KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2023 Mathematics 2024

Unit 22 Booklet

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

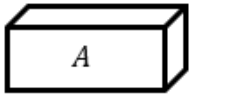
Similarity with Area and Volume

Scale factors

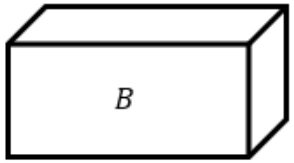
Length Scale Factor	Area Scale Factor	Volume Scale Factor
$\times 2$	$\times 4$	$\times 8$
$\times 3$	$\times 9$	$\times 27$
$\times 4$	$\times 16$	$\times 64$
…	…	…
$\times k$	$\times k^2$	$\times k^3$

Worked Example

Cuboids A and B are similar.



4 cm



12 cm

Write down the scale factor for:

Length $A \rightarrow B$

Length $B \rightarrow A$

Surface Area $A \rightarrow B$

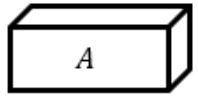
Surface Area $B \rightarrow A$

Volume $A \rightarrow B$

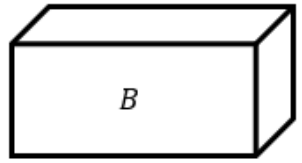
Volume $B \rightarrow A$

Your Turn

Cuboids A and B are similar.



4 cm



8 cm

Write down the scale factor for:

Length $A \rightarrow B$

Length $B \rightarrow A$

Surface Area $A \rightarrow B$

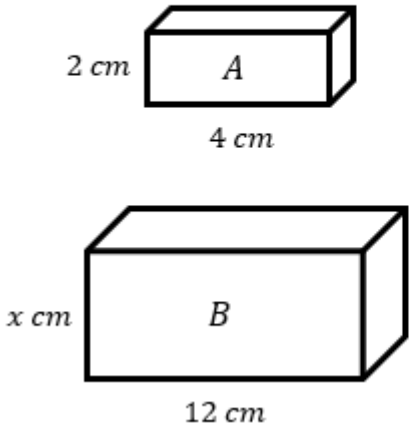
Surface Area $B \rightarrow A$

Volume $A \rightarrow B$

Volume $B \rightarrow A$

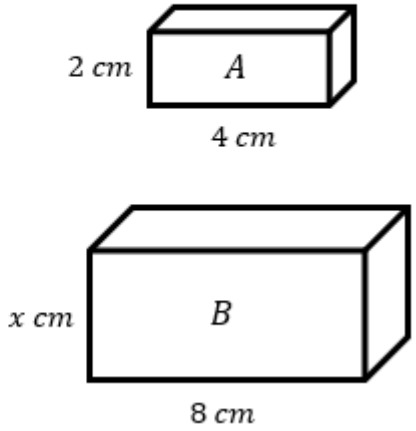
Worked Example

Cuboids A and B are similar.
Find x.



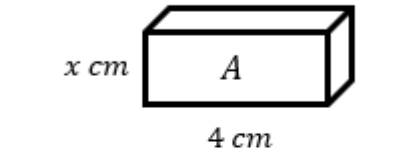
Your Turn

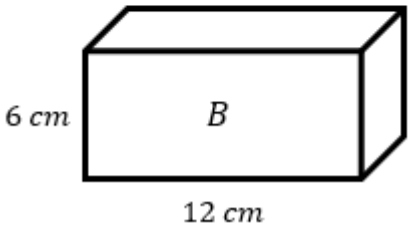
Cuboids A and B are similar.
Find x.



Worked Example

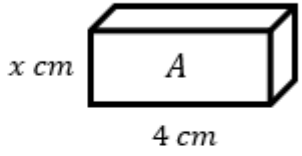
Cuboids A and B are similar.
Find x.

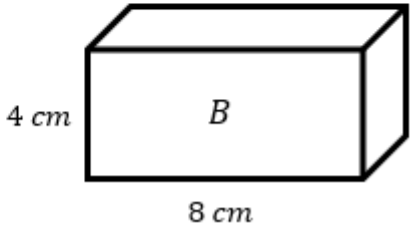




Your Turn

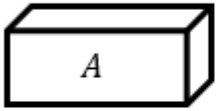
Cuboids A and B are similar.
Find x.



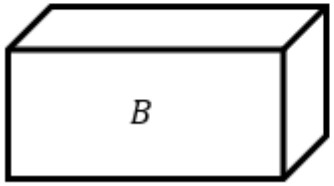


Worked Example

Cuboids A and B are similar.
The surface area of cuboid A is $72\ cm^2$. What is the surface area of cuboid B?



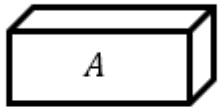
$4\ cm$



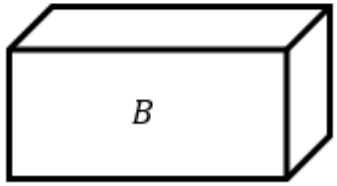
$12\ cm$

Your Turn

Cuboids A and B are similar.
The surface area of cuboid A is $72\ cm^2$. What is the surface area of cuboid B?



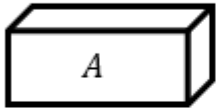
$4\ cm$



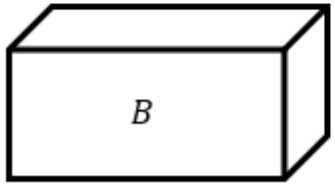
$8\ cm$

Worked Example

Cuboids A and B are similar. The volume of cuboid A is $432\ cm^3$. What is volume of cuboid B?



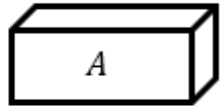
$4\ cm$



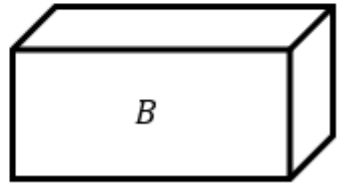
$12\ cm$

Your Turn

Cuboids A and B are similar. The volume of cuboid A is $432\ cm^3$. What is volume of cuboid B?



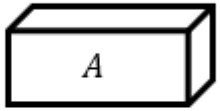
$4\ cm$



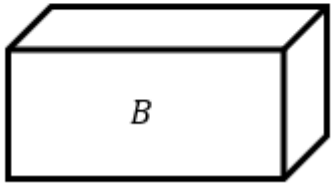
$8\ cm$

Worked Example

Cuboids A and B are similar.
The volume of cuboid B is $432\ cm^3$. What is volume of cuboid A?



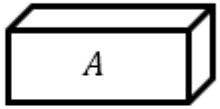
$4\ cm$



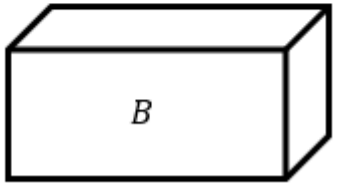
$12\ cm$

Your Turn

Cuboids A and B are similar.
The volume of cuboid B is $432\ cm^3$. What is volume of cuboid A?



$4\ cm$



$8\ cm$

Worked Example	Your Turn
A and B are mathematically similar solids. The surface area of A is $100\ cm^2$. The surface area of B is $64\ cm^2$. Work out the ratio of the volume of A to the volume of B.	A and B are mathematically similar solids. The surface area of A is $120\ cm^2$. The surface area of B is $480\ cm^2$. Work out the ratio of the volume of A to the volume of B.

Worked Example	Your Turn
A and B are mathematically similar solids. The volume of A is $500\ cm^3$. The volume of B is $256\ cm^3$. Work out the ratio of the surface area of A to the surface area of B.	A and B are mathematically similar solids. The volume of A is $120\ cm^3$. The volume of B is $960\ cm^3$. Work out the ratio of the surface area of A to the surface area of B.

Worked Example	Your Turn
The surface area of two mathematically similar solids are in the ratio $16:49$. The volume of the smaller solid is $128\ cm^3$. Work out the volume of the larger solid.	The surface area of two mathematically similar solids are in the ratio $9:25$. The volume area of the smaller solid is $108\ cm^3$. Work out the volume of the larger solid.

Worked Example	Your Turn
The volume of two mathematically similar solids are in the ratio $64: 343$. The surface area of the smaller solid is $32\ cm^2$. Work out the surface area of the larger solid.	The volume of two mathematically similar solids are in the ratio $27: 125$. The surface area of the smaller solid is $36\ cm^2$. Work out the surface area of the larger solid.

FILL IN THE GAPS

Ratio of Lengths	Ratio of Areas	Ratio of Volumes
$2 : 5$	$4 : 25$	$8 : 125$
$5 : 3$		
$1 : \_\_$		$\_\_ : 1000$
	$16 : \_\_$	$\_\_ : 27$
$0.5 : 0.3$		
$0.4 : \_\_$	$\_\_ : 0.01$	$\_\_ : 0.001$
		$0.008 : 0.000001$
$a : \_\_ : \_\_$	$\_\_ : b^2 : \_\_$	$\_\_ : \_\_ : c^3$
$2a : \_\_ : \_\_$	$\_\_ : 9b^2 : \_\_$	$\_\_ : \_\_ : 64c^3$
$a^2 : b^3 : c^4$		
	$a : b^2 : c^3$	
$a : \_\_ : \_\_$	$\_\_ : b : \_\_$	$\_\_ : \_\_ : c$
$\_\_ : \_\_ : 4\sqrt{c}$	$\_\_ : 9b : \_\_$	$8a^{\frac{3}{2}} : \_\_ : \_\_$

FILL IN THE GAPS

	Length	Area	Volume
Scale Factor		4	8
Shape A	3 cm	10 cm^2	25 cm^3
Shape B			

	Length	Area	Volume
Scale Factor		9	
Shape A	4 cm	20 cm^2	70 cm^3
Shape B			

	Length	Area	Volume
Scale Factor			125
Shape A	0.5 m	2 m^2	5 m^3
Shape B			

	Length	Area	Volume
Scale Factor			
Shape A	4.5 mm	20 mm^2	35 mm^3
Shape B		180 mm^2	

	Length	Area	Volume
Scale Factor			
Shape A	2.5 cm	8 cm^2	20 cm^3
Shape B			67.5 cm^3

	Length	Area	Volume
Scale Factor			
Shape A	0.6 m	2.8 m^2	
Shape B		0.7 m^2	1.4 m^2

	Length	Area	Volume
Scale Factor			
Shape A		1.8 cm^2	
Shape B	2 cm	5 cm^2	25 cm^3

	Length	Area	Volume
Scale Factor			
Shape A	7.5 cm		135 cm^3
Shape B		22 cm^2	40 cm^3

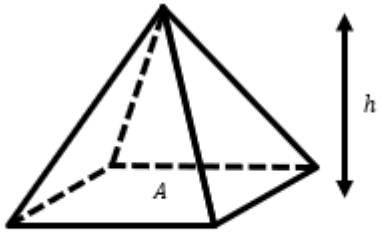
Extra Notes

Volume and Surface Area of Pyramids

Volume and Surface Area of Pyramids

$$\text{Volume of Pyramid} = \frac{1}{3} \times \text{Base Area} \times \text{Height}$$

$$\text{Volume of Pyramid} = \frac{1}{3}Ah$$



Pyramids

DEFINITION

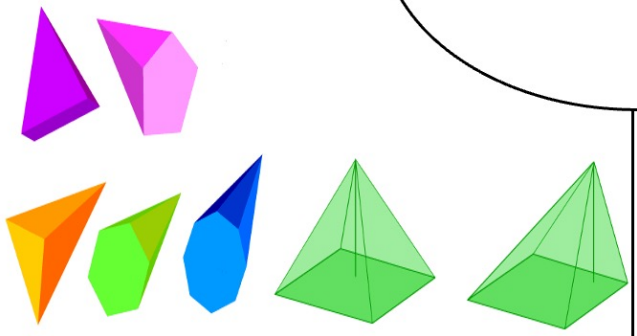
A 3-D shape created by connecting the vertices of a polygon to a single point outside the plane of the polygon (the apex) using straight edges.

CHARACTERISTICS

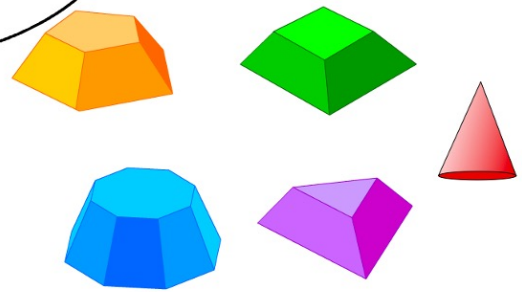
- At most one face that isn't a triangle
- Half the edges all meet at one point

Pyramid

EXAMPLES

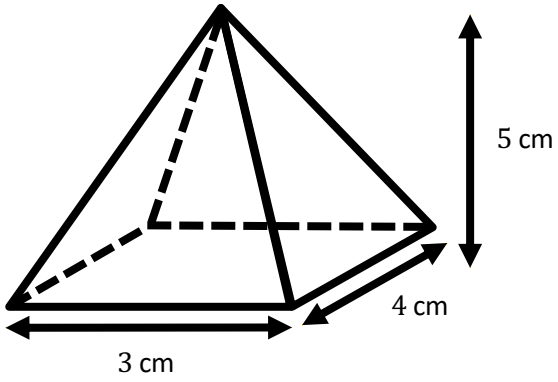


NON-EXAMPLES



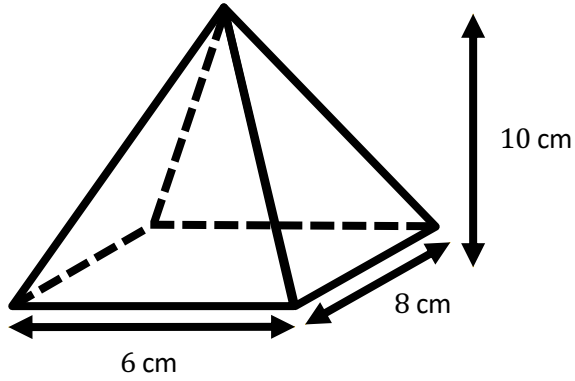
Worked Example

Calculate the volume of the following rectangular-based pyramid.



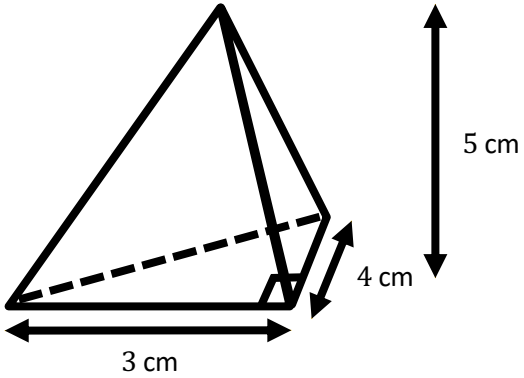
Your Turn

Calculate the volume of the following rectangular-based pyramid.



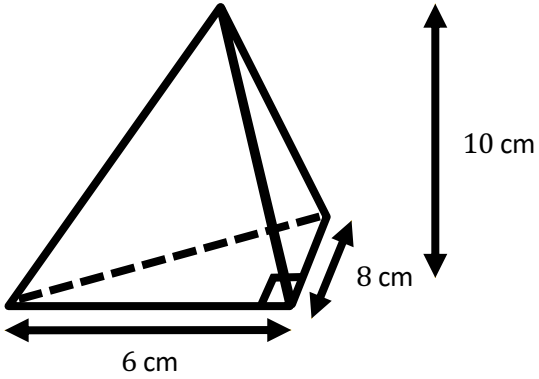
Worked Example

Calculate the volume of the following triangular-based pyramid.



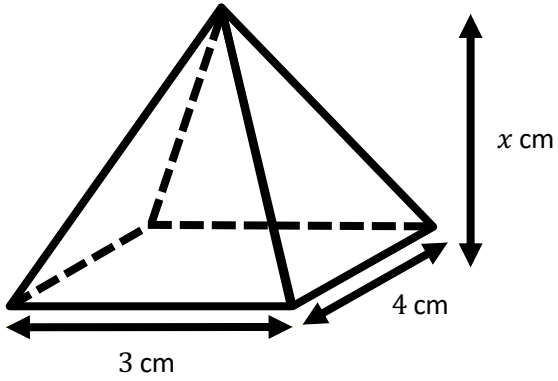
Your Turn

Calculate the volume of the following triangular-based pyramid.



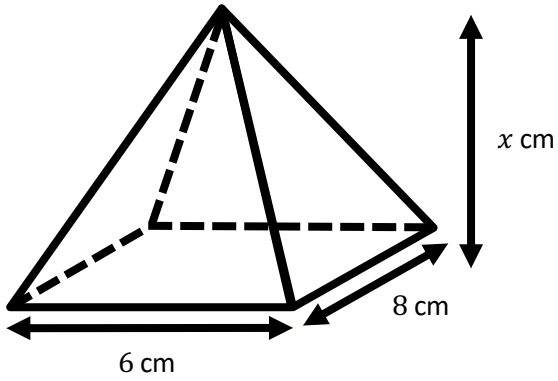
Worked Example

Find the height, x, given that the volume of the following rectangular-based pyramid is $20\ cm^3$.



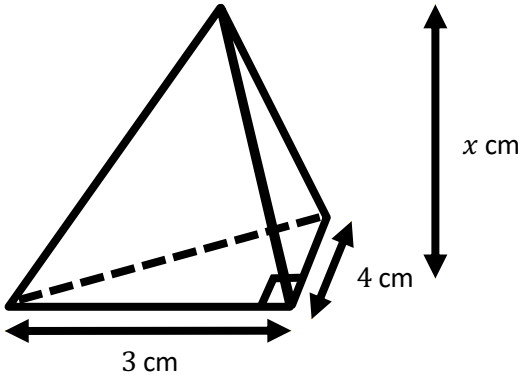
Your Turn

Find the height, x, given that the volume of the following rectangular-based pyramid is $160\ cm^3$.



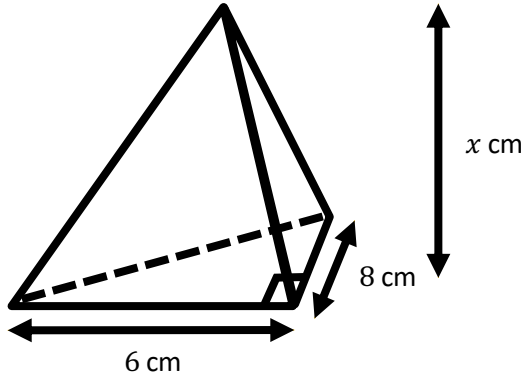
Worked Example

Find the height, x, given that the volume of the following triangular-based pyramid is $10\ cm^3$.



Your Turn

Find the height, x, given that the volume of the following triangular-based pyramid is $80\ cm^3$.



Worked Example

The diagram shows a pyramid.

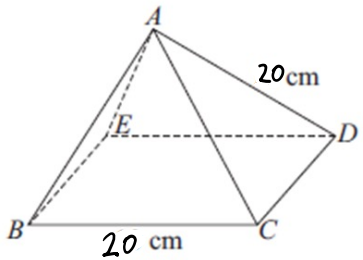


Diagram **NOT** accurately drawn

BCDE is a square with sides of length 20 cm.
The other faces of the pyramid are equilateral triangles with sides of length 20 cm.

(a) Calculate the volume of the pyramid.
Give your answer correct to 3 significant figures.

Your Turn

The diagram shows a pyramid.

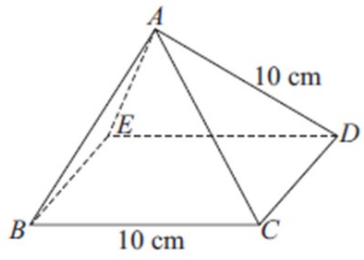


Diagram **NOT** accurately drawn

BCDE is a square with sides of length 10 cm.
The other faces of the pyramid are equilateral triangles with sides of length 10 cm.

(a) Calculate the volume of the pyramid.
Give your answer correct to 3 significant figures.

