



KING EDWARD VI  
HANDSWORTH GRAMMAR  
SCHOOL FOR BOYS



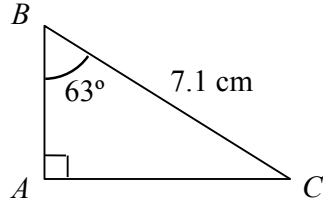
KING EDWARD VI  
ACADEMY TRUST  
BIRMINGHAM

**2023**      **Year 11**      **2024**  
**Mathematics**  
**Unit 21 Tasks**

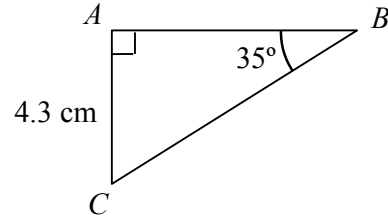
**DO NOT WRITE INSIDE**

# Fluency Practice

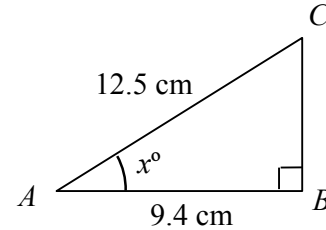
**A1** Find length  $AB$



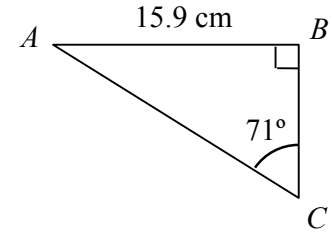
**A2** Find length  $AB$



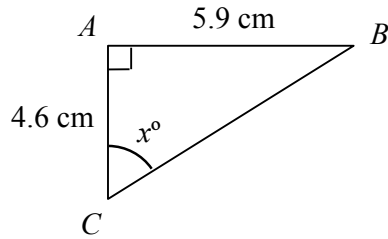
**A3** Find angle  $BAC$



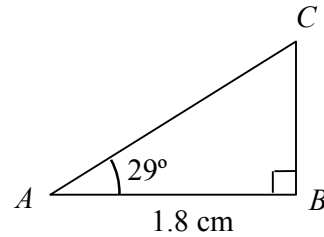
**A4** Find length  $AC$



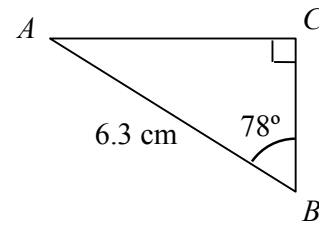
**B1** Find angle  $ACB$



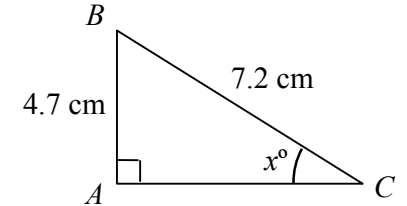
**B2** Find length  $BC$



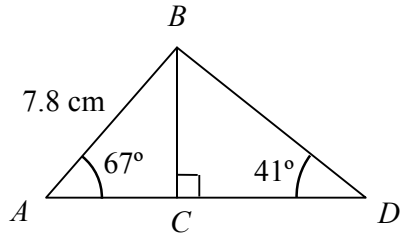
**B3** Find length  $AC$



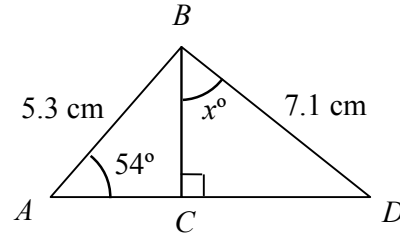
**B4** Find angle  $ACB$



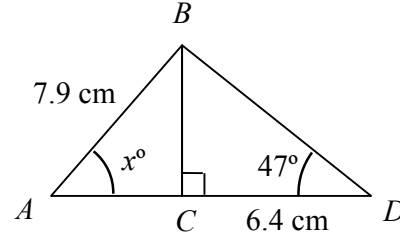
**C1** Find length  $CD$



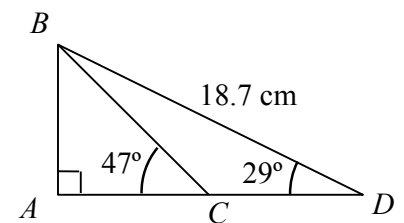
**C2** Find angle  $CBD$



**C3** Find angle  $BAC$

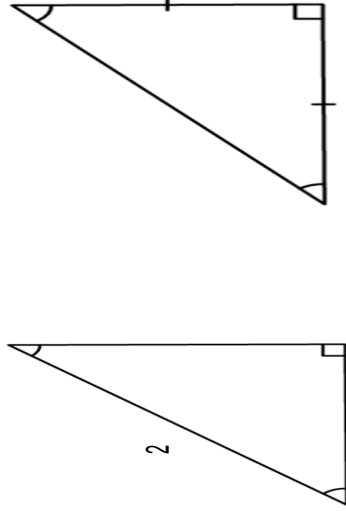


**C4** Find length  $CD$



## exact trig values

Complete the missing sides and angles for these special triangles:



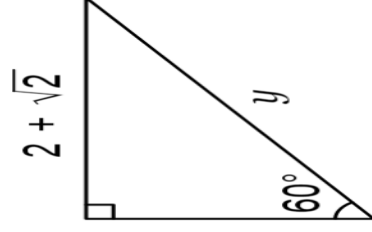
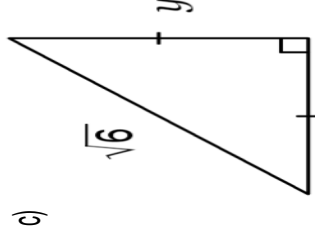
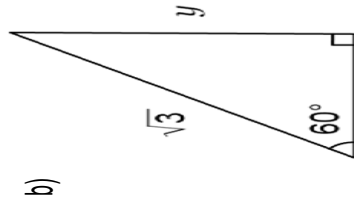
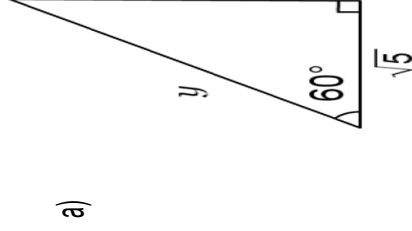
Complete the table to show the exact values of  $\sin$ ,  $\cos$  and  $\tan$  for these angles:

	$30^\circ$	$45^\circ$	$60^\circ$
$\sin$			
$\cos$			
$\tan$			

In this exercise, assume  $\theta$  is an acute angle and rationalise the denominator of your answers, where appropriate.

- Given that  $\sin(\theta) = \frac{1}{\sqrt{2}}$ , what is  $\theta$ ?
- Given that  $\tan(\theta) = \sqrt{3}$ , what is  $\cos(\theta)$ ?

3. Calculate the exact length of  $y$  in each triangle:



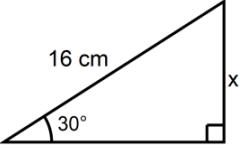
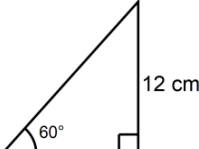
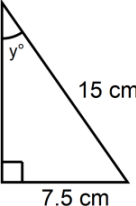
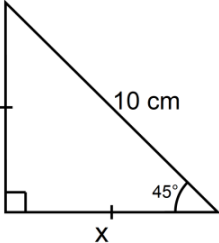
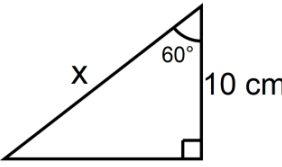
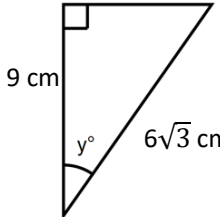
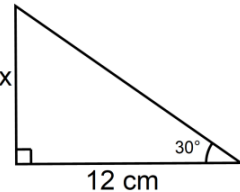
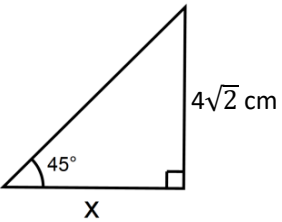
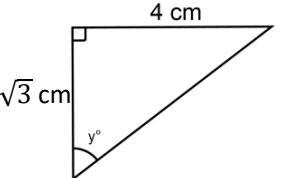
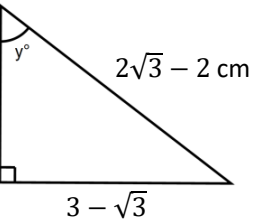
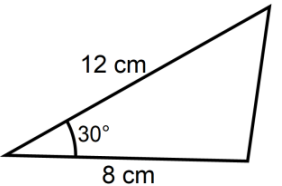
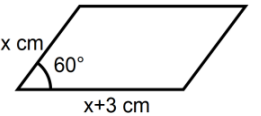
4. Simplify  $4\sin(60^\circ)\cos(60^\circ)$

5. Simplify  $2\sin(45^\circ)\cos(45^\circ)$

6. Write  $6\tan(60^\circ) + 3\tan(30^\circ)$  in the form  $a\sqrt{b}$

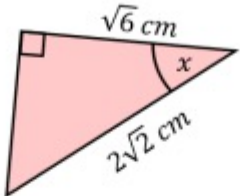
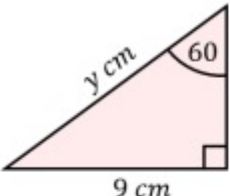
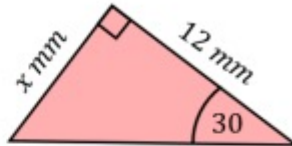
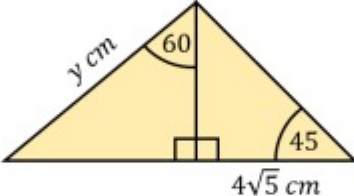
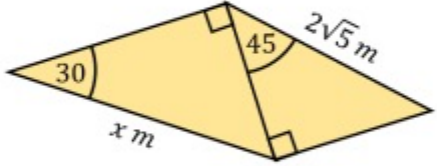
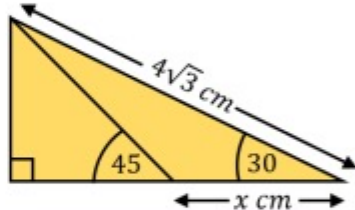


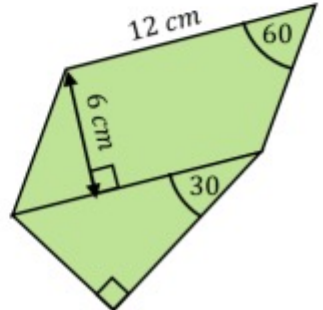
7. Show that  $\tan(45^\circ) = \frac{\sin(45^\circ)}{\cos(45^\circ)}$

8. Show that  $\tan(30^\circ) = \frac{\sin(30^\circ)}{\cos(30^\circ)}$

<p>Sine</p>	<p>State the value of sine of</p> <table border="1"> <thead> <tr> <th><math>0^\circ</math></th> <th><math>30^\circ</math></th> <th><math>45^\circ</math></th> <th><math>60^\circ</math></th> <th><math>90^\circ</math></th> </tr> </thead> <tbody> <tr> <td> </td> <td> </td> <td> </td> <td> </td> <td> </td> </tr> </tbody> </table>	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$						<p>Work out the exact value of <math>x</math></p> 	<p>Work out the exact value of <math>x</math></p> 	<p>Work out the size of angle <math>y</math></p> 
$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$										
<p>Cosine</p>	<p>State the value of cosine for</p> <table border="1"> <thead> <tr> <th><math>0^\circ</math></th> <th><math>30^\circ</math></th> <th><math>45^\circ</math></th> <th><math>60^\circ</math></th> <th><math>90^\circ</math></th> </tr> </thead> <tbody> <tr> <td> </td> <td> </td> <td> </td> <td> </td> <td> </td> </tr> </tbody> </table>	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$						<p>Work out the exact value of <math>x</math></p> 	<p>Work out the exact value of <math>x</math></p> 	<p>Work out the size of angle <math>y</math></p> 
$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$										
<p>Tangent</p>	<p>State the value of tan for</p> <table border="1"> <thead> <tr> <th><math>0^\circ</math></th> <th><math>30^\circ</math></th> <th><math>45^\circ</math></th> <th><math>60^\circ</math></th> </tr> </thead> <tbody> <tr> <td> </td> <td> </td> <td> </td> <td> </td> </tr> </tbody> </table>	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$					<p>Work out the exact value of <math>x</math></p> 	<p>Work out the exact value of <math>x</math></p> 	<p>Work out the size of angle <math>y</math></p> 		
$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$											
<p>Challenge Questions</p>	<p>Show that <math>20 \cos 30^\circ + 4 \sin 60^\circ - 2 \tan 60^\circ</math> can be written in the form <math>\sqrt{k}</math> where <math>k</math> is an integer</p>	<p>Work out the size of angle <math>y</math></p> 	<p>Work out the area of the triangle</p> 	<p>The parallelogram has an area of <math>20\sqrt{3} \text{ cm}^2</math>. Find the value of <math>x</math>.</p> 										



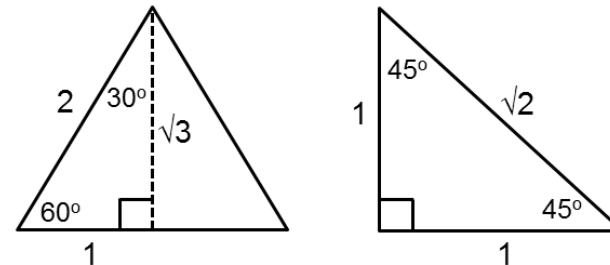
## Exact Trigonometric Values in Right-Angled Triangles

(a)	(b)	(c)
<p>Find the exact value of <math>x</math>.</p> 	<p>Find the exact length of <math>y</math>.</p> 	<p>Find the exact length of <math>x</math>.</p> 
(d)	(e)	(f)
<p>Find the exact length of <math>y</math>.</p> 	<p>Find the exact length of <math>x</math>.</p> 	<p>Find the exact length of <math>x</math>.</p> 
(g)	(h)	(i)
<p>Find the area of the parallelogram.</p> 	<p>Find the exact value of the shaded area.</p> 	<p>A triangle is joined to a parallelogram. Find the exact perimeter of the shape.</p> 

using exact trigonometric values for some angles

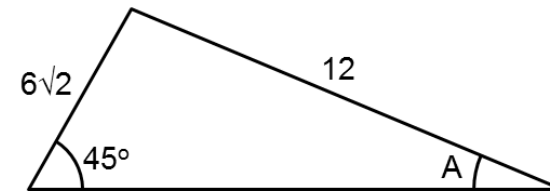
what are the values of these:

- (1)  $5\sin 30^\circ - 11\cos 60^\circ + 4\tan 45^\circ$
- (2)  $\sin 45^\circ \cdot \cos 45^\circ + \cos 30^\circ \cdot \tan 30^\circ$
- (3)  $\sin 30^\circ \cdot \tan 45^\circ + \tan 30^\circ \cdot \sin 60^\circ$
- (4)  $2\sin 45^\circ \cdot \cos 45^\circ$
- (5)  $\cos^2 45^\circ + \sin^2 45^\circ$
- (6)  $\cos 30^\circ \cdot \sin 60^\circ + \sin 30^\circ \cdot \cos 60^\circ$
- (7)  $\sin^2 30^\circ + \cos^2 30^\circ$
- (8)  $3\cos^2 30^\circ - 5\sin^2 30^\circ$
- (9)  $\frac{1}{\cos^2 30^\circ} - \tan^2 30^\circ$
- (10)  $\frac{2\tan^2 45^\circ}{\tan^2 60^\circ} + 2\tan^2 30^\circ$

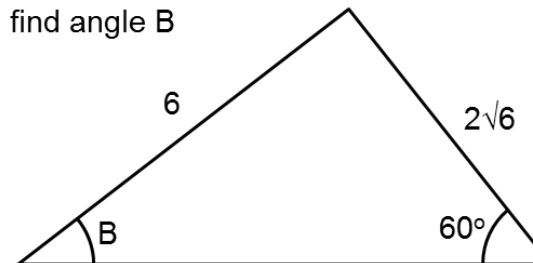


note that  $\sin^2 30^\circ$  is  $(\sin 30^\circ)^2$

(11) find angle A



(12) find angle B





## Using Surds in Trigonometry

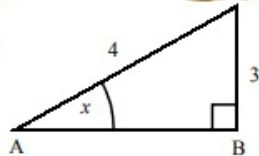


### Example

If  $\sin x = \frac{3}{4}$ , find  $\tan x$  and  $\cos x$ .

This can be done without calculating the angle  $x$ . Just remember

$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$  So, we could draw a triangle like this:



We can now use Pythagoras' theorem to calculate AB.

$$\begin{aligned} AB^2 &= 4^2 - 3^2 \\ &= 7 \\ AB &= \sqrt{7} \end{aligned}$$

So,  $\tan x = \frac{3}{\sqrt{7}}$  and  $\cos x = \frac{\sqrt{7}}{4}$

### Exercise

Copy and complete the table below using a similar method to the one above. Give all answers in surd form.

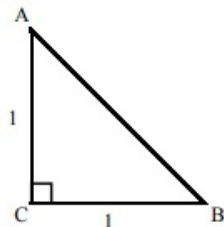
Sin $x$	Cos $x$	Tan $x$
$\frac{3}{5}$		
	$\frac{1}{3}$	
		$\frac{5}{2}$
		$\frac{\sqrt{7}}{2}$
$\frac{1}{\sqrt{5}}$		
	$\frac{2}{\sqrt{13}}$	
		$\frac{\sqrt{5}}{\sqrt{2}}$
	$\frac{\sqrt{7}}{3}$	
$\frac{4\sqrt{3}}{9}$		
		$\frac{\sqrt{3}+1}{8-\sqrt{5}}$



## Trigonometric Ratios for $45^\circ$ , $30^\circ$ and $60^\circ$ in Surd Form



### Task 1



This is a right-angled, isosceles triangle. Two sides have length 1 unit.

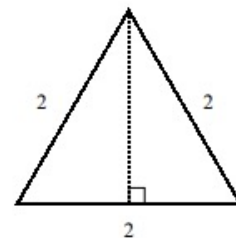
Use Pythagoras' theorem to find the length of the hypotenuse AB. Leave your answer as a surd.

Using surds where necessary, write down the ratios for  $\sin 45^\circ$ ,  $\cos 45^\circ$  and  $\tan 45^\circ$ .

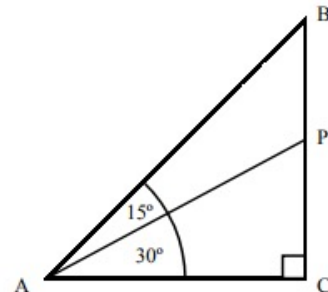
Check your answers using a calculator.

### Task 2

Here is an equilateral triangle. Each side is 2 units in length. Find the height of the triangle (in surd form) using Pythagoras' theorem and use this to express the sine, cosine and tangent of  $60^\circ$  and  $30^\circ$ .



### Task 3



In triangle ABC,  $AC = BC = 1$  unit.

Use your value of  $\tan 30^\circ$  from task 2 to find the lengths of CP and BP.

Show that  $AP = \frac{2}{\sqrt{3}}$  units.

Using the Sine rule for triangle ABP, show that

$$\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

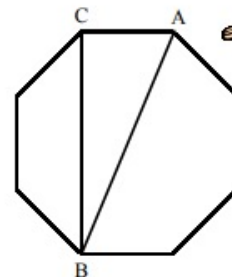
Use the value of  $\sin 15^\circ$  to find cosine and tangent of  $15^\circ$  in surd form.

### Task 4

This is a unit octagon. Each side is 1 unit.

Show that  $\angle BAC = 67\frac{1}{2}^\circ$  and  $\angle ABC = 22\frac{1}{2}^\circ$

Try to find the tangent, sine and cosine of  $67\frac{1}{2}^\circ$  and  $22\frac{1}{2}^\circ$  in surd form.

