KING EDWARD VI HANDSWORTH GRAMMAR SCHOOL FOR BOYS

KING EDWARD VI ACADEMY TRUST BIRMINGHAM

Year 11

2023   Mathematics   2024

Unit 23 Booklet

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

Contents Page

1 Completing the Square

Worked Example	Your Turn
a) Express $x^2 + 12x + 36$ in the form $(x + a)^2$ b) Express $4x^2 + 12x + 9$ in the form $(ax + b)^2$	a) Express $x^2 - 14x + 49$ in the form $(x + a)^2$ b) Express $9x^2 - 12x + 4$ in the form $(ax + b)^2$

Worked Example	Your Turn
a) What must be added to $x^2 + 10x$ to make it into a perfect square? b) What must be added to $9x^2 + 30x$ to make it into a perfect square?	a) What must be added to $x^2 - 14x$ to make it into a perfect square? b) What must be added to $4x^2 - 12x$ to make it into a perfect square?

Worked Example	Your Turn
Complete the square on the following expressions: a) $x^2 + 10x$ b) $x^2 - 10x$	Complete the square on the following expressions: a) $x^2 + 6x$ b) $x^2 - 6x$

Worked Example	Your Turn
K266a: **Complete the square for quadratics of the form** $x^2 + bx + c$ **where** b **is even.** Write $\quad x^2 + 4x + 6$ in the form $(x + a)^2 + b$ where a and b are integers.	K266a: **Complete the square for quadratics of the form** $x^2 + bx + c$ **where** b **is even.** Write $\quad x^2 - 2x + 5$ in the form $(x + a)^2 + b$ where a and b are integers.

Worked Example	Your Turn
K266b: Complete the square for quadratics of the form $x^2 + bx + c$ **where** b **is odd.**	**K266b: Complete the square for quadratics of the form** $x^2 + bx + c$ **where** b **is odd.**
Write $$x^2 + 5x - 7$$ in the form $(x+a)^2 + b$ where a and b are constants to be found.	Write $$x^2 - 11x - 4$$ in the form $(x+a)^2 + b$ where a and b are constants to be found.

Worked Example	Your Turn
K266c: **Complete the square for quadratics of the form** $x^2 + bx + c$, **where** b **is algebraic.** Write $x^2 + 2px + 4$ in the form $(x + a)^2 + b$, where a and b are in terms of p.	K266c: **Complete the square for quadratics of the form** $x^2 + bx + c$, **where** b **is algebraic.** Express $x^2 + 8kx + 5$ in the form $(x + a)^2 + b$, where a and b are in terms of k.

Fill in the Blanks   Completing the Square $(x^2 + bx + c)$

Quadratic Expression	$\left(x+\frac{b}{2}\right)^2$	$\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$	$\pm c$	Completed Square
$x^2 + 10x$	$(x+5)^2$	$(x+5)^2 - 25$	$(x+5)^2 - 25$	$(x+5)^2 - 25$
$x^2 + 8x - 2$	$(x+4)^2$	$(x+4)^2 - 16$	$(x+4)^2 - 16 - 2$	
$x^2 + 2x + 5$	$(x+1)^2$	$(x+1)^2 - 1$		
$x^2 + 4x + 7$	$(x+2)^2$			
$x^2 + 6x - 11$				
$x^2 + 18x + 50$				
$x^2 - 12x$	$(x-6)^2$	$(x-6)^2 - 36$	$(x-6)^2 - 36$	$(x-6)^2 - 36$
$x^2 - 2x + 5$	$(x-1)^2$	$(x-1)^2 - 1$	$(x-1)^2 - 1 + 5$	
$x^2 - 6x - 1$	$(x-3)^2$	$(x-3)^2 - 9$		
$x^2 - 8x + 16$	$(x-4)^2$			
$x^2 - 4x + 5$				
$x^2 + 7x + 1$	$\left(x+\frac{7}{2}\right)^2$	$\left(x+\frac{7}{2}\right)^2 - \frac{49}{4}$	$\left(x+\frac{7}{2}\right)^2 - \frac{49}{4} + 1$	$\left(x+\frac{7}{2}\right)^2 - \frac{45}{4}$
$x^2 - 5x - 3$				
$x^2 + x + 6$				
$x^2 - 3x + 2$				$(x+5)^2 - 7$
				$\left(x-\frac{3}{2}\right)^2 + \frac{3}{4}$

Worked Example	Your Turn

K270a: **Complete the square for quadratics of the form** $ax^2 + bx + c$.

Write the expression in the form $r(x + p)^2 + q$ where r, p, and q are integers.

$$2x^2 - 4x - 2$$

K270a: **Complete the square for quadratics of the form** $ax^2 + bx + c$.

Write the expression in the form $r(x + p)^2 + q$ where r, p, and q are integers.

$$3x^2 + 12x + 19$$

Harder Completing the Square $(ax^2 + bx + c)$

Quadratic Expression	Take out Common Factor	Complete the Square	Multiply by Common Factor	Completed Square
$2x^2 + 16x$	$2[x^2 + 8x]$	$2[(x + 4)^2 - 16]$	$2(x + 4)^2 - 32$	$2(x + 4)^2 - 32$
$3x^2 - 18x$	$3[x^2 - 6x]$			
$2x^2 + 12x + 1$	$2[x^2 + 6x] + 1$	$2[(x + 3)^2 - 9] + 1$	$2(x + 3)^2 - 18 + 1$	$2(x + 3)^2 - 17$
$2x^2 - 20x - 7$	$2[x^2 - 10x] - 7$	$2[(x - 5)^2 - 25] - 7$		
$3x^2 + 6x - 5$	$3[x^2 + 2x] - 5$			
$4x^2 + 16x - 1$				
$5x^2 - 30x + 11$				
$2x^2 - 10x + 3$	$2[x^2 - 5x] + 3$	$2\left[\left(x - \dfrac{5}{2}\right)^2 - \dfrac{25}{4}\right] + 3$	$2\left(x - \dfrac{5}{2}\right)^2 - \dfrac{25}{2} + 3$	$2\left(x - \dfrac{5}{2}\right)^2 - \dfrac{19}{2}$
$2x^2 + 6x - 1$				
$3x^2 - 9x + 2$				

Worked Example	Your Turn
K270b: **Complete the square for quadratics of the form** $ax^2 + bx + c$, **where** a **is negative.**	K270b: **Complete the square for quadratics of the form** $ax^2 + bx + c$, **where** a **is negative.**
Write $-3y^2 + 2y + 6$ in the form $a(y+b)^2 + c$, where a, b and c are rational numbers.	Express $-x^2 + 2x + 8$ in the form $a(x+b)^2 + c$, where a, b and c are rational numbers.

Worked Example	Your Turn
Solve the equation: $(x + 1)^2 = 4$	Solve the equation: $(x - 4)^2 = 9$

Worked Example	Your Turn
Solve the equation: $(2x + 3)^2 = 4$	Solve the equation: $(3x - 2)^2 = 9$

Worked Example	Your Turn
K267a: **Solve a quadratic equation to get exact solutions.**	K267a: **Solve a quadratic equation to get exact solutions.**
Solve the following quadratic equation, leaving your answer in exact form:	Solve the following quadratic equation, leaving your answer in exact form:
$$3k^2 - 7k + 1 = 0$$	$$4b^2 - 7b + 3 = 0$$

Worked Example	Your Turn
K267b: **Solve a quadratic equation using the formula or completing the square to get numerical solutions.**	K267b: **Solve a quadratic equation using the formula or completing the square to get numerical solutions.**
Solve the following quadratic equation, giving your answer accurate to 2 decimal places: $2x^2 - 7x - 7 = 0$	Solve the following quadratic equation, giving your answer accurate to 2 decimal places: $2x^2 + 7x + 3 = 0$

