



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2023 Mathematics 2024

Unit 17 Booklet – Part 1

HGS Maths



Tasks



Dr Frost Course



Name: _____

Class: _____



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2023 Mathematics 2024

Unit 17 Booklet – Part 2

HGS Maths



Tasks



Dr Frost Course



Name: _____

Class: _____

Contents Page

- 1 [Bounds and Error Intervals](#)
- 2 [Basic Circle Theorems](#)
- 3 [Direct and Inverse Proportion](#)
- 4 [Constructions and Loci](#)

1 Bounds and Error Intervals

When someone says that a distance is 50 metres, what do they mean? Measurements in real life can never be made with absolute accuracy – there is always a certain amount of error. So 50 metres could be accurate to the nearest metre, or to the nearest 10 metres, for example. Knowing within what interval the true distance lies can be very important in many applications of mathematics. When measurements are combined in a calculation, and each value has a certain amount of error, things can get complicated – and sometimes the result can be counterintuitive.

A number has been rounded to 30 to the nearest 10.

What could the number be?

What is the lowest and highest possible value it could be?

This smallest possible value is called the lower bound. The largest possible value is called the upper bound.

When a measure is expressed to a given unit, the maximum error is half of this unit.

For a value x , the error interval is:

least possible value $\leq x <$ greater possible value

Worked Example

A number z , when rounded to the nearest 100, is equal to 6700. Find the upper and lower bound of z .

Your Turn

A number z , when rounded to the nearest 10, is equal to 740. Find the upper and lower bound of z .

Worked Example

A number x , when rounded to 3 decimal places, is equal to 0.007. Find the upper and lower bound of x .

Your Turn

A number x , when rounded to 2 decimal places, is equal to 0.03. Find the upper and lower bound of x .

Worked Example

A number x , when rounded to 3 significant figures, is equal to 612000. Find the upper and lower bound of x .

Your Turn

A number x , when rounded to 2 significant figures, is equal to 35000. Find the upper and lower bound of x .

Worked Example

A number y , when rounded to 1 decimal place, is equal to 8.2.
Find the error interval for y .

Your Turn

A number y , when rounded to the nearest 10, is equal to 680.
Find the error interval for y .

Worked Example

The number of people on a bus is given as 50, correct to the nearest 10.
What is the lowest and highest possible number of people on the bus?

Your Turn

There are 9500 Red pandas left in the wild.
This number is accurate to the nearest 500.
What are the smallest and largest number of Sumatran orangutans that can be left?

Fill in the Gaps

x	Level of Accuracy	Lower Bound	Upper Bound	Error Interval
30	<i>to the nearest 10</i>	25		$25 \leq x < 35$
700	<i>to the nearest 100</i>		750	
25	<i>to the nearest integer</i>		25.5	
25	<i>to the nearest 5</i>	22.5		
24000	<i>to the nearest 1000</i>			
7.8	<i>to the nearest 0.1</i>			
360	<i>to the nearest 20</i>			
360	<i>to the nearest integer</i>			
6000	<i>to the nearest 100</i>			
200	<i>to the nearest 5</i>			
200	<i>to the nearest 10</i>			
13	<i>to the nearest tenth</i>			
	<i>to the nearest integer</i>		8.5	
		49500	50500	
8				$7 \leq x <$

Fill in the Gaps

x	Level of Accuracy	Lower Bound	Upper Bound	Error Interval
6.4	<i>to 1 decimal place</i>	6.35		$6.35 \leq x < 6.45$
7	<i>to the nearest integer</i>		7.5	
7.3	<i>to 1 decimal place</i>	7.25		
5.19	<i>to 2 decimal places</i>			
12.3	<i>to the nearest 0.1</i>			
40	<i>to 1 significant figure</i>		45	
1.5	<i>to 2 significant figures</i>			
0.76	<i>to 2 decimal places</i>			
10	<i>to 1 significant figure</i>			
27	<i>to the nearest integer</i>			
27.9	<i>to 1 decimal place</i>			
654	<i>to 3 significant figures</i>			
	<i>to 1 significant figure</i>	75		
		5.35	5.45	
				$95 \leq x < 150$


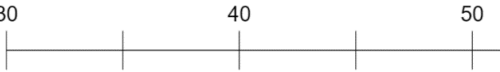
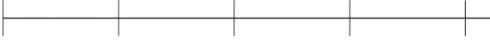

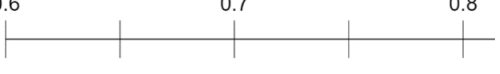



Fill in the Gaps

Value	Rounded to	Lower Bound	Upper Bound	Error Interval	Inequality on a number line
4.2	1 <i>dp</i>	4.15	4.25	$4.15 \leq x < 4.25$	
3.2	1 <i>dp</i>			$\leq x <$	
3.6	1 <i>dp</i>			$\leq x <$	
3.68	2 <i>dp</i>	3.675	3.685	$\leq x <$	
8.63	2 <i>dp</i>			$\leq x <$	
8.43	2 <i>dp</i>				
	2 <i>dp</i>	8.815	8.825		
	2 <i>dp</i>	9.615	9.625		

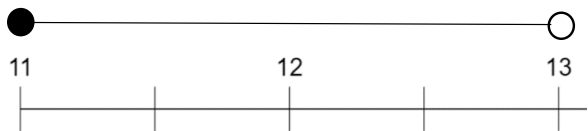
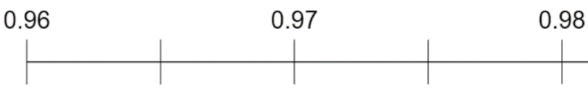



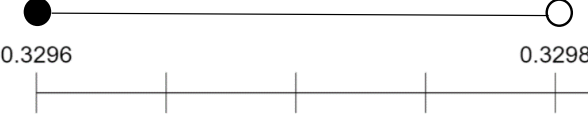


Fill in the Gaps

				$9.685 \leq x < 9.695$	
9.685	3 dp	9.6845	9.6855		
90.685	3 dp				
58.690	3 dp				
	3 dp				
	3 dp			$\leq x < 812.3275$	
	3 dp			$42.3795 \leq x <$	

Fill in the Gaps

Value	Rounded to	Lower Bound	Upper Bound	Error Interval	Inequality on a number line
4	$1\ sf$	3.5	4.5	$3.5 \leq x < 4.5$	
40	$1\ sf$	35		$\leq x <$	
30	$1\ sf$			$\leq x <$	
200	$1\ sf$			$\leq x <$	
0.7	$1\ sf$		0.75	$\leq x <$	
0.08	$1\ sf$				
	$1\ sf$			$8.5 \leq x < 9.5$	
	$1\ sf$				

Fill in the Gaps

12	2 sf	11.5	12.5	$11.5 \leq x < 12.5$	
0.97	2 sf				
760	2 sf				
7.68	3 sf				
9.61	3 sf				
					
	1 sf			$\leq x < 7.5$	
	2 sf			$435 \leq x <$	

Fill in the Gaps

Number	Rounding	Lower bound	Upper bound	Error interval
4	Nearest integer	3.5	4.5	
40	Nearest ten			$35 \leq x < 45$
40	Nearest integer	39.5	40.5	
50	Nearest integer	49.5		
50	Nearest ten		55	
550		545		
5.5	1 decimal place			
55.5	1 decimal place			
89.6	1 decimal place			
50	1 significant figure			

Worked Example

$$p = 5qr$$

$q = 0.709$ correct to 3 significant figures.

$r = 0.071$ correct to 3 decimal places.

Work out the lower bound for the value of p

Give your answer correct to 3 decimal places when appropriate.

Your Turn

$$a = 5bc$$

$b = 0.124$ correct to 3 decimal places.

$c = 98000$ correct to 2 significant figures.

Work out the lower bound for the value of a

Give your answer correct to 3 decimal places when appropriate.

Worked Example

$$p = 4q + 2r$$

$q = 907000$ correct to 3 significant figures.

$r = 8.88$ correct to 2 decimal places.

Work out the lower bound for the value of p

Give your answer correct to 3 decimal places when appropriate.

Your Turn

$$a = 4b + 3c$$

$b = 55.4$ correct to 1 decimal place.

$c = 3.1$ correct to 2 significant figures.

Work out the lower bound for the value of a

Give your answer correct to 3 decimal places when appropriate.

Worked Example

$$p = 5q - 5r$$

$q = 0.003$ correct to 1 significant figure.

$r = 1.93$ correct to 2 decimal places.

Work out the lower bound for the value of p

Give your answer correct to 3 decimal places when appropriate.

Your Turn

$$a = 3b - 2c$$

$b = 98.9$ correct to 1 decimal place.

$c = 26.5$ correct to 3 significant figures.

Work out the lower bound for the value of a

Give your answer correct to 3 decimal places when appropriate.

Worked Example

$$p = \frac{2q}{r}$$

$q = 0.9$ correct to 1 significant figure.

$r = 0.075$ correct to 3 decimal places.

Work out the lower bound for the value of p

Give your answer correct to 3 decimal places when appropriate.

Your Turn

$$a = \frac{4b}{c}$$

$b = 78.4$ correct to 1 decimal place.

$c = 4150$ correct to 3 significant figures.

Work out the lower bound for the value of a

Give your answer correct to 3 decimal places when appropriate.

Worked Example

$$p = \frac{q}{r-s}$$

$p = 5$ correct to 1 significant figure.

$r = 0.002$ correct to 1 significant figure.

$s = 0.645$ correct to 3 decimal places.

Work out the lower bound for the value of p

Give your answer correct to 3 decimal places when appropriate.

Your Turn

$$x = \frac{y}{z-w}$$

$y = 0.786$ correct to 3 decimal places.

$z = 702$ correct to 3 significant figures.

$w = 0.5$ correct to 1 significant figure.

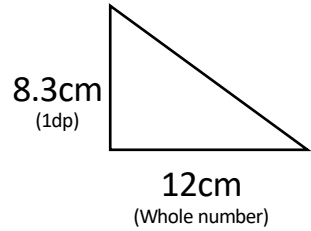
Work out the lower bound for the value of x

Give your answer correct to 3 decimal places when appropriate.

Worked Example

The height and width of the triangle below have been rounded as shown in brackets.

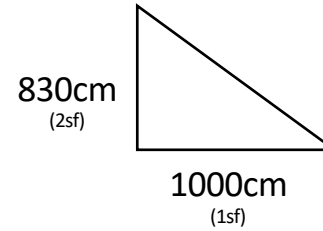
Work out the LB and UB for the **area** of the triangle.



Your Turn

The height and width of the triangle below have been rounded as shown in brackets.

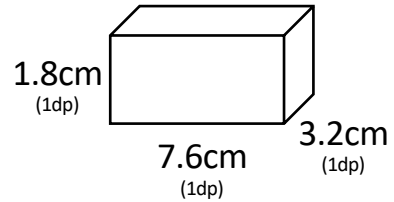
Work out the LB and UB for the **area** of the triangle.



Worked Example

The dimensions of the cuboid below have been rounded as shown in brackets.

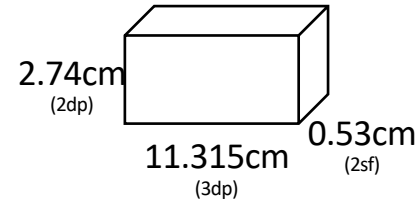
Work out the LB and UB for the **volume** of the cuboid.



Your Turn

The dimensions of the cuboid below have been rounded as shown in brackets.

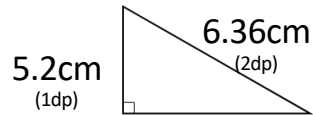
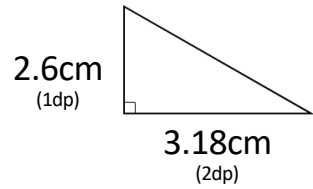
Work out the LB and UB for the **volume** of the cuboid.



Worked Example

Use Pythagoras' Theorem to find the LB and UB of the missing sides below.

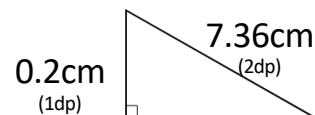
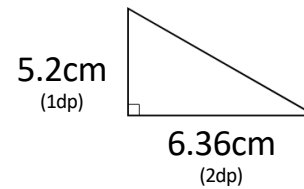
All lengths have been rounded as shown in brackets.



Your Turn

Use Pythagoras' Theorem to find the LB and UB of the missing sides below.

All lengths have been rounded as shown in brackets.



Fill in the Gaps

a , b and c are all rounded to the degree of accuracy stated. Find the maximum and minimum values for x .
 Values given for x_{\max} are exact.

a	b	c	Equation	x_{\max}	x_{\min}
10 (1 sig fig)	12.1 (3 sig fig)	3.4 (2 sig fig)	$\sqrt{ax} = b - c$		
0.5 (1 sig fig)	4.5 (2 sig fig)	-2.0 (2 sig fig)	$\frac{a}{x} = b^2 + 3c$		
5.2 (2 sig fig)	3.4 (2 sig fig)	5 (1 sig fig)	$\frac{ax^2}{b} = c$		
3 (1 sig fig)	4 (1 sig fig)	8 (1 sig fig)	$ax + c = b$		
5 (1 sig fig)	-3 (1 sig fig)	<input type="text"/> (2 sig fig)	$ax = bc$	-3.25	
<input type="text"/> (2 sig fig)	4.3 (2 sig fig)	0.3 (1 sig fig)	$a + x = \frac{b}{c}$	9.35	

Considering Bounds

$$m = \frac{\sqrt{s}}{t}$$

$s = 3.47$ correct to 2 decimal places. $t = 8.132$ correct to 3 decimal places. By considering bounds, work out the value of m to a suitable degree of accuracy. You must show all your working and give a reason for your final answer.

$$s_{lower} = 3.465 \quad s_{upper} = 3.475$$

$$t_{lower} = 8.1315 \quad t_{upper} = 8.1325$$

$$m_{lower} = \frac{\sqrt{s_{lower}}}{t_{upper}} = \frac{\sqrt{3.465}}{8.1325} = 0.2288903 \dots$$

$$m_{upper} = \frac{\sqrt{s_{upper}}}{t_{lower}} = \frac{\sqrt{3.475}}{8.1315} = 0.2292486 \dots$$

If we had to only choose a single value for m , what would be most sensible?

We don't know where m is between the 0.2289... and 0.2292... Ideally we want to quote a value of m such that we would round to this same value regardless of what m actually was, but still give as much accuracy as possible..

$m = 0.229$ "as both the lower bound and upper bound are this to 3dp".

Worked Example

$$a = \frac{\sqrt{b}}{c}$$

$b = 0.24$ correct to 2 decimal places.

$c = 57.2$ correct to 3 significant figures.

By considering bounds, work out the value of a , giving your answer to a suitable degree of accuracy.

Your Turn

$$a = \frac{b}{\sqrt{c}}$$

$b = 0.359$ correct to 3 significant figures.

$c = 0.64$ correct 2 decimal places.

By considering bounds, work out the value of a , giving your answer to a suitable degree of accuracy.

Truncation

When we truncate a number, we find an estimate for the number without doing any rounding. To truncate a number, we miss off digits past a certain point in the number, filling-in zeros if necessary to make the truncated number approximately the same size as the original number.

To truncate a number to 1 decimal place, miss off all the digits after the first decimal place.

To truncate a number to 2 decimal places, miss off all the digits after the second decimal place.

To truncate a number to 3 significant figures, miss off all the digits after the first 3 significant figures (the first non-zero digit and the next two digits). Fill in any spaces with zeros to make the number approximately the same size as the original value.

Worked Example

Truncate 41.53681 to:

- a) 1 decimal place
- b) 2 decimal places
- c) 3 decimal places

Your Turn

Truncate 11.95291 to:

- a) 1 decimal place
- b) 2 decimal places
- c) 3 decimal places

Worked Example

A number z , when truncated to 2 decimal places, is equal to 4.97. Find the upper and lower bound of z .

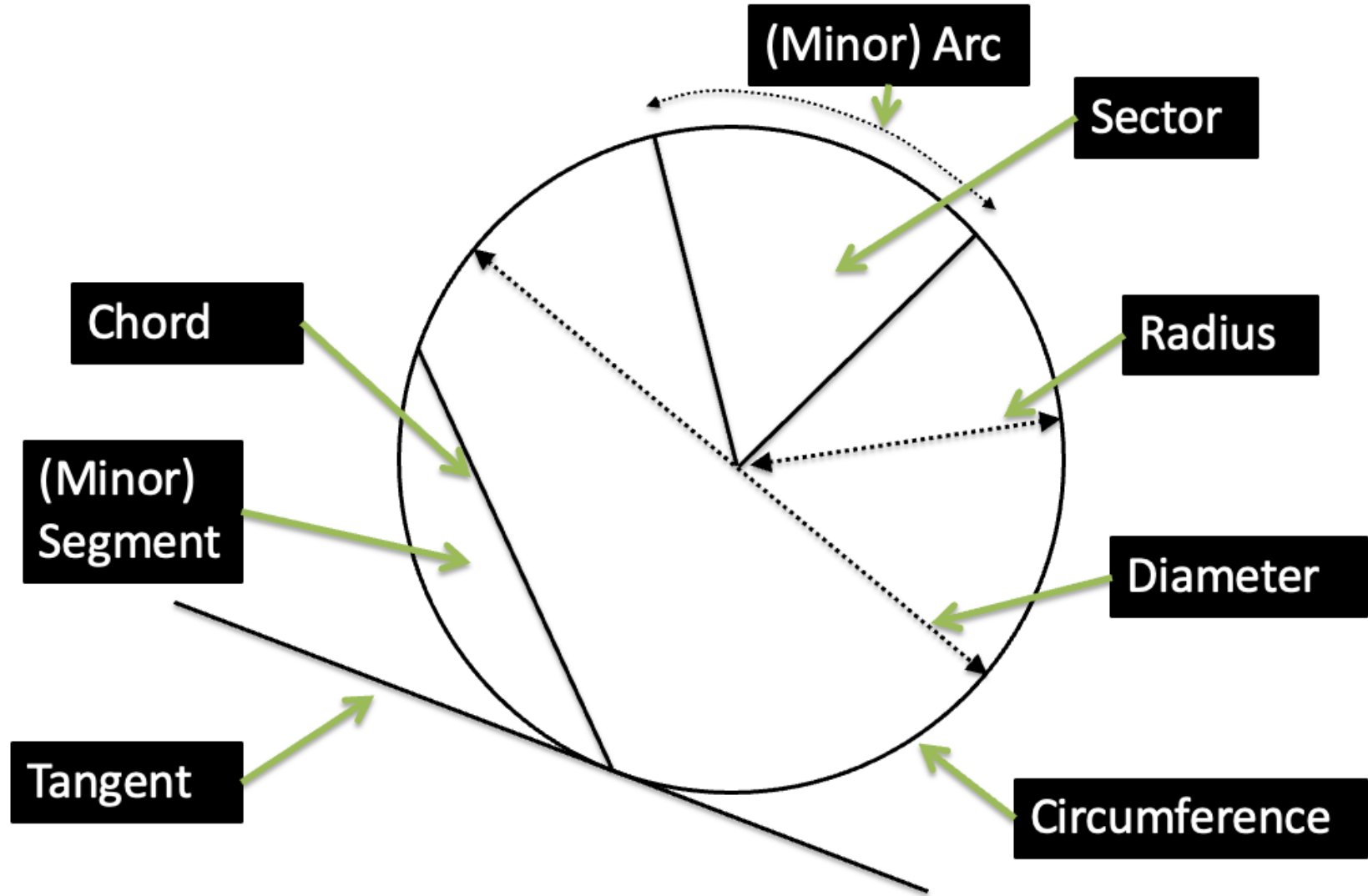
Your Turn

A number x , when truncated to 3 decimal places, is equal to 0.545. Find the upper and lower bound of x .

