Year 10

2023   Mathematics   2024

Unit 17 Tasks – Part 1

DO NOT WRITE INSIDE

Year 10

2023   Mathematics   2024

Unit 17 Tasks – Part 2

DO NOT WRITE INSIDE

Contents Page

1 Bounds and Error Intervals

When someone says that a distance is 50 metres, what do they mean? Measurements in real life can never be made with absolute accuracy – there is always a certain amount of error. So 50 metres could be accurate to the nearest metre, or to the nearest 10 metres, for example. Knowing within what interval the true distance lies can be very important in many applications of mathematics. When measurements are combined in a calculation, and each value has a certain amount of error, things can get complicated – and sometimes the result can be counterintuitive.

A number has been rounded to 30 to the nearest 10.

What could the number be?

What is the lowest and highest possible value it could be?

This smallest possible value is called the lower bound. The largest possible value is called the upper bound.

When a measure is expressed to a given unit, the maximum error is half of this unit.

For a value x, the error interval is:
least possible value $\leq x <$ greater possible value

Fluency Practice

1) A number q, when rounded to the nearest 10, is equal to 440
   Find the upper and lower bound of q

2) A number p, when rounded to the nearest whole, is equal to 65
   Find the upper and lower bound of p

3) A number x, when rounded to the nearest 100, is equal to 4000
   Find the upper and lower bound of x

4) A number x, when rounded to the nearest whole, is equal to 39
   Find the upper and lower bound of x

5) A number y, when rounded to the nearest 10, is equal to 420
   Find the upper and lower bound of y

6) A number z, when rounded to the nearest 100, is equal to 8300
   Find the upper and lower bound of z

7) A number x, when rounded to the nearest 100, is equal to 200
   Find the upper and lower bound of x

8) A number x, when rounded to the nearest 10, is equal to 380
   Find the upper and lower bound of x

9) A number x, when rounded to the nearest 10, is equal to 970
   Find the upper and lower bound of x

10) A number p, when rounded to the nearest 10, is equal to 30
    Find the upper and lower bound of p

Fluency Practice

1) A number x, when rounded to 3 decimal places, is equal to 0.007
   Find the upper and lower bound of x

2) A number x, when rounded to 1 decimal place, is equal to 18.4
   Find the upper and lower bound of x

3) A number q, when rounded to 3 decimal places, is equal to 0.008
   Find the upper and lower bound of q

4) A number p, when rounded to 3 decimal places, is equal to 0.055
   Find the upper and lower bound of p

5) A number x, when rounded to 1 decimal place, is equal to 6.8
   Find the upper and lower bound of x

6) A number z, when rounded to 1 decimal place, is equal to 44.9
   Find the upper and lower bound of z

7) A number y, when rounded to 2 decimal places, is equal to 0.08
   Find the upper and lower bound of y

8) A number x, when rounded to 3 decimal places, is equal to 0.007
   Find the upper and lower bound of x

9) A number y, when rounded to 1 decimal place, is equal to 9.8
   Find the upper and lower bound of y

10) A number z, when rounded to 1 decimal place, is equal to 78.9
    Find the upper and lower bound of z

Fluency Practice

1) A number x, when rounded to 3 significant figures, is equal to 6.33
   Find the upper and lower bound of x

2) A number q, when rounded to 1 significant figure, is equal to 8
   Find the upper and lower bound of q

3) A number p, when rounded to 3 significant figures, is equal to 535
   Find the upper and lower bound of p

4) A number y, when rounded to 3 significant figures, is equal to 5.61
   Find the upper and lower bound of y

5) A number x, when rounded to 1 significant figure, is equal to 80
   Find the upper and lower bound of x

6) A number x, when rounded to 3 significant figures, is equal to 9770
   Find the upper and lower bound of x

7) A number p, when rounded to 2 significant figures, is equal to 0.86
   Find the upper and lower bound of p

8) A number q, when rounded to 3 significant figures, is equal to 6.09
   Find the upper and lower bound of q

9) A number q, when rounded to 3 significant figures, is equal to 0.988
   Find the upper and lower bound of q

10) A number q, when rounded to 1 significant figure, is equal to 0.7
    Find the upper and lower bound of q

Fluency Practice

1) A number y, when rounded to the nearest 10, is equal to 20
   Find the error interval for y

2) A number x, when rounded to 2 decimal places, is equal to 4.23
   Find the error interval for x

3) A number x, when rounded to 1 decimal place, is equal to 12.5
   Find the error interval for x

4) A number y, when rounded to the nearest 100, is equal to 7500
   Find the error interval for y

5) A number y, when rounded to 1 decimal place, is equal to 0.8
   Find the error interval for y

6) A number y, when rounded to 1 decimal place, is equal to 88.4
   Find the error interval for y

7) A number y, when rounded to the nearest 10, is equal to 480
   Find the error interval for y

8) A number y, when rounded to the nearest 100, is equal to 8800
   Find the error interval for y

9) A number x, when rounded to 1 decimal place, is equal to 9.9
   Find the error interval for x

10) A number y, when rounded to 3 decimal places, is equal to 0.138
   Find the error interval for y

Intelligent Practice

Value	Rounded to	Lower Bound	Upper Bound	Error Interval
6000	Nearest 1000			
6000	Nearest 100			
600	Nearest 100			
600	Nearest 10			
6000	Nearest 10			
60	Nearest 10			
60	To nearest whole			
6	To the nearest whole			
6000	To the nearest whole			
600	To the nearest whole			

Intelligent Practice

Value	Rounded to	Lower Bound	Upper Bound	Error Interval
400	1 significant figure			
400	3 significant figures			
400	2 significant figures			
40	2 significant figures			
4	2 significant figures			
4	1 significant figure			
0.4	1 significant figure			
0.3	1 significant figure			
30	2 significant figures			
0.03	1 significant figure			

Fluency Practice

Write these error intervals.

(a) 50 cm to the nearest 10 cm

(b) 150 cm to the nearest 10 cm

(c) 800 g to the nearest 10 g

(d) 800 g to the nearest 100 g

(e) 500 m to the nearest 100 m

(f) 500 m to the nearest 10 m

(g) 500 m to the nearest m

(h) 5 m to the nearest m

(i) 55 g to the nearest g

(j) 87 g to the nearest g

Write these error intervals.

(a) 4.5 cm to 1 decimal place

(b) 7.5 cm to 1 decimal place

(c) 37.5 cm to 1 decimal place

(d) 37.4 m to 1 decimal place

(e) 30.4 g to 1 decimal place

(f) 30.9 g to 1 decimal place

(g) 30.0 g to 1 decimal place

Write the error intervals.

(a) 500 g to 1 significant figure

(b) 100 cm to 1 significant figure

(c) 50 cm to 1 significant figure

(d) 20 cm to 1 significant figure

(e) 5 m to 1 significant figure

(f) 2 m to 1 significant figure

Write the upper and lower bounds.

(a) 347 cm to the nearest cm

(b) 80 g to 1 significant figure

(c) 54.6 m to 1 decimal place

(d) 45.86 g to 2 decimal places

(e) 52 cm to 2 significant figures

Fluency Practice

Maximum & Minimum : SF

Lower Bound	Rounded Number	Upper Bound
	200 (to the nearest 100)	
	500 (1 significant figure)	
	7000 (to the nearest 1000)	
	12 000 (2 significant figures)	
	80 (to the nearest 10)	
	80 (to the nearest 5)	
	40 (to the nearest 20)	
	450 (to the nearest 50)	
	830 (2 significant figures)	
	17 (to the nearest integer)	
	5280 (3 significant figures)	
	174 000 (3 sf)	
	0.1 (to the nearest tenth)	
	3.42 (3 sf)	

Maximum & Minimum : DP

Lower Bound	Rounded Number	Upper Bound
	5 (to the nearest integer)	
	0.2 (to the nearest tenth)	
	0.8 (to 1 decimal place)	
	7.1 (to 1 decimal place)	
	0.01 (nearest hundredth)	
	54.9 (to 1 decimal place)	
	0.17 (to 2 decimal places)	
	0.228 (to 3 dp)	
	5.0 (to the nearest tenth)	
	5.00 (nearest hundredth)	
	10.84 (to 2 dp)	
	2.000 (nearest thousandth)	
	9.40 (to 2 dp)	
	16.901 (to 3 dp)	

Fluency Practice

Here are some species of animal, and their numbers left in the wild (according to the WWF). Find their upper and lower bounds.

Animal	Number left	Rounding	Lower Bound	Upper Bound
African elephant	415,000	Nearest 1000		415,499
Amur leopard	60	Nearest 10		
Black rhino	5,500	Nearest 100		
Galápagos penguin	2,000	Nearest 1000		
Greater sage-grouse	100,000	Nearest 100,000		
Vaquita dolphin	10	Nearest 5		
Sundra tiger	"Under 400"	?!?		

Question 1: Write down the lower bound and the upper bound for each of the following:

(a) 4g measured to the nearest gram

(b) 12cm correct to the nearest centimetre

(c) 75 miles given to the nearest mile

(d) 50kg measured to the nearest 10kg

(e) 130 seconds given the nearest 10 seconds

(f) 225km given to the nearest 5km

(g) 400ml given to the nearest 100ml

(h) 1900 hours correct to the nearest 10 hours

(i) 2700mm measured to the nearest 100mm

(j) 5000km correct to the nearest 100km

(k) 28000kg measured to the nearest 10kg

(l) 30000km/h given to the nearest 10000km/h
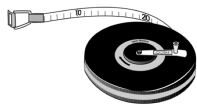
Question 2: Write down the lower bound and the upper bound for each of the following:

(a) 80 people given to the nearest 10 people

(b) £10 given to the nearest pound

(c) 500 chairs correct to the nearest 100 chairs

(d) 14000 flights given to the nearest 1000 flights

(e) £29000 given to the nearest £100

Question 3: Write down the lower bound and the upper bound for each of the following:

(a) 3.8cm measured to the nearest 0.1 centimetre

(b) 15.2 seconds to the nearest tenth of a second

(c) 6.4g rounded to one decimal place

(d) 515.9kg correct to one decimal place

(e) 0.07 seconds rounded to two decimal places

(f) 5.26mm measured to the nearest 0.01mm

(g) 24.091kg correct to three decimal places

(h) 8cm measured to the nearest 0.1cm

Question 4: Write down the lower bound and the upper bound for each of the following:

(a) 4 miles correct to 1 significant figure

(b) 30cm rounded to 1 significant figure

(c) 900ml given to 1 significant figure

(d) 0.2m given to 1 significant figure

(e) 14 hours given to 2 significant figures

(f) 280g correct to 2 significant figures

(g) 42000km rounded to 2 significant figures

(h) 748 gallons correct to 3 significant figures

(i) 400m rounded to 2 significant figures

(j) 8000kg given to 2 significant figures

(k) 290000km/h given to 4 significant figures

(l) 0.024 correct to 2 significant figures

(m) 15.1 kilometres rounded to 3 significant figures

(n) 100g correct to 1 significant figure

Extension

Limits of Accuracy
Video 183 on www.corbettmaths.com

Question 1: Declan is considering buying a sofa that is 207cm long.
The space that Declan wants to the sofa is 210cm to the nearest 10cm.
Should Declan buy the sofa?

Question 2: Mr Jones wants to buy a notebook for every student in year 11.
He knows there are 300 students in year 11 to the nearest 100.
What is the greatest possible number of notebooks that he would have to buy?

Question 3: The length of a field is 400m to the nearest 10m.
Rebecca says the lower bound is 350m and the upper bound is 450m.
Is she correct?

Question 4: The table shows the prices of posting large letters.
Gerard wants to post two large letters:

- a large letter weighing 230g to the nearest 10 grams
- a large letter weighing 500g to the nearest 10 grams

1st Class		
Size	Weight up to and including	Price
Large Letter	100g	95p
	250g	£1.26
	500g	£1.68
	750g	£2.42

(a) What is the smallest possible price for posting both large letters?

(b) What is the greatest possible price for posting both large letters?

Question 5: Below is a question that was posted online and the results.
Explain which answer you agree with.

A number has been rounded to 10, correct to 1 significant figure. What are the lower and upper bounds?

30%	5 and 15
55%	9.5 and 10.5
12%	9.5 and 15
3%	7.5 and 12.5

Answers

learn by heart

Upper & Lower Bounds: *The maximum & minimum values a measured quantity could have taken before rounding*

examples

A length is measured to be 49.5cm to 1 d.p. Write an inequality to show the error interval for this length.

$49.45 \leq length < 49.55$

The number of people at a football match was 300, correct to 1 significant figure. What are the upper and lower bounds for the actual number of people who attended?

$250 \leq people \leq 349$

For continuous data, add and subtract half of the value to which the number has been rounded to find the upper and lower bounds.

E.g. if the number is rounded to the tenths column, i.e. nearest 0.1, add half of this, 0.05, to find the upper bound

questions

1. Shade all the numbers in this grid which round to 600, to 1 significant figure:

64	521	653	580	700	550	599.99
679	620	500	60	593	650	549.5
6000	595	545	521	650.8	742	608.8

2. Each of these numbers has been rounded to the degree of accuracy shown. Write down the error interval for each measurement.

a) 40.8 (1 d.p.)

b) 560 (nearest 10)

c) 201 (nearest integer)

d) 400 (nearest 100)

e) 90.9 (nearest tenth)

f) 540 (nearest 20)

g) 6000 (nearest 1000)

h) 500 (nearest 10)

Fluency Practice

Questions

3. Shade all the numbers in this grid which round to 0.5 to 1 significant figure.

0.8	0.56	0.407	0.55	0.491	0.4500	0.9
0.42	0.7	0.48	0.45	0.445	0.53	0.05
0.50	0.40	5	0.3689	0.500	0.545	0.491

4. Each of these numbers has been rounded to the degree of accuracy shown. Write down the error interval for each measurement.

a) 55 (2 significant figures) e) 82.6 (3 significant figures)

b) 0.8 (1 significant figure) f) 0.90 (2 significant figures)

c) 4000 (1 significant figure) g) 2000 (3 significant figures)

d) 8.23 (2 d.p.) h) 2000 (4 significant figures)

5. Which of these numbers has the largest upper bound?

a) 500 (1 s.f.) b) 510 (2 s.f.) c) 516 (3 s.f.) d) 500 (2 s.f.)

6. Which of these numbers has the smallest lower bound?

a) 30 (1 s.f.) b) 24 (2 s.f.) c) 22.5 (3 s.f.) d) 20 (2 s.f.)

7. The number of people attending a county fair was 1600, correct to two significant figures. What is the upper bound for the actual number?

a) 1650 b) 1550 c) 1649 d) 1700

8. Which of these numbers is the same when rounded to either 1 significant figure or 2 significant figures?

a) 452 b) 0.314 c) 495 d) 196

Problem Solving

To ride the SuperTwister rollercoaster you must be at least 120cm tall.

12 children measured their heights and recorded their findings. Use the statements below to work out which children definitely can ride the SuperTwister, which ones definitely can't, and which ones might be able to.

Paula is 122cm, to the nearest cm.

Sheena is 130cm, to the nearest 10cm.

Bob is 100cm, to the nearest 10cm.

Rakan is 125cm, to the nearest 5cm.

Caitlin is 1 meter, to the nearest meter.

Sarah is 120cm, to the nearest integer

Emma is 120cm tall, measured to the nearest cm.

Ali is 121cm, to the nearest cm.

Jenny is 115cm, to the nearest 5cm.

Hayden is 1.2 meters tall, rounded to 1 d.p.

Carl is 110cm, to the nearest 10cm.

Harry is 150cm, to the nearest 50cm.

Definitely Can	Can't be Sure	Definitely Can't

Applying Bounds

*Q. Choose two of these four numbers to make the **biggest** possible answer in each of the calculations below:*

5.5   6.5   9.5   10.5

$$\square + \square \qquad \square - \square \qquad \square \times \square \qquad \frac{\square}{\square}$$

*Q. Now do it again, but this time try to make the **smallest** possible answer each time.*

Fluency Practice

Maximization & Minimization
Re-write each expression, but replace each term with either **MIN** or **MAX**.

$a + b$	To find the maximum result:
$a - b$	Minimum:
$a \times b - c$	Maximum:
$\dfrac{a}{b}$	Max.:
$a(b - c)$	Max.:
$\dfrac{a + b}{c}$	Min.:
$a - (b - c)$	Min.:
$\dfrac{a - b}{c + d}$	Max.:
$a - b + \sqrt{c - d}$	Min.:
$\dfrac{a}{b} - \dfrac{c}{d}$	Min.:
$\dfrac{ab - c}{d(e - f)} - (-g)$	Max:

$a + b$	To find the maximum result:
$a - b$	Minimum:
$a \times b - c$	Maximum:
$\dfrac{a}{b}$	Max.:
$a(b - c)$	Max.:
$\dfrac{a + b}{c}$	Min.:
$a - (b - c)$	Min.:
$\dfrac{a - b}{c + d}$	Max.:
$a - b + \sqrt{c - d}$	Min.:
$\dfrac{a}{b} - \dfrac{c}{d}$	Min.:
$\dfrac{ab - c}{d(e - f)} - (-g)$	Max:

Fluency Practice

1) $p = qr$
   $q = 0.095$ correct to 3 decimal places.
   $r = 2000$ correct to 1 significant figure.
   Work out the upper bound for the value of p
   Give your answer correct to 3 decimal places when appropriate.

2) $a = 2b^2c$
   $b = 0.2$ correct to 1 significant figure.
   $c = 7.04$ correct to 3 significant figures.
   Work out the upper bound for the value of a
   Give your answer correct to 3 decimal places when appropriate.

3) $a = 2bc$
   $b = 480$ correct to 2 significant figures.
   $c = 9.73$ correct to 2 decimal places.
   Work out the upper bound for the value of a
   Give your answer correct to 3 decimal places when appropriate.

4) $a = 5bc$
   $b = 63$ correct to 2 significant figures.
   $c = 581$ correct to 3 significant figures.
   Work out the lower bound for the value of a
   Give your answer correct to 3 decimal places when appropriate.

5) $p = 5q^2r$
   $q = 84.4$ correct to 1 decimal place.
   $r = 0.48$ correct to 2 decimal places.
   Work out the lower bound for the value of p
   Give your answer correct to 3 decimal places when appropriate.

Fluency Practice

1) $a = 2b + 5c$
   $b = 0.042$ correct to 3 decimal places.
   $c = 0.7$ correct to 1 significant figure.
   Work out the upper bound for the value of a
   Give your answer correct to 3 decimal places when appropriate.

2) $x = y + 2z$
   $y = 1.1$ correct to 2 significant figures.
   $z = 3.6$ correct to 1 decimal place.
   Work out the lower bound for the value of x
   Give your answer correct to 3 decimal places when appropriate.

3) $p = 5q + 3r$
   $q = 0.963$ correct to 3 decimal places.
   $r = 77.9$ correct to 1 decimal place.
   Work out the upper bound for the value of p
   Give your answer correct to 3 decimal places when appropriate.

4) $p = 2q + 5r$
   $q = 0.064$ correct to 3 decimal places.
   $r = 2.35$ correct to 2 decimal places.
   Work out the lower bound for the value of p
   Give your answer correct to 3 decimal places when appropriate.

5) $p = 2q + r$
   $q = 0.89$ correct to 2 decimal places.
   $r = 590$ correct to 2 significant figures.
   Work out the lower bound for the value of p
   Give your answer correct to 3 decimal places when appropriate.

Fluency Practice

1) $a = 2b - 4c$
   $b = 0.8$ correct to 1 significant figure.
   $c = 9.69$ correct to 2 decimal places.
   Work out the upper bound for the value of a
   Give your answer correct to 3 decimal places when appropriate.

2) $x = 4y - z$
   $y = 0.7$ correct to 1 significant figure.
   $z = 0.4$ correct to 1 significant figure.
   Work out the upper bound for the value of x
   Give your answer correct to 3 decimal places when appropriate.

3) $a = 5b - 5c$
   $b = 0.058$ correct to 2 significant figures.
   $c = 8.4$ correct to 1 decimal place.
   Work out the lower bound for the value of a
   Give your answer correct to 3 decimal places when appropriate.

4) $a = 3b - 5c$
   $b = 2.53$ correct to 2 decimal places.
   $c = 3700$ correct to 2 significant figures.
   Work out the upper bound for the value of a
   Give your answer correct to 3 decimal places when appropriate.

5) $a = b - 4c$
   $b = 0.64$ correct to 2 decimal places.
   $c = 0.416$ correct to 3 decimal places.
   Work out the lower bound for the value of a
   Give your answer correct to 3 decimal places when appropriate.

Fluency Practice

1) $p = \frac{3q}{r}$

   $q = 0.321$ correct to 3 decimal places.

   $r = 0.007$ correct to 1 significant figure.

   Work out the upper bound for the value of p

   Give your answer correct to 3 decimal places when appropriate.

2) $x = \frac{4y}{z}$

   $y = 0.188$ correct to 3 decimal places.

   $z = 77.3$ correct to 1 decimal place.

   Work out the upper bound for the value of x

   Give your answer correct to 3 decimal places when appropriate.

3) $x = \frac{3y}{z^2}$

   $y = 7.42$ correct to 2 decimal places.

   $z = 500$ correct to 1 significant figure.

   Work out the lower bound for the value of x

   Give your answer correct to 3 decimal places when appropriate.

4) $p = \frac{5q}{r}$

   $q = 0.32$ correct to 2 significant figures.

   $r = 0.4$ correct to 1 significant figure.

   Work out the upper bound for the value of p

   Give your answer correct to 3 decimal places when appropriate.

5) $x = \frac{5y^3}{z}$

   $y = 7.81$ correct to 2 decimal places.

   $z = 0.37$ correct to 2 decimal places.

   Work out the upper bound for the value of x

   Give your answer correct to 3 decimal places when appropriate.

Fluency Practice

1) $a = \dfrac{b}{c+d}$

   $b = 6.3$ correct to 1 decimal place.
   $c = 7.49$ correct to 2 decimal places.
   $d = 2.22$ correct to 2 decimal places.
   Work out the lower bound for the value of a
   Give your answer correct to 3 decimal places when appropriate.

2) $x = \dfrac{y}{z-w}$

   $y = 45.5$ correct to 3 significant figures.
   $z = 0.05$ correct to 1 significant figure.
   $w = 1.73$ correct to 2 decimal places.
   Work out the lower bound for the value of x
   Give your answer correct to 3 decimal places when appropriate.

3) $x = \dfrac{y}{z-w}$

   $y = 5.21$ correct to 2 decimal places.
   $z = 2.7$ correct to 2 significant figures.
   $w = 3.29$ correct to 2 decimal places.
   Work out the lower bound for the value of x
   Give your answer correct to 3 decimal places when appropriate.

4) $a = \dfrac{1}{bc}$

   $b = 0.082$ correct to 3 decimal places.
   $c = 0.081$ correct to 3 decimal places.
   Work out the lower bound for the value of a
   Give your answer correct to 3 decimal places when appropriate.

5) $p = \dfrac{5}{qr}$

   $q = 383$ correct to 3 significant figures.
   $r = 918$ correct to 3 significant figures.
   Work out the lower bound for the value of p
   Give your answer correct to 3 decimal places when appropriate.

Fluency Practice

The values for *a* & *b* are correct to the nearest ...(   )...

Calculations with Error Intervals

	a	*b*	Sum		Difference	
			Lower Bound	Upper Bound	Lower Bound	Upper Bound
1	2 (integer)	4 (integer)				
2	20 (10)	50 (10)				
3	35 (5)	120 (10)				
4	8 (1 sf)	40 (20)				
5	300 (100)	350 (50)				
6	7 (1 sf)	14 (2 sf)				
7	1.4 (1 dp)	1 (integer)				
8	6.0 (1 dp)	5.2 (2 sf)				
9	1.48 (2 dp)	0.17 (2 dp)				
10	2.00 (3 sf)	1.509 (3 dp)				

Fluency Practice

a = 30 (nearest 10)	b = 7 (nearest integer)	c = 2000 (to 1 sf)	d = 4.5 (to 2 sf)	e = 1.2 (to 1 dp)	f = 0.86 (to 2 dp)

Find the LOWER BOUND of…   MIN.   Find the UPPER BOUND of…
                                             MAX.

1) $a + b$

2) $c - b$

3) $a - (b + d)$

4) $\dfrac{c}{a}$

5) $a - \dfrac{d}{e}$

6) $\dfrac{e}{b - f}$

1) $b \times c$

2) $d + e$

3) $c - a$

4) $\dfrac{a + b}{e}$

5) $\dfrac{c}{ef}$

6) $\dfrac{de(c - f)}{b}$

Page 47

Calculating with Upper and Lower Bounds

Given that a is 40 to the nearest 10 and b is 8 correct to the nearest integer:	Given that e is 20 correct to the nearest five and f is 2.5 correct to 1 decimal place:	Given that p is 200 to 1 significant figure and q is 25 to 2 significant figures:	Given that x is 3 to the nearest integer, y is 1.5 to the nearest tenth and z is 12 to 2 significant figures:
(a) Find the upper and lower bounds of $10a$	**(d)** Find the upper and lower bounds of $e - f$	**(g)** Find the upper and lower bounds of $\sqrt{p}$	**(j)** Find the upper and lower bounds of $2(x + z - y)$
(b) Find the upper and lower bounds of $a + b$	**(e)** Find the upper and lower bounds of $\dfrac{e}{f}$	**(h)** Find the upper and lower bounds of $\dfrac{1000}{pq}$	**(k)** Find the upper and lower bounds of $\dfrac{z}{x-y}$
(c) Find the upper and lower bounds of $a \times b$	**(f)** Find the upper and lower bounds of e^2	**(i)** Find the upper and lower bounds of $\sqrt{\dfrac{1}{p-q}}$	**(l)** Find the upper and lower bounds of $z - x \times 2^y$

Problem Solving

Make each calculation as small as possible

| 2 | 5 | 10 | 20 |

$\boxed{} + \boxed{}$

$\boxed{} - \boxed{}$

$\boxed{} \times \boxed{}$

$\dfrac{\boxed{}}{\boxed{}}$

$\left(\boxed{}\right)^2$

$\sqrt{\dfrac{\boxed{}}{\boxed{}}}$

$\left(\boxed{} - \boxed{}\right)^2$

$\dfrac{\boxed{}}{\boxed{}}$

More-Same-Less – Lower Bound

Instructions: Calculate the lower bound and under bound of middle box. Complete the remaining boxes changing as little as possible.

Lower Bound of x

Lower boud of $x + y$

	Less	Same	More
More			
Same		$2x + y$ $x = 2.3 \ to \ 1 \ d.p.$ $y = 4.9 \ to \ 1 \ d.p.$	
Less			

Question 1:   India has pieces of ribbon, each measuring 7cm to the nearest centimetre.

(a)   find the least total length of 10 pieces of ribbon.
(b)   find the greatest total length of 10 pieces of ribbon.

Question 2:   Leon has four barrels, each with a mass of 30kg to the nearest 10 kg.

(a)   find the the minimum possible mass of the four barrels.
(b)   find the maximum possible mass of the four barrels.

Question 3:   A rectangle has a length of 14cm and width of 5cm, both to nearest centimetre.

Find   (a)   the maximum possible area and
         (b)   the minimum possible area.

14cm

5cm
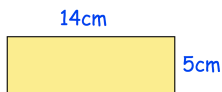
Question 4:   The classes in a primary school have 20 students to the nearest 10.
               There are 7 classes in the primary school.
               Work out the greatest possible number of students that attend the school.

Question 5:   Harry and Peter take part in a race.
               It took Harry 30 seconds to the nearest 10 seconds to finish the race.
               It took Peter 43 seconds to the nearest second to finish the race.
               Work out the minimum possible difference between their finishing times.

Question 6:   An average orange weighs 131g to the nearest gram.
               A net contains 8 oranges.
               The net weighs 10g to the nearest gram

               What is the maximum possible weight of the net of oranges?

Question 7:   Megan has 2 litres of fruit juice to the nearest litre.
               She pours the fruit juice into glasses that hold 100ml to the nearest 10ml.
               Work out the lowest possible number of glasses she can fill.

Question 8:   A rectangular football pitch has a width of 72m, measured to the nearest metre.
               The length of the pitch is 105m, measured to the nearest 5 metres.

               Work out the lower bound for the perimeter of the pitch.

Question 9:   The lengths of time taken for 4 people to complete a puzzle are listed below.
               Each time is given to one decimal place.

               20.8 seconds   35.1 seconds   19.7 seconds   41.3 seconds

               (a) Work out the greatest possible range
               (b) Work out the smallest possible mean.

Question 10:   Mr Rodgers wants to keep 28 new maths textbooks on a shelf in his classroom.
               Each book has a mass of 700g correct to 1 significant figure.
               The shelf can hold up to 20kg to the nearest kilogram.
               Can the shelf safely hold the textbooks?

Question 11:   The base of a triangle is 30cm, correct to 2 significant figures.
               The height of the triangle is 40cm, correct to 1 significant figure.
               Calculate the upper bound for the area of the triangle

Question 12:   Kelly drove a distance of 120 miles, to the nearest 10 miles, in a time of 2 hours,
               to the nearest hour.
               Work out the difference between Kelly's greatest possible and lowest possible
               average speed.

