KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2023 Mathematics 2024

Unit 21 Booklet

HGS Maths

Tasks

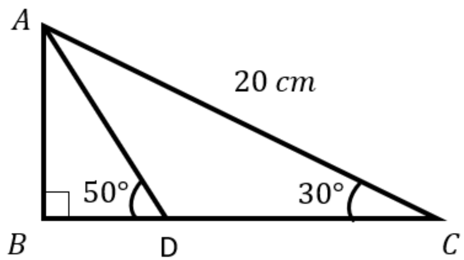
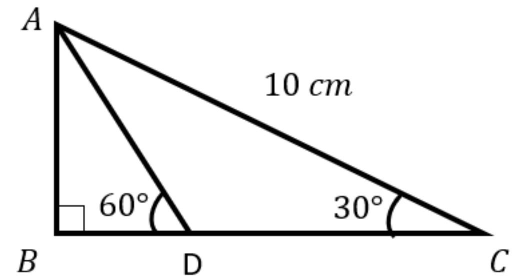
Dr Frost Course

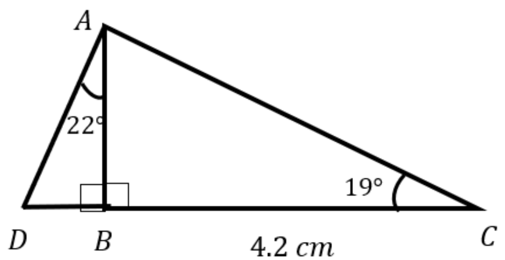
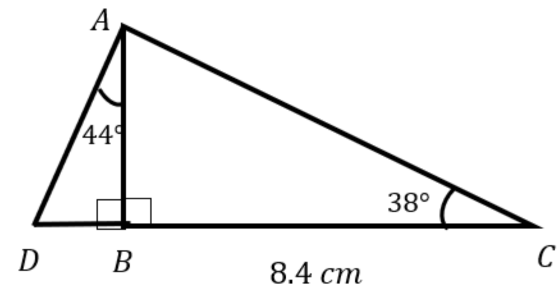
Name: ______________________________

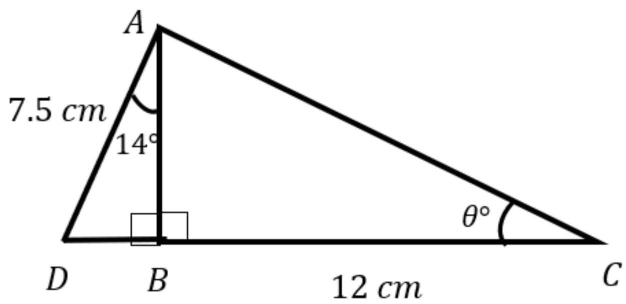
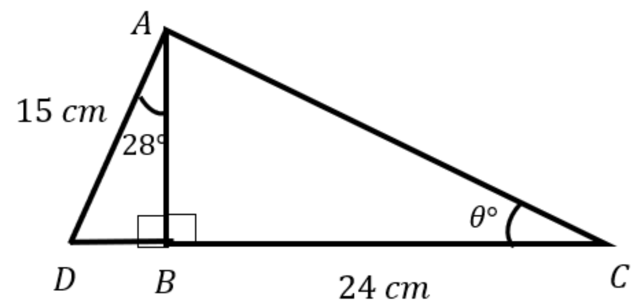
Class: ______________________________

Unit 21

Recap of Right-Angled Trigonometry

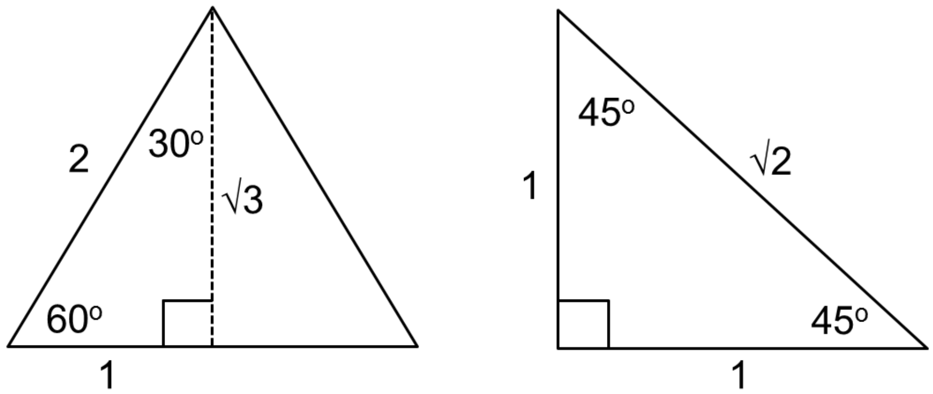
Worked Example	Your Turn
Calculate the length of BD: 	Calculate the length of BD: 

Worked Example	Your Turn
BC is $4.2\ cm$. Calculate the length of AD: 	BC is $8.4\ cm$. Calculate the length of AD: 

Worked Example	Your Turn
BC is $12\ cm$. Calculate θ 	BC is $24\ cm$. Calculate θ 

Exact Trigonometric Values

exact values in trigonometry



angle	sin	cos	tan
0°			
30°			
45°			
60°			
90°			

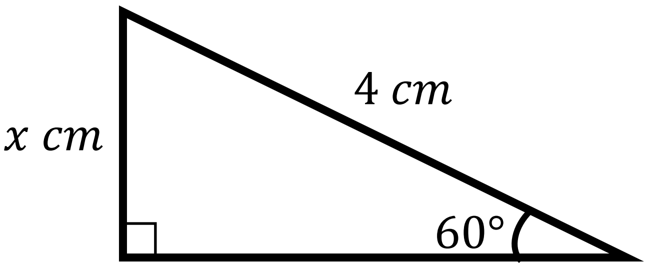
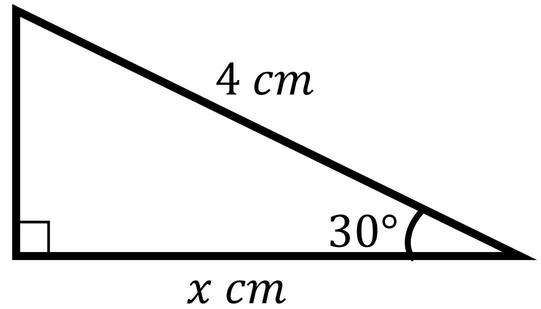
TIP:

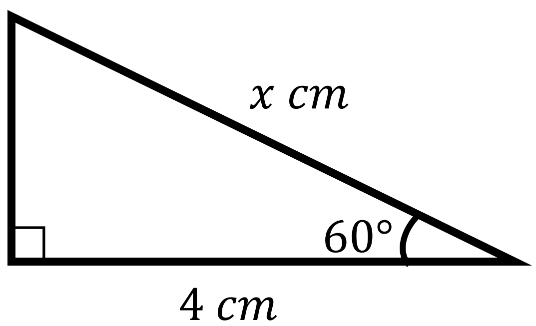
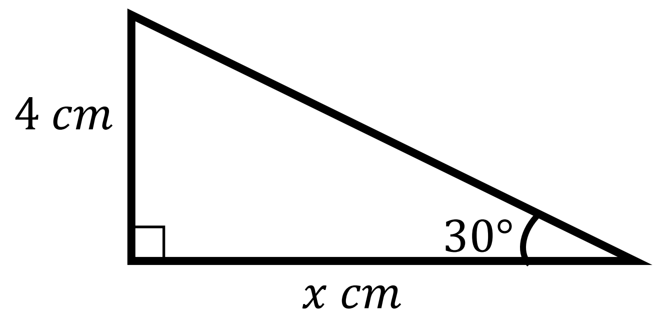
Use the general expression:

$$\frac{\sqrt{n}}{2}$$

For sine ***n*** *goes from 0 to 4, for cosine it's 4 to 0 and tan is the numerator of sine over cosine (simplified)*

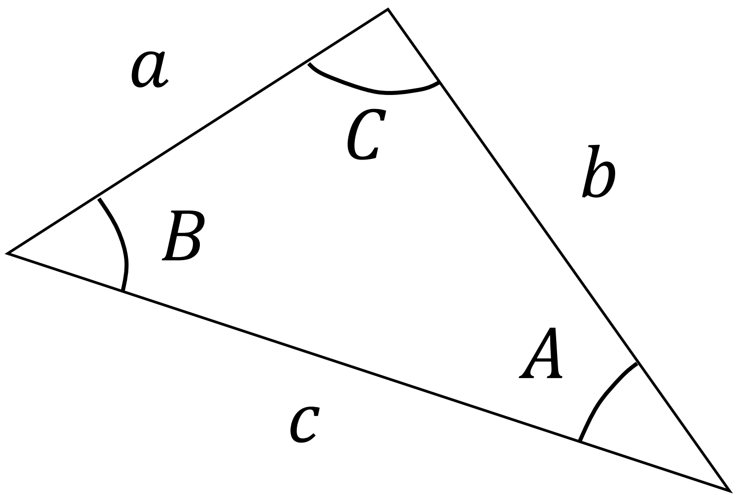
Worked Example	Your Turn
Show that $5\sin 30° \times \cos 30° \times 8\tan 30°$ is an integer	Show that $2\sin 60° \times 5\cos 60° \times 6\tan 60°$ is an integer

Worked Example	Your Turn
Without a calculator, calculate x:  	Without a calculator, calculate x: 

Worked Example	**Your Turn**
Without a calculator, calculate x:	Without a calculator, calculate x:
	

