KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

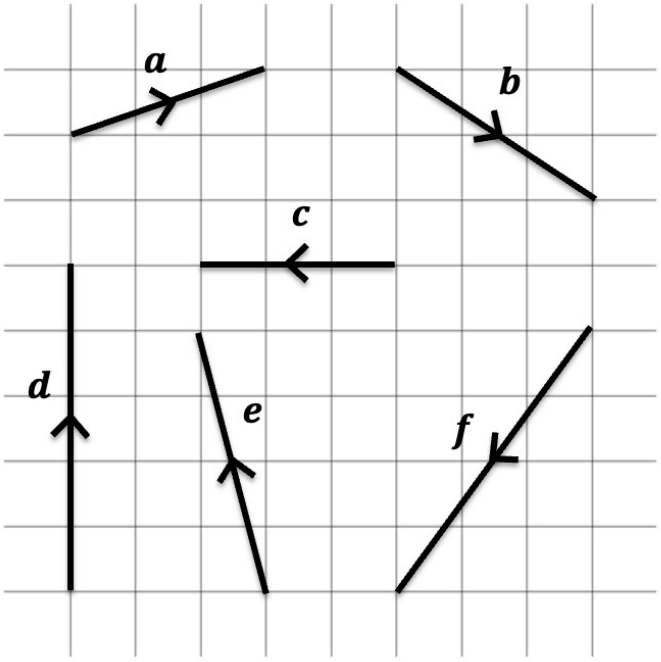
Year 11

2023 Mathematics 2024

Unit 24 Tasks

DO NOT WRITE INSIDE

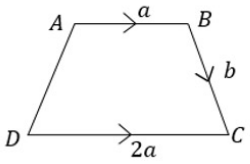
1 State the value of each vector.



$\boldsymbol{a} =$? $\boldsymbol{b} =$? $\boldsymbol{c} =$?

$\boldsymbol{d} =$? $\boldsymbol{e} =$? $\boldsymbol{f} =$?

2



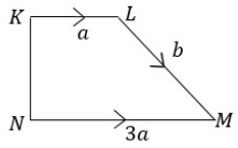
a. $\overrightarrow{BA} =$?

b. $\overrightarrow{AC} =$?

c. $\overrightarrow{DB} =$?

d. $\overrightarrow{AD} =$?

3



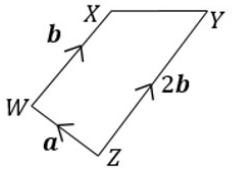
a. $\overrightarrow{MK} =$?

b. $\overrightarrow{NL} =$?

c. $\overrightarrow{NK} =$?

d. $\overrightarrow{KN} =$?

4



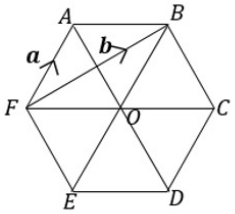
a. $\overrightarrow{ZX} =$?

b. $\overrightarrow{YW} =$?

c. $\overrightarrow{XY} =$?

d. $\overrightarrow{XZ} =$?

5



a. $\overrightarrow{AB} =$?

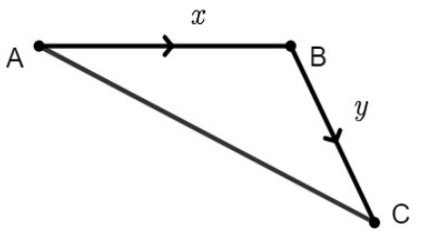
b. $\overrightarrow{FO} =$?

c. $\overrightarrow{AO} =$?

d. $\overrightarrow{FD} =$?

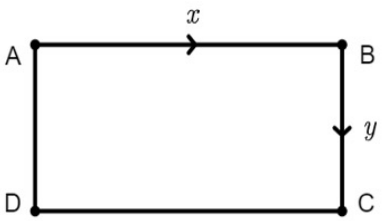
1. ABC is a triangle. Find, in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$:

$(a)\ \overrightarrow{BA}\ \ (b)\ \overrightarrow{CB}$
$(c)\ \overrightarrow{AC}\ (d)\ \overrightarrow{CA}$



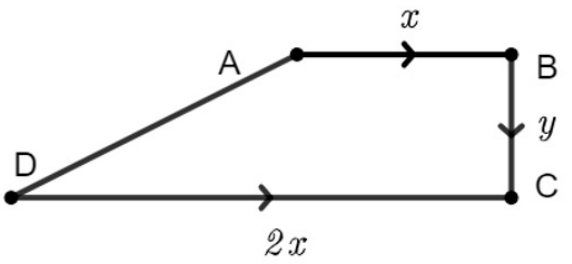
2. ABCD is a rectangle. Find, in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$:

$(a)\ \overrightarrow{DA}\ \ (b)\ \overrightarrow{AC}$
$(c)\ \overrightarrow{CA}\ (d)\ \overrightarrow{BD}$



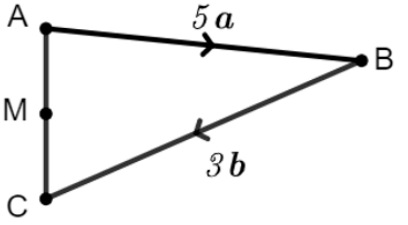
3. ABCD is a trapezium. Find, in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$:

$(a)\ \overrightarrow{AC}\ \ (b)\ \overrightarrow{DB}$
$(c)\ \overrightarrow{AD}\ (d)\ \overrightarrow{DA}$



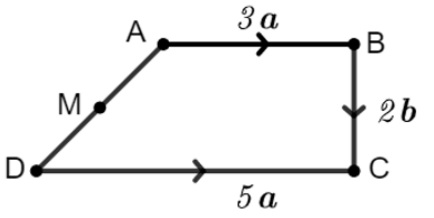
1. ABC is a triangle. M is the midpoint of AC. Find, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$:

$(a)\ \overrightarrow{AC}\ \ (b)\ \overrightarrow{CA}$
$(c)\ \overrightarrow{AM}\ (d)\ \overrightarrow{BM}$



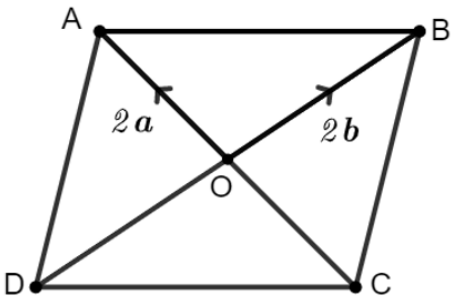
2. ABCD is a trapezium. M is the midpoint of AD. Find, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$:

$(a)\ \overrightarrow{DA}\ \ (b)\ \overrightarrow{DM}$
$(c)\ \overrightarrow{AD}\ (d)\ \overrightarrow{MC}$

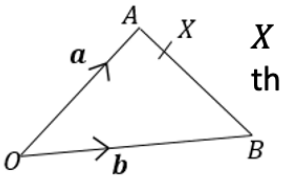


3. ABCD is a parallelogram. Find, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$:

$(a)\ \overrightarrow{OC}\ \ (b)\ \overrightarrow{AC}$
$(c)\ \overrightarrow{AB}\ (d)\ \overrightarrow{AD}$



1



X is a point such that $AX : XB = 1 : 4$

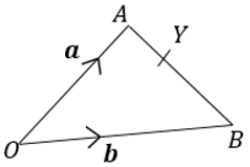
a. $\overrightarrow{AB} =$?

b. $\overrightarrow{AX} =$?

c. $\overrightarrow{OX} =$?

d. $\overrightarrow{BX} =$?

2



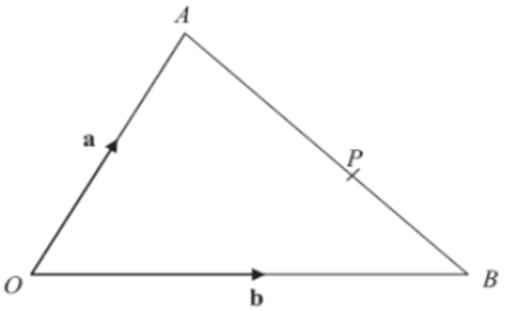
Y is a point such that $YB = 2AY$

a. $\overrightarrow{AY} =$?

b. $\overrightarrow{OY} =$?

c. $\overrightarrow{YO} =$?

3



[June 2009 2H Q23]

a) Find $\overrightarrow{AB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

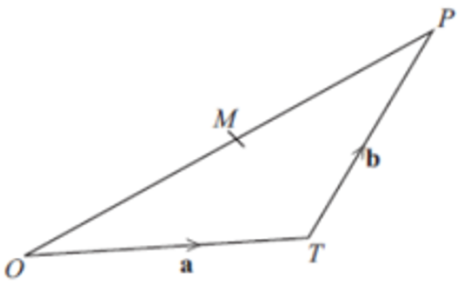
?

b) P is on AB such that $AP : PB = 3 : 2$.

Show that $\overrightarrow{OP} = \frac{1}{5}(2\boldsymbol{a} + 3\boldsymbol{b})$

?

4 [Nov 2010 1H Q27] M is the midpoint of OP.



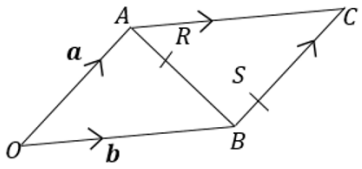
a) Express $\overrightarrow{OM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

?

b) Express $\overrightarrow{TM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ giving your answer in its simplest form.

?

5



$OACB$ is a parallelogram. R is a point such that $AR : RB = 2 : 3$. S is a point such that $BS : SC = 1 : 3$.

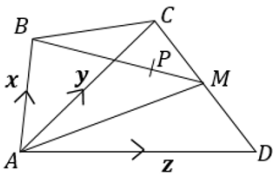
a. $\overrightarrow{OR} =$?

b. $\overrightarrow{BS} =$?

c. $\overrightarrow{OS} =$?

d. $\overrightarrow{RS} =$?

6



M is the midpoint of CD, $BP : PM = 2 : 1$

a. $\overrightarrow{DC} =$?

b. $\overrightarrow{DM} =$?

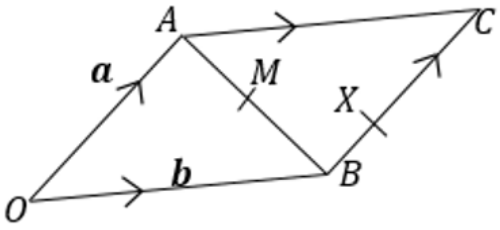
c. $\overrightarrow{AM} =$?

d. $\overrightarrow{BM} =$?

e. $\overrightarrow{BP} =$?

f. $\overrightarrow{AP} =$?

7



a. $\overrightarrow{AB} =$?

b. $\overrightarrow{BC} =$?

c. $\overrightarrow{MB} =$?

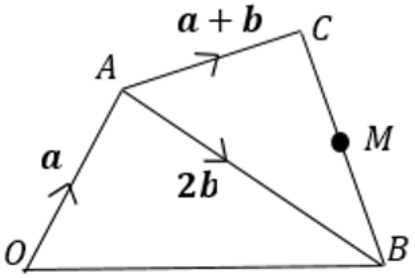
d. $\overrightarrow{MX} =$?

e. $\overrightarrow{XA} =$?

f. $\overrightarrow{CM} =$?

g. $\overrightarrow{XO} =$?

8



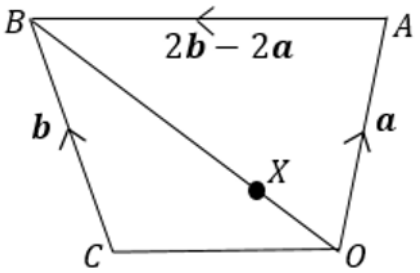
a. $\overrightarrow{OB} =$?

b. $\overrightarrow{BC} =$?

c. $\overrightarrow{AM} =$?

d. $\overrightarrow{OM} =$?

9



a. $\overrightarrow{OC} =$?

b. $\overrightarrow{XC} =$?

c. $\overrightarrow{AX} =$?

Fluency Practice

Q1.