Question 13:   Rosie is buying strawberries, apples and grapes for a picnic.
               She buys 4kg of strawberries and 3kg of grapes, both to the nearest kilogram.
               Rosie buys 50 apples to the nearest 10 apples.
               A kilogram of strawberries costs £1.20 to the nearest 10p
               A kilogram of grapes costs £1.30 to the nearest 10p
               An apple costs 20p each to the nearest 10p.
               Work out the upper bound for the amount of money Rosie would have to pay

Question 14:   A circle has an area of 600cm^2 to 2 significant figures.
               Work out the lower bound of the radius.

Question 15: $w = aT$
               Given a = 15 correct to 2 significant figures
               and w = 700 correct to 2 significant figures
               Calculate the upper bound for T

Question 16: Shane estimated the distance between Cardiff and Swansea is 40 miles and that his average driving speed would be 60 mph.

He estimated the distance to the nearest 5 miles and the speed to the nearest 10mph.

Calculate the upper bound for the time the journey should take.
Give your answer to the nearest minute.

Question 17: A solid metal sphere has a radius of 4cm to 1 significant figure.
The sphere has a mass of 1200g to 2 significant figures.

Work out the lower bound for the density of the metal.

Question 18: The final velocity of a traveling object is given by the formula,   v = u + at

where v is the final velocity
u is the initial velocity
a is the acceleration
and t is the time.

Given u = 5.4m/s correct to 1 decimal place
a = 4.9m/s² correct to 1 decimal place
v = 25.32 correct to 2 decimal places

Calculate the upper bound for t.

Question 19: The population of a country is 6.4×10^6 to the nearest hundred thousand

The area of country is $8.4 \times 10^4 \, km^2$ to the nearest 100km²

Calculate the lower bound of the population density.

Answers

Page 55

Fluency Practice
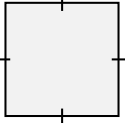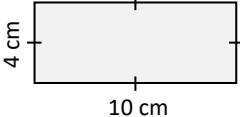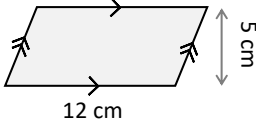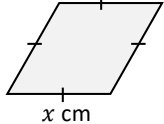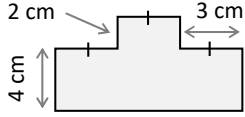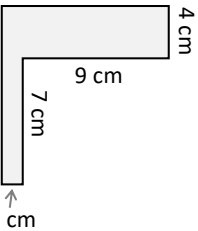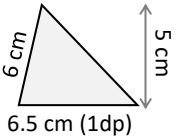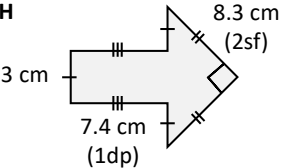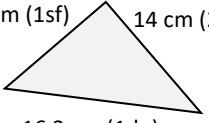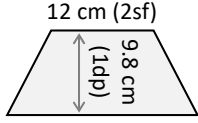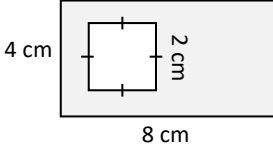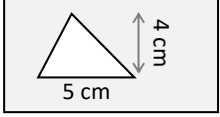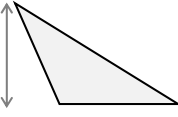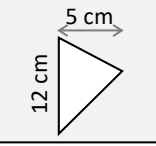
Calculations with Error Intervals

A = Area, P = Perimeter

Unless stated otherwise, each number has been rounded to the nearest centimetre.
Round your answers to 2dp. Remember! The rounded answer must be **within** the error interval!

not drawn accurately

A

6 cm

$\leqslant P <$

B

4 cm

10 cm

$\leqslant A <$

C

5 cm

12 cm

$\leqslant P <$

D

x cm

Perimeter = 280 cm (2sf)

$\leqslant x <$

E

2 cm   3 cm

4 cm

$\leqslant A <$

F

4 cm

9 cm

7 cm

1 cm

$\leqslant A <$

G

6 cm

5 cm

6.5 cm (1dp)

$\leqslant A <$

H

8.3 cm (2sf)

3 cm

7.4 cm (1dp)

$\leqslant A <$

I

10 cm (1sf)   14 cm (2sf)

16.2 cm (1dp)

$\leqslant P <$

J

12 cm (2sf)

9.8 cm (1dp)

20 cm (1sf)

$\leqslant A <$

K

4 cm

2 cm

8 cm

$\leqslant A <$

L

6.1 cm (1dp)

4 cm

5 cm

11 cm (2sf)

$\leqslant A <$

M

6 cm

x cm

Area = 51 cm^2 (2sf)

$\leqslant x <$

N

x cm

9.3 cm (1dp)

Area = 30 cm^2 (1sf)

$\leqslant x <$

O

14 cm (2sf)

12 cm

5 cm

x cm

Area = 150 cm^2 (2sf)

$\leqslant x <$

Fluency Practice

Complete the error interval for the speed of each journey.
Each measurement is rounded. Give answers in m/s or km/h to 1 dp, **within the actual error interval**.

		Min ≤	Distance	< Max	Min ≤	Time	< Max	Min ≤ Speed < Max
a			5 km			1 hour		
b			200 m (1 sf)			20 seconds (1 sf)		
c			10 km (nearest 10)			2 hours		
d			300 km (1 sf)			6 hours		
e			120 m (nearest 10)			15 seconds (2 sf)		
f			2 km (1 sf)			2 minutes (1 sf)		
g			15 km (nearest 5)			3 minutes (exact)		
h			450 km (2 sf)			0.5 hours (1 dp)		
i			0.8 km (1 dp)			9 seconds (1 sf)		
j			0.24 m (2 dp)			4.0 seconds (1 dp)		
k			2.2 km (1 dp)			1m:13s (integer)		
l			42.20 km (2 dp)			3h:12m (integer)		

Fluency Practice

example

Given a = 3.4 (correct to 1 d.p.) and b = 4.6 (correct to 1 d.p.), work out the upper bound of a ÷ b

$\frac{a}{b}$ is greatest when a is largest and b is smallest.

$$Upper\ bound = \frac{3.45}{4.55} = 0.75824...$$

questions

1. Complete the table to show the upper and lower bounds of each of these calculations:

a	b	calculation	upper bound	lower bound
2.8 (2 s.f.)	1.5 (2 s.f.)	a + b		
50 (1 s.f.)	30 (1 s.f.)	a - b		
0.5 (1 d.p.)	0.4 (1 d.p.)	a - b		
9.15 (2 d.p.)	3 (1 s.f.)	a × b		
18 (2 s.f.)	5.1 (2 s.f.)	a ÷ b		
20 (1 s.f.)	0.8 (1 d.p)	a ÷ b		

2. The measurements on this trapezium are all correct to the nearest cm.
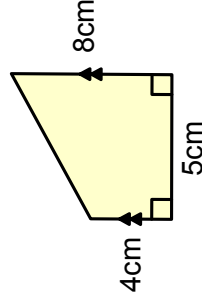
Work out the upper bound for the area of the trapezium.

8cm

4cm

5cm

3. The radius of a circle is 3.84cm, correct to 3 significant figures. Calculate the lower bound for its area. Give your answer to 3 d.p.

4. The area of a square is 42cm^2, correct to 2 significant figures. What is the smallest possible side length it could have? Give your answer to 2 d.p.

Fluency Practice

questions

In these questions, give your answers to 2 d.p. unless otherwise stated.

5. Given the formula $p = \dfrac{t}{s}$, work out the maximum value of p when t is measured to be 4.9 (1 d.p.) and s is measured as 1.65 (2 d.p.)

6. Given the formula $g = \dfrac{5}{tb}$, work out the maximum value of g when t is measured to be 0.8 (1 d.p.) and b is measured as 1.3 (1 d.p.)

7. Emma is painting a house. She is using tins of paint that cover $11m^2$ to the nearest m^2. If she has a total area of $300m^2$ to paint, correct to 1 s.f., how many tins of paint should she buy to be certain she will cover the required area?

8. Dejaun is cutting small pieces from a length of rope which is 4m, correct to the nearest metre. Each small piece of rope is 2.8cm, to the nearest mm. What is the maximum number of pieces of rope that Dejaun might be able to cut from the rope?

9. 4800 people attended a football match, correct to 2 significant figures. Of these, 1800 were children correct to 2 significant figures. What is the maximum number of adults who could have attended?

errors in the error interval?

Which of these error intervals are wrong for the measurements given?

	Measurement	Error Interval
a)	A length, l, is 42.7cm (1 d.p.)	$42.65\text{cm} \leq l < 42.75\text{cm}$
b)	A weight, w, is 160kg (2 s.f.)	$159.5\text{kg} \leq w < 160.5\text{kg}$
c)	The number of people, p, is 400 (1 s.f.)	$350 \leq p \leq 449$
d)	A height, h, is 140cm (2 s.f.)	$135 \leq h < 144$
e)	An area, a, is $0.08m^2$ (1 s.f.)	$0.075 \leq a < 0.085$
f)	A perimeter, p, is 100cm (2 s.f.)	$99.5 \leq p < 105\text{cm}$

Fluency Practice

Steve measures the length and width of a rectangle as 600 mm and 400 mm, both correct to 10 mm.

(a) Find the upper and lower bounds of the **perimeter** of the rectangle.

(b) Find the upper and lower bounds of the **area** of the rectangle.

Milly measures the length and width of a field as 25 m and 20 m, both to the nearest m.

(a) Find the upper and lower bounds of the **perimeter** of the field.

(b) Find the upper and lower bounds of the **area** of the field.

Ola's weight is 47 kg, correct to the nearest kg. Tia's weight is 55 kg, also correct to the nearest kg.

(a) Find the upper and lower bounds of the **total weight** of the two girls.

(b) Find the upper and lower bounds of the **difference** in their weights.

A car travels 240 km in 4 hours, both measured to the nearest unit. Find the upper and lower bounds of the car's speed.

A rock has a mass of 5 kg to the nearest 0.5 kg, and a volume of 2.1 m^3 to 1 decimal place. Find the upper and lower bounds of the density of the rock.

A cuboid has dimensions of 8 cm by 10 cm by 12 cm, all measured to the nearest cm. Find the upper and lower bounds of the volume of the cuboid.

Fluency Practice

(a)	(b)	(c)	(d)
Find the upper and lower bounds of 286 metres to the nearest metre.	Find the upper and lower bounds of 21 cm to the nearest cm.	Find the upper and lower bounds of 7.8 cm to 1 decimal place.	Find the upper and lower bounds of 5.24 kg to 2 decimal places.
(e)	**(f)**	**(g)**	**(h)**
Find the upper and lower bound of 80 cm to 1 significant figure.	Find the upper and lower bound of 5.6 kg to 2 significant figures.	A square has a side length of 4.1 cm to 1 decimal place. Find the lower bound of the perimeter of the square.	A rectangle measures 10 cm by 15 cm, both to the nearest cm. Find the upper bound of the area of the rectangle.
(i)	**(j)**	**(k)**	**(l)**
$a = b - c$ $c = 18$ correct to 2 significant figures. $b = 4.7$ correct to 1 decimal place. Find the upper and lower bounds of a.	$p = \dfrac{q}{r}$ $q = 20$ correct to 1 significant figure. $r = 6.3$ correct to 1 decimal place. Find the lower bound of p to 3 significant figures.	$c = \dfrac{d - e}{f}$ $d = 46, e = 8.5, f = 15,$ all correct to 2 significant figures. Find the upper bound of c to 2 decimal places.	$x = \dfrac{3a}{g - b}$ $a = 28, b = 12, g = 18,$ all correct to 2 significant figures. Find the lower bound of x to 3 significant figures.

Fluency Practice

A1 Zoe weighs 62 kg, correct to the nearest kilogram. Write down the lower bound for Zoe's weight.	**A2** The length of line AB = 8.3 cm, correct to 2 significant figures. Write down the upper bound for the length of AB.	**A3** Anu weighs 83 kg, correct to the nearest <u>half</u> kilogram. Write down the upper bound for Anu's weight.	**A4** The length of line CD = 27 cm, correct to the nearest 0.5 cm Write down the lower bound for the length of CD.
B1 Correct to the nearest millimetre, the length of a side of a regular hexagon is 3.6 cm Calculate the upper bound for the perimeter of the hexagon.	**B2** The perimeter of a square is 24 cm, correct to the nearest half centimetre. Work out the lower bound for the length of a side.	**B3** Correct to 1 significant figure, the area of a rectangle is 80 cm^2. Correct to 2 significant figures, the length of the rectangle is 12 cm. Calculate the upper bound for the width.	**B4** Correct to 2 significant figures the area of a square is 230 cm^2. Calculate the lower bound for the perimeter of the square.
C1 x = 1.8 correct to 1 decimal place. Calculate the lower bound for the value of $4x + 1$	**C2** Correct to 1 significant figure, a = 20 and b = 5 Work out the upper bound of $$5(a - b)$$	**C3** $x = p(q - r)$ p = 42, q = 24 and r = 14 all correct to 2 significant figures. Work out the lower bound for the value of x.	**C4** Correct to 2 significant figures, w = 58, x = 28 and y = 18 Calculate the upper bound of $$\frac{w}{x - y}$$
D1 Jada has 100 litres of oil, correct to the nearest litre. The oil is poured into tins of volume 1.5 litres, correct to one decimal place. Calculate the upper bound for the number of tins that can be filled.	**D2** There are 300 sheets of paper in a pile, correct to the nearest 10 sheets. The height of the pile is 160 mm, correct to the nearest 10 mm. Calculate the upper bound for the thickness of one sheet.	**D3** The distance to school is 2.8 km, correct to the nearest 0.1 km. Sam walks at a speed of 5 km/h, correct to the nearest km/h. Calculate the upper bound for the time Sam takes to walk to school.	**D4** Correct to 2 decimal places, the volume of a solid cube is 42.88 cm^3 Calculate the lower bound for the surface area of the cube.

Extension

1.

The inside of a mug is cylindrical with height 8.5cm and diameter 7.2cm, both correct to the nearest mm.

a) What is the maximum capacity of the mug?

b) What is the minimum capacity?

2.

Wood flooring is made in the form of rectangles measuring 24cm by 11cm, measurements correct to the nearest cm. When arranged as shown on the left:

a) What is the largest possible gap?

b) What is the smallest possible gap?

Gap

3.

A restaurant provides a stick of butter to each table. The dimensions required by the restaurant chain's management are 3cm by 3cm by 8cm, correct to the nearest cm.

a) What is the largest possible volume?

b) What is the smallest possible volume?

c) What percentage saving would be made on butter if the smallest possible volume were provided compared to the largest?

Extension

The diagram below shows a room which is to be painted (walls and ceiling). Rough measurements have been taken and are correct to the nearest metre.

3m

4m

5m
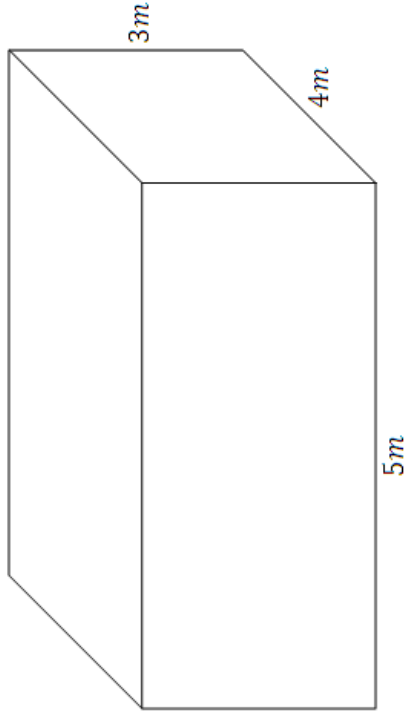
1. Calculate an estimate for the area to be painted, using the measurements given.

2. Paint comes in 2.5 litre tins which cost £18 and cover $25m^2$ of area. Find an upper bound for the cost of painting this room.

3. Calculate a minimum value for the amount of paint, in litres, needed for the room.

4. Enough tins of paint are bought to be certain of having enough. What is the maximum possible quantity of paint left over?

Fluency Practice

In each case you're trying to find a single suitable value of m to use (with justification), and you've calculated the upper and lower bounds.

1) $m_{lower} = 15.6241$
   $m_{upper} = 15.5935$

2) $m_{lower} = 148343$
   $m_{upper} = 151033$

3) $m_{lower} = 4.95013$
   $m_{upper} = 4.94502$

4) $m_{lower} = 1.29584$
   $m_{upper} = 1.29545$

Fluency Practice

1) $a = \frac{\sqrt{b}}{c}$

 $b = 0.018$ correct to 3 decimal places.
 $c = 0.032$ correct to 3 decimal places.
 By considering bounds, work out the value of a, giving your answer to a suitable degree of accuracy.

2) $a = \frac{5}{bc}$

 $b = 5.54$ correct to 3 significant figures.
 $c = 0.437$ correct to 3 significant figures.
 By considering bounds, work out the value of a, giving your answer to a suitable degree of accuracy.

3) $x = \frac{y}{z+w}$

 $y = 0.047$ correct to 3 decimal places.
 $z = 0.007$ correct to 1 significant figure.
4) $w = 0.016$ correct to 3 decimal places.
 By considering bounds, work out the value of x, giving your answer to a suitable degree of accuracy.

5) $x = \frac{y^2}{z}$

 $y = 0.163$ correct to 3 decimal places.
 $z = 0.571$ correct to 3 significant figures.
 By considering bounds, work out the value of x, giving your answer to a suitable degree of accuracy.

6) $p = \frac{q}{r-s}$

 $q = 8.7$ correct to 1 decimal place.
 $r = 0.02$ correct to 1 significant figure.
 $s = 0.894$ correct to 3 significant figures.
 By considering bounds, work out the value of p, giving your answer to a suitable degree of accuracy.

Fluency Practice

example

Given that $p = \dfrac{t}{v}$ and $t = 42.8$ (1 d.p.) and $v = 16.6$ (1 d.p.), work out the value of p to a suitable degree of accuracy.

solution

1. Work out the upper bound of p, which is $\dfrac{42.85}{16.55} = 2.58912386...$

2. Work out the lower bound of p, which is $\dfrac{42.75}{16.65} = 2.567567567...$

3. Look at which digits agree, i.e. they are both *2.6 (1 d.p.)*

questions

1. Given that $a = x + c$ and $x = 109.31$ (2 d.p.) and $c = 1.42$ (2 d.p.), work out a to a suitable degree of accuracy.

2. Given that $t = \dfrac{p}{v}$, and $p = 47.3$ (3 s.f.) and $v = 27.6$ (2 s.f.), work out t to a suitable degree of accuracy.

3. Sally is 1.8m tall, correct to 2 significant figures. Asha is 1.65m tall, correct to 3 significant figures. What is the maximum possible difference between their heights?

4. Given that $g = s^4$ and $s = 1.14$ (2 d.p.), work out the value of g to a suitable degree of accuracy.

5. Given that $p = t - q$, and $t = 4.6$ (1 d.p.) and $q = 3$ (nearest integer), use an inequality to write down an error interval for p.

6. Milo weighs 48.2kg and Ana 42.1kg, both correct to 3 significant figures. Work out their maximum possible total combined weight.

7. Given that $b = \dfrac{\sqrt{a}}{4}$ and $a = 9.74$ (2 d.p.) work out the value of b to a suitable degree of accuracy.

Fluency Practice

1) Truncate 38.25369 to 1 decimal place.

2) Truncate 78.15659 to 1 decimal place.

3) Truncate 16.85692 to 3 decimal places.

4) Truncate 86.13870 to 1 decimal place.

5) Truncate 67.61322 to 1 decimal place.

6) Truncate 77.36573 to 1 decimal place.

7) Truncate 20.93772 to 3 decimal places.

8) Truncate 32.43570 to 3 decimal places.

9) Truncate 74.97334 to 1 decimal place.

10) Truncate 5.37433 to 2 decimal places.

Fluency Practice

learn by heart

Truncation: *to shorten a decimal number without rounding*

example

Truncate 42.89464 to 3 decimal places.

= 42.894

notice that this is different to ROUNDING to 3 decimal places!

questions

1. Truncate 2.98456 to 3 d.p.

2. Truncate 43.00956 to 2 d.p.

3. Truncate 409.98 to the integer.

4. Truncate 5.03953 to 2 d.p.

5. Complete the table to compare different methods of rounding:

Number	Rounded to 1 significant figure	Rounded to 2 significant figures	Rounded to 2 decimal places	Truncated to 2 decimal places
408.6577				
0.48759				
989.00865				
500.506033				

6. Raul has chosen one of the numbers from the list below. Can you use the clues to work out which number he chose?

Clue 1: When rounded to 1 significant figure, Raul's number is 3

Clue 2: When rounded to 1 decimal place, Raul's number is 2.6

Clue 3: When truncated to 1 decimal place, Raul's number is 2.5

2.68	2.50

2.63	2.56

2.52	3.54

Fluency Practice

1) A number p , when truncated to 2 decimal places, is equal to 0.77
   Find the upper and lower bound of p

2) A number p , when truncated to 1 decimal place, is equal to 1.7
   Find the upper and lower bound of p

3) A number z , when truncated to 1 decimal place, is equal to 1.4
   Find the upper and lower bound of z

4) A number z , when truncated to 3 decimal places, is equal to 0.031
   Find the upper and lower bound of z

5) A number x , when truncated to 3 decimal places, is equal to 0.555
   Find the upper and lower bound of x

6) A number x , when truncated to 1 decimal place, is equal to 64.4
   Find the upper and lower bound of x

7) A number x , when truncated to 3 decimal places, is equal to 0.026
   Find the upper and lower bound of x

8) A number p , when truncated to 1 decimal place, is equal to 44.5
   Find the upper and lower bound of p

9) A number p , when truncated to 1 decimal place, is equal to 81.3
   Find the upper and lower bound of p

10) A number y , when truncated to 2 decimal places, is equal to 0.52
    Find the upper and lower bound of y

Fluency Practice

Section A – Truncating Numbers

1. Truncate the following numbers to the unit:

   a) 25.3   b) 76.25   c) 10.9   d) 10.99   e) 10.999   f) 11.0001

2. Truncate the following numbers to the tens:

   a) 12   b) 58   c) 125   d) 972   e) 3684   f) 47209

3. Truncate the following numbers to the hundreds:

   a) 305   b) 395   c) 420   d) 4329   e) 670934   f) 1000001

4. Truncate the following numbers to 1 d.p.:

   a) 0.51   b) 0.58   c) 0.55   d) 1.452   e) 1.9538   f) 13.999999

5. Truncate the following numbers to 2 d.p.:

   a) 12.4534   b) 534.9872   c) 0.01684   d) 0.008452   e) 4.43920   f) 10.999123

Section B – The Error Interval for a Truncated Number

If each of the following numbers has been truncated as described, give the corresponding error interval:

1. 1 (to the unit)
2. 68 (to the unit)
3. 945 (to the unit)
4. 380 (to the tens)
5. 210 (to the tens)
6. 800 (to the hundreds)
7. 5400 (to the hundreds)
8. 10000 (to the hundreds)
9. 56.7 (to 1 d.p.)
10. 99.1 (to 1 d.p.)
11. 234.6 (to 1 d.p.)
12. 0.45 (to 2 d.p.)
13. 0.01 (to 2 d.p.)
14. 10.301 (to 3 d.p.)
15. 10.300 (to 3 d.p.)

Fluency Practice

example

When truncated to 2 decimal places, p = 4.28. Write down the error interval for p.

$$4.28 \leq p < 4.29$$

The smallest this number could have been was 4.28000...

At its largest it could have been 4.289999....

questions

1. Truncate 43.9859 to 2 decimal places.

2. Truncate 109.49 to the integer.

3. A distance, d, when truncated to 3 decimal places is 46.507. Write down its upper and lower bounds.

4. A weight, w, when truncated to 2 decimal places is 2.84. Write down an inequality to show its error interval.

5. A length, l, when truncated to 2 decimal places is 0.94. Which of the following is the correct error interval for l?

   a) $0.94 < l < 0.949$
   b) $0.935 \leq l < 0.945$
   c) $0.94 \leq l < 0.95$
   d) $0.94 < l < 0.95$

6. Shade all of the numbers in this grid that truncate to 0.45.

0.448	0.4512	0.4	0.5445	0.457	0.045	4.457	0.4501
0.4567	0.4499	0.458	0.4508	0.4051	0.544	0.4491	0.4
$0.\dot{4}\dot{5}$	$0.\dot{4}$	$0.\dot{5}$	$0.\dot{4}\dot{5}$	$0.4\dot{5}1$	$0.\dot{5}\dot{4}$	$0.1\dot{4}\dot{5}$	$0.0\dot{4}\dot{5}$

7. True or False: 1.85 rounded to 1 decimal place is greater than 1.85 truncated to 1 decimal place.

8. True or False: 4.743 rounded to 1 decimal place is greater than 4.743 truncated to 2 decimal places.

Fluency Practice

Use inequalities to write down the error in these calculators with broken screens

a)

$$96\_\_\_\_\_$$

b)

$$63\_\_\_\_\_$$

c)

$$951\_$$

d)

$$0.05\_\_$$

e)

$$1.0\_\_$$

f)

$$81\_.\_\_$$

(g)

$$6.2\_\_\_$$

(h)

$$0.\_\_\_$$

(i)

$$5\_2\_$$

(j)

$$5\_\_2\_\_$$

(k)

$$5\_\_.2\_\_$$

(l)

$$0.05\_\_2\_\_$$

Fluency Practice

I've broken some calculators.
For each screen: what's the biggest the number could have been? What's the smallest? Write you answer as **error interval**.

1 `45`▯

2 `59`▯▯

3 `1`▯▯

4 `159`▯▯▯

5 159000 has been truncated to 3 significant figures. Write down the error interval of the number.
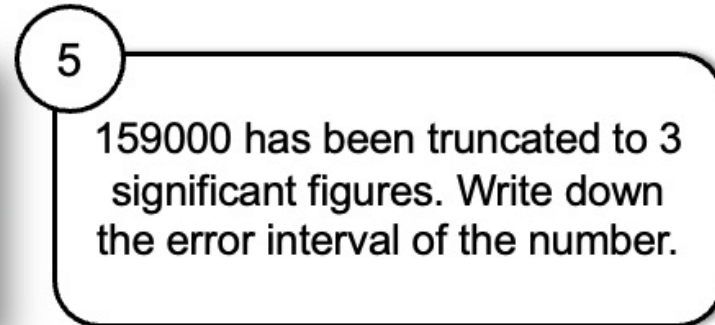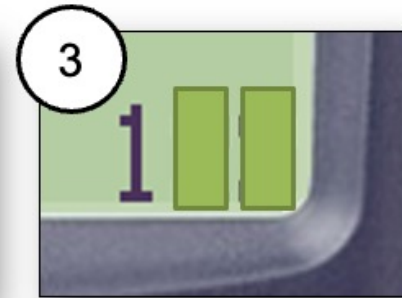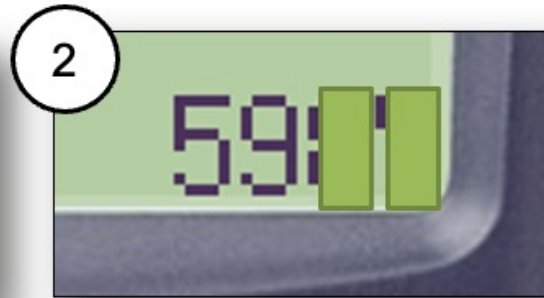
2 Basic Circle Theorems

Fluency Practice

rules

The angle in a semicircle is a right angle.
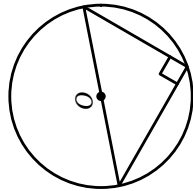
The angle at the centre is twice the angle at the circumference.

