KING EDWARD VI HANDSWORTH GRAMMAR SCHOOL FOR BOYS

KING EDWARD VI ACADEMY TRUST BIRMINGHAM

Year 10

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Unit 19 Booklet

HGS Maths

Tasks

Dr Frost Course

Name: ______________________________

Class: ______________________________

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1 Advanced Indices

Indices Recap

Multiplication Law:
$y^a \times y^b = y^{a+b}$

Division Law:
$y^a \div y^b = y^{a-b}$

Power Law:
$(y^a)^b = y^{ab}$

Worked Example	**Your Turn**
Simplify:	Simplify:
1)	1)
a) $y^{11} \times y^5$	a) $x^5 \times x^{-2}$
b) $6y^3 \times 2y^5$	b) $7x^5 \times 8x^{-3}$
c) $y^5 \div y^2$	c) $y^5 \div y^4$
d) $8y^3 \div 2y$	d) $15y^3 \div 3y$
e) $(y^3)^7$	e) $(y^7)^8$
f) $(3y^4)^2$	f) $(5y^4)^3$
2)	2)
a) $\frac{a^6 \times a^4}{a^2}$	a) $\frac{a^6 \times a^{-4}}{a^2}$
b) $(4a^6b^3)^2$	b) $(2a^6b^3)^4$
c) $\frac{8a^5b^3}{4ab^7}$	c) $\frac{12a^2b^3}{4ab^7}$

Power Zero

$2^4 = 16$

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

Any non-zero number divided by itself equals 1, i.e. $2 \div 2 = 1$

Using the exponent rule for division:

$$\frac{2^1}{2^1} = 2^{1-1} = 2^0 = 1$$

Worked Example	Your Turn
Simplify: a) $4x^0$ b) $x^4 \times x^0$ c) $\frac{x^9}{x^0}$ d) $x^0 \div x^{-2}$	Simplify: a) $8x^0$ b) $x^0 \times x^8$ c) $\frac{x^0}{x^{18}}$ d) $x^{-4} \div x^0$

Negative Indices

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

$2^{-3} = \frac{1}{8}$

$$\frac{2^3}{2^7} = \frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2 \times 2 \times 2 \times 2} = \frac{1}{2^4}$$

Using the exponent rule for division:

$$\frac{2^3}{2^7} = 2^{3-7} = 2^{-4}$$

Therefore

$$\frac{1}{2^4} = 2^{-4}$$

Worked Example	Your Turn
Evaluate: a) 3^{-2} b) -3^{-2} c) $(-3)^{-2}$	Evaluate: a) 5^{-3} b) -5^{-3} c) $(-5)^{-3}$

Worked Example	Your Turn
Write $\frac{1}{4^2}$ in index form	Write $\frac{1}{5^3}$ in index form

Worked Example	Your Turn
Simplify: a) $\left(\frac{3}{10}\right)^{-2}$ b) $\left(-\frac{3}{10}\right)^{-2}$	Simplify: a) $\left(\frac{2}{5}\right)^{-3}$ b) $\left(-\frac{2}{5}\right)^{-3}$

Worked Example	Your Turn
Rewrite the following with a negative index: a) $\frac{1}{x^5}$ b) $\frac{3}{x^5}$ c) $\frac{1}{3x^5}$	Rewrite the following with a negative index: a) $\frac{1}{d^{10}}$ b) $\frac{9}{d^{10}}$ c) $\frac{9}{18d^{10}}$

Expanding Brackets with Indices

Worked Example	Your Turn
Simplify: a) $2a^3(3a^2 + 5a^{-4})$ b) $p^{\frac{1}{2}}\left(2p^{\frac{1}{2}} - p^{-\frac{3}{2}}\right)$ c) $x^2\left(x^{\frac{1}{3}} - x^{\frac{1}{4}}\right)$	Simplify: a) $3a^{-2}(4a^5 + 2a)$ b) $2p^{\frac{1}{3}}\left(3p^{\frac{2}{3}} - p^{-\frac{1}{3}}\right)$ c) $n^{\frac{3}{5}}\left(n^{\frac{1}{2}} + \frac{1}{n^{\frac{1}{2}}}\right)$