Extra Notes

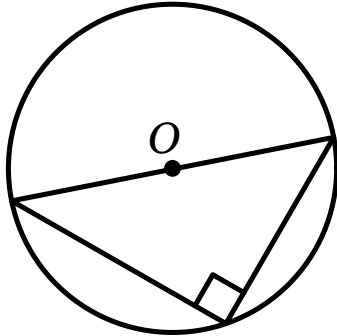
2 Basic Circle Theorems

Parts of a Circle

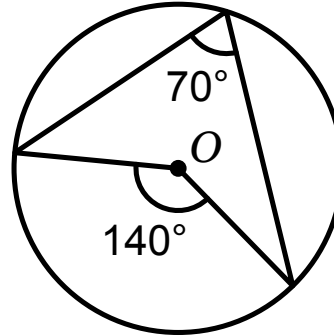


Circle Theorems 1

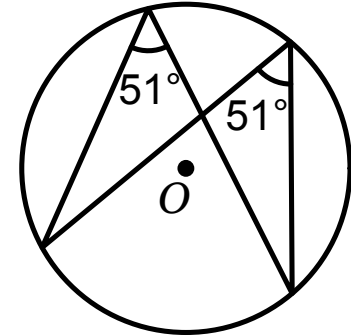
The angle in a semicircle is a right angle.



The angle at the centre is twice the angle at the circumference.

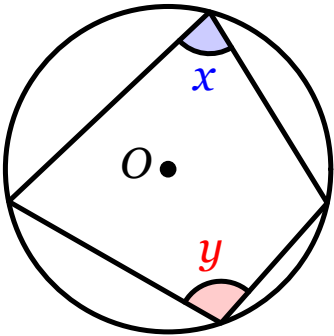


Angles in the same segment are equal.



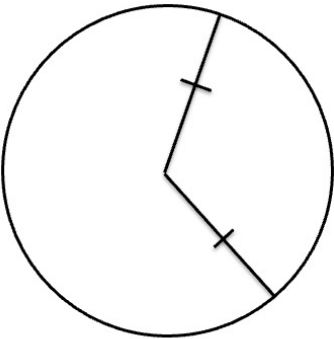
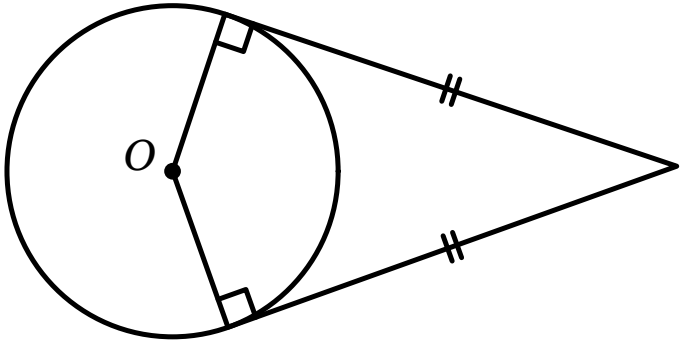
Circle Theorems 2

Opposite angles of a cyclic quadrilateral sum to 180° .



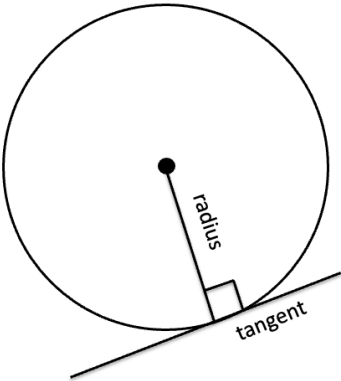
$$x + y = 180^\circ$$

Tangents to a point are equal in length.



Radius is of constant length

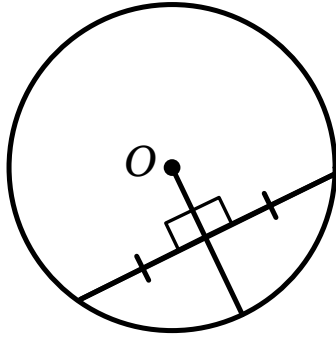
Tip: When you have multiple radii, put a mark on each of them to remind yourself they are the same length.



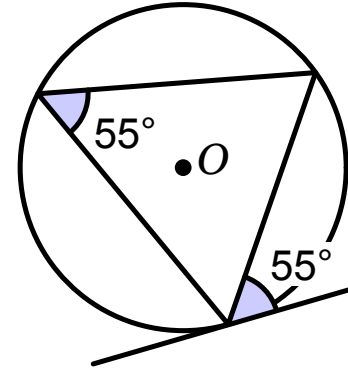
"Angle between radius and tangent is 90° ".

Circle Theorems 3

The perpendicular from the centre to a chord bisects the chord.

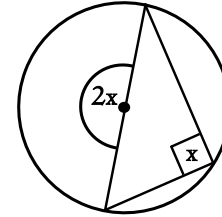
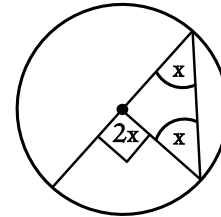
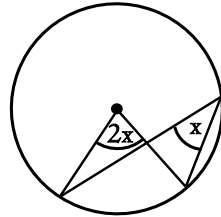
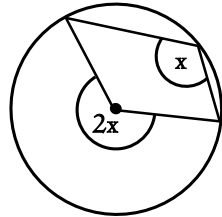
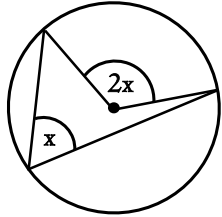


The angle between a chord and a tangent equals the angle in the **alternate segment**.



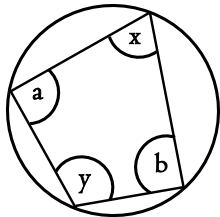
Circle Theorems Summary

What you need to remember when answering Circle Theorems questions

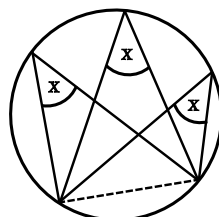


The angle at the centre of a circle is twice the angle at the circumference, so...

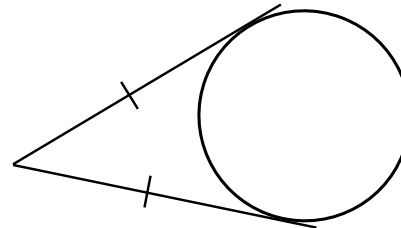
...the angle in a semi-circle is 90°



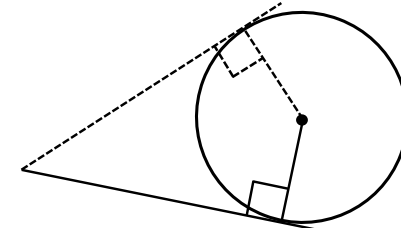
Opposite angles in a cyclic quadrilateral add up to 180°



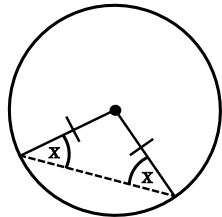
Angles subtended by the same arc [or chord] are equal in size



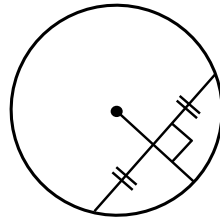
Tangents from the same point are equal in length



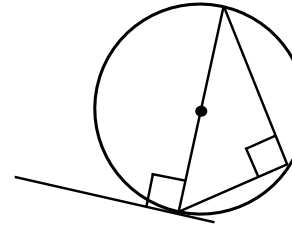
A radius and a tangent form a right angle



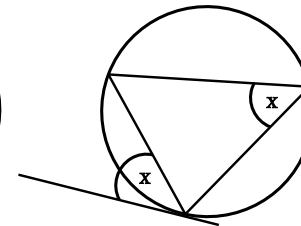
Two radii make an isosceles triangle



The perpendicular bisector of a chord passes through the centre of the circle

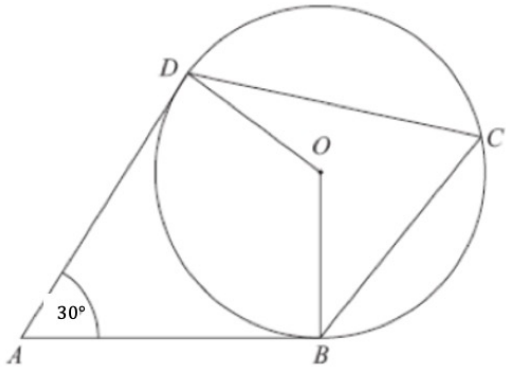


The angle between a tangent and a chord [or diameter] is equal to the angle in the alternate segment



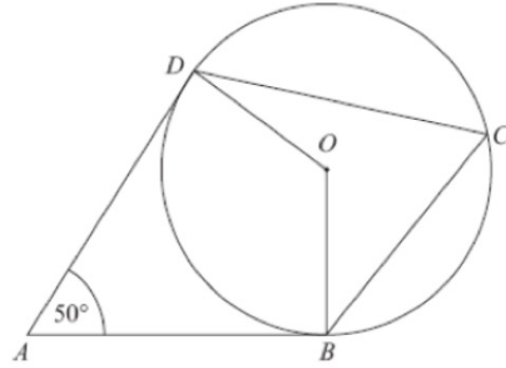
Worked Example

Calculate the size of angle BCD



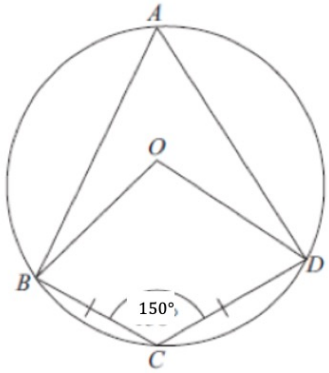
Your Turn

Calculate the size of angle BCD



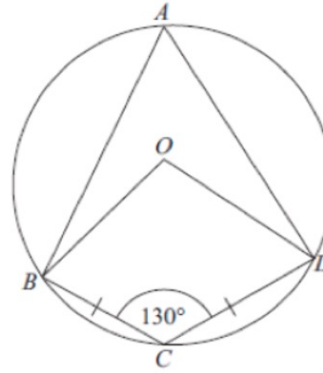
Worked Example

Calculate the size of angle CDO



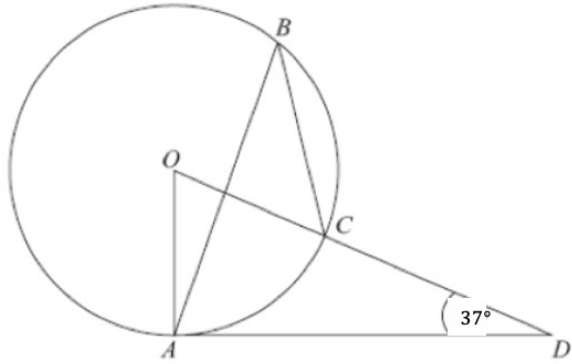
Your Turn

Calculate the size of angle CDO



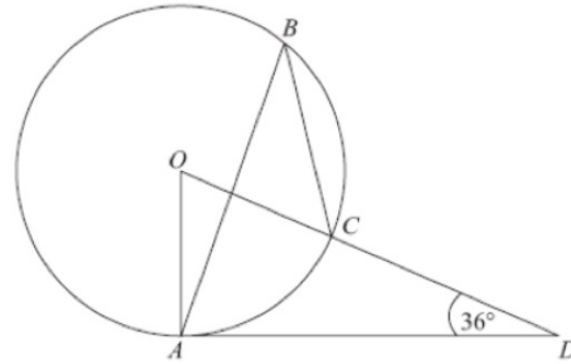
Worked Example

Calculate the size of angle ABC



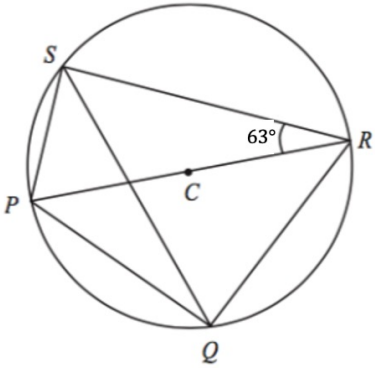
Your Turn

Calculate the size of angle ABC



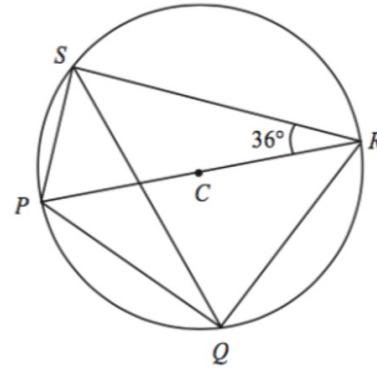
Worked Example

Calculate the size of angle RQS



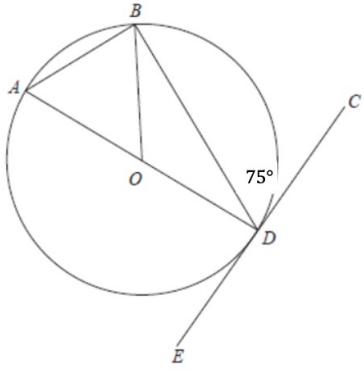
Your Turn

Calculate the size of angle RQS



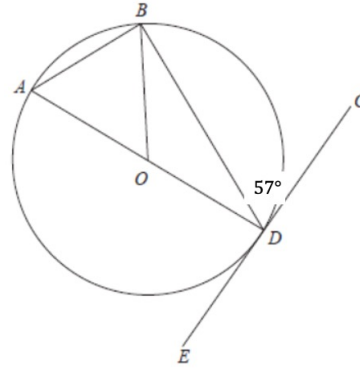
Worked Example

Calculate the size of angle AOB

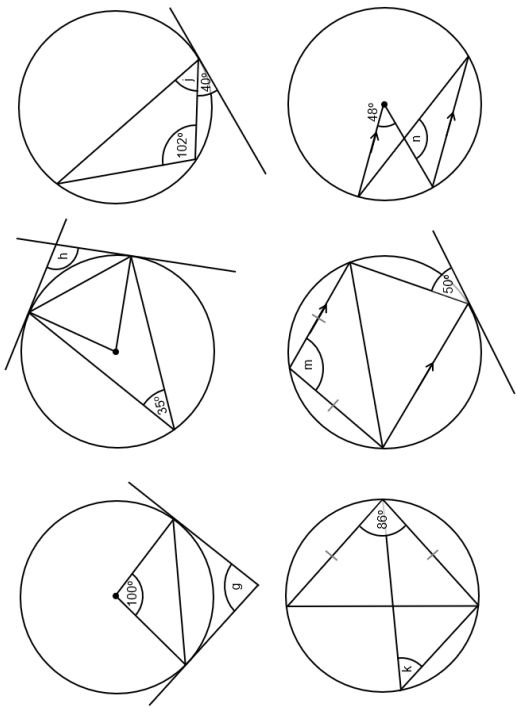


Your Turn

Calculate the size of angle AOB



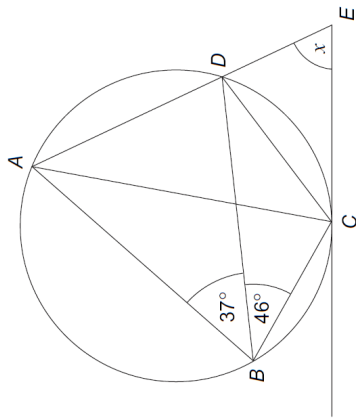
Fluency Practice



Incomplete Example:

For this question, the steps are given below, but the reasons are missing. Complete the table with the correct reasons, and label the calculated angles on the diagram.

The line joining C and E is a tangent to the circle



Not drawn accurately

Work out the size of angle x .

Step	Reason
Angle DCE = 46°	
Angle ACD = 37°	
Angle CAE = 46°	
$x = 180 - 46 - 46 - 37$	
$x = 51^\circ$	

Extra Notes

3 Direct and Inverse Proportion

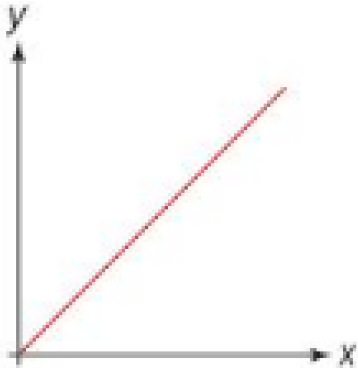
Direct Proportion

y is directly proportional to x
 y is proportional to x
 y varies directly to x

$$y \propto x$$

$$y = kx$$

k is called the constant of proportionality



The graph of $y = kx$ is a straight line that passes through the origin.

Worked Example

y is directly proportional to x

When $y = 20, x = 2$

- a) Find y when $x = 5$
- b) Find x when $y = 200$

Your Turn

b is directly proportional to a

When $b = 30, a = 5$

- a) Find b when $a = 2$
- b) Find a when $b = 3000$

Worked Example

y is directly proportional to the square of x

When $y = 36, x = 3$

- a) Find y when $x = 5$
- b) Find x when $y = 400$

Your Turn

b is directly proportional to the square of a

When $b = 12, a = 2$

- a) Find b when $a = 3$
- b) Find a when $b = 300$

Worked Example

y is directly proportional to the cube of x

When $y = 32, x = 2$

- a) Find y when $x = 5$
- b) Find x when $y = 108$

Your Turn

b is directly proportional to the cube of a

When $b = 54, a = 3$

- a) Find b when $a = 4$
- b) Find a when $b = 16$

Worked Example

y is directly proportional to the square root of x

When $y = 36, x = 16$

- a) Find y when $x = 25$
- b) Find x when $y = 900$

Your Turn

b is directly proportional to the square root of a

When $b = 36, a = 144$

- a) Find b when $a = 49$
- b) Find a when $b = 243$

Fill in the Gaps

General Statement	General Equation	Table of Values	Value of k	Specific Equation	When $x = 5$, $y = ?$	When $y = 24$, $x = ?$								
$y \propto x$	$y = kx$	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;">3</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;"></td></tr> </table>	x	1	2	10	y	3			$k = 3$	$y = 3x$	$y = 3 \times 5$ $y = 15$	$24 = 3 \times x$ $x = 8$
x	1	2	10											
y	3													
$y \propto x$	$y = kx$	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;">8</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">80</td></tr> </table>	x	1	2	10	y	8		80				$x = 3$
x	1	2	10											
y	8		80											
		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;"></td></tr> </table>	x	1	2	10	y					$y = 2.5x$		$24 = 2.5 \times x$ $x = 9.6$
x	1	2	10											
y														
$y \propto x$		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">10</td><td style="padding: 2px 5px;"></td></tr> </table>	x	1	2	10	y		10					
x	1	2	10											
y		10												
$y \propto x^2$	$y = kx^2$	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">600</td></tr> </table>	x	1	2	10	y			600	$k = 6$			$24 = 6 \times x^2$ $x = 2$
x	1	2	10											
y			600											
$y \propto x^2$		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">150</td></tr> </table>	x	1	2	10	y			150				
x	1	2	10											
y			150											
		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;"></td></tr> </table>	x	1	2	10	y		1		$k = 0.5$			
x	1	2	10											
y		1												

