



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

2023 **Year 11** **2024**
Mathematics
Unit 23 Tasks

DO NOT WRITE INSIDE

Contents Page

- 1 [Completing the Square](#)
- 2 [Quadratic Graphs](#)
- 3 [Quadratic Inequalities](#)
- 4 [Kinematics](#)
- 5 [Iterations](#)

1 Completing the Square

Fluency Practice

Express each of the following expressions in the form $(x + a)^2$.

1. $x^2 + 6x + 9$

5. $x^2 - 5x + \frac{25}{4}$

9. $x^2 - \frac{1}{2}x + \frac{1}{16}$

2. $a^2 + 4a + 4$

6. $b^2 + 3b + \frac{9}{4}$

10. $x^2 + 8x + 16$

3. $p^2 - 10p + 25$

7. $x^2 + 9x + \frac{81}{4}$

11. $x^2 + x + \frac{1}{4}$

4. $s^2 - 12s + 36$

8. $a^2 - a + \frac{1}{4}$

12. $x^2 + \frac{2}{3}x + \frac{1}{9}$

13. $p^2 + 18p + 81$

15. $t^2 - \frac{3}{2}t + \frac{9}{16}$

17. $x^2 - 2cx + c^2$

14. $a^2 - \frac{4}{5}a + \frac{4}{25}$

16. $x^2 + 2bx + b^2$

18. $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}$

Express the following expressions in the form $(ax + b)^2$.

19. $9x^2 + 6x + 1$

22. $9x^2 - 24x + 16$

25. $9x^2 - 6x + 1$

20. $4x^2 - 12x + 9$

23. $4x^2 - 4x + 1$

26. $4x^2 + 2x + \frac{1}{4}$

21. $100x^2 - 60x + 9$

24. $25x^2 + 20x + 4$

27. $\frac{9x^2}{4} + 2x + \frac{4}{9}$

Fluency Practice

What must be added to each of the following expressions to make it into a perfect square?

1. $x^2 + 4x$

5. $x^2 - 3x$

9. $a^2 - \frac{3}{2}a$

2. $a^2 + 8a$

6. $x^2 + 20x$

10. $x^2 + x$

3. $x^2 - 12x$

7. $c^2 + 7c$

11. $x^2 + 2hx$

4. $p^2 - 14p$

8. $b^2 - \frac{1}{2}b$

12. $x^2 + \frac{b}{a}x$

What must be added to each of the following expressions to make it into a perfect square?

1. $9x^2 + 12x$

4. $100x^2 - 60x$

7. $49a^2 - 28a$

2. $4x^2 + 12x$

5. $25x^2 - 20x$

8. $\frac{a^2}{4} - 2a$

3. $36a^2 + 60a$

6. $4x^2 + 20x$

9. $\frac{4}{9}a^2 - \frac{2a}{3}$

Intelligent Practice

Complete the square on the following expressions:

1) $x^2 + 4x$

2) $x^2 + 8x$

3) $x^2 + 12x$

4) $x^2 - 4x$

5) $x^2 - 8x$

6) $x^2 - 12x$

7) $x^2 + 3x$

8) $x^2 + 5x$

9) $x^2 - 7x$

Problem Solving

The missing values a, b, c, d, e and f are the digits 1, 2, 3, 4, 5 and 6 in some order (no repeats!)

$$x^2 + \overset{a}{\square}x + \overset{b}{\square} \overset{c}{\square}$$

$$= \left(x + \overset{d}{\square} \right) \overset{e}{\square} + \overset{f}{\square}$$

Fluency Practice

Complete the square.

- (a) $x^2 + 10x$
- (b) $x^2 + 8x$
- (c) $x^2 - 8x$
- (d) $x^2 - 6x + 3$
- (e) $x^2 - 6x - 3$
- (f) $x^2 + 4x - 3$

Write in completed square format.

- (a) $x^2 - 5x$
- (b) $x^2 + 7x$
- (c) $x^2 + 7x - 5$
- (d) $x^2 - 9x - 5$
- (e) $x^2 - 9x + 5$
- (f) $x^2 - 9x + 13$

Complete the square.

- (a) $2x^2 + 12x$
- (b) $2x^2 + 8x$
- (c) $2x^2 - 8x$
- (d) $2x^2 - 8x + 3$
- (e) $2x^2 - 6x + 3$
- (f) $3x^2 - 6x + 3$

Write in completed square format.

- (a) $3x^2 - 6x + 5$
- (b) $3x^2 + 6x + 5$
- (c) $4x^2 + 16x + 5$
- (d) $4x^2 - 16x - 5$
- (e) $5x^2 - 30x + 9$
- (f) $5x^2 + 30x - 9$

Fluency Practice

Solve the equations:

1. $(x + 1)^2 = 9$

2. $(x - 2)^2 = 16$

3. $(x - 3)^2 = 25$

4. $(x + 6)^2 = 100$

5. $(x + 7)^2 = 1$

6. $(x - 1)^2 = 25$

7. $(x + 2)^2 = 49$

8. $(x - 5)^2 = 16$

9. $(x - 7)^2 = 4$

10. $(x + 4)^2 = 16$

11. $(x + 3)^2 = 25$

12. $(x - 9)^2 = 36$

13. $(x + 1)^2 = \frac{1}{4}$

14. $(x - 2)^2 = \frac{9}{4}$

15. $(x - \frac{1}{2})^2 = \frac{25}{4}$

Solve the equations:

16. $(2x - 1)^2 = 16$

17. $(3x + 2)^2 = 25$

18. $(5x - 1)^2 = 36$

19. $(3x - 4)^2 = 1$

20. $(7x + 2)^2 = 100$

21. $(2x + 1)^2 = 36$

22. $(3x - 4)^2 = 49$

23. $(5x + 2)^2 = 25$

24. $(4x - 3)^2 = 1$

25. $(9x - 5)^2 = 4$

26. $(5x + 3)^2 = 16$

27. $(7x - 5)^2 = 81$

Fluency Practice

Solve the following equations by completing the square.

1. $x^2 + 4x = 5$

2. $x^2 - 6x = 7$

3. $x^2 + 10x = 11$

4. $x^2 + 8x + 3 = 0$

5. $x^2 - 4x + 1 = 0$

6. $x^2 + 8x = 3$

7. $x^2 - 4x = 9$

8. $x^2 + 9x + 4 = 0$

9. $x^2 - 7x + 5 = 0$

10. $x^2 - x - 4 = 0$

11. $x^2 + 9x - 3 = 0$

12. $x^2 + 8x + 4 = 0$

Solve the following equations by completing the square.

13. $2x^2 + 6x = 9$

14. $6x^2 - 12x = 5$

15. $4x^2 + 8x = 3$

16. $2x^2 - 3x - 4 = 0$

17. $3x^2 + 12x - 8 = 0$

18. $3x^2 - 5x = 1$

19. $5x^2 - 5x = 4$

20. $5x^2 + 8x + 2 = 0$

21. $4x^2 - 7x - 3 = 0$

22. $6x^2 - 5x - 1 = 0$

23. $7x^2 + 7x - 4 = 0$

24. $3x^2 - 9x = 2$

Fluency Practice

Question 1: Write the following expressions in the form $(x + a)^2 + b$

(a) $x^2 + 8x + 1$

(b) $x^2 + 10x + 3$

(c) $x^2 + 2x - 1$

(d) $x^2 - 6x - 10$

(e) $x^2 - 4x - 13$

(f) $x^2 - 12x + 3$

(g) $x^2 + 14x + 3$

(h) $x^2 - 2x - 15$

(i) $x^2 + 4x - 11$

(j) $x^2 + x - 8$

(k) $x^2 + 3x + 1$

(l) $x^2 - 7x - 2$

(m) $x^2 - 9x - 1$

(n) $x^2 + 11x + 3$

(o) $x^2 - 100x - 25$

Question 2: Solve the following equations (use completing the square).

(a) $x^2 + 4x + 1 = 0$

(b) $x^2 + 8x - 10 = 0$

(c) $x^2 + 14x - 4 = 0$

(d) $x^2 - 8x - 2 = 0$

(e) $x^2 - 10x + 10 = 0$

(f) $x^2 + 18x + 7 = 0$

(g) $x^2 + 12x + 3 = 19$

(h) $x^2 = 2x + 10$

(i) $x^2 - 7x - 3 = 0$

(j) $x^2 + x - 7 = 0$

(k) $x^2 + 3x + 8 = 0$

(l) $2x^2 - 10x - 30 = x^2 - 4x$

Question 3: Write the following expressions in the form $a(x + b)^2 + c$

(a) $2x^2 + 8x + 2$

(b) $2x^2 + 12x - 3$

(c) $3x^2 - 12x + 2$

(d) $4x^2 + 12x - 5$

(e) $2x^2 - 3x - 5$

(f) $5x^2 - 20x + 30$

Question 4: Solve the following equations (use completing the square).

(a) $3x^2 + 12x + 3 = 0$

(b) $2x^2 + 16x - 20 = 0$

(c) $3x^2 - 6x + 1 = 0$

(d) $5x^2 + 10x - 9 = 0$

(e) $2x^2 - 5x - 3 = 0$

(f) $2x^2 - 7x + 1 = 0$

Fluency Practice

Question 1: Solve each of the equations below using completing the square

(a) $x^2 + 6x + 8 = 0$

(b) $x^2 + 10x + 24 = 0$

(c) $x^2 + 14x + 40 = 0$

(d) $x^2 - 4x - 45 = 0$

(e) $x^2 - 12x + 35 = 0$

(f) $x^2 - 2x - 3 = 0$

(g) $x^2 + 14x - 51 = 0$

(h) $x^2 - 6x - 16 = 0$

(i) $x^2 - 2x + 1 = 0$

Question 2: Solve each of the following equations using completing the square

(a) $x^2 + 5x + 4 = 0$

(b) $x^2 - 3x - 18 = 0$

(c) $x^2 + x - 12 = 0$

(d) $x^2 - 7x + 12 = 0$

(e) $x^2 - 11x + 24 = 0$

(f) $x^2 - 7x - 30 = 0$

Question 3: Solve each of the following equations using completing the square
Write each answer in simplified surd form

(a) $x^2 + 4x - 3 = 0$

(b) $x^2 + 6x - 10 = 0$

(c) $x^2 - 2x - 5 = 0$

(d) $x^2 - 10x + 1 = 0$

(e) $x^2 + 8x + 3 = 0$

(f) $x^2 - 8x - 22 = 0$

(g) $x^2 + 20x + 7 = 0$

(h) $x^2 - 12x + 1 = 0$

(i) $x^2 - 30x - 100 = 0$

Question 4: Solve each of the following equations using completing the square
Write each answer in simplified surd form

(a) $x^2 + x - 7 = 0$

(b) $x^2 - 3x + 1 = 0$

(c) $x^2 + 11x - 5 = 0$

(d) $x^2 - 7x + 9 = 0$

(e) $x^2 - x - 50 = 0$

(f) $x^2 + 13x + 1 = 0$

Question 5: Solve each of the following equations using completing the square
Write each answer in simplified surd form

(a) $x^2 - 4x + 1 = 10$

(b) $x^2 + 2x + 5 = 20 - 8x$

(c) $3x^2 = 2x^2 + x + 7$

(d) $x^2 + 6x + 4 = 2x^2 + 8x - 1$

(e) $\frac{12}{x} = 10 + x$

(f) $\frac{x-1}{2} = \frac{5}{x+3}$

Fluency Practice

Question 6: Solve each of the following equations using completing the square

(a) $5x^2 + 30x - 10 = 0$

(b) $2x^2 + 7x + 3 = 0$

(c) $3x^2 + 12x - 2 = 0$

(d) $2x^2 - 3x - 7 = 0$

(e) $5x^2 + 2x - 8 = 0$

(f) $10x^2 - 2x - 1 = 0$

Apply

Question 1: Find the points where the curve $y = x^2 + 10x + 3$ meets:

(a) the y-axis

(b) the x-axis

Question 2: The length of a rectangle is 20cm longer than its width.

The area of the rectangle is 1000cm^2

Find the width and length of the rectangle.

Give your answers in surd form.



Question 3: Abby is trying to solve $x^2 + 4x + 15 = 0$

By using completing the square, explain why there are no (real) solutions

Question 4: The curve $y = x^2 + 8x - 1$ meets the x-axis at the points A and B

The point C is (2, 5)

Find the area of triangle ABC

Question 5: James has solved the equation $x^2 + ax + b = 0$
His solutions are $x = -3 + \sqrt{17}$ and $x = -3 - \sqrt{17}$

Find a and b

Question 6: By using completing the square on $ax^2 + bx + c = 0$ to establish the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Fluency Practice

Solve these quadratic equations by completing the square.

(a) $x^2 - 2x - 1 = 0$

(b) $x^2 - 4x - 1 = 0$

(c) $x^2 + 4x + 2 = 0$

(d) $x^2 + 4x - 2 = 0$

(e) $x^2 + 10x + 2 = 0$

(f) $x^2 + 12x - 5 = 0$

Solve these quadratic equations by completing the square.

(a) $x^2 + 11x - 1 = 0$

(b) $x^2 + 11x - 3 = 0$

(c) $x^2 + 5x - 3 = 0$

(d) $x^2 - 5x - 3 = 0$

(e) $x^2 - x - 3 = 0$

Solve these quadratic equations by completing the square.

(a) $2x^2 - 8x + 3 = 0$

(b) $2x^2 - 4x - 3 = 0$

(c) $3x^2 + 12x + 3 = 0$

(d) $3x^2 - 18x - 2 = 0$

(e) $4x^2 + 16x - 2 = 0$

(f) $5x^2 + 20x - 5 = 0$

Solve these quadratic equations by completing the square.

(a) $2x^2 + 3x - 1 = 0$

(b) $2x^2 + 5x - 1 = 0$

(c) $2x^2 - 5x - 3 = 0$

(d) $2x^2 - 11x - 3 = 0$

Fluency Practice

(a) Solve $x^2 = 9$	(b) Solve $x^2 - 7x + 10 = 0$	(c) Solve $x^2 - 5x + 6 = 0$	(d) Solve $x^2 + 8x + 12 = 0$
(e) Solve $x^2 + 2x - 8 = 0$	(f) Solve $x^2 + 10x + 21 = 0$	(g) Solve $x^2 - 3x - 18 = 0$	(h) Solve $x^2 - 1 = x + 5$
(i) Solve $3x^2 - 7x + 2 = 0$	(j) Solve $x^2 + 5x + 2 = 0$, giving your solutions to 3 significant figures.	(k) Solve $x^2 + 3x - 8 = 0$, giving your solutions to 3 significant figures.	(l) Solve $3x^2 + 2x - 9 = 0$, giving your solutions in surd form.

Fluency Practice

There are three algebraic methods for solving quadratic equations: (a) By factorising (b) Using the quadratic formula (c) By completing the square

Solve each of the following quadratic equations using each of the three methods, remembering that sometimes it is not possible to solve by factorising. When using the quadratic formula, give your answers to 2 decimal places.

