KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2023 Mathematics 2024

Unit 24 Booklet

HGS Maths

Tasks

Dr Frost Course

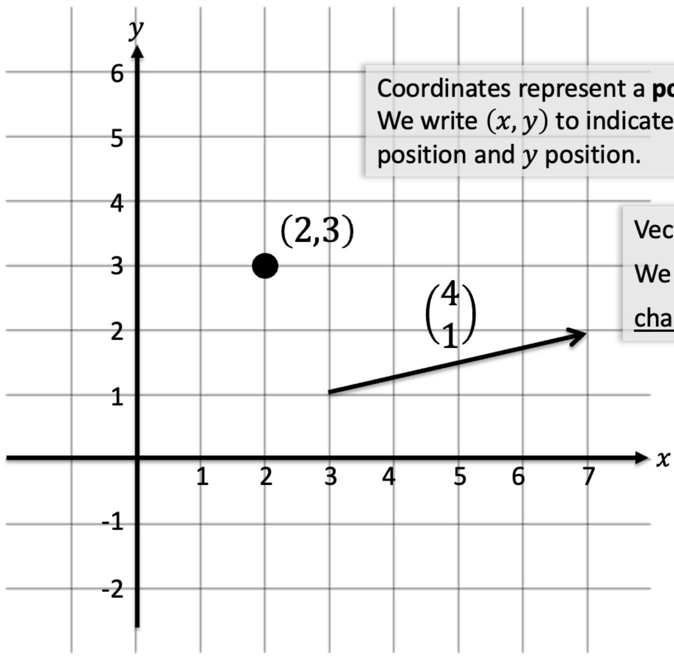
Name: ______________________________

Class: ______________________________

Vectors – the basics:

A vector has magnitude (how long it is) and direction.

Column Vector: $\binom{x}{y}$ where x is movement right or left and y is movement up or down. Right and up are taken to be positive.



Coordinates represent a **position**. We write (x, y) to indicate the x position and y position.

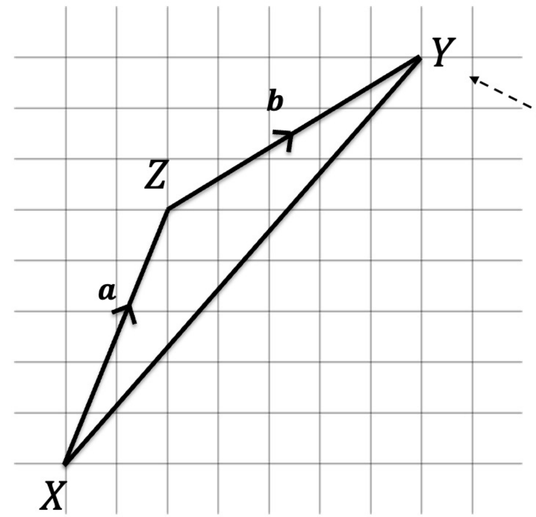
Vectors represent a **movement**. We write $\binom{x}{y}$ to indicate the <u>change</u> in x and <u>change</u> in y.

Note we put the numbers vertically. Do **NOT** write $\left(\frac{4}{1}\right)$; there is no line and vectors are very different to fractions!

You may have seen vectors briefly before if you have done transformations; we can use them to describe the movement of a shape in a **translation**.

Writing Vectors

You are used to variables representing numbers in maths. They can also represent vectors!

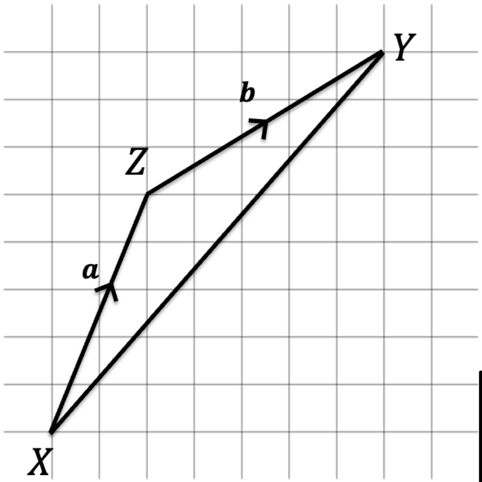


What can you say about how we use variables for vertices (points) vs variables for vectors?
*We use **capital letters** for vertices and **lower case letters** for vectors.*

There's 3 ways in which can represent the vector from point X to Z:

1. $\boldsymbol{a}$ (in **bold**)
2. $\underline{a}$ (with an 'underbar')
3. $\overrightarrow{XZ}$

Vectors – the basics:



$\overrightarrow{XZ} = \boldsymbol{a} = \binom{2}{5}$

$\overrightarrow{ZY} = \boldsymbol{b} = \binom{5}{3}$

$\overrightarrow{XY} = \binom{7}{8}$

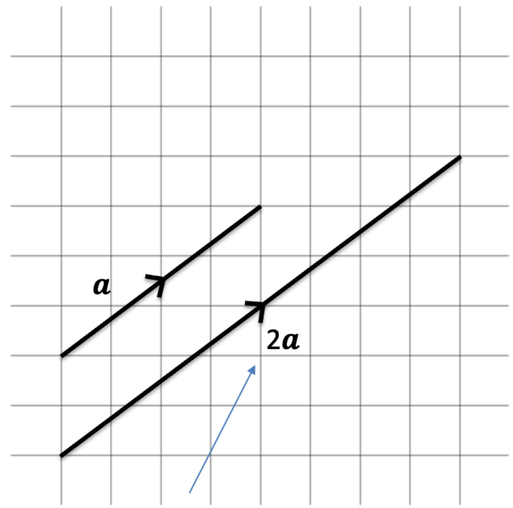
What do you notice about the numbers in $\binom{7}{8}$ when compared to $\binom{2}{5}$ and $\binom{5}{3}$?
We have simply added the x values and y values to describe the combined movement.

i.e.

$$\boldsymbol{a} + \boldsymbol{b} = \binom{2}{5} + \binom{5}{3} = \binom{7}{8}$$
$$\overrightarrow{XZ} + \overrightarrow{ZY} = \overrightarrow{XY}$$

Important Note: The point is that we can use <u>any route</u> to get from the start to finish, and the vector will always be the same.

- Route 1: We go from X to Y via Z.
  $\overrightarrow{XZ} = \binom{2}{5} + \binom{5}{3} = \binom{7}{8}$
- Route 2: Use the direct line from X to Y: $\binom{7}{8}$



Note: Note that vector letters are bold but scalars are not.

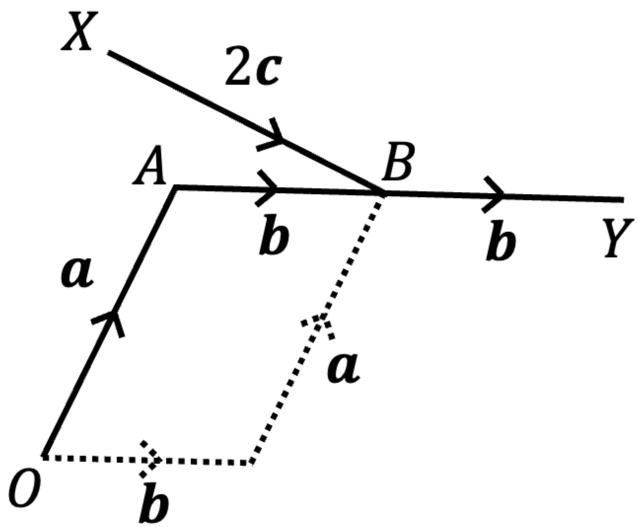
We can 'scale' a vector by multiplying it by a normal number, aptly known as a **scalar**.

If $\boldsymbol{a} = \binom{4}{3}$, then
$$2\boldsymbol{a} = 2\binom{4}{3} = \binom{8}{6}$$

What is the same about $\boldsymbol{a}$ and $2\boldsymbol{a}$ and what is different?

Same: Same direction / Parallel
Different: The length of the vector, known as the **magnitude**, is longer.

Vectors – the basics:



If $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{AB} = \boldsymbol{b}$ and $\overrightarrow{XB} = 2\boldsymbol{c}$, then find the following in terms of a, b and c:

$\overrightarrow{OB} =$

$\overrightarrow{OY} =$

$\overrightarrow{AX} =$

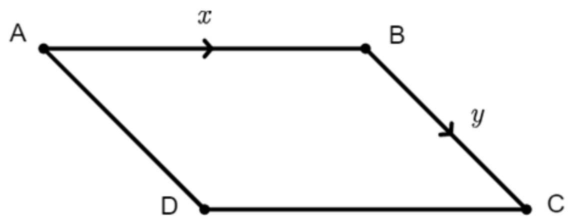
$\overrightarrow{XO} =$

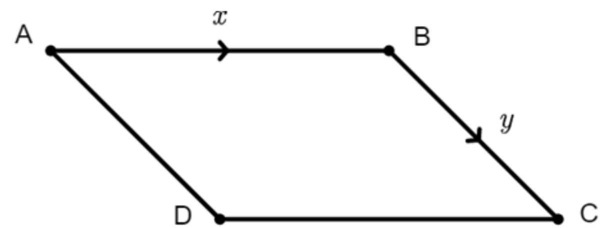
$\overrightarrow{YX} =$

Note: Since $b + a$ would end up at the same finish point, we can see $\boldsymbol{b} + \boldsymbol{a} = \boldsymbol{a} + \boldsymbol{b}$ (i.e. vector addition, like normal addition, is 'commutative')

Note: Since $-\binom{x}{y} = \binom{-x}{-y}$, subtracting a vector goes in the opposite direction.

Example	Your Turn
ABCD is a parallelogram. Express $\overrightarrow{DB}$ in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$.	ABCD is a parallelogram. Express $\overrightarrow{CA}$ in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$.

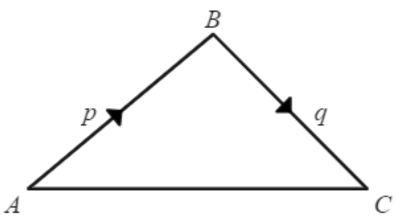




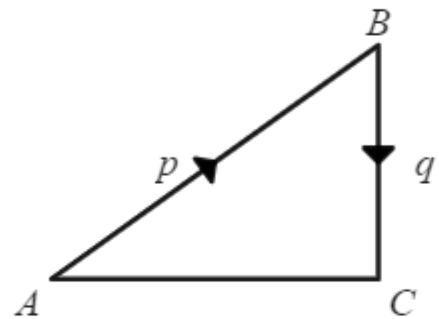
Example	Your Turn

473a: **Determine a vector between two points using an appropriate path.**

Express $\vec{AC}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.