Extra Notes

2 Quadratic Graphs

Recap
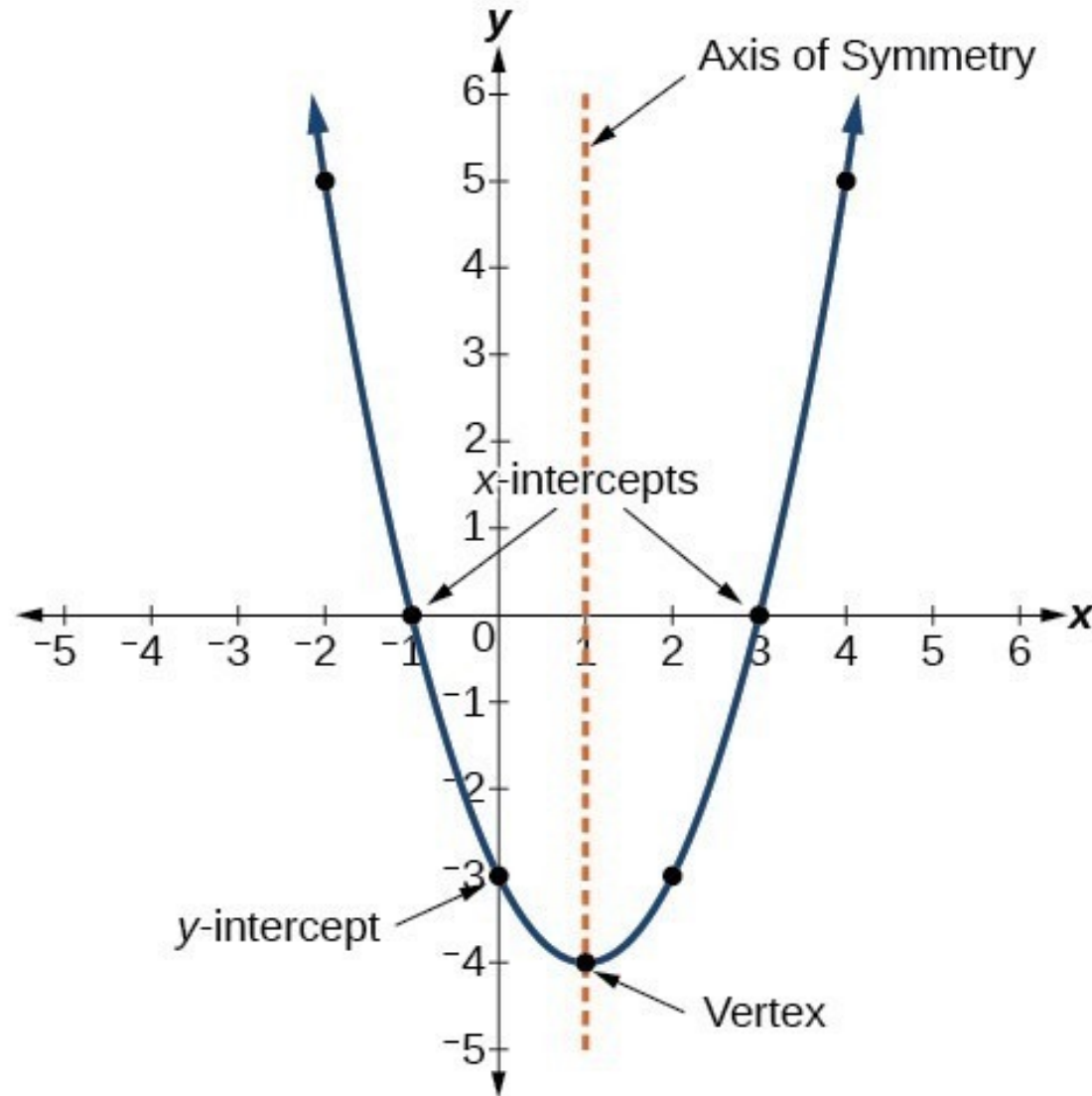
Worked Example	Your Turn
Sketch the following graph: $y = (x - 7)(x + 10)$	Sketch the following graph: $y = (x + 3)(x + 8)$

Worked Example	Your Turn
Sketch the following graph: $y = x(x + 2)$	Sketch the following graph: $y = x(x - 3)$

Worked Example	Your Turn
K205d: **Sketch a quadratic graph in the form** $y = (x-a)(x-b)$**, including its intercepts only.**	K205d: **Sketch a quadratic graph in the form** $y = (x-a)(x-b)$**, including its intercepts only.**
A quadratic graph has equation $y = (x+1)(x+5)$ Find the values of a, b and c.	A quadratic graph has equation $y = (x-7)(x-1)$ Find the values of a, b and c.
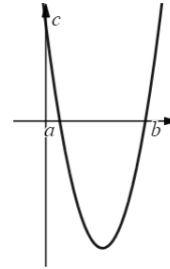
Worked Example	Your Turn
Sketch the following graph: $y = x^2 + 5x - 36$	Sketch the following graph: $y = x^2 + 6x + 8$
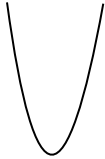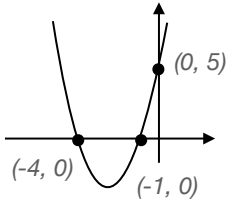
Worked Example	Your Turn
Sketch the following graph: $y = 2x^2 - 13x + 15$	Sketch the following graph: $y = 5x^2 + 13x + 6$

Fill in the Gaps

Function	Roots	Y intercept	Shape	Graph
$y = x^2 + 5x + 4$	$x^2 + 5x + 4 = 0$ $(x + 4)(x + 1) = 0$ $x = -4, = x - 1$ (-1, 0) and (-4, 0)	$y = (0)^2 + 5(0) + 4$ $y = 4$ (0,5)	*Positive x²* 	
$y = x^2 + 7x + 6$				
$y = x^2 - 6x + 5$				
$y = 6 - x - x^2$				

Fill in the Gaps

Function	Roots	Y intercept	Shape	Graph
$y = x^2 + 8x$				
$y = x^2 - 25$				
$y = 9 - x^2$				
$y = 35 - 2x - x^2$				

Fill in the Gaps

Function	Roots	Y intercept	Shape	Graph
$y = (x-5)(x-2)$				
$y = (x-3)(x+3)$				
$y = -(x+4)(x-1)$				
$y = (2-x)(3+x)$				

Worked Example	Your Turn
K271a: **Find the minimum point of a quadratic graph when given in completed the square form.** $x^2 + 8x + 11$ can be expressed as $(x+4)^2 - 5$. Hence, or otherwise, state the coordinates of the minimum point of the graph $y = x^2 + 8x + 11$.	K271a: **Find the minimum point of a quadratic graph when given in completed the square form.** $x^2 + 6x + 14$ can be expressed as $(x+3)^2 + 5$. Hence, or otherwise, state the coordinates of the minimum point of the graph $y = x^2 + 6x + 14$.

Worked Example	Your Turn

K271b: State quadratic given the maximum or minimum point.

The graph below shows part of the parabola with equation of the form $y = (x + a)^2 + b$

$(-2, -5)$
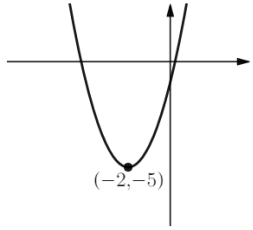
The minimum turning point $(-2, -5)$ is shown on the diagram.

State the values of a and b.

K271b: State quadratic given the maximum or minimum point.

The graph below shows part of the parabola with equation of the form $y = -(x + a)^2 + b$

$(-5, 5)$
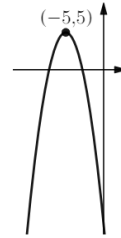
The maximum turning point $(-5, 5)$ is shown on the diagram.

State the values of a and b.

Worked Example	Your Turn

K271c: Find the maximum point of a quadratic graph when given in completed the square form.

$-x^2 - 6x - 10$ can be expressed as $-(x+3)^2 - 1$.

Hence, or otherwise, state the coordinates of the maximum point of the graph $y = -x^2 - 6x - 10$.

K271c: Find the maximum point of a quadratic graph when given in completed the square form.

$-x^2 + 8x - 19$ can be expressed as $-(x-4)^2 - 3$.

Hence, or otherwise, state the coordinates of the maximum point of the graph $y = -x^2 + 8x - 19$.

Worked Example	Your Turn
K271d: **Find the minimum point of a quadratic graph.** Find the minimum point of the graph with equation $y = x^2 - 2x - 3$	K271d: **Find the minimum point of a quadratic graph.** Find the minimum point of the graph with equation $y = x^2 + 10x + 26$

Worked Example	Your Turn
K271e: **Find the maximum point of a quadratic graph.** Find the maximum point of the graph with equation $y = -x^2 + 2x - 2$	K271e: **Find the maximum point of a quadratic graph.** Find the maximum point of the graph with equation $y = -x^2 + 2x - 5$

Worked Example	Your Turn
K271f: **Find the line of symmetry of a quadratic graph.** A graph has equation $y = x^2 + 10x - 9$ Find the equation of its line of symmetry.	K271f: **Find the line of symmetry of a quadratic graph.** A graph has equation $y = x^2 - 9x - 16$ Find the equation of its line of symmetry.

Worked Example	Your Turn

K271g: Sketch a quadratic graph, including its intercepts and turning point.

A quadratic graph has equation $y = x^2 + 6x + 5$

Find the values of a, b, c, d and e.

A quadratic graph has equation $y = x^2 - 4x - 5$
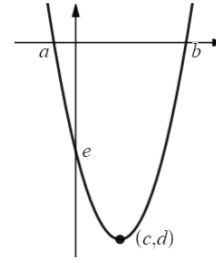
Find the values of a, b, c, d and e.