Equation	By Factorising	By Formula	By Completing the Square	Equation	By Factorising	By Formula	By Completing the Square
$x^2 + 4x + 3 = 0$	$(x + 3)(x + 1) = 0$ $x = -3$ or $x = -1$	$a = 1, b = 4, c = 3$ $x = \frac{-4 \pm \sqrt{16 - 12}}{2}$ $x = -3$ or $x = -1$	$(x + 2)^2 - 4 + 3 = 0$ $(x + 2)^2 = 1$ $x = -2 \pm \sqrt{1}$ $x = -3$ or $x = -1$	$3x^2 - 12x = 0$			
$x^2 - 6x + 8 = 0$				$2x^2 - 9x - 5 = 0$			
$x^2 - x - 12 = 0$				$2x^2 + 8x - 1 = 0$			
$x^2 + 4x - 2 = 0$				$x^2 + 3x = 18$			
$x^2 + 2x - 5 = 0$				$2x^2 = 4x + 1$			

Problem Solving

In each row, explain which equation is the odd one out.

$(x + 5)^2 + 10 = 0$	$x^2 + 10x + 35 = 0$	$x^2 + 10x + 25 = 10$	$(x + 5)^2 - 25 + 35 = 0$
$5x^2 + 10x = 10$	$(x^2 + 2x) = 2$	$(x + 1)^2 = 3$	$(x + 1)^2 - 2 = 2$
$2x^2 + 12x - 20 = 0$	$2(x^2 + 6x) = 20$	$(x + 3)^2 = 19$	$x = 3 \pm \sqrt{19}$
$6x^2 + 36x = 18$	$x^2 + 6x = 3$	$(x + 3)^2 + 9 = 3$	$(x + 3)^2 = 12$

* Challenge *

How many equations can you write that are equivalent to : $(x + 3)^2 = 30$

Can you write some equations that are equivalent to $x^2 + 20x = 26$?

2 Quadratic Graphs

Fluency Practice

For each of these, sketch a graph marking clearly any points where it crosses the axes.

1. $y = (x - 4)(x - 1)$

11. $y = x^2 + 2x + 1$

2. $y = (x - 2)(x - 1)$

12. $y = x^2 - 2x + 1$

3. $x(x - 2)$

13. $y = x^2 + 5x + 4$

4. $y = (x + 1)(x - 1)$

14. $y = x^2 + 7x + 12$

5. $x(x - 5)$

15. $y = x^2 - 3x - 10$

6. $y = (x - 10)(x + 2)$

16. $y = x^2 + 3x - 10$

7. $y = (-x + 10)(x + 2)$

17. $y = x^2 + 5x - 14$

8. $y = (x + 3)(x - 4)$

18. $y = x^2 - 7x - 30$

9. $y = (x - 3)(x - 2)$

19. $y = x^2 - 17x - 30$

10. $y = (x + 15)(x - 4)$

20. $y = x^2 - 6x - 8$

Don't forget that a line crosses the y -axis when $x = 0$ and crosses the x -axis when $y = 0$.

If $x = 0$ then put a zero in where there is an x and then find out what y equals.

If $y = 0$ then put a zero in where there is a y and rearrange the equation to find out what y equals.

For example, to find out where $y = (x + 7)(x + 2)$ crosses the y -axis, put in 0 instead of x and get $y = (0 + 7)(0 + 2) = 7 \times 2 = 14$. To find where it crosses the x -axis, put in 0 instead of y and get $(x + 7)(x + 2) = 0$. If two things multiplied make zero, then at least one of them must be zero, so either $x + 7 = 0 \Rightarrow x = -7$ or $x + 2 = 0 \Rightarrow x = -2$.

Fluency Practice

For each of these, sketch a graph marking clearly any points where it crosses the axes.

1. $y = 3x^2 + 4x + 1$

6. $y = 4x^2 - 31x + 21$

2. $y = 2x^2 + 7x + 6$

7. $y = -2x^2 - 7x + 4$

3. $y = 2x^2 - 5x - 3$

8. $y = 14x^2 - 3x - 2$

4. $y = 6x^2 - x - 2$

9. $y = -12x^2 - x + 1$

5. $y = 2x^2 - x - 15$

10. $y = 4x^2 - 1$

Don't forget that a line crosses the y -axis when $x = 0$ and crosses the x -axis when $y = 0$.

If $x = 0$ then put a zero in where there is an x and then find out what y equals.

If $y = 0$ then put a zero in where there is a y and rearrange the equation to find out what y equals.

For example, to find out where $y = (x+7)(x+2)$ crosses the y -axis, put in 0 instead of x and get $y = (0+7)(0+2) = 7 \times 2 = 14$. To find where it crosses the x -axis, put in 0 instead of y and get $(x+7)(x+2) = 0$. If two things multiplied make zero, then at least one of them must be zero, so either $x+7=0 \Rightarrow x=-7$ or $x+2=0 \Rightarrow x=-2$.

$2x^2 - x - 6$ Look for factors of -12 (2 times -6)

$$2x^2 - 4x + 3x - 6$$

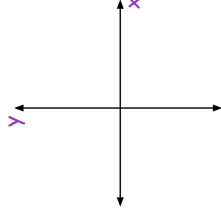
$$2x(x - 2) + 3(x - 2)$$

$$(2x + 3)(x - 2)$$

Fluency Practice

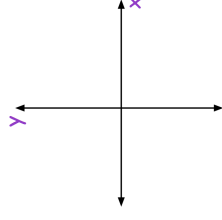
Question 1: Emilio wants to sketch the graph of $y = x^2 - 7x + 10$

- (a) Find the value of y when $x = 0$
- (b) Use your answer to (a) to plot where the graph crosses the y -axis.
- (c) Solve the equation $x^2 - 7x + 10 = 0$
- (d) Use your answers to (c) to help you plot where the graph crosses the x -axis.
- (e) Sketch the graph of $y = x^2 - 7x + 10$



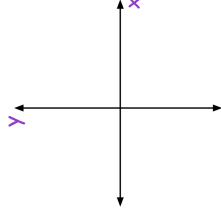
Question 2: Rebecca wants to sketch the graph of $y = x^2 + 7x - 8$

- (a) Find the value of y when $x = 0$
- (b) Use your answer to (a) to plot where the graph crosses the y -axis.
- (c) Solve the equation $x^2 + 7x - 8 = 0$
- (d) Use your answers to (c) to help you plot where the graph crosses the x -axis.
- (e) Sketch the graph of $y = x^2 + 7x - 8$



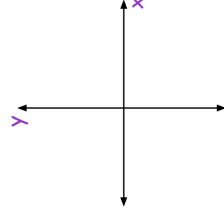
Question 3: Michael wants to sketch the graph of $y = x^2 + 16x + 64$

- (a) Find the value of y when $x = 0$
- (b) Use your answer to (a) to plot where the graph crosses the y -axis.
- (c) Solve the equation $x^2 + 16x + 64 = 0$
- (d) Use your answers to (c) to help you plot where the graph meets the x -axis.
- (e) Sketch the graph of $y = x^2 + 16x + 64$



Question 4: James wants to sketch the graph of $y = x^2 + 4x + 10$

- (a) Find the value of y when $x = 0$
- (b) Use your answer to (a) to plot where the graph crosses the y -axis.
- (c) Show that the equation $x^2 + 4x + 10 = 0$ has no real roots.
- (d) Explain why your answer to (c) means that the graph does not cross the x -axis.
- (e) Find the value of y when $x = 1$
- (e) Sketch the graph of $y = x^2 + 4x + 10$



Fluency Practice

Question 5: Sketch the following graphs.

(a) $y = x^2 + 6x + 8$

(b) $y = x^2 - x - 6$

(c) $y = x^2 + 6x + 9$

(d) $y = x^2 - 13x + 42$

(e) $y = x^2 + 5x - 36$

(f) $y = x^2 - 2x + 1$

(g) $y = x^2 + 5x + 11$

(h) $y = x^2 - 4x + 7$

Question 6: Sketch the following graphs.

(a) $y = (x - 7)(x + 10)$

(b) $y = (x + 3)(x + 8)$

(c) $y = (x - 2)^2$

Question 7: Sketch the following graphs.

(a) $y = x^2 - 49$

(b) $y = x^2 - 1$

(c) $y = x^2 - 196$

Question 8: Michael wants to sketch the graph of $y = -x^2 + 5x + 14$

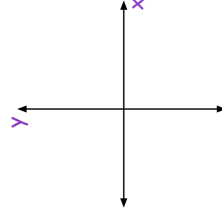
(a) Find the value of y when $x = 0$

(b) Use your answer to (a) to plot where the graph crosses the y -axis.

(c) Solve the equation $-x^2 + 5x + 14 = 0$

(d) Use your answers to (c) to help you plot where the graph crosses the x -axis.

(e) Sketch the graph of $y = -x^2 + 5x + 14$



Question 9: Sketch the following graphs.

(a) $y = -x^2 - 5x - 4$

(b) $y = -x^2 + 9x - 18$

(c) $y = 84 - 5x - x^2$

(d) $y = (3 - x)(x + 8)$

(e) $y = -x^2 - 8x - 16$

(f) $y = 144 - x^2$

Question 10: Robyn wants to sketch the graph of $y = 2x^2 + 9x + 4$

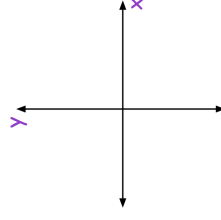
(a) Find the value of y when $x = 0$

(b) Use your answer to (a) to plot where the graph crosses the y -axis.

(c) Solve the equation $2x^2 + 9x + 4 = 0$

(d) Use your answers to (c) to help you plot where the graph crosses the x -axis.

(e) Sketch the graph of $y = 2x^2 + 9x + 4$



Fluency Practice

Question 10: Sketch the following graphs

- (a) $y = 5x^2 + 13x + 6$ (b) $y = 3x^2 - 16x - 12$ (c) $y = 2x^2 - 13x + 15$
 (d) $y = (2x + 5)(2x - 1)$ (e) $y = 3 - 20x - 7x^2$ (f) $y = 4x^2 - 28x + 49$

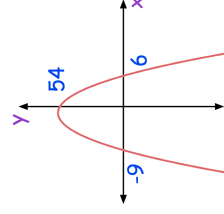
Question 11: Sketch the following graphs.

Label exactly where each graph crosses the coordinates axes.

- (a) $y = x^2 + 4x + 1$ (b) $y = x^2 - 10x + 10$ (c) $y = x^2 - 8x - 2$
 (d) $y = x^2 + 18x + 7$ (e) $y = x^2 - 7x - 3$ (f) $y = 2x^2 + 8x + 2$

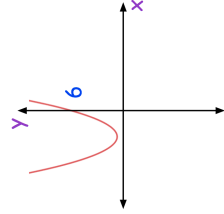
Apply

Question 1: Dominic sketches the graph of $y = x^2 + 3x - 54$



Can you spot any mistakes?

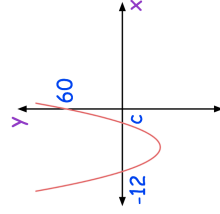
Question 2: Hannah sketches the graph of $y = x^2 - 2x + 6$



- (a) Can you spot any mistakes?
 (b) Use the discriminant, $b^2 - 4ac$, to explain why the graph of $y = x^2 - 2x + 6$ does not cross the x-axis.

Question 3: Shown is the graph of $y = x^2 + ax + b$

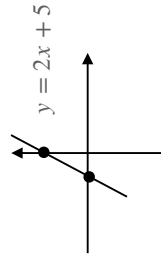
Find the values of a, b and c.



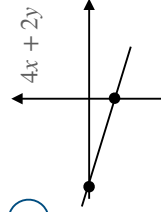
Fluency Practice

Find the coordinates of where each of the following cut the x-axis and y -axis.

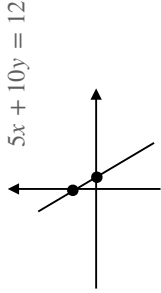
1



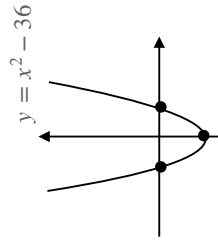
2



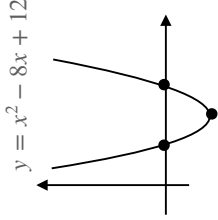
3



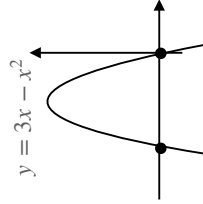
4



5



6



Find the coordinates of where each of the following cut the x-axis and y -axis.
State if the equation represents a line or parabola.

7

$$y = 12x - 24$$

8

$$y = x^2 + 10x$$

9

$$y = 5 - x$$

10

$$y = x^2 + 5x + 4$$

11

$$3y = 6x + 12$$

12

$$y = x^2 + 7x + 6$$

13

$$y = (x - 5)(x - 2)$$

14

$$4y + 8x - 12 = 0$$

15

$$y = 35 - 2x - x^2$$

16

$$y = 5x^2 + 2x - 7$$

17

$$y = x^2 - 49$$

18

$$8y + 4x - 12 = 0$$

19

$$y = 9 - x^2$$

20

$$y = 21x^2 - 35x - 14$$

21

$$y = 6 - x - x^2$$

22

$$2x + 3y - 7 = 0$$

23

$$y = (x - 3)(x + 3)$$

24

$$y = 12x - 6x^2$$

Intelligent Practice

Find the coordinates of the turning point of the following graphs:

1) $y = x^2 + 10x + 4$

1) $y = 2x^2 + 4x + 4$

2) $y = x^2 + 10x + 3$

2) $y = 2x^2 + 4x + 3$

3) $y = x^2 + 10x + 2$

3) $y = 2x^2 + 4x + 2$

4) $y = x^2 + 10x + 1$

4) $y = 2x^2 + 4x + 1$

5) $y = x^2 + 10x - 1$

5) $y = 2x^2 + 4x - 1$

6) $y = x^2 + 10x - 2$

6) $y = 2x^2 + 4x - 2$

7) $y = x^2 + 10x - 3$

7) $y = 2x^2 + 4x - 3$

8) $y = x^2 + 10x - 4$

8) $y = 2x^2 + 4x - 4$

Fluency Practice

Apply

Question 1: Write $(x + 3)^2 - 4$ in the form $x^2 + bx + c$

Question 2: Write $(x - 2)^2 - 9$ in the form $x^2 + bx + c$

Question 3: Write $(x - 7)^2 + 11$ in the form $x^2 + bx + c$

Question 4: Use completing the square to find the minimum point for each graph below

(a) $y = x^2 + 10x + 12$

(b) $y = x^2 + 4x + 1$

(c) $y = x^2 + 6x + 8$

(d) $y = x^2 - 2x + 3$

(e) $y = x^2 - 6x - 3$

(f) $y = x^2 - x - 4$

(g) $y = x^2 + 9x + 1$

(h) $y = x^2 - 6x - 2$

(i) $y = x^2 + 22x + 100$

Question 5: By using completing the square to solve $ax^2 + bx + c = 0$, prove the quadratic formula.

Question 6: Can you spot any mistakes?

$$\text{Solve } x^2 + 10x + 2 = 0$$

$$(x + 5)^2 - 10 + 2 = 0$$

$$(x + 5)^2 - 12 = 0$$

$$(x + 5)^2 = 12$$

$$x + 5 = \sqrt{12}$$

$$x = -5 + \sqrt{12}$$

Fluency Practice

1. By writing the following in the form $y = (x + a)^2 + b$, where a and b are integers, write down the coordinates of the turning point of the curve. Hence sketch the curve.

(a) $y = x^2 - 8x + 20$

(c) $y = x^2 + 4x - 6$

(e) $y = -x^2 + 6x + 10$

(b) $y = x^2 - 10x - 1$

(d) $y = 2x^2 - 12x + 8$

(f) $y = 5 - 2x - x^2$

2. Given the following minimum turning points of quadratic curves, find an equation of the curve in the form $y = x^2 + ax + b$. Hence sketch each curve.