EXTRA NOTES

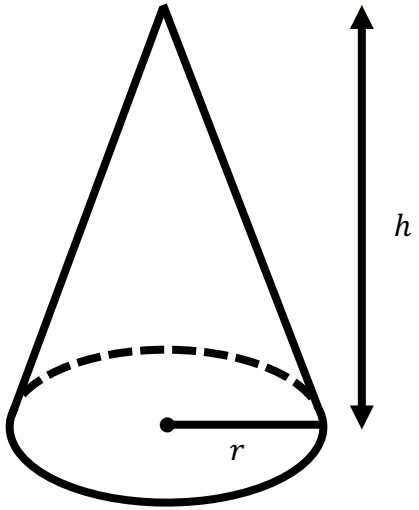
Volume of a cone

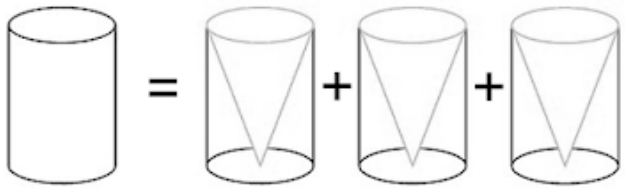
Volume of a cone

$$\text{Volume of Cone} = \frac{1}{3} \times \text{Area of circle} \times \text{height}$$

$$\text{Volume of Cone} = \frac{1}{3} \times \pi \times \text{radius}^2 \times \text{height}$$

$$\text{Volume of Cone} = \frac{1}{3}\pi r^2 h$$





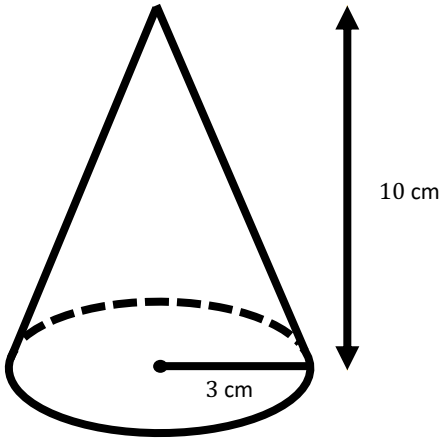
3 cones make up the cylinder

volume of a cone =

$\frac{1}{3}$ base circle × height

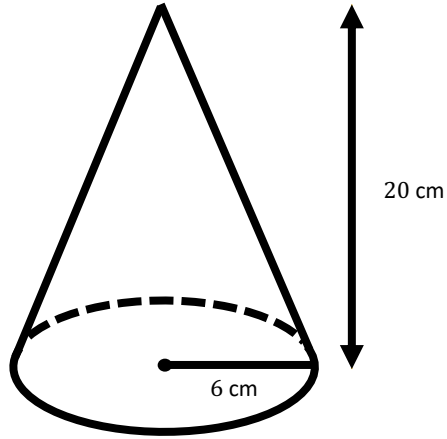
Worked Example

Calculate the volume of the following cone. Give your answer in terms of π and to 1 decimal place.



Your Turn

Calculate the volume of the following cone. Give your answer in terms of π and to 1 decimal place.



Worked Example

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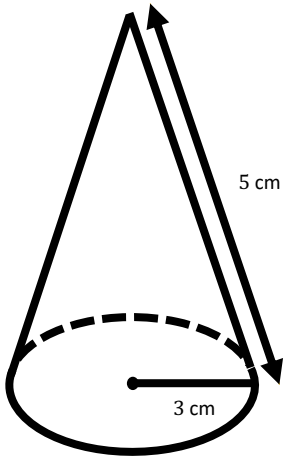
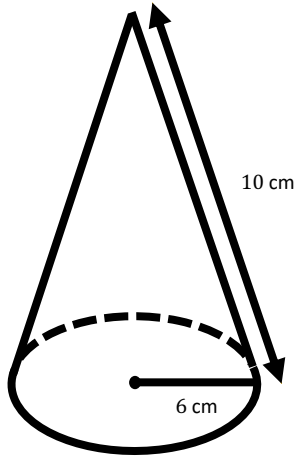


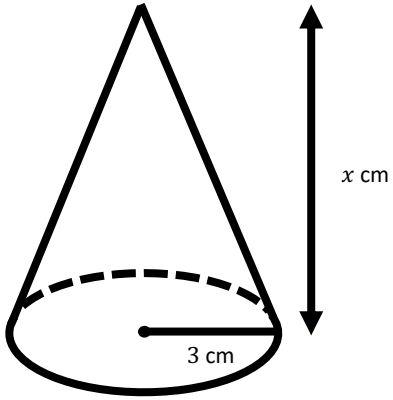
Your Turn

Calculate the volume of the following cone. Give your answer in terms of π and to 1 decimal place.



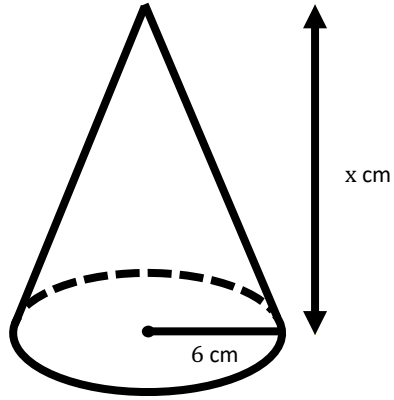
Worked Example

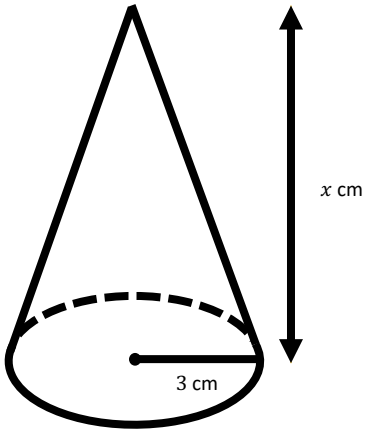
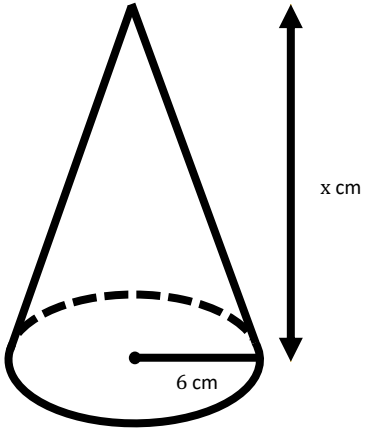
Find the height, x, given that the volume of the following cone is $30\pi\ cm^3$.

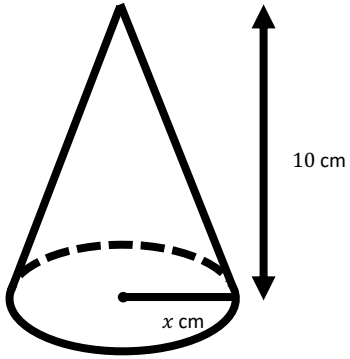
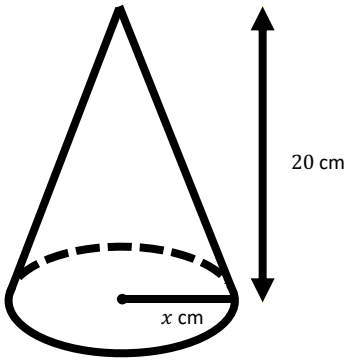


Your Turn

Find the height, x, given that the volume of the following cone is $240\pi\ cm^3$.

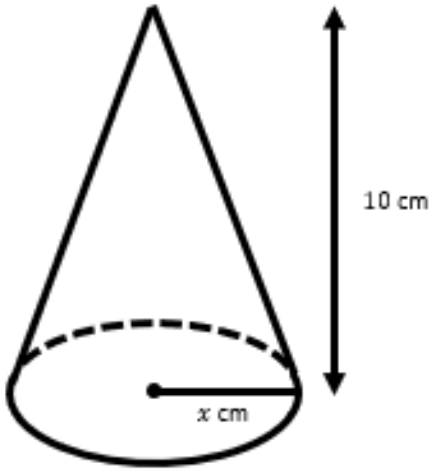


Worked Example	Your Turn
Find the height, x, given that the volume of the following cone is $94.2\ cm^3$. Give your answer to 1 decimal place.  	Find the height, x, given that the volume of the following cone is $754.0\ cm^3$. Give your answer to 1 decimal place. 

Worked Example	Your Turn
Find the radius, x, given that the volume of the following cone is $30\pi\ cm^3$.  	Find the radius, x, given that the volume of the following cone is $240\pi\ cm^3$. 

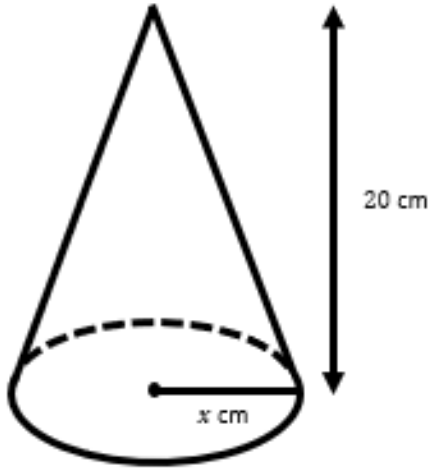
Worked Example

Find the radius, x, given that the volume of the following cone is $94.2\ cm^3$. Give your answer to 1 decimal place.



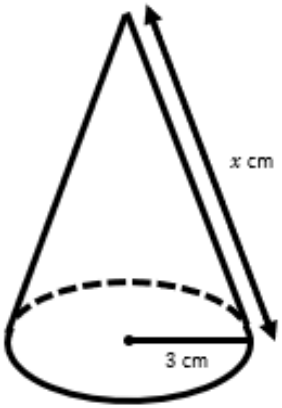
Your Turn

Find the radius, x, given that the volume of the following cone is $754.0\ cm^3$. Give your answer to 1 decimal place.



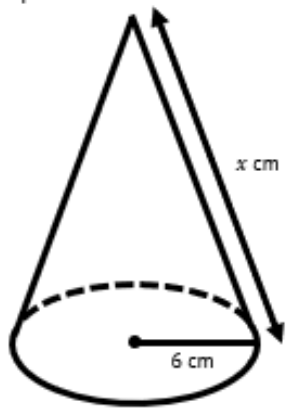
Worked Example

Find the slanted height, x, given that the volume of the following cone is $37.7\ cm^3$. Give your answer to 1 decimal place.



Your Turn

Find the slanted height, x, given that the volume of the following cone is $301.6\ cm^3$. Give your answer to 1 decimal place.



Surface Area of a cone

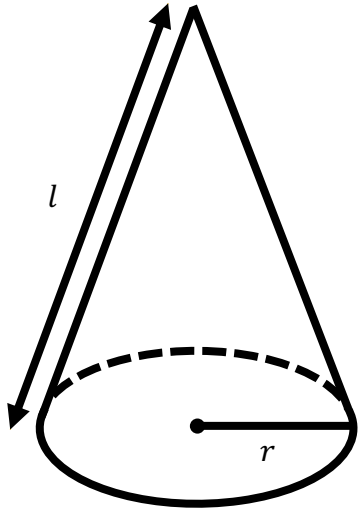
Surface Area of a cone

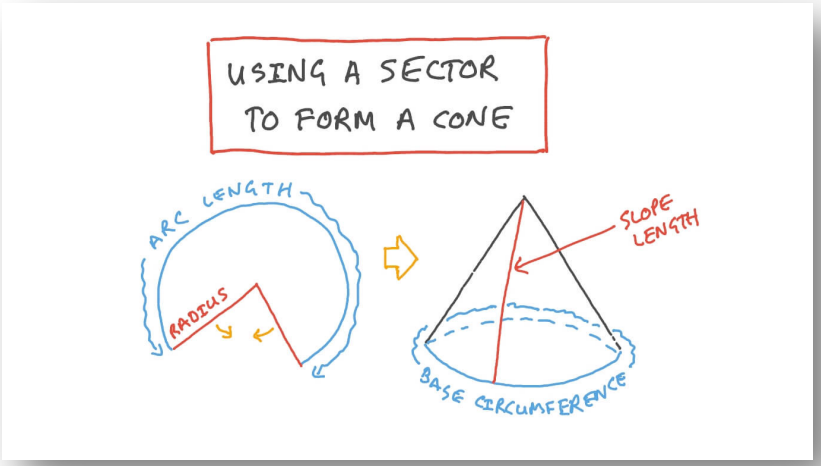
Curved Surface Area of Cone = $\pi \times$ radius $\times$ length

Curved Surface Area of Cone = πrl

Total Surface Area of Cone = $\pi \times$ radius $\times$ length + $\pi \times$ radius^2

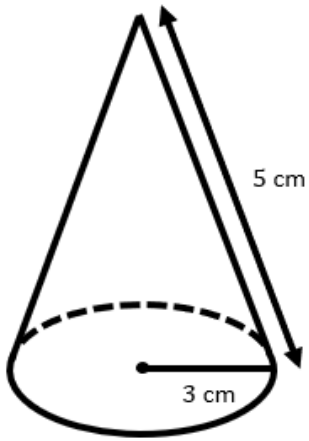
Total Surface Area of Cone = $\pi rl + \pi r^2$





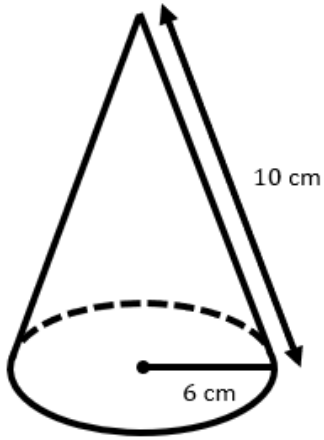
Worked Example

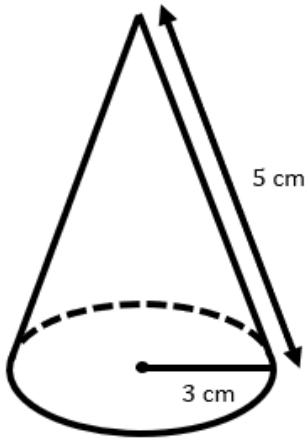
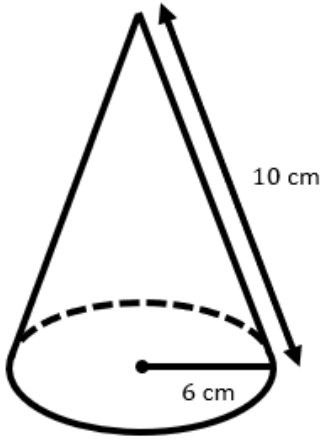
Calculate the curved surface area of the following cone. Give your answer in terms of π and to 1 decimal place.



Your Turn

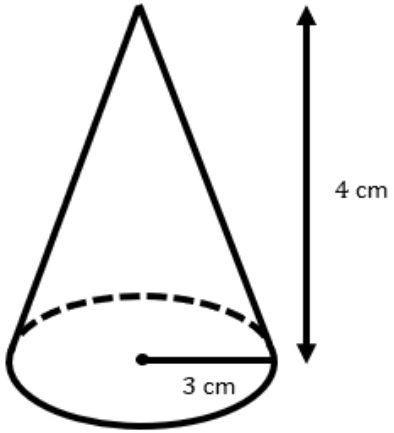
Calculate the curved surface area of the following cone. Give your answer in terms of π and to 1 decimal place.



Worked Example	Your Turn
Calculate the total surface area of the following cone. Give your answer in terms of π and to 1 decimal place.  	Calculate the total surface area of the following cone. Give your answer in terms of π and to 1 decimal place. 

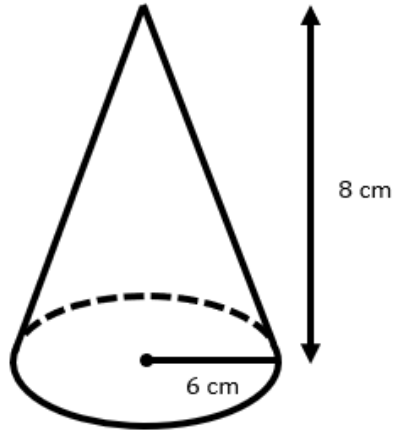
Worked Example

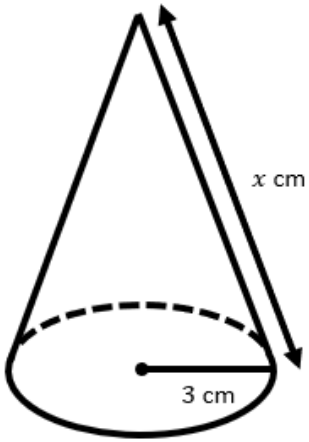
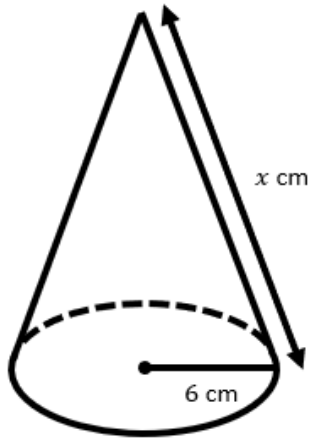
Calculate the total surface area of the following cone. Give your answer in terms of π and to 1 decimal place.



Your Turn

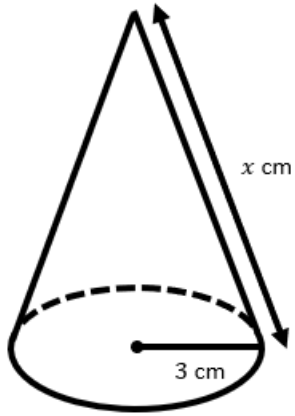
Calculate the total surface area of the following cone. Give your answer in terms of π and to 1 decimal place.



Worked Example	Your Turn
Find the slanted height, x, given that the total surface area of the following cone is $24\pi\ cm^2$.  	Find the slanted height, x, given that the total surface area of the following cone is $96\pi\ cm^2$. 

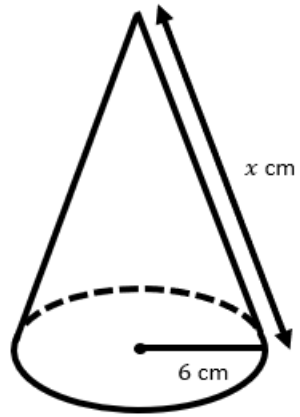
Worked Example

Find the slanted height, x, given that the total surface area of the following cone is $75.4\ cm^2$. Give your answer to 1 decimal place.



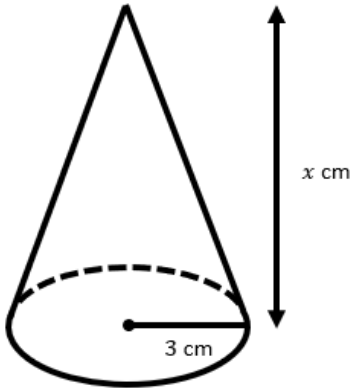
Your Turn

Find the slanted height, x, given that the total surface area of the following cone is $301.6\ cm^2$. Give your answer to 1 decimal place.



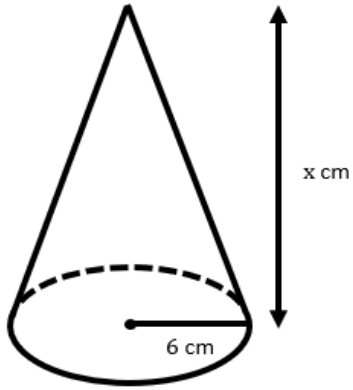
Worked Example

Find the perpendicular height, x, given that the total surface area of the following cone is $75.4\ cm^2$. Give your answer to 1 decimal place.



Your Turn

Find the perpendicular height, x, given that the total surface area of the following cone is $301.6\ cm^2$. Give your answer to 1 decimal place.