Worked Example	Your Turn
$x^2 + 2x - 15$	$x^2 + 2x - 24$
Factorise:	Factorise:
Complete the square:	Complete the square:
Intercepts:	Intercepts:
Turning point:	Turning point:
Sketch:	Sketch:
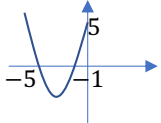
Fill in the Gaps
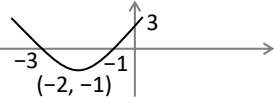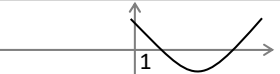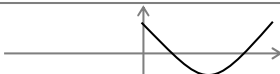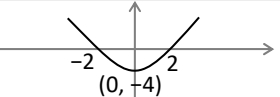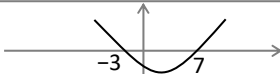
	$x^2 + bx + c$	$(x+d)(x+e)$	$(x+p)^2 + q$	x intercepts	y intercept	Turning point	Sketch
1.	$x^2 - 2x - 3$						
2.		$(x-1)(x+3)$					
3.							(sketch: parabola, -5, 1, -5)
4.			$(x+1)^2 - 9$				
5.				$(-2,0)$ $(-4,0)$			
6.		$(x-4)(x+4)$					
7.				$(4,0)$ $(0,0)$			

Fill in the Gaps

	$x^2 + bx + c$	$(x+d)(x+e)$	$(x+p)^2+q$	x intercepts	y intercept	Turning point	Sketch
8.	$x^2 + 4x$						
9.						$(-2, 0)$	
10.		$(x+3)(x+3)$					
11.							
12.			$(x+3)^2+4$				
13.		$-(x+1)(x+5)$					
14.					$(0, 5)$		

Fill in the Gaps

① **Significant Points of a Quadratic Curve Using Factorisation & Symmetry**

Equation	Factorisation	1st Root	Turning Point	2nd Root	y-intercept	Sketch
$y = x^2 + 4x + 3$	$(\quad)(x + 1) = 0$			$(-1, 0)$	$+3$	
$y = x^2 + 8x + 7$	$(x + 1) = 0$	$(-7, 0)$	$(x, -9)$			
$y = x^2 - 6x + 5$	$= 0$		$(3, y)$			
$y = x^2 - 8x + 12$						
	$(x + 3)(x - 5) = 0$				-15	
$y = x^2 + 2x - 24$						
		$(-2, 0)$		$(2, 0)$	-4	
$y = x^2 + 5x$			$(x, -6.25)$			
$y = x^2 + 8x + 16$						

Fill in the Gaps

Standard	Factorised	Vertex	Intercepts			Values			Vertex	
			x	x	y	at $x=1$	at $x=2$	at $x=3$	x-coordinate	y-coordinate
$x^2 - 8x + 15$	$(x-3)(x-5)$	$(x-4)^2 - 1$	3	5	15	8	3	0	4	-1
	$(x-2)(x-3)$									
$x^2 + 6x + 5$		$(x+2)^2 - 25$	-4	3	-12					
						-4	-3	0		
	$(2x-3)(4x-5)$									
$6x^2 - 13x + 6$		$35\left(x - \frac{1}{35}\right)^2 - \frac{36}{35}$								

Fill in the Gaps

Fill in the blanks:

1

$y = \boxed{}x^2 + \boxed{}x + \boxed{}$

10, 5, 1, $(\boxed{}, \boxed{})$

2

$y = \boxed{}x^2 + \boxed{}x + \boxed{}$

-4, $(\boxed{}, -4)$, $(-3, -22)$

3

$y = \boxed{}x^2 + \boxed{}x + \boxed{}$

$(3, 34)$, $(\boxed{}, 34)$, $(11, 2)$

4

$y = -x^2 + \boxed{}x + \boxed{}$

10, 6, $(\boxed{}, \boxed{})$

5

$y = 3x^2 + \boxed{}x + \boxed{}$

$(\boxed{}, 4)$, $(\boxed{}, 4)$, $(-3, 1)$

6

$y = x^2 + 16x + 5$

$(\boxed{}, 5)$, $(4, \boxed{})$, 5
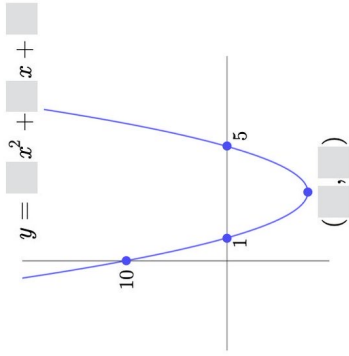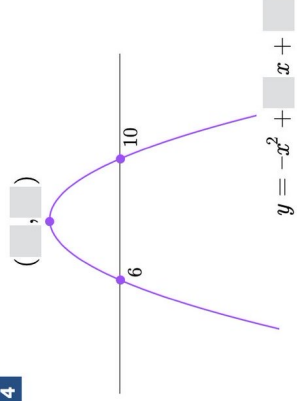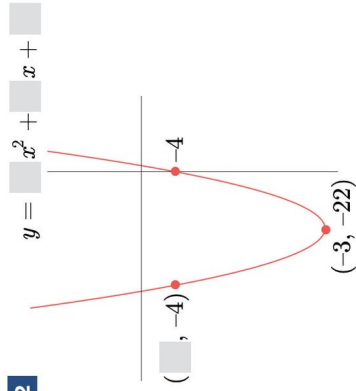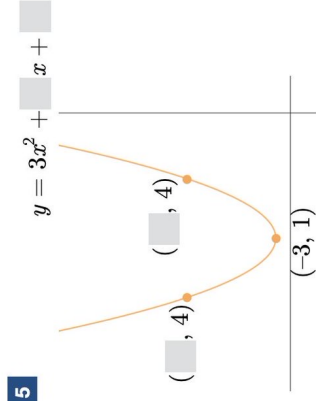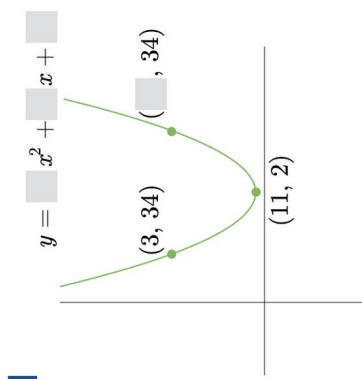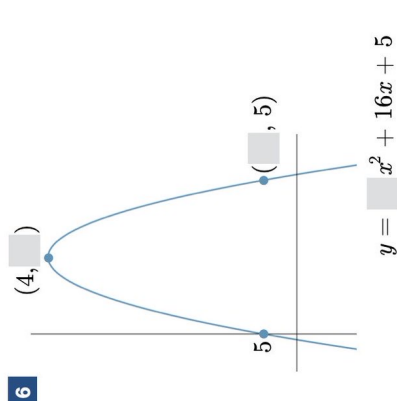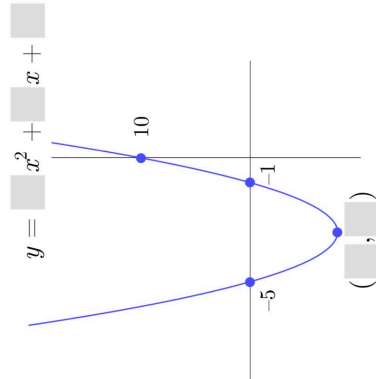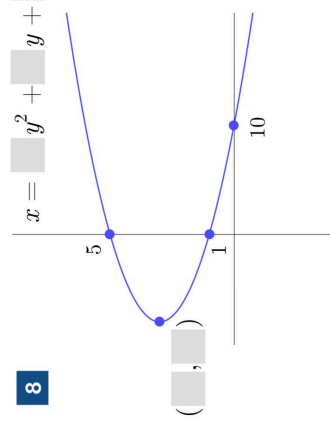
Fill in the blanks. It may help to compare questions 7 and 8 to question 1.

7

$y = x^2 + \boxed{}x + \boxed{}$

10, -1, -5, $(\boxed{}, \boxed{})$

8

$x = \boxed{}y^2 + \boxed{}y + \boxed{}$

5, 1, 10, $(\boxed{}, \boxed{})$

Extra Notes

3 Quadratic Inequalities

Worked Example	Your Turn
Shade in the region on the x-axis which represents the inequality.	Shade in the region on the x-axis which represents the inequality.
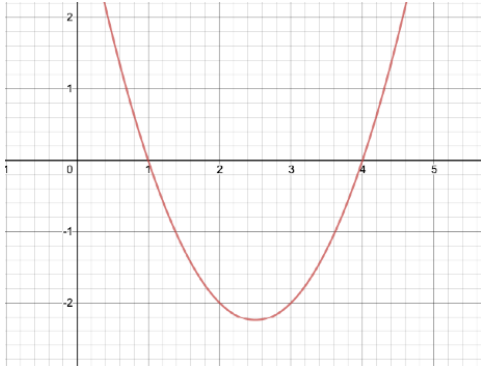
Worked Example

$$x^2 - 4x + 5 < 0$$

$$x^2 - 4x + 5 \geq 0$$
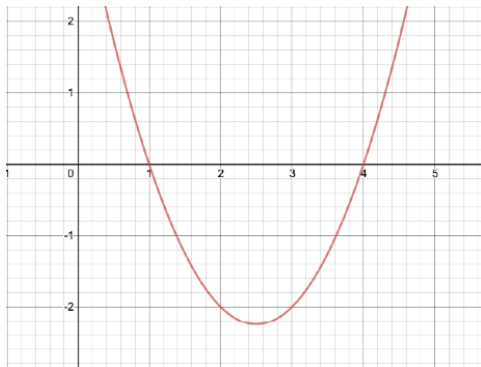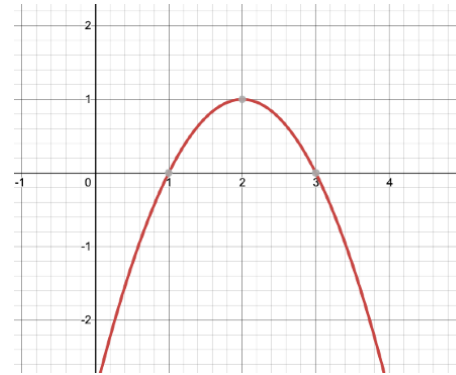
Your Turn