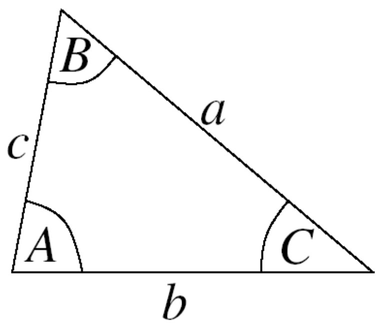
Non Right-Angled Triangles and Trigonometry

Labelling Non Right-Angled Triangles and Trigonometry



We label the sides a, b, c and their corresponding OPPOSITE angles A, B, C

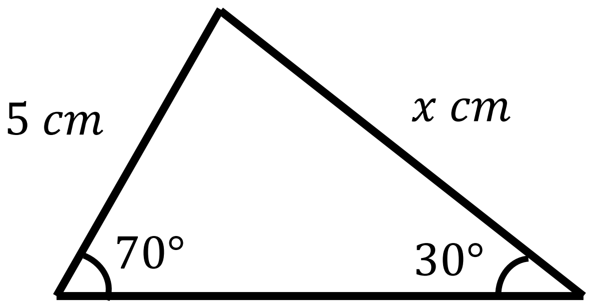
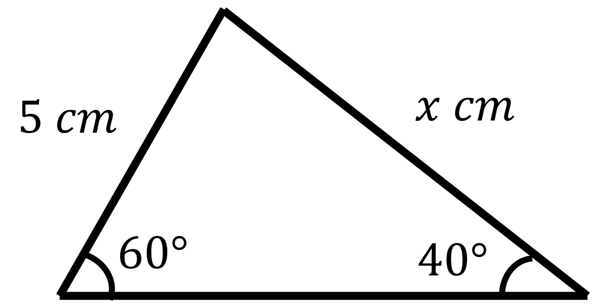
The Sine Rule

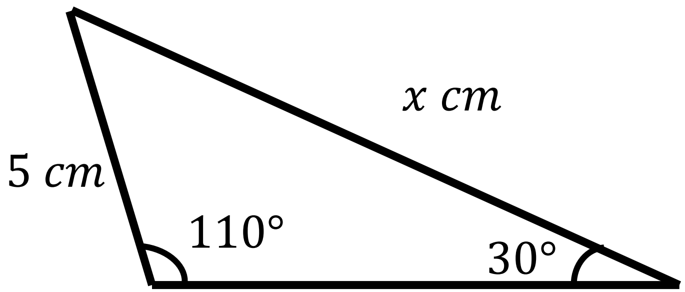
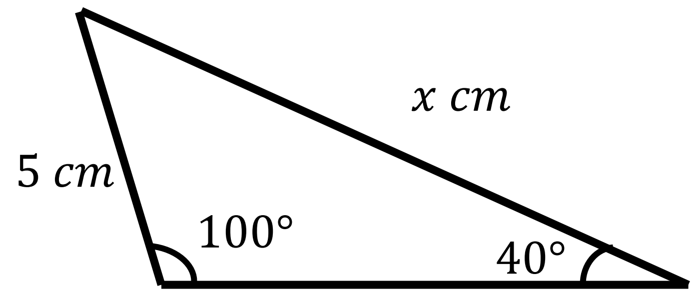


Sine Rule

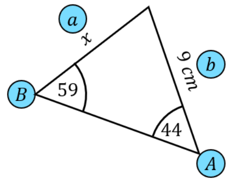
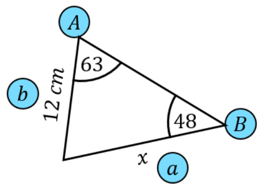
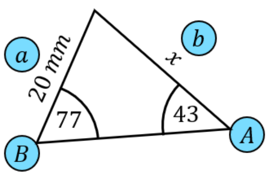
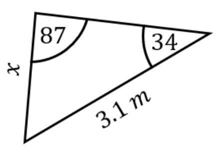
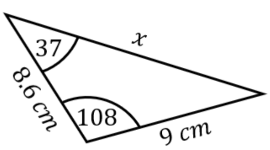
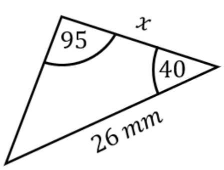
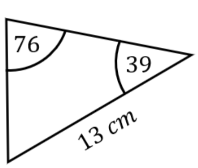
$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \quad \text{or} \quad \frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

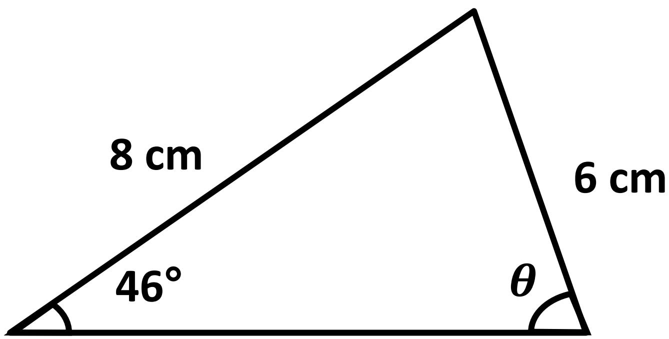
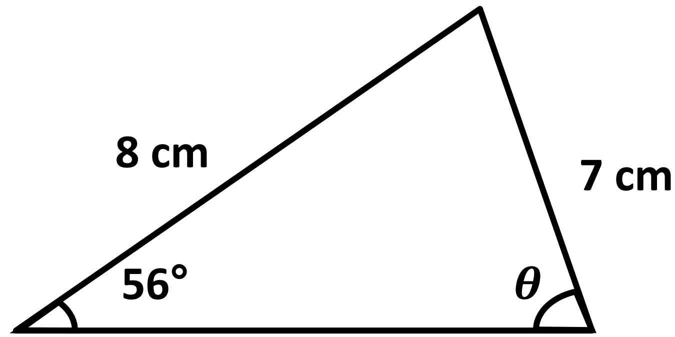
(for finding sides) (for finding angles)

Worked Example	Your Turn
Find the value of x  	Find the value of x 

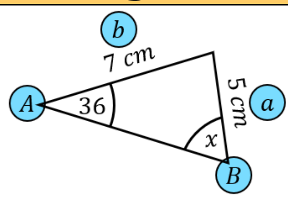
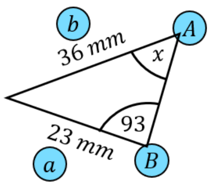
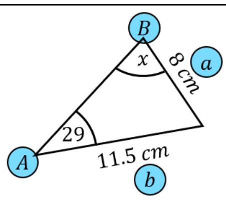
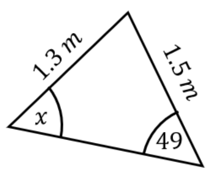
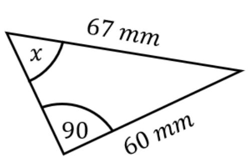
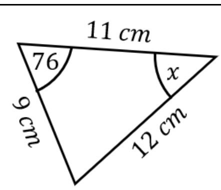
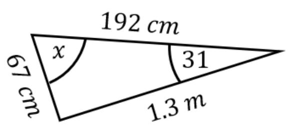
Worked Example	Your Turn
Find the value of x 	Find the value of x 

FILL IN THE GAPS

Labelled diagram	Substitute into formula	Rearrange formula	Length (1dp)
	$\frac{x}{\sin 44} = \frac{9}{\sin 59}$	$x = \frac{9 \times \sin 44}{\sin 59}$	
	$\frac{x}{\sin 63} = \frac{12}{\sin 48}$		
			
			
			
			
	$\frac{x}{\sin 65} = \frac{13}{\sin 76}$		
		$x = \frac{3.5 \times \sin 36}{\sin 68}$	

Worked Example	Your Turn
Find the value of θ 	Find the value of θ 

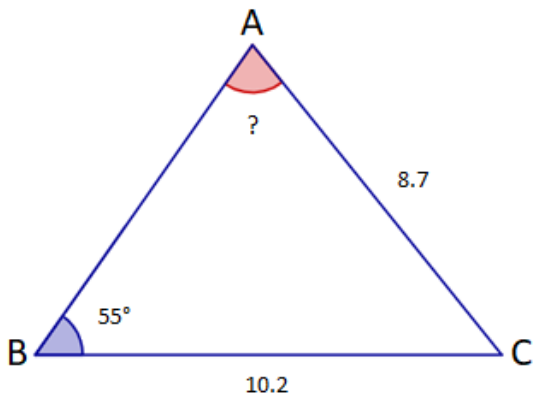
FILL IN THE GAPS

Labelled diagram	Substitute into formula	Rearrange formula	Acute Angle (1dp)
	$\frac{\sin 36}{5} = \frac{\sin x}{7}$	$\sin x = \frac{7 \times \sin 36}{5}$	$x = 55.4°$
	$\frac{\sin x}{23} = \frac{\sin 93}{36}$		
			
			
			
			
			
		$\sin x = \frac{5 \times \sin 47}{10}$	

Ambiguous Sine Rule

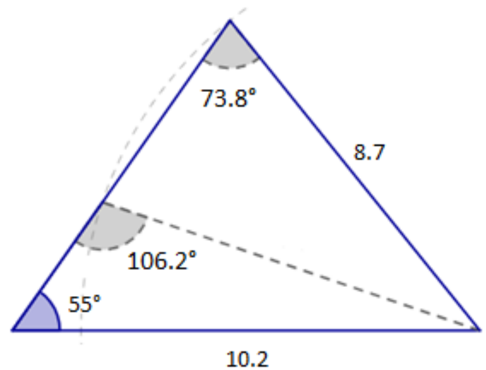
The sine rule can be used to determine the unknown sides or angles of a triangle given some of its sides and angles.

