



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2023 Mathematics 2024

Unit 19 Tasks – Part 1

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Unit 19 Tasks – Part 2

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Unit 19 Tasks – Part 3

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Contents Page

- 1 [Advanced Indices](#)
- 2 [Calculating with Surds](#)
- 3 [Algebraic Fractions](#)

1 Advanced Indices

Fluency Practice

For the following terms,

a) Write with a positive exponent

b) Evaluate:

- | | | | | | | | |
|-----|--------------|-----|-------------|-----|--------------|-----|--------------|
| 1. | 26^{-1} | 2. | 2^{-1} | 3. | 10^{-2} | 4. | 2^{-2} |
| 5. | -26^{-1} | 6. | -2^{-1} | 7. | -10^{-2} | 8. | -2^{-2} |
| 9. | $(-26)^{-1}$ | 10. | $(-2)^{-1}$ | 11. | $(-10)^{-2}$ | 12. | $(-2)^{-2}$ |
| 13. | 2^{-5} | 12. | -7^{-3} | 13. | $(-8)^{-2}$ | 14. | $(-10)^{-5}$ |
| 15. | $(-4)^{-3}$ | 16. | 9^{-4} | 17. | -11^{-2} | 18. | $(-3)^{-3}$ |
| 19. | -3^{-4} | 20. | 25^{-2} | 21. | $(-2)^{-6}$ | 22. | 15^{-2} |

Decide if there are mistakes in the following and explain how to fix the answer:

a) $4^{-2} = -16$ b) $10^{-3} = \frac{1}{30}$

Fluency Practice

1. Write in index form:

a) $\frac{1}{5^2}$

b) $\frac{1}{3^4}$

c) $\frac{1}{8^3}$

d) $\frac{1}{4^5}$

e) $\frac{1}{10^3}$

f) $\frac{1}{2^6}$

2. Write in the form 2^n :

a) $\frac{1}{2}$

b) $\frac{1}{4}$

c) $\frac{1}{8}$

d) $\frac{1}{32}$

e) $\frac{1}{64}$

f) $\frac{1}{256}$

3. Write in the form 5^n :

a) $\frac{1}{5}$

b) $\frac{1}{125}$

c) $\frac{1}{625}$

d) $\frac{1}{3125}$

e) $\frac{1}{78,125}$

f) $\frac{1}{390,625}$

4. Arrange in ascending order:

$$\frac{1}{50}$$

$$5^{-2}$$

$$\frac{3}{10}$$

$$2^{-3}$$

Fluency Practice

Simplify the following:

1. $\left(\frac{1}{5}\right)^{-1}$

2. $\left(\frac{2}{5}\right)^{-1}$

3. $\left(\frac{3}{5}\right)^{-1}$

4. $\left(\frac{4}{5}\right)^{-1}$

5. $\left(\frac{1}{4}\right)^{-1}$

6. $\left(\frac{3}{4}\right)^{-1}$

7. $\left(\frac{1}{3}\right)^{-2}$

8. $\left(\frac{2}{3}\right)^{-2}$

9. $\left(\frac{1}{5}\right)^{-2}$

10. $\left(\frac{2}{5}\right)^{-2}$

11. $\left(\frac{3}{5}\right)^{-2}$

12. $\left(\frac{4}{5}\right)^{-2}$

13. $\left(\frac{2}{3}\right)^{-3}$

14. $\left(\frac{4}{3}\right)^{-3}$

15. $\left(\frac{7}{8}\right)^{-2}$

16. $\left(-\frac{1}{10}\right)^{-4}$

17. $\left(-\frac{4}{9}\right)^{-3}$

18. $\left(-\frac{9}{10}\right)^{-2}$

19. $\left(-\frac{5}{3}\right)^{-3}$

20. $\left(-\frac{3}{2}\right)^{-4}$

21. $\left(-\frac{3}{10}\right)^{-3}$

22. $\left(-\frac{3}{2}\right)^{-3}$

23. $\left(-\frac{8}{5}\right)^{-3}$

24. $\left(-\frac{3}{8}\right)^{-2}$

25. $\left(-\frac{6}{5}\right)^{-2}$

Fluency Practice

Write down the reciprocal of:

- (a) 4 (b) 10 (c) 15
(d) -3 (e) 0.4 (f) 1
(g) $\frac{3}{4}$ (h) $\frac{1}{8}$ (i) $\frac{2}{7}$

Evaluate:

- (a) 4^{-1} (b) 10^{-1}
(c) 15^{-1} (d) $(-3)^{-1}$
(e) a^{-1} (f) $\left(\frac{2}{3}\right)^{-1}$
(g) $\left(\frac{8}{5}\right)^{-1}$ (h) $\left(1\frac{3}{7}\right)^{-1}$

Evaluate:

- (a) 4^{-2} (b) 10^{-3}
(c) 15^{-2} (d) $(-3)^{-2}$
(e) b^{-3} (f) $\left(\frac{1}{3}\right)^{-2}$
(g) $\left(\frac{2}{5}\right)^{-3}$ (h) 0.1^{-2}

Find x .

- (a) $\frac{1}{10} = 10^x$ (b) $\frac{1}{1000} = 10^x$
(c) $\frac{1}{2} = 2^x$ (d) $\frac{1}{8} = 2^x$
(e) $\frac{1}{16} = 2^x$ (f) $\frac{1}{256} = 2^x$

Fluency Practice

Negative Powers

startingpointsmaths.com

1 Complete each collection of equivalent expressions.

Example

(a)

5^{-2}	$\frac{1}{25}$	$\frac{1}{64}$
$1 \div 5 \div 5$	$\frac{1}{5} \times \frac{1}{5}$	
	25^{-1}	
	$\frac{1}{5^2}$	

(b)

2^{-3}		

(c)

(d)

(e)

(f)

(g)

Fluency Practice

positive, negative & zero indices

Match these calculations to their answers, which are jumbled up below. Record your answers in the table at the bottom.

A $3^2 \times 2^0$	B $3^{-1} \times 3$	C $(-2)^2$	D $6^2 \div 6^1$	E 30×3^{-1}
F 2×3^{-1}	G $2^{-1} \times 6^2$	H $5^{-1} \times 5^{-1}$	I 10×5^{-1}	J $-2^1 \times 2^1$
K $2^2 \times 2^3$	L $25 \div 5^0$	M 11×2^0	N $3^1 \times 3^0$	O $5^{-1} \times 2^{-2}$
P $2^{-1} \div 2^1$	Q $(3^2)^{-1}$	R $2^{-1} \times 2 \times 2^3$	S $(-2 \times -3)^{-1}$	T $2^{-2} \times 3$
U $2 \times 5^2 \times 10^{-1}$	V 63×3^{-2}	W $12 \div 6^0$	X $5^{-1} + 5^{-1}$	Y $(4^{-2} \times 4^1)^2$

$\frac{2}{3}$ 1 $\frac{1}{25}$ 6 $\frac{1}{6}$ $\frac{1}{9}$ $\frac{1}{16}$ 10 8 $\frac{3}{4}$ 5 2 $\frac{2}{5}$
 32 7 18 9 3 25 11 $\frac{1}{20}$ 4 -4 12 $\frac{1}{4}$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Fluency Practice

TASK

Evaluate:

Calculations with Negative Indices

(a) $\left(\frac{3}{4}\right)^{-1} + 5^{-1}$

(d) $\left(\frac{2}{3} + \frac{4}{5}\right)^{-1}$

(g) $6^{-2} + 3^{-3}$

(b) $\left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{12}\right)^{-2}$

(e) $\left(2\frac{1}{3}\right)^{-2} - \left(1\frac{3}{4}\right)^{-2}$

(h) $\left(\frac{2}{x}\right)^{-3} + \left(\frac{5}{x}\right)^{-1}$

(c) $\left(\frac{4}{7}\right)^{-2} + \left(1\frac{5}{11}\right)^{-1}$

(f) $0.4^{-1} + 0.75^{-2}$

(i) $\left(\frac{1}{3x}\right)^{-2} + \left(\frac{3}{2x}\right)^{-3}$

Fluency Practice

TASK

Show that

Calculations with Negative Indices

(a) $\left(\frac{3}{4}\right)^{-1} + 5^{-1} = 1\frac{8}{15}$

(d) $\left(\frac{2}{3} + \frac{4}{5}\right)^{-1} = \frac{15}{22}$

(g) $6^{-2} + 3^{-3} = \frac{7}{108}$

(b) $\left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{12}\right)^{-2} = 152$

(e) $\left(2\frac{1}{3}\right)^{-2} - \left(1\frac{3}{4}\right)^{-2} = -\frac{1}{7}$

(h) $\left(\frac{2}{x}\right)^{-3} + \left(\frac{5}{x}\right)^{-1} = \frac{5x^3 + 8x}{40}$

(c) $\left(\frac{4}{7}\right)^{-2} + \left(1\frac{5}{11}\right)^{-1} = 3\frac{3}{4}$

(f) $0.4^{-1} + 0.75^{-2} = 4\frac{5}{18}$

(i) $\left(\frac{1}{3x}\right)^{-2} + \left(\frac{3}{2x}\right)^{-3} = \frac{8x^3 + 243x^2}{27}$

Fluency Practice

Rewrite the following with positive indices:

1. x^{-6}

2. x^{-7}

3. a^{-8}

4. a^{-10}

5. p^{-11}

6. k^{-14}

7. $2s^{-3}$

8. $7z^{-3}$

9. $3d^{-4}$

10. $3d^{-7}$

11. $14x^{-1}$

12. $\frac{12}{3}x^{-2}$

13. $\frac{16x^{-5}}{4}$

14. $\frac{a^{-9}}{2}$

15. $\frac{e^{-5}}{6}$

16. $\frac{y^{-8}}{3}$

17. $\frac{g^{-7}}{10}$

18. $\frac{2x^{-4}}{3}$

19. $\frac{4f^{-5}}{7}$

20. $\frac{5x^{-8}}{9}$

21. $(3x)^{-3}$

22. $(-3x)^{-3}$

23. $(5x)^{-2}$

24. $(-5x)^{-2}$

25. $(-7a)^{-3}$

Fluency Practice

Task 1

Rewrite the following with negative indices:

- | | | | | | | | | | |
|-----|------------------|-----|--------------------|-----|--------------------|-----|---------------------|-----|--------------------|
| 1. | $\frac{1}{x^4}$ | 2. | $\frac{1}{x}$ | 3. | $\frac{1}{d^{10}}$ | 4. | $\frac{1}{b^{13}}$ | 5. | $\frac{1}{y^x}$ |
| 6. | $\frac{1}{m^n}$ | 7. | $\frac{3}{x^{13}}$ | 8. | $\frac{5}{x^9}$ | 9. | $\frac{7}{y^{10}}$ | 10. | $\frac{3}{x^{11}}$ |
| 11. | $\frac{4}{x^3}$ | 12. | $\frac{12}{3x^2}$ | 13. | $\frac{a}{x^n}$ | 14. | $\frac{1}{2x^4}$ | 15. | $\frac{1}{3x^7}$ |
| 16. | $\frac{1}{8a^5}$ | 17. | $\frac{1}{7b^6}$ | 18. | $\frac{3}{4x^5}$ | 19. | $\frac{2}{5x^{12}}$ | 20. | $\frac{1}{8a^5}$ |

Task 2

Match the equivalent pairs of expressions

$$(3x)^{-1} \quad (3x)^{-2} \quad 3x^{-1} \quad x^3 \quad x^{-3} \quad 3x^{-2}$$

$$\frac{1}{x^3} \quad \frac{1}{x^{-3}} \quad \frac{1}{9x^2} \quad \frac{3}{x} \quad \frac{1}{3x} \quad \frac{3}{x^2}$$

Fluency Practice

Task 1

Expand and fully simplify:

- | | | | |
|---|--|--|--|
| 1. $x(x^3 - 4)$ | 2. $x(x^2 - x^{-2})$ | 3. $2y^{-1}(3y^2 - 2y^3)$ | |
| 4. $2y^2(3y^4 + 5y^{-2})$ | 5. $m(2m^{-1} - 4m^{-4})$ | 6. $3c(4c^3 - 6c^{-4})$ | |
| 7. $4a^2(2a^{-1} + 3a^{-2})$ | 8. $t^{-2}(3t^{-2} - t^2)$ | 9. $4d^{\frac{1}{2}}(3d^{\frac{1}{2}} - d^{-\frac{1}{2}})$ | |
| 10. $a^{-2}(a + a^{-1})$ | 11. $2w^5\left(\frac{1}{w} + 4w^{-2}\right)$ | 12. $x^2\left(x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)$ | |
| 13. $x^{\frac{1}{2}}(x^4 + x^3)$ | 14. $u^{\frac{1}{2}}(3u + u^3)$ | 15. $x^{-\frac{1}{4}}(x^8 + x^6)$ | |
| 16. $3m^{\frac{3}{2}}\left(m^{\frac{3}{2}} + \frac{3}{m^{\frac{1}{2}}}\right)$ | 17. $n^{\frac{1}{3}}\left(2n^{\frac{4}{3}} - \frac{1}{n^{\frac{2}{3}}}\right)$ | 18. $b^{\frac{1}{3}}(b^2 + 2b^{-1})$ | |
| 19. $x^{\frac{1}{3}}(x^{-2} + x^4)$ | 20. $x^{\frac{1}{2}}\left(\frac{3}{x^4} - \frac{2}{x^5}\right)$ | 21. $p^3(p^{-2} + p^3)$ | |
| 22. $x^{-3}(x^5 + x^2)$ | 23. $5x^{\frac{1}{2}}\left(2x^{\frac{1}{2}} + 3x^{\frac{3}{2}}\right)$ | 24. $3a^{-1}(4a^3 + 2a)$ | |
| 25. $2u^{-5}(u + 2u^5)$ | 26. $3m^2(2m^2 + 7m^{-4})$ | 27. $a^{\frac{1}{2}}\left(a^{\frac{1}{2}} + a^{-\frac{1}{2}}\right)$ | |
| 28. $p^{\frac{1}{3}}\left(p^{\frac{2}{3}} + p^{\frac{1}{3}}\right)$ | 29. $e^{-\frac{2}{3}}\left(e^{\frac{7}{3}} - 2e^{\frac{2}{3}}\right)$ | 30. $5n^4\left(n^{-2} + \frac{2}{n^3}\right)$ | |
| 31. $p^4\left(3p^{-4} - \frac{2}{p^3}\right)$ | 32. $3a\left(a^{\frac{1}{2}} + 2a^{-2}\right)$ | 33. $x^{\frac{1}{2}}(2x - 3)$ | |
| 34. $2p^4\left(p^{\frac{1}{4}} - p\right)$ | 35. $x^{\frac{2}{3}}\left(x^{\frac{1}{2}} + x^{\frac{1}{4}}\right)$ | 36. $x^{\frac{1}{5}}\left(2x^{\frac{9}{5}} + \frac{3}{x^{\frac{3}{5}}}\right)$ | |
| 37. $2a^{\frac{5}{3}}\left(\frac{1}{a^{\frac{2}{3}}} - 4a^{\frac{4}{3}}\right)$ | 38. $x^{-4}\left(x^{-\frac{1}{2}} - x^2\right)$ | 39. $x^2\left(\frac{1}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{1}{3}}}\right)$ | |
| 40. $x^{-\frac{1}{2}}\left(\frac{1}{x^{-\frac{1}{4}}} - \frac{3}{x^{\frac{1}{6}}}\right)$ | | | |

Task 2

Lauren writes down the following statement:

$$p^{\frac{1}{3}}\left(p^{\frac{2}{3}} - p^{-\frac{1}{3}}\right) = p - 1$$

Is this statement true? Justify your answer.

Fluency Practice

Expand and fully simplify:

1. $(m^{-2} + m^3)(m^{-2} + m)$

2. $(x^{-3} + x^{-6})(x^7 + x^{-2})$

3. $(x + x^{-1})^2$

4. $(b^{\frac{1}{2}} + 1)^2$

5. $(3k^{\frac{1}{2}} - 2)^2$

6. $\left(x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{2}}}\right)\left(x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{2}}}\right)$

7. $\left(c^{\frac{1}{2}} + \frac{1}{c^{\frac{1}{2}}}\right)^2$

8. $\left(x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{4}}}\right)\left(x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{4}}}\right)$

9. $\left(x^{\frac{2}{3}} + y^{\frac{1}{2}}\right)\left(x^{\frac{1}{3}} - y^{\frac{1}{2}}\right)$

10. $\left(5c^{\frac{3}{4}} + 9c^{\frac{1}{2}}\right)\left(6c^{\frac{1}{3}} - 2c^{\frac{1}{4}}\right)$

Fluency Practice

Expand and simplify

a $x(x^2 - x^{-1})$

b $2x^3(x^{-1} + 3)$

c $x^{-1}(3x - x^3)$

d $4x^{-2}(3x^5 + 2x^3)$

e $\frac{1}{2}x^2(6x + 4x^{-1})$

f $3x^{\frac{1}{2}}(x^{-\frac{1}{2}} - x^{\frac{3}{2}})$

g $x^{-\frac{3}{2}}(5x^2 + x^{\frac{7}{2}})$

h $x^{\frac{1}{3}}(3x^{\frac{5}{3}} - x^{-\frac{4}{3}})$

i $(x^2 + 1)(x^4 - 3)$

j $(2x^5 + x)(x^4 + 3)$

k $(x^2 - 2x^{-1})(x - x^{-2})$

l $(x^2 - x^{\frac{3}{2}})(x - x^{\frac{1}{2}})$

Fluency Practice

Task 1

Rewrite the following using the radical sign:

1. $x^{\frac{1}{5}}$
2. $x^{\frac{1}{6}}$
3. $x^{\frac{1}{7}}$
4. $x^{\frac{1}{8}}$
5. $x^{\frac{1}{9}}$
6. $x^{\frac{1}{10}}$
7. $x^{\frac{1}{m}}$
8. $x^{-\frac{1}{2}}$
9. $x^{-\frac{1}{3}}$
10. $x^{-\frac{1}{4}}$
11. $x^{-\frac{1}{5}}$
12. $x^{-\frac{1}{6}}$
13. $x^{-\frac{1}{7}}$
14. $x^{-\frac{1}{a}}$

Task 2

Rewrite the following using the radical sign:

1. $4x^{\frac{1}{3}}$
2. $4x^{-\frac{1}{3}}$
3. $\frac{1}{4}x^{\frac{1}{3}}$
4. $\frac{1}{4}x^{-\frac{1}{3}}$
5. $15y^{\frac{1}{10}}$
6. $15y^{-\frac{1}{10}}$
7. $\frac{1}{15}y^{\frac{1}{10}}$
8. $\frac{1}{15}y^{-\frac{1}{10}}$
9. $-14x^{\frac{1}{8}}$
10. $-14x^{-\frac{1}{8}}$
11. $-\frac{1}{14}x^{\frac{1}{8}}$
12. $-\frac{1}{14}x^{-\frac{1}{8}}$
13. $18c^{\frac{1}{2}}$
14. $18c^{-\frac{1}{2}}$
15. $\frac{1}{18}c^{\frac{1}{2}}$
16. $\frac{1}{18}c^{-\frac{1}{2}}$
17. $-20k^{\frac{1}{9}}$
18. $-20k^{-\frac{1}{9}}$
19. $-\frac{k^{\frac{1}{9}}}{20}$
20. $-\frac{k^{-\frac{1}{9}}}{20}$
21. $3c^{\frac{1}{7}}$
22. $3c^{-\frac{1}{7}}$
23. $\frac{c^{\frac{1}{7}}}{3}$
24. $\frac{1}{3}c^{-\frac{1}{7}}$

Task 3

Rewrite the following using fractional exponents:

1. $\sqrt[9]{a}$
2. $\sqrt[4]{k}$
3. $\sqrt[5]{p}$
4. $\sqrt[8]{x}$
5. $\sqrt[3]{m}$
6. $\sqrt[7]{z}$
7. $\sqrt[10]{w}$
8. $\sqrt[4]{t}$
9. $\frac{1}{\sqrt[9]{a}}$
10. $\frac{1}{\sqrt[4]{k}}$
11. $\frac{1}{\sqrt[10]{l}}$
12. $\frac{1}{\sqrt[7]{h}}$
13. $\frac{1}{\sqrt[13]{p}}$
14. $\frac{1}{\sqrt[7]{m}}$
15. $\frac{1}{\sqrt[10]{v}}$

Fluency Practice

Task 1

Evaluate

- | | | | |
|---|--|--|---|
| 1. $25^{\frac{1}{2}}$ | 2. $25^{-\frac{1}{2}}$ | 3. $16^{\frac{1}{2}}$ | 4. $16^{-\frac{1}{2}}$ |
| 5. $9^{\frac{1}{2}}$ | 6. $9^{-\frac{1}{2}}$ | 7. $100^{\frac{1}{2}}$ | 8. $100^{-\frac{1}{2}}$ |
| 9. $36^{\frac{1}{2}}$ | 10. $36^{-\frac{1}{2}}$ | 11. $8^{\frac{1}{3}}$ | 12. $8^{-\frac{1}{3}}$ |
| 13. $125^{\frac{1}{3}}$ | 14. $125^{-\frac{1}{3}}$ | 15. $1000^{\frac{1}{3}}$ | 16. $1000^{-\frac{1}{3}}$ |
| 17. $16^{\frac{1}{4}}$ | 18. $16^{-\frac{1}{4}}$ | 19. $32^{\frac{1}{5}}$ | 20. $32^{-\frac{1}{5}}$ |
| 21. $216^{\frac{1}{3}}$ | 22. $216^{-\frac{1}{3}}$ | 23. $512^{\frac{1}{3}}$ | 24. $512^{-\frac{1}{3}}$ |
| 25. $\left(\frac{25}{49}\right)^{\frac{1}{2}}$ | 26. $\left(\frac{25}{49}\right)^{-\frac{1}{2}}$ | 27. $\left(\frac{36}{121}\right)^{\frac{1}{2}}$ | 28. $\left(\frac{36}{121}\right)^{-\frac{1}{2}}$ |
| 29. $\left(\frac{625}{1296}\right)^{\frac{1}{4}}$ | 30. $\left(\frac{625}{1296}\right)^{-\frac{1}{4}}$ | 31. $\left(\frac{216}{343}\right)^{\frac{1}{3}}$ | 32. $\left(\frac{216}{343}\right)^{-\frac{1}{3}}$ |
| 33. $\left(\frac{343}{1000}\right)^{\frac{1}{3}}$ | 34. $\left(\frac{343}{1000}\right)^{-\frac{1}{3}}$ | 35. $\left(\frac{125}{729}\right)^{\frac{1}{3}}$ | 36. $\left(\frac{125}{729}\right)^{-\frac{1}{3}}$ |

Task 2

- Given that $y = x^{\frac{1}{3}}$, find y when $x = 64$.
- Given that $y = x^{\frac{1}{2}}$, find y when $x = 16$.
- Given that $y = x^{\frac{1}{4}}$, find y when $x = 81$.
- Given that $y = x^{\frac{1}{3}}$, find y when $x = 125$.

Fluency Practice

Task 3

For $a > 1$, put the following in order of size from smallest to largest:

$$a^0, a^2, a, a^{-2}, a^{\frac{1}{2}}$$

Task 4

Solve for a :

$$\frac{x \times x^4 \times x^a}{\sqrt{x}} = \sqrt{\frac{(x^a)^8}{\left(\frac{1}{x^4}\right)}}$$

Task 5

The statement $x > x^{\frac{1}{2}}$ is:

Always True

Sometimes True

Never True

Fluency Practice

Find the value of:

- (a) $4^{1/2}$ (b) $100^{1/2}$
- (c) $196^{1/2}$ (d) $49^{1/2}$
- (e) $64^{1/3}$ (f) $512^{1/3}$
- (g) $1000^{1/3}$ (h) $16^{1/4}$

Evaluate:

- (a) $\left(\frac{4}{9}\right)^{1/2}$ (b) $\left(\frac{1}{36}\right)^{1/2}$
- (c) $\left(\frac{225}{16}\right)^{1/2}$ (d) $\left(\frac{25}{169}\right)^{1/2}$
- (e) $\left(\frac{64}{27}\right)^{1/3}$ (f) $\left(\frac{8}{729}\right)^{1/3}$
- (g) $\left(\frac{1}{1000}\right)^{1/3}$ (h) $\left(\frac{16}{10000}\right)^{1/4}$

Evaluate:

- (a) $9^{-1/2}$ (b) $225^{-1/2}$
- (c) $25^{-1/2}$ (d) $512^{-1/3}$
- (e) $343^{-1/3}$ (f) $8^{-1/3}$
- (g) $\left(\frac{8}{343}\right)^{-1/3}$ (h) $\left(\frac{25}{9}\right)^{-1/2}$

Solve these equations.

- (a) $x^{1/2} = 9$ (b) $x^{1/2} = 11$
- (c) $x^{1/3} = 5$ (d) $x^{-1/2} = \frac{1}{4}$

Problem Solving

Which is bigger?

$$10^{\frac{1}{2}} \quad \text{or} \quad 20^{\frac{1}{3}}$$

$$20^{\frac{1}{2}} \quad \text{or} \quad 40^{\frac{1}{3}}$$

$$40^{\frac{1}{2}} \quad \text{or} \quad 80^{\frac{1}{3}}$$

$$5^{\frac{1}{2}} \quad \text{or} \quad 10^{\frac{1}{3}}$$

Fluency Practice

Task 1

Write in index form:

1. $\sqrt{a^3}$

2. $5\sqrt{a^3}$

3. $\frac{1}{\sqrt{a^3}}$

4. $\frac{5}{\sqrt{a^3}}$

5. $\sqrt{w^5}$

6. $4\sqrt{w^5}$

7. $\frac{1}{\sqrt{w^5}}$

8. $\frac{4}{\sqrt{w^5}}$

9. $\sqrt[3]{x^2}$

10. $9\sqrt[3]{x^2}$

11. $\frac{1}{\sqrt[3]{x^2}}$

12. $\frac{9}{\sqrt[3]{x^2}}$

13. $\sqrt[3]{w^4}$

14. $-6\sqrt[3]{w^4}$

15. $\frac{1}{\sqrt[3]{w^4}}$

16. $-\frac{6}{\sqrt[3]{w^4}}$

17. $\sqrt[5]{m^2}$

18. $-2\sqrt[5]{m^2}$

19. $\frac{1}{\sqrt[5]{m^2}}$

20. $-\frac{2}{\sqrt[5]{m^2}}$

21. $\sqrt[9]{k^4}$

22. $-12\sqrt[9]{k^4}$

23. $\frac{1}{\sqrt[9]{k^4}}$

24. $-\frac{12}{\sqrt[9]{k^4}}$

25. $\sqrt[6]{x^5y^7}$

26. $\sqrt[4]{a^7b^3c}$

27. $\sqrt[7]{p^2q^3r^4}$

28. $\sqrt[\frac{1}{7}]{p^2q^3r^4}$

Fluency Practice

Task 2

Write with a radical sign in simplest form:

- | | | | | |
|---------------------------|---------------------------|----------------------------|---------------------------|----------------------------|
| 1. $x^{\frac{3}{4}}$ | 2. $m^{\frac{7}{5}}$ | 3. $p^{\frac{3}{2}}$ | 4. $y^{\frac{5}{8}}$ | 5. $c^{\frac{4}{3}}$ |
| 6. $4(x^{\frac{3}{4}})$ | 7. $9(m^{\frac{7}{5}})$ | 8. $16(p^{\frac{3}{2}})$ | 9. $5(y^{\frac{5}{8}})$ | 10. $10(c^{\frac{4}{3}})$ |
| 11. $x^{-\frac{3}{4}}$ | 12. $m^{-\frac{7}{5}}$ | 13. $p^{-\frac{3}{2}}$ | 14. $y^{-\frac{5}{8}}$ | 15. $c^{-\frac{4}{3}}$ |
| 16. $4(x^{-\frac{3}{4}})$ | 17. $9(m^{-\frac{7}{5}})$ | 18. $16(p^{-\frac{3}{2}})$ | 19. $5(y^{-\frac{5}{8}})$ | 20. $10(c^{-\frac{4}{3}})$ |

Task 3

Find equivalent terms from the following:

x^{-3}	x^3	x^2	\sqrt{x}	$\sqrt[3]{x}$	x^{-1}	$2x^{-1}$	$2\sqrt{x}$
$x^{\frac{3}{2}}$	$x^{\frac{1}{2}}$	$\sqrt{x^3}$	$x\sqrt{x}$	$x^{-\frac{1}{3}}$	x^{-2}	$\frac{1}{2x^2}$	$x^{\frac{2}{3}}$
$x^{-\frac{1}{2}}$	$2x^{-2}$	$\frac{\sqrt{x}}{x}$	x^4	x^{-4}	$\frac{1}{2\sqrt{x}}$	x^0	$x^{-\frac{3}{2}}$
$\frac{1}{2}x^{-\frac{1}{2}}$	$\frac{1}{2}x^{-2}$	$x^{\frac{1}{3}}$	x	$\frac{2}{\sqrt{x}}$	$\frac{1}{x}$	$\frac{1}{2x}$	$\sqrt[3]{x^2}$

Fluency Practice

Task 1

Evaluate:

- | | | | | | | | | | |
|-----|--|-----|---|-----|---|-----|---|-----|--|
| 1. | $8^{\frac{4}{3}}$ | 2. | $27^{\frac{5}{3}}$ | 3. | $64^{\frac{5}{6}}$ | 4. | $81^{\frac{3}{2}}$ | 5. | $625^{\frac{3}{4}}$ |
| 6. | $9^{\frac{3}{2}}$ | 7. | $100^{\frac{3}{2}}$ | 8. | $3125^{\frac{2}{5}}$ | 9. | $8^{\frac{5}{3}}$ | 10. | $4^{\frac{5}{2}}$ |
| 11. | $81^{\frac{3}{4}}$ | 12. | $125^{-\frac{2}{3}}$ | 13. | $243^{-\frac{2}{5}}$ | 14. | $32^{-\frac{2}{5}}$ | 15. | $16^{-\frac{3}{4}}$ |
| 16. | $4^{-\frac{5}{2}}$ | 17. | $64^{-\frac{2}{3}}$ | 18. | $8^{-\frac{4}{3}}$ | 19. | $32^{-\frac{4}{5}}$ | 20. | $125^{-\frac{2}{3}}$ |
| 21. | $1000^{-\frac{2}{3}}$ | 22. | $81^{-\frac{3}{4}}$ | 23. | $625^{-\frac{3}{4}}$ | 24. | $(-8)^{\frac{4}{3}}$ | 25. | $(-27)^{\frac{5}{3}}$ |
| 26. | $(-3125)^{\frac{2}{5}}$ | 27. | $(-8)^{\frac{5}{3}}$ | 28. | $(-125)^{-\frac{2}{3}}$ | 29. | $(-243)^{-\frac{2}{5}}$ | 30. | $(-32)^{-\frac{2}{5}}$ |
| 31. | $(-64)^{-\frac{2}{3}}$ | 32. | $(-8)^{-\frac{4}{3}}$ | 33. | $(-32)^{-\frac{4}{5}}$ | 34. | $(-125)^{-\frac{2}{3}}$ | 35. | $(-1000)^{-\frac{2}{3}}$ |
| 36. | $\left(\frac{4}{25}\right)^{\frac{3}{2}}$ | 37. | $\left(\frac{4}{25}\right)^{-\frac{3}{2}}$ | 38. | $\left(\frac{8}{125}\right)^{\frac{2}{3}}$ | 39. | $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$ | 40. | $\left(\frac{49}{100}\right)^{\frac{3}{2}}$ |
| 41. | $\left(\frac{49}{100}\right)^{-\frac{3}{2}}$ | 42. | $\left(\frac{8}{125}\right)^{\frac{2}{3}}$ | 43. | $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$ | 44. | $\left(\frac{16}{100}\right)^{\frac{5}{2}}$ | 45. | $\left(\frac{16}{100}\right)^{-\frac{5}{2}}$ |
| 46. | $\left(\frac{64}{1000}\right)^{\frac{4}{3}}$ | 47. | $\left(\frac{64}{1000}\right)^{-\frac{4}{3}}$ | 48. | $\left(\frac{16}{25}\right)^{\frac{3}{2}}$ | 49. | $\left(\frac{16}{25}\right)^{-\frac{3}{2}}$ | 50. | $\left(\frac{32}{243}\right)^{\frac{6}{5}}$ |

Fluency Practice

Task 2

a) Arrange the following in ascending order:

$$25^{\frac{1}{2}}, 8^{\frac{2}{3}}, 27^{\frac{1}{3}}$$

b) Which is the odd one out? Explain your answer.

$$64^{\frac{1}{2}}, 16^{\frac{3}{4}}, 9^{\frac{2}{3}}, 4^{\frac{3}{2}}$$

Task 3

Gina has completed her homework.