$$-x^2 + 4x - 3 \leq 0$$

$$-x^2 + 4x - 3 > 0$$
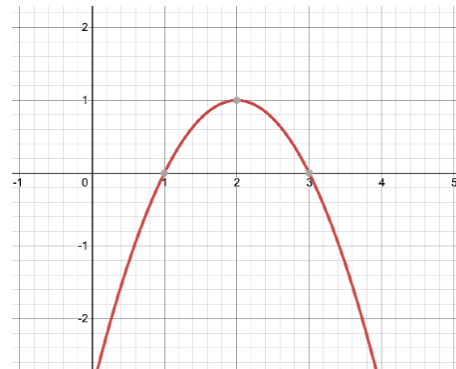
Worked Example	Your Turn
Shade in the region on the x-axis which represents the inequality.	Shade in the region on the x-axis which represents the inequality.
$$x^2 + x - 6 \leq 0$$	$$x^2 + x - 6 > 0$$
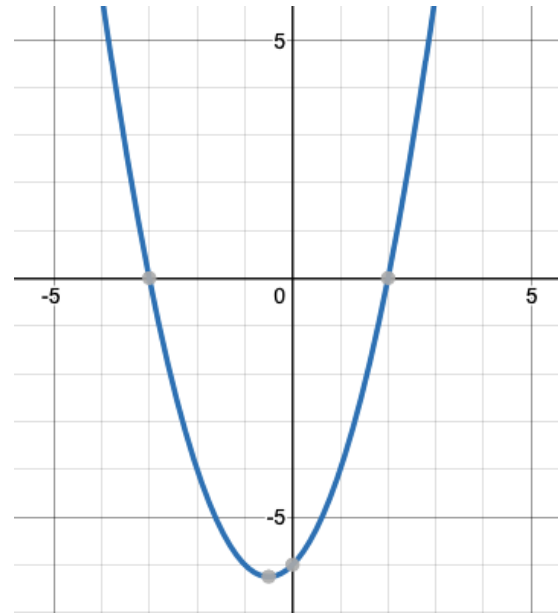
Worked Example	Your Turn
K292a: **Solve quadratic inequalities of the form** $x^2 + bx + c > 0$ **or** $x^2 + bx + c < 0.$	K292a: **Solve quadratic inequalities of the form** $x^2 + bx + c > 0$ **or** $x^2 + bx + c < 0.$
Solve:	Solve:
$z^2 - z \leq 0$	$m^2 - 7m + 12 \geq 0$

Worked Example	Your Turn
K292b: **Solve quadratic inequalities of the form** $x^2 + bx + c > 0$ **or** $x^2 + bx + c < 0$, **requiring rearrangement.**	K292b: **Solve quadratic inequalities of the form** $x^2 + bx + c > 0$ **or** $x^2 + bx + c < 0$, **requiring rearrangement.**
Solve the inequality	Solve the inequality
$$m(m-8) + 15 < 0$$	$$x(x-4) \leq 2(x-4)$$

Worked Example	Your Turn
K292c: **Solve quadratic inequalities of the form** $ax^2 > b$ **or** $ax^2 < b$	K292c: **Solve quadratic inequalities of the form** $ax^2 > b$ **or** $ax^2 < b$
Solve the inequality	Solve the inequality
$$9x^2 \geq 4$$	$$3k^2 > 27$$

Worked Example	Your Turn
K292d: **Solve quadratic inequalities of the form** $ax^2 + bx + c > 0$ **or** $ax^2 + bx + c < 0$, **requiring rearrangement.**	K292d: **Solve quadratic inequalities of the form** $ax^2 + bx + c > 0$ **or** $ax^2 + bx + c < 0$, **requiring rearrangement.**
Solve the inequality	Solve the inequality
$$(5m + 4)(m + 4) \leq 21$$	$$(5m + 2)(m - 2) + 7 > 0$$

Worked Example	Your Turn
K292e: **Solve quadratic inequalities of the form** $-x^2 + bx + c > 0$ **or** $-x^2 + bx + c < 0$	K292e: **Solve quadratic inequalities of the form** $-x^2 + bx + c > 0$ **or** $-x^2 + bx + c < 0$
Solve: $$4x - x^2 > 0$$	Solve: $$3x - 2 - x^2 < 0$$

Worked Example	Your Turn
K292f: Solve quadratic inequalities involving a division by a positive algebraic expression.	K292f: Solve quadratic inequalities involving a division by a positive algebraic expression.
Solve	Solve
$$2 \le \frac{5x}{x^2 + 1}$$	$$1 > \frac{7x}{x^2 + 6}$$

Worked Example	Your Turn
K292g: **Combine a linear and a quadratic inequality.** n is an integer such that $7n + 4 < 18$ and $\dfrac{5n}{n^2 + 1} \geq 2$ Find all the possible values of n.	K292g: **Combine a linear and a quadratic inequality.** n is an integer such that $7n + 1 > 8$ and $2n^2 - 7n + 5 \leq 0$ Find all the possible values of n.
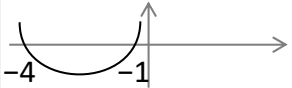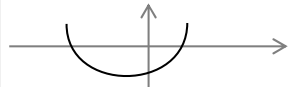
Fill in the Gaps

Quadratic Inequality	Factorised Quadratic	Critical Values	Sketch and Shade	Solution
$x^2 - 3x - 10 < 0$	$(x-5)(x+2) < 0$	$x = 5,$ $x = -2$	(−2, 5)	$-2 < x < 5$
$x^2 + 4x - 12 < 0$	$(x+6)(x-2) < 0$			
$x^2 - 6x + 5 < 0$				
$x^2 + 5x + 6 \leq 0$				
$x^2 - 4x - 12 > 0$	$(x-6)(x+2) > 0$	$x = 6,$ $x = -2$	(−2, 6)	$x < -2,$ $x > 6$
$x^2 + 4x + 3 \geq 0$				
$x^2 - x - 6 \geq 0$				$-4 < x < 1$

Fill in the Gaps

Quadratic Inequalities

Inequality	Factorisation	Critical Values		Sketch	Solution(s)	Integer Answers (?)
$x^2 + 5x + 4 > 0$	$(x+1)(x+4) = 0$	-1			$x >$ $x <$	
$x^2 + 7x + 10 > 0$		-2				
$x^2 + 2x - 15 > 0$					$x > 3$	
	$(x+3)(x-4) = 0$					
$x^2 + 7x + 6 < 0$						
	$(x-2)(x+4) = 0$					
$x^2 + x - 20 \leqslant 0$						
					$3 < x < 4$	
$x^2 - 8x + 16 \leqslant 0$						
$x^2 - 12x + 35 \geqslant 0$						

Quadratic Inequality	Rearrange	Factorise	Critical Values	Sketch and Shade	Solution
$x^2 < 7x - 12$	$x^2 - 7x + 12 < 0$				
$3x^2 + 5 \geq 16x$					
$x^2 > 3x$					
$2x^2 < 6 + 11x$					
$\dfrac{4x^2 + 5x}{3} \leq 2$					

Extra Notes

4 Kinematics

Worked Example	Worked Example

Dave goes to visit his friend Bob who lives just a few doors away.
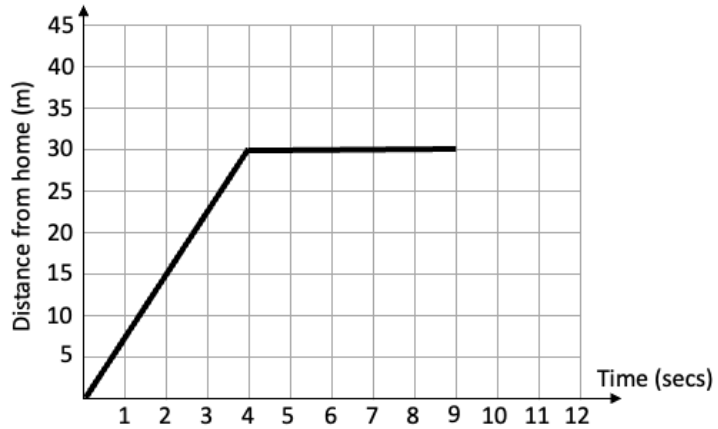
1) How many metres do he travel per second on his journey there?
2) Therefore, how in general can we interpret the gradient of a distance-time graph?