70° 140°
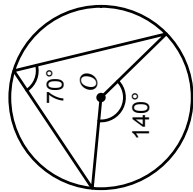
Angles in the same segment are equal.
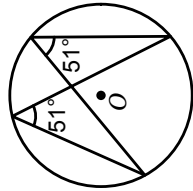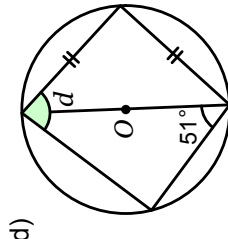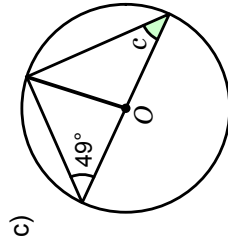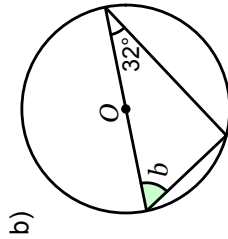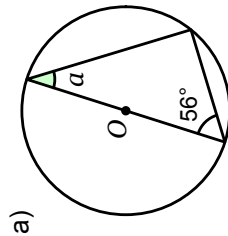
51° 51° 51°

exercise

1. Work out the angles marked with letters.

a) a, 56°

b) b, 32°

c) 49°, c

d) d, 51°

2. Work out the angles marked with letters.

a) a, 55°

b) 144°, b

c) c, 208°

d) 70°, d

3. Work out the angles marked with letters.

a) a, 46°

b) 50°, b, 48°

c) 45°, c, 71°

d) 45°, 120°, d

Page 95

Fluency Practice

4. Work out the angles marked θ.

a)

b)

c)

d)

e)

f)

g)

h)

i)
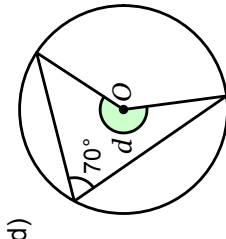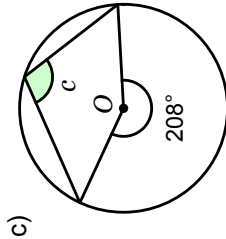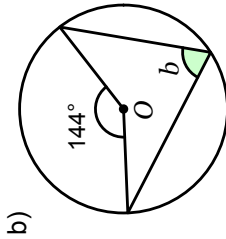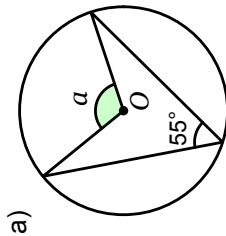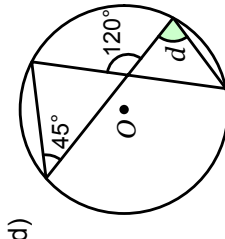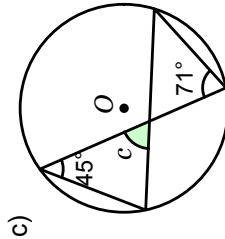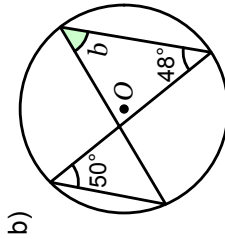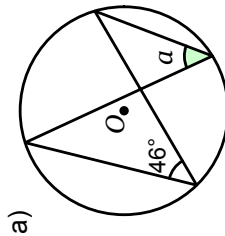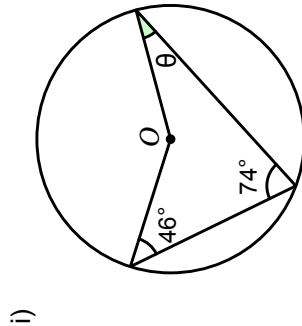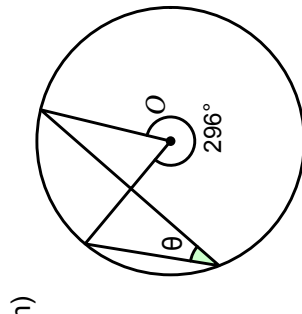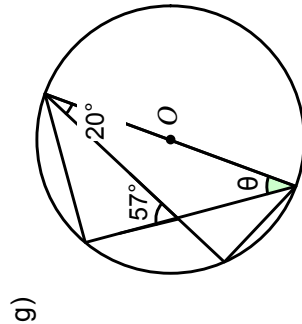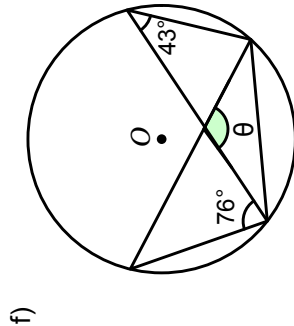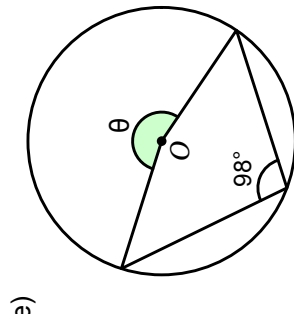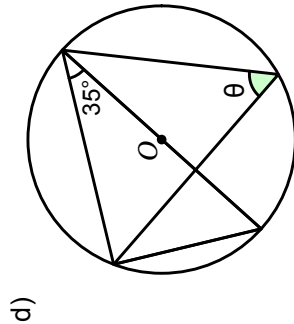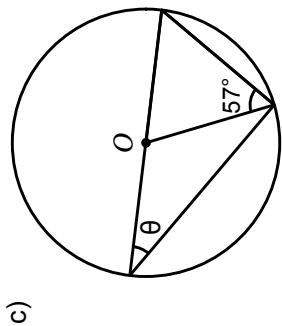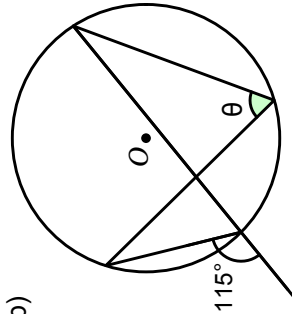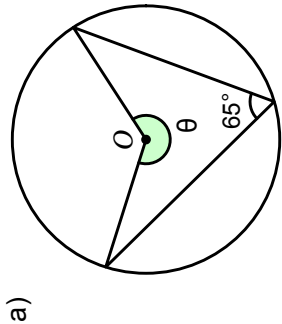
5. Points A, B, C and D lie on the circumference of a circle. AEC and DEB are straight lines.
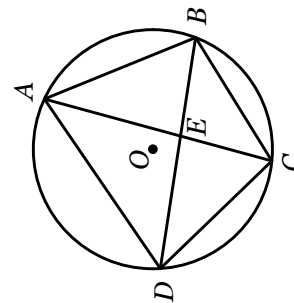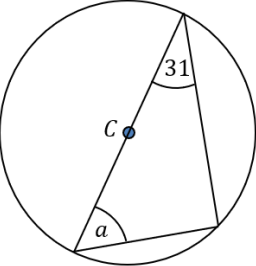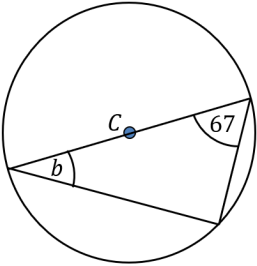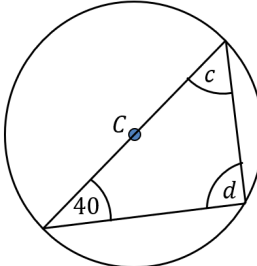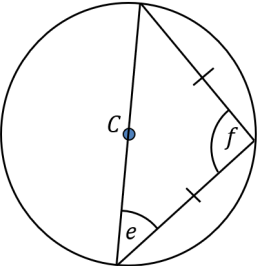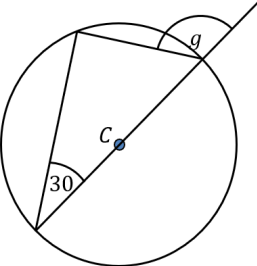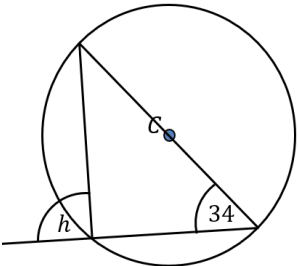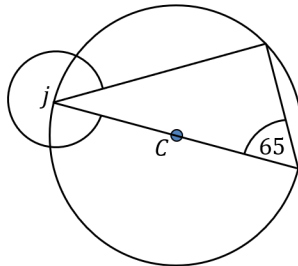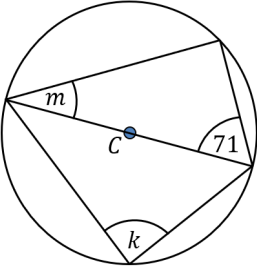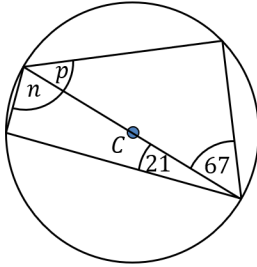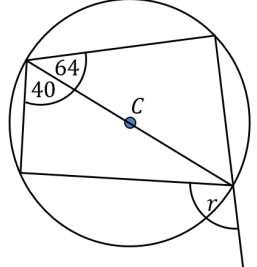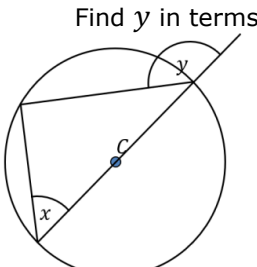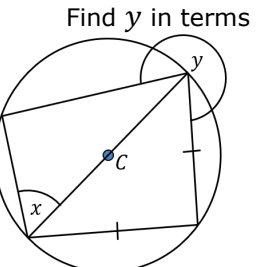
Prove that triangles ABE and DCE are similar.

6. A, B and C are points on a circle with centre O.
$\angle BAC = x$

Show that: $\angle OBC = 90 - x$

Page 96
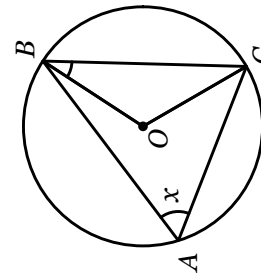
Fluency Practice

Right-Angle in a Semi-Circle Practice Grid

(a)	(b)	(c)	(d)

(e)	(f)	(g)	(h)

(i)	(j)	(k)	(l)

(k) Find y in terms of x

(l) Find y in terms of x

Fluency Practice

Angle at the Centre

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)
Find y in terms of x

(l)
Find y in terms of x
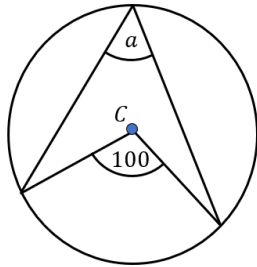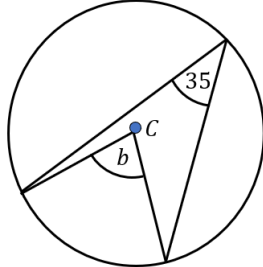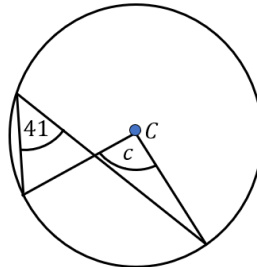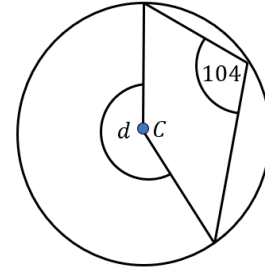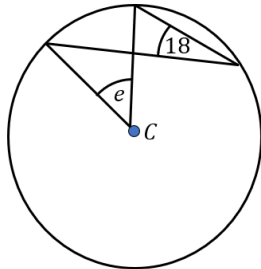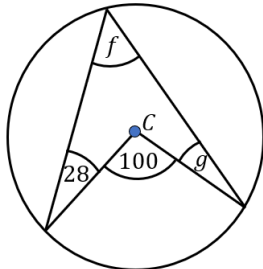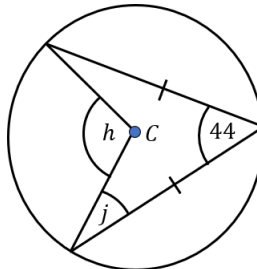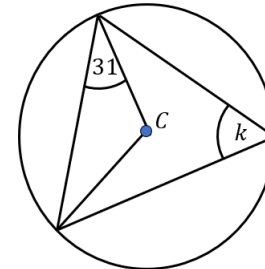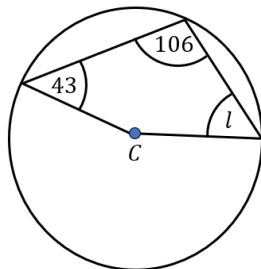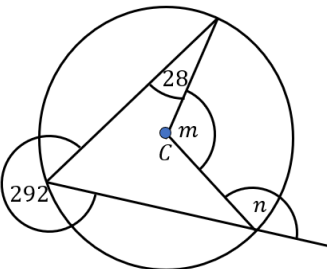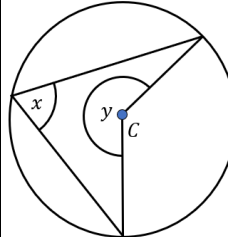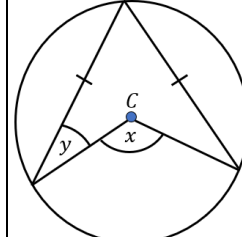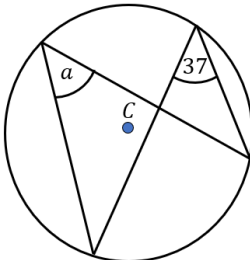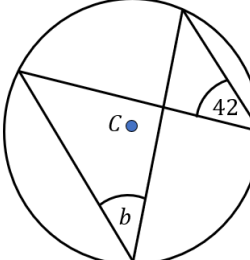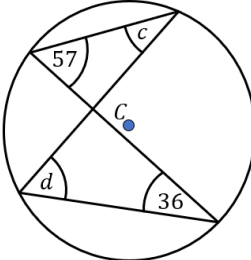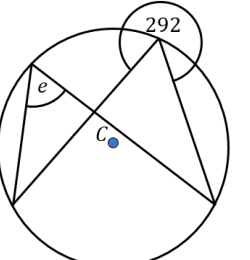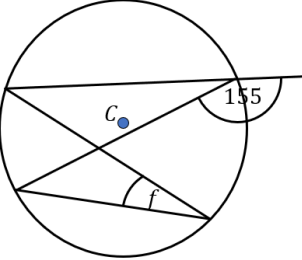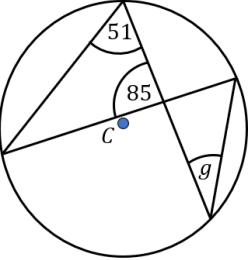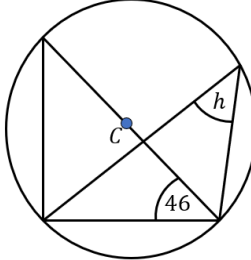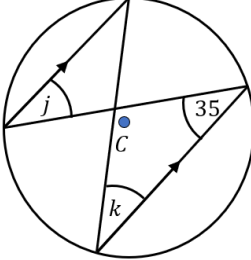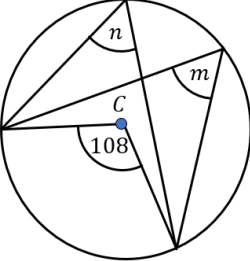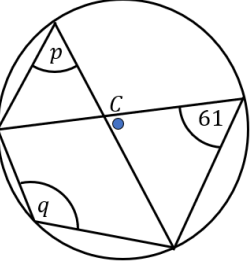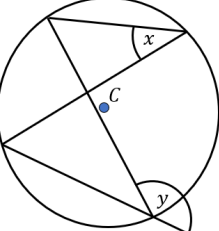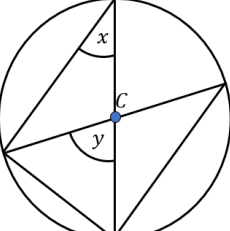
Fluency Practice

Angles from the Same Segment

(a)	(b)	(c)	(d)

(e)	(f)	(g)	(h)

(i)	(j)	(k)	(l)

Find y in terms of x

Find y in terms of x

Fluency Practice

rules

Opposite angles of a cyclic quadrilateral sum to 180°.
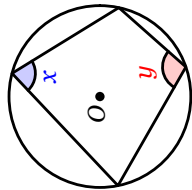
$x + y = 180°$

Tangents to a point are equal in length.
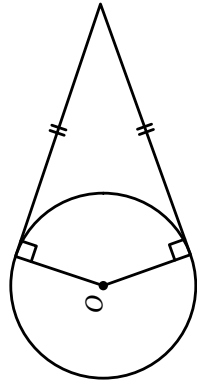
exercise

1. Work out the angles marked with letters.
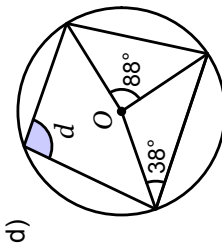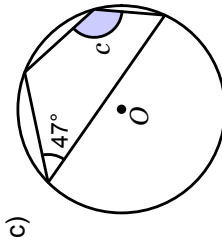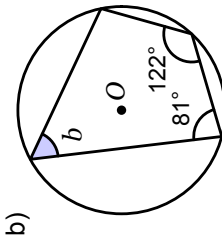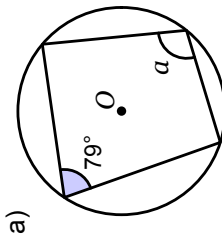
a)

b)

c)

d)

2. Work out the angles marked with letters.

a)

b)

c)

3. Work out the angles marked θ.

a)

b)

Fluency Practice

4. Work out the angles marked θ.

a)

b)

c)

d)

e)

f)

g)

h)

i)
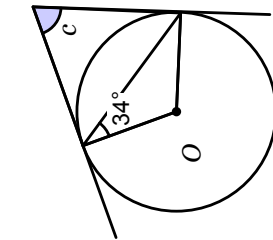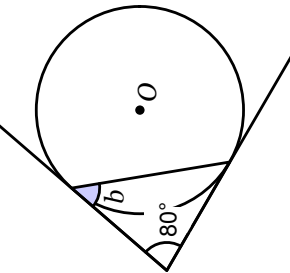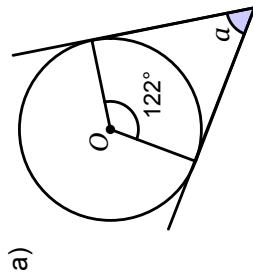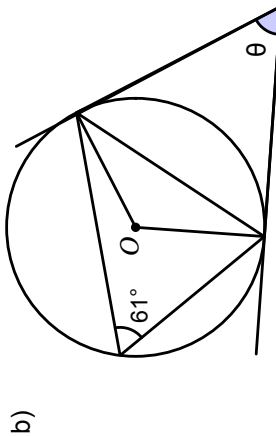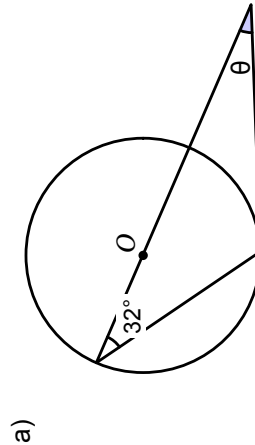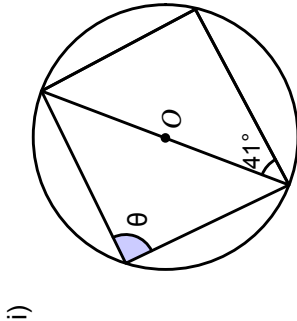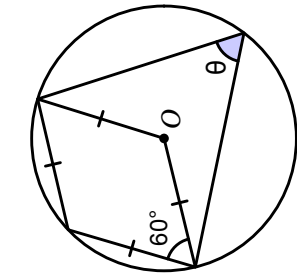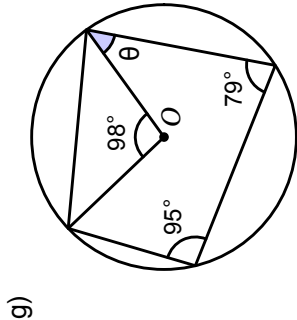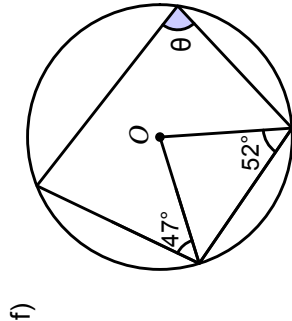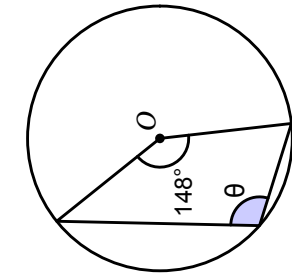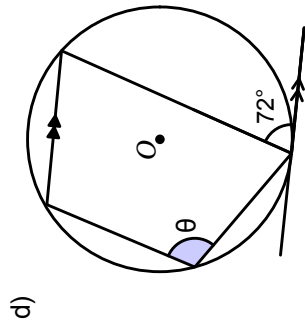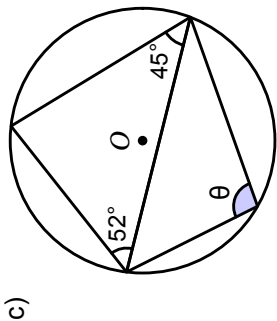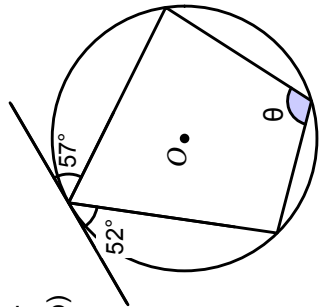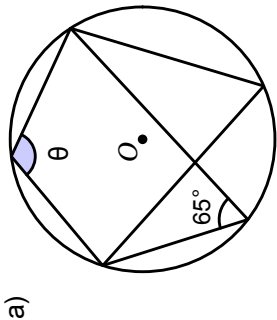
5. $a : b : c = 4 : 6 : 5$.

Work out the values of a, b, c and d.
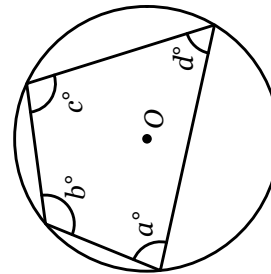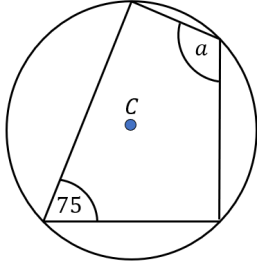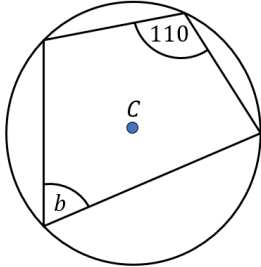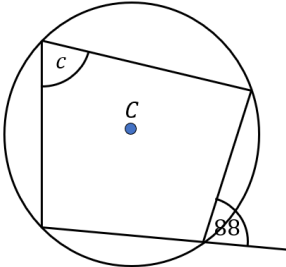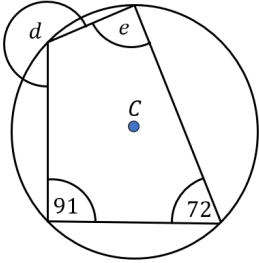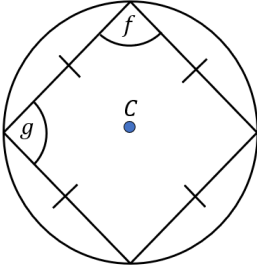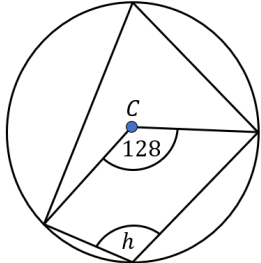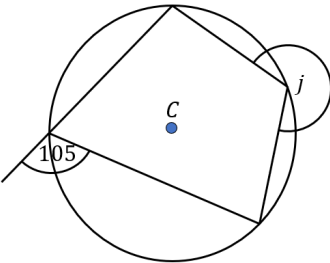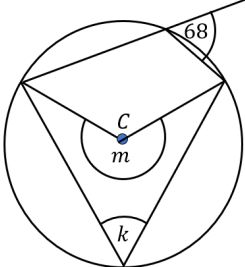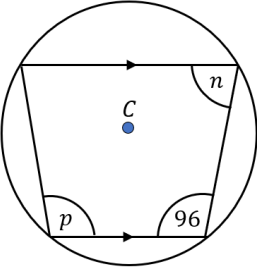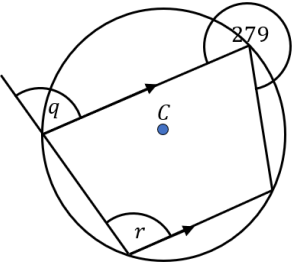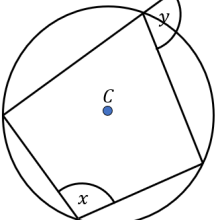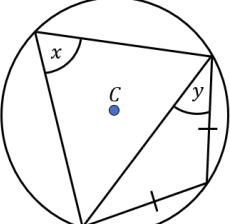
6. A, B and C are points on the circumference of a circle with centre O.
    AD and CD are tangents to the circle.
    $\angle ADC = 48°$.

Work out $\angle ABC$.

Fluency Practice

Cyclic Quadrilaterals

(a)	(b)	(c)	(d)

(e)	(f)	(g)	(h)

(i)	(j)	(k)	(l)

(k) Find y in terms of x

(l) Find y in terms of x

Fluency Practice

Cyclic Quadrilaterals We have enough information to find **some** of the missing angles. Which ones can we find?

①

A =
B =
C =

100° B A C

②
D =
E =
F =

E D F
80°
65°

G =
H =
I =

③
45°
130°
G
I H

④
J =
K =

35°
J 290°
K

⑤
40° 25°
110° L
N M

L =
M =
N =

⑥
O =
P =
Q =

O
115° P
Q

⑦
R S 50°
30°
T

R =
S =
T =

⑧
U =
V =
W =
X =

U 28°
X V
W

Page 111
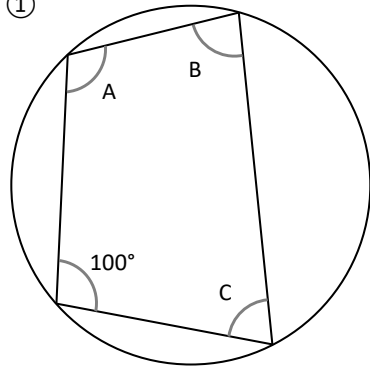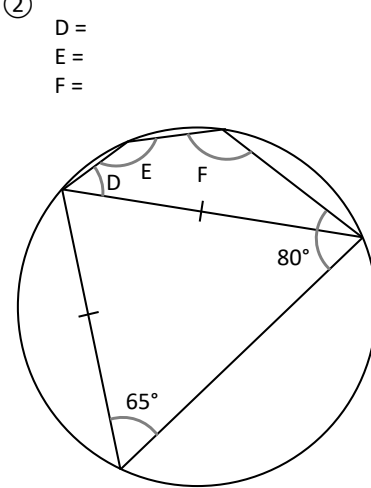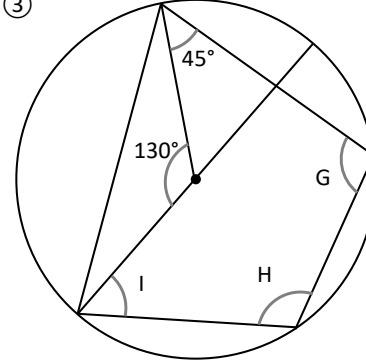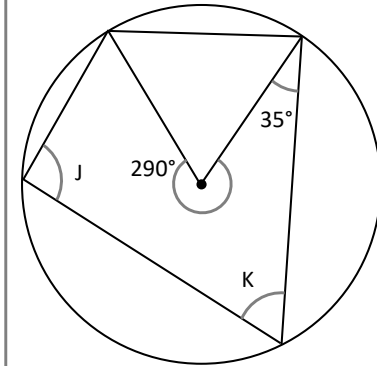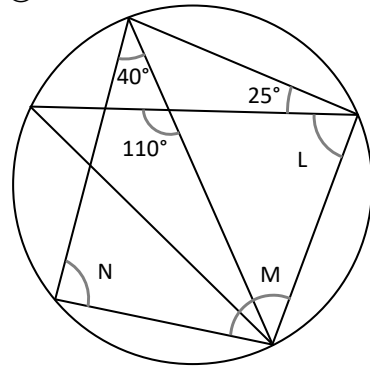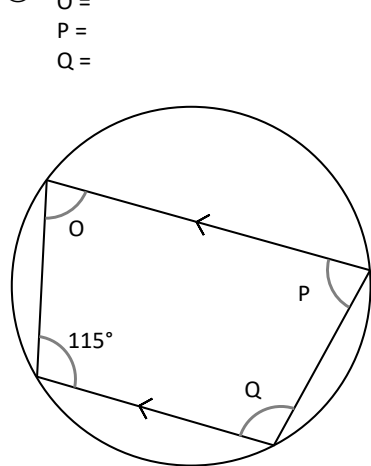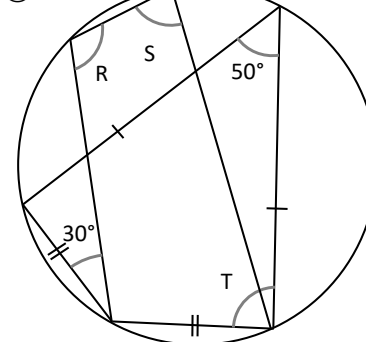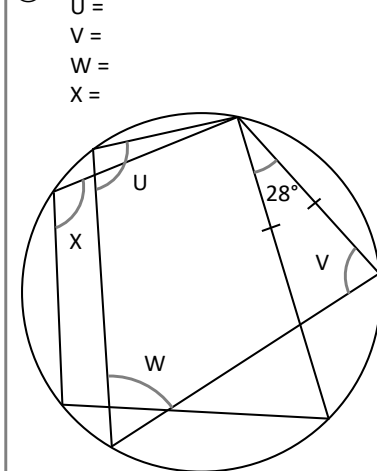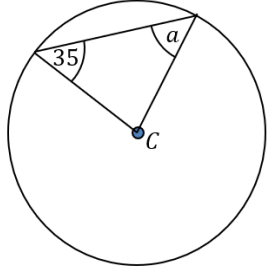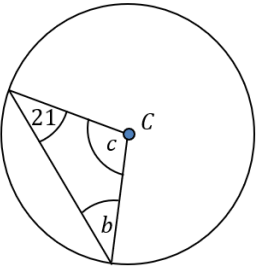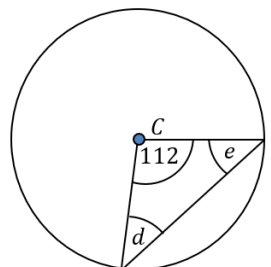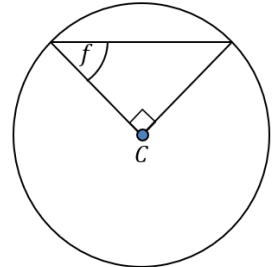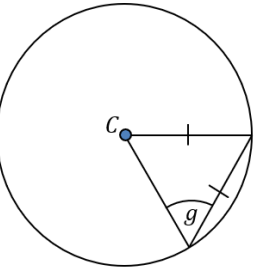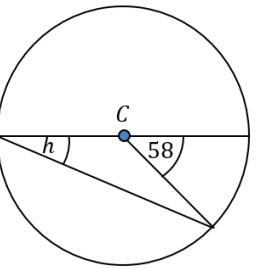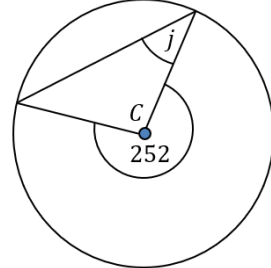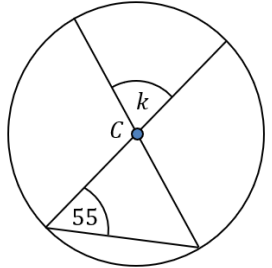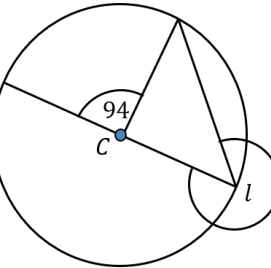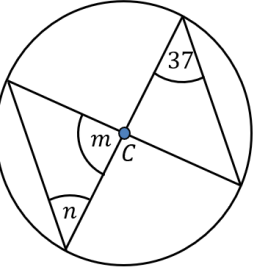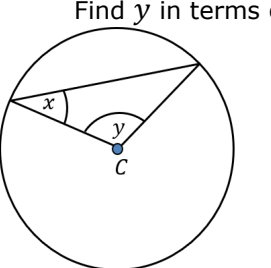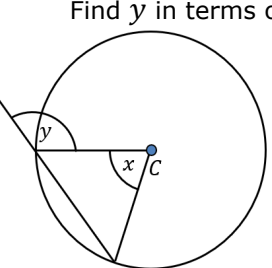
Fluency Practice

Isosceles Triangle in a Circle Practice Grid

(a)	(b)	(c)	(d)

(e)	(f)	(g)	(h)

(i)	(j)	(k)	(l)

(k) Find y in terms of x

(l) Find y in terms of x

Fluency Practice

The perpendicular from the centre to a chord bisects the chord.
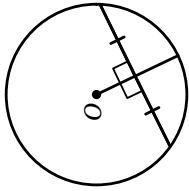
The angle between a chord and a tangent equals the angle in the **alternate segment**.