The *ambiguous case* can occur when we are given the angle-side-side, as shown in the diagram below:

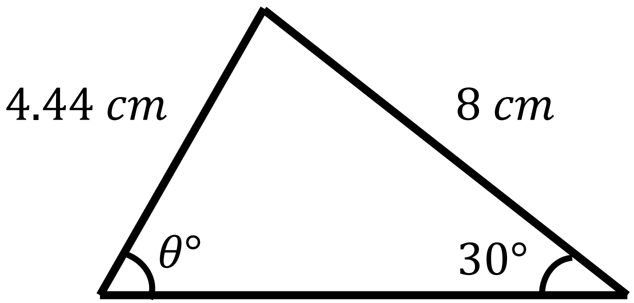
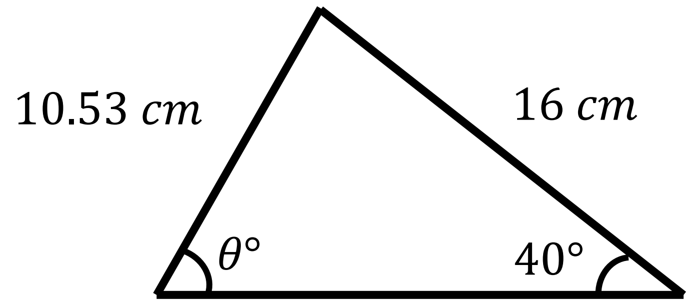


$$\frac{\sin A}{10.2} = \frac{\sin 55}{8.7}$$

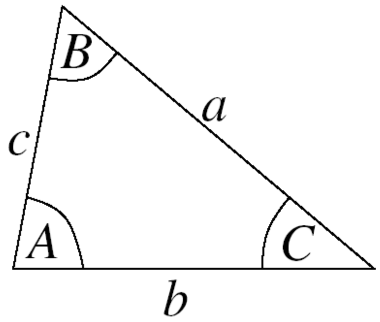
$$A = 73.8° \text{ or } 106.2°$$



Angle *A* can be acute or obtuse resulting in 2 possible triangles.

Worked Example	Your Turn
Find the possible values of θ 	Find the possible values of θ 

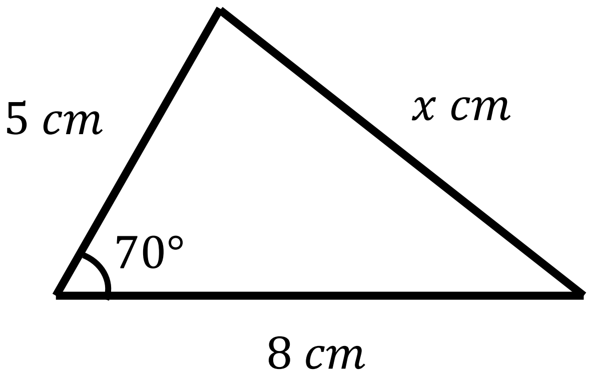
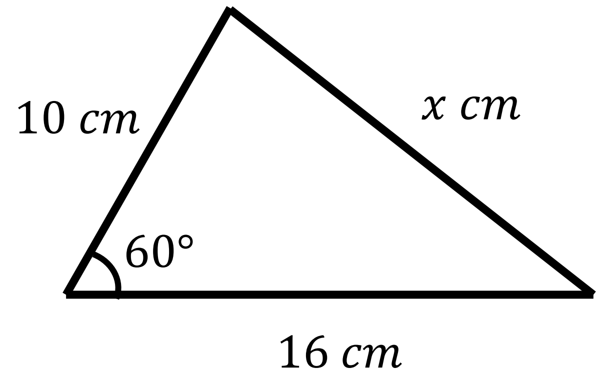
Cosine Rule



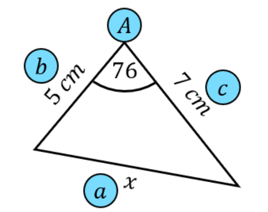
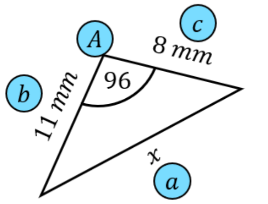
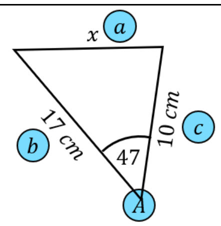
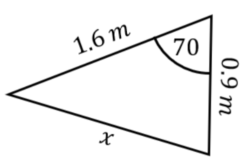
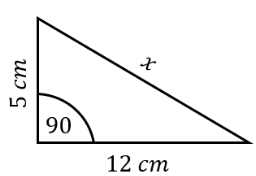
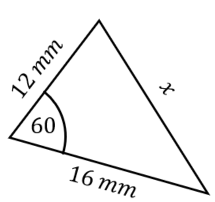
Cosine Rule

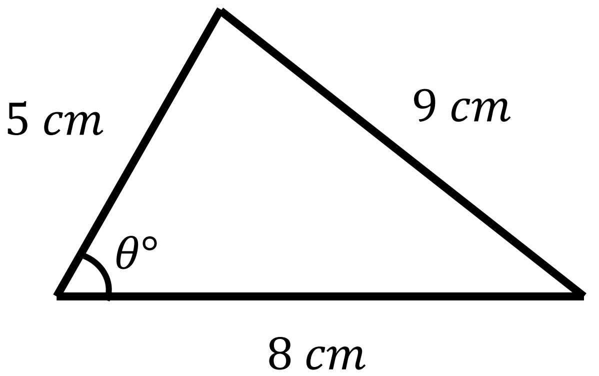
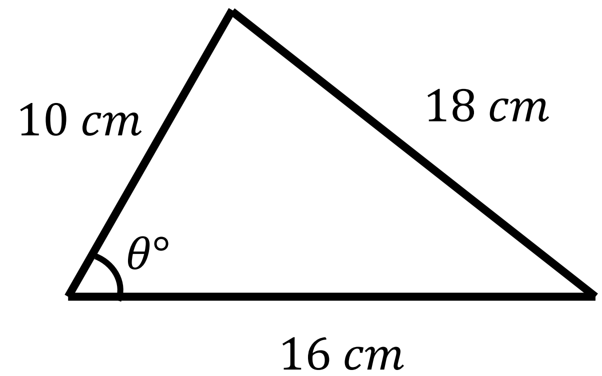
$a^2 = b^2 + c^2 - 2bc\cos(A)$ or $\cos(A) = \dfrac{b^2 + c^2 - a^2}{2bc}$

(for finding sides) (for finding angles)

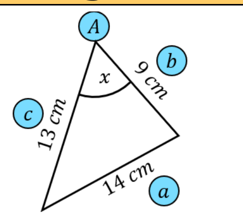
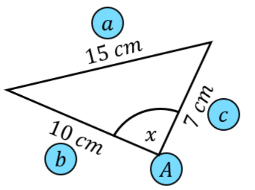
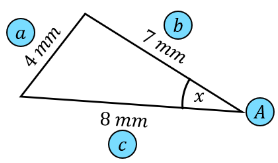
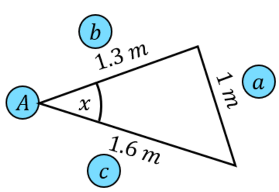
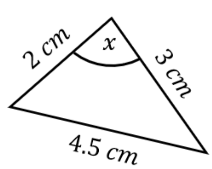
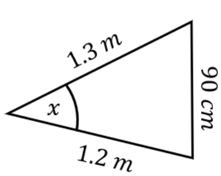
Worked Example	Your Turn
Find the value of x 	Find the value of x 

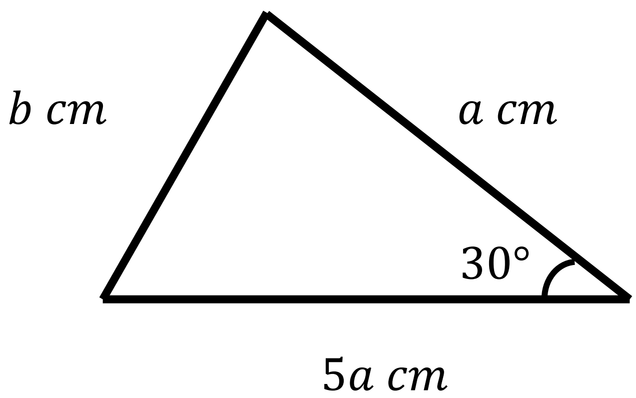
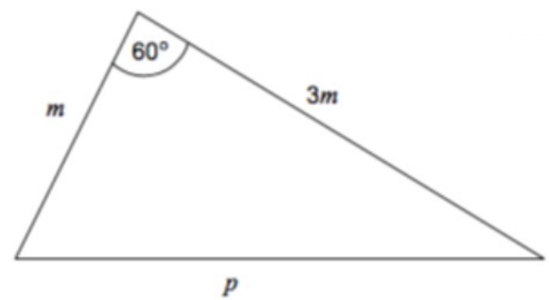
FILL IN THE GAPS

Labelled diagram	Substitute into formula	x^2	x to 1dp
	$x^2 = 7^2 + 5^2$ $-2 \times 7 \times 5 \times \cos 76$	$x^2 = 57.065..$	
	$x^2 = 11^2 + 8^2$ $-2 \times 11 \times 8 \times \cos 96$		
			
			
			
