KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2023 Mathematics 2024

Unit 25 Booklet and Tasks

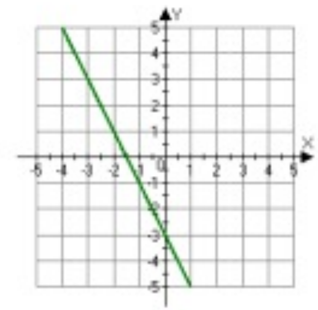
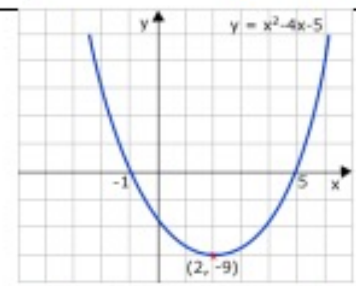
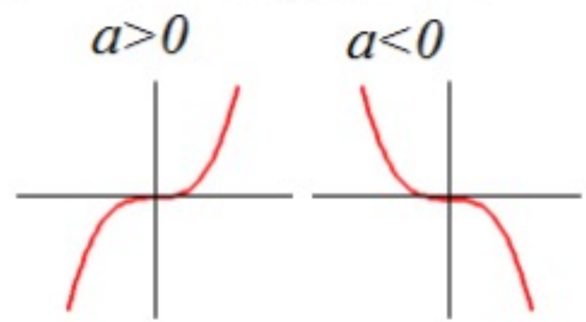
HGS Maths

Dr Frost Course

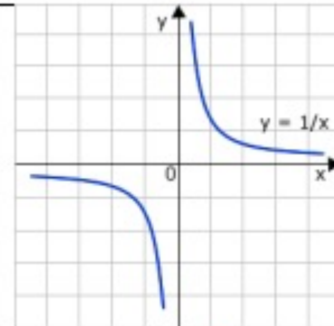
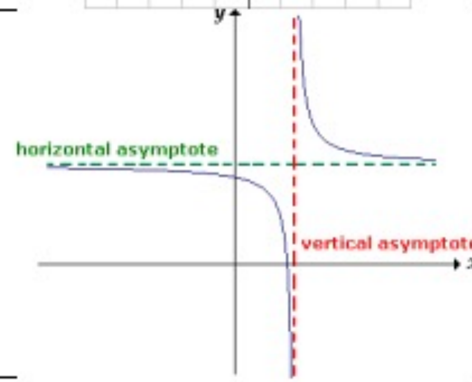
Name: ______________________________

Class: ______________________________

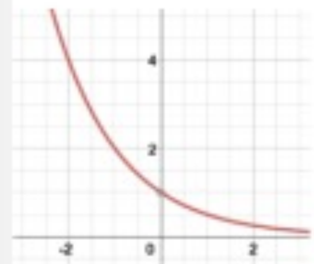
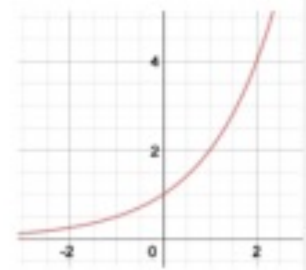
Graphs you should already recognise the shape of:

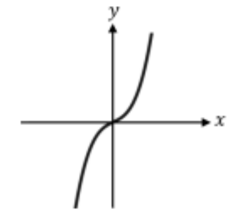
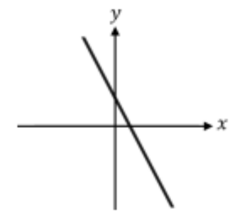
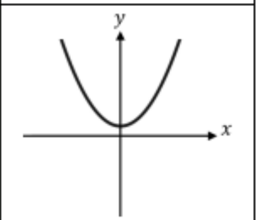
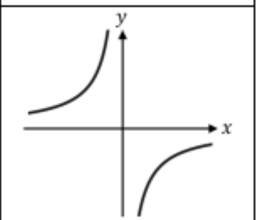
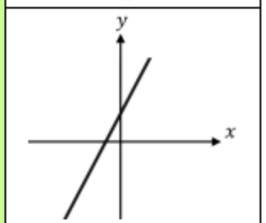
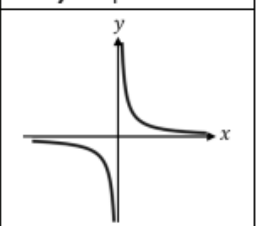
2. Linear Graph	**Straight line** graph. The **equation** of a linear graph can contain an **x-term**, a **y-term** and a **number**.	Example:  Other examples: $x = y$ $y = 4$ $x = -2$ $y = 2x - 7$ $y + x = 10$ $2y - 4x = 12$
3. Quadratic Graph	A **'U-shaped'** curve called a **parabola**. The equation is of the form $\boldsymbol{y = ax^2 + bx + c}$, where a, b and c are numbers, $\boldsymbol{a \neq 0}$. If $\boldsymbol{a < 0}$, the parabola is **upside down**.	
4. Cubic Graph	The equation is of the form $\boldsymbol{y = ax^3 + k}$, where $\boldsymbol{k}$ **is an number**. If $\boldsymbol{a > 0}$, the curve is **increasing**. If $\boldsymbol{a < 0}$, the curve is **decreasing**.	