Fill in the Gaps

General Statement	General Equation	Table of Values	Value of k	Specific Equation								
$y \propto x^3$	$y = kx^3$	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px;">x</td> <td style="padding: 2px;">1</td> <td style="padding: 2px;">2</td> <td style="padding: 2px;">4</td> </tr> <tr> <td style="padding: 2px;">y</td> <td style="padding: 2px;">3</td> <td style="padding: 2px;"></td> <td style="padding: 2px;"></td> </tr> </table>	x	1	2	4	y	3			$k = 3$	
x	1	2	4									
y	3											
$y \propto \sqrt{x}$		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px;">x</td> <td style="padding: 2px;">1</td> <td style="padding: 2px;">4</td> <td style="padding: 2px;">25</td> </tr> <tr> <td style="padding: 2px;">y</td> <td style="padding: 2px;"></td> <td style="padding: 2px;">24</td> <td style="padding: 2px;"></td> </tr> </table>	x	1	4	25	y		24			
x	1	4	25									
y		24										
$y \propto x$		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px;">x</td> <td style="padding: 2px;">1</td> <td style="padding: 2px;">4</td> <td style="padding: 2px;">10</td> </tr> <tr> <td style="padding: 2px;">y</td> <td style="padding: 2px;"></td> <td style="padding: 2px;">3</td> <td style="padding: 2px;"></td> </tr> </table>	x	1	4	10	y		3			
x	1	4	10									
y		3										
	$y = k\sqrt[3]{x}$	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px;">x</td> <td style="padding: 2px;">1</td> <td style="padding: 2px;">8</td> <td style="padding: 2px;">125</td> </tr> <tr> <td style="padding: 2px;">y</td> <td style="padding: 2px;"></td> <td style="padding: 2px;">20</td> <td style="padding: 2px;"></td> </tr> </table>	x	1	8	125	y		20			
x	1	8	125									
y		20										
		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px;">x</td> <td style="padding: 2px;">1</td> <td style="padding: 2px;">4</td> <td style="padding: 2px;">10</td> </tr> <tr> <td style="padding: 2px;">y</td> <td style="padding: 2px;"></td> <td style="padding: 2px;">32</td> <td style="padding: 2px;">200</td> </tr> </table>	x	1	4	10	y		32	200		
x	1	4	10									
y		32	200									
		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px;">x</td> <td style="padding: 2px;">1</td> <td style="padding: 2px;">4</td> <td style="padding: 2px;"></td> </tr> <tr> <td style="padding: 2px;">y</td> <td style="padding: 2px;"></td> <td style="padding: 2px;">3</td> <td style="padding: 2px;">7.5</td> </tr> </table>	x	1	4		y		3	7.5	$k = 1.5$	
x	1	4										
y		3	7.5									
		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px;">x</td> <td style="padding: 2px;">1</td> <td style="padding: 2px;">4</td> <td style="padding: 2px;"></td> </tr> <tr> <td style="padding: 2px;">y</td> <td style="padding: 2px;"></td> <td style="padding: 2px;">$\frac{32}{3}$</td> <td style="padding: 2px;">24</td> </tr> </table>	x	1	4		y		$\frac{32}{3}$	24	$k = \frac{2}{3}$	
x	1	4										
y		$\frac{32}{3}$	24									
$y \propto x^3$		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px;">x</td> <td style="padding: 2px;">2</td> <td style="padding: 2px;">$\sqrt{5}$</td> <td style="padding: 2px;"></td> </tr> <tr> <td style="padding: 2px;">y</td> <td style="padding: 2px;"></td> <td style="padding: 2px;">25</td> <td style="padding: 2px;">$27\sqrt{5}$</td> </tr> </table>	x	2	$\sqrt{5}$		y		25	$27\sqrt{5}$		
x	2	$\sqrt{5}$										
y		25	$27\sqrt{5}$									
		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px;">x</td> <td style="padding: 2px;">1</td> <td style="padding: 2px;">8</td> <td style="padding: 2px;"></td> </tr> <tr> <td style="padding: 2px;">y</td> <td style="padding: 2px;"></td> <td style="padding: 2px;">$2a$</td> <td style="padding: 2px;">$4a$</td> </tr> </table>	x	1	8		y		$2a$	$4a$	$k = a$	
x	1	8										
y		$2a$	$4a$									

Fill in the Gaps

Direct Proportion – Method Breakdown

Complete the table. Use the equation with the known constant (k) to answer the question.

Relationship in Words	Equation	Known Values	Substitution	Constant of Proportionality (k)	Equation Re-write	Question
y is directly proportional to x	$y = kx$	When $x = 9$, $y = 45$	$45 = k(9)$			When $x = 10$, $y =$
y is directly proportional to x squared	$y = kx^2$	When $x = 3$, $y = 36$	$36 = k(3)^2$			When $x = 5$, $y =$
y is directly proportional to x cubed		When $x = 4$, $y = 128$				When $x = 3$, $y =$
y is directly proportional to the square root of x	$y = k\sqrt{x}$	When $x = 25$, $y = 15$	$15 = k\sqrt{25}$			When $x = 100$, $y =$
	$y = k\sqrt[3]{x}$	When $x = 8$, $y = 20$				When $x = 64$, $y =$
	$y = kx$	When $x = 5$, $y = 40$				When $x = 2.5$, $y =$
y is directly proportional to x squared		When $x = 4$, $y = 96$				When $x = 10$, $y =$
y is directly proportional to the square root of x		When $x = 81$, $y = 81$				When $x = 36$, $y =$
		When $x = 5$, $y = 500$	$500 = k(5)^3$			When $x = 3$, $y =$
y is directly proportional to the cube root of x		When $x = 1000$, $y = 70$				When $x = 8$, $y =$
y is directly proportional to x		When $x = 16$, $y = 56$				When $y = 49$, $x =$
y is directly proportional to x squared		When $x = 3$, $y = 4.5$				When $y = 72$, $x =$
y is directly proportional to x cubed		When $x = 2$, $y = 1.6$				When $y = 12.8$, $x =$

Worked Example

a) y is directly proportional to $x + 2$
When $y = 20$, $x = 2$
Find y when $x = 5$

b) y is directly proportional to $x^2 + 4$
When $y = 52$, $x = 3$
Find y when $x = 5$

Your Turn

a) y is directly proportional to $x + 2$
When $y = 12$, $x = 2$
Find y when $x = 8$

b) y is directly proportional to $2x^2$
When $y = 36$, $x = 3$
Find y when $x = 5$

Inverse Proportion

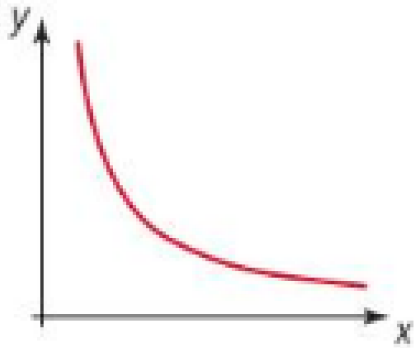
y is inversely proportional to x
 y varies inversely or indirectly to x

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

k is called the constant of proportionality

The graph of $y = \frac{k}{x}$ is a reciprocal graph.



Worked Example

y is inversely proportional to x

When $y = 5, x = 2$

- a) Find y when $x = 5$
- b) Find x when $y = 0.5$

Your Turn

b is inversely proportional to a

When $b = 10, a = 3$

- a) Find b when $a = 5$
- b) Find a when $b = 0.25$

Worked Example

y is inversely proportional to the square of x

When $y = 6, x = 10$

- a) Find y when $x = 5$
- b) Find x when $y = 1.5$

Your Turn

b is inversely proportional to the square of a

When $b = 6, a = 5$

- a) Find b when $a = 10$
- b) Find a when $b = 6$

Worked Example

y is inversely proportional to the cube of x

When $y = 8, x = 10$

- a) Find y when $x = 2$
- b) Find x when $y = 93.75$

Your Turn

b is inversely proportional to the cube of a

When $b = 5, a = 2$

- a) Find b when $a = 10$
- b) Find a when $b = 0.625$

Worked Example

y is inversely proportional to the square root of x

When $y = 4, x = 25$

- a) Find y when $x = 4$
- b) Find x when $y = 2.5$

Your Turn

b is inversely proportional to the square root of a

When $b = 4, a = 9$

- a) Find b when $a = 16$
- b) Find a when $b = 6$

Fill in the Gaps

General Statement	General Equation	Table of Values	Value of k	Specific Equation	When $x = 6$, $y = ?$	When $y = 10$, $x = ?$								
$y \propto \frac{1}{x}$	$y = \frac{k}{x}$	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">4</td><td style="padding: 2px 5px;">8</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;">48</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;"></td></tr> </table>	x	1	4	8	y	48			$k = 48$	$y = \frac{48}{x}$	$y = \frac{48}{6} = 8$	$x = \frac{48}{10} = 4.8$
x	1	4	8											
y	48													
$y \propto \frac{1}{x}$	$y = \frac{k}{x}$	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">5</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;">120</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">24</td></tr> </table>	x	1	2	5	y	120		24				$x = \frac{120}{10} = 12$
x	1	2	5											
y	120		24											
		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">5</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;"></td></tr> </table>	x	1	5	10	y					$y = \frac{30}{x}$		$x = \frac{30}{10} = 3$
x	1	5	10											
y														
$y \propto \frac{1}{x}$		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">5</td><td style="padding: 2px 5px;">20</td><td style="padding: 2px 5px;">100</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">30</td><td style="padding: 2px 5px;"></td></tr> </table>	x	5	20	100	y		30					
x	5	20	100											
y		30												
$y \propto \frac{1}{x^2}$	$y = \frac{k}{x^2}$	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">3</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">40</td></tr> </table>	x	1	2	3	y			40	$k = 360$			$x = \sqrt{\frac{360}{10}} = 6$
x	1	2	3											
y			40											
$y \propto \frac{1}{x^2}$		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">3</td></tr> </table>	x	1	2	10	y			3				
x	1	2	10											
y			3											
		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr><td style="padding: 2px 5px;">x</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">5</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;">y</td><td style="padding: 2px 5px;"></td><td style="padding: 2px 5px;">4</td><td style="padding: 2px 5px;"></td></tr> </table>	x	1	5	10	y		4		$k = 20$			
x	1	5	10											
y		4												

Fill in the Gaps

General Statement	General Equation	Table of Values	Value of k	Specific Equation												
$y \propto \frac{1}{x^2}$	$y = \frac{k}{x^2}$	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;">x</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">5</td> </tr> <tr> <td style="padding: 2px 5px;">y</td> <td colspan="3" style="padding: 2px 5px;">100</td> </tr> </table>	x	1	2	5	y	100			$k = 100$					
x	1	2	5													
y	100															
$y \propto \frac{1}{\sqrt{x}}$		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;">x</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">25</td> </tr> <tr> <td style="padding: 2px 5px;">y</td> <td colspan="3" style="padding: 2px 5px;">5</td> </tr> </table>	x	1	4	25	y	5								
x	1	4	25													
y	5															
$y \propto \frac{1}{x^3}$		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;">x</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">10</td> </tr> <tr> <td style="padding: 2px 5px;">y</td> <td colspan="3" style="padding: 2px 5px;">125</td> </tr> </table>	x	1	2	10	y	125								
x	1	2	10													
y	125															
	$y = \frac{k}{\sqrt[3]{x}}$	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;">x</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">8</td> <td style="padding: 2px 5px;">125</td> </tr> <tr> <td style="padding: 2px 5px;">y</td> <td colspan="3" style="padding: 2px 5px;">20</td> </tr> </table>	x	1	8	125	y	20								
x	1	8	125													
y	20															
		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;">x</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">10</td> </tr> <tr> <td style="padding: 2px 5px;">y</td> <td colspan="3" style="padding: 2px 5px;">7.5</td> </tr> <tr> <td style="padding: 2px 5px;"></td> <td colspan="3" style="padding: 2px 5px;">3</td> </tr> </table>	x	1	4	10	y	7.5				3				
x	1	4	10													
y	7.5															
	3															
		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;">x</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;"></td> </tr> <tr> <td style="padding: 2px 5px;">y</td> <td colspan="3" style="padding: 2px 5px;">0.75</td> </tr> <tr> <td style="padding: 2px 5px;"></td> <td colspan="3" style="padding: 2px 5px;">0.12</td> </tr> </table>	x	1	2		y	0.75				0.12			$k = 3$	
x	1	2														
y	0.75															
	0.12															
		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;">x</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;"></td> </tr> <tr> <td style="padding: 2px 5px;">y</td> <td colspan="3" style="padding: 2px 5px;">$\frac{1}{12}$</td> </tr> <tr> <td style="padding: 2px 5px;"></td> <td colspan="3" style="padding: 2px 5px;">$\frac{1}{30}$</td> </tr> </table>	x	1	4		y	$\frac{1}{12}$				$\frac{1}{30}$			$k = \frac{1}{6}$	
x	1	4														
y	$\frac{1}{12}$															
	$\frac{1}{30}$															
$y \propto \frac{1}{x^3}$		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;">x</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">$\sqrt{3}$</td> <td style="padding: 2px 5px;">2</td> </tr> <tr> <td style="padding: 2px 5px;">y</td> <td colspan="3" style="padding: 2px 5px;">3</td> </tr> </table>	x	1	$\sqrt{3}$	2	y	3								
x	1	$\sqrt{3}$	2													
y	3															
		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;">x</td> <td style="padding: 2px 5px;">0.5</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;"></td> </tr> <tr> <td style="padding: 2px 5px;">y</td> <td style="padding: 2px 5px;">$4a$</td> <td style="padding: 2px 5px;"></td> <td style="padding: 2px 5px;">$\frac{a}{16}$</td> </tr> </table>	x	0.5	2		y	$4a$		$\frac{a}{16}$	$k = a$					
x	0.5	2														
y	$4a$		$\frac{a}{16}$													

Fill in the Gaps

Inverse Proportion – Method Breakdown Complete the table. Use the equation with the known constant (k) to answer the question.

Relationship in Words	Equation	Known Values	Substitution	Constant of Proportionality (k)	Equation Re-write	Question
y is inversely proportional to x	$y = \frac{k}{x}$	When $x = 8$, $y = 2$	$2 = \frac{k}{8}$			When $x = 2$, $y =$
y is inversely proportional to x squared		When $x = 4$, $y = 0.5$	$0.5 = \frac{k}{(4)^2}$			When $x = 2$, $y =$
	$y = \frac{k}{x^3}$	When $x = 2$, $y = 5$				When $x = 1$, $y =$
	$y = \frac{k}{\sqrt{x}}$	When $x = 25$, $y = 4$				When $x = 100$, $y =$
		When $x = 8$, $y = 4$	$4 = \frac{k}{\sqrt[3]{8}}$			When $x = 64$, $y =$
y is inversely proportional to x		When $x = 2$, $y = 2.5$				When $x = 20$, $y =$
y is inversely proportional to x cubed		When $x = 4$, $y = 0.25$				When $x = 10$, $y =$
y is inversely proportional to the square root of x		When $x = 100$, $y = 3$				When $x = 9$, $y =$
y is inversely proportional to the cube root of x		When $x = 125$, $y = 10$				When $x = 8$, $y =$
y is inversely proportional to x squared		When $x = 10$, $y = 2$				When $x = 5$, $y =$

Worked Example

y is inversely proportional to $x + 3$

When $y = 52$, $x = 3$

Find y when $x = 5$

Your Turn

y is inversely proportional to $2x + 1$

When $y = 30$, $x = 4$

Find y when $x = 7$

Fill in the Gaps

Type	Statement	k-Formula	k value $x = 2, y = 4$	Final Formula
y is proportional to x	$y \propto x$	$y = kx$		
x is proportional to y				
y is inversely proportional to x	$y \propto \frac{1}{x}$	$y = \frac{k}{x}$		
x is inversely proportional to y				
y is proportional to the square of x				
x is proportional to the square of y				
x is proportional to \sqrt{y}				
Y is inversely proportional to \sqrt{x}				
Y is proportional to x^3				
x is proportional to 3 more than y				

Fill in the Gaps

Direct & Inverse Proportion – Method Breakdown Complete the table. Use the equation with the known constant (k) to answer the question.

Relationship in Words	Equation	Known Values	Substitution	Constant of Proportionality (k)	Equation Re-Write	Question
y is directly proportional to x	$y = kx$	When $x = 9$, $y = 45$	$45 = k(9)$			When $x = 10$, $y =$
y is inversely proportional to x	$y = \frac{k}{x}$	When $x = 8$, $y = 2$				When $x = 2$, $y =$
y is directly proportional to x squared		When $x = 3$, $y = 36$	$36 = k(3)^2$			When $x = 5$, $y =$
	$y = kx^3$	When $x = 4$, $y = 128$				When $x = 3$, $y =$
	$y = \frac{k}{x^2}$	When $x = 4$, $y = 0.5$				When $x = 2$, $y =$
y is inversely proportional to x cubed		When $x = 2$, $y = 5$				When $x = 1$, $y =$
y is directly proportional to the square root of x	$y = k\sqrt{x}$	When $x = 25$, $y = 15$				When $x = 100$, $y =$
	$y = k\sqrt[3]{x}$	When $x = 8$, $y = 20$				When $x = 64$, $y =$
	$y = \frac{k}{\sqrt{x}}$	When $x = 25$, $y = 4$				When $x = 100$, $y =$
y is inversely proportional to the cube root of x		When $x = 8$, $y = 4$				When $x = 64$, $y =$
y is directly proportional to x squared		When $x = 3$, $y = 4.5$				When $y = 72$, $x =$
y is inversely proportional to the square root of x		When $x = 100$, $y = 3$				When $x = 9$, $y =$

Worked Example

x is inversely proportional to y^2

y is directly proportional to $\sqrt[3]{z}$

Given that $x = 10$ and $z = 512$ when $y = 7$ find a formula for x in terms of z

Your Turn

x is directly proportional to y^3

y is inversely proportional to \sqrt{z}

Given that $x = 10$ and $z = 36$ when $y = 5$ find a formula for x in terms of z

Worked Example

q is proportional to t^3

t is decreased by 30%

Work out the percentage decrease in q

Your Turn

y is proportional to z^2

z is decreased by 80%

Work out the percentage decrease in y

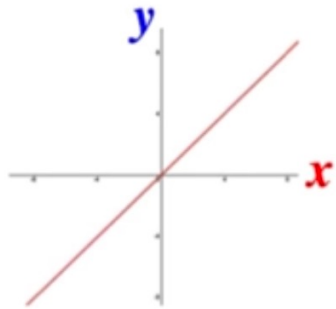
Worked Example

t is inversely proportional to z^3
 z is decreased by 50%
Find the percentage increase in t

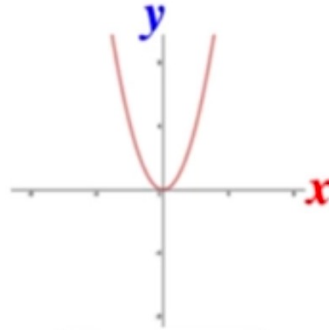
Your Turn

y is inversely proportional to p^2
 p is decreased by 50%
Find the percentage increase in y

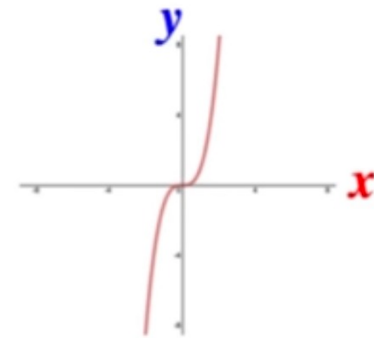
Graphs



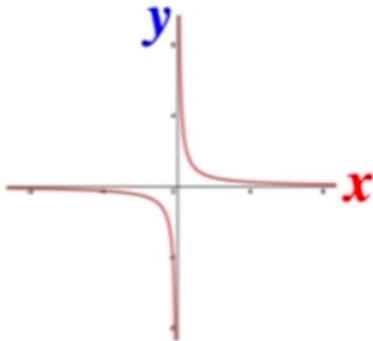
$$y = kx$$



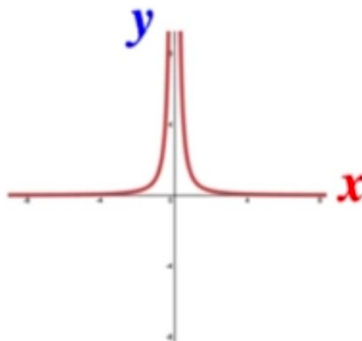
$$y = kx^2$$



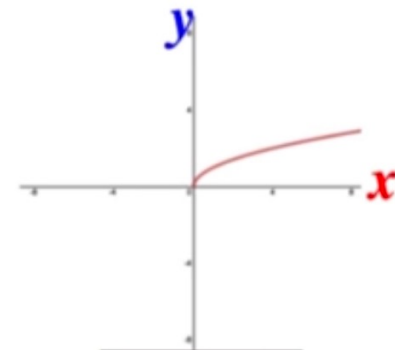
$$y = kx^3$$



$$y = \frac{k}{x}$$



$$y = \frac{k}{x^2}$$

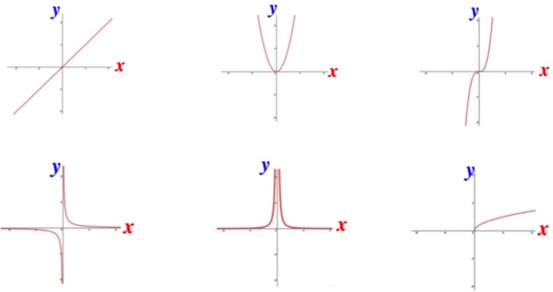


$$y = k\sqrt{x}$$

Fluency Practice

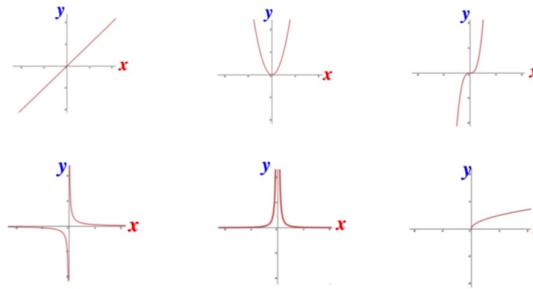
y is proportional to the square of x

Which of the following could be the graph demonstrating between y and x ?