Express $\vec{CA}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

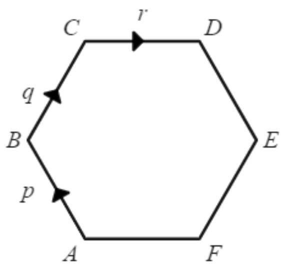


Example	Your Turn
473a: **Determine a vector between two points using an appropriate path.**	473a: **Determine a vector between two points using an appropriate path.**

Example:

473a: **Determine a vector between two points using an appropriate path.**

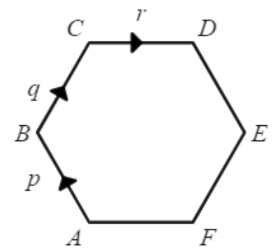
Express $\vec{CF}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$.



Your Turn:

473a: **Determine a vector between two points using an appropriate path.**

Express $\vec{AE}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$.

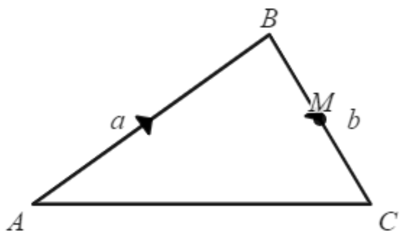


Example	Your Turn

473b: **Determine a vector involving a midpoint.**

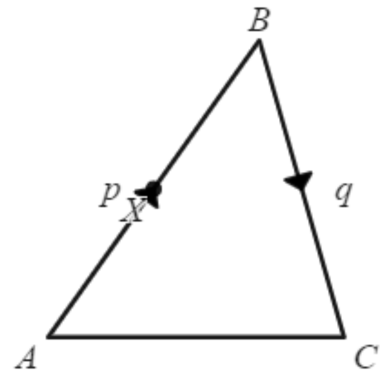
The point M is the midpoint of BC.

Express $\vec{AM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.



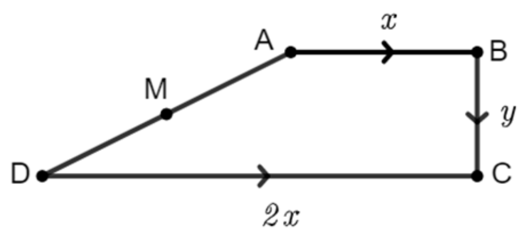
The point X is the midpoint of AB.

Express $\vec{CX}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

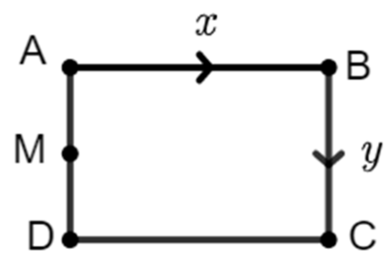


Example	Your Turn

ABCD is a trapezium.
M is the midpoint of AD.
Find $\overrightarrow{MA}$ in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$.



ABCD is a rectangle.
M is the midpoint of AD.
Find $\overrightarrow{MA}$ in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$.

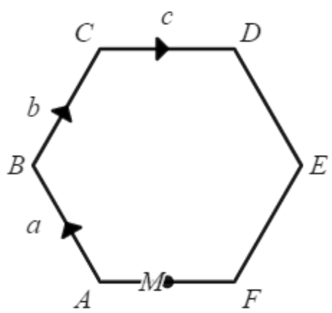


Example	Your Turn

473b: **Determine a vector involving a midpoint.**

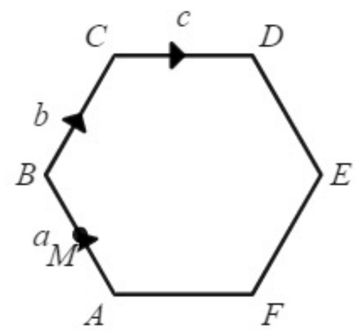
The point M is the midpoint of FA.

Express $\vec{AM}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.



The point M is the midpoint of AB.

Express $\vec{FM}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.



The ‘Two Parter’ Exam Question

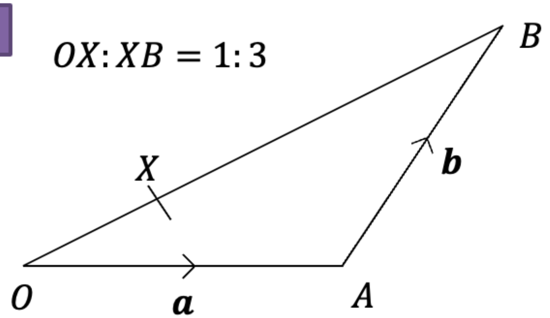
Many exams questions follow a two-part format:

a) **Find a relatively easy vector using skills from Lesson 1.**
b) **Find a harder vector that uses a fraction of your vector from part (a).**

Your Turn

A

$OX : XB = 1 : 3$

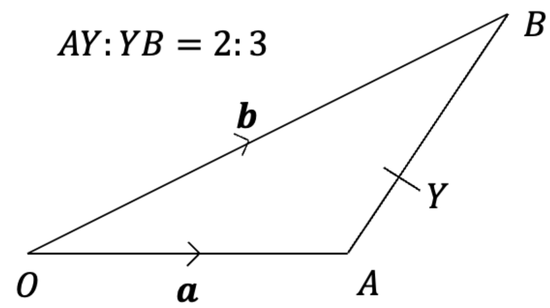


$\overrightarrow{AX} =$

Your Turn

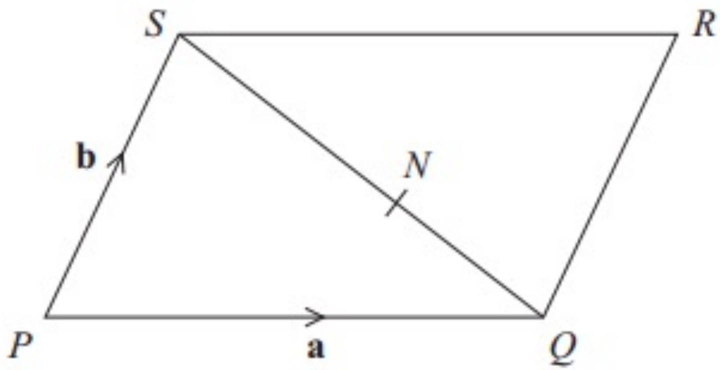
B

$AY : YB = 2 : 3$



$\overrightarrow{OY} =$

Example



PQRS is a parallelogram.
N is the point on *SQ* such that $SN : NQ = 3 : 2$
$\overrightarrow{PQ} = \mathbf{a}$ $\overrightarrow{PS} = \mathbf{b}$

(a) Write down, in terms of **a** and **b**, an expression for $\overrightarrow{SQ}$.

(b) Express $\overrightarrow{NR}$ in terms of **a** and **b**.

Your Turn

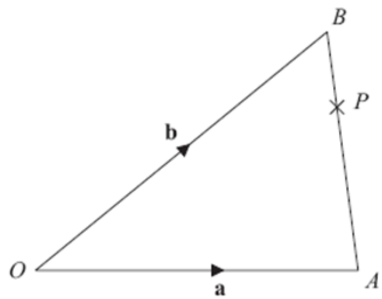


Diagram **NOT** accurately drawn

OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$
$\overrightarrow{OB} = \mathbf{b}$

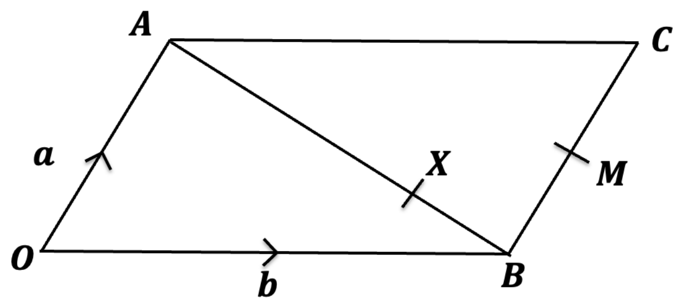
(a) Find $\overrightarrow{AB}$ in terms of **a** and **b**.

....
(1)

P is the point on *AB* such that *AP* : *PB* = 3 : 1

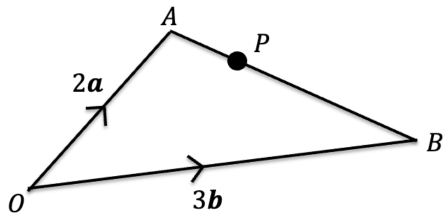
(b) Find $\overrightarrow{OP}$ in terms of **a** and **b**.
Give your answer in its simplest form.

Example



X is a point on AB such that $AX: XB = 3: 1$. M is the midpoint of BC.
Show that $\overrightarrow{XM}$ is parallel to $\overrightarrow{OC}$.

Your Turn

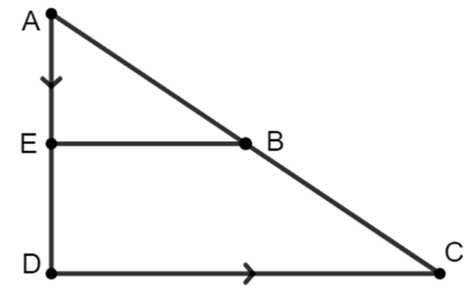


a) Find $\overrightarrow{AB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

b) P is the point on AB such that $AP:PB = 2:3$.
Show that $\overrightarrow{OP}$ is parallel to the vector $\boldsymbol{a}+\boldsymbol{b}$.

Your Turn

$\overrightarrow{AE} = 3\boldsymbol{a} - 2\boldsymbol{b}, \overrightarrow{DC} = 2\boldsymbol{a} + 4\boldsymbol{b},$
E and B are the midpoints of AD and AC.
Is $\overrightarrow{EB}$ parallel to $\overrightarrow{DC}$?