Dave pushes his friend Bob, who is on a wonky skateboard, down a steep hill.
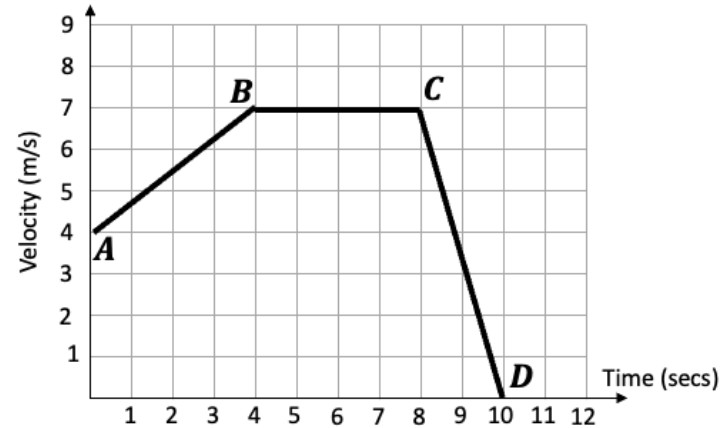
1) What does the gradient, i.e. the rate of change of speed, represent now?
2) State and describe the acceleration between:
a) A and B
b) B and C
c) C and D

Your Turn	Your Turn
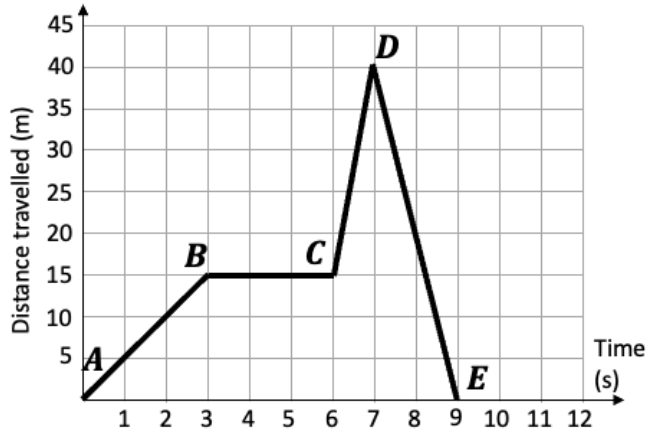
Determine the speed from:
a) A to B
b) B to C
c) D to E
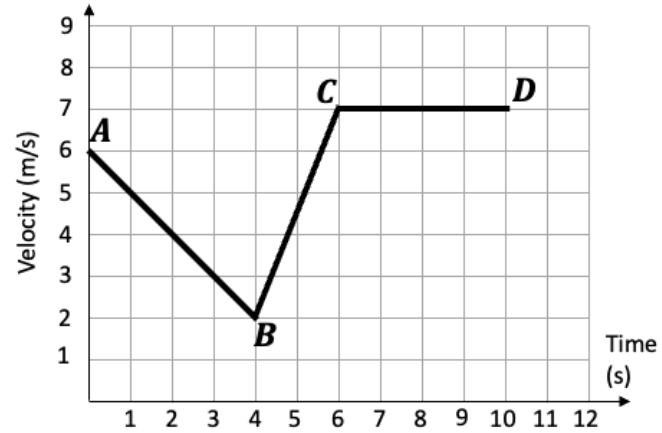
Determine the acceleration from:
a) A to B
b) B to C
c) C to D

Worked Example	Your Turn
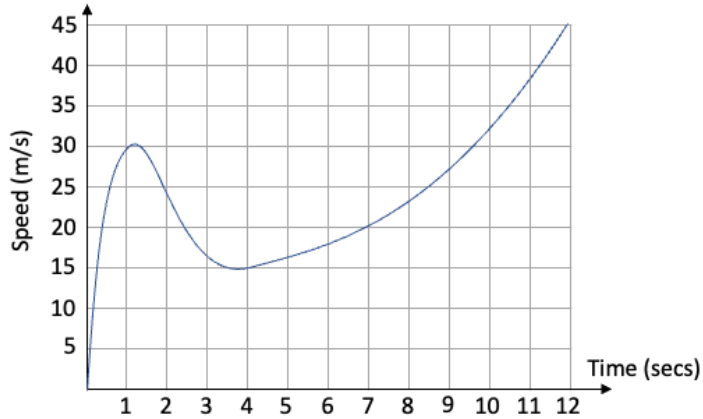
Suppose we wish to find the **acceleration** of a car after 7 seconds.

The line is curved this time, so how do we find the gradient at the instant when $t = 7$?
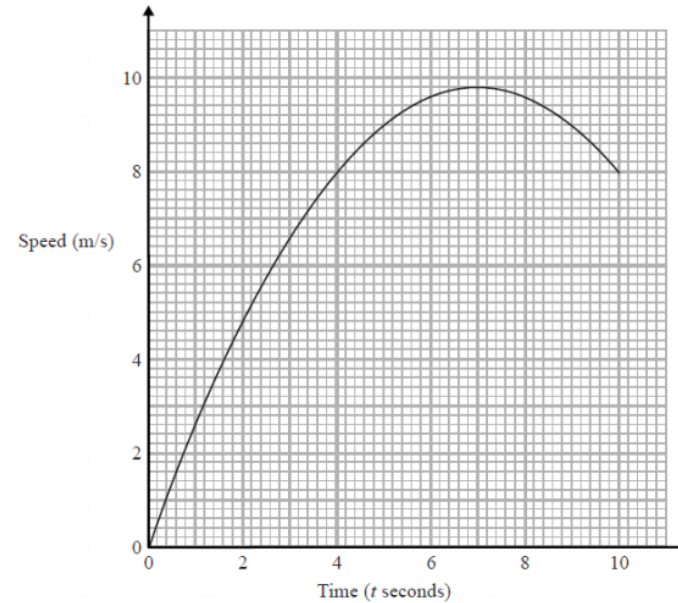
Karol runs in a race. The graph shows her speed, in metres per second, t seconds after the start of the race.

Calculate an estimate for the gradient of the graph when $t = 4$. You must show how you get your answer.

Temperature °C

Time (*t* seconds)
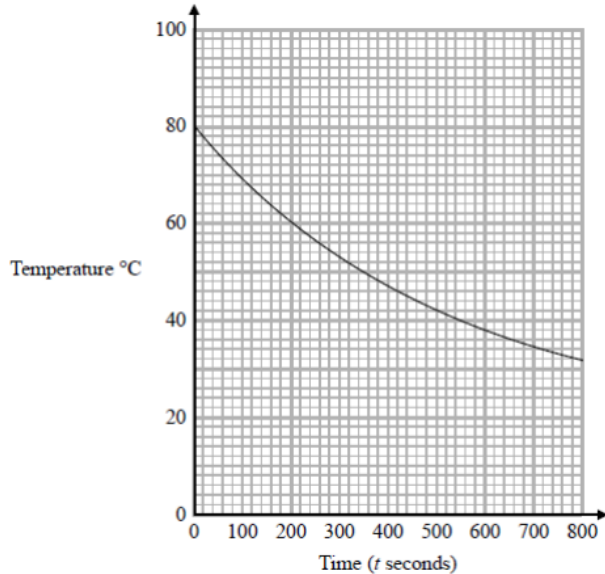
The graph gives information about the variation in the temperature, in $°C$, of an amount of water that is allowed to cool from $80°C$.

a) Work out the average rate of decrease of the temperature of the water between $t = 0$ and $t = 800$.

b) The instantaneous rate of decrease of the temperature of the water at time T seconds is equal to the average rate of decrease of the temperature of the water between $t = 0$ and $t = 800$. Find an estimate for the value of T.

Fluency Practice

1
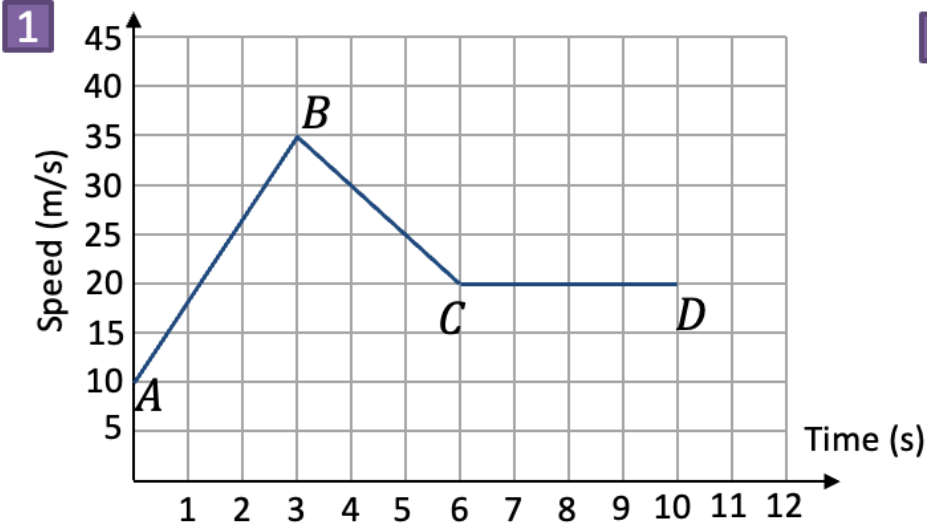
The speed-time graph shows the motion of a toboggan as it descends a hill. Determine the acceleration of the toboggan between:
a) A and B = ?
b) B and C = ?
c) C and D = ?