Volume and Surface Area of Cones

Radius r	Vertical Height h	Slanted Height l	Volume in terms of π	Volume to 3 s.f.	Curved Surface Area in terms of π	Total Surface Area in terms of π	Volume : Total Surface Area
$5\ cm$	$12\ cm$	$13\ cm$	$100\pi\ cm^3$			$90\pi\ cm^2$	$10 : 9$
$6\ cm$	$8\ cm$	$10\ cm$			$60\pi\ cm^2$		
	$30\ mm$	$34\ mm$		$8040\ mm^3$			
$0.7\ m$	$2.4\ m$						
$9\ cm$		$15\ cm$					
$2\ m$			$\frac{14}{5}\pi\ cm^3$				
		$20\ mm$			$240\pi\ mm^2$		
					$15\pi\ cm^2$	$24\pi\ cm^2$	
		$17\ cm$	$320\pi\ cm^3$				$8 : 5$

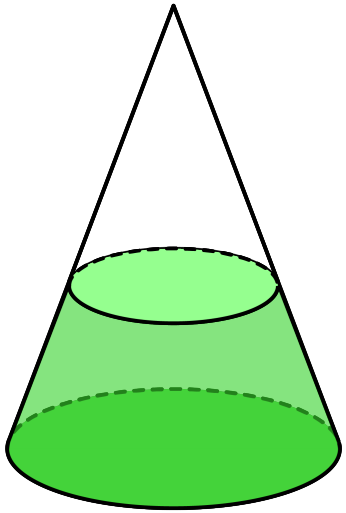
EXTRA NOTES

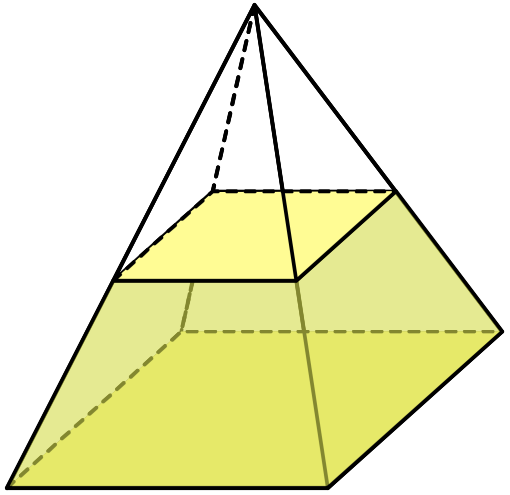
Volume of Frustums

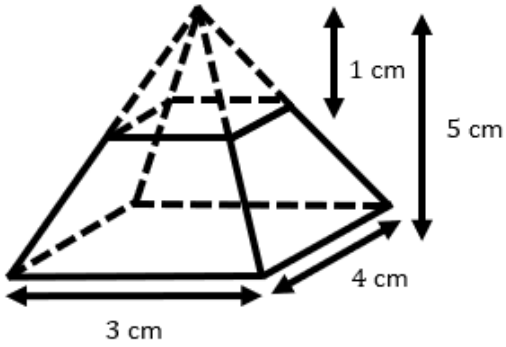
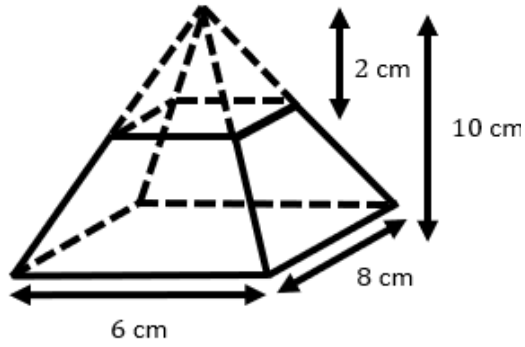
Volume of Frustums

Volume is the amount of space an object takes up.

A **frustum** is a pyramid/cone with part of the top chopped off.

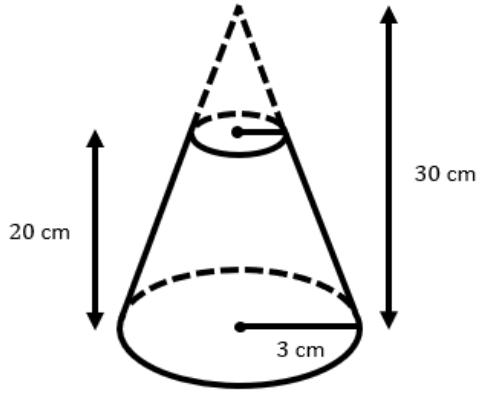




Worked Example	Your Turn
Calculate the volume of the following frustum.  	Calculate the volume of the following frustum. 

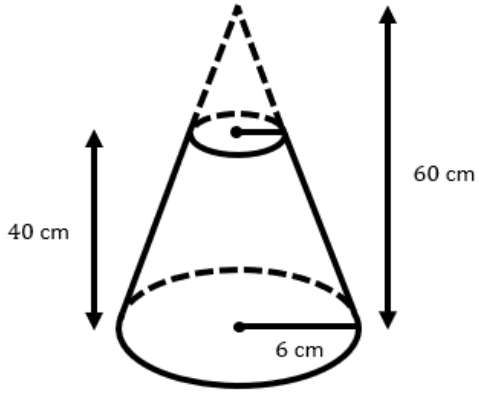
Worked Example

Calculate the volume of the following frustum. Give your answer in terms of π and to 1 decimal place.



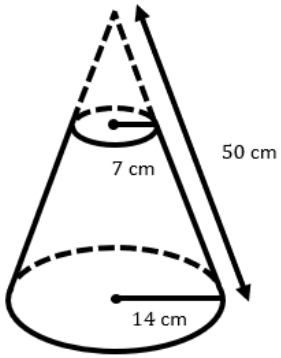
Your Turn

Calculate the volume of the following frustum. Give your answer in terms of π and to 1 decimal place.



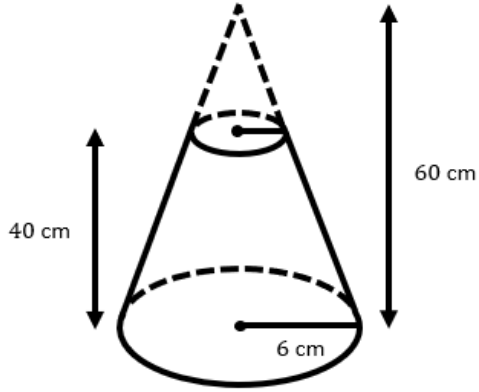
Worked Example

Calculate the volume of the following frustum. Give your answer in terms of π and to 1 decimal place.



Your Turn

Calculate the volume of the following frustum. Give your answer in terms of π and to 1 decimal place.



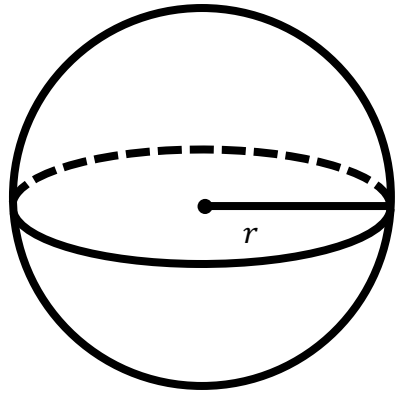
EXTRA NOTES

Volume of Spheres

Volume of Spheres

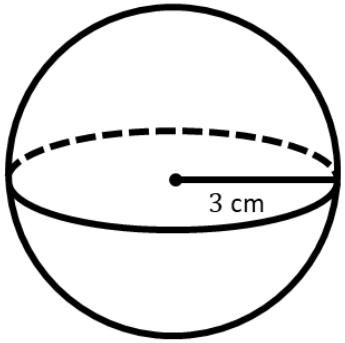
$$\text{Volume of Sphere} = \frac{4}{3} \times \pi \times \text{radius}^3$$

$$\text{Volume of Sphere} = \frac{4}{3}\pi r^3$$



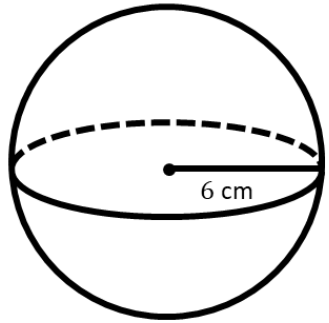
Worked Example

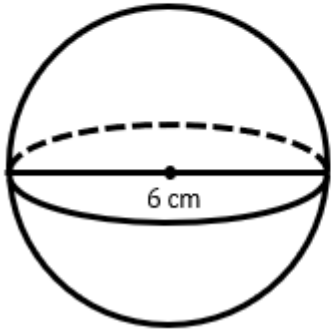
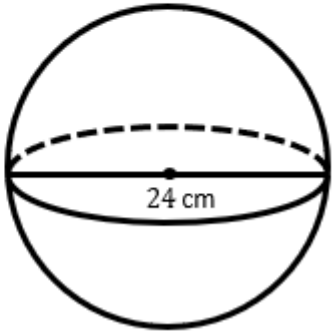
Calculate the volume of the following sphere. Give your answer in terms of π and to 1 decimal place.

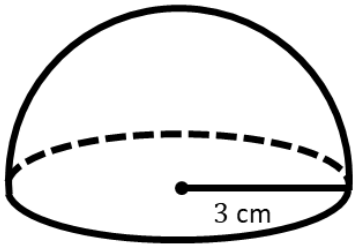
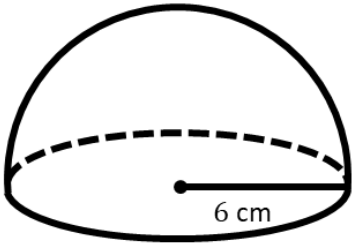


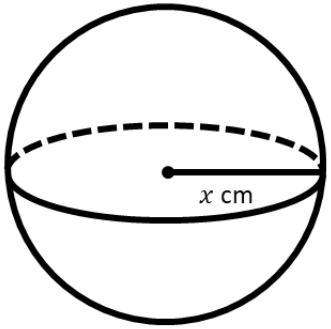
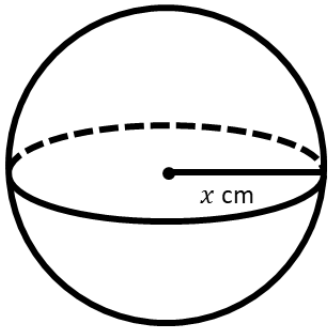
Your Turn

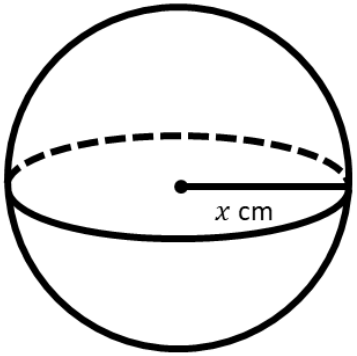
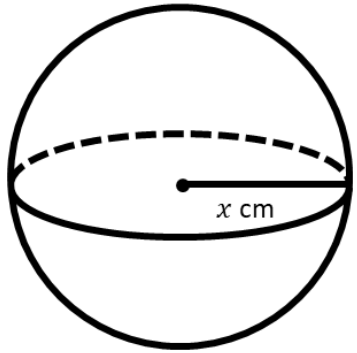
Calculate the volume of the following sphere. Give your answer in terms of π and to 1 decimal place.

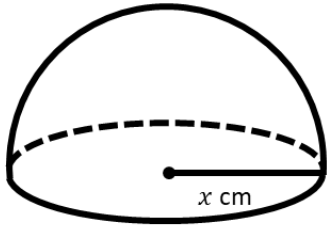
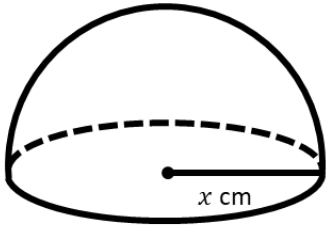


Worked Example	Your Turn
Calculate the volume of the following sphere. Give your answer in terms of π and to 1 decimal place.  	Calculate the volume of the following sphere. Give your answer in terms of π and to 1 decimal place. 

Worked Example	Your Turn
Calculate the volume of the following hemisphere. Give your answer in terms of π and to 1 decimal place.  	Calculate the volume of the following hemisphere. Give your answer in terms of π and to 1 decimal place. 

Worked Example	Your Turn
Find the radius, x, given that the volume of the following sphere is $36\pi\ cm^3$.  	Find the radius, x, given that the volume of the following sphere is $288\pi\ cm^3$. 

Worked Example	Your Turn
Find the radius, x, given that the volume of the following sphere is $113.1\ cm^3$. Give your answer to 1 decimal place.  	Find the radius, x, given that the volume of the following sphere is $904.8\ cm^3$. Give your answer to 1 decimal place. 

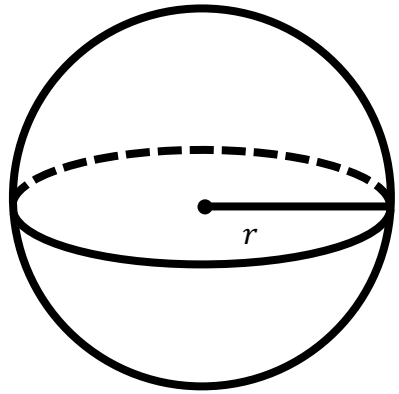
Worked Example	Your Turn
Find the radius, x, given that the volume of the following hemisphere is $56.5\ cm^3$. Give your answer to 1 decimal place.  	Find the radius, x, given that the volume of the following hemisphere is $452.4\ cm^3$. Give your answer to 1 decimal place. 

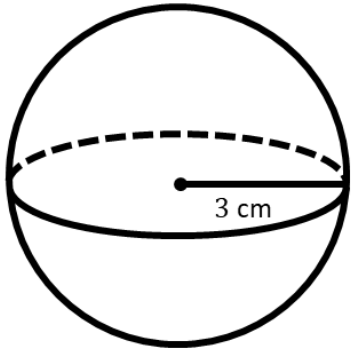
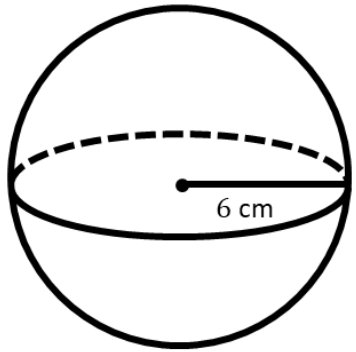
Surface Area of Spheres

Surface Area of Spheres

$$\text{Surface Area of Sphere} = 4 \times \pi \times \text{radius}^2$$

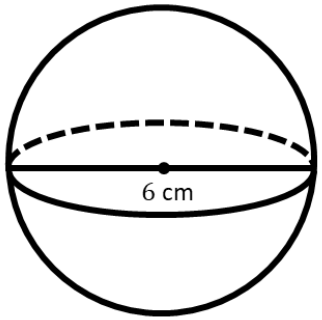
$$\text{Surface Area of Sphere} = 4\pi r^2$$



Worked Example	Your Turn
Calculate the surface area of the following sphere. Give your answer in terms of π and to 1 decimal place.  	Calculate the surface area of the following sphere. Give your answer in terms of π and to 1 decimal place. 

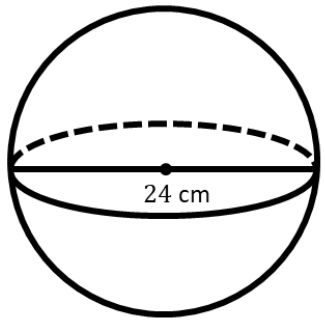
Worked Example

Calculate the surface area of the following sphere. Give your answer in terms of π and to 1 decimal place.



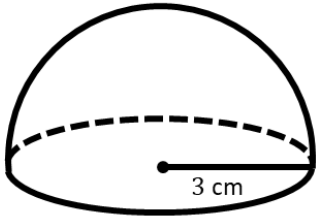
Your Turn

Calculate the surface area of the following sphere. Give your answer in terms of π and to 1 decimal place.



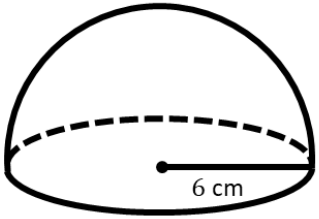
Worked Example

Calculate the curved surface area of the following hemisphere. Give your answer in terms of π and to 1 decimal place.



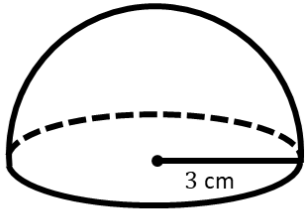
Your Turn

Calculate the curved surface area of the following hemisphere. Give your answer in terms of π and to 1 decimal place.



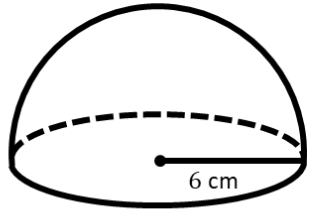
Worked Example

Calculate the total surface area of the following hemisphere. Give your answer in terms of π and to 1 decimal place.



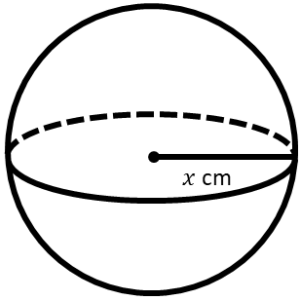
Your Turn

Calculate the total surface area of the following hemisphere. Give your answer in terms of π and to 1 decimal place.



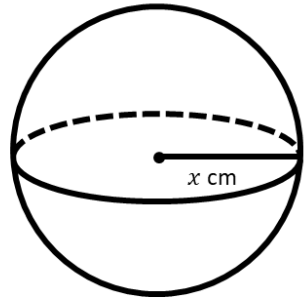
Worked Example

Find the radius, x, given that the surface area of the following sphere is $36\pi\ cm^2$.



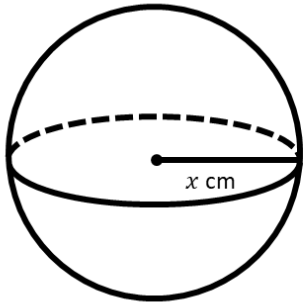
Your Turn

Find the radius, x, given that the surface area of the following sphere is $144\pi\ cm^2$.



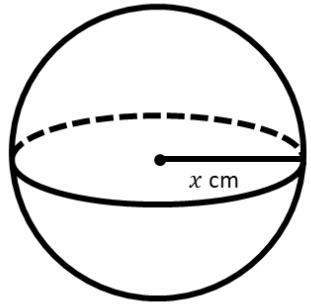
Worked Example

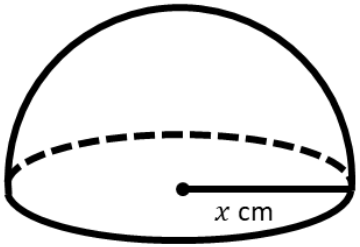
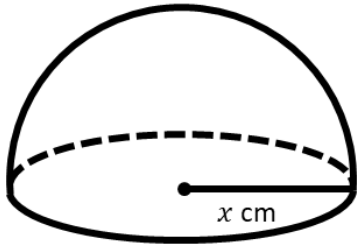
Find the radius, x, given that the surface area of the following sphere is $113.1\ cm^2$. Give your answer to 1 decimal place.



Your Turn

Find the radius, x, given that the surface area of the following sphere is $452.4\ cm^2$. Give your answer to 1 decimal place.



Worked Example	Your Turn
Find the radius, x, given that the total surface area of the following hemisphere is $84.8\ cm^2$. Give your answer to 1 decimal place.  	Find the radius, x, given that the total surface area of the following hemisphere is $339.3\ cm^2$. Give your answer to 1 decimal place. 

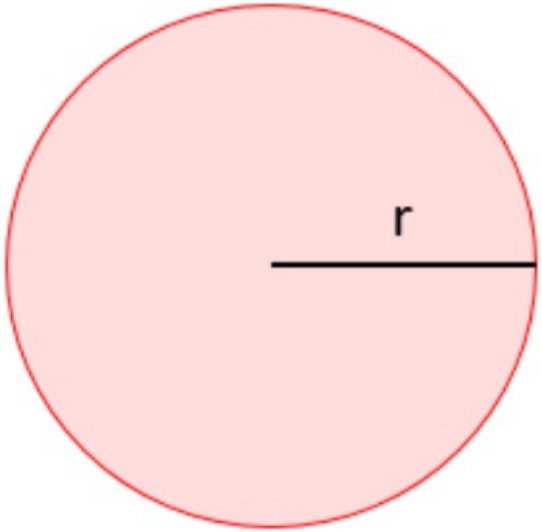
Worked Example	Your Turn
A sphere has a surface area of $36\pi\ cm^2$. Work out the volume of the sphere. Give your answer in terms of π and to 1 decimal place.	A sphere has a surface area of $144\pi\ cm^2$. Work out the volume of the sphere. Give your answer in terms of π and to 1 decimal place.

Worked Example	Your Turn
A sphere has a volume of $36\pi\ cm^3$. Work out the surface area of the sphere. Give your answer in terms of π and to 1 decimal place.	A sphere has a volume of $288\pi\ cm^3$. Work out the surface area of the sphere. Give your answer in terms of π and to 1 decimal place.

