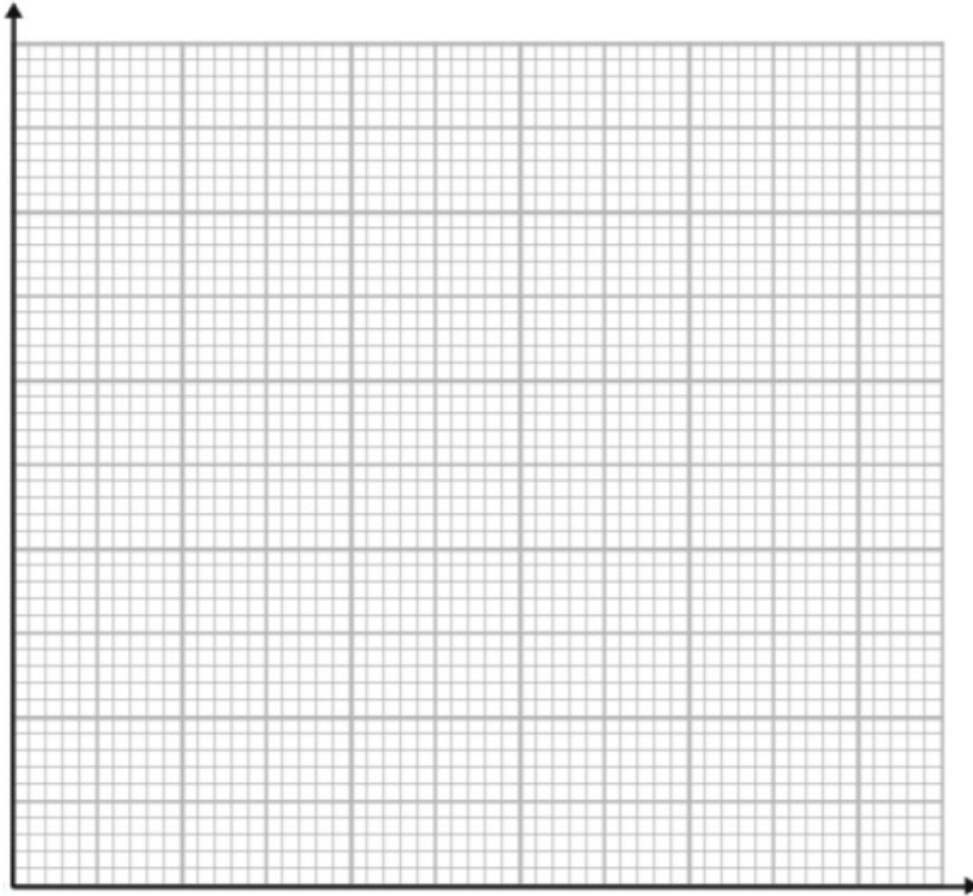
Day	Customers
Monday	42
Tuesday	47
Wednesday	3
Thursday	36
Friday	40
Saturday	31



## Your Turn

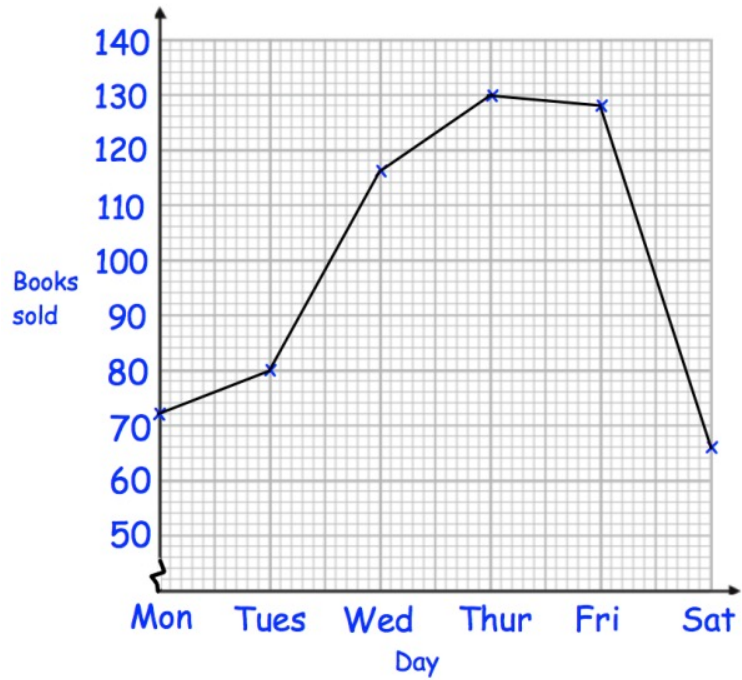
The table shows the value of a share in a mobile phone company over one day. Draw a line graph for the data.

Time	Value
9am	30.2p
11am	31.4p
1pm	29.6p
3pm	25.8p
5pm	24.2p
7pm	25.6p



## Worked Example

The line graph shows the number of books sold in a charity shop over one week.