2
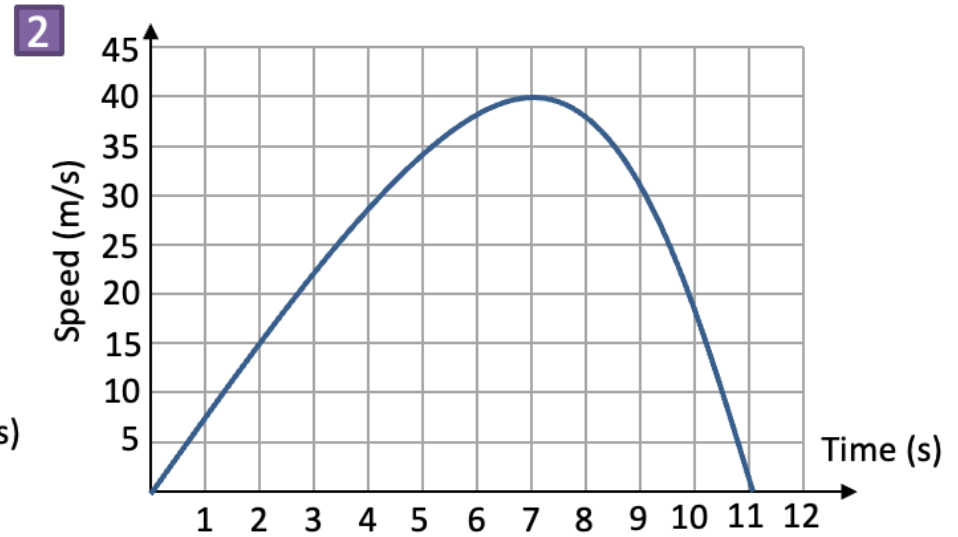
The graph shows the speed of a car over time.
Estimate the acceleration of the car when:
a) $t = 0$?
b) $t = 6$?
c) $t = 10$?

3
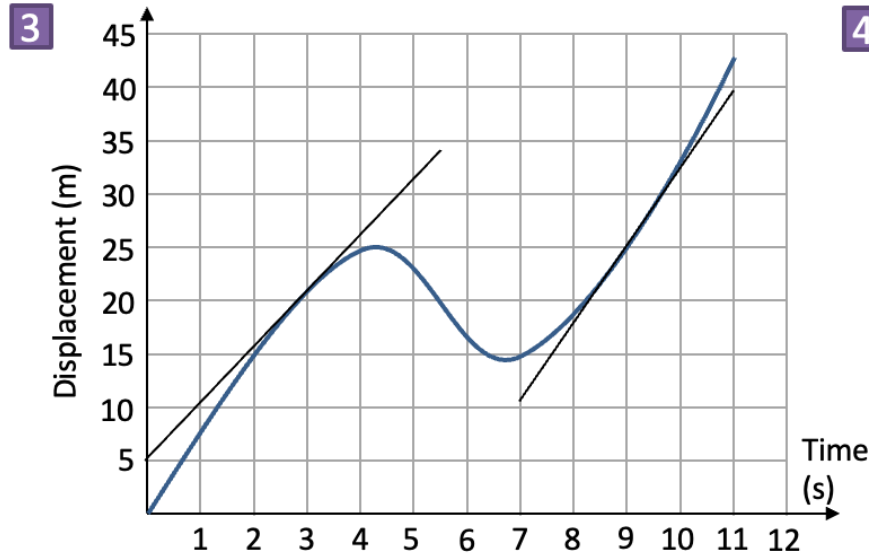
The graph shows Sheila's displacement from a wall during a game of dodgeball. Estimate her velocity when:

a) $t = 3$?

b) $t = 10$?

Note: The difference between 'distance' and 'displacement' is that displacement can be negative, and can decrease, whereas distance travelled always increases.

4
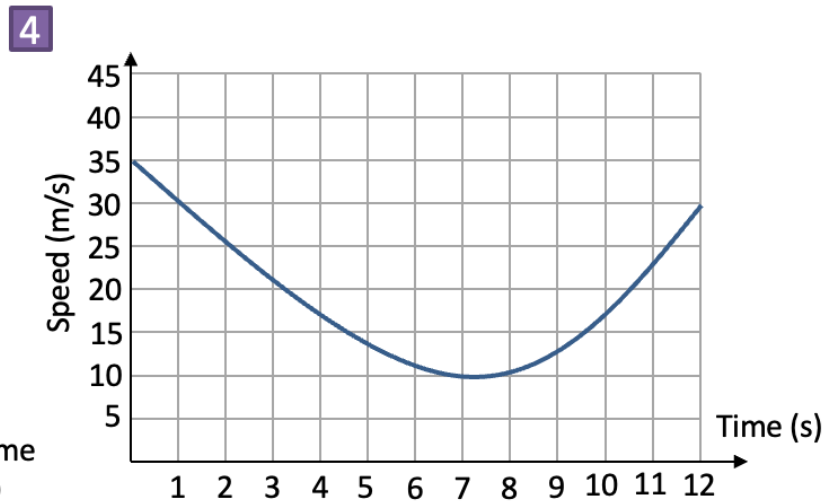
The graph shows the motion of a car before and after the driver sees a speed camera. Estimate the acceleration of the car when:

a) $t = 6$?

b) $t = 12$?

5
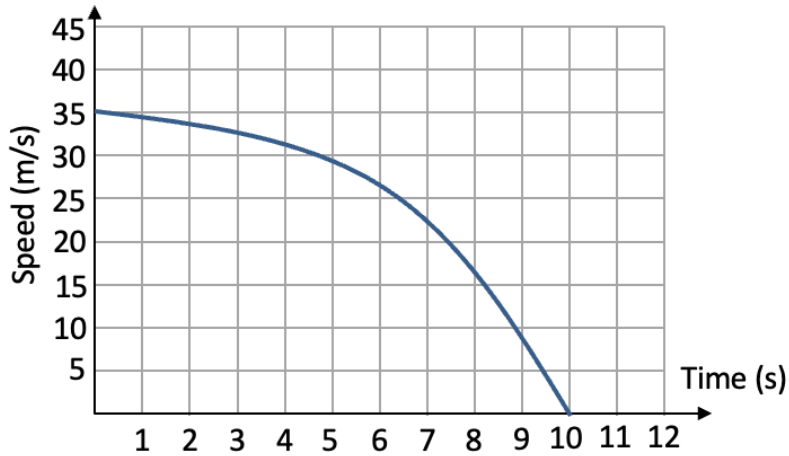
The graph shows the speed of a train after applying its breaks.

a) Determine the average acceleration of the train.

?

b) Estimate the time at which the acceleration of the train was this average.

?
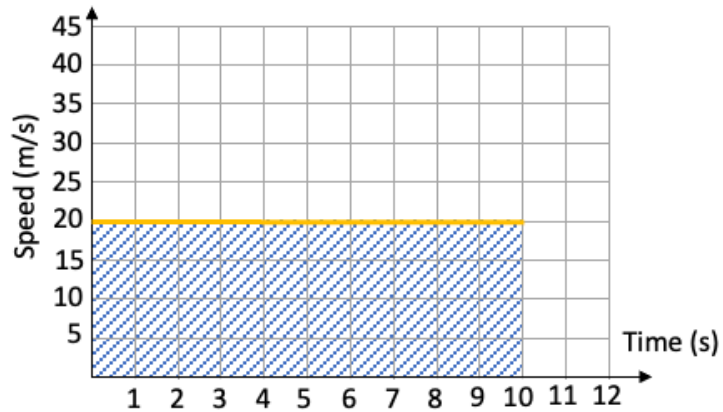
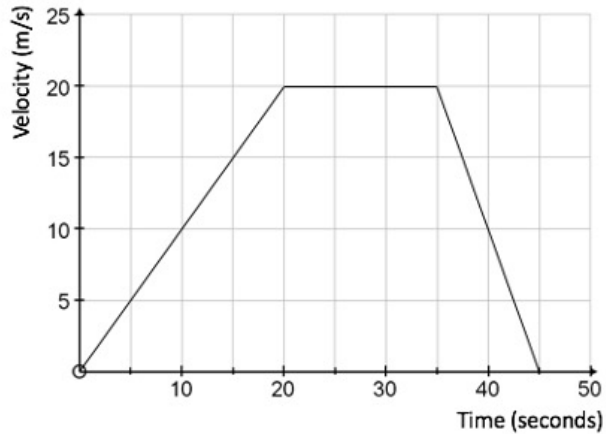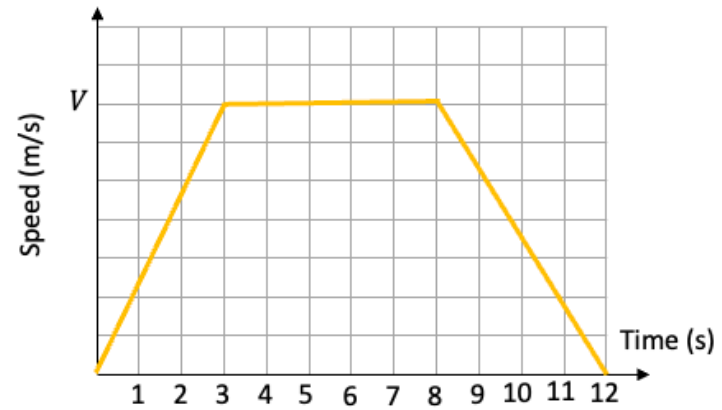| Worked Example	Worked Example
a) A cat runs at a constant speed of 20 m/s for 10 seconds, as indicated by the graph. What distance did it travel? b) What is the area of the shaded region under the graph up to 10 seconds? How does it relate to the distance?	What is the distance travelled?
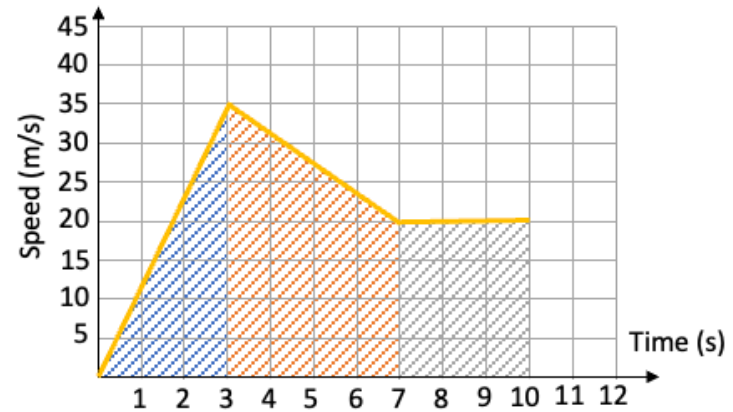
Your Turn	**Your Turn**

The velocity-time graph drawn below shows the journey of a car. Find the total distance travelled.