Can you spot any mistakes?

Question 1

Work out $9^{\frac{1}{2}}$

4.5

Question 2

Work out $27^{\frac{2}{3}}$

18

Task 4

a) Given that $y = x^{\frac{3}{5}}$, find y when $x = 32$.

b) Given that $y = x^{\frac{3}{2}}$, find y when $x = 49$.

c) Given that $y = x^{\frac{2}{3}}$, find y when $x = 1000$.

d) Given that $y = x^{\frac{3}{4}}$, find y when $x = 81$.

e) Given that $y = x^{\frac{5}{6}}$, find y when $x = 64$.

f) Given that $y = x^{\frac{3}{4}}$, find y when $x = 10,000$.

Fluency Practice

Task 5

Find equivalent pairs from the following:

$8^{\frac{2}{3}}$	$(-1)^{\frac{1}{3}}$	$9^{\frac{3}{2}}$	$16^{\frac{1}{4}}$	$\frac{2}{3}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{64}{125}$
$\left(\frac{1}{2}\right)^{-3}$	$\left(\frac{2}{5}\right)^{-2}$	$\left(\frac{1}{9}\right)^{-\frac{1}{2}}$	$36^{-\frac{1}{2}}$	$\frac{1}{3}$	4	1	$\frac{1}{2}$
3^{-2}	$\left(\frac{16}{25}\right)^{\frac{3}{2}}$	3^{-1}	$4^{-\frac{3}{2}}$	8	-1	$\frac{1}{9}$	3
$8^{-\frac{1}{3}}$	$(-1)^{-2}$	$\left(\frac{1}{5}\right)^{-1}$	$\left(\frac{4}{9}\right)^{\frac{1}{2}}$	5	$\frac{25}{4}$	27	2

Task 6

- | | | | | |
|--|--|--|--------------------------|----------------------------|
| 1. $(4x)^{\frac{3}{4}}$ | 2. $(9m)^{\frac{7}{5}}$ | 3. $(16p)^{\frac{3}{2}}$ | 4. $(5y)^{\frac{5}{8}}$ | 5. $(10c)^{\frac{4}{3}}$ |
| 6. $(4x)^{-\frac{3}{4}}$ | 7. $(9m)^{-\frac{7}{5}}$ | 8. $(16p)^{-\frac{3}{2}}$ | 9. $(5y)^{-\frac{5}{8}}$ | 10. $(10c)^{-\frac{4}{3}}$ |
| 11. $\left(\frac{4x^6}{9y^4}\right)^{\frac{3}{5}}$ | 12. $\left(\frac{243x^5}{32y^{20}}\right)^{\frac{3}{5}}$ | 13. $\left(\frac{64x^3}{27y^6}\right)^{\frac{2}{3}}$ | | |

Fluency Practice

Task 7

Match up each expression from the first column with its partner from the second column, and write the answers in the table below

First column		Second column			
1	\sqrt{x}	A	$x^{\frac{3}{2}}$	K	$x^{\frac{1}{2}}$
2	$\frac{1}{x}$	B	$\frac{1}{2}x^{-3}$	L	$x^{\frac{9}{2}}$
3	$\frac{1}{\sqrt{x}}$	C	$\frac{1}{2}x^{-1}$	M	x^{-2}
4	$\sqrt[3]{x}$	D	x^{-4}	N	$x^{\frac{3}{2}}$
5	$\frac{1}{x^2}$	E	x^{-1}	O	$2x^{-1}$
6	$\frac{1}{2x^3}$	F	$\frac{1}{2}x$	P	$x^{\frac{1}{2}}$
7	$\frac{2}{x^3}$	G	x^4	Q	x^{-5}
8	$x\sqrt{x}$	H	x^2	R	$x^{\frac{2}{3}}$
9	$\frac{\sqrt{x}}{x^2}$	I	$2x$	S	$2x^{-3}$
10	$\frac{1}{x^{-2}}$	J	$x^{\frac{1}{5}}$	T	$x^{\frac{7}{2}}$

Table for Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	

Fluency Practice

2^{\square} \square^2 16	$\square^{\frac{3}{\square}}$ $\square^{\frac{2}{\square}}$ 8
3^{\square} $\square^{\frac{3}{\square}}$ 27	4^{\square} $\square^{\frac{2}{\square}}$ 4

2^{\square} \square^2 16	$\square^{\frac{3}{\square}}$ $\square^{\frac{3}{\square}}$ 8
3^{\square} $\square^{\frac{3}{\square}}$ 27	10^{\square} $\square^{\frac{2}{\square}}$ 10,000

Fluency Practice

Find the value of each of the following.

$$\left(\frac{25}{81}\right)^{\frac{1}{-2}}$$

$$\left(\frac{125}{1000}\right)^{\frac{2}{-3}}$$

$$\left(\frac{16}{81}\right)^{\frac{3}{-4}}$$

$$\left(\frac{216}{512}\right)^{\frac{2}{3}}$$

$$\left(2\frac{1}{4}\right)^{\frac{1}{-2}}$$

$$\left(3\frac{3}{8}\right)^{\frac{1}{3}}$$

1. Fill in the blanks

$$1. 100^{\frac{3}{2}} = (\sqrt{\square})^{\square} = 10^{\square} = ?$$

$$2. 16^{\frac{5}{4}} = (\sqrt{\square})^{\square} = \square^5 = ?$$

$$3. 64^{\frac{\square}{\square}} = (\sqrt[3]{64})^4 = \square^{\square} = ?$$

$$4. \square^{\frac{\square}{5}} = (\sqrt{32})^{\square} = \square^{\square} = 8$$

$$5. \square^{\frac{\square}{\square}} = (\sqrt[3]{\square})^{\square} = 3^5 = ?$$

$$6. \square^{\frac{3}{2}} = (\sqrt{\square})^{\square} = \square^{\square} = 216$$



[Edexcel GCSE June 2003-6H Q17b]

Let $x = 2^p$, $y = 2^q$,

If $xy = 32$ and $2xy^2 = 32$, find the value of p and the value of q .

Fluency Practice

Evaluate:

(a) $27^{2/3}$ (b) $125^{2/3}$

(c) $8^{2/3}$ (d) $(-8)^{2/3}$

(e) $49^{3/2}$ (f) $100^{3/2}$

(g) $16^{3/4}$ (h) $4^{5/2}$

Evaluate:

(a) $64^{-2/3}$ (b) $343^{-2/3}$

(c) $64^{-3/2}$ (d) $1000^{-2/3}$

(e) $25^{-3/2}$ (f) $81^{-3/2}$

(g) $10000^{-3/4}$ (h) $32^{-2/5}$

Evaluate:

(a) $\left(\frac{8}{343}\right)^{-2/3}$ (b) $\left(\frac{1}{125}\right)^{-2/3}$

(c) $\left(\frac{343}{64}\right)^{-2/3}$ (d) $\left(\frac{4}{9}\right)^{-3/2}$

(e) $\left(\frac{100}{9}\right)^{-3/2}$ (f) $\left(\frac{27}{8}\right)^{-4/3}$

(g) $\left(\frac{25}{4}\right)^{-3/2}$ (h) $\left(\frac{16}{10000}\right)^{-3/4}$

Find x .

(a) $x^{2/3} = 4$ (b) $x^{3/2} = 64$

(c) $y^x \times y^{2/3} = y^3$

Fluency Practice

Fractional Indices

Patrons are reminded that $\sqrt{x} \equiv x^{\frac{1}{2}}$ and $\sqrt[3]{x} \equiv x^{\frac{1}{3}}$.
 Also note that $\sqrt{4} = 2$. ($\sqrt{4} \neq \pm 2$.)

1. Without a calculator evaluate the following:

- | | |
|--|--|
| <p>(a) $4^{\frac{1}{2}}$.</p> <p>(b) $16^{\frac{1}{2}}$.</p> <p>(c) $9^{\frac{1}{2}}$.</p> <p>(d) $8^{\frac{1}{3}}$.</p> <p>(e) $64^{\frac{1}{6}}$.</p> <p>(f) $125^{\frac{1}{3}}$.</p> <p>(g) $16^{\frac{1}{4}}$.</p> <p>(h) $81^{\frac{1}{4}}$.</p> <p>(i) $32^{\frac{1}{5}}$.</p> <p>(j) $27^{\frac{1}{3}}$.</p> <p>(k) $4^{\frac{2}{2}}$.</p> <p>(l) $8^{\frac{2}{3}}$.</p> <p>(m) $4^{-\frac{1}{2}}$.</p> <p>(n) $4^{\frac{1}{2}}$.</p> | <p>(o) $9^{-\frac{3}{2}}$.</p> <p>(p) $8^{-\frac{2}{3}}$.</p> <p>(q) $32^{\frac{1}{5}}$.</p> <p>(r) $27^{-\frac{5}{3}}$.</p> <p>(s) $(\frac{1}{4})^{\frac{1}{2}}$.</p> <p>(t) $(\frac{1}{27})^{-\frac{1}{3}}$.</p> <p>(u) $(\frac{1}{64})^{\frac{1}{3}}$.</p> <p>(v) $(\frac{8}{27})^{-\frac{1}{3}}$.</p> <p>(w) $(\frac{64}{9})^{\frac{2}{2}}$.</p> <p>(x) $(\frac{27}{8})^{-\frac{5}{3}}$.</p> <p>(y) $(\frac{4}{9})^{-\frac{1}{2}}$.</p> <p>(z) $(64 \times 5^{-3})^{\frac{1}{3}}$.</p> |
|--|--|

2. Evaluate:

- | | |
|---|---|
| <p>(a) $\frac{1}{(\frac{2}{3})^2 + (\frac{4}{9})^{\frac{1}{2}}}$.</p> <p>(b) $\frac{2}{(\frac{4}{9})^{\frac{2}{3}} + (\frac{2}{3})^3}$.</p> <p>(c) $\frac{1}{(\frac{8}{27})^{\frac{2}{3}} - (\frac{81}{4})^{-\frac{1}{2}}}$.</p> <p>(d) $\frac{4}{(\frac{81}{16})^{-\frac{3}{4}} + (\frac{4}{3})^3}$.</p> | <p>(e) $\frac{\frac{2}{3} + (\frac{4}{9})^{\frac{1}{2}}}{1 + (\frac{8}{27})^{-\frac{1}{3}}}$.</p> <p>(f) $\frac{4 + (\frac{9}{4})^{\frac{2}{2}}}{2 + (\frac{1}{9})^{\frac{1}{2}}}$.</p> <p>(g) $\frac{(\frac{27}{8})^{-\frac{1}{3}} + \frac{1}{6}}{(\frac{1}{4})^{\frac{1}{2}} - 9^{-\frac{1}{2}}}$.</p> <p>(h) $\frac{(\frac{9}{4})^{\frac{1}{2}} - (\frac{9}{16})^{-\frac{1}{2}}}{3 - (\frac{27}{8})^{\frac{1}{3}}}$.</p> |
|---|---|

3. Simplify the following:

- | | |
|---|--|
| <p>(a) $\frac{x}{\sqrt{x}}$.</p> <p>(b) $\frac{x^2 \times \sqrt{x}}{x^{-1}}$.</p> | <p>(c) $x^{-2} \times x^{\frac{1}{2}}$.</p> <p>(d) $\frac{x^2 \times x^{0.5}}{\sqrt[3]{x}}$.</p> |
|---|--|

Problem Solving

always, sometimes, never true?

$$a^{\frac{1}{2}} \geq a^{\frac{1}{4}}$$

$$a^{\frac{4}{3}} = a^{\frac{2}{3}} \times a^{\frac{2}{3}}$$

$$4a^{\frac{1}{2}} = 2\sqrt{a}$$

$$2a^{\frac{1}{2}} \geq a^{\frac{1}{2}}$$

$$a^{\frac{1}{4}} = \sqrt{a^{\frac{1}{2}}}$$

$$(a^{\frac{1}{6}})^{\frac{1}{6}} = a^{\frac{1}{12}}$$

$$(a^{\frac{1}{2}})^2 = 1$$

$$a \neq 0$$

$$a > a^{\frac{1}{2}}$$

$$a^{\frac{1}{2}} > (a+1)^{\frac{1}{2}}$$

$$a < 0$$

$$a^{\frac{1}{2}} =$$

undefined

$$a^{1.25} = \sqrt[4]{a^5}$$

$$a^3 \geq a^2$$

Fluency Practice

A1 State the value of 5^0	A2 State the value of 7^1	A3 Evaluate 4^{-3} Give your answer as a fraction in its simplest terms.	A4 Evaluate $\sqrt[3]{125}$
B1 Evaluate $49^{\frac{1}{2}}$	B2 Evaluate $27^{\frac{1}{3}}$	B3 Evaluate $64^{\frac{2}{3}}$	B4 Evaluate $16^{\frac{3}{4}}$
C1 Evaluate $8^{\frac{1}{3}}$ Give your answer as a fraction in its simplest terms.	C2 Evaluate $36^{\frac{1}{2}}$ Give your answer as a fraction in its simplest terms.	C3 Evaluate $81^{\frac{3}{4}}$ Give your answer as a fraction in its simplest terms.	C4 Evaluate $216^{\frac{2}{3}}$ Give your answer as a fraction in its simplest terms.
D1 Evaluate $\left(\frac{9}{16}\right)^{\frac{1}{2}}$ Give your answer as a fraction in its simplest terms.	D2 Evaluate $\left(\frac{27}{343}\right)^{\frac{2}{3}}$ Give your answer as a fraction in its simplest terms.	D3 Evaluate $\left(\frac{125}{8}\right)^{-\frac{1}{3}}$ Give your answer as a fraction in its simplest terms.	D4 Evaluate $\left(\frac{49}{36}\right)^{-\frac{3}{2}}$ Give your answer as a fraction in its simplest terms.

Fluency Practice

square and cube roots: give your answer as a fraction

$$(1) \quad \sqrt{0.36}$$

$$(2) \quad \sqrt{1^{24}/25}$$

$$(3) \quad \sqrt{2^{14}/25}$$

$$(4) \quad \sqrt{3.24}$$

$$(5) \quad \sqrt{4.84}$$

$$(6) \quad \sqrt{5^{19}/25}$$

$$(7) \quad \sqrt[3]{1^{91}/125}$$

$$(8) \quad (1^{61}/64)^{-1/3}$$

$$(1) \quad \sqrt{2^{1/4}}$$

$$(2) \quad \sqrt{1^{15}/49}$$

$$(3) \quad \sqrt{1^{11}/25}$$

$$(4) \quad \sqrt[3]{1^{271}/729}$$

$$(5) \quad \frac{1}{2} \sqrt[3]{15.625}$$

$$(6) \quad 2(2^{46}/49)^{-1/2}$$

$$(7) \quad \sqrt{2/7 \times 3^{3/8} \times 1^{5/16}}$$

$$(8) \quad 3(3.375)^{-1/3}$$

no calculator

Fluency Practice

$$(2\frac{1}{4})^{-\frac{1}{2}} \times (3\frac{3}{8})^{\frac{1}{3}}$$

$$(1.25)^{-\frac{1}{3}} \times (10)^{\frac{1}{3}}$$

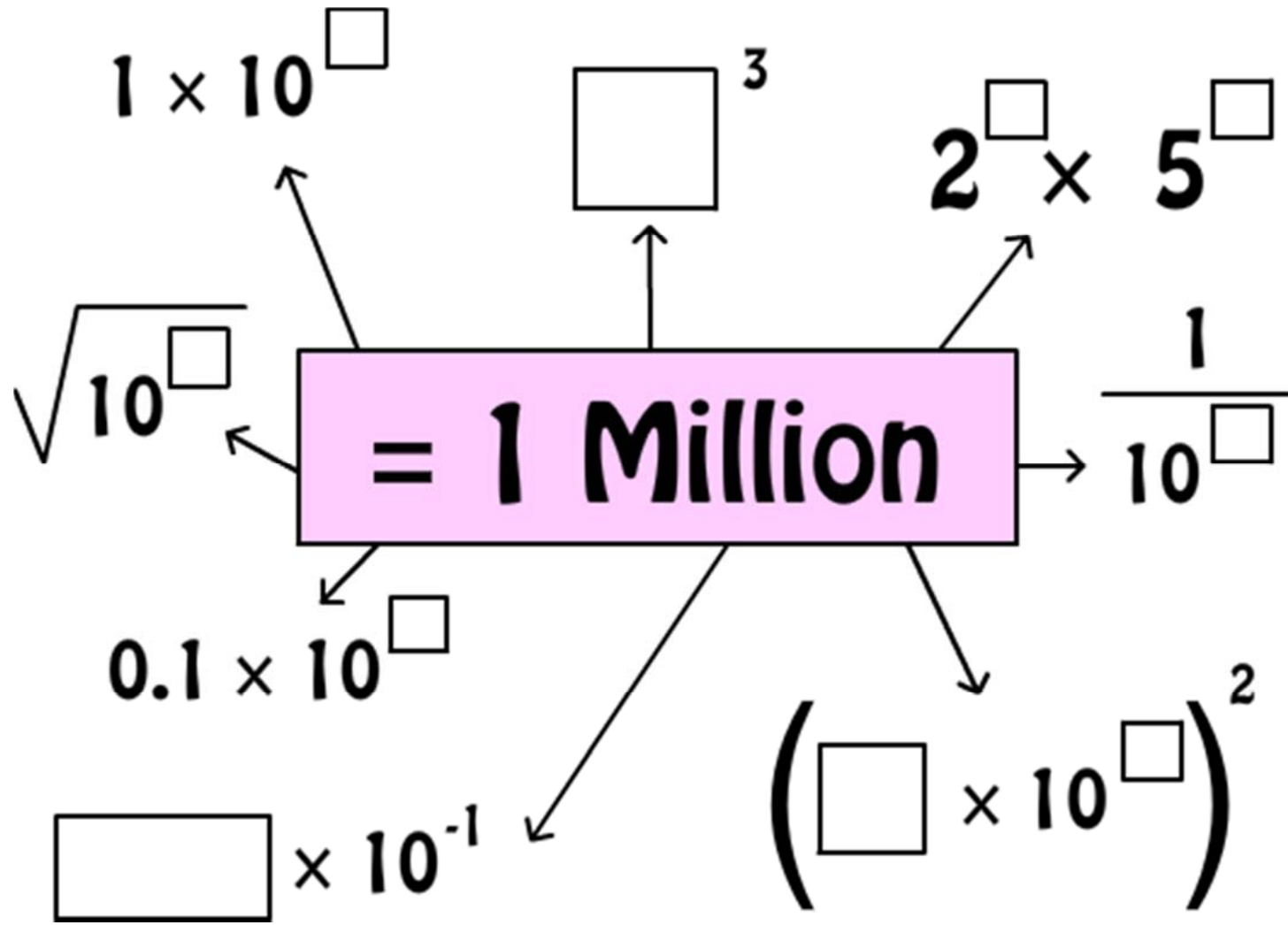
$$(1.5)^{-2} \times (16)^{-\frac{1}{2}} \times (81)^{\frac{3}{4}}$$

$$(48)^{\frac{1}{2}} \times (9)^{\frac{1}{6}} \times (3)^{-\frac{5}{6}}$$

$$(20)^{\frac{2}{3}} \div (100)^{\frac{1}{6}} \times \left(\frac{5}{2\sqrt{2}}\right)^{\frac{2}{3}}$$

$$(12)^{\frac{1}{3}} \div (81)^{\frac{1}{6}} \times (162)^{\frac{1}{3}}$$

Fluency Practice



Fluency Practice

Write the following without negative or fraction powers.

① x^{-3}

② $3x^{-1}$

③ $5x^{-2}$

④ $\frac{2}{3}x^{-4}$

⑤ $\frac{4}{5}x^{-3}$

⑥ $\frac{1}{5}x^{-3}$

⑦ $\frac{x^{-3}}{5}$

⑧ $x^{\frac{1}{2}}$

⑨ $x^{\frac{3}{2}}$

⑩ $3x^{\frac{5}{2}}$

⑪ $x^{-\frac{1}{2}}$

⑫ $x^{-\frac{3}{4}}$

⑬ $3x^{-\frac{2}{3}}$

⑭ $\frac{1}{6}x^{-\frac{3}{4}}$

Write the following without roots and, if the x is on the denominator, with a negative power:

① $\frac{4}{x}$

② $\frac{4}{x^2}$

③ $\frac{3}{4x^2}$

④ $\frac{1}{4x^2}$

⑤ $\frac{4}{3x^2}$

⑥ $\frac{1}{5x}$

⑦ $\frac{5}{x^3}$

⑧ \sqrt{x}

⑨ $\sqrt[3]{x}$

⑩ $\sqrt[3]{x^2}$

⑪ $5\sqrt{x^3}$

⑫ $\frac{1}{\sqrt{x}}$

⑬ $\frac{1}{\sqrt[4]{x}}$

⑭ $\frac{3}{\sqrt[3]{x^2}}$

⑮ $\frac{2}{3\sqrt{x}}$

⑯ $\frac{1}{3\sqrt[3]{x^2}}$

⑰ $\frac{4}{5\sqrt{x^3}}$

⑱ $\frac{1}{4\sqrt{x}}$

Fluency Practice

Write as a single power of x :

- (a) $\sqrt{x^3}$ (b) $(\sqrt{x})^3$
(c) $\sqrt[3]{x^2}$ (d) $\sqrt[4]{x}$
(e) $(\sqrt[3]{x})^4$ (f) $\sqrt{x^5}$

Write as a single power of x :

- (a) $\frac{1}{3\sqrt{x}}$ (b) $\left(\frac{1}{\sqrt[3]{x}}\right)^2$
(c) $\frac{1}{\sqrt{x^3}}$ (d) $\left(\frac{1}{\sqrt{x}}\right)^3$
(e) $\left(\frac{1}{\sqrt[3]{x}}\right)^5$ (f) $\frac{1}{\sqrt[2]{x^7}}$

Write as a single power of x :

- (a) $x^2 \times \sqrt{x}$ (b) $\sqrt[3]{x} \times x$
(c) $\frac{x^4}{\sqrt{x}}$ (d) $\frac{\sqrt[3]{x}}{x}$
(e) $\sqrt{\frac{1}{x^5}}$ (f) $\frac{1}{x\sqrt{x}}$

(a) Given that

$$\frac{y^4 \times \sqrt{y}}{\sqrt{y^5}} = y^a$$

find the value of a .

(b) Given that

$$\frac{1}{\sqrt[3]{y^2}} \times (y\sqrt{y})^4 = y^b$$

find the value of b .

(c) Given that

$$\left(\sqrt[2]{y^3}\right)^3 \times \frac{1}{y^c} = \left(y^2 \times \sqrt[4]{y^3}\right)^{-2}$$

find the value of c .

Fluency Practice

Solve

(a) $\sqrt{x} = 2$ (b) $5\sqrt[3]{x} = 3$

(c) $\sqrt{x} = \frac{16}{\sqrt{x}}$ (d) $x = \frac{27}{\sqrt{x}}$

(e) $2\sqrt{x} = \frac{64}{x^2}$ (f) $\frac{4}{\sqrt[3]{x}} = \frac{x}{4}$

Solve

(a) $x^{1/3} = -2$ (b) $5x^{1/2} - 2 = 13$

(c) $\frac{x^{3/2}}{20} = 6\frac{1}{4}$ (d) $x^{1/2} = \frac{9}{4x^{3/2}}$

(e) $x^{-2} = \frac{1}{36}$ (f) $48x^{-1/4} - 3 = 0$

Solve

(a) $(\sqrt{x} - 1)(\sqrt{x} - 5) = 0$

(b) $(\sqrt{x})^2 - 4\sqrt{x} + 3 = 0$

(c) $x - 7\sqrt{x} + 10 = 0$

(d) $8\sqrt{x} = x + 12$

(e) $8 - \frac{15}{\sqrt{x}} = \sqrt{x}$

(f) $4 - 2\sqrt{x} = \frac{2}{\sqrt{x}}$

Solve

(a) $(x^2 - 9)(x^2 - 1) = 0$

(b) $x^4 - 6x^2 + 8 = 0$

(c) $x^2 + \frac{4}{x^2} = 5$

(d) $x^6 - 27 = 26x^3$

(e) $x^3 = 7 + \frac{8}{x^3}$

Intelligent Practice

Write in terms of the given number:

- 1) 16 as a power of 2
- 2) 16 as a power of 4
- 3) 64 as a power of 4
- 4) 125 as a power of 5
- 5) 4^2 as a power of 2
- 6) 4^3 as a power of 2
- 7) 4^4 as a power of 2
- 8) 27^3 as a power of 3
- 9) 27^{3x} as a power of 3
- 10) 25^{3x} as a power of 5

Fluency Practice

Find the value of each of the following.

$$\sqrt[3]{4 \times 2 \times 10^6}$$

$$\sqrt[4]{9 \times 9 \times 10^8}$$

$$\sqrt[3]{4 \times 16 \times 10^6}$$

$$\sqrt[2]{3 \times 27 \times 10^4}$$

$$\sqrt[4]{125 \times 5 \times 10^{12}}$$

$$\sqrt[5]{16 \times 64 \times 10^{15}}$$

Intelligent Practice

Solve the following equations:

1) $3^x = 243$

2) $3^x = \frac{1}{243}$

3) $4^x = 64$

4) $4^x = \frac{1}{64}$

5) $5^x = 625$

6) $5^x = \frac{1}{625}$

Intelligent Practice

Solve the following equations:

1) $\left(\frac{1}{3}\right)^x = 81$

2) $\left(\frac{1}{3}\right)^x = \frac{1}{81}$

3) $\left(\frac{1}{3}\right)^x = \frac{1}{27}$

4) $\left(\frac{1}{3}\right)^x = 27$

5) $\left(\frac{1}{4}\right)^x = 64$

6) $\left(\frac{1}{4}\right)^x = 256$

7) $\left(\frac{1}{4}\right)^x = \frac{1}{256}$

8) $\left(\frac{5}{25}\right)^x = 625$

Intelligent Practice

Find the value of x that satisfies:

1) $2^{3x} \times 2^{x-3} = 32$

2) $2^{2x} \times 2^{x+3} = 64$

3) $3^{2x} \times 3^{x-6} = 81$

4) $3^{x+2} \times 3^{x-2} = 243$

5) $2^{4x} \div 2^{x-2} = 32$

6) $3^{6x} \div 3^{x-5} = 243$

Intelligent Practice

Find the value of x that satisfies:

1) $16^{\frac{1}{4}} \times 2^x = 8^{\frac{2}{3}}$

2) $81^{\frac{1}{4}} \times 3^x = 27^{\frac{2}{3}}$

3) $25^{\frac{1}{2}} \times 125^x = 5^{\frac{4}{2}}$

4) $27^{\frac{1}{4}} \times 3^x = 9^{\frac{2}{3}}$

Fluency Practice

Repeat addition Index problems

1) Simplify

a) $3^5 + 3^5 + 3^5$

b) $3^y + 3^y + 3^y$

c) $3^{2y+4} + 3^{2y+4} + 3^{2y+4}$

d) $2^3 + 2^3$

e) $2^3 + 2^3 + 2^3 + 2^3$

f) $2^3 + 2^3 + 3^3 + 3^3 + 3^3$

g) $2^3 + 2^3 + 3^3$

h) $2^x + 2^x + 2^y$

i) $2^x + 2^y + 2^x + 2^y$

j) $2^3 \times 2^3 + 2^6$

k) $2^5 + 2^{10} \div 2^5$

l) $(2^x)^2 + 2^{x^2} + 2^{2x}$

2) Solve

a) $4^5 + 4^5 + 4^5 + 4^5 = 4^x$

b) $4^5 + 4^5 + 4^5 + 4^5 = 4^{3x-5}$

c) $4^x + 4^x + 4^x + 4^x = 4^{10}$

d) $4^{3x+1} + 4^{3x+1} + 4^{3x+1} + 4^{3x+1} = 4^{10}$

e) $2^{3x+1} + 2^{3x+1} + 2^{3x+1} + 2^{3x+1} = 2^{10}$

f) $2^{3x+1} + 2^{3x+1} + 2^{3x+1} + 2^{3x+1} = 4^{10}$

g) $2^{5x-2} + 2^{5x-2} = 4^{10x-1}$

h) $2^{5x+3} + 2^5 \times 2^{5x-2} = 4^{10x-1}$

i) $4^3 + 4^3 = 2^x$

j) $8^3 + 8^3 + 8^3 + 8^3 = 2^x$

j) $8^3 + 8^3 + 8^3 + 8^3 = 4^x$

k) $8^{3x+2} + 8^{3x+2} + 8^{3x+2} + 8^{3x+2} = 2^x$

3) Solve

a) $2^3 + 2^3 = \frac{1}{2^x}$

b) $\frac{1}{2^3} + \frac{1}{2^3} = \frac{1}{2^x}$

c) $\frac{1}{8} + \frac{1}{4} \div 2 = \frac{1}{2^x}$

d) $\frac{1}{4^{3x}} + \frac{1}{4^{3x}} = \frac{1}{2^{7(x-2)}}$

e) $(\sqrt{2})^3 + (\sqrt{2})^3 = 2^x$

f) $\sqrt[3]{2^5} + \sqrt[3]{2^5} = 2^x$

g) $\sqrt[3]{4^5} + \sqrt[3]{4^5} = 8^x$

4) $2^{3x-2} + 2^{3x-2} = 4^{y+1}$

and $3^x \times (3^{x-4})^4 = \frac{1}{3^{3y}}$ find the value of x and y.

Fluency Practice

Find the value of m in each question

$$8^2 = 2^{2m-4}$$

$$16^5 = 2^{2m+4}$$

$$27^4 = 3^{5m-3}$$

$$81^5 = 3^{3m+2}$$

$$25^6 = 5^{7m-9}$$

$$125^7 = 5^{9m+3}$$

Fluency Practice

Find the value of x in each question

$$3^{-n} = 0.2$$

$$(3^4)^n = x$$

$$5^{-n} = 0.25$$

$$(5^3)^n = x$$

$$6^{-n} = 0.5$$

$$(6^5)^n = x$$

$$7^{-n} = 0.1$$

$$(7^3)^n = x$$

$$9^{-n} = 0.4$$

$$(9^2)^n = x$$

$$3^{-n} = 0.04$$

$$(3^{0.5})^n = x$$

Fluency Practice

- (a) Write 4^5 as a power of 2
- (b) Write 8^3 as a power of 2.
- (c) Write 16^3 as a power of 4.
- (d) Write 27^4 as a power of 3.
- (e) Write 125^3 as a power of 5.

- (a) $2^6 \times 2^4 = 4^n$. Find n .
- (b) $8^n = 2^3 \times 2^9$. Find n .
- (c) $3^n = 3^3 \times 9^2$. Find n .
- (d) $2^n = \frac{16}{2^3}$. Find n .
- (e) $\frac{3^8}{81} = 3^n$. Find n .

- (a) $4^3 \times 16 = 2^n$. Find n .
- (b) $243 \times 9^2 = 3^n$. Find n .
- (c) $2^n \times 4^3 = 1024$. Find n .
- (d) $\frac{9^2 \times 81^{1/2}}{3^2} = 3^n$. Find n .
- (e) $\frac{2^{2n} \times 64}{4^2} = 2^3$. Find n .

- (a) $128 = 4^{2x} \times 2^x$. Find x .
- (b) $\frac{1}{\sqrt[3]{9^4}} = 3^x$. Find x .
- (c) $16^{1/5} \times 2^x = 8^{3/4}$. Find x .
- (d) $\left(2^{1/2}\right)^x = \frac{32}{2^2}$. Find x .
- (e) $243 = \frac{3^x}{\sqrt[3]{81}}$. Find x .

Fluency Practice

Change of Base Jumbled Answers

Write each question on the left as a single power of 5. Find your answer on the right.