			
	$x^2 = 32^2 + 14^2$ $-2 \times 32 \times 14 \times \cos 53$		

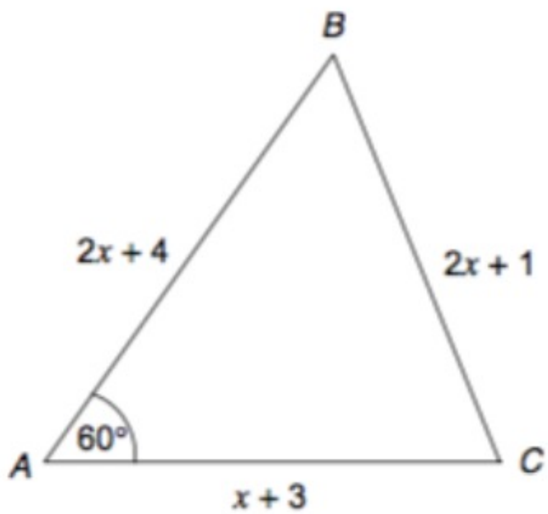
Worked Example	Your Turn
Find the value of θ 	Find the value of θ 

FILL IN THE GAPS

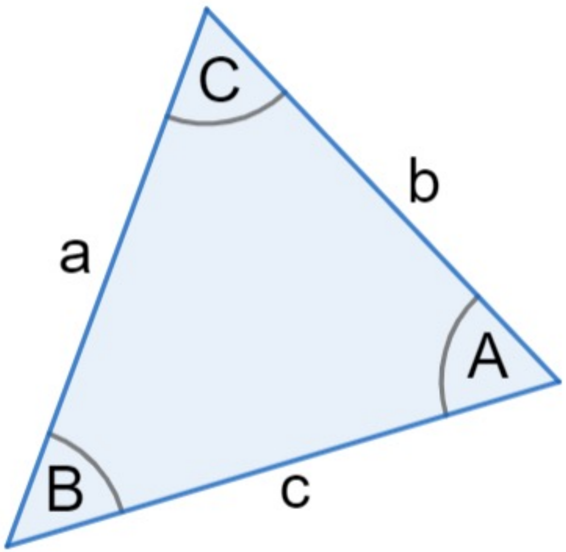
Labelled diagram	Substitute into formula	Rearrange formula	Angle (1dp)
	$14^2 = 9^2 + 13^2$ $-2 \times 9 \times 13 \times \cos x$	$\cos x = \dfrac{9^2 + 13^2 - 14^2}{2 \times 9 \times 13}$	$x = 76.7°$
	$15^2 = 10^2 + 7^2$ $-2 \times 10 \times 7 \times \cos x$	$\cos x = \dfrac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$	
	$4^2 = 7^2 + 8^2$ $-2 \times 7 \times 8 \times \cos x$		
			
			
			
		$\cos x = \dfrac{6^2 + 5^2 - 3^2}{2 \times 6 \times 5}$	

Worked Example	Your Turn
Use the cosine rule to express b in terms of a 	Use the cosine rule to express p in terms of m 

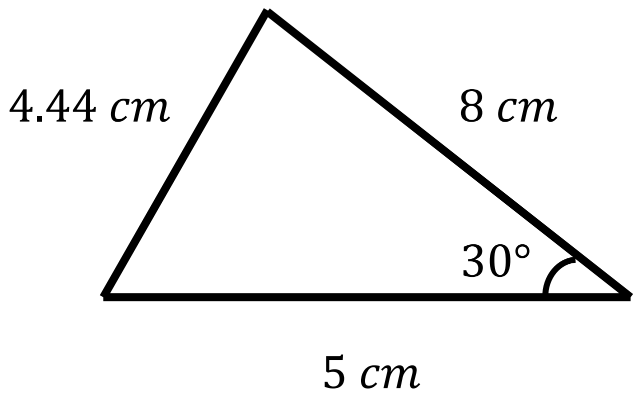
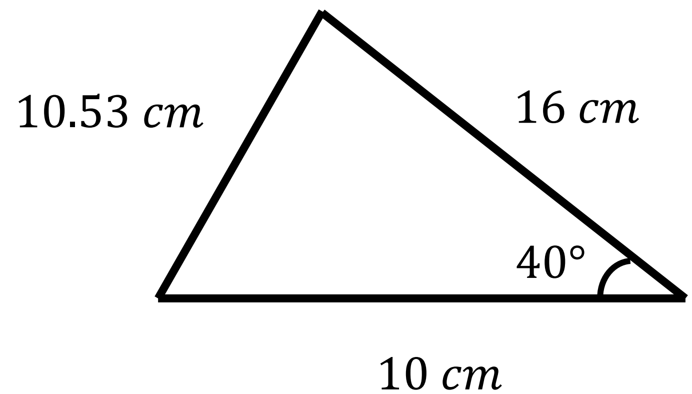
Worked Example	Your Turn
A clock's hands are $5\ cm$ and $3.5\ cm$. Find the distance between the tips of the hands at 4 o'clock	A clock's hands are $10\ cm$ and $7\ cm$. Find the distance between the tips of the hands at 4 o'clock

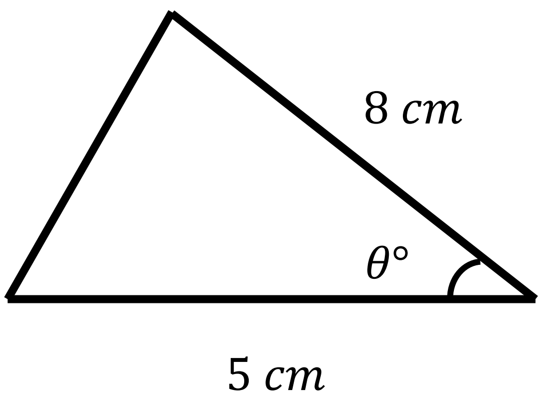
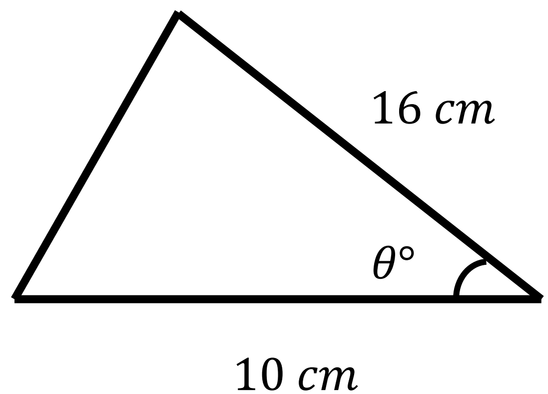
Worked Example	Thinking
Use the cosine rule to find the exact value of x 	

Area of a Triangle



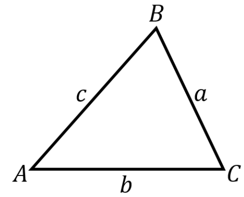
$$\text{Area} = \frac{1}{2}ab\sin(C)$$

Worked Example	Your Turn
Calculate the area of the triangle:  	Calculate the area of the triangle: 

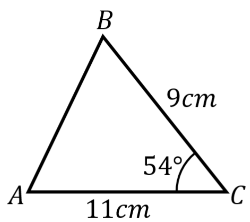
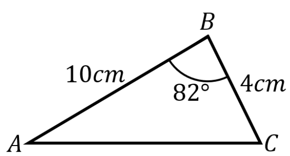
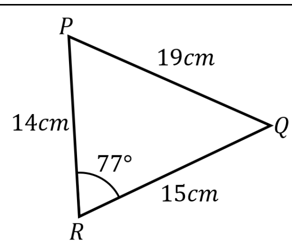
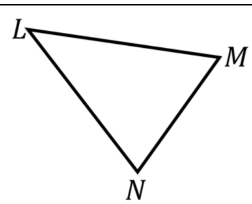
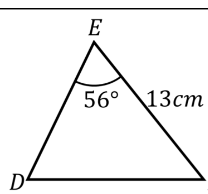
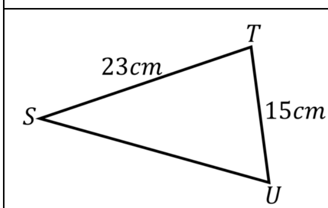
Worked Example	**Your Turn**
The area is $10\ cm^2$ Calculate θ 	The area is $51.42\ cm^2$ Calculate θ 

FILL IN THE GAPS

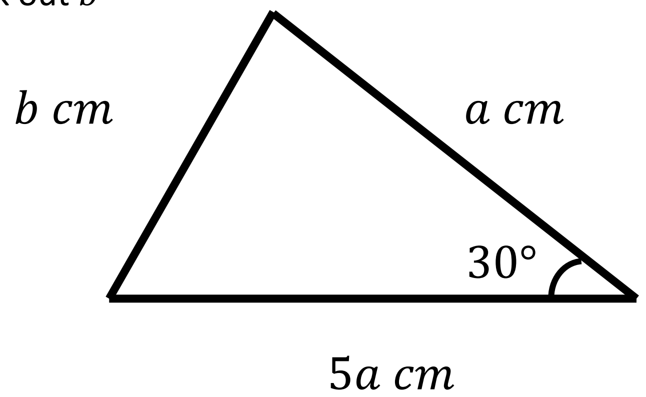
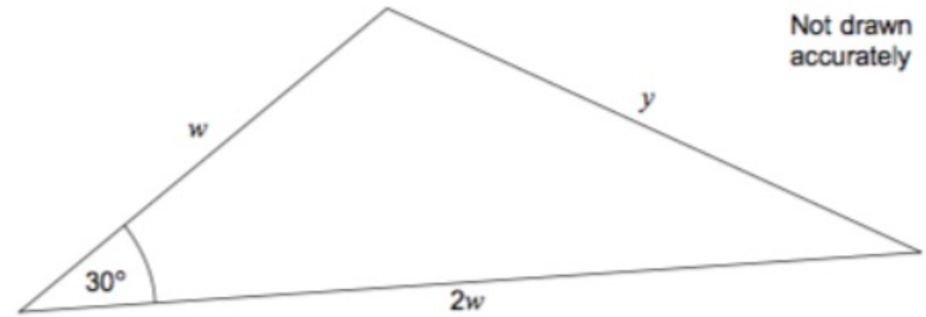
Fill in the blanks for each triangle and calculation (to 1dp) below using the area formula:

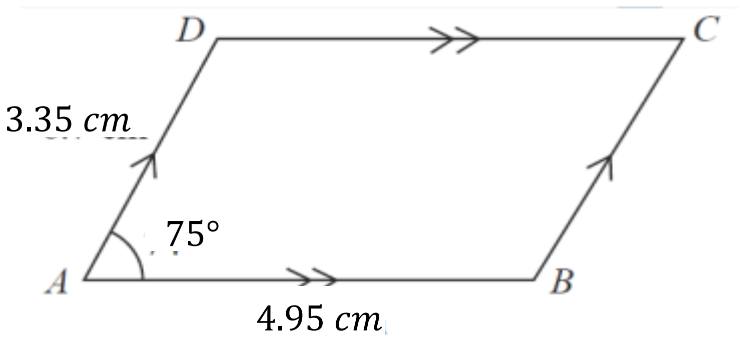
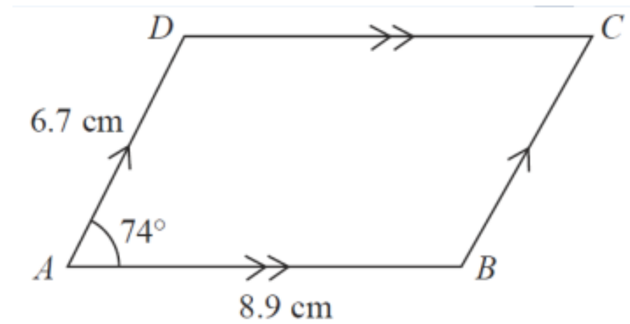


$$Area = \frac{1}{2}ab\sin C$$

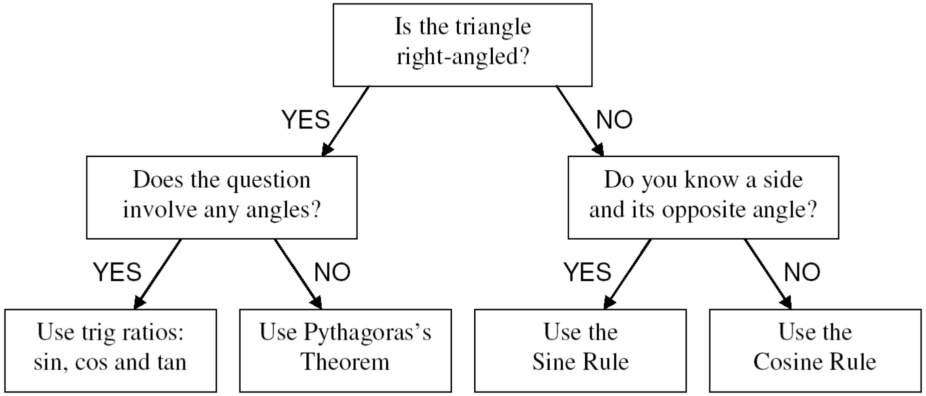
Shape	Calculation	Answer
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad ^\circ$	$Area = \quad cm^2$
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad ^\circ$	$Area = \quad cm^2$
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad ^\circ$	$Area = \quad cm^2$
	$A = \frac{1}{2} \times 8 \times 5 \sin 63^\circ$	$Area = \quad cm^2$
	$A = \frac{1}{2} \times 13 \times \quad \sin 56^\circ$	$Area = 38.8cm^2$
	$A = \frac{1}{2} \times 23 \times 15 \sin \quad ^\circ$	$Area = 172.3cm^2$

Worked Example	Your Turn
A triangle has sides $5.1\ cm$, $3.4\ cm$ and $2.85\ cm$. Work out the area of the triangle.	A triangle has sides $10.2\ cm$, $6.8\ cm$ and $5.7\ cm$. Work out the area of the triangle.

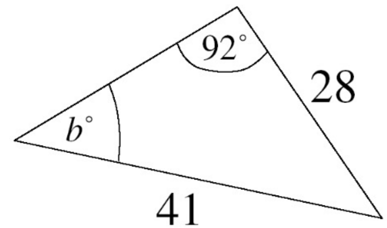
Worked Example	Your Turn
The area of the triangle is $10\ cm^2$. Work out b 	The area of the triangle is $18\ cm^2$. Work out y 

Worked Example	Your Turn
Calculate the area of the parallelogram  	Calculate the area of the parallelogram 

REVIEW

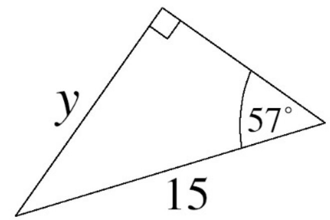


e.g. 1



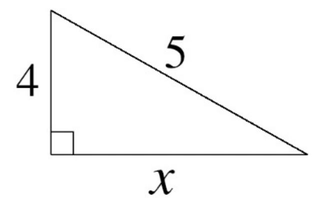
1. The triangle is not right-angled.
2. We do know a side and its opposite angle.
3. Therefore we use the Sine Rule.

e.g. 2



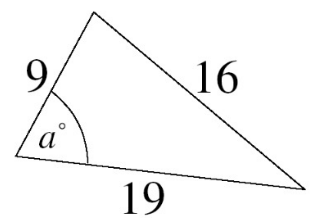
1. The triangle is right-angled.
2. The question involves angles.
3. Therefore we use trig ratios - sin, cos and tan.

e.g. 3



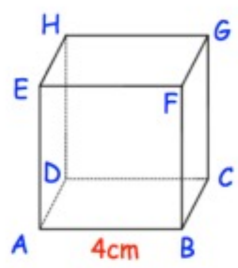
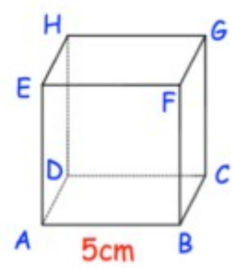
1. The triangle is right-angled.
2. The question does not involve angles.
3. Therefore we use Pythagoras's Theorem.

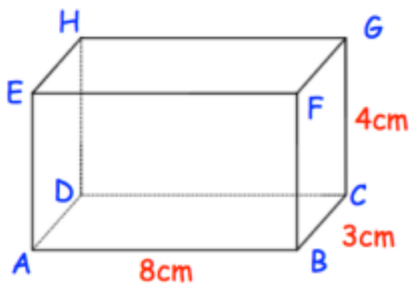
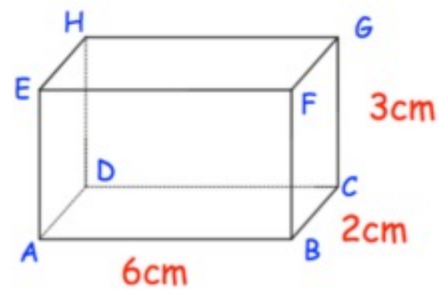
e.g. 4

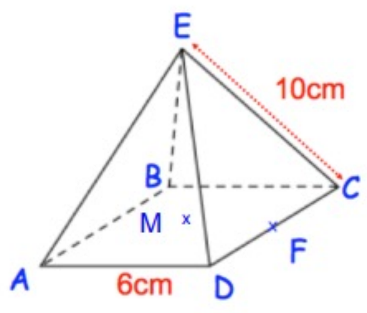
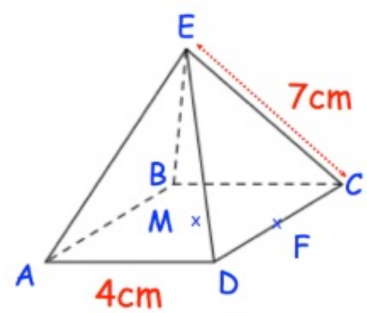


1. The triangle is not right-angled.
2. We do not know a side and its opposite angle.
3. Therefore we use the Cosine Rule.

3D Pythagoras' Theorem

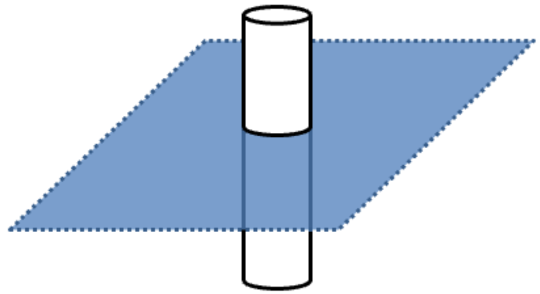
Worked Example	Your Turn
Shown below is a cube. (a) Calculate the length AC. (b) Calculate the length AG. 	Shown below is a cube. (a) Calculate the length AC. (b) Calculate the length AG. 

Worked Example	Your Turn
Shown below is a cuboid. (a) Find the length AC. (b) Find the length AG.  	Shown below is a cuboid. (a) Find the length AC. (b) Find the length AG. 

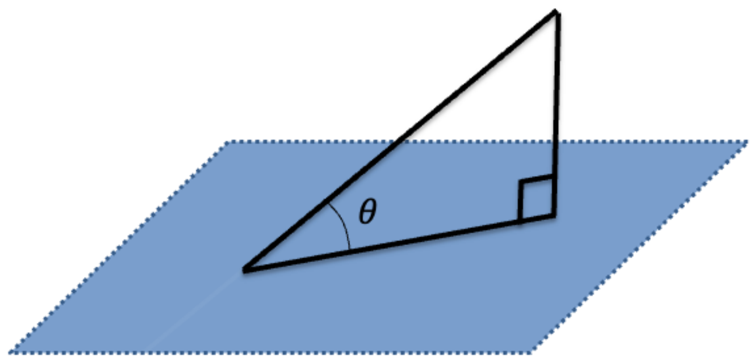
Worked Example	Your Turn
Shown below is a square based pyramid. (a) Find the length BD. (b) Find the length EM. (c) Find the length EF. 	Shown below is a square based pyramid. (a) Find the length BD. (b) Find the length EM. (c) Find the length EF. 