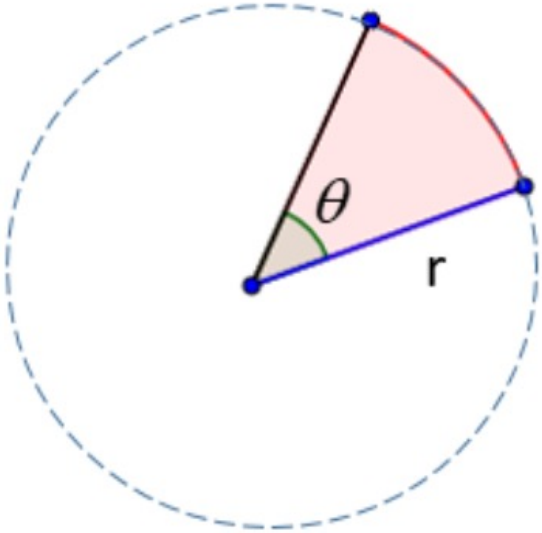
EXTRA NOTES

Area of a sector

Area of Circle and Sector



$$\text{area of circle} = \pi r^2$$

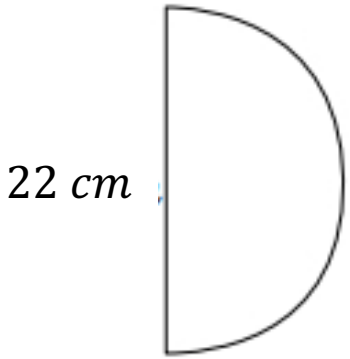


If θ is measured in degrees then

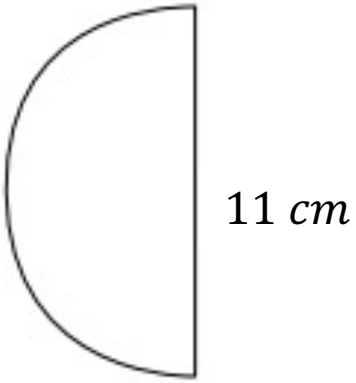
$$\text{area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

Worked example

Calculate the area of the sector:



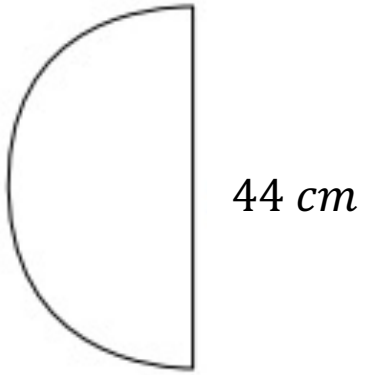
With calculator to 3 s.f.



Without calculator in terms of π

Your turn

Calculate the area of the sector:

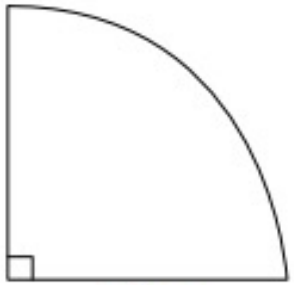


With calculator to 3 s.f.

Without calculator in terms of π

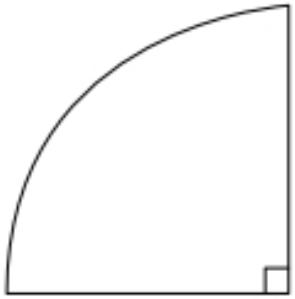
Worked example

Calculate the area of the sector:



$34\ cm$

With calculator to 3 s.f.

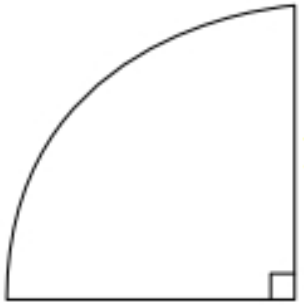


$68\ cm$

Without calculator in terms of π

Your turn

Calculate the area of the sector:



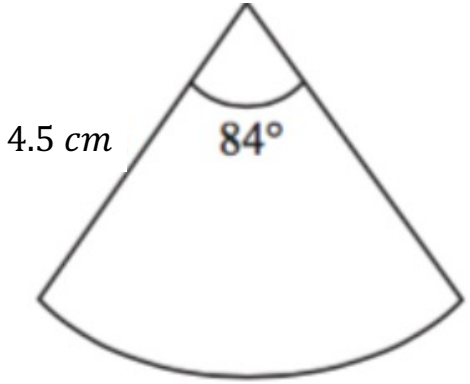
$17\ cm$

With calculator to 3 s.f.

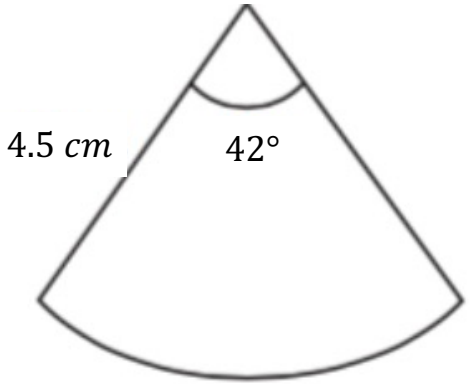
Without calculator in terms of π

Worked example

Calculate the area of the sector:

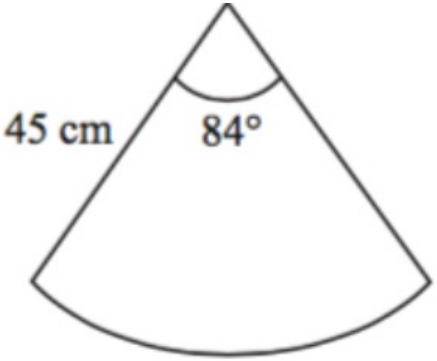


With calculator to 3 s.f.



Your turn

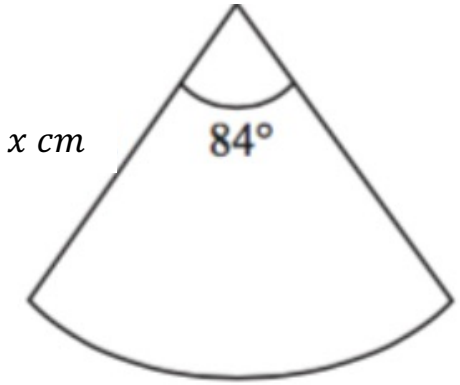
Calculate the area of the sector:



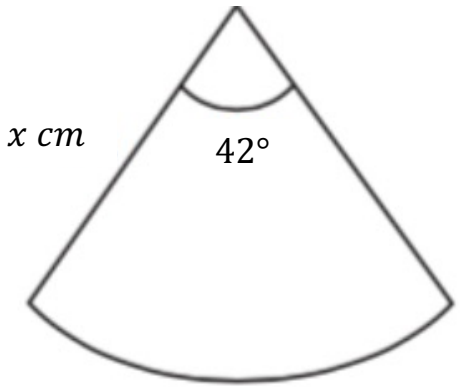
With calculator to 3 s.f.

Worked example

The area of the sector is $14.844\ cm^2$. Find x

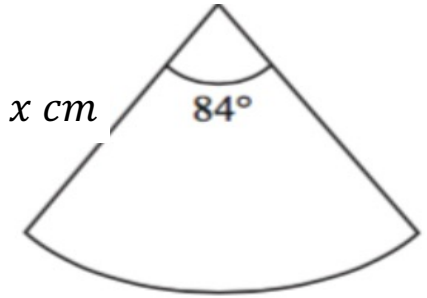


The area of the sector is $7.422\ cm^2$. Find x



Your turn

The area of the sector is $1484.402529\ cm^2$. Find x



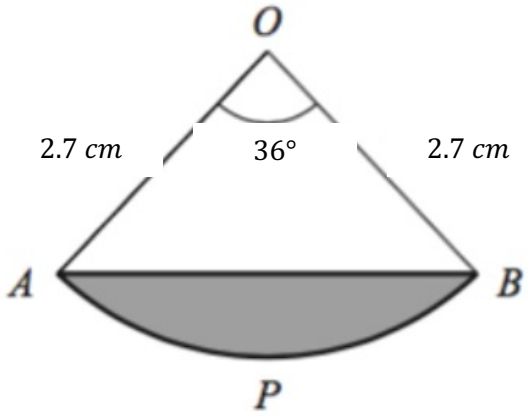
Fill In The Blanks...

Area of a Sector

Radius	Angle	Fraction	Area
$8\ cm$	$90°$	$\frac{90}{360} = \frac{1}{4}$	$\frac{90}{360} \times \pi \times 8^2 = 50.3\ cm^2$
$7\ cm$	$45°$	$\frac{45}{360} = \frac{1}{8}$	
$15\ mm$	$60°$		
$4\ cm$	$75°$		
$1.8\ m$	$130°$		
$11\ cm$	$275°$		
$9\ mm$		$\frac{\square}{360} = \frac{5}{36}$	
$10\ cm$		$\frac{\square}{360} = \frac{7}{9}$	
$25\ mm$			$\frac{\square}{360} \times \pi \times 25^2 = 327.2\ mm^2$
$2\ m$			$\frac{\square}{360} \times \pi \times 2^2 = 4.712\ m^2$
	$35°$		$\frac{35}{360} \times \pi \times \square^2 = 2.75\ cm^2$
	$315°$		$\frac{315}{360} \times \pi \times \square^2 = 464.6\ mm^2$
	$58°$		$\frac{58}{360} \times \pi \times \square^2 = 50.61\ cm^2$

Worked example

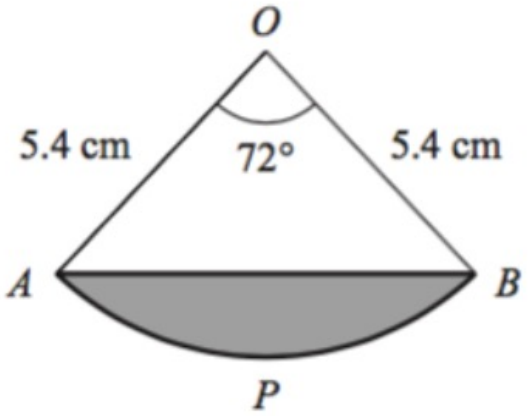
Calculate the area of the shaded segment APB



With calculator to 3 s.f.

Your turn

Calculate the area of the shaded segment APB

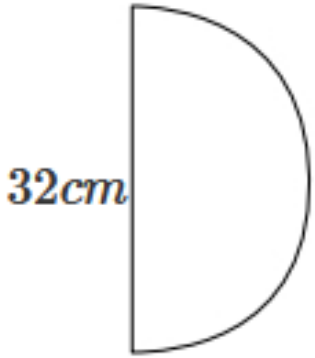


With calculator to 3 s.f.

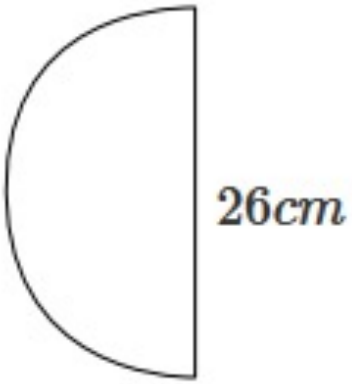
EXTENSION answer in terms of π

Worked example

Calculate the perimeter of the sector:



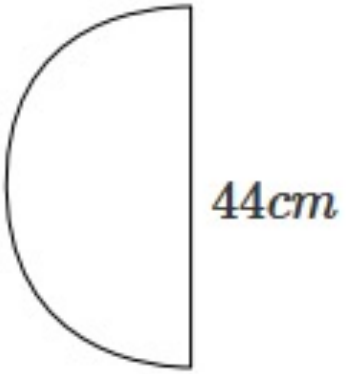
With calculator to 3 s.f.



Without calculator in terms of π

Your turn

Calculate the perimeter of the sector:

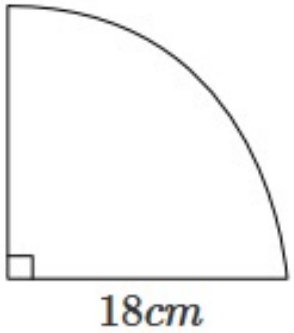


With calculator to 3 s.f.

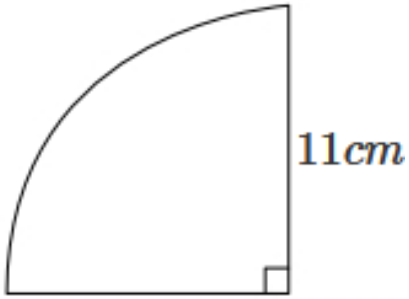
Without calculator in terms of π

Worked example

Calculate the perimeter of the sector:



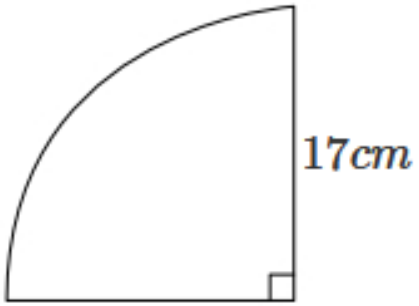
With calculator to 3 s.f.



Without calculator in terms of π

Your turn

Calculate the perimeter of the sector:

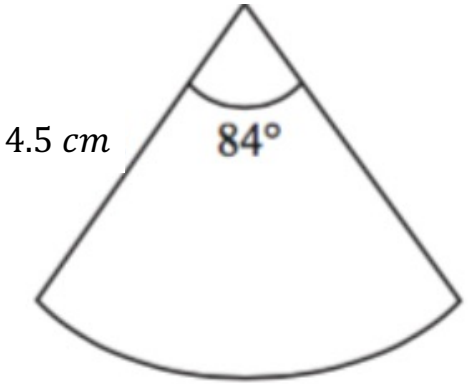


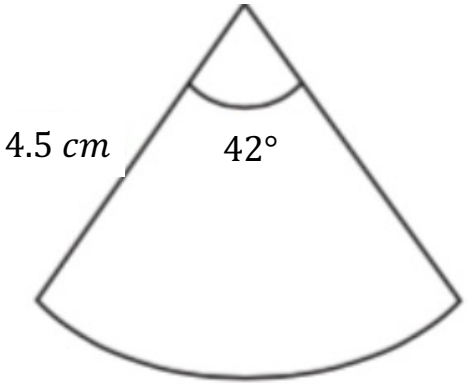
With calculator to 3 s.f.

Without calculator in terms of π

Worked example

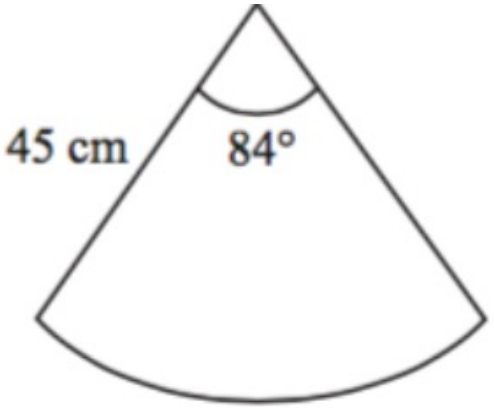
Calculate the **arc length** of the sector:





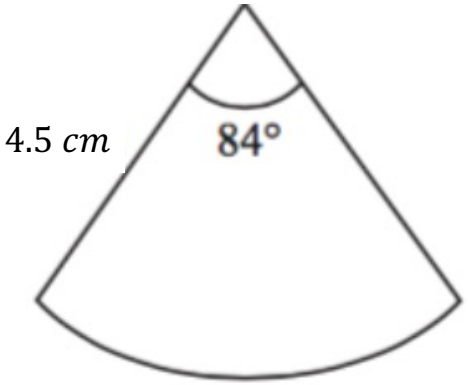
Your turn

Calculate the **arc length** of the sector·

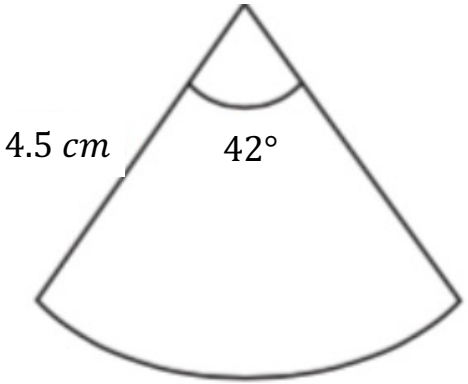


Worked example

Calculate the **perimeter** of the sector:

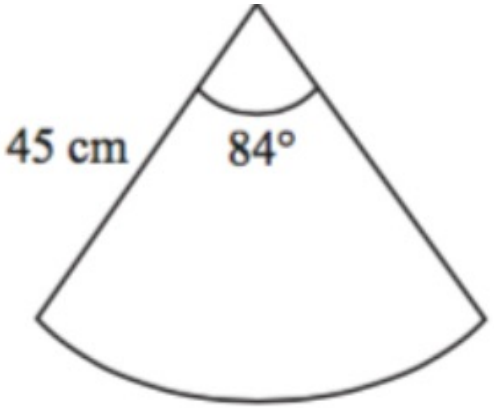


With calculator to 3 s.f.



Your turn

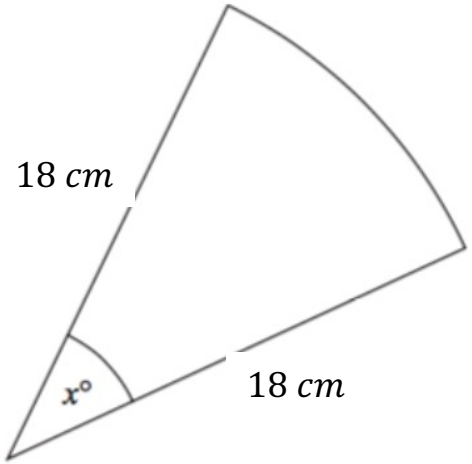
Calculate the **perimeter** of the sector.



With calculator to 3 s.f.

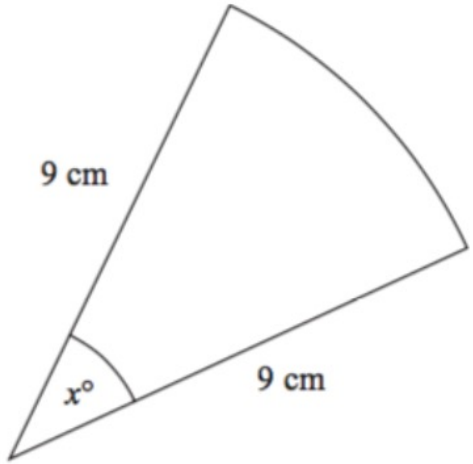
Worked example

The perimeter of the sector is 50 cm. Find x



Your turn

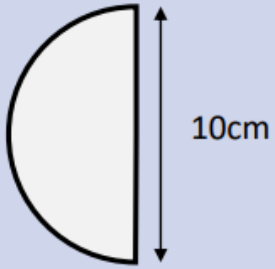
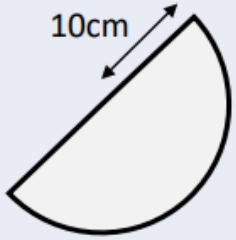
The perimeter of the sector is 25 cm. Find x

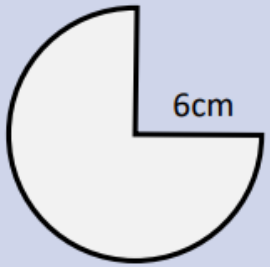


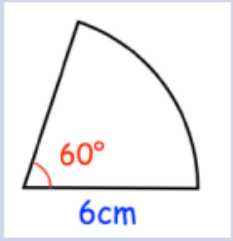
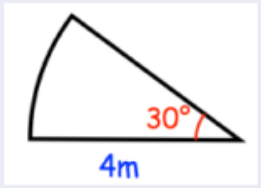
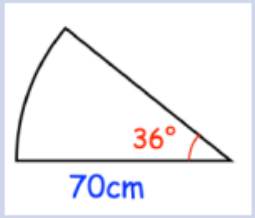
Fill In The Blanks...