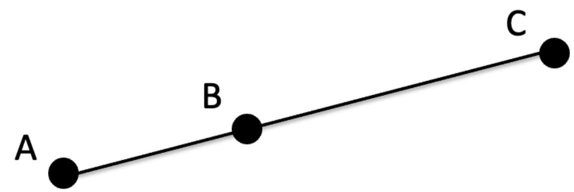


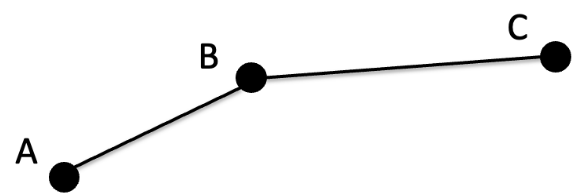
Proving Three Points form a Straight Line

Points A, B and C form a straight line if:

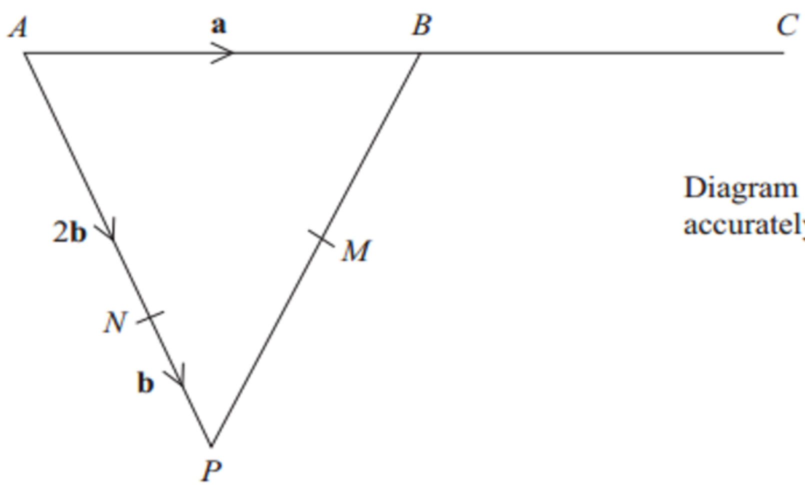
$\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel (and B is a common point).

Alternatively, we could show $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel. This tends to be easier.





Example



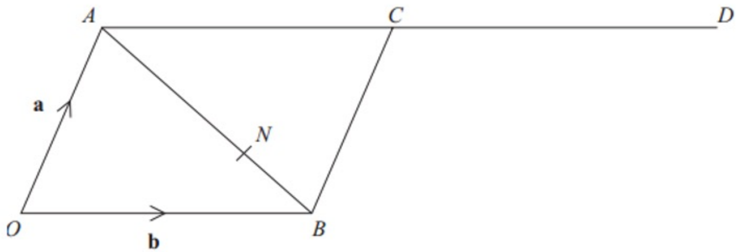
$$\overrightarrow{AN} = 2\boldsymbol{b}, \qquad \overrightarrow{NP} = \boldsymbol{b}$$

B is the midpoint of AC. M is the midpoint of PB.

a) Find $\overrightarrow{PB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

b) Show that NMC is a straight line.

Your Turn



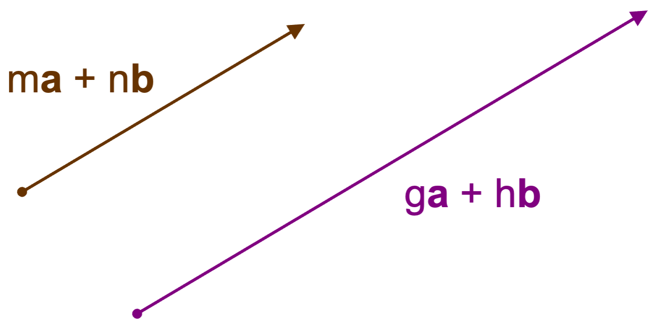
$\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$
D is the point such that $\overrightarrow{AC} = \overrightarrow{CD}$
The point N divides AB in the ratio $2:1$.

(a) Write an expression for $\overrightarrow{ON}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(b) Prove that OND is a straight line.

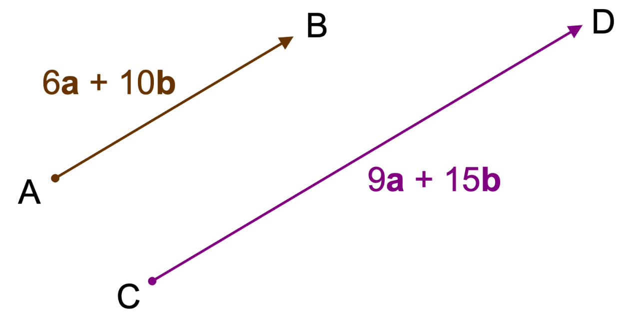
Vector Proofs

if two vectors are parallel
(or are sections of the same line)



then $\frac{g}{m} = \frac{h}{n}$

if two vectors are parallel
(or sections of the same line)



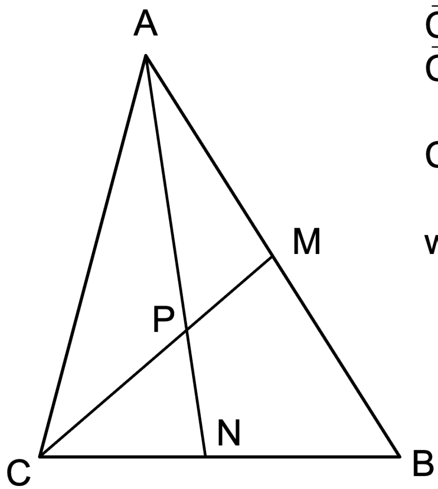
$\frac{9}{6} = \frac{15}{10} = 1\frac{1}{2}$

$\overrightarrow{CD} = 1\frac{1}{2}\,\overrightarrow{AB}$

$\overrightarrow{AB} = \frac{2}{3}\,\overrightarrow{CD}$

Example

Edexcel GCSE
November 2018, 1H, Q21



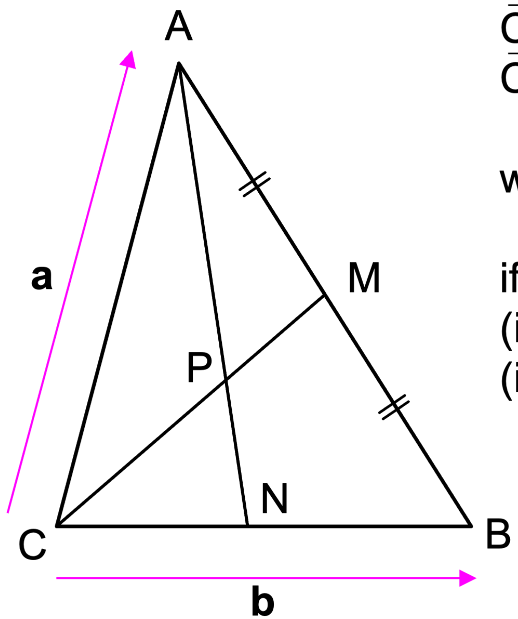
M = midpoint of AB

$\overrightarrow{CA} = \mathbf{a}$
$\overrightarrow{CB} = \mathbf{b}$

CP : PM = 3 : 2

work out the ratio CN : NB

Your Turn



M = midpoint of AB

$\overrightarrow{CA} = \mathbf{a}$
$\overrightarrow{CB} = \mathbf{b}$

work out the ratio CN : NB

if
(i) CP : PM = 2 : 1
(ii) CP : PM = 3 : 1

Example

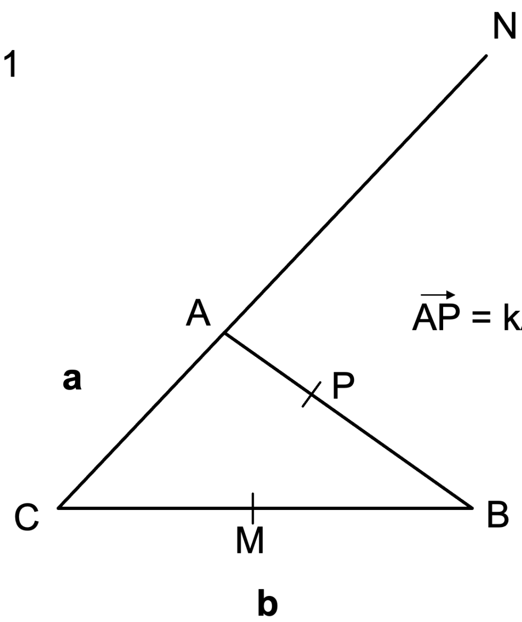
Edexcel
November 2017, 3H, Q21

M = midpoint of CB

$\overrightarrow{CA} = \mathbf{a}$
$\overrightarrow{CB} = \mathbf{b}$

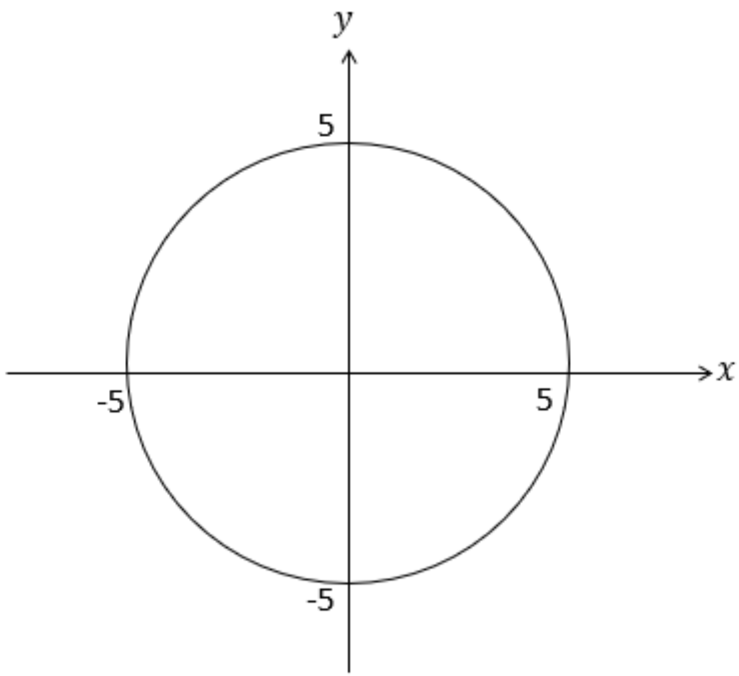
AN : CA = 2 : 1

MPN is a straight line
work out the ratio AP : PB



$\overrightarrow{AP} = k\overrightarrow{AB}$

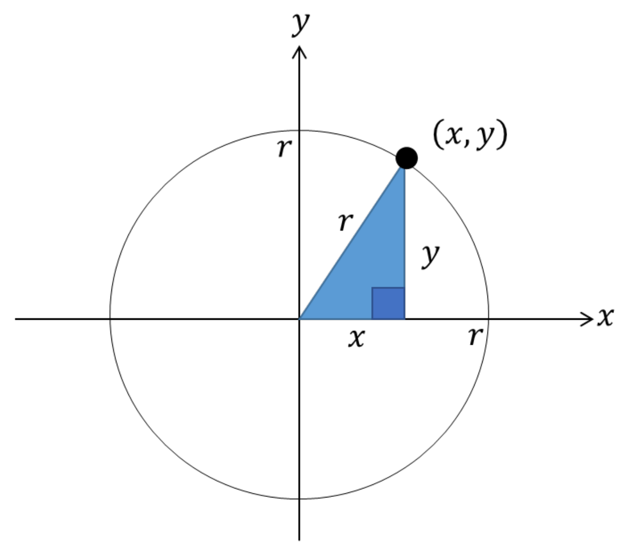
Equation of a circle



The equation of this circle is:
$x^2 + y^2 = 25$

The equation of a circle with centre at the origin and radius r is:
$x^2 + y^2 = r^2$

Proof



How could we show how r, x and y are related?
(Draw a right-angled triangle inside your circle, with one vertex at the origin and another at the circumference)

$x^2 + y^2 = r^2$

Example	Your Turn

446a: **Appreciate that a point on the circumference of a circle satisfies its equation.**

Determine which of the following points lies on the circle with the equation $x^2 + y^2 = 98$.