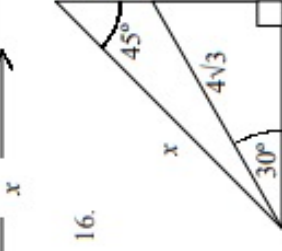
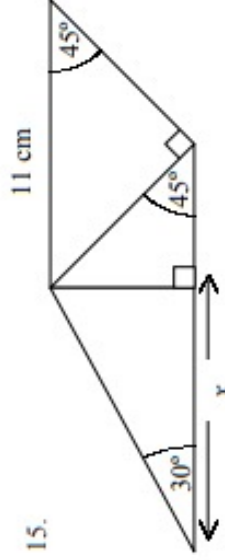
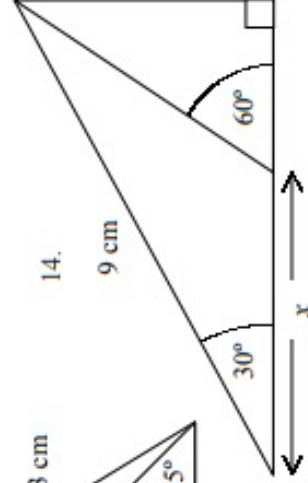
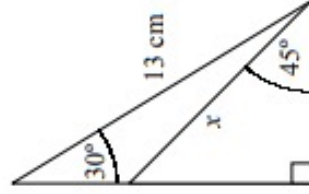
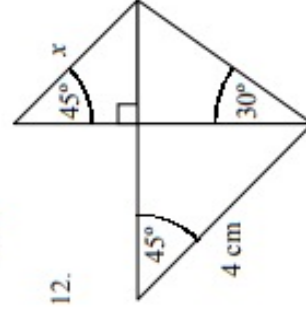
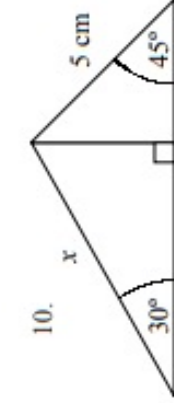
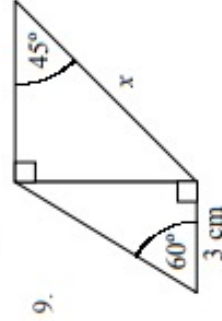
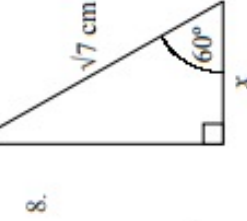
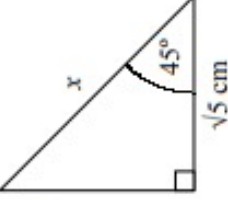
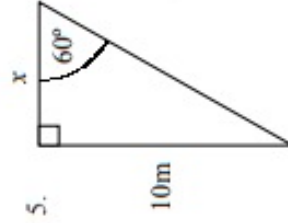
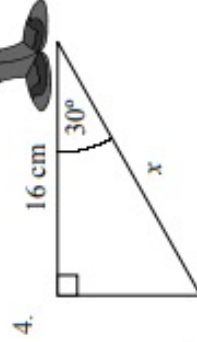
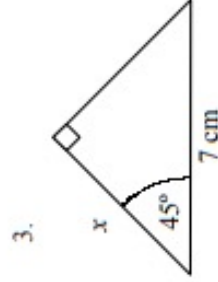
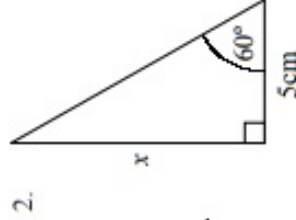
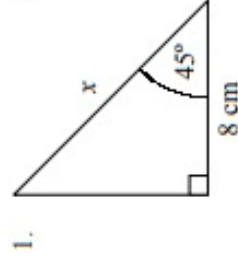




## Using Trigonometric ratios for $45^\circ$ , $30^\circ$ and $60^\circ$ in Surd Form



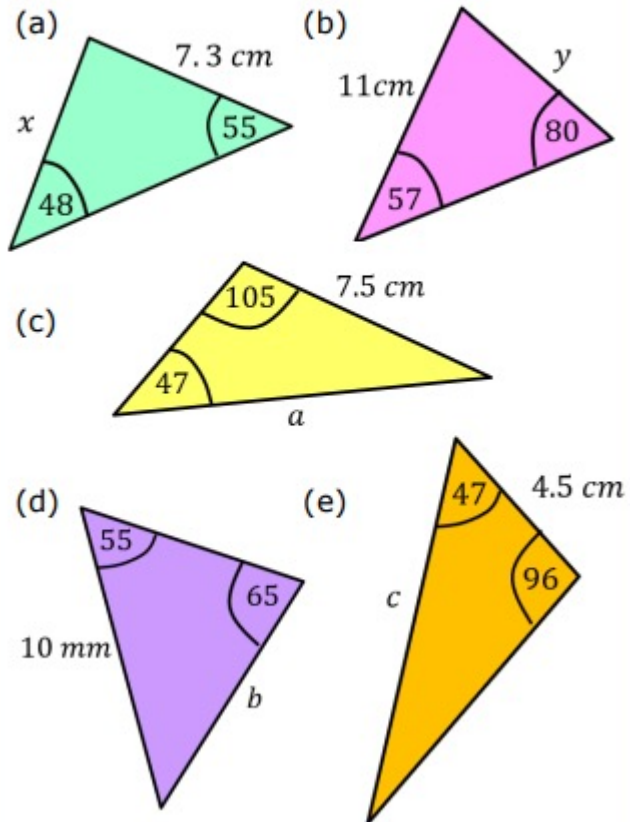
Find the missing length marked  $x$  in each diagram. Express your answers in surd form.



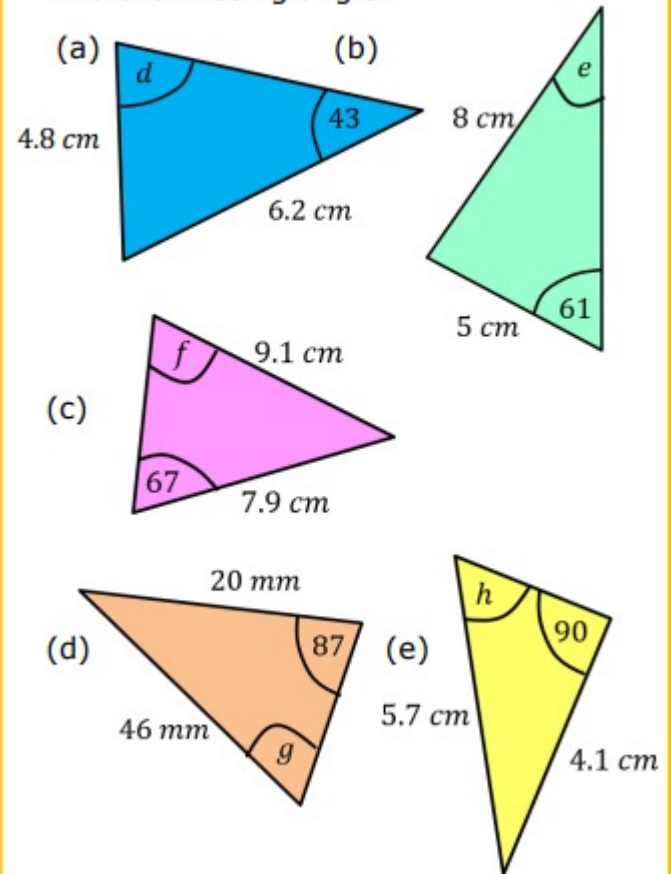
# Fluency Practice

## Sine Rule

Find the missing length.

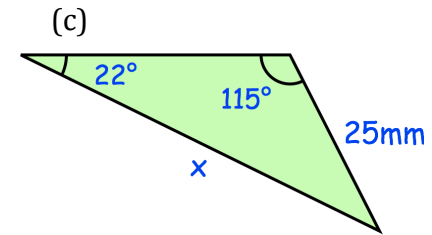
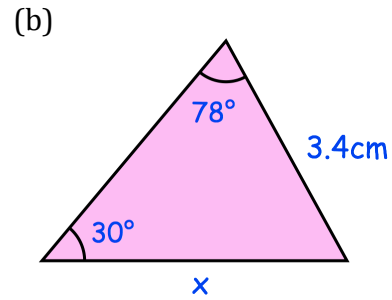
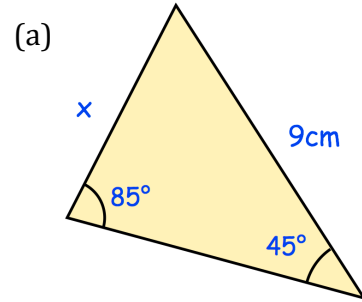


Find the missing angle.

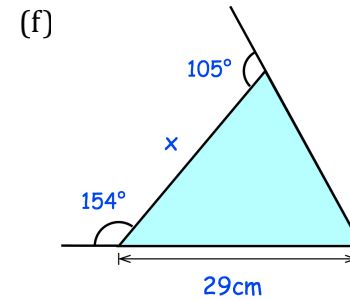
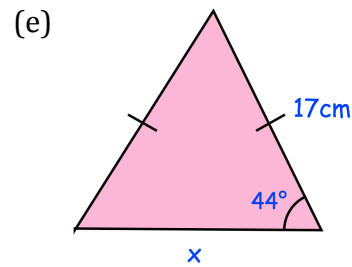
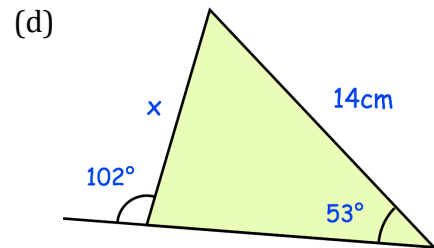
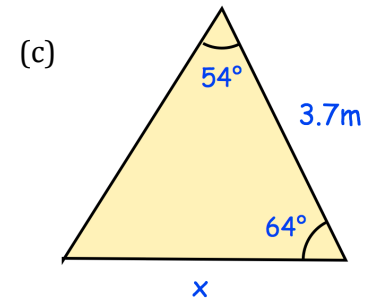
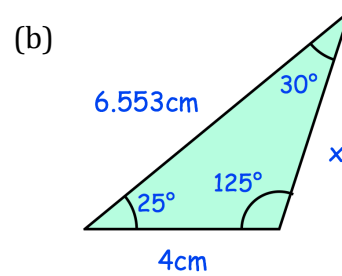
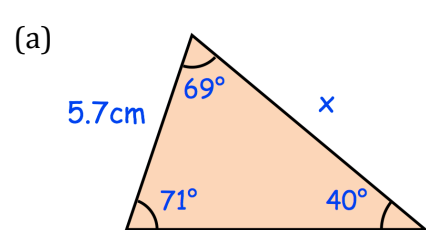


# Fluency Practice

Question 1: Find  $x$  for each of the triangles below.

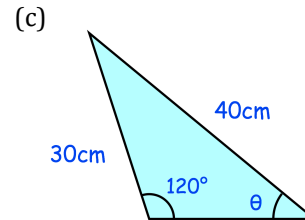
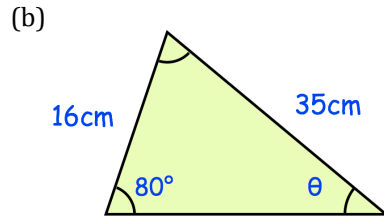
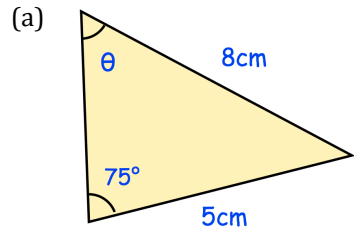


Question 2: Find  $x$  for each of the triangles below.

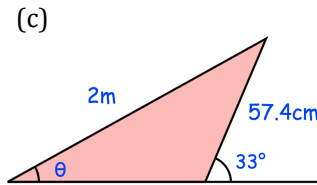
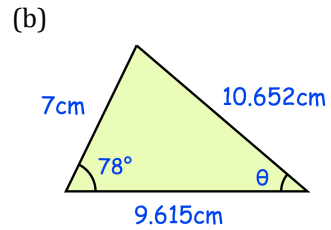
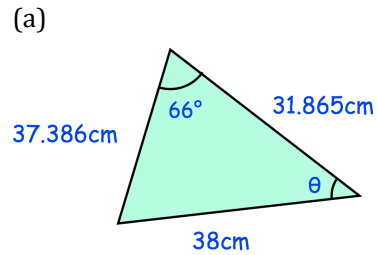


# Fluency Practice

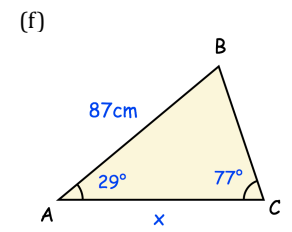
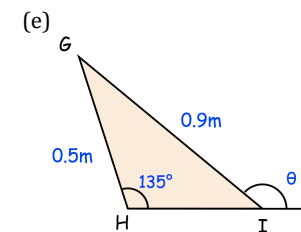
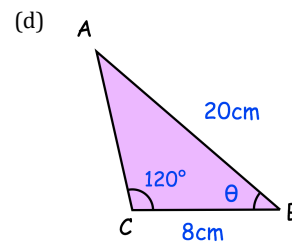
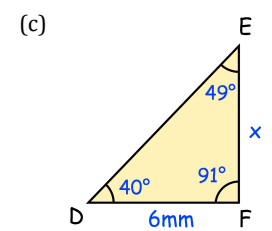
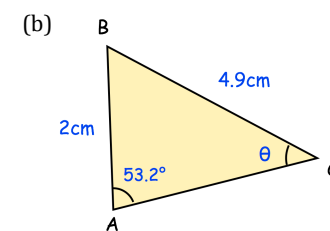
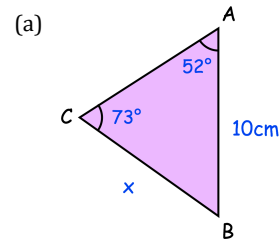
Question 3: Find the size of  $\theta$  for each of these triangles.



Question 4: Find the size of  $\theta$  for each of these triangles.



Question 5: Find the missing angle  $\theta$  or side,  $x$ , for each of the triangles below.

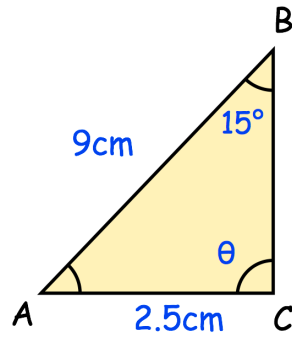




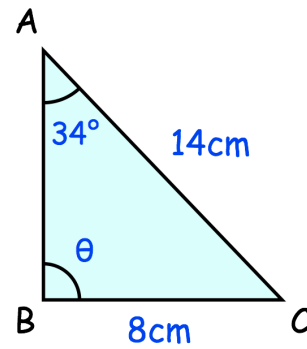
## Fluency Practice

Question 6: The triangles below have two possible values for the angle  $\theta$ . Find the size of these possible angles.

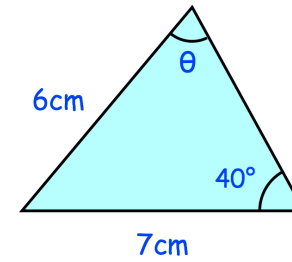
(a)



(b)



(c)

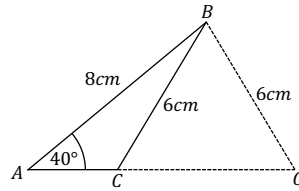




# Fluency Practice

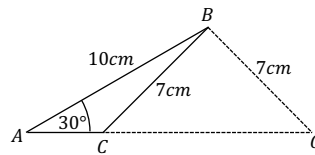
1. In triangle  $ABC$ ,  $AB = 8\text{cm}$ ,  $BC = 6\text{cm}$  and angle  $CAB = 40^\circ$  (see diagram)

Find the two possible sizes of angle  $BCA$ .



2. In triangle  $ABC$ ,  $AB = 10\text{cm}$ ,  $BC = 7\text{cm}$  and angle  $CAB = 30^\circ$  (see diagram)

Find the two possible sizes of angle  $BCA$ .



3. In triangle  $ABC$ ,  $AB = 12\text{cm}$ ,  $BC = 9\text{cm}$  and angle  $CAB = 47^\circ$

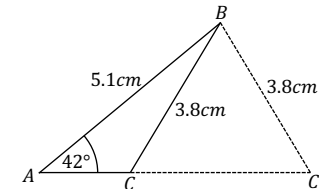
Find the two possible sizes of angle  $BCA$ .

4. In triangle  $ABC$ ,  $AB = 13\text{cm}$ ,  $BC = 11\text{cm}$  and angle  $CAB = 53^\circ$

Find the two possible sizes of angle  $BCA$ .

5. In triangle  $ABC$ ,  $AB = 5.1\text{cm}$ ,  $BC = 3.8\text{cm}$  and angle  $CAB = 42^\circ$  (see diagram)

- (i) Find the two possible sizes of angle  $BCA$ .



- (ii) Find the two possible lengths of  $AC$ .

6. In triangle  $ABC$ ,  $AB = 17.4\text{cm}$ ,  $BC = 10.7\text{cm}$  and angle  $CAB = 23^\circ$

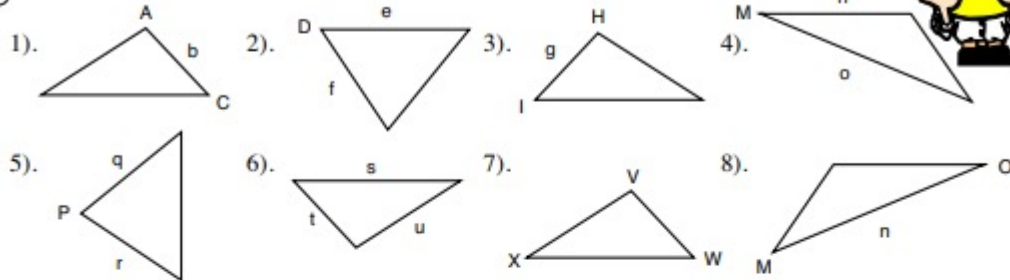
- (i) Find the two possible sizes of angle  $BCA$ .

- (ii) Find the two possible lengths of  $AC$ .



# The Sine and Cosine Rules 1.

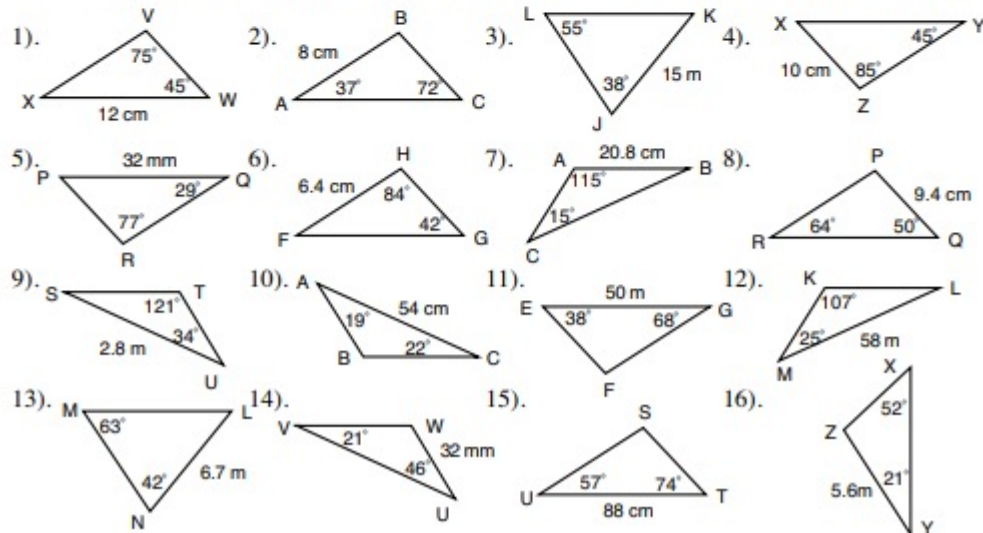
Finish off correctly labelling these diagrams.



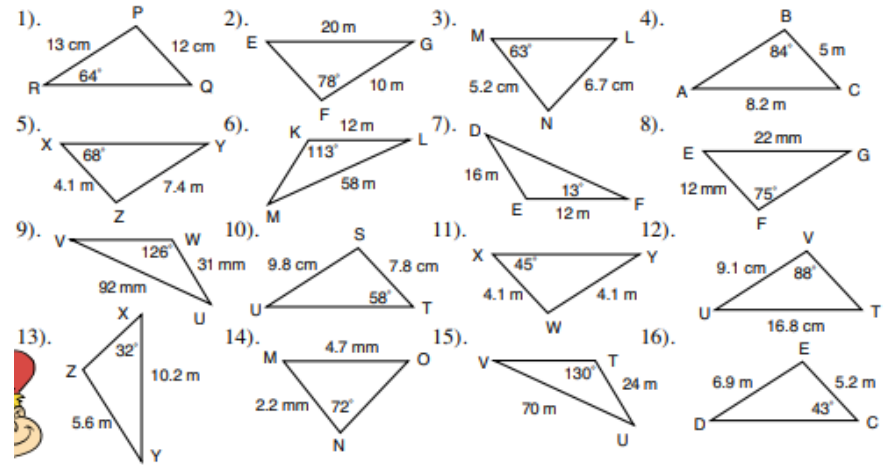
## The Sine rule.

Diagrams not to scale.

A). Find **all** the missing sizes in each of these diagrams. Give the answer to 1 decimal place.

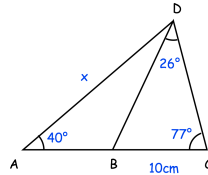


B). Find **all** the missing angles in each of these diagrams. Give the answer to 1 decimal place.



# Extension

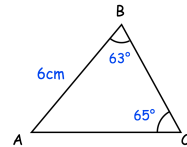
Question 1: ACD is a triangle and B is a point on AC.  
 BC = 10cm  
 Angle ACD =  $77^\circ$     Angle BDC =  $26^\circ$



Angle DAC =  $40^\circ$

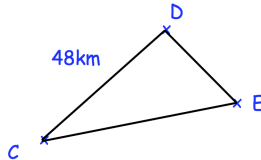
- (a) Find the length of BD
- (b) Find the size of angle ABD
- (c) Find the length of AD

Question 2: Calculate the perimeter of triangle ABC



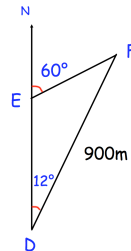
Question 3: Donhampton is 48km from Castletown on a bearing of  $050^\circ$ .  
 Eastville is on a bearing of  $075^\circ$  from Castletown and on a bearing of  $150^\circ$  from Donhampton.

Calculate the distance of Eastville from Castletown.



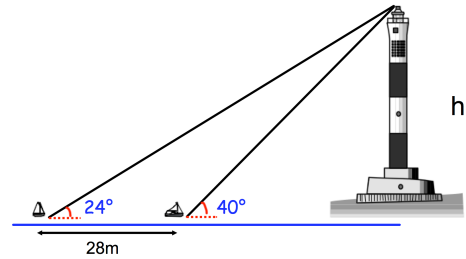
Question 4: The diagram shows 3 helicopters D, E and F.  
 Helicopter D is due South of helicopter E.  
 The bearing of helicopter F from helicopter E is  $060^\circ$   
 The bearing of helicopter F from helicopter D is  $012^\circ$   
 The distance of helicopter F from helicopter D is 900m

- (a) Find the distance of helicopter F from helicopter E
- (b) Find the distance of helicopter D from helicopter E.



Question 5: Two small boats are 28m apart.  
 The angle of elevation of the boats to the top of the lighthouses are  $24^\circ$  and  $40^\circ$  respectively.

Calculate the height of the lighthouse.

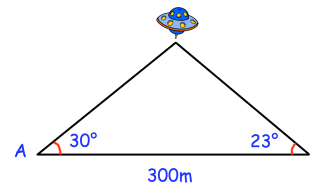


Question 6: There are two possible triangles DEF, where DE = 6.2cm, EF = 7.9cm and  $\angle DFE = 30^\circ$

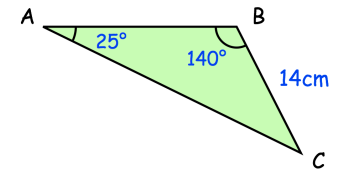
Sketch both possible triangles with all sides and angles found.