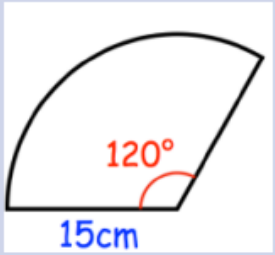
Arc Length and Perimeter of a Sector

Radius	Angle	Arc Length	Perimeter
$8\ cm$	$90°$	$\frac{90}{360} \times \pi \times 2 \times 8 = 12.6\ cm$	$28.6\ cm$
$7\ cm$	$45°$	$\frac{45}{360} \times \pi \times 2 \times 7 = 5.5\ cm$	
$15\ mm$	$60°$	$\frac{60}{360} \times \pi \times 2 \times 15 = 15.7\ mm$	
$4\ cm$	$75°$		
$1.8\ m$	$130°$		
$11\ cm$	$275°$		
$9\ mm$	$32°$		
$10\ cm$		$\frac{\square}{360} \times \pi \times 2 \times 10 = 13.96\ cm$	
$25\ mm$		$\frac{\square}{360} \times \pi \times 2 \times 25 = 93.81\ mm$	
$2\ m$		$\frac{\square}{360} \times \pi \times 2 \times 2 = \square\ m$	$5.05\ m$
$8.9\ cm$		$\frac{\square}{360} \times \pi \times 2 \times 8.9 = \square\ cm$	$35.2\ m$
		$\frac{\square}{360} \times \pi \times 2 \times \square = 4.61\ cm$	$15.61\ cm$
		$\frac{\square}{360} \times \pi \times 2 \times \square = 55.29\ mm$	$99.29\ mm$

Sector.	Radius.	Diameter.	Fraction of the whole circle.	Arc length.	Length of straight sides.	Perimeter in terms of pi.	Perimeter to 1 decimal place.
	$5cm$	$10cm$	$\frac{1}{2}$	$5\pi = 15.7cm$	$10cm$	$(5\pi + 10)\ cm$	$25.7cm$
							
							

Sector.	Radius.	Diameter.	Fraction of the whole circle.	Arc length.	Length of straight sides.	Perimeter in terms of pi.	Perimeter to 1 decimal place.
							
		$4cm$	$\frac{1}{4}$				
	$3cm$		$\frac{1}{2}$				

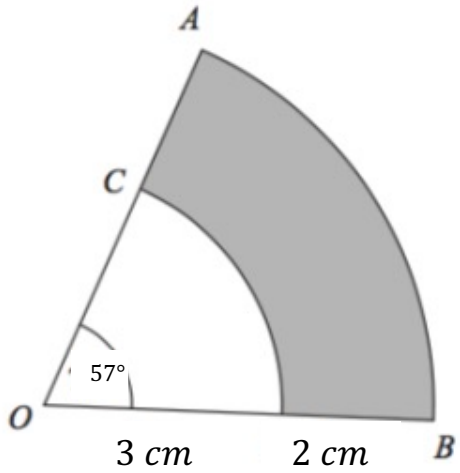
Sector.	Radius.	Diameter.	Fraction of the whole circle.	Arc length.	Length of straight sides.	Perimeter in terms of pi.	Perimeter to 1 decimal place.
							
							
							

Sector.	Radius.	Diameter.	Fraction of the whole circle.	Arc length.	Length of straight sides.	Perimeter in terms of pi.	Perimeter to 1 decimal place.
							
		$10cm$	$\frac{1}{3}$				
	$10cm$		$\frac{1}{8}$				

Sector.	Radius.	Diameter.	Fraction of the whole circle.	Arc length.	Length of straight sides.	Perimeter in terms of pi.	Perimeter to 1 decimal place.
			$\frac{1}{2}$		$7cm$		
			$\frac{1}{3}$			$(4\pi + 12)\ cm$	
			$\frac{1}{10}$		$40cm$		$46.3cm$

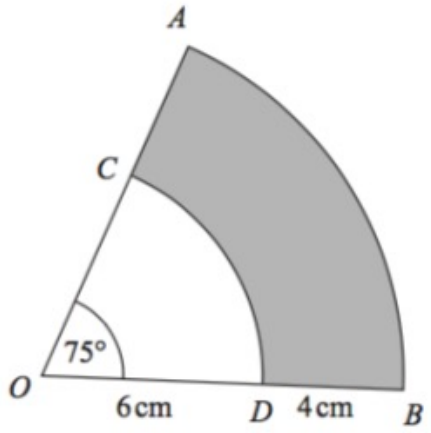
Worked example

Calculate the perimeter of the shaded region



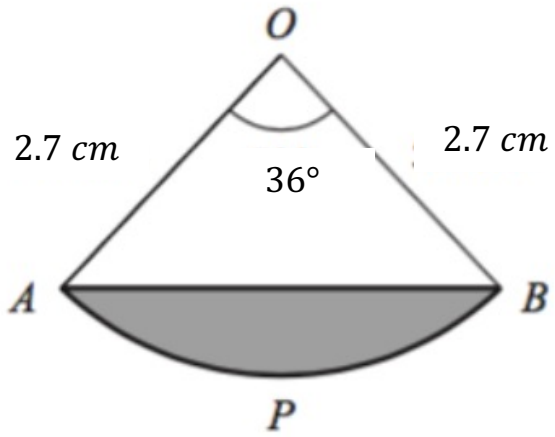
Your turn

Calculate the perimeter of the shaded region



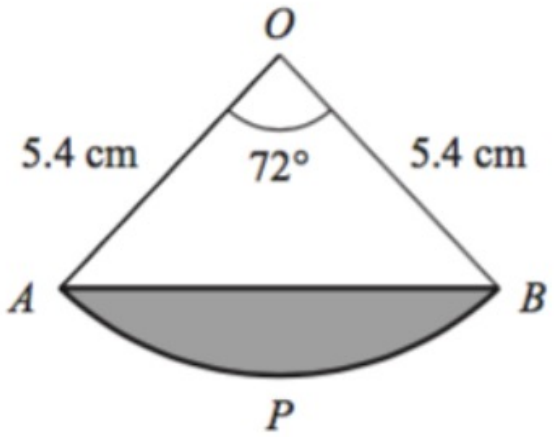
Worked example

Calculate the perimeter of the shaded segment APB



Your turn

Calculate the perimeter of the shaded segment APB



EXTRA NOTES

Worked example

80 students visited the library over three days.
The two-way table shows some information about these students.

	Monday	Tuesday	Wednesday	Total
Year 7			20	64
Year 8	9			
Total		25	36	100

(a) Complete the two-way table.

(b) Write down the probability that the student is in Year 7.

(c) Write down the probability that the student visited the library on Tuesday.

Your turn

80 students visited the library over three days.
The two-way table shows some information about these students.

	Monday	Tuesday	Wednesday	Total
Year 7			13	38
Year 8	14			
Total		33	26	80

(a) Complete the two-way table.

(b) Write down the probability that the student is in Year 7.

(c) Write down the probability that the student visited the library on Tuesday.

Worked example

K55c: Determine a probability from a two-way table.

78 people were asked if they prefer to go on holiday in Croatia or in Portugal or in France.

The responses are shown in the two-way table below.

	Croatia	Portugal	France	Total
Female	11	22	10	43
Male	7	9	19	35
Total	18	31	29	78

One of the people is chosen at random.

What is the probability that this person is a male that said Portugal?

Your turn

K55c: Determine a probability from a two-way table.

60 students each attended one revision lesson at the weekend.

Each student went to English, History or Science.

The two-way table below shows the attendance of each revision lesson.

	English	History	Science	Total
Saturday	5	17	7	29
Sunday	9	15	7	31
Total	14	32	14	60

One of the students that attended on Sunday is picked at random.

Find the probability that this student attended History.

Venn Diagrams and Probability

A set is a collection of numbers, or letters, or symbols, or objects, etc., which are related in some way.

The items in a set are called '**members**' or '**elements**'

Curly brackets (often called 'braces') are usually used when listing or describing sets – this helps to distinguish sets from lists of unrelated items.

The elements within a set are usually described in words or listed

Examples:

Description in words	List of elements
{even numbers less than 11}	{2, 4, 6, 8, 10}
{the first five prime numbers}	{2, 3, 5, 7, 11}
{multiples of three between 10 and 20}	{12, 15, 18}
{factors of 27 which are even}	{ }

More examples of sets:

Description in words	List of elements
{quadrilaterals with four equal length sides}	{square, rhombus}
{vowels}	{a, e, i, o, u}
{letters in the word 'banana'}	{a, b, n}
{yellow fruit}	{grapefruit, banana, lemon, …}

Notes:
Elements are only ever included once – as shown with {letters in the word 'banana'} = {a, b, n}

{yellow fruits} is an imprecise description and the list of elements contains only examples.

What is a set?

In mathematics, it is often useful to represent **a collection of items**.
We use curly braces to indicate a **set** of items...

$\{-4, 1, 3\}$

A set is a collection of items with 2 properties:

a) It does not contain duplicates.
b) The order of the elements does not matter. (but we usually write the items in ascending order)

Is it a set?

a) $\{-3.5, 2, 9\}$
b) $\{4, 5, 5, 6\}$
c) $\{1\}$
d) $\{\{1,2\}, \{3,4\}\}$
e) $\{red, blue, green\}$

Are these sets the same?

$\{3,1,2\} = \{1,2,3\}$

Venn Diagrams and Probability

Finite Sets vs Infinite Sets

The examples with seen have been **finitely** large sets.

- $\{-4, 1, 3\}$

But it is also possible to have sets which are **infinitely** large…

- "the set of all positive integers (whole numbers)"
- "the set of all odd numbers"

The Universal set is the set of all elements under consideration.

Elements that can be in other sets are restricted to those within the Universal set. For example, if the Universal set was {integers less than 10}, then {prime numbers} would be limited to {2, 3, 5, 7}.

Likewise if the Universal set was {even numbers}, then {factors of 18} would be {2, 6, 18}

Notation

In Britain the special symbol '$\mathcal{E}$' is used to represent the Universal set but in some countries, such as America, the letter 'U' is used.

Thus we could write

$\mathcal{E}$ = {integers less than 10} or $\mathcal{E}$ = {prime numbers}

When we have more than one set, capital letters are usually used to represent them.

Examples:

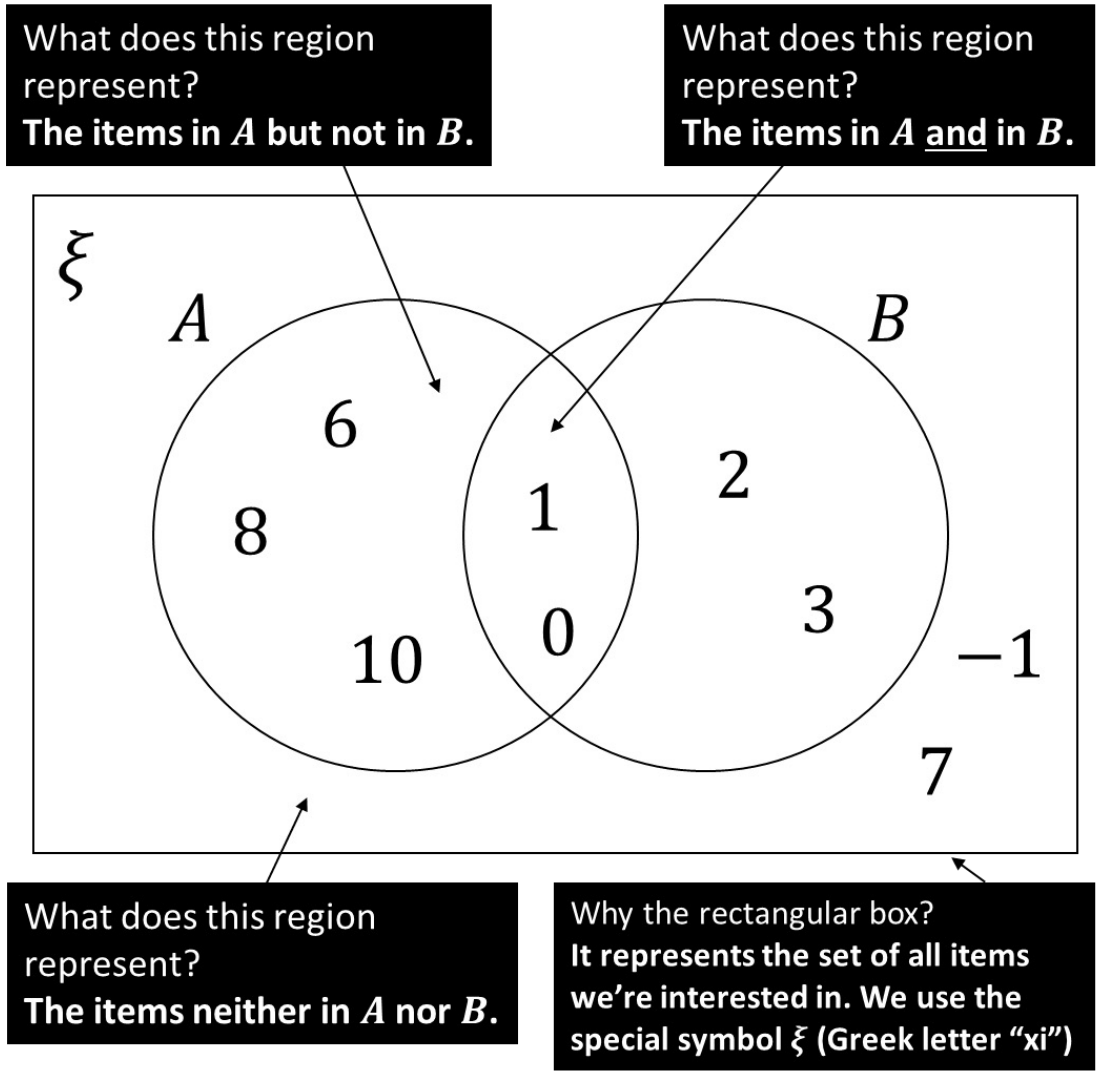
Description in words	List of elements
A = {prime numbers between 10 and 20}	A = {11, 13, 17, 19}
B = {factors of 24}	B = {1, 2, 3, 4, 6, 8, 12, 24}
C = {vowels}	C = {a, e, i, o, u}

Note that it is often convenient to use letters that are in some way connected to the description of the set.

e.g. P = {prime numbers between 10 and 20}, F = {factors of 24} and V = {vowels}

Venn Diagrams and Probability

Venn Diagrams are a way of showing the items in each set.



Worked example

List the following sets:

a) {factors of 15}

b) {the first four square numbers}

c) {letters in the word LONDON}

d) {possible outcomes when an ordinary coin is thrown}

Your turn

List the following sets:

a) {the first four multiples of 15}

b) {the first four cube numbers}

c) {letters in the word BIRMINGHAM}

d) {possible outcomes when an ordinary dice is thrown}

Worked example

a) U = {odd numbers less than 15}
A = {prime numbers}
B = {multiples of 3}
List:

i) A

ii) B

b) U = {first 10 letters of the alphabet}
X = {vowels}
Y = {letters in the word 'ENGLISH'}
List:

i) X

ii) Y

c) U = {factors of 24}
P = {prime numbers}
E = {even numbers}
O = {odd numbers}
List:

i) P

ii) E

iii) O

Your turn

a) U = {even numbers less than 15}
A = {prime numbers}
B = {multiples of 3}
List:

i) A

ii) B

b) U = {first 10 letters of the alphabet}
X = {vowels}
Y = {letters in the word 'FRENCH'}
List:

i) X

ii) Y

c) U = {factors of 30}
P = {prime numbers}
E = {even numbers}
O = {odd numbers}
List:

i) P

ii) E

iii) O

Worked example

Represent as a Venn diagram:
$\xi = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$A = \{0, 1, 3, 5, 8\}$
$B = \{2, 5, 8, 9\}$

Your turn

Represent as a Venn diagram:
$\xi = \{2, 3, 4, 5, 7, 11, 13, 17, 19\}$
$A = \{2, 3, 5, 11, 13\}$
$B = \{5, 7, 13, 17, 19\}$

Worked example

Represent as a Venn diagram:

ξ =Positive integers between 1 and 10 inclusive

$A = \{$Prime numbers$\}$

$B = \{$Even numbers$\}$

Your turn

Represent as a Venn diagram:

$\xi =$ Integers between 0 and 5 inclusive

$A = \{$Prime numbers$\}$

$B = \{$Odd numbers$\}$

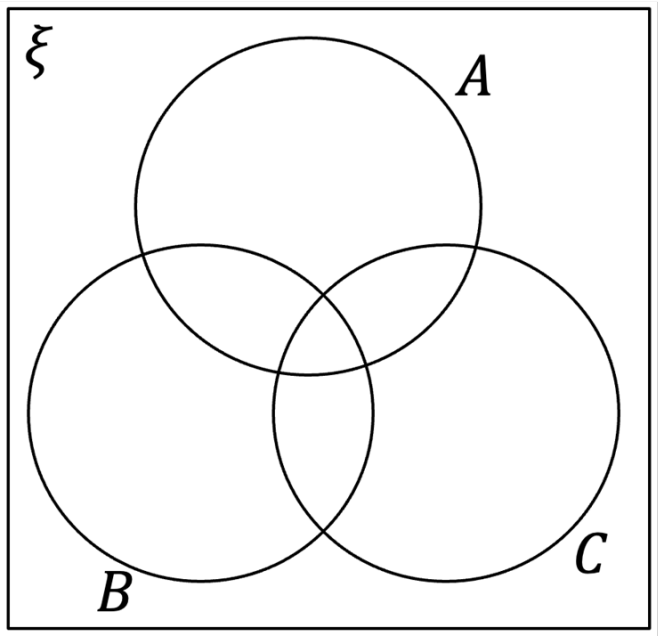
Worked example

ξ = whole numbers from 1 to 15

A = set of all prime numbers

B = set of all numbers one less than a power of 2

C = set of all square numbers



Your turn

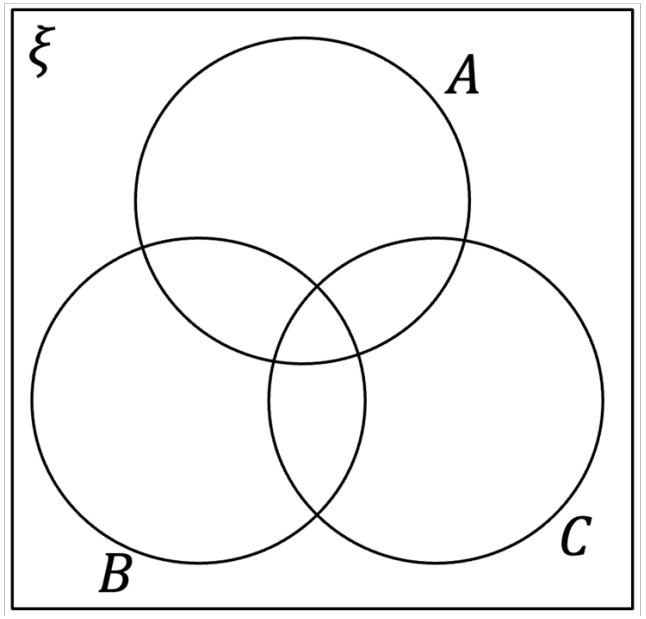
ξ = whole numbers from 1 to 10

A = set of all cube numbers

B = set of all odd numbers

C = set of all multiples of 3

Bonus: If we extended ξ to include more positive integers, what is the smallest number that would appear in all three of A, B, C?