- ○ $(-6, 7)$
- ○ $(-7, 7)$
- ○ $(-7, 6)$

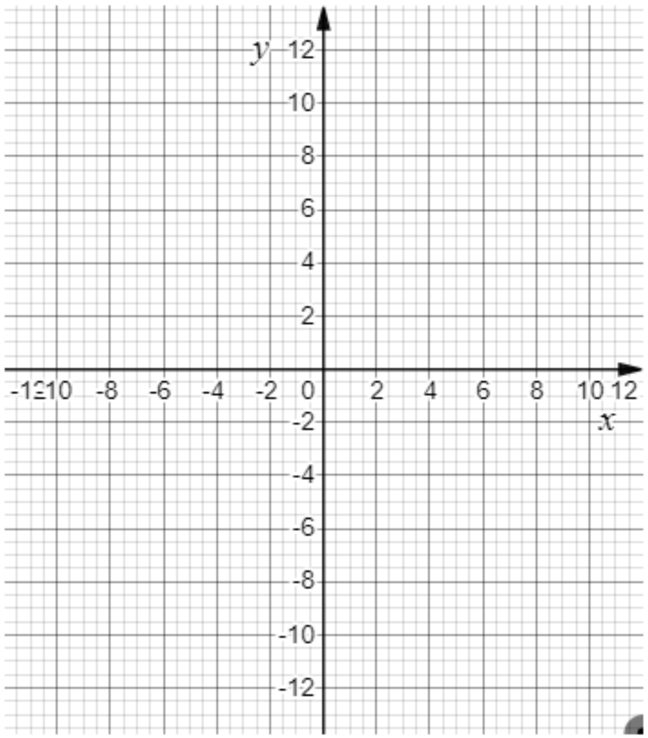
Determine which of the following points lies on the circle with the equation $x^2 + y^2 = 58$.

- ○ $(7, 3)$
- ○ $(8, 3)$
- ○ $(7, 4)$

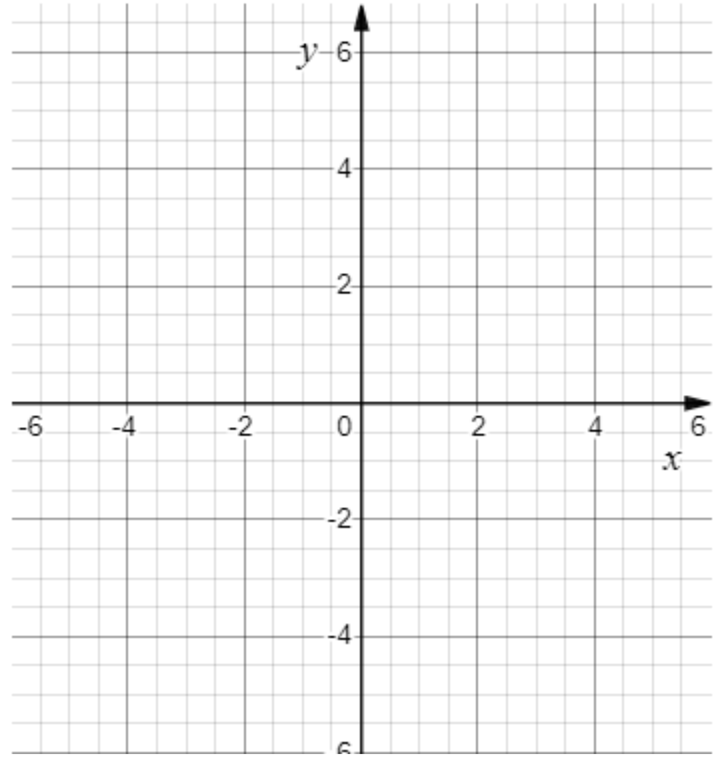
Example	Your Turn

446b: **Plot the graph of a circle centred at the origin.**

Draw the graph of $x^2 + y^2 = 100$



Draw the graph of $x^2 + y^2 = 16$



KS446 c, d , g and h – Fill in the blanks

#	Equation	Radius (simplified exact form)	Circumference (exact)	Area (exact)
1	$x^2 + y^2 = 9$			
2	$x^2 + y^2 = 9^2$			
3	$x^2 + y^2 = 144$			
4	$x^2 + y^2 = 14$			
5	$x^2 + y^2 = 180$			
6	$x^2 + y^2 = r^2$			
7	$x^2 + y^2 =$	8		
8	$x^2 + y^2 =$	$\sqrt{11}$		
9	$x^2 + y^2 =$	$2\sqrt{3}$		
10	$x^2 + y^2 =$	a		

KS446 c, d , g and h – Fill in the blanks

#	Equation	Radius (simplified exact form)	Circumference (exact)	Area (exact)
11	$x^2 + y^2 =$		10π	
12	$x^2 + y^2 =$		28π	
13	$x^2 + y^2 =$		100π	
14	$x^2 + y^2 =$		π	
15	$x^2 + y^2 =$		$k\pi$	
16	$x^2 + y^2 =$			49π
17	$x^2 + y^2 =$			100π
18	$x^2 + y^2 =$			400π
19	$x^2 + y^2 =$			50π
20	$x^2 + y^2 =$			$k^2\pi$

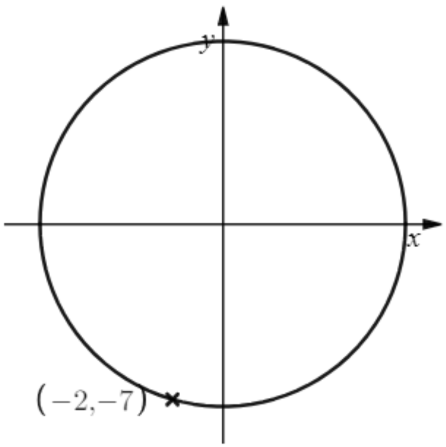
Fill in the Gaps

Equation	Radius	Area	Point 1	Point 2	Where is $(3, 7)$?
$x^2 + y^2 = 25$			$(3,$ ____$)$	$($____$, 0)$	Outside
$x^2 + y^2 = 50$			$(-5,$ ____$)$	$($____$, 7)$	
$x^2 + y^2 = 65$			$(1,$ ____$)$	$($____$, 7)$	
	15		$(9,$ ____$)$	$($____$, 0)$	
	$5\sqrt{5}$		$(-5,$ ____$)$	$($____$, 11)$	
		130π	$(-7,$ ____$)$	$($____$, 11)$	
		2042	$(19,$ ____$)$	$($____$, 11)$	
			$(-4,$ ____$)$	$(8, 11)$	
			$(1,$ ____$)$	$(-7, 11)$	
			$(-7,$ ____$)$	$($____$, \sqrt{22})$	On the circle

Example	Your Turn

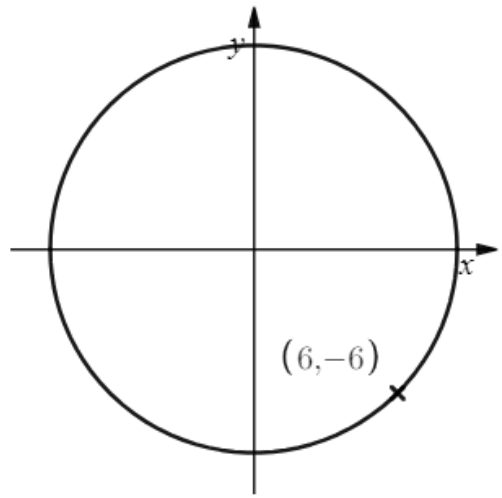
446f: **Determine the equation of a circle centred at the origin, using a point given on the circumference.**

The point $(-2, -7)$ lies on a circle centered on the origin.



Find an equation for this circle.

The point $(6, -6)$ lies on a circle centered on the origin.



Find an equation for this circle.

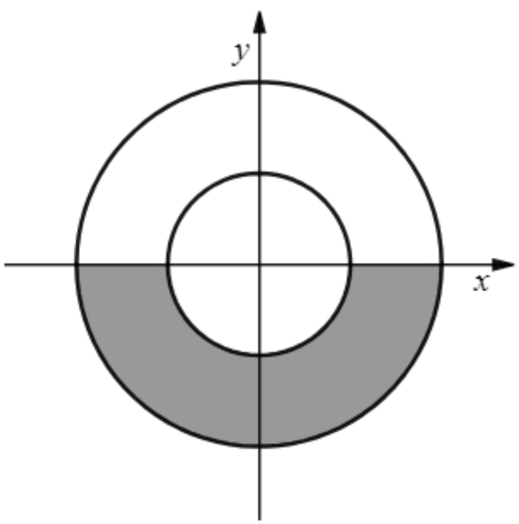
Example

446i: Determine an area or perimeter of a portion of an annulus, given the equations of the circles centred at the origin.

The annulus below is formed of two circles centred on the origin.

The equations of the circles are:

$$x^2 + y^2 = 16$$
$$x^2 + y^2 = 4$$



Calculate the perimeter of the shaded shape.