(a) $(2, -3)$

(c) $(-1, 5)$

(e) $(1, -7)$

(b) $(-4, 1)$

(d) $(3, -12)$

(f) $(-4, -1)$

3. Find the maximum or minimum value of the following curves and sketch each curve.

(a) $y = x^2 + 4x + 2$

(c) $y = -x^2 + 2x - 3$

(e) $y = x^2 - 3x - 1$

(b) $y = 1 - 6x - x^2$

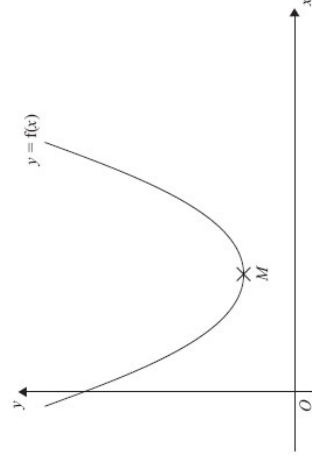
(d) $y = x^2 - 8x + 8$

(f) $y = -3x^2 + 12x - 9$

4. The expression $x^2 - 3x + 8$ can be written in the form $(x - a)^2 + b$ for all values of x .

(i) Find the value of a and the value of b .

The equation of a curve is $y = f(x)$ where $f(x) = x^2 - 3x + 8$. The diagram shows part of a sketch of the graph of $y = f(x)$.



The minimum point of the curve is M .

(ii) Write down the coordinates of M .

Fluency Practice

5. (i) Sketch the graph of $f(x) = x^2 - 6x + 10$, showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.
- (ii) Hence, or otherwise, determine whether $f(x) - 3 = 0$ has any real roots. Give reasons for your answer.
- *6. The minimum point of a quadratic curve is $(1, -4)$. The curve cuts the y -axis at -1 . Show that the equation of the curve is $y = 3x^2 - 6x - 1$
- *7. The maximum point of a quadratic curve is $(-2, -5)$. The curve cuts the y -axis at -13 . Find the equation of the curve. Give your answer in the form $ax^2 + bx + c$.

* = extension

Fluency Practice

Plotting Quadratics

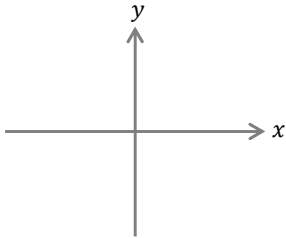
Factorise each quadratic expression. Use this factorisation to sketch the **x-intercepts**.



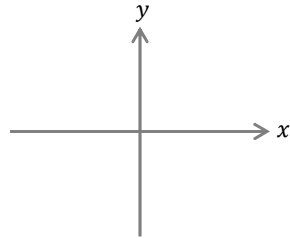
Every quadratic graph is **symmetrical**. Use this fact to find the **turning point** of each graph & its coordinates.

How can we calculate the y-intercept?

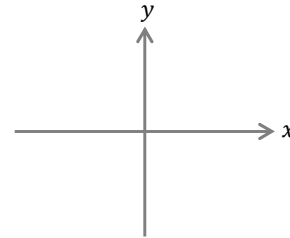
A) $y = x^2 - 4x$



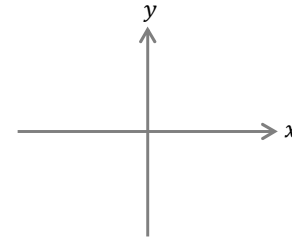
B) $y = x^2 + 8x$



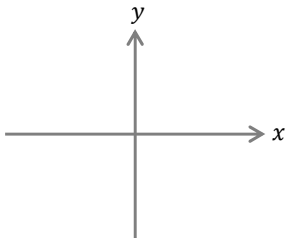
C) $y = x^2 + 6x + 8$



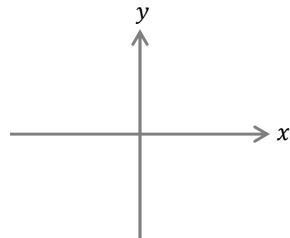
D) $y = x^2 - 8x + 12$



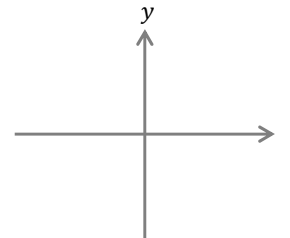
E) $y = x^2 - 2x - 3$



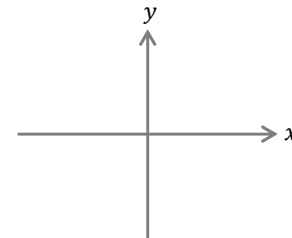
F) $y = x^2 - 9$



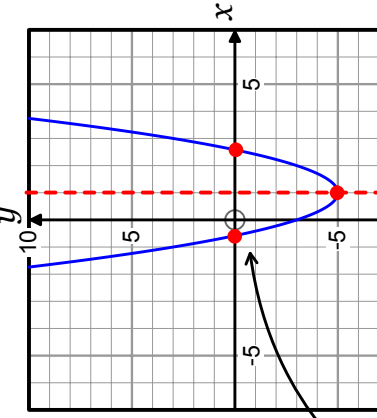
G) $y = x^2 - 5x + 6$



H) $y = x^2 + x - 20$



Fluency Practice



A Quadratic Graph has:

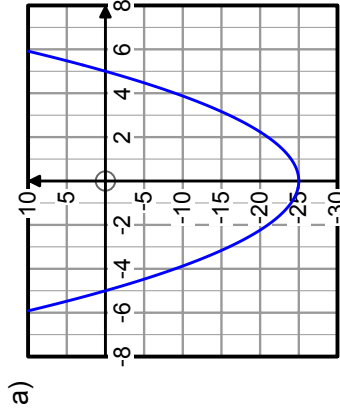
- a **vertex or turning point** $(-1, 1)$ on the graph shown, where the graph 'turns'

- a **line of symmetry** $x = -1$ on the graph it passes through the vertex

- **0, 1 or 2 roots** (x intercepts) $x \approx -0.7$ and 2.7 for this graph this is where the graph crosses the x axis

exercise

- Write down estimations or exact values of the y -intercept, roots, vertex and line of symmetry for each of these graphs:

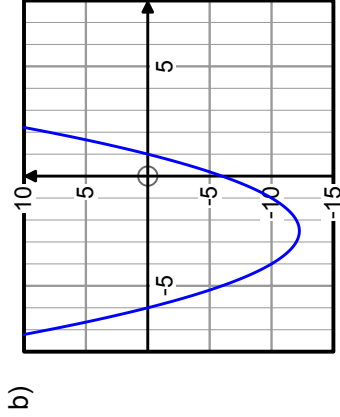


y - intercept:

turning point:

line of symmetry:

roots:



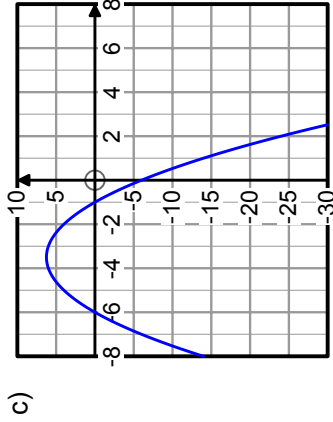
y - intercept:

turning point:

line of symmetry:

roots:

Fluency Practice

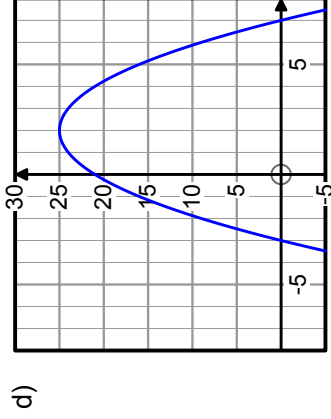


y - intercept:

turning point:

line of symmetry:

roots:



y - intercept:

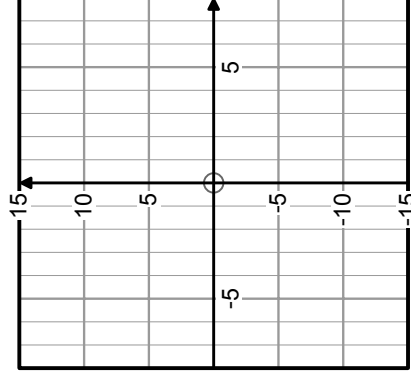
turning point:

line of symmetry:

roots:

2. Consider the graph $y = (x + 4)(x - 2)$.

- What are the roots of the function?
- Can you work out the line of symmetry without drawing the graph?
- Can you work out the y-intercept without drawing the graph?
- Sketch the graph of $y = (x + 4)(x - 2)$.



3. Write down the roots, y-intercept and line of symmetry for each of these functions:

a) $y = (x + 4)(x - 4)$

y - intercept:

line of symmetry:

roots:

b) $y = (x + 1)(x - 4)$

y - intercept:

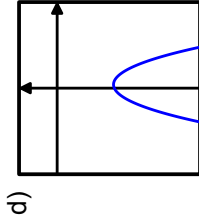
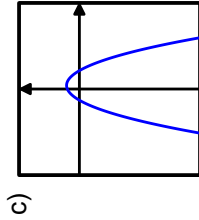
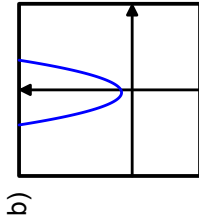
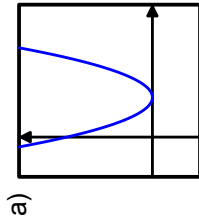
line of symmetry:

roots:

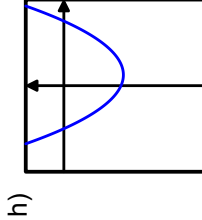
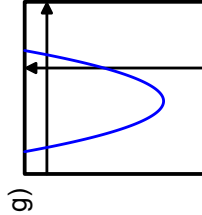
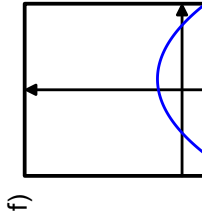
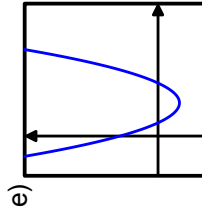
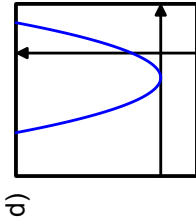
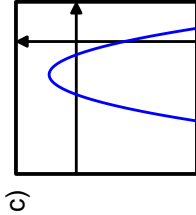
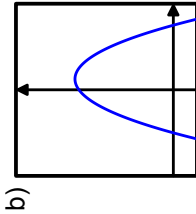
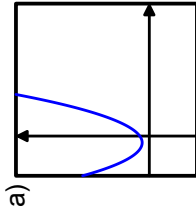
Fluency Practice

4. By first factorising, write down the roots of $y = x^2 + 8x + 12$

5. Which of these graphs show functions with no roots? Circle all that apply.

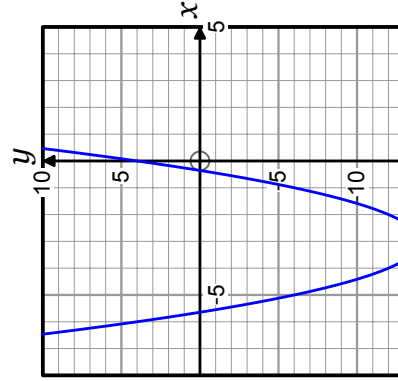


6. Three of the following are graphs of functions with roots at $x = 3, x = -2$. Which three?



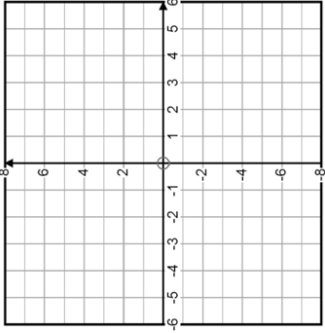
7. *Challenge!*

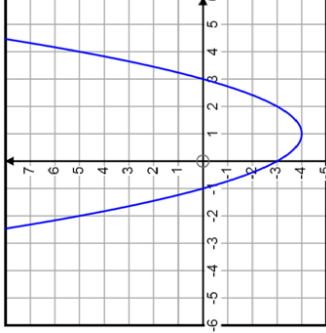
The graph shows the quadratic function $y = 2x^2 + 12x + 4$.



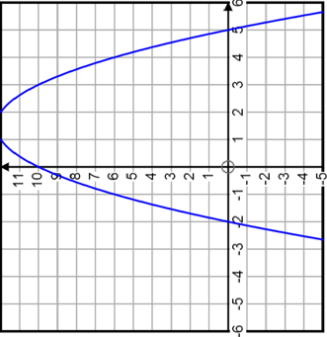
- a) Can you work out the values of the roots of the function, correct to 2 decimal places?
- b) The line of symmetry is at $x = -3$. Can you work out the coordinates of the turning point?

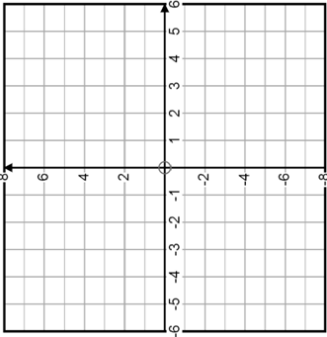
Fluency Practice

<p>A graph of this quadratic would look like:</p> 	<p>The factors of this quadratic are:</p>	<p>The roots of this quadratic are:</p>
<p>The y-intercept of the graph is:</p>	<p>The quadratic is: $x^2 - 5x + 4$</p>	<p>In completed square form this quadratic would be:</p>
<p>This quadratic crosses the x axis at:</p>	<p>The minimum point of this quadratic is:</p>	<p>This quadratic crosses the x axis at:</p>

<p>A graph of this quadratic would look like:</p> 	<p>The factors of this quadratic are:</p>	<p>The roots of this quadratic are:</p>
<p>The y-intercept of the graph is:</p>	<p>The quadratic is:</p>	<p>In completed square form this quadratic would be:</p>
<p>This quadratic crosses the x axis at:</p>	<p>The minimum point of this quadratic is:</p>	<p>This quadratic crosses the x axis at:</p>

Fluency Practice

<p>A graph of this quadratic would look like:</p> 	<p>The factors of this quadratic are:</p>	<p>The roots of this quadratic are:</p>
<p>The y-intercept of the graph is:</p>	<p>The quadratic is:</p>	<p>In completed square form this quadratic would be:</p>
<p>This quadratic crosses the x axis at:</p>	<p>The maximum point of this quadratic is:</p>	

<p>A graph of this quadratic would look like:</p> 	<p>The factors of this quadratic are: 2() ()</p>	<p>The roots of this quadratic are:</p>
<p>The y-intercept of the graph is: (0,8)</p>	<p>The quadratic is:</p>	<p>In completed square form this quadratic would be:</p>
<p>This quadratic crosses the x axis at: x = 1, x = 4</p>	<p>The minimum point of this quadratic is:</p>	

More-Same-Less

Instructions: Calculate the turning point in the middle box. Then complete the remaining boxes trying to make the minimal change possible.