Question 7: A UFO is flying above two people, standing on the ground at points A and B.  
 A and B are 300m apart.

The angle of elevation of the UFO from A is  $30^\circ$   
 The angle of elevation of the UFO from B is  $23^\circ$   
 Find the height of the UFO above the ground.



Question 9: Shown below is triangle ABC.  
 All values are correct to 2 significant figures.  
 Calculate the upper bound for the length of AC.



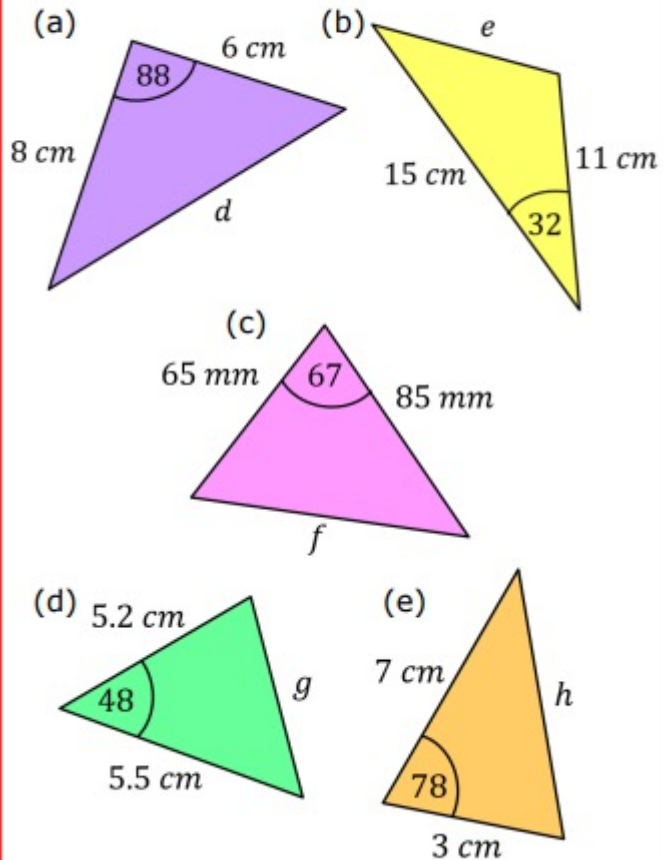
Question 8: Two ships, A and B, leave a port at midday.  
 A travels on a bearing of  $085^\circ$  at a speed of 18km/h.  
 B travels on a bearing of  $113^\circ$  at a speed of  $y$  km/h.  
 At 14:00 the distance between A and B is 30km.  
 Boat B was travelling at a slower speed than boat A  
 Work out  $y$ , the speed of boat B.

Question 10: Prove the Sine Rule.

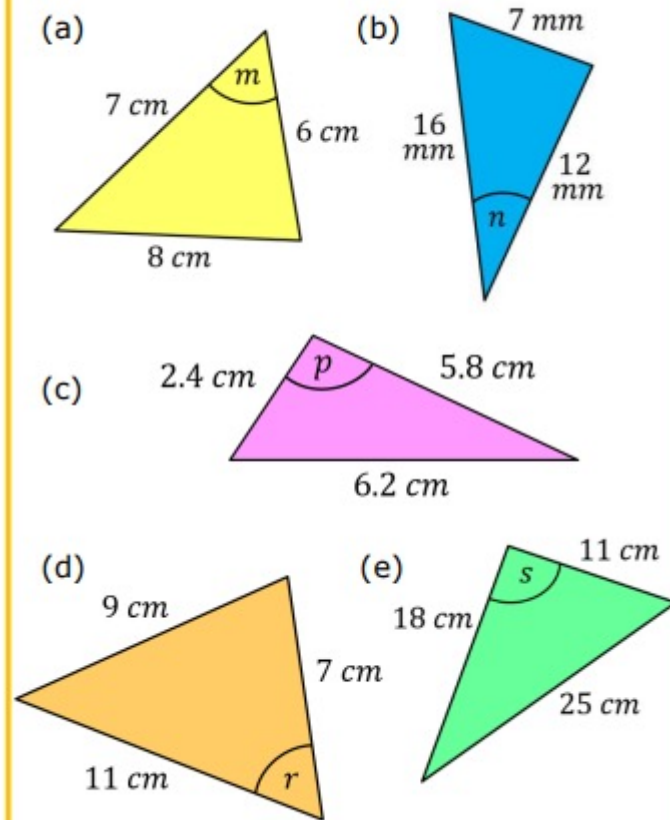
# Fluency Practice

## Cosine Rule

Find the missing length.

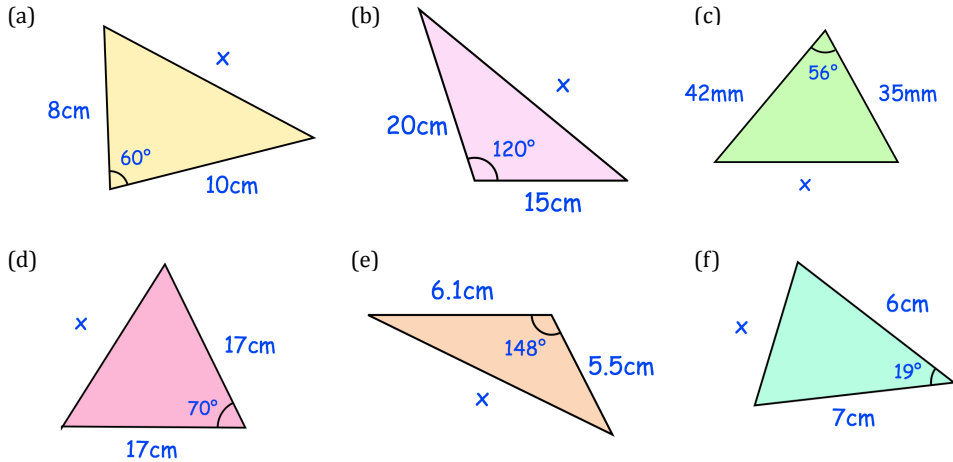


Find the missing angle.



# Fluency Practice

Question 1: Find  $x$  for each of the triangles below.

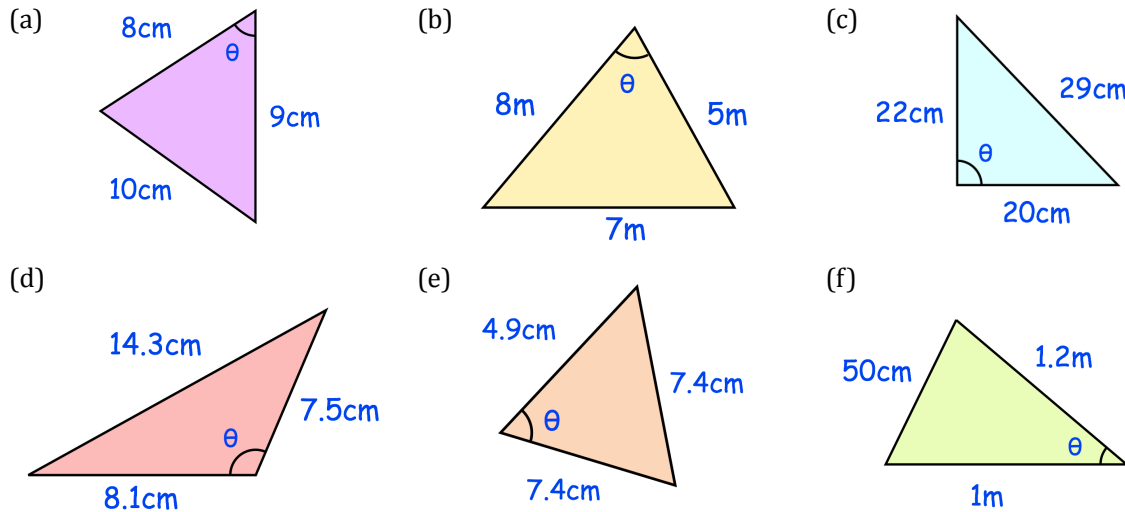


Question 3: In triangle ABC, the side AB = 6cm, the side BC = 8cm and angle ABC =  $100^\circ$ . Find the length of side AC. Give your answer to 1 decimal place.

Question 4: In triangle DEF, the side DE = 30cm, the side DF = 40cm and the side EF = 45cm. Find the size of angle DFE. Give your answer to 3 significant figures.

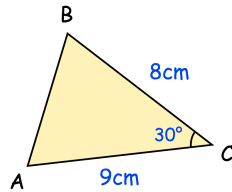
Question 5: In triangle GHI, the side GH = 3cm, the side HI = 18cm and the side GI = 20.5cm. Find the size of angle HGI. Give your answer to 3 significant figures.

Question 2: Find the size of  $\theta$  for each of these triangles.



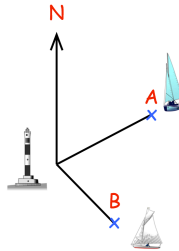
# Extensions

Question 1: Calculate the perimeter of triangle ABC

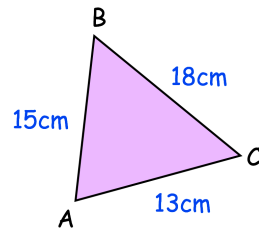


Question 2: Boat A is 16km from a lighthouse on a bearing of 055°  
Boat B is 11km from the same lighthouse on a bearing of 152°

Calculate the distance between the two boats.



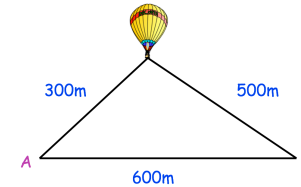
Question 3: Find the size of the smallest angle in this triangle.



Question 4: A hot air balloon is flying above two points, standing on the ground at points A and B, 600m apart.

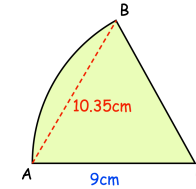
The hot air balloon is 300m from A and 500m from B.

- Work out the angle of elevation from point B
- How high is the hot air balloon from the ground?



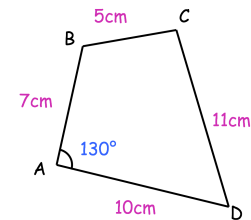
Question 5: Shown is sector OAB.  
O is the centre of the circle with radius 9cm  
A and B are points on the circle.  
The length of the chord AB is 10.35cm

Find the area of sector OAB



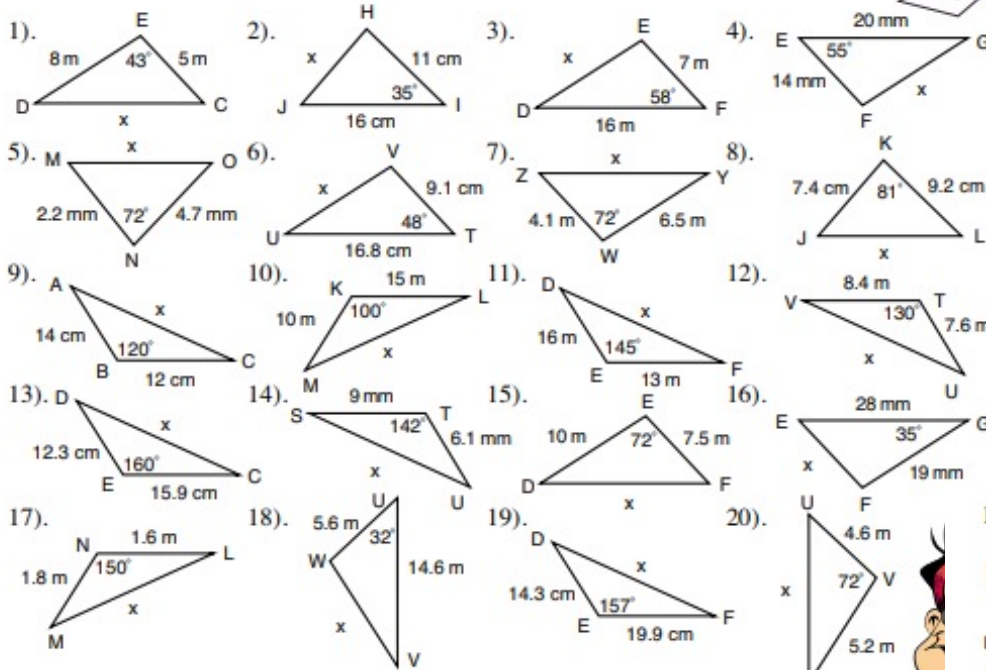
Question 6: ABCD is a quadrilateral.  
AB = 7cm BC = 5cm BC = 11cm AD = 10cm Angle BAD = 130°

Work out the size of angle BCD.

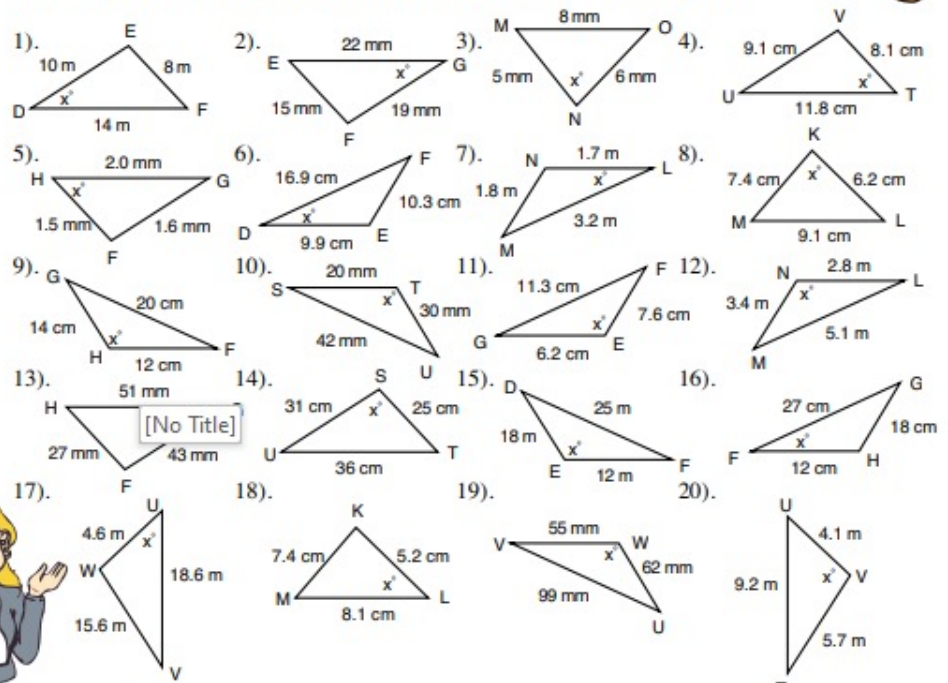


# The Cosine rule.

A). Find the missing length marked x. Give the answer to 1 decimal place.



B). Find the missing angle marked x. Give the answer to 1 decimal place.

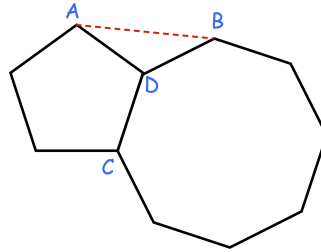


# Extensions

Question 7: A is a vertex of a regular pentagon.  
B is a vertex of a regular octagon.  
C and D are vertices of both polygons.

The perimeter of the octagon is 40cm.

Work out the length AB



Question 8: A clock has two hands.  
A minute hand which is 5.5cm long and an hour hand which is 4cm long.

Find the distance between the tips of the two hands at 7:15am

Question 9: Two ships, A and B, leave a port at 10:30

Ship A travels on a bearing of  $196^\circ$  at a speed of 30km/h.  
Ship B travels on a bearing of  $244^\circ$  at a speed of 24km/h.

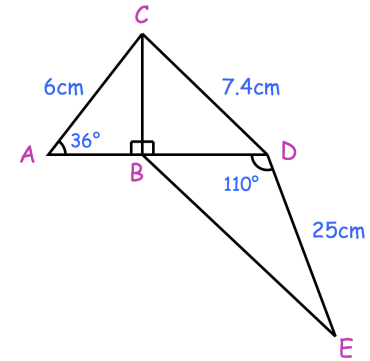
(a) Work out the distance between A and B at 14:00

(b) Work out the bearing of B from A at 14:00

Question 10: In the diagram:

ABD is a straight line.  
AC = 6cm CD = 7.4cm DE = 25cm  
Angle BAC =  $36^\circ$  Angle BDE =  $110^\circ$

Calculate the length of BE

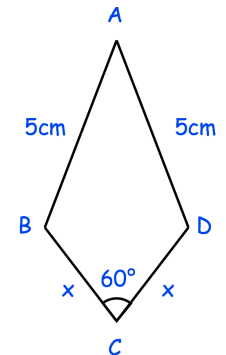


Question 11: The Cosine Rule is  $a^2 = b^2 + c^2 - 2bc\cos A$

Make  $\cos A$  the subject.

Question 12: Shown is kite ABCD

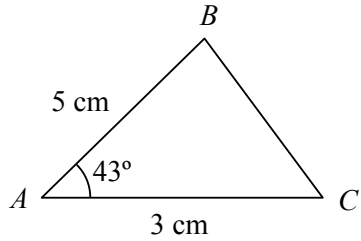
Prove  $\cos BAD = 1 - \frac{x^2}{50}$



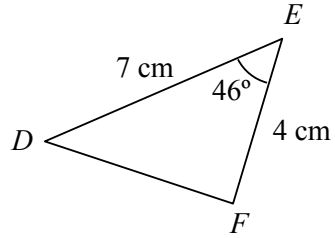


# Fluency Practice

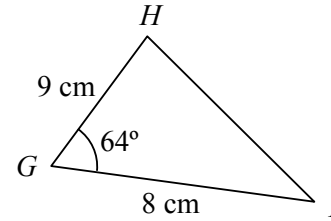
**A1** Find the length of  $BC$



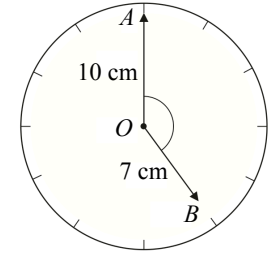
**A2** Find the length of  $DF$



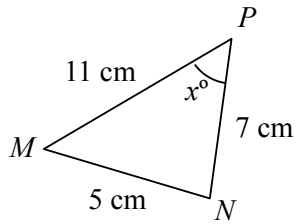
**A3** Find the length of  $HI$



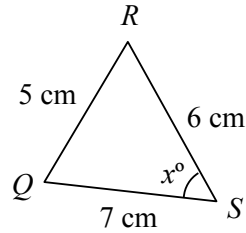
**A4** Find the distance  $AB$



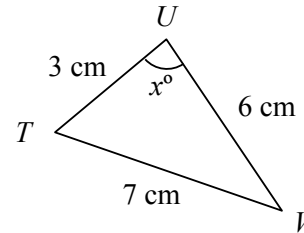
**B1** Find angle  $MPN$



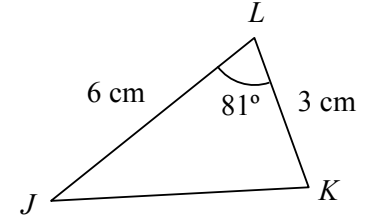
**B2** Find angle  $QSR$



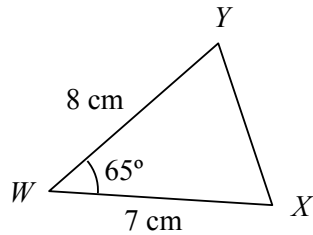
**B3** Find angle  $TUV$



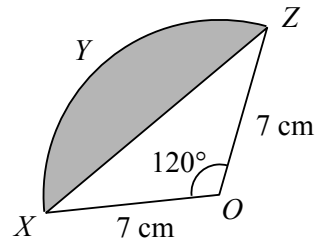
**B4** Find angle  $JKL$



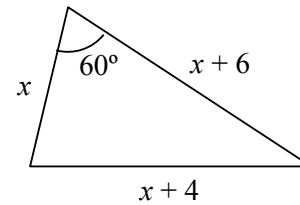
**C1** Find angle  $XYW$



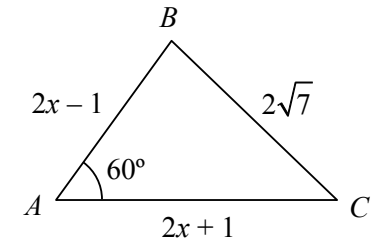
**C2** Find the perimeter of  $XYZ$



**C3** Find  $x$



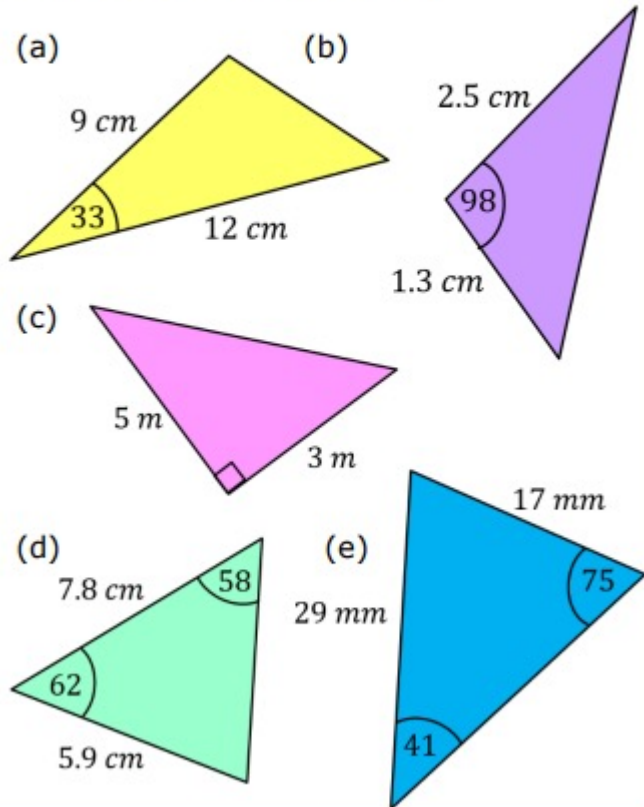
**C4** Find  $x$



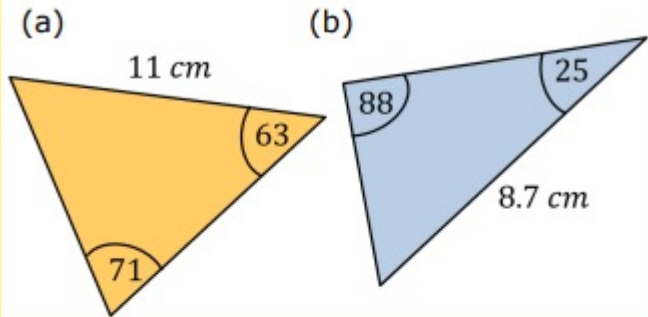
# Fluency Practice

**Area of a Triangle** ( $A = \frac{1}{2}ab \sin C$ )

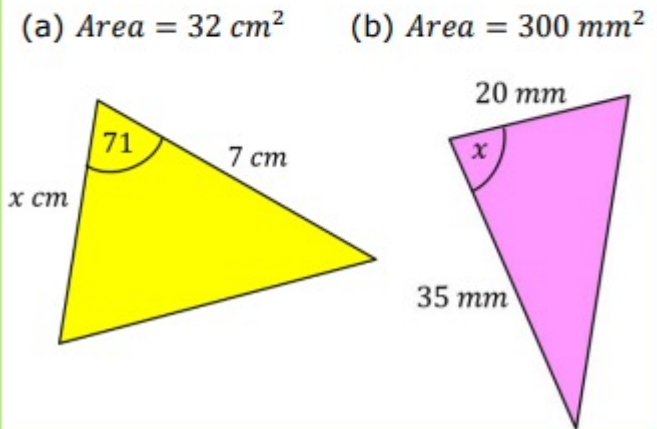
Find the area of the triangle.