A $125 = \underline{\hspace{2cm}}$

B $\frac{1}{25} = \underline{\hspace{2cm}}$

5^{14} 5^{13}

C $25^6 = \underline{\hspace{2cm}}$

D $125^{-3} = \underline{\hspace{2cm}}$

5^{12} 5^{10}

E $\frac{1}{25^2} = \underline{\hspace{2cm}}$

F $25^{\frac{1}{2}} = \underline{\hspace{2cm}}$

5^8 5^7

G $\left(\frac{1}{25}\right)^3 = \underline{\hspace{2cm}}$

H $(125^2)^{-4} = \underline{\hspace{2cm}}$

5^4 5^3

I $25 \times 25 = \underline{\hspace{2cm}}$

J $25^4 \times 25^3 = \underline{\hspace{2cm}}$

5^2 $5^{1.5}$

K $25^6 \times 5^{-5} = \underline{\hspace{2cm}}$

L $\frac{1}{125} \times \frac{1}{25} = \underline{\hspace{2cm}}$

5^1 $5^{\frac{3}{4}}$

M $25^7 \times \frac{1}{5} = \underline{\hspace{2cm}}$

N $\frac{1}{5^{-9}} = \underline{\hspace{2cm}}$

5^{-1} 5^1

O $125^{\frac{1}{3}} = \underline{\hspace{2cm}}$

P $\frac{1}{25^{-4}} = \underline{\hspace{2cm}}$

5^{-2} 5^{-3}

more challenging

Q $\sqrt{5} \times 5 = \underline{\hspace{2cm}}$

R $\frac{25^2}{5^4} \times 5^{-3} = \underline{\hspace{2cm}}$

5^{-4} 5^{-5}

S $\sqrt{25^{10}} = \underline{\hspace{2cm}}$

T $\frac{1}{\sqrt[3]{125}} = \underline{\hspace{2cm}}$

5^{-6} 5^{-9}

U $\frac{5}{\sqrt{5}} = \underline{\hspace{2cm}}$

V $\sqrt{625} = \underline{\hspace{2cm}}$

5^{-24} 5^{\square}

Fluency Practice

$2^1 = 2$	$2^{11} = 2,048$	$2^{21} = 2,097,152$
$2^2 = 4$	$2^{12} = 4,096$	$2^{22} = 4,194,304$
$2^3 = 8$	$2^{13} = 8,192$	$2^{23} = 8,388,608$
$2^4 = 16$	$2^{14} = 16,384$	$2^{24} = 16,777,216$
$2^5 = 32$	$2^{15} = 32,768$	$2^{25} = 33,554,432$
$2^6 = 64$	$2^{16} = 65,536$	$2^{26} = 67,108,864$
$2^7 = 128$	$2^{17} = 131,072$	$2^{27} = 134,217,728$
$2^8 = 256$	$2^{18} = 262,144$	$2^{28} = 268,435,456$
$2^9 = 512$	$2^{19} = 524,288$	$2^{29} = 536,870,912$
$2^{10} = 1024$	$2^{20} = 1,048,576$	$2^{30} = 1,073,741,824$

Without using a calculator or paper, can you mentally compute these expressions?

- 32×16
- $4 \times 64 \times 1024$
- $4 \times 16 \times 32 \times 64$
- $1024 \div 64$
- $1,048,576 \div 32,768$
- $1,073,741,824 \div 64$
- 128^2
- $(1,024)^3$
- $\sqrt{65,536}$
- $\sqrt[3]{134,217,728}$
- $\sqrt[5]{1,073,741,824}$
- $(1,048,576)^{1/2}$
- $(262,144)^{2/3}$
- $\frac{524,288}{(16,384)^{\frac{1}{2}}}$
- $\frac{(67,108,864)^{1/2}}{16^3(256)^{1/4}}$
- $\frac{1}{4} \cdot (65,536)$

Fluency Practice

Equating Powers

Whenever you see 4 you should think 2^2 .

Whenever you see 8 you should think 2^3 .

Whenever you see 27 you should think 3^3 .

Whenever you see 64 you should think 4^3 or 8^2 or 2^6 .

Etcetera...

Also, pro tip; when you see 1 you should think 2^0 or 3^0 or 5^0 or ...

All of these problems reduce to

$$k^{\text{fish}} = k^{\text{dog}} \Rightarrow \text{fish} = \text{dog}.$$

Questions

1. Solve $2^{x+1} = 4^x$.

2. Solve $8^{3x} = 2^{x-2}$.

3. Solve $3^{2x-1} = 9^{4x}$.

4. Solve $27^{3-x} = 81^{2x}$.

5. Solve $2 \times 2^{2x+1} = 8^{x-1}$.

6. Solve $5^x \times 25^3 = 5^{2x-1}$.

7. Solve $4^x \times 8^{x-1} = 2 \times 4^{3x+1}$.

8. Solve $8 \times 2^{x-1} = 4^{2x-1}$.

9. Solve $16 \times 8^{2x+1} = 2 \times 16^x$.

10. Solve $32^x = \frac{8^x}{2^{x+1}}$.

11. Solve $\frac{2^{x+1}}{4} = \frac{4^{x+3}}{8^x}$.

12. Solve $\frac{9^x}{27^{x+1}} = \frac{3^x}{81^{1-x}}$.

13. Solve $\frac{5^{-2x+1}}{25^{3-x}} = \frac{125^x}{5^{x-4}}$.

14. Solve $8 \times \frac{2^{2-x}}{4^{3-2x}} = \frac{8^x}{4^{2+x}}$.

15. Solve $1 = 8^2 \times 4^{x-1} \times 2^{x+1}$.

16. Solve $\frac{7^x}{49^{6-x}} = \frac{343^{1-x}}{7^{2x-3}}$.

17. Solve $8^{ax} = 4^{bx+1}$.

18. Solve $27^{a+b} = 3^{cx-a}$.

19. Solve $9^{ax} \times 27^x \times 3^{ax} = 1$.

20. Solve $\frac{5^{ax-2}}{25^{b-x}} = 125^{cx+d}$.

21. Solve $\frac{2^{ax}}{2^{3-bx}} = \frac{4^{4+cx}}{8^{k-x}}$.

Only do the following if you've studied solving quadratics by factorisation.

22. Solve $4^{x+2} = 2^{x^2+5}$.

23. Solve $3^{x^2+2} = 27^x$.

24. Solve $5^{2x^2} = 25^{2x+3}$.

25. Solve $(x^2 + 5x + 5)^{x^2+11x+30} = 1$.

Fluency Practice

Indices

1. Simplify fully:

(a) $x^2 \times x^3 \times x^5$.

(b) $2 \times (x^2)^3$.

(c) $(x^{-3})^{-5}$.

(d) $(3x^2)^3$.

(e) $(12x^9) \div (4x^3)$.

2. Simplify fully:

(a) $\frac{(2x^{-2})^3 \times (2x^3)^2}{6^2 \times x^{-4}}$.

(b) $\frac{2x^8 \times (2x)^8}{2x \times (2x)^5}$.

(c) $\frac{(3x^{-1})^3 \times (2x^{-2})^4}{(6x^2)^3}$.

(d) $\frac{3x^3(2xy)^2}{6x^5y^2}$.

(e) $\frac{a^3b^7}{a^{-2}b^4}$.

(f) $\frac{64(x^{-3}y^7)^2x^6y}{(4x^4y^2)^2}$.

(g) $\frac{7pq^{-1}(2pq^3)^3}{14p(2p^2q)^2}$.

3. Evaluate without a calculator:

(a) 3^{-1} .

(b) 5^{-2} .

(c) 4^{-3} .

(d) 7^0 .

(e) $(\frac{2}{3})^{-1}$.

(f) $(\frac{4}{3})^{-2}$.

(g) $(2\frac{1}{3})^{-1}$.

(h) $(1\frac{3}{4})^{-3}$.

4. Evaluate without a calculator:

(a) $\frac{2^8 \times 2^{-3}}{2^4}$.

(b) $8^4 \div 4^6$.

5. Solve for x

(a) $2^x = 8$.

(b) $3^x = 81$.

(f) $4x \times (3x^{-2})^2$.

(g) $\frac{(3x^2)^3}{3x^{-2}}$.

(h) $\frac{12x^8}{6x^4} + 6x^4$.

(i) $2x^5(2x^5 + 2x^5)$.

(h) $\frac{(4m^7n)^3}{8n^3m^{-7}}$.

(i) $\frac{(uv)^2(2u^2v)^3}{(u^3v^8)^2(uv^{-1})^4}$.

(j) $\frac{3x(x^4y^8)^3}{x^3(3x^5y)^2}$.

(k) $\frac{6u^2v^3}{3u^{-2}v^5} \times \frac{4(u^3v^4)^2}{2uv^{-1}}$.

(l) $\frac{(2x^3)^3y}{3x^{-2}y^3} \div \frac{4(xy)^{-8}}{(3x^{-4})^2}$.

(m) $\frac{4x^3y}{7xy^2} \div \frac{8x(yz)^2}{14xz^2y^3}$.

(n) $\frac{6x}{y^2} \div \frac{6x}{(3xy^3)}$.

(i) $(\frac{1}{2})^{-2}$.

(j) $(0.6)^{-2}$.

(k) $(\frac{2}{3})^{-1} + (\frac{5}{2})^{-1}$.

(l) $(\frac{3}{2})^{-2} + (\frac{3}{2})^2$.

(m) $(\frac{1}{5})^{-2} - (\frac{2}{7})^{-1}$.

(n) $(2\frac{1}{2})^{-3} + (3\frac{2}{3})^{-1}$.

(o) $(0.4)^{-2} - (0.2)^{-1}$.

(c) $27^3 \div 9^4$.

(d) $\frac{64^2}{8 \times 16}$.

(c) $5^x = \frac{1}{25}$.

(d) $3^{-x} = \frac{1}{27}$.

Fluency Practice

Simplify:

- (a) $4^5 \times 4^2$
- (b) $4^5 \times 4^3$
- (c) $4^3 \times 4^5$
- (d) 4×4^5
- (e) $4^5 \times 4^{-2}$
- (f) $4^0 \times 4^3$
- (g) $4^{2.5} \times 4^{0.5}$
- (h) $4^5 \times 5^2$
- (i) $5^{-3} \times 5^2$
- (j) $5^{-5} \times 5^{-3}$

Simplify:

- (a) $4^5 \div 4^2$
- (b) $4^5 \div 4^3$
- (c) $4^2 \div 4^5$
- (d) $4^5 \div 4$
- (e) $4^2 \div 4^0$
- (f) $4^5 \div 4^{-2}$
- (g) $4^5 \div 5^4$
- (h) $5^{2.5} \div 5^{0.5}$
- (i) $\frac{5^7}{5^2}$
- (j) $\frac{(-5)^7}{(-5)^2}$

Simplify:

- (a) $(3^4)^5$
- (b) $(3^5)^4$
- (c) $(3^2)^5$
- (d) $(3^{-2})^5$
- (e) $(3^4)^1$
- (f) $3^3 \times (3^4)^{0.5}$

Simplify:

- (a) $\frac{2^3 \times 2^8}{2^5}$
- (b) $\frac{2^{-3} \times 2^8}{2^1}$

Find x :

- (a) $5^x \times 5^4 = 5^7$
- (b) $\frac{3^x \times 3^{-2}}{3^4} = 3^{10}$
- (c) $10^2 \times 10^x = 1000000$

Fluency Practice

(a)	(b)	(c)	(d)
Write down the cube root of 27	Work out $3^5 - \sqrt{441}$	Write down the value of 5^0	Simplify $y^5 \times y^4$
(e)	(f)	(g)	(h)
Simplify $(x^{-3})^5$	Write as a power of 2 $\frac{2^{12}}{2^3}$	Simplify $(3a^2b^4)^3$	Simplify $\frac{a^5 \times a^2}{a^{-3}}$
(i)	(j)	(k)	(l)
Write $2\sqrt{2}$ as a single power of 2	Evaluate $\left(\frac{4}{9}\right)^{3/2}$	Evaluate $8^{-4/3}$	$\frac{4^{10} \times 4^x}{4^6} = 4^{-1}$ Find the value of x .
(m)	(n)	(o)	(p)
$\frac{2^{10}}{64} = 2^n$ Find the value of n .	Write $\frac{1}{\sqrt[3]{4}}$ as a single power of 2	$4^a = 16 \times 8^{2a}$ Find the value of a .	Given that $9^x = (27^a)^{1/2} \times 3^b$ find an expression for x in terms of a and b .

Fluency Practice

A1 Write 5^8 as a power of 25	A2 Write 8^6 as a power of 2	A3 Write 8^6 as a power of 4	A4 Write $8^2 \times 4^3$ as a power of 2
B1 Write 2 as a power of 8	B2 Write 4 as a power of 8	B3 Write $\sqrt{8}$ as a power of 2	B4 Express $3\sqrt{3}$ as a power of 9
C1 Express $\frac{1}{81}$ as a single power of 3	C2 Express $\frac{1}{\sqrt{3}}$ as a single power of 9	C3 Express $\frac{1}{4\sqrt{2}}$ as a single power of 2	C4 Express $3^7 + 9^4 + 15 \times 27^2$ as a power of 3
D1 Solve: $32^x = \frac{1}{16}$	D2 Solve: $2^x = 8^{\frac{1}{4}} \times 16^{\frac{1}{3}}$	D3 Find the value of x if: $9^{\frac{3}{4}} \times 27^x = 81^{\frac{2}{3}}$	D4 Find the value of m and n if: $6 \times 12^m = 9^4 \times 2^n$

Fluency Practice

Section a:

Simplify the following, giving your answer in index notation, or as a fraction or integer where possible:

- | | | |
|---------------------------|-------------------------------|------------------------------------|
| A) $3^4 \times 3^5$ | H) $(2c^2)^3$ | O) $6a^3 \div 2a$ |
| B) $4^4 \times 4^2$ | I) $(a^4b^2)^2$ | P) $15m^3k^3 \div 3m^2k^5$ |
| C) $y^0 \times y^{-3}$ | J) $(2b^2)^3 \times (2b^2)^4$ | Q) $(ab^2)^3 \div ab^2$ |
| D) $(-2)^3 \times (-2)^2$ | K) $(5^2)^a \times (5^a)^4$ | R) $c^2 \times c^0 \div c$ |
| E) $3^{2a} \times 3^{4a}$ | L) $z^3 \div z^2$ | S) $27c^2 \times c^{-1} \div 9c^4$ |
| F) $(a^2)^5$ | M) $8^3 \div 8^{-3}$ | T) $3ab^4 \times 6a^{-5}b^2$ |
| G) $(2^2)^3$ | N) $z^3 \div z^{-4}$ | U) $-2x^3y^0 \times -2x^2y^{-5}$ |

Section b:

Simplify the following, giving your answer in index notation

- | | | |
|---------------------------|--|--|
| A) $12ab^3 \div 6a^2b^2$ | D) $\frac{2a^4b}{(3a^2)^2}$ | G) $\frac{(2xy)^3}{(6x^3y)^2} \times \frac{2xy}{3x^2}$ |
| B) $\frac{6x^3y}{3x^2}$ | E) $\frac{1}{a^2} \times \frac{2}{ab^2}$ | H) $\left(\frac{2}{x^2}\right)^3$ |
| C) $\frac{12a^2b}{4ab^2}$ | F) $\frac{2b^3c}{3ab} \times \frac{4ab}{2c}$ | I) $\left(\frac{ab^2}{8a}\right)^2$ |

Section c (hard):

Determine the value of the letters:

- | | | |
|-------------------------|-------------------------|--|
| A) $2^a = 4^2$ | F) $3^f \div 3^2 = 3^8$ | L) $\frac{2^3}{2^{-1}} = 2^l$ |
| B) $2^b = 8^2$ | G) $2^g \div 2^3 = 16$ | M) $\frac{2^m}{(2^m)^2} = \frac{1}{4}$ |
| C) $2^c \times 4 = 2^5$ | H) $(3^h)^2 = 3^6$ | N) $\frac{2 \times 3^{2n}}{2^2 \times 3^n} = \frac{18}{4}$ |
| D) $2^{3d} = 64$ | I) $26^i = 1$ | |
| E) $2^e \times 2 = 32$ | J) $(-4)^j = -64$ | |
| | K) $(2k)^2 = 100$ | |

Fluency Practice

1. Find the value of ...

a. $144^{\frac{1}{2}}$ b. $625^{\frac{1}{4}}$ c. $27^{\frac{1}{3}}$ d. $\left(\frac{1}{16}\right)^{0.25}$ e. $0.01^{\frac{1}{2}}$ f. $\left(\frac{125}{64}\right)^{\frac{1}{3}}$ g. $\left(\frac{1}{64}\right)^{-\frac{1}{6}}$ h. $81^{-\frac{1}{4}}$

2. Find the value of ...

a. $16^{\frac{3}{2}}$ b. $1000^{\frac{2}{3}}$ c. $32^{\frac{3}{5}}$ d. $\left(\frac{1}{4}\right)^{\frac{5}{2}}$ e. $\left(\frac{1}{81}\right)^{\frac{3}{4}}$ f. $\left(\frac{27}{125}\right)^{\frac{2}{3}}$ g. $\left(\frac{3125}{243}\right)^{\frac{4}{5}}$ h. $625^{-\frac{3}{4}}$

3. Find the missing values

a. $16^{\frac{3}{\square}} = 64$ b. $\square^{\frac{2}{5}} = 25$ c. $\left(\frac{\square}{81}\right)^{\frac{3}{4}} = \frac{8}{\square}$ d. $\left(\frac{100}{\square}\right)^{\frac{\square}{2}} = \frac{1000}{27}$ e. $\left(\frac{\square}{125}\right)^{\frac{2}{\square}} = 25$

4. Write in the form 9^n

a. 81 b. 3 c. 27 d. 243

5. Write in the form 64^n

a. 8 b. 4 c. 16 d. 512

6. Find the value of x

a. $9^{\frac{x}{4}} = 3$ b. $16^{\frac{x+2}{4}} = 2$ c. $125^{\frac{2x-1}{6}} = 5$ d. $27^{\frac{5x-7}{6}} = 81$

e. $32^{\frac{2x-3}{10}} = 4$ f. $\left(\frac{4}{25}\right)^{5x-2} = \frac{2}{5}$ g. $2 \times 9^{\frac{3x-7}{4}} = 6$ h. $3 \times 4^{2x+8} = 24$

Fluency Practice

Part One - Consolidate

$$1) 5^6 \times 5^3 = 5^? \quad 4) b^6 \times b^{-3} = b^?$$

$$2) 5^{-3} \div 5^3 = 5^? \quad 5) h^{-4} \div h^{-2} = h^?$$

$$3) 5^2 \times 5^{-4} = 5^? \quad 6) y^{\frac{1}{2}} \times y^2 = y^?$$

Part Two – Develop

$$1) y^5 \times y^? = y^3 \quad 4) y^{-3} \div y^? = 1$$

$$2) 6^2 \div ?^2 = 9 \quad 5) h^2 \times h = 1$$

$$3) 5^0 \times 5^? = 5 \quad 6) y^{\frac{1}{2}} \times y^? = y^2$$

Part Three – Enhance

$$1) \frac{1}{36} \times \sqrt{6} = 6^?$$

$$2) 8^x \div 8^{3x} = 8^4$$

$$3) \sqrt[3]{125} \times 5^x = 25$$

$$4) \left(\frac{1}{b}\right)^2 \times \sqrt[3]{b} = b^?$$

$$5) \left(\sqrt[3]{(h)^2}\right)^{\frac{3}{2}} = h^?$$

$$6) y^x \times y^{\frac{2}{3}} = y^3$$

Problem Solving

Part Four - Investigate

x^2

x^{-3}

\sqrt{x}^3

$x^{-\frac{1}{2}}$

$x^{\frac{2}{3}}$

$\left(\frac{1}{x}\right)^2$

$x\sqrt{x}$

Here are seven cards: Assume that x is a positive integer greater than 1

Find:

Two different pairs of cards that when multiplied, equal x

A matching pair of cards

Two cards which multiply and give an answer that is on a different card also in the list!

The two cards who give the largest product

Finally – create one more card similar to above, and use three more from the list, so that you can complete this sum:

$$\boxed{} \times \boxed{} = \boxed{} \div \boxed{}$$

Fluency Practice

Complete the tables below! The first row has been completed for you.

2^x	Evaluate
2^3	8
2^0	
2^4	32
2^4	
	$\sqrt{2}$
2^{-1}	
	2
	$1\frac{1}{8}$
$2^{\frac{3}{2}}$	
2^{-6}	
	$\sqrt{128}$
	$1\frac{1}{\sqrt[3]{2}}$
$2^{-\frac{1}{2}}$	
	$8\sqrt{2}$
$2^{-\frac{5}{2}}$	

3^x	Evaluate
3^2	9
	1
3^4	
	729
3^{-3}	
	$\sqrt[3]{3}$
	$1\frac{1}{3}$
$3^{0.5}$	
	$9\sqrt{3}$
$3^{0.2}$	
	$\sqrt{27}$
$3^{-1.5}$	
	$\frac{1}{243}$
	$\frac{81}{\sqrt{3}}$
	$27\sqrt[5]{3}$

Exam Questions

Complete the following exam style questions.

1) Find the value of the missing constant in the following equations.

a) $3^a = \frac{1}{27}$

b) $5^b = \frac{1}{\sqrt{5}}$

c) $7^c = \sqrt[3]{49}$ **(3)**

2) Solve the following equations to find x .

a) $\frac{8}{2^{3x}} = \sqrt{2}$

b) $4^x = \frac{\sqrt[3]{2}}{32}$

c) $125 = \frac{(\sqrt{5})^3}{25^{1-x}}$ **(9)**

3) Solve the equations below to find x and y .

$$\frac{16^x}{8^y} = \frac{1}{4}$$

$$4^x 2^y = 16$$
 (6)

Problem Solving

$x\sqrt{x}$

$(x - 2)$

$(x + 3)$

$(x + 5)$

$(x + 2)$

$(x - 3)$

$(x + 1)$

$(x - 1)$

x^2

\sqrt{x}

$x^{\frac{3}{2}}$

$\frac{1}{x}$

$x^2 + 2x + 4$

Select expressions from above which multiply to given the answers listed below.

You may not use an expression more than once in each product.

a) $x^2 + 8x + 15$

b) $x^3 + 2x^2$

c) $x^2 - 9$

d) x^2

e) $x^{\frac{3}{2}} - 3x^{\frac{1}{2}}$

f) 1

g) \sqrt{x}

h) $\frac{4}{x} + x + 2$

i) $x^2 + 6x + 5$

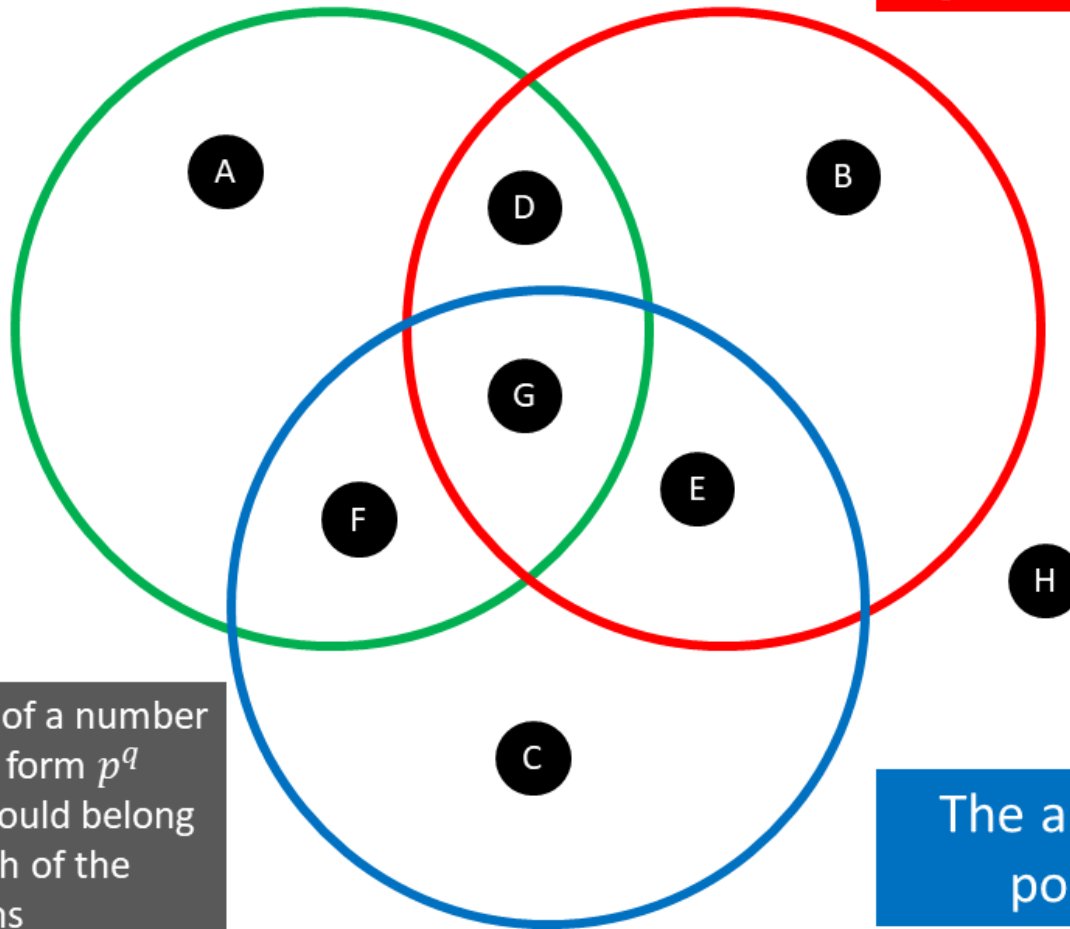
j) $x^3 + 2x^2 - 5x - 6$

k) $x^3 - 2x^2 - 5x + 6$

Maths Venns

The power (q) is negative

The base number (p) is a fraction



If you think a region is impossible to fill, convince me why!

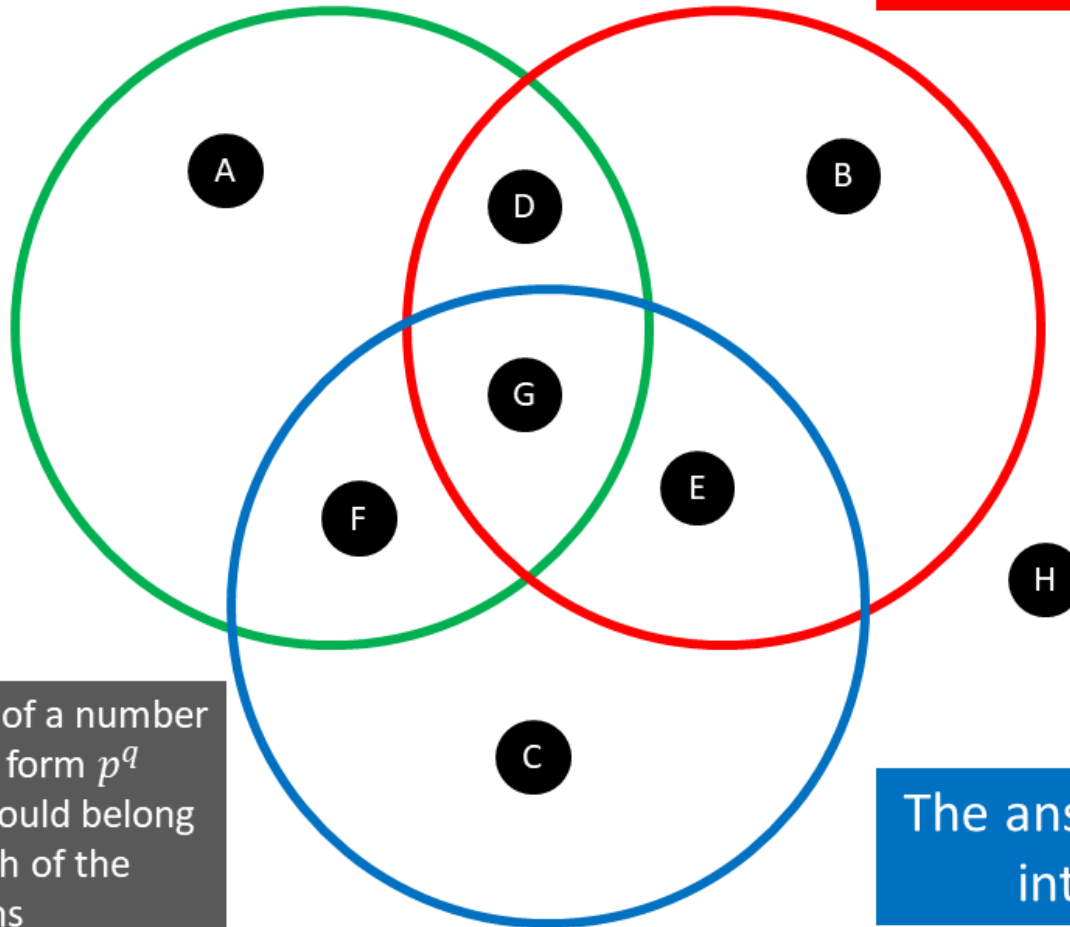
Think of a number in the form p^q that could belong in each of the regions

The answer is positive

Maths Venns

The power (q) is a fraction

The base number (p) is a fraction



If you think a region is impossible to fill, convince me why!

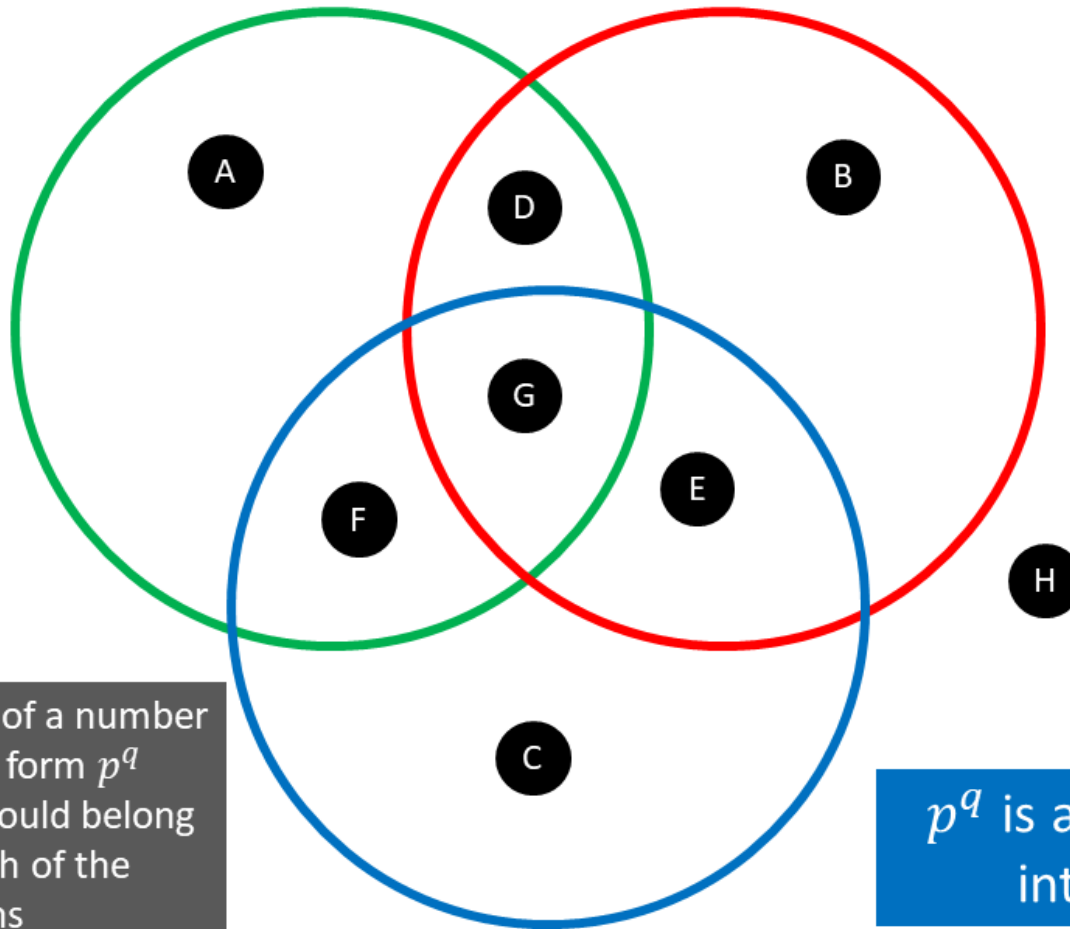
Think of a number in the form p^q that could belong in each of the regions

The answer is an integer

Maths Venns

The power (q) is negative

The power (q) cannot be written as a unit fraction



If you think a region is impossible to fill, convince me why!