Worked Example	Your Turn
Simplify: $(2m^9 - m^{-2})(6m^{-3} + m^5)$	Simplify: $(7x^3 - x^{-4})(4x^{-2} + x^9)$

Fractional Indices

$x^{\frac{1}{2}} \times x^{\frac{1}{2}} = \left(x^{\frac{1}{2}}\right)^2 = x^1$ $x^{\frac{1}{2}}$ squared is x therefore the square root of x is $x^{\frac{1}{2}}$ i.e. $\sqrt{x}$

$x^{\frac{1}{3}} \times x^{\frac{1}{3}} \times x^{\frac{1}{3}} = \left(x^{\frac{1}{3}}\right)^3 = x^1$ $x^{\frac{1}{3}}$ cubed is x therefore the cubed root of x is $x^{\frac{1}{3}}$ i.e. $\sqrt[3]{x}$

$x^{\frac{1}{4}} \times x^{\frac{1}{4}} \times x^{\frac{1}{4}} \times x^{\frac{1}{4}} = \left(x^{\frac{1}{4}}\right)^4 = x^1$

The fourth power of $x^{\frac{1}{4}}$ is x therefore the fourth root of x is $x^{\frac{1}{4}}$ i.e. $\sqrt[4]{x}$

$x^{\frac{1}{n}} \times x^{\frac{1}{n}} \times x^{\frac{1}{n}} \times x^{\frac{1}{n}} \times \ldots = \left(x^{\frac{1}{n}}\right)^n = x^1$

The n^{th} power of $x^{\frac{1}{n}}$ is x therefore the n^{th} root of x is $x^{\frac{1}{n}}$ i.e. $\sqrt[n]{x}$

Worked Example	Your Turn
Evaluate:	Evaluate:
a) $64^{\frac{1}{2}}$	a) $64^{\frac{1}{3}}$
b) $64^{-\frac{1}{2}}$	b) $64^{-\frac{1}{3}}$
c) $\left(\frac{81}{16}\right)^{\frac{1}{4}}$	c) $\left(\frac{81}{16}\right)^{\frac{1}{2}}$
d) $\left(\frac{81}{16}\right)^{-\frac{1}{4}}$	d) $\left(\frac{81}{16}\right)^{-\frac{1}{2}}$

Fractional Indices

$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$

$8^{\frac{3}{3}} = \left(8^{\frac{1}{3}}\right)^3 = \left(\sqrt[3]{8}\right)^3 = (2)^3 = 8$

$8^{\frac{4}{3}} = \left(8^{\frac{1}{3}}\right)^4 = \left(\sqrt[3]{8}\right)^4 = (2)^4 = 16$

$8^{\frac{5}{3}} = \left(8^{\frac{1}{3}}\right)^5 = \left(\sqrt[3]{8}\right)^5 = (2)^5 = 32$

$8^{\frac{m}{3}} = \left(8^{\frac{1}{3}}\right)^m = \left(\sqrt[3]{8}\right)^m = (2)^m$