A car accelerates to V m/s in 3 seconds, maintains this speed for 6 seconds, then decelerates until coming to rest at 12 seconds. The car travels 340 m in total.
Determine the value of V.

Worked Example	Your Turn

Worked Example
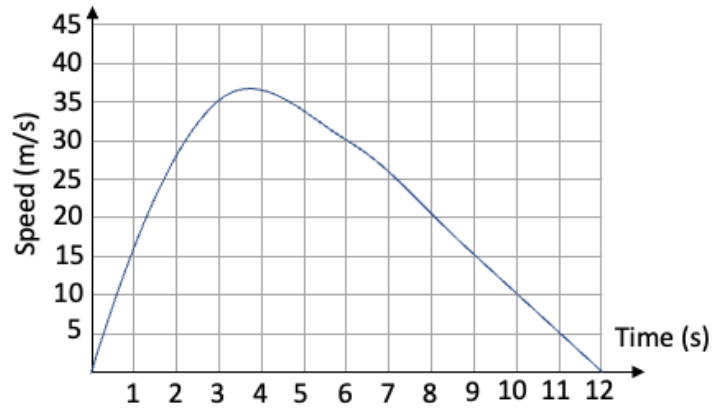
a) Suppose we want to estimate the area under the graph in order to determine the distance travelled. What might be a good strategy?

b) Work out an estimate for the distance the car travelled in the first 12 seconds. Use <u>4 strips</u> of equal width.
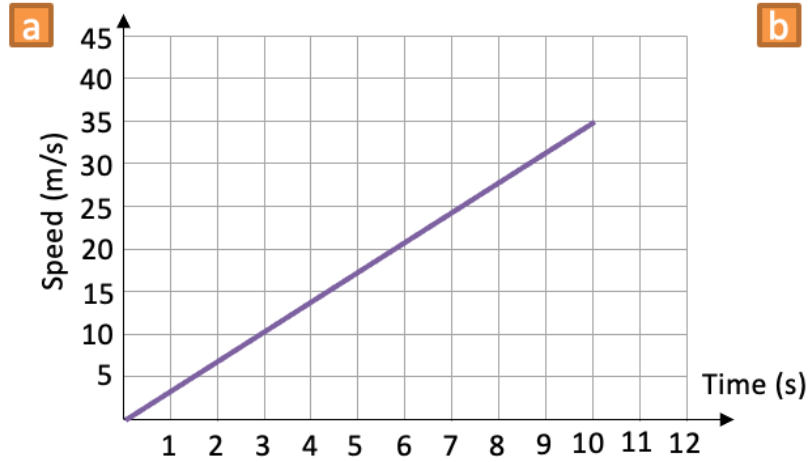
Your Turn
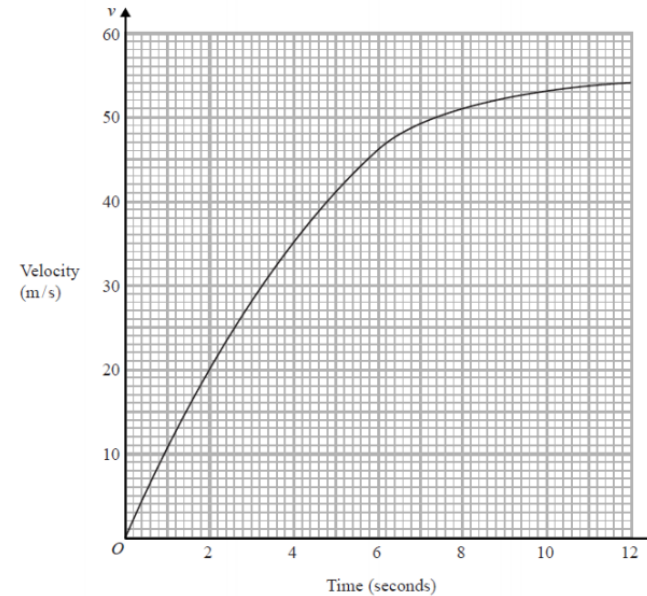
The graph shows information about the velocity, v m/s, of a parachutist t seconds after leaving a plane.
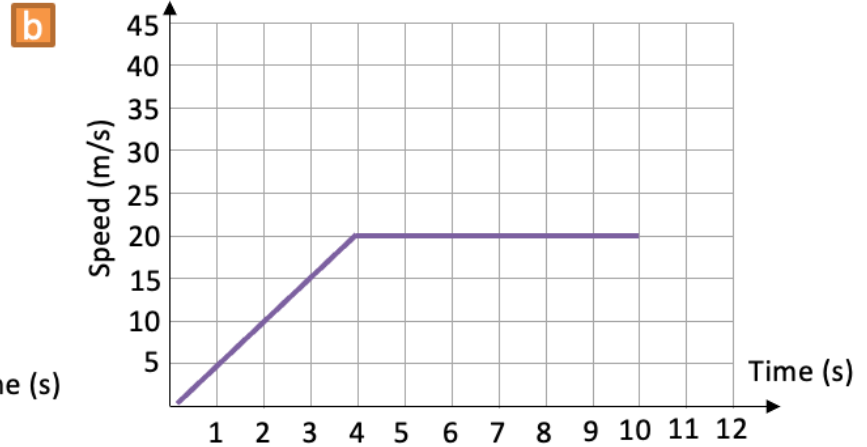
Work out an estimate for the distance fallen by the parachutist in the first 12 seconds after leaving the plane.
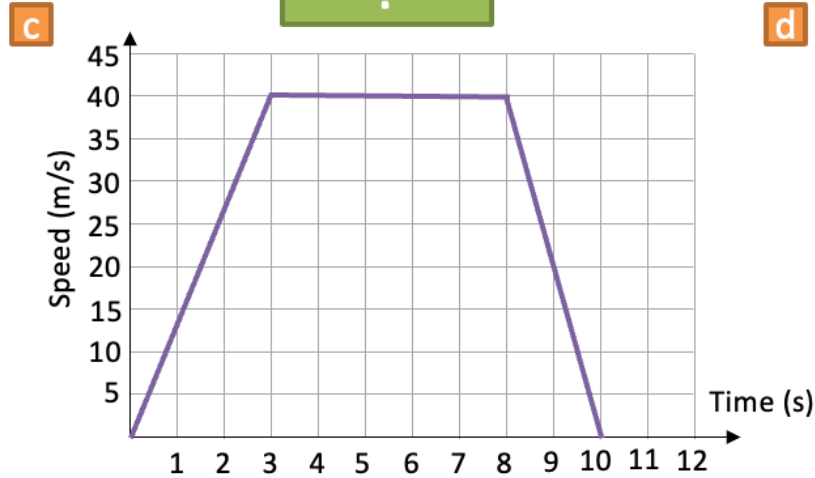Use 3 strips of equal width.

Fluency Practice

1 Calculate the distance travelled by these cars in their first 10 seconds.
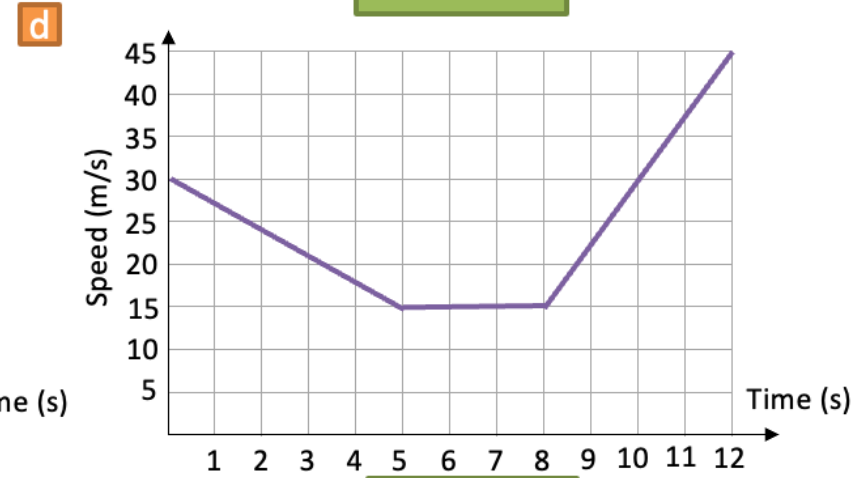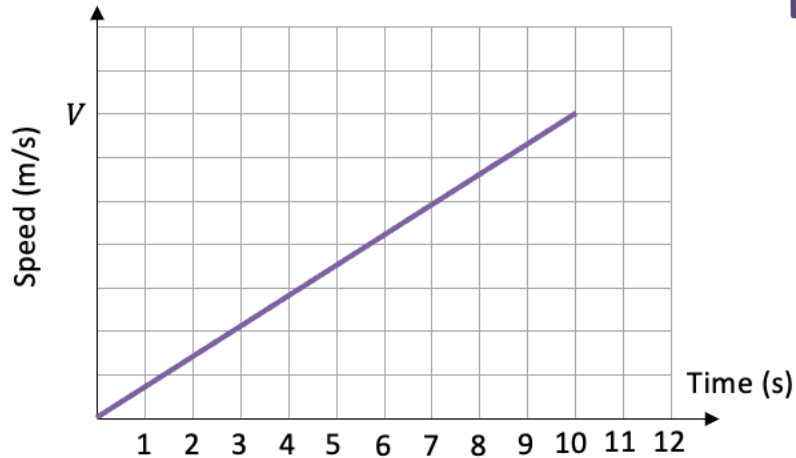
a