Find the area of the triangle.



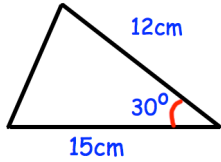
Find the missing length or angle.



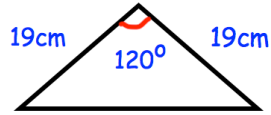
# Fluency Practice

Question 1: Find the area of each of these triangles.

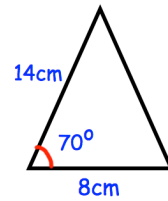
(a)



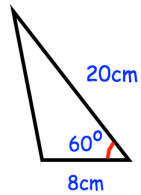
(b)



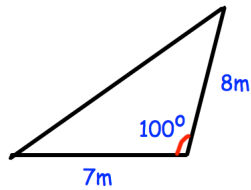
(c)



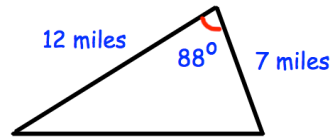
(d)



(e)

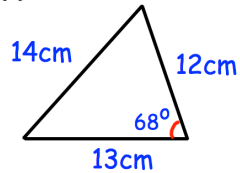


(f)

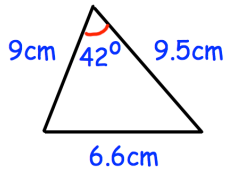


Question 2: Find the area of each of these triangles.

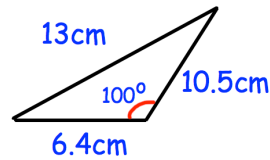
(a)



(b)



(c)



Question 3: Find the area of each of these triangles

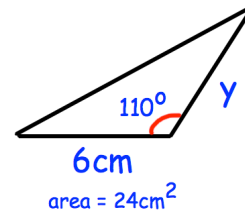
(a) ABC with AB = 10cm, BC = 9cm and angle ABC =  $44^\circ$ .

(b) DEF with EF = 28cm, DF = 34cm and angle DFE =  $81^\circ$ .

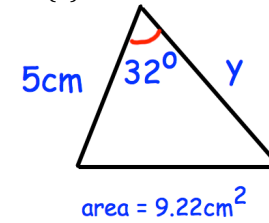
(c) XYZ with YZ = 9mm, XY = 13mm and angle XYZ =  $121^\circ$ .

Question 4: Find the length of the missing side in each of these triangles. Give each answer to one decimal place.

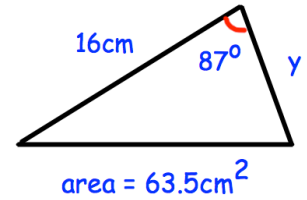
(a)



(b)

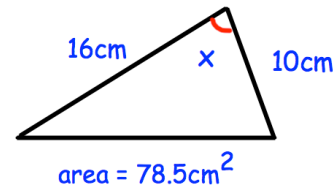


(c)

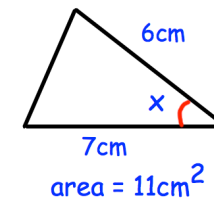


Question 5: Find the size of the missing acute angles below. Give each answer to one decimal place.

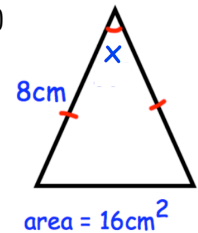
(a)



(b)



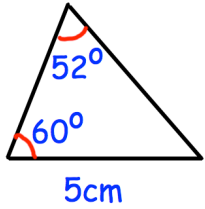
(c)



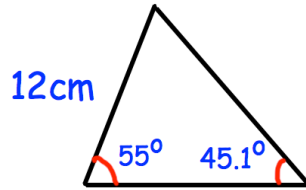
## Extension

Question 1: Find the area of these triangles.

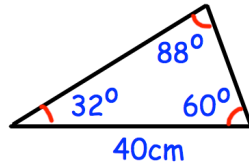
(a)



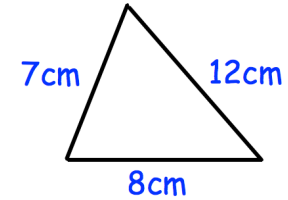
(b)



(c)

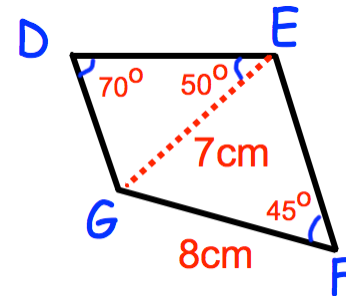


Question 2: Find the area of this triangle.



Question 3: A triangular field has three fences. One is 40m long, another is 50m and the other is 60m.  
Find the area of the field.

Question 4: Find the area of quadrilateral DEFG.

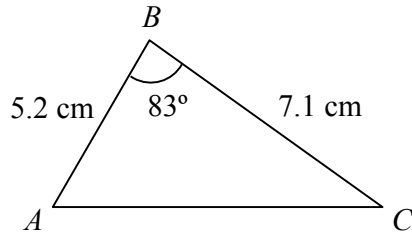


Question 5: Parallelogram ABCD has side lengths  $AB = 5\text{ cm}$  and  $BC = 9\text{ cm}$ .  
Angle  $BCD = 55^\circ$ .  
Find the area of the parallelogram.

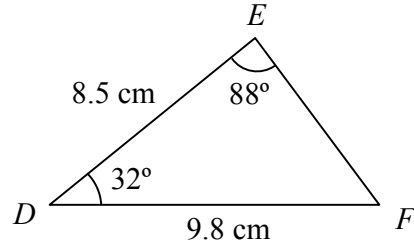
Question 6: Prove the area of any triangle is given by the formula  $A = \frac{1}{2}ab\sin C$ .

# Fluency Practice

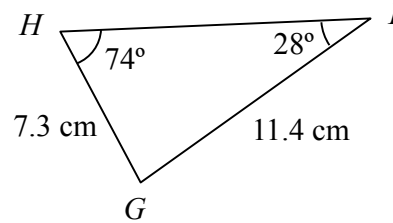
**A1** Find the area of triangle  $ABC$



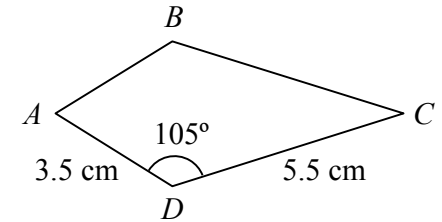
**A2** Find the area of triangle  $DEF$



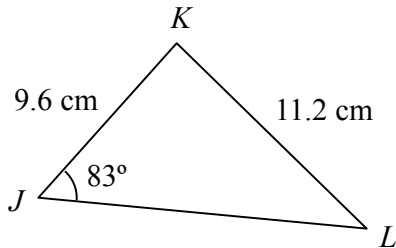
**A3** Find the area of triangle  $GHI$



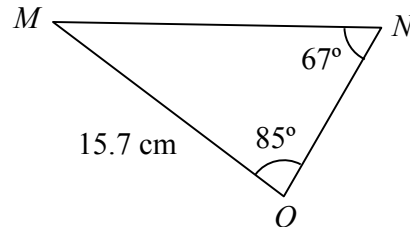
**A4** Find the area of the kite



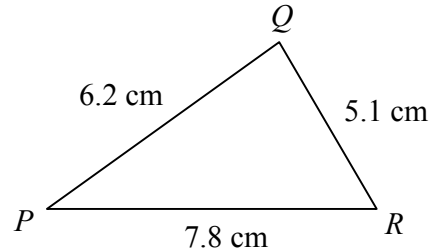
**B1** Find the area of triangle  $JKL$



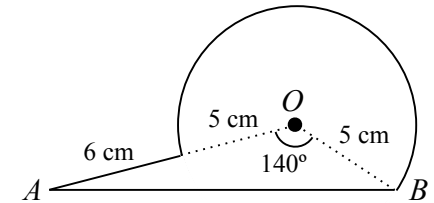
**B2** Find the area of triangle  $MNO$



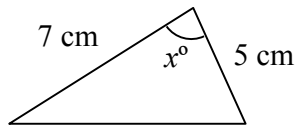
**B3** Find the area of triangle  $PQR$



**B4** Find the area of the shape

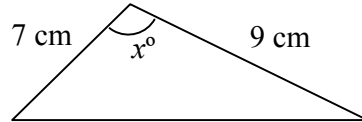


**C1** The area of the triangle is  $16.5 \text{ cm}^2$ .



The angle  $x^\circ$  is **acute**.  
Find the value of  $x$ .

**C2** The area of the triangle is  $20 \text{ cm}^2$ .



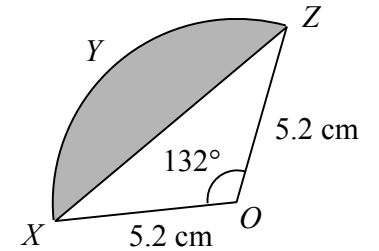
The angle  $x^\circ$  is **obtuse**.  
Find the value of  $x$ .

**C3**

$ABC$  is a triangle.  
 $AB = 11 \text{ cm}$   
 $AC = 7 \text{ cm}$

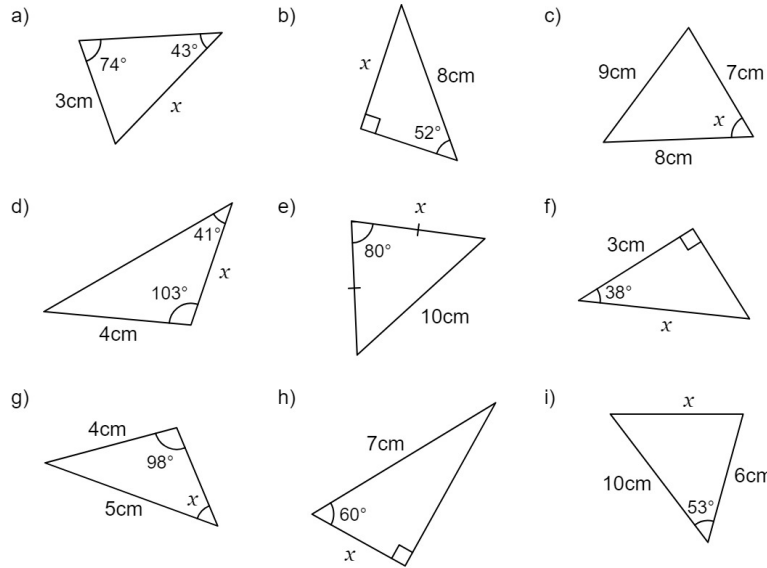
The area of triangle  $ABC$  is  $32 \text{ cm}^2$   
Find, in degrees, the **two** possible sizes of angle  $BAC$ .

**C4** Find the **shaded** area

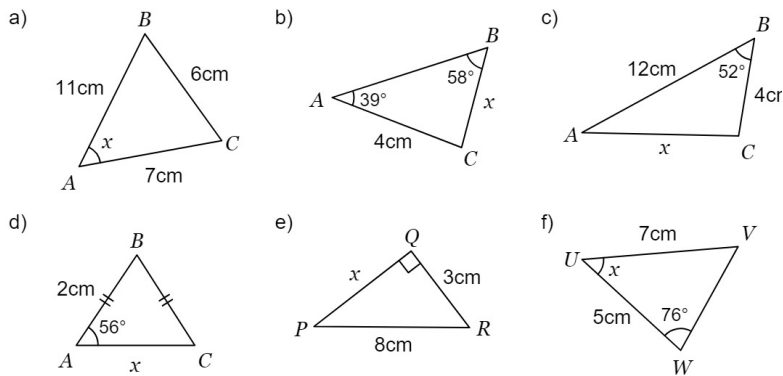


# Fluency Practice

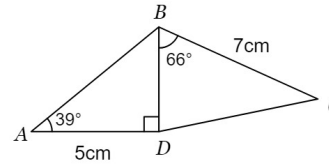
1. Work out the value of  $x$  in each triangle. Give your answers correct to 2 significant figures.



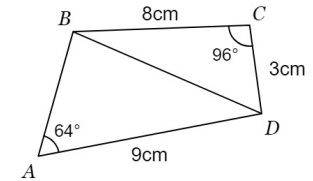
2. Work out the value of  $x$  in each triangle. Give your answers correct to 2 significant figures.



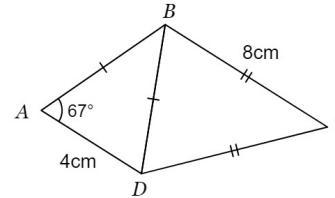
3. Work out the length  $CD$ . Round your answer to the nearest 0.1cm.



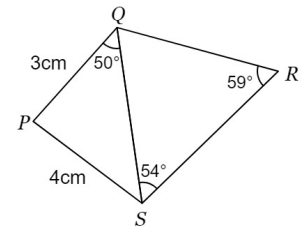
4. Work out  $\hat{ABD}$ . Round your answer to the nearest degree.



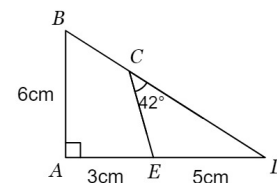
5. Work out the  $\hat{BCD}$  correct to the nearest degree.



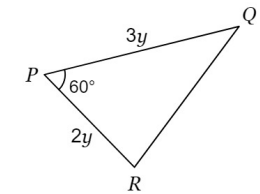
6. Work out  $RS$ . Round your answer to the nearest 0.1cm.



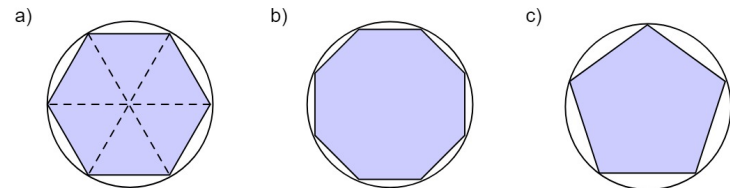
7.  $BCD$  and  $AED$  are straight lines.  $AE = 3\text{cm}$  and  $ED = 5\text{cm}$ . Work out  $CE$  correct to 2 significant figures.



8. Show that  $QR = y\sqrt{7}$ .



9. Each regular polygon is inscribed in a circle with radius 1cm. Work out the area of each polygon, correct to the nearest  $0.01\text{cm}^2$ .

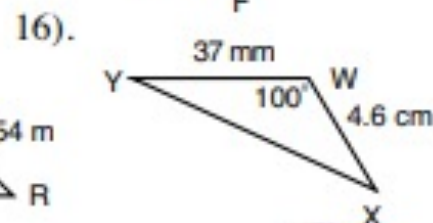
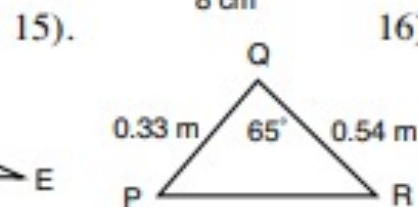
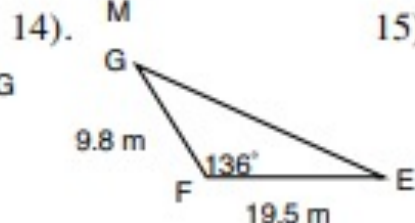
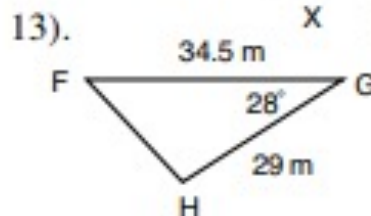
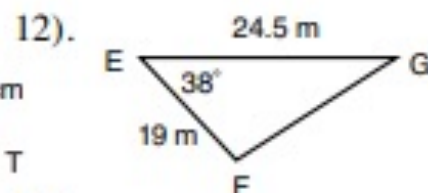
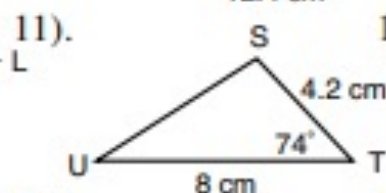
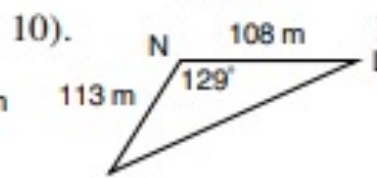
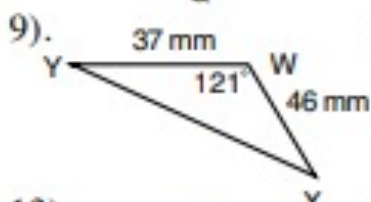
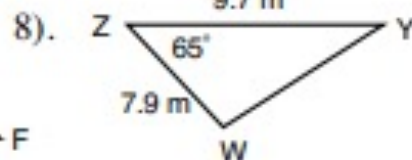
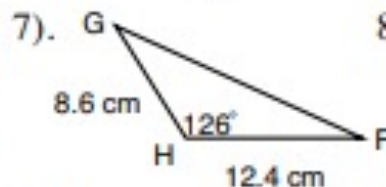
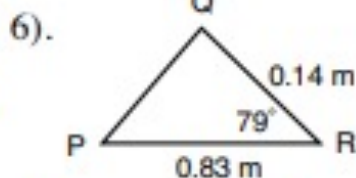
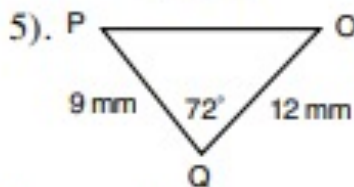
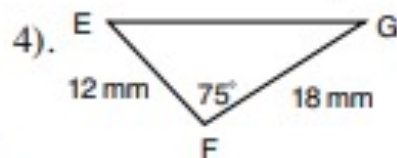
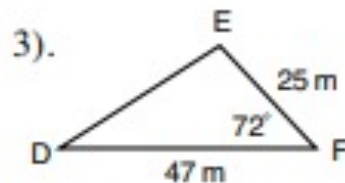
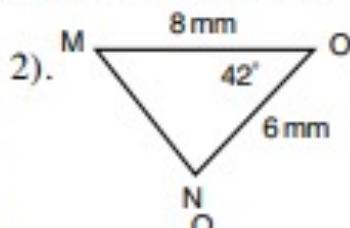
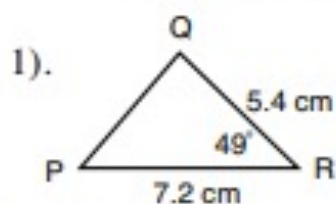




## Areas and the Sine Rule.



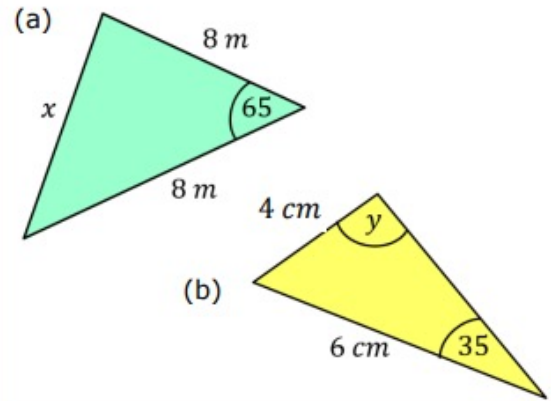
Find the **perimeter** and the **area** of each of these triangles. Leave the answer to an appropriate degree of accuracy.