Think of a number in the form p^q that could belong in each of the regions

p^q is a positive integer

Problem Solving

$$2^3 + 2^3 + 3^4 + 3^4 + 3^4 = ?$$

- $2^3 + 3^4$
 4^5
 $2^4 + 3^5$
 $2^6 + 3^{12}$

$$\sqrt{9^{16}x^2}$$

- | | |
|--------------------------------|----------------------------------|
| <input type="radio"/> 9^{4x} | <input type="radio"/> 9^{8x^2} |
| <input type="radio"/> 3^{4x} | <input type="radio"/> 3^{8x^2} |

$$\frac{6^{2007} - 6^{2006}}{30} = ?$$

- 30
 0.2
 6^{2005}
 6^{2007}
 36

$$27^{-\frac{x}{3}} + 81^{\frac{1-x}{4}}$$

If the expression above can be stated in the form of $\frac{a}{b^x}$ for positive integers a and b , what is the value of $a + b$?

$$\begin{cases} ab = a^b \\ \frac{a}{b} = a^{3b} \end{cases}$$

a and b are real numbers, such that $a > 1$ and $b \neq 0$, satisfying the above system.

Find b^{-a} .

2 Calculating with Surds

Intelligent Practice

Simplify:

1) $\sqrt{2} \times \sqrt{3}$

2) $\sqrt{4} \times \sqrt{3}$

3) $\sqrt{5} \times \sqrt{3}$

4) $\sqrt{6} \times \sqrt{3}$

5) $\sqrt{24} \times \sqrt{3}$

6) $\sqrt{12} \times \sqrt{3}$

7) $\sqrt{12} \times \sqrt{4}$

8) $\sqrt{11} \times \sqrt{4}$

9) $\sqrt{10} \times \sqrt{4}$

10) $\sqrt{10} \times \sqrt{8}$

11) $\sqrt{20} \times \sqrt{4}$

12) $\sqrt{20} \times \sqrt{5}$

13) $\sqrt{20} \times \sqrt{10}$

14) $\sqrt{20} \times \sqrt{20}$

15) $\sqrt{20} \times \sqrt{40}$

16) $\sqrt{20} \times \sqrt{40} \times \sqrt{2}$

17) $\sqrt{20} \times \sqrt{40} \times \sqrt{3}$

18) $(\sqrt{20} \times \sqrt{40} \times \sqrt{3})^2$

Intelligent Practice

Simplify:

1) $2\sqrt{2} \times \sqrt{3}$

2) $2\sqrt{4} \times \sqrt{3}$

3) $\sqrt{5} \times 2\sqrt{3}$

4) $\sqrt{6} \times 2\sqrt{3}$

5) $\sqrt{24} \times 3\sqrt{3}$

6) $2\sqrt{12} \times 3\sqrt{3}$

7) $2\sqrt{12} \times 3\sqrt{4}$

8) $2\sqrt{11} \times 3\sqrt{4}$

9) $\sqrt{10} \times 2\sqrt{4}$

10) $\sqrt{10} \times 2\sqrt{8}$

11) $\sqrt{20} \times 2\sqrt{4}$

12) $5\sqrt{20} \times 3\sqrt{5}$

13) $5\sqrt{20} \times 3\sqrt{10}$

14) $5\sqrt{20} \times 3\sqrt{20}$

15) $2\sqrt{20} \times 3\sqrt{40}$

16) $2\sqrt{20} \times 2\sqrt{40} \times 2\sqrt{2}$

17) $2\sqrt{20} \times 2\sqrt{40} \times \sqrt{8}$

18) $2\sqrt{20} \times 2\sqrt{40} \times 2\sqrt{8}$

Intelligent Practice

Simplify:

1) $\sqrt{6} \div \sqrt{3}$

2) $\sqrt{6} \div \sqrt{2}$

3) $\sqrt{12} \div \sqrt{2}$

4) $\sqrt{12} \div \sqrt{3}$

5) $\sqrt{15} \div \sqrt{3}$

6) $\sqrt{18} \div \sqrt{3}$

7) $\sqrt{72} \div \sqrt{3}$

8) $\sqrt{36} \div \sqrt{3}$

9) $\sqrt{36} \div \sqrt{4}$

10) $\sqrt{40} \div \sqrt{4}$

11) $\sqrt{40} \div \sqrt{40}$

12) $\sqrt{40} \div \sqrt{5}$

13) $\sqrt{100} \div \sqrt{5}$

14) $\sqrt{200} \div \sqrt{10}$

15) $\sqrt{400} \div \sqrt{20}$

16) $\sqrt{800} \div \sqrt{40}$

17) $\sqrt{800} \div \sqrt{40} \div \sqrt{2}$

18) $\sqrt{800} \div \sqrt{40} \div \sqrt{20}$

Intelligent Practice

Simplify:

1) $2\sqrt{6} \div \sqrt{3}$

2) $4\sqrt{6} \div \sqrt{2}$

3) $4\sqrt{12} \div 2\sqrt{2}$

4) $4\sqrt{12} \div 2\sqrt{3}$

5) $6\sqrt{15} \div 3\sqrt{3}$

6) $6\sqrt{18} \div 3\sqrt{3}$

7) $24\sqrt{72} \div 3\sqrt{3}$

8) $12\sqrt{36} \div 3\sqrt{3}$

9) $12\sqrt{36} \div 4\sqrt{4}$

10) $20\sqrt{40} \div 4\sqrt{4}$

11) $20\sqrt{40} \div 20\sqrt{40}$

12) $20\sqrt{40} \div 5\sqrt{5}$

13) $10\sqrt{100} \div 5\sqrt{5}$

14) $20\sqrt{200} \div 10\sqrt{10}$

15) $40\sqrt{400} \div 20\sqrt{20}$

16) $80\sqrt{800} \div 40\sqrt{40}$

17) $80\sqrt{800} \div 40\sqrt{40} \div 2\sqrt{2}$

18) $80\sqrt{800} \div 40\sqrt{40} \div 2\sqrt{20}$

Fluency Practice

Multiplication Madness

Think of this like a normal multiplication table, just with terms missing everywhere. Fill in all blanks.

	$-\sqrt{5}$	$\sqrt{2}$			$-\sqrt{6}$		$2\sqrt{3}$	$-\sqrt{2}$			-2
3											
		$\sqrt{10}$			$3\sqrt{5}$		$2\sqrt{15}$				
				$16\sqrt{2}$							$8\sqrt{2}$
											4
$\sqrt{7}$	$-\sqrt{35}$						$2\sqrt{14}$				
								$2\sqrt{6}$			
		6				$9\sqrt{2}$					
					$-9\sqrt{2}$						
			$4\sqrt{6}$				$8\sqrt{3}$				
$-2\sqrt{5}$											-20
$\sqrt{2}$			$2\sqrt{3}$		$-2\sqrt{3}$				$-\sqrt{6}$		
		4									

Fluency Practice

Surds: Multiplication Squares

Can you fill in the missing numbers in these multiplication squares?

x	$\sqrt{10}$	$\sqrt{6}$
$\sqrt{8}$	$\sqrt{\quad}$	$2\sqrt{\quad}$
$\sqrt{6}$	$2\sqrt{\quad}$	\quad

x	$\sqrt{6}$	$\sqrt{3}$
$\sqrt{12}$	$\sqrt{\quad}$	$\sqrt{\quad}$
$\sqrt{7}$	$\sqrt{3} \times \sqrt{\quad}$	$\sqrt{\quad}$

x	$3\sqrt{6}$	$2\sqrt{8}$
$\sqrt{6}$	\quad	$\sqrt{\quad}$
$\sqrt{3}$	$\sqrt{\quad}$	$4\sqrt{\quad}$

x	$\sqrt{9}$	$2\sqrt{3}$
$2\sqrt{3}$	$\sqrt{\quad}$	\quad
$3\sqrt{3}$	$\sqrt{\quad}$	\quad

x	$\sqrt{\quad}$	$\sqrt{\quad}$
$\sqrt{6}$	$\sqrt{48}$	$\sqrt{30}$
\quad	$\sqrt{72}$	$\sqrt{45}$

x	$2\sqrt{5}$	10
\quad	$\sqrt{60}$	\quad
\quad	$4\sqrt{6}$	$4\sqrt{\quad}$

* Challenge: Make up one of your own!*

Intelligent Practice

Simplify:

1) $2\sqrt{3} + 5\sqrt{3}$

2) $2\sqrt{3} + 5\sqrt{6}$

3) $2\sqrt{3} + 5\sqrt{12}$

4) $2\sqrt{3} + 5\sqrt{27}$

5) $2\sqrt{3} + 6\sqrt{27}$

6) $2\sqrt{30} + 6\sqrt{27}$

7) $2\sqrt{30} + 6\sqrt{120}$

8) $6\sqrt{30} + 2\sqrt{120}$

9) $6\sqrt{60} + 2\sqrt{240}$

10) $6\sqrt{6} + 2\sqrt{24}$

11) $6\sqrt{6} + 2\sqrt{24} + 5\sqrt{36}$

12) $6\sqrt{16} + 2\sqrt{24} + 5\sqrt{36}$

Intelligent Practice

Simplify:

$$1) \frac{2\sqrt{3}+5\sqrt{3}}{\sqrt{3}}$$

$$2) \frac{2\sqrt{3}+5\sqrt{12}}{\sqrt{3}}$$

$$3) \frac{2\sqrt{3}+5\sqrt{27}}{\sqrt{3}}$$

$$4) \frac{5\sqrt{27}-2\sqrt{3}}{\sqrt{3}}$$

$$5) \frac{10\sqrt{27}-4\sqrt{3}}{2\sqrt{3}}$$

$$6) \frac{10\sqrt{27}-30\sqrt{3}}{2\sqrt{3}}$$

$$7) \frac{10\sqrt{270}+30\sqrt{30}}{2\sqrt{30}}$$

$$8) \frac{10\sqrt{120}+30\sqrt{30}}{2\sqrt{30}}$$

$$9) \frac{10\sqrt{120}+30\sqrt{30}}{50\sqrt{30}}$$

Fluency Practice

1) True or False?

Are these calculations correct?

Adding & Subtracting Surds

a) $\sqrt{8} + \sqrt{18} =$
 $2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$

b) $\sqrt{8} + \sqrt{50} =$
 $2\sqrt{2} + 5\sqrt{2} = 7\sqrt{2}$

c) $\sqrt{12} + \sqrt{27} =$
 $2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$

d) $\sqrt{90} - \sqrt{40} =$
 $3\sqrt{10} - 2\sqrt{10} = \sqrt{10}$

e) $\sqrt{80} - \sqrt{20} =$
 $8\sqrt{10} - 2\sqrt{10} = 6\sqrt{10}$

f) $\sqrt{125} + \sqrt{45} =$
 $5\sqrt{5} + 3\sqrt{5} = 8\sqrt{5}$

g) $\sqrt{63} + \sqrt{28} =$
 $3\sqrt{7} + 4\sqrt{7} = 7\sqrt{7}$

h) $\sqrt{150} - \sqrt{54} =$
 $5\sqrt{6} - 3\sqrt{6} = 2\sqrt{6}$

i) $\sqrt{75} - \sqrt{108} =$
 $5\sqrt{3} - 6\sqrt{3} = \sqrt{3}$

j) $\sqrt{180} + \sqrt{18} =$
 $10\sqrt{18} + \sqrt{18} = 11\sqrt{18}$

k) $\sqrt{72} + 2\sqrt{8} =$
 $6\sqrt{2} + 4\sqrt{2} = 10\sqrt{2}$

l) $2\sqrt{18} - \sqrt{32} =$
 $5\sqrt{2} - 4\sqrt{2} = \sqrt{2}$

2) Free Practice

a) $\sqrt{48} + \sqrt{27} =$

b) $\sqrt{360} + \sqrt{40} =$

c) $\sqrt{150} - \sqrt{96} =$

d) $\sqrt{200} - \sqrt{32} =$

e) $\sqrt{20} + \sqrt{40} =$

f) $\sqrt{48} - \sqrt{12} + \sqrt{27} =$

g) $\sqrt{8} + \sqrt{8} =$

h) $\sqrt{45} + \sqrt{45} =$

i) $\sqrt{18} + \sqrt{18} + \sqrt{32} =$

j) $\sqrt{90} + (2 \times \sqrt{40}) =$

k) $\sqrt{18} + 2\sqrt{18} =$

l) $3\sqrt{20} - \sqrt{180} =$

3) Extension

a) $2\sqrt{24} + \sqrt{54} =$

b) $2\sqrt{28} - \sqrt{63} =$

c) $2\sqrt{32} + 2\sqrt{18} =$

d) $2\sqrt{48} - 2\sqrt{117} =$

e) $3\sqrt{50} - \sqrt{200} =$

f) $2\sqrt{180} - 2\sqrt{125} =$

g) $3\sqrt{175} + 4\sqrt{28} =$

h) $3\sqrt{500} - 3\sqrt{245} =$

i) $2\sqrt{192} - 4\sqrt{75} =$

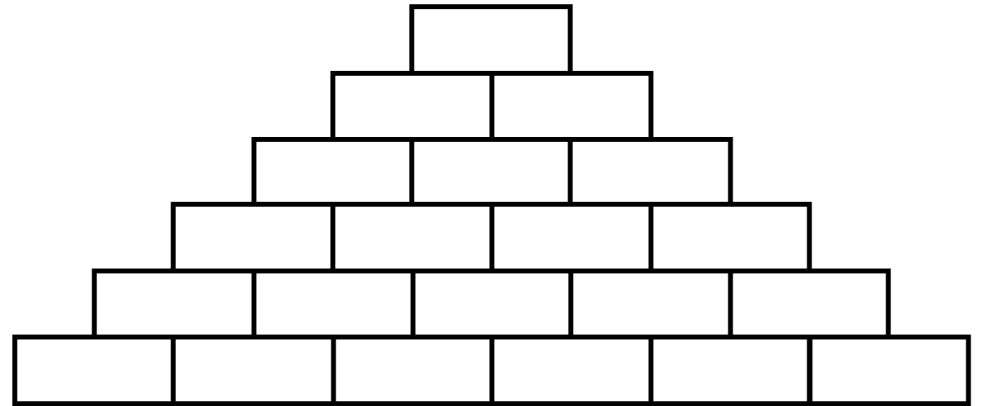
j) $2\sqrt{448} + 3\sqrt{640} =$

Problem Solving

Pyramid II

Cut out the bricks below. Place the bricks in a pyramid shape so that each brick is the sum of the two bricks beneath it.

$7\sqrt{2} + \sqrt{147}$	$\sqrt{2} - \sqrt{75}$	$\sqrt{3}$
$2\sqrt{2} - \sqrt{432}$	$-5\sqrt{3}$	$\sqrt{3} + \sqrt{8}$
$-9\sqrt{3}$	$\sqrt{18} + \sqrt{48}$	$-3\sqrt{3} + \sqrt{722}$
$2\sqrt{2}$	$-\sqrt{2} - \sqrt{48}$	$12\sqrt{2} + 8\sqrt{3}$
$5\sqrt{2} + \sqrt{3}$	$2(\sqrt{3} + \sqrt{2})$	$-\sqrt{2} + \sqrt{3}$
$\sqrt{98} - \sqrt{363}$	$2\sqrt{3}$	$\sqrt{2} + \sqrt{12}$
$\sqrt{2}$	$\sqrt{8} - \sqrt{27}$	$\sqrt{27} + 4\sqrt{2}$

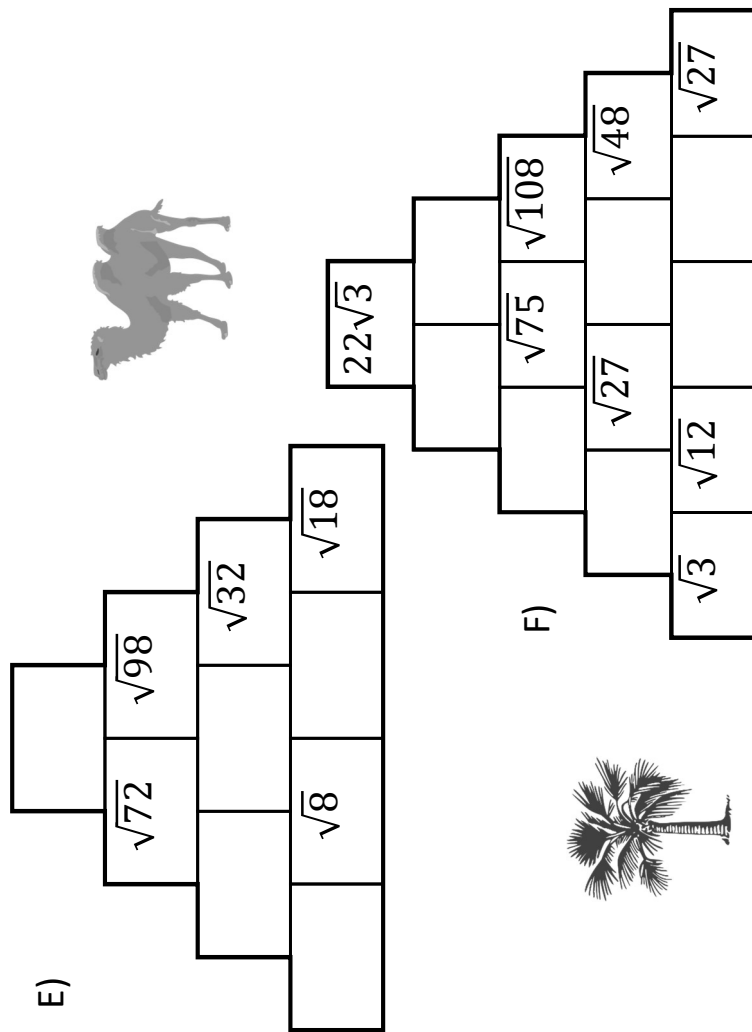
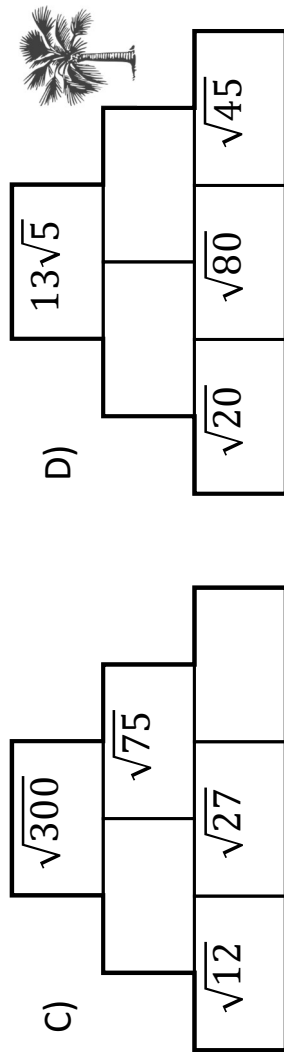
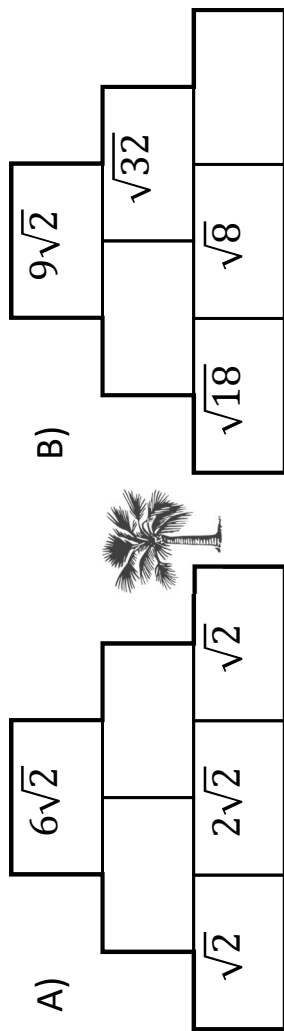


Fluency Practice

Surd Pyramids

1

Each brick is the two bricks below it added together.

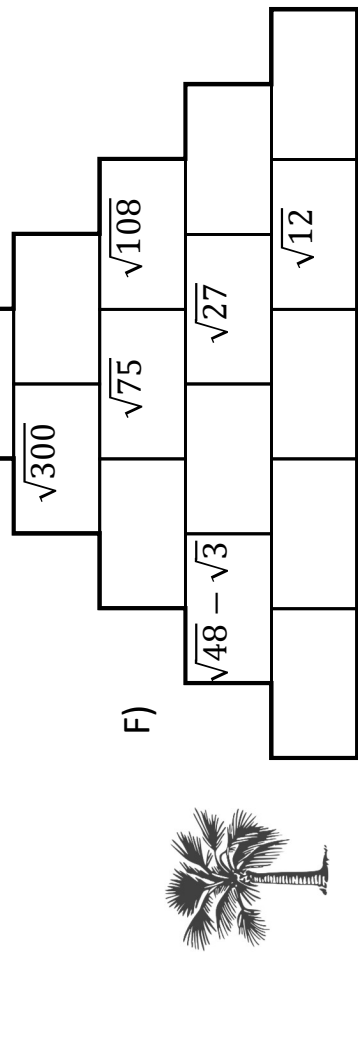
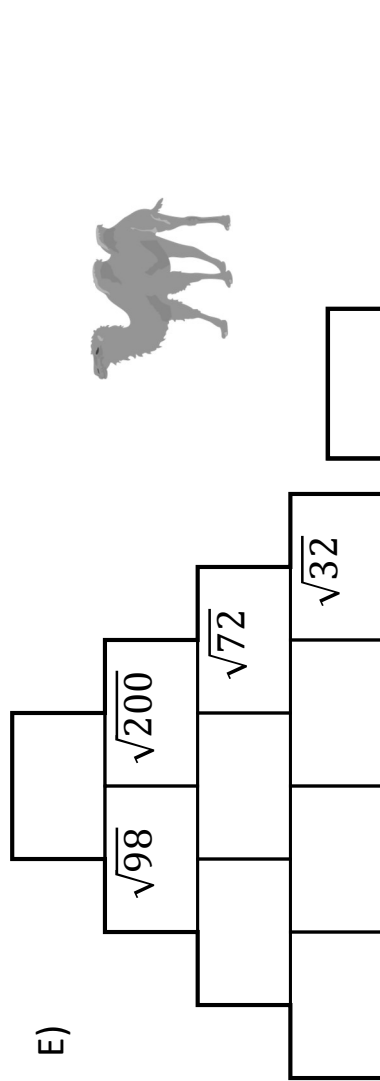
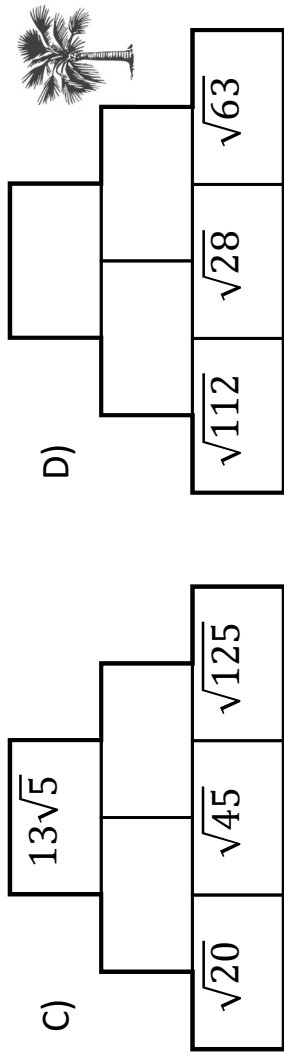
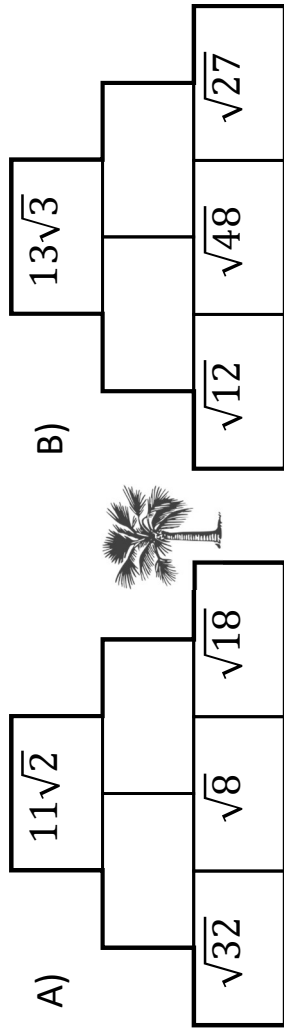


Fluency Practice

Surd Pyramids

2

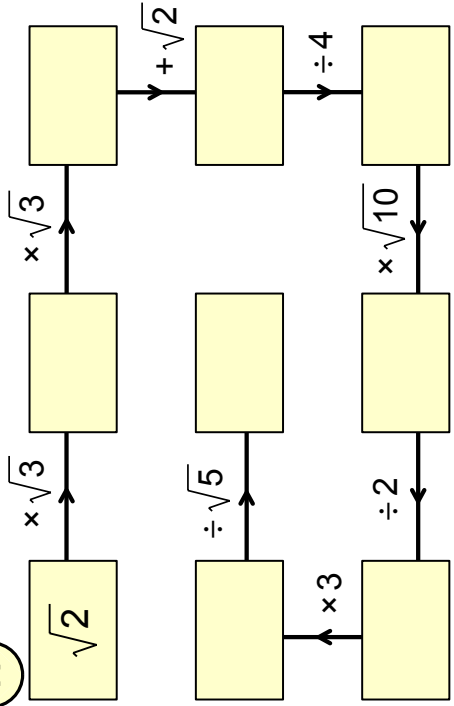
Each brick is the two bricks below it added together.



Problem Solving

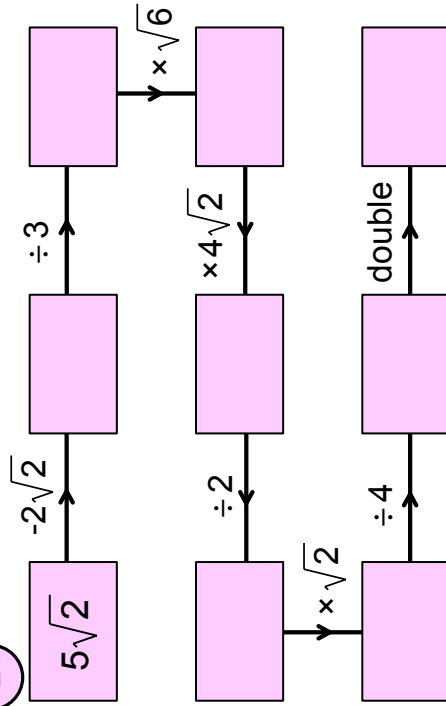
surds puzzles

A



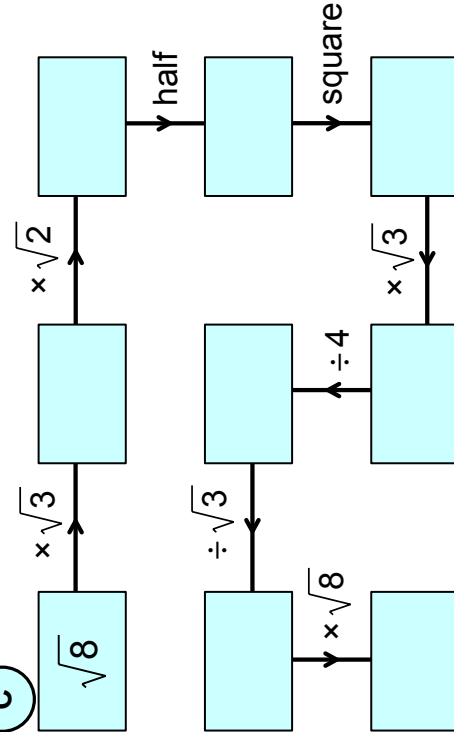
$\sqrt{5}$	$\sqrt{2}$
$4\sqrt{2}$	$\sqrt{6}$
$\sqrt{20}$	3
$3\sqrt{2}$	$3\sqrt{5}$

B



$8\sqrt{6}$	$3\sqrt{2}$
$2\sqrt{3}$	$4\sqrt{3}$
$\sqrt{2}$	$4\sqrt{6}$
$8\sqrt{3}$	$2\sqrt{3}$

C

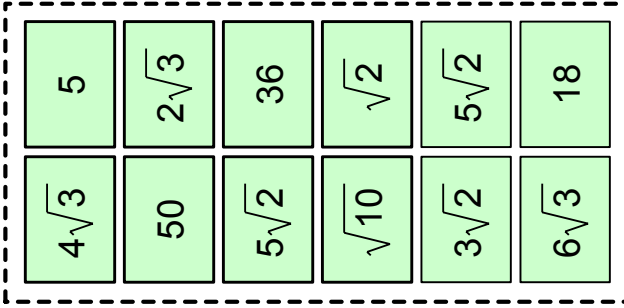
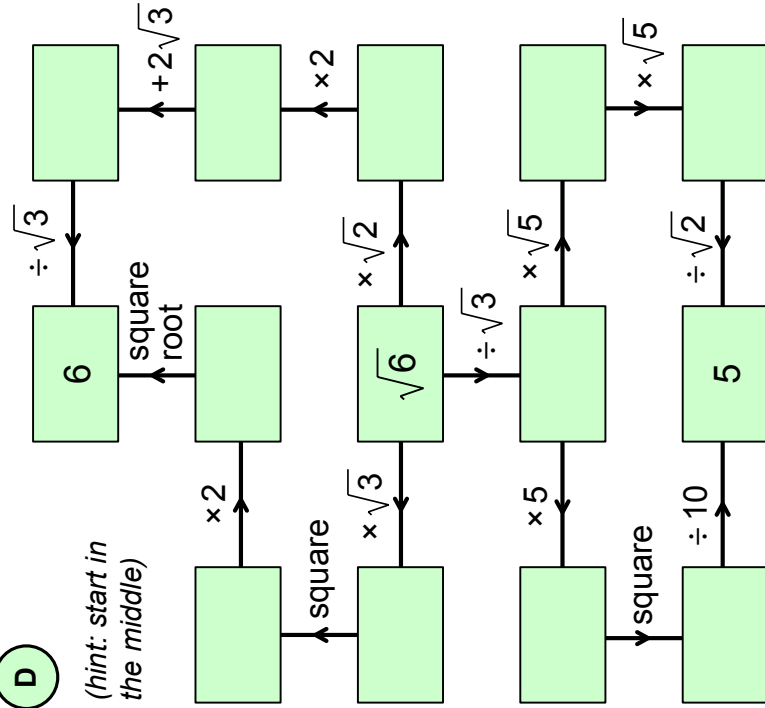


3	$2\sqrt{3}$
$2\sqrt{6}$	$12\sqrt{3}$
$3\sqrt{3}$	$4\sqrt{3}$
12	$6\sqrt{2}$

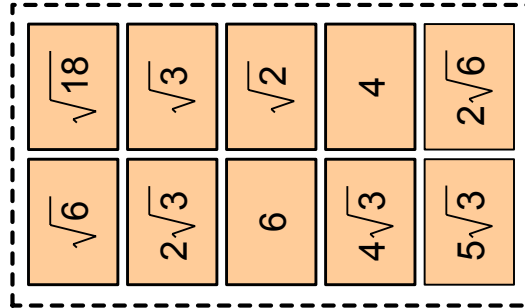
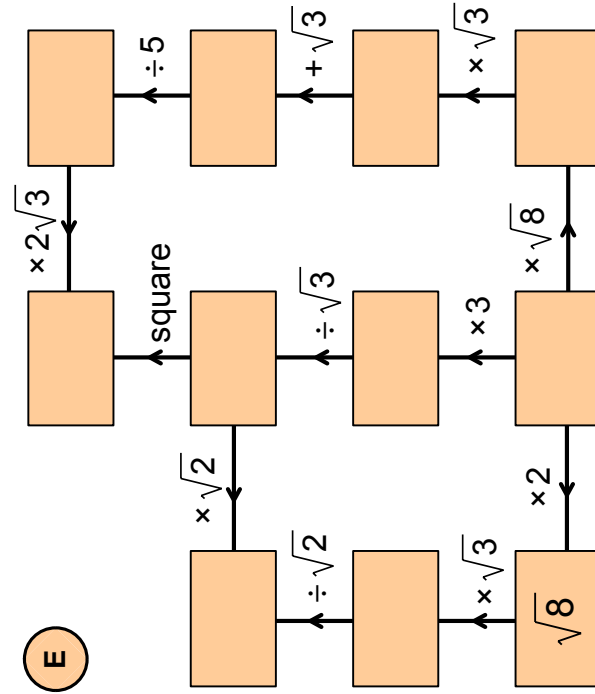
Problem Solving

D

(hint: start in the middle)

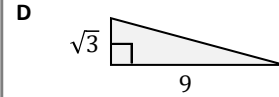
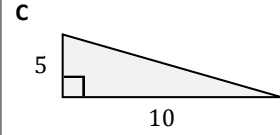
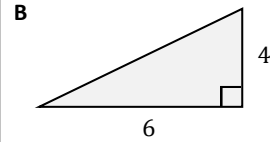
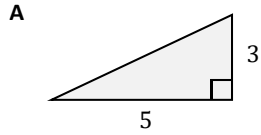


E

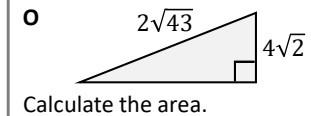
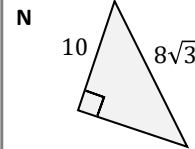
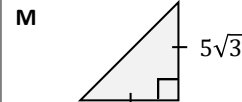
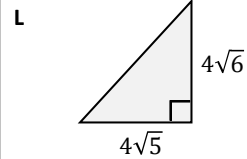
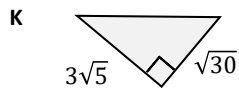
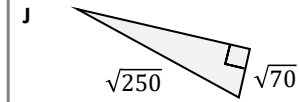
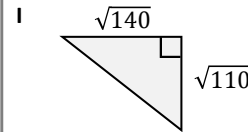
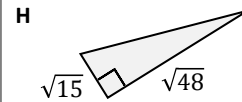
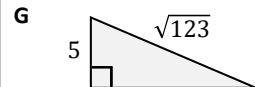
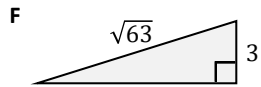
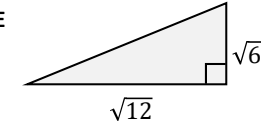


Fluency Practice

Find the missing lengths & simplify your answers.