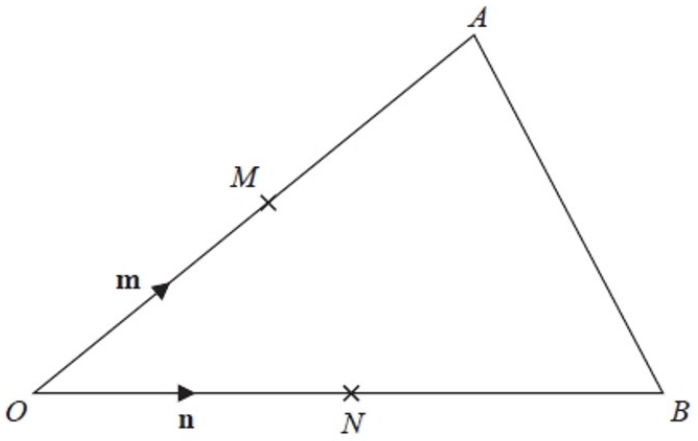


Diagram **NOT** accurately drawn

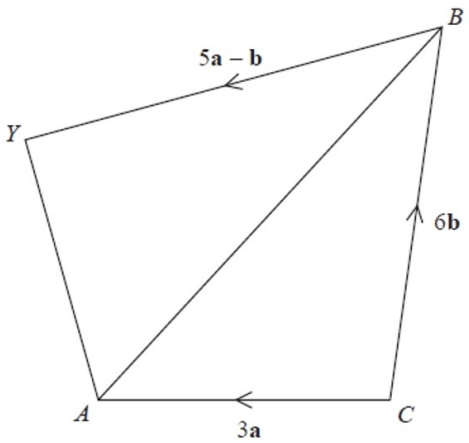
OAB is a triangle.
M is the midpoint of *OA*.
N is the midpoint of *OB*.

$\overrightarrow{OM}$ = **m**

$\overrightarrow{ON}$ = **n**

Show that *AB* is parallel to *MN*.

Q2.



CAYB is a quadrilateral.

$\overrightarrow{CA} = 3\mathbf{a}$
$\overrightarrow{CB} = 6\mathbf{b}$
$\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

X is the point on *AB* such that *AX* : *XB* = 1 : 2

Prove that $\overrightarrow{CX} = \frac{2}{5}\overrightarrow{CY}$

Q3.

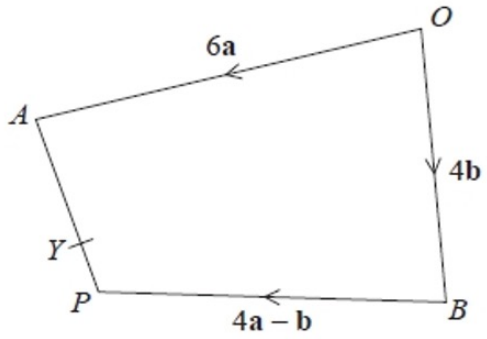


Diagram **NOT** accurately drawn

OBPA is a quadrilateral.

$\overrightarrow{OA} = 6\mathbf{a}$

$\overrightarrow{OB} = 4\mathbf{b}$

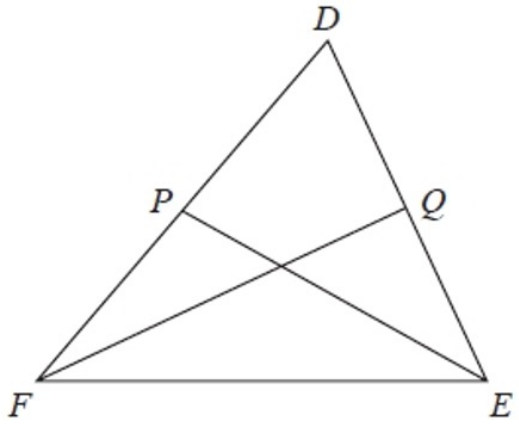
$\overrightarrow{BP} = 4\mathbf{a} - \mathbf{b}$

Y is the point on *AP* such that *AY* : *YP* = 2 : 1

Show that $\overrightarrow{OY}$ is parallel to the vector 7**a** + 3**b**

Q4.

DEF is a triangle.

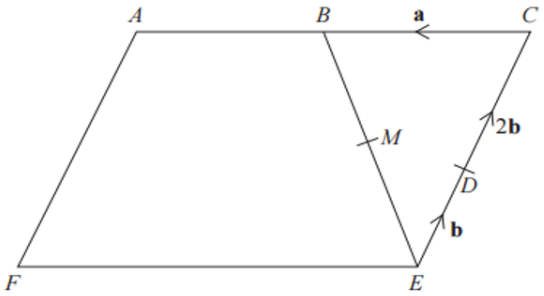


P is the midpoint of *FD*.
Q is the midpoint of *DE*.

$\overrightarrow{FD} = \mathbf{a}$ and $\overrightarrow{FE} = \mathbf{b}$

Use a vector method to prove that *PQ* is parallel to *FE*.

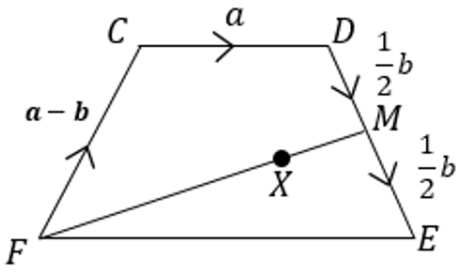
3



$ACEF$ is a parallelogram. B is the midpoint of AC. M is the midpoint of BE. Show that AMD is a straight line.

?

4



$\overrightarrow{CD} = \boldsymbol{a}$, $\overrightarrow{DE} = \boldsymbol{b}$ and $\overrightarrow{FC} = \boldsymbol{a} - \boldsymbol{b}$

i) Express $\overrightarrow{CE}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

?

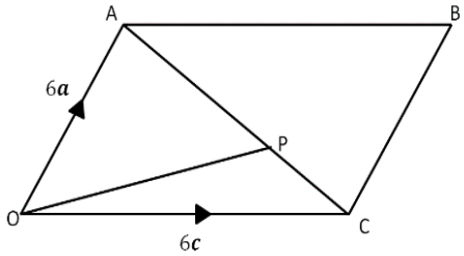
ii) Prove that $\overrightarrow{FE}$ is parallel to $\overrightarrow{CD}$.

?

iii) X is the point on FM such that such that $FX:XM = 4:1$. Prove that C, X and E lie on the same straight line.

?

5



$OABC$ is a parallelogram. P is the point on AC such that $AP = \frac{2}{3}AC$.

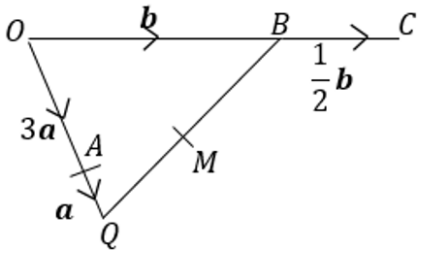
i) Find the vector $\overrightarrow{OP}$. Give your answer in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$.

?

ii) Given that the midpoint of CB is M, prove that OPM is a straight line.

?

6



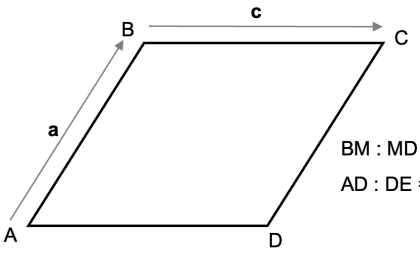
$\overrightarrow{OA} = 3\boldsymbol{a}$ and $\overrightarrow{AQ} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{BC} = \frac{1}{2}\boldsymbol{b}$. $M$is the midpoint of QB. Prove that AMC is a straight line.

?

Fluency Practice

vector proofs

(1) a **parallelogram** (ABCD)

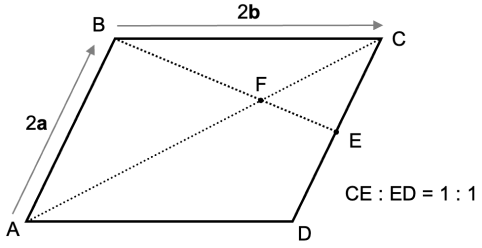


BM : MD = 1 : 1

AD : DE = k : 1

$\overrightarrow{ME} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$ find the value of k

(2) a **parallelogram** (ABCD)



CE : ED = 1 : 1

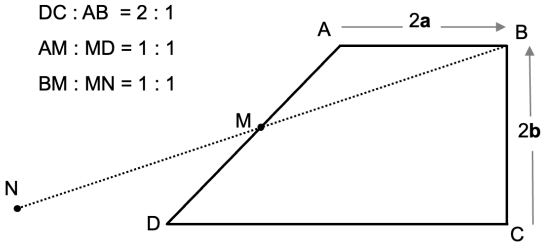
show that AF : FC = 2 : 1

(3) a **trapezium** (ABCD)

DC : AB = 2 : 1

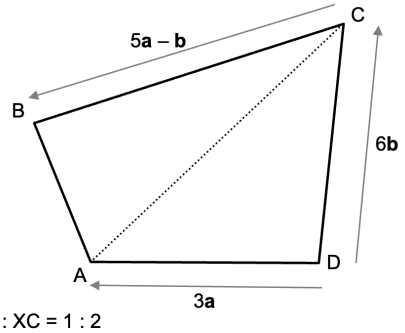
AM : MD = 1 : 1

BM : MN = 1 : 1



show that CDN is a straight line

(4)

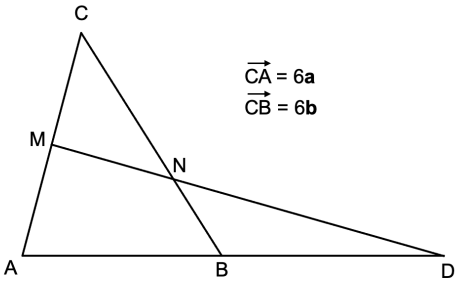


AX : XC = 1 : 2

DX : DB = k : 1 find the value of k

vector proofs

(1)

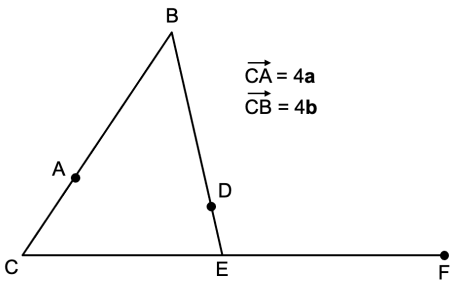


CM : MA = 1 : 1

AB : BD = 1 : 1

$\overrightarrow{CN} = k\mathbf{b}$ find the value of k

(2)



CA : AB = 1 : 2

ED : DB = 1 : 3

CE : EF = 1 : 2

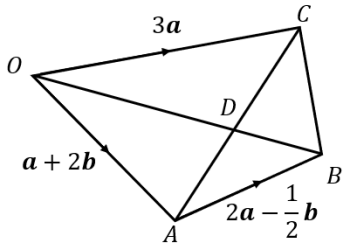
show that ADF is a straight line

(3) choose some other ratios to explore

Vector Proof – Equating Coefficients

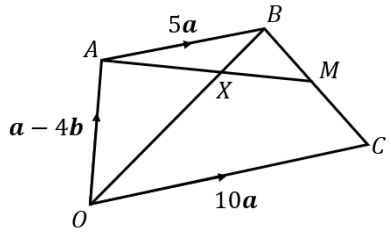
(a)

$OABC$ is a quadrilateral, where $\overrightarrow{OC} = 3\boldsymbol{a}$, $\overrightarrow{OA} = \boldsymbol{a} + 2\boldsymbol{b}$ and $\overrightarrow{AB} = 2\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$. The point D is on OB and AC such that $OD : OB = \lambda : 1$ and $AD : AC = \mu : 1$. By finding two ways to express the vector $\overrightarrow{OD}$, find the values of λ and μ.



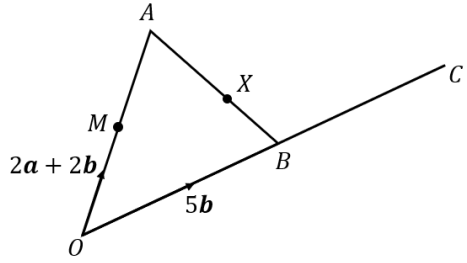
(b)

$OABC$ is a trapezium, where $\overrightarrow{OC} = 10\boldsymbol{a}$, $\overrightarrow{OA} = \boldsymbol{a} - 4\boldsymbol{b}$ and $\overrightarrow{AB} = 5\boldsymbol{a}$. M is the midpoint of the line BC. The point X is on OB and AM such that $OX : OB = \lambda : 1$ and $AX : AM = \mu : 1$. Find the values of λ and μ and the vector $\overrightarrow{OX}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.



(c)

In the triangle OAB, $\overrightarrow{OB} = 5\boldsymbol{b}$ and $\overrightarrow{OM} = 2\boldsymbol{a} + 2\boldsymbol{b}$, where M is the midpoint of OB. OC is the line OB produced and $\overrightarrow{OB} = \overrightarrow{BC}$. The point X is on the line AB such that $AX : AB = \lambda : 1$. Given that MXC is a straight line, find the value of λ and the vector $\overrightarrow{MX}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.