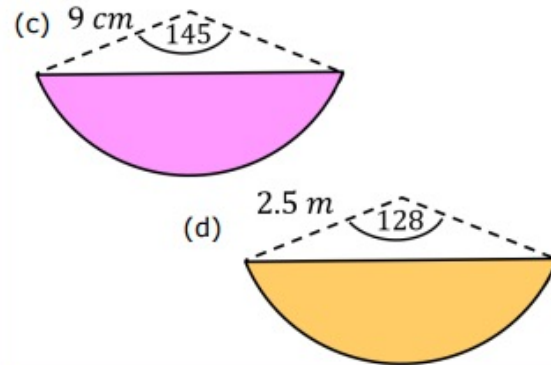
# Fluency Practice

## Mixed Sine and Cosine Rule

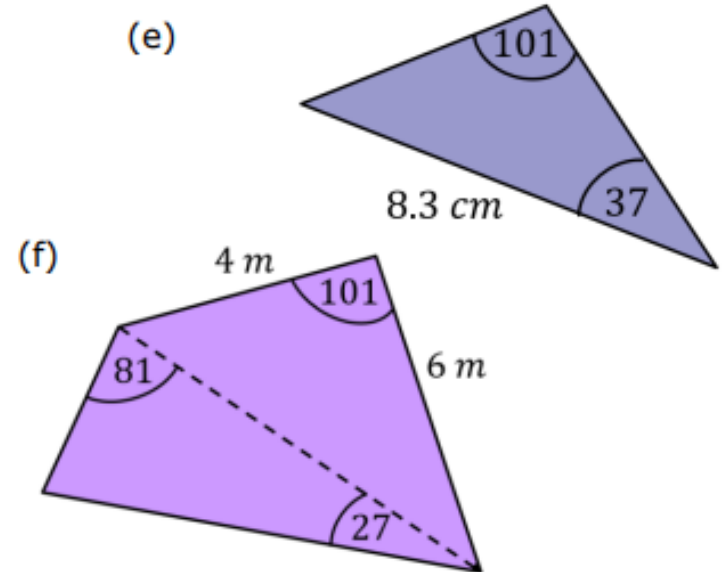
Find the missing length or angle.



Find the area of the segment.



Find the perimeter of these shapes.



(g) The hands of a clock have lengths 8 cm and 10 cm. The time on the clock is 5 o'clock. Find the distance between the tips of the two hands.





## Areas and the Sine Rule.

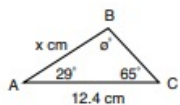


Find the **perimeter** and the **area** of each of these triangles. Leave the answer to an appropriate degree of accuracy.

- 1). 2). 3). 4). 5). 6). 7). 8). 9). 10). 11). 12). 13). 14). 15). 16).

### Examination Style Questions.

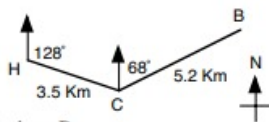
- 1). In  $\triangle ABC$ ,  $AC = 12.4$  cm,  $\angle BAC = 29^\circ$ ,  $\angle ACB = 65^\circ$ .  
The length of  $AB$  is  $x$  cm.  $\angle ABC$  is  $\phi$ .  
a). Write down the value of  $\phi$ .  
b). Hence calculate the value of  $x$ .



- 2). The diagram shows a church clock at 1240 hours.  
The hour hand is 0.8 m long and the minute hand is 1.4 m long.  
a). Calculate the angle the **hour** hand has moved through since 1200.  
b). Calculate the distance, in cm, between the tips of the hands at 1240 hours.



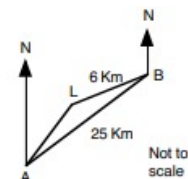
- 3). A helicopter leaves a heliport H and, its measuring instruments show that it flies 3.5 Km on a bearing of  $128^\circ$  to a checkpoint C. It then flies 5.2 Km on a bearing of  $068^\circ$  to its base B.  
a). Show that  $\angle HCB$  is  $120^\circ$ .  
b). Calculate the direct distance from the heliport H to the base B.



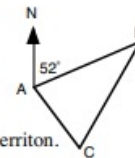
- 4). A ship sails south west from A at  $16 \text{ Km h}^{-1}$ .  
At midnight it is at B when flashes from a lighthouse are seen on a bearing of  $230^\circ$ .  
90 minutes later, flashes from the same lighthouse are seen due west of the ship.  
a). Sketch a diagram showing this information, clearly indicating North.  
b). Calculate the distance between the lighthouse and the ship at 01.30 am.



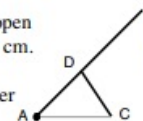
- 5). A port, B, is 25 Km north east of a port, A.  
A lighthouse, L, is 6 Km from B on a bearing of  $255^\circ$  from B.  
a). Calculate the distance AL.  
b). Calculate, to the nearest degree, the bearing of L from A.



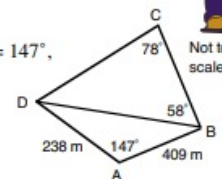
- 6). The sketch shows the flight paths between three airports at Appleton (A), Berriton (B) and Cherriton (C). Berriton is 90 Km from Appleton on a bearing of  $052^\circ$ . Cherriton is 110 Km from Appleton on a bearing of  $165^\circ$ .  
a). i). What is the size of  $\angle BAC$ ?  
ii). Calculate the distance from Berriton to Cherriton.  
iii). By finding the  $\angle ACB$ , find the bearing of Berriton from Cherriton.  
b). Find the area of land enclosed by the flight path.



- 7). The diagram represents a car bonnet, AB, hinged at A and propped open by a stay, CD.  $AC = 29$  cm,  $AB = 58$  cm,  $AD = 21$  cm, and  $CD = 25$  cm.  
a). Calculate the size of  $\angle CAD$ , correct to the nearest degree.  
b). Find the height of B above the level of AC, giving your answer correct to the nearest centimetre.

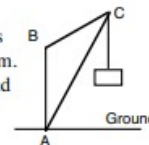


- 8). ABCD is a horizontal field.  $AB = 409$  m,  $DA = 238$  m,  $\angle DAB = 147^\circ$ ,  $\angle BCD = 78^\circ$  and  $\angle DBC = 58^\circ$ .

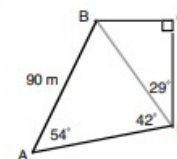


Calculate the area, in hectares, of the field, giving your answer correct to 2 significant figures. ( $10000 \text{ m}^2 = 1$  hectare).

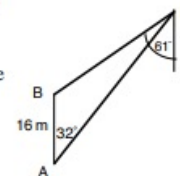
- 9). The diagram shows a crane lifting a load. The three girders forming the crane are  $AB = 9$  m,  $BC = 12$  m and  $AC = 16$  m. If AB is vertical, calculate the angle between the jib AC and the vertical. Give your answer to the nearest degree.



- 10). The diagram shows a field in the form of a quadrilateral ABCD.  $AB = 90$  metres,  $\angle BAD = 54^\circ$ ,  $\angle ADB = 42^\circ$ ,  $\angle BDC = 29^\circ$  and  $\angle BCD = 90^\circ$ .  
a). Find the lengths of the other three sides of the field.  
b). Find the area of the field to 2 significant figures.



- 11). A batsman strikes a ball from A directly towards a fielder F in a direction making an angle  $32^\circ$  with the line of the wickets. The fielder picks up the ball and throws it at an angle of  $61^\circ$  with the line of the wickets, to the other wicket B. If the distance between A and B is 16 metres, how far is the fielder  
a). from A,  
b). from B.

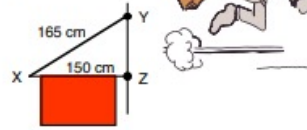




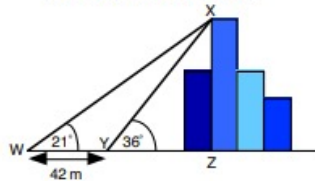
## Trigonometry, Sine and Cosine Rule. Exam Style Questions 1.



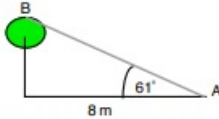
- 1). Two metal bars are hinged at X. Y and Z are attached to a vertical pole. XZ is horizontal.
- Calculate YZ (to 2 s.f.).
  - Find  $\angle YXZ$  to the nearest degree.
  - Find  $\angle XYZ$  to the nearest degree.
  - XZ is examined and it is found **not** to be horizontal, because  $YZ = 74$  cm. Now calculate  $\angle XZY$  to the nearest degree.



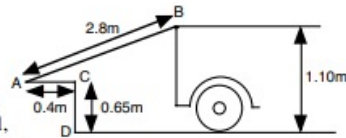
- 2). a). Find the height of the tree in the diagram, and then the distance AB.



- b). Find the distance XY in this diagram showing the height of a building. Find the height of the building XZ.



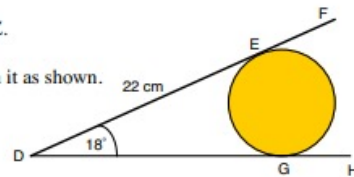
- 3). A van unloads at a factory. Its trailer door is opened onto a loading bay as shown. From the diagram



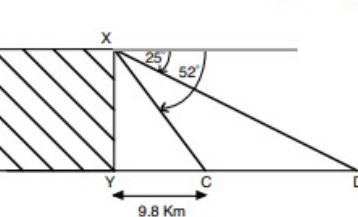
- find the vertical distance between C and B,
- find the angle AB makes with the horizontal,
- find the horizontal distance between B and the vertical wall, CD, of the factory loading bay.

- 4). At 5.00 pm a ship is observed at X, 9 Km due south of a lighthouse L. The ship travels at a constant speed on a bearing of  $054^\circ$  and reaches Y, a point due east of the lighthouse, at 5.50 pm. Z is a point on the ship's path which is nearest to L.

- Draw a sketch showing X, L, Y and Z. Calculate LZ to 2 s.f.
- Find XZ.
- Find the speed of the ship.
- Find the time at which the ship reaches Z.

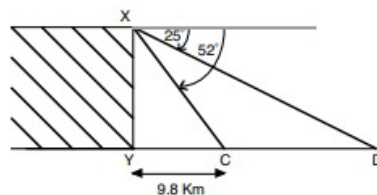


- 5). A tennis ball is on a table, with a ruler resting on it as shown.  $DE = 22$  cm and  $\angle EDG = 18^\circ$ . Calculate the radius of the tennis ball.

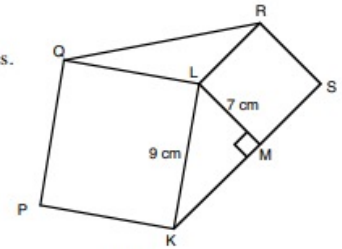


- 6). A woman stands on a cliff at X. She looks out to sea and can see a power boat at C. The angle of depression from where she stands to the boat is  $52^\circ$ . Calculate

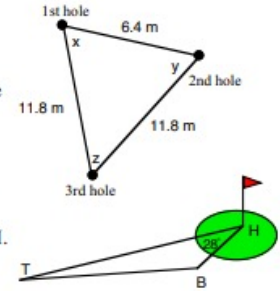
- the height of the cliff,
  - CX to 3 s.f.
- C sails in a straight line away from Y to D. It takes 16 minutes to travel this distance. The angle of depression from X is now  $25^\circ$ .
- Calculate the average speed in Km/h.



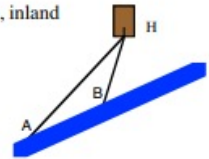
- 7). KLM is a triangle. KLQP and MLRS are squares. Calculate
- $\angle KLM$  to 1 d.p.,
  - $\angle QLR$  to 1 d.p.,
  - Explain why the area of  $\triangle KLM =$  the area of  $\triangle QLR$ .
  - Find the area of the pentagon KSRQP.
  - Calculate the length of QR.



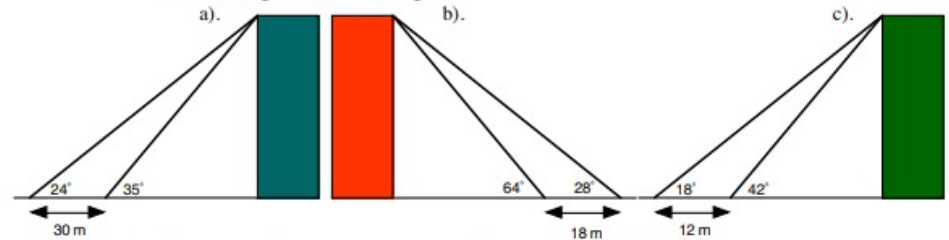
- 8). a). A positions of the 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> holes on a putting green are shown.
- Calculate the angles x, y and z.
  - A path is laid from the first to second hole in the shape of an arc of a circle, with the centre the third hole. How long is the path?
- b). On a pitch and putt range the distance from the Tee, T, to the hole, H is 108 m. Alan hits his ball at an angle of  $12^\circ$  to the line TH. His ball now lies at B and  $\angle THB = 28^\circ$ . Find how far his ball is now from the hole.



- 9). A man walks down the edge of a straight river and notices a hut, H, inland on his side of the river. At point A he notices that the angle AH made with the river is  $19^\circ$ . After walking another 350 m to point B, he notices that the angle BH makes with the river is  $42^\circ$ .
- Find the distance BH.
  - Find the shortest distance from the hut to the river edge.

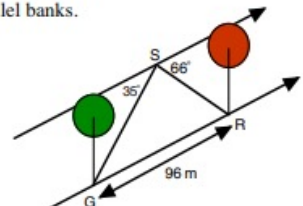


- 10). A surveyor has to find the height of three buildings. Her measurements are shown in each diagram. (Diagrams not to scale). Find the height of each building.



- 11). The surveyor then moves on to a straight river with parallel banks. On the opposite side of the river are a green and red tree. She knows that they are exactly 96 m apart. She stands at S and takes the angle readings shown.

- Find the distance
  - SR,
  - SG.
- Calculate the width of the river.







## Trigonometry, Sine and Cosine Rule.

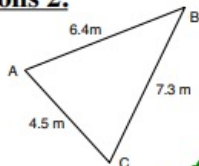
### Exam Style Questions 2.



- 1). A gardener pegs out a triangular flower bed with the dimensions shown.

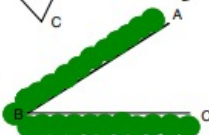
Calculate

- $\angle ABC$ ,
- the area of the flower bed.



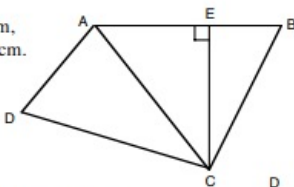
- 2). The diagram represents the corner of a farmer's field. AB and BC are straight hedges. AB = 53 m and BC = 72 m.

- The farmer has a temporary straight fence that joins A to C. The area of this enclosure is 1090 m<sup>2</sup>. Find  $\angle ABC$ .
- The farmer wants to make a different enclosure by erecting a straight fence from A to the nearest point on BC.
  - What is the shortest length of fence required?
  - How far along BC from B will this meet?
  - What is the area of the new enclosure?



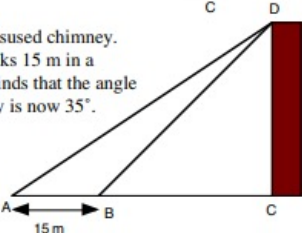
- 3). In the diagram ABCD is a quadrilateral. CD = 9.3 cm,  $\angle EAC = 50^\circ$ ,  $\angle ECD = 78^\circ$ , AE = 6.4 cm, BC = 8.5 cm. Calculate

- AC,
- CE,
- $\angle ECB$ ,
- the area of the quadrilateral ABCD.



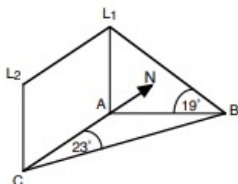
- 4). a). A surveyor has to calculate the height of a disused chimney. The angle of elevation from A is  $28^\circ$ . He walks 15 m in a straight line towards the chimney, to B. He finds that the angle of elevation from B to the top of the chimney is now  $35^\circ$ . What is the height of the chimney?

- b). A second surveyor uses the same technique to calculate the height of a church spire. The angle of elevation from a point to the top of the spire is  $18^\circ$ . She walks 24 m in a straight line towards the spire and finds that the angle of elevation this point to the top of the spire is now  $25^\circ$ . What is the height of the church spire?



- 5). C is due south of A. B is north east of A and on a heading of  $023^\circ$  from C. AB is 1450 m. A plane flies south at a constant height and passes over A at L<sub>1</sub>, its angle of elevation from B is  $19^\circ$ . Calculate

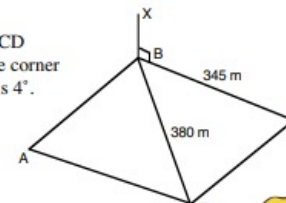
- its height above A,
- the angle of elevation of the aircraft from B when at L<sub>2</sub>, to the nearest degree,
- the area of the triangle ABC.



- 6). The diagram represents 2 triangular fields ABD and BCD in a horizontal plane. A vertical mast, BX, stands in the corner of the fields at B. The angle of elevation of X from C is  $4^\circ$ . BC = 345 m, DB = 380 m,  $\angle CBD = 42^\circ$ ,  $\angle ABD = 73^\circ$  and  $\angle DAB = 58^\circ$ .

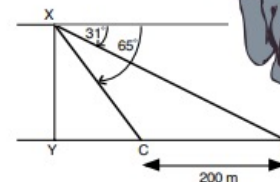
Calculate

- the height of the mast,
- the length of CD,
- the length AB,
- the total area of the 2 fields in hectares (1 hectare = 10000 m<sup>2</sup>).



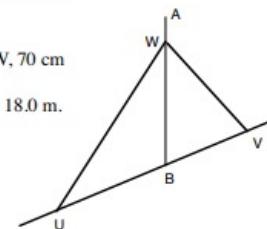
- 7). A girl stands on a cliff at X. She looks out to sea and can see a dinghy at D. The angle of depression from where she stands to the boat is  $31^\circ$ . Calculate

- CX,
  - the height of the cliff.
- D sails in a straight line towards Y to C. It takes 2 minutes to travel this distance. The angle of depression from X is now  $65^\circ$ .
- Calculate the average speed in Km/h.
  - The dinghy carries on at this speed until it reaches Y. How many seconds does it take to travel from C to Y?



- 8). A pylon, AB, is erected on the flat side of a hill. The pylon is 8.4 m. It is secured by wires attached at W, 70 cm below the top of the pylon A. UV is a straight line. The length of wire WU is 15.2 m. UB = 10.2 m, UV = 18.0 m. Calculate

- $\angle UBV$ ,
- $\angle AUB$ ,
- the length of the wire WV,
- the area of triangle WUV.



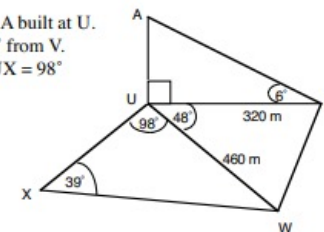
- 9). A surveyor marks out the boundary of a new triangular field. From a start point, A, he marks out a line on a bearing of  $042^\circ$  for 700m to B. At B he then continues on a bearing of  $152^\circ$  for 1400m to C. From C he travels back to A.

- Calculate the total perimeter of the field.
- To mark out the boundary AC on what bearing did he walk from C to A?
- What is the area of the field?

- 10). Two fields UVW and UWX have a vertical mast A built at U. The angle of elevation to the top of the mast is  $6^\circ$  from V. UV = 320 m, UW = 460 m,  $\angle VUW = 48^\circ$ ,  $\angle WUX = 98^\circ$  and  $\angle UXW = 39^\circ$ .

Calculate

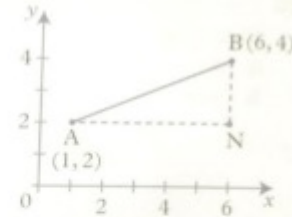
- the height of the mast,
- the length UX,
- the area of the field UVWX in hectares (1 hectare = 10000 m<sup>2</sup>).



# Extension

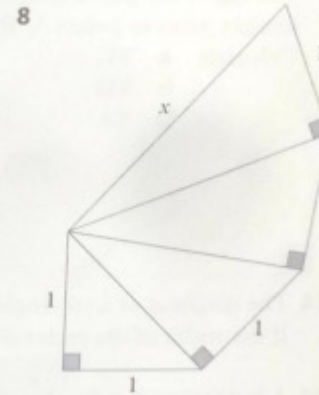
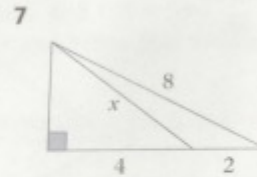
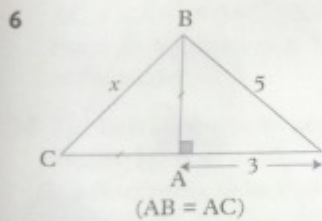
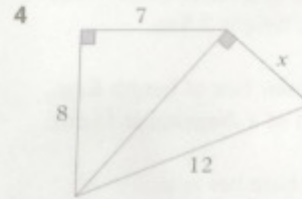
## Exercise 11 3

- 1 In the diagram A is (1, 2) and B is (6, 4).  
Work out the length AB. (First find the lengths of AN and BN.)



- 2 On square grid paper plot P(1, 3), Q(6, 0), R(6, 6). Find the lengths of the sides of triangle PQR. Is the triangle isosceles?

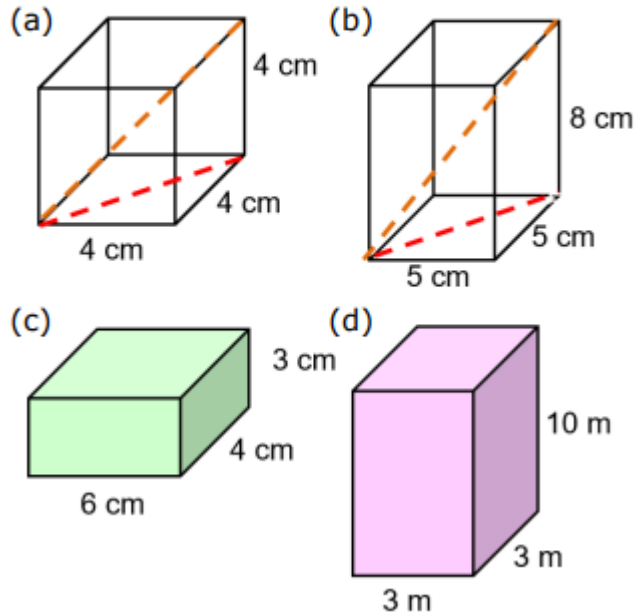
In questions 3 to 8, find  $x$ .