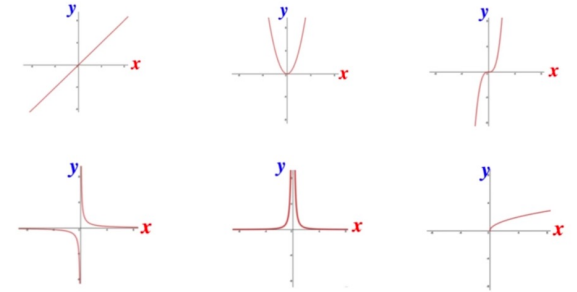
$y \propto x$

Which of the following could be the graph demonstrating between y and x ?



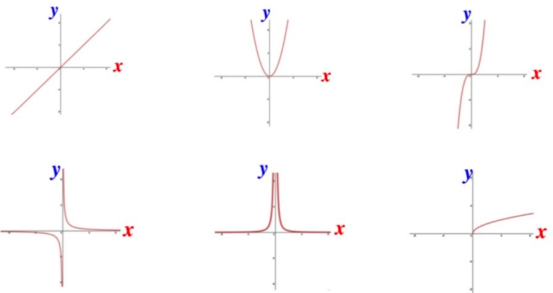
y is inversely proportional to x

Which of the following could be the graph demonstrating between y and x ?



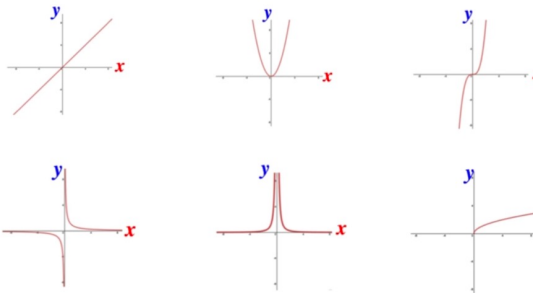
$y \propto x^3$

Which of the following could be the graph demonstrating between y and x ?



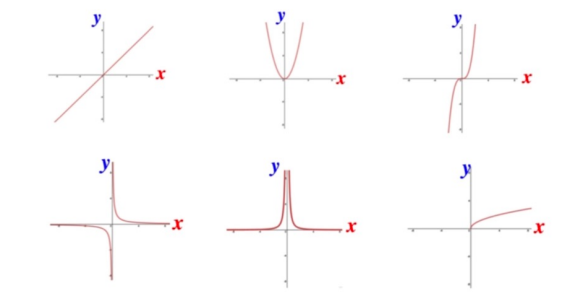
y is inversely proportional to the square of x

Which of the following could be the graph demonstrating between y and x ?



$y \propto \sqrt{x}$

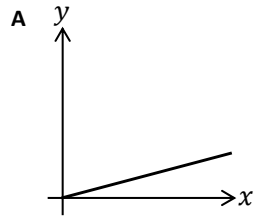
Which of the following could be the graph demonstrating between y and x ?



Fluency Practice

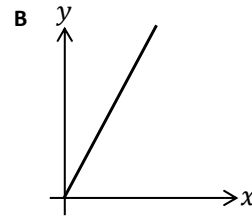
Proportional Graphs

For each **black** graph, select the correct proportional relationship & complete the table of values.
(x & y axes have equal scales)



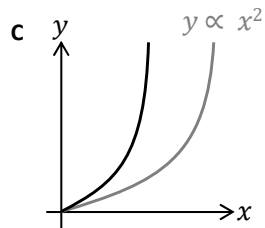
$y \propto 2x + 5$ $y \propto \frac{x}{4}$ $y \propto 4x$

x	0	2	4	8
y				



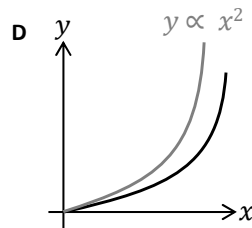
$y \propto 0.5x$ $y \propto \frac{x}{5}$ $y \propto 5x$

x	1	2	4	8
y				



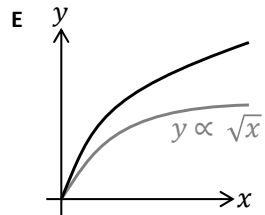
$y \propto 0.5x^2$ $y \propto 3x^2$ $y \propto \frac{5}{x^2}$

x	1	2	3	4
y				



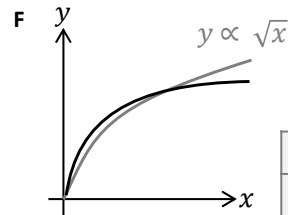
$y \propto 0.2x^2$ $y \propto 3x^3$ $y \propto 4x^2$

x	1	5	10	20
y				



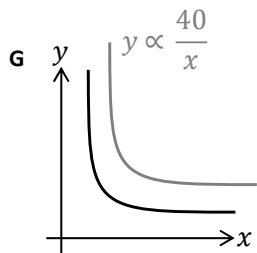
$y \propto 2\sqrt[3]{x}$ $y \propto 3\sqrt{x}$ $y \propto \frac{5}{x^2}$

x	1	4	9	16
y				



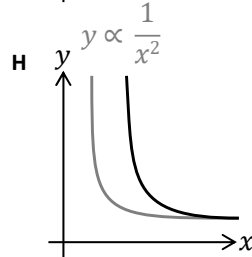
$y \propto 4\sqrt[3]{x}$ $y \propto \sqrt[3]{x}$ $y \propto 2\sqrt{x}$

x	1	8	64	125
y				



$y \propto \frac{60}{x}$ $y \propto \frac{1}{x^2}$ $y \propto \frac{20}{x}$

x	1	2	4	10
y				



$y \propto \frac{5}{x^2}$ $y \propto \frac{1}{x^3}$ $y \propto \frac{1}{\sqrt{x}}$

x	1	2	5	10
y				

Extra Notes

4 Constructions and Loci

To 'construct' something in the strictest sense means to draw it using only two things:

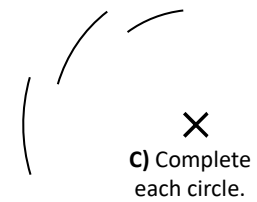
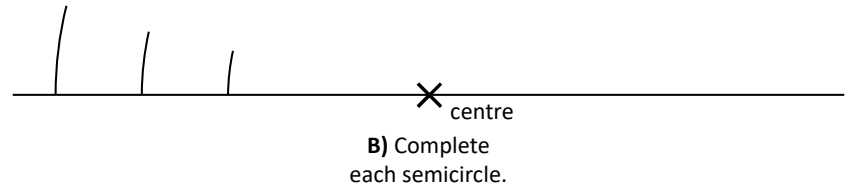
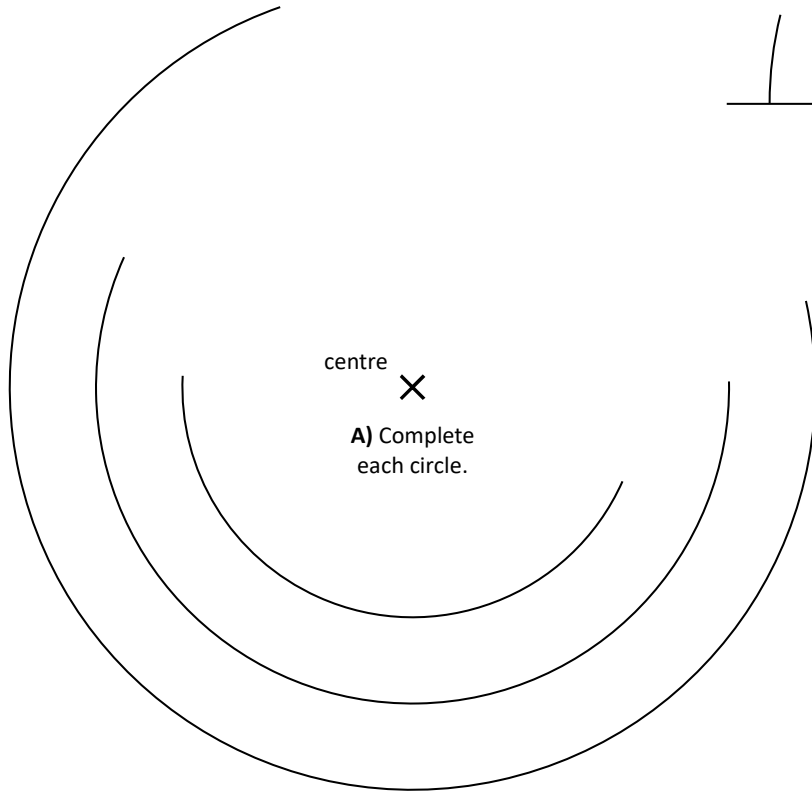
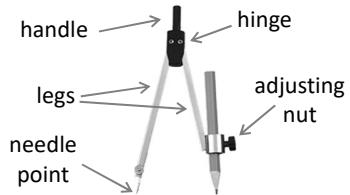
- Compass
- Straight Edge (Apart from where a length is specified, you are not allowed to measure lengths)

Bisect means cut into two equal parts.

Equidistant means equal distance from

Fluency Practice

Compass Skills



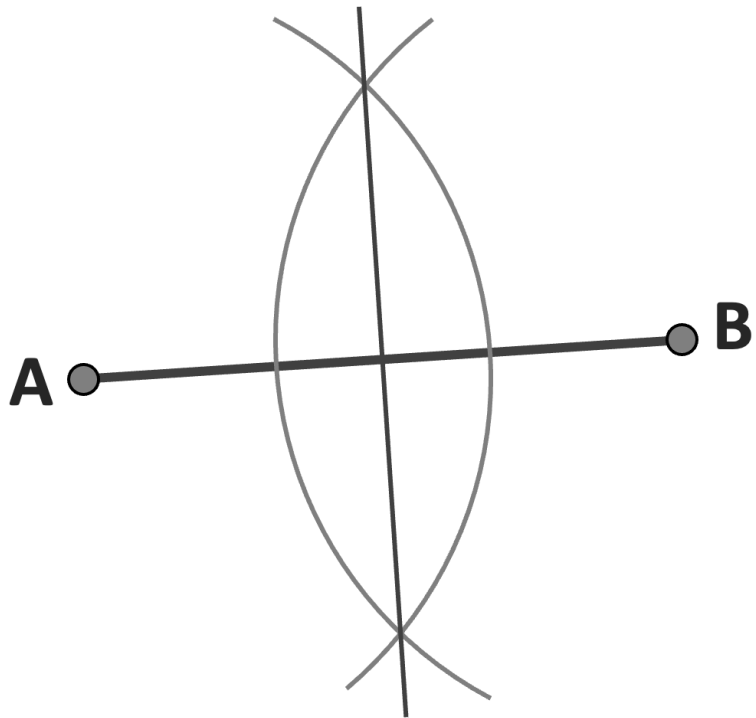
Quick Tips:

- When the compass is closed, the point and the pencil point should meet.
- Make sure the legs are not loose (this needs tight hinge screws).
- Place paper underneath the worksheet to make the point hold.
- When drawing, try to only hold the handle with a finger and thumb.

Perpendicular Bisector

Draw two points on your page and label them A and B.
Join them with a straight line.
Construct its perpendicular bisector.

- 1) Draw two equal arcs.
- 2) Connect the intersections with a straight line.
- 3) This line is the perpendicular bisector and contains all the points equidistant from A and B.



Worked Example

Construct the perpendicular bisector of the line:



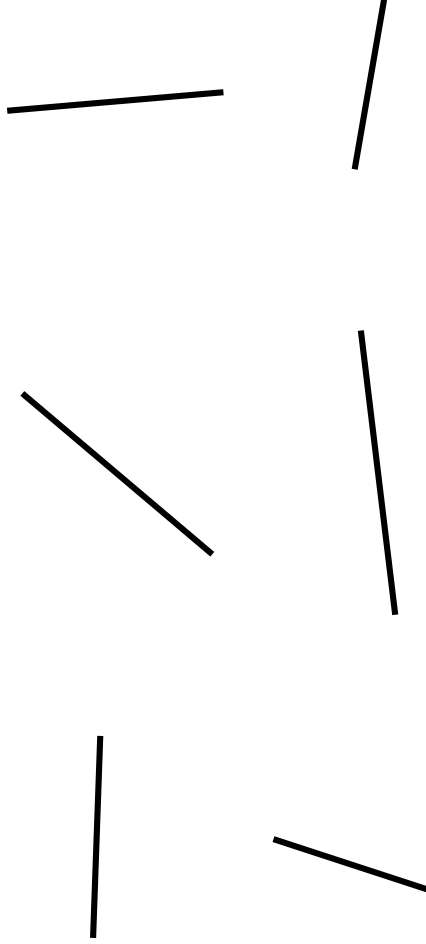
Your Turn

Construct the perpendicular bisector of the line:

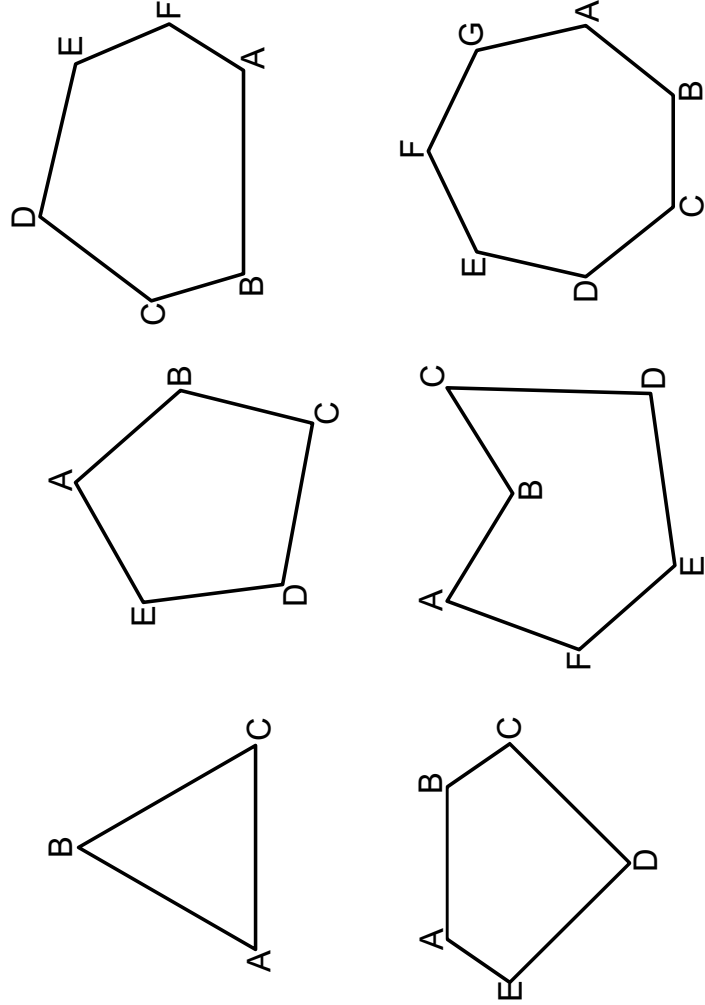


Fluency Practice

Q1 Construct the perpendicular bisector of the following lines.



Q2 Construct the perpendicular bisector of the line **AB** in each of the following.

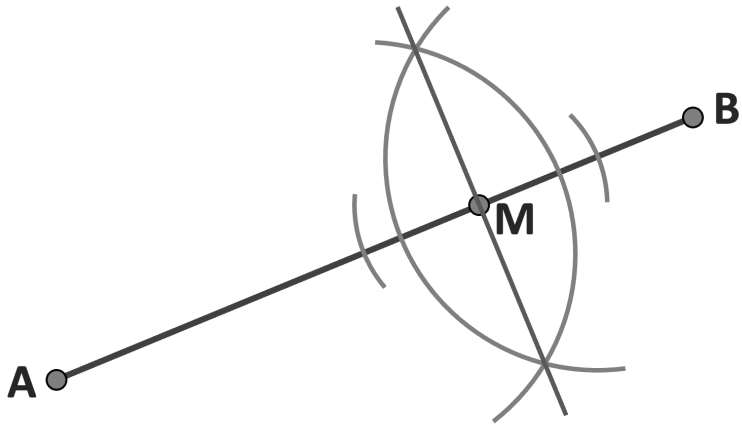


Perpendicular Line at a Point 1

M is a point on the line AB.

Construct a line perpendicular to AB through M.

- 1) Use your compass to find two points on the line equidistance from M.
- 2) Construct a perpendicular bisector of these two points.



Worked Example

Construct a perpendicular to the line which passes through the marked point:



Your Turn

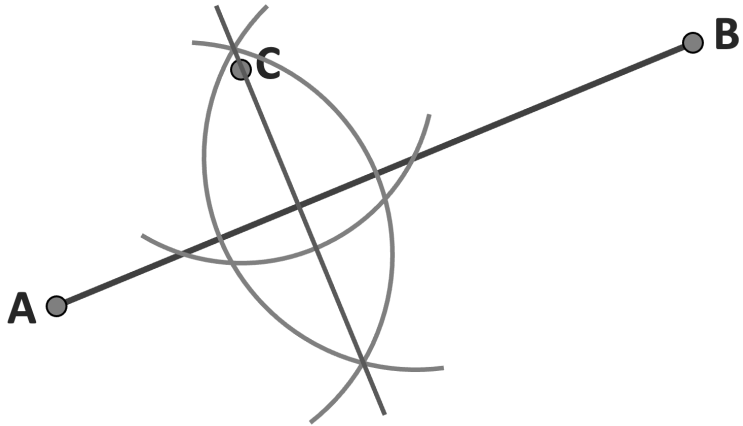
Construct a perpendicular to the line which passes through the marked point:



Perpendicular Line at a Point 2

Construct a line perpendicular to AB through C, which is a point not on AB.

- 1) Use your compass to find two points on the line equidistance from C.
- 2) Construct a perpendicular bisector of these two points.