Value of the y coordinate of the turning point

		Less	Same	More
Value of the x coordinate of the turning point	More			
	Same		$y = x^2 + 8x + 5$	
	Less			

3 Quadratic Inequalities

Fluency Practice

Question 1: Solve the following inequalities

- (a) $(x - 4)(x - 1) < 0$ (b) $(x - 2)(x + 1) < 0$ (c) $(x + 7)(x + 3) \leq 0$
(d) $(x - 5)(x + 4) \leq 0$ (e) $x(x - 9) > 0$ (f) $(x + 6)(x - 5) > 0$
(g) $(x + 10)(x + 1) \geq 0$ (h) $(x - 7)(x + 7) \geq 0$ (i) $(x + 8)(x + 2) < 0$
(j) $(x - 4)(x + 7) \geq 0$ (k) $(x + 1)(x - 5) \leq 0$ (l) $(x - 12)(x - 11) > 0$

Question 2: Solve the following inequalities

- (a) $x^2 + 5x + 6 > 0$ (b) $x^2 + 7x + 10 < 0$ (c) $x^2 - 4x - 5 \leq 0$
(d) $x^2 + 2x - 24 > 0$ (e) $x^2 - 6x + 8 \geq 0$ (f) $x^2 + 3x - 4 < 0$
(g) $x^2 - x - 56 > 0$ (h) $x^2 + 9x + 18 < 0$ (i) $x^2 - 13x + 22 \leq 0$
(j) $x^2 - 4x - 32 < 0$ (k) $x^2 - 64 \geq 0$ (l) $x^2 - 14x + 48 > 0$

Question 3: Solve the following inequalities

- (a) $x^2 - 2x < 15$ (b) $x^2 + 6x > x - 4$ (c) $x^2 < 36$
(d) $x^2 > 121$ (e) $2x^2 - x - 12 \leq x^2 - 2x$ (f) $6x > x^2 - 8x + 40$
(g) $x^2 + 6x < 36 - 10x$ (h) $x^2 + 5x + 1 \geq 7x + 25$

Question 4: Solve the following inequalities

- (a) $-x^2 + 8x - 15 < 0$ (b) $-x^2 + 3x + 10 \geq 0$ (c) $-x^2 - 6x - 5 \leq 0$
(d) $18x - x^2 - 32 > 0$ (e) $7x + 44 - x^2 < 0$ (f) $-3x^2 + 4x > -4x^2 + 3x$

Fluency Practice

Question 5: Solve the following inequalities

- (a) $(2x - 1)(x - 4) < 0$ (b) $(x + 4)(5x + 1) > 0$ (c) $(2x + 7)(x - 8) \leq 0$
(d) $(3x - 2)(x + 1) \leq 0$ (e) $(4x - 3)(2x - 9) > 0$ (f) $(3x + 2)(3x - 5) \geq 0$

Question 6: Solve the following inequalities

- (a) $5x^2 + 7x + 2 > 0$ (b) $3x^2 + 8x - 3 < 0$ (c) $2x^2 - 9x + 4 > 0$
(d) $4x^2 - 3x - 1 \geq 0$ (e) $6x^2 - 13x + 7 < 0$ (f) $2x^2 + x - 6 \leq 0$
(g) $4x^2 - 11x + 6 > 0$ (h) $4x^2 - 27x + 18 > 0$ (i) $15x^2 + 4x - 35 < 0$

Apply

Question 1: Tia has attempted her maths homework.
Can you spot any mistakes?

Solve $x^2 - 8x - 33 > 0$

$(x - 11)(x + 3) = 0$
 $x = 11$ or $x = -3$

$x > -3$
 $x > 11$

Question 2: (a) Solve $x^2 + 6x + 3 = 0$ giving your answers in surd form.

(b) Solve the inequality $x^2 + 6x + 3 < 0$

Question 3: The set of values for x that satisfies a quadratic inequality is $-5 < x < -2$
Write down a possible quadratic inequality.

Question 4: The set of values for x that satisfies a quadratic inequality is $x < -3$ or $x > 6$
Write down a possible quadratic inequality.

Fluency Practice

Question 5: Find the set of values of x that satisfy both

$$2x - 6 > 6 - 6x \quad \text{and} \quad x^2 - 6x + 2 < 42$$

Question 6: The set of values for x that satisfies a quadratic inequality is $x < -0.5$ or $x > 1.5$.
Write down a possible quadratic inequality.

Question 7: The width of a rectangular field is x metres.
The length of the field is 30m longer than the width.
The perimeter of the field is less than 500m.
The area of the field is greater than 4000m^2 .

By writing suitable inequalities, find the possible values of x .

Fluency Practice

Solve these inequalities.

- (a) $(x - 3)(x - 6) > 0$
- (b) $(x - 3)(x - 6) < 0$
- (c) $(x + 3)(x - 6) < 0$
- (d) $(x + 3)(x + 4) \geq 0$
- (e) $x(x + 3) > 0$
- (f) $(2x - 1)(x + 3) < 0$

Solve these inequalities.

- (a) $x^2 - 6x + 16 > 0$
- (b) $x^2 + 7x + 12 \geq 0$
- (c) $x^2 - 7x + 12 < 0$
- (d) $x^2 - 9x + 20 > 0$
- (e) $x^2 - 16 < 0$
- (f) $x^2 - 9x < 0$

Solve these inequalities.

- (a) $x^2 - 2x > 35$
- (b) $x^2 + 2x < 48$
- (c) $2x^2 > 11x - 12$
- (d) $16x - x^2 \leq 6x$

Find the solution sets for these inequalities.

- (a) $\frac{x^2+12}{2} > 4x$
- (b) $(x - 3)(2x + 3) < 2x(1 - 2x) - 5$
- (c) $(x + 5)^2 \geq 1$
- (d) $(5 - x)(x + 3) \leq 1$

Fluency Practice

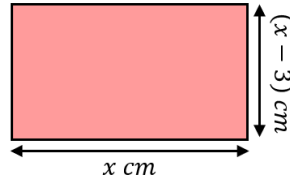
A1 Solve $x + 5 > 11$	A2 Solve $4x + 11 \leq 29$	A3 Solve $7 - x \geq 15$	A4 Work out the integer values of x that satisfy both the inequalities $3x - 4 \leq 11$ and $2x + 3 > 9$
B1 Solve $\frac{2x+5}{3} > 7$	B2 Solve $6x + 3 \leq 2x + 19$	B3 Solve $3x + 9 < 4x + 5$	B4 Work out the lowest integer which satisfies the inequality $5x - 2 \geq 3x + 7$
C1 List the integer values for x if: $-3 \leq x < 4$	C2 List the integer values for x if: $-5 \leq 5x \leq 15$	C3 List the integer values for x if: $4 \leq 3x + 1 < 12$	C4 List the integer values for x if: $2x < 3x + 1 < 13$
D1 Solve $x^2 - 7 < 42$	D2 Solve $3x^2 - 17 < 31$	D3 Solve $5x^2 - 13 \leq 32$	D4 List the integer values for x if: $2x + 3 < 4x + 5 \leq 3x + 7$

Fluency Practice

Solving Quadratic Inequalities in Context

(a)

A rectangle has sides of length x cm and width $(x - 3)$ cm, as shown. If the area of the rectangle is greater than 10 cm²:

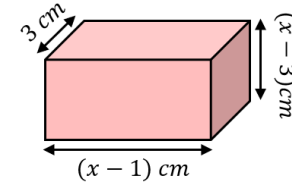


(i) Show that $x^2 - 3x - 10 > 0$

(ii) Find the range of possible values of x .

(b)

A cuboid has dimensions of 3 cm, $(x - 1)$ cm and $(x - 3)$ cm, as shown. If the volume of the cuboid is greater than 45 cm³:

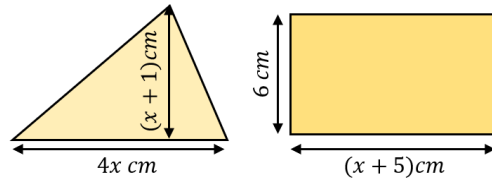


(i) Show that $x^2 - 4x - 12 > 0$

(ii) Find the range of possible values of x .

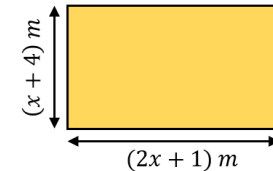
(c)

Given that the area of the rectangle is greater than the area of the triangle, find the range of possible values of x .



(d)

A rectangular lawn has a length of $(2x + 1)$ m and a width of $(x + 4)$ m, as shown. Given that the area of the lawn is less than 49 m², find the range of possible values of x .



4 Kinematics

Fluency Practice

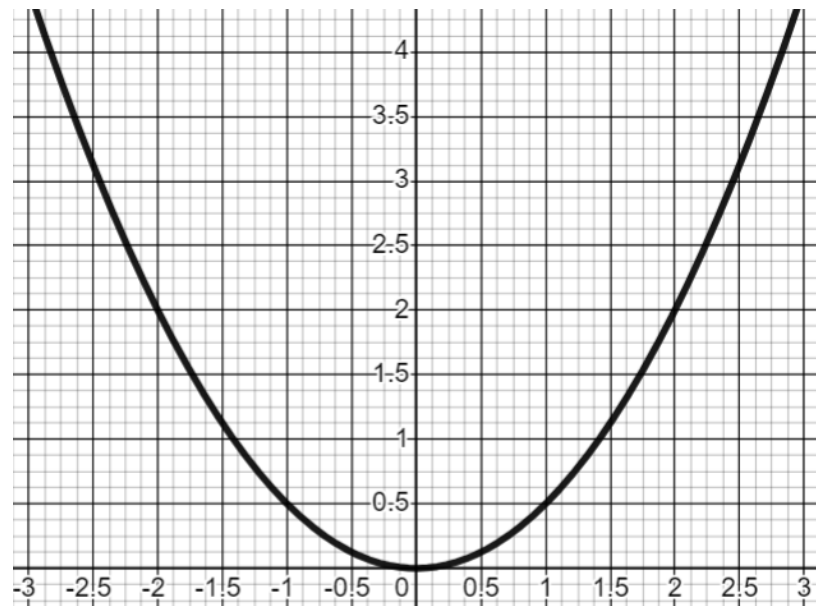
Estimating the Gradient of a Curve

(a)

Estimate the gradient of the curve at

(a) $x = 2$

(b) $x = -1$

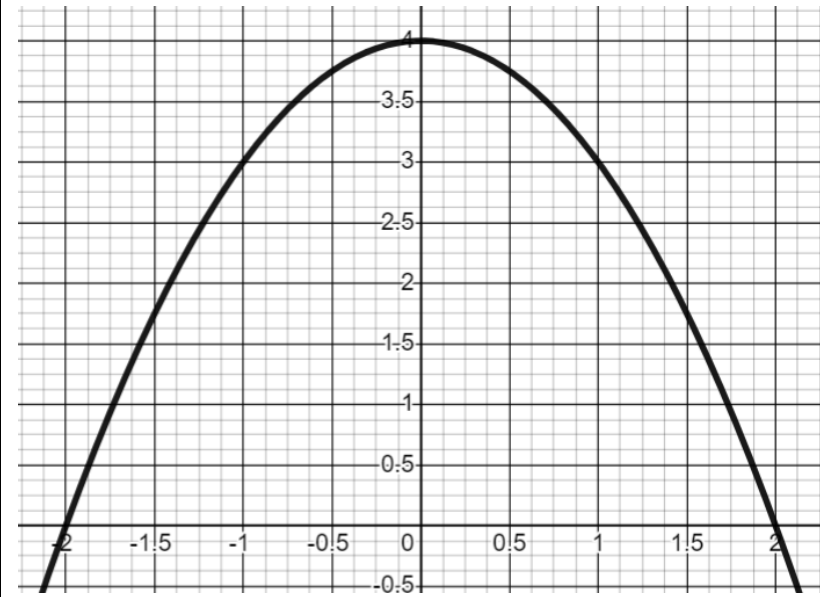


(b)

Estimate the gradient of the curve at

(a) $x = -1.5$

(b) $x = 1$



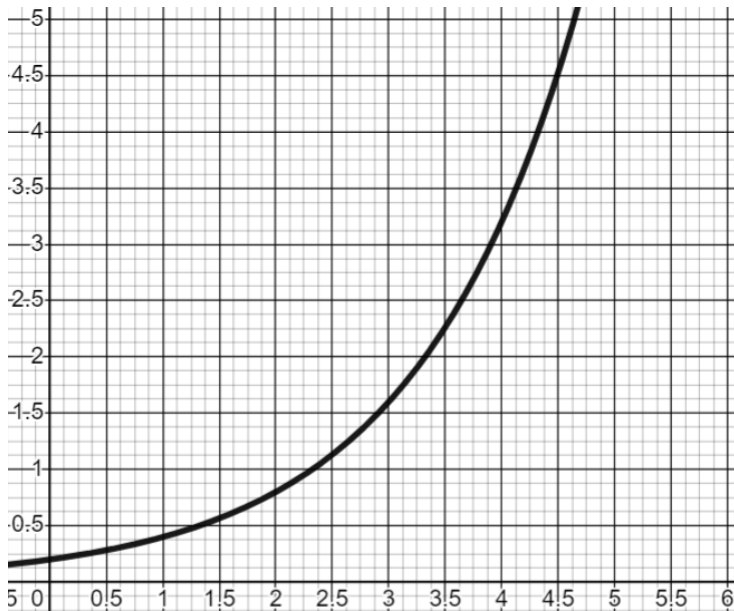
Fluency Practice

(c)

Estimate the gradient of the curve at

(a) $x = 2$

(b) $x = 4$

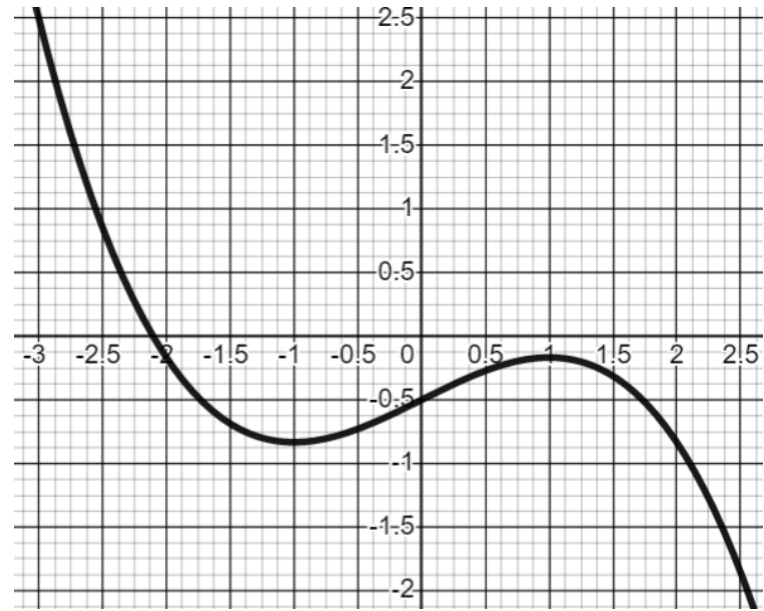


(d)

Estimate the gradient of the curve at

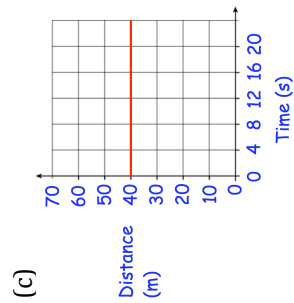
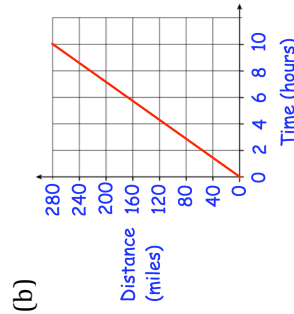
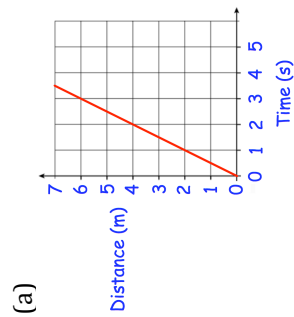
(a) $x = 2$