Diagrams not drawn accurately.



14 Answers	$3\sqrt{7}$	$3\sqrt{2}$	$5\sqrt{6}$	$\sqrt{34}$	$2\sqrt{23}$	$5\sqrt{5}$	$5\sqrt{3}$
	$2\sqrt{13}$	$4\sqrt{11}$	$2\sqrt{21}$	$5\sqrt{10}$	$7\sqrt{2}$	$6\sqrt{5}$	$3\sqrt{6}$

Averages with... Surds

For each set of numbers find: (i) the mean, (ii) the median, (iii) the range.

Leave your answers in their simplest form.

a) $5\sqrt{3}$ $4\sqrt{3}$ $6\sqrt{3}$

b) $\sqrt{3}$ $\sqrt{108}$ $\sqrt{12}$

c) $2\sqrt{12}$ $4\sqrt{12}$ $3\sqrt{27}$

d) $2\sqrt{27}$ $3\sqrt{12}$ $\sqrt{243}$

e) $2\sqrt{75}$ $\sqrt{48}$ $5\sqrt{12}$

f) $\sqrt{243}$ $5\sqrt{27}$ $3\sqrt{75}$

g) $\sqrt{20}$ $\sqrt{20}$ $\sqrt{80}$ $\sqrt{320}$

h) $\sqrt{28}$ $\sqrt{63}$ $3\sqrt{63}$ $4\sqrt{7}$

i) $-\sqrt{99}$ $2\sqrt{44}$ $-\sqrt{11}$ $4\sqrt{44}$

j) 0 $-2\sqrt{44}$ $-4\sqrt{44}$ $\sqrt{44}$

k) $\sqrt{2}$ $-\sqrt{18}$ $\sqrt{48}$ $\sqrt{8}$

l) $2\sqrt{150}$ $-2\sqrt{54}$ $2\sqrt{24}$ $-2\sqrt{96}$

Intelligent Practice

Expand and simplify:

1) $3(4 + \sqrt{5})$

2) $3(\sqrt{5} + 4)$

3) $6(\sqrt{5} + 4)$

4) $6(\sqrt{5} - 4)$

5) $6(4 - \sqrt{5})$

6) $-6(4 - \sqrt{5})$

7) $-6(-4 - \sqrt{5})$

8) $6(4 + \sqrt{5})$

9) $\sqrt{5}(4 + \sqrt{5})$

10) $\sqrt{5}(4 - \sqrt{5})$

11) $2\sqrt{5}(4 - \sqrt{5})$

12) $\sqrt{20}(4 - \sqrt{5})$

13) $\sqrt{20}(\sqrt{4} - \sqrt{5})$

14) $\sqrt{20}(\sqrt{6} - \sqrt{5})$

15) $\sqrt{20}(\sqrt{6} - \sqrt{10})$

16) $\sqrt{20}(\sqrt{12} - \sqrt{10})$

17) $\sqrt{10}(\sqrt{6} - \sqrt{5})$

18) $\sqrt{10}(\sqrt{6} - \sqrt{5} + \sqrt{24})$

Intelligent Practice

Expand and simplify:

1) $(\sqrt{5} + 2)(\sqrt{5} + 1)$

2) $(2 + \sqrt{5})(1 + \sqrt{5})$

3) $(2 - \sqrt{5})(1 + \sqrt{5})$

4) $(2 - \sqrt{5})(1 - \sqrt{5})$

5) $(2 + 2\sqrt{5})(1 + \sqrt{5})$

6) $(\sqrt{5} + 2)(\sqrt{5} - 1)$

7) $(\sqrt{7} + 2)(\sqrt{7} - 1)$

8) $(2\sqrt{7} + 2)(3\sqrt{7} - 1)$

9) $(2\sqrt{7} + 2)(3\sqrt{7} - 6)$

10) $(\sqrt{7} + 6)(\sqrt{7} - 6)$

11) $2(\sqrt{7} + 6)(\sqrt{7} - 6)$

12) $(\sqrt{7} - 6)(\sqrt{7} - 6)$

13) $(\sqrt{7} - 6)^2$

14) $(\sqrt{7} - 6)^3$

Intelligent Practice

Expand and simplify:

1) $(2 + \sqrt{3})(4 + \sqrt{12})$

2) $(2 + \sqrt{3})(4 - \sqrt{12})$

3) $(2 - \sqrt{3})(4 + \sqrt{12})$

4) $(2 - \sqrt{3})(4 - \sqrt{12})$

5) $(\sqrt{3} - 2)(\sqrt{12} - 4)$

6) $(\sqrt{12} - 2)(\sqrt{3} - 4)$

7) $(\sqrt{27} - 2)(\sqrt{3} - 4)$

8) $(\sqrt{27} - 3)(\sqrt{3} - 5)$

9) $(\sqrt{54} - 3)(\sqrt{6} - 5)$

10) $(2\sqrt{54} - 3)(\sqrt{6} - 5)$

11) $(2\sqrt{54} - 3)(4\sqrt{6} - 5)$

12) $(2\sqrt{44} - 3)(4\sqrt{11} - 5)$

13) $(3\sqrt{44} - 2)(5\sqrt{11} - 4)$

14) $(3\sqrt{44} - 2)^2$

Fluency Practice

① 5×5

② $5 \times \sqrt{5}$

③ $\sqrt{5} \times \sqrt{5}$

④ $5 \times 5\sqrt{5}$

⑤ $\sqrt{5} \times 5\sqrt{5}$

⑥ $5\sqrt{5} \times 5\sqrt{5}$

⑦ $5(5 + \sqrt{5})$

⑧ $\sqrt{5}(5 + \sqrt{5})$

⑨ $5(5 + 5\sqrt{5})$

⑩ $5\sqrt{5}(5 + 5\sqrt{5})$

⑪ $5\sqrt{5}(\sqrt{5} + 5\sqrt{5})$

⑫ $(5 + \sqrt{5})(5 + \sqrt{5})$

⑬ $(5 + \sqrt{5})(5 - \sqrt{5})$

⑭ $(5 + \sqrt{5})(5 + 5\sqrt{5})$

⑮ $(5 + 5\sqrt{5})(5 + 5\sqrt{5})$

⑯ $(\sqrt{5} + 5\sqrt{5})(\sqrt{5} + 5\sqrt{5})$

Fluency Practice

(a)	(b)	(c)	(d)
Simplify $2\sqrt{5} + 3\sqrt{5}$	Simplify $6\sqrt{5} + 5\sqrt{5}$	Simplify $2\sqrt{5} - 3\sqrt{5}$	Simplify $7\sqrt{3} - 3\sqrt{3}$
(e)	(f)	(g)	(h)
Simplify $2\sqrt{3} + 3\sqrt{2}$	Simplify $7\sqrt{10} + 3\sqrt{10} - \sqrt{10}$	Expand $3(6 + \sqrt{5})$	Expand $\sqrt{2}(7 - \sqrt{3})$
(i)	(j)	(k)	(l)
Expand $5\sqrt{2}(3 - \sqrt{7})$	Expand and simplify $(3 + \sqrt{2})(6 + \sqrt{2})$	Expand and simplify $(3 + \sqrt{7})(5 - \sqrt{7})$	Expand and simplify $(4 + 3\sqrt{5})(2 - \sqrt{5})$

Fluency Practice

Simplify

(a) $4\sqrt{3} + 2\sqrt{3}$ (b) $4\sqrt{3} - 2\sqrt{3}$

(c) $2\sqrt{3} - 4\sqrt{3}$ (d) $2\sqrt{3} - 4\sqrt{5}$

(e) $-4\sqrt{2} + 2\sqrt{2}$ (f) $\sqrt{2} - 2\sqrt{2}$

(g) $6\sqrt{5} + 2\sqrt{5} - 3\sqrt{5}$

(h) $\sqrt{3} - 2\sqrt{3} + 7\sqrt{3}$

Expand and simplify where possible

(a) $5(2 + \sqrt{3})$ (b) $\sqrt{5}(2 + \sqrt{3})$

(c) $\sqrt{5}(\sqrt{2} - \sqrt{3})$ (d) $5(\sqrt{2} - \sqrt{3})$

(e) $\sqrt{3}(\sqrt{3} - 7)$ (f) $\sqrt{3}(2 + \sqrt{3})$

(g) $5\sqrt{2}(2 + \sqrt{3})$ (h) $\sqrt{5}(2\sqrt{3} + \sqrt{5})$

(i) $\sqrt{5}(\sqrt{2} + 2\sqrt{3})$ (j) $3\sqrt{5}(2\sqrt{2} + 3\sqrt{3})$

Expand and simplify where possible

(a) $(2 + \sqrt{3})(1 + \sqrt{3})$

(b) $(2 + \sqrt{3})(4 + \sqrt{3})$

(c) $(5 - \sqrt{5})(4 + \sqrt{5})$

(d) $(2 + \sqrt{5})(4 - \sqrt{5})$

(e) $(1 + 2\sqrt{3})(4 - \sqrt{3})$

(f) $(2 + 3\sqrt{5})(4 - 2\sqrt{5})$

Calculate the areas of these shapes, giving answers in their simplest form

(a) A square with side length $2 + \sqrt{5}$ cm

(b) A rectangle with length $\sqrt{7}$ cm and width $1 + \sqrt{3}$ cm

(c) A triangle with base $\sqrt{8}$ cm and height $2\sqrt{8}$ cm

Fluency Practice

- (a) Show that $(5 - \sqrt{12})(2 + \sqrt{3})$ can be written in the form $a + \sqrt{b}$ where a and b are integers.
- (b) Show that $(6 + \sqrt{2})(\sqrt{8} - 4)$ can be written in the form $a\sqrt{2} + b$ where a and b are integers.

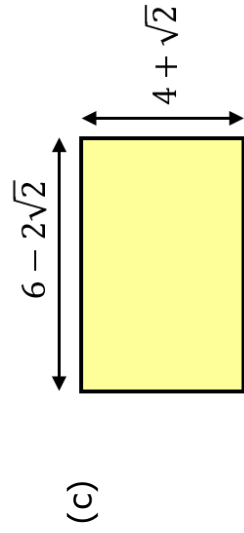
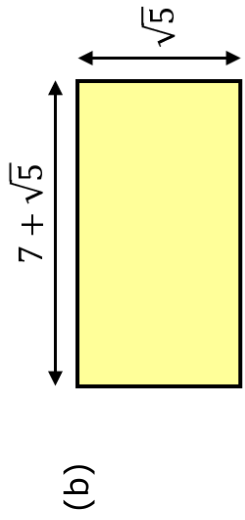
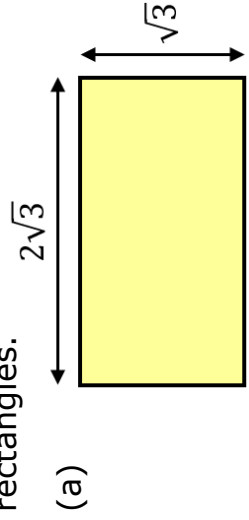
- (c) Show that $\sqrt{3}(\sqrt{12} - 2)^2$ can be written in the form $a\sqrt{3} + b$ where a and b are integers.
- (d) Show that $(5 + \sqrt{5})(\sqrt{20} - 2)(3 - \sqrt{5})$ can be written in the form $a\sqrt{5} + b$ where a and b are integers.
- (e) Show that $(1 + \sqrt{8})^3$ can be written in the form $p + \sqrt{q}$ where p and q are integers.

- (f) Given that
$$(a + \sqrt{6})(10 - \sqrt{6}) = 24 + b\sqrt{6}$$
find the values of the integers a and b .
- (g) Given that
$$(5 - \sqrt{8})(3 - a\sqrt{2}) = b - 21\sqrt{2}$$
find the values of the integers a and b .

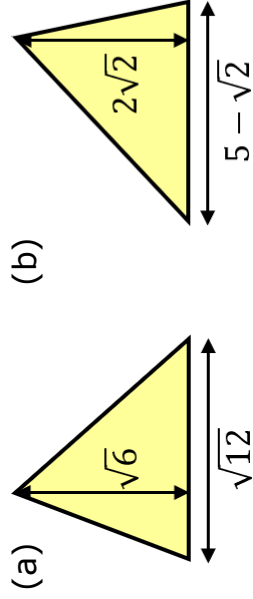
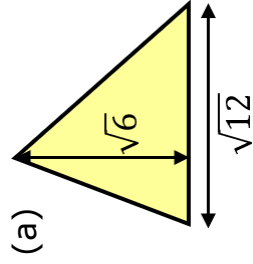
- (h) Given that
$$(p + 2\sqrt{q})^2 = 40 + 16\sqrt{q}$$
find the values of the integers p and q .
- (i) Express $(6 + \sqrt{3})(a - 2\sqrt{3})(4 - \sqrt{12})$ in the form $b - 76\sqrt{3}$ where a and b are integers to be found.
- (j) Given that
$$(p + \sqrt{q})(p - 3\sqrt{q}) = 13 - 14\sqrt{q}$$
find the values of the integers p and q .

Fluency Practice

Find the area and perimeter of these rectangles.



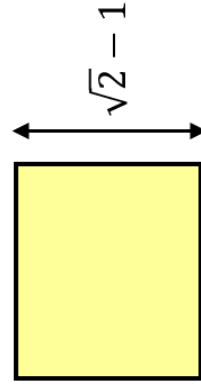
Find the area of these triangles.



(a) Given that $(3 + \sqrt{a})(4 + \sqrt{a}) = 17 + b\sqrt{a}$ find the values of a and b .

(b) Given that $(5 - \sqrt{x})^2 = y - 20\sqrt{2}$ find the values of x and y .

(a) The perimeter of this rectangle is $8\sqrt{2}$. Find the area of the rectangle.



Intelligent Practice

Rationalise:

1) $\frac{5}{\sqrt{2}}$

2) $\frac{6}{\sqrt{2}}$

3) $\frac{6}{\sqrt{3}}$

4) $\frac{4}{\sqrt{3}}$

5) $\frac{4}{\sqrt{6}}$

6) $\frac{5}{\sqrt{6}}$

7) $\frac{5}{2\sqrt{6}}$

8) $\frac{5}{3\sqrt{6}}$

9) $\frac{5}{10\sqrt{6}}$

10) $\frac{5}{10\sqrt{5}}$

11) $\frac{5}{\sqrt{5}}$

12) $\frac{8}{\sqrt{8}}$

13) $\frac{10}{\sqrt{8}}$

14) $\frac{10}{\sqrt{12}}$

15) $\frac{10}{2\sqrt{3}}$

16) $\frac{10+\sqrt{3}}{2\sqrt{3}}$

17) $\frac{10-\sqrt{3}}{2\sqrt{3}}$

18) $\frac{12-\sqrt{3}}{2\sqrt{3}}$

Fluency Practice

Simplify each fraction, leaving the denominator as a rational number. Any surds in the numerator should also be fully simplified.

$\frac{2}{\sqrt{2}} =$	$\frac{3}{\sqrt{2}} =$	$\frac{4}{\sqrt{2}} =$	$\frac{5}{\sqrt{2}} =$	$\frac{6}{\sqrt{2}} =$	$\frac{7}{\sqrt{2}} =$	$\frac{8}{\sqrt{2}} =$
$\frac{\sqrt{2}}{\sqrt{2}} =$	$\frac{\sqrt{3}}{\sqrt{2}} =$	$\frac{\sqrt{4}}{\sqrt{2}} =$	$\frac{\sqrt{5}}{\sqrt{2}} =$	$\frac{\sqrt{6}}{\sqrt{2}} =$	$\frac{\sqrt{7}}{\sqrt{2}} =$	$\frac{\sqrt{8}}{\sqrt{2}} =$
$\frac{2}{2\sqrt{2}} =$	$\frac{3}{2\sqrt{2}} =$	$\frac{4}{2\sqrt{2}} =$	$\frac{5}{2\sqrt{2}} =$	$\frac{6}{2\sqrt{2}} =$	$\frac{7}{2\sqrt{2}} =$	$\frac{8}{2\sqrt{2}} =$
$\frac{2}{\sqrt{8}} =$	$\frac{3}{\sqrt{8}} =$	$\frac{4}{\sqrt{8}} =$	$\frac{5}{\sqrt{8}} =$	$\frac{6}{\sqrt{8}} =$	$\frac{7}{\sqrt{8}} =$	$\frac{8}{\sqrt{8}} =$
$\frac{2\sqrt{6}}{2\sqrt{2}} =$	$\frac{3\sqrt{6}}{2\sqrt{2}} =$	$\frac{4\sqrt{6}}{2\sqrt{2}} =$	$\frac{5\sqrt{6}}{2\sqrt{2}} =$	$\frac{6\sqrt{6}}{2\sqrt{2}} =$	$\frac{7\sqrt{6}}{2\sqrt{2}} =$	$\frac{8\sqrt{6}}{2\sqrt{2}} =$
$\frac{2\sqrt{6}}{2\sqrt{3}} =$	$\frac{3\sqrt{6}}{2\sqrt{3}} =$	$\frac{4\sqrt{6}}{2\sqrt{3}} =$	$\frac{5\sqrt{6}}{2\sqrt{3}} =$	$\frac{6\sqrt{6}}{2\sqrt{3}} =$	$\frac{7\sqrt{6}}{2\sqrt{3}} =$	$\frac{8\sqrt{6}}{2\sqrt{3}} =$

Fluency Practice

Express each fraction below as either:

- a rational number
- a fraction with a rational denominator, or
- a surd in simplest form

$$\frac{\sqrt{8}}{\sqrt{1}} \quad \frac{\sqrt{8}}{\sqrt{2}} \quad \frac{\sqrt{8}}{\sqrt{3}} \quad \frac{\sqrt{8}}{\sqrt{4}} \quad \frac{\sqrt{8}}{\sqrt{5}} \quad \frac{\sqrt{8}}{\sqrt{6}} \quad \frac{\sqrt{8}}{\sqrt{7}} \quad \frac{\sqrt{8}}{\sqrt{8}} \quad \frac{\sqrt{8}}{\sqrt{9}} \quad \frac{\sqrt{8}}{\sqrt{10}} \quad \frac{\sqrt{8}}{\sqrt{11}} \quad \frac{\sqrt{8}}{\sqrt{12}}$$

Intelligent Practice

Rationalise:

1) $\frac{2}{\sqrt{2}+3}$

2) $\frac{2}{3+\sqrt{2}}$

3) $\frac{2}{3-\sqrt{2}}$

4) $\frac{7}{3-\sqrt{2}}$

5) $\frac{7}{\sqrt{2}-3}$

6) $\frac{7}{\sqrt{7}-3}$

7) $\frac{\sqrt{7}}{\sqrt{7}-3}$

8) $\frac{\sqrt{5}}{\sqrt{5}-3}$

9) $\frac{\sqrt{5}}{5-\sqrt{3}}$

10) $\frac{2\sqrt{5}}{5-\sqrt{3}}$

11) $\frac{4\sqrt{5}}{5-\sqrt{3}}$

12) $\frac{4\sqrt{5}}{5-2\sqrt{3}}$

13) $\frac{4\sqrt{5}}{5-3\sqrt{3}}$

14) $\frac{4\sqrt{3}}{5-3\sqrt{3}}$

Fill in the Gaps

Question	Working		Answer
$\frac{5}{\sqrt{3}}$	$\times \frac{\sqrt{3}}{\sqrt{3}}$	$= \frac{5\sqrt{3}}{\sqrt{9}}$	$= \frac{5\sqrt{3}}{3}$
$\frac{\sqrt{3}}{\sqrt{7}}$	$\times \frac{\sqrt{7}}{\sqrt{7}}$		
$\frac{5\sqrt{5}}{\sqrt{6}}$			
$\frac{2 + \sqrt{3}}{\sqrt{5}}$	$\times \frac{\sqrt{5}}{\sqrt{5}}$	$= \frac{\sqrt{5}(2 + \sqrt{3})}{\sqrt{25}}$	$= \frac{2\sqrt{5} + \sqrt{15}}{5}$
$\frac{3 - \sqrt{5}}{\sqrt{2}}$			
$\frac{1 + \sqrt{2}}{2\sqrt{3}}$			
$\frac{\sqrt{2} - 3\sqrt{5}}{5\sqrt{2}}$			

Question	Working		Answer
$\frac{3}{2 + \sqrt{2}}$	$\times \frac{2 - \sqrt{2}}{2 - \sqrt{2}}$	$= \frac{3(2 - \sqrt{2})}{4 - \sqrt{4}}$	$= \frac{6 - 3\sqrt{2}}{2}$
$\frac{8}{4 - \sqrt{3}}$			
$\frac{\sqrt{5}}{6 + \sqrt{5}}$			
$\frac{3\sqrt{5}}{3 - \sqrt{7}}$			
$\frac{7 + \sqrt{2}}{3 - \sqrt{2}}$	$\times \frac{3 + \sqrt{2}}{3 + \sqrt{2}}$	$= \frac{(7 + \sqrt{2})(3 + \sqrt{2})}{9 - \sqrt{4}}$	$= \frac{23 + 10\sqrt{2}}{7}$
$\frac{1 - \sqrt{8}}{5 + \sqrt{2}}$			
$\frac{a + \sqrt{b}}{a\sqrt{b}}$			

Fluency Practice

Rationalise the denominator.

(a) $\frac{1}{\sqrt{5}}$

(b) $\frac{2}{\sqrt{5}}$

(c) $\frac{2}{\sqrt{7}}$

(d) $\frac{\sqrt{2}}{\sqrt{7}}$

(e) $\frac{2\sqrt{2}}{\sqrt{5}}$

(f) $\frac{2}{3\sqrt{5}}$

Rationalise the denominator.

(a) $\frac{6\sqrt{2}}{\sqrt{6}}$

(b) $\frac{5\sqrt{2}}{\sqrt{10}}$

(c) $\frac{5\sqrt{3}}{\sqrt{6}}$

(d) $\frac{2\sqrt{6}}{\sqrt{15}}$

Rationalise the denominator.

(a) $\frac{1+\sqrt{2}}{\sqrt{5}}$

(b) $\frac{2-\sqrt{2}}{\sqrt{5}}$

(c) $\frac{2-3\sqrt{2}}{\sqrt{7}}$

(d) $\frac{5+\sqrt{5}}{\sqrt{3}}$

(e) $\frac{10-\sqrt{2}}{\sqrt{3}}$

(f) $\frac{2+\sqrt{3}}{3\sqrt{5}}$

Rationalise the denominator.

(a) $\frac{1}{2+\sqrt{5}}$

(b) $\frac{2}{2-\sqrt{5}}$

(c) $\frac{\sqrt{2}}{3+\sqrt{7}}$

(d) $\frac{2\sqrt{5}}{3-\sqrt{7}}$

(e) $\frac{10\sqrt{2}}{5+\sqrt{3}}$

(f) $\frac{2+\sqrt{3}}{3-\sqrt{5}}$

(g) $\frac{2+\sqrt{3}}{5+\sqrt{3}}$

(h) $\frac{2+3\sqrt{3}}{3-4\sqrt{5}}$

Fluency Practice

Simplifying surds rationalising the denominator

Write each expression in the form $a + b\sqrt{2}$, where $a, b \in \mathbb{Q}$.

$\frac{2 + \sqrt{2}}{4}$	
$\frac{2 + \sqrt{8}}{4}$	
$\frac{2 + 2\sqrt{8}}{4}$	
$\frac{2(2 + \sqrt{200})}{8}$	
$\frac{4 + \sqrt{32}}{4}$	
$\frac{(2 + \sqrt{2})^2}{4}$	
$\frac{\sqrt{50}}{20}$	
$\frac{1}{4\sqrt{2}}$	
$\frac{1}{1 + \sqrt{2}}$	
$\frac{1}{2 - \sqrt{2}}$	
$\frac{4}{4 - \sqrt{8}}$	
$\frac{4 - \sqrt{2}}{\sqrt{2} - 1}$	

$$\frac{1}{2} + \frac{5}{2}\sqrt{2}$$

$$\sqrt{2} - 1$$

$$2 + 3\sqrt{2}$$

$$\frac{1}{2} + \frac{1}{2}\sqrt{2}$$

$$\frac{1}{2} + \sqrt{2}$$

$$1 + \sqrt{2}$$

$$\frac{1}{4}\sqrt{2}$$

$$\sqrt{2} - 2$$

$$1 + \frac{1}{2}\sqrt{2}$$

$$2 + \sqrt{2}$$

$$\frac{1}{8}\sqrt{2}$$

$$\frac{3}{2} + \sqrt{2}$$

$$1$$

$$\frac{1}{2} + \frac{1}{4}\sqrt{2}$$

$\frac{\sqrt{50} - \sqrt{18}}{\sqrt{8}}$	
$\frac{\sqrt{8} - \sqrt{18}}{1 + \sqrt{2}}$	

Fluency Practice

Yay or Nay?

Each fraction has been standardised by
rationalising the denominator.

Which have been completed incorrectly?

$$\text{a) } \frac{1}{4 + \sqrt{2}} \times \frac{4 - \sqrt{2}}{4 - \sqrt{2}} = \frac{1(4 - \sqrt{2})}{16 + 4\sqrt{2} - 4\sqrt{2} - 2} = \frac{4 - \sqrt{2}}{14}$$

$$\text{b) } \frac{2}{3 + \sqrt{3}} \times \frac{3 - \sqrt{3}}{3 - \sqrt{3}} = \frac{2(3 - \sqrt{3})}{9 + 3\sqrt{3} - 3\sqrt{3} - 3} = \frac{6 - \sqrt{6}}{6}$$

$$\text{c) } \frac{3}{6 - \sqrt{5}} \times \frac{6 + \sqrt{5}}{6 + \sqrt{5}} = \frac{3(6 + \sqrt{5})}{36 - 6\sqrt{5} + 6\sqrt{5} - 5} = \frac{18 - 3\sqrt{5}}{30}$$

$$\text{d) } \frac{\sqrt{5}}{2 - \sqrt{2}} \times \frac{2 + \sqrt{2}}{2 + \sqrt{2}} = \frac{\sqrt{5}(2 + \sqrt{2})}{4 - 2\sqrt{2} + 2\sqrt{2} - 2} = \frac{2\sqrt{5} + \sqrt{10}}{2}$$

$$\text{e) } \frac{\sqrt{3}}{4 + \sqrt{3}} \times \frac{4 - \sqrt{3}}{4 - \sqrt{3}} = \frac{\sqrt{3}(4 - \sqrt{3})}{16 + 4\sqrt{3} - 4\sqrt{3} - 3} = \frac{3\sqrt{4} - 3}{13}$$

$$\text{f) } \frac{\sqrt{3}}{5 + \sqrt{6}} \times \frac{5 - \sqrt{6}}{5 - \sqrt{6}} = \frac{\sqrt{3}(5 - \sqrt{6})}{25 - 5\sqrt{6} + 5\sqrt{6} - 6} = \frac{5\sqrt{3} - \sqrt{18}}{19}$$

$$\text{g) } \frac{\sqrt{7}}{6 - \sqrt{7}} \times \frac{6 + \sqrt{7}}{6 + \sqrt{7}} = \frac{\sqrt{7}(6 + \sqrt{7})}{36 - 6\sqrt{7} + 6\sqrt{7} - 7} = \frac{6\sqrt{7} + 7}{29}$$

$$\text{h) } \frac{2 + \sqrt{2}}{3 + \sqrt{2}} \times \frac{3 - \sqrt{2}}{3 - \sqrt{2}} = \frac{(2 + \sqrt{2})(3 - \sqrt{2})}{9 + 3\sqrt{2} - 3\sqrt{2} - 2} = \frac{4 + \sqrt{2}}{7}$$

$$\text{i) } \frac{4 - \sqrt{5}}{3 - \sqrt{5}} \times \frac{3 + \sqrt{5}}{3 + \sqrt{5}} = \frac{(4 - \sqrt{5})(3 + \sqrt{5})}{9 - 3\sqrt{5} + 3\sqrt{5} - 5} = \frac{7 - \sqrt{5}}{4}$$

Fluency Practice

Rationalising the Denominator

To rationalise these denominators we will multiply by one term.

$$A) \frac{2 + \sqrt{3}}{\sqrt{3}}$$

$$B) \frac{5 + \sqrt{2}}{\sqrt{2}}$$

$$C) \frac{\sqrt{5} + 3}{\sqrt{5}}$$

$$D) \frac{6 + \sqrt{2}}{\sqrt{3}}$$

$$E) \frac{\sqrt{5} + 8}{\sqrt{2}}$$

$$F) \frac{\sqrt{3} - 2}{\sqrt{3}}$$

$$G) \frac{3 + 2\sqrt{5}}{\sqrt{5}}$$

$$H) \frac{\sqrt{10} + 3}{\sqrt{2}}$$

$$I) \frac{3\sqrt{6} - 5}{\sqrt{3}}$$

$$J) \frac{4 - 3\sqrt{14}}{\sqrt{2}}$$

To rationalise these denominators we will multiply by two terms within a bracket.

$$K) \frac{5}{\sqrt{2} + 3}$$

$$L) \frac{8}{1 + \sqrt{3}}$$

$$M) \frac{1}{\sqrt{5} - 2}$$

$$N) \frac{\sqrt{2}}{-3 + \sqrt{2}}$$

$$O) \frac{2\sqrt{3}}{\sqrt{5} - 1}$$

$$P) \frac{5\sqrt{2}}{2\sqrt{5} - 3}$$

$$Q) \frac{3\sqrt{7}}{5\sqrt{3} - 4}$$

$$R) \frac{\sqrt{3} + 2}{\sqrt{3} - 1}$$

$$S) \frac{\sqrt{5} + 2\sqrt{3}}{4\sqrt{2} + 3}$$

$$T) \frac{5\sqrt{5} - 5}{-1 + 2\sqrt{3}}$$

$$U) \frac{5}{\sqrt{2}} + \frac{2}{3}$$

$$V) \frac{3}{4} + \frac{5}{\sqrt{3}}$$

$$W) \frac{\sqrt{2}}{\sqrt{3}} - \frac{3\sqrt{5}}{2}$$

X) Work out the missing length. ?

$$\sqrt{3} - 1$$

Area
= $2\sqrt{5} + 5\sqrt{3}$

Fluency Practice

(a) Show that $\frac{5+\sqrt{3}}{2-\sqrt{3}}$ can be written in the form $a + b\sqrt{3}$, where a and b are integers to be found.

(b) Show that $\frac{8-\sqrt{2}}{2+\sqrt{2}}$ can be written in the form $a + b\sqrt{2}$, where a and b are integers to be found.