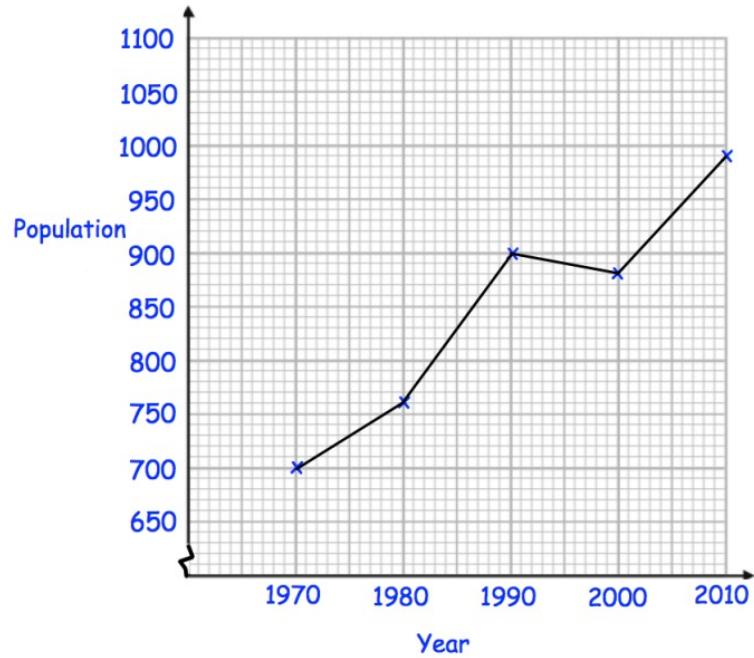


When did the charity shop sell:

- The most books
- The least books
- How many books were sold on Friday?

## Your Turn

The line graph shows the population of a village over time.



When was the population:

- Highest?
- Lowest?
- What was the population in 2000?

# Time Series

## Key Points

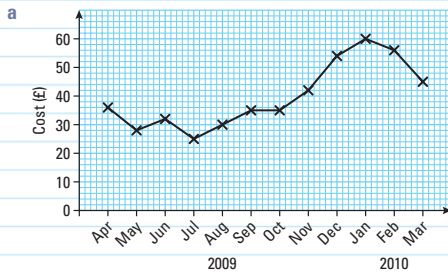
- A graph showing how a given value changes over time is called a time series graph.
- You can use a time series graph to identify whether there is any seasonal variation in the data – for example, if there is a peak or a trough at the same time each year.
- A time series can help you to identify whether there is any trend in the data.

## Example 1

The table below gives information about the cost of the gas Angela used each month between April 2009 and March 2010.

Month	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Jan	Feb	Mar
Cost (£)	36	28	32	25	30	35	35	42	54	60	56	45

- Draw a time series graph to show this information.
- In which month did Angela spend most on gas?
- Explain how the cost of gas changes over the year.



Plot the points on the grid.

Join the points with straight lines.

Find when the highest value occurs.

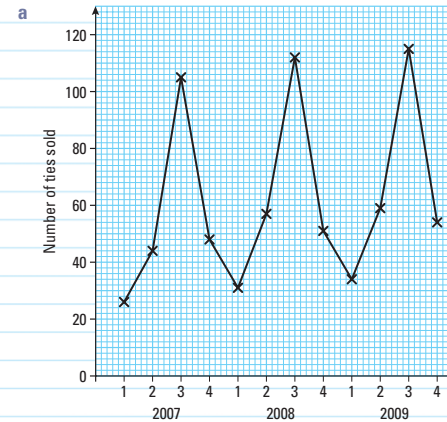
- January
- The cost of gas decreases during the first half of the year then increases in the second half of the year.

## Example 2

The table shows the number of ties sold in a school shop in each quarter of three successive years.

Year	Quarter			
	1	2	3	4
2007	26	44	105	48
2008	31	57	112	51
2009	34	59	115	54

- Plot the time series graph.
- In which quarter is the sale of ties highest?
- Describe the trend in the number of ties sold.

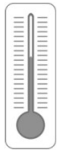


Find the quarter in which most ties are sold each year.

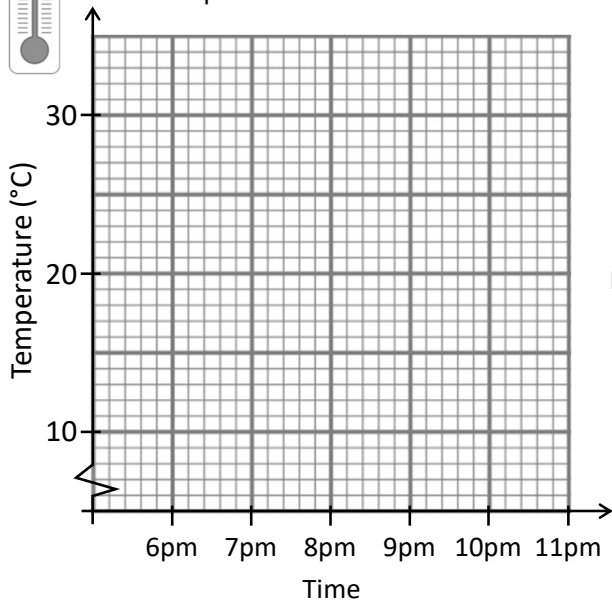
Note that there is a seasonal variation in the number of ties sold. The greatest number of ties sold is always in Quarter 3.