(b) $x = -1$

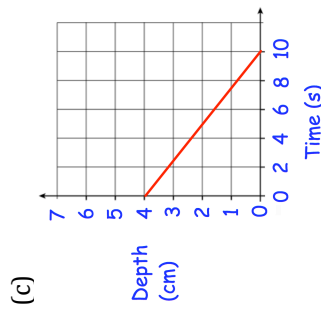
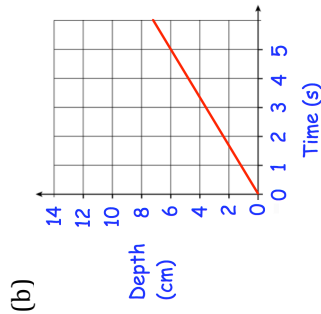
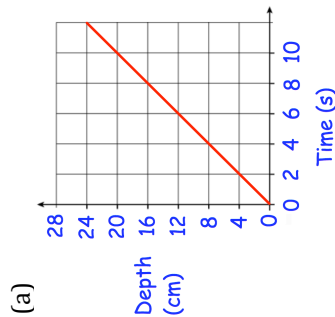


Fluency Practice

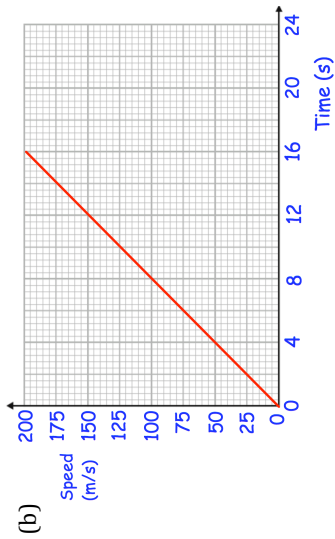
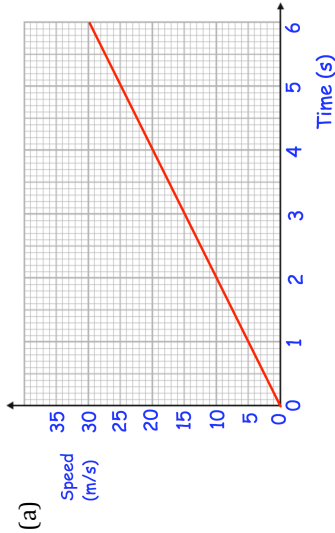
Question 1: For each graph below, work out the speed.



Question 2: For each graph below, work out the rate of change of depth.
Give each answer in cm/s

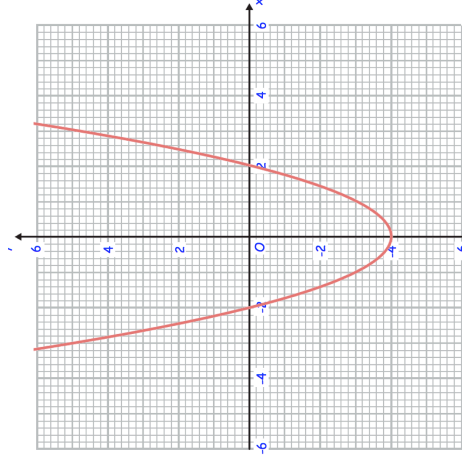


Question 3: For each graph below, work out the acceleration.
Give each answer in m/s^2



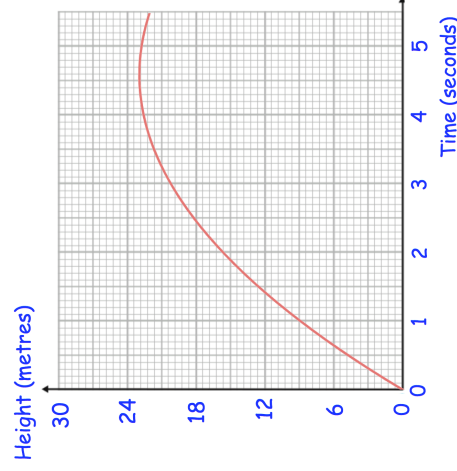
Fluency Practice

Question 4: Here is part of a quadratic graph



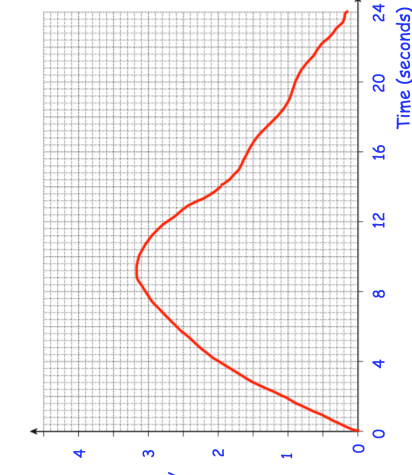
- (a) What is the gradient of the graph at the point $(0, -4)$?
- (b) Calculate an estimate of the gradient of the graph at the point $(2, 0)$
- (c) Calculate an estimate of the gradient of the graph at the point $(-1, -3)$

Question 5: The graph shows the height of a ball above the ground.



- (a) Use the graph to work out an estimate of the speed of the ball at 1 second.
- (b) When was the speed 0 m/s ?
- (c) Use the graph to work out an estimate of the speed of the ball at 4 seconds.

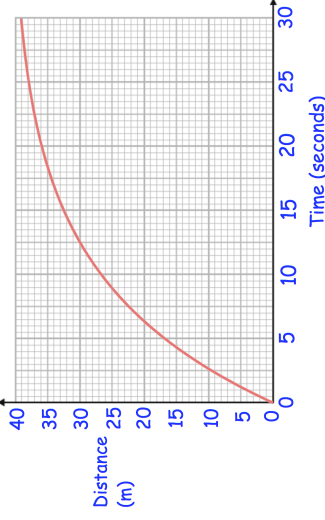
Question 6: The graph shows the velocity of object.



- (a) Use the graph to work out an estimate of the acceleration of the object at 2 seconds.
- (b) Use the graph to work out an estimate of the acceleration of the object at 8 seconds.
- (c) Use the graph to work out an estimate of the deceleration of the object at 16 seconds

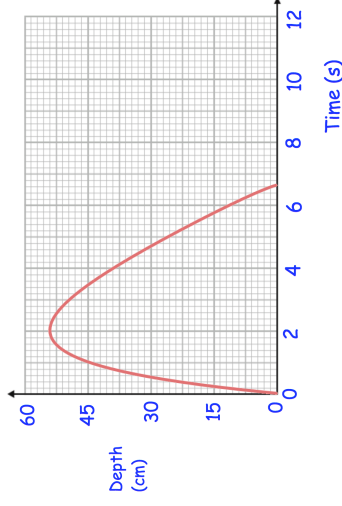
Fluency Practice

Question 7: Shown is a distance-time graph of part of a journey.



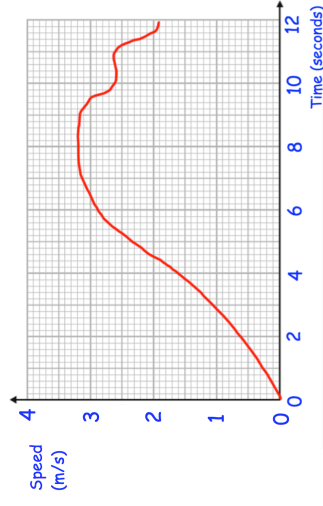
- (a) Work out the average speed over the first 5 seconds of the journey.
- (b) Work out the average speed between 15 and 30 seconds.

Question 8: Hugh has a bucket with holes in it. Hugh fills the bucket with water and records the depth of water. The graph shows the depth of water in the bucket.



- (a) Work out the average rate of change of depth of water between 0 and 2 seconds.
- (b) Work out the average rate of change of depth of water between 2 and 6 seconds.

Question 9: Here is the speed of a toy car during 12 seconds.



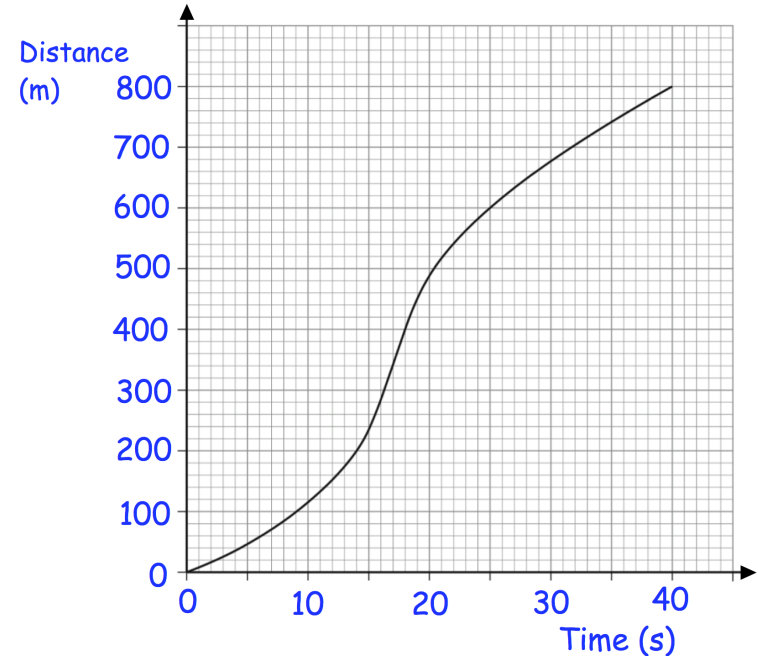
- (a) Work out the average acceleration of the toy car between 1 and 5 seconds.
- (b) Work out the average acceleration of the toy car between 8 and 12 seconds.

Fluency Practice

Apply

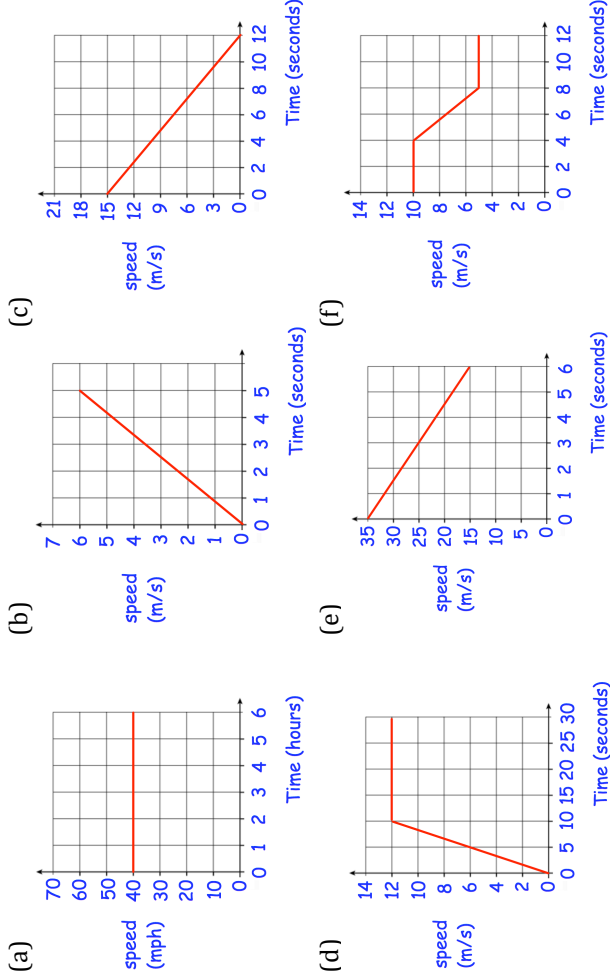
Question 1: Below is the distance-time graph for the first 40 seconds of a train journey.

- Use the graph to calculate an estimate for the speed of the train at 30 seconds.
- Explain why your answer to (a) is only an estimate.
- Estimate the highest speed reached by the train on the journey.

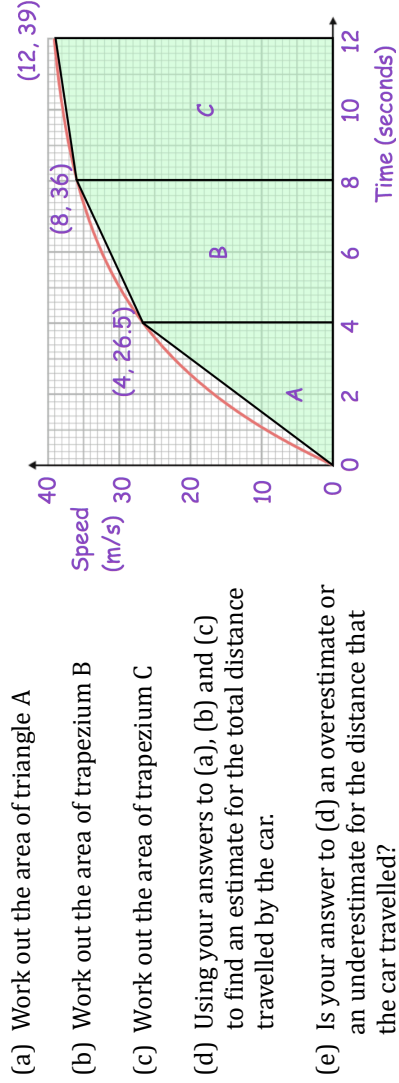


Fluency Practice

Question 1: Shown below are speed-time graphs for some journeys.
For each journey, calculate the total distance travelled.



Question 2: Here is the speed-time graph for a car's journey.



(a) Work out the area of triangle A

(b) Work out the area of trapezium B

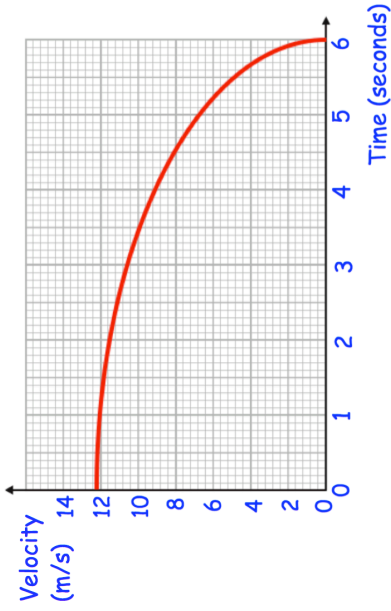
(c) Work out the area of trapezium C

(d) Using your answers to (a), (b) and (c) to find an estimate for the total distance travelled by the car.

(e) Is your answer to (d) an overestimate or an underestimate for the distance that the car travelled?

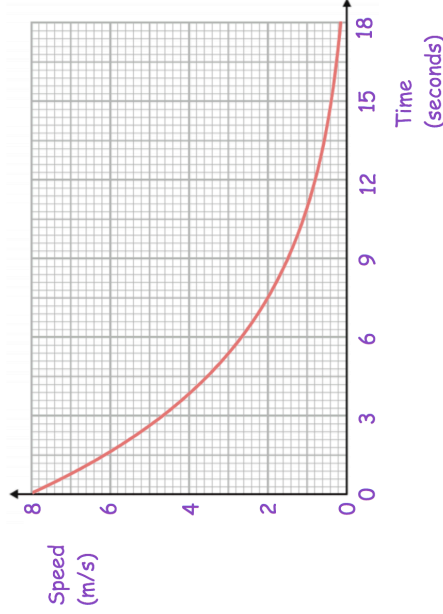
Fluency Practice

Question 3: Here is a velocity-time graph for 6 seconds of a journey.