Worked Example

Construct a perpendicular to the line which passes through the marked point:



Your Turn

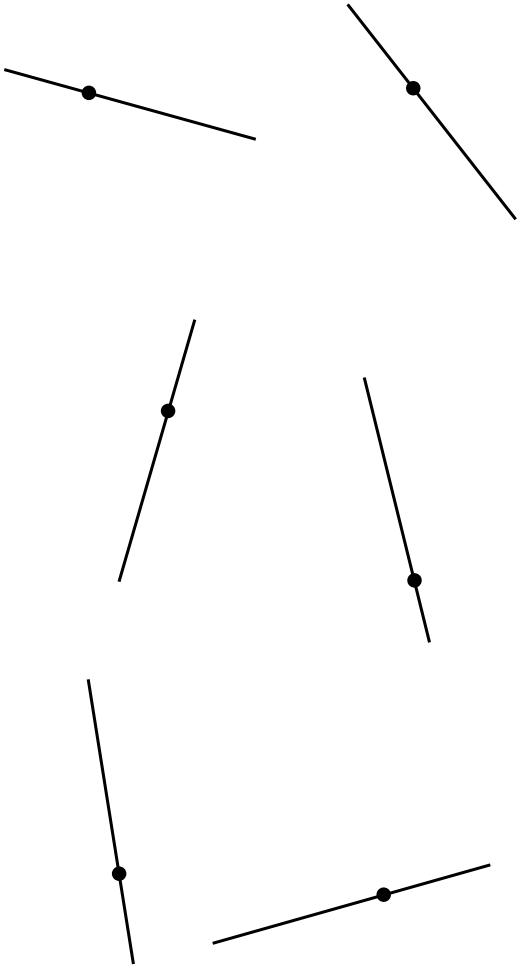
Construct a perpendicular to the line which passes through the marked point:

X

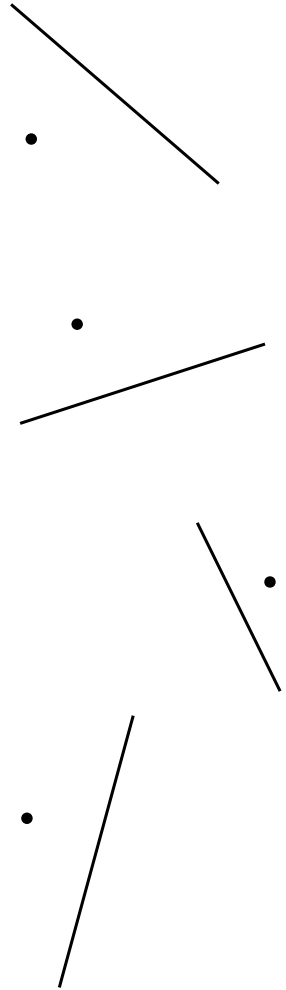


Fluency Practice

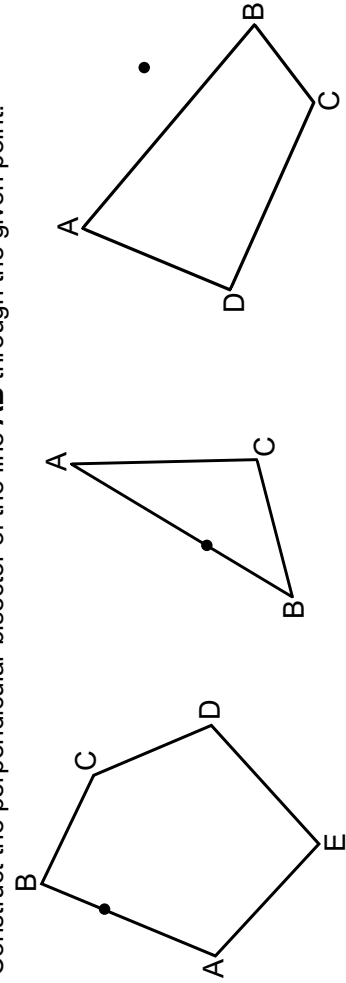
Q3 Construct the perpendicular bisector of the following lines through the given point.



Q4 Construct the perpendicular bisector of the following lines through the given point.



Q5 Construct the perpendicular bisector of the line **AB** through the given point.

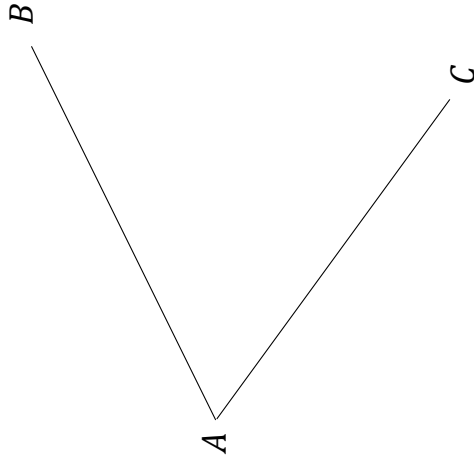


Fluency Practice

Constructing Perpendiculars and Bisectors

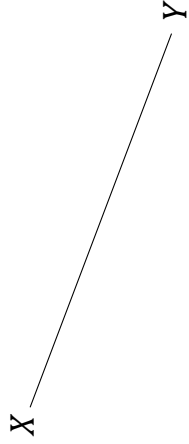
(a)

Construct the bisector of the angle BAC.



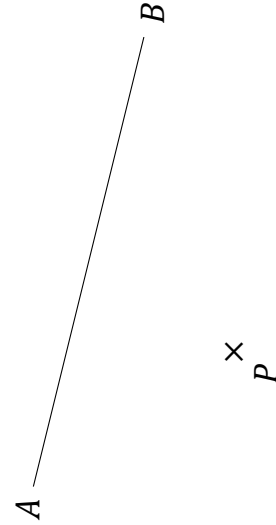
(b)

Construct the perpendicular bisector of the line XY .



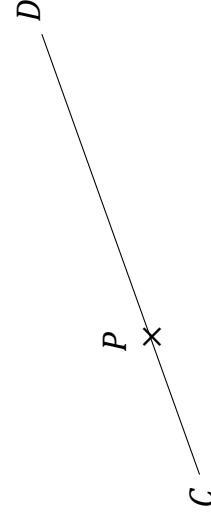
(c)

Construct the perpendicular to the line AB that passes through point P .



(d)

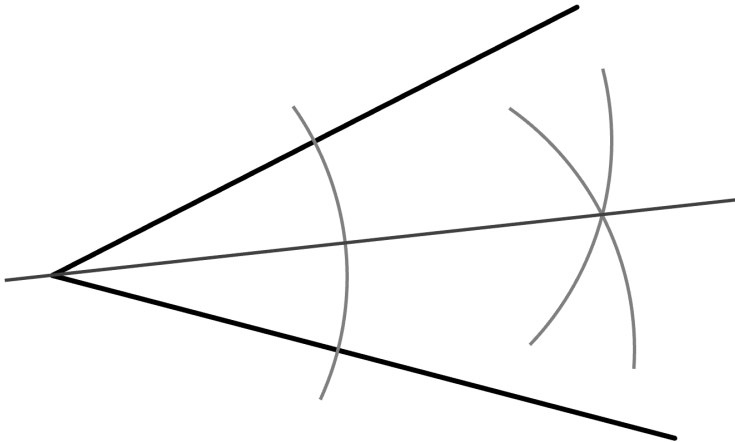
Construct the perpendicular to the line CD that passes through point P .



Angle Bisector

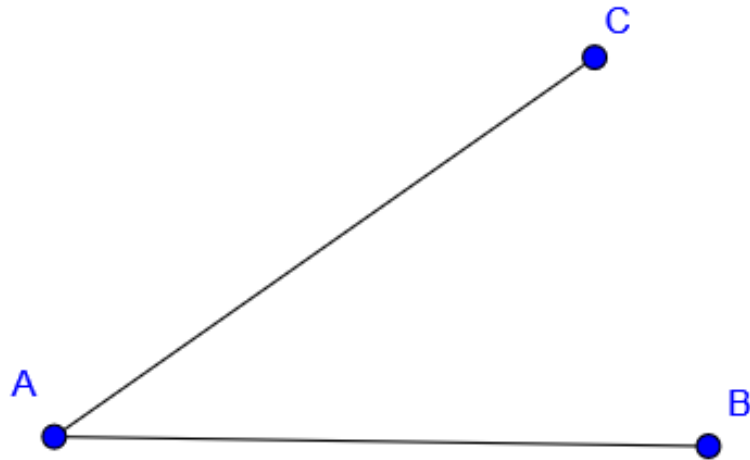
Draw an acute angle on your page.
Construct its angle bisector.

- 1) Draw an arc from the vertex.
- 2) Draw two more equal arcs from the intersections.
- 3) Join the new intersection up to the vertex.
- 4) This line is the angle bisector and contains all points equidistant from both arms of the angle.



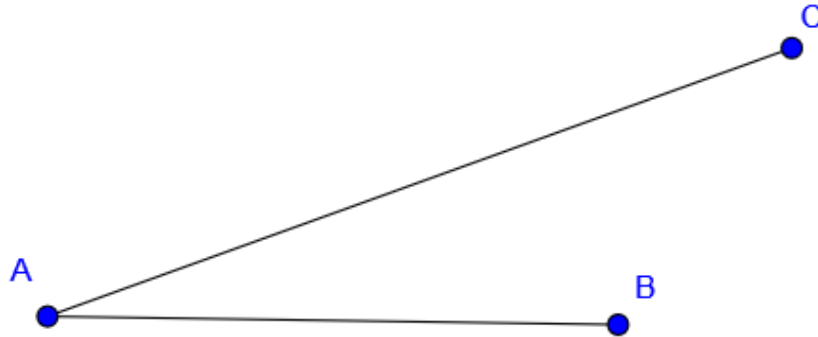
Worked Example

Bisect angle BAC:



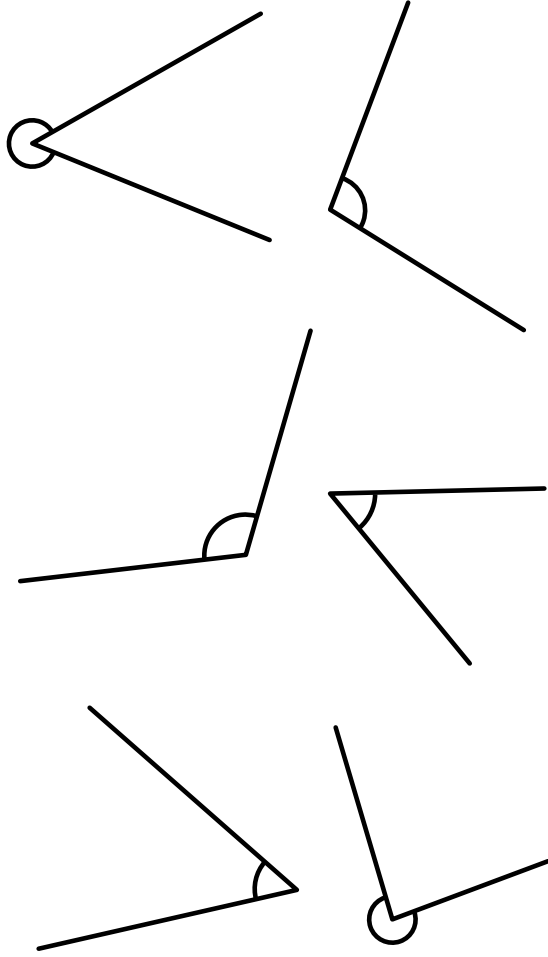
Your Turn

Bisect angle BAC:

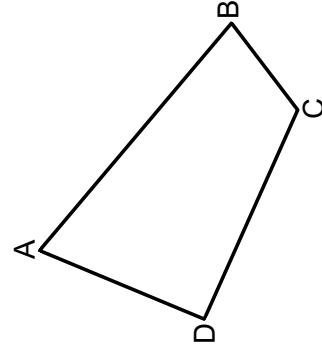
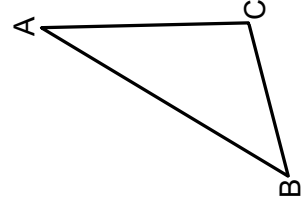
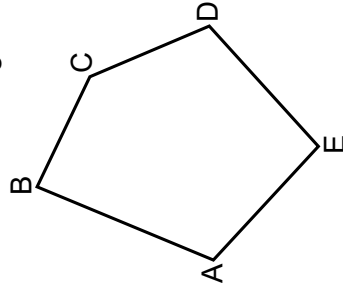


Fluency Practice

Q6 Construct the angle bisector of the following angles.



Q7 Construct the angle bisector of $\angle ABC$ for each of the following shapes..



Q8 Construct a 90° angle.

Q9 Construct a 45° angle.

Constructing Triangles

You can construct a unique triangle when you know:

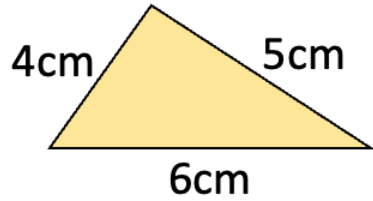
Two sides and the angle between them (**SAS**)

Two angles and a side (**ASA**)

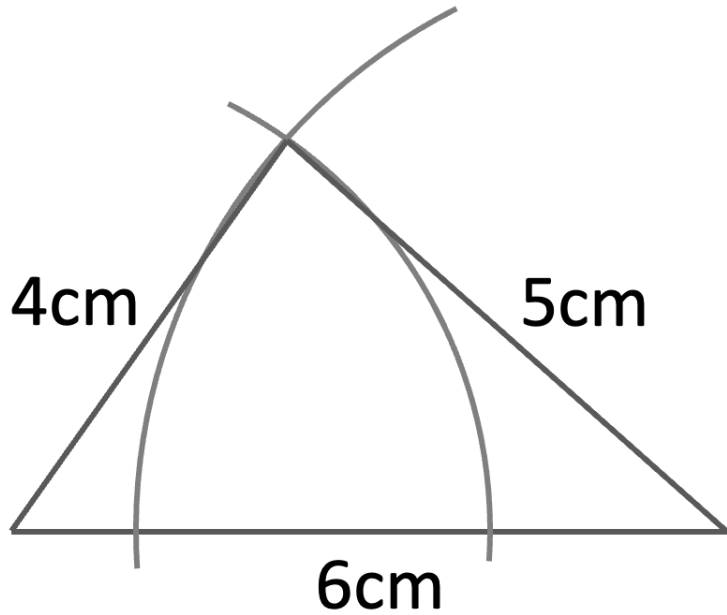
Three sides (**SSS**)

SSS

Using a ruler and compass only, construct the following SSS triangle accurately.



- 1) Draw a 6cm line with a ruler.
- 2) Draw two arcs with lengths 4cm and 5cm from each end of the line.
- 3) Join the ends of the line to the intersection.



Worked Example

Construct a triangle with:

- A side length of 10 *cm*
- A side length of 6 *cm*
- A side length of 8 *cm*

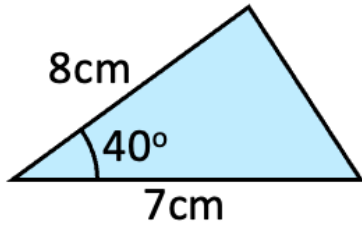
Your Turn

Construct a triangle with:

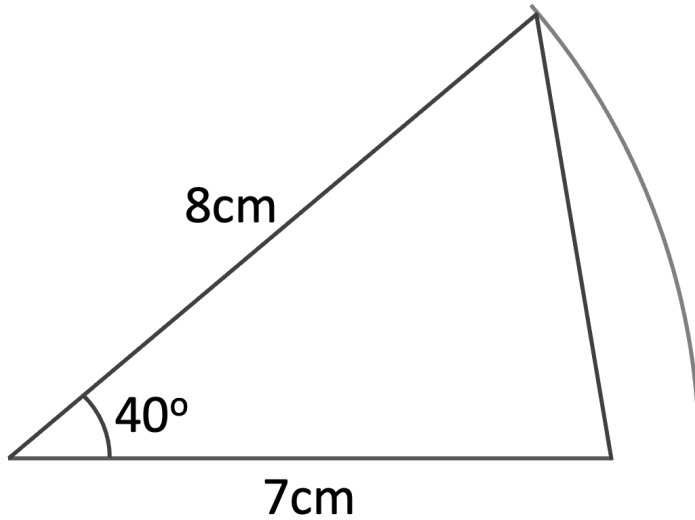
- A side length of 5 *cm*
- A side length of 3 *cm*
- A side length of 4 *cm*

SAS

Using a ruler, compass and protractor, construct the following SAS triangle accurately.



- 1) Draw a 7cm line with a ruler.
- 2) Draw an arc with length 8cm .
- 3) Measure an angle of 40° .
- 4) Draw a line through the angle to the arc.
- 5) Join up the end of the lines.



Worked Example

Construct a triangle with:

- A side length of 10 *cm*
- An angle of 30°
- A side length of 8 *cm*

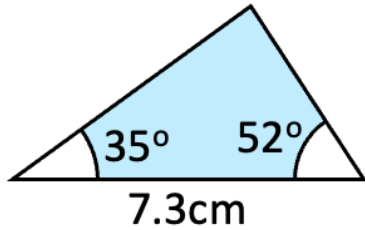
Your Turn

Construct a triangle with:

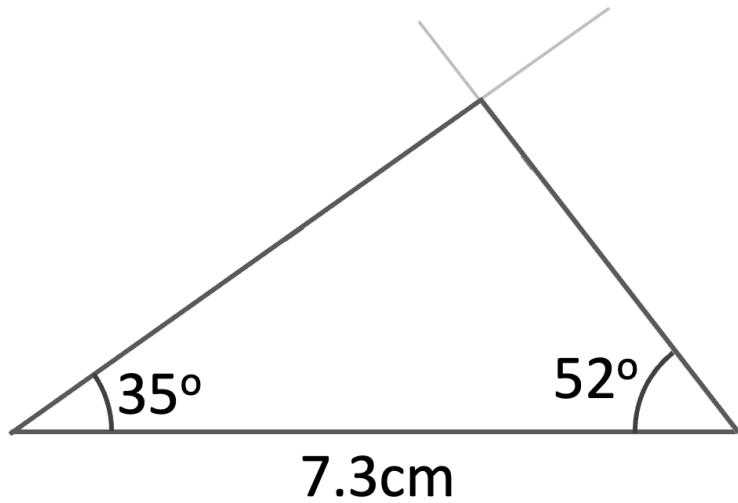
- A side length of 5 *cm*
- An angle of 30°
- A side length of 4 *cm*

ASA

Using a ruler, compass and protractor, construct the following ASA triangle accurately.



- 1) Draw a 7.3cm line with a ruler.
- 2) Measure both angles.
- 3) Draw a faint line through each angle and label them.
- 4) Draw a solid line over each faint line up to the intersection.



Worked Example

Construct a triangle with:

- An angle of 30°
- A side length of 10 cm
- An angle of 45°

Your Turn

Construct a triangle with:

- An angle of 30°
- A side length of 5 cm
- An angle of 60°

Loci

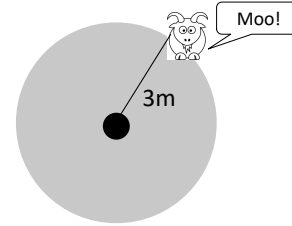
A **locus** is a path formed by a point which moves according to a rule. The plural is **loci**.

A **locus of points** is a set of points satisfying a certain condition.

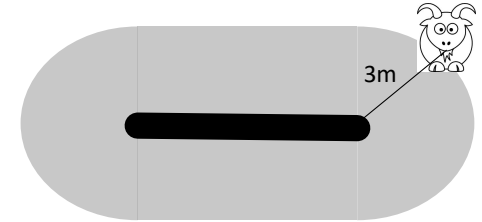
We can use our constructions from last lesson to find the loci satisfying certain conditions...