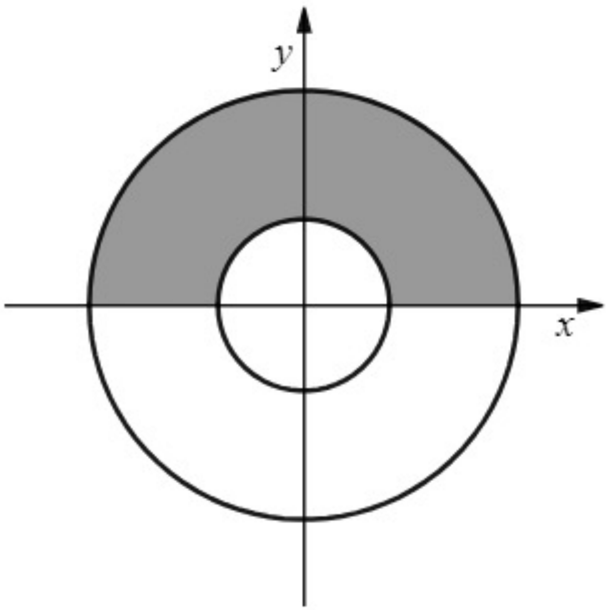
Give your answer correct to 2 decimal places.

Your Turn

The annulus below is formed of two circles centred on the origin.

The equations of the circles are:

$$x^2 + y^2 = 25$$
$$x^2 + y^2 = 4$$



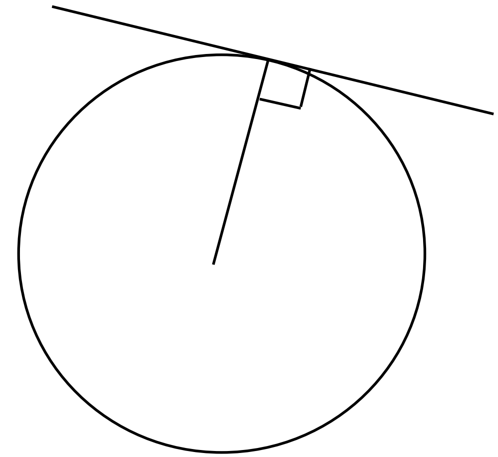
Calculate the perimeter of the shaded shape.

Give your answer correct to 2 decimal places.

Tangent to Circles

As always, to get an equation of a line we need:

- A point (we have that!)
- The gradient.

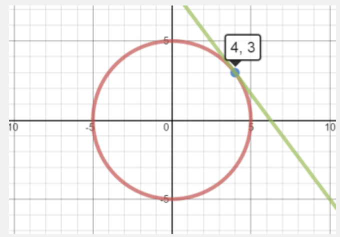


There's only ONE thing you need to remember for this topic, related to finding the gradient of the tangent:

The tangent is perpendicular to the radius.

Gradient of line	Gradient of perpendicular line
3	
-4	
$\frac{1}{2}$	
$-\frac{3}{4}$	

Equations of Tangents to Circles

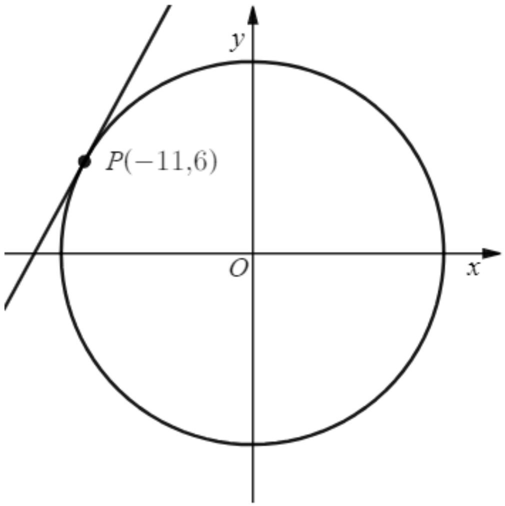
Question	Radius	Sketch	Gradient of radius at point	Gradient of tangent at point	Equation of tangent at point
Example: Find the equation of the tangent to $x^2 + y^2 = 25$ at $(4, 3)$	5		$\frac{3}{4}$	$-\frac{4}{3}$	$y = -\frac{4}{3}x + \frac{25}{3}$
1. Find the equation of the tangent to $x^2 + y^2 = 5$ at $(2, 1)$					
2. Find the equation of the tangent to $x^2 + y^2 = 100$ at the point on the circumference with x-coordinate 6 and a positive y-coordinate					
3. Find the equation of the tangent to $x^2 + y^2 = 45$ at $(-6, 3)$					

Question	Radius	Sketch	Gradient of radius at point	Gradient of tangent at point	Equation of tangent at point
4. Find the equation of the tangent to $x^2 + y^2 - 20 = 0$ at $(-4, -2)$					
5. Find the equation of the tangent to $x^2 + y^2 = 13$ at the point on the circumference with x-coordinate 3 and a negative y-coordinate					
6. Find the equation of the tangent to the circle with centre $(0, 0)$ and diameter $\sqrt{32}$ at the point $(2, 2)$					
7. Find the equation of the tangent to $x^2 + y^2 = 25$ at the point $(5, 0)$					

Example

447a: **Determine the equation of a tangent to a circle centred at the origin.**

The diagram shows the circle with equation $x^2 + y^2 = 157$

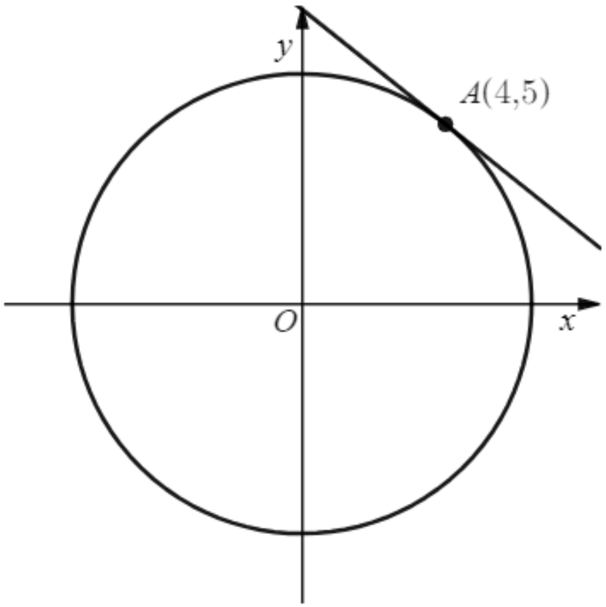


A tangent to the circle is drawn at point P with coordinates $(-11, 6)$

Find an equation of the tangent at P.

Your Turn

The diagram shows the circle with equation $x^2 + y^2 = 41$



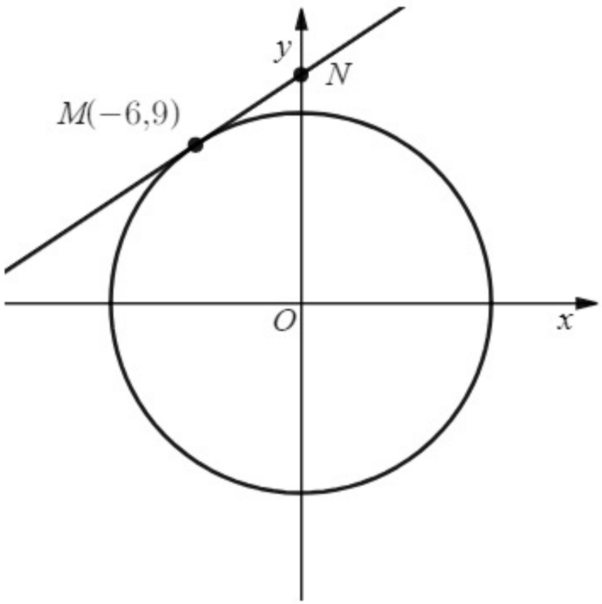
A tangent to the circle is drawn at point A with coordinates $(4, 5)$

Find an equation of the tangent at A.

Example

447b: Determine the x-intercept or y-intercept of a tangent to a circle centred at the origin.

The diagram shows a circle with centre $(0, 0)$ and a tangent at the point $M\,(-6, 9)$



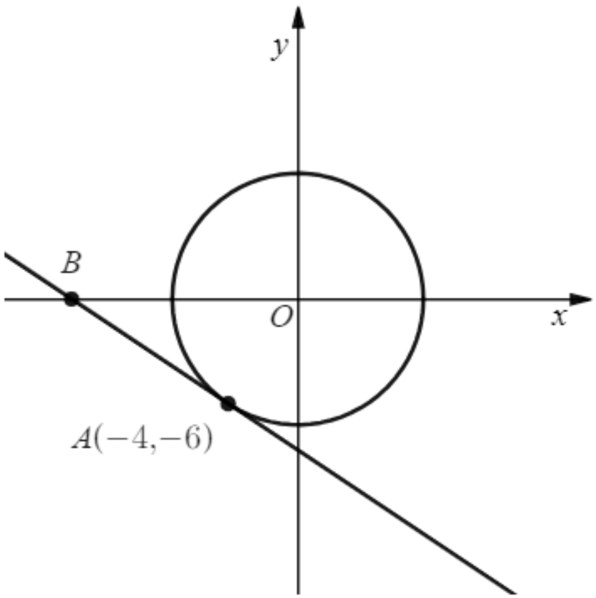
The tangent to the circle at M intersects the y-axis at point N.

Work out the y-coordinate of N.

Your Turn

A circle has equation $x^2 + y^2 = 52$

A is the point on the circle with coordinates $(-4, -6)$



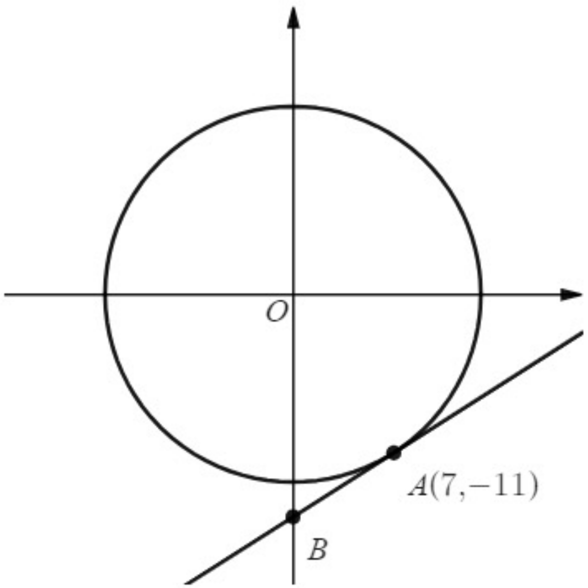
The tangent to the circle at A intersects the x-axis at point B.