3D Trigonometry

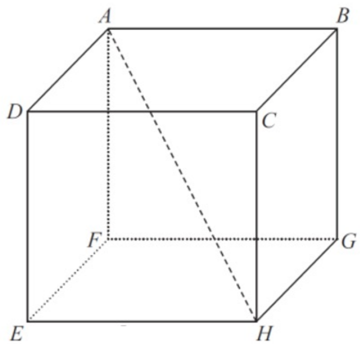
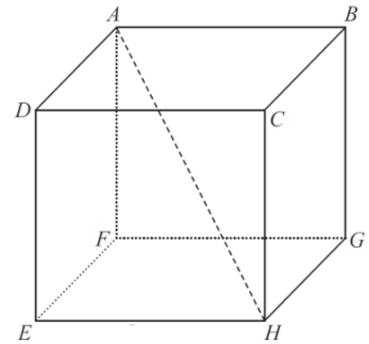
Angles Between Lines and Planes

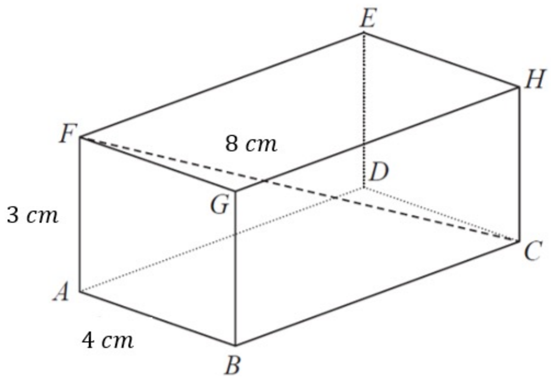
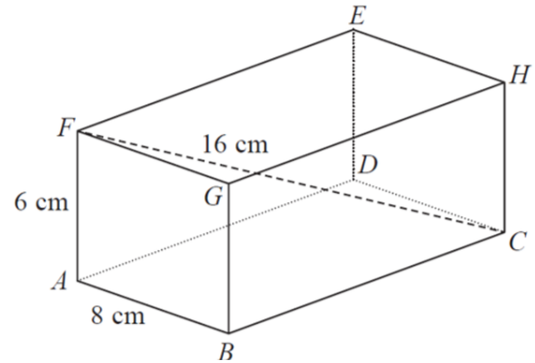


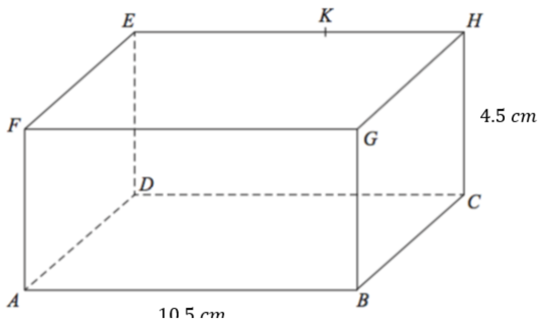
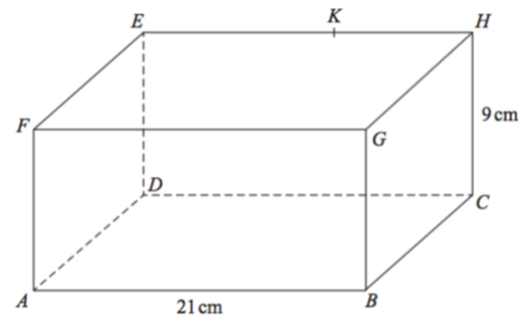
A plane is:
A flat 2D surface (not necessary horizontal).

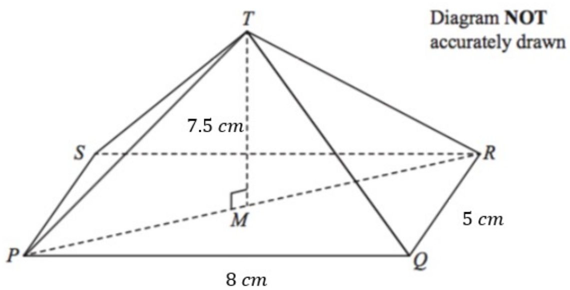
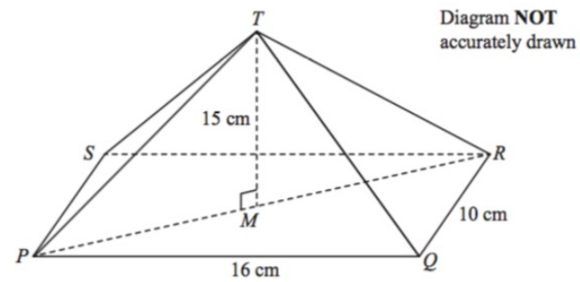


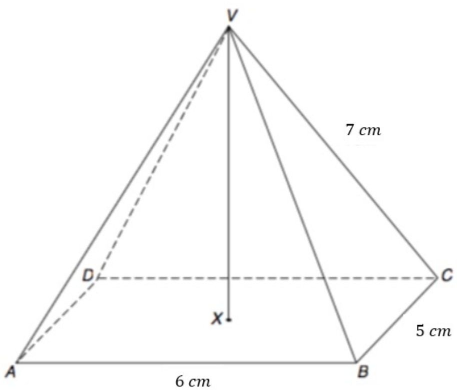
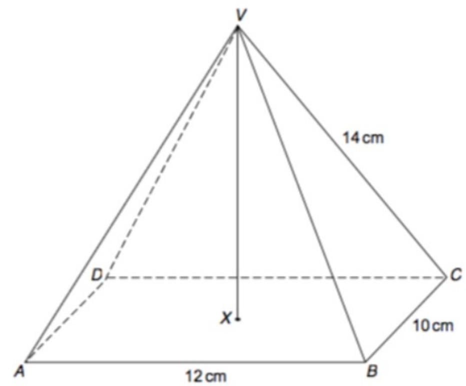
When we want to find the angle between a **line** and a **plane**, use the "**drop method**" – imagine the line is a pen which you drop onto the plane. The angle you want is between the original and dropped lines.

Worked Example	Your Turn
A cube ABCDEFGH has side lengths of 10 cm. Find the angle between the diagonal AH and the base EFGH. 	A cube ABCDEFGH has side lengths of 5 cm. Find the angle between the diagonal AH and the base EFGH. 

Worked Example	Your Turn
Calculate the angle between the line FC and the plane $ABGF$.  	Calculate the angle between the line FC and the plane $ABGF$. 

Worked Example	Your Turn
K is the point on EH such that angle AKB $= 68°$ and $BK = 8.25\ cm$. Calculate the size of angle BAK. 	K is the point on EH such that angle AKB $= 68°$ and $BK = 16.5\ cm$. Calculate the size of angle BAK. 

Worked Example	Your Turn
M is the midpoint of PR. Calculate the size of the angle between TP and the base $PQRS$.  	M is the midpoint of PR. Calculate the size of the angle between TP and the base $PQRS$. 

Worked Example	Your Turn
VABCD is a rectangular based pyramid. Calculate the angle between VC and the plane ABCD. 	VABCD is a rectangular based pyramid. Calculate the angle between VC and the plane ABCD. 

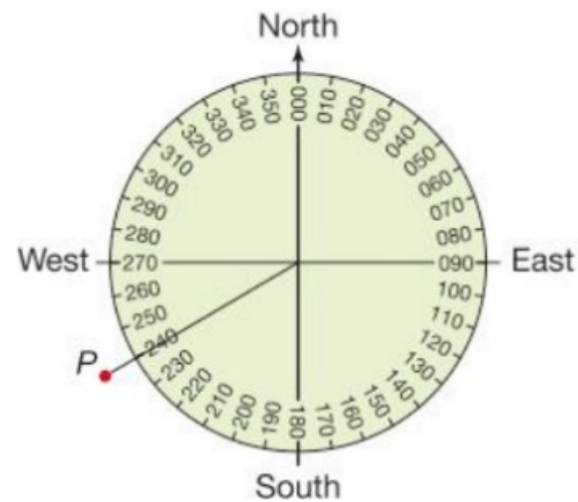
Bearings

North, east, south or west are often not enough to give an accurate direction.

- A **bearing** is an angle measured clockwise from north.

You use a 360° **scale** or a **bearing** to give a direction accurately.

- To give a bearing accurately you measure from north, measure clockwise and use three figures.

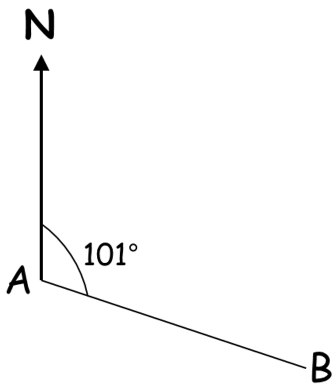
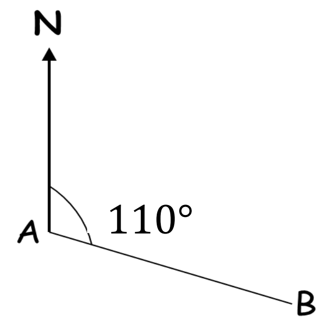


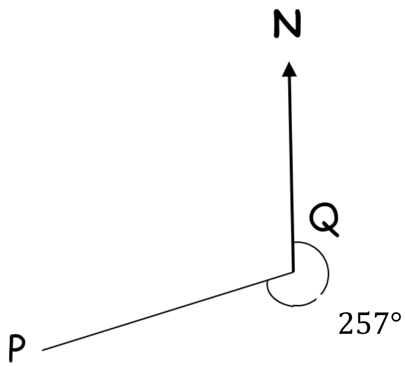
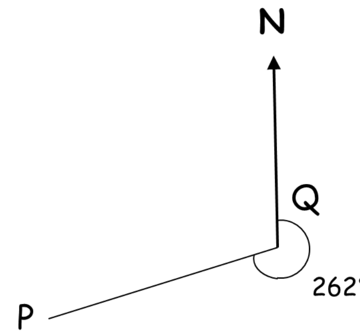
The bearing of P is 240°.