Corbett maths

Equation of a Circle

Video 12 on www.corbettmaths.com

Examples

Workout

Click here

Scan here

Question 1: Write down the (i) centre and the (ii) radius for each of these circles.

(a) $(x - 3)^2 + (y - 2)^2 = 100$

(b) $(x - 1)^2 + (y + 3)^2 = 16$

(c) $(x + 5)^2 + (y - 7)^2 = 4$

(d) $(x + 6)^2 + (y + 8)^2 = 1$

(e) $x^2 + y^2 = 25$

(f) $(x + 4)^2 + y^2 = 9$

(g) $(x + 2)^2 + (y - 7)^2 = 5$

(h) $(x - 6)^2 + (y - 1)^2 = 20$

Question 2: Write the equations of each of these circles.

(a) Centre is (2, 7) and has a radius of 6.

(b) Centre is (4, −2) and has a radius of 3.

(c) Centre is (−1, 3) and has a radius of 4.

(d) Centre is (−1, −2) and has a radius of 9.

(e) Centre is (0, 5) and has a radius of 5.

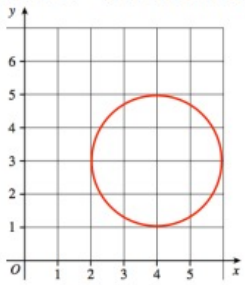
(f) Centre is the origin and has radius 8.

(g) Centre is (11, 0) and has a radius of $\sqrt{3}$.

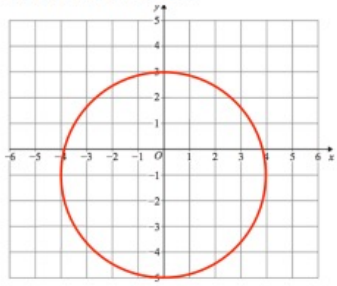
(h) Centre is (−3, −7) and has a radius of $2\sqrt{2}$.

Question 3: Write the equations of each of these circles.

(a)



(b)



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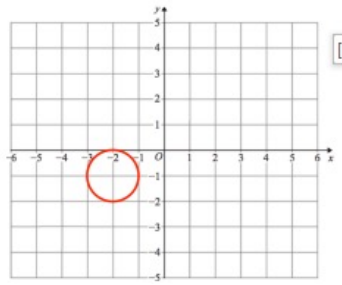

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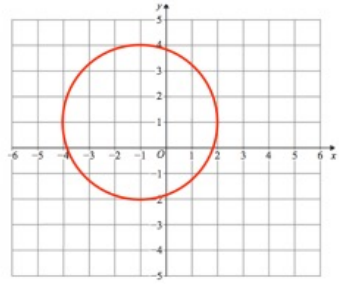


Question 4: Find if the coordinates given lie on the circles below:

(a) Does (4, 2) lie on the circle with equation $(x - 1)^2 + (y + 2)^2 = 25$

(b) Does (3, 9) lie on the circle with equation $(x - 4)^2 + (y - 3)^2 = 36$

(c) Does (− 16, 4) lie on the circle with equation $(x + 4)^2 + (y - 9)^2 = 169$

(d) Does (2, −2) lie on the circle with radius 8 and centre (5, 5)?

(e) Does (0, 7) lie on the circle with radius $\sqrt{10}$ and centre (−3, 8)?

Question 5: Find where the circle $(x - 3)^2 + (y - 5)^2 = 106$ meets the x-axis.

Question 6: Find where the circle $(x - 1)^2 + (y + 2)^2 = 20$ meets the x-axis.

Question 7: Find where the circle $(x + 2)^2 + (y + 3)^2 = 45$ meets the x-axis.

Question 8: Find where the circle $(x - 2)^2 + (y - 1)^2 = 68$ meets the x-axis.

Question 9: Find where the circle $(x - 4)^2 + (y - 6)^2 = 32$ meets the x-axis.

Apply

Question 1: A circle has centre (7, 2).
The point (1, −6) lies on the circle.
Find the equation of the circle.

Question 2: A circle has centre A.
The points C (0, 4) and D (10, 4) lie on the diameter of the circle.

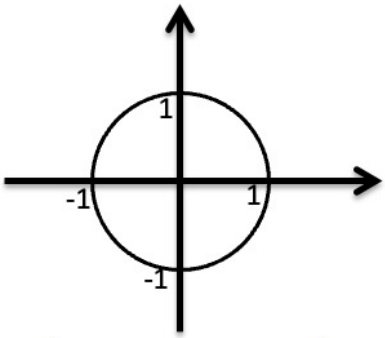
(a) Find the coordinates of A.
(b) Find the equation of the circle.

Answers

Click here

Scan here

Fluency Practice



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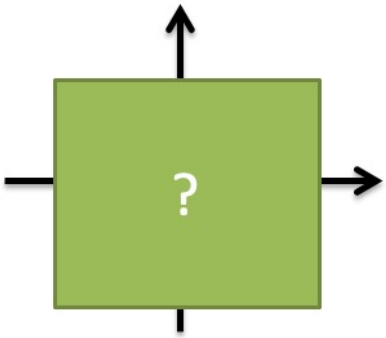
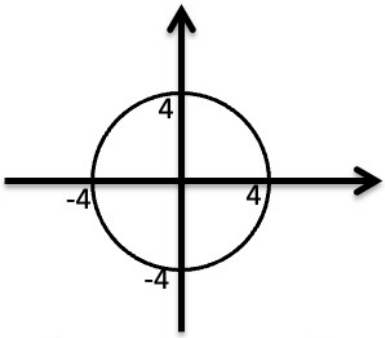


$x^2 + y^2 = 9$



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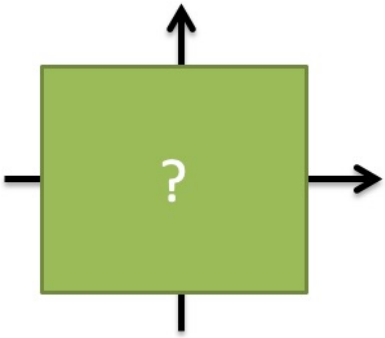
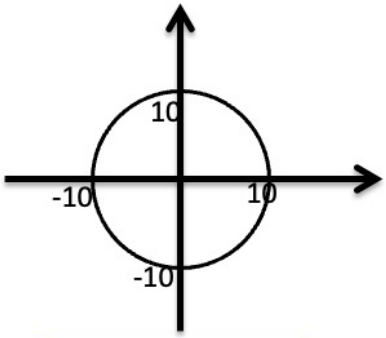


$x^2 + y^2 = 64$



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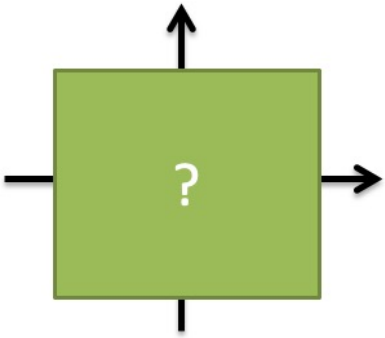


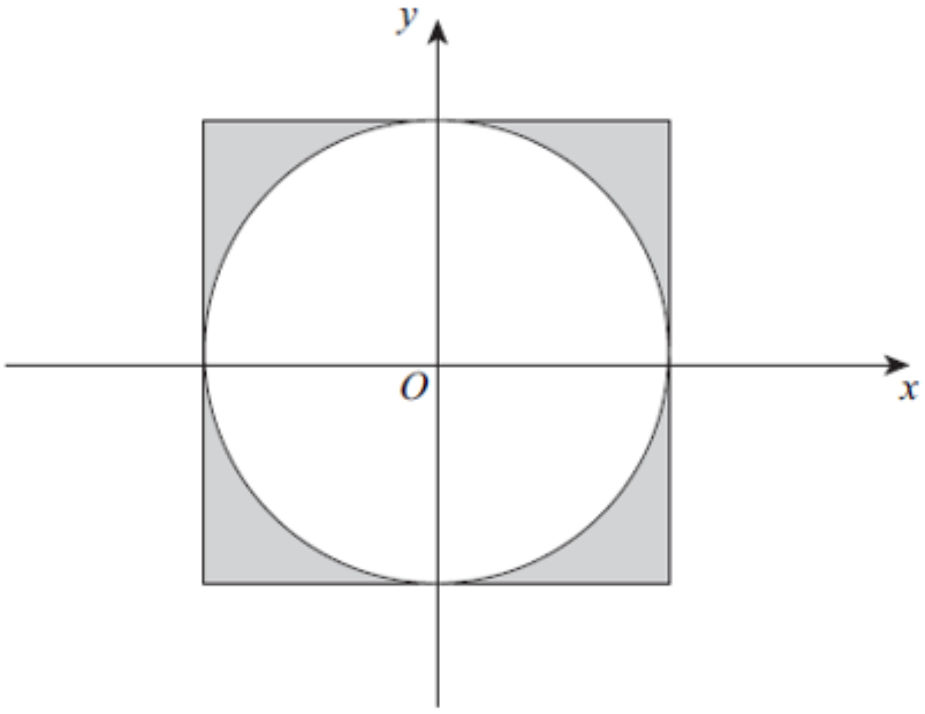
$x^2 + y^2 = 36$

Fluency Practice

1) Write down the centre and radius of the circle with equation $x^2 + y^2 = 36$

2) A circle has the equation $x^2 + y^2 = 36$. Work out its circumference.

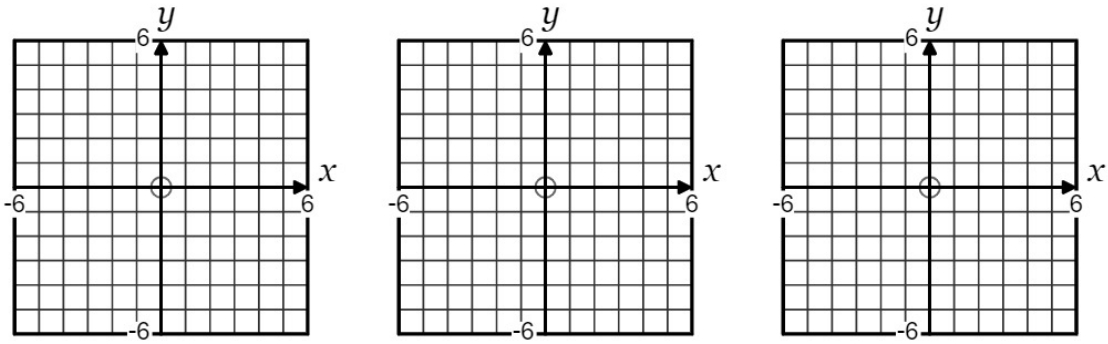
3) The circle $x^2 + y^2 = 25$ touches each side of the square as shown. Work out the total shaded area.



Fluency Practice

1. Draw the graph of each function:

   a) $x^2 + y^2 = 4$ b) $x^2 + y^2 = 25$ c) $x^2 + y^2 = 4^2$



2. A circle has equation $x^2 + y^2 = 64$. Work out the diameter of this circle.

3. Write down the equation of a circle centred on the origin with:

   a) a radius of 7 b) a diameter of 20 c) an area of 36π

4. Which of the following points lie on the circle with equation $x^2 + y^2 = 100$? Select all that apply.

   a) (10, 1) b) (-8, 6) c) (7, -7) d) $(5, 5\sqrt{3})$

5. Which of the following points lie on the circle with equation $x^2 + y^2 = 80$? Select all that apply.

   a) $(0, 4\sqrt{5})$ b) $(3\sqrt{10}, -\sqrt{10})$ c) $(-7, -\sqrt{31})$

6. Ravi draws the graphs with equations $x^2 + y^2 = 169$ and $y = -5$.
   Work out the coordinates of the points where Ravi's two graphs intersect.

7. Circle **C** has equation $x^2 + y^2 = 81$.
   Write down the equations of the two horizontal lines that are tangent to **C**.

8. Which of the following are equations of circles? Select all that apply.

   a) $x^2 + y = 121$ b) $(x + y)^2 = 9$ c) $x^2 + y^2 - 24 = 0$

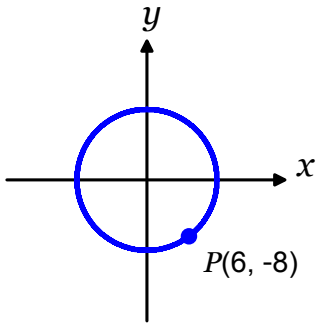
   d) $x = 16 - y^2$ e) $20 = y^2 + x^2$ f) $x^2 + y^2 = \sqrt{5}$

Fluency Practice

Circle graphs: find the radius

Work out the radius of each circle graph centred on the origin. Lines that look like tangents *are* tangents.