$x^{\frac{1}{5}} = \sqrt[5]{x}$

$x^{\frac{2}{5}} = \left(x^{\frac{1}{5}}\right)^2 = (\sqrt[5]{x})^2$

$x^{\frac{3}{5}} = \left(x^{\frac{1}{5}}\right)^3 = (\sqrt[5]{x})^3$

$x^{\frac{4}{5}} = \left(x^{\frac{1}{5}}\right)^4 = (\sqrt[5]{x})^4$

$x^{\frac{m}{5}} = \left(x^{\frac{1}{5}}\right)^m = (\sqrt[5]{x})^m$

$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = (\sqrt[n]{x})^m$

Worked Example	Your Turn
Evaluate: a) $25^{\frac{3}{2}}$ b) $25^{-\frac{3}{2}}$ c) $\left(\frac{36}{25}\right)^{\frac{3}{2}}$ d) $\left(\frac{36}{25}\right)^{-\frac{3}{2}}$	Evaluate: a) $81^{\frac{3}{4}}$ b) $81^{-\frac{3}{4}}$ c) $\left(\frac{81}{256}\right)^{\frac{3}{4}}$ d) $\left(\frac{81}{256}\right)^{-\frac{3}{4}}$

Laws of Indices

$y^a \times y^b = y^{a+b}$

$y^a \div y^b = y^{a-b}$

$(y^a)^b = y^{ab}$

$(yz)^a = y^a z^a$

$$\left(\frac{y}{z}\right)^a = \frac{y^a}{z^a}$$

$y^0 = 1$

$$y^{-a} = \frac{1}{y^a}$$

$$y^{\frac{1}{b}} = \sqrt[b]{y}$$

$$y^{\frac{a}{b}} = (\sqrt[b]{y})^a$$

$$y^{-\frac{1}{b}} = \frac{1}{\sqrt[b]{y}}$$

$$y^{-\frac{a}{b}} = \frac{1}{\left(\sqrt[b]{y}\right)^a}$$

Change of Base

What do you notice about all of the numbers: 1, 10, 100, 1000, …

They are all powers of 10.

What do you notice about all of the numbers: $2, 8, 4$,16….

They are all powers of 2.

We could replace the numbers with 2^1, 2^3 and 2^2 so that we have a consistent base.

Worked Example	Your Turn
a) Write 27 as a power of 3	a) Write 8 as a power of 2
b) Write 27^x as a power of 3	b) Write 8^x as a power of 2
c) Write 8^{2x} as a power of 2	c) Write 8^{3x} as a power of 2

Worked Example	Your Turn
Find the value of each of the following: a) $\sqrt{3^6 \times 16}$ b) $\sqrt[3]{3^6 \times 8}$ c) $\sqrt[4]{3^8 \times 16}$	Find the value of each of the following: a) $\sqrt{2^4 \times 9}$ b) $\sqrt[3]{64 \times 3^3}$ c) $\sqrt[4]{81 \times 256}$

Worked Example	Your Turn
Solve the equation: $3^x = \frac{1}{9}$	Solve the equation: $4^x = \frac{1}{64}$

Worked Example	Your Turn
Solve the equation: $\left(\frac{1}{3}\right)^x = 27$	Solve the equation: $\left(\frac{1}{4}\right)^x = 64$

Worked Example	Your Turn
Find the value of x that satisfies: a) $2^x \times 2^{x-3} = 32$ b) $2^{2x} \div 2^{x-3} = 32$	Find the value of x that satisfies: a) $3^x \times 3^{x-2} = 81$ b) $3^{3x} \div 3^{x-2} = 81$

Worked Example	Your Turn
Find the value of x that satisfies: $125^{\frac{1}{4}} \times 5^{2x+3} = 25^{\frac{2}{3}}$	Find the value of x that satisfies: $64^{\frac{1}{4}} \times 4^{3x+1} = 16^{\frac{2}{3}}$

Extra Notes

2 Calculating with Surds

Multiplying Surds

Worked Example	Your Turn
Simplify: a) $\sqrt{5} \times \sqrt{6}$ b) $\sqrt{3} \times \sqrt{6}$	Simplify: a) $\sqrt{5} \times \sqrt{7}$ b) $\sqrt{3} \times \sqrt{8}$

Worked Example	Your Turn
Simplify: a) $2\sqrt{5} \times \sqrt{6}$ b) $3\sqrt{3} \times 2\sqrt{6}$	Simplify: a) $2\sqrt{5} \times \sqrt{7}$ b) $3\sqrt{3} \times 2\sqrt{8}$