55°
55°
55°

exercise

1. Work out the angles marked θ.

a)
θ
62°

b)
58°
47°
θ

c)
θ
52°

d)
θ
46°

e)
50°
θ
66°

f)
θ
66°
109°

g)
65°
θ
32°

h)
60°
65°
θ

i)
226°
θ
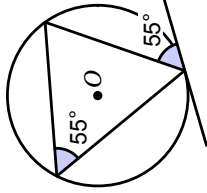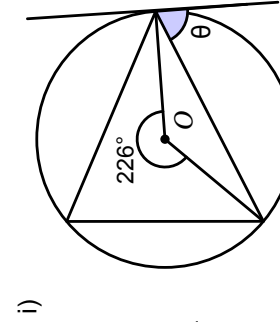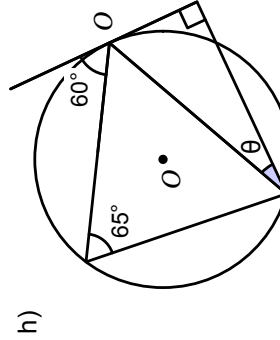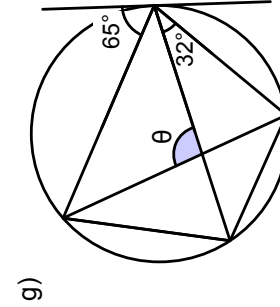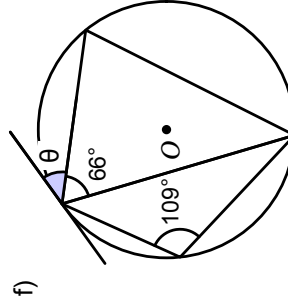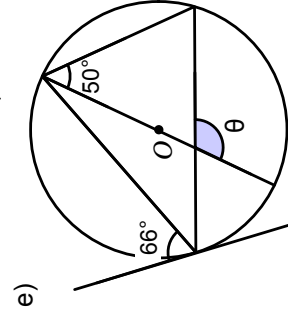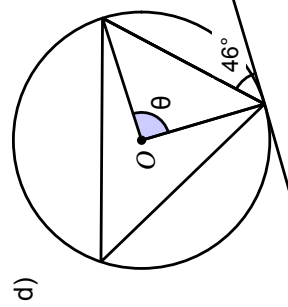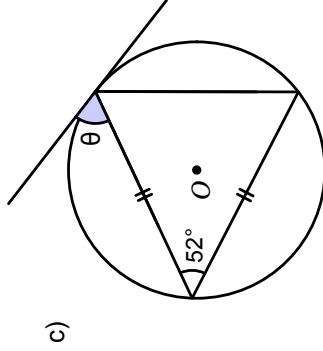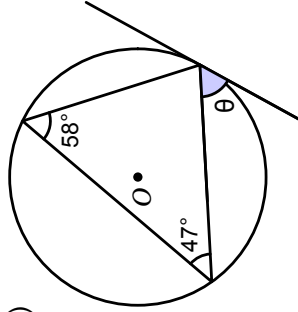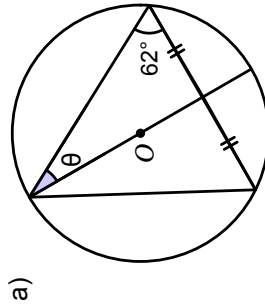
Fluency Practice

Perpendicular from the Centre to a Chord

(a)	(b)	(c)	(d)

(e)	(f)	(g)	(h)

(i)	(j)	(k)	(l)

Find y in terms of x

Find y in terms of x

Fluency Practice

Alternate Segment Theorem

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

Find y in terms of x

(l)

Find y in terms of x

Fluency Practice

Alternate Segment Theorem   We have enough information to find **some** of the missing angles. Which ones can we find?

①

A =
B =

②

30°

C =
D =
E =

③

H

F

G

50°

F =
G =
H =

④

I =
J =
K =

60°

J

30°

I

K

⑤

L =
M =

M

55°

L

⑥

30°

N

P   O

20°

N =
O =
P =

⑦

S

Q

R

T

55°

U

Q =
R =
S =
T =
U =

⑧

V =
W =
X =
Y =
Z =

Y

X

Z

V

W

35°

Page 121
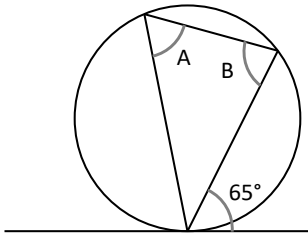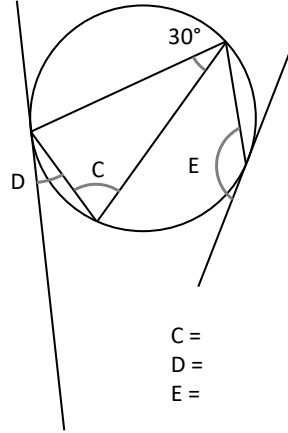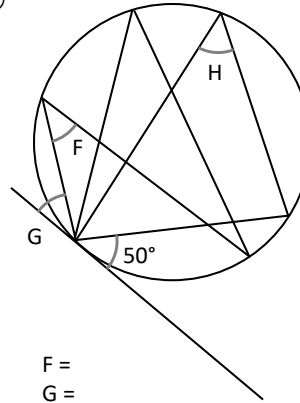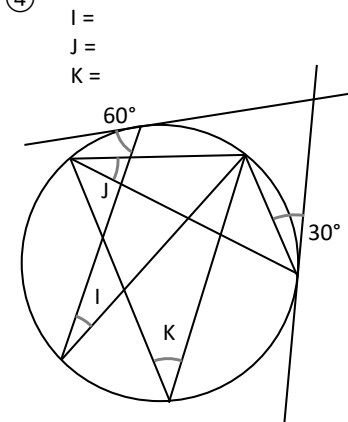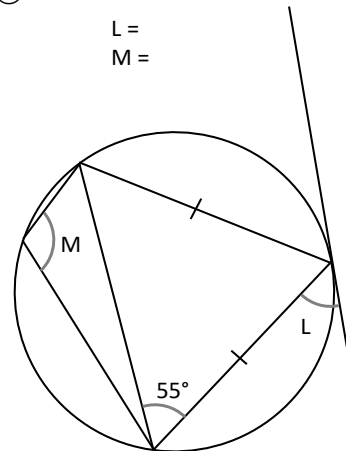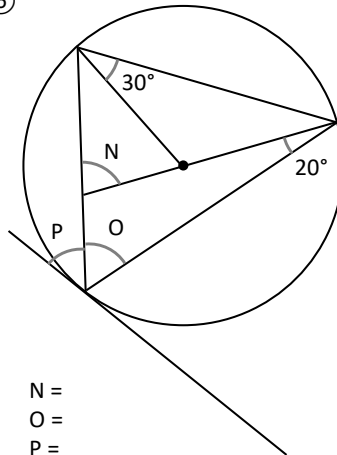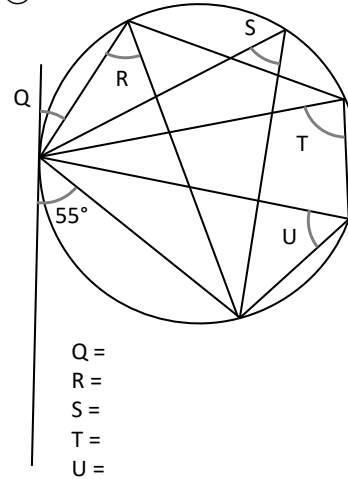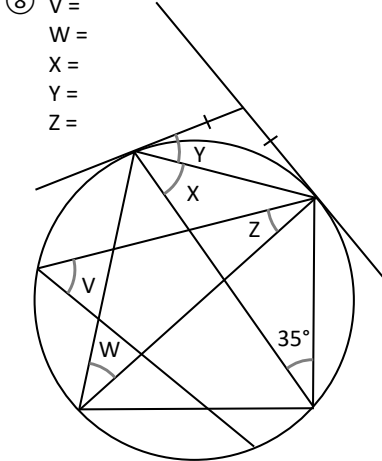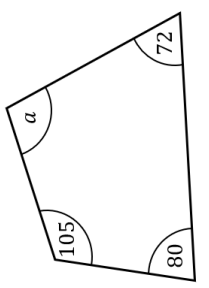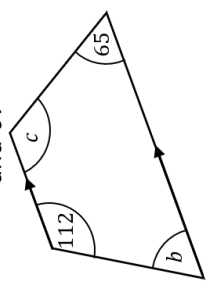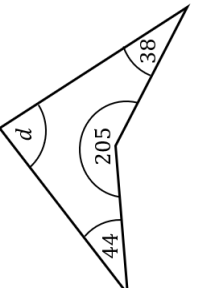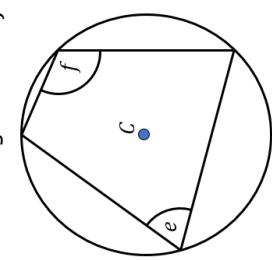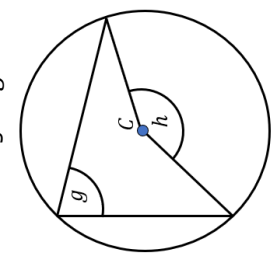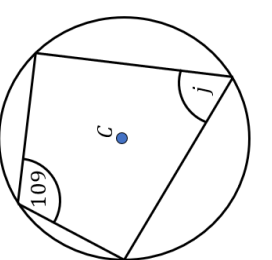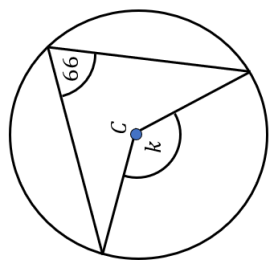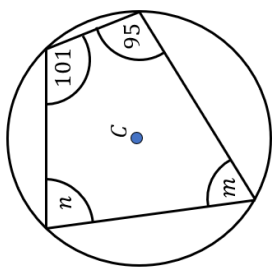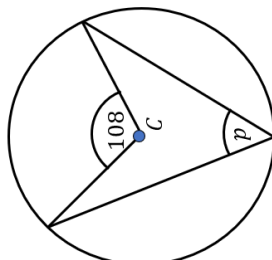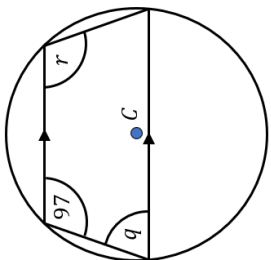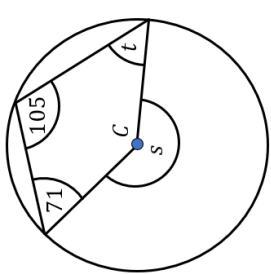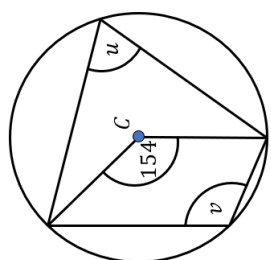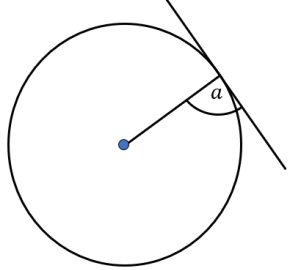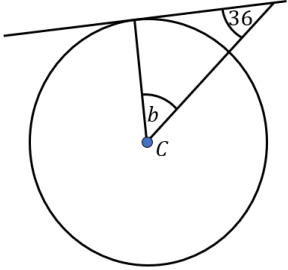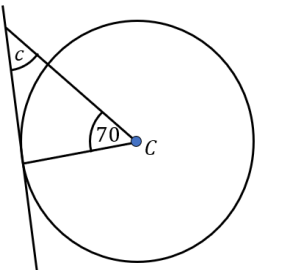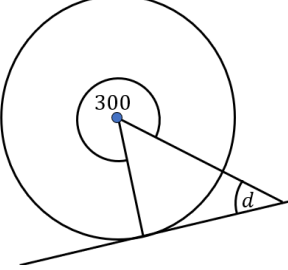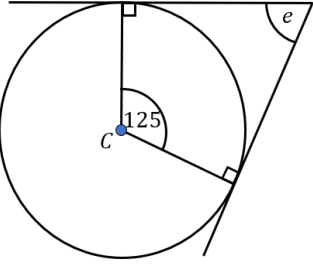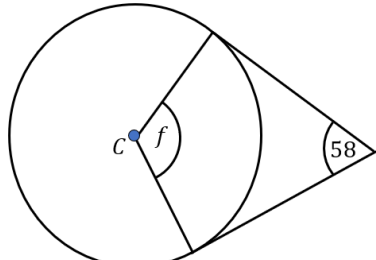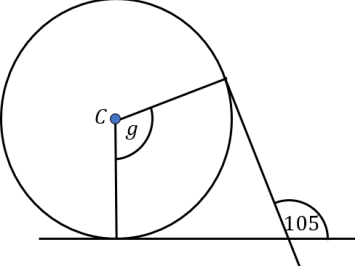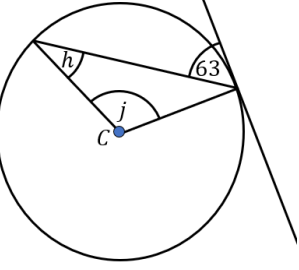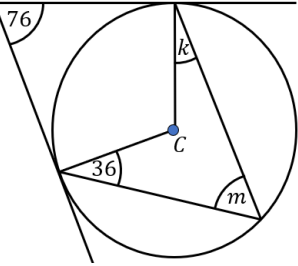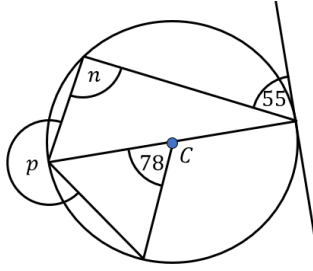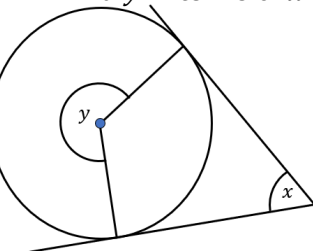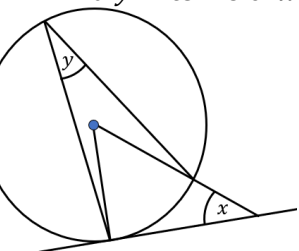
Fluency Practice

Circle Theorems and Quadrilaterals

(a) Find the missing angle a.

(b) Find the missing angles b and c.

(c) Find the missing angle d.

(d) What is the relationship between angles e and f?

(e) What is the relationship between angles g and h?

(f) Find the value of j.

(g) Find the value of k.

(h) Find the values of m and n.

(i) Find the value of p.

(j) Find the values of q and r.

(k) Find the values of s and t.

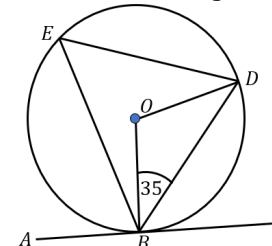**(l)** Find the values of u and v.

Fluency Practice
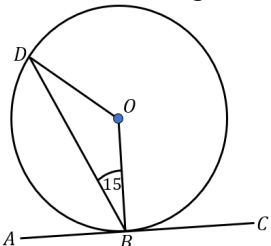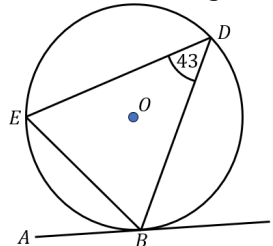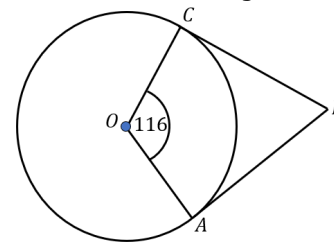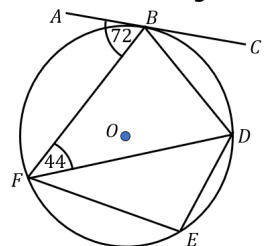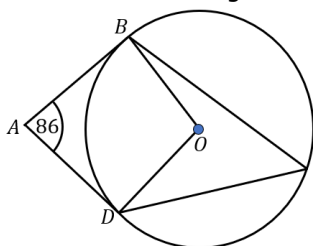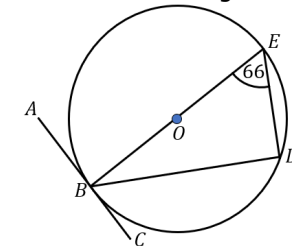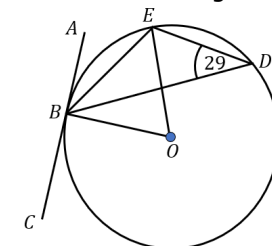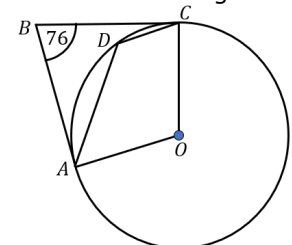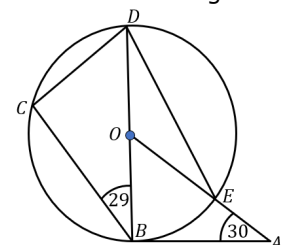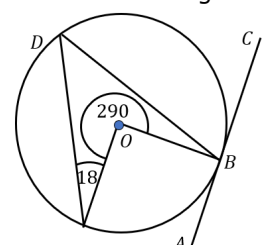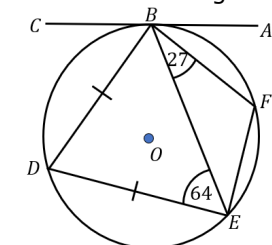
Circle Theorems and Tangents

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

Find y in terms of x

(l)

Find y in terms of x

Fluency Practice

Mixed Circle Theorems

(a)
Find the size of angle BCD

(b)
Find the size of angle CBD

(c)
Find the size of angle DCB

(d)
Find the size of angle BAC

(e)
Find the size of angle ABC

(f)
Find the size of angle OBC

(g)
Find the size of angle BCD

(h)
Find the size of angle AED

(i)
Find the size of angle ODC

(j)
Find the size of angle ODA

(k)
Find the size of angle CAD

(l)
Find the size of angle CED

Fluency Practice

Mixed Circle Theorems with Tangents

(a)
Find the size of angle ABD

(b)
Find the size of angle EBA

(c)
Find the size of angle ABC

(d)
Find the size of angle BED

(e)
Find the size of angle DEF

(f)
Find the size of angle BCD

(g)
Find the size of angle DBC

(h)
Find the size of angle ABE

(i)
Find the size of angle ADC

(j)
Find the size of angle EDC

(k)
Find the size of angle DBC

(l)
Find the size of angle FEB

Fluency Practice

Angles at the Circumference   We have enough information to find **some** of the missing angles. Which ones can we find?

①

C
B
A
30°

A =
B =
C =

②

D =
E =
F =

E
F
D
20°

③

120°
I
H
G

G =
H =
I =

④

J =
K =
L =

J
130°
K
L

⑤

M =
N =
O =
P =

M
N
O
P
30°

⑥

Q
R
S
35°

Q =
R =
S =

⑦

T =
U =
V =
W =
X =

X
U
V
T
W
55°

⑧

AA
CC
BB
30°
Y
85°
55°
Z
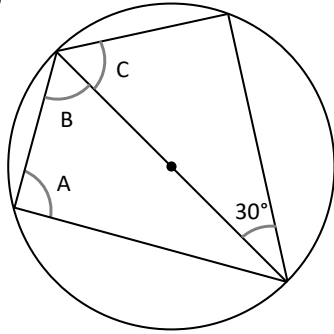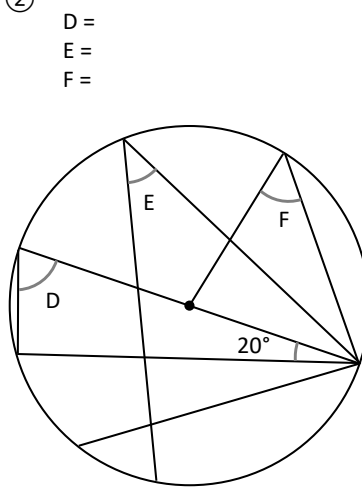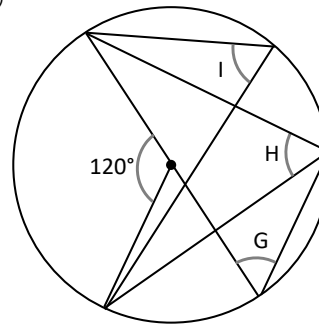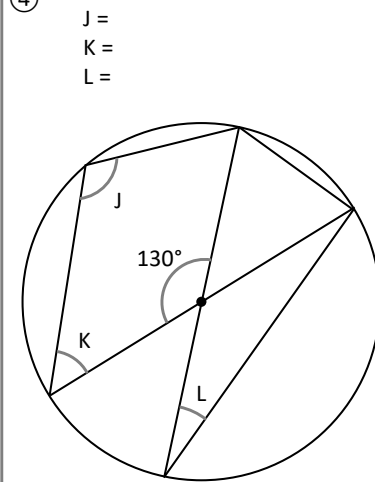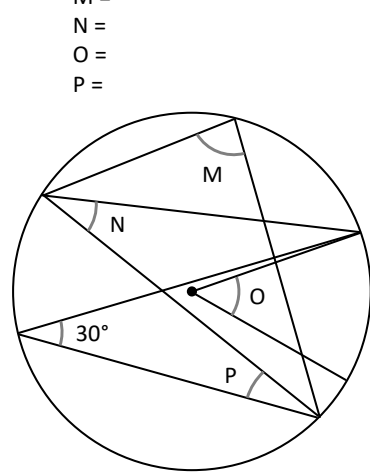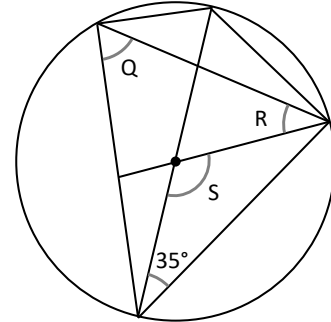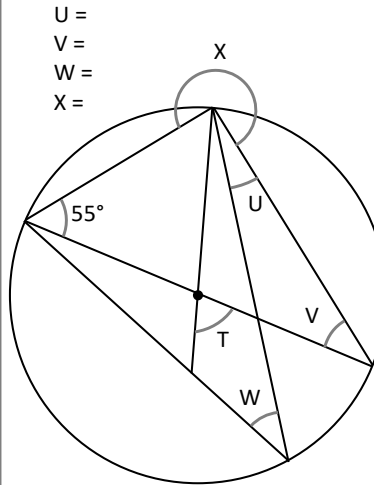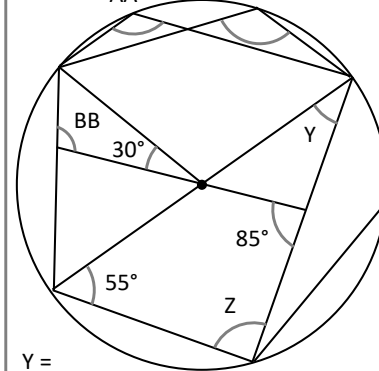
Y =
Z =
AA =
BB =
CC =

Fluency Practice

Tangents & Chords   We have enough information to find **some** of the missing angles & lengths. Which ones can we find?
(Assume lines that *look* like tangent are tangents.)