Loci involving:		Interpretation	Resulting Locus
Thing A	Thing B		
Point	-	A given distance from point A	
Line	-	A given distance from line A	
Point	Point	Equidistant from 2 points or given distance from each point.	
Line	Line	Equidistant from 2 lines	
Point	Line	Equidistant from point A and line B	

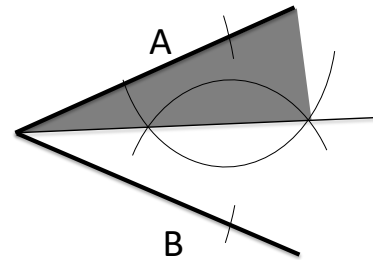
Loci can also be **regions** satisfying certain descriptions.



A goat is attached to a post, by a rope of length 3m. Shade the locus representing the points the goat can reach.



A goat is now attached to a metal bar, by a rope of length 3m. The rope is attached to the bar by a ring, which is allowed to move freely along the bar. Shade the locus representing the points the goat can reach.



Shade the region consisting of points which are closer to line A than to line B.

Common schoolboy error: Thinking the locus will be oval in shape.

As always, you **MUST** show construction lines or you will be given no credit.

Fluency Practice

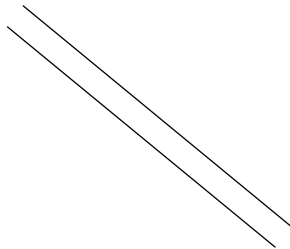
Complete as many of the following challenges as you can, as a group, making a note of the shapes you produce for each one. You will also be expected to demonstrate one of these shapes to the rest of the class.

1. In your group, stand **exactly 2m** from one member of your group.
Draw and describe the shape you have created:



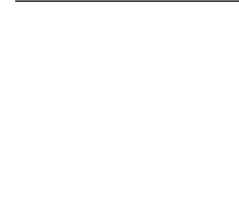
This is the locus of points a fixed distance from a point.

2. In your group, stand **exactly 1m** away from a straight wall.
Draw and describe the shape you have created:



This is the locus of points a fixed distance from a line.

3. In your group, stand **exactly 2m** a wall around a corner.
Draw and describe the shape you have created:



This can give the locus of points a fixed distance from a rectangle.

4. In your group, stand **exactly the same distance** away from two members of your group.
Draw and describe the shape you have created:



This is the locus of points equidistant from two fixed points.

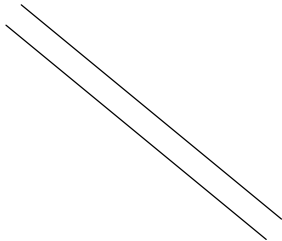
Fluency Practice

5. In your group, stand **within 2m** of one member of your group.
Draw and describe the area you have created:



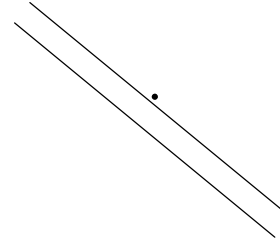
This is the locus of points within a given distance of a point.

6. In your group, stand **no further than 1m** away from a straight wall.
Draw and describe the area you have created:



This is the locus of points within a given distance of a line.

7. In your group, stand **at least 1m** away from a straight wall, and **within 2m** of a person standing beside the wall.
Draw and describe the area you have created:



This is the locus of points which satisfy both conditions.

8. Design your own conditions, either by combining those used in these challenges or creating new ones altogether.
Draw and describe the area you have created:

Worked Example

Construct the locus of points 1 *cm* away from a point.

Your Turn

Construct the locus of points 2 *cm* away from a point.

Worked Example

Construct the locus of points which is:

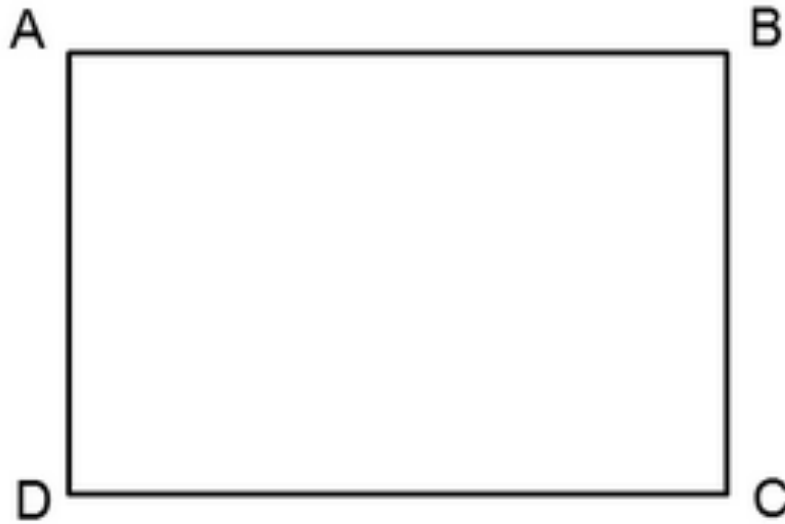
- More than 3 *cm* from A



Your Turn

Construct the locus of points which is:

- More than 5 *cm* from B



Worked Example

Construct the locus of points equidistant from two points.

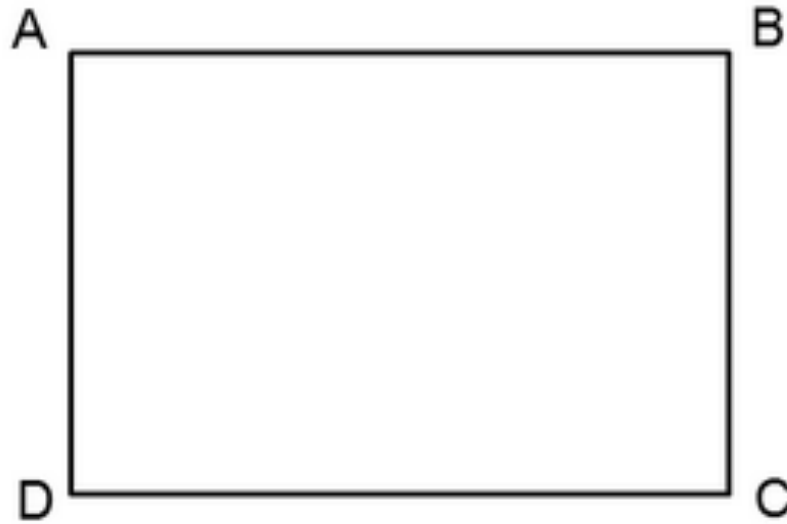
Your Turn

Construct the locus of points equidistant from two points.

Worked Example

Construct the locus of points which are:

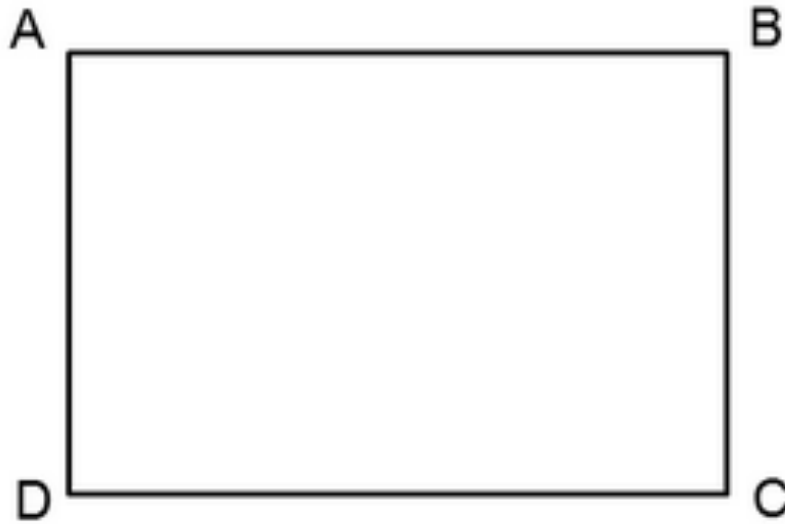
- Closer to B than A
- Closer to C than D



Your Turn

Construct the locus of points which are:

- Closer to C than B
- Closer to D than A



Worked Example

Construct the locus of points equidistant from two intersecting lines.

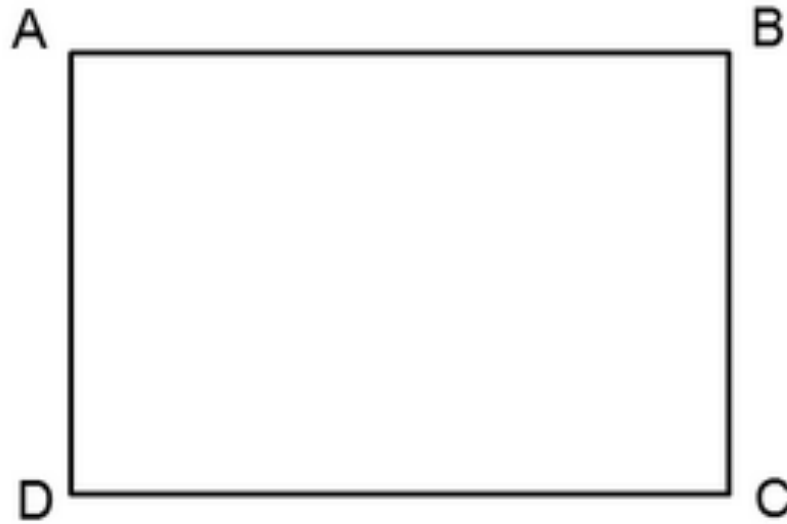
Your Turn

Construct the locus of points equidistant from two intersecting lines.

Worked Example

Construct the locus of points which is:

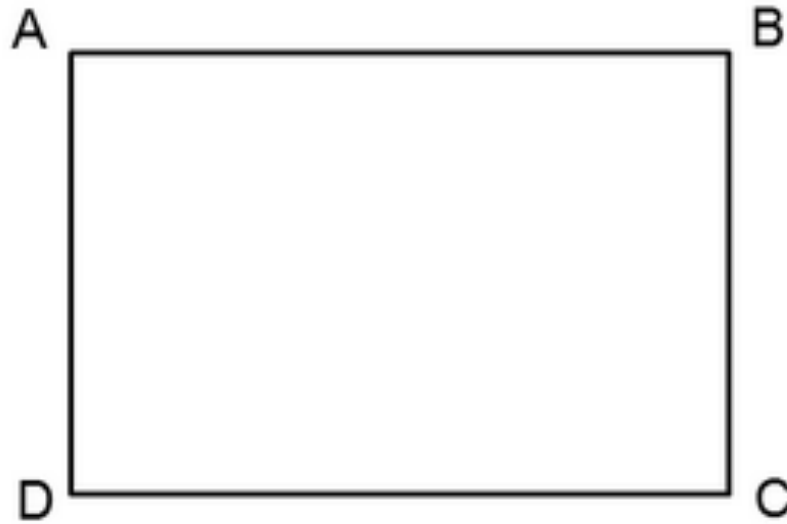
- Closer to AD than AB



Your Turn

Construct the locus of points which is:

- Closer to BC than DC



Worked Example

Construct the locus of points 1 *cm* away from the line.



Your Turn

Construct the locus of points 1 *cm* away from the line.



Worked Example

Construct the locus of points equidistant from a line.

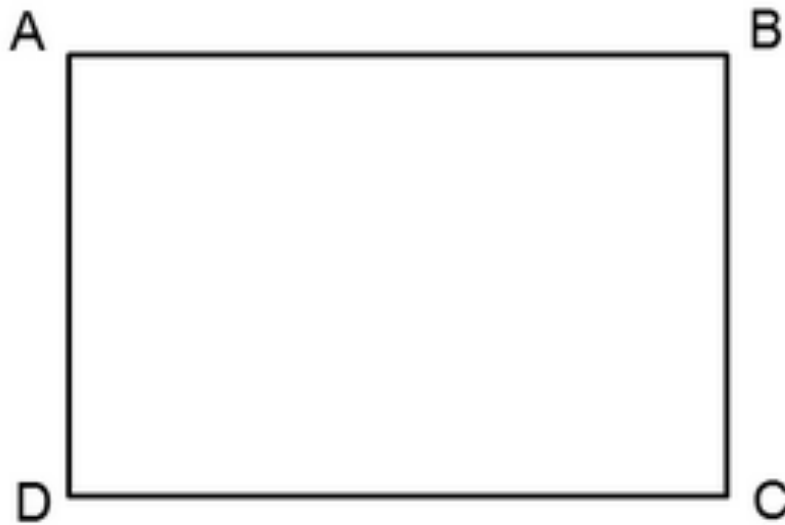
Your Turn

Construct the locus of points equidistant from a line.

Worked Example

Construct the locus of points which are:

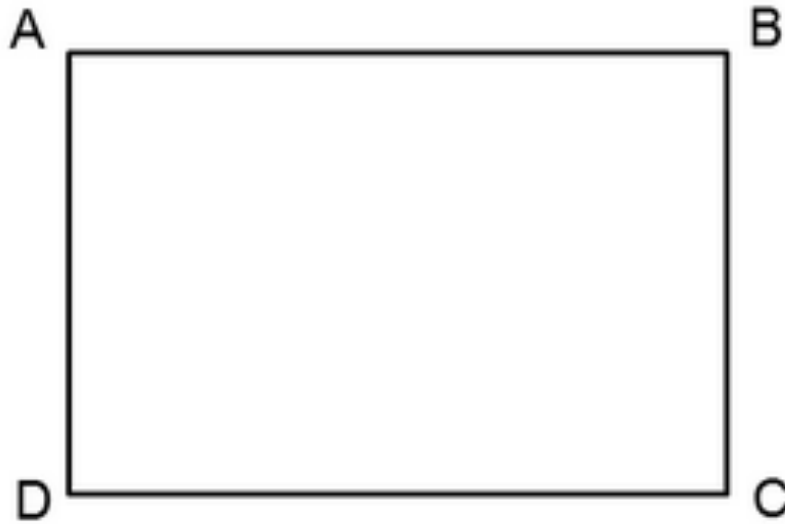
- More than 3 *cm* from AB
- More than 4 *cm* from AD



Your Turn

Construct the locus of points which are:

- More than 5 *cm* from AB
- More than 3 *cm* from AD



Worked Example

Construct the locus of points which are:

- Closer to B than C
- More than 3 *cm* from A



Your Turn

Construct the locus of points which are:

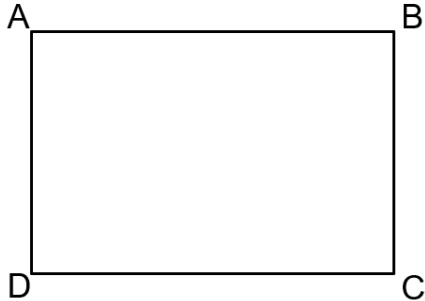
- Closer to C than A
- Less than 5 *cm* from B



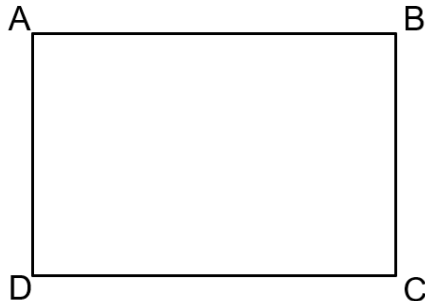
Fluency Practice

Loci Practice Grid – Shade the region inside the rectangle which satisfies the conditions given.

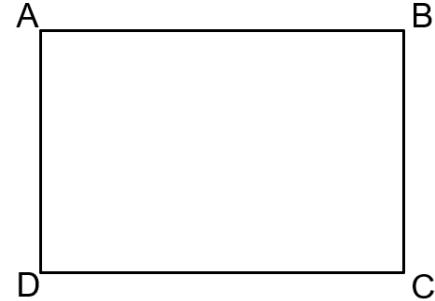
(a) Less than 3 cm from A



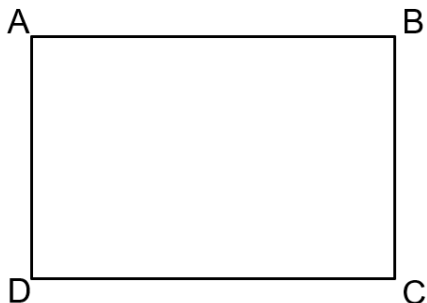
(b) More than 3 cm from D



(c) More than 2 cm from AB



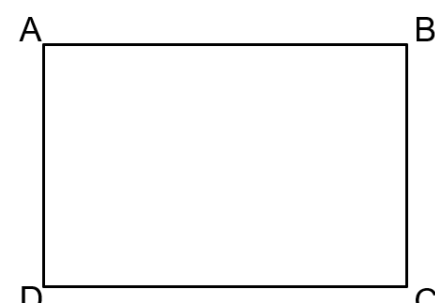
(d) Less than 3 cm from BC



(e) Closer to AB than AD



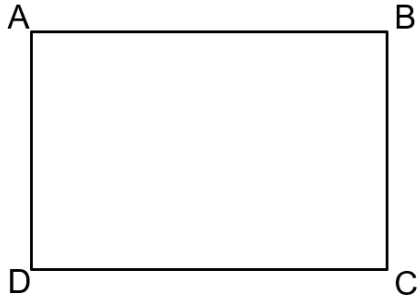
(f) Closer to A than B



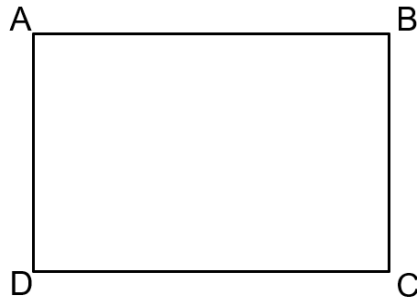
Fluency Practice

Harder Loci Practice Grid – Shade the region inside the rectangle which satisfies the conditions given.