(c) Show that $\frac{4\sqrt{2}-1}{3+\sqrt{8}}$ can be written in the form $a\sqrt{2} + b$, where a and b are integers to be found.

(d) Show that $\frac{2+\sqrt{20}}{3-\sqrt{5}} \times 3\sqrt{5}$ can be written in the form $a + b\sqrt{5}$, where a and b are integers to be found.

(e) Show that $\frac{(2+2\sqrt{7})^2}{3-\sqrt{7}}$ can be written in the form $a + b\sqrt{7}$, where a and b are integers to be found.

(f) Show that $\frac{\sqrt{8}(4-3\sqrt{2})}{\sqrt{2}+1} + 5(4-\sqrt{2})$ can be written in the form $a + b\sqrt{2}$, where a and b are integers to be found.

(g) Given that $\frac{a+\sqrt{12}}{2-\sqrt{3}} = b + 10\sqrt{3}$ find the values of a and b .

(h) Given that $\frac{a}{(1+\sqrt{3})^2} + (a\sqrt{3})^3 = 2 + b\sqrt{3}$ find the values of a and b .

Fluency Practice

(a)	(b)	(c)	(d)
Write $\sqrt{108}$ in the form $k\sqrt{3}$	Write $\sqrt{45} + \sqrt{20}$ in the form $k\sqrt{5}$	Write $\sqrt{96} - \sqrt{24}$ in the form $k\sqrt{6}$	Expand $\sqrt{2}(5 + \sqrt{8})$
(e)	(f)	(g)	(h)
Expand and simplify $(7 + \sqrt{3})(4 - \sqrt{3})$	Expand and simplify $(5 + 2\sqrt{2})(6 - \sqrt{2})$	Expand and simplify $(5 + 3\sqrt{2})^2$	Rationalise the denominator and simplify fully $\frac{15}{\sqrt{18}}$
(i)	(j)	(k)	(l)
Rationalise the denominator and simplify fully $\frac{5 + 4\sqrt{3}}{\sqrt{3}}$	Express $\frac{\sqrt{3} + \sqrt{27}}{\sqrt{2}}$ as a single surd.	Rationalise the denominator and simplify fully $\frac{\sqrt{3} + 5}{2 - \sqrt{3}}$	$(4 + \sqrt{a})(7 - \sqrt{a}) = 23 + k\sqrt{a}$ Find the values of the positive integers a and k .

Fluency Practice

<p>A1</p> <p>Express $\sqrt{48}$ in the form $a\sqrt{b}$ where b is a prime number.</p>	<p>A2</p> <p>Express $4\sqrt{5}$ in the form \sqrt{a} where a is an integer.</p>	<p>A3</p> <p>Express $\sqrt{175} - \sqrt{63}$ in the form $a\sqrt{7}$ where a is an integer.</p>	<p>A4</p> <p>Express $\sqrt{75} + \sqrt{27}$ in the form $a\sqrt{b}$ where a and b are integers</p>
<p>B1</p> <p>Expand $(3 + \sqrt{5})(2 + \sqrt{5})$</p> <p>Give your answer in the form $a + b\sqrt{5}$ where a and b are integers</p>	<p>B2</p> <p>Expand $(2 + 3\sqrt{7})^2$</p> <p>Give your answer in the form $a + b\sqrt{7}$ where a and b are integers</p>	<p>B3</p> <p>Show that</p> $(4 + 2\sqrt{3})(5 - \sqrt{3}) = 14 + 6\sqrt{3}$ <p>Show clear working out.</p>	<p>B4</p> <p>Show that</p> $(2 + \sqrt{8})(9 - 3\sqrt{2}) = 6 + 12\sqrt{2}$ <p>Show clear working out.</p>
<p>C1</p> <p>Show that</p> $\frac{12}{\sqrt{45}} = \frac{4\sqrt{5}}{5}$ <p>Show clear working out.</p>	<p>C2</p> <p>Rationalise the denominator and simplify fully</p> $\frac{9 + 2\sqrt{3}}{\sqrt{3}}$	<p>C3</p> <p>Simplify $\frac{2 + \sqrt{20}}{\sqrt{5} - 1}$ giving your answer in the form $a + b\sqrt{5}$</p>	<p>C4 A rectangle has an area of $\sqrt{80}$ cm² and a width of $1 + \sqrt{5}$ cm. Calculate the exact height of the rectangle, leaving your answer as a simplified surd.</p>
<p>D1 Given that</p> $(a + \sqrt{b})^2 = 49 + 12\sqrt{b}$ <p>Find the value of a and the value of b.</p>	<p>D2 Given that</p> $(1 + \sqrt{e})(3 + \sqrt{e}) = f + 4\sqrt{5}$ <p>Find the value of e and the value of f.</p>	<p>D3 Given that</p> $(5 + 3\sqrt{2})^2 = p + \frac{q}{\sqrt{8}}$ <p>Find the value of p and the value of q.</p>	<p>D4 Given that</p> $(\sqrt{x} + \sqrt{8x})^2 = 54 + y\sqrt{2}$ <p>Find the value of x and the value of y.</p>

Fluency Practice

Question 4: A shed has dimensions, in metres, of height $\sqrt{5}$, width $\sqrt{6}$ and length $\sqrt{10}$. Find the volume of the shed.
Give your answer as a simplified surd.

Question 5: Mrs Jenkins is making decorations for a wedding.

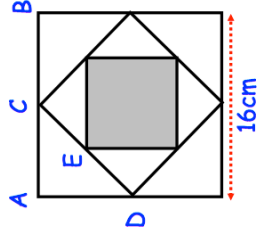
She needs $18\sqrt{5}$ metres of ribbon in total.

Mrs Jenkins has 40 metres of ribbon.

Does she have enough ribbon?



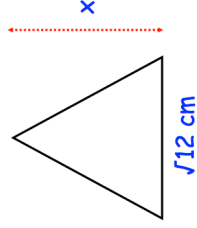
Question 6: The midpoints of the sides of a square of side 16cm are joined to form another square. This process is then repeated to create the shaded square.



Find the area of the shaded square.

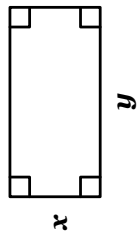
Question 7: The area of a rectangle is $\sqrt{125}$ cm².
The length of the rectangle is $(2 + \sqrt{5})$ cm.
Calculate the width of the rectangle.
Express your answer in the form $a + b\sqrt{5}$, where a and b are integers.

Question 8: The triangle below has an area of $2\sqrt{6}$ cm².
Find the height of the triangle, x .
Give your answer as a simplified surd.



Fluency Practice

surds & rectangles 1



Write your answers in the simplest form, without fractions.

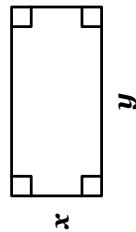
Complete each row with the side lengths x and y , the perimeter and area of a rectangle.

	Side length x	Side length y	Perimeter	Area
A	$3\sqrt{2}$	$\sqrt{2}$		
B	$\sqrt{3}$	$\sqrt{48}$		
C	$\sqrt{5}$		$6\sqrt{5}$	
D		$\sqrt{8}$	$\sqrt{200}$	
E		$2\sqrt{3}$		$8\sqrt{3}$
F	3			$\sqrt{27}$
G	$\sqrt{2}$	$\sqrt{2} + 3$		
H	$2\sqrt{5}$	$\sqrt{5} - 2$		
I		$\sqrt{3} + 2$	$4(\sqrt{3} + 1)$	
J	$2\sqrt{2}$		$6\sqrt{2} - 2$	
K	2			$4\sqrt{3} + 6$
L		$\sqrt{3}$		$\sqrt{75}$
M	$\sqrt{5}$			15
N		$2\sqrt{2}$		12
O	$\sqrt{3}$			$2\sqrt{6}$
P		$3\sqrt{3}$		18

Fluency Practice

Surds & rectangles 2

Complete each row with the side lengths x and y , the perimeter and area of a rectangle.



Write your answers in the simplest form, without fractions.

	Side length x	Side length y	Perimeter	Area
A	$\sqrt{5} + \sqrt{2}$	$\sqrt{2}$		
B	$3\sqrt{2}$	$2\sqrt{3} - \sqrt{2}$		
C	$\sqrt{3} + 2\sqrt{2}$		$2\sqrt{3} + 8\sqrt{2}$	
D		$2\sqrt{7} - \sqrt{5}$	$4\sqrt{7}$	
E	$\sqrt{2} + 5$	$\sqrt{2} + 1$		
F	$2 + \sqrt{3}$	$2 - \sqrt{3}$		
G	$\sqrt{3} + 4$		$6\sqrt{3} + 6$	
H	$5 - 2\sqrt{2}$		$2(5 + \sqrt{2})$	
I		$\sqrt{3}$		$\sqrt{3} + 6$
J	$\sqrt{5}$			$4\sqrt{5} - 5$
K		$4\sqrt{3} - 2$	$16\sqrt{3}$	
L	$\sqrt{7} - 2$			3
M		$2 - \sqrt{2}$		4
N	$\sqrt{3}$			$3 + \sqrt{3}$
O		$3 + \sqrt{3}$		$2\sqrt{3}$
P	$\sqrt{5} + 3$			$2 + 2\sqrt{5}$

Sequences with... Surds

- 1) Assuming that each pair of numbers is the start of an arithmetic sequence, find:
 - (i) the next three terms, (ii) the n th term rule, (iii) the 200th term.
- 2) Assuming that each pair of numbers is the start of a geometric sequence, find:
 - (i) the next three terms, (ii) the ratio between the first and third terms, (iii) the ratio between the second and fifth terms.

a) $\sqrt{2}, 3\sqrt{2}$

b) $\sqrt{2}, \sqrt{32}$

c) $\sqrt{2}, 2$

d) $\sqrt{2}, 1$

e) $\sqrt{2}, 2 + \sqrt{2}$

f) $\sqrt{27}, \sqrt{108}$

g) $\sqrt{48}, \sqrt{108}$

h) $\sqrt{3}, \sqrt{108}$

i) $6, \sqrt{108}$

j) $-\sqrt{75}, \sqrt{108}$

Challenge

Show clearly that the mean and median of $\sqrt{3}$, $\sqrt{12}$, $\sqrt{48}$ and $\sqrt{75}$ are equal.

Simplify

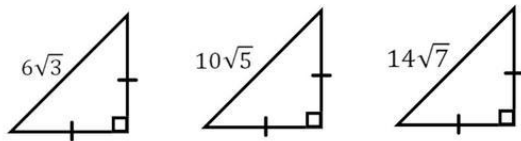
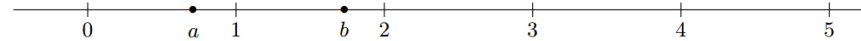
$$\sqrt{1 + \frac{1}{2}} \sqrt{1 + \frac{1}{3}} \sqrt{1 + \frac{1}{4}} \sqrt{1 + \frac{1}{5}} \sqrt{1 + \frac{1}{6}} \sqrt{1 + \frac{1}{7}}$$

Complete the multiplication table.

×		
	$2\sqrt{6}$	$\sqrt{6} - 2$
	$2\sqrt{6} + 6$	1

On the number line shown below, a is a number between 0 and 1, and b is a number between 1 and 2. Mark possible positions on this line for

$$\sqrt{a}, \sqrt{b}, a^2, b^2 \text{ and } \sqrt{\frac{b}{a}}$$



These triangles all share a rather wonderful property. Find the shorter sides and you'll see!

Can you find other triangles with this property?

Find the sum of

$$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \text{and so on up to } + \frac{1}{\sqrt{99} + \sqrt{100}}$$

Fluency Practice

For each question express the answer as a surd in the form $a\sqrt{x}$ where a is an integer

$$\frac{6}{\frac{1}{\sqrt{5}} + \sqrt{5}}$$

$$\frac{8}{\sqrt{3} - \frac{1}{\sqrt{3}}}$$

$$\frac{18}{\frac{2}{\sqrt{7}} + \sqrt{7}}$$

$$\frac{5\sqrt{2}}{\frac{3}{\sqrt{2}} + \sqrt{2}}$$

$$\frac{3\sqrt{5}}{\sqrt{5} - \frac{2}{\sqrt{5}}}$$

$$\frac{4\frac{1}{\sqrt{3}} + 2\sqrt{3}}{\sqrt{3} + \frac{2}{\sqrt{3}}}$$

Fluency Practice

Write each ratio in its simplest form.

$$\sqrt{20} : \sqrt{45}$$

$$\sqrt{63} : \sqrt{28} : \sqrt{175}$$

$$a = \sqrt{11} : \sqrt{c}$$
$$b = \sqrt{44} : \sqrt{d}$$

$$a = \sqrt{3} : \sqrt{c}$$
$$b = \sqrt{48} : \sqrt{d}$$

Given that c:d is in the
ratio 1:4

Find the ratio a:b

Given that c:d is in the
ratio 1:16

Find the ratio a:b

$$a = \sqrt{7} : \sqrt{c}$$
$$b = \sqrt{63} : \sqrt{d}$$

$$a = \sqrt{12} : \sqrt{c}$$
$$b = \sqrt{192} : \sqrt{d}$$

Given that c:d is in the
ratio 1:9

Find the ratio a:b

Given that c:d is in the
ratio 1:4

Find the ratio a:b

3 Algebraic Fractions

Intelligent Practice

Simplify:

1) $\frac{3}{12}$

2) $\frac{3x}{12}$

3) $\frac{3}{12x}$

4) $\frac{12x}{3}$

5) $\frac{12x}{6}$

6) $\frac{12x}{6x}$

7) $\frac{12xy}{6x}$

8) $\frac{6x}{12xy}$

9) $\frac{10x}{12xy}$

10) $\frac{10x}{12xy^2}$

11) $\frac{10xy}{12xy^2}$

12) $\frac{10x^2y}{12xy^2}$

13) $\frac{10x^2y^2}{12xy^2}$

14) $\frac{10x^2y^3}{12xy^2}$

15) $\frac{100x^2y^4}{120xy^2}$

Fill in the Gaps

Question	Write Each Term Separately	Simplify Each Variable	Answer
$\frac{10a^2}{2a}$	$\frac{10 \times a^2}{2 \times a}$	$5 \times a$	$5a$
$\frac{9b}{3b}$	$\frac{9 \times b}{3 \times b}$		
$\frac{2a}{4b}$	$\frac{2 \times a}{4 \times b}$		
$\frac{12a^3}{3a}$			
$\frac{5ab}{a}$			
$\frac{15a^2}{5ab}$			
$\frac{24ab}{4bc}$	$\frac{24 \times a \times b}{4 \times b \times c}$		
$\frac{6ab^2}{3a^2b}$			

Question	Write Each Term Separately	Simplify Each Variable	Answer
$\frac{4a^7}{2a^3}$	$\frac{4 \times a^7}{2 \times a^3}$	$2 \times a^4$	
$\frac{16b^5}{2b^2}$			
$\frac{2a^9}{10a^2}$			
$\frac{5a^6}{2a}$			
$\frac{25ab^7}{5b^2}$	$\frac{25 \times a \times b^7}{5 \times b^2}$		
$\frac{14a^4b^8}{2a^3b^6}$			
$\frac{9ab^5}{3a^2b^3}$			
$\frac{15a^4b}{25a^2b^2}$			

Intelligent Practice

Simplify:

$$1) \frac{2(x+3)}{3(x+3)}$$

$$2) \frac{2(x+3)}{(x+3)}$$

$$3) \frac{(x+3)}{2(x+3)}$$

$$4) \frac{x+3}{2x+6}$$

$$5) \frac{(x+3)(x+4)}{2x+6}$$

$$6) \frac{(x+3)(x+4)}{2x+8}$$

$$7) \frac{x^2+7x+12}{2x+8}$$

$$8) \frac{x^2+7x+12}{(x+4)(x-9)}$$

$$9) \frac{x^2+7x+12}{x^2-5x-36}$$

$$10) \frac{2x^2+14x+24}{x^2-5x-36}$$

$$11) \frac{2x^2+14x+24}{3x^2-15x-108}$$

$$12) \frac{2x^2+14x+24}{3x^2+4x-15}$$

$$13) \frac{14x-24-2x^2}{3x^2+4x-15}$$

$$14) \frac{14x-24-2x^2}{3x^2-4x-15}$$

$$15) \frac{14x-24-2x^2}{4x+15-3x^2}$$

Intelligent Practice

Simplify:

$$1) \frac{x+2}{(x+2)(x-1)}$$

$$2) \frac{x-1}{(x+2)(x-1)}$$

$$3) \frac{(x-1)^2}{(x+2)(x-1)}$$

$$4) \frac{(x+2)^2}{(x+2)(x-1)}$$

$$5) \frac{x^2-4}{(x+2)(x-1)}$$

$$6) \frac{x^2-4}{(x+2)(2x-5)}$$

$$7) \frac{4x^2-25}{2x^2-x-10}$$

$$8) \frac{4x^2-25}{4x^2+20x+25}$$

$$9) \frac{4x^2+8x-5}{4x^2+20x+25}$$

$$10) \frac{4ax^2+8ax-5a}{4a^2x^2+20a^2x+25a^2}$$

$$11) \frac{14x-24-2x^2}{3x^2+4x-15}$$

$$12) \frac{14x-24-2x^2}{3x^2-4x-15}$$

Fluency Practice

Q1 Simplify the following algebraic fractions.

$$[\text{a}] \quad \frac{6x + 10}{2}$$

$$[\text{e}] \quad \frac{21x + 14}{7}$$

$$[\text{i}] \quad \frac{12x + 18}{9}$$

$$[\text{b}] \quad \frac{8x - 12}{4}$$

$$[\text{f}] \quad \frac{8x - 12}{6}$$

$$[\text{j}] \quad \frac{8x - 28}{12}$$

$$[\text{c}] \quad \frac{5x + 15}{5}$$

$$[\text{g}] \quad \frac{5x + 20}{10}$$

$$[\text{k}] \quad \frac{22x - 44}{33}$$

$$[\text{d}] \quad \frac{20x + 24}{4}$$

$$[\text{h}] \quad \frac{6x - 12}{9}$$

$$[\text{l}] \quad \frac{18x + 45}{27}$$

Fluency Practice

Q2 Simplify the following algebraic fractions.

$$[\mathbf{a}] \quad \frac{3(x + 4)}{x + 4}$$

$$[\mathbf{e}] \quad \frac{5(x - 3)}{x - 3}$$

$$[\mathbf{i}] \quad \frac{5x + 10}{x + 2}$$

$$[\mathbf{m}] \quad \frac{2x + 6}{3x + 9}$$

$$[\mathbf{b}] \quad \frac{3(x + 4)}{5(x + 4)}$$

$$[\mathbf{f}] \quad \frac{5(x - 3)}{6(x - 3)}$$

$$[\mathbf{j}] \quad \frac{x + 2}{5x + 10}$$

$$[\mathbf{n}] \quad \frac{3x - 21}{4x - 28}$$

$$[\mathbf{c}] \quad \frac{x + 4}{3(x + 4)}$$

$$[\mathbf{g}] \quad \frac{x - 3}{5(x - 3)}$$

$$[\mathbf{k}] \quad \frac{4x + 24}{x + 6}$$

$$[\mathbf{o}] \quad \frac{4x - 20}{3x - 15}$$

$$[\mathbf{d}] \quad \frac{x + 4}{3x + 12}$$

$$[\mathbf{h}] \quad \frac{x - 3}{5x - 15}$$

$$[\mathbf{l}] \quad \frac{x + 6}{4x + 24}$$

$$[\mathbf{p}] \quad \frac{2x + 12}{3x + 18}$$

Fluency Practice

Q3 Simplify the following algebraic fractions.

$$[a] \frac{4x + 8}{7x + 14}$$

$$[e] \frac{3x + 12}{2x + 8}$$

$$[i] \frac{2x + 2}{7x + 7}$$

$$[m] \frac{5x - 20}{8x - 32}$$

$$[b] \frac{5x + 25}{9x + 45}$$

$$[f] \frac{2x + 6}{10x + 30}$$

$$[j] \frac{2x - 20}{7x - 70}$$

$$[n] \frac{10x + 40}{9x + 36}$$

$$[c] \frac{5x - 40}{3x - 24}$$

$$[g] \frac{2x - 14}{9x - 63}$$

$$[k] \frac{8x + 24}{10x + 30}$$

$$[o] \frac{8x + 96}{9x + 108}$$

$$[d] \frac{3x - 15}{5x - 25}$$

$$[h] \frac{5x - 30}{7x - 42}$$

$$[k] \frac{10x - 80}{7x - 56}$$

$$[p] \frac{4x - 24}{6x - 36}$$

Fluency Practice

Q4 Simplify the following algebraic fractions.

$$[\text{a}] \quad \frac{6x^2 + 5x}{x}$$

$$[\text{e}] \quad \frac{12x + 6}{2x + 1}$$

$$[\text{i}] \quad \frac{4x^4 + 8x^5}{12x^3}$$

$$[\text{b}] \quad \frac{2x - 7x^3}{x}$$

$$[\text{f}] \quad \frac{4x + 8}{x + 2}$$

$$[\text{j}] \quad \frac{9x^5 - 6x^3}{18x^3}$$

$$[\text{c}] \quad \frac{5t^2 - 4t^4}{t^2}$$

$$[\text{g}] \quad \frac{6x + 10}{9x + 15}$$

$$[\text{k}] \quad \frac{6x^2 + 15x}{2x + 5}$$

$$[\text{d}] \quad \frac{9y^5 + 2y^3}{y^3}$$

$$[\text{h}] \quad \frac{6x + 9}{8x + 12}$$

$$[\text{l}] \quad \frac{(x - 3)(x + 5)}{(x + 5)}$$

Fluency Practice

Q5 Simplify the following algebraic fractions.

$$[\mathbf{a}] \quad \frac{x^2 + 4x}{2x + 8}$$

$$[\mathbf{b}] \quad \frac{6x^3 - x^2}{12x - 2}$$

$$[\mathbf{c}] \quad \frac{x + 3x^2}{5 + 15x}$$

$$[\mathbf{d}] \quad \frac{x^4 - 5x^3}{6x - 30}$$

$$[\mathbf{e}] \quad \frac{2x^2 + 6x^3}{6x + 2}$$

$$[\mathbf{f}] \quad \frac{x^4 - 4x^3}{x^3 - 4x^2}$$

$$[\mathbf{g}] \quad \frac{3x^5 - 9x^4}{x^4 - 3x^3}$$

$$[\mathbf{h}] \quad \frac{4x^3 + 8x^2}{2x^3 + 4x^2}$$

$$[\mathbf{i}] \quad \frac{(x + 2)(x + 3)}{4x + 12}$$

$$[\mathbf{j}] \quad \frac{5x - 20}{(x + 5)(x - 4)}$$

$$[\mathbf{k}] \quad \frac{9x + 27}{(x + 3)(x + 2)}$$

$$[\mathbf{l}] \quad \frac{(x - 3)(x + 5)}{(x + 5)(x + 4)}$$

Q1 Simplify the following algebraic fractions.

$$[\mathbf{a}] \quad \frac{(x - 5)(x + 2)}{(x + 2)(x + 5)}$$

$$[\mathbf{b}] \quad \frac{(x + 4)(x + 3)}{(x + 5)(x + 3)}$$

$$[\mathbf{c}] \quad \frac{(x - 1)(x + 1)}{(x + 6)(x - 1)}$$

$$[\mathbf{d}] \quad \frac{(x + 7)(x + 5)}{(x - 9)(x + 5)}$$

$$[\mathbf{e}] \quad \frac{(x + 6)(x - 8)}{(x - 8)(x - 2)}$$

$$[\mathbf{f}] \quad \frac{(x + 3)(x - 9)}{(x - 5)(x + 3)}$$

$$[\mathbf{g}] \quad \frac{(x + 4)(x - 4)}{(x - 4)(x + 6)}$$

$$[\mathbf{h}] \quad \frac{(x + 2)(x - 4)}{(x - 4)(x + 3)}$$

Fluency Practice

Q2 Simplify the following algebraic fractions.

$$[a] \frac{(x - 2)^2}{x(x - 2)}$$

$$[b] \frac{x^2 + 4x}{(x + 4)(x + 3)}$$

$$[c] \frac{(x - 1)(x + 1)}{(x - 1)^2}$$

$$[d] \frac{x(x + 6)^2}{x(x - 9)(x + 6)}$$

$$[e] \frac{x - 8}{(x - 8)^2(x - 2)}$$

$$[f] \frac{(x + 3)^3(x + 2)}{(x + 2)(x + 3)^2}$$

$$[g] \frac{(x + 4)^2(x - 4)}{(x - 4)(x + 4)^3}$$

$$[h] \frac{(x + 2)(x - 3)(x + 5)}{(x + 5)(x - 6)(x - 3)}$$

Q3 Simplify the following algebraic fractions.

$$[a] \quad \frac{x^2 + 6x + 8}{x + 2}$$

$$[b] \quad \frac{x^2 + 5x - 24}{x + 8}$$

$$[c] \quad \frac{x^2 - 1}{x - 1}$$

$$[d] \quad \frac{x + 4}{x^2 - 2x - 24}$$

$$[e] \quad \frac{x + 2}{x^2 - x - 6}$$

$$[f] \quad \frac{x^2 - 3x - 10}{x - 5}$$

$$[g] \quad \frac{x + 4}{x^2 + 13x + 36}$$

$$[h] \quad \frac{x^2 + 8x - 9}{x + 9}$$

Q4 Simplify the following algebraic fractions.

$$[\mathbf{a}] \quad \frac{x^2 + 3x + 2}{x^2 + 5x + 4}$$

$$[\mathbf{b}] \quad \frac{x^2 + 4x + 4}{x^2 + 6x + 8}$$

$$[\mathbf{c}] \quad \frac{x^2 + 6x + 5}{x^2 + 3x - 10}$$

$$[\mathbf{d}] \quad \frac{x^2 + 2x - 3}{x^2 - 6x + 5}$$

$$[\mathbf{e}] \quad \frac{x^2 + 4x - 45}{x^2 + 10x + 9}$$

$$[\mathbf{f}] \quad \frac{x^2 - 16}{x^2 + 7x + 12}$$

$$[\mathbf{g}] \quad \frac{x^2 - x - 20}{x^2 - 7x + 10}$$

$$[\mathbf{h}] \quad \frac{x^2 + 15x + 56}{x^2 + 3x - 40}$$

Q5 Simplify the following algebraic fractions.

$$[a] \frac{2x^2 - 18x}{x^2 - 2x - 63}$$

$$[b] \frac{4x^2 + 16x}{x^2 + 12x + 32}$$

$$[c] \frac{3x^2 + 24x}{x^2 + 2x - 48}$$

$$[d] \frac{x^2 + 7x}{2x^2 + 17x + 21}$$

$$[e] \frac{2x^2 + 4x}{2x^2 + 8x + 8}$$

$$[f] \frac{2x^2 + 8x}{2x^2 + 20x + 48}$$

$$[g] \frac{6x^2 + 40x - 14}{6x^2 - 2x}$$

$$[h] \frac{6x^2 + 16x - 32}{6x^2 - 8x}$$

Q6 Simplify the following algebraic fractions.

$$[a] \frac{3x^2 + 6x + 3}{x^2 - 4x - 5}$$

$$[b] \frac{2x^2 + 19x + 24}{x^2 + 4x - 32}$$

$$[c] \frac{2x^2 + 15x + 25}{x^2 - x - 30}$$

$$[d] \frac{3x^2 + 25x + 28}{x^2 - 49}$$

$$[e] \frac{5x^2 + 28x - 12}{x^2 + 4x - 12}$$

$$[f] \frac{3x^2 - 25x + 42}{x^2 - 12x + 36}$$

$$[g] \frac{5x^2 + 7x - 6}{x^2 - 5x - 14}$$

$$[h] \frac{4x^2 + 40x - 64}{x^2 - 64}$$

Fluency Practice

A1 Simplify $\frac{x+3}{x^2+x-6}$	A2 Simplify $\frac{x+6}{x^2+10x+24}$	A3 Simplify $\frac{2x-18}{x^2-12x+27}$	A4 Simplify $\frac{x^2-7x}{x^2-2x-35}$
B1 Simplify $\frac{x^2-49}{x^2+2x-35}$	B2 Simplify $\frac{x^2-4}{x^2+6x-16}$	B3 Simplify $\frac{x^2-16}{x^2-2x-24}$	B4 Simplify $\frac{x^2-36}{x^2+3x-18}$
C1 Simplify $\frac{x^2+6x-16}{2x^2-x-6}$	C2 Simplify $\frac{x^2-7x+10}{3x^2-14x-5}$	C3 Simplify $\frac{x^2+3x}{5x^2+19x+12}$	C4 Simplify $\frac{x^2+4x-21}{4x^2-17x+15}$
D1 Simplify $\frac{2x^2-5x-12}{2x^2-11x+12}$	D2 Simplify $\frac{5x^2-24x-5}{5x^2-14x-3}$	D3 Simplify $\frac{4x^2-9}{2x^2+11x+12}$	D4 Simplify $\frac{6x^2-11x+4}{4x^2+8x-5}$

Fluency Practice

Simplifying Algebraic Fractions

a $\frac{3x + 6}{4x + 8} \rightarrow$

b $\frac{12x + 4}{15x + 5} \rightarrow$

c $\frac{2x^2 + 6}{x^2 + 5x + 6} \rightarrow$

d $\frac{x^2 + 7x + 12}{x^3 + 4x^2} \rightarrow$

e $\frac{x^2 - 7x + 10}{2x^3 - 10x^2} \rightarrow$

f $\frac{x^2 - 2x - 8}{x^2 - 6x + 8} \rightarrow$

g $\frac{x^2 - 5x - 6}{x^2 + 5x + 4} \rightarrow$

h $\frac{8x^2 - 24x}{2x^2 + 4x - 30} \rightarrow$

i $\frac{3x^2 - 3x - 18}{3x^2 - 9x - 30} \rightarrow$

j $\frac{x^2 - 1}{2x - 2} \rightarrow$

k $\frac{x^2 - 4}{4x + 8} \rightarrow$

l $\frac{5x + 25}{x^2 - 25} \rightarrow$

m $\frac{x^2 - 9}{x^2 - 3x} \rightarrow$

n $\frac{3x^2 + 6x}{x^2 - 4} \rightarrow$

o $\frac{12 - 4x}{9 - x^2} \rightarrow$

p $\frac{x^2 - 16}{3x^3 + 12x^2} \rightarrow$

q $\frac{2x^2 - 8}{2x^2 - 4x} \rightarrow$

r $\frac{x^2 + 2x - 3}{x^2 - 1} \rightarrow$

s $\frac{x^2 - 16}{x^2 - 5x + 4} \rightarrow$

t $\frac{x^2 + 5x + 6}{x^2 - 9} \rightarrow$

u $\frac{x^2 + 12x + 36}{x^2 - 36} \rightarrow$

v $\frac{4x^2 - 1}{6x^2 + 3x} \rightarrow$

w $\frac{3x^3 + 4x^2}{9x^2 - 16} \rightarrow$

x $\frac{2x^2 - x - 3}{4x^2 - 9} \rightarrow$

y $\frac{9x^2 - 4}{3x^2 - 8x + 4} \rightarrow$

z $\frac{16x^2 - 25}{8x^2 + 2x - 15} \rightarrow$

aa $\frac{6x^2 - 7x - 20}{9x^2 - 16} \rightarrow$

Fluency Practice

Question 1: Simplify the following algebraic fractions

(a) $\frac{42xyz}{56}$ (b) $\frac{45ab}{60abc}$ (c) $\frac{16mn}{18n}$ (d) $\frac{40x^2y}{32xy}$

(e) $\frac{17cf}{34c^3}$ (f) $\frac{8x^4}{2x^2}$ (g) $\frac{33a^2b^2}{44a^3b}$ (h) $\frac{12x^3}{20x^7}$

Question 2: Simplify the following algebraic fractions

(a) $\frac{6x + 8}{2}$ (b) $\frac{9x - 12}{3}$ (c) $\frac{35x^2 + 20}{5}$

(d) $\frac{7m - 70n^3}{7}$ (e) $\frac{10c + 25}{15}$ (f) $\frac{8w + 2 - 4x}{12}$

(g) $\frac{9x^2 + 12x + 33}{6}$ (h) $\frac{3x^2 + 5x}{x}$ (i) $\frac{3x^3 - 7x^2}{x}$