Worked example

Represent in a Venn diagram:

$\xi = \{Integers\ between\ 1\ and\ 10\ inclusive\}$

$A = \{odd\ numbers\}$

$B = \{numbers\ greater\ than\ 4\}$

$C = \{numbers\ less\ than\ 3\}$

Your turn

Represent in a Venn diagram:

$\xi = \{Integers\ between\ 1\ and\ 20\ inclusive\}$

$A = \{prime\ numbers\}$

$B = \{square\ numbers\}$

$C = \{even\ numbers\}$

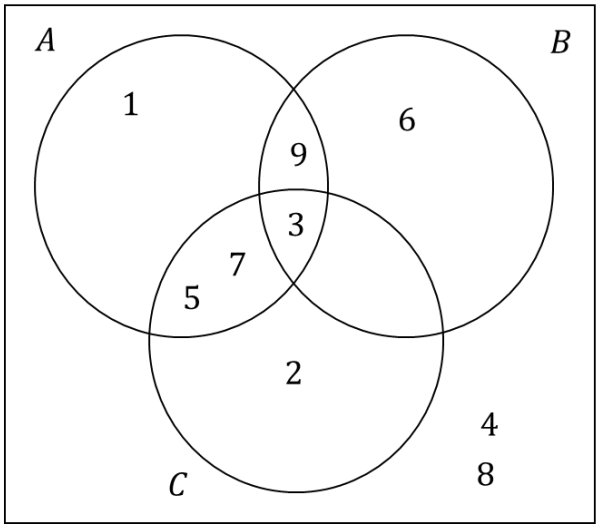
Worked example

From the Venn diagram below, write in roster notation:

$\xi =$

$A =$

$B =$



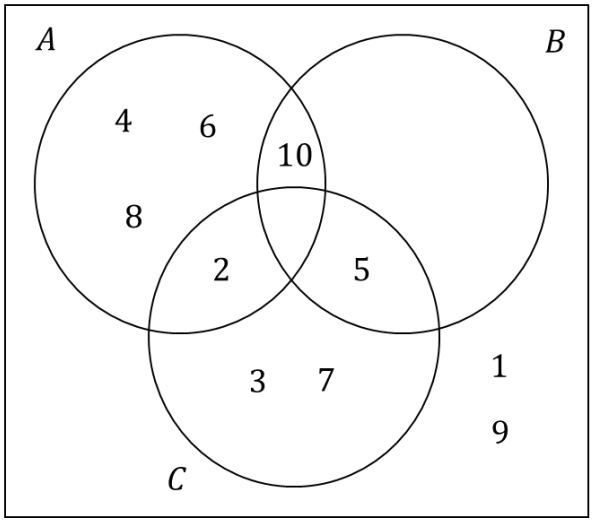
Your turn

From the Venn diagram below, write in roster notation:

$\xi =$

$A =$

$B =$



Worked example

There are 150 pupils. The examinations available are: English, Maths and Science.

- 15 pupils are sitting English and Maths but not science.
- 20 pupils are sitting Science and Maths but not English.
- 18 pupils are sitting Science and English but not Maths.
- 8 pupils are sitting all three exams.
- 55 are sitting English in total.
- 72 are sitting Maths in total.
- 65 are sitting Science in total.

A pupil is chosen at random. What is the probability that they are sitting no exams?

Your turn

There are 130 pupils. The examinations available are: English, Maths and Science.

- 10 pupils are sitting English and Maths but not science.
- 20 pupils are sitting Science and Maths but not English.
- 9 pupils are sitting Science and English but not Maths.
- 13 pupils are sitting all three exams.
- 49 are sitting English in total.
- 83 are sitting Maths in total.
- 62 are sitting Science in total.

A pupil is chosen at random. What is the probability that they are sitting no exams?

Worked example

In a group of 28 scientists:

- 20 have degrees in Physics.
- 18 have degrees in Chemistry.
- Some have degrees in both.
- 4 scientists have degrees which are neither Physics nor Chemistry.

A scientist is chosen at random. Find the probability that the scientist has a degree in:

a) Physics

b) Chemistry

c) Both Physics and Chemistry

d) Neither Physics nor Chemistry

Your turn

In a group of 30 mathematicians:

- 15 have studied Calculus.
- 22 have studied Topology.
- Some have studied both.
- 3 mathematicians have not yet studied either Calculus or topology.

A mathematician is chosen at random. Find the probability that the mathematician has studied:

a) Calculus

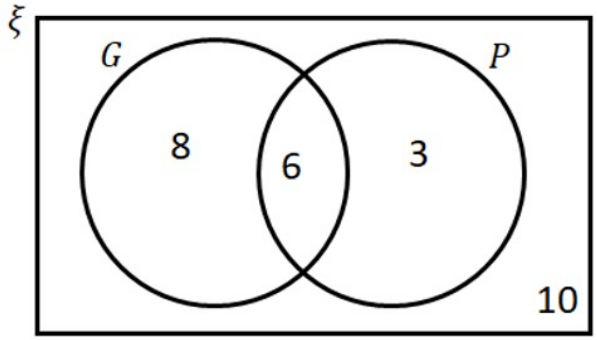
b) Topology

c) Both Calculus and Topology

d) Neither Calculus nor topology

Worked example

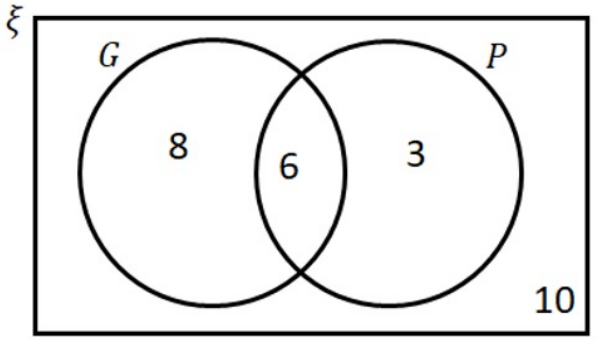
The Venn diagram shows a sample of people who play the guitar (G) or piano (P).



Find the probability that a student plays the guitar, given that they play the piano.

Your turn

The Venn diagram shows a sample of people who play the guitar (G) or piano (P).



Find the probability that a student plays the piano, given that they play the guitar.

Worked example

A vet surveys 100 of her clients. She finds that
25 own dogs, 15 own dogs and cats, 11 own dogs and tropical fish, 53 own cats, 10 own cats and tropical fish, 7 own dogs, cats and tropical fish, 40 own tropical fish.

Draw a Venn Diagram, and hence answer the following questions:

a) $\mathrm{P}(\text{owns dog only})$
b) $\mathrm{P}(\text{does not own tropical fish})$
c) $\mathrm{P}(\text{does not own dogs, cats, or tropical fish})$
d) Given that a randomly chosen person owns a cat, what's the probability they own a dog?

Your turn

The following shows the results of a survey on the types of exercise taken by a group of 100 people.
65 run, 8 swim, 60 cycle, 40 run and swim, 30 swim and cycle, 35 run and cycle and 25 do all three

a) Draw a Venn Diagram to represent these data.

Find the probability that a randomly selected person from the survey

b) takes none of these types of exercise,
c) swims but does not run,
d) takes at least two of these types of exercise.

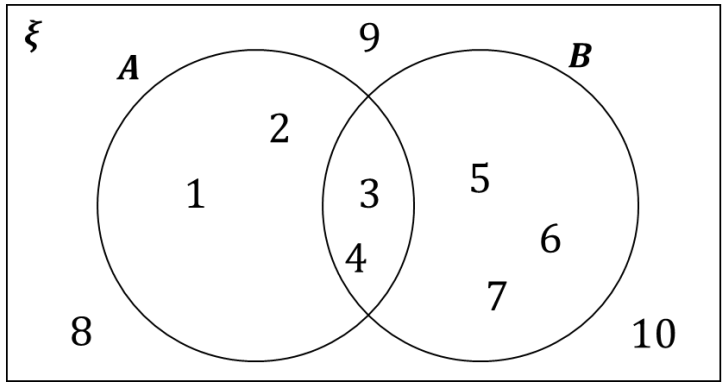
Jason is one of the above group. Given that Jason runs,

e) find the probability that he swims but does not cycle.

Combining Sets

We have various operations on numbers, such as addition:
$1 + 2 = 3$ and multiplication: $2 \times 3 = 6$

So are there similar operations on sets? Yes!



$A \cap B = \{3,4\}$

$A \cap B$ is the **intersection** of A and B
It means "the things in A **and** in B"

$A \cup B = \{1,2,3,4,5,6,7\}$

$A \cup B$ is the **union** of A and B
It means "the things in A **or** in B"*

* Things in A or B also includes things in both.

$A' = \{5,6,7,8,9,10\}$

A' is the **complement** of A
It means "the things **not in A**"

Notation

Complement: ‘

The <u>opposite</u> of a set.

B’ = everywhere not in B

Intersection: ∩

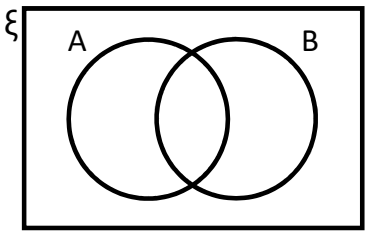
The <u>overlap</u> of regions.

A ∩ **B** = everywhere A and B overlap

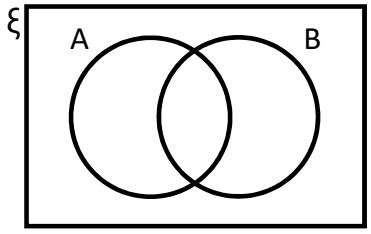
Union: ∪

The <u>sum</u> of regions.

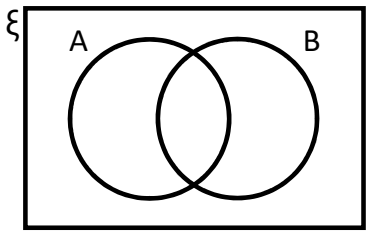
A ∪ **B** = A added to B



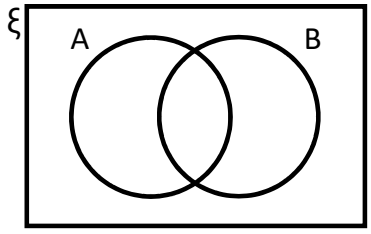
A



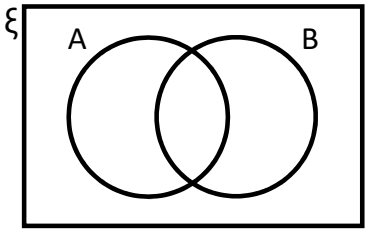
B



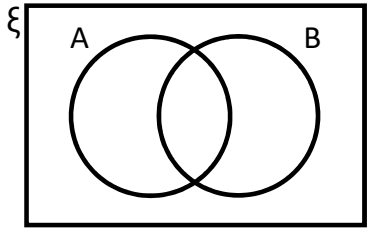
A’



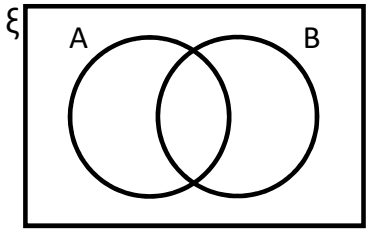
B’



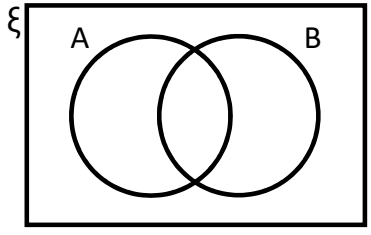
A ∪ B



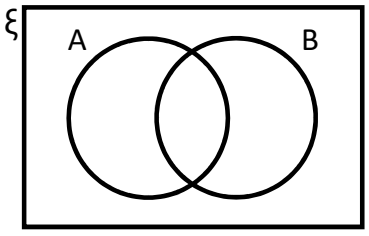
A ∪ B’



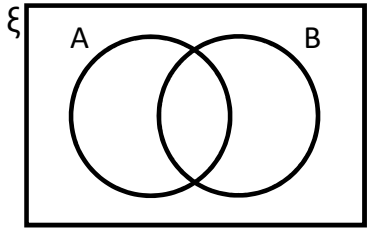
A’ ∪ B



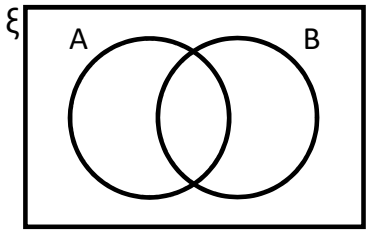
A’ ∪ B’



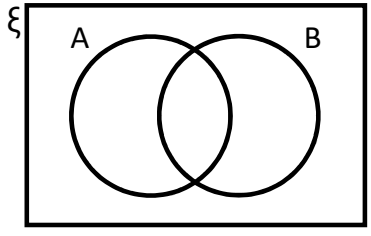
A ∩ B



A ∩ B’



A’ ∩ B



A’ ∩ B’

Worked example

$\xi = \{1,2,3,\ldots,10\}$
$A = \{2,4,6,8,10\}$
$B = \{3,6,9\}$

a) $A \cap B =$

b) $A \cup B =$

c) $A' =$

d) $B' =$

e) $A \cap B' =$

f) $A' \cap B =$

g) $A' \cap B' =$

Your turn

$\xi =$ { all whole numbers }
$A =$ { factors of 60 }
$B =$ { multiples of 3 }

a) $A \cap B =$

b) $A \cup B =$

c) $A' =$

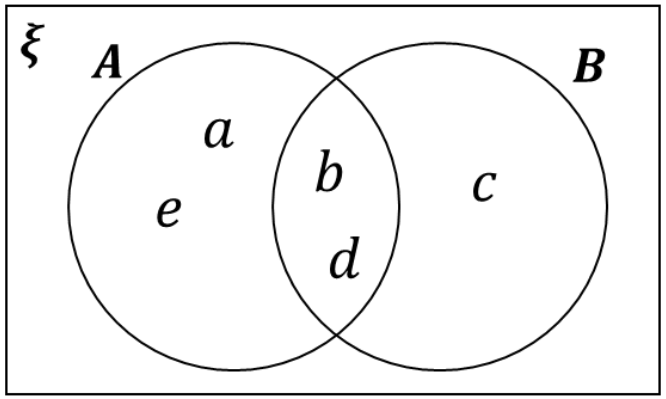
d) $B' =$

e) $A \cap B' =$

f) $A' \cap B =$

g) $A' \cap B' =$

Worked example



a) $A \cap B =$

b) $A \cup B =$

c) $A' =$

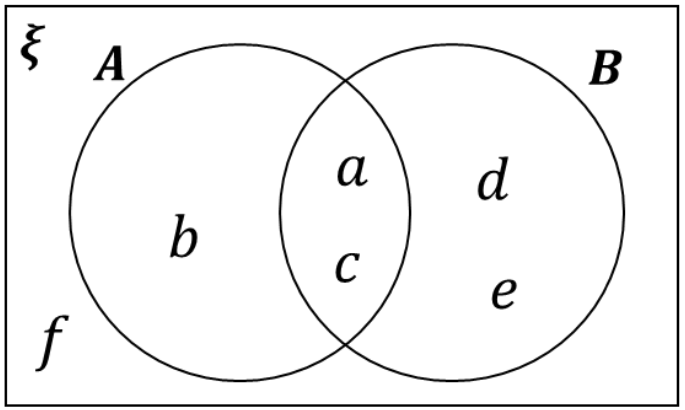
d) $B' =$

e) $A \cap B' =$

f) $A' \cap B =$

g) $A' \cap B' =$

Your turn



a) $A \cap B =$

b) $A \cup B =$

c) $A' =$

d) $B' =$

e) $A \cap B' =$

f) $A' \cap B =$

g) $A' \cap B' =$

Fill In The Blanks...

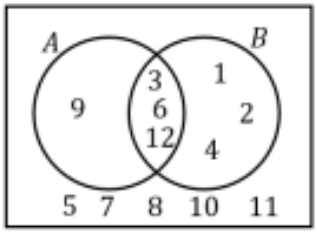
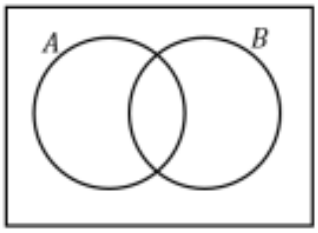
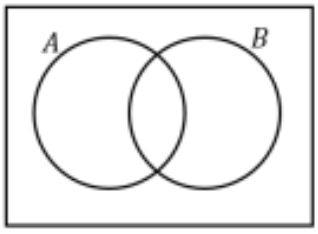
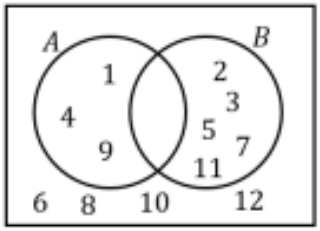
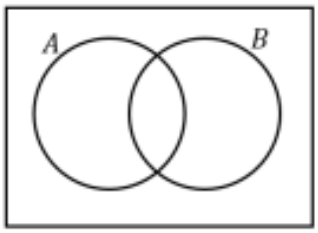
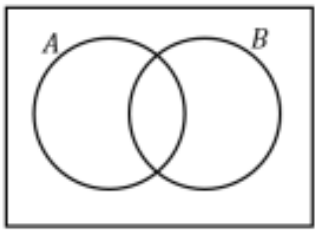
Basic Set Notation

A	B	$A \cap B$	$A \cup B$
$\{1, 2, 3, 4, 5\}$	$\{4, 5, 6, 7, 8\}$	$\{4, 5\}$	$\{1, 2, 3, 4, 5, 6, 7, 8\}$
$\{1, 3, 5, 7\}$	$\{5, 6, 7, 8, 9\}$		
$\{a, b, c, d, e\}$	$\{b, c, d, e, f\}$		
$\{0, 1, 2, 3\}$	$\{4, 5, 6, 7, 8\}$		
Odd numbers from 1 to 9 inclusive	*Prime numbers less than 10*		
Square numbers less than 20	*Multiples of 4 from 4 to 20 inclusive*		
Even numbers from 2 to 12 inclusive	*Multiples of 3 less than 15*		
$\{1, 4, 7, 10, 13\}$	*Square numbers less than 20*		
Odd numbers from 1 to 9 inclusive	*Even numbers from 2 to 10 inclusive*		
$\{5, 6, 7, 8, 9\}$		$\{5, 6\}$	$\{3, 4, 5, 6, 7, 8, 9\}$
	$\{2, 4, 6, 8\}$	$\{2, 4, 6\}$	$\{1, 2, 3, 4, 5, 6, 8\}$
$\{11, 12, 13, 14\}$		$\{13\}$	$\{11, 12, 13, 14\ 17, 19, 23\}$
	$\{4, 5, 6, 7\}$	$\{\}$	$\{0, 1, 2, 3, 4, 5, 6, 7\}$
Square numbers less than 20		$\{1, 4, 16\}$	$\{1, 2, 4, 8, 9, 16\}$
	Factors of 10	$\{5, 10\}$	$\{1, 2, 5, 10, 15, 20\}$

Fill In The Blanks...

Venn Diagrams and Set Notation

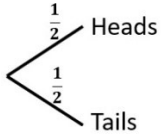
In all questions $\xi = \{Integers\ from\ 1\ to\ 12\ inclusive\}$

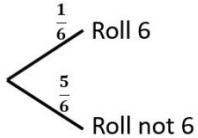
Sets	Venn Diagram	$A \cup B$	$A \cap B$	$A' \cap B$	$(A \cup B)'$
$A =$ $\{multiples\ of\ 3\}$ $B =$ $\{factors\ of\ 12\}$			$\{3, 6, 12\}$		$\{5, 7, 8, 10, 11\}$
$A =$ $\{prime\ numbers\}$ $B =$ $\{odd\ numbers\}$					
$A =$ $\{integers\ less\ than\ 7\}$ $B =$ $\{square\ numbers\}$					
					
			$\{1, 8\}$	$\{\}$	$\{3, 5, 6, 7, 9, 10, 11, 12\}$
		$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$	$\{6, 9, 12\}$	$\{5, 7, 8, 10, 11\}$	

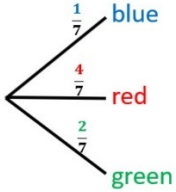
EXTRA NOTES

Tree Diagrams

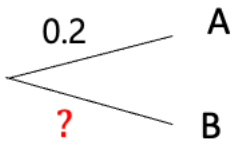
Structuring Probability Trees for Single Events



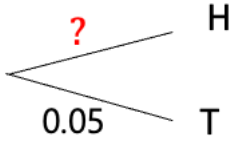




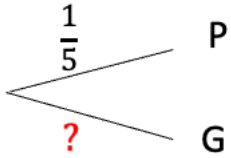
a.



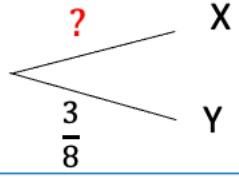
b.



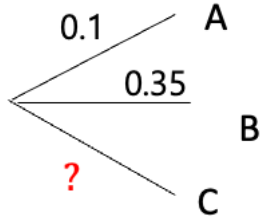
c.



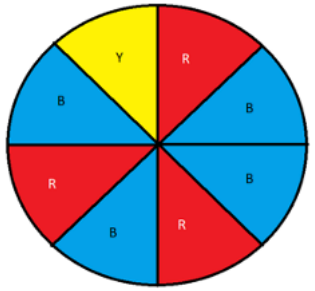
d.