Dividing Surds

Worked Example	Your Turn
Simplify: a) $\sqrt{60} \div \sqrt{2}$ b) $\sqrt{60} \div \sqrt{3}$	Simplify: a) $\sqrt{90} \div \sqrt{3}$ b) $\sqrt{90} \div \sqrt{2}$

Worked Example	Your Turn
Simplify: a) $2\sqrt{60} \div \sqrt{2}$ b) $12\sqrt{60} \div 2\sqrt{3}$	Simplify: a) $3\sqrt{90} \div \sqrt{3}$ b) $12\sqrt{90} \div 3\sqrt{2}$

Adding and Subtracting Surds

Worked Example	Your Turn
Simplify: a) $2\sqrt{5} + 5\sqrt{5}$ b) $2\sqrt{20} + 5\sqrt{5}$ c) $2\sqrt{20} + 5\sqrt{10}$	Simplify: a) $2\sqrt{6} + 5\sqrt{6}$ b) $2\sqrt{54} + 5\sqrt{6}$ c) $2\sqrt{20} + 5\sqrt{15}$

Worked Example	Your Turn
Simplify: $\dfrac{2\sqrt{20}+5\sqrt{5}}{\sqrt{5}}$	Simplify: $\dfrac{2\sqrt{54}-5\sqrt{6}}{\sqrt{6}}$

Expanding Brackets with Surds

Worked Example	Your Turn
Expand and simplify: a) $2\left(4+\sqrt{3}\right)$ b) $-\sqrt{3}\left(4+\sqrt{3}\right)$ c) $\sqrt{12}(4+\sqrt{3})$	Expand and simplify: a) $-2\left(\sqrt{3}+4\right)$ b) $\sqrt{3}\left(\sqrt{3}+4\right)$ c) $\sqrt{27}(\sqrt{3}+4)$

Worked Example	Your Turn
Expand and simplify: a) $\left(2-\sqrt{3}\right)\left(4+\sqrt{3}\right)$ b) $\left(2-\sqrt{3}\right)^2$	Expand and simplify: a) $\left(\sqrt{3}-2\right)\left(\sqrt{3}+4\right)$ b) $\left(\sqrt{3}-2\right)^2$

Worked Example	Your Turn
Expand and simplify: a) $\left(2-\sqrt{20}\right)\left(4+\sqrt{5}\right)$ b) $(2-2\sqrt{20})(4+5\sqrt{5})$	Expand and simplify: a) $\left(\sqrt{54}-2\right)\left(\sqrt{6}+4\right)$ b) $(2\sqrt{54}-2)(5\sqrt{6}+4)$

Worked Example	Your Turn
Express b and c in terms of a: $\left(a+\sqrt{12}\right)^2 = b + c\sqrt{3}$	Express b and c in terms of a: $\left(a+\sqrt{8}\right)^2 = b + c\sqrt{2}$

Worked Example	Your Turn
Find the value of a and b: $\left(a-3\sqrt{5}\right)^2 = b-42\sqrt{5}$	Find the value of a and b: $\left(a-2\sqrt{3}\right)^2 = b-20\sqrt{3}$

Rationalising Surds

Worked Example	Your Turn
Rationalise: a) $\frac{3}{\sqrt{5}}$ b) $\frac{3}{2\sqrt{5}}$ c) $\frac{3+\sqrt{5}}{\sqrt{5}}$	Rationalise: a) $\frac{10}{\sqrt{5}}$ b) $\frac{3}{2\sqrt{6}}$ c) $\frac{10+\sqrt{5}}{\sqrt{5}}$

Worked Example	Your Turn
A rectangle has area $64\ cm^2$ and a width of $\sqrt{32}\ cm$. Find the length of the rectangle in the form $a\sqrt{b}$	A rectangle has area $60\ cm^2$ and a width of $\sqrt{12}\ cm$. Find the length of the rectangle in the form $a\sqrt{b}$

Conjugates

Is $\sqrt{3} - 1$ the conjugate of $\sqrt{3} + 1$?