Examples/Non-Examples

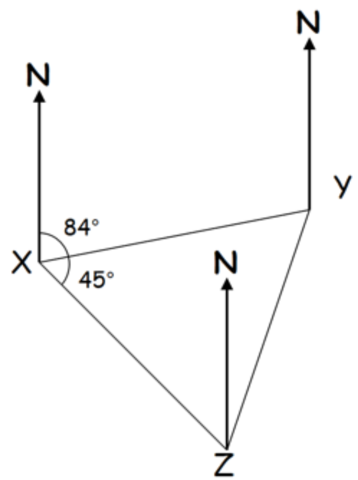
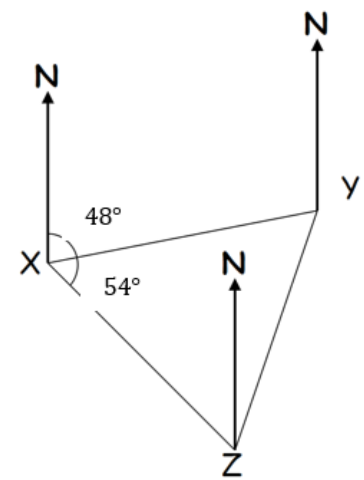
Bearings

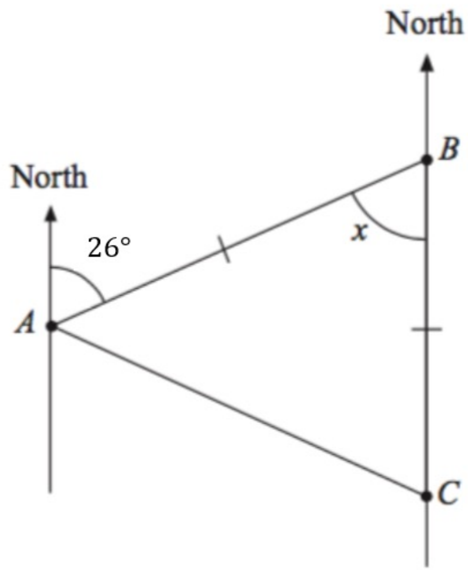
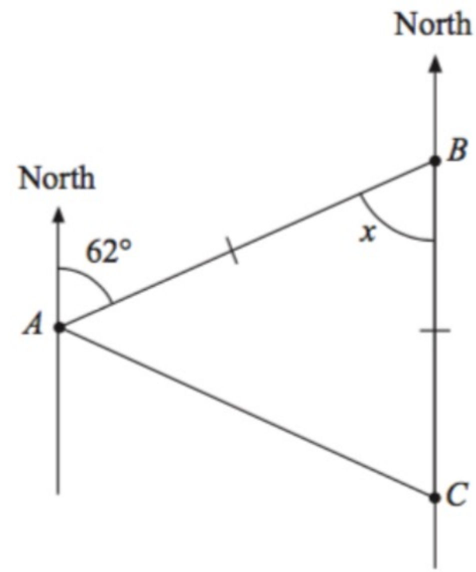
1) 045 Yes / No
2) 090 Yes / No
3) 45 Yes / No
4) 360 Yes / No
5) 361 Yes / No
6) 450 Yes / No
7) 30 Yes / No
8) 030 Yes / No
9) -145 Yes / No
10) 260 Yes / No
11) 365 Yes / No
12) 180 Yes / No
13) 27 Yes / No
14) -049 Yes / No
15) 049.5 Yes / No
16) 0180 Yes / No
17) 045 Yes / No
18) 145 Yes / No
19) -260 Yes / No
20) 0100 Yes / No
21) 80 Yes / No
22) 080 Yes / No
23) 0005 Yes / No
24) 000.5 Yes / No
25) 100.005 Yes / No

Worked Example	Your Turn
Find the bearing of B from A  Find the bearing of A from B	Find the bearing of B from A  Find the bearing of A from B

Worked Example	Your Turn
Find the bearing of P from Q  Find the bearing of Q from P	Find the bearing of P from Q  Find the bearing of Q from P

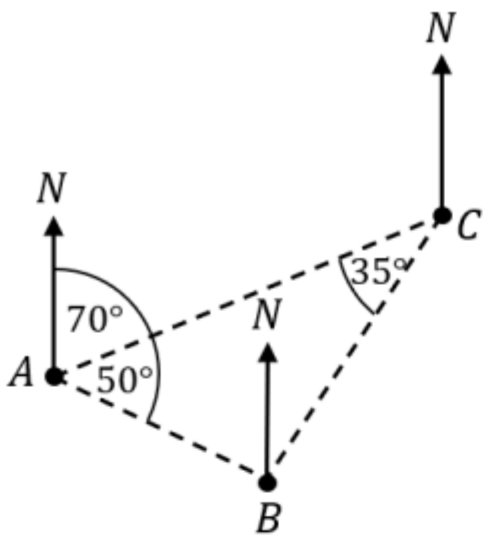
Worked Example	Your Turn
The bearing of B from A is $030°$. What is the bearing of A from B?	The bearing of B from A is $050°$. What is the bearing of A from B?
The bearing of B from A is $130°$. What is the bearing of A from B?	The bearing of B from A is $150°$. What is the bearing of A from B?

Worked Example	**Your Turn**
Find the bearing of:  Y from X X from Y Z From X X from Z	Find the bearing of:  Y from X X from Y Z From X X from Z

Worked Example	Your Turn
Calculate the bearing of C from A 	Calculate the bearing of C from A 

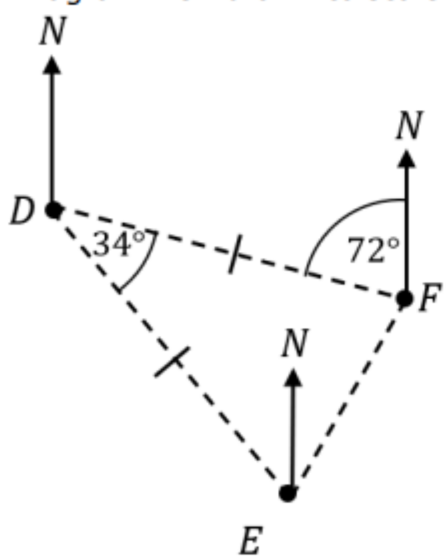
Calculating Bearings

Diagram NOT drawn to scale

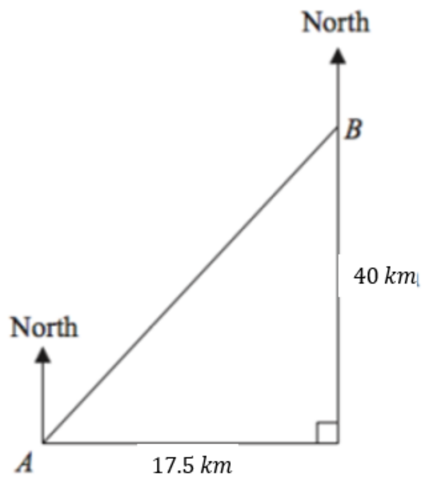
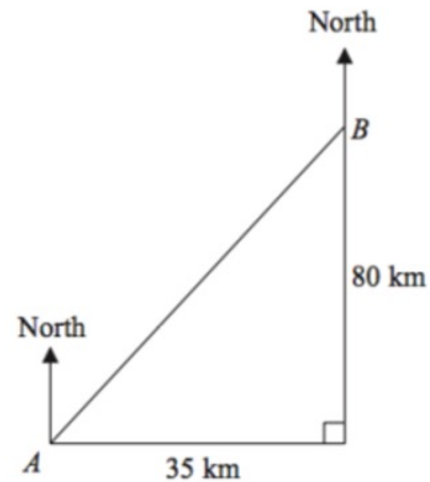


(a)	(b)	(c)
Find the bearing of C from A	Find the bearing of B from A	Find the bearing of A from C
(d)	**(e)**	**(f)**
Find the bearing of C from B	Find the bearing of A from B	Find the bearing of B from C

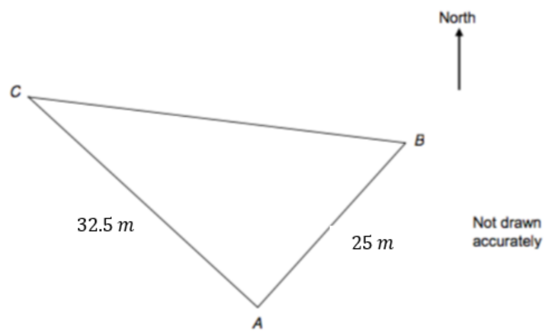
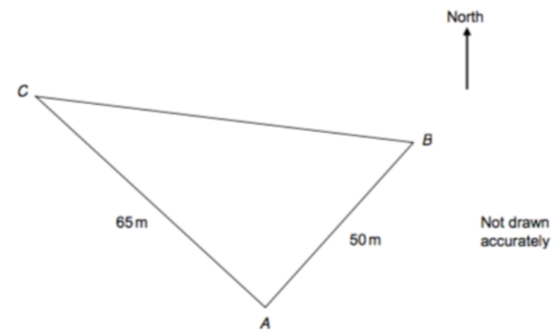
Diagram NOT drawn to scale