①

B

A

C

D

A =
B =
C =
D =

②

E

86°

F

G

40°

E =
F =
G =

③

H

135°

J   6 cm

I

H =
I =
J =

④

325°

K =
L =
M =

K

L   8 cm

M

⑤

6 cm

N

O   P

Q

N =
O =
P =
Q =

⑥

R =
S =
T =

R

S

4 cm

T

⑦

U

70°

W

V

U =
V =
W =

⑧

X

Y

4 cm

Z

115°

7 cm

AA

X =
Y =
Z =
AA =
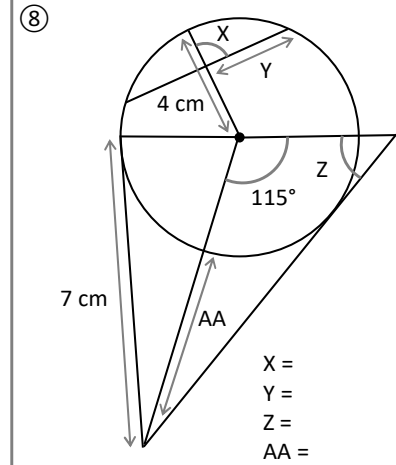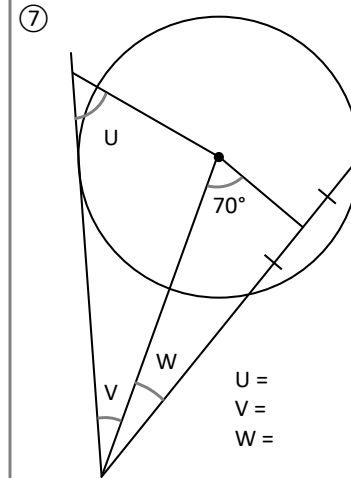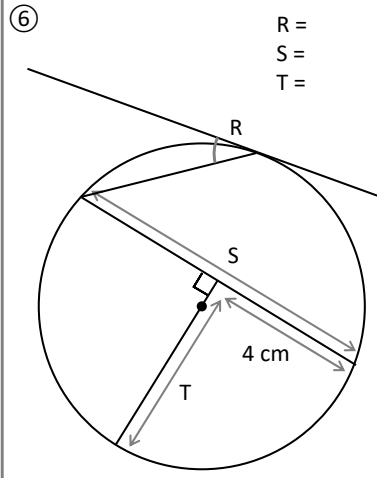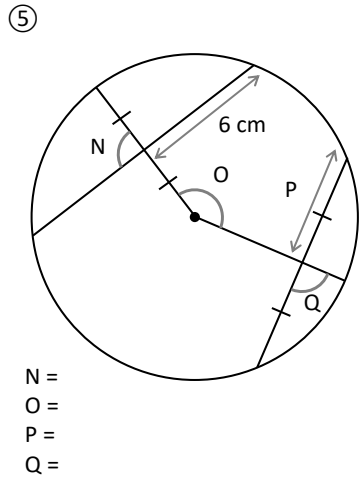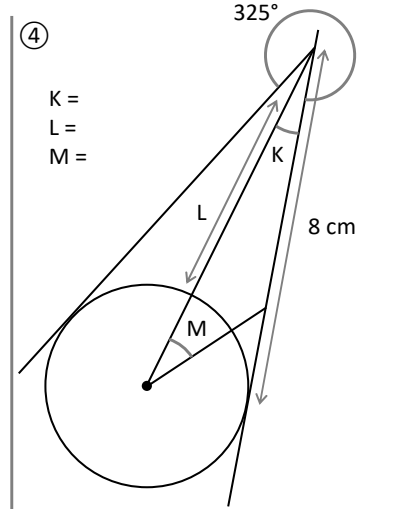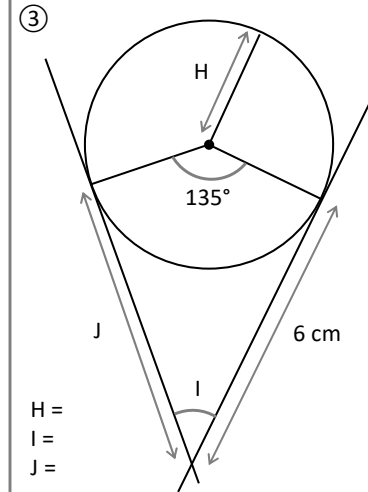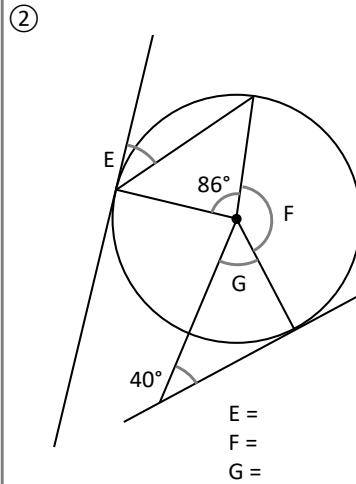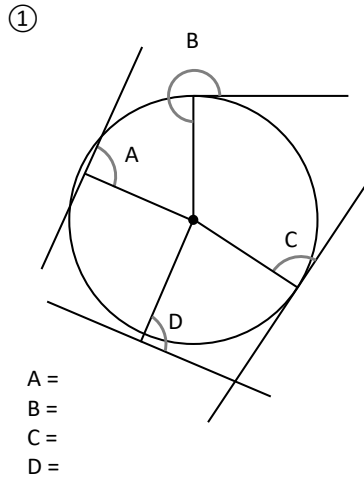
Fluency Practice

1. Work out the angles marked θ.

a)

b)

c) 104°

d) 42°

e) 68°

f) 116°

g) 47°, 75°

h) 42°, 57°

i) 68°, 61°

j) 50°, 32°

k) 30°

1. Work out the angles marked θ.

a)

b)

c)

d)

e)

f)

g)

h)
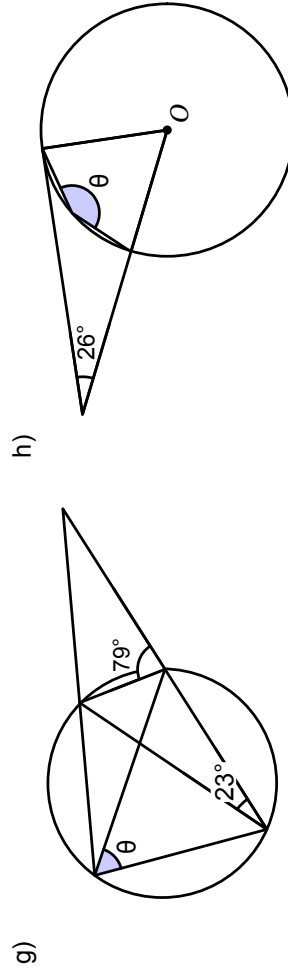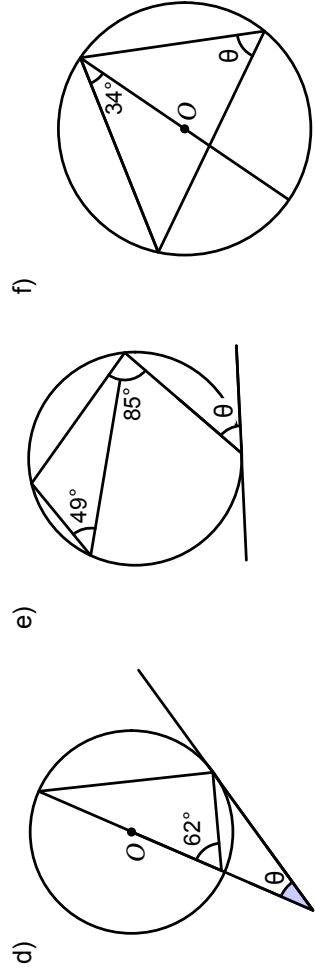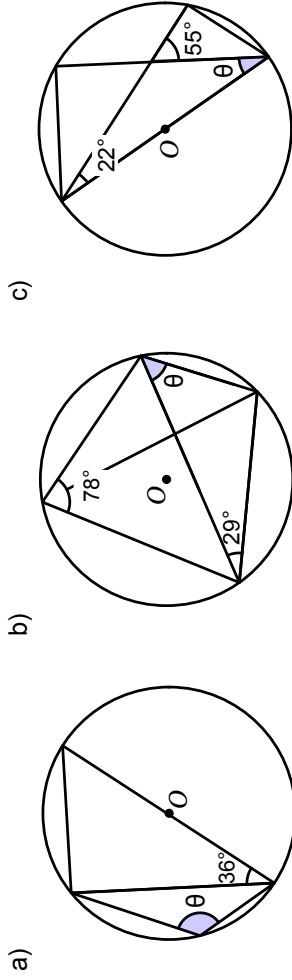
i) *A* and *B* are the centres of the circles.
EDC is a straight line.
D lies on both circles.

j)

Fluency Practice

1 a

A, B, D (85°), C, x

$x =$ **?**

b

x, 60°, 51°

$x =$ **?**

c

x, O, 150°

$x =$ **?**

2 a

A, O, x, B, D, 110°, C

$x =$ **?**

b

A, O, x, 80°, B, C

$x =$ **?**

d

O, x

$x =$ **?**

e

O, x, 80°

$x =$ **?**

f

O, x, 50°

$x =$ **?**

c

A, O, x, 70°, C, B

$x =$ **?**

d

A, D, 85°, 30°, B, x, C

$x =$ **?**

e

A, O, 80°, x, C, B

$x =$ **?**
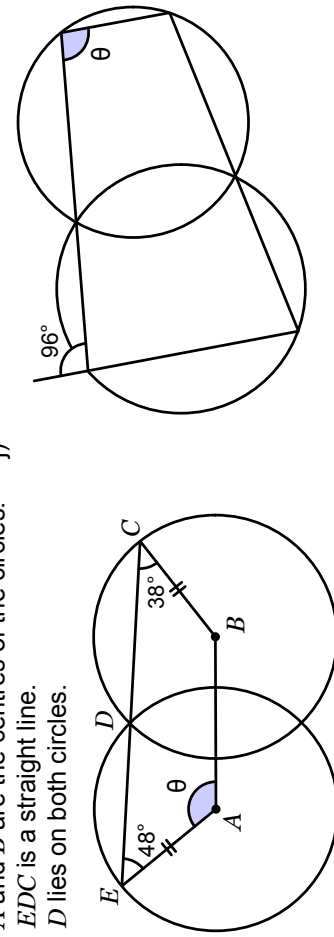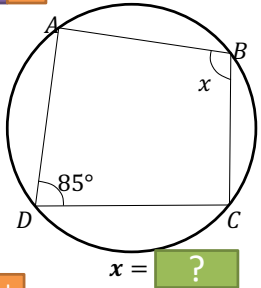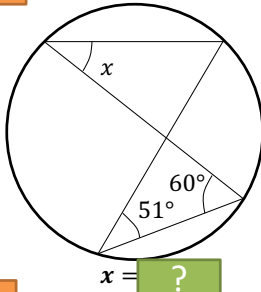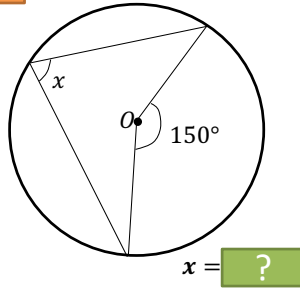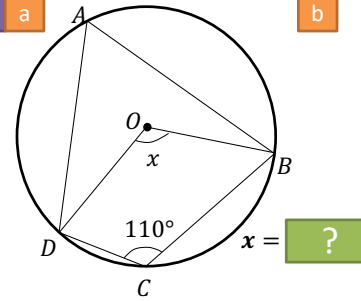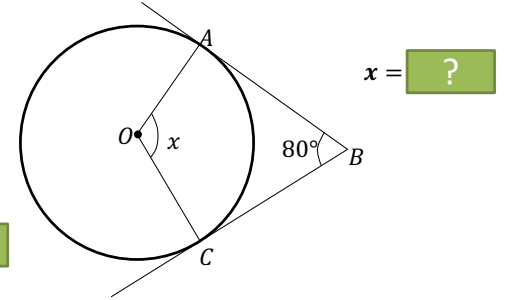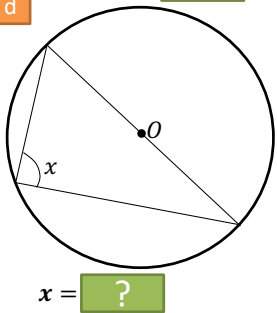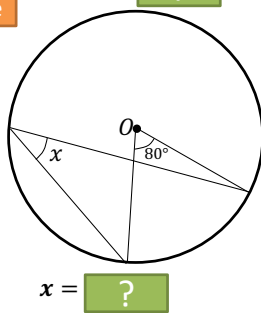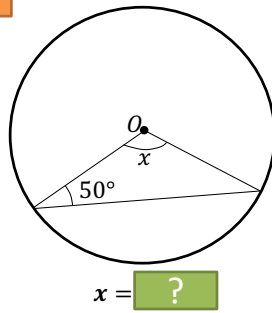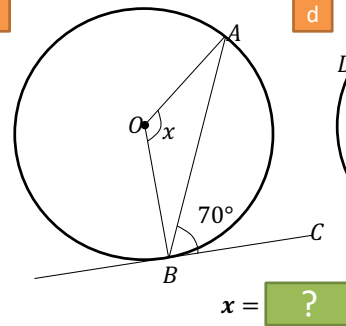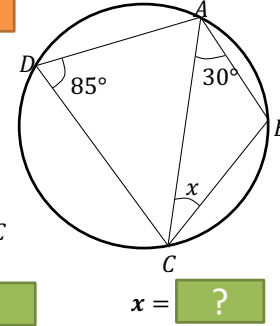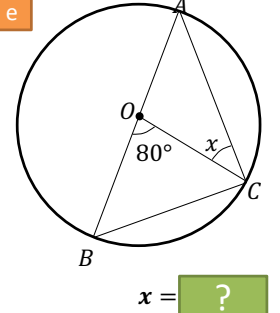
Fluency Practice

3 **a**

$82°$

$x =$ [?]

b

y

x

$78°$

$x =$ [?]
$y =$ [?]

c

x

$x =$ [?]

d

$80°$

x
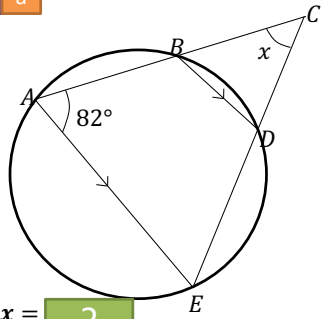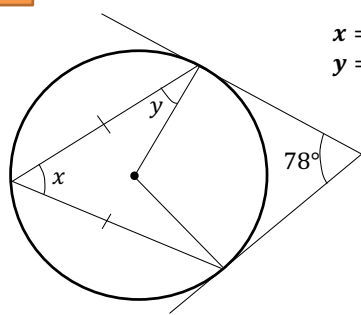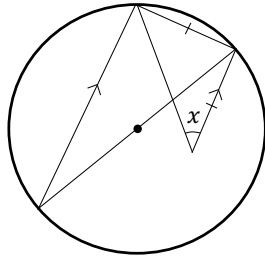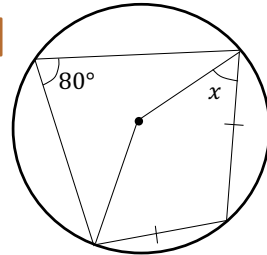
4 Determine each indicated angle in terms of x.

x

$\angle AOB =$ [?]

x

$\angle AOB =$ [?]

x

$\angle BAC =$ [?]

Fluency Practice

a

B

A 70°

D *x*

C

$x =$?

b

A

x

B

$x =$?

c

P

Q

100°

150° *S*

V

x

U

T

$x =$?

Fluency Practice

1

55°

B

A

x

y

65°

C

D

x = ?
y = ?

2

x

70°

y

x = ?
y = ?

3

B

49°

A

O

x

71°

C

D

x = ?

4

B

A

121°

C

x

63°

D

E

x = ?

5

A

x

O

70°

B

62°

C

D

x = ?

6

A

x

O

B

81°

58°

C

D

x = ?
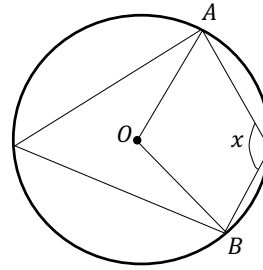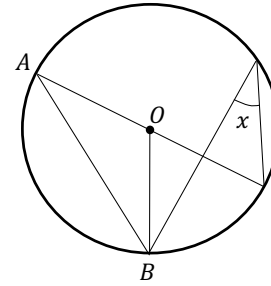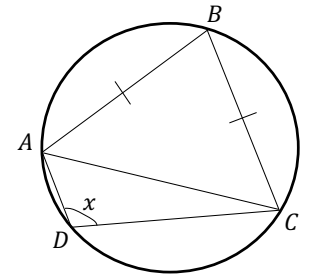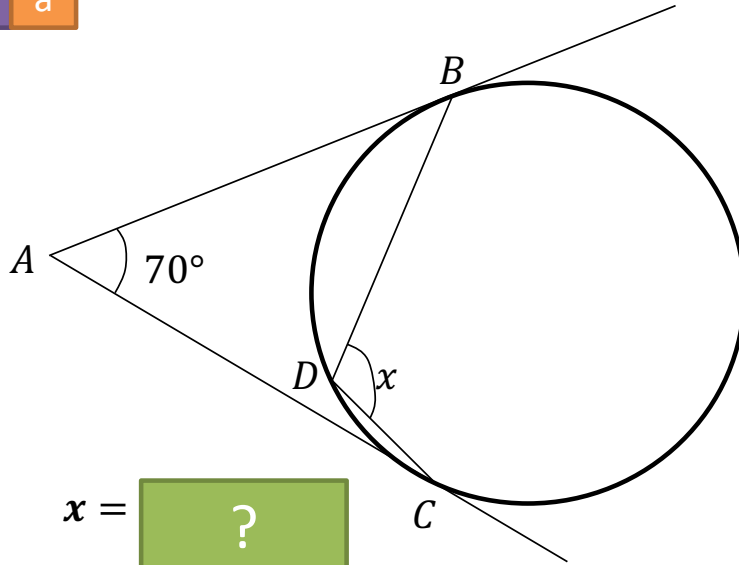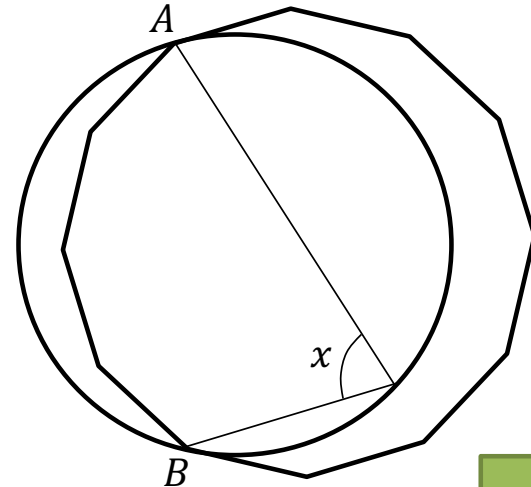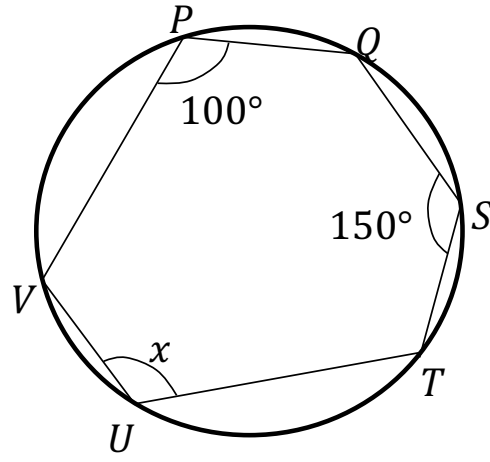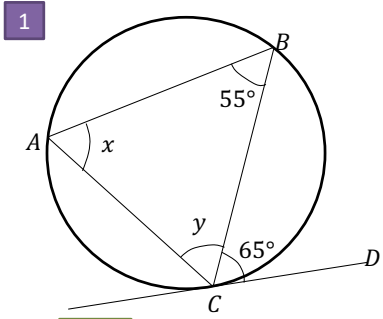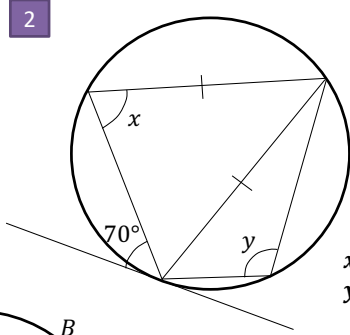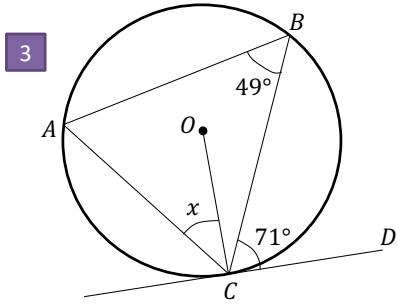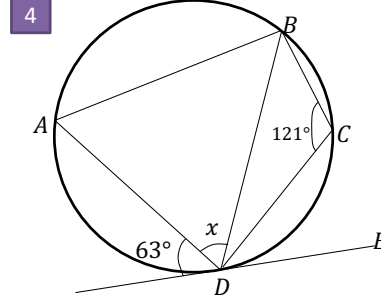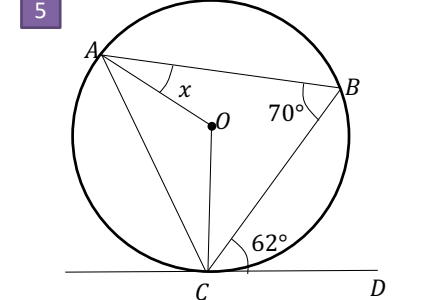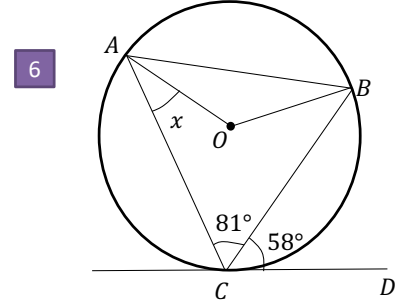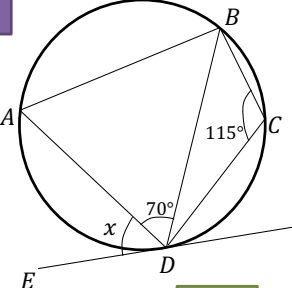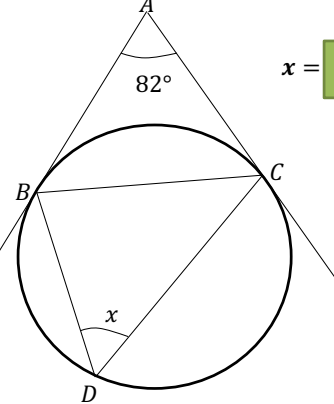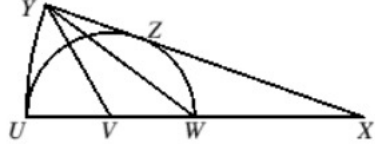
Fluency Practice

7

$x = $ [?]

8

$x = $ [?]

☠2
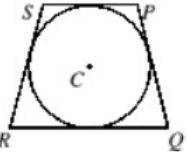
[SMC 2012 Q22] A semicircle of radius r is drawn with centre V and diameter UW. The line UW is then extended to the point X, such that UW and WX are of equal length. An arc of the circle with centre X and radius $4r$ is then drawn so that the line XY is a tangent to the semicircle at Z, as shown. What, in terms of r, is the area of triangle YVW?

[?]

☠1

[SMC 2012 Q20] In trapezium $PQRS$, $SR = PQ = 25$cm and SP is parallel to RQ. All four sides of $PQRS$ are tangent to a circle with centre C. The area of the trapezium is $600cm^2$. What is the radius of the circle?

$r = $ [?]

Fluency Practice

Trigonometry with Circle Theorems (assume tangents)

A
61°
x cm
10 cm

B
12 cm
x
15 cm

C
11 cm
49°
41°
x cm

D
x cm
48°
7 cm

E
39°
9 cm
x cm

F
x
7 cm
4 cm

G
45°
8 cm
29°
x cm

H
x
12 cm
13 cm

I
23°
15 cm
x cm

J
x
12 cm
7 cm

K
x cm
14 cm
54°

L
x
9 cm
11 cm

M
81°
x cm
57°
8 cm

N
17 cm
39°
74°
x cm

O
11 cm
24°
126°
x cm
39°

14 Answers	14.3	12.7	8.7	7.1	16.8	47.3	78.6
	53.1	16.5	40.0	9.4	299.5	8.9	34.8

Fluency Practice

Unless otherwise stated, find the value of y

35°

35°

35°

y

2y

3y

y

y

35°

y

35°

35°

10cm

35°
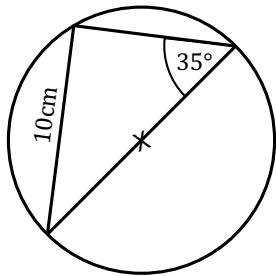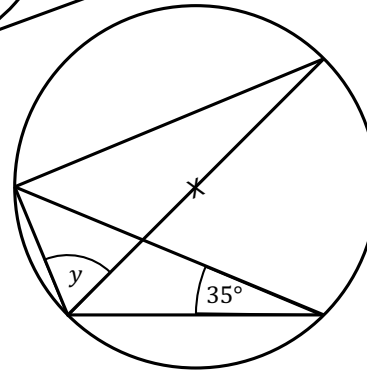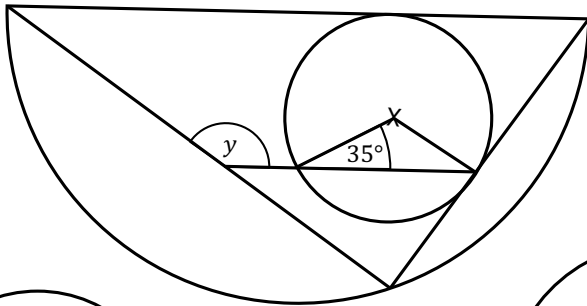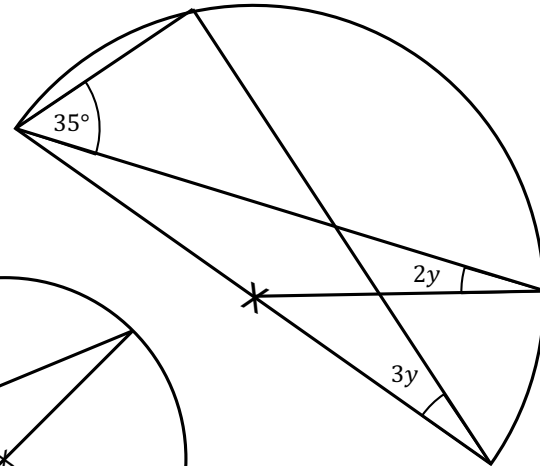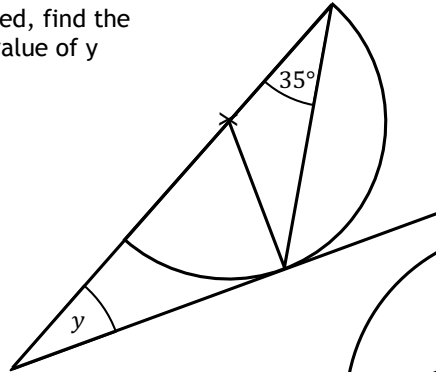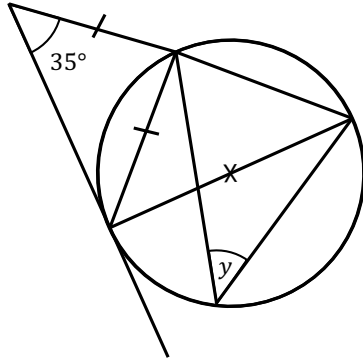
Find the area of the circle to 3sf

35°

y

35°

65°
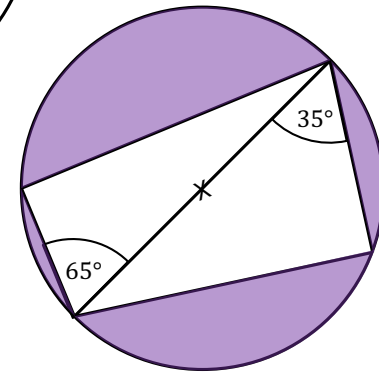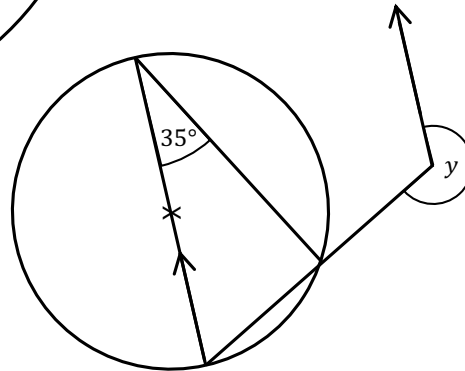
The area of the circle is 49π cm^2

Find the shaded area to 3sf

Fluency Practice
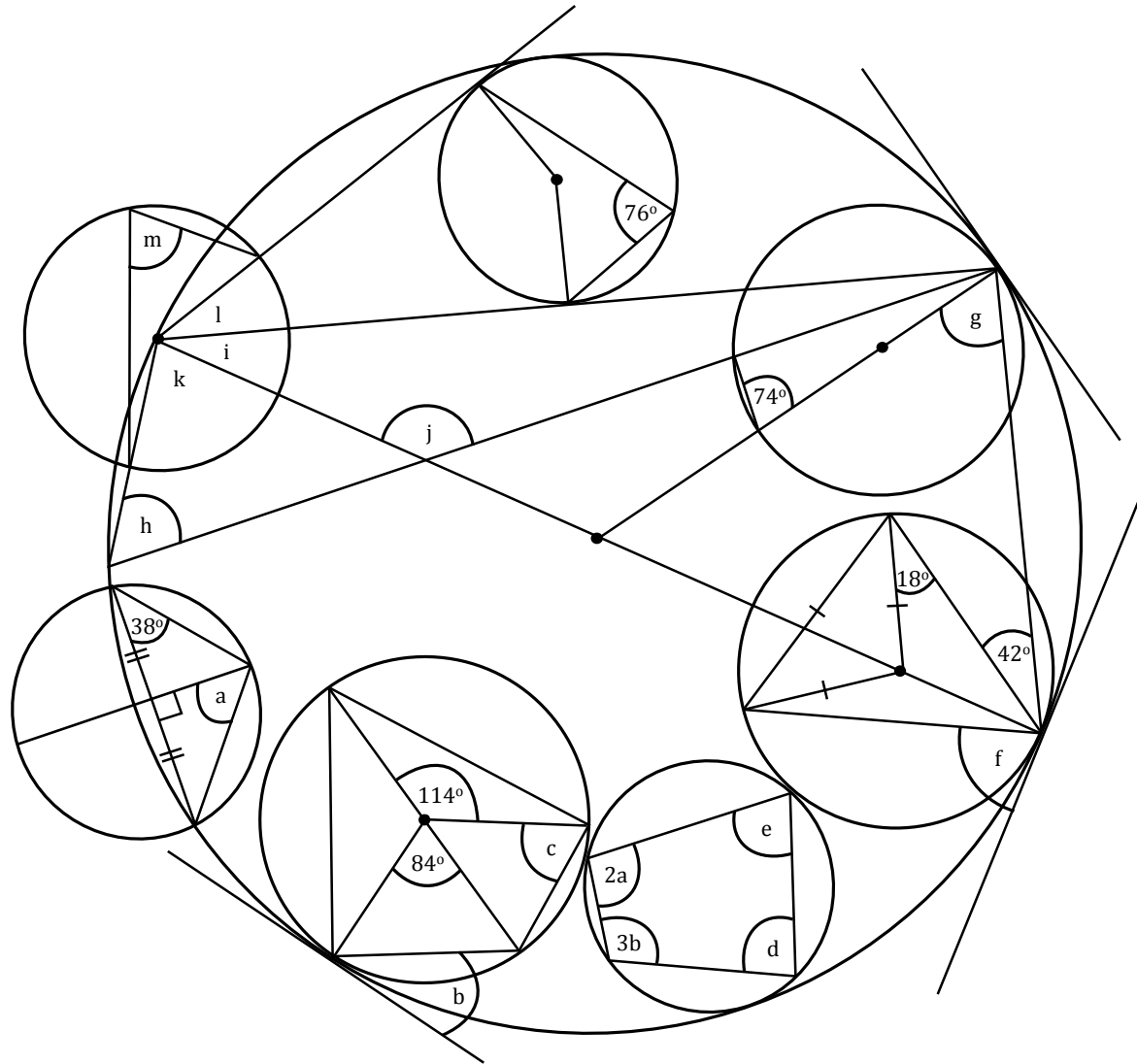
Fluency Practice

1. Work out value of x in each diagram:

a)

b)

c)

d)

e)

f)
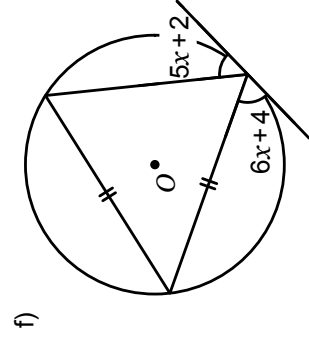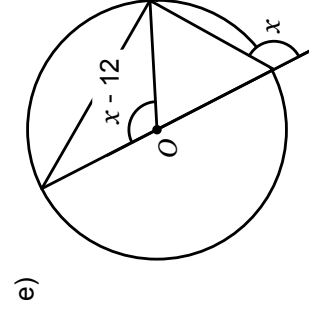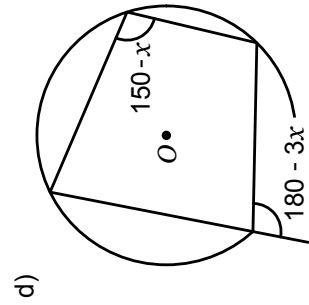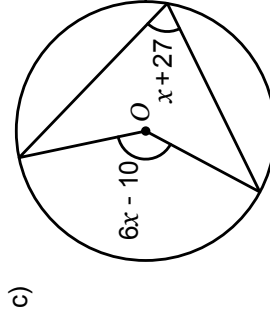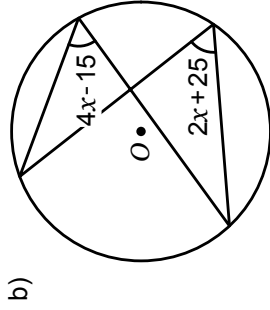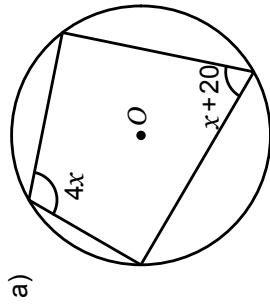
2. Work out values of x and y in each diagram:

a)

b)
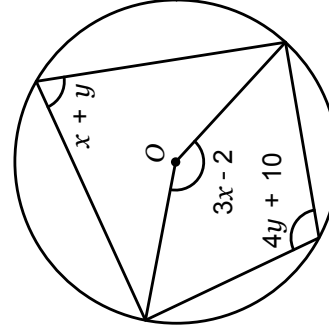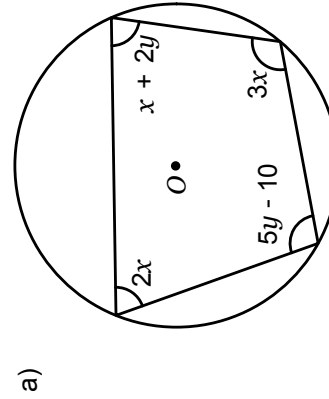
3. BC is a tangent to the circle.
   Work out the value of x.