A



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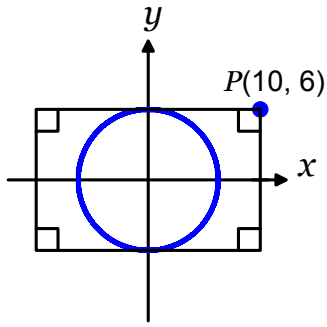

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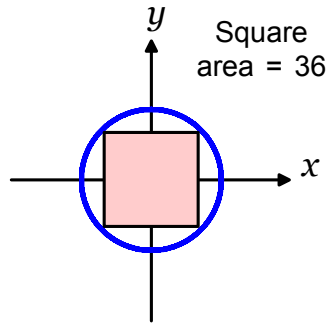

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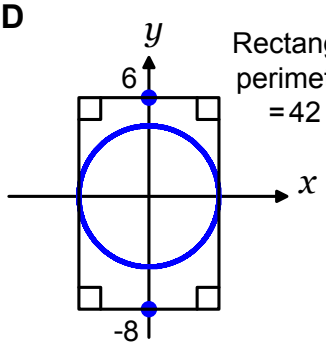

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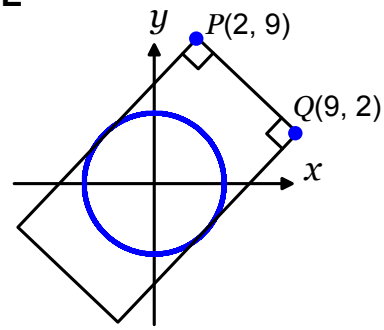

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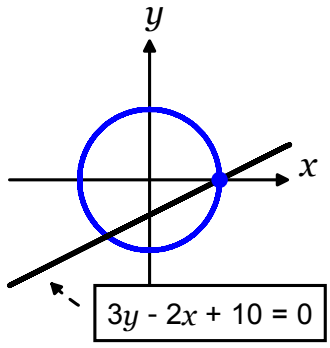

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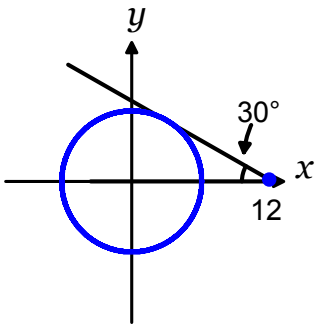
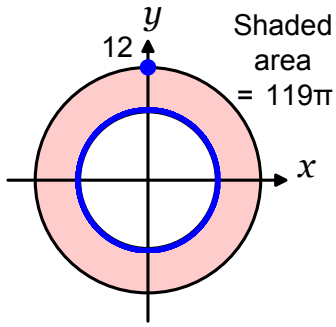
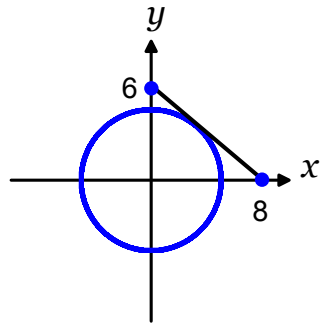


H *Radius of inner circle?*



I *Challenge*

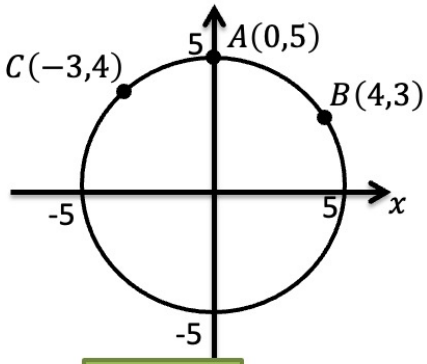


Extension

Find the radius of the circle: $x^2 + y^2 = 36$	Find the radius of the circle: $x^2 + y^2 = 45$
Find the diameter of the circle: $x^2 + y^2 = 100$	Find the area of the circle: $x^2 + y^2 = 144$
Find the circumference of this circle: $x^2 + y^2 = 9$	Find the area of the circle: $x^2 + y^2 = 48$
Write down the equation of the circle with centre $(0, 0)$ and diameter 800.	Write down the equation of the circle with centre $(0, 0)$ and area 201.
Write the equation of a circle with centre $(0, 0)$ and radius $\sqrt{11}$.	Write the equation of a circle with centre $(0, 0)$ and radius $2\sqrt{3}$.
The point $(3, a)$ is on the circle $x^2 + y^2 = 25$ Find the possible values of a.	The point $(b, 12)$ is on the circle $x^2 + y^2 = 169$ Find the possible values of b.

Fluency Practice

1 The diagram shows the circle with equation $x^2 + y^2 = 25$. What is the equation of the tangent at the following points?

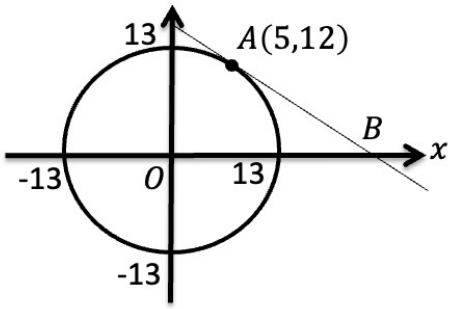


a $A(0,5)$: ?

b $B(4,3)$: ?

c $C(-3,4)$: ?

2



The tangent to the above circle at the point $A(5,12)$ intersects the x axis at the point B.

a Find the equation of the tangent to the circle at the point A.

?

b) Find the area of triangle OAB.

b ?

The line l is tangent at the point $P(x, y)$ to the circle with equation $x^2 + y^2 = 1$. The gradient of l is $-\frac{1}{2}$. Determine the point $P(x, y)$.

?

Fluency Practice

WARM-UP

1. Work out the gradient of the line from the origin to each of these points:

   a) (5, 8) b) (-3, 7) c) $(-2, \sqrt{3})$ d) $(-2\sqrt{2}, 4)$

2. In each case, the point P lies on the circle C. Work out a, b and c.

   a) C: $x^2 + y^2 = 100$
   P: $(-6, a)$
   $a > 0$

   b) C: $x^2 + y^2 = 4^2$
   P: $(2, b)$
   $b > 0$

   c) C: $x^2 + y^2 - 25 = 0$
   P: $(-2, c)$
   $c < 0$

3. Work out the gradient of a line **perpendicular** to a line with gradient:

   a) -4 b) $\frac{2}{5}$ c) $-\frac{3}{4}$ d) $\frac{1}{5}$

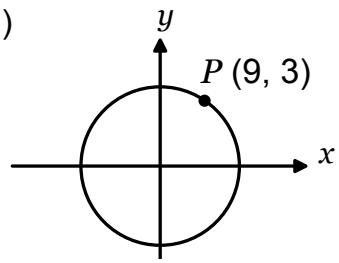
4. In each case, the point P lies on a line with gradient m. Work out the equation of the line in the form $y = mx + c$.

   a) P(2, 5), $m = 3$ b) P(-3, 4), $m = -1$ c) P(-3, -2), $m = \frac{1}{4}$

QUESTIONS

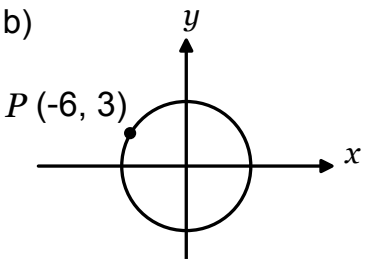
1. Work out the equation of the tangents to these circles at the given point P. Give your answers in the form $y = mx + c$.

   a)



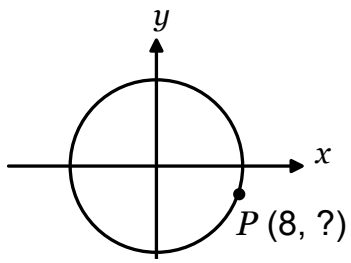
   $x^2 + y^2 = 90$

   b)



   $x^2 + y^2 = 45$

   c)



   $x^2 + y^2 = 80$

2. Circle C has equation $x^2 + y^2 = 400$.
   Point P(a, b) lies on C such that $a = -12$ and $b < 0$.

   a) Work out the value of b.

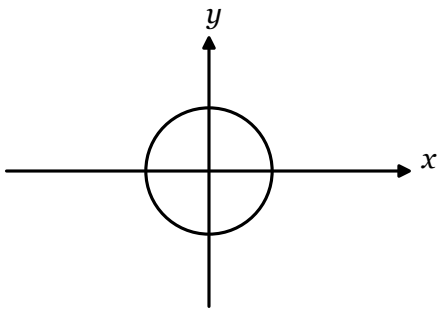
   b) Work out the equation of the tangent to C at P in the form $y = mx + c$.

Fluency Practice

3. Circle C has equation $x^2 + y^2 = 72$.
   Two points on C have a tangent with a gradient of 0.

   Work out the coordinates of these two points.

4. Circle C is centred on the origin and has a radius of 15 units.
   Point P(a, b) lines on C, such that $a = 12$ and $b > 0$.
   Line T is the tangent to C at P.



   a) Work out the equation of T.

   b) Work out the coordinates of the point at which T meets the y-axis.

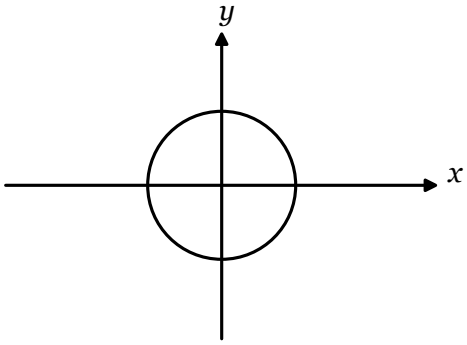
   c) Work out the coordinates of the point at which T meets the x-axis.

   d) Work out the area of the triangle formed by the x-axis, the y-axis and T.

5. Circle C has equation $x^2 + y^2 = 32$.
   Two different lines are both tangents to C and have a gradient of 1.
   Work out the coordinates of the points where the tangents meet C.

6. A circle is centred on the origin.
   The points P(-15, -20) and Q(-24, c) lie on the circle, where $c > 0$.
   Work out the equation of the tangent to the circle at Q.

7. Circle C has equation $x^2 + y^2 = 50$.
   O is the origin (0, 0).
   P(5, s) is a point on the circle.
   Q(1, t) is a point on the circle.
   $s > 0$ and $t < 0$.



   a) Work out the values of s and t.

   b) Work out the equations of the tangents to C at P and Q.

   c) Work out the coordinates of the point R, where the two tangents meet.

   d) Given that $P\hat{O}Q = 127°$, work out $P\hat{R}Q$.