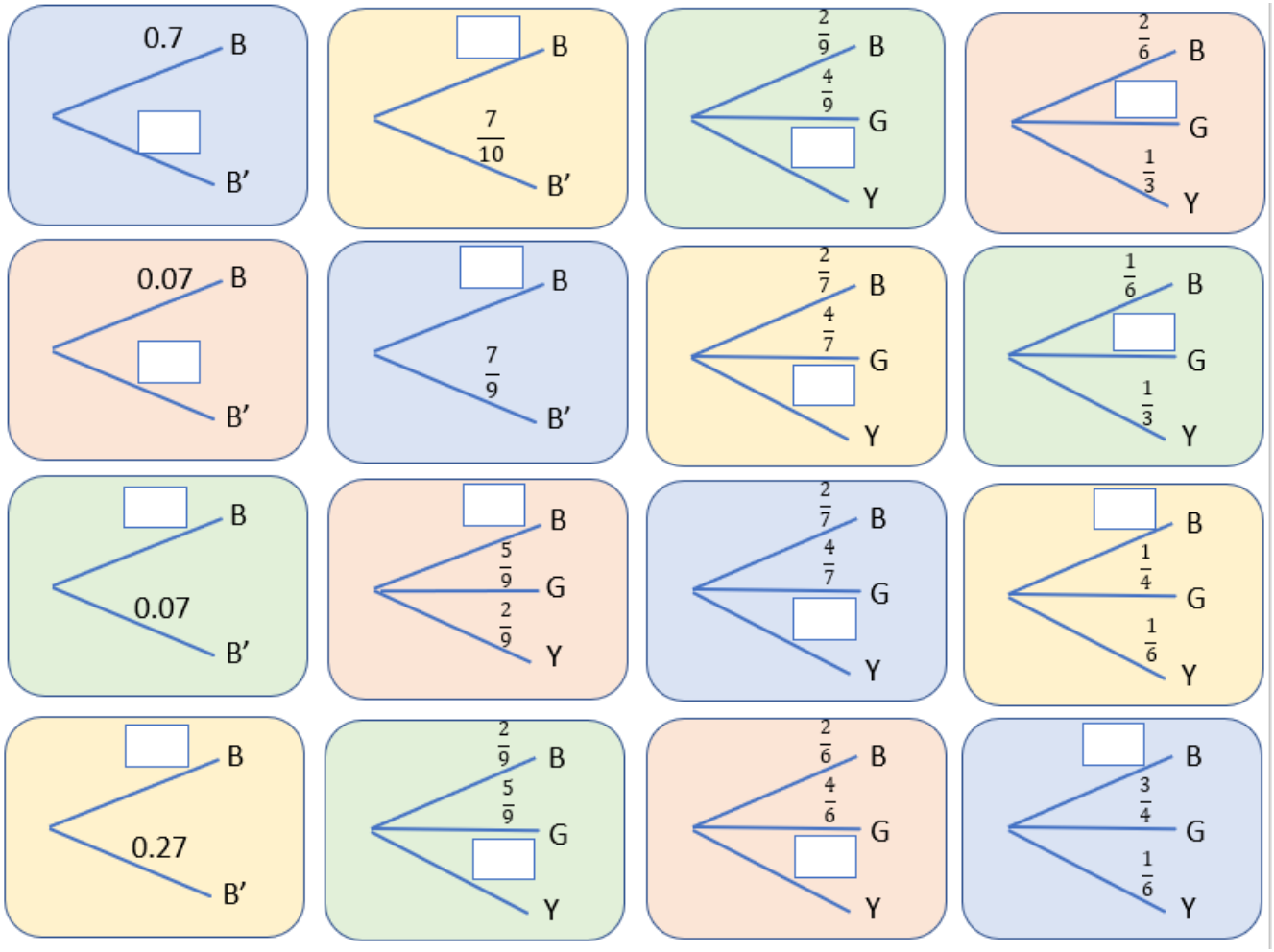
e.



f. Draw a probability tree for this spinner.



Fluency Practice

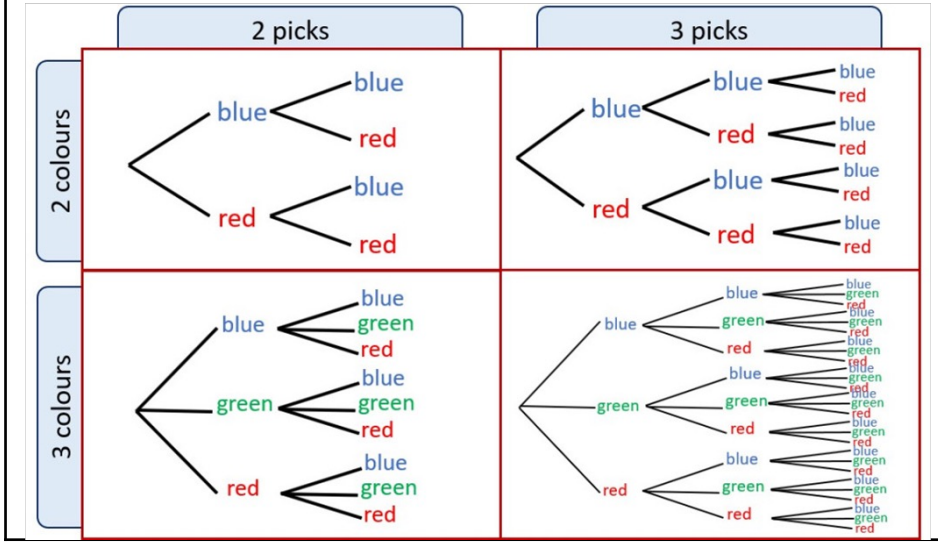


Structuring Probability Trees

Represent each scenario using a probability tree.

(1) There are some blue and red marbles in a bag. I pick **two** marbles.	(2) There are some blue and red marbles in a bag. I pick **three** marbles.
(3) There are some blue, green and red marbles in a bag. I pick **two** marbles.	(4) There are some blue, green and red marbles in a bag. I pick **three** marbles.

- How do we know how many branches to draw at each intersection?
- How do we know how many layers of branches to draw?



Represent each scenario using a probability tree.

(A) You play a game three times.
Each time, you can either win, lose or draw.

(B) You flip a coin three times.

(c) Three people are travelling to work separately.
Each person is either on time or late.

- Why do the probability trees for 'Marble Scenario 4' and 'Contextual Scenario A' have the same structure?
- Which of the marble scenarios has the same structure as Contextual Scenario C?
- How could I alter one of the contextual scenarios to make it have a tree like Marble Scenario 3?

Represent each scenario using a probability tree.

One bag contains red, blue and green marbles.
Another bag contains only red and blue marbles.

I pick a marble from each bag.

There are some blue marbles and 1 red marble in a bag.

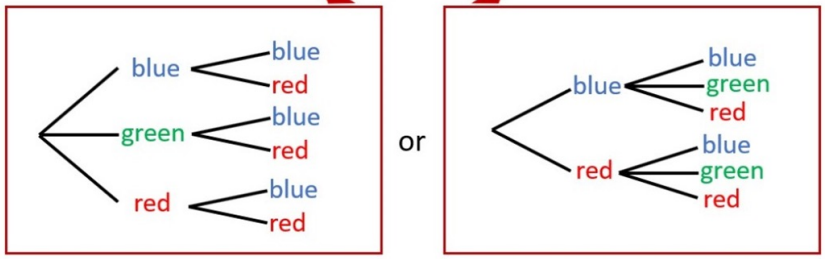
I pick **two** marbles.

I'm going to keep flipping a coin until I get a tails.

Structuring Probability Trees

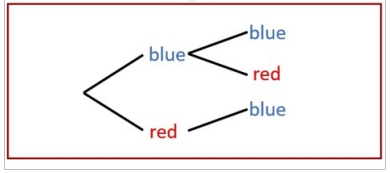
One bag contains red, blue and green marbles.
Another bag contains only red and blue marbles.

I pick a marble from each bag.

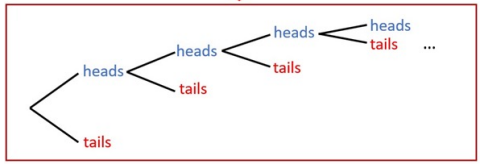


There are some blue marbles and 1 red marble in a bag.

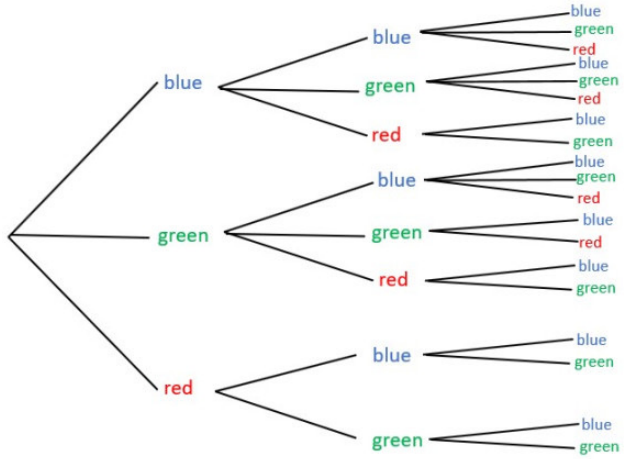
I pick **two** marbles.



I'm going to keep flipping a coin until I get a tails.



The probability tree represents drawing three marbles from the same bag, without replacement.

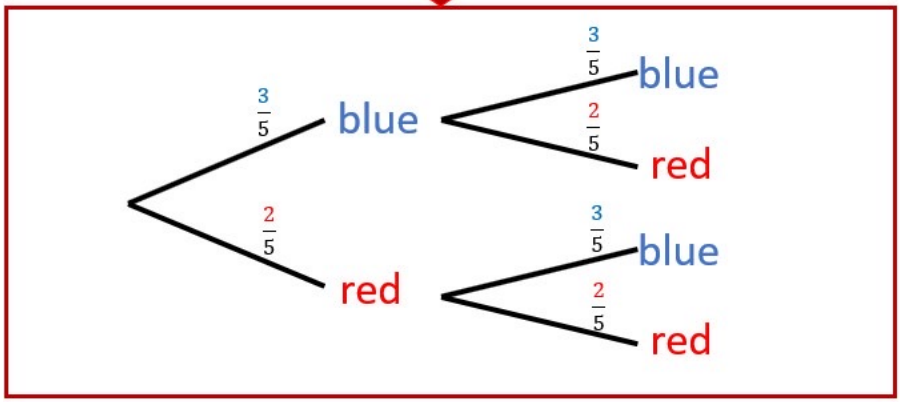


How many marbles of each colour could there be in the bag?

Labelling and Calculating Probabilities

There are **3 blue** marbles and **2 red** marbles in a bag.

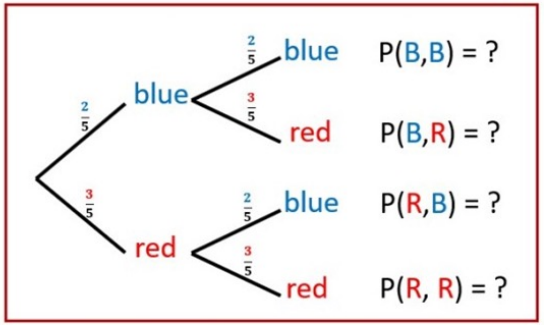
I pick a marble, replace it and pick another.

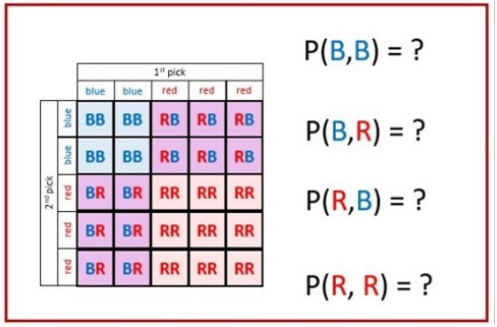


How do we calculate probabilities for combined events?
Let's compare two different representations to deduce a rule.

There are 2 blue marbles and 3 red marbles in a bag.

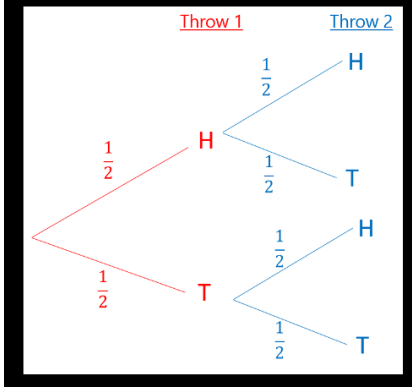
I pick a marble, replace it and then pick another marble.



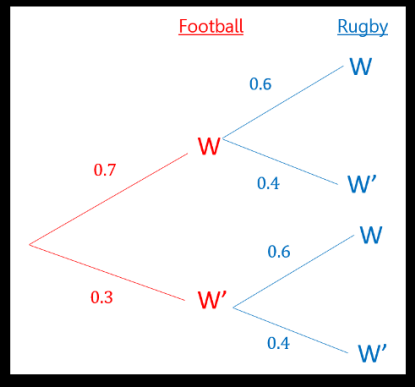


Types

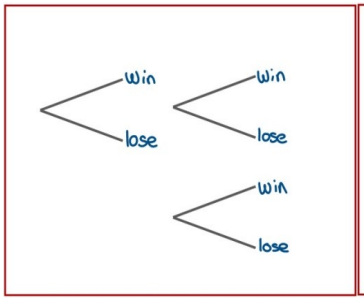
Do the same thing twice

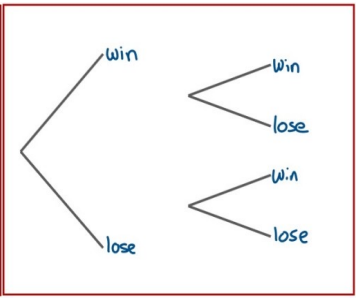


Do two different things



Mistakes

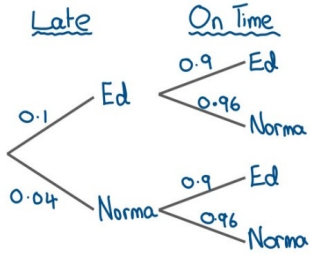




Based on previous records:

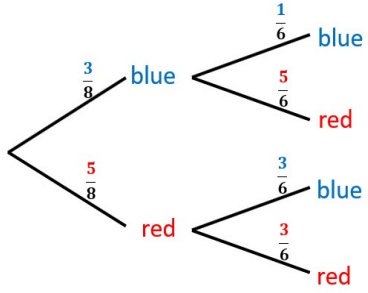
- the probability that Ed is late to work is 0.1;
- the probability that Norma is late to work is 0.04.

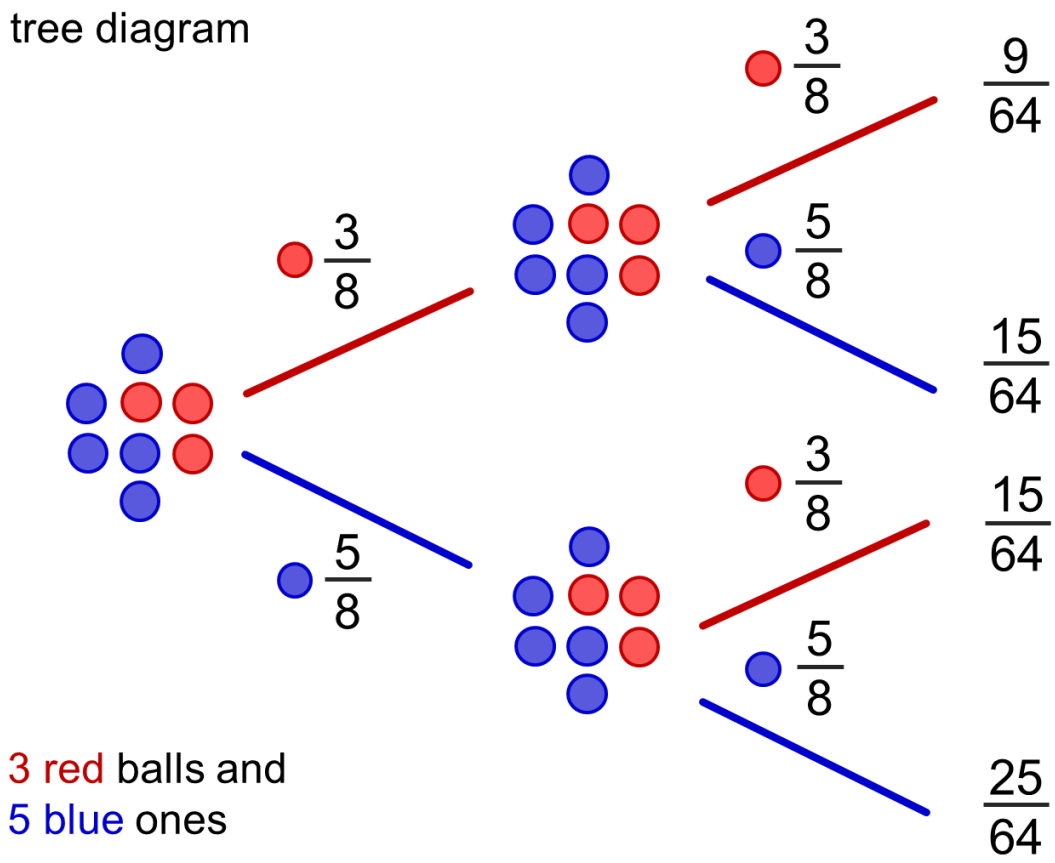
Use this to find the probability that at least one of them is late for work.



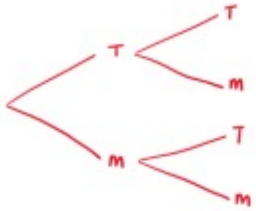
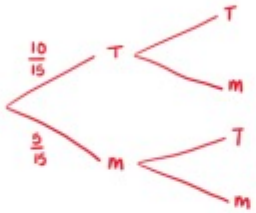
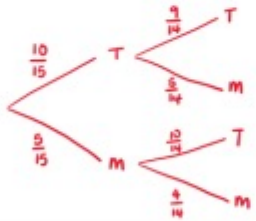
There are 3 blue marbles and 5 red marble in a bag.

I pick **two** marbles.

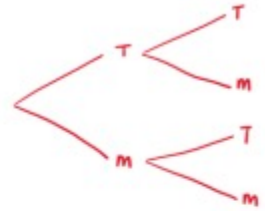
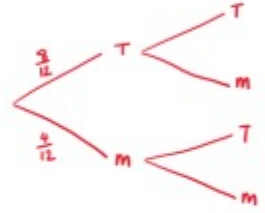
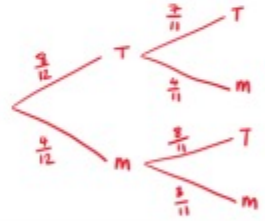



tree diagram
3/8
5/8
3/8
5/8
3/8
5/8
9/64
15/64
15/64
25/64
3 red balls and
5 blue ones

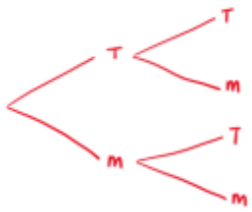
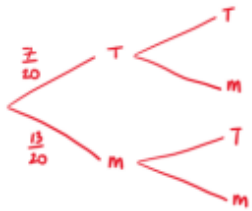
a) There are 15 sweets in a bag.
10 of the sweets are toffee and 5 are mint.
Reece takes two of the sweets at random.
Work out the probability that Reece takes one of each type of sweet.

Draw a tree diagram.	
What is the probability of choosing each type of sweet first?	
What is the probability of choosing each type of sweet second?	
What is the probability of choosing one of each type of sweets?	$P(T,M) = \frac{10}{15} \times \frac{5}{14} = \frac{50}{210}$ $P(M,T) = \frac{5}{15} \times \frac{10}{14} = \frac{50}{210}$ $\frac{50}{210} + \frac{50}{210} = \frac{100}{210} = \frac{10}{21}$

b) There are 12 sweets in a bag.
8 of the sweets are toffee and 4 are mint.
Reece takes two of the sweets at random.
Work out the probability that Reece takes one of each type

Draw a tree diagram.	
What is the probability of choosing each type of sweet first?	
What is the probability of choosing each type of sweet second?	
What is the probability of choosing one of each type of sweets?	

c) There are 20 sweets in a bag.
7 of the sweets are toffee and 13 are mint.
Reece takes two of the sweets at random.
Work out the probability that Reece takes one of each type of sweet.