(a) Less than 3 cm from A and more than 2 cm from AB



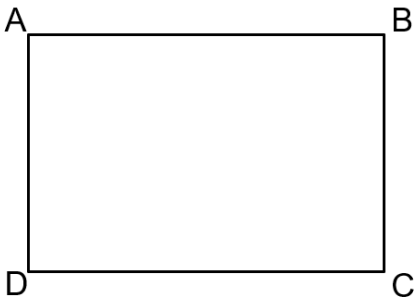
(b) More than 3 cm from B and closer to AB than BC



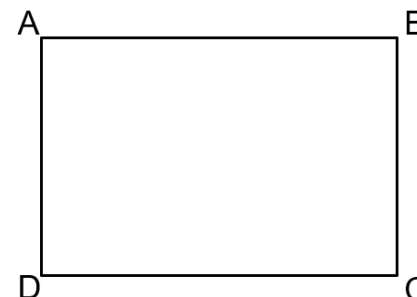
(c) Closer to AD than AB and less than 2 cm from AB



(d) Less than 3 cm from BC and less than 2 cm from CD



(e) Closer to A than B and more than 4 cm from A




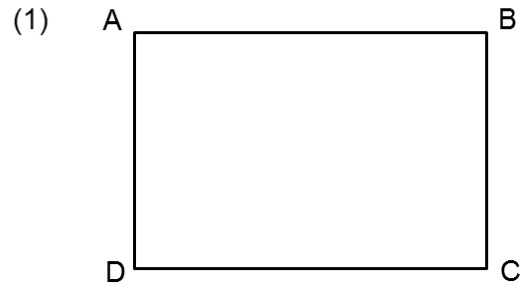
(f) More than 3 cm from D and more than 4 cm from B



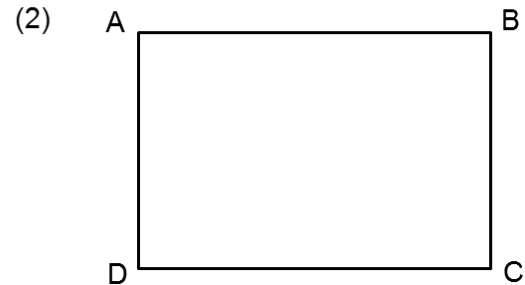
Fluency Practice

loci and regions (inside the rectangle)

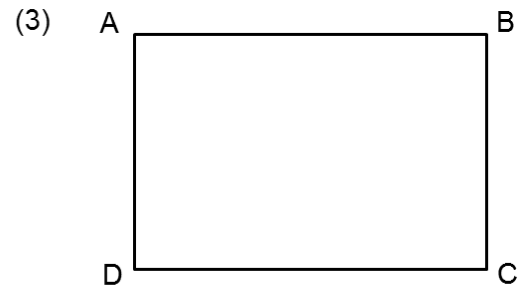
shade the region (hatching: )



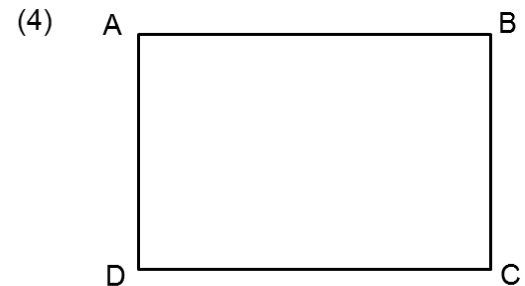
more than 3 cm from AB
more than 4 cm from AD



less than 2 cm from AB
less than 3 cm from C

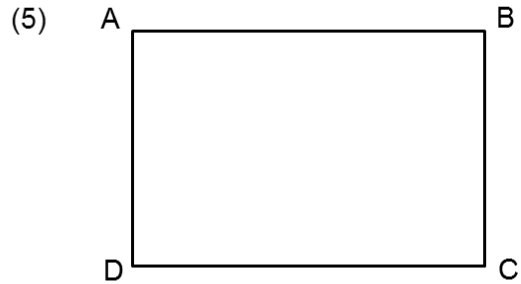


more than 4 cm from A
closer to B than C

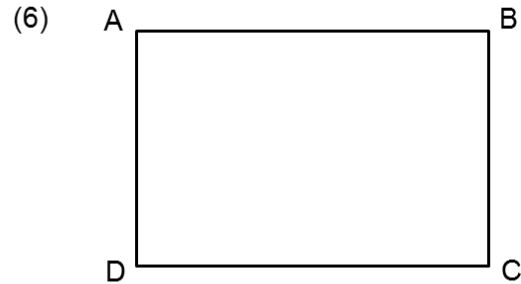


closer to AD than AB
more than 3 cm from AD

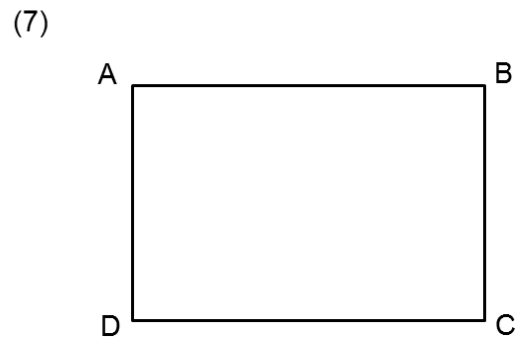
Fluency Practice



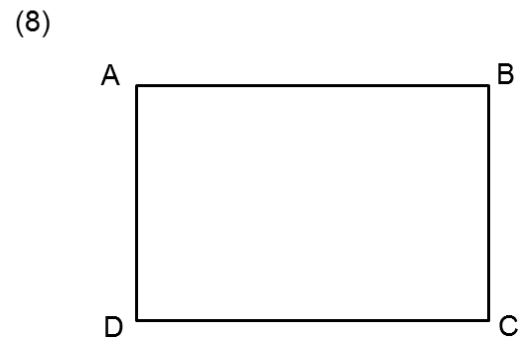
closer to AB than AD
more than 4 cm from C



closer to B than A
closer to BC than BA

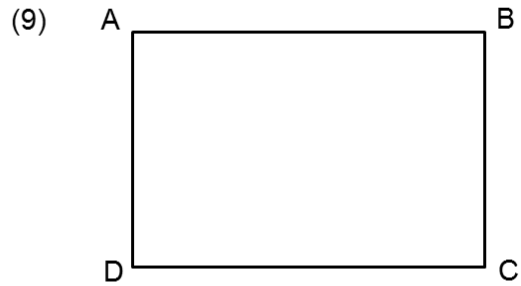


closer to CB than CD
closer to C than A

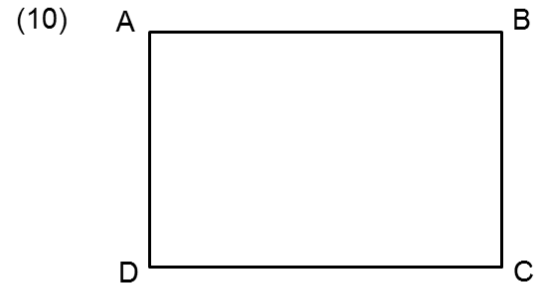


closer to CB than CD
less than 5 cm from D

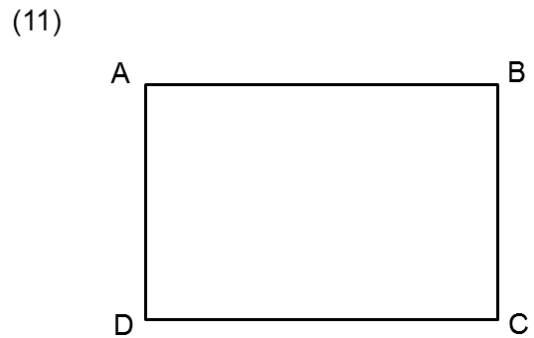
Fluency Practice



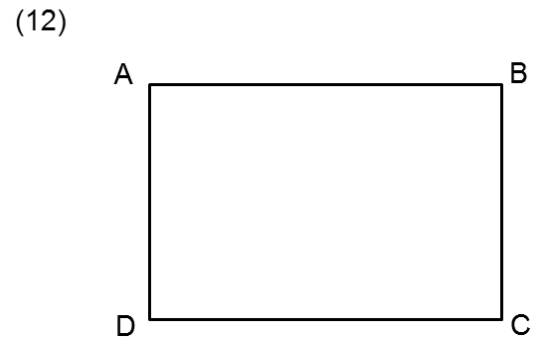
less than $3\frac{1}{2}$ cm from D
less than 5 cm from B



closer to D than B
more than 4 cm from C



closer to AB than AD
less than 4 cm from A
closer to B than A



closer to AB than BC
less than 6 cm from D
closer to AB than DC

Intelligent Practice

1. Show all the points that are 4cm from point A



2. Show all the points that are 4cm from point A and 6cm from point B



3. Show all the points that are the same distance from points A and B



4. Show all the points that are closer to point A than point B

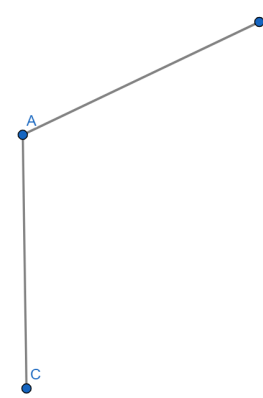


Intelligent Practice

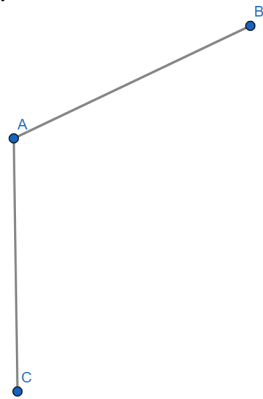
5. Show all the points that are 4cm from the line segment AB



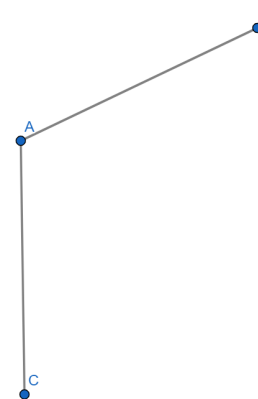
6. Show all the points that are the same distance from the line segments AB and AC



7. Show all the points that are the same distance from the line segments AB and AC, and the same distance from points A and B



8. Show all the points that are closer to line segment AB than AC, and less than 4cm from point A



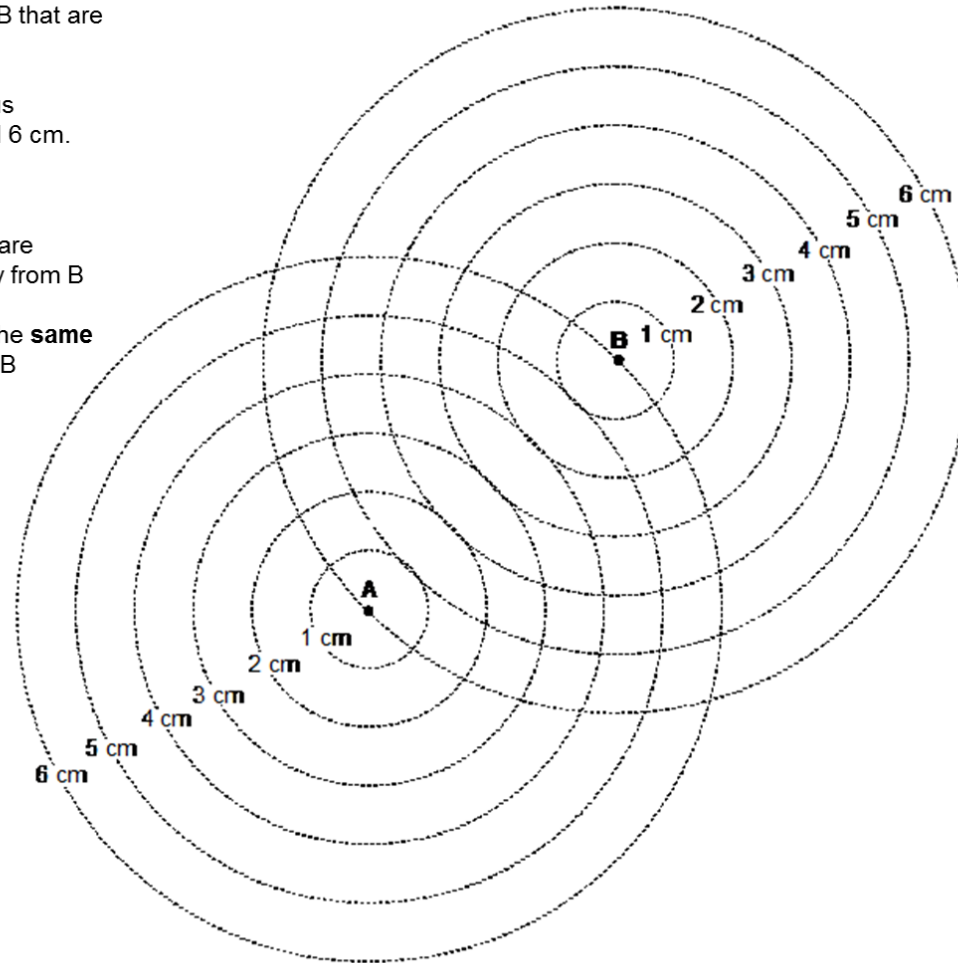
Fluency Practice

1. whirls

the diagram shows two points A and B that are 6 cm. apart

around each point are circles of radius 1 cm., 2 cm., 3 cm., 4 cm., 5 cm. and 6 cm.

- (a) mark with a cross two points that are 4 cm away from A **and** 4 cm away from B
- (b) draw the locus of points that are the **same distance** from A as they are from B



Fluency Practice

2. tree (i)

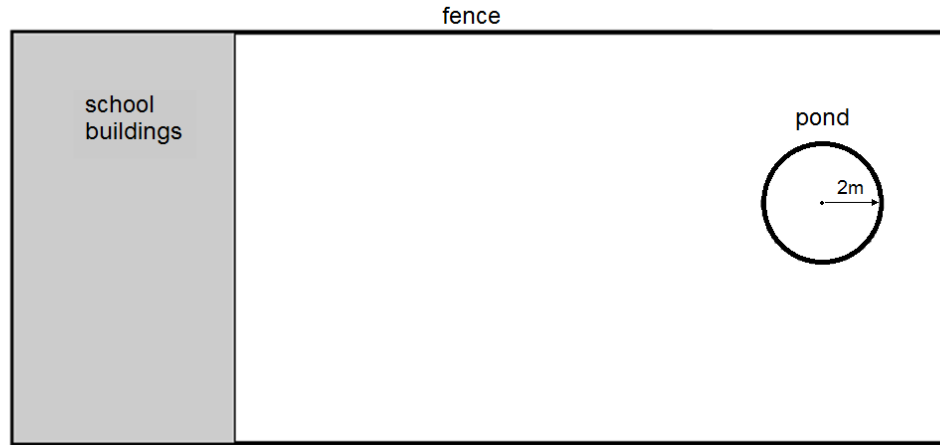
students are planning to plant a tree in the school garden

it must be at least **10m.** from the school buildings and

it must also be at least **8m.** from the centre of the circular pond

shade in the **region** to show accurately where the tree could be planted

scale 1 cm. = 2 m.



3. tree (ii)

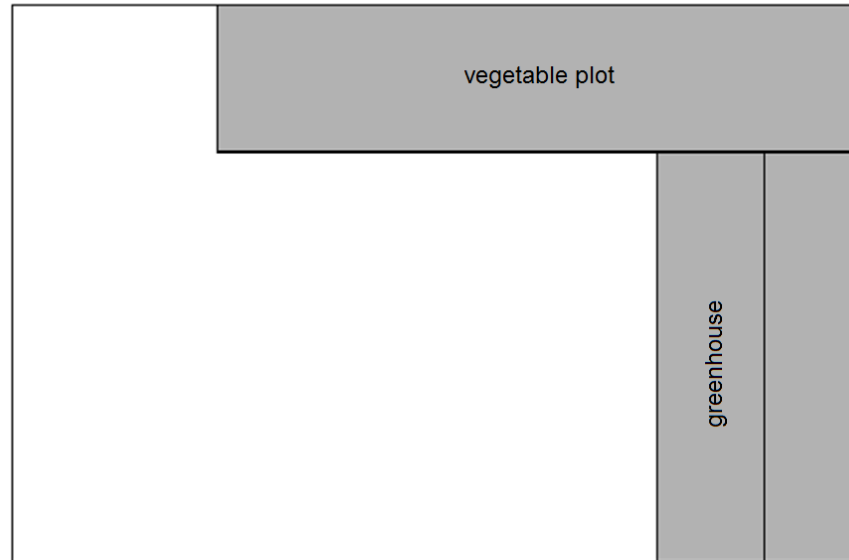
a gardener wants to plant a tree

they want it to be **more than 8m.** away from the **vegetable plot**

they want it to be **more than 18m.** away from the **greenhouse**

the plan shows part of the garden
the scale is **1cm.** to **4m.**

show **accurately** on the plan the region of the garden where she can plant the tree and label this region **R**



Fluency Practice

4. fence

in the scale drawing, the shaded area is a semi-circular lawn

there is a fence **all around** the lawn

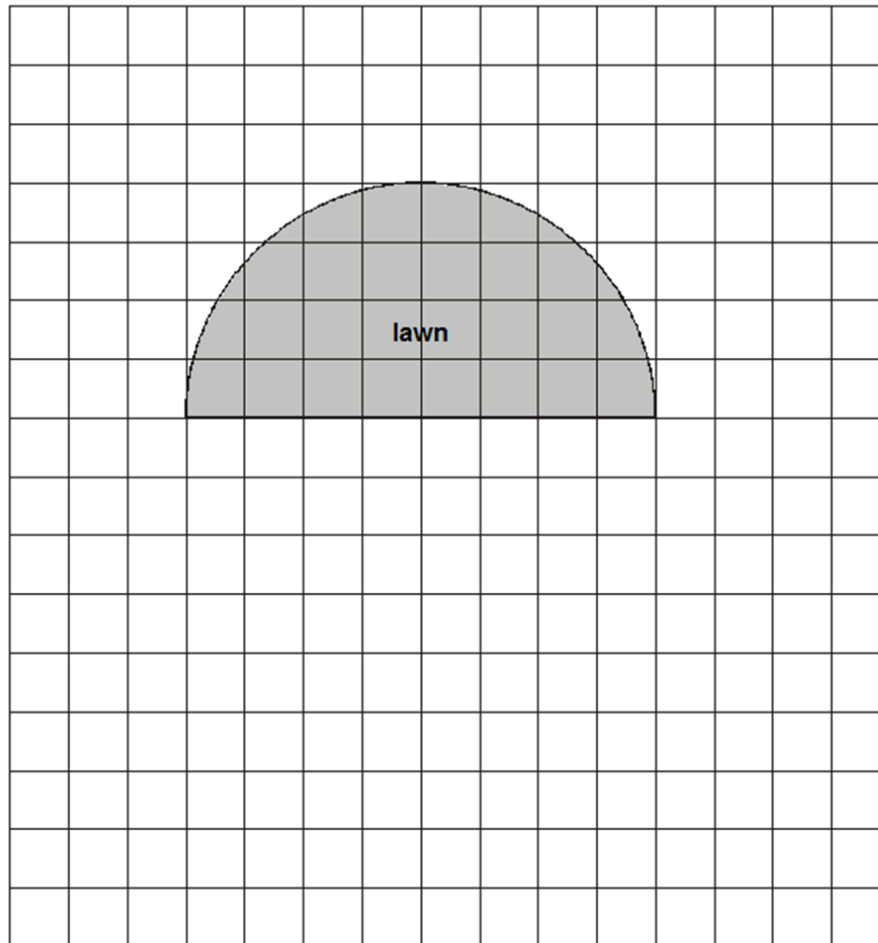
the shortest distance from the fence to the edge of the lawn is **always 6m**.

on the diagram, draw the fence **accurately**

scale: 1 cm = 3 m

3 m

3 m



Fluency Practice

5. mast

the plan shows the position of three towns,
each marked with an ×

the scale of the plan is **1 cm. to 10 km.**

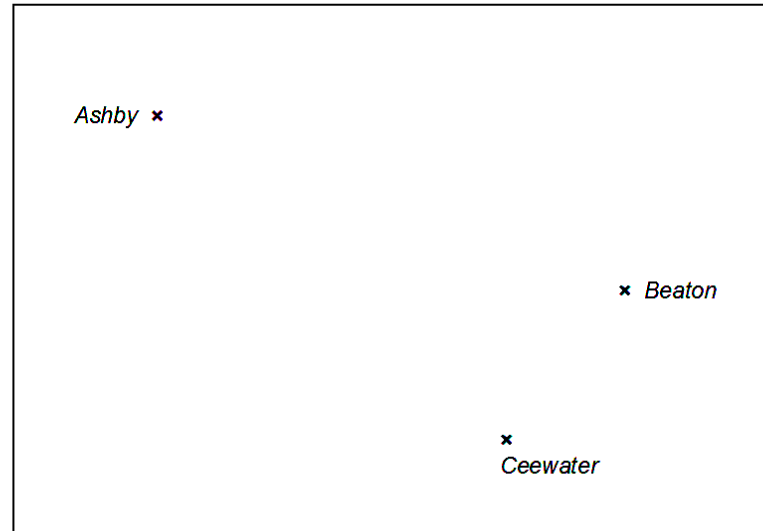
the towns need a new phone mast

the new mast must be:

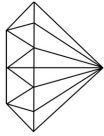
- nearer to *Ashby* than *Ceewater*
- less than 45 km from *Beaton*

show on the plan the region where the new mast
can be placed

leave in your construction lines to show how you
found the region



Fluency Practice



Diamond HEIST!