Work out the x-coordinate of B.

Example

447c: Determine the area of a triangle formed by a tangent to a circle centred at the origin.

The diagram shows a circle with centre $(0, 0)$ and a tangent at the point $A\,(7, -11)$

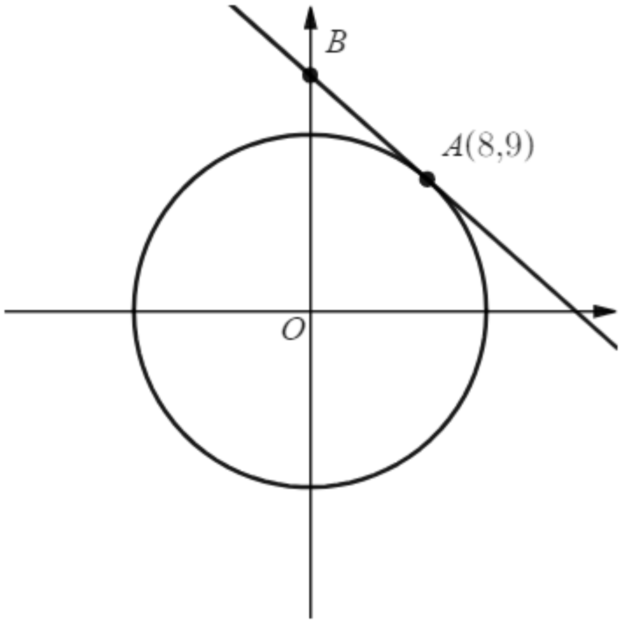


The tangent to the circle at A intersects the y-axis at point B.

Work out the area of triangle OAB.

Your Turn

The diagram shows a circle with centre $(0, 0)$ and a tangent at the point $A\,(8, 9)$



The tangent to the circle at A intersects the y-axis at point B.

Work out the area of triangle OAB.

Non-Linear Simultaneous Equations

Example

Solve the following pair of simultaneous equations:

$$xy = 2$$
$$y = x + 1$$

Your Turn

Solve the following pair of simultaneous equations:

$$xy = 2$$
$$y = x - 1$$

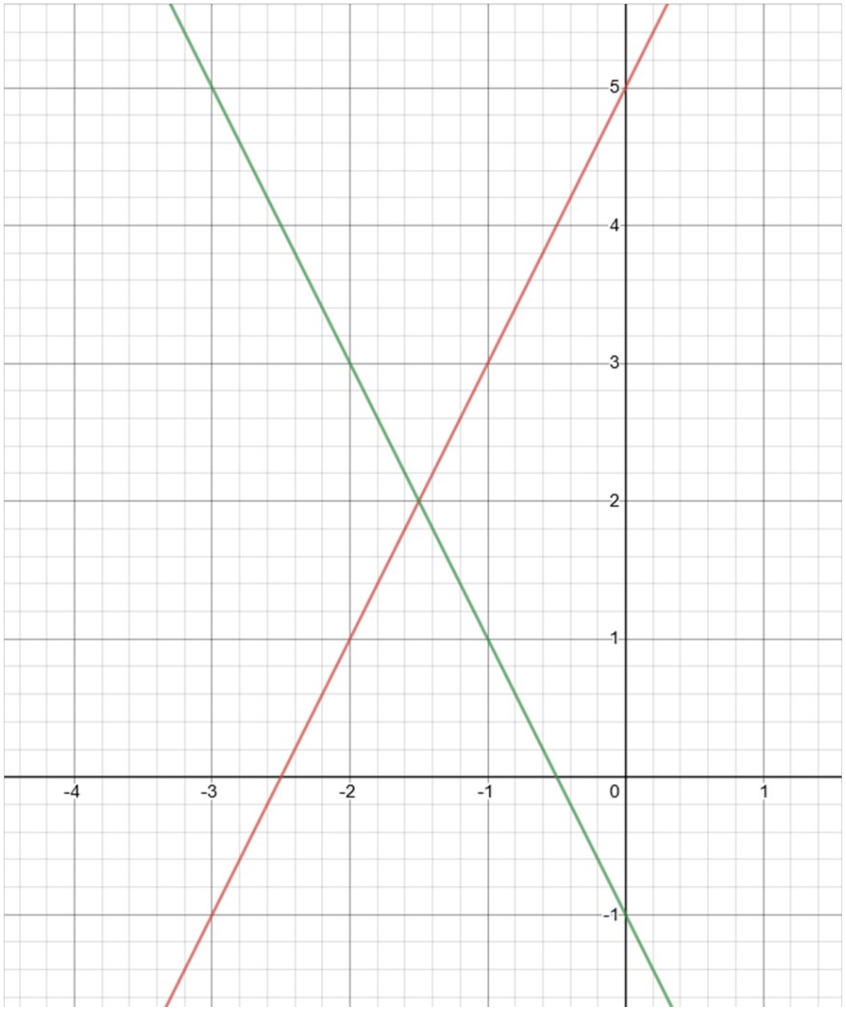
Example	Your Turn
420a: **Solve non-linear simultan where y is the subject of both e equation to be solved.** Solve the following simultaneous equations. $\begin{cases} y = x^2 + 3x - 28 \\ y = 2x + 2 \end{cases}$	Solve the following simultaneous equations. $\begin{cases} y = x^2 + 10x + 21 \\ y = x + 3 \end{cases}$

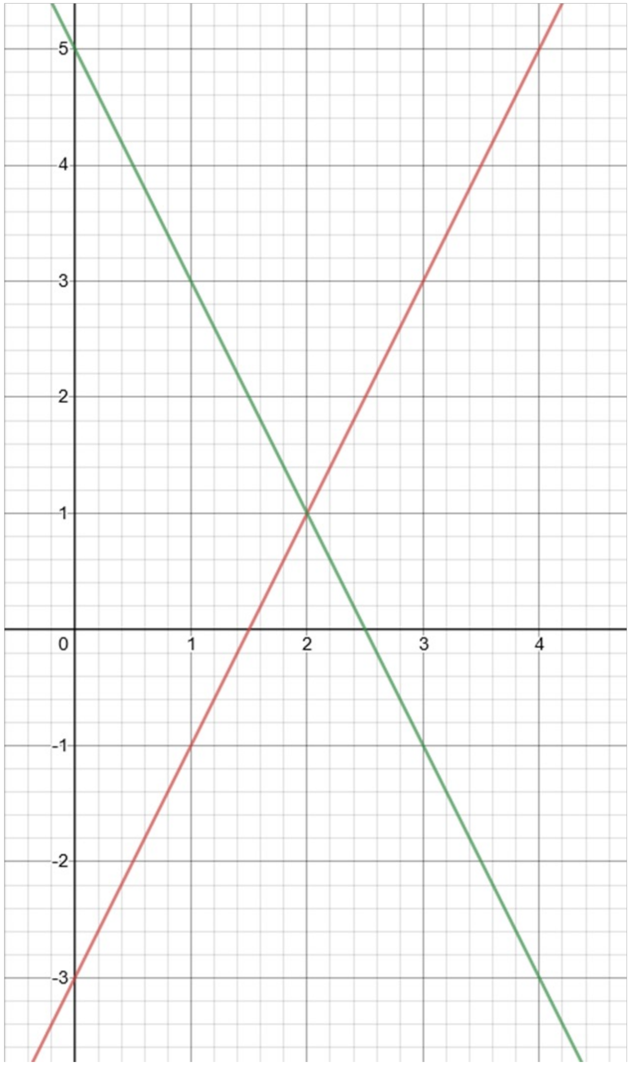
Example	Your Turn
420c: **Solve non-linear simultaneous equations/systems of equations with one equation given in the form $x^2 + y^2 = a$ and the other where x or y is the subject.** Solve the following simultaneous equations. $\begin{cases} y = x + 4 \\ x^2 + y^2 = 58 \end{cases}$	Solve the following simultaneous equations. $\begin{cases} y = 2x + 1 \\ x^2 + y^2 = 2 \end{cases}$

Example	Your Turn
420d: **Solve simultaneous equations/systems of equations given in the form $ax + by = c$ and $x^2 + y^2 = d$** Solve the following simultaneous equations. $\begin{cases} 5x + 5y = 2 \\ x^2 + y^2 = 4 \end{cases}$	Solve the following simultaneous equations. $\begin{cases} 3x + y = 3 \\ x^2 + y^2 = 1 \end{cases}$

Graphical Simultaneous Equations

Example	Your Turn
Solve: $y = 2x + 5$ $y = -2x - 1$	Solve: $y = 2x - 3$ $y = -2x + 5$



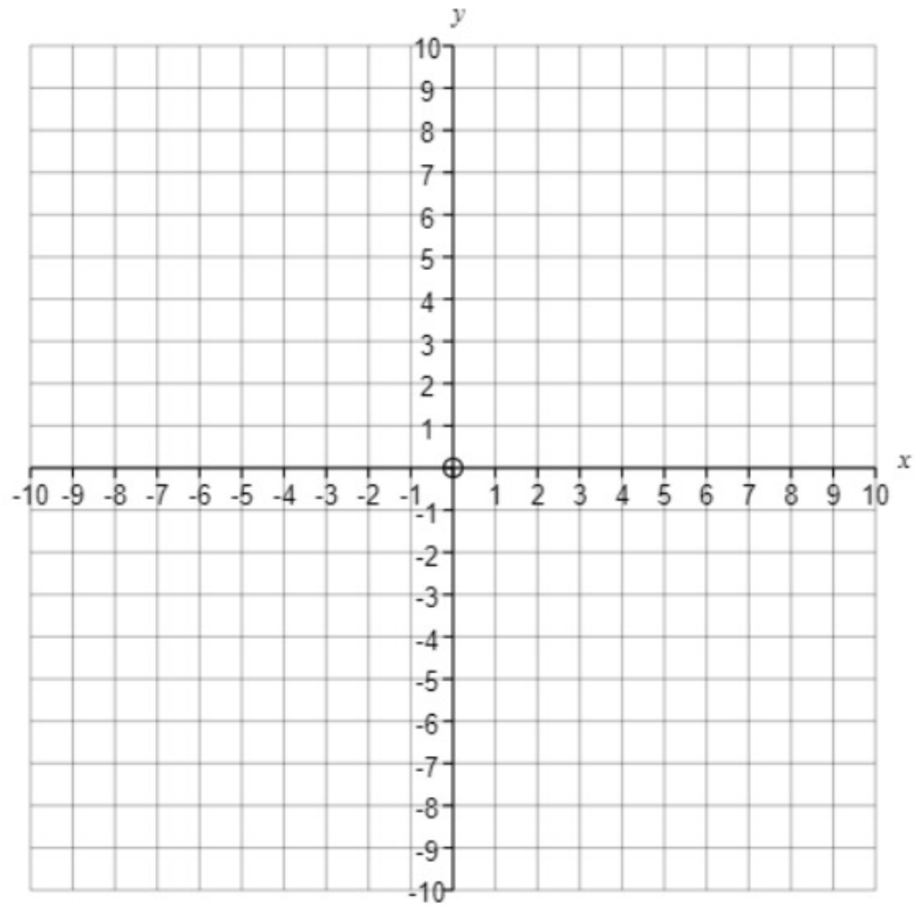


Example

Solve, using a graphical approach, the simultaneous equations:

$2x - y = 8$

$x - y = 1$

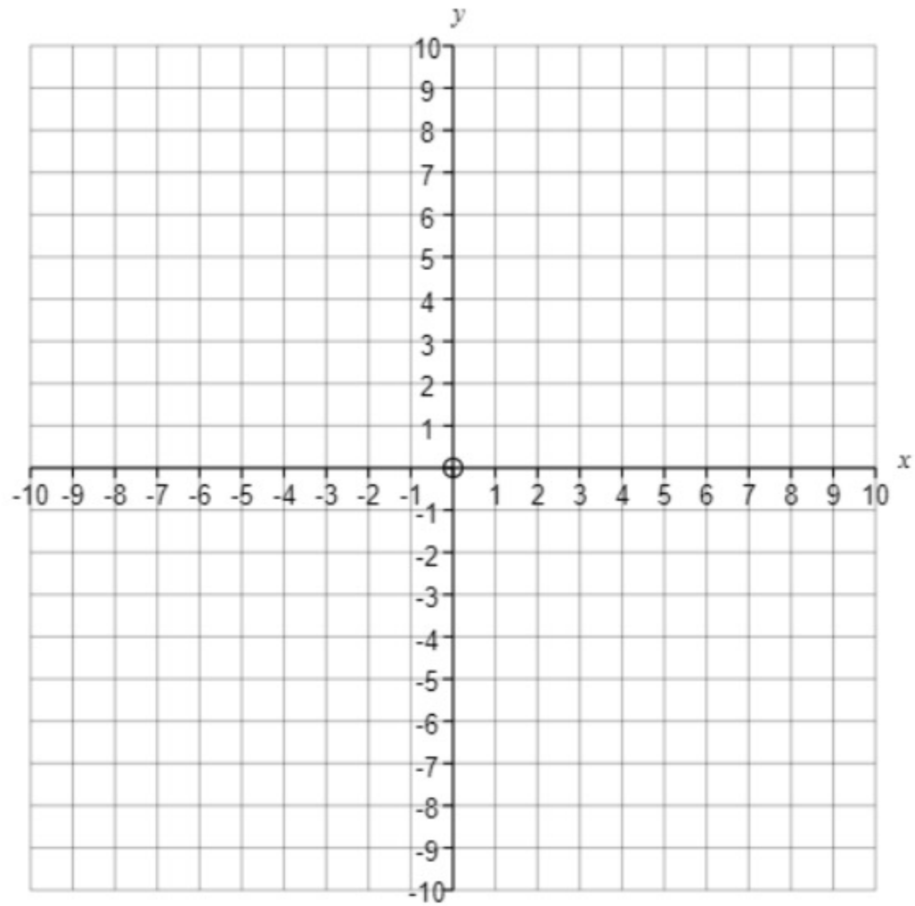


Your Turn

Solve, using a graphical approach, the simultaneous equations:

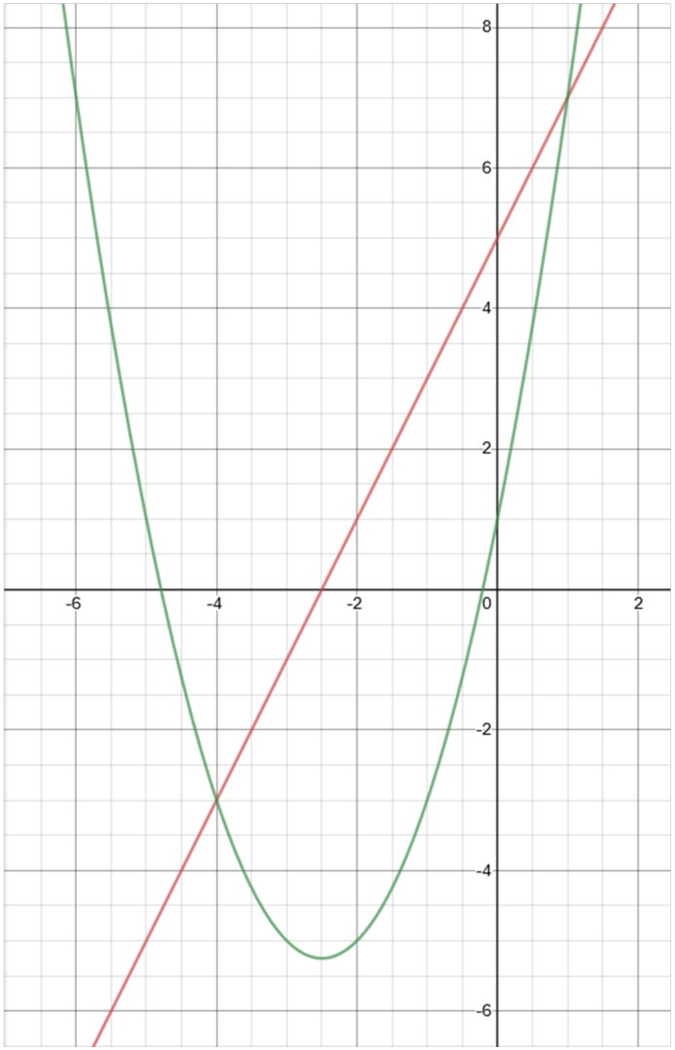
$-2x + y = 1$

$x + y = 10$



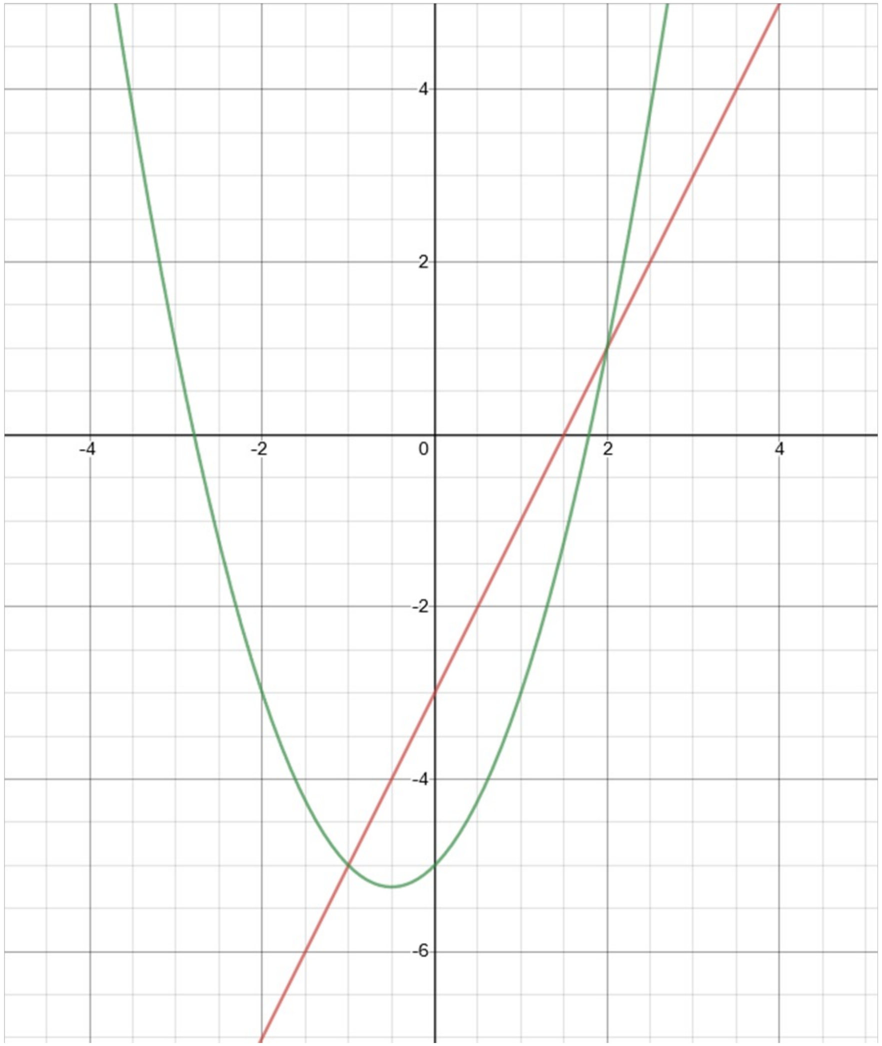
Example

Solve:
$y = 2x + 5$
$y = x^2 + 5x + 1$



Your Turn

Solve:
$y = 2x - 3$
$y = x^2 + x - 5$



Algebraic Proof

KEY SKILLS WHICH YOU NEED FOR THIS TOPIC

YOU NEED TO GO THROUGH ALL OF THESE

454a: Identify expressions that represent a multiple of an integer.

m and n are integers.

Select the expressions that represent a multiple of 3.

- ☐ $6n+1$
- ☐ $3m^2-6$
- ☐ $9m+15$
- ☐ $9n^2-3$
- ☐ $5(7n-3)-3$
- ☐ $3(10n-8)+9$

454b: Identify expressions that represent odd/even integers.

m and n are integers.

Select the expressions that represent even numbers for all m or n.

- ☐ $5n-5$
- ☐ $9m-1$
- ☐ $3m+8$
- ☐ $10m+12$
- ☐ $2m-19$
- ☐ $20(6n+6)-3$

454c: Use expressions for consecutive integers.

A number is given as $-2n-9$ where n is an integer.

Write down the expression for the next consecutive integer.

454d: Use expressions for consecutive odd/even integers.

An odd number is given as $4n+15$ where n is an integer.

Write down the expression for the next consecutive odd number.

454e: Use expressions for different odd/even integers, not necessarily consecutive.

Given that m and n are integers.