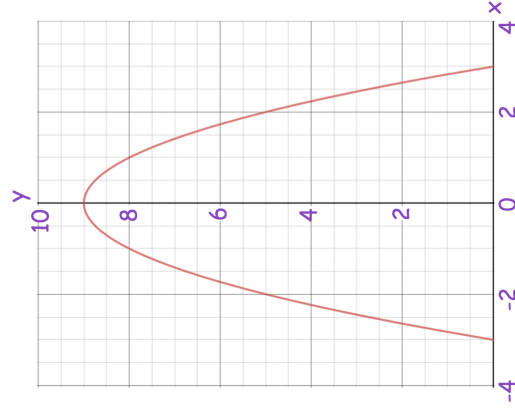
- (a) Work out an estimate for the distance travelled over 6 seconds. Use 3 strips of equal width.
- (b) Is your answer to (a) an overestimate or an underestimate of the actual distance travelled?

Question 4: Here is a speed-time graph for a remote-controlled car



- (a) Work out an estimate for the distance travelled over the first 12 seconds of the journey. Use 4 strips of equal width.
- (b) Is your answer to (a) an overestimate or an underestimate of the actual distance travelled?

Question 5: Here is a sketch of $y = 9 - x^2$

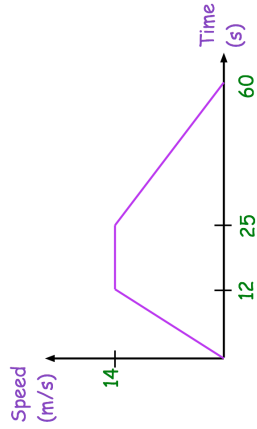


The graph is used to model the cross section of a tunnel.

Calculate an estimate of the area under the graph.

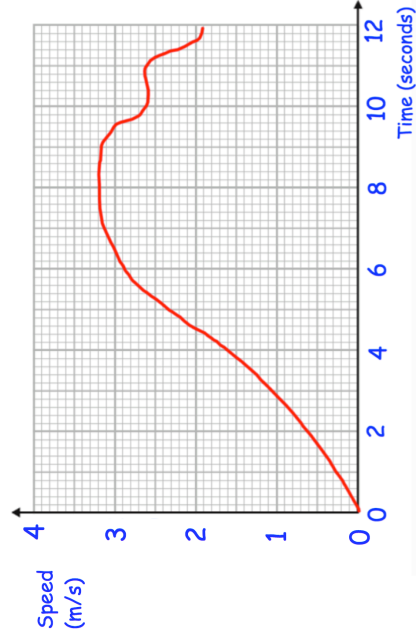
Fluency Practice

Question 1: The graph shows the speed of a bicycle between two houses.



Calculate the distance between the houses.

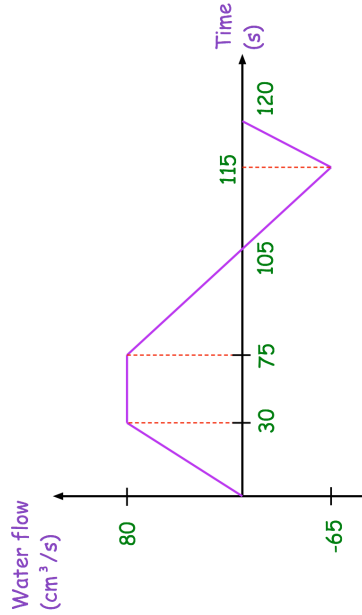
Question 2: Here is a speed-time graph for the first 12 seconds of a journey.



Calculate an estimate for the total distance travelled over the 12 seconds.

Question 3: The graph below shows information on how an empty container is being filled with water.

How much water is in the container after 120 seconds?

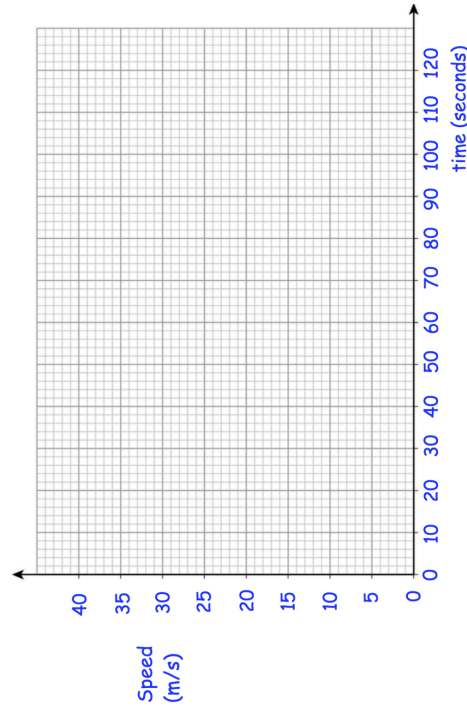


Fluency Practice

Question 4: Finn is driving his car in a straight line.

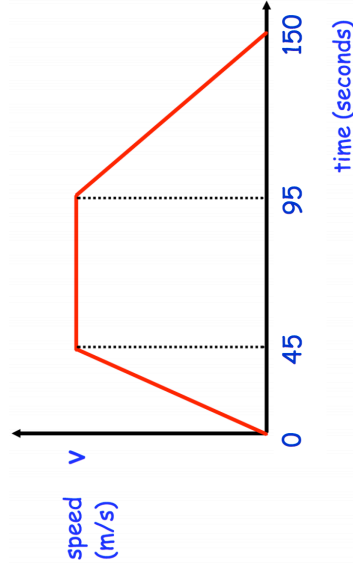
The car begins at rest.
He accelerates uniformly at 2m/s^2 for 15 seconds.
Finn drives at the same speed for the next 25 seconds.
He then accelerates uniformly to a speed of 40m/s by 80 seconds.
The remainder of the 2 minute journey is spent decelerating to rest.

(a) Draw a speed-time graph for his journey.



(b) Write down the average speed for the total journey.

Question 5: Here is a speed-time graph for a train journey.



The journey took 150 seconds.
The train travelled 1.53km in the 150 seconds.

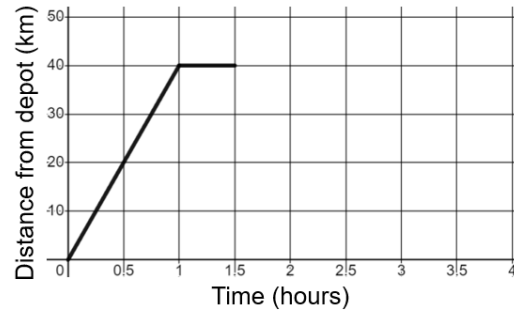
Work out the value of v .

Fluency Practice

Completing Distance-Time Graphs

(a)

A delivery driver sets off from the depot to deliver a parcel.



(a) Calculate the speed the driver travels at over the first part of the journey.

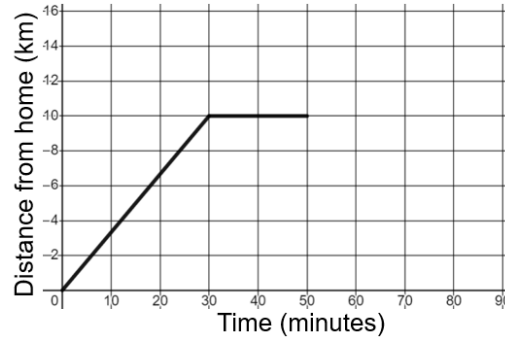
(b) How long does the driver stop for while delivering the parcel?

(c) The driver returns to the depot at a constant speed of 20 km/h. Complete the graph.

(d) How far has the delivery driver travelled in total?

(b)

Karen sets off on her bike to visit a friend. A graph showing her journey is shown.



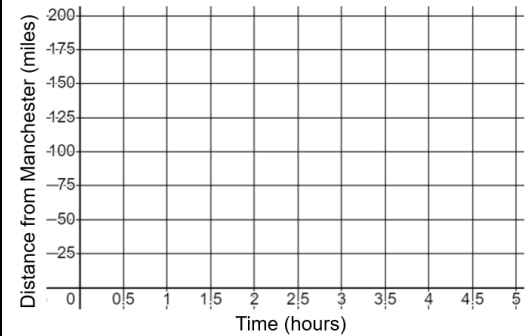
(a) At what speed does Karen ride during the first stage of her journey?

(b) How far away from home is Karen when she stops for a rest?

(c) After a rest, Karen continues on to her friend's house, which is 16 km away from her home. She travels at 12 km/h. Complete the graph.

(c)

A bus travels from Manchester to London, a distance of 200 miles.



(a) The bus sets off and travels at 50 mph for 90 minutes. It then stops at the services for 30 minutes, before setting off again. The bus continues its journey, again at 50 mph, for the next two hours. It then gets stuck in slow-moving traffic, travelling the last 25 miles in one hour. Draw a distance-time graph to represent this journey.

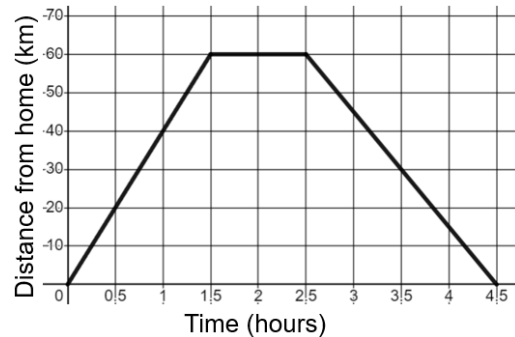
(b) Calculate the bus's average speed across the whole journey.

Fluency Practice

Reading Distance-Time Graphs

(a)

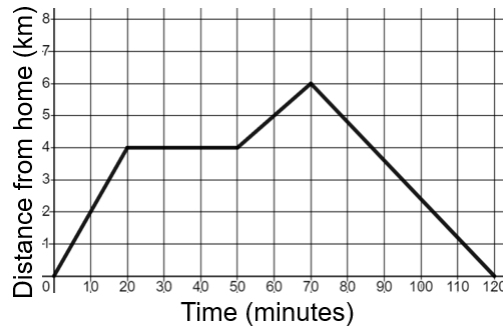
The distance-time graph shows Jamil's journey as he goes to visit a friend.



- (a) How long after Jamil has set off from home does he stop to visit his friend?
- (b) Calculate Jamil's speed as he travels to his friend's house.
- (c) How long does Jamil stay at his friend's house?
- (d) Calculate the speed Jamil travels at as he returns home.

(b)

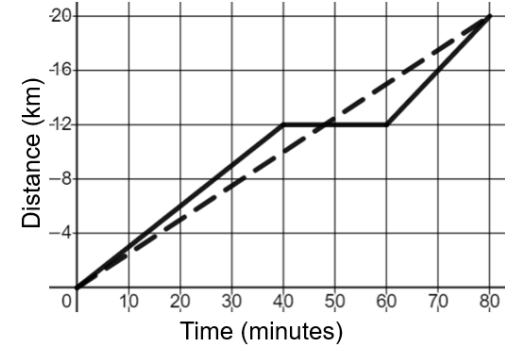
The travel graph shows Natalie's journey as she goes for a walk.



- (a) Natalie sets off from home and arrives at her friend's house 20 minutes later. How long does Natalie stay at her friend's house?
- (b) Natalie then walks for a further 20 minutes to the post box, before returning home. How far does she walk in total?
- (c) Calculate Natalie's speed in km/h as she walks home from the post box.

(c)

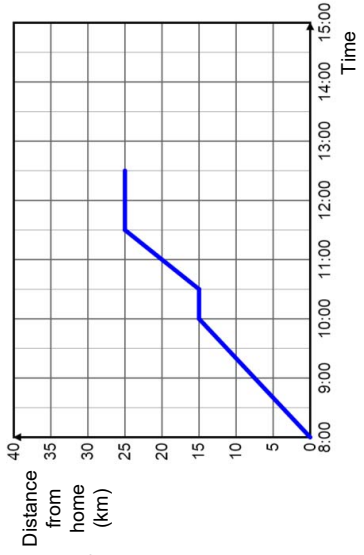
The graph shows the journey of two runners, Pol and Pat, during a 20 km race.



- (a) Pol runs the race at a constant speed over 80 minutes. Calculate Pol's speed in km/h.
- (b) Describe Pat's run, calculating any speeds in km/h.
- (c) Pol runs past Pat 12 km into the race. At what time does this happen?

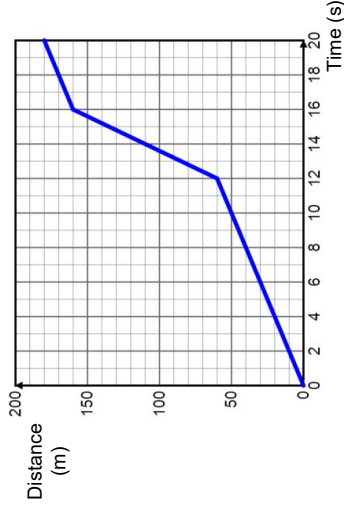
Fluency Practice

1. Sarah spent a day on a cycling trip. The distance-time graph shows her journey.



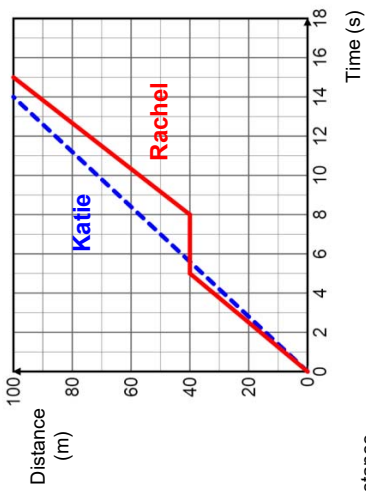
- (a) At what time did Sarah first stop for a break?
- (b) What was Sarah's average speed between 8:00 and 10:00?
- (c) What was Sarah's average speed between 10:30 and 11:30?
- (d) At 12:30, Sarah started cycling home. She did not take another break and arrived home 2 hours later. Use this information to complete the graph of Sarah's trip.

2. The distance-time graph shows the motion of a go-kart moving along a track.



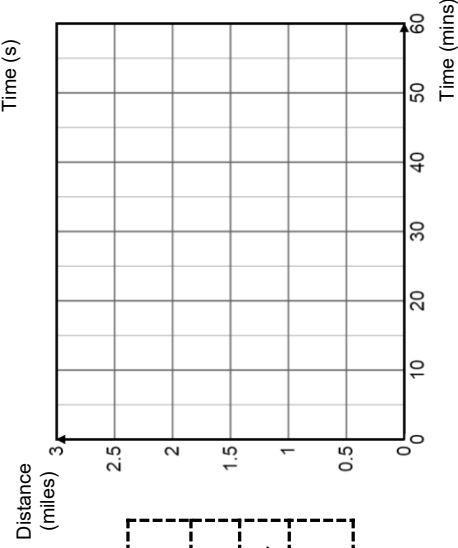
- (a) What was the average speed of the go-kart over the entire 20 seconds?
- (b) True or false? The go-kart was travelling at the same speed for the first twelve seconds as it was for the last 4 seconds.
- (c) State the time interval during which the go-kart was travelling fastest.