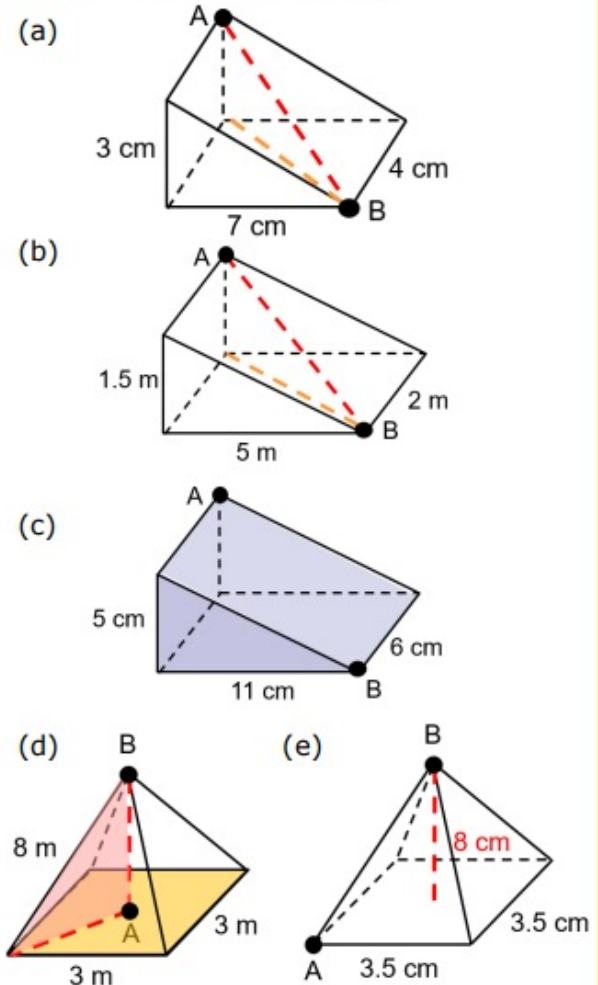
# Fluency Practice

## Pythagoras in 3D

Find the length of the diagonal to 1dp.



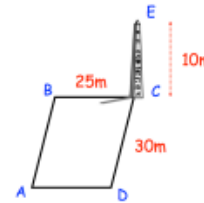
Find the distance AB to 1dp.



# Fluency Practice

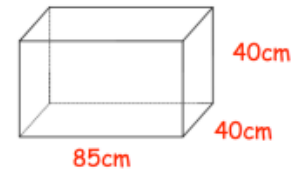
## Apply

Question 1: A rectangular field is 30m long and 25m wide. An obelisk is situated in the corner of the field.



Calculate the length AE.

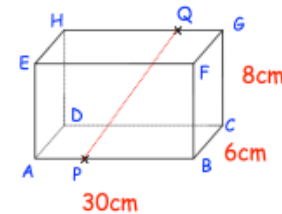
Question 2: Can a 1m rod fit inside this box?  
Explain your answer.



Question 3: Calculate the volume of this cone.



Question 4: ABCDEFGH is a cuboid  
P is a point on AB, such that AP:PB is 1:2  
Q is a point on GH, such that GQ:QH is 2:3  
Calculate the distance PQ.



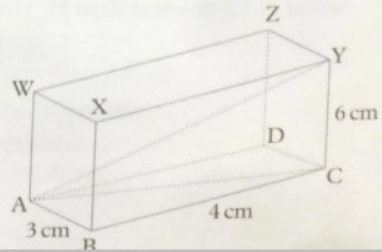
# Fluency Practice

9 The diagram shows a rectangular block.

Calculate

a AC

b AY.

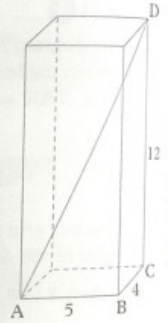


10 The diagram shows a cuboid 5 cm by 4 cm by 12 cm.

Calculate

a AC

b AD.



11 Find the length of a diagonal of a rectangular room of length 5 m, width 3 m and height 2.5 m.

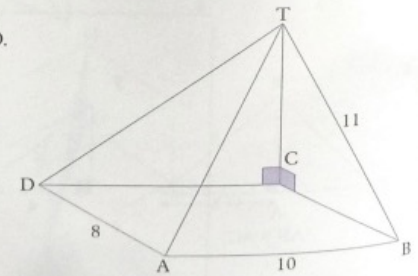
12 Find the height of a rectangular box of length 8 cm, width 6 cm where the length of a diagonal is 11 cm.

13 TC is a vertical pole whose base lies at a corner of the horizontal rectangle ABCD. The top of the pole T is connected by straight wires to points A, B and D.

Calculate a TC

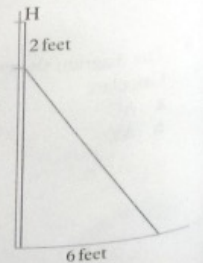
b TD

\*c TA.



14 The diagonal of a rectangle exceeds the length by 2 cm. If the width of the rectangle is 10 cm, find the length.

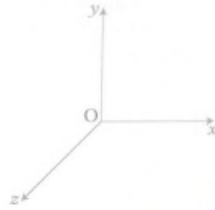
15 A ladder reaches H when held vertically against a wall. When the base is 6 feet from the wall, the top of the ladder is 2 feet lower than H. How long is the ladder?



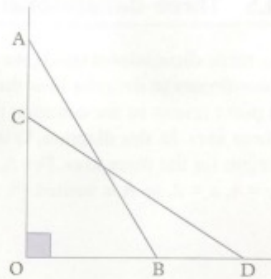


# Fluency Practice

- 16 a Sketch axes in three dimensions as shown in the diagram.  
 b Find the length of the line from  $O(0, 0, 0)$  to  $A(3, 4, 7)$ .  
 c Find the length of the line from  $O(0, 0, 0)$  to  $B(1, 10, 13)$ .

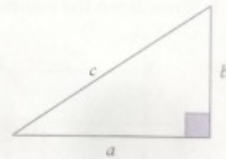


- 17 The diagram represents the starting position (AB) and the finishing position (CD) of a ladder as it slips. The ladder is leaning against a vertical wall.



$AC = x$ ,  $OC = 4AC$ ,  $BD = 2AC$  and  $OB = 5m$ .  
 Form an equation in  $x$ , find  $x$  and hence find the length of the ladder.

- 18 The best known right-angled triangle is the 3, 4, 5 triangle [ $3^2 + 4^2 = 5^2$ ].  
 It is interesting to look at other right-angled triangles where all the sides are whole numbers.



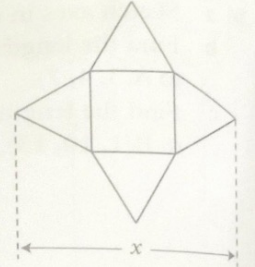
- a i Find  $c$  if  $a = 5$ ,  $b = 12$ .  
 ii Find  $c$  if  $a = 7$ ,  $b = 24$ .  
 iii Find  $a$  if  $c = 41$ ,  $b = 40$ .

b Write the results in a table.

a	b	c
3	4	5
5	12	?
7	24	?
?	40	41

- c Look at the sequences in the 'a' column and in the 'b' column. Then write the connection between  $b$  and  $c$  for each triangle.  
 d Predict the next three sets of values of  $a$ ,  $b$ ,  $c$ . Check to see if they really do form right-angled triangles.

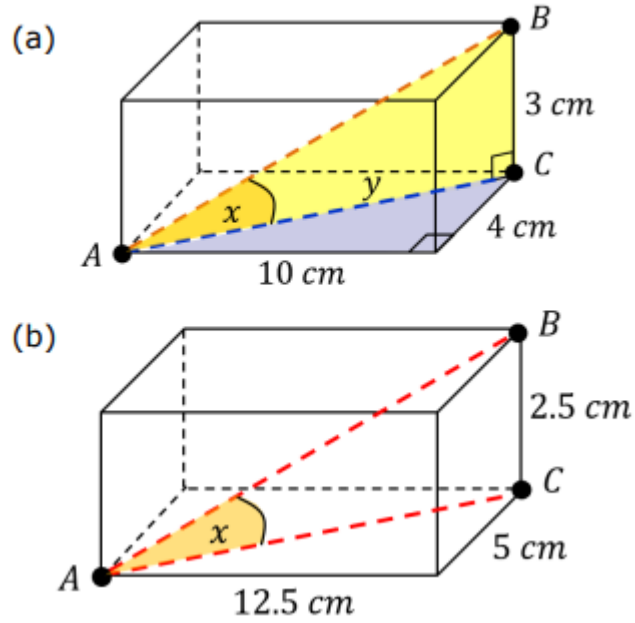
- 19 The diagram shows the net of a square-based pyramid. The base is  $8\text{ cm} \times 8\text{ cm}$  and the vertical height is  $10\text{ cm}$ . Find  $x$ .



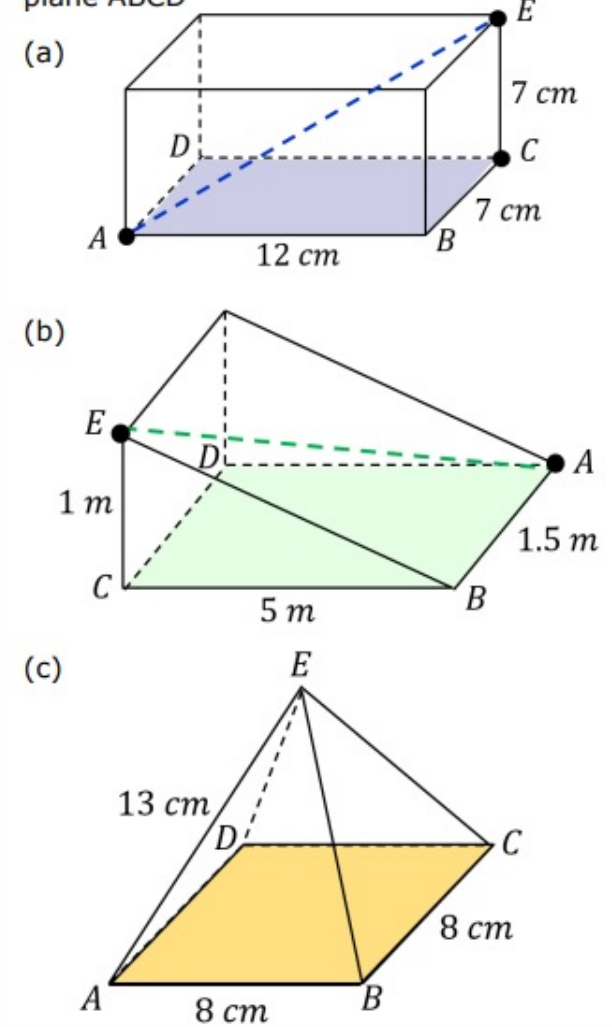


# Fluency Practice

Find the angle  $\widehat{BAC}$



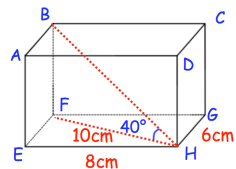
Find the angle between the line EA and the plane ABCD



# Fluency Practice

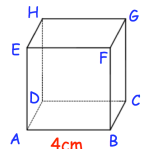
Question 1: Here is a cuboid.

$EH = 8\text{cm}$ ,  $GH = 6\text{cm}$ ,  $FH = 10\text{cm}$  and  $\angle FHB = 40^\circ$



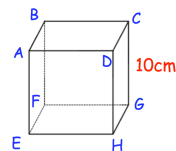
- Work out the length BF
- Work out the length BH
- Work out the size of angle FBH

Question 2: Here is a cube with side length 4cm.



- Calculate the length AC
- Calculate the size of angle CAG

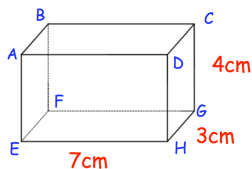
Question 3: Shown is a cube with side length 10cm



- Calculate the length of FH.
- Calculate the size of angle DFH

Question 4: Shown is a cuboid.

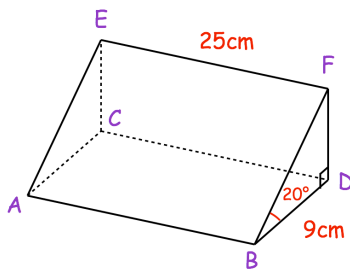
- Work out the size of angle GEH
- Work out the size of angle CHG
- Work out the length EG
- Work out the size of angle CEG



Question 5: ABCDEF is a triangular prism

$BD = 9\text{cm}$ ,  $EF = 25\text{cm}$ ,  $\angle BDF = 90^\circ$  and  $\angle DBF = 20^\circ$

- Work out the length DF
- Work out the length BF
- Work out the length BC
- Work out the size of angle CBE

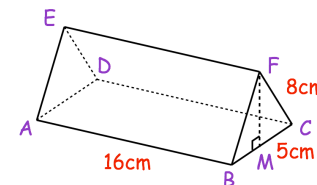


Question 6: Shown is a triangular prism.

M is the midpoint of BC.  
 $AB = 16\text{cm}$ ,  $CM = 5\text{cm}$ ,  $CF = 8\text{cm}$  and  $\angle BMF = 90^\circ$

Calculate

- Length FM
- Length BE
- Length BD
- Angle CBD
- Angle ABE
- Angle DMF

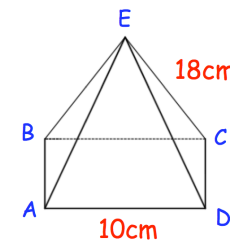


Question 7: Shown is a square-based pyramid.

The apex E is directly over the centre of the base.

$AD = 10\text{cm}$  and  $CE = 18\text{cm}$

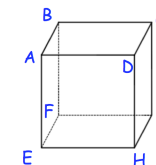
- Work out the length of AC
- Calculate angle CAE
- Work out the height of the pyramid.



Question 8: Shown is a cuboid.

$FG = 5.5\text{cm}$   $DH = 6.2\text{cm}$   $\text{Angle FHG} = 47^\circ$

Calculate the angle between DF and the plane EFGH.

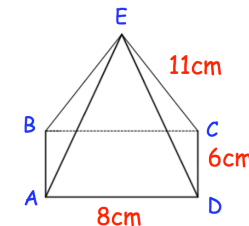


Question 9: Shown below is a rectangular-based pyramid.

The apex E is directly over the centre of the base.

$AD = 8\text{cm}$   $CD = 6\text{cm}$   $CE = 11\text{cm}$

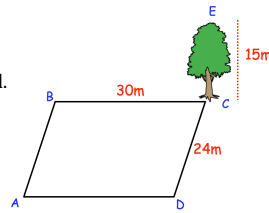
- Calculate the height of the pyramid.
- Calculate the angle between face ABE and the base ABCD.



# Extensions

## Apply

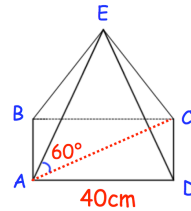
Question 1: A tree is located in the corner of a rectangular field. The field is 30 metres long and 24 metres wide. The tree is 15 metres tall.



Calculate angle CAE.

Question 2: Here is a square-based pyramid. The apex E is directly over the centre of the base.

Calculate the volume of the pyramid.



Question 3: The diagram shows a cuboid and a pyramid. The apex of the pyramid, I, is directly above the centre, M, of ABCD.

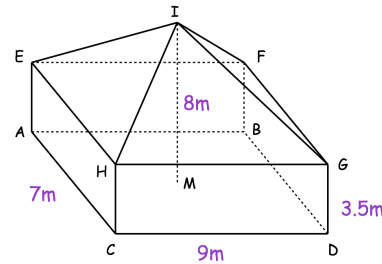
AC = 7m    CD = 9m    DG = 3.5m  
IM = 8m

(a) Calculate the angle between EI and the plane EFGH.

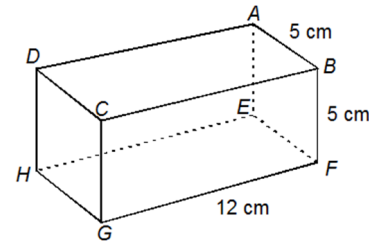
(b) Calculate the angle between AI and plane ABCD.

(c) Calculate the angle between the planes FGI and EFGH.

(d) Calculate the angle between the planes EHI and ACEH.

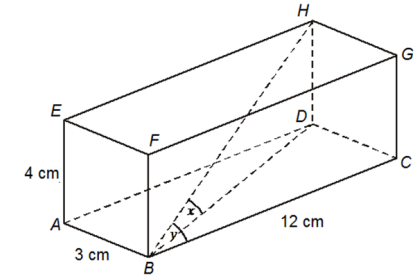


(1)



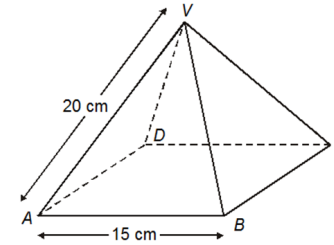
calculate angle  $DFH$  for a cuboid with sides of 5 cm, 5 cm and 12 cm

(2)



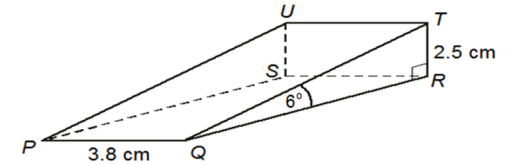
what are the sizes of the angles marked  $x$  (angle  $HBD$ ) and  $y$  (angle  $HBC$ ) for a cuboid with sides of 3 cm, 4 cm and 12 cm?

(3)



for the right, square-based pyramid shown, calculate the angle between the edge  $VA$  and the base,  $ABCD$

(4)



for the door wedge shape (triangular prism) calculate the length of the diagonal  $PT$



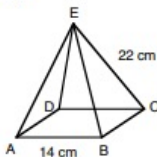
## Trigonometry in Three dimensions.



- 1). The base of a right pyramid, vertex E, is a square of side 14 cm. The length of a slant edge of the pyramid is 22 cm.

Calculate

- the length of a diagonal on the base,
  - the height of the pyramid,
  - the volume of the pyramid,
  - the angle that the slant edge EC makes with the base of the pyramid.
- X is the midpoint of BC.
- Find the length EX.
  - Calculate the angle that EX makes with the base of the pyramid.

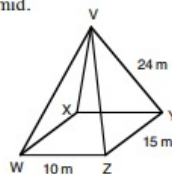


- 2). The base of a right pyramid, vertex V, is a rectangle of sides 10 m and 15 m as shown in the diagram.

The length of a slant edge of the pyramid is 24 cm.

Calculate

- the height of the pyramid,
  - the volume of the pyramid,
  - the angle that the slant edge VY makes with the base of the pyramid.
- A is the midpoint of WZ.
- Find the length AV.
  - Calculate the angle that AV makes with the base of the pyramid.
- B is the midpoint of YZ.
- Find the length VB.
  - Calculate the angle that VB makes with the base of the pyramid.

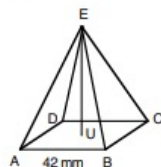


- 3). The base of a right pyramid, vertex E, is a square of side 42 mm. The height of the pyramid UE is 68 mm.

V is the midpoint of BC.

Calculate

- the volume of the pyramid,
  - the distance EV,
  - $\angle EVU$ ,
  - the length of one of the slant edges,
  - the angle that the slant edge makes with the base of the pyramid.
- f). The pyramid is made of solid metal. It is melted down and recast as a cube. Calculate the side length of the cube.



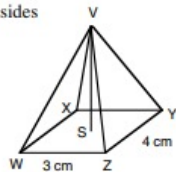
- 4). The base of a right pyramid, vertex V, is a rectangle of sides 3 cm and 4 cm as shown in the diagram.

The height of the pyramid SV is 6 cm.

A is the midpoint of WZ. B is the midpoint of YZ.

Calculate

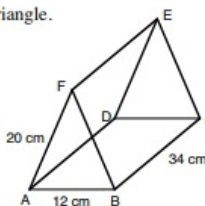
- the volume of the pyramid,
  - the distance VA,
  - the distance VB,
  - $\angle VAS$ ,
  - $\angle VBS$ ,
  - the length of the slant edge VZ,
  - $\angle VZS$ .
- h). The pyramid is made of solid metal. It is melted down and recast as a sphere. Calculate the radius of the sphere.



- 5). ABCDEF is a triangular prism. The triangle ABF is an isosceles triangle. X is the midpoint of AB.

Calculate

- the height of  $\triangle ABF$ ,
- the volume of the prism,
- the angle that the face FECB makes with the base ABCD.
- the length XC,
- Calculate the angle between FC and the base ABCD.



- 6). A box has dimensions 10 cm by 14 cm by 32 cm.

A length of straight wire is to be placed in the box.

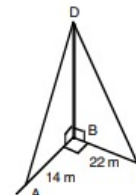
Calculate the largest length of wire that the box can hold.

- 7). A vertical mast, BD, is held in place by a number of cables.

Two are shown on the diagram.  $\angle BAD = 54^\circ$ .

Calculate

- the height of the mast,
- the length AD,
- the angle DC makes with the ground,
- the distance between A and C,
- $\angle BCA$ .



- 8). A room is the shape of a cuboid. It has dimensions 5.2 m by 4.1 m x 2.6 m high. What is the diagonal distance, top to bottom across the room ?

- 9). A, B and C represent 3 points on horizontal ground.

A is due South of B. The point C is 400 m due East of B and on a bearing of  $028^\circ$  from A.

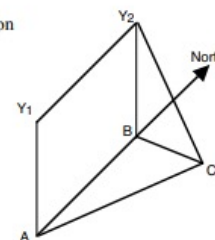
A helicopter flies due South at constant height and speed, passing over B at  $Y_2$  and heading towards  $Y_1$ .

$Y_1$  is directly over A.

The helicopter flies at a height of 600 m above the ground.

Calculate

- the distance  $CY_2$ ,
- the angle of elevation of the helicopter at  $Y_2$  from C,
- the distance AB,
- the angle of elevation of the helicopter at  $Y_2$  from A,
- the distance AC,
- the angle of elevation of the helicopter at  $Y_1$  from C,
- the helicopter takes 20 s to pass from  $Y_2$  to  $Y_1$ . At what speed is the helicopter travelling in Km/h ?

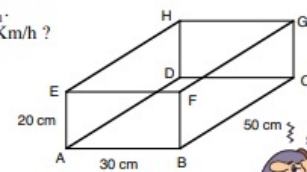


- 10). ABCDEFGH is a cuboid with measurements shown in the diagram.

X is the midpoint of AB, Y is the midpoint of BC and Z is the midpoint of FB.