Select the expressions that represent two consecutive even numbers.

- ☐ $2m$ and $2n$
- ☐ $2n-2$ and $2n$
- ☐ $2m+1$ and $2n-1$
- ☐ $2m$ and $2n-1$
- ☐ $2m$ and $2n+5$
- ☐ $2m-2$ and $2n+2$

Example	Your Turn
Prove that $4n - 3 + 2n - 9$ is a multiple of 3 for all real integers n	Prove that $4n - 3 + 10n - 11$ is a multiple of 7 for all real integers n

Example	Your Turn
Prove that the sum of five consecutive integers is a multiple of 5.	Prove that the sum of three consecutive integers is a multiple of 3.

Example	Your Turn
Prove that the product of two odd numbers is an odd number.	Prove that the product of two even numbers is an even number.

Example	Your Turn
Prove algebraically that $n^2 - 2 - (n - 2)^2$ is always even, given n is an integer greater than 1	Prove algebraically that $(2n + 1)^2 - (2n + 1)$ is an even number

Example	Your Turn
Prove that $(n-1)^2 + n^2 + (n+1)^2$ is two more than a multiple of 3 for all positive integer values of n	Prove that $(n+1)^2 - n^2$ is one more than a multiple of 2 for all positive integer values of n

Example	Your Turn
Prove that $(2n+3)^2-(2n-3)^2$ is a multiple of 8 for all positive integer values of n	Prove that $(3n+2)^2-(3n-2)^2$ is a multiple of 8 for all positive integer values of n

Example	Your Turn
Prove algebraically that the difference between two different odd numbers is an even number.	Prove algebraically that the difference between two different even numbers is an even number.

Example	Your Turn
Prove that the product of four consecutive integers is always a multiple of 8	Prove that the product of three consecutive integers is always a multiple of 6

Example	Your Turn
Prove that, for all positive values of n, $\frac{(n+3)^2-(n-2)^2}{2n^2+n} = \frac{a}{b}$ where a and b are integers or variables.	Prove that, for all positive values of n, $\frac{(n+2)^2-(n+1)^2}{2n^2+3n} = \frac{a}{b}$ and find the integers a and b

Example	Your Turn

455a: **Equate coefficients to determine constants in an identity.**

Given that

$$4ax + 5b + x + x \equiv 18x - 25$$

Find the values of a and b.

Given that

$$3px + 1q - 7x \equiv -4x + 2$$

Find the values of p and q.

Example	Your Turn
455b: Equate coefficients to determine constants in an identity requiring two single bracket expansions. Given that $5(4qx + p) + 3(3qx - p) \equiv 29x + 12$ Find the values of p and q.	Given that $3(ax + 2b) + 3(2ax + b) \equiv -54x + 54$ Find the values of a and b.

Example	Your Turn

455d: **Equate coefficients to determine constants in an identity, involving a combination of single and double bracket expansions.**

Given that

$$y^2 + cy + 9 = (y + d)^2 + 2y$$

where c and d are positive integers, find the value of c and the value of d.

Given that

$$(2x + e)^2 - 4x = 4x^2 + fx + 49$$

where e and f are positive integers, find the value of e and the value of f.

Geometric Sequences

A **geometric sequence**, also known as a **geometric progression**, is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the common ratio.

nth term of Geometric Sequences

$$\boldsymbol{a \times r^{n-1}}$$

$a =$ first term
$r =$ common ratio

Example

Write down the first term and common ratio of the following geometric sequences:

a) $4, 12, 36, 108$
b) $4, -12, 36, -108$
c) $108, 36, 12, 4$
d) $\sqrt{7}, 7, 7\sqrt{7}, 49$

Your Turn

Write down the first term and common ratio of the following geometric sequences:

a) $5, 20, 80, 320$
b) $5, -20, 80, -320$
c) $320, 80, 20, 5$
d) $\sqrt{3}, 3, 3\sqrt{3}, 9$

Example

Generate the first 5 terms of the following geometric sequences:

a) 3^{n-1}
b) $4 \times 3^{n-1}$

Your Turn

Generate the first 5 terms of the following geometric sequences:

a) 4^{n-1}
b) $5 \times 4^{n-1}$

Geometric Sequences			
(a)	**(b)**	**(c)**	**(d)**
Find the next two terms in the sequence $7, 14, 28, 56, \ldots$	Find the next two terms in the sequence $40, 20, 10, 5, \ldots.$	Find the first four terms of the sequence with first term 2 and common ratio 3	Find the first term and common ratio for the sequence: $3, 15, 75, 375, \ldots$
(e)	**(f)**	**(g)**	**(h)**
Find the first term and common ratio for the sequence: $160, 80, 40, 20, \ldots$	Find the next two terms in the sequence $2, -4, 8, -16, \ldots.$	Find the first four terms of the sequence with first term 120 and common ratio 0.5	Find the first term and common ratio for the sequence: $4, -8, 16, -32, \ldots$
(i)	**(j)**	**(k)**	**(l)**
Find the first four terms of the sequence with first term 5 and common ratio -2	Find the first four terms of the sequence with nth term $6 \times 3^{n-1}$	Find the nth term of the sequence with first term 10 and common ratio 4	Find the nth term of the sequence with first term 250 and common ratio 0.2
(m)	**(n)**	**(p)**	
Find the first four terms of the sequence with nth term $400 \times \left(\frac{1}{2}\right)^{n-1}$	Find the nth term of the sequence with first term 8 and common ratio -5	A tree starts with four branches. Every month each branch splits into two. How many branches will the tree have after 5 months? Find a formula for the number of branches b after n months.	

Quadratic Sequences

A quadratic sequence is a sequence of numbers where the second difference is constant.

The n^{th} term of a quadratic sequence has the form $an^2 + bn + c$ where a, b and c are numbers and $a \neq 0$

Example

Generate the first 5 terms of the following quadratic sequence:

a) $n^2 - 5$
b) $3n^2 - 5$
c) $3n^2 + 2n - 5$

Your Turn

Generate the first 5 terms of the following quadratic sequence:

a) $n^2 + 5$
b) $3n^2 + 5$
c) $3n^2 - 2n + 5$

The nth term method

Sequence	5	8	13	20
First Difference	3	5	7	
Second Difference		2	2	
[1] n^2	1	4	9	16
Sequence minus n^2	4	4	4	4
Linear nth term	$+4$			
Quadratic nth term	n^2+4			

Sequence	12	15	20	27
First Difference	3	5	7	
Second Difference				
[ ] n^2				
Sequence minus n^2				
Linear nth term				
Quadratic nth term				

Sequence	0	3	8	15
First Difference				
Second Difference				
[ ] n^2				
Sequence minus n^2				
Linear nth term				
Quadratic nth term				

Sequence	7	13	23	37
First Difference	6	10	14	
Second Difference		4	4	
[2] n^2	2	8	18	32
Sequence minus n^2				
Linear nth term				
Quadratic nth term				

Sequence	-1	5	15	29
First Difference				
Second Difference				
[ ] n^2				
Sequence minus n^2				
Linear nth term				
Quadratic nth term				

Sequence	11	20	35	56
First Difference				
Second Difference				
[ ] n^2				
Sequence minus n^2				
Linear nth term				
Quadratic nth term				

Sequence	2	7	14	23
First Difference	5	7	9	
Second Difference		2	2	
[1] n^2	1	4	9	16
Sequence minus n^2	1	3	5	7
Linear nth term	$2n-1$			
Quadratic nth term	n^2+2n-1			

Sequence	7	14	25	40
First Difference				
Second Difference				
[ ] n^2				
Sequence minus n^2				
Linear nth term				
Quadratic nth term				

Sequence	-1	10	27	50
First Difference				
Second Difference				
[ ] n^2				
Sequence minus n^2				
Linear nth term				
Quadratic nth term				

Sequence	6	8	12	18
First Difference				
Second Difference				
[ ] n^2				
Sequence minus n^2				
Linear nth term				
Quadratic nth term				

Sequence	2.5	8	14.5	22
First Difference				
Second Difference				
[0.5] n^2	0.5	2	4.5	8
Sequence minus n^2				
Linear nth term				
Quadratic nth term				

Sequence	3	3	1	-3
First Difference				
Second Difference				
[ ] n^2				
Sequence minus n^2				
Linear nth term				
Quadratic nth term				

Example

Find the nth term of the following quadratic sequence:

a) $4, 7, 12, 19, 28, \ldots$
b) $4, 16, 36, 64, 100, \ldots$

Your Turn

Find the nth term of the following quadratic sequence:

a) $-2, 1, 6, 13, 22, \ldots$
b) $0.5, 2, 4.5, 8, 12.5, \ldots$

Why is ‘a’ half the second difference?

Consider the general quadratic sequence $an^2 + bn + c$

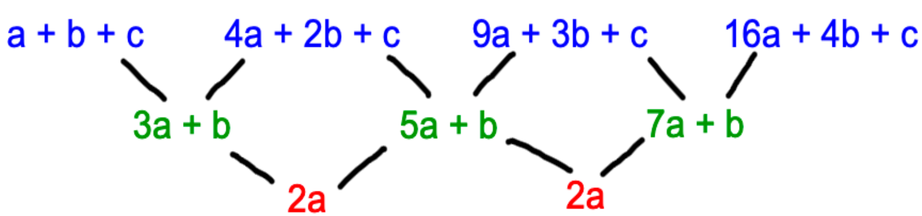
- We substitute $n = 1$ to get the first term:
  $$a(1)^2 + b(1) + c = a + b + c$$

- We substitute $n = 2$ to get the first term:
  $$a(2)^2 + b(2) + c = 4a + 2b + c$$

- We substitute $n = 3$ to get the first term:
  $$a(3)^2 + b(3) + c = 9a + 3b + c$$

- We substitute $n = 4$ to get the first term:
  $$a(4)^2 + b(4) + c = 16a + 4b + c$$

Finding



- a is half the second difference.
- c is the zeroth term.
- b can be found by substitution.

Example	Your Turn
Find the n^{th} term of the following sequence: $-4, -1, 4, 11, 20$	Find the n^{th} term of the following sequence: $6, 9, 14, 21, 30$

Example	Your Turn
Find the n^{th} term of the following sequence: $-2, 7, 22, 43, 70$	Find the n^{th} term of the following sequence: $8, 17, 32, 53, 80$

Example

Find the n^{th} term of the following sequence:

0, 11, 28, 51, 80

Your Turn

Find the n^{th} term of the following sequence:

6, 13, 26, 45, 70