Is $-\sqrt{3} + 1$ the conjugate of $\sqrt{3} + 1$?

Is $-\sqrt{3} + 1$ the conjugate of $1 + \sqrt{3}$?

Is $1 - \sqrt{3}$ the conjugate of $1 + \sqrt{3}$?

Is $-1 - \sqrt{3}$ the conjugate of $1 - \sqrt{3}$?

Is $1 + \sqrt{3}$ the conjugate of $1 - \sqrt{3}$?

Is $1 + \sqrt{5}$ the conjugate of $1 - \sqrt{5}$?

Is $1 - 3\sqrt{5}$ the conjugate of $1 + 3\sqrt{5}$?

Is $3\sqrt{5} - 1$ the conjugate of $1 + 3\sqrt{5}$?

Is $3\sqrt{5} - 1$ the conjugate of $3\sqrt{5} + 1$?

Is $-3\sqrt{5} - 1$ the conjugate of $3\sqrt{5} + 1$?

Is $-3\sqrt{5} - 1$ the conjugate of $3\sqrt{5} - 1$?

Worked Example	Your Turn
Rationalise: a) $\frac{6}{4+\sqrt{3}}$ b) $\frac{6}{\sqrt{3}+5}$	Rationalise: a) $\frac{6}{4-\sqrt{3}}$ b) $\frac{6}{\sqrt{3}+4}$

Worked Example	Your Turn
Rationalise: a) $\frac{6}{4+2\sqrt{3}}$ b) $\frac{6}{2\sqrt{3}+5}$	Rationalise: a) $\frac{6}{4-2\sqrt{3}}$ b) $\frac{6}{2\sqrt{3}+4}$

Worked Example	Your Turn
A rectangle has an area of $(2+\sqrt{2})\ cm^2$ and a width of $(3\sqrt{2}-4)\ cm$. Find the length of the rectangle in the form $a+b\sqrt{2}$	A rectangle has an area of $(15-6\sqrt{3})\ cm^2$ and a width of $(2\sqrt{3}-3)\ cm$. Find the length of the rectangle in the form $a+b\sqrt{3}$

Worked Example	Your Turn
Rationalise: $\dfrac{4}{\frac{1}{\sqrt{3}}+\sqrt{3}}$	Rationalise: $\dfrac{3}{\sqrt{2}+\frac{1}{\sqrt{2}}}$

Worked Example	Your Turn
Find in its simplest form $a : b$, given: $a = \sqrt{5} + \sqrt{c}$ $b = \sqrt{80} + \sqrt{d}$ c and d are positive integers $c : d = 1 : 16$	Find in its simplest form $a : b$, given: $a = \sqrt{7} + \sqrt{c}$ $b = \sqrt{63} + \sqrt{d}$ c and d are positive integers $c : d = 1 : 9$

Extra Notes

3 Algebraic Fractions

Simplifying Algebraic Fractions

Worked Example	Your Turn
Simplify: $\frac{6x}{10x^2}$	Simplify: $\frac{6x}{10x^3}$

Worked Example	Your Turn
Simplify: a) $\frac{5x+10}{x+2}$ b) $\frac{x+2}{x^2+5x+6}$ c) $\frac{2x^2+14+24}{3x^2-15x-108}$	Simplify: a) $\frac{3x+12}{x+4}$ b) $\frac{x+3}{x^2+7x+12}$ c) $\frac{2x^2+14x+24}{3x^2+15x+18}$

Multiplying and Dividing Algebraic Fractions

Worked Example	Your Turn
Simplify: a) $\frac{6x}{2y} \times \frac{4y}{5}$ b) $\frac{6x}{2y} \div \frac{4y}{5}$	Simplify: a) $\frac{5a}{2b} \times \frac{5b}{30}$ b) $\frac{5a}{2b} \div \frac{5b}{30}$