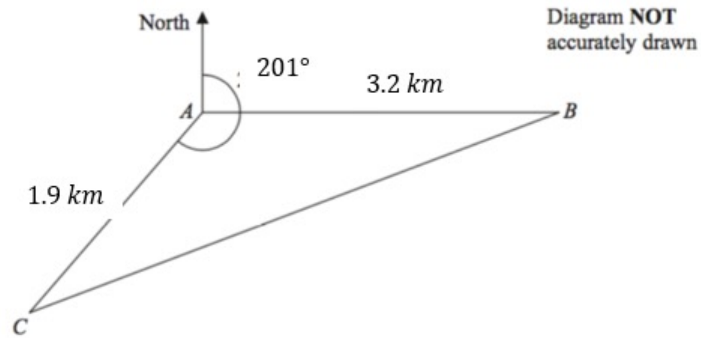
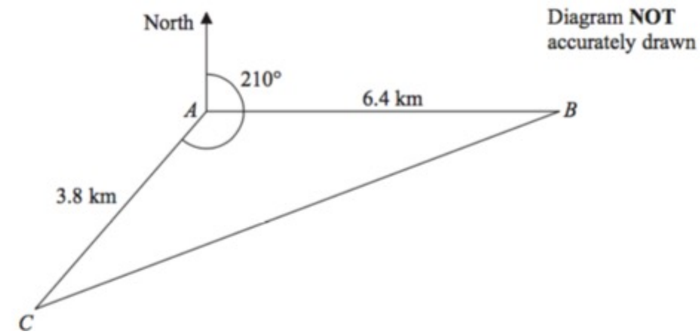


(g)	(h)	(i)
Find the bearing of D from F	Find the bearing of F from D	Find the bearing of E from D
(j)	**(k)**	**(l)**
Find the bearing of E from F	Find the bearing of D from E	Find the bearing of F from E

Worked Example	Your Turn
Work out the bearing of town B from town A  	Work out the bearing of town B from town A 

Worked Example	Your Turn
A ship sails on a bearing of $120°$ for $50\ km$. How far east has it travelled?	A ship sails on a bearing of $130°$ for $25\ km$. How far east has it travelled?

Worked Example	Your Turn
B is 25 m from A on a bearing of 020° C is 32.5 m from A on a bearing of 342° Angle CAB is 75° Work out distance BC  	B is 50 m from A on a bearing of 040° C is 65 m from A on a bearing of 325° Angle CAB is 75° Work out distance BC 

Worked Example	Your Turn
Calculate the bearing of B from C 	Calculate the bearing of B from C 

Worked Example	Your Turn
A, B and C are three points. The bearing of A from B is $045º$. The bearing of C from A is $135º$. AB = $10\ km$ and AC $=\ 6\ km$. Find the distance BC and the bearing of C from B.	A, B and C are three points. The bearing of A from B is $054º$. The bearing of C from A is $153º$. AB $=\ 6\ km$ and AC $=\ 10\ km$. Find the distance BC and the bearing of C from B.

Advanced Ratio

Worked Example *K107a*	Your Turn
Given that $p : q = 7 : 3$ and that $q : r = 6 : 11$, find the ratio $p : q : r$ Give your ratio in its simplest form with integer parts.	Given that $x : y = 3 : 25$ and that $y : z = 5 : 4$, find the ratio $x : y : z$ Give your ratio in its simplest form with integer parts.

Worked Example *K107b*	Your Turn
A bag contains only blue, purple and pink marbles. The ratio of blue marbles to purple marbles is $5:3$. The ratio of purple marbles to pink marbles is $1:4$. Calculate the percentage of marbles that are pink.	A bag contains only black, purple and orange marbles. The ratio of black marbles to purple marbles is $28:9$. The ratio of purple marbles to orange marbles is $1:7$. Calculate the percentage of marbles that are black.

Worked Example *K319a*	**Your Turn**
There are blue counters and white counters in a bag in the ratio $4 : 3$ 10 blue counters are added and the ratio becomes $2 : 1$ Work out how many white counters there are in the bag.	There are black counters and red counters in a bag in the ratio $3 : 4$ 20 black counters are removed and the ratio becomes $1 : 3$ Work out how many red counters there are in the bag.

Worked Example *K107c*	Your Turn
A pencil case contains pens, pencils and crayons. The ratio of pens to pencils is $3n : 11$. The ratio of pencils to crayons is $2 : 9n$. Work out the ratio of pens to crayons. Give your answer in its simplest form.	A picnic box contains sandwiches, cakes and apples. The ratio of sandwiches to cakes is $2n : 3$. The ratio of cakes to apples is $6 : 11n$. Work out the ratio of sandwiches to apples. Give your answer in its simplest form.

Worked Example *K107d*	Your Turn
In a box, number of red buttons : purple buttons $= 1 : 5$ number of purple buttons : orange buttons $= 1 : 3$ There are 15 orange buttons in the box. Work out the number of red buttons in the box.	In a box, number of red pens : green pens $= 1 : 5$ number of green pens : blue pens $= 6 : 1$ There are 36 red pens in the box. Work out the number of blue pens in the box.

Worked Example *K319b*	**Your Turn**
There are black counters and red counters in a bag in the ratio $3 : 7$ 5 black counters are removed and 10 red counters are added to the bag, and the ratio becomes $2 : 5$. Work out the original number of red counters in the bag.	There are white counters and red counters in a bag in the ratio $3 : 4$ 10 white counters are removed and 1 red counter is added to the bag, and the ratio becomes $2 : 3$. Work out the original number of red counters in the bag.

Worked Example *K107e*	**Your Turn**
The ratio $a : b : c = 6 : 7 : 6$. The ratio $c : d : e = 5 : 7 : 3$. Find the ratio $a : d$. Give your ratio in its simplest form.	The ratio $a : b : c = 6 : 5 : 3$. The ratio $c : d : e = 1 : 8 : 3$. Find the ratio $b : d$. Give your ratio in its simplest form.

Worked Example *K107f*	Your Turn
A biscuit tin contains shortbread, cookies and bourbons. The ratio of shortbread to cookies is $6 : 5$. The ratio of cookies to bourbons is $1 : 3$. There are more than 107 biscuits in the biscuit tin. Find the least possible number of cookies in the biscuit tin.	A pencil case contains pens, pencils and crayons. The ratio of pens to pencils is $5 : 6$. The ratio of pencils to crayons is $5 : 2$. There are less than 270 items in the pencil case. Find the greatest possible number of pens in the pencil case.

Worked Example *K107j*	Your Turn
a, b, c and d are integers with no common factors. $a : b = 4 : 3$ $c : d = 1 : 6$ $2a = 3d$ Find $a : b : c : d$	a, b, c and d are integers with no common factors. $3a = 5b$ $5c = 7d$ $a = \frac{1}{7}d$ Find $a : b : c : d$

Worked Example *K105f*	**Your Turn**
The ratio $p + 4 : 3q - 2$ is equal to $1 : 2$. Express p in terms of q.	The ratio $3a : 6b + 4$ is equal to $1 : 4$. Express a in terms of b.

Worked Example *K105g*	Your Turn
Given that $8x = y$, work out the ratio $x : y$	Given that $10p = q$, work out the ratio $p : q$

Worked Example *K107g*	**Your Turn**
The points A, B, C and D lie in order on a straight line. $AB : BD = 1 : 3$ $AC : CD = 11 : 5$ Work out $AB : BC : CD$	The points A, B, C and D lie in order on a straight line. $AB : BD = 1 : 3$ $AC : CD = 9 : 11$ Work out $AB : BC : CD$

Worked Example *K107h*	Your Turn
Green shapes and purple shapes are used in a game. Some of the shapes are triangles. All the other shapes are hexagons. The ratio of triangles to hexagons is $3 : 1$ The ratio of green triangles to purple triangles is $2 : 3$ Work out the fraction of shapes that are purple triangles.	White shapes and black shapes are used in a game. Some of the shapes are triangles. All the other shapes are hexagons. The ratio of triangles to hexagons is $3 : 4$ The ratio of white triangles to black triangles is $5 : 1$ Work out the fraction of shapes that are white triangles.

Worked Example *K107i*	Your Turn
Green shapes and purple shapes are used in a game. Some of the shapes are circles. All of the other shapes are squares. The ratio of the number of green shapes to the number of purple shapes is $3:1$ The ratio of the number of green circles to the number of green squares is $4:1$ The ratio of the number of purple circles to the number of purple squares is $3:1$ Work out what fraction of all the shapes are circles.	Green shapes and purple shapes are used in a game. Some of the shapes are triangles. All of the other shapes are hexagons. The ratio of the number of green shapes to the number of purple shapes is $5:2$ The ratio of the number of green triangles to the number of green hexagons is $1:1$ The ratio of the number of purple triangles to the number of purple hexagons is $1:4$ Work out what fraction of all the shapes are triangles.

Worked Example *K166a*	Your Turn
Find the midpoint of the line segment AB where $A(8,3)$ and $B(15,19)$	Find the midpoint of $(10,9)$ and $(20,14)$

Worked Example *K166b*	Your Turn
$M(2, 0.5)$ is the midpoint of the line segment AB where $A(5, -4)$. Find the coordinates of B.	$M(4, 1)$ is the midpoint of the line segment AB where $A(5, 5)$. Find the coordinates of B.

Worked Example *K166c*	Your Turn
The point M lies on the line segment AB where $A(-4,-2)$ and $B(-1,4)$. Given that $AM : MB = 1 : 2$, find the coordinates of M.	The point M lies on the line segment AB where $A(4,3)$ and $B(10,15)$. Given that $AM : MB = 2 : 1$, find the coordinates of M.