Although the number of ties sold varies greatly from quarter to quarter the trend in the number of ties sold is upwards.

- Quarter 3
- The number of ties sold is increasing over time.



A weather station recorded the temperature in the Atacama Desert.



Time	Temp. (°C)
6 pm	30
7 pm	28
8 pm	19
9 pm	14
10 pm	11
11 pm	8

Plot a line graph with this data.

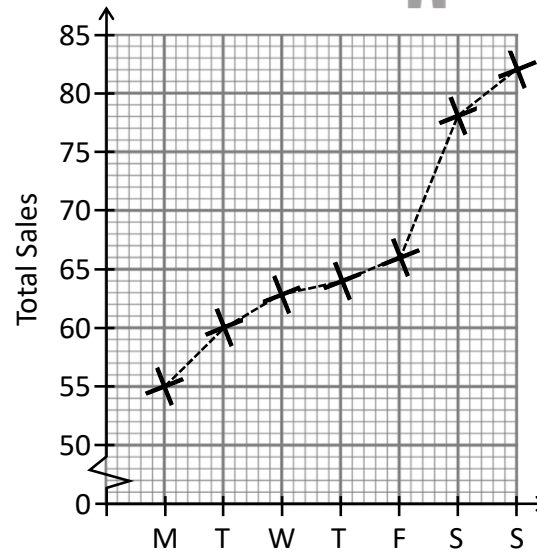
What trend does the line graph show? Why?

Will this trend continue?

Estimate the temperature at 20:30

## Time Series / Line Graphs

A clothes shop records **total sales** over one week.



How many sales were made on Tuesday?

On what day were...

...the most sales made?

...the least sales made?

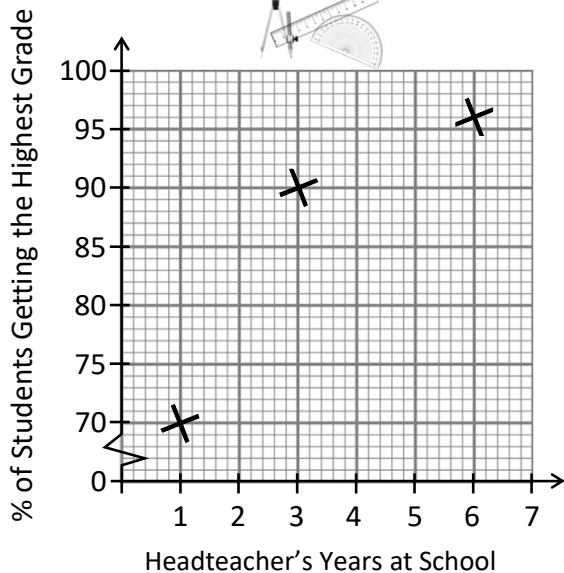
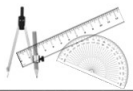
What trend does the line graph have?

Why **must** it have this trend?

Why is the y-axis truncated?

Why do you think this line is dashed and not solid?

A headteacher creates a line graph for the results during their 6 years at a school.



Estimate the amount of students that got the highest grade...

...after 2 years.

...after 5 years.

*"In the next two years, 100% of students will get the highest grade."*

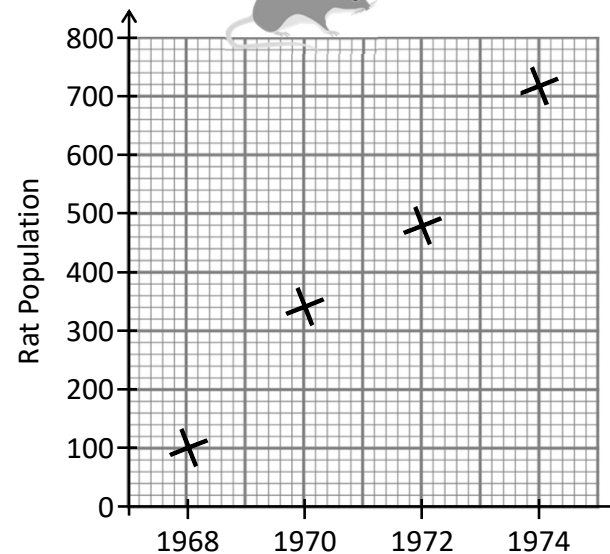
*"More students are getting the highest grade."*

*"Results have always been improving."*

Do you agree with the headteacher's statements?

Is the graph misleading?

In 1967 some rats were accidentally introduced to small McGuffin Island. Every 2 years a scientist came to the island and estimated the rat population.



Find an estimate for the rat population in...

...1969

...1971

...1973

What trend does the line graph have?

Do we expect the population to keep increasing?

## Extra Notes