3. Katie and Rachel race each other over 100m. The graph illustrates the race.



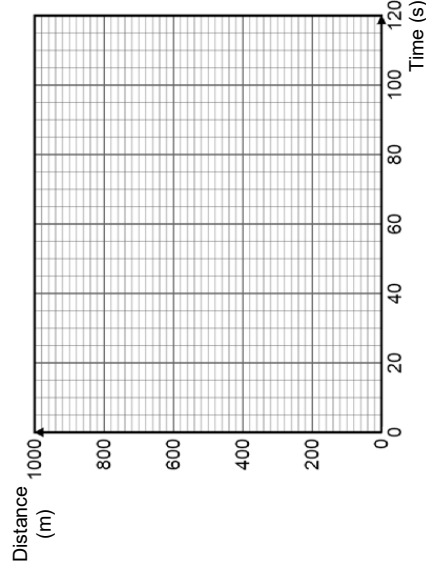
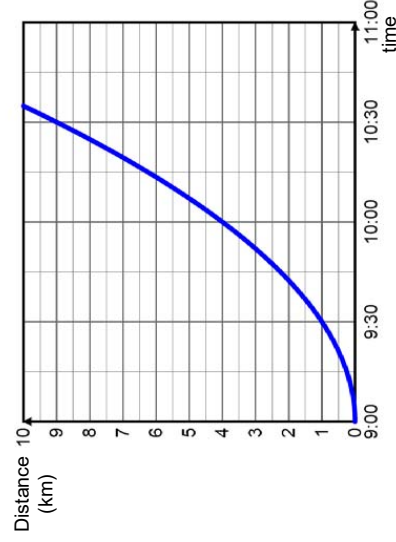
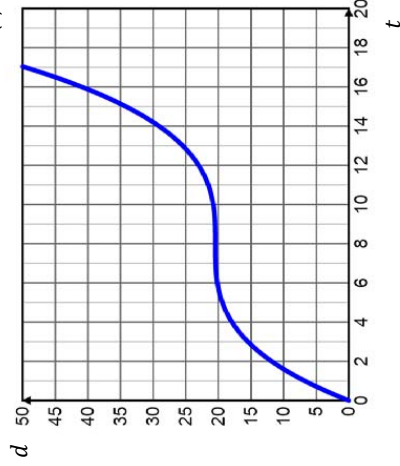
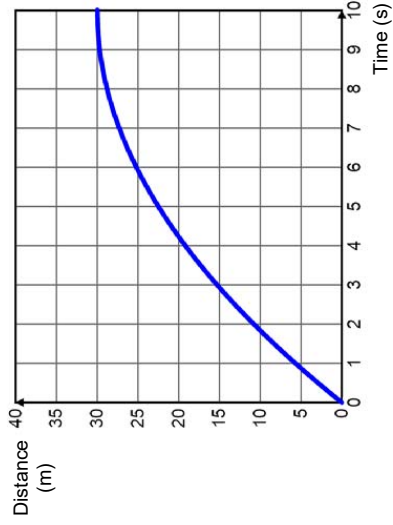
- (a) Katie maintained a steady pace. What was her average speed?
- (b) What was Rachel's speed between the 5th and 8th second of the race?
- (c) Who won the race, and by how many seconds?

4. John drives from his home to a shop and back. Use the clues to complete the distance-time graph for his journey.



- John left home and travelled to the shop at a constant speed, arriving after 20 minutes.*
- John spent 20 minutes at the shop.*
- The shop is two and a half miles from John's home.*
- John travelled home at twice the speed that he travelled to the shop.*

Fluency Practice



5. The graph shows the motion of a car.
 - (a) Work out the average speed of the car over the 10 second interval.
 - (b) Work out the instantaneous speed of the car at 6 seconds.
 - (c) Work out the time at which the instantaneous speed is equal to the average speed over the 10 second interval.

6. The graph shows the distance, d metres, travelled by a canoeist, t seconds after passing a marker point.
 - (a) Work out the canoeist's instantaneous speed when $t = 4$.
 - (b) Work out the canoeist's average speed over the interval $10 \leq t \leq 15$.
 - (c) Explain, without calculating, how you can tell that the canoeist is moving faster when $t = 16$ than at $t = 12$.

7. Mark and Sophie take part in a 10km run. Mark's progress is shown on the graph.
 - (a) Work out Mark's instantaneous speed at 9:30.
 - (b) Sophie started the run at the same time as Mark. She maintained a steady speed of 6km/h throughout her run. Draw Sophie's progress on the graph.
 - (c) At one point one of the two runners overtook the other. At what time did this happen, and who overtook who?

8. Four stops on a tramline in Sheffield are illustrated, with the distance between stops shown in metres.

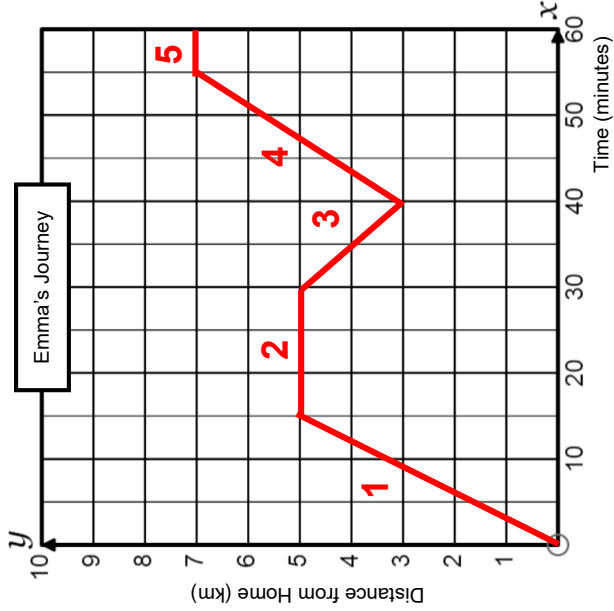

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                    ●-----●-----●-----●
                    |         |         |
                    350m    270m    210m
                    |         |         |
                    West Street  City Hall  Cathedral  Castle Square
                