Intelligent Practice

Solve the following pairs of simultaneous equations:

1. $xy = 12$
$y = x + 1$

2. $xy = 12$
$y = x - 1$

3. $xy = 12$
$y = x - 4$

4. $xy = 12$
$y = x + 4$

5. $xy = 12$
$y = 2x + 5$

6. $xy = 12$
$y = 2x - 5$

7. $xy = 12$
$2y = x + 5$

8. $xy = 12$
$2y = x - 5$

9. $4xy = 12$
$y = x - 2$

10. $4xy = 12$
$y = x + 2$

Intelligent Practice

Solve the following pairs of simultaneous equations:

1. $x^2 + y^2 = 25$
$y = x + 5$

2. $x^2 + y^2 = 25$
$y = x - 5$

3. $x^2 + y^2 = 25$
$y = 2x + 5$

4. $x^2 + y^2 = 25$
$y = 2x - 5$

5. $x^2 + y^2 = 17$
$y = x + 5$

6. $x^2 + y^2 = 17$
$y = x - 5$

7. $2x^2 + y^2 = 17$
$y = x + 5$

8. $2x^2 + y^2 = 17$
$y = x - 5$

Intelligent Practice

Solve the following pairs of simultaneous equations:

1. $y = x^2 + 5x - 2$
$y = x + 3$

2. $y = x^2 - 3x - 2$
$y = x + 3$

3. $y = x^2 - 5x - 2$
$y = -x + 3$

4. $y = x^2 + 3x - 2$
$y = -x + 3$

5. $y = x^2 - 3x - 2$
$y = 2x - 8$

6. $y = x^2 - 3$
$y = x - 3$

7. $y = x^2 - 3$
$y = x + 3$

Fluency Practice

Question 1: Solve the following simultaneous equations

(a) $y = x + 3$
$y = x^2 + 5x - 2$

(b) $y = x^2 + x - 14$
$y = x - 5$

(c) $y = 2x - 1$
$y = x^2 - 2x + 2$

(d) $y = 2x^2 + 9x + 1$
$y = 3x + 9$

(e) $y = 2x^2 + x + 1$
$y = x^2 - 5x - 7$

(f) $y = -x^2 + 5x + 2$
$y = 3x^2 - x - 2$

Question 2: Solve the following simultaneous equations

(a) $x + y = 4$
$y = x^2 + 3x - 1$

(b) $x + y = 7$
$xy = 10$

(c) $x^2 + y^2 = 13$
$x + y = 5$

(d) $2x - y + 4 = 0$
$y = x^2 + x - 2$

(e) $x^2 + y^2 = 29$
$7 + x + y = 0$

(f) $xy = -6$
$x + 2y = -4$

(g) $y = x^2 - 3x + 3$
$y = 10x - 39$

(h) $x^2 + y^2 = 1$
$x + 2y = 1$

(i) $5x + y = 5$
$2x^2 - 9x - y = 11$

(j) $y = 9x^2 + 11x + 3$
$5x - y + 2 = 0$

(k) $2x + y = 5$
$2x^2 + y^2 = 11$

(l) $x - 4y + 1 = 0$
$x^2 - 4xy + y^2 = 13$

Question 3: Solve the following simultaneous equations

(a) $2x + y = 7$
$x^2 - y^2 = 8$

(b) $x^2 + y^2 = 20$
$y = x + 3$

(c) $y = x^2 - 9x - 3$
$y = x$

(d) $2x^2 + y^2 = 10$
$2x - y = 5$

(e) $y = x^2 + x - 7$
$4x + 2y + 1 = 0$

(f) $y = x - 2$
$2x^2 - xy = 11$

Extension

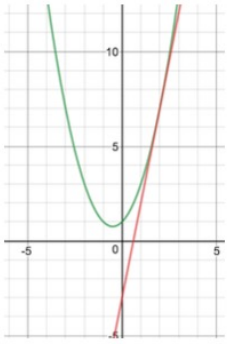
Question 1: Find the coordinates where the line $x + y = 3$ and the curve $x^2 + 3y = 27$ intersect

Question 2: How many points of intersection does the curve $y = (x - 3)(x + 4)$ have with the line $y = x - 8$?

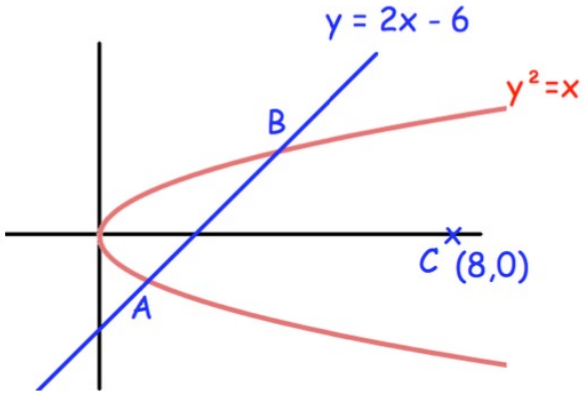
Question 3: The line $y = x + 4$ and the curve $y = x^2 + 3x + 4$ intersect at the points A and B. Find the distance between the points A and B.

Question 4: Find the coordinates of the points where the line $x + 5y = 37$ and the curve $y = x^2 + x + 2$ meet.

Question 5: Show, by using simultaneous equations, that the line $y = 5x - 3$ is a tangent to the curve $y = x^2 + x + 1$

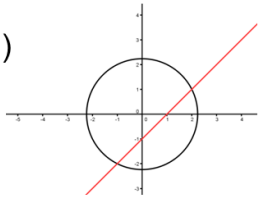


Question 6: Shown below is the curve $y^2 = x$ and the line $y = 2x - 6$
The curve and the line meet at the points A and B
The point C is (8, 0)
Show ABC is a right angled triangle.



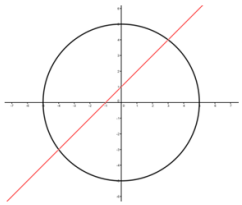
Extension

solving simultaneous equations: one linear, one quadratic (i)
find the coordinates of the points of intersection

(1) 

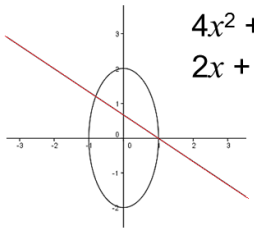
$x^2 + y^2 = 5$
$x - y = 1$

(2) $x^2 + y^2 = 58$
$y - x = 4$

(3) 

$x^2 + y^2 = 25$
$-x + y = 1$

(4) $x^2 + y^2 = 2$
$x + 3y = 2$

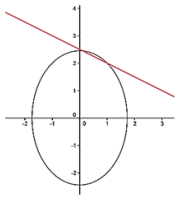
(5) 

$4x^2 + y^2 = 4$
$2x + 3y = 2$

(6) $2x^2 + xy + 1 = 0$
$y - x = 4$

(7) $4x^2 + y^2 = 37$
$2y + x = 1$

(8) $2x^2 + y^2 = 6$
$x + 2y = 5$



factorise any quadratic equations …

solving simultaneous equations: one linear, one quadratic (ii)
find the coordinates of the points of intersection

(1) $y = 3x^2 - 14x - 5$
$y = 4x - 32$

(2) $x^2 + 8y = 13$
$2y + x = 2$

(3) $4x^2 + y^2 = 10$
$2x - y = 4$

(4) $2x^2 + y^2 = 57$
$2y + x = 6$

(5) $2y^2 + x^2 = 48$
$y + x = 8$

(6) $x^2 + y^2 + 2x - 4y = 8$
$x + y = 6$

(7) $x^2 + y^2 + 6x - 4y = 4$
$y = 3x + 4$

(8) $2x^2 - y^2 + xy = 14$
$4x + 5y = 0$

Fluency Practice

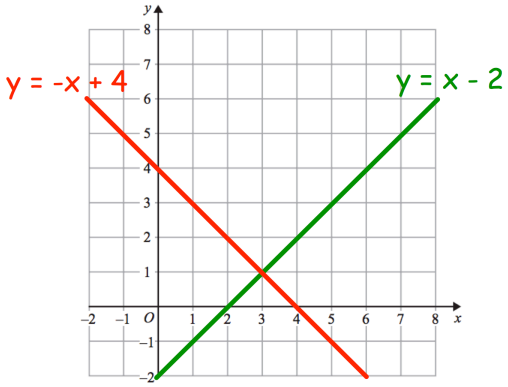
Question 1: Shown below are the graphs of $y = -x + 4$ and $y = x - 2$

(a) Write down the coordinates of the point where the graphs of $y = -x + 4$ and $y = x - 2$ intersect.

(b) Use your answer to (a) to solve the simultaneous equations.

$y = -x + 4$

$y = x - 2$



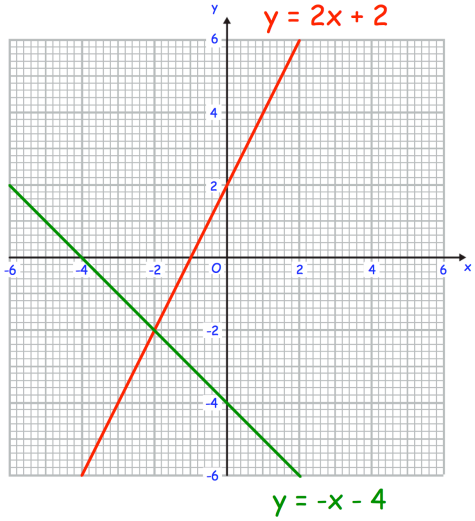
Question 2: Shown below are the graphs of $y = 2x + 2$ and $y = -x - 4$

(a) Write down the coordinates of the point where the graphs of $y = 2x + 2$ and $y = -x - 4$ intersect.

(b) Use your answer to (a) to solve the simultaneous equations.

$y = -x - 4$

$y = x - 2$



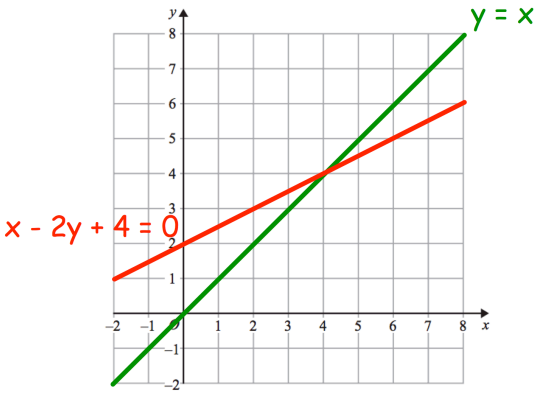
Question 3: Shown below are the graphs of $y = x$ and $x - 2y + 4 = 0$

(a) Write down the coordinates of the point where the graphs of $y = x$ and $x - 2y + 4 = 0$ intersect.

(b) Use your answer to (a) to solve the simultaneous equations.

$y = x$

$x - 2y + 4 = 0$

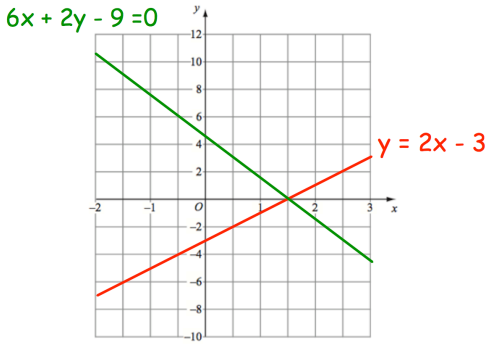


Fluency Practice

Question 4: Shown below are the graphs of $6x + 2y - 9 = 0$ and $y = 2x - 3$

Use the graphs to solve the simultaneous equations

$6x + 2y - 9 = 0$
$y = 2x - 3$.

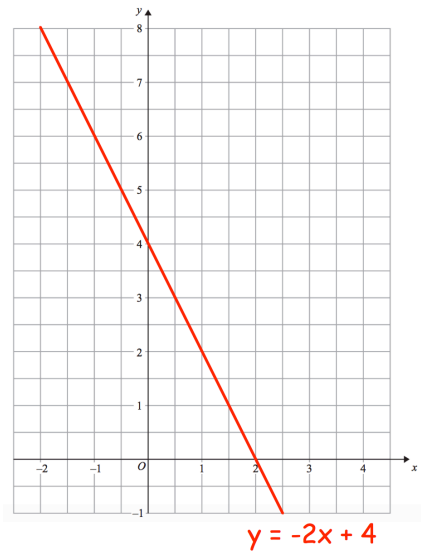


Question 5: The straight line $y + 2x = 4$ has been drawn on the grid.

(a) On the same grid, draw the graph of $y = x + 1$

(b) Use the graphs to solve the simultaneous equations

$y + 2x = 4$
$y = x + 1$.

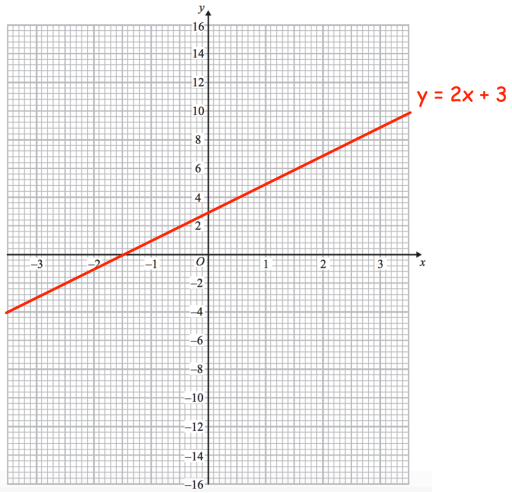


Question 6: The straight line $y = 2x + 3$ has been drawn on the grid.

(a) On the same grid, draw the graph of $y = -3x + 8$

(b) Use the graphs to solve the simultaneous equations

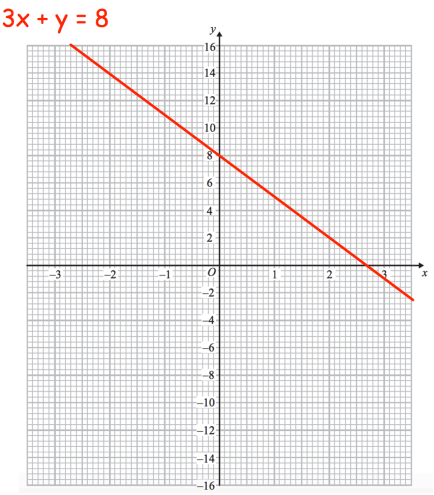
$y = 2x + 3$
$y = -3x + 8$



Fluency Practice

Question 7: The straight line $3x + y = 8$ has been drawn on the grid.