Worked Example	Your Turn
Simplify fully: $\frac{2x^2+7x-15}{x^2-36} \times \frac{2x+12}{2x^3-3x^2}$	Simplify fully: $\frac{2x^2-17x+21}{x^2-49} \times \frac{5x^2+15x}{2x^2-3x}$

Worked Example	Your Turn
Simplify fully: $$\frac{3x^2+8x+5}{x^2-25} \div \frac{3x^2+5x}{5x^2-25x}$$	Simplify fully: $$\frac{3x^2-x-14}{9x^2-4} \div \frac{x+2}{3x^2+2x}$$

Adding and Subtracting Algebraic Fractions

Worked Example	Your Turn
Simplify: $\frac{x}{5}+\frac{3x}{8}$	Simplify: $\frac{5}{x}+\frac{8}{3x}$

Worked Example	Your Turn
Write the following expression as a single fraction in its simplest form: $\frac{8}{2y}+\frac{3}{3x^2y^2}$	Write the following expression as a single fraction in its simplest form: $\frac{5}{6b}+\frac{3}{4a^3b}$

Worked Example	Your Turn
Simplify: a) $\frac{5x+2}{3}+\frac{x-3}{2}$ b) $\frac{5x+2}{3}-\frac{x-3}{2}$	Simplify: a) $\frac{4x+5}{2}+\frac{x-1}{3}$ b) $\frac{4x+5}{2}-\frac{x-1}{3}$

Worked Example	Your Turn
Write the following expression as a single fraction in its simplest form: $\frac{1}{x^2 - 1} + \frac{1}{x + 1}$	Write the following expression as a single fraction in its simplest form: $\frac{1}{a^2 - 9} + \frac{1}{a - 1}$

Worked Example	Your Turn
Write the following expression as a single fraction in its simplest form: $\frac{6}{x^2 - 4} - \frac{14}{x + 2}$	Write the following expression as a single fraction in its simplest form: $\frac{4}{a^2 - 9} - \frac{5}{a - 3}$

Worked Example	Your Turn
Write as a single simplified fraction: $3-(x-4)\div\frac{x^2-16}{x-5}$	Write as a single simplified fraction: $5-(x-2)\div\frac{x^2-4}{x+3}$

Solving Equations with Algebraic Fractions

Worked Example	Your Turn
Solve $\frac{x+4}{2}+\frac{x+1}{5}=5$	Solve $\frac{x-4}{2}+\frac{x-1}{5}=2$

Worked Example	Your Turn
Solve $\frac{x+1}{3} - \frac{x-3}{5} = 1$	Solve $\frac{x+2}{3} - \frac{x-6}{5} = 2$

Worked Example	Your Turn
Solve $\frac{4}{x+6}+\frac{5}{x+8}=1$	Solve $\frac{4}{x+3}+\frac{5}{x+4}=2$

Worked Example	Your Turn
Solve $\frac{3}{x-6}+\frac{4}{x-9}=1$	Solve $\frac{3}{x-2}+\frac{4}{x-3}=3$

Rearranging Formulae with Algebraic Fractions

Worked Example	Your Turn
Make x the subject: $\frac{y}{a}+\frac{3y}{x-2}=5$	Make x the subject: $\frac{5p}{x+3}+\frac{p}{b}=2$

Worked Example	Your Turn
Make x the subject: $\frac{1}{x} - \frac{1}{y} = \frac{1}{z}$	Make p the subject: $\frac{1}{p} + \frac{1}{q} = \frac{1}{r}$

Worked Example	Your Turn
Make x the subject: $\frac{5x}{A} - \frac{b}{c} = \frac{4x - d}{a}$	Make p the subject: $\frac{3x}{E} - \frac{f}{g} = \frac{5x - h}{F}$

Extra Notes