Fluency Practice
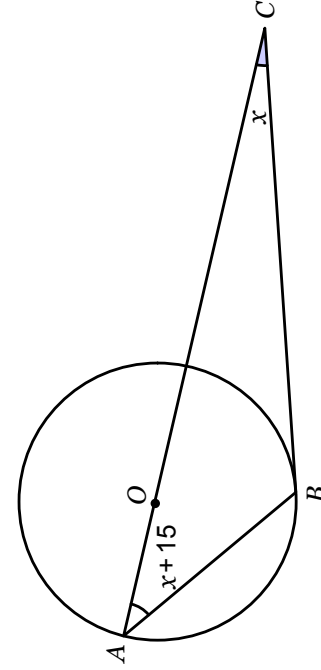
Solving Linear Equations from...

Find the value of each letter.

All angles are given in degrees.

$5a - 25$

$8a - 34$

$10 - 2b$

$45 + 3b$

$3c + 5$

c

$3d + 13$

$8d + 13$

$3e + 10$

$5e + 48$

$\dfrac{f}{2} + 36$

$\dfrac{3f}{2} + 88$

$7g + 85$

$g + 15$

$30h + 24$

$3h + 6$

$4i - 36$

$7i + 5$

$3 + 2i$

$4j - 8$

$3j$

Fluency Practice
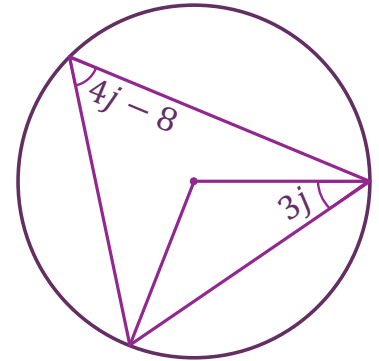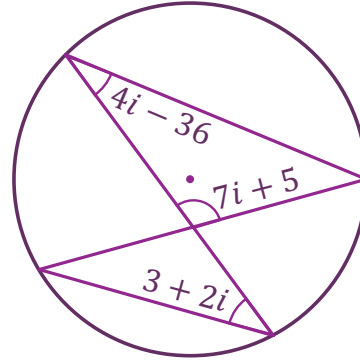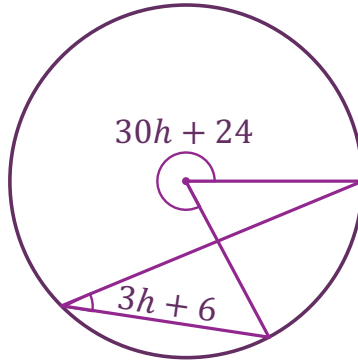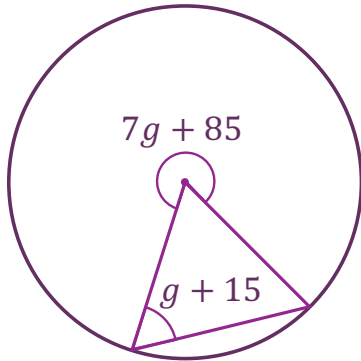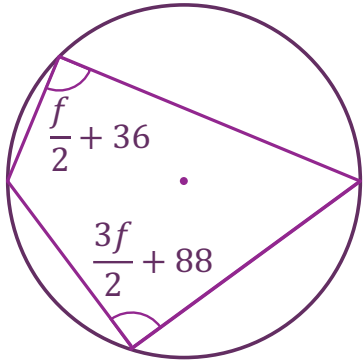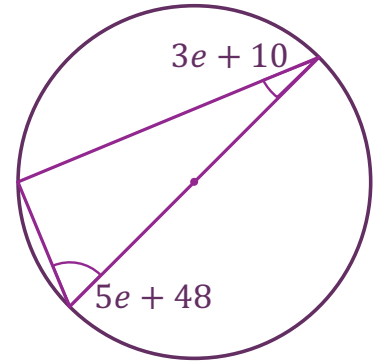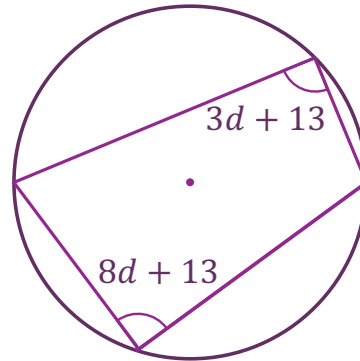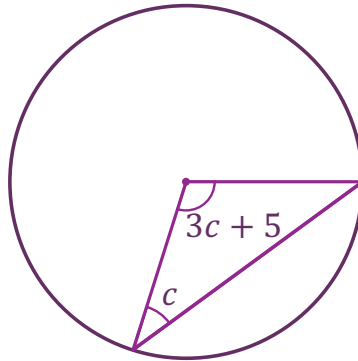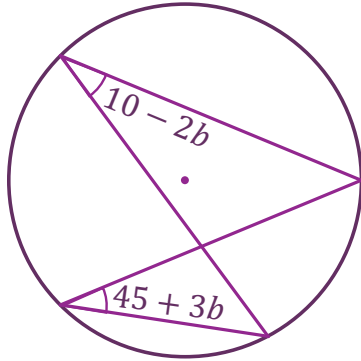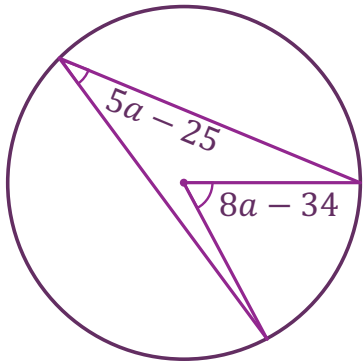
Solving further equations with… # Circle Theorems

a) Find x and y

$3y$

$2y$

$3x$

$2x + 10°$

b) $x + y = 150°$
Find x and y

x

y

c) $x : y = 2 : 7$
Find x and y

y

x

d) Find x

$40°$ x

e) Find x

$3x$

$40°$

$2x$

f) Find x

5

x

12

g) Find x and y

$2x$

$y + 20°$

$4x + y$

h) Find h

10 cm

h

$70°$
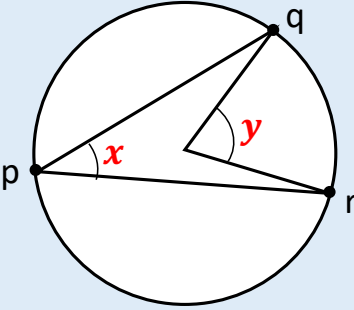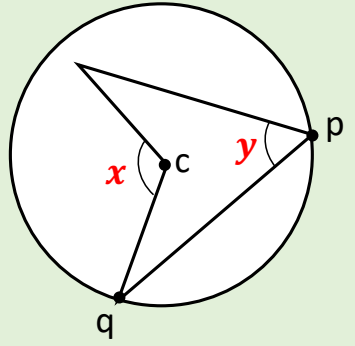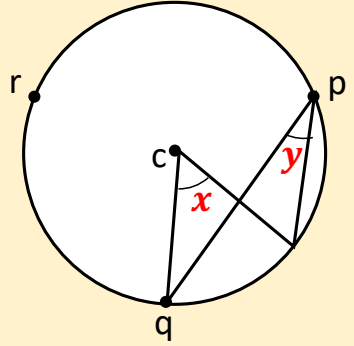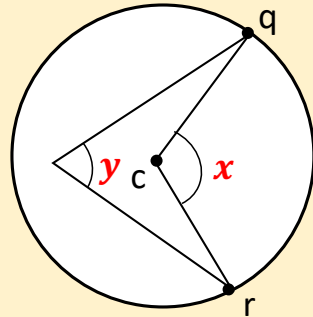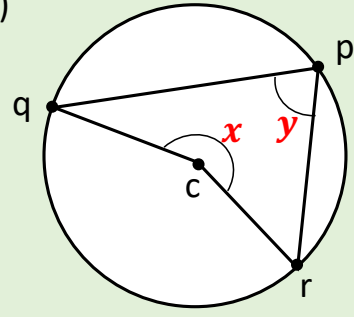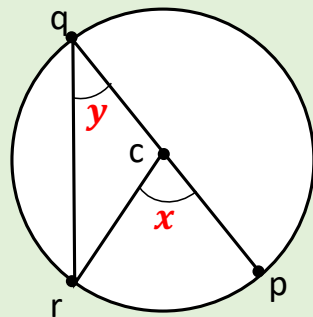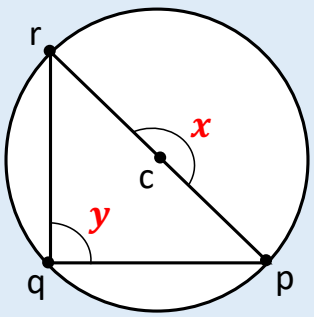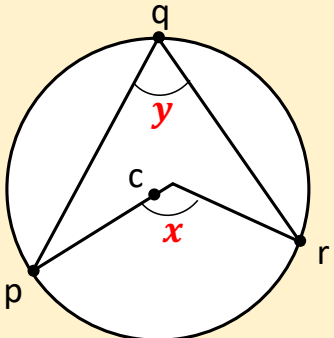
Problem Solving

(1)

(2)

(3)

(4)

c is the centre of the circle
p, q and r are all on the
circumference of the circle

Determine in which of the
diagrams; x IS ALWAYS
TWICE y

(5)

(6)

(7)

(8)

$x°$

Create a Circle Theorem Question

Draw an question that needs knowledge of [...] to solve.

angles within semicircles

radii & tangents

angles at the centre & at the circumference

tangents & alternate segments

circle theorems & vertically opposite angles

circle theorems & interior angles of a polygon

circle theorems & Pythagoras' theorem

angles subtended by the same arc

cyclic quadrilaterals

tangents to a circle & external points

radii & chords

circle theorems & isosceles triangles

2 circle theorems

circle theorems & parallel lines

Diagrams don't need to be drawn accurately!

$x°$

3 Direct and Inverse Proportion

Workout 1

Question 1: A is directly proportional to B.
When A = 12, B = 3
(a) Find a formula for A in terms of B.
(b) Find the value of A when B = 5
(c) Find the value of B when A = 36

Question 2: C is directly proportional to D.
When C = 125, D = 5
(a) Find an equation for C in terms of D.
(b) Find the value of C when D = 10
(c) Find the value of D when C = 75

Question 3: E is directly proportional to F.
When E = 2, F = 8
(a) Find an equation for E in terms of F.
(b) Find the value of E when F = 30
(c) Find the value of F when E = 100

Question 4: y is directly proportional to x.
When x = 400, y = 10
(a) Find a formula for y in terms of x.
(b) Calculate the value of y when x = 450
(c) Find the value of x when y = 200

Question 5: y is directly proportional to x.
Complete the table.

x	4	9	12
y			72

Question 6: y is directly proportional to x
Complete the table.

x	2.5	8	
y	4		50

Question 7: The cost, C pounds, of hiring a car is directly proportional to the number of days, d, it is hired.
When d = 5, C = 180

(a) Find the value of C when d = 3
(b) Find the value of d when C = 252

Question 8: In a spring, the tension (T newtons) is directly proportional to the extension of the spring (y cm).
When the tension is 180 newtons, the extension is 4cm.

(a) Find a formula for T in terms of y.
(b) Work out the tension when the extension is 3cm
(c) Work out the extension, when the tension is 585 newtons.

Workout 2

x	16	8	
y		10	20

Page 163

Fluency Practice

Question	General Equation	Find k	New Equation	Find Value using Equation
A is directly proportional to B, when $A = 10, B = 2$. Find A when $B = 12$.	$A = kB$	$10 = k \times 2$ so $k = 5$	$A = 5B$	$A = 5 \times 12 = 60$
(a) y is directly proportional to x, when $y = 55, x = 5$. Find y when $x = 9$				
(b) N is directly proportional to L, when $N = 1.8, L = 0.6$. Find N when $L = 2.5$				
(c) y is directly proportional to x. If $y = 5$ when $x = 10$, find y when $x = 60$				

(d) A is directly proportional to B and when $A = 12, B = 3$. Find A when $B = 20$	(e) h is directly proportional to V and $h = 36$ when $V = 8$. Find h when $V = 44$	(f) y is directly proportional to the x, and $y = 250$ when $x = 5$. Find x when $y = 7.5$	(g) y is directly proportional to x. When $x = 2, y = 64$. Find x when $y = 80$

Fluency Practice

1. For each table of values, state whether or not $y \propto x$.

(a)

x	0	1	2	3
y	0	4	8	12

(b)

x	2	4	6	10
y	10	20	36	50

(c)

x	0	1	2	3
y	2	5	8	11

(d)

x	2	4	8	16
y	16	8	4	2

(e)

x	1	2	3	4
y	2.5	5	7.5	10

(f)

x	0	2	4	10
y	0	$\frac{n}{2}$	n	$\frac{5n}{2}$

2. x and y are variables such that $x \propto y$ and $y = 5$ when $x = 2$.
Which is the correct graph for these variables?

A   B   C   D
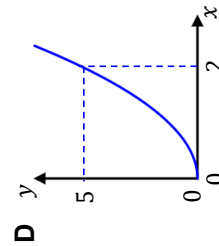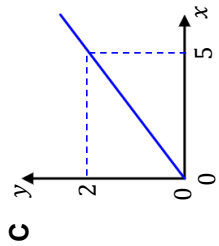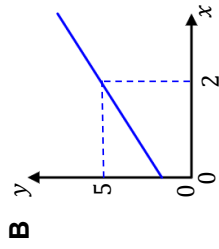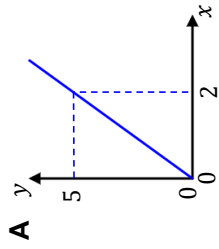
3. x and y are variables such that $x \propto y$ and $y = 6$ when $x = 5$.
Which is the correct graph for these variables?

A   B   C   D
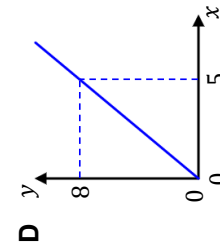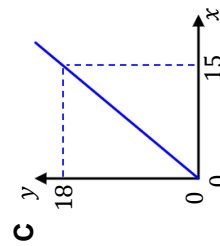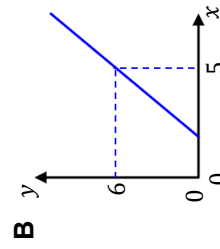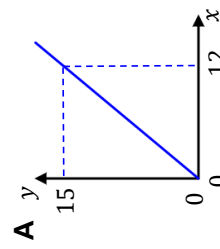
4. In each table, $x \propto y$. Work out the missing values.

(a)

x	0	1	2.5
y	0	6	30

(b)

x	2	4	6	8
y				

(c)

x	0	4	5
y	0	0.6	1.5

(d)

x	4	16	32
y	$\frac{1}{6}$	$\frac{1}{3}$	

Fluency Practice

5. John's weekly wage (£w) is directly proportional to the number of hours he has worked in that week (h). One week, John works for 9 hours and earns £58.50.

(a) Work out a formula for w in terms of h.
(b) How much would John earn if he worked for 15 hours?
(c) How many hours would John need to work to earn £130 ?

6. Sophie is filling a container from a tap. Water is pouring from a tap at a constant rate. The amount of water poured (V litres) is directly proportional to the time (t s) that the tap has been on. After 20 seconds, 1.6 litres of water have been poured.

(a) Work out a formula for V in terms of t.
(b) How many litres of water will have been poured after 3 minutes?
(c) How long does the tap need to be on to fill a 50 litre container? Give your answer in minutes and seconds.

7. Fabric is sold from a roll. Customers can specify the length of fabric they would like to have cut from the roll. The cost of a patterned fabric (£C) is directly proportional to the length of the fabric (x m). Rachel buys 1.5m of the fabric for £2.70.

(a) Work out a formula for C in terms of x.
(b) How much would it cost to buy 2.5m of the fabric?
(c) What length of fabric could be bought for £6 ?

8. The distance travelled on a bicycle is directly proportional to the number of turns of its rear wheel. With 5 turns of the wheel, the bicycle travels 9.75m.

(a) Work out a formula for D, the distance travelled by the bicycle in m, in terms of t, the number of turns of the wheel.
(b) How far will the bicycle travel with 12.5 turns of the wheel?
(c) How many times must the wheel turn to travel a distance of 100m?

9. Q is directly proportional to P. $Q = 40$ when $P = 15$.

(a) Work out a formula for Q in terms of P.
(b) Given $P = 6$, work out the value of Q.
(c) Given $Q = 84$, work out the value of P.

10. y is directly proportional to x. $y = 54$ when $x = 36$.

(a) Work out a formula for y in terms of x.
(b) Given $x = 30$, work out the value of y.
(c) Given $y = 27$, work out the value of x.

11. In the table, $x \propto y$.
Decide whether each statement is true or false.

x	2	3	q
y	5	p	r

(a) $2y = 5x$
(b) $\frac{q}{3} = \frac{r}{p}$
(c) $\frac{q}{r} = 2.5$
(d) The ratio $3 : p$ is equivalent to $2 : 5$

Corbett
mαths

Direct/Inverse Proportion
Videos 254 & 255 on Corbettmaths

F Corbett cy Pr
maths

Workout 3

Question 1: A is directly proportional to B^2
When A = 50, B = 5
(a) Find a formula for A in terms of B.
(b) Find the value of A when B = 3
(c) Find the value of B when A = 200

Question 2: y is directly proportional to the square of x
When y = 6.4, x = 4
(a) Find a formula for y in terms of x
(b) Find the value of y when x = 8
(c) Find the value of x when y = 78.4

Question 3: W is directly proportional to P^3.
When P = 2, W = 32
(a) Express W in terms of P
(b) What is the value of W when P = 4?
(c) What is the value of P when W = 4000?

Question 4: Z is directly proportion to $\sqrt{x}$
When Z = 12, x = 36
(a) Express Z in terms of x
(b) Work out the value of Z when x = 121
(c) Work out the value of x when Z = 18

Question 5: C is directly proportional to the cube of D
When D = 5, C = 175
(a) Work out the value of C when D = 6
(b) Work out the value of D when C = 4725

Question 6: y is directly proportional to the cube root of x
When y = 7600, x = 4096
(a) Find an equation connecting y and x.
(b) Calculate the value of y when x = 125
(c) Calculate the value of x when y = 9975

Question 7: The table shows a set of values for x and y.
y is directly proportional the the square root of x.

x	25	
y	9	36

Question 8: An object when dropped, falls d metres in t seconds.
d is directly proportional to the square of t.
The object falls 80 metres in 4 seconds.
Work out how far the object falls in 9 seconds.

Question 9: The cost of a circular table is directly proportional to the square of its radius.
A table with a radius of 40cm cost £90.

What is the cost of a table with a radius of 60cm?

Question 10: The mass of a paperweight is m grams.
The length of the paperweight is L centimetres.
m is directly proportional to the cube of L.
m = 4968 when L = 12

(a) Work out an equation connecting m and L
(b) Work out the mass of a paperweight with a length of 4 centimetres

Workout 4

Fluency Practice

Question	General Equation	Find k	New Equation	Find Value using Equation
A is directly proportional to B^2, when $A = 45$, $B = 3$. Find A when $B = 7$	$A = k \times B^2$	$45 = k \times 3^2$ so $k = 5$	$A = 5B^2$	$A = 5 \times 7^2 = 245$
(a) y is directly proportional to x^2, and $y = 270$ when $x = 3$. Find y when $x = 5$				
(b) N is directly proportional to L^3, when $N = 1280$, $L = 4$. Find N when $L = 3$				
(c) A is directly proportional to $\sqrt{B}$ and when $A = 90$, $B = 9$. Find A when $B = 25$				

(d) A is directly proportional to B^2 and when $A = 8$, $B = 4$. Find A when $B = 0.5$

(e) h is directly proportional to $\sqrt{w}$ and $h = 15$ when $w = 4$. Find h when $w = 64$

(f) A is directly proportional to V^3 and when $A = 400$, $V = 2$. Find V when $A = 6250$

(g) y is directly proportional to $\sqrt[3]{x}$. When $x = 8$, $y = 64$. Find x when $y = 128$

Intelligent Practice

1) y is directly proportional to x.
When y = 80, x = 2.
Find y when x = 4

2) y is directly proportional to 2x.
When y = 80, x = 2.
Find y when x = 4

3) y is directly proportional to 8x.
When y = 80, x = 2.
Find y when x = 4

4) y is directly proportional to x^2.
When y = 80, x = 2.
Find y when x = 4

5) y is directly proportional to x + 2.
When y = 80, x = 2.
Find y when x = 4

6) y is directly proportional to $(x + 2)^2$
When y = 80, x = 2.
Find y when x = 4

7) x is directly proportional to y
When y = 80, x = 2.
Find y when x = 4

8) x is directly proportional to 2y
When y = 80, x = 2.
Find y when x = 4

9) y is directly proportional to $\sqrt{x + 2}$
When y = 80, x = 2.
Find y when x = 4

1. For each table of values, state whether $y \propto x$, $y \propto x^2$, $y \propto x^3$, or $y \propto \sqrt{x}$ and work out an equation for y in terms of x.

(a)

x	1	2	3	4
y	3	12	27	48

(b)

x	1	2	3	4
y	2	16	54	128

(c)

x	2	3	4	10
y	8	12	16	40

(d)

x	1	4	9	16
y	2	4	6	8

(e)

x	1	2	3	4
y	0.5	4	13.5	32

(f)

x	1	3	6	12
y	$\frac{1}{3}$	3	12	48

2. In each table, use the given proportional relationship to work out the missing values.

(a) $y \propto x^3$

x	0	1	2	
y			24	648

(b) $y \propto \sqrt{x}$

x	0	1	4	9
y				15

(c) $y \propto x^2$

x	1	2	6
y		10	13.5

(d) $y \propto x^3$

x	2	3		6
y		18	$\frac{250}{3}$	

3. The mass of dough required to make a pizza is directly proportional to the square of the diameter of the pizza. A pizza with a diameter of 10 inches requires 500g of dough.

(a) Work out a formula for m, the mass of dough required in grams, in terms of d, the diameter of the pizza in inches.

(b) How much dough is required for a 12 inch pizza?

(c) Amy has 320g of dough to make a pizza with. What diameter should her pizza have?

4. A stone is dropped from a hot air balloon. The distance travelled by the stone is proportional to the square of the time is has been falling. After 2 seconds, the stone has fallen a distance of 19.6m.

(a) Work out a formula for the distance travelled by the stone (d m), in terms of time (t s).

(b) How far will the stone have fallen after 4 seconds?

(c) How long will it take for the stone to fall a total of 100m? Give your answer to the nearest tenth of a second.

5. David makes a range of similar pots, of varying sizes. The capacity of each pot is directly proportional to the cube of its height. One of the pots has a height of 20cm and a capacity of 400ml.

(a) Work out a formula for the capacity of a pot (c ml), in terms of its height (h cm).

(b) What is the capacity, in litres, of a pot with a height of 30cm?

(c) What is the height of a pot with a capacity of 878.8ml?

Fluency Practice

6. The distance visible from a point to the horizon is directly proportional to the square root of the height above sea level. From a point on a lighthouse 9m above sea level, it is possible to see 10.8km to the horizon.

(a) Work out a formula for d, the distance to the horizon in km, in terms of h, the height above sea level in m.

(b) How far is it possible to see to the horizon at a height of 16m above sea level?

(c) At what height above sea level is the distance to the horizon 18km?

7. A pendulum's period is the time taken for it to complete one full swing. The period is directly proportional to the square root of the pendulum's length. A pendulum with a length of 36cm has a period of 1.2s

(a) Work out a formula for T, the period of a pendulum in s, in terms of l, the length of the pendulum in cm.

(b) Work out the period of a pendulum with a length of 42cm, correct to the nearest tenth of a second.

(c) What length of pendulum has a period of 1 second?

8. B is directly proportional to the square of A. $B = 128$ when $A = 4$.

(a) Work out a formula for A in terms of B.

(b) Given $A = 5$, work out the value of B.

(c) Given $B = 32$, work out the value of A.

9. q is directly proportional to the cube of p. $q = 6$ when $p = 2$.

(a) Work out a formula for q in terms of p.

(b) Given $p = 6$, work out the value of q.

(c) Given $q = 750$, work out the value of p.

10. y is directly proportional to the square root of x. $y = 12.5$ when $x = 25$.

(a) Work out a formula for y in terms of x.

(b) Given $x = 80$, work out the value of y as a surd in its simplest form.

(c) Given $y = 5\sqrt{3}$, work out the value of x.

11. P is directly proportional to the cube of t. $P = 1$ when $t = \frac{2}{3}$.

(a) Work out a formula for P in terms of t.

(b) Given $t = 2$, work out the value of P.

(c) Given $P = \frac{8}{3}$, work out the value of t.

12. A cuboid has dimensions x cm, x cm and $2x$ cm, where x is a variable.

(a) Find a formula in terms of x for V, the volume of the cuboid.

(b) Complete the statement: $V \propto$ …...

(c) Find a formula in terms of x for A, the surface area of the cuboid.

(d) Complete the statement: $A \propto$ …...

(e) (Challenge) Find the proportional relationship between V and A.