(a) On the same grid, draw the graph of $x + y = 9$



(b) Use the graphs to solve the simultaneous equations

$3x + y = 8$
$x + y = 9$

Question 8: By drawing the graphs of $y = 3x + 1$ and $x + y = 7$

Solve the simultaneous equations
$y = 3x + 1$
$x + y = 7$

Question 9: By drawing the graphs of $y = 3x + 5$ and $x - 2y + 6 = 0$

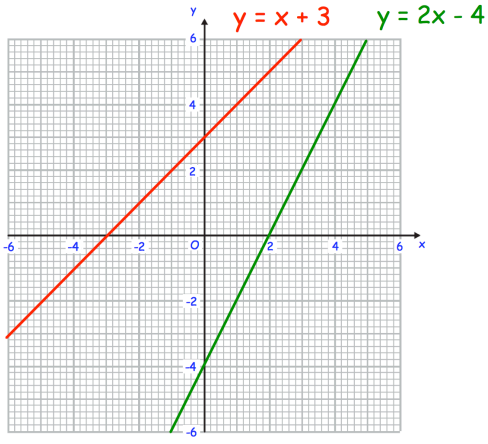
Solve the simultaneous equations
$y = 3x + 5$
$x - 2y + 6 = 0$

Extension

Question 1: Jesse has been asked to graphically solve the simultaneous equations

$y = x + 3$
$y = 2x - 4$

He has drawn the graph shown.



Jesse says that there is no answer to the simultaneous equations.

Explain why Jesse is incorrect.

Question 2: Harry and Trevor are trying to solve the simultaneous equations

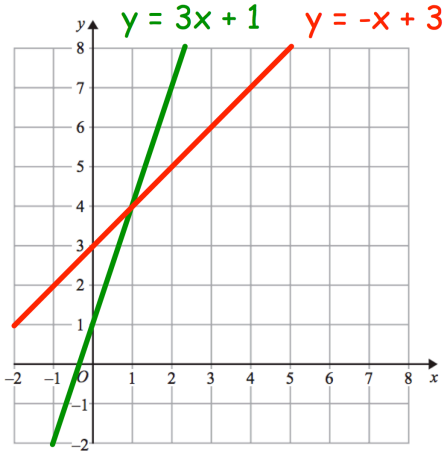
$y = 3x + 1$
$y = -x + 3$

Harry's answer is $x = 1$ and $y = 4$
Trevor's answer is $x = 0.5$ and $y = 2.5$

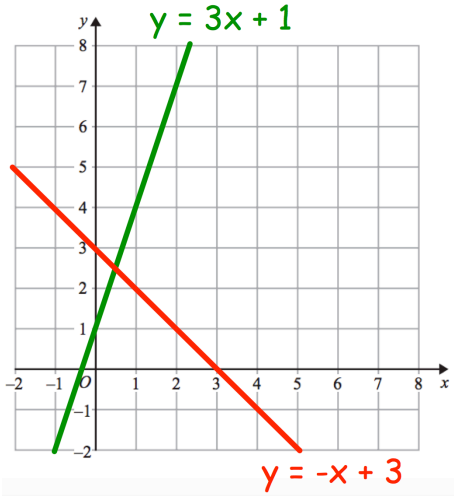
(a) By looking at the graphs below, decide who is correct

(b) What mistake was made by the other boy?

Harry



Trevor



Fluency Practice

1. n is a positive integer. Write each expression in the correct box.

$6n$ | $n + 2$ | $2n - 3$ | $5n$ | $4n + 8$ | $2n + 1$ | $(2n)^2$ | $4(n + 1)$ | $2n^2 + 4n - 3$ | $n^2 + 1$

Always an even number	**Sometimes** an even number	**Never** an even number

	Example	***Your turn***
2.	n is a positive integer. Prove algebraically that $6(n + 3) - n + 2$ is **always** a **multiple of 5**.	n is a positive integer. Prove algebraically that $n(2n + 1) + n^2 + 2n$ is **always** a **multiple of 3**.
3.	n is a positive integer. Prove algebraically that $(n + 2)^2 - n^2$ is **always** a **multiple of 4**.	n is a positive integer. Prove algebraically that $(4n + 1)^2 - 1$ is **always** a **multiple of 8**.
4.	n is a positive integer. Prove algebraically that $(4n + 3)^2 + 2(n + 1)$ is **always** an **odd number**.	n is a positive integer. Prove algebraically that $(n + 1)(n + 2) - n^2$ is **never** a **multiple of 3**.

Fluency Practice

1. n is a positive integer. Find a counter-example to show that these statements are **false**:

   a) $3n + 1$ is always even.

   b) $6n - 1$ is always prime.

	Example	*Your turn*
2.	Prove algebraically that for any three consecutive integers, the difference between the square of the middle number and the product of the two outer numbers is always 1.	Prove algebraically that the sum of any pair of consecutive odd numbers is always a multiple of 4.
3.	Prove algebraically that the sum of any two odd numbers is an even number.	Prove algebraically that the difference between any two odd numbers is an even number.

4. n is a positive integer. Prove algebraically that $(n + 3)^2 - (n + 1)^2$ is a multiple of 4.

5. Prove that the difference between the squares of two consecutive odd numbers is always a multiple of 8.

Prove it!

Prove that $(a + b)^2$ is not equal to $a^2 + b^2$

Show that $5a + 2a + 4 + 3$ is always a multiple of 7

$n - 1 + n + 1$ will be even. Why?

The sum of three consecutive numbers is 3 times the middle number

(hint – call your first number a, your second number a + 1)

The sum of two consecutive odd numbers is always even

For any three consecutive numbers, the difference between the square of the middle number and the product of the largest and smallest is always 1

Show that $5(2x + 5) + 11(x - 2) = 3(7x + 1)$

Fluency Practice

Question 1: Prove the following

(a) $(n+7)^2-(n+1)^2=12(n+4)$ (b) $(n+1)^2-(n-3)^2=8(n-1)$

(c) $(n+1)^2+(n+5)^2-(n+9)^2=(n+5)(n-11)$

Question 2: Prove the following

(a) $(n+4)^2-(n+2)^2$ is always a multiple of 4 for all positive integer values of n.

(b) $(n+10)^2-(n+2)^2$ is always a multiple of 16 for all positive integer values of n.

(c) $(2n+3)^3-(2n+1)$ is always even for all positive integer values of n.

(d) $(5n+2)^2-(5n-1)^2$ is always a multiple of 3 for all positive integer values of n.

(e) $(2n+9)^2-(2n+5)^2$ is always a multiple of 4 for all positive integer values of n.

(f) $(n+2)^2-(n-2)^2+3$ is always odd for all positive integer values of n.

Question 3: n is a positive integer.

(a) Write an expression for an even number.
(b) Write an expression of an odd number.

Question 4: Prove the following

(a) The sum of any three consecutive integers is divisible by 3.
(b) The sum of any three consecutive even numbers is always a multiple of 6.
(c) The sum of two consecutive odd numbers is even.
(d) The sum of three consecutive odd numbers is always a multiple of 3.
(e) The sum of four consecutive odd numbers is always a multiple of 8.
(f) The sum of two consecutive integers is always odd.
(g) The sum of four consecutive integers is **not** a multiple of 4

Question 5: Prove the following

(a) Prove the product of two odd numbers is always odd.

(b) Prove the product of two even consecutive numbers is always a multiple of 4.

(c) The difference between the squares of any two consecutive integers is equal to the sum of the two integers.

(d) Prove the sum of the squares of any two consecutive even numbers is always a multiple of 4.

(e) Prove that when any odd integer is squared, the result is always one more than a multiple of 8.

Extension

Question 1: The first five terms of a linear sequence are 2, 7, 12, 17, 22

(a) Find the nth term of the sequence

A new sequence is generated by squaring each term of the linear sequence and then adding 1.

(b) Prove that all terms in the new sequence are divisible by 5.

Question 2: The first two terms of a fibonacci sequence are ***a*** and ***b***.

(a) Show the 4th term of the sequence is ***a+2b***
(b) Prove that the sum of the first 10 terms is equal to 11 times the 7th term.

Question 3: Cara writes down a 3-digit number where the first digit is greater than the last. e.g. 681
She then reverses the number to give 186.
Cara then subtracts this number from her starting number. 681 − 186 = 495
She then reverses her answer to give 594.
Cara then adds these number 495 + 594 = 1089.

$$\begin{array}{r} 681 \\ -186 \\ \hline 495 \end{array} \qquad \begin{array}{r} 594 \\ +495 \\ \hline 1089 \end{array}$$

Cara repeats this several times and always gets 1089 as her answer.
Prove algebraically that the answer is always 1089.

proof KS455 Qs

Question 1

Skill involved: 455a: Equate coefficients to determine constants in an identity.

Given that

$$aaa + 3bb + 7aa \equiv 3a + 30$$

Find the values of a and b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Question 2

Skill involved: 455a: Equate coefficients to determine constants in an identity.

Given that

$$6aaa + 6bb + 10a + 7baa \equiv 45a + 58$$

Find the values of a and b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Question 3

Skill involved: 455a: Equate coefficients to determine constants in an identity.

Given that

$$5baa - 4a + 3baa \equiv 32a - 8$$

Find the values of a and b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Question 4

Skill involved: 455a: Equate coefficients to determine constants in an identity.

Given that

$$2baa + 4a + 10baa + 9a \equiv 24a + 26$$

Find the values of a and b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Question 5

Skill involved: 455a: Equate coefficients to determine constants in an identity.

Given that

$6px + 5q + x \equiv 7x + 15$

Find the values of p and q.

$p = \ldots\ldots\ldots, q = \ldots\ldots\ldots$

Question 6

Skill involved: 455a: Equate coefficients to determine constants in an identity.

Given that

$px + 1q - 1p + 8 \equiv 4x + 9$

Find the values of p and q.

$p = \ldots\ldots\ldots, q = \ldots\ldots\ldots$

Question 7

Skill involved: 455a: Equate coefficients to determine constants in an identity.

Given that

$4axx + 5b - 8axx + 7b \equiv -12x + 48$

Find the values of a and b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Question 8

Skill involved: 455a: Equate coefficients to determine constants in an identity.

Given that

$2bax + 2x - 5a \equiv -x + 8$

Find the values of a and b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Question 9

Skill involved: 455b: Equate coefficients to determine constants in an identity requiring two single bracket expansions.

Given that

$$4(3ax + b) + 3(ax + 2b) \equiv -45x + 60$$

Find the values of a and b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Question 10

Skill involved: 455b: Equate coefficients to determine constants in an identity requiring two single bracket expansions.

Given that

$$5(qx + 2p) + 3(2qx + 3p) \equiv 55x + 114$$

Find the values of p and q.

$p = \ldots\ldots\ldots, q = \ldots\ldots\ldots$

Question 11

Skill involved: 455b: Equate coefficients to determine constants in an identity requiring two single bracket expansions.

Given that

$$2(qx + 3p) + 4(3qx - p) \equiv 56x - 2$$

Find the values of p and q.

$p = \ldots\ldots\ldots, q = \ldots\ldots\ldots$

Question 12

Skill involved: 455b: Equate coefficients to determine constants in an identity requiring two single bracket expansions.

Given that

$$3(3px + q) + 5(4px + q) \equiv 145x + 40$$

Find the values of p and q.

$p = \ldots\ldots\ldots, q = \ldots\ldots\ldots$

Question 13

Skill involved: 455b: Equate coefficients to determine constants in an identity requiring two single bracket expansions.

Given that

$$2(3px - 2p) - 3(4px + 3p) \equiv -6x - 52$$

Find the values of p and q.

$p = \ldots\ldots\ldots, q = \ldots\ldots\ldots$

Question 14

Skill involved: 455b: Equate coefficients to determine constants in an identity requiring two single bracket expansions.

Given that

$$3(3ax + 2b) - 5(2ax - b) \equiv 2x - 44$$

Find the values of a and b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Question 15

Skill involved: 455b: Equate coefficients to determine constants in an identity requiring two single bracket expansions.

Given that

$$3(px + 4q) + 4(2px + q) \equiv 66x - 16$$

Find the values of pp and qq.

$pp = \ldots\ldots\ldots\ldots, qq = \ldots\ldots\ldots\ldots$

Question 16

Skill involved: 455b: Equate coefficients to determine constants in an identity requiring two single bracket expansions.

Given that

$$5(4bbaa + 3aa) - 4(2bbaa + 3aa) \equiv 36aa - 18$$

Find the values of aa and bb.