?
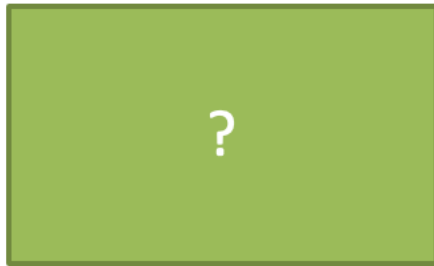
b

?

c

?

d
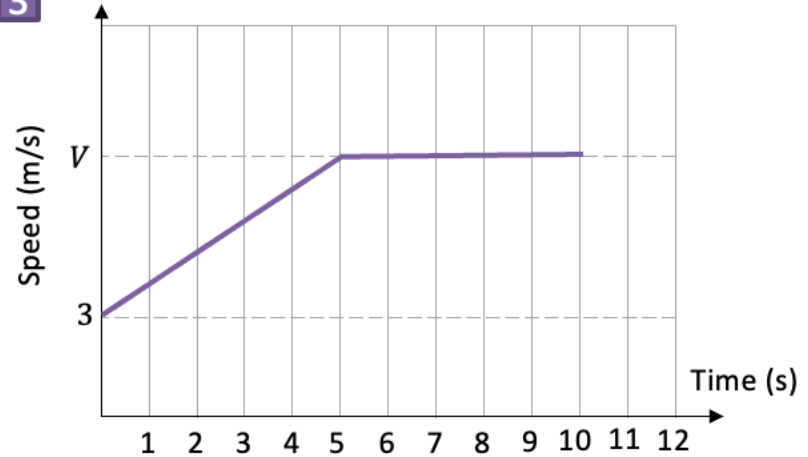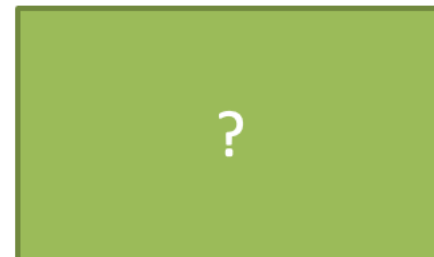

?

Fluency Practice

2

A car accelerates from rest with constant acceleration, until it reaches a speed of V m/s after 10 seconds. The distance travelled is 150m.
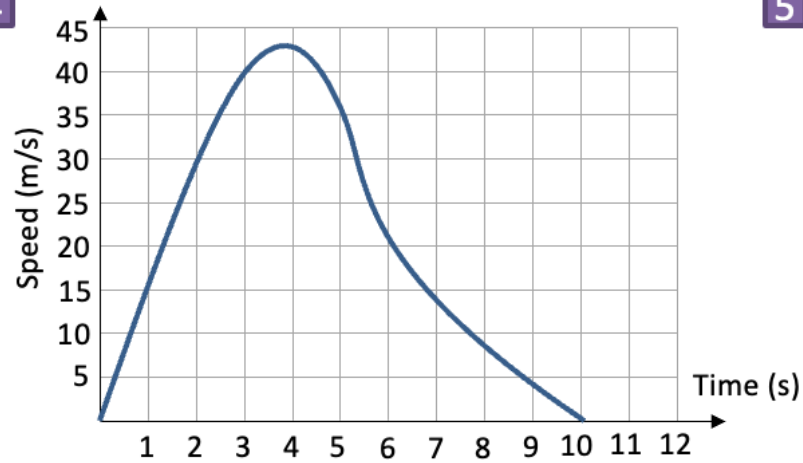Determine the value of V.

?

3

A tortoise starts running at a speed of 3 m/s, increasing to V m/s after 5 seconds. He maintains this speed for the next 5 seconds.
Over the 10 seconds he has travelled 155m. Determine V.
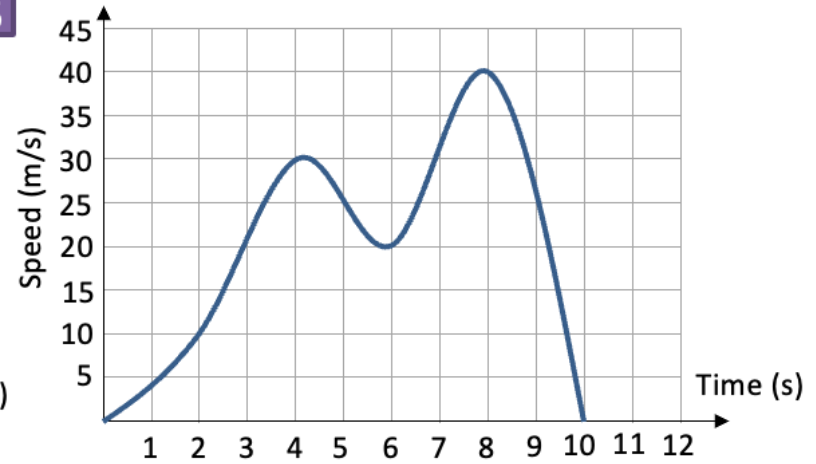
?

Fluency Practice

4

a) Using 2 strips, estimate the distance the car has travelled in the first 10 seconds.

?

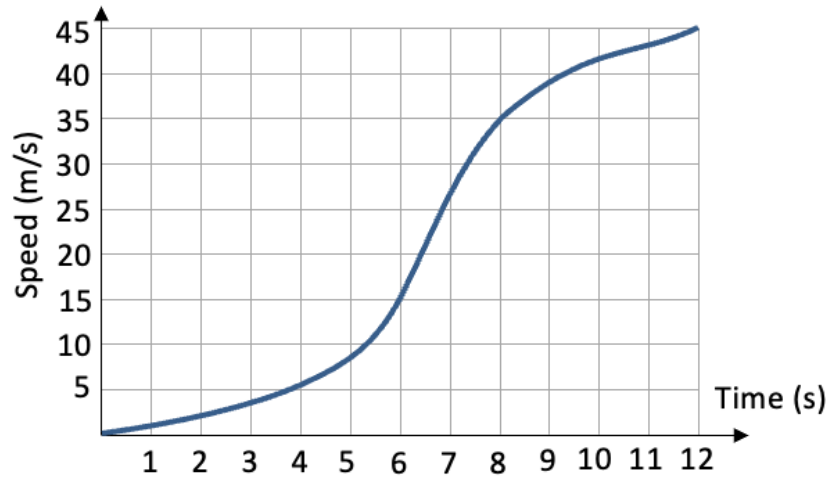b) Does this underestimate or overestimate the true area?

?

5

Using 5 strips, estimate the distance the car has travelled in the first 10 seconds.

?

Fluency Practice

Estimate the distance travelled in the first 12 seconds when using:

a) 1 strip (i.e. a single triangle)

?

b) 2 strips

?

c) 3 strips.

?

Extra Notes

5 Iterations

Worked Example	Your Turn
$x_1 = 3, x_{n+1} = x_n + 2$ Find the first four terms of the sequence.	$x_1 = 5, x_{n+1} = x_n - 3$ Find the first four terms of the sequence.

Worked Example	Your Turn
a) Show that the equation $x^2 - x - 1 = 0$ can be arranged to give $x = \sqrt{x+1}$ b) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \sqrt{x_n + 1}$ three times, to find an estimate for the solution of $x^2 - x - 1 = 0$. Write all the digits on your calculator display.	a) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \dfrac{1}{4} - \dfrac{x^3}{4}$ b) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \dfrac{1}{4} - \dfrac{(x_n)^3}{4}$ two times, to find an estimate for the solution of $x^3 + 4x = 1$. Write all the digits on your calculator display.

Worked Example	Your Turn
a) Show that $2x^2 - 5x + 1 = 0$ can be written in the form $x = \sqrt{\dfrac{5x-1}{2}}$ b) Use the iteration formula $x_{n+1} = \sqrt{\dfrac{5x_n - 1}{2}}$, to find x_4 to 3 decimal places. Start with $x_0 = 2$	a) Show that $x^2 - x - 19 = 0$ can be written in the form $x = \sqrt{19 + x}$ b) Use the iteration formula $x_{n+1} = \sqrt{19 + x_n}$ to find x_4 to 3 decimal places. Start with $x_0 = 0$

Worked Example	Your Turn
Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$.	Show that the equation $x^3 - 6x = 5$ has a solution between $x = 2$ and $x = 3$.

Extra Notes