```

A tram spends 30 seconds at each stop, and when it is moving has an average speed of 8 m/s.

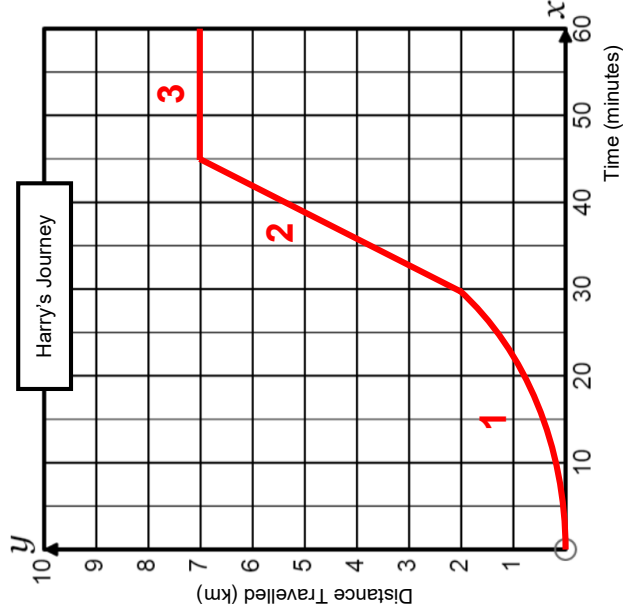
Complete the graph to show the journey of a tram from the moment it leaves West Street to the moment it arrives at Castle Square.

Fluency Practice



- A. During stage 1, Emma travelled at 3 km/minute.
- B. Emma was stationary during stage 2.
- C. The graph shows that Emma reached the top of two hills.
- D. Emma travelled 4000m during Stage 4.
- E. The slowest part of Emma's journey was during stage 3.
- F. Emma's average speed over the whole journey was 7 km/h.
- G. Emma was travelling faster during stage 4 than stage 1.
- H. When she was 5 minutes into her journey, Emma had travelled 3 km.

True Statements: _____



- A. It took Harry half an hour to travel the first 2km of his journey.
- B. Harry travelled at a constant speed during section 1.
- C. Harry's speed 35 minutes into his journey was 1½ km/h.
- D. During section 3 of his journey, Harry's speed was 0 mph.
- E. Harry was accelerating during section 1 of the journey
- F. After 45 minutes, Harry had travelled 7 km.
- G. During section 1, Harry travelled at an average speed of 15 km/h.
- H. Harry's speed was fastest during section 1 of the journey.

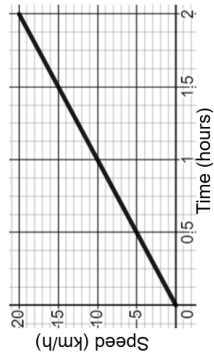
True Statements: _____

Fluency Practice

Speed-Time Graphs

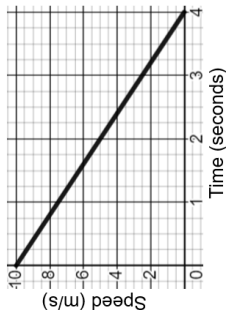
(a)

Find the acceleration.



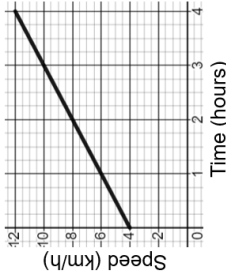
(b)

Find the deceleration.



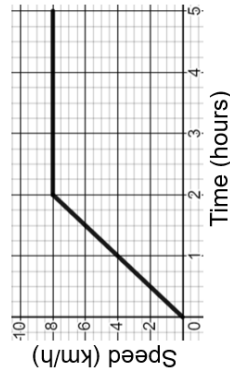
(c)

Find the acceleration.



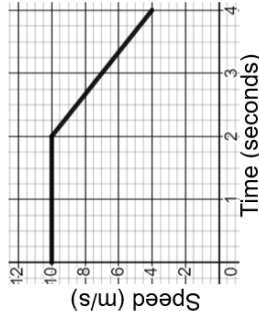
(d)

Describe each step of the journey, stating the values of any acceleration or deceleration.



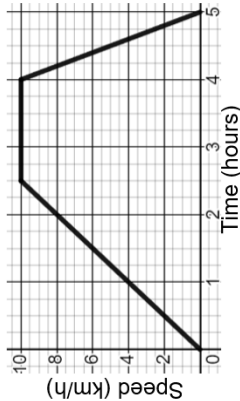
(e)

Describe each step of the journey, stating the values of any acceleration or deceleration.



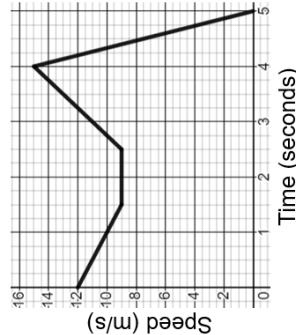
(f)

Describe each stage of the speed-time graph, calculating the values of any acceleration and deceleration.



(g)

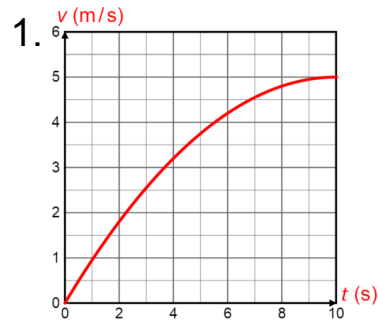
Describe each stage of the speed-time graph, calculating the values of any acceleration and deceleration.



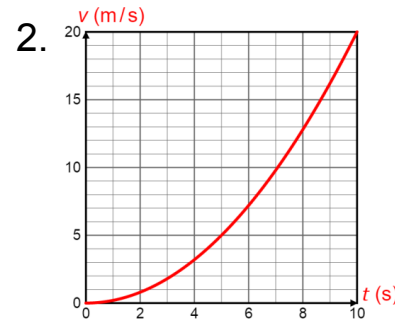
Fluency Practice

velocity-time graphs: tangents & areas

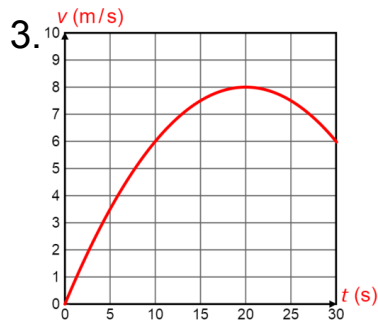
Work out the best estimates you can.



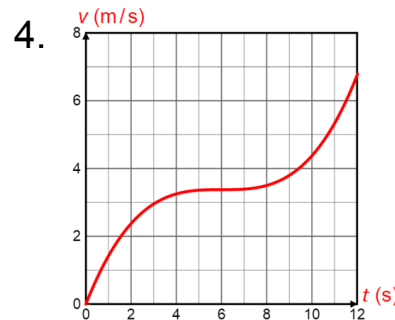
- Maximum speed.
- Acceleration at $t = 4$.
- Total distance travelled.



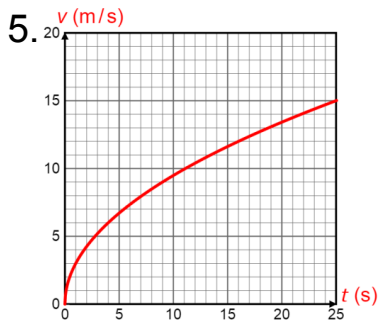
- Average acceleration.
- Acceleration at $t = 4$.
- Total distance travelled.



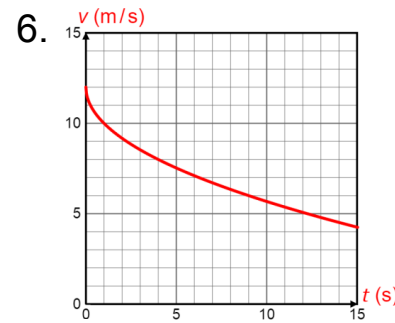
- Time at which acceleration is zero.
- Acceleration at $t = 10$.
- Total distance travelled.



- Time at which speed is 2m/s.
- Acceleration at $t = 4$.
- Distance travelled in first 6 seconds.



- Average acceleration.
- Time at which: instantaneous acceleration = average acceleration

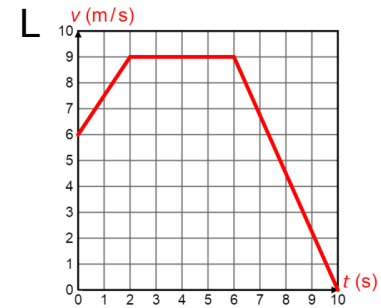
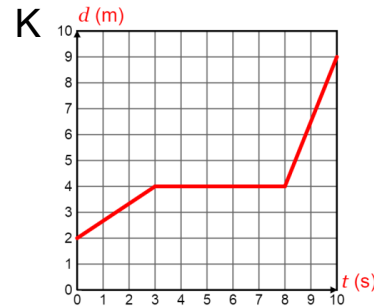
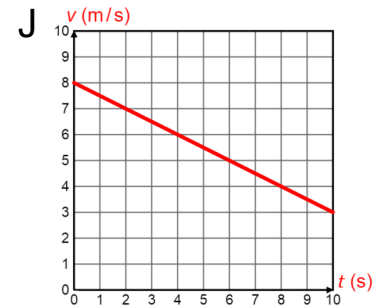
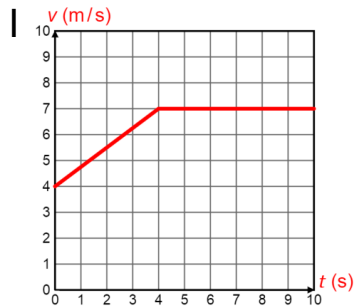
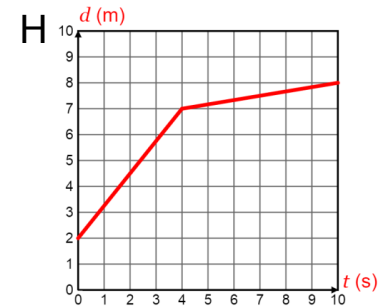
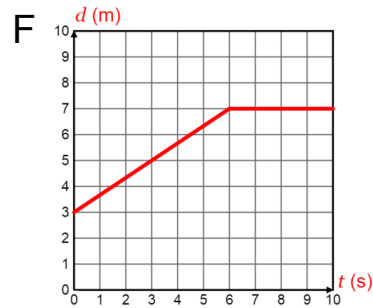
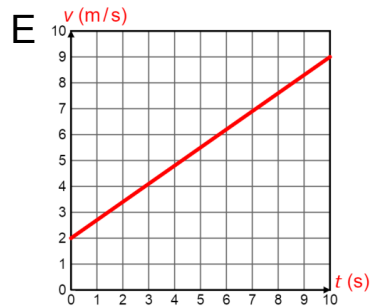
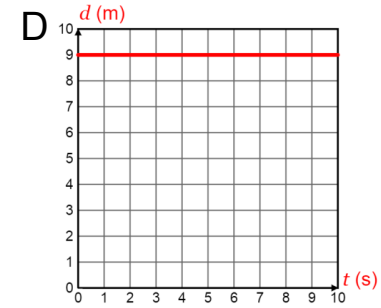
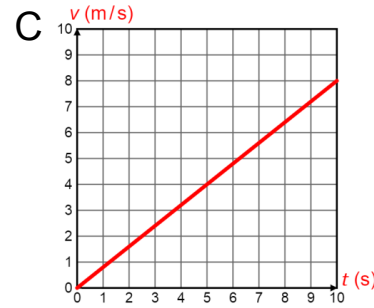
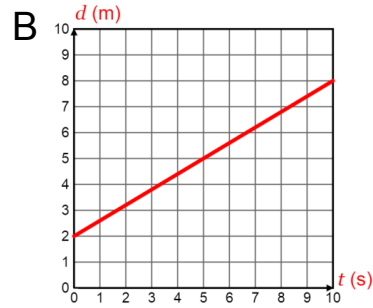
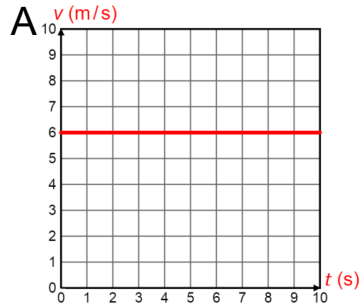


- Average acceleration.
- Total distance travelled.
- Is your distance an under-estimate or an over-estimate?

Fluency Practice

what distance has been travelled?

For each graph, work out the total distance travelled during the 10 seconds. Watch out – some are **distance-time** graphs and some are **velocity-time** graphs.



5 Iterations

Fluency Practice

1

[Edexcel Specimen Papers Set 2, Paper 2H Q13]

The number of slugs in a garden t days from now is p , where

$$p_0 = 100$$

$$p_{t+1} = 1.06p_t$$

Work out the number of slugs in the garden 3 days from now.



slugs

Why do you think the sequence refers to the first term as p_0 rather than p_1 ?
Because p_0 indicates the number of slugs 'after 0 days', i.e. the initial value.

2

A sequence is defined as:

$$x_1 = 5, \quad x_2 = 2$$

$$x_{n+2} = 2x_{n+1} - x_n$$

Determine x_5 .

Fluency Practice

1 For each iterative formula, find x_1, x_2, x_3 and x_4 correct to three decimal places

(a) $x_{n+1} = \frac{1}{x_n} + 1, x_0 = 1.5$

?

(b) $x_{n+1} = 2 - \frac{x_n^3}{8}, x_0 = 1.5$

?

(c) $x_{n+1} = \sqrt{\frac{5x_n - 1}{2}}, x_0 = 2$

?

(d) $x_{n+1} = \frac{1}{5 - 2x_n}, x_0 = 0.5$

?

2 For the equation $x - \sqrt{x + 3} = 0$, we can use the iterative formula $x_{n+1} = \sqrt{x_n + 3}$ and $x_0 = 2$. Explain the relationship between the values of x_1, x_2, x_3 and the equation $x - \sqrt{x + 3} = 0$

?

3 a) Show that $x^2 + 4x - 29 = 0$ can be written in the form $x = \sqrt{29 - 4x}$.

?

b) Use the iteration formula $x_{n+1} = \sqrt{29 - 4x_n}$ to find x_4 to 3 decimal places. Start with $x_0 = 0$.

?

4 a) Show that $x - \sqrt{x} - 1 = 0$ has a root between 2.6 and 2.7

?

b) Using $x_{n+1} = \sqrt{x_n} + 1$ and $x_0 = 2.6$, find x_3 to 3 decimal places.

?

Fluency Practice

Solving Quadratic Equations using Iteration

Quadratic Equation: $x^2 - 2x - 1 = 0$

(rearrange)

Iterative Formula: $x_{n+1} = \sqrt{2x_n + 1}$

Initial Value: $x_1 = 3$

x_2	2.64575...
x_3	2.50828...
x_4	2.45287...
x_5	2.43017...
x_6	2.42081...
x_7	2.41694...
x_8	2.41534...
x_9	2.41468...
x_{10}	2.41440...
x_{11}	2.41429...
x_{12}	2.41424...

$x = 2.4$
Both solutions
are the same
(to 1 dp).

$x = 2.41$
Both solutions
are the same
(to 2 dp).

- A** For this quadratic equation find the solutions, to 1 dp & to 2 decimal places.

$$x^2 - 3x - 1 = 0$$

Iterative Formula: $x_{n+1} = \sqrt{3x_n + 1}$

Initial Value: $x_1 = 4$

x_2	
x_3	3.437...
x_4	
x_5	
x_6	
x_7	
x_8	
x_9	3.303...
x_{10}	
x_{11}	
x_{12}	

- B** Find a solution for this equation to 2 dp & to 3 dp.

$$x^2 - 5x + 2 = 0$$

Iterative Formula: $x_{n+1} = \sqrt{5x_n - 2}$

Initial Value: $x_1 = 5$

x_2	
x_3	
x_4	
x_5	4.5991...
x_6	
x_7	
x_8	
x_9	
x_{10}	
x_{11}	
x_{12}	
x_{13}	

- C** Find a solution for this equation to 3 dp.
Choose your own initial value.

$$x^2 - 4x - 7 = 0$$

- D** Find a solution for this equation to 1 dp.

$$x^2 + 4x - 13 = 0$$

Why do only some initial values work?

- E** Find a solution for this cubic equation to 3 dp.

$$x^3 - 5x + 3 = 0$$

(Rearrange so x^3 is by itself, then take the cube root of both sides).

Fluency Practice

iteration 1: roots & intervals

1. Show that there is a root of the equation $x^3 + x - 3 = 0$ in the interval (1, 2).

2. Show that there is a root of the equation $x^2 - 5x + 2 = 0$ in the interval (0.4, 0.5).

3. Here is a table of values for the continuous function $f(x)$:

x	1	2	3	4	5	6
$f(x)$	1.9	1.6	1.1	0.4	-0.5	-1.6

State two consecutive values of x between which there is a solution to the equation $f(x) = 0$.

4. Here is a table of values for the continuous function $g(x)$:

x	-3	-2	-1	0	1	2	3
$g(x)$	2.33	3.52	2.16	-0.24	-2.20	-2.20	1.24

State **two pairs** of consecutive values of x between which there are solutions to the equation $g(x) = 0$.

5. $f(x)$ is a continuous function. Choose the correct statement for each of the following:

(a) $f(3) = 2.34$
 $f(4) = -0.01$

(b) $f(6) = -4$
 $f(7) = -1.52$

(c) $f(-1) = 0.62$
 $f(0.5) = 0.74$

(d) $f(7) < 0$
 $f(8) > 0$

Statements

① There is a root of $f(x) = 0$ in the interval.

② There is **not** a root of $f(x) = 0$ in the interval.

③ It is not possible to tell if there is a root of $f(x) = 0$ in the interval.

6. $f(x) = x^2 - 4x - 3$.

(a) Show that there is a root of the equation $f(x) = 0$ in the interval (4, 5).

(b) Work out $f(4.5)$.

(c) State an interval that is smaller than (4, 5) and contains the root.

7. (a) $a = 2.2$ correct to 1 decimal place. State the lower and upper bounds of a .

(b) Use the bounds as an interval to prove that a is root of the equation $x^3 - x - 9 = 0$, correct to 1 decimal place.

Fluency Practice

Iteration 2: recurrence relations

1. Given the recurrence relation $x_{n+1} = 3x_n - 2$ and $x_0 = 2$, work out x_1, x_2, x_3 and x_4 .

2. Given the recurrence relation $x_{n+1} = (x_n)^2 - 5$ and $x_0 = 3$, work out x_1, x_2, x_3 and x_4 .

3. Given that $P_{n+1} = 1.05P_n$ and $P_0 = 2000$, work out P_4 correct to 3 significant figures.

4. Given that $U_{n+1} = 1.02(U_n - 50)$ and $U_0 = 1500$, work out U_3 correct to 3 significant figures.

5. Match the recurrence relation rules to the correct sequence of values (x_0, x_1, x_2, x_3):

A: $x_{n+1} = 3x_n - 1$	P: 3, 5, 7, 9
B: $x_{n+1} = x_n + 2$	Q: 2, 5, 14, 41
C: $x_{n+1} = (x_n)^2 + 1$	R: 2, 5, 8, 11
D: $x_{n+1} = x_n + 3$	S: 3, 6, 15, 42
E: $x_{n+1} = 3(x_n - 1)$	T: 2, 5, 26, 677

6. An approximate solution to the equation $x^3 - 8x + 4 = 0$ can be found using the iterative formula:

$$x_{n+1} = \frac{x_n^3 + 4}{8}, \quad x_0 = 0$$

Work out x_1, x_2, x_3 and x_4 . Round each value to 4 decimal places where necessary.

7. (a) Show that the equation $x^3 - 5x - 2 = 0$ has a root between $x = -1$ and $x = 0$.

(b) Starting with $x_0 = 0$, use the iterative formula:

$$x_{n+1} = \frac{x_n^3 - 2}{5}$$

three times to find an estimate for the root of $x^3 - 5x - 2 = 0$. Round your answer to 2 decimal places.

8. (a) Show that the equation $x^3 + x^2 = 10x - 6$ has a root in the interval $(0, 1)$.

(b) Starting with $x_0 = 1$, use the iterative formula

$$x_{n+1} = \frac{x_n^3 + x_n^2 + 6}{10}$$

three times to find an estimate for the root of $x^3 + x^2 = 10x - 6$. Round your answer to 3 significant figures.

Fluency Practice

iteration 3: rearrangement

1. Show that the equation $x^3 - 8x - 5 = 0$ can be rearranged to each of these forms:

(a) $x = \sqrt[3]{8x + 5}$ (b) $x = \frac{x^3 - 5}{8}$

(c) $x = \frac{5}{x^2 - 8}$ (d) $x = \pm \sqrt{\frac{8x + 5}{x}}$

2. (a) Show that the equation $x^3 - 4x - 5 = 0$ has a root in the interval (2, 3).

(b) Show that the equation $x^3 - 4x - 5 = 0$ can be rearranged to give $x = \sqrt[3]{4x + 5}$.

- (c) Starting with $x_0 = 2$, use the iteration formula

$$x_{n+1} = \sqrt[3]{4x_n + 5}$$

three times to find an estimate for the root of $x^3 - 4x - 5 = 0$. Round your estimate to 3 significant figures.

3. (a) Show that the equation $x^3 + 3x - 2 = 0$ has a root between $x = 0$ and $x = 1$.

(b) Show that the equation $x^3 + 3x - 2 = 0$ can be rearranged to give $x = \frac{2}{x^2 + 3}$.

- (c) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{2}{x_n^2 + 3}$$

twice to find an estimate for the root of $x^3 + 3x - 2 = 0$. Round your answer to 2 decimal places.

4. (a) Show that the equation $x^3 - 4x - 2 = 0$ has a root in the interval (-1, 0).

(b) Show that the equation $x^3 - 4x - 2 = 0$ can be rearranged to give $x = \frac{x^3 - 2}{4}$.

- (c) Starting with $x_0 = -0.5$, use the iteration formula

$$x_{n+1} = \frac{x_n^3 - 2}{4}$$

three times to find an estimate for the root of $x^3 - 4x - 2 = 0$. Round your answer to 3 significant figures.

Fluency Practice

iteration 4: review

1. The expected number of birds in a colony t years from now is P_t , where

$$P_0 = 600$$
$$P_{t+1} = 1.025P_t$$

Work out the expected number of birds in the colony 3 years from now.

2. The value of an investment t years from now is $£V_t$, where

$$V_0 = 20,000$$
$$V_{t+1} = 1.02V_t + 2000$$

Work out the value of the investment 2 years from now.

3. The expected value of a car n years from now is $£c_n$, where

$$c_0 = 18,000$$
$$c_{n+1} = 0.85c_n - 500$$

Work out the expected value of the car 2 years from now.

4. (a) Show that the equation $x^3 - 6x - 6 = 0$ has a root in the interval $(2, 3)$.

(b) Show that the equation $x^3 - 6x - 6 = 0$ can be rearranged to give $x = \sqrt{\frac{6x+6}{x}}$.

- (c) Starting with $x_0 = 3$, use the iteration formula

$$x_{n+1} = \sqrt{\frac{6x_n+6}{x_n}}$$

three times to find an estimate for the root of $x^3 - 6x - 6 = 0$. Round your answer to 2 decimal places.

5. (a) Show that the equation $x^3 + 2x + 8 = 0$ has a root between $x = -2$ and $x = -1$.

(b) Show that the equation $x^3 + 2x + 8 = 0$ can be rearranged to give $x = \sqrt[3]{-2x - 8}$.

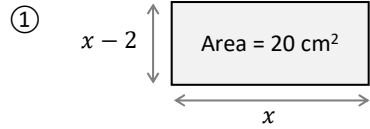
- (c) Starting with $x_0 = -1.5$, use the iteration formula

$$x_{n+1} = \sqrt[3]{-2x_n - 8}$$

three times to find an estimate for the root of $x^3 + 2x + 8 = 0$. Round your answer to 2 decimal places.

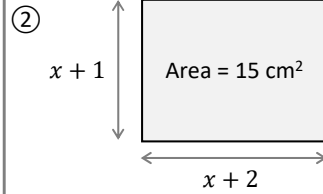
Fluency Practice

Forming & Solving Equations using Iteration



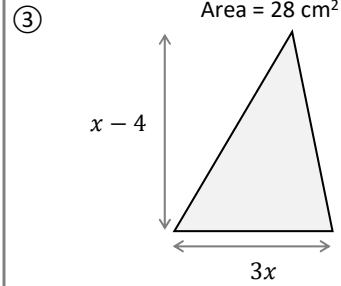
- 1) Form a quadratic equation ($= 0$).
- 2) Rearrange to form an iterative function.
(Use a square root to form $x_{n+1} =$)
- 3) Use the function to find the value of x (to 2 dp)

How can we check our answer?

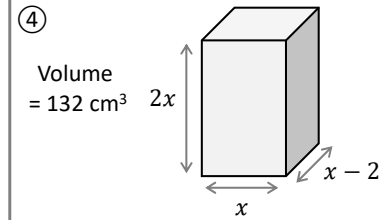


Use iteration to find the value of x to 2 dp

Why do we need to be careful choosing an initial value?



Use iteration to find the value of x to 3 dp



Use iteration to find the value of x to 3 dp