(j) $\frac{8x^6 + x^4 + 3x}{x}$ (k) $\frac{10x^7 + 15x^5 - 30x^4}{5x}$ (l) $\frac{3c^6 - 15c^4}{6c}$

(m) $\frac{-8x^5 - 12x^4 + 2x^3}{-4x}$ (n) $\frac{6c^9 - 12c^3}{3c^2}$ (o) $\frac{6c^6 + 2c^2}{4c^4}$

Fluency Practice

Question 3: Simplify the following algebraic fractions

$$(a) \frac{(x+6)(x+3)}{(x+3)}$$

$$(b) \frac{(x-1)(x+1)}{(x-1)}$$

$$(c) \frac{(x-3)}{(x-4)(x-3)}$$

$$(d) \frac{(x+7)^2}{(x+7)}$$

$$(e) \frac{(x-3)(x+2)}{(x+2)(x+9)}$$

$$(f) \frac{(x+2)(x+4)^2}{(x+4)}$$

$$(g) \frac{(x+1)(x+2)(x+3)}{(x+2)(x+3)(x+4)}$$

$$(h) \frac{x(x+3)^2}{x(x+1)(x+3)}$$

Question 4: Simplify the following algebraic fractions

$$(a) \frac{x^2 + 5x + 4}{x^2 + 4x + 3}$$

$$(b) \frac{x^2 + 6x + 9}{x^2 - 2x - 15}$$

$$(c) \frac{x^2 - 2x}{x^2 + 2x - 8}$$

$$(d) \frac{x^2 - 7x + 10}{x^2 + 3x - 10}$$

$$(e) \frac{x^2 + 8x + 15}{x^2 - x - 12}$$

$$(f) \frac{x^2 + 13x + 40}{x^2 + 14x + 48}$$

$$(g) \frac{x^2 - 2x - 8}{x^2 + 6x - 40}$$

$$(h) \frac{x^2 + 10x + 24}{x^2 - 36}$$

$$(i) \frac{x^2 + 4x - 45}{x^2 + 10x + 9}$$

$$(j) \frac{x^2 + 11x}{x^2 - 121}$$

$$(k) \frac{x^2 - 1}{x^2 + x}$$

$$(l) \frac{x^2 - 15x + 44}{x^2 - 16}$$

$$(m) \frac{x^2 - x - 6}{x^2 - 2x - 3}$$

Fluency Practice

Question 5: Simplify the following algebraic fractions

$$(a) \frac{3x^2 + 7x + 4}{x^2 + 3x + 2}$$

$$(b) \frac{x^2 - 2x - 8}{3x^2 + 7x + 2}$$

$$(c) \frac{5x^2 - 13x - 6}{x^2 - 9}$$

$$(d) \frac{2x^2 + 3x - 2}{2x^2 - 15x + 7}$$

$$(e) \frac{9x^2 - 1}{3x^2 - 13x + 4}$$

$$(f) \frac{x^2 + 17x + 70}{5x^2 + 38x + 21}$$

$$(g) \frac{3x^2 + 5x - 12}{12x^2 - 19x + 4}$$

$$(h) \frac{3x^2 + 11x + 6}{9x^2 + 21x + 10}$$

$$(i) \frac{4x^2 + x - 3}{4x^2 + 9x + 5}$$

$$(j) \frac{9x^2 - 30x + 25}{6x^2 + 5x - 25}$$

$$(k) \frac{10x^2 - 23x + 12}{4x^2 + 4x - 15}$$

$$(l) \frac{20x^2 + 21x + 4}{16x^2 - 1}$$

Fluency Practice

Simplify

(a) $\frac{4x^2}{8x}$ (b) $\frac{4xy}{12y^2}$

(c) $\frac{10x^2y}{20xy}$ (d) $\frac{9x^2yz}{27y^2}$

(e) $\frac{15ab}{18a^2}$ (f) $\frac{36ab^2c}{6abd}$

Simplify

(a) $\frac{x(x+9)}{6x}$ (b) $\frac{2x(x-4)}{5(x-4)}$

(c) $\frac{x^2+6x}{5x}$ (d) $\frac{x^2-5x}{3x-15}$

(e) $\frac{3x-x^2}{6-2x}$ (f) $\frac{12x^2-6x}{14x-7}$

Simplify

(a) $\frac{x(x-5)}{(x+2)(x-5)}$ (b) $\frac{2x(x+1)}{4(x+2)(x+1)}$

(c) $\frac{x(x+5)}{x^2+7x+10}$ (d) $\frac{x^2+2x+1}{(x+1)(x-4)}$

Simplify

(a) $\frac{x^2+6x}{x^2+5x-6}$ (b) $\frac{x^2-x-6}{x^2+x-12}$

(c) $\frac{2x^2+14x}{x^2+10x+21}$ (d) $\frac{x^2-4}{x^2+2x-8}$

Simplify

(a) $\frac{x^2+7x+6}{2x^2+3x+1}$ (b) $\frac{3x^2+7x+2}{x^2+9x+14}$

(c) $\frac{5x^2+28x-12}{x^2+4x-12}$ (d) $\frac{2x^2+15x+25}{x^2-x-30}$

(e) $\frac{5x^2+7x-6}{x^2-5x-14}$ (f) $\frac{2x^2-32}{2x^2+9x+4}$

Fluency Practice

tough algebra questions

- (1) simplify $(a + b)^2 - (a - b)^2$
- (2) expand $(a - b)(a^2 + ab + b^2)$
- (3) expand (i) $(n - 1)(n^2 + n + 1)$
(ii) $(n - 1)(n^3 + n^2 + n + 1)$
- (4) show that $(n + 1)(n + 2) + (n + 2)^2 - (n + 1)(n + 6)$ is a perfect square
- (5) if $\frac{1}{2}(\frac{2}{3}(2a + b) + a - 3b) = \frac{1}{4}(a + 2b)$ show that $b = \frac{7}{8}a$
- (6) simplify $\frac{a^2b + a}{b^2a + b}$
- (7) simplify $\frac{k^2 - 7k + 10}{k^2 - 5k}$
- (8) simplify $\frac{d^2 - 8d + 15}{2d^2 - 7d - 15}$
- (9) if $a = t - \frac{1}{t}$ and $b = t + \frac{1}{t}$ show that $b^2 - a^2 = 4$
- (10) if $a = \frac{b}{b + c}$ show that $\frac{a}{1 - a} = \frac{b}{c}$

Problem Solving

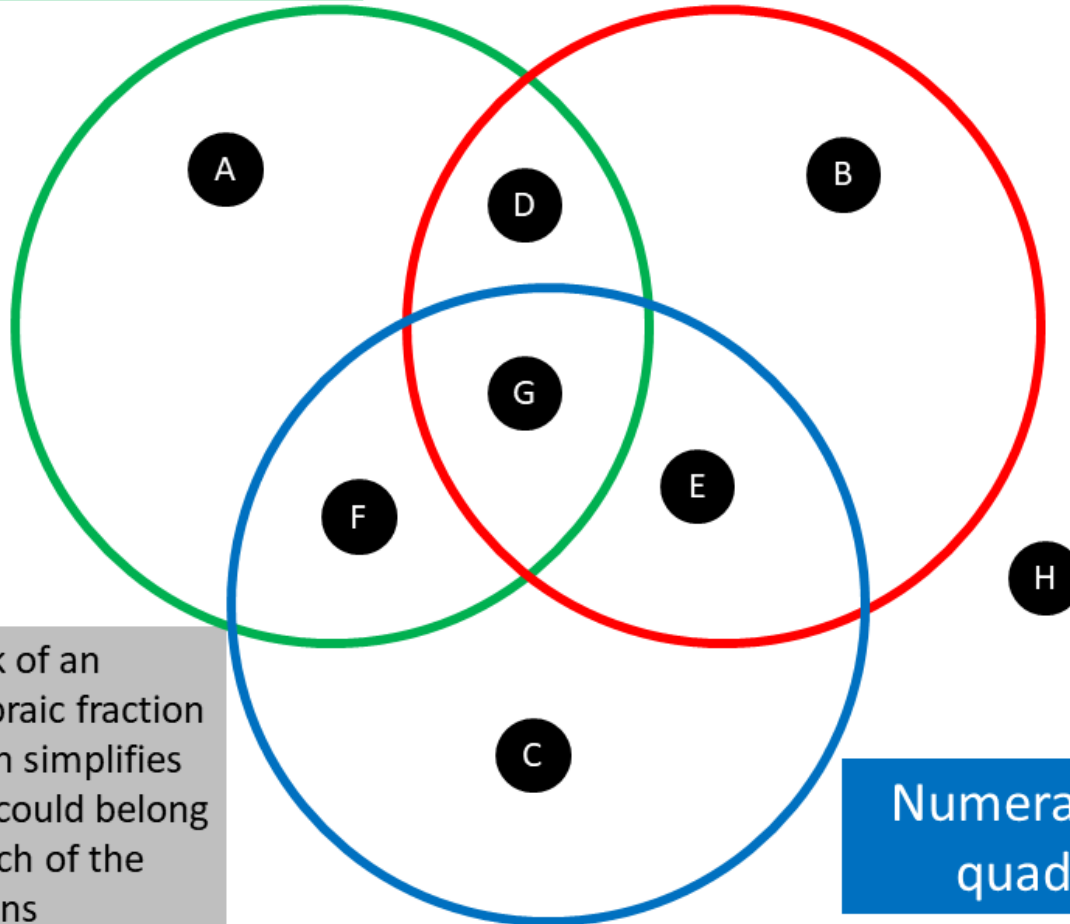
Use the digits 0-9 (at most one time each) to fill in the boxes below so that the rational expression on the left simplifies to the rational expression on the right.

$$\frac{x^2 + \boxed{}x + \boxed{}\boxed{}}{x^2 - \boxed{}\boxed{}} = \frac{x + \boxed{}}{x - \boxed{}}$$

Maths Venns

Denominator is
 $x^2 + 6x + 8$

Equivalent to $\frac{1}{x+4}$



If you think a region is impossible to fill, convince me why!

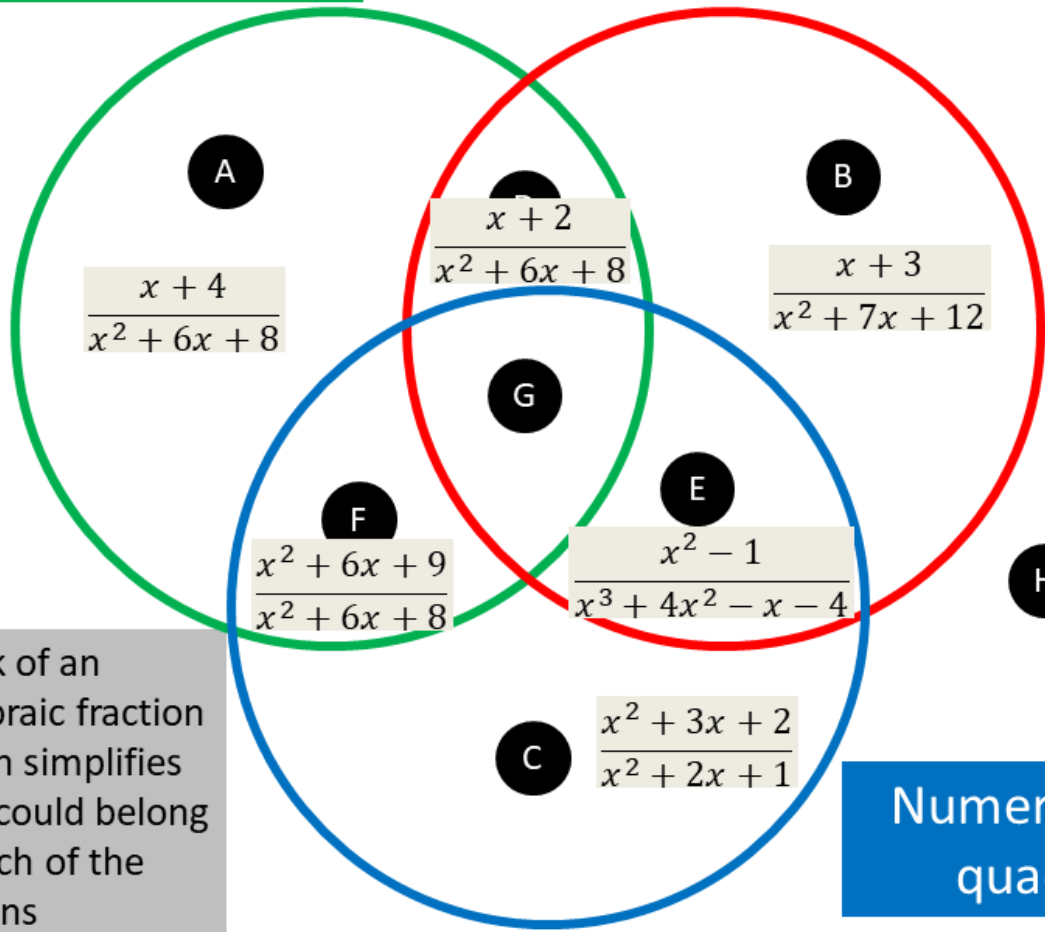
Think of an algebraic fraction which simplifies that could belong in each of the regions

Numerator is a quadratic

Maths Venns

Denominator is $x^2 + 6x + 8$

Equivalent to $\frac{1}{x+4}$



A $\frac{x + 4}{x^2 + 6x + 8}$

D $\frac{x + 2}{x^2 + 6x + 8}$

B $\frac{x + 3}{x^2 + 7x + 12}$

F $\frac{x^2 + 6x + 9}{x^2 + 6x + 8}$

E $\frac{x^2 - 1}{x^3 + 4x^2 - x - 4}$

If you think a region is impossible to fill, convince me why!

H $\frac{x + 1}{x^2 + 2x + 1}$

Think of an algebraic fraction which simplifies that could belong in each of the regions

C $\frac{x^2 + 3x + 2}{x^2 + 2x + 1}$

Numerator is a quadratic

Intelligent Practice

Simplify:

$$1) \quad \frac{3}{5} \times \frac{x}{y}$$

$$2) \quad \frac{x}{y} \times \frac{3}{5}$$

$$3) \quad \frac{x}{y} \times \frac{3}{6}$$

$$4) \quad \frac{x}{y} \times \frac{3}{6x}$$

$$5) \quad \frac{x}{xy} \times \frac{3}{6}$$

$$6) \quad \frac{7x}{xy} \times \frac{3}{6}$$

$$7) \quad \frac{7x}{xy} \times \frac{3}{6y}$$

$$1) \quad \frac{7x}{xy} \div \frac{3}{6y}$$

$$2) \quad \frac{7x}{xy} \div \frac{6y}{3}$$

$$3) \quad \frac{7x}{xy} \times \frac{6y}{3}$$

$$4) \quad \frac{7x}{xy} \times \frac{6y}{3} \times \frac{4y}{14}$$

$$5) \quad \frac{7x}{xy} \div \frac{6y}{3} \times \frac{4y}{14}$$

$$6) \quad \frac{7x}{xy} \div \frac{6y}{3} \div \frac{4y}{14}$$

$$7) \quad \frac{7x^2}{xy} \div \frac{6y}{3} \div \frac{4y}{14}$$

$$1) \quad \frac{7x^2}{xy} \div \left(\frac{6y}{3} \div \frac{4y}{14} \right)$$

$$2) \quad \frac{7x^2}{xy} \div \left(\frac{6y}{3} \times \frac{4y}{14} \right)$$

$$3) \quad \frac{7x^2}{xy} \div \left(\frac{6y}{9} \times \frac{4y}{14} \right)$$

Fill in the Gaps

Question	Write as a Single Fraction	Simplify Numerator and Denominator	Simplified Answer (where possible)
$\frac{x}{4} \times \frac{2x}{3}$	$\frac{x \times 2x}{4 \times 3}$	$\frac{2x^2}{12}$	$\frac{x^2}{6}$
$\frac{x}{6} \times \frac{4y}{5}$	$\frac{x \times 4y}{6 \times 5}$	$\frac{4xy}{30}$	
$\frac{2}{x} \times \frac{3xy}{5}$	$\frac{2 \times 3xy}{x \times 5}$		
$\frac{4x}{3y} \times \frac{2y}{x}$			
$\frac{2}{7x} \times \frac{3xy}{4}$			
$\frac{x^2}{8} \times \frac{4y}{x}$			
$\frac{2y}{x} \times \frac{9x^2y}{4}$			
$\frac{10y}{x^2} \times \frac{3xy^2}{5}$			
$\frac{4yz}{3} \times \frac{x^2}{6y^3}$			
$\frac{2x^3}{15yz} \times \frac{5x^2y^2}{z^3}$			
$\frac{\square}{5y^2} \times \frac{4x^2y}{\square}$	$\frac{\square \times 4x^2y}{5y^2 \times \square}$	$\frac{24x^3y}{15y^2}$	
$\frac{12x}{5yz} \times \frac{\square}{\square}$			$\frac{6x^3y}{25z}$

Fill in the Gaps

Question	Write as a Multiplication	Simplify Numerator and Denominator	Simplified Answer (where possible)
$\frac{x}{4} \div \frac{1}{2y}$	$\frac{x}{4} \times \frac{2y}{1}$	$\frac{2xy}{4x}$	$\frac{y}{2}$
$\frac{3x}{y} \div \frac{1}{xy}$	$\frac{3x}{y} \times \frac{xy}{1}$	$\frac{3x^2y}{y}$	
$\frac{2y}{3} \div \frac{4}{y}$	$\frac{2y}{3} \times \frac{y}{4}$		
$\frac{xy}{5} \div \frac{3y}{10}$			
$\frac{4y}{3x} \div \frac{xy}{6}$			
$\frac{x^2}{8} \div \frac{3x}{4y}$			
$\frac{6xy}{5} \div \frac{x}{y}$			
$\frac{5x}{3} \div \frac{10}{xy}$			
$\frac{2x}{y} \div \frac{4xy}{9}$			
$\frac{4}{5xy} \div \frac{2y^2}{x^2}$			
$\frac{\square}{y} \div \frac{\square}{3y^2}$	$\frac{\square}{y} \times \frac{3y^2}{\square}$	$\frac{6xy^2}{8y}$	
$\frac{5x}{2y} \div \frac{\square}{\square}$			10y

Fluency Practice

Simplify:

(a) $\frac{4x^2}{8x}$

(b) $\frac{4xy}{12y^2}$

(c) $\frac{9abc}{12bc}$

(d) $\frac{5a}{8a^2b}$

(e) $\frac{10x^2y}{20xy}$

(f) $\frac{9x^2yz}{27y^2}$

Simplify:

(a) $\frac{x}{2} \times \frac{x}{5}$

(b) $\frac{2x}{5} \times \frac{10}{3y}$

(c) $\frac{2x^2}{7} \times \frac{x}{5}$

(d) $\frac{5xy}{11} \times \frac{33}{2x}$

(e) $\frac{4ab}{9} \times \frac{3a}{8b^2}$

(f) $\frac{x^2}{y^2} \times \frac{y}{x}$

(g) $\frac{4}{9} \times \frac{abc}{8}$

(h) $\frac{2x^3}{5y} \times y$

Simplify:

(a) $\frac{x}{3} \div \frac{x}{7}$

(b) $\frac{2x}{5} \div \frac{10y}{3}$

(c) $\frac{2x^2}{7} \div \frac{x}{5}$

(d) $\frac{5x}{11y} \div \frac{3}{2x}$

(e) $\frac{3}{x} \div \frac{6y}{11}$

(f) $\frac{4}{5ab} \div \frac{10b}{3a}$

(g) $\frac{8a}{7b} \div 4$

(h) $\frac{2x^2y}{5z} \div \frac{7x}{2yz}$

Simplify:

(a) $\frac{a^3b^4cd^2}{5a} \times \frac{10d}{ab^2c^3}$

(b) $\frac{20a^2b^5c^3}{15} \times \frac{3c^2}{a^5b^2}$

Intelligent Practice

Simplify the following as far as possible:

$$1) \frac{5}{x^2+x-2} \times \frac{x-1}{2}$$

$$2) \frac{9}{x^2-4x+3} \times \frac{x-3}{2}$$

$$3) \frac{7}{x-2} \div \frac{8}{x^2-x-2}$$

$$4) \frac{3}{x-4} \div \frac{8}{x^2+x+20}$$

Simplify the following as far as possible:

$$1) \frac{x^2-4x+3}{4x} \times \frac{3}{x^2-2x+1}$$

$$2) \frac{x^2+3x-4}{7x} \times \frac{3x}{x^2+7x+12}$$

$$3) \frac{x^2-2x-3}{2x} \div \frac{x^2-7x+12}{3}$$

$$4) \frac{x^2+4x+3}{3x} \div \frac{x^2+3x+2}{5x}$$

Simplify the following as far as possible:

$$1) \frac{x^2-2x+1}{x^2-1} \times \frac{x^2+4x+3}{x^2+2x-3}$$

$$2) \frac{x^2+4x-5}{x^2-x-2} \times \frac{x^2-x-2}{x^2+6x+5}$$

$$3) \frac{x^2+4x-5}{x^2+x-2} \div \frac{x^2+10x+25}{x^2+8x+15}$$

$$4) \frac{x^2+x-2}{x^2-2x-8} \div \frac{x^2-4x+3}{x^2-6x+9}$$

Fluency Practice

Question 1: Express the following as a single fraction.

(a) $\frac{2}{g} \times \frac{3}{h}$

(b) $\frac{3}{c} \times \frac{a}{4}$

(c) $\frac{w}{x} \times \frac{3}{a}$

(d) $\frac{3a}{7} \times \frac{2c}{9}$

(e) $\frac{a}{e} \times \frac{f}{b}$

(f) $\frac{e}{8} \times \frac{d}{8}$

(g) $\frac{x}{2} \times \frac{x}{5}$

(h) $\frac{7}{y} \times \frac{2}{y}$

(i) $\frac{3}{w} \times \frac{x}{4} \times \frac{y}{w}$

(j) $\frac{2x}{5} \times \frac{3x}{7}$

(k) $\frac{x}{y} \times \frac{x}{y}$

(l) $\frac{6a}{7c} \times \frac{5a}{c}$

Question 2: Express the following as a single **simplified** fraction.

(a) $\frac{2x}{y} \times \frac{y}{4}$

(b) $\frac{3a}{c} \times \frac{5}{6}$

(c) $\frac{4}{5a} \times \frac{5w}{8}$

(d) $\frac{3a}{7} \times \frac{2c}{9}$

(e) $\frac{10g}{w} \times \frac{w}{5}$

(f) $\frac{4x}{5y} \times \frac{3y}{8x}$

(g) $\frac{2y}{3} \times \frac{2y}{wy}$

(h) $\frac{6x}{5y} \times \frac{4x}{3y}$

(i) $\frac{x^2}{a} \times \frac{a^2}{x^2}$

(j) $\frac{ab}{c} \times \frac{c}{ae}$

(k) $\frac{6c}{w^2} \times \frac{15w^3}{2c^2}$

(l) $\frac{2a^4}{3b^3} \times \frac{6b^2}{5a}$

(m) $\frac{2a^3b}{3} \times \frac{6}{ab^2}$

(n) $\frac{x^4y^4}{z^2} \times \frac{z}{x^6y}$

(o) $\frac{14a^2bc^3}{9} \times \frac{6b^3}{21a^3c}$

Fluency Practice

Question 3: Express the following as a single fraction. **Simplify** if possible.

$$(a) \frac{x}{4} \times \frac{x-3}{2}$$

$$(b) \frac{x}{9} \times \frac{6}{x+7}$$

$$(c) \frac{x+1}{15} \times \frac{5}{x}$$

$$(d) \frac{1}{x+3} \times \frac{2}{x+1}$$

$$(e) \frac{3x+2}{3} \times \frac{x+1}{3}$$

$$(f) \frac{x+4}{x-4} \times \frac{x-2}{x+5}$$

$$(g) \frac{x+1}{x-7} \times \frac{x-5}{x+1}$$

$$(h) \frac{7}{2x+8} \times \frac{x+4}{14}$$

$$(i) \frac{4}{2x-1} \times \frac{6x-3}{x+7}$$

$$(j) \frac{x+8}{15} \times \frac{10}{x^3+8x^2}$$

$$(k) \frac{4}{x-2} \times \frac{x^2-2x}{8}$$

$$(l) \frac{x^2+5x+6}{4} \times \frac{2}{x+2}$$

$$(m) \frac{x^2+2x-8}{x^2+5x+6} \times \frac{x+2}{x+4}$$

$$(n) \frac{x^2+x-6}{x^2-25} \times \frac{x^2+10x+25}{x^2-4}$$

$$(o) \frac{3x^2+8x-3}{25} \times \frac{30}{6x^2+13x-5}$$

Fluency Practice

Question 1: Express the following as a single **simplified** fraction.

(a) $\frac{x}{2} \div \frac{2}{3}$

(b) $\frac{a}{c} \div \frac{d}{5}$

(c) $\frac{3}{w} \div \frac{2}{a}$

(d) $\frac{c}{4} \div \frac{3}{c}$

(e) $\frac{3a}{4} \div \frac{6c}{7}$

(f) $\frac{4x}{9y} \div \frac{6x}{7}$

(g) $\frac{10x}{3y} \div \frac{15x}{y}$

(h) $\frac{ab}{3} \div \frac{2a}{b}$

(i) $\frac{4fg}{h} \div \frac{f}{2h}$

Question 2: Express the following as a single fraction. **Simplify** if possible.

(a) $\frac{x-4}{8} \div \frac{3x-12}{2}$

(b) $\frac{x+3}{x+2} \div \frac{x+1}{x+2}$

(c) $\frac{x+1}{2} \div \frac{2x+2}{3}$

(d) $\frac{3x+9}{2} \div \frac{x+3}{4}$

(e) $\frac{4}{x-2} \div \frac{3}{x^2-2x}$

(f) $\frac{11}{4x^2+2x} \div \frac{3}{2x+1}$

(g) $\frac{x+3}{x+1} \div \frac{x}{(x+1)^2}$

(h) $\frac{x^2}{7} \div \frac{6x^3+8x^2}{x^2-7x}$

(i) $\frac{x}{x+6} \div \frac{x+6}{x^2}$

(j) $\frac{x^2+7x+10}{2} \div \frac{x^2+4x-5}{4}$

(k) $\frac{14}{x^2-5x+6} \div \frac{7}{x^2+3x-10}$

(l) $\frac{x+4}{x+5} \div \frac{3x+12}{x^2-25}$

(m) $\frac{x^3-x}{x+2} \div \frac{x^2-x}{x^2-5x-14}$

Fluency Practice

Q7 Simplify the following algebraic fractions.

$$[\text{a}] \frac{1}{2x^2 + x - 10} \div \frac{1}{6x^2 + 15x}$$

$$[\text{b}] \frac{1}{4x^2 + 11x - 3} \div \frac{1}{2x^2 + 6x}$$

$$[\text{c}] \frac{1}{2x^2 + 5x + 2} \div \frac{1}{x^2 + 2x}$$

$$[\text{d}] \frac{1}{3x^2 - 5x + 2} \div \frac{1}{6x^2 - 4x}$$

$$[\text{e}] \frac{4}{2x^2 + 5x - 3} \div \frac{2}{10x^2 - 5x}$$

$$[\text{f}] \frac{6}{3x^2 + 10x - 8} \div \frac{3}{2x^2 + 8x}$$

$$[\text{g}] \frac{4}{5x^2 - 19x - 4} \div \frac{1}{3x^2 - 12x}$$

$$[\text{h}] \frac{1}{4x^2 + 4x - 15} \div \frac{9}{4x^2 + 10x}$$

Fluency Practice

Q9 Simplify the following algebraic fractions.

[a]
$$\frac{x^3 + 7x^2 + 12x}{x^2 + 4x}$$

[b]
$$\frac{x^3 + 9x^2 + 8x}{x^2 + x}$$

[c]
$$\frac{x^2 + 7x}{x^3 + 11x^2 + 28x}$$

Q10 Show that,

$$\frac{(x^2 - x - 6)(x^2 - 3x - 4)(x^2 + 5x)}{(x^2 - 2x - 8)(x^2 + 2x - 15)(3x + 3)} = \frac{x}{3}$$

Fill in the Gaps

Fill in the table. One row has multiple possible correct answers.

First Expression	Second Expression	Highest Common Factor	Lowest Common Multiple
$3x^2y^3$	$4x^4$	x^2	
$(x + 3)^2(y + 1)$	$(x + 3)(x + 4)(y + 1)$		$(x + 3)^2(x + 4)(y + 1)$
$9x^3yz$	$6x^2y^2z^2$		
$x^2 + 7x + 6$	$(x + 6)(x + 2)$		
$x^2 - 9$		$x - 3$	$(x + 3)(x - 3)$
	$2x^2y$	$2xy$	$14x^2y^3$
$2y^2 - 7y - 4$	$3y^2 - 11y - 4$		
		$5a^2b$	$5a^2b$
		1	$10(x - 3)(x + 2)(x - 1)$
$12x^3y^2$		$6xy$	$36x^3y^2z$
$96ab^3$		24	$288ab^3$

Intelligent Practice

Simplify:

1) $\frac{3x}{2} + \frac{4x}{5}$

2) $\frac{x}{2} + \frac{4x}{5}$

3) $\frac{x}{2} + \frac{x}{5}$

4) $\frac{x}{2} - \frac{x}{5}$

5) $\frac{x}{5} - \frac{x}{2}$

6) $\frac{5x}{5} - \frac{x}{2}$

7) $\frac{5}{x} + \frac{2}{x}$

8) $\frac{5}{x^2} + \frac{2}{x}$

9) $\frac{5}{2x^2} - \frac{2}{x}$

10) $\frac{6}{2x^2} - \frac{2}{x}$

11) $\frac{6}{2xy} + \frac{2}{x}$

12) $\frac{6}{2xy} + \frac{2}{3x}$

13) $\frac{6}{2xy} - \frac{2}{3x^2}$

14) $\frac{6y}{2xy} + \frac{2}{3x^2}$

15) $\frac{y}{2xy} - \frac{2}{3x^3}$

Fill in the Gaps

Question	With a Common Denominator	Unsimplified Answer	Simplified Answer (where possible)
$\frac{x}{4} + \frac{7x}{20}$	$\frac{5x}{20} + \frac{7x}{20}$	$\frac{12x}{20}$	
$\frac{7x}{18} - \frac{2x}{9}$	$\frac{7x}{18} - \frac{4x}{18}$		
$\frac{2x}{3} + \frac{x}{4}$	$\frac{\square}{12} + \frac{\square}{12}$		
$\frac{17x}{30} + \frac{x}{10}$	$\frac{17x}{\square} + \frac{3x}{\square}$		
$\frac{x}{6} + \frac{11x}{24}$			
$\frac{3x}{4} - \frac{7x}{36}$			
$\frac{7}{2x} + \frac{3}{x}$	$\frac{\square}{2x} + \frac{\square}{2x}$		
$\frac{6}{5x} - \frac{9}{20x}$			
$\frac{5}{x} + \frac{2}{x^2}$	$\frac{\square}{x^2} + \frac{2}{x^2}$		
	$\frac{\square}{xy} - \frac{\square}{xy}$	$\frac{3-y}{xy}$	
	$\frac{\square}{4x^2} + \frac{\square}{4x^2}$	$\frac{7x+6}{4x^2}$	
$\frac{3}{10xy} - \frac{2}{x^2}$			

Fluency Practice

Simplify:

$$\begin{array}{ll} \text{(a)} & \frac{x}{5} + \frac{x}{6} \\ \text{(b)} & \frac{x}{3} + \frac{x}{8} \\ \text{(c)} & \frac{x}{2} - \frac{x}{3} \\ \text{(d)} & \frac{x}{4} - \frac{x}{9} \\ \text{(e)} & \frac{2x}{3} + \frac{x}{4} \\ \text{(f)} & \frac{3x}{5} + \frac{2x}{9} \\ \text{(g)} & \frac{3x}{5} - \frac{2x}{7} \\ \text{(h)} & \frac{4x}{3} - \frac{5x}{8} \end{array}$$

Simplify:

$$\begin{array}{ll} \text{(a)} & \frac{3}{x} + \frac{5}{x} \\ \text{(b)} & \frac{6}{y} - \frac{4}{y} \\ \text{(c)} & \frac{2}{x} + \frac{5}{2x} \\ \text{(d)} & \frac{6}{xy} - \frac{4}{x} \\ \text{(e)} & \frac{2}{x} + \frac{7}{x^2} \\ \text{(f)} & \frac{4}{xy} - \frac{2}{3x} \\ \text{(g)} & \frac{5}{4a} - \frac{1}{b} \\ \text{(h)} & \frac{7}{ab} - \frac{3}{2a} \end{array}$$

Simplify:

$$\begin{array}{ll} \text{(a)} & \frac{3y}{6x} + \frac{5}{9x} \\ \text{(b)} & \frac{5}{xy} - \frac{3x}{y} \\ \text{(c)} & \frac{3}{ab} + \frac{6b}{5a} \\ \text{(d)} & \frac{7}{x^2} - \frac{2}{x} \end{array}$$

Simplify:

$$\begin{array}{ll} \text{(a)} & \frac{3}{x} + \frac{5}{3x} + \frac{2}{3} \\ \text{(b)} & \frac{8}{xy} + \frac{2y}{x} - \frac{x}{y} \\ \text{(c)} & \frac{9}{a^2b} - \frac{3}{ab} - \frac{1}{ab^2} \end{array}$$

Fluency Practice

Q1 Write the following as a fraction in its simplest form.