$aa = \ldots\ldots\ldots\ldots, bb = \ldots\ldots\ldots\ldots$

Question 17

Skill involved: 455d: Equate coefficients to determine constants in an identity, involving a combination of single and double bracket expansions.

Given that

$$(2aa + ee)^2 - 3aa = 4aa^2 + 9aa + ff$$

where ee and ff are positive integers, find the value of ee and the value of ff.

$ee = \ldots\ldots\ldots\ldots, ff = \ldots\ldots\ldots\ldots$

Question 18

Skill involved: 455d: Equate coefficients to determine constants in an identity, involving a combination of single and double bracket expansions.

Given that

$$(2aa + cc)^2 + 6aa = 4aa^2 + 34aa + dd$$

where cc and dd are positive integers, find the value of cc and the value of dd.

$cc = \ldots\ldots\ldots\ldots, dd = \ldots\ldots\ldots\ldots$

Question 19

Skill involved: 455d: Equate coefficients to determine constants in an identity, involving a combination of single and double bracket expansions.

Given that

$(yy + aa)^2 - 6 = yy^2 + 14yy + bb$

where aa and bb are positive integers, find the value of aa and the value of bb.

$aa = \ldots\ldots\ldots\ldots, bb = \ldots\ldots\ldots\ldots$

Question 20

Skill involved: 455d: Equate coefficients to determine constants in an identity, involving a combination of single and double bracket expansions.

Given that

$(4aa - 2)(5aa + 4) - aaa(10aa + 12) = dda - 8$

where cc and dd are integers, find the value of cc and the value of dd.

$cc = \ldots\ldots\ldots\ldots, dd = \ldots\ldots\ldots\ldots$

Question 21

Skill involved: 455d: Equate coefficients to determine constants in an identity, involving a combination of single and double bracket expansions.

Given that

$(aa + 4)(aa + cc) = (aa + 2)^2 + dda - 12$

where cc and dd are integers, find the value of cc and the value of dd.

$cc = \ldots\ldots\ldots\ldots, dd = \ldots\ldots\ldots\ldots$

Question 22

Skill involved: 455d: Equate coefficients to determine constants in an identity, involving a combination of single and double bracket expansions.

Given that

$(2aa - 5)^2 + maa - 7 = (aa - 6)(4aa + ss)$

where r and s are integers, find the value of r and the value of s.

$r = \ldots\ldots\ldots, s = \ldots\ldots\ldots$

Question 23

Skill involved: 455d: Equate coefficients to determine constants in an identity, involving a combination of single and double bracket expansions.

Given that

$$(a + 8)(a + a) = (a + 5)^2 + a + b$$

where a and b are integers, find the value of a and the value of b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Question 24

Skill involved: 455d: Equate coefficients to determine constants in an identity, involving a combination of single and double bracket expansions.

Given that

$$(2a - 2)^2 + aa + 6 = (a + b)(4a - 2)$$

where a and b are integers, find the value of a and the value of b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Generating Quadratic Sequences

By finding the first and second differences, decide whether each of these sequences is quadratic.

(a) 1, 5, 11, 19, 29, 41

(b) 2, 5, 8, 11, 14, 17

(c) 0, 8, 22, 41, 68, 98

(d) 2, 9, 20, 35, 54, 77

(e) 4, 1, 0, 1, 4, 9

(f) 6, 17, 36, 65, 98, 141

(g) 18, 37, 62, 93, 130, 173

(h) 3, 9, 23, 43, 75, 113

(i) −10, −4, 12, 38, 74, 120

(j) 17, 39, 69, 107, 153, 207

Generate the first five terms of each of these quadratic sequences.

(a) $n^2 + 10$ (b) $n^2 - 1$

(c) $n^2 + n$ (d) $n^2 + 2n + 1$

(e) $n^2 - 3n$ (f) $n^2 - n - 2$

(g) $2n^2 + 5$ (h) $3n^2 - 7$

(i) $2n^2 + n - 5$ (j) $4n^2 + 3n - 1$

Find the 10^{th} and 50^{th} term of the following quadratic sequences.

(a) $n^2 + 5$ (b) $n^2 - 2$

(c) $n^2 - n$ (d) $n^2 + 2n$

(e) $n^2 - 3n + 1$ (f) $n^2 - n - 2$

(g) $4n^2 + 1$ (h) $3n^2$

(i) $2n^2 + n - 1$ (j) $5n^2 + 3n$

Fluency Practice

1. Below is the first number in a geometric sequence. Write the first 4 terms of 2 possible sequences that could start with that number:

a. 2

b. 4

c. 8

d. 10

e. 7

f. 0.4

g. -4

h. ¼

i. -0.3

j. $\sqrt{2}$

k. ab

l. $\frac{a^2b}{2}$

m. $\sqrt{a}$

n. ab^2c^3

o. $\frac{\pi}{2}$

Easy

Medium

Hard

2. Calculate the first 5 terms of these sequences:

a. 2^n

b. 2^{n+1}

c. $(-2)^n$

d. $-(2)^n$

e. $0.2^{(2n)}$

f. 3×3^n

g. $0.4 \times 3^{(n-1)}$

h. $\frac{3}{5} \times 1.1^n$

i. $9.1 \times 10^{(\frac{1}{n})}$

j. $0.03 \times 0.9^{(3n)}$

k. $a \times 3^{bn}$

l. $\frac{h}{3^n}$

m. $n \times a^n p^2$

n. $m \times 0.03^{(b+n)}$

o. $\frac{1}{f} \times ad^{(n-3)}$

Easy

Medium

Hard

3. Work out T(n) for the following geometric series:

a. 4, 8, 16, 32

b. 4, 2, 1, 0.5

c. 5, 15, 45, 135

d. 3, 18, 108, 648

e. -5, -10, -20, -40

f. -2, 4, -8, 16

g. 4, -4, 4, -4

h. $\frac{1}{2}, \frac{1}{8}, \frac{1}{32}, \frac{1}{128}$

i. $\sqrt{2}, 2, 2\sqrt{2}, 4$

j. $5, 5\sqrt{5}, 25, 25\sqrt{5}$

k. ab, ab^2, ab^3, ab^4

l. $\frac{a}{b^2}, \frac{a^2}{b^3}, \frac{a^3}{b^4}, \frac{a^4}{b^5}$

m. $-a^2, a^2, -a^2, a^2$

n. $-ab, ab^2, -ab^3, ab^4$

o. $a^2b^2, a^2b, a^2, \frac{a^2}{b}$

Easy

Medium

Hard

gemoetric ex

Question 1

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$8, 32, 128, 512, \ldots$

nth term = ...

Question 2

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$1, 5, 25, 125, \ldots$

nth term = ...

Question 3

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$8, 16, 32, 64, \ldots$

nth term = ...

Question 4

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$2, 6, 18, 54, \ldots$

nth term = ...

Question 5

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$3, 12, 48, 192, \ldots$

nth term = ...

Question 6

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$2, 6, 18, 54, \ldots$

nth term = ...

Question 7

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$7, 56, 448, 3584, \ldots$

nth term = ...

Question 8

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$5, 20, 80, 320, \ldots$

nth term = ...

Question 9

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$6, 30, 150, 750, \ldots$

nth term = ...

Question 10

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$6, 42, 294, 2058, \ldots$

nth term = ...

Question 11

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$2, 12, 72, 432, \ldots$

nth term = ...

Question 12

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$3, 21, 147, 1029, \ldots$

nth term = ...

Question 13

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$5, 10, 20, 40, \ldots$

nth term = ...

Question 14

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$6, 18, 54, 162, \ldots$

nth term = ...

Question 15

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$3, 24, 192, 1536, \ldots$

nth term = ...

Question 16

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$6, 12, 24, 48, \ldots$

nth term = ...

Question 17

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$2, 8, 32, 128, \ldots$

nth term = ...

Question 18

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$3, 18, 108, 648, \ldots$

nth term = ...

Question 19

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$3, 6, 12, 24, \ldots$

nth term = ...

Question 20

Skill involved: 371a: Determine the nth term for a geometric sequence where first term and common ratio are distinct positive integers, both greater than one.

Find the nth term of the sequence.

$1, 3, 9, 27, \ldots$

nth term = ...

Quadratic Sequences

Quadratic sequences take the form

$$an^2 + bn + c$$

Section 1

For each of the following quadratic sequences, identify the values of a, b and c:

1. $2n^2 + 3n + 4$
2. $4n^2 + 8n + 5$
3. $5n^2 - 11n$
4. $n^2 + n - 2$
5. $\frac{n^2}{2} - 4$
6. $-n^2$

Section 2

Find the first 3 terms for these quadratic sequences:

7. $n^2 + 3$
8. $2n^2 + 2$
9. $3n^2$
10. $4n^2 - 30$
11. $n^2 + n - 9$
12. $2n^2 - 2n - 4$
13. $\frac{n^2}{2} + 3n + 9$
14. $n(n + 3)$
15. $(n + 3)(2n + 6)$

Find the 10th and 100th term of these sequences:

16. $n^2 + 3$
17. $2n^2 - 4n$
18. $-8n^2$
19. $\frac{n^2}{3} + 20$

Section 3

Which of these terms are **not** in the sequences provided in **bold:**:

20. $\{12, 39, 55, 103\}$ $(\boldsymbol{n^2 + 3})$
21. $\{15, 35, 63, 82\}$ $(\boldsymbol{n^2 - 1})$
22. $\{108, 116, 128, 162\}$ $(\boldsymbol{2n^2})$
23. $\{1, 38, 56, 119\}$ $(\boldsymbol{n^2 + 10n})$

Match the sequences to the descriptions below (assume n is always a whole number):

24. $n^2 + 1$
25. $n^2 + n$
26. $n^2 \div 3$
27. $2n^2$
28. $n^2 + 3n + \frac{3}{5}$

A: Always even
B: Always odd
C: Sometimes even, sometimes odd
D: Sometimes an integer
E: Never an integer

Section 4

Find the nth term of these quadratic sequences:

29. 1, 4, 9, 16
30. -3, 0, 5, 12
31. 3, 12, 27, 48
32. 2, 5, 10, 17
33. 8, 11, 16, 23
34. 0, 6, 16, 30
35. 8, 14, 24, 38
36. 11, 20, 35, 56
37. 0.5, 2, 4.5, 8
38. $9.\dot{3}$, $10.\dot{3}$, 12, $14.\dot{3}$
39. 3, 7, 13, 21
40. 5, 13, 25, 41
41. 7, 16, 27, 40
42. 0, 1, 4, 9
43. - 5, 8, 27, 52

Easiest | Medium | Hardest

Fluency Practice

Question 1: Find the next two terms for each quadratic sequence

(a) 4, 6, 10, 16, 24 … …

(b) 1, 2, 4, 7, 11 … …

(c) 2, 5, 10, 17, 26 … …

(d) 3, 9, 19, 33, 51 … …

(e) 50, 48, 44, 38, 30 … …

(f) 3, 14, 29, 48, 71 … …

Question 2: List the first 5 terms of the sequences with n^{th} term:

(a) n^2

(b) $n^2 + 1$

(c) $n^2 + 4$

(d) $n^2 - 2$

(e) $2n^2$

(f) $5n^2$

(g) $\frac{1}{2}n^2$

(h) $\frac{1}{4}n^2$

(i) $3n^2 + 10$

(j) $\frac{3}{5}n^2$

Question 3: The quadratic n^{th} term of the sequence below is n^2

1, 4, 9, 16, 25, 36, 49 …

Find the n^{th} term of each of these sequences

(a) 4, 7, 12, 19, 28, 39, 52 …

(b) 51, 54, 59, 66, 75, 86, 99 …

(c) −5, −2, 3, 10, 19, 30…

(d) 3, 12, 27, 48, 75, 108 …

(e) 20, 80, 180, 320, 500, 720 …

(f) 0.2, 0.8, 1.8, 3.2, 5 …

(g) 3, 9, 19, 33, 51, 73, 99 …

(h) 2.5, 4, 6.5, 10, 14.5, 20 …

Question 4: For each n^{th} term, work out the first five terms of the sequence.

(a) $n^2 + n$

(b) $n^2 + 2n$

(c) $n^2 - n$

(d) $n^2 - 3n$

(e) $n^2 + n + 2$

(f) $n^2 - 2n + 5$

(g) $n^2 + 4n - 10$

(h) $2n^2 + n$

(i) $3n^2 - n + 6$

(j) $10n^2 + 5n - 7$

Question 5: For each n^{th} term, work out the first five terms of the sequence.

(a) $-n^2$

(b) $-2n^2$

(c) $-4n^2 + 2$

(d) $-n^2 + 3n$

(e) $50 - n^2$

(f) $6n - n^2$

(g) $-n^2 - 7n - 2$

Fluency Practice

Question 6: For each n^{th} term, work out the first five terms of the sequence.