Draw a tree diagram.	
What is the probability of choosing each type of sweet first?	
What is the probability of choosing each type of sweet second?	
What is the probability of choosing one of each type of sweets?	

d) There are 35 sweets in a bag.
20 of the sweets are toffee and 15 are mint.
Reece takes two of the sweets at random.
Work out the probability that Reece takes one of each type of sweet.

Worked example

In bag A there are 2 white and 5 red counters. In bag B there are 7 white counters and 3 red counters. A person takes at random one counter from A and one counter from B.

a) Draw a probability tree diagram to represent the situation.
b) Find the probability that the counters are the same colour.
c) Find the probability that the counters are different colours.

Your turn

In bag A there are 4 white and 7 red counters. In bag B there are 9 white counters and 5 red counters. A person takes at random one counter from A and one counter from B.

a) Draw a probability tree diagram to represent the situation.
b) Find the probability that the counters are the same colour.
c) Find the probability that the counters are different colours.

Worked example

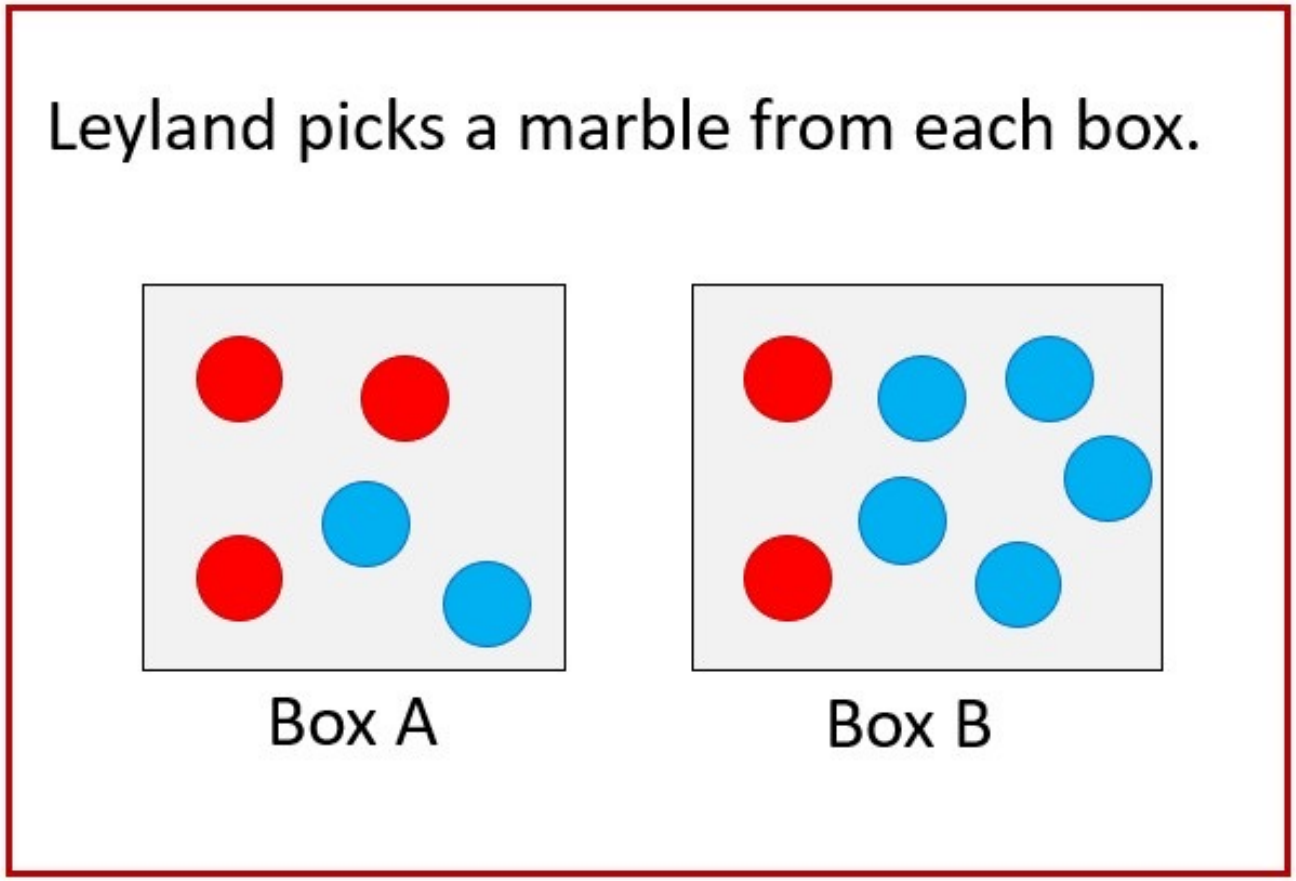
A person plays a game of tennis and then a game of golf. They can only win or lose each game. The probability of winning tennis is 0.3. The probability of winning golf is 0.7. The results of each game are independent of each other.

a) Draw a probability tree to represent this information.
b) Calculate the probability that the person win both games.
c) Calculate the probability that the person wins one and loses one.
d) Calculate the probability that the person wins at least one game.

Your turn

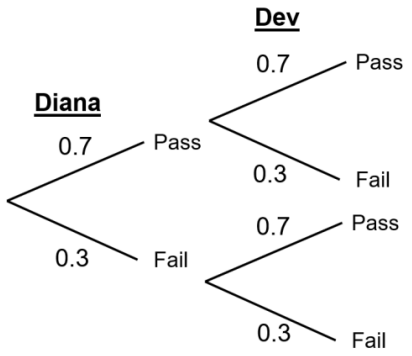
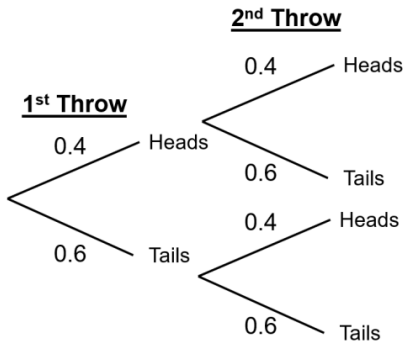
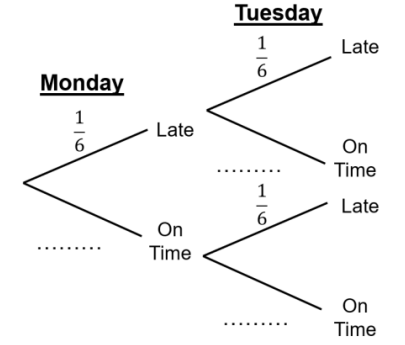
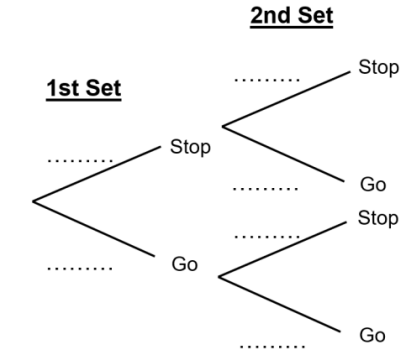
A person plays a game of tennis and then a game of golf. They can only win or lose each game. The probability of winning tennis is 0.6. The probability of winning golf is 0.35. The results of each game are independent of each other.

a) Draw a probability tree to represent this information.
b) Calculate the probability that the person loses both games.
c) Calculate the probability that the person wins one and loses one.
d) Calculate the probability that the person loses at least one game.

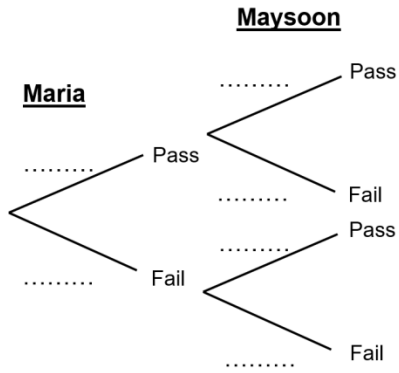
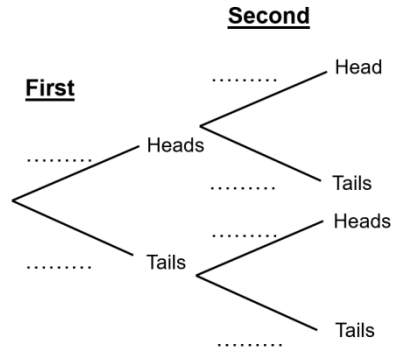


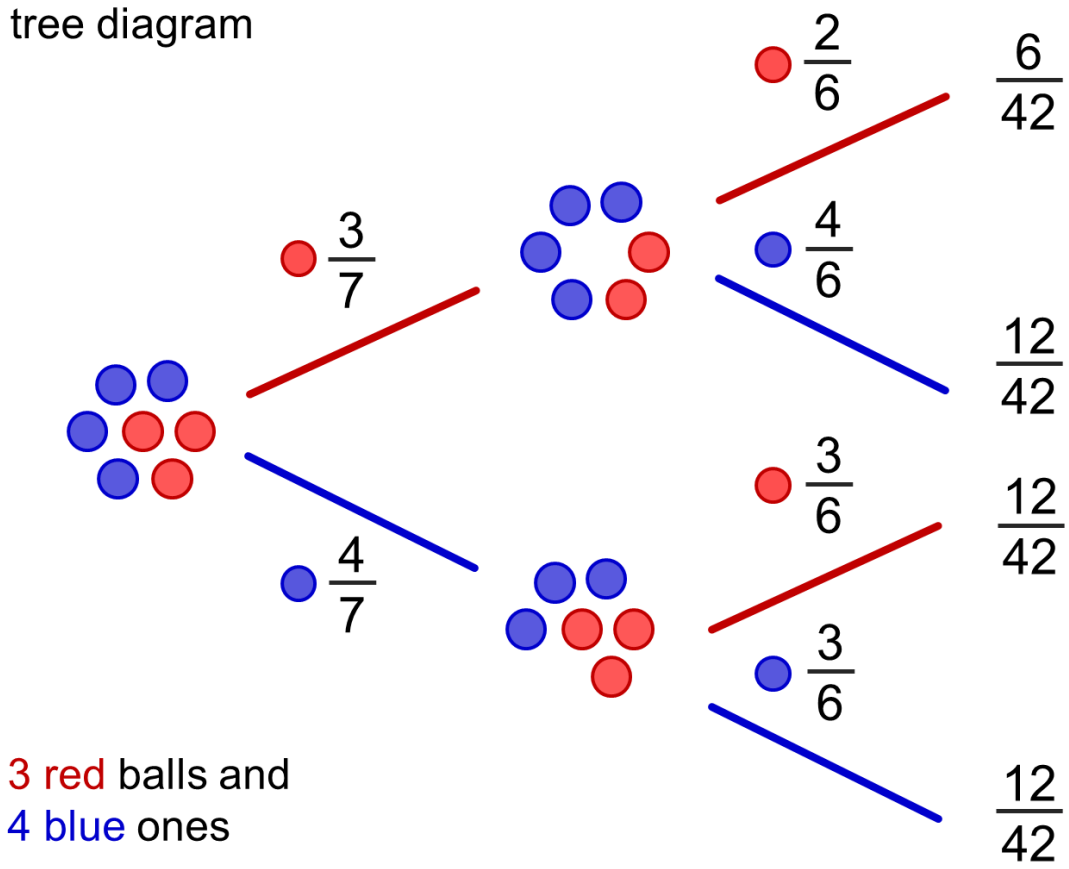
- Does it matter which box Leyland chooses from first?
- Will the order affect the probabilities of the combined outcomes?

FILL IN THE GAPS

Question	Tree Diagram	Probability	
The probability of passing a music exam is 0.7. Diana and Dev both sit the music exam. Complete the tree diagram and calculate the probability of each outcome.		$P(PF) = 0.7 \times 0.7 =$	0.49
		$P(PF) = 0.7 \times 0.3 =$	
		$P(FP) = 0.3 \times 0.7 =$	
		$P(FF) = 0.3 \times 0.3 =$	
The probability of a biased coin landing on tails is 0.4. The coin is tossed twice. Complete the tree diagram and calculate the probability of each outcome.		$P(HH) = 0.4 \times 0.4 =$	
		$P(HT) =$ × =	
		$P(TH) =$ × =	
		$P(TT) =$ × =	
The probability of Abby being late for work is $\frac{1}{6}$. Abby works Monday and Tuesday. Complete the tree diagram and calculate the probability of each outcome.		$P(LL) =$ × =	
		$P(LO) =$ × =	
		$P(OL) =$ × =	
		$P(OO) =$ × =	
The probability of stopping at traffic lights is $\frac{3}{8}$. Jameela drives through two sets of traffic lights. Complete the tree diagram and calculate the probability of each outcome.			

FILL IN THE GAPS

Question	Tree Diagram	Probability	
Two students, Maria and Maysoon each sit their driving theory exam. Complete the tree diagram and calculate the probability of each outcome.		$P(PP) =$ $\times$ $=$	
		$P(PF) =$ $\times$ $=$	
		$P(FP) =$ $\times$ $=$	
		$P(FF) = 0.6 \times 0.6 =$	0.36
A biased coin is tossed once and then tossed again for a second time. Complete the tree diagram and calculate the probability of each outcome.		$P(HH) = 0.2 \times$ $=$	0.04
		$P(HT) =$ $\times$ $=$	
		$P(TH) =$ $\times$ $=$	
		$P(TT) =$ $\times$ $=$	
A car travels through two sets of traffic lights. The probability of stopping at each set is the same. Complete the tree diagram and calculate the probability of each outcome.		$P(SS) =$ $\times$ $=$	
		$P(SG) = \frac{3}{7} \times$ $=$	
		$P(GS) =$ $\times$ $=$	
		$P(GG) =$ $\times$ $=$	
There are 12 red or blue balls in a box. There are more blue balls than red balls. A ball is removed at random, the colour recorded, then replaced. A second ball is then removed. Complete the tree diagram and probabilities.		$P(RR) =$ $\times$ $=$	
		$P(RB) =$ $\times$ $=$	$\frac{35}{144}$
		$P(BR) =$ $\times$ $=$	
		$P(BB) =$ $\times$ $=$	


tree diagram
3/7
4/7
2/6
4/6
3/6
3/6
6/42
12/42
12/42
12/42
3 red balls and
4 blue ones

Worked example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	15

One counter is taken out the bag. It is not replaced. Then another counter is taken out the bag. Find the probability that:

a) Both counters are red
b) Neither counter is red
c) The counters are different colours

Your turn

There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	5

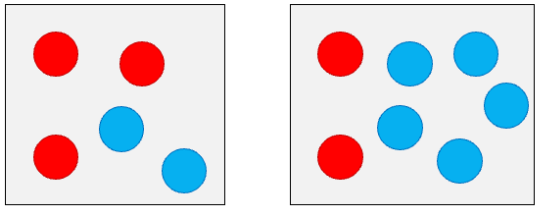
One counter is taken out the bag. It is not replaced. Then another counter is taken out the bag. Find the probability that:

a) Both counters are purple
b) Neither counter is purple
c) The counters are different colours

Reasoning (Dependent Events)

Laura picks a marble from Box A and puts it into Box B.

She then picks a marble from Box B.

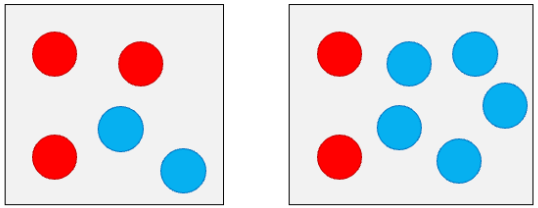


Box A Box B

vs

Laura picks a marble from Box B and puts it into Box A.

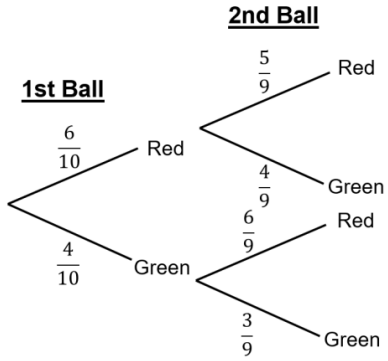
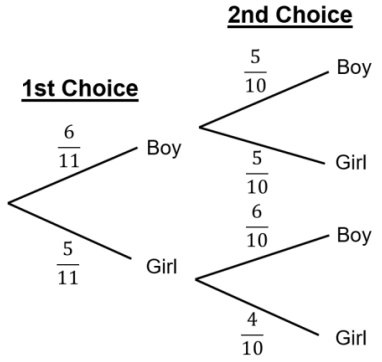
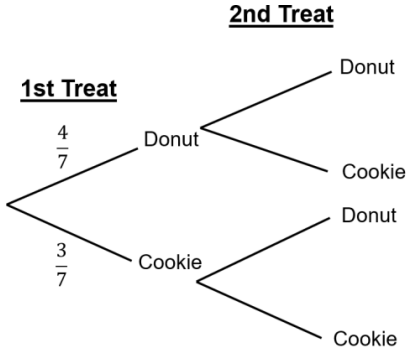
She then picks a marble from Box A.



Box A Box B

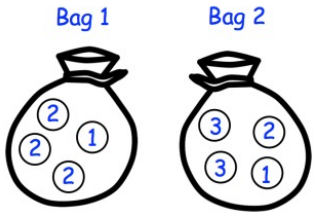
- Does it matter which box Laura chooses from first?
- Will the order affect the probabilities of the combined outcomes?

FILL IN THE GAPS

Question	Tree Diagram	Probability	
There are 6 red balls and 4 green balls in a bag. Two balls are chosen at random. Complete the tree diagram and calculate the probability of each outcome.		$P(RR) = \frac{6}{10} \times \frac{5}{9} =$	$\frac{30}{90}$
		$P(RG) = \frac{6}{10} \times \frac{4}{9} =$	$\frac{24}{90}$
		$P(GR) =$ × =	
		$P(GG) =$ × =	
There are 6 boys and 5 girls in a football team. Two team members are chosen at random. Complete the tree diagram and calculate the probability of each outcome.		$P(BB) =$ × =	
		$P(BG) =$ × =	
		$P(GB) =$ × =	
		$P(GG) =$ × =	
There are 4 donuts and 3 cookies in a tin. Riaz chooses two treats at random. Complete the tree diagram and calculate the probability of each outcome.		$P(DD) =$ × =	
		$P(DC) =$ × =	
		$P(CD) =$ × =	
		$P(CC) =$ × =	
There are 7 blue pens and 5 red pens in a pencil case. Two pens are chosen at random. Complete the tree diagram and calculate the probability of each outcome.		$P(BB) =$	
		$P(BR) =$	
		$P(RB) =$	
		$P(RR) =$	

Exam Q

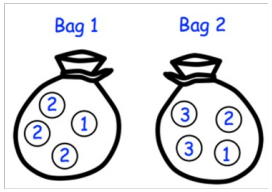
There are two bags with numbered discs as shown.



A person chooses a disc at random from bag 1.
If it is labelled 2, he puts the disc in bag 2.
If it is labelled 1, he does not put the disc in bag 2.
He then chooses a disc at random from bag 2.
He then adds the numbers of the two discs he selected to give his score.
Find the probability that his score is 4.

Your Turn

There are two bags with numbered discs as shown.



A person chooses a disc at random from bag 1.
If it is labelled 2, he puts the disc in bag 2.
If it is labelled 1, he does not put the disc in bag 2.
He then chooses a disc at random from bag 2.
He then adds the numbers of the two discs he selected to give his score.
Find the probability that his score is 5.

Worked Example *K297a*

Neha has 6 sweets, of which k are blue. The remainder of the sweets are green.

Neha eats a sweet, does not regurgitate it, and then eats another sweet.

The probability that she eats two blue sweets is $\frac{1}{5}$.

Show that $k^2 + ak + b = 0$, where a and b are constants to be found.

Your Turn

Hannah has n marbles, of which 7 are red. The remainder of the marbles are blue.

Hannah takes a marble, does not replace it, and then takes another marble.

The probability that she takes two red marbles is $\frac{3}{4}$.

Show that $n^2 + an + b = 0$, where a and b are constants to be found.

...with replacement:
The item is returned before another is chosen. The probability of each event on each trial is fixed.

...without replacement:
The item is not returned.
•Total balls decreases by 1 each time.
•Number of items of this type decreases by 1.

Note that if the question doesn't specify which, e.g. "You pick two balls from a bag", then PRESUME WITHOUT REPLACEMENT.

EXTRA NOTES