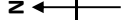
You're planning to steal the diamond from the Grand Casino Vault!
Find a route from the vault door to the diamond avoiding the lasers and motion detectors!

Lasers

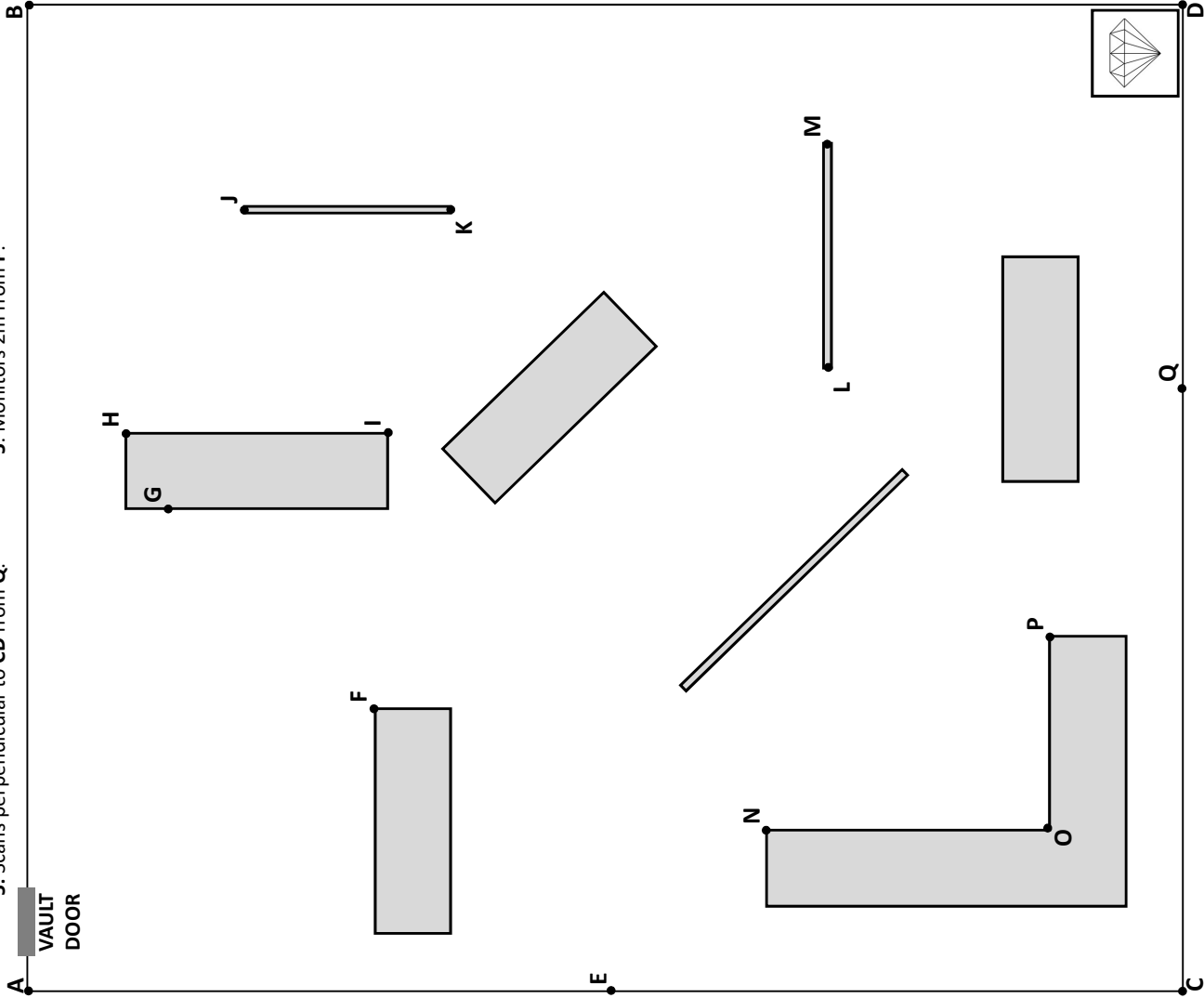
- 1: Scans 3m from AC from the north wall.
- 2: Scans from E perpendicular to AC.
- 3: Scans east equidistant from HI.
- 4: Scans from O, equidistant between ON & OP.
- 5: Scans perpendicular to CD from Q.

Motion Detectors

- 1: Monitors 3m from F.
- 2: Monitors 2m from G.
- 3: Monitors 3.5m from the wall JK.
- 4: Monitors 3m from the wall LM.
- 5: Monitors 2m from P.



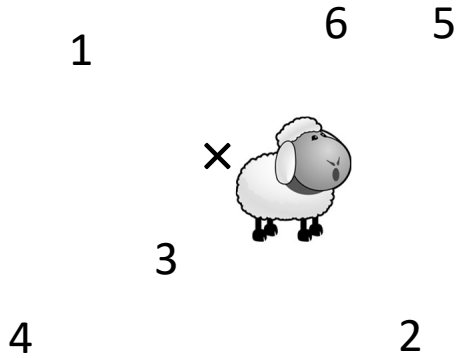
1 cm = 1m



Fluency Practice

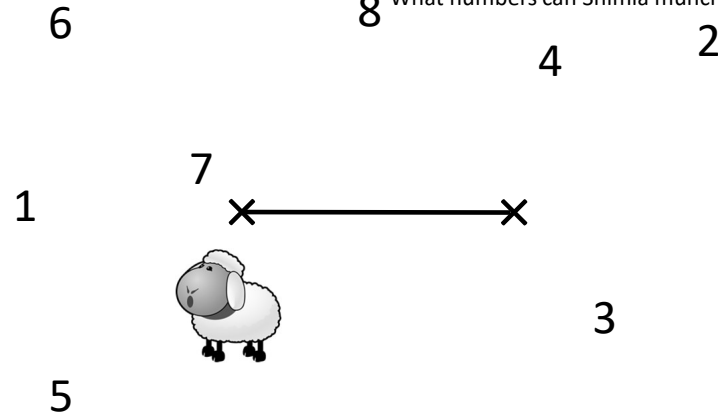
1. 1 cm = 1 m

Shimla is attached to a 4 metre rope that is attached to a stake in the ground. What numbers can Shimla munch?



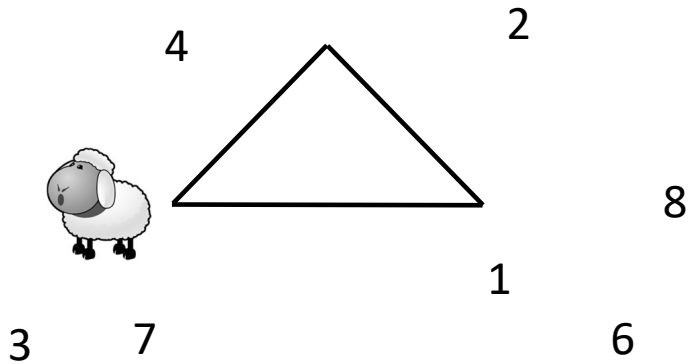
2.

Shimla is attached to a 4 metre rope that is attached to a rail on top of a fence. What numbers can Shimla munch?



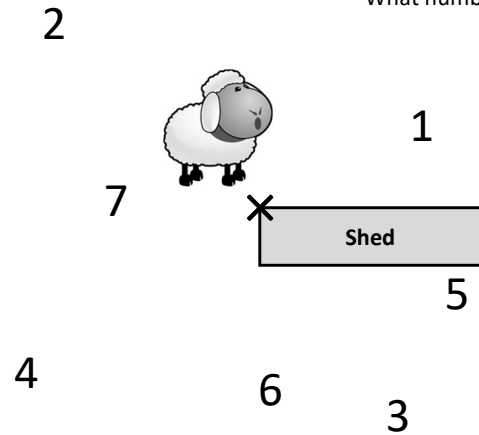
3.

Shimla is attached to a 3 metre rope that is attached to a rail on top of a triangular fence. What numbers can Shimla munch?



4.

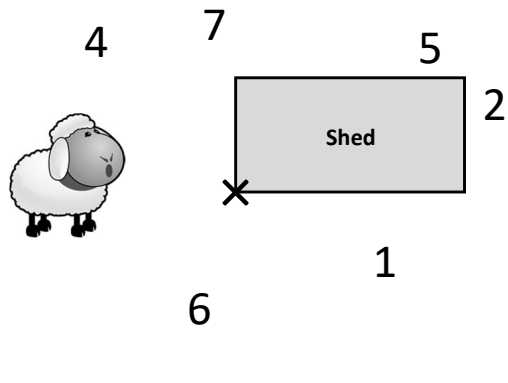
Shimla is attached to a 4 metre rope that is attached to the corner of a shed. What numbers can Shimla munch?



Fluency Practice

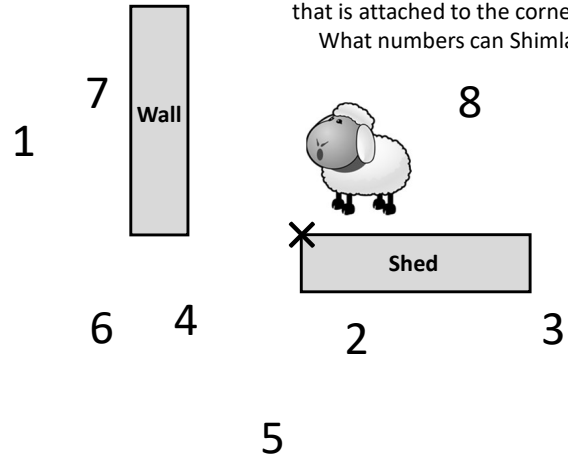
5. $1 \text{ cm} = 1 \text{ m}$

Shimla is attached to a 5 metre rope that is attached to the corner of a shed. What numbers can Shimla munch?



6.

Shimla is attached to a 5 metre rope that is attached to the corner of a shed. What numbers can Shimla munch?



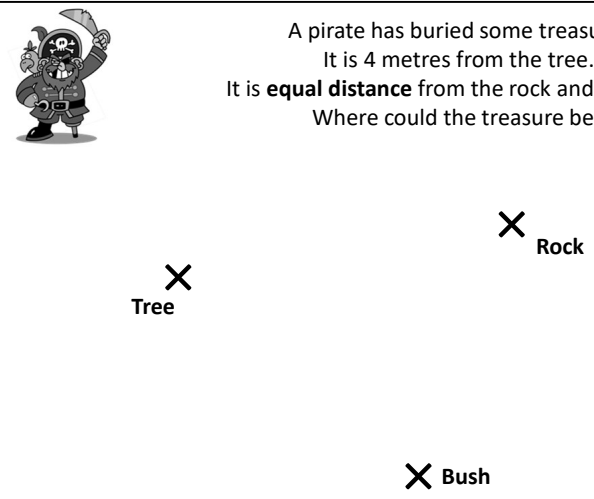
7.

Shimla is attached to a 4 metre rope and Shawn is attached to a 4 metre rope. Where can they **both** graze?



8.

A pirate has buried some treasure! It is **equal distance** from the rock and the bush. Where could the treasure be?

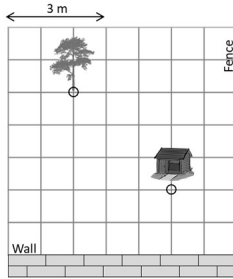


Fluency Practice

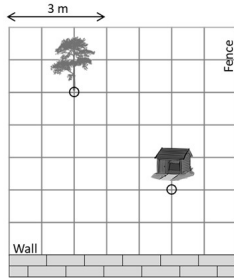
Locs

Max is deciding where to plant a new tree.
On each diagram, mark or shade **all** the possible locations according to the rules.

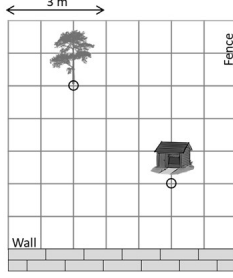
A) 4m away from the wall.



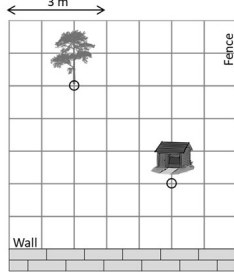
B) 2m or less from the fence.



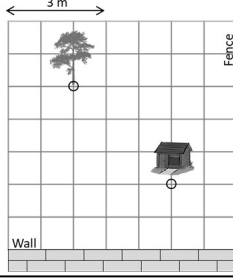
C) 1m from the wall & 5m from the fence.



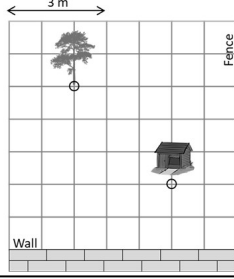
D) 4m away from the wall & 2m from the shed.



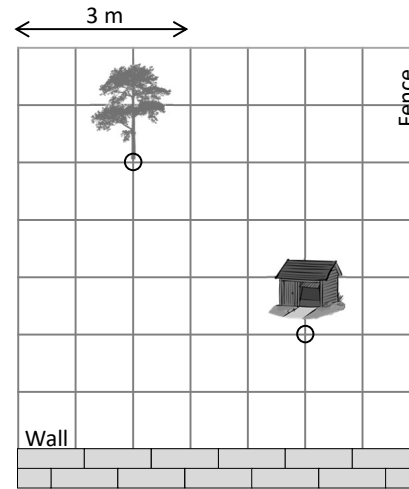
E) 4m or more from the wall & 3m or less from the fence.



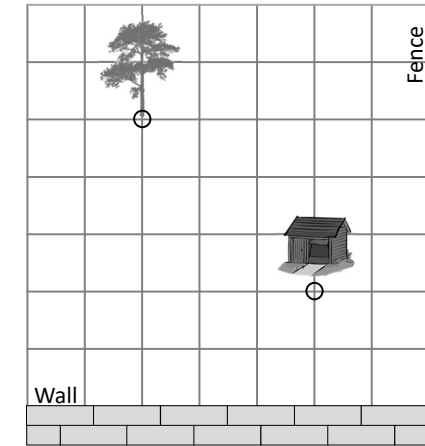
F) Equidistance from the fence & the wall.



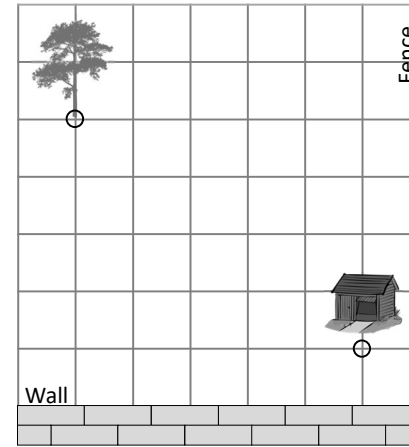
G) 5m or less from the shed.



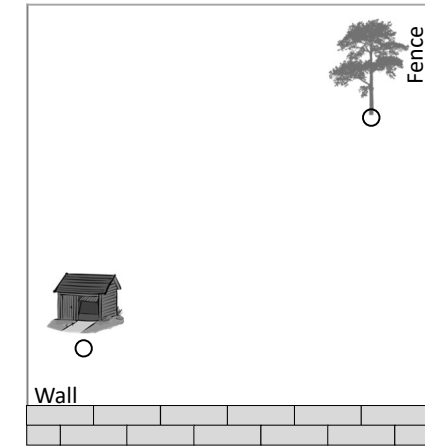
H) 5m away from the tree & 1m from the fence.



I) 4m away from both the tree & the shed.



J) Equidistant from the tree and the shed.



1

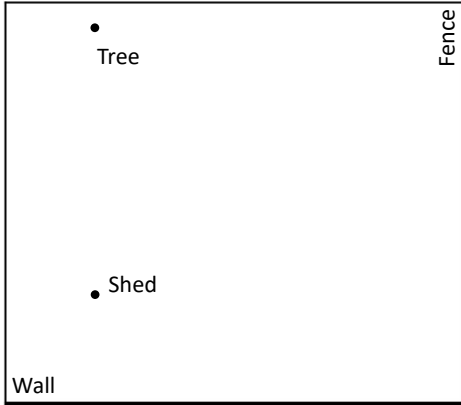
Fluency Practice

2

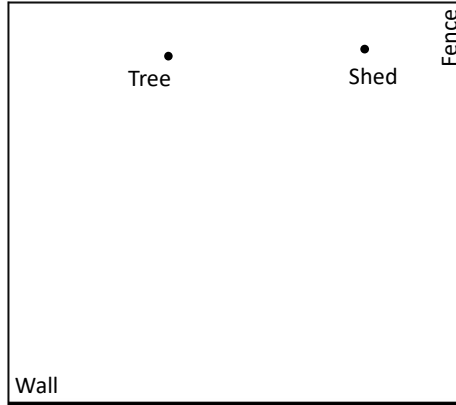
Loci

Jenny is deciding where to dig a well. On each diagram (1cm = 2m), mark or shade **all** the possible locations according to the rules. Use a **dotted line** if those points are **not** included.

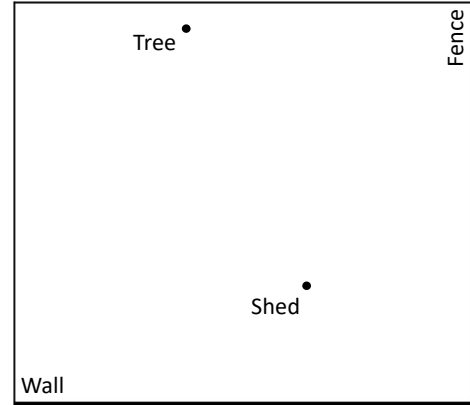
- A) 4m or less from the fence.
12m or less from the tree.



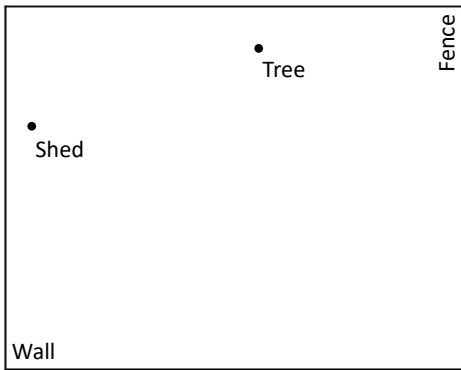
- B) 10m or more from the shed.
Less than 6m from the wall.



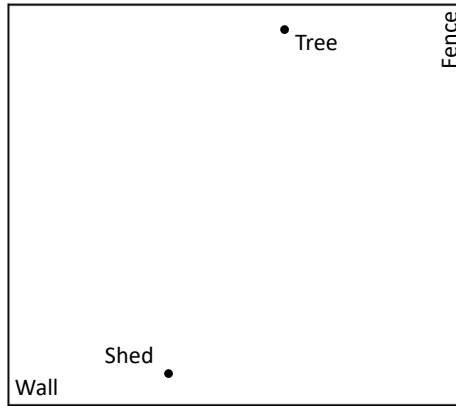
- C) Equidistant from the tree and the shed
and more than 5m from the fence.



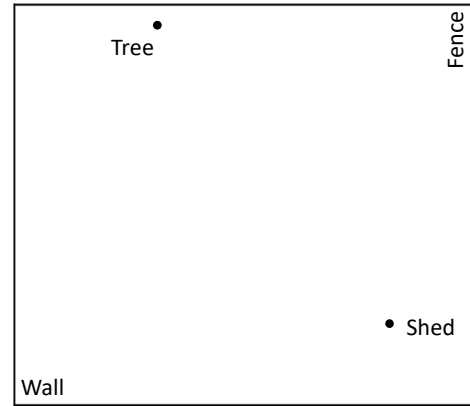
- D) Equidistant from the fence and the wall.
11m or less from the shed.



- E) Less than 8m from the tree,
less than 9m from the shed and
more than 8m from the fence.



- E) 2 metres closer to the wall
than to the fence and equidistant from
the tree and the shed.



Extra Notes