Question 3:   W is directly proportional to the cube of y.
Harry says that when y is halved, the value of W is divided by 10.

Explain why Harry is wrong.

Question 7:   A is directly proportional to the cube root of B.
B is increased by 60%.
Work out the percentage increase in A.

Fluency Practice

The weight of a piece of wire is directly proportional to its length. A piece of wire is 25 cm long and has a weight of 6 grams. Another piece of the same wire is 30 cm long. Calculate the weight of the 30 cm piece of wire.

In a spring, the tension, T Newtons, is directly proportional to its extension, x cm. When the tension is 300 Newtons, the extension is 12 cm.
(a) Find a formula for T in terms of x.
(b) Calculate the tension, in Newtons, when the extension is 15 cm.

The time, T seconds, it takes a kettle to boil some water is directly proportional to the mass of water, m kg, in the kettle. When $m = 250$, $T = 300$. Find T when $m = 400$.

In a factory, chemical reactions are carried out in cylindrical containers. The time, T minutes, the chemical reaction takes is directly proportional to the square of the radius, R cm, of the cylindrical container. When $R = 12$, $T = 72$. Find the value of T when $R = 15$.

The amount of clay used to make a statue is directly proportional to the cube of the height of the statue. A statue which is 10 cm tall requires 500 cm^3 of clay. How much clay is required for a similar statue which is twice as tall?

Extension

direct proportion

(1) the distance travelled by a Citroen 2CV6 in km (d, travelling at a steady speed)
is directly proportional to the time taken (h hrs)
it travels 75 km in 2½ hours
how far will it travel in 4 hours at the same steady speed?

(2) the amount of water a pig uses when taking a shower (w litres, at a steady flow) is
directly proportional to the time it takes to shower (t minutes)
a pig uses 25.2 litres in 9 minutes
how much water would it use in half an hour?

(3) the distance a cow falls (f metres) is directly proportional to the square of the
time taken (t secs)
it falls 432m in 6 secs
how far will it fall in 10 secs?

(4) the pressure (p g/cm^2) required to blow up a balloon at any time is directly proportional
to the cube of its radius (r cm)
the pressure is 80 g/cm^2 when the radius is 5cm
what pressure is required at a time when this radius has doubled?

(5) the slowest speed a bird can fly (f mph) is directly proportional to the square
root of its length (L cm).
the slowest speed of a particular bird, 23cm long, is 24.5 mph
what are the slowest speeds of:
(i) a chaffinch, 15 cm long and (ii) a curlew, 56cm long?

Fluency Practice

Question 1:   T is inversely proportional to N.
When T = 30, N = 5.
(a) Find an equation connecting T and N.
(b) Work out the value of T when N = 10
(c)   Work out the value of N when T = 25

Question 2: w is inversely proportional to f
When f = 12, w = 40
(a) Find a formula connecting w and f
(b) Find the value of w when f = 60

Question 3:   B is inversely proportional to y
When B = 0.8, y = 13
(a) Find an equation for B in terms of y.
(b) Work out the value of B when y = 5

Question 4:   y is inversely proportional to x
Complete the table.

x	16	8	
y		10	20

Question 5:   The number of days, D, to complete research is inversely proportional to the
number of researchers, R, who are working.
The research takes 125 days to complete when 24 people work on it.
Find out how many people are needed to complete the research in 60 days.

Question 6:   The volume, V litres, which a fixed mass of gas occupies is inversely proportional
to its pressure, P pascals.

When the pressure is 200000 pascals, its volume is 6 litres.

(a) Find an equation connecting V and P.
(b) Find the volume of gas when the pressure is 150000
(c)   Find the pressure when the volume of gas is 20 litres.

Fluency Practice

Question	Equation	Find k	New Equation	Find Value using Equation
A is inversely proportional to B, when $A = 5, B = 6$. Find A when $B = 10$.	$A = \dfrac{k}{B}$	$5 = \dfrac{k}{6}$ so $k = 30$	$A = \dfrac{30}{B}$	$A = \dfrac{30}{10} = 3$
(a) y is inversely proportional to x, when $y = 12, x = 5$. Find y when $x = 4$.				
(b) N is inversely proportional to L, when $N = 2.5$, $L = 8$. Find N when $L = 4$.				
(c) y is inversely proportional to x. If $y = 5$ when $x = 8$, find y when $x = 20$				

(d) A is inversely proportional to B and when $A = 12, B = 3$. Find A when $B = 10$	(e) h is inversely proportional to V and $h = 36$ when $V = 8$. Find h when $V = 20$	(f) y is inversely proportional to x, and $y = 0.2$ when $x = 5$. Find x when $y = 25$	(g) y is inversely proportional to x. When $x = 2, y = 64$. Find x when $y = 80$.

Question 1: y is inversely proportional to the square of x.
When y = 200, x = 2
(a) Find an equation connecting y and x.
(b) Work out the value of y when x = 5
(c) Work out the value of x when y = 50

Question 2: A is inversely proportional to the square of B.
When A = 2, B = 3
Find the value of A when B = 2

Question 3: W is inversely proportional to √B
When B = 9, W = 9
(a) Express W in terms of B
(b) Work out the value of W when B = 4
(c) Work out the value of B when W = 1

Question 4: T is inversely proportional to the cube of L
When L = 0.2, T = 5
(a) Write a formula connecting T and L.
(b) Work out the value of T when L = 0.5
(c) Work out the value of L when T = 2

Question 5: q is inversely proportional to the square of t.
When q = 7.5, t = 1.6
(a) Calculate the value of q when t = 8
(b) Calculate the value of t when q = 1.875

Question 6: y is inversely proportional to the cube root of x
When y = 2500, x = 64
(a) Find the value of y when x = 8
(b) Find the value of x when y = 2000

Question 7: The table shows a set of values for x and y.
y is inversely proportional the the square of x.

x	1	2	3
y	7	$1\frac{3}{4}$	

(a) Find the equation for y in terms of x
(b) Find the missing value

Question 8: The force, *F* newtons, exerted by a magnet on a metal object is inversely proportional to the square of the distance *d* cm

When the *d* = 2cm, *F* = 60N

(a) Express *F* in terms of *d*
(b) Find the force when the distance between the magnet and the metal object is 10cm
(c) Find the distance between the magnet and the metal object when the force is 15N

Question 9: The time taken, *t*, for passengers to be checked-in for a flight is inversely proportional to the square of the number of staff, *s*, working

It takes 30 minutes for passengers to be checked-in when 5 staff are working.

(a) Find an equation connecting *t* and *s*.
(b) How long would it take to check-in the passengers with 3 staff working?

Apply

Fluency Practice

Question	Equation	Find k	New Equation	Find Value using Equation
A is inversely proportional to B^2, and when $A = 6, B = 5$. Find A when $B = 2$	$A = \dfrac{k}{B^2}$	$6 = \dfrac{k}{5^2}$ so $k = 150$	$A = \dfrac{150}{B^2}$	$A = \dfrac{150}{2^2} = 37.5$
(a) y is inversely proportional to x^2 and when $y = 10, x = 2$. Find y when $x = 5$				
(b) y is inversely proportional to x^3, and $y = 5$ when $x = 3$. Find y when $x = 10$				
(c) A is inversely proportional to $\sqrt{B}$ and when $A = 90, B = 9$. Find A when $B = 25$				

(d) h is inversely proportional to V^2 and $h = 3$ when $V = 8$. Find h when $V = 4$	(e) B is inversely proportional to $\sqrt{C}$, and when $B = 18, C = 16$. Find B when $C = 0.36$	(f) y is inversely proportional to x^3, and $y = 20$ when $x = 6$. Find x when $y = 67.5$	(g) y is inversely proportional to $\sqrt[3]{x}$. When $x = 8, y = 4$, find x when $y = 0.8$

Intelligent Practice

'y is inversely proportional to x.' unless otherwise stated.

A

1) When y = 5, x = 12
Find y when x = 3

2) When y = 12, x = 5
Find y when x = 3

3) When y = 5, x = 12
Find y when x = 6

4) When y = 10, x = 12
Find y when x = 3

5) When y = 10, x = 24
Find y when x = 3

6) When y = 10, x = 24
Find y when x = 1.5

7) When y = 10, x = 24
Find y when x = 30

8) When y = 5, x = 12
Find x when y = 15

9) When y = 5, x = 12
Find x when y = 30

10) When y = 12, x = 5
Find y when x = 120

B

1) $$y \propto \frac{k}{x}$$

If y = 6 when x = 2, find the value of y when x = 4

2) When y = 6, x = 12
Find y when x = 4

3) $$y \propto \frac{k}{x}$$

If y = 6 when x = 2, find the value of y when x = 1

4) $$y \propto \frac{k}{x}$$

If y = 6 when x = 2, find the value of x when y = 2

5) When y = 12, x = 12
Find y when x = 12

6) $$y \propto \frac{k}{x}$$

If y = 12 when x = 4, find the value of y when x = 2

C

1) $$y \propto \frac{k}{x^2}$$

If y = 15 when x = 2, find the value of y when x = 4

2) $$y \propto \frac{k}{x^2}$$

If y = 15 when x = 4, find the value of y when x = 8

3) When y = 10, x = 1.5
Find y when x = 3

4) $$y \propto \frac{k}{x}$$

If y = 6 when x = 2, find the value of x when y = 7

5) $$y \propto \frac{k}{x^3}$$

If y = 15 when x = 2, find the value of y when x = 4

6) When y = 1, x = 1
Find y when x = 576

1) y varies inversely with x
When y = 5, x = 6.
Find y when x = 3.

2) y varies inversely with $2x$
When y = 5, x = 6.
Find y when x = 3.

3) y varies inversely with x^2
When y = 5, x = 6.
Find y when x = 3.

4) y varies inversely with x^2
When y = 5, x = 6.
Find y when x = 1.

5) y varies inversely with $x + 3$
When y = 5, x = 6.
Find y when x = 3.

6) y varies inversely with $x + 3$
When y = 5, x = 6.
Find y when x = 1.

7) y varies inversely with $(x + 3)^2$
When y = 5, x = 6.
Find y when x = 3.

8) y varies inversely with $(x + 3)^2$
When y = 5, x = 3.
Find y when x = 0.

9) y varies inversely with $(2x)^2$
When y = 5, x = 6.
Find y when x = 3.

Fluency Practice

1. For each table of values, $y \propto \frac{1}{x}$. Work out the missing values.

(a)

x	2	3	6	8
y	24		8	

(b)

x	2	4	5	10
y		15		

(c)

x	0.5	1	1.5	2
y		12		

(d)

x		4	5	6
y	5		3	2.5

2. For each table of values, $y \propto \frac{1}{x}$. Work out an equation for y in terms of x.

(a)

x	2	4	5	8
y	12	6	4.8	3

(b)

x	1	2	3	4
y		1	$\frac{2}{3}$	$\frac{1}{2}$

(c)

x	$\frac{1}{2}$	2	4	$\frac{1}{10}$
y	$\frac{2}{5}$			$\frac{1}{25}$

(d)

x	$\frac{1}{2}$	$\frac{3}{2}$	3	12
y	$\frac{3}{2}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{16}$

3. Which is the correct graph for variables that are inversely proportional?

A   B   C   D
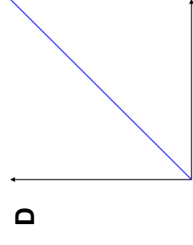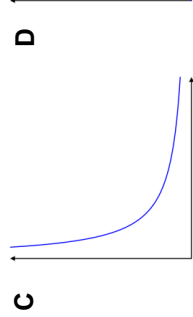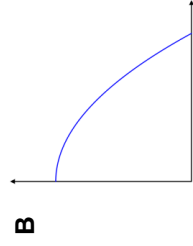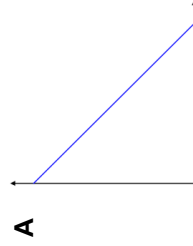
4. Tim has a wheelbarrow full of sand. He can fill a 12 litre bucket with the sand 15 times.

(a) How many times could he fill a bucket with a capacity of 18 litres, with the sand from the wheelbarrow?

(b) Work out a formula for the number of times a bucket can be filled n, in terms of the capacity of the bucket c (litres).

(c) Tim has a large bucket which he is able to fill $7\frac{1}{2}$ times with the sand. Work out the capacity of this bucket.

5. The cost of renting a house is to be shared equally amongst its occupants. If there are 4 people living at the house, each of them will pay £360 per month.

(a) Work out a formula for the amount each person will pay per month $£C$, in terms of the number of people living in the house, n.

(b) If there were 5 people living in the house, how much would each person have to pay per month?

(c) If the amount each person paid for rent was £240, how many people would be living in the house?

6. P and Q are variables that are inversely proportional. $P = 3$ when $Q = \frac{1}{9}$.

(a) Work out a formula for Q in terms of P.

(b) Work out the value of Q when $P = 5$.

(c) Work out the value of Q when $P = \frac{1}{2}$.

(d) Work out the value of P when $Q = \frac{1}{21}$.

7. X and Y are variables that are inversely proportional. $Y = 6$ when $X = \frac{1}{8}$.

(a) Work out a formula for Y in terms of X.

(b) Work out the value of Y when $X = 3$.

(c) Work out the value of X when $Y = \frac{9}{10}$.

8. a is inversely proportional to b. Shade the correct statement(s):

If b is doubled, a is halved.	If 4 is added to a, 4 is subtracted from b.
If a is increased by 25%, b is decreased by 25%.	If a is increased by 25%, b is decreased by 20%.
a is directly proportional to the reciprocal of b.	The graph for a and b is a parabolic curve.

inverse proportion: $y \propto \dfrac{1}{x^2}$, $y \propto \dfrac{1}{x^3}$, $y \propto \dfrac{1}{\sqrt{x}}$

9. The force between two magnets is inversely proportional to the square of the distance between them. When the magnets are 0.5m apart, the force between them is 400 newtons.

(a) Work out a formula for the force between the magnets, F newtons, in terms of the distance between them, d metres.

(b) Work out F when $d = 2$.

(c) Work out d when $F = 2500$.

10. B is inversely proportional to the cube of A. $B = 4$ when $A = 3$.

(a) Work out a formula for B in terms of A.

(b) Work out B when $A = 2$.

(c) Work out A when $B = 5$.

11. y is inversely proportional to the square root of x. $y = 3$ when $x = 36$.

(a) Work out a formula for y in terms of x.

(b) Work out y when $x = 4$.

(c) Work out x when $y = 2$.

12. q is inversely proportional to the square of p. Shade the correct statement(s):

If p is doubled, q is halved.	If p is doubled, q is divided by 4.
The constant of proportionality is equal to pq^2.	The constant of proportionality is equal to $(pq)^2$.

Apply

Exten
Corbett
maths

Direct/Inverse Proportion
Videos 254 & 255 on Corbettmaths

Question 1: C a
C is
Wh
Find

Question 2: The force, F newtons, exerted by a magnet on a metal object is inversely
proportional to the square of the distance d cm
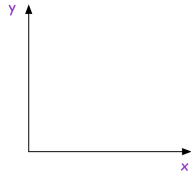When d = 4cm, F = 300N.
Explain what happens to F, when d is halved.

Question 3: W is directly proportional to the cube of y.
Harry says that when y is halved, the value of W is divided by 10.

Explain why Harry is wrong.

Question 4: y is inversely proportional to x.
y and x are positive quantities.
When y = 200, x = 64

(a) Express y in terms of x
(b) Sketch a graph of the relationship between y and x

(c) Calculate the value of x, when y is twice as big as x.

Question 6: The time taken, t, for the waiters in a restaurant to serve food to the guest at a
wedding is inversely proportional to the square of the number of waiters, w,
working

It takes 90 minutes for the food to be served when 6 waiters are working.

What is the minimum number of waiters needed so that the time taken to serve
the food is under 30 minutes?

Question 9: x is directly proportional to w^2
When w = 4, x = 48

y is inversely proportional to x^3
When x = 2, y = 14

Find a formula for y in terms of w.
Give your answer in its simplest form.

Answers

Click here

Scan here

Graph 1   Graph 2   Graph 3   Graph 4

Answers

© CORBETTMATHS 2

Click here

Scan here

e 202

Graph   Type of

Fluency Practice

The shutter speed, S, of a camera varies inversely as the size of the aperture setting, f. When $f = 8$, $S = 125$.

(a) Find a formula for S in terms of f.

(b) Hence, or otherwise, calculate the value of S when $f = 4$.

The pressure of water from a hose is inversely proportional to the hose radius. For a hose of radius 2 cm, the water pressure is 40 Pa. What hose radius do you need for a pressure of 50 Pa?

The amount of diesel a van uses is inversely proportional to the number of miles it travels. When a van travels 320 miles, it uses 36 litres of diesel. How much diesel will it need to travel 200 miles?

In a science experiment, p is found to be inversely proportional to t. When $p = 42.8$, $t = 0.8$. Find t when $p = 23.6$. Give your answer to 2 decimal places.

The light intensity I on a surface is inversely proportional to the square of the distance x from the light source. When the surface is 6 cm from the light source, the intensity is 2400.
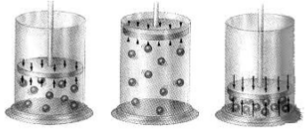
(a) Find the light intensity when the surface is 15 cm from the light source.

(b) If the light intensity is 600, how far is the surface from the light source?
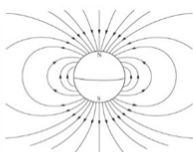
Extension

inverse proportion

(1) the amount of time (t, hours) it takes to travel to work is inversely
proportional to the average speed (v, mph)
it takes 30 minutes in a car at an average speed of 27.4 mph
how long would it take on a bike at an average speed of 9.13 mph?

(2) at a constant temperature the volume of a gas (V) varies inversely as the
pressure (P)
the pressure is 80 gm/cm^2 for a volume of 45 cc
what will the volume of gas be for a pressure of 25 gm/cm^2?
what will the pressure be for a volume of 75 cc?

(3) the pull on a satellite (P) is inversely proportional to the square of the average
distance from the centre of the Earth (d)
the pull on an 'Explorer' satellite is 900N when it is 9600km away
what will the pull be if it was 8000km away?
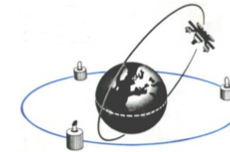
(4) the magnetic force (B) between two magnets is inversely proportional to the cube of the
distance between them (d cm)
the force is 10 gauss for a distance of 10.7cm
what is the force for double this distance?

(5) a person's pulse rate (p) is inversely proportional to the square root of their
height (h cm).
for someone of height 50cm the pulse rate is 134 counts/minute
what is the pulse rate for someone of height 178cm?

Fluency Practice

A is directly proportional to B If A = 5 and B = 100 Find B when A = 4	C is inversely proportional to D If C = 3 and D = 10 Find C when D = 6	E is directly proportional to F If E = 14 and F = 56 Find F when E = 10	G is inversely proportional to H If G = 8 and H = 5 Find H when G = 10
J is directly proportional to K If J = 7 and K = 56 Find J when K = 80	L is directly proportional to M If L = 10 and M = 12 Find M when L = 2	N is directly proportional to P If N = 3 and P = 300 Find P when N = 0.07	Q is inversely proportional to R If Q = 0.1 and R = 80 Find R when Q = 4
S is directly proportional to T If S = 6 and T = 2.4 Find S when T = 10	U is inversely proportional to V If U = 0.8 and V = 0.4 Find U when V = 0.2	W is directly proportional to X If W = 0.5 and X = 16 Find W when X = 64	Y is inversely proportional to Z If Y = 0.01 and Z = 4000 Find Z when Y = 0.2

Fluency Practice

(a)	(b)	(c)	(d)
y is directly proportional to x. When $x = 8, y = 40$. Find a formula for y in terms of x.	F is inversely proportional to t. When $F = 2.5, t = 4$. Find a formula for F in terms of t.	p is directly proportional to the square of q. When $q = 3$, $p = 90$. Find a formula linking p and q.	y is directly proportional to x^3. When $x = 5, y = 2500$. Find a formula for y in terms of x.

(e)	(f)	(g)	(h)
Sketch the graph showing y is inversely proportional to x.	y is directly proportional to $\sqrt{x}$. When $x = 4, y = 0.5$. Find the value of y when $x = 64$.	d is inversely proportional to w^2. When $w = 0.5, d = 12$. Find a formula for d in terms of w.	T is inversely proportional to $\sqrt{L}$. When $L = 16, T = 25$. Find the value of L when $T = 10$.

(i)

The distance d travelled by a ball is proportional to the square of the time taken, t. After 4 seconds the ball has travelled 40 m.
(i) Find a formula linking d and t.
(ii) Find the distance travelled after 7 seconds.

(j)

x	1	2	5	10	20
y	100	25	4		

(i) Find a formula for y in terms of x.
(ii) Complete the table.

Extension

direct and inverse proportion

(1) A is directly proportional to the square of L
when L is 8, A is 160

    a) find A when L = 6
    b) find L when A = 250

(2) surface area (A) of a solid is directly
proportional to the square of its length (L)
L = 5, A is 150

    a) find A when L = 3
    b) find L when A = 2400

(3) the radius (r) of a circle is directly proportional
to the square root of the area (A)
when the area is 100, the radius is 5.642

    a) find r when A = 400
    b) find A when r = 4

(4) T is directly proportional to the cube of K
when K is 4, T is 320

    a) find T when K = 2
    b) find K when T = 40000

(5) the force between objects (F) is inversely
proportional to the square of the distance
between them (D)
when the distance is 5, the force is 4
    a) find F when D = 2
    b) find D when F = 400

(6) P is inversely proportional to R
when P is 8, R is 10

    a) find P when R = 16
    b) find R when P = 2½

(7) Q is inversely proportional to N
when Q is 8, N is 5

    a) find Q when N = 16
    b) find N when Q = 80

(8) the length (L) of a rectangle of a fixed area is
inversely proportional to the width (W)
when the length is 16, the width is 1.5

    a) what is the length when the width is 32?
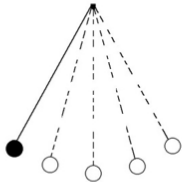   b) what is the width when the length is 15?

Extension

direct and inverse proportion in Physics

(1) the kinetic energy (E) of a saloon car is directly proportional to
the square of the speed of the car (S)
when the speed is 48 km/hr the energy is 141.51 Kjoules

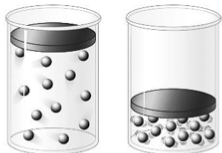what is the kinetic energy when the speed is doubled, to 96 km/hr?

(2) the period (T) of a pendulum swing is directly proportional to
the square root of the length of the pendulum (L)
when the length is 25cm the period is 10.03 seconds

a) what is the period when the length is 100cm?
b) what is the length for a period of 22.43 seconds?

(3) in a circuit, the resistance (R ohms) is inversely proportional to the current (A amps)
when the resistance is 12 ohms the current is 9.6 amps
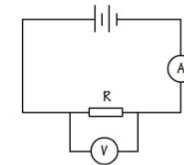
a) what is the current when the resistance is 9 ohms?
b) what is the current when the resistance is 6.4 ohms?
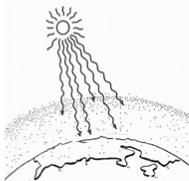
(4) at a constant temperature the volume of a gas (V) varies inversely as the
pressure (P)
the pressure is 1×10^5 Pa for a volume of 12×10^{-6} m^3
a) what will the volume be for a pressure of 5×10^4 Pa?
b) what will the pressure be for a volume of 4×10^{-6} m^3?

Extension

direct and inverse proportion in Physics
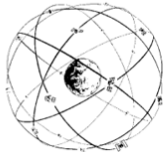
(4) Coulomb's law states that the magnetic force (F) between two dipoles is inversely
proportional to the square of the distance (D) between them
when the distance is 1.5m the force is 2.4×10^{10} N
what is the magnetic force when the distance between the dipoles is 3m?
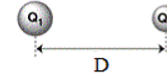
(5)

Stefan's law states that the total energy (E) radiated by a black body is directly
proportional to the 4th power of the temperature (T) in degrees Kelvin
the temperature of the Earth is 288K; the energy is 390 Watts/m^2
what is the energy for the Sun, with a temperature of 6000K?

(6) the escape velocity of a rocket is inversely proportional to the square root of its
distance from the centre of Earth
at the surface of the Earth (6378km from the centre) this velocity is 11.2 km/sec
what is the escape velocity 9000 km above the Earth's surface?
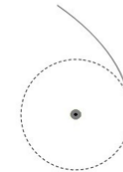
(7)

the period of a satellite squared (T^2) is directly proportional to the cube of its
distance from the centre of Earth
100 km above the surface of the Earth (i.e. 6478km from the centre) this period is
1.44 hours
what is the period of a satellite, 200 km above the Earth's surface?

(8) in an experiment, measurements of 'h' are taken after time
2 secs, 5 secs and 6 secs with the results shown in the table

t	2	5	6
h	10.1	62.4	89.9

which rule fits the data best?
(a) h $\propto$ t   (b) h $\propto t^2$ (c) h $\propto t^3$

Fluency Practice

1) x is directly proportional to y^2
   y is directly proportional to z^3
   Given that $x = 2$ and $z = 5$ when $y = 6$ find a formula for x in terms of z

2) x is inversely proportional to y^3
   y is inversely proportional to z^3
   Given that $x = 5$ and $z = 7$ when $y = 4$ find a formula for x in terms of z

3) x is inversely proportional to y
   y is inversely proportional to z
   Given that $x = 5$ and $z = 4$ when $y = 6$ find a formula for x in terms of z

4) x is inversely proportional to y^3
   y is inversely proportional to z
   Given that $x = 4$ and $z = 6$ when $y = 5$ find a formula for x in terms of

5) x is inversely proportional to $\sqrt[3]{y}$
   y is inversely proportional to $\sqrt{z}$
   Given that $x = 3$ and $z = 9$ when $y = 216$ find a formula for x in terms of z

6) x is inversely proportional to y^3
   y is directly proportional to z^3
   Given that $x = 7$ and $z = 3$ when $y = 4$ find a formula for x in terms of z

7) x is directly proportional to y^2
   y is inversely proportional to z
   Given that $x = 7$ and $z = 6$ when $y = 3$ find a formula for x in terms of z

8) x is directly proportional to $\sqrt{y}$
   y is directly proportional to z^3
   Given that $x = 10$ and $z = 7$ when $y = 4$ find a formula for x in terms of z

9) x is directly proportional to $\sqrt{y}$
   y is inversely proportional to z
   Given that $x = 3$ and $z = 3$ when $y = 16$ find a formula for x in terms of z

10) x is directly proportional to y^3
    y is directly proportional to z^3
    Given that $x = 10$ and $z = 4$ when $y = 8$ find a formula for x in terms of z

A is proportional to B^2 and A is proportional to C^3. When $A = 3, B = 1, C = 2$.
(a) Find the value of C when $B = 8$.   (b) Find the exact value of B when $C = 1$.

A is proportional to the square root of B. A is proportional to C^3.
When A =17, B= 16, C=4.
(a) Find B when $C = 1$.   (b) Find C when $B = 9$ (to 1 decimal place)

A is proportional to square root of B. A is inversely proportional to C.
When $A = 5, B = 4, C = 3$.
(a) Find B when $C = 2$.   (b) Find C when $B = 1$.

A is inversely proportional to B. A is inversely proportional to the square root of C.
(a) Find B when C=4.   (b) Find C when $B = 10$

A is proportional to B^2, and B is proportional to C^3.
Show that A is proportional to C^d, where d is an integer value to be found. ($d = 6$)

A^2 is proportional to B^3 and B^2 is proportional to C. Show that A^n is proportional to C^m where n and m are integers to be found.   ($n = 4, m = 3$)

The square root of A is proportional to cube of B. The cube root of B is proportional to C.
Show that A is proportional to C^n, where n is an integer to be found.   ($n = 6$)

Fluency Practice

a is directly proportional to b^2. b varies directly with c. When $a = 320, b = 8$ when $c = 4$. Write a formula for a in terms of c	$d \propto e, e \propto f^3$ When $d = 48, e = 32$ when $f = 2$. Write a formula for d in terms of f	g varies directly with the square of h. g also varies directly with the cube of i. When $g = 9, h = \frac{3}{2}$ when $i = 3$. Write a formula for h in terms of i	$m \propto \sqrt{n}, n \propto p,\,^2$ When $m = 10, n = 4$ when $p = 6$. Write a formula for m in terms of p
$a \propto b^2$ and $b \propto \frac{1}{\sqrt{c}}$ Given that $a = 3$. $b = 1$ when $c = 4$. Write a formula for a in terms of c	d varies inversely with the square of e. e is inversely proportional to f. When $d = 5, e = 2$ when $f = 15$. Write a formula for d in terms of f	$g \propto \frac{1}{\sqrt{h}}$ and $h \propto \frac{1}{i^2}$. Given that $g = 12,\ h = 9$ when $i = 2$. Write a formula for g in terms of i	m varies inversely with p^2. m is also inversely proportional to the cube of n. $m = 8$ when $n = 5$ and $p = 3$. Write a formula for p in terms of n
$a^2 \propto b^3$ and $b \propto \frac{1}{c}$ Given that $a = 9, b = 3$ when $c = 4$. Write a formula for a in terms of c	$d^2 \propto e$ and $d \propto \frac{1}{\sqrt{f}}$ Given that $d = 10, e = 20$ when $f = 49$. Write a formula for e in terms of f.	$a \propto b^2$, $b \propto \frac{1}{\sqrt{c}}$, $c \propto \frac{1}{d^3}$, $d \propto \sqrt[3]{e}$ Given that $a = 9, b = 30, c = \frac{1}{36}, d = 12$ when $e = 27$. Write a formula for a in terms of e	

Extension

1 $x \propto yz$. When $y = 2$ and $z = 3$, $x = 30$.
   (a) Find the relation between x, y and z.
   (b) Find x when $y = 4$ and $z = 6$.