Calculate

- |                     |                     |                     |
|---------------------|---------------------|---------------------|
| i). AF,             | ii). ED,            | iii). AC,           |
| iv). $\angle FXB$ , | v). $\angle GYC$ ,  | vi). $\angle EZF$ , |
| b). i). GX,         | ii). $\angle GXC$ , |                     |
| c). i). HY,         | ii). $\angle HYD$ , |                     |
| d). i). DZ,         | ii). $\angle ZDB$ , |                     |
| e). AG,             |                     |                     |



# Fluency Practice

Question 5: Give these directions of travel as three figure bearings

- (a) North            (b) South-east        (c) West            (d) North-east  
 (e) East            (f) South-west        (g) South            (h) North-west

Question 6: A dolphin is on a bearing of  $100^\circ$  from the island.  
 The same dolphin is on a bearing of  $015^\circ$  from the lighthouse.  
 On a sketch of the diagram below, mark the location of the dolphin.



Question 7: A hot-air balloon is on a bearing of  $140^\circ$  from the radar A.  
 The same hot-air balloon is on a bearing of  $065^\circ$  from the radar B.  
 On a sketch of the diagram below, mark the location of the hot-air balloon.



Question 8: A UFO is on a bearing of  $015^\circ$  from the radar A.  
 The same UFO is on a bearing of  $315^\circ$  from the radar B.  
 On a sketch of the diagram below, mark the location of the UFO.



Question 9:

- (a) The bearing of A from B is  $025^\circ$ , find the bearing of B from A.  
 (b) The bearing of A from B is  $061^\circ$ , find the bearing of B from A.  
 (c) The bearing of A from B is  $098^\circ$ , find the bearing of B from A.  
 (d) The bearing of A from B is  $102^\circ$ , find the bearing of B from A.  
 (e) The bearing of A from B is  $193^\circ$ , find the bearing of B from A.  
 (f) The bearing of A from B is  $222^\circ$ , find the bearing of B from A.  
 (g) The bearing of A from B is  $315^\circ$ , find the bearing of B from A.

Question 10: Make a copy of the diagram below into your book.



- (a) Find the bearing of B from A.  
 (b) Find the bearing of A from B.

Use the scale 1cm represents 20miles.

- (c) From your diagram, work out the real distance between A and B.  
 C is 140 miles from B on a bearing of  $110^\circ$ .  
 (d) On your diagram, mark C with a cross.



# Fluency Practice

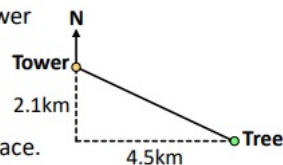
**Q1** A scout troop are hiking in a forest. Starting from their base, they walk 4.2km south followed by 7.1km west. They want to walk the shortest distance back to their base.  
On what bearing should the scouts walk?

**Q2** A dragon has been wreaking havoc in a local village. It is then chased away by a knight. The dragon flies 3 miles due south followed by 4 miles due west.

**[a]** Work out the distance between the dragon and the village.

**[b]** Find the bearing of the dragon from the village to the nearest degree.

**Q3** The diagram shows the positions of a tower and a tree. The tree is 2.1km South of the tower and 4.5km East of the tower.



**[a]** Work out the distance between the tower and the tree to one decimal place.

**[b]** Work out the bearing of the tree from the tower to the nearest degree.

**Q4** A aeroplane flies 22 miles due south followed by 34 miles due east. On what bearing is the aeroplane's starting position from its current position to the nearest degree?

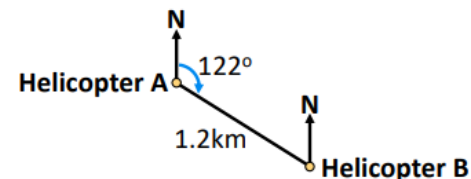
**Q5** A buoy is located 214m west and 185m south of a ship. Calculate the bearing of the buoy from the ship to the nearest degree.

**Q6** Two ships sail away from port. Ship A sails 6.4miles due south followed by 12.4miles due west. Ship B sails 3miles due south followed by 10.1 miles due east.

**[a]** Work out the distance between the two ships to one decimal place.

**[b]** Work out the bearing of Ship B from Ship A to the nearest degree.

**Q1** The following diagram represents the positions and bearings of two helicopters. Helicopter B is 1.2km away from helicopter A on a bearing of  $122^\circ$ . How far north is helicopter A from helicopter B?



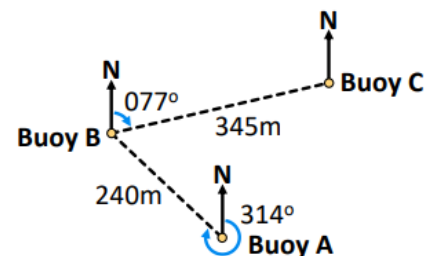
**Q2** Animal conservationists are tracking the position of a polar bear. The polar bear's position is 3km in an eastward direction and is on a bearing of  $104^\circ$ . What is the shortest distance between the polar bear and the conservationists.

**Q3** The following diagram represents the positions and bearings of three buoys floating in the ocean.

**[a]** How far west is Buoy B from Buoy C.

**[b]** How far east is Buoy A from Buoy B.

**[c]** How far north is Buoy C from Buoy A.



**Q4** A ship sails on a bearing of  $074^\circ$  for 10 miles followed by a bearing of  $131^\circ$  for 15 miles. Work out the bearing of the ship from its starting position to the nearest degree.

## Fluency Practice

**Ex1** A hotel is located 3km east and 2km north of the beach. Calculate the bearing of the hotel from the beach to the nearest degree.

**Ex2** A ship sets off from port. It sails 3 miles due south and then 5 miles east. On what bearing is the port from the ship to the nearest degree?

**Ex3** An aeroplane flies 50 miles on a bearing of  $112^\circ$ . How far due East has it travelled to the nearest one decimal place?

**Ex4** Ground troops are on a bearing of  $072^\circ$  from their base. They know that they are a distance of 6km due East of the base but don't know their distance due North. Work out the shortest distance between the base and the ground troops to the nearest one decimal place.

# Key Skills LEVEL 1 Qs

## Question 1

**Skill involved: K107a: Combine two ratios to form one.**

Given that  $x : y = 5 : 9$  and that  $y : z = 18 : 5$ , find the ratio  $x : y : z$

Give your ratio in its simplest form with integer parts.

## Question 2

**Skill involved: K107a: Combine two ratios to form one.**

There are only green counters, red counters and white counters in a bag.

The ratio of green counters to red counters is  $6 : 7$

The ratio of red counters to white counters is  $28 : 3$

Find the ratio of green counters : red counters : white counters

Give your ratio in its simplest form with integer parts.

## Question 3

**Skill involved: K107a: Combine two ratios to form one.**

There are only yellow balls, green balls and white balls in a bag.

The ratio of yellow balls to green balls is  $3 : 35$

The ratio of green balls to white balls is  $7 : 9$

Find the ratio of yellow balls : green balls : white balls

Give your ratio in its simplest form with integer parts.

## Question 4

**Skill involved: K107a: Combine two ratios to form one.**

There are only Year 9 pupils, Year 10 pupils and Year 11 pupils in a group

The ratio of Year 9 to Year 10 is  $11 : 8$

The ratio of Year 10 to Year 11 is  $24 : 7$

Find the ratio Year 9 : Year 10 : Year 11

Give your ratio in its simplest form with integer parts.

## Question 5

**Skill involved: K107a: Combine two ratios to form one.**

Given that  $x : y = 11 : 9$  and that  $y : z = 18 : 1$ , find the ratio  $x : y : z$

Give your ratio in its simplest form with integer parts.

## Question 6

**Skill involved: K107a: Combine two ratios to form one.**

Given that  $x : y = 11 : 6$  and that  $y : z = 24 : 7$ , find the ratio  $x : y : z$

Give your ratio in its simplest form with integer parts.

## Question 7

**Skill involved: K107a: Combine two ratios to form one.**

Given that  $a : b = 6 : 5$  and that  $b : c = 1 : 11$ , find the ratio  $a : b : c$

Give your ratio in its simplest form with integer parts.

## Question 8

**Skill involved: K107a: Combine two ratios to form one.**

There are only Year 11 pupils, Year 12 pupils and Year 13 pupils in a group

The ratio of Year 11 to Year 12 is  $11 : 7$

The ratio of Year 12 to Year 13 is  $28 : 5$

Find the ratio Year 11 : Year 12 : Year 13

Give your ratio in its simplest form with integer parts.



---

### Question 9

**Skill involved: K107a: Combine two ratios to form one.**

There are only Year 10 pupils, Year 11 pupils and Year 12 pupils in a group

The ratio of Year 10 to Year 11 is 9 : 20

The ratio of Year 11 to Year 12 is 5 : 4

Find the ratio Year 10 : Year 11 : Year 12

Give your ratio in its simplest form with integer parts.

---

### Question 10

**Skill involved: K107a: Combine two ratios to form one.**

There are only red balls, blue balls and green balls in a bag.

The ratio of red balls to blue balls is 8 : 1

The ratio of blue balls to green balls is 5 : 8

Find the ratio of red balls : blue balls : green balls

Give your ratio in its simplest form with integer parts.

---

### Question 11

**Skill involved: K107b: Combine two ratios to determine what fraction or percentage of items are a particular type.**

A box contains only blue, purple and pink pens.

The ratio of blue pens to purple pens is 46 : 9.

The ratio of purple pens to pink pens is 1 : 5.

Calculate the percentage of pens that are blue.

---

### Question 12

**Skill involved: K107b: Combine two ratios to determine what fraction or percentage of items are a particular type.**

A pencil case contains only blue, purple and pink pencils.

The ratio of blue pencils to purple pencils is 22 : 7.

The ratio of purple pencils to pink pencils is 1 : 3.

Calculate the percentage of pencils that are blue.

---

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### Question 13

**Skill involved: K107b: Combine two ratios to determine what fraction or percentage of items are a particular type.**

A pencil case contains only blue, purple and pink pencils.

The ratio of blue pencils to purple pencils is 20 : 9.

The ratio of purple pencils to pink pencils is 3 : 7.

Calculate the percentage of pencils that are pink.

---

### Question 14

**Skill involved: K107b: Combine two ratios to determine what fraction or percentage of items are a particular type.**

A pencil case contains only blue, purple and pink pencils.

The ratio of blue pencils to purple pencils is 15 : 7.

The ratio of purple pencils to pink pencils is 1 : 4.

Calculate the percentage of pencils that are purple.

---

### Question 15

**Skill involved: K107b: Combine two ratios to determine what fraction or percentage of items are a particular type.**

A bag contains only blue, purple and pink marbles.

The ratio of blue marbles to purple marbles is 1 : 7.

The ratio of purple marbles to pink marbles is 1 : 6.

Calculate the percentage of marbles that are purple.

---

### Question 16

**Skill involved: K107b: Combine two ratios to determine what fraction or percentage of items are a particular type.**

A bag contains only black, purple and orange marbles.

The ratio of black marbles to purple marbles is 37 : 9.

The ratio of purple marbles to orange marbles is 1 : 6.

Calculate the percentage of marbles that are orange.

---

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### Question 17

**Skill involved: K107b: Combine two ratios to determine what fraction or percentage of items are a particular type.**

A bag contains only red, green and blue marbles.  
The ratio of red marbles to green marbles is 23 : 7.  
The ratio of green marbles to blue marbles is 1 : 10.

Calculate the percentage of marbles that are red.

---

### Question 18

**Skill involved: K107b: Combine two ratios to determine what fraction or percentage of items are a particular type.**

A pencil case contains only black, purple and orange pencils.  
The ratio of black pencils to purple pencils is 5 : 3.  
The ratio of purple pencils to orange pencils is 1 : 4.

Calculate the percentage of pencils that are black.

---

### Question 19

**Skill involved: K107b: Combine two ratios to determine what fraction or percentage of items are a particular type.**

A bag contains only black, purple and orange marbles.  
The ratio of black marbles to purple marbles is 23 : 3.  
The ratio of purple marbles to orange marbles is 1 : 8.

Calculate the percentage of marbles that are purple.

---

### Question 20

**Skill involved: K107b: Combine two ratios to determine what fraction or percentage of items are a particular type.**

A bag contains only red, green and blue marbles.  
The ratio of red marbles to green marbles is 5 : 9.  
The ratio of green marbles to blue marbles is 1 : 4.

Calculate the percentage of marbles that are blue.

---

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### Question 21

**Skill involved: K319a: Determine an amount given a change in a ratio where one part is changing.**

The ratio boys : girls in a theatre is 1 : 4

15 girls leave and the ratio becomes 2 : 5

Work out how many boys there are in the theatre.

---

### Question 22

**Skill involved: K319a: Determine an amount given a change in a ratio where one part is changing.**

The ratio boys : girls in a theatre is 2 : 1

3 boys leave and the ratio becomes 5 : 3

Work out how many girls there are in the theatre.

---

### Question 23

**Skill involved: K319a: Determine an amount given a change in a ratio where one part is changing.**

There are green counters and red counters in a bag in the ratio 2 : 3

28 green counters are removed and the ratio becomes 1 : 5

Work out how many red counters there are in the bag.

---

### Question 24

**Skill involved: K319a: Determine an amount given a change in a ratio where one part is changing.**

The ratio boys : girls in a stadium is 2 : 3

5 girls arrive and the ratio becomes 1 : 2

Work out how many boys there are in the stadium.

---

### Question 25

**Skill involved: K319a: Determine an amount given a change in a ratio where one part is changing.**

There are blue counters and green counters in a bag in the ratio 4 : 3

4 green counters are removed and the ratio becomes 2 : 1

Work out how many blue counters there are in the bag.

---

### Question 26

**Skill involved: K319a: Determine an amount given a change in a ratio where one part is changing.**

There are green counters and yellow counters in a bag in the ratio 4 : 1

20 green counters are removed and the ratio becomes 2 : 3

Work out how many yellow counters there are in the bag.

---

### Question 27

**Skill involved: K319a: Determine an amount given a change in a ratio where one part is changing.**

The ratio boys : girls in a theatre is 1 : 5

72 boys arrive and the ratio becomes 5 : 1

Work out how many girls there are in the theatre.

---

### Question 28

**Skill involved: K319a: Determine an amount given a change in a ratio where one part is changing.**

There are red counters and black counters in a bag in the ratio 1 : 4

28 red counters are added and the ratio becomes 2 : 1

Work out how many black counters there are in the bag.

---

### Question 29

**Skill involved: K319a: Determine an amount given a change in a ratio where one part is changing.**

There are yellow counters and white counters in a bag in the ratio 5 : 3

44 white counters are added and the ratio becomes 3 : 4

Work out how many yellow counters there are in the bag.

---

---

### Question 30

**Skill involved: K319a: Determine an amount given a change in a ratio where one part is changing.**

There are black counters and blue counters in a bag in the ratio 4 : 1

76 black counters are removed and the ratio becomes 1 : 5

Work out how many blue counters there are in the bag.

---

## LEVEL 2 QUESTIONS

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### Question 1

**Skill involved: K107c: Combine two ratios involving algebraic terms.**

A biscuit tin contains shortbread, cookies and bourbons.

The ratio of shortbread to cookies is  $7n : 4$ .

The ratio of cookies to bourbons is  $5 : 8n$ .

Work out the ratio of shortbread to bourbons.  
Give your answer in its simplest form.

---

### Question 2

**Skill involved: K107c: Combine two ratios involving algebraic terms.**

A picnic box contains sandwiches, cakes and apples.

The ratio of sandwiches to cakes is  $10a : 9$ .

The ratio of cakes to apples is  $6 : 5a$ .

Work out the ratio of sandwiches to apples.  
Give your answer in its simplest form.

---

### Question 3

**Skill involved: K107c: Combine two ratios involving algebraic terms.**

A pencil case contains pens, pencils and crayons.

The ratio of pens to pencils is  $2m : 5$ .

The ratio of pencils to crayons is  $7 : 6m$ .

Work out the ratio of pens to crayons.  
Give your answer in its simplest form.

---

### Question 4

**Skill involved: K107c: Combine two ratios involving algebraic terms.**

A bag of sweets contains jellies, mints and toffees.

The ratio of jellies to mints is  $11n : 10$ .

The ratio of mints to toffees is  $7 : 2n$ .

Work out the ratio of jellies to toffees.  
Give your answer in its simplest form.

---

### Question 5

**Skill involved: K107c: Combine two ratios involving algebraic terms.**

A pencil case contains pens, pencils and crayons.

The ratio of pens to pencils is  $9n : 11$ .

The ratio of pencils to crayons is  $9 : 7n$ .

Work out the ratio of pens to crayons.  
Give your answer in its simplest form.

---

### Question 6

**Skill involved: K107c: Combine two ratios involving algebraic terms.**

A pencil case contains pens, pencils and crayons.

The ratio of pens to pencils is  $9n : 4$ .

The ratio of pencils to crayons is  $3 : 8n$ .

Work out the ratio of pens to crayons.  
Give your answer in its simplest form.

---

### Question 7

**Skill involved: K107c: Combine two ratios involving algebraic terms.**

A picnic box contains sandwiches, cakes and apples.

The ratio of sandwiches to cakes is  $8n : 11$ .

The ratio of cakes to apples is  $1 : 7n$ .

Work out the ratio of sandwiches to apples.  
Give your answer in its simplest form.

---

### Question 8

**Skill involved: K107c: Combine two ratios involving algebraic terms.**

A biscuit tin contains shortbread, cookies and bourbons.

The ratio of shortbread to cookies is  $7n : 11$ .

The ratio of cookies to bourbons is  $1 : 2n$ .

Work out the ratio of shortbread to bourbons.  
Give your answer in its simplest form.

---

### Question 9

**Skill involved: K107c: Combine two ratios involving algebraic terms.**

A picnic box contains sandwiches, cakes and apples.

The ratio of sandwiches to cakes is  $10n : 3$ .

The ratio of cakes to apples is  $7 : 2n$ .

Work out the ratio of sandwiches to apples.  
Give your answer in its simplest form.

---

### Question 10

**Skill involved: K107c: Combine two ratios involving algebraic terms.**

A picnic box contains sandwiches, cakes and apples.

The ratio of sandwiches to cakes is  $3n : 4$ .

The ratio of cakes to apples is  $5 : 9n$ .

Work out the ratio of sandwiches to apples.  
Give your answer in its simplest form.

---

### Question 11

**Skill involved: K107d: Determine a value in a ratio when another ratio and an amount is given.**

In a pencil case,

number of blue pencils : purple pencils =  $2 : 1$   
number of purple pencils : green pencils =  $5 : 2$

There are 30 blue pencils in the pencil case.

Work out the number of green pencils in the pencil case.

---

### Question 12

**Skill involved: K107d: Determine a value in a ratio when another ratio and an amount is given.**

In a pencil case,

number of red pencils : green pencils =  $3 : 5$   
number of green pencils : blue pencils =  $2 : 1$

There are 15 blue pencils in the pencil case.

Work out the number of red pencils in the pencil case.

---

### Question 13

**Skill involved: K107d: Determine a value in a ratio when another ratio and an amount is given.**

In a box,

number of red buttons : green buttons =  $2 : 5$   
number of green buttons : blue buttons =  $2 : 3$

There are 28 red buttons in the box.

Work out the number of blue buttons in the box.

---

### Question 14

**Skill involved: K107d: Determine a value in a ratio when another ratio and an amount is given.**

In a pencil case,

number of blue pencils : purple pencils =  $4 : 3$   
number of purple pencils : green pencils =  $5 : 1$

There are 160 blue pencils in the pencil case.

Work out the number of green pencils in the pencil case.



---

### Question 15

**Skill involved: K107d: Determine a value in a ratio when another ratio and an amount is given.**

In a box,

number of red buttons : green buttons = 1 : 3  
number of green buttons : blue buttons = 5 : 4

There are 5 red buttons in the box.

Work out the number of blue buttons in the box.

---

### Question 16

**Skill involved: K107d: Determine a value in a ratio when another ratio and an amount is given.**

In a box,

number of red buttons : green buttons = 4 : 1  
number of green buttons : blue buttons = 5 : 4

There are 40 blue buttons in the box.

Work out the number of red buttons in the box.

---

### Question 17

**Skill involved: K107d: Determine a value in a ratio when another ratio and an amount is given.**

In a pencil case,

number of blue pencils : purple pencils = 2 : 1  
number of purple pencils : green pencils = 5 : 2

There are 20 blue pencils in the pencil case.

Work out the number of green pencils in the pencil case.

---

### Question 18

**Skill involved: K107d: Determine a value in a ratio when another ratio and an amount is given.**

In a box,

number of red buttons : green buttons = 1 : 5  
number of green buttons : blue buttons = 1 : 4

There are 4 red buttons in the box.

Work out the number of blue buttons in the box.

---

### Question 19

**Skill involved: K107d: Determine a value in a ratio when another ratio and an amount is given.**

In a bag,

number of red marbles : purple marbles = 2 : 3  
number of purple marbles : orange marbles = 1 : 2

There are 54 orange marbles in the bag.

Work out the number of red marbles in the bag.

---

### Question 20

**Skill involved: K107d: Determine a value in a ratio when another ratio and an amount is given.**

In a pencil case,

number of red pencils : green pencils = 3 : 2  
number of green pencils : blue pencils = 1 : 3

There are 18 red pencils in the pencil case.

Work out the number of blue pencils in the pencil case.

---

### Question 41

**Skill involved: K319b: Determine an amount given a change in a ratio where two parts are changing.**

Lucy and Evie have money in their bank account in the ratio 4 : 1

Lucy gives £3 to Evie, and the ratio becomes 7 : 2.

Work out the original amount of money that Lucy had.

---

### Question 42

**Skill involved: K319b: Determine an amount given a change in a ratio where two parts are changing.**

Lucy and Luke have money in their bank account in the ratio 8 : 3

Lucy gives £6 to Luke, and the ratio becomes 5 : 6.

Work out the original amount of money that Lucy had.

---

### Question 43

**Skill involved: K319b: Determine an amount given a change in a ratio where two parts are changing.**

Charlie and Tim have money in their bank account in the ratio 7 : 6

Charlie gives £8 to Tim, and the ratio becomes 6 : 7.