[a] $\frac{x}{3} + \frac{x}{9}$

[e] $\frac{x}{8} + \frac{2x}{3}$

[b] $\frac{x}{2} - \frac{x}{8}$

[f] $\frac{3x}{4} - \frac{x}{5}$

[c] $\frac{x}{4} - \frac{x}{6}$

[g] $\frac{5x}{6} - \frac{x}{9}$

[d] $\frac{x}{10} + \frac{x}{4}$

[h] $\frac{3x}{7} + \frac{2x}{5}$

Q2 Write the following as a fraction in its simplest form.

[a] $\frac{3}{x} + \frac{9}{x}$

[e] $\frac{8}{x} + \frac{3}{2x}$

[b] $\frac{8}{x} - \frac{2}{x}$

[f] $\frac{5}{x} - \frac{4}{3x}$

[c] $\frac{6}{x} - \frac{4}{x}$

[g] $\frac{9}{x} - \frac{6}{5x}$

[d] $\frac{10}{x} + \frac{4}{x}$

[h] $\frac{7}{3x} + \frac{5}{2x}$

Fluency Practice

Q3 Work out the following.

$$[\text{a}] \frac{3}{x} + \frac{5}{x^2}$$

$$[\text{e}] \frac{5}{x^2} - \frac{5}{2x^2y}$$

$$[\text{i}] \frac{5}{3s} + \frac{2}{s^2t}$$

$$[\text{b}] \frac{6}{x^2} + \frac{2}{x}$$

$$[\text{f}] \frac{1}{ab^2} - \frac{2}{b}$$

$$[\text{j}] \frac{3}{u^2v^2} - \frac{2}{uv}$$

$$[\text{c}] \frac{4}{xy} - \frac{3}{x}$$

$$[\text{g}] \frac{2}{3x} + \frac{1}{3t^2}$$

$$[\text{k}] \frac{3}{2r} - \frac{2}{3st}$$

$$[\text{d}] \frac{10}{xy} - \frac{8}{xyz}$$

$$[\text{h}] \frac{8}{p^2} + \frac{3}{2pq}$$

$$[\text{l}] \frac{2}{5x^3} + \frac{3x}{2}$$

Fluency Practice

Q4 Work out the following.

[a] $\frac{1}{x} + \frac{2}{y} + \frac{3}{z}$

[b] $\frac{5}{m} + \frac{4}{n} + \frac{1}{p}$

[c] $\frac{1}{2r} + \frac{2}{s} + \frac{3}{2t}$

[d] $\frac{1}{2a} + \frac{2}{3b} + \frac{3}{4c}$

Q5 Gemma says that,

$$\frac{2x}{5} + \frac{x}{7} = \frac{3x}{12}$$

Why is Gemma wrong?

Work out the correct answer.

Q6 Find the value of,

$$\frac{x}{2} + \frac{x}{3} + \frac{x}{6}$$

Q7 Write the following as a single fraction in its simplest form.

$$[a] \frac{x+2}{7} + \frac{x}{3}$$

$$[b] \frac{x+5}{4} + \frac{2x}{3}$$

$$[c] \frac{x+5}{2} + \frac{x+2}{5}$$

$$[d] \frac{x+1}{6} - \frac{x-1}{7}$$

$$[e] \frac{x+1}{2} - \frac{x+2}{4}$$

$$[f] \frac{x+8}{2} - \frac{x+3}{9}$$

$$[g] \frac{3x+2}{10} + \frac{5x-4}{6}$$

$$[h] \frac{3x+1}{6} + \frac{4x-1}{4}$$

Fluency Practice

Simplify the following as far as possible:

1) $\frac{6}{2xy} + \frac{2}{x}$

2) $\frac{6}{2xy} + \frac{2}{3x}$

3) $\frac{6}{2xy} - \frac{2}{3x^2}$

4) $\frac{6y}{2xy} + \frac{2}{3x^2}$

5) $\frac{y}{2xy} - \frac{2}{3x^3}$

Simplify the following as far as possible:

1) $\frac{1}{x^2-4} - \frac{1}{x-2}$

2) $\frac{x}{x^2-4} - \frac{1}{x-2}$

3) $\frac{x}{x^2-x-2} - \frac{1}{x-2}$

4) $\frac{x-3}{x^2-x-2} - \frac{1}{x-2}$

5) $\frac{x-3}{x^2-x-2} - \frac{2x}{x-2}$

Simplify the following as far as possible:

1) $\frac{2x+1}{x+4} - \frac{x-5}{x-2}$

2) $\frac{11x+27}{2x^2+11x-6} - \frac{3}{x+6}$

Challenge: Express the following as a fraction in its simplest form:

$$\frac{3}{x-1} + \frac{2}{x+1} + \frac{4}{x-3}$$

Fluency Practice

Question 1: Express the following as a single simplified fraction.

(a) $\frac{x}{3} + \frac{x}{5}$

(b) $\frac{c}{2} + \frac{c}{7}$

(c) $\frac{w}{3} + \frac{w}{9}$

(d) $\frac{x}{2} - \frac{x}{3}$

(e) $\frac{a}{5} - \frac{a}{9}$

(f) $\frac{m}{2} - \frac{m}{8}$

(g) $\frac{m}{3} + \frac{2m}{7}$

(h) $\frac{3x}{5} + \frac{x}{2}$

(i) $\frac{3c}{4} + \frac{5c}{9}$

(j) $\frac{m}{2} - \frac{2m}{5}$

(k) $\frac{3n}{4} - \frac{5n}{9}$

(l) $\frac{7h}{8} - \frac{5h}{12}$

Question 2: Express the following as a single simplified fraction.

(a) $2 - \frac{5}{x^2} + \frac{5}{x}$

(b) $5 - \frac{1}{6x} - \frac{1}{3x}$

(c) $2 - \frac{4}{fg} - \frac{4}{9}$

(d) $\frac{6}{ac} + \frac{2}{3}$

(e) $\frac{9}{w} + \frac{wx}{4}$

(f) $\frac{d}{3} + \frac{2}{d^2}$

(g) $\frac{m^2}{6} - \frac{9}{4m}$

(h) $3 - \frac{1}{4b^2} - \frac{1}{2b}$

(i) $\frac{ac}{5} + \frac{4}{c}$

(j) $x^3 - \frac{2}{w} - \frac{2}{wx^2}$

(k) $2 - \frac{3}{ab^2} - \frac{3}{b^3}$

(l) $\frac{xy}{5} - \frac{1}{x}$

Fluency Practice

Question 3: Express the following as a single simplified fraction.

(a) $\frac{x+4}{3} + \frac{x+1}{2}$ (b) $\frac{2x+1}{2} + \frac{x+3}{5}$ (c) $\frac{5x-2}{3} + \frac{2x+7}{4}$

(d) $\frac{3x+11}{2} + \frac{x-20}{3}$ (e) $\frac{5x-4}{2} + \frac{x+1}{3}$ (f) $\frac{x-4}{8} + \frac{2x-3}{2}$

(g) $\frac{7x+4}{2} - \frac{x+1}{3}$ (h) $\frac{9x+1}{5} - \frac{x+2}{2}$ (i) $\frac{3x+11}{2} - \frac{2x-3}{6}$

(j) $\frac{x-8}{3} - \frac{x-5}{7}$ (k) $\frac{4x-7}{10} - \frac{2x-9}{3}$ (l) $\frac{5-x}{6} - \frac{8-3x}{10}$

Question 4: Express the following as a single simplified fraction.

(a) $\frac{2}{x+5} + \frac{3}{x+1}$ (b) $\frac{2}{x+1} + \frac{1}{x+3}$ (c) $\frac{4}{x+5} - \frac{2}{x-1}$

(d) $\frac{x+1}{x-2} + \frac{x+3}{x+5}$ (e) $\frac{x+3}{2x+1} - \frac{x-2}{x-1}$ (f) $\frac{x}{x+7} + \frac{2x+5}{3x+1}$

(g) $\frac{3}{x+1} + \frac{x+7}{(x+1)(x+2)}$ (h) $\frac{1-x}{(x-7)(x+1)} - \frac{2}{x-7}$

Question 5: Solve the following equations

(a) $\frac{x+5}{3} + \frac{x+1}{2} = 8$ (b) $\frac{x+6}{5} + \frac{x-3}{2} = 13$

(c) $\frac{2x-1}{4} + \frac{x-3}{5} = 3$ (d) $\frac{x+5}{10} - \frac{x+4}{2} = 1$

(e) $\frac{x-3}{4} - \frac{x-8}{3} = 4$

Fluency Practice

Q8 Write the following as a single fraction in its simplest form.

$$[\text{a}] \frac{2}{x+2} + \frac{5}{x}$$

$$[\text{b}] \frac{1}{x+5} + \frac{5}{2x}$$

$$[\text{c}] \frac{3}{x+5} + \frac{4}{x+2}$$

$$[\text{d}] \frac{6}{x+1} - \frac{3}{x-1}$$

$$[\text{e}] \frac{2}{x+1} - \frac{4}{x+2}$$

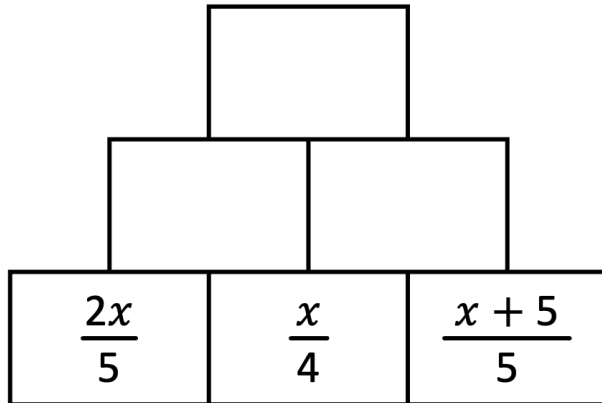
$$[\text{f}] \frac{5}{x+3} - \frac{9}{x+11}$$

$$[\text{g}] \frac{10}{3x+2} + \frac{2}{5x-4}$$

$$[\text{h}] \frac{2}{2x+1} + \frac{5}{4x-1}$$

Fluency Practice

Q9 Fill in the missing boxes so that each block is the sum of the two below it.



Q10 Fill in the empty squares to make all the equations in this grid correct both horizontally and vertically.

	+		=	$\frac{2x}{3}$
+		+	=	+
$\frac{2x}{3}$	+	$\frac{2x}{5}$	=	
=		=	=	=
	+	$\frac{14x}{15}$	=	

Q8 Simplify the following algebraic fractions.

$$[a] \frac{1}{3x^2 + 11x - 4} \div \frac{1}{3x + 12} + \frac{2}{3x + 1}$$

$$[b] \frac{1}{5x^2 + 14x - 3} \div \frac{1}{2x + 6} + \frac{3}{2x - 5}$$

$$[c] \frac{1}{2x^2 + 5x - 12} \div \frac{1}{5x + 20} - \frac{1}{4x + 3}$$

$$[d] \frac{1}{6x^2 + x - 1} \div \frac{1}{9x - 3} - \frac{2}{3x - 2}$$

Fluency Practice

A1 Simplify

$$\frac{x}{5} + \frac{3x}{4}$$

A2 Simplify

$$\frac{3x}{2} - \frac{5x}{6}$$

A3 Simplify

$$\frac{x+3}{5} + \frac{x-5}{4}$$

A4 Simplify

$$\frac{2x-1}{3} - \frac{2}{5}$$

B1 Simplify

$$\frac{4}{x-1} + \frac{3}{x+1}$$

B2 Simplify

$$\frac{3}{x+1} - \frac{6}{2x+3}$$

B3 Simplify

$$\frac{3}{2x-3} - \frac{1}{x+2}$$

B4 Simplify

$$\frac{3}{x-1} - \frac{2}{x}$$

C1 Simplify

$$\frac{2}{x-1} + \frac{x-11}{x^2+3x-4}$$

C2 Simplify

$$\frac{2}{x+2} + \frac{x}{x^2+5x+6}$$

C3 Simplify

$$\frac{2}{x-1} + \frac{x}{x^2-4x+3}$$

C4 Simplify

$$\frac{5}{2x-6} - \frac{x+2}{x^2-4x+3}$$

D1 Simplify

$$1 + \frac{4}{(x+3)(x-2)}$$

D2 Simplify

$$1 + \frac{x^2+x-6}{(x+4)(x-2)}$$

D3 Simplify

$$2 + \frac{5-x}{x^2-2x-15}$$

D4 Simplify

$$5 - (x+2) \div \left(\frac{x^2-4}{x-3} \right)$$

Fluency Practice

1. Combine the following algebraic fractions, fully simplifying your answer.

(a) $\frac{2}{x+4} + \frac{3}{x+1}$.

$\frac{5x+14}{(x+1)(x+4)}$

(l) $\frac{5}{x^2+3x} + \frac{2}{x+3}$.

$\frac{2x+5}{x(x+3)}$

(b) $\frac{3}{x-2} + \frac{5}{x+4}$.

$\frac{8x+2}{(x+4)(x-2)}$

(m) $\frac{x}{x^2-x-6} - \frac{3}{2x+4}$.

$\frac{9-x}{2(x-3)(x+2)}$

(c) $\frac{3}{x-1} - \frac{x+2}{x+3}$.

$\frac{11-2x}{(x-1)(x+3)}$

(n) $\frac{x+1}{x^2-4x+3} - \frac{x-3}{x^2-1}$.

$\frac{8}{(x+1)(x-3)}$

(d) $\frac{1}{x-5} - \frac{x-7}{x-1}$.

$\frac{8-2x}{(x-5)(x-1)}$

(o) $\frac{2}{x} - \frac{3}{x^2-x} - \frac{4}{x-1}$.

$-\frac{2x+5}{x(x-1)}$

(e) $\frac{2x+4}{3x+6} - \frac{3x+6}{a}$.

$\frac{7}{6(x+2)}$

(p) $\frac{1}{x-1} - \frac{1}{x^2+x-2} + \frac{4}{x+2}$.

$\frac{5x-3}{(x-1)(x+2)}$

(f) $\frac{a}{x+b} + \frac{x+c}{x+c}$.

$\frac{2ax+ab+ac}{(x+b)(x+c)}$

(q) $\frac{2}{x^2-x} - \frac{1}{x^2-1}$.

$\frac{5x-3}{(x-1)(x+2)}$

(g) $2 + \frac{3}{x-5}$.

$\frac{2x-7}{x-5}$

(r) $\frac{3}{2x-3} + \frac{1}{x-4} - \frac{5x}{2x^2-11x+12}$.

$\frac{15}{-(2x-3)(x-4)}$

(h) $\frac{5}{2-x} + 3 + \frac{1}{x+1}$.

$\frac{3x^2-7x-13}{(x+1)(x-2)}$

(s) $\frac{a}{x+1} - \frac{b}{x^2+2x+1} - \frac{c}{(x+1)^3}$.

$\frac{ax^2+2ax-bx+a-b-c}{(x+1)^3}$

(i) $\frac{a}{x+k} + \frac{b}{x+2k}$.

$\frac{ax+bk+2ak+bk}{(x+k)(x+2k)}$

(t) $\frac{5}{2x+3} - 2x$.

$\frac{5-6x-4x^2}{2x+3}$

(j) $\frac{1}{(x-1)^2} - \frac{2}{x-1}$.

$\frac{3-2x}{(x-1)^2}$

(u) $\frac{3}{x-2} + 2$.

$\frac{x(2x+1)}{(x-2)(x-4)}$

2. Write the following as a single fraction.

(a) $\frac{1}{\frac{2}{x-4} - 3}$.

$\frac{x-4}{14-3x}$

(g) $\frac{3}{x-2} + 2$.

$\frac{x(2x+1)}{(x-2)(x-4)}$

(b) $\frac{3}{\frac{1}{x-2} + \frac{2}{x}}$.

$\frac{3x(x-2)}{3x-4}$

(h) $\frac{4}{\frac{1}{x} - \frac{2}{x+1}}$.

$\frac{4(x+1)}{3(1-x)}$

(c) $2 - \frac{3}{x}$.

$\frac{1}{2x-3}$

(i) $\frac{1}{\frac{x-1}{x-1} + \frac{3}{x-2}}$.

$\frac{(4x-5)(x+3)}{(x-2)(5-x)}$

(d) $\frac{\frac{4}{x-1}}{1 - \frac{3}{x-1}}$.

$\frac{4}{x-4}$

(j) $\frac{1-x-3}{\frac{1}{x+2} - 1}$.

$-\frac{(x-5)(x+2)}{(x-3)(x+1)}$

(e) $2 + \frac{5}{x+3} - \frac{7}{x+3}$.

$-\frac{2x+11}{7x+20}$

(k) $\frac{a-b}{b} + \frac{c-b}{b}$.

$-\frac{(x-5)(x+2)}{(x-3)(x+1)}$

(f) $\frac{a}{e} + \frac{c}{x+b}$.

$\frac{ax+ab-c}{ex+eb+f}$

(l) $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}$.

$\frac{3x+2}{2x+1}$

Fluency Practice

$$(m) \quad 3 - \frac{\frac{5}{x-1}}{x - \frac{4}{x-1}}$$

$$\frac{3x^2 - 3x - 17}{x^2 - x - 4}$$

$$(o) \quad \frac{2}{3 + \frac{4}{5 + \frac{6}{x}}} - \frac{1}{2 + \frac{3}{4 + \frac{5}{x}}}$$

$$\frac{34x^2 + 65x + 30}{(19x + 18)(11x + 10)}$$

$$(n) \quad \frac{1}{\frac{1}{x} + 1} - \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}$$

$$0 \text{ (ho ho)}$$

3. Cancel the following to their simplest form.

$$(a) \quad \frac{x^2 + 3x}{2x + 6} \quad \square$$

$$(h) \quad \frac{x^2 + 6x + 8}{x^2 - 4x + 3} \div \frac{2x + 8}{x - 3}$$

$$\frac{x+2}{2(x-1)}$$

$$(b) \quad \frac{x^2 - 1}{x^2 + 5x + 4}$$

$$\frac{x-1}{x+4}$$

$$(i) \quad \frac{\pi x^2 - \pi x - 2\pi}{x^3 + x^2 - 6x} \div \frac{2x + 2}{3x + 9}$$

$$\frac{3\pi}{2x}$$

$$(c) \quad \frac{2x^2 + 7x - 4}{2x^2 + 9x - 5}$$

$$\frac{x+4}{x+5}$$

$$(j) \quad \frac{12x^2 + 10x + 2}{2x - 6} \div \frac{18x^2 + 15x + 3}{5x - 15}$$

$$\frac{5}{3}$$

$$(d) \quad \frac{4x^2 - 8x - 12}{2x^2 - 2x - 12}$$

$$\frac{2(x+1)}{(x+2)}$$

$$(k) \quad \frac{2x^2 + 2x - 4}{3x^3 - 3x} \div \frac{4x + 8}{9 + 9x}$$

$$\frac{3}{2x}$$

$$(e) \quad \frac{4\pi x^2 - \pi}{12\pi x^2 - 10\pi x + 2\pi}$$

$$\frac{2x+1}{2(3x-1)}$$

$$(f) \quad \frac{2a^2x^2 + a^2x - 10a^2}{2ax^2 - 2ax - 4a}$$

$$\frac{a(2x+5)}{2(x+1)}$$

$$(l) \quad \frac{4x^2 - 1}{2x^2 - 7x - 4} \div \frac{2x^2 + 5x - 3}{2x^2 - 2x - 24}$$

$$2$$

$$(g) \quad \frac{12x^4 + 17x^3 - 5x^2}{9x^3 - 25x}$$

$$\frac{x(4x-1)}{3x-5}$$

$$(m) \quad \frac{6\pi x^2 + 19\pi x + 10\pi}{3\pi x^3 + 4\pi x^2 - 7\pi x} \div \frac{6x + 15}{18x + 42}$$

$$\frac{2(3x+2)}{x(x-1)}$$

4. Find the value of the capital letters which make the following identities correct.

$$(a) \quad \frac{A}{x} + \frac{B}{x+1} \equiv \frac{5x+2}{x(x+1)}$$

$$A = 2, B = 3$$

Fluency Practice

Q1

Solve the following equations.

$$[\text{a}] \frac{x+3}{4} + \frac{x+4}{5} = 2$$

$$[\text{e}] \frac{x+2}{2} + \frac{7x-7}{4} = 6$$

$$[\text{b}] \frac{x-1}{2} + \frac{x+3}{6} = 4$$

$$[\text{f}] \frac{x+5}{3} + \frac{2x+3}{4} = 2$$

$$[\text{c}] \frac{x-1}{4} - \frac{x+1}{6} = 1$$

$$[\text{g}] \frac{x+3}{6} + \frac{3x+8}{5} = 9$$

$$[\text{d}] \frac{x+2}{2} + \frac{x+3}{3} = 1$$

$$[\text{h}] \frac{x+2}{6} + \frac{7x+6}{2} = 6$$

Q2

Solve the following equations.

$$[\text{a}] \frac{3x+1}{2} + \frac{5x+7}{6} = 2$$

$$[\text{e}] \frac{8x-1}{5} + \frac{4x-5}{2} = 3$$

$$[\text{b}] \frac{6x-1}{3} - \frac{3x+1}{7} = 10$$

$$[\text{f}] \frac{3x-2}{6} + \frac{5x+6}{7} = 7$$

$$[\text{c}] \frac{2x+5}{2} - \frac{3x+1}{5} = 2$$

$$[\text{g}] \frac{9x+8}{6} + \frac{4x+3}{5} = 5$$

$$[\text{d}] \frac{2x+4}{7} + \frac{2x-3}{2} = 1$$

$$[\text{h}] \frac{2x+7}{6} + \frac{2x-8}{3} = 5$$

Fluency Practice

Question 5: Solve the following equations

$$(a) \quad \frac{x+5}{3} + \frac{x+1}{2} = 8$$

$$(b) \quad \frac{x+6}{5} + \frac{x-3}{2} = 13$$

$$(c) \quad \frac{2x-1}{4} + \frac{x-3}{5} = 3$$

$$(d) \quad \frac{x+5}{10} - \frac{x+4}{2} = 1$$

$$(e) \quad \frac{x-3}{4} - \frac{x-8}{3} = 4$$

Fluency Practice

Q3

Solve the following equations.

$$[\text{a}] \frac{2}{x+6} + \frac{4}{x+8} = 2$$

$$[\text{b}] \frac{4}{x-5} + \frac{2}{x-8} = 3$$

$$[\text{c}] \frac{2}{x-9} + \frac{4}{x-7} = 2$$

$$[\text{d}] \frac{3}{x-1} + \frac{2}{x-3} = 1$$

$$[\text{e}] \frac{1}{2x-7} + \frac{6}{x+4} = 1$$

$$[\text{f}] \frac{1}{2x-5} + \frac{4}{x+2} = 1$$

$$[\text{g}] \frac{5}{2x-3} + \frac{2}{x-6} = 1$$

$$[\text{h}] \frac{2}{5x+2} + \frac{5}{x+4} = 1$$

Q4

Solve the following equations.

$$[\text{a}] \frac{4}{2x-1} + \frac{6}{4x-7} = 2$$

$$[\text{b}] \frac{5}{2x-6} + \frac{3}{2x+6} = 1$$

$$[\text{c}] \frac{6}{4x+1} + \frac{4}{2x+3} = 2$$

$$[\text{d}] \frac{2}{4x-9} + \frac{4}{2x-3} = 2$$

$$[\text{e}] \frac{6}{2x-3} + \frac{4}{3x-7} = 2$$

$$[\text{f}] \frac{3}{4x+7} + \frac{6}{2x+5} = 3$$

$$[\text{g}] \frac{3}{4x+9} + \frac{2}{2x+7} = 1$$

$$[\text{h}] \frac{3}{2x-1} + \frac{5}{2x-7} = 2$$

Fluency Practice

Question 6: Solve the following equations

$$(a) \quad \frac{4}{x+1} + \frac{2}{x-2} = 3$$

$$(b) \quad \frac{2}{x-3} + \frac{1}{x-4} = 2$$

$$(c) \quad \frac{2}{x-5} - \frac{2}{x-4} = 1$$

$$(d) \quad \frac{2}{x+1} - \frac{10}{x+4} = -1$$

$$(e) \quad \frac{3}{x-2} + \frac{3}{x+2} = 2$$

$$(f) \quad \frac{11}{(x-1)(x+4)} + \frac{5}{x-1} = 1$$

Fluency Practice

Solve

(a) $\frac{x-1}{3} = 2$ (b) $\frac{2x-3}{5} = 7$

(c) $\frac{x+1}{10} = \frac{3x}{5}$ (d) $\frac{x-8}{6} = \frac{3-x}{4}$

Solve

(a) $\frac{x}{2} + \frac{x}{3} = 5$ (b) $\frac{8x}{15} - \frac{x}{5} = 6$

(c) $\frac{x+3}{8} + \frac{x+4}{3} = 4$

(d) $\frac{x+1}{3} + \frac{x}{4} = 5$

(e) $\frac{x+1}{2} + \frac{3x-1}{4} = 4$

(f) $\frac{x+13}{2} - \frac{12-3x}{3} = 1$

Solve

(a) $\frac{4}{x+1} + \frac{5}{x+2} = 2$

(b) $\frac{3}{x+2} + \frac{4}{x-3} = 2$

(c) $\frac{5}{3x+2} - \frac{3}{2x-3} = 4$

(d) $\frac{18}{4x-1} - \frac{1}{x+1} = 1$

(e) $\frac{3}{2x-1} - \frac{4}{3x-1} = 1$

Fluency Practice

(a)	(b)	(c)	(d)
Simplify $\frac{x(x-5)}{x^2}$	Simplify $\frac{x^2+2x}{x^2+7x+10}$	Simplify $\frac{x^2-x-6}{x^2+6x+8}$	Simplify $\frac{3x^2-13x+4}{x^2-16}$
(e)	(f)	(g)	(h)
Simplify $\frac{x^2}{x-2} \times \frac{3x-6}{4x}$	Simplify $\frac{x(x-1)}{x^2+2x} \div \frac{x^2-1}{3x}$	Write as a single fraction $\frac{x}{3} + \frac{2x}{5}$	Write as a single fraction $\frac{5x}{6} - \frac{x}{4}$
(i)	(j)	(k)	(l)
Simplify fully $\frac{5}{x^2} + \frac{4}{3x}$	Simplify fully $\frac{3}{x+2} - \frac{2}{x-1}$	Write as a single fraction $\frac{2x-1}{5} + \frac{x+3}{4}$	Simplify fully $\frac{x}{2x+4} - \frac{3}{x^2+5x+6}$
(m)	(n)	(o)	
Solve $\frac{x}{3} + \frac{2x-1}{4} = 1$	Solve $\frac{2(x+1)}{5} = \frac{8+x}{6}$	Solve $\frac{5}{x+3} + \frac{3}{x-1} = 4$	

Fluency Practice

A1 Solve

$$\frac{3x}{5} + 3 = 7$$

A2 Solve

$$\frac{x+4}{3} = 11$$

A3 Solve

$$\frac{5x-11}{3} = x$$

A4 Solve

$$\frac{7x-16}{2} = 2x-3$$

B1 Solve

$$\frac{x+4}{2} = \frac{x+10}{3}$$

B2 Solve

$$\frac{x-2}{3} = \frac{x+4}{5}$$

B3 Solve

$$\frac{x-10}{10} = \frac{10-x}{3}$$

B4 Solve

$$\frac{2}{5x-2} = \frac{3}{6x+1}$$

C1 Solve

$$\frac{x-1}{2} + \frac{2x+3}{4} = 1$$

C2 Solve

$$\frac{2x-1}{4} + \frac{x-1}{5} = 2$$

C3 Solve

$$\frac{6x-1}{4} - \frac{5-2x}{2} = 1$$

C4 Solve

$$\frac{2x+1}{3} + \frac{x-5}{2} = 4$$

D1 Solve

$$\frac{6}{x-2} - \frac{6}{x+1} = 1$$

D2 Solve

$$\frac{3}{x+2} + \frac{4}{x-3} = 2$$

D3 Solve

$$\frac{3}{x+1} + \frac{2}{2x-3} = 1$$

D4 Solve

$$\frac{4}{2x+1} + \frac{1}{4x^2-1} = 3$$

Fluency Practice

Simplify

(a) $\frac{x}{2} \times \frac{x}{5}$ (b) $\frac{2x}{5} \times \frac{10}{3y}$

(c) $\frac{2x^2}{7} \times \frac{x}{5}$ (d) $\frac{5xy}{11} \times \frac{33}{2x}$

(e) $\frac{x}{3} \div \frac{x}{7}$ (f) $\frac{2x}{5} \div \frac{10y}{3}$

(g) $\frac{2x^2}{7} \div \frac{x}{5}$ (h) $\frac{5x}{11y} \div \frac{3}{2x}$

Simplify

(a) $\frac{x}{5} + \frac{x}{6}$ (b) $\frac{3x}{5} - \frac{2x}{9}$

(c) $\frac{3}{x} + \frac{5}{2x}$ (d) $\frac{6}{xy} - \frac{4}{3x}$

(e) $\frac{2}{x} + \frac{7}{x^2}$ (f) $\frac{x}{2} + \frac{x+1}{4}$

(g) $\frac{2x}{3} + \frac{x+2}{9}$ (h) $\frac{x+4}{5} - \frac{x}{7}$

Simplify

(a) $\frac{x-3}{5} \times \frac{15}{2x-6}$ (b) $\frac{x}{6} \div \frac{2x^2+x}{3}$

(c) $\frac{x^2-1}{8} \div \frac{x+1}{4}$ (d) $\frac{5x+5}{4} \times \frac{x^2+2x+1}{2}$

Simplify

(a) $\frac{x+2}{2} + \frac{x+3}{5}$ (b) $\frac{x-3}{3} + \frac{x+4}{7}$

(c) $\frac{2x+1}{2} - \frac{3x+1}{5}$ (d) $\frac{7x+4}{2} - \frac{2x-1}{7}$

Simplify

(a) $\frac{2}{x+1} + \frac{3}{x+2}$ (b) $\frac{5}{x-3} - \frac{3}{x+2}$

(c) $\frac{5}{x-3} - \frac{3}{2x+1}$ (d) $\frac{8}{x-1} + \frac{2}{3x-2}$

Intelligent Practice

1) Make r the subject of $\frac{6}{p} = \frac{7}{q} - \frac{6}{r}$

2) Make b the subject of $\frac{9}{a} = \frac{10}{b} - \frac{2}{c}$

3) Make z the subject of $\frac{1}{x} = \frac{3}{y} - \frac{3}{z}$

4) Make b the subject of $\frac{1}{a} = \frac{10}{b} + \frac{6}{c}$

5) Make q the subject of $\frac{6}{p} = \frac{1}{q} + \frac{9}{r}$

1) Make p the subject of $\frac{1}{10p} - \frac{4}{q} = \frac{7}{s}$
Give your answer in its simplest form.

2) Make x the subject of $\frac{1}{9x} - \frac{1}{y} = \frac{4z}{w}$
Give your answer in its simplest form.

3) Make a the subject of $\frac{4}{2a} - \frac{1}{b} = \frac{1}{d}$
Give your answer in its simplest form.

4) Make a the subject of $\frac{1}{2a} + \frac{5}{b} = \frac{9}{d}$
Give your answer in its simplest form.

5) Make a the subject of $\frac{3}{5a} + \frac{1}{b} = \frac{c}{d}$
Give your answer in its simplest form.