(a) $n(n + 1)$ (b) $n(n + 3)$ (c) $(n + 1)(n + 5)$ (d) $n(n - 2)$

(e) $(n - 3)(n + 1)$ (f) $(n - 8)(n - 3)$

Question 7: Work out the n^{th} term for each quadratic sequence

(a) 7, 12, 19, 28, 39 … (b) 7, 16, 31, 52, 79 … (c) 6, 13, 24, 39, 58 …

(d) 3, 13, 27, 45, 67 … (e) 9, 20, 35, 54, 77 … (f) 9, 24, 45, 72, 105 …

(g) −6, −1, 6, 15, 26 … (h) −5, −4, −1, 4, 11 … (i) 7, 10, 17, 28, 43 …

(j) 2.5, 5, 8.5, 13, 18.5 … (k) −0.5, 1, 4.5, 10, 17.5 …

Question 8: Calculate the 10^{th} term of each sequence in question 7

Question 9: Work out the n^{th} term for each quadratic sequence

(a) 3, 1, −3, −9, −17… (b) −4, −12, −24, −40, −60 … (c) 6, 5, 2, −3, −10 …

(d) 100, 96, 90, 82, 72 … (e) −17, −30, −49, −74, −105 … (f) 6, 5.5, 4.5, 3, 1 …

Question 10: Calculate the 10^{th} term of each sequence in question 9

Question 11: A sequence has an n^{th} term of $n^2 + n - 20$
Work out which term in the sequence has a value of 52.

Question 12: A sequence has an n^{th} term of $n^2 + 2n - 5$
Work out which term in the sequence has a value of 58.

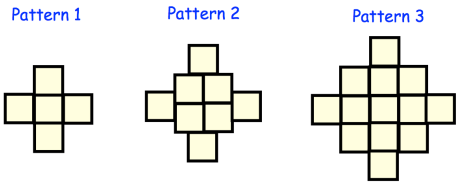
Question 13: A sequence has an n^{th} term of $n^2 - 6n + 7$
Work out which term in the sequence has a value of 23.

Extension

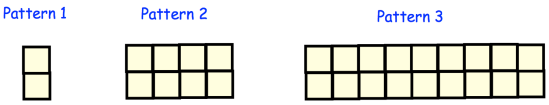
Question 1: The first 5 terms of a quadratic sequence are: 4, 10, 18, 28, 40
Work out the difference between the 10th and 20th terms.

Question 2: Below are patterns of tiles.
The number of tiles in each form quadratic sequences.
Find the number of tiles in pattern n for each.

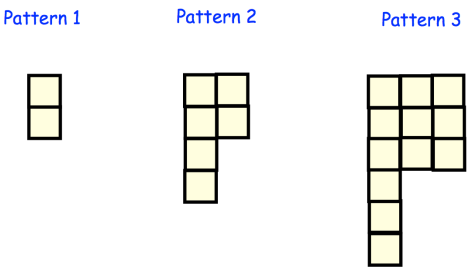
(a)



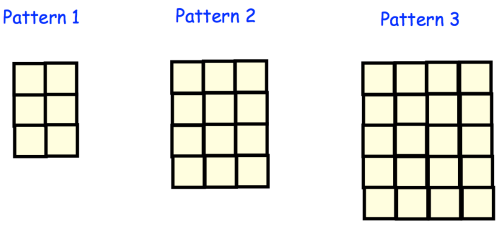
(b)



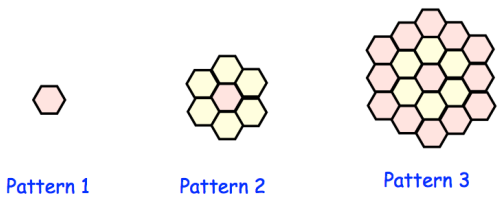
(c)



(d)



Question 3: Here is a pattern made from tiles.
How many tiles are needed to make Pattern 20?



Question 4: The first 4 terms of a sequence are: 400, 390, 375, 355 ...
Which term is the first to be negative?

Question 5: The n^{th} term of a quadratic sequence is $n^2 + 4n$
Two consecutive terms have a difference of 25.
Work out the two terms.

Question 6: Prove every term in the sequence $n^2 - 8n + 21$ is positive

Nth Term of Quadratic Sequences

Find the nth term of these sequences:

(a) $8, 11, 16, 23, \ldots$

(b) $-4, -1, 4, 11, \ldots$

(c) $6, 12, 22, 36, \ldots$

(d) $-1, 5, 15, 29, \ldots$

(e) $4, 13, 28, 49, \ldots$

(f) $2, 14, 34, 62, \ldots$

Find the nth term of these sequences:

(a) $3, 7, 13, 21, \ldots$

(b) $2, 7, 14, 23, \ldots$

(c) $9, 18, 31, 48, \ldots$

(d) $1, 5, 13, 25, \ldots$

(e) $7, 23, 49, 85, \ldots$

(f) $7, 15, 29, 49, \ldots$

Find the nth term of these sequences:

(a) $10.5, 12, 14.5, 18, \ldots$

(b) $5.5, 9, 13.5, 19, \ldots$

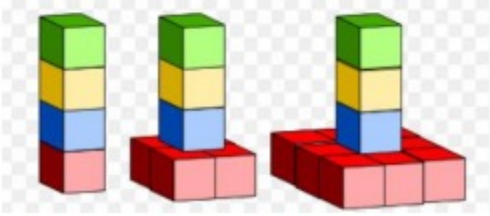
(c) $1.5, 5, 11.5, 21, \ldots$

(d) $3.5, 10, 19.5, 32, \ldots$

(e) $19, 16, 11, 4, -5, \ldots$

(f) $5, 6, 5, 2, -3, \ldots$

Here are some patterns made from cubes. Find an expression for the nth term of this sequence.



quad seqns ex for unit 24

Question 1

Skill involved: 370e: Determine the nth term formula of a quadratic sequence in the form $\mathbf{n^2 + bn + c}$

Here are the first five terms of a quadratic sequence

$$1322334661$$

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 2

Skill involved: 370e: Determine the nth term formula of a quadratic sequence in the form $\mathbf{n^2 + bn + c}$

Here are the first five terms of a quadratic sequence

$$-2 \quad -5 \quad -6 \quad -5 \quad -2$$

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 3

Skill involved: 370e: Determine the nth term formula of a quadratic sequence in the form $\mathbf{n^2 + bn + c}$

Here are the first five terms of a quadratic sequence

$$-6 \quad -12 \quad -16 \quad -18 \quad -18$$

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 4

Skill involved: 370e: Determine the nth term formula of a quadratic sequence in the form $\mathbf{n^2 + bn + c}$

Here are the first five terms of a quadratic sequence

1421304154

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 5

Skill involved: 370f: Determine the nth term formula of a quadratic sequence in the form $an^2 + c$

Here are the first five terms of a quadratic sequence.

−54194067

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 6

Skill involved: 370f: Determine the nth term formula of a quadratic sequence in the form $an^2 + c$

Here are the first five terms of a quadratic sequence.

713233755

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 7

Skill involved: 370f: Determine the nth term formula of a quadratic sequence in the form $an^2 + c$

Here are the first five terms of a sequence.

413284976

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 8

Skill involved: 370f: Determine the nth term formula of a quadratic sequence in the form $an^2 + c$

Here are the first five terms of a quadratic sequence.

39193351

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 9

Skill involved: 370g: Determine the nth term formula of a quadratic sequence in the form $\mathbf{an^2 + bn}$

Here are the first five terms of a quadratic sequence

210244470

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 10

Skill involved: 370g: Determine the nth term formula of a quadratic sequence in the form $\mathbf{an^2 + bn}$

Here are the first five terms of a quadratic sequence

618366090

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 11

Skill involved: 370g: Determine the nth term formula of a quadratic sequence in the form $\mathbf{an^2 + bn}$

Here are the first five terms of a quadratic sequence

618366090

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 12

Skill involved: 370g: Determine the nth term formula of a quadratic sequence in the form $\mathbf{an^2 + bn}$

Here are the first five terms of a quadratic sequence

18214065

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 13

Skill involved: 370h: Determine the nth term formula of a quadratic sequence in the form $\mathbf{an^2 + bn + c}$

Here are the first five terms of a quadratic sequence

19213757

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 14

Skill involved: 370h: Determine the nth term formula of a quadratic sequence in the form $\mathbf{an^2 + bn + c}$

Here are the first five terms of a quadratic sequence

719355579

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 15

Skill involved: 370h: Determine the nth term formula of a quadratic sequence in the form $\mathbf{an^2 + bn + c}$

Here are the first five terms of a quadratic sequence

618345478

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 16

Skill involved: 370h: Determine the nth term formula of a quadratic sequence in the form $\mathbf{an^2 + bn + c}$

Here are the first five terms of a quadratic sequence

413264364

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 17

Skill involved: 370i: Determine the nth term formula of a quadratic sequence in the form $\mathbf{an^2 + bn + c}$ where a is negative

Here are the first five terms of a quadratic sequence

$-10 - 14 - 20 - 28 - 38$

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 18

Skill involved: 370i: Determine the nth term formula of a quadratic sequence in the form $\mathbf{an^2 + bn + c}$ where a is negative

Here are the first five terms of a quadratic sequence

$-4 - 8 - 18 - 34 - 56$

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 19

Skill involved: 370i: Determine the nth term formula of a quadratic sequence in the form $\mathbf{an^2 + bn + c}$ where a is negative

Here are the first five terms of a quadratic sequence

$-12 - 24 - 40 - 60 - 84$

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 20

Skill involved: 370i: Determine the nth term formula of a quadratic sequence in the form $\mathbf{an^2 + bn + c}$ where a is negative

Here are the first five terms of a quadratic sequence

$-2 - 8 - 18 - 32 - 50$

Find an expression, in terms of n, for the nth term of the sequence.

.....

Question 21

Skill involved: 370j: Determine the coefficients of a quadratic nth term rule given information about the terms in the sequence.

The nth term of a sequence is given by

$$an^2 + b$$

The fifth term is 74 and the seventh term is 146.

Find the values of a and b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Question 22

Skill involved: 370j: Determine the coefficients of a quadratic nth term rule given information about the terms in the sequence.

The nth term of a sequence is given by

$an^2 + bn$

The first term is -1 and the second term is 4.

Find the values of a and b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Question 23

Skill involved: 370j: Determine the coefficients of a quadratic nth term rule given information about the terms in the sequence.

The nth term of a sequence is given by

$an^2 + bn$

The fifth term is 95 and the tenth term is 340.

Find the values of a and b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Question 24

Skill involved: 370j: Determine the coefficients of a quadratic nth term rule given information about the terms in the sequence.

The nth term of a sequence is given by

$an^2 + bn$

The second term is 6 and the third term is 15.

Find the values of a and b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Question 25

Skill involved: 370k: Determine the position of a term in a quadratic sequence.

A sequence has an nth term of

$-n^2 + 4n - 5$

A term in this sequence is equal to -37.

Find the position of this term.

$n =$ .....

Question 26

Skill involved: 370k: Determine the position of a term in a quadratic sequence.

A sequence has an nth term of

$$n^2 + n - 1$$

A term in this sequence is equal to 239.

Find the position of this term.

$n =$ .....

Question 27

Skill involved: 370k: Determine the position of a term in a quadratic sequence.

A sequence has an nth term of

$$3n^2 + n - 3$$

A term in this sequence is equal to 599.

Find the position of this term.

$n =$ .....

Question 28

Skill involved: 370k: Determine the position of a term in a quadratic sequence.

A sequence has an nth term of

$$3n^2 + n + 2$$

A term in this sequence is equal to 522.

Find the position of this term.

$n =$ .....

Question 29

Skill involved: 370l: Use an existing formula for a quadratic sequence to determine the formula for a similar sequence.

Here are the first five terms of a sequence.

$$5 \quad 4 \quad 5 \quad 8 \quad 13$$

An expression for the nth term of this sequence is $n^2 - 4n + 8$.

Find an expression for the nth term of a sequence whose first five terms are

$$-3 \quad -4 \quad -3 \quad 5$$

.....

Question 30

Skill involved: 370l: Use an existing formula for a quadratic sequence to determine the formula for a similar sequence.

Here are the first five terms of a sequence.

$$-2 \quad 2 \quad 8 \quad 16 \quad 26$$

An expression for the nth term of this sequence is $n^2 + n - 4$.

Find an expression for the nth term of a sequence whose first five terms are

$$2 \quad 8 \quad 16 \quad 26 \quad 38$$

.....