2 $x \propto \dfrac{y}{z}$, $x = 27$ when $y = 9$ and $z = 2$.
   (a) Find the relation between x, y and z.
   (b) Find x when $y = 14$ and $z = 12$.

3 $p \propto \dfrac{q}{r^2}$ and $p = 3\frac{1}{3}$ when $q = 5$ and $r = 3$.
   (a) Find the equation connecting p, q and r.
   (b) Find p when $q = 9$ and $r = 1.2$.

4 The height (h) of a cone varies directly as its volume (V) and inversely as the square of its radius (r). Use the constant k to show the relationship between h, V and r.

5 $A \propto BC$ and $A = 6$ when $B = 4$ and $C = 9$.
   (a) Find A when $B = 3$ and $C = 10$.
   (b) Find C if $A = 20$ and $B = 15$.
   (c) By what percentage does A change if B is increased by 10% and C is decreased by 10%?

6 x varies directly with the square of y and with z. When $y = 2$ and $z = 3$, $x = 4\frac{1}{2}$.
   (a) Find x when $y = 5$ and $z = 4$.
   (b) Find y when $x = 21$ and $x = 3\frac{1}{2}$.
   (c) What happens to x if y is doubled and z halved?

7 x, y and z are related quantities such that x varies directly as y and inversely as the square root of z. When $x = 300$ and $y = 65$, $z = 25$. Calculate the value of x when $y = 468$ and $z = 144$.

8 If $P \propto \dfrac{1}{V}$ and $V \propto R^2$, how does P vary with R?

9 $x \propto y$ and $y \propto z^3$. How does x vary with z?

10 $x \propto y^2$ and $y \propto z^2$. How does x vary with z?

11 $A \propto BC$ and $B \propto \dfrac{1}{C^2}$. How does A vary with C?

12 y varies directly as x and inversely as z. x varies inversely as y^2. Prove that z^2 varies directly as x^3.

Fluency Practice

1) y is inversely proportional to p^2
   p is decreased by 80%
   Work out the percentage increase in y

2) z is proportional to $\sqrt[3]{x}$
   x is decreased by 78.4%
   Work out the percentage decrease in z

3) y is inversely proportional to $\sqrt{z}$
   z is decreased by 96%
   Find the percentage increase in y

4) z is proportional to q^3
   q is increased by 10%
   Work out the percentage increase in z

5) p is proportional to z^2
   z is increased by 20%
   Find the percentage increase in p

1) z is proportional to $\sqrt{p}$
   p is decreased by 96%
   Work out the percentage decrease in z

2) t is proportional to x^2
   x is increased by 20%
   Work out the percentage increase in t

3) x is proportional to p^3
   p is decreased by 60%
   Work out the percentage decrease in x

4) p is proportional to z^3
   z is increased by 10%
   Find the percentage increase in p

5) q is proportional to y^2
   y is increased by 30%
   Find the percentage increase in q

c is proportional to d^3. If d increases by 12%, what is the percentage increase of c (to 1dp)?

a is proportional to b^2. If a increases by 21%, what is the percentage increase of b?

p is proportional to the square of q. If q decreases by 3%, what is the percentage decrease of p (to 1dp)?

x is proportional to y^2. If x decreases by 36%, what is the percentage decrease of y?

c is proportional to $\sqrt{d}$. If c increases by 20%, what is the percentage increase of d?

p^2 is proportional to q^3. If p increases by 12%, what is the percentage increase of q (to the nearest percent)?

A is proportional to B^2 and B is proportional to C^3. A increases by 50%. What is the percentage increase of C (to the nearest percent)?

Fluency Practice

learn by heart

If y is **directly** proportional to x,

the graph is a **straight line through the origin**.

the equation is $y = kx$ where k is a number.
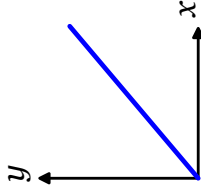
If y is **inversely** proportional to x,

the graph is a **curve as shown**.

the equation is $y = \dfrac{k}{x}$ where k is a number.
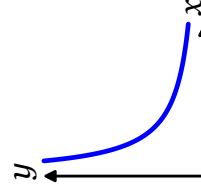
exercise

1. Which **one** of the following shows a graph where x and y are **directly** proportional?

    a)

    b)

    c)
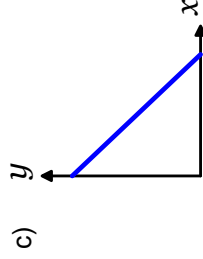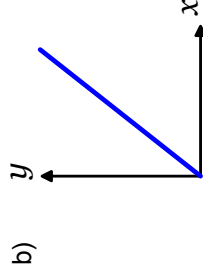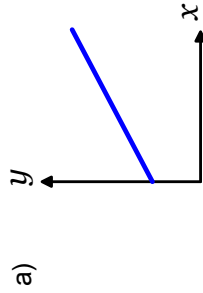
2. Which **one** of the following shows a graph where x and y are **inversely** proportional?

    a)

    b)

    c)
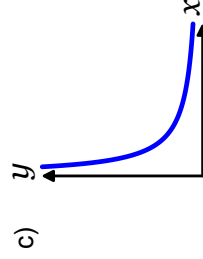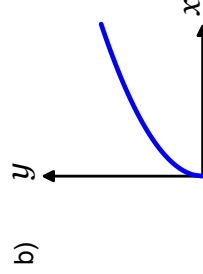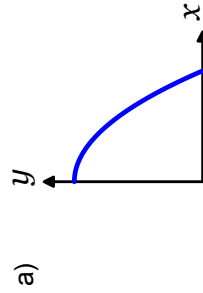
3. For each equation, decide whether x and y are **directly proportional, inversely proportional**, or **neither**.

    a) $y = 3x + 1$

    b) $y = \dfrac{4}{x}$

    c) $y = 0.5x$

    d) $y = \dfrac{1}{2}x$

    e) $y = 6x^2$

    f) $y = \dfrac{2}{x}$

    g) $y = \dfrac{3}{x} + 5$

    h) $y = \dfrac{9}{x}$

    i) $y = 5x$

Page 224

Fluency Practice

4. A graph is shown to the right.
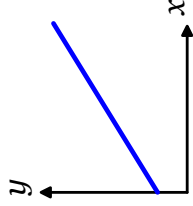
   Jessica says
   "*y* is directly proportional to *x*"

   Explain how you can tell Jessica is wrong.

5. Four equations are shown.
   k is a constant in each equation.

$y = \dfrac{k}{x}$	$y = \dfrac{1}{x+k}$	$y = kx$	$y = k + x$

   a) In which equation are x and y directly proportional?

   b) In which equation are x and y inversely proportional?

6. For each graph, decide whether x and y are **directly proportional**, **inversely proportional**, or **neither**.

   a)

   b)

   c)

   d)

   e)

   f)
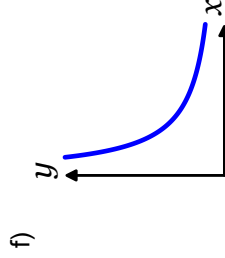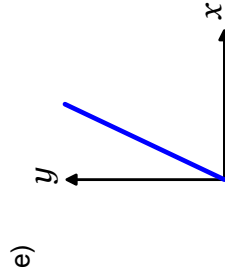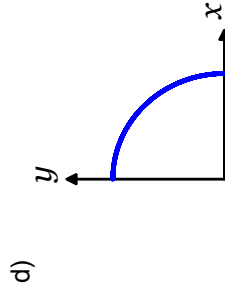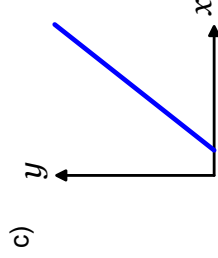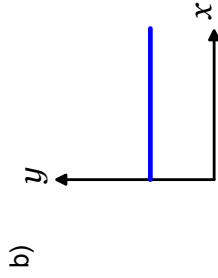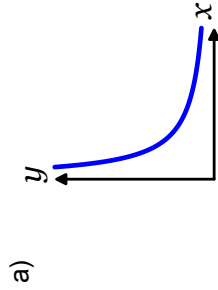
G Proportional limits

example

At a playgroup, at least 1 adult is needed for every 6 children.

a) If there are 5 adults, work out the greatest possible number of children.

1 adult for up to 6 children
5 adults for up to 30 children

30 children

b) If there are 20 children, work out the least number of adults required.

3 adults for up to 18 children
4 adults for up to 24 children

4 adults

4 Constructions and Loci

To 'construct' something in the strictest sense means to draw it using only two things:
- Compass
- Straight Edge (Apart from where a length is specified, you are not allowed to measure lengths)

Bisect means cut into two equal parts.
Equidistant means equal distance from

Q10 Draw a near vertical line of length 6cm. Using a compass and ruler only, construct its perpendicular bisector.

Q11 Draw a line 8cm in length. Using a compass and ruler only, construct its perpendicular bisector.

Q12 Using a protractor, draw an angle of 64°. Using a compass and ruler only, construct its angle bisector. Check your answer by measuring the two angles formed.

Q13 Using a protractor, draw an angle of 120°. Using a compass and ruler only, construct its angle bisector. Check your answer by measuring the two angles formed.

Q14 Draw a triangle and construct the perpendicular bisector of each side. Draw the smallest possible circle that **does not enter** the triangle.

Q15 Draw another triangle and construct the angle bisector of each vertex. Draw the largest possible circle that **does not exit** the triangle.

Fluency Practice

Q1 Use your compass and ruler only to construct the following **SSS** triangles.

[a]

5cm

[b]

4.2cm

[c]

7.6cm

[d]

6cm   6cm

8cm

[e]

5cm   7cm

8cm

[f]

7.4cm   4.2cm

6.2cm

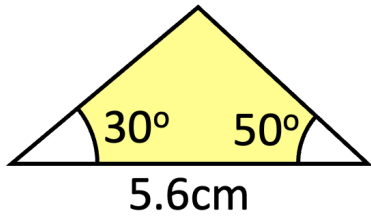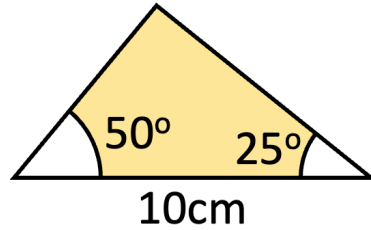**[g]**

2cm   10cm

8.5cm

[h]

8.6cm   4.3cm

5.5cm
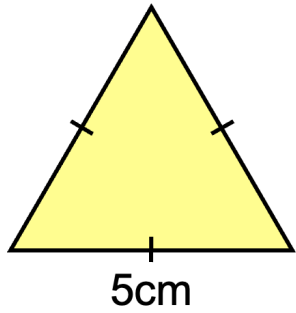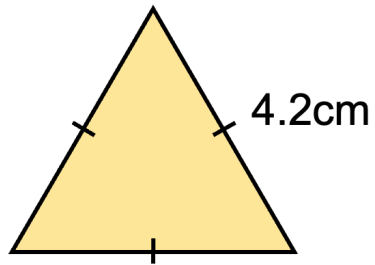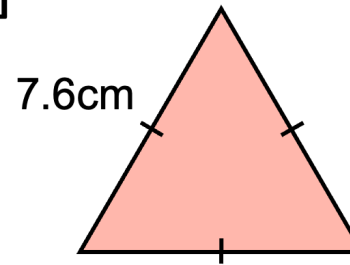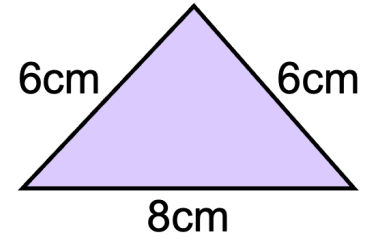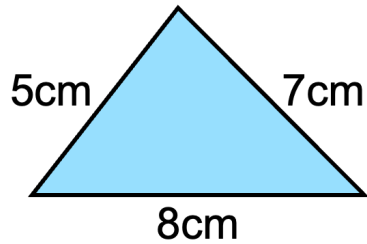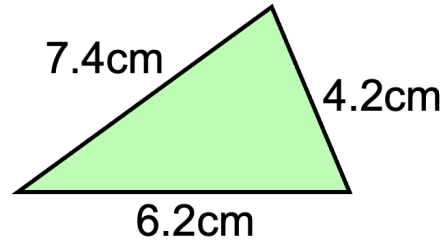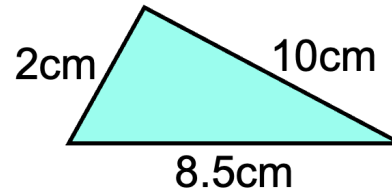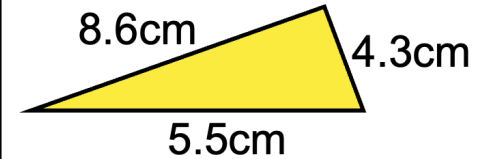
Fluency Practice

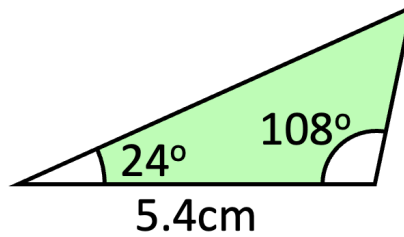**Q2** Use your compass and ruler only to construct the following triangles.

[a]
4cm
36°
5cm
5cm

[b]
5cm
100°
6cm

[c]
4cm
60°
8cm

[d]
8.2cm
25°
8.2cm

[e]
4.5cm
36°
7.8cm

[f]
7.3cm
42°
8.2cm

[g]
6.3cm
3.5cm

[h]
5.4cm
7.5cm
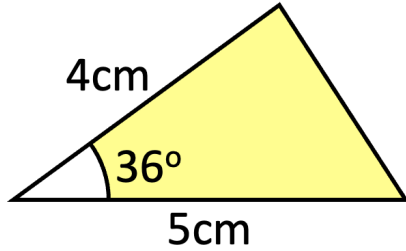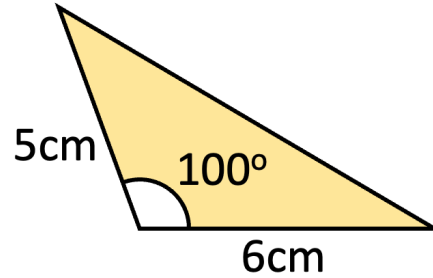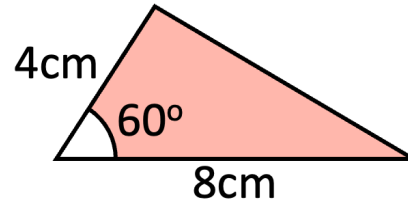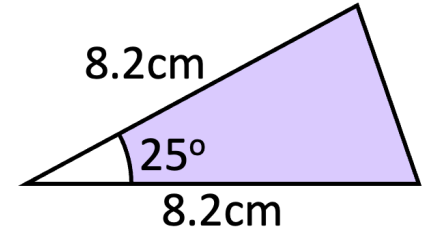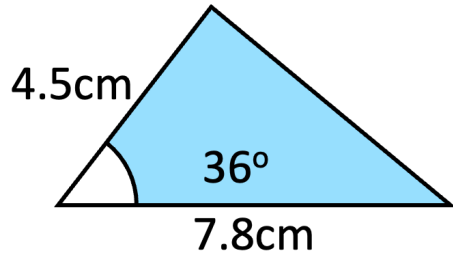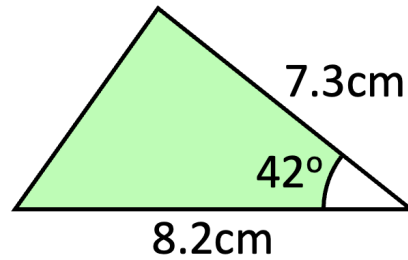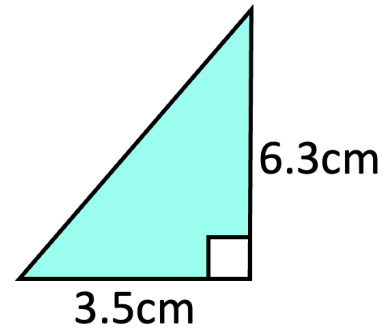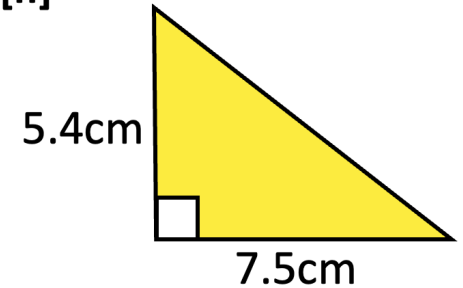
Fluency Practice

Q3 Use your compass and ruler only to construct the following triangles.

[a]

30°   50°
5.6cm

[b]

50°   25°
10cm

[c]

45°   45°
8.1cm

[d]

80°   18°
6.3cm

[e]

100°   30°
3.2cm

[f]

24°   108°
5.4cm

[g]

15°   55°
6.5cm

[h]

95°   12°
4.2cm
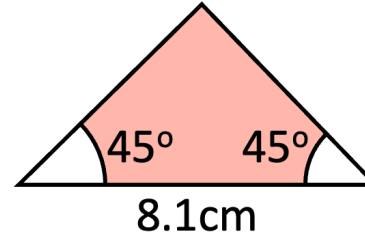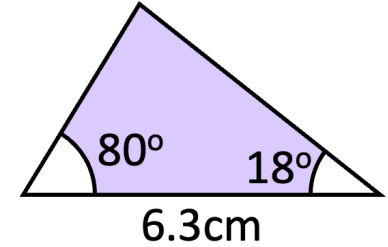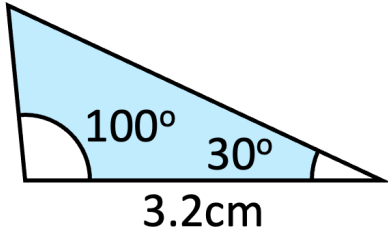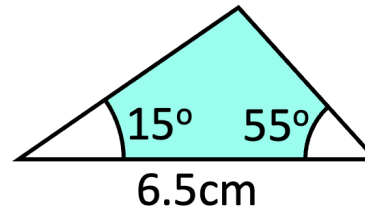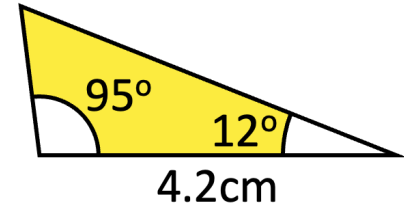
Extension

Q4 Use the scale of 1cm = 50m, construct the following triangles.

[a]

150m — 175m — 250m

[b]

0.7km — 0.2km — 0.4km
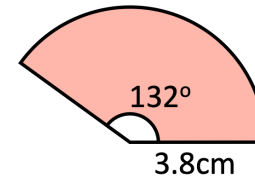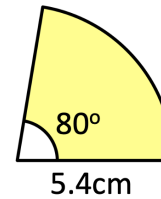
[c]

175m — 45° — 300m

[d]

0.25km — 58° — 425cm
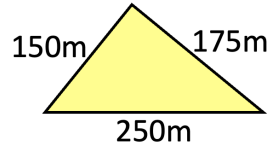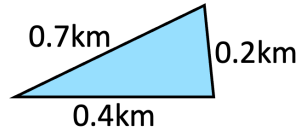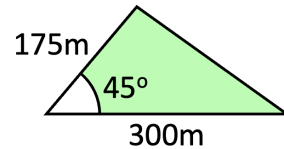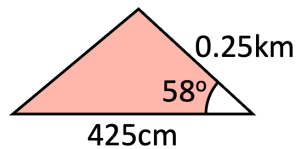
Q5 Accurately draw two different isosceles triangles with an angle of 40°.

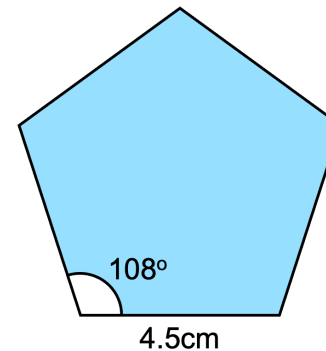**Q6** Construct an equilateral triangle with side length of 7cm. By measuring its height, work out its area.

Q7 Accurately draw the sectors below.

80° — 5.4cm

132° — 3.8cm

Q8 Accurately construct the following quadrilateral and find the length of the missing side.

5cm — 112° — 100° — 4.2cm — 7.5cm
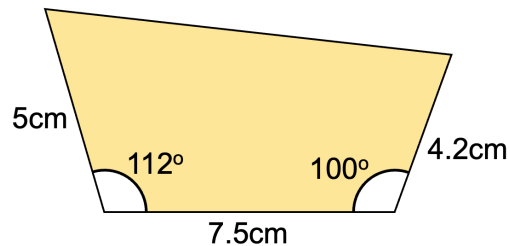
Q9 Use the information below to accurately construct a regular pentagon.

108° — 4.5cm

tice

Construct the following triangles.

(a)

7 cm
55°
8 cm

(b)

76°
7 cm
58°

(c)

8 cm
36°
98°
9 cm

Construct the following triangles.

(a)

10 cm
9 cm
6 cm

(b)

11 cm
8 cm
5 cm
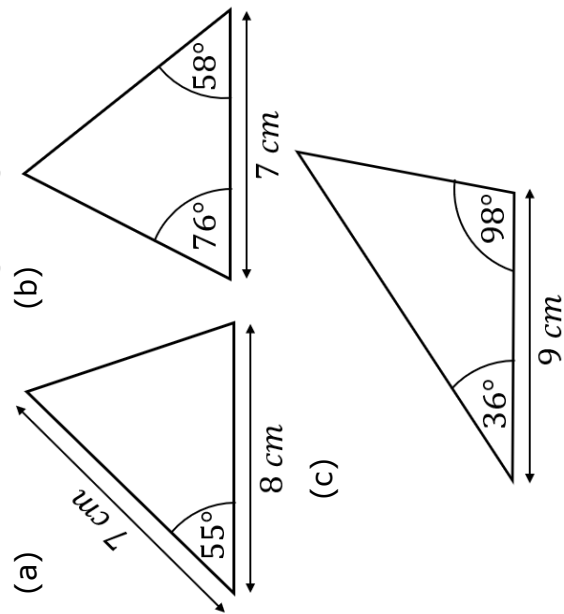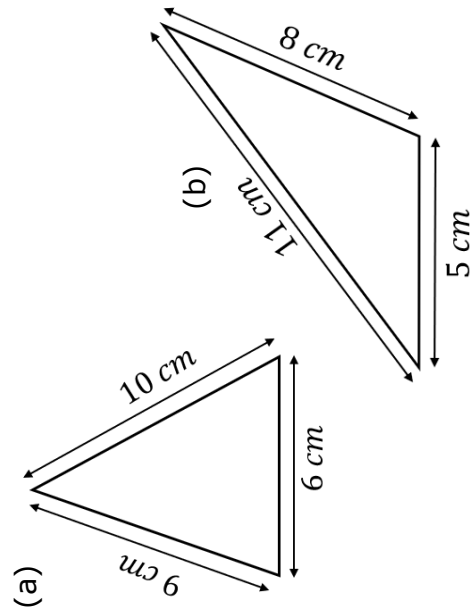
(a) In the triangle ABC, AB is 7 cm, BC is 6 cm and angle ABC is 65°. Make and accurate construction of the triangle.

(b) In the triangle DEF, DE is 6 cm, EF is 8 cm and DF is 10 cm. Make an accurate construction of the triangle. Measure the angle DEF.

In the triangle KLM, KL is 7 cm, LM is 8 cm and angle LKM is 40°. Construct the two possible triangles accurately. Measure the angle KLM in both triangles. What do you notice?

(Triangles at top:)

7 cm
55°
8 cm

36°
9 c

10 cm
9 cm
6 cm

Constructing Triangles

Determine which triangle matches with
which information below and draw in the sides of
the triangles and label them. Also decide which
triangles are Equilateral, Isosceles, Scalene or right-
angled and which is triangle is impossible.

AB: 5cm, AC: 4cm, BC: 3cm	AB: 5cm, AC: 6cm, BC: 6cm
AB: 5cm, AC: 4cm, BC: 6cm	AB: 5cm, AC: 5cm, BC: 5cm
AB: 5cm, AC: 5cm, BC: 6cm	AB: 5cm, AC: 2cm, BC: 2cm
AB: 5cm, AC: 4cm, BC: 4cm	
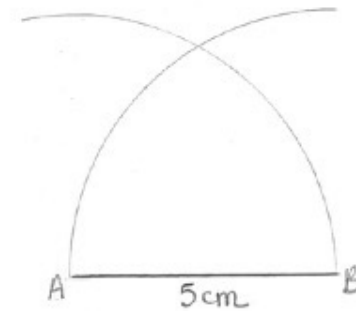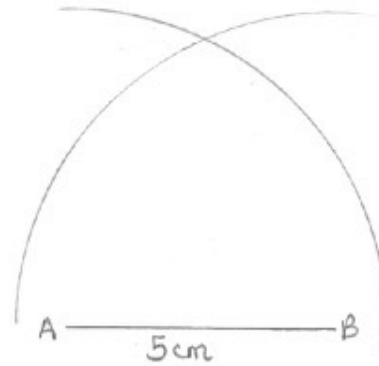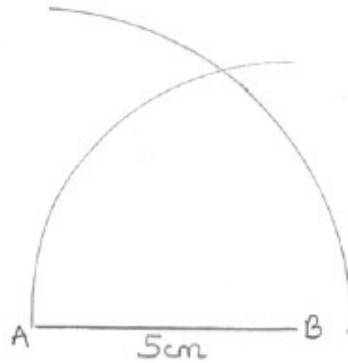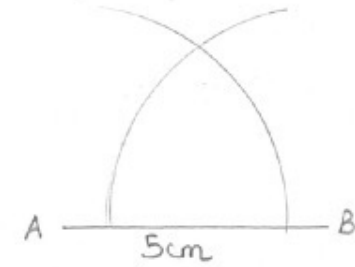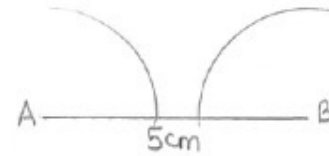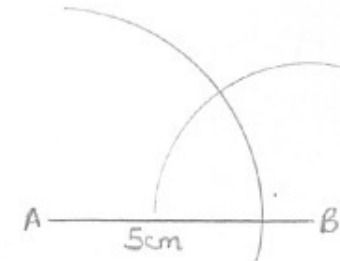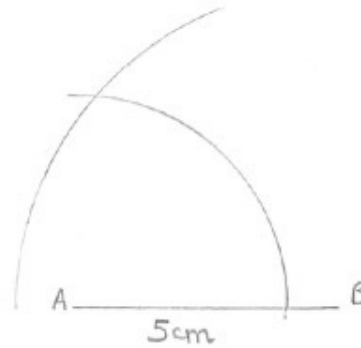
constructions and
congruence in triangles

three sides the same [SSS].
(1) 9 cm, 8 cm and 5 cm.
(2) 11 cm, 6 cm and 4.5 cm.
Will everyone's triangles look
the same?
Reasons?

 three angles the same [AAA].
(3) 45°, 70°, 65°.
(4) 80°, 40°, 70°.
Will all the triangles that people
have drawn be identical
(be congruent)?
Reasons?

two angles and an included side (between them) the same [ASA]
(5) 60°, 9cm, 40°.
(6) 100°, 8cm, 30°.
Will all the triangles drawn be congruent?
Reasons?

two sides and an angle [SAS] and [ASS]
a triangle PQR with:
(7) PQ = 9cm, QR = 8cm, angle P = 50°.
(8) PQ = 9cm, PR = 11cm, angle P = 50°.
(9) PQ = 8cm, PR = 6cm, angle R = 90°.
(10) PQ = 10cm, PR = 7cm, angle P = 50°.
Will all the triangles drawn be congruent?
Reasons?

Problem Solving

A treasure chest is buried somewhere on a plot of land surrounded by fencing. The land is a square 6m in length. Four people each have a different treasure map detailing where the treasure is buried. Which map has smallest area that needs to be checked?

B   C

A ⟷ D

6m
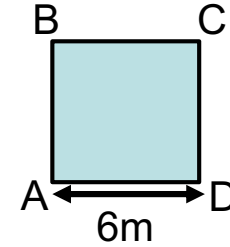
The treasure is closer to the fence AB than the fence AD	**The treasure is exactly 5m from corner D**
The treasure is no further than 3.2m from the fence CD	**The treasure is exactly 2m from the centre**