Work out the original amount of money that Charlie had.

---

### Question 44

**Skill involved: K319b: Determine an amount given a change in a ratio where two parts are changing.**

Evie and Lucy have money in their bank account in the ratio 2 : 1

Evie gives £2 to Lucy, and the ratio becomes 7 : 4.

Work out the original amount of money that Evie had.

---

### Question 45

**Skill involved: K319b: Determine an amount given a change in a ratio where two parts are changing.**

There are yellow counters and blue counters in a bag in the ratio 5 : 2

3 yellow counters are removed and 10 blue counters are added to the bag, and the ratio becomes 3 : 4.

Work out the original number of yellow counters in the bag.

---

### Question 46

**Skill involved: K319b: Determine an amount given a change in a ratio where two parts are changing.**

Evie and Lucy have money in their bank account in the ratio 7 : 6

Evie gives £9 to Lucy, and the ratio becomes 6 : 7.

Work out the original amount of money that Lucy had.

---

### Question 47

**Skill involved: K319b: Determine an amount given a change in a ratio where two parts are changing.**

Tim and Evie have money in their bank account in the ratio 4 : 3

Tim gives £6 to Evie, and the ratio becomes 3 : 4.

Work out the original amount of money that Evie had.

---

### Question 48

**Skill involved: K319b: Determine an amount given a change in a ratio where two parts are changing.**

There are white counters and red counters in a bag in the ratio 2 : 7

9 white counters are removed and 7 red counters are added to the bag, and the ratio becomes 1 : 4.

Work out the original number of red counters in the bag.

---

### Question 49

**Skill involved: K319b: Determine an amount given a change in a ratio where two parts are changing.**

There are black counters and yellow counters in a bag in the ratio 3 : 4

1 black counter is removed and 6 yellow counters are added to the bag, and the ratio becomes 5 : 7.

Work out the original number of black counters in the bag.

## K166b & c

### Question 21

**Skill involved:** K166b: Find a coordinate given another and their midpoint.

$M(1.5, 2.5)$  is the midpoint of  $A(2, 5)$  and  $B$ . Find the coordinates of  $B$ .

---

### Question 22

**Skill involved:** K166b: Find a coordinate given another and their midpoint.

$M(3, -1)$  is the midpoint of the line segment  $AB$  where  $A(5, -4)$ . Find the coordinates of  $B$ .

---

### Question 23

**Skill involved:** K166b: Find a coordinate given another and their midpoint.

$M(-0.5, -2)$  is the midpoint of  $A(0, -4)$  and  $B$ . Find the coordinates of  $B$ .

---

### Question 24

**Skill involved:** K166b: Find a coordinate given another and their midpoint.

$M(-1, -2.5)$  is the midpoint of the line segment  $AB$  where  $A(-1, -5)$ . Find the coordinates of  $B$ .

---

### Question 25

**Skill involved:** K166b: Find a coordinate given another and their midpoint.

$M(0.5, -1.5)$  is the midpoint of  $A(-4, -5)$  and  $B$ . Find the coordinates of  $B$ .

---

### Question 26

**Skill involved:** K166b: Find a coordinate given another and their midpoint.

$M(-1.5, -2)$  is the midpoint of  $A(1, -5)$  and  $B$ . Find the coordinates of  $B$ .

---

### Question 27

**Skill involved:** K166b: Find a coordinate given another and their midpoint.

$M(0, 4)$  is the midpoint of  $A(-2, 4)$  and  $B$ . Find the coordinates of  $B$ .

---

### Question 28

**Skill involved:** K166b: Find a coordinate given another and their midpoint.

$M(0.5, -2)$  is the midpoint of  $A(3, -3)$  and  $B$ . Find the coordinates of  $B$ .

---

### Question 29

**Skill involved:** K166b: Find a coordinate given another and their midpoint.

$M(-0.5, -1.5)$  is the midpoint of  $A(2, 0)$  and  $B$ . Find the coordinates of  $B$ .

---

### Question 30

**Skill involved:** K166b: Find a coordinate given another and their midpoint.

$M(0, -2.5)$  is the midpoint of the line segment  $AB$  where  $A(-3, 0)$ . Find the coordinates of  $B$ .

---

### Question 31

**Skill involved:** K166c: Find the coordinates of a point that shares a line in a ratio.

The point  $M$  lies on the line segment  $AB$  where  $A(-3, -4)$  and  $B(5, 12)$ .

Given that  $AM : MB = 1 : 3$ , find the coordinates of  $M$ .

---

### Question 32

**Skill involved:** K166c: Find the coordinates of a point that shares a line in a ratio.

The point  $M$  lies on the line segment  $AB$  where  $A(-1, -1)$  and  $B(3, 3)$ .

Given that  $AM : MB = 3 : 1$ , find the coordinates of  $M$ .



---

**Question 33**

**Skill involved: K166c: Find the coordinates of a point that shares a line in a ratio.**

The point  $M$  lies on the line segment  $AB$  where  $A(-3, 4)$  and  $B(3, 16)$ .

Given that  $AM : MB = 2 : 1$ , find the coordinates of  $M$ .

---

**Question 34**

**Skill involved: K166c: Find the coordinates of a point that shares a line in a ratio.**

The point  $M$  lies on the line segment  $AB$  where  $A(-4, 1)$  and  $B(12, 5)$ .

Given that  $AM : MB = 3 : 1$ , find the coordinates of  $M$ .

---

**Question 35**

**Skill involved: K166c: Find the coordinates of a point that shares a line in a ratio.**

The point  $M$  lies on the line segment  $AB$  where  $A(0, -2)$  and  $B(12, 10)$ .

Given that  $AM : MB = 3 : 1$ , find the coordinates of  $M$ .

---

**Question 36**

**Skill involved: K166c: Find the coordinates of a point that shares a line in a ratio.**

The point  $M$  lies on the line segment  $AB$  where  $A(1, -5)$  and  $B(17, 3)$ .

Given that  $AM : MB = 3 : 1$ , find the coordinates of  $M$ .

---

**Question 37**

**Skill involved: K166c: Find the coordinates of a point that shares a line in a ratio.**

The point  $M$  lies on the line segment  $AB$  where  $A(1, 3)$  and  $B(4, 15)$ .

Given that  $AM : MB = 2 : 1$ , find the coordinates of  $M$ .

---

**Question 38**

**Skill involved: K166c: Find the coordinates of a point that shares a line in a ratio.**

The point  $M$  lies on the line segment  $AB$  where  $A(-2, 3)$  and  $B(4, 15)$ .

Given that  $AM : MB = 2 : 1$ , find the coordinates of  $M$ .

---

**Question 39**

**Skill involved: K166c: Find the coordinates of a point that shares a line in a ratio.**

The point  $M$  lies on the line segment  $AB$  where  $A(-5, -3)$  and  $B(7, 9)$ .

Given that  $AM : MB = 1 : 2$ , find the coordinates of  $M$ .

---

**Question 40**

**Skill involved: K166c: Find the coordinates of a point that shares a line in a ratio.**

The point  $M$  lies on the line segment  $AB$  where  $A(-2, -5)$  and  $B(2, 7)$ .

Given that  $AM : MB = 3 : 1$ , find the coordinates of  $M$ .

## Level 3 and 4 Qs

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### Question 1

**Skill involved: K107e: Combine two three-part ratios to form one.**

In a school,

The ratio of Year 7 to Year 8 to Year 9 is  $5 : 4 : 1$

The ratio of Year 9 to Year 10 to Year 11 is  $4 : 1 : 8$

Find the ratio Year 7 : Year 11

Give your ratio in its simplest form.

---

### Question 2

**Skill involved: K107e: Combine two three-part ratios to form one.**

The ratio  $a : b : c = 2 : 1 : 7$ .

The ratio  $c : d : e = 8 : 1 : 3$ .

Find the ratio  $a : e$ .

Give your ratio in its simplest form.

---

### Question 3

**Skill involved: K107e: Combine two three-part ratios to form one.**

In a school,

The ratio of Year 7 to Year 8 to Year 9 is  $1 : 3 : 1$

The ratio of Year 9 to Year 10 to Year 11 is  $7 : 1 : 5$

Find the ratio Year 7 : Year 11

Give your ratio in its simplest form.

---

### Question 4

**Skill involved: K107e: Combine two three-part ratios to form one.**

In a school,

The ratio of Year 7 to Year 8 to Year 9 is  $4 : 7 : 5$

The ratio of Year 9 to Year 10 to Year 11 is  $6 : 2 : 7$

Find the ratio Year 7 : Year 11

Give your ratio in its simplest form.

---

### Question 5

**Skill involved: K107e: Combine two three-part ratios to form one.**

The ratio  $a : b : c = 3 : 2 : 7$ .

The ratio  $c : d : e = 8 : 7 : 1$ .

Find the ratio  $a : d$ .

Give your ratio in its simplest form.

---

### Question 6

**Skill involved: K107e: Combine two three-part ratios to form one.**

There are only red balls, green balls, blue balls, white balls and yellow balls in a bag.

The ratio of red balls to green balls to blue balls is  $2 : 3 : 4$

The ratio of blue balls to white balls to yellow balls is  $6 : 3 : 2$

Find the ratio of green balls : yellow balls

Give your ratio in its simplest form.

---

### Question 7

**Skill involved: K107f: Combine ratios in order to reflect on the total possible quantity of items.**

A bag contains jellies, mints and toffees.

The ratio of jellies to mints is  $2 : 1$ .

The ratio of mints to toffees is  $6 : 5$ .

There are more than 51 sweets in the bag.

Find the least possible number of mints in the bag.

---

### Question 8

**Skill involved: K107f: Combine ratios in order to reflect on the total possible quantity of items.**

A biscuit tin contains shortbread, cookies and bournons.

The ratio of shortbread to cookies is 2 : 5.  
The ratio of cookies to bourbons is 6 : 5.

There are less than 339 biscuits in the biscuit tin.

Find the greatest possible number of bourbons in the biscuit tin.

---

### Question 9

**Skill involved: K107f: Combine ratios in order to reflect on the total possible quantity of items.**

A biscuit tin contains shortbread, cookies and bourbons.

The ratio of shortbread to cookies is 5 : 3.  
The ratio of cookies to bourbons is 5 : 4.

There are more than 211 biscuits in the biscuit tin.

Find the least possible number of shortbread in the biscuit tin.

---

### Question 10

**Skill involved: K107f: Combine ratios in order to reflect on the total possible quantity of items.**

A biscuit tin contains shortbread, cookies and bourbons.

The ratio of shortbread to cookies is 4 : 3.  
The ratio of cookies to bourbons is 2 : 3.

There are more than 50 biscuits in the biscuit tin.

Find the least possible number of cookies in the biscuit tin.

---

### Question 11

**Skill involved: K107f: Combine ratios in order to reflect on the total possible quantity of items.**

A bag contains jellies, mints and toffees.

The ratio of jellies to mints is 2 : 5.  
The ratio of mints to toffees is 4 : 5.

There are less than 216 sweets in the bag.

Find the greatest possible number of jellies in the bag.

---

### Question 12

**Skill involved: K107f: Combine ratios in order to reflect on the total possible quantity of items.**

A bag contains jellies, mints and toffees.

The ratio of jellies to mints is 5 : 2.  
The ratio of mints to toffees is 3 : 4.

There are more than 90 sweets in the bag.

Find the least possible number of toffees in the bag.

---

### Question 13

**Skill involved: K107g: Problem solve involving combining ratios involving lengths.**

The points  $A, B, C$  and  $D$  lie in order on a straight line.

$$\begin{aligned}AB : BD &= 2 : 5 \\AC : CD &= 13 : 8\end{aligned}$$

Work out  $AB : BC : CD$

---

### Question 14

**Skill involved: K107g: Problem solve involving combining ratios involving lengths.**

The points  $A, B, C$  and  $D$  lie in order on a straight line.

$$\begin{aligned}AB : BD &= 11 : 10 \\AC : CD &= 5 : 2\end{aligned}$$

Work out  $AB : BC : CD$

---

### Question 15

**Skill involved: K107g: Problem solve involving combining ratios involving lengths.**

The points  $A, B, C$  and  $D$  lie in order on a straight line.

$$AB : BD = 11 : 9$$
$$AC : CD = 4 : 1$$

Work out  $AB : BC : CD$

---

### Question 16

**Skill involved: K107g: Problem solve involving combining ratios involving lengths.**

The points  $A, B, C$  and  $D$  lie in order on a straight line.

$$AB : BD = 15 : 13$$
$$AC : CD = 5 : 2$$

Work out  $AB : BC : CD$

---

### Question 17

**Skill involved: K107g: Problem solve involving combining ratios involving lengths.**

The points  $A, B, C$  and  $D$  lie in order on a straight line.

$$AB : BD = 2 : 3$$
$$AC : CD = 18 : 7$$

Work out  $AB : BC : CD$

---

### Question 18

**Skill involved: K107g: Problem solve involving combining ratios involving lengths.**

The points  $A, B, C$  and  $D$  lie in order on a straight line.

$$AB : BD = 17 : 11$$
$$AC : CD = 6 : 1$$

Work out  $AB : BC : CD$

---

### Question 19

**Skill involved: K107h: Combine ratios where the second ratio is a subdivision of one of the parts of the first.**

Green shapes and purple shapes are used in a game.  
Some of the shapes are triangles.  
All the other shapes are hexagons.

The ratio of triangles to hexagons is  $1 : 5$   
The ratio of green triangles to purple triangles is  $2 : 5$

Work out the fraction of shapes that are green triangles.

---

### Question 20

**Skill involved: K107h: Combine ratios where the second ratio is a subdivision of one of the parts of the first.**

On a farm the ratio of the number of pigs to the number of horses is  $4 : 3$

The ratio of the number of male horses to the number of female horses is  $3 : 2$

Work out what percentage of all the pigs and horses on the farm that are male horses.  
Give your answer correct to the nearest whole number.

---

### Question 21

**Skill involved: K107h: Combine ratios where the second ratio is a subdivision of one of the parts of the first.**

There are some clothes in a shop.

The shop only sells t-shirts and trousers.  
Each item of clothing is either black or blue.

For these clothes

$$\text{number of trousers} : \text{number of t-shirts} = 2 : 5$$
$$\text{number of black trousers} : \text{number of blue trousers} = 3 : 1$$

Work out the proportion of clothes that are blue trousers.  
Give your answer as a fraction in its simplest form.

---

### Question 22

**Skill involved: K107h:** Combine ratios where the second ratio is a subdivision of one of the parts of the first.

Blue shapes and red shapes are used in a game.  
Some of the shapes are triangles.  
All the other shapes are hexagons.

The ratio of triangles to hexagons is 5 : 3  
The ratio of blue triangles to red triangles is 3 : 5

Work out the fraction of shapes that are red triangles.

---

### Question 23

**Skill involved: K107h:** Combine ratios where the second ratio is a subdivision of one of the parts of the first.

Green shapes and purple shapes are used in a game.  
Some of the shapes are triangles.  
All the other shapes are hexagons.

The ratio of triangles to hexagons is 3 : 1  
The ratio of green triangles to purple triangles is 1 : 1

Work out the fraction of shapes that are purple triangles.

---

### Question 24

**Skill involved: K107h:** Combine ratios where the second ratio is a subdivision of one of the parts of the first.

Blue shapes and red shapes are used in a game.  
Some of the shapes are stars.  
All the other shapes are hearts.

The ratio of stars to hearts is 1 : 5  
The ratio of blue stars to red stars is 6 : 1

Work out the fraction of shapes that are blue stars.

---

### Question 25

**Skill involved: K107i:** Combine ratios where the second two ratios are subdivisions of each part of the first.

White shapes and black shapes are used in a game.  
Some of the shapes are circles.  
All of the other shapes are squares.

The ratio of the number of white shapes to the number of black shapes is 2 : 1  
The ratio of the number of white circles to the number of white squares is 2 : 3  
The ratio of the number of black circles to the number of black squares is 1 : 1

Work out what fraction of all the shapes are squares.

---

### Question 26

**Skill involved: K107i:** Combine ratios where the second two ratios are subdivisions of each part of the first.

White shapes and black shapes are used in a game.  
Some of the shapes are circles.  
All of the other shapes are squares.

The ratio of the number of white shapes to the number of black shapes is 4 : 5  
The ratio of the number of white circles to the number of white squares is 1 : 2  
The ratio of the number of black circles to the number of black squares is 1 : 3

Work out what fraction of all the shapes are squares.

---

### Question 27

**Skill involved: K107i:** Combine ratios where the second two ratios are subdivisions of each part of the first.

Green shapes and purple shapes are used in a game.  
Some of the shapes are stars.  
All of the other shapes are hearts.

The ratio of the number of green shapes to the number of purple shapes is 4 : 5  
The ratio of the number of green stars to the number of green hearts is 2 : 1  
The ratio of the number of purple stars to the number of purple hearts is 3 : 2

Work out what fraction of all the shapes are hearts.



---

### Question 28

**Skill involved: K107i:** Combine ratios where the second two ratios are subdivisions of each part of the first.

White shapes and black shapes are used in a game.  
Some of the shapes are stars.  
All of the other shapes are hearts.

The ratio of the number of white shapes to the number of black shapes is 2 : 1  
The ratio of the number of white stars to the number of white hearts is 4 : 3  
The ratio of the number of black stars to the number of black hearts is 4 : 3

Work out what fraction of all the shapes are stars.

---

### Question 29

**Skill involved: K107i:** Combine ratios where the second two ratios are subdivisions of each part of the first.

Green shapes and purple shapes are used in a game.  
Some of the shapes are circles.  
All of the other shapes are squares.

The ratio of the number of green shapes to the number of purple shapes is 5 : 4  
The ratio of the number of green circles to the number of green squares is 1 : 4  
The ratio of the number of purple circles to the number of purple squares is 2 : 1

Work out what fraction of all the shapes are squares.

---

### Question 30

**Skill involved: K107i:** Combine ratios where the second two ratios are subdivisions of each part of the first.

Blue shapes and red shapes are used in a game.  
Some of the shapes are circles.  
All of the other shapes are squares.

The ratio of the number of blue shapes to the number of red shapes is 3 : 4  
The ratio of the number of blue circles to the number of blue squares is 3 : 1  
The ratio of the number of red circles to the number of red squares is 1 : 3

Work out what fraction of all the shapes are squares.

---

### Question 31

**Skill involved: K107j:** Form a four-part ratio.

$a, b, c$  and  $d$  are integers with no common factors.

$$a = \frac{2}{7}b$$

$$c = \frac{2}{5}d$$

$$a = 4d$$

Find  $a : b : c : d$

---

### Question 32

**Skill involved: K107j:** Form a four-part ratio.

$a, b, c$  and  $d$  are integers with no common factors.

$$2a = b$$

$$c : d = 4 : 1$$

$$a = \frac{7}{2}d$$

Find  $a : b : c : d$

---

### Question 33

**Skill involved: K107j:** Form a four-part ratio.

$a, b, c$  and  $d$  are integers with no common factors.

$$a = \frac{5}{3}b$$

$$c : d = 2 : 3$$

$$a : d = 4 : 1$$

Find  $a : b : c : d$

---

### Question 34

**Skill involved: K107j:** Form a four-part ratio.

$a, b, c$  and  $d$  are integers with no common factors.

$$a = \frac{4}{7}b$$

$$c : d = 7 : 1$$

$$5a = 2d$$

Find  $a : b : c : d$

---

### Question 35

**Skill involved: K107j: Form a four-part ratio.**

$a, b, c$  and  $d$  are integers with no common factors.

$$a : b = 1 : 7$$

$$c = \frac{2}{3}d$$

$$7a = 5d$$

Find  $a : b : c : d$

---

### Question 36

**Skill involved: K107j: Form a four-part ratio.**

$a, b, c$  and  $d$  are integers with no common factors.

$$a : b = 3 : 2$$

$$c = \frac{3}{4}d$$

$$a : d = 4 : 5$$

Find  $a : b : c : d$

---

### Question 37

**Skill involved: K105f: Write a ratio as a linear function.**

The ratio  $4p + 3 : 3q - 5$  is equal to  $4 : 7$ .

Express  $p$  in terms of  $q$ .

---

### Question 38

**Skill involved: K105f: Write a ratio as a linear function.**

The ratio  $5y - 4 : 6z + 2$  is equal to  $2 : 5$ .

Express  $y$  in terms of  $z$ .

---

### Question 39

**Skill involved: K105f: Write a ratio as a linear function.**

The ratio  $4y + 3 : 4z + 4$  is equal to  $6 : 7$ .

Express  $z$  in terms of  $y$ .

---

### Question 40

**Skill involved: K105f: Write a ratio as a linear function.**

The ratio  $4p - 1 : q + 5$  is equal to  $4 : 3$ .

Express  $p$  in terms of  $q$ .

---

### Question 41

**Skill involved: K105f: Write a ratio as a linear function.**

The ratio  $5p : 2q + 3$  is equal to  $4 : 3$ .

Express  $p$  in terms of  $q$ .

---

### Question 42

**Skill involved: K105f: Write a ratio as a linear function.**

The ratio  $4x - 4 : 3y$  is equal to  $3 : 2$ .

Express  $y$  in terms of  $x$ .

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**Question 43**

Skill involved: K105g: Convert an equation involving two variables into a ratio.

Given that  $x = 2y$ , work out the ratio  $x:y$

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**Question 44**

Skill involved: K105g: Convert an equation involving two variables into a ratio.

Given that  $9y = 8x$ , work out the ratio  $x:y$

---

**Question 45**

Skill involved: K105g: Convert an equation involving two variables into a ratio.

Given that  $7q = 8p$ , work out the ratio  $p:q$

---

**Question 46**

Skill involved: K105g: Convert an equation involving two variables into a ratio.

Given that  $2p = 3q$ , find the ratio  $p:q$

---

**Question 47**

Skill involved: K105g: Convert an equation involving two variables into a ratio.

Given that  $3y = 8x$ , find the ratio  $x:y$

---

**Question 48**

Skill involved: K105g: Convert an equation involving two variables into a ratio.

Given that  $3q = 7p$ , find the ratio  $p:q$

---