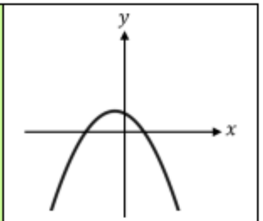
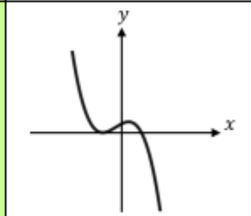
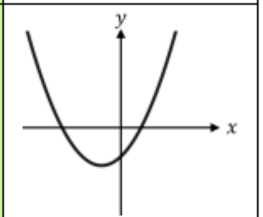
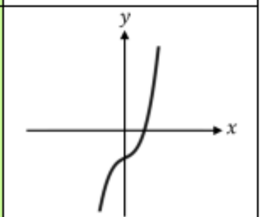
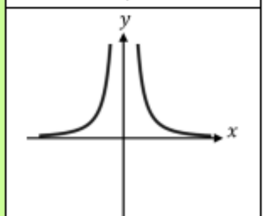
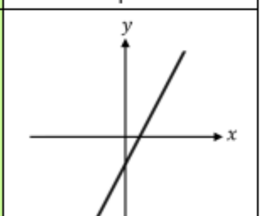
Reciprocal graphs and asymptotes

5. Reciprocal Graph	The equation is of the form $y = \frac{A}{x}$, where A **is a number** and $x \neq 0$. The graph has **asymptotes** on the **x-axis and y-axis**.	
6. Asymptote	A **straight line** that a graph **approaches** but **never touches**.	

Exponential graphs

7. Exponential Graph	The equation is of the form $\boldsymbol{y = a^x}$, where a is a number called the **base**. If $\boldsymbol{a > 1}$ the graph **increases**. If $\boldsymbol{0 < a < 1}$, the graph **decreases**. The graph has an **asymptote** which is the **x-axis**.	 

1	
2	
3	
4	
5	
6	

7	
8	
9	
10	
11	
12	

A	$y = x^2 + 2x - 3$
B	$y = \frac{2}{x}$
C	$y = x^3 + x$
D	$y = -\frac{10}{x}$
E	$y = 3 - 2x$
F	$y = 2 - x - x^2$
G	$y = 2x^3 + x - 3$
H	$y = 2x - 3$
I	$y = \frac{5}{x^2}$
J	$y = x^2 + 1$
K	$y = 1 + x - x^2 - x^3$
L	$y = 2x + 3$

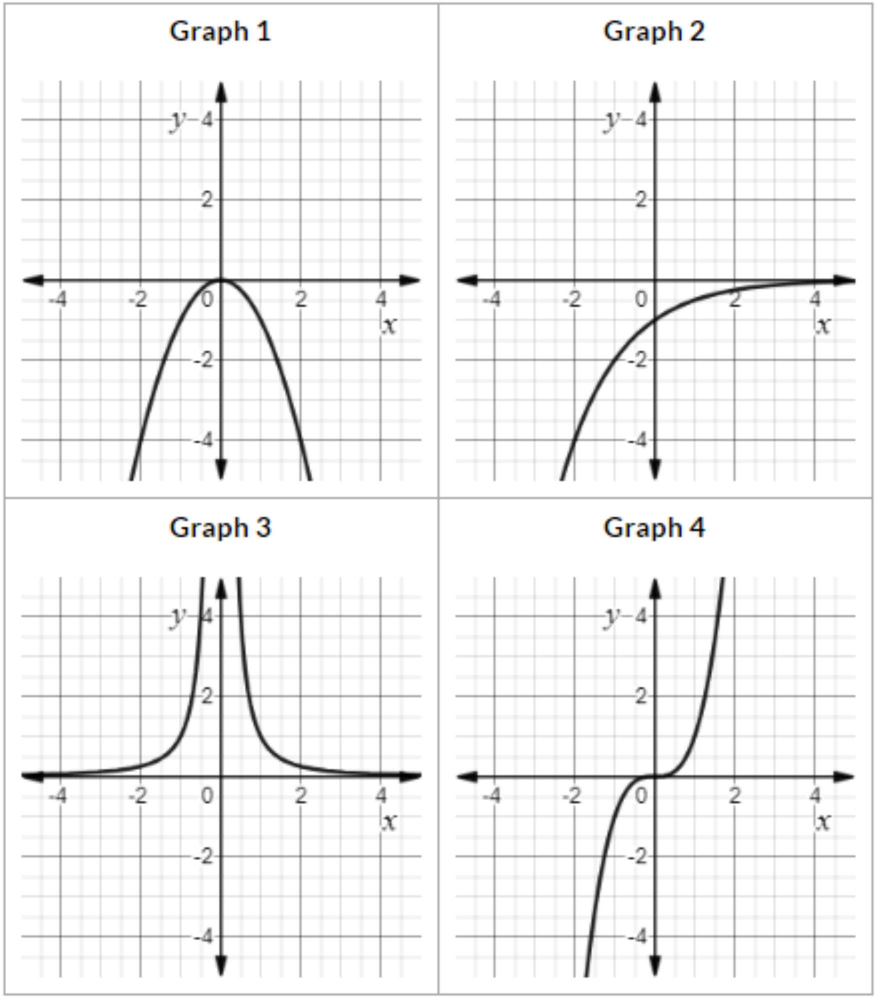
1	2	3	4	5	6

7	8	9	10	11	12

Secondary → Algebra → Curved Graphs

426b: Recognise the shape of simple quadratic, cubic, reciprocal and exponential graphs.

Four graphs are sketched below.



Match each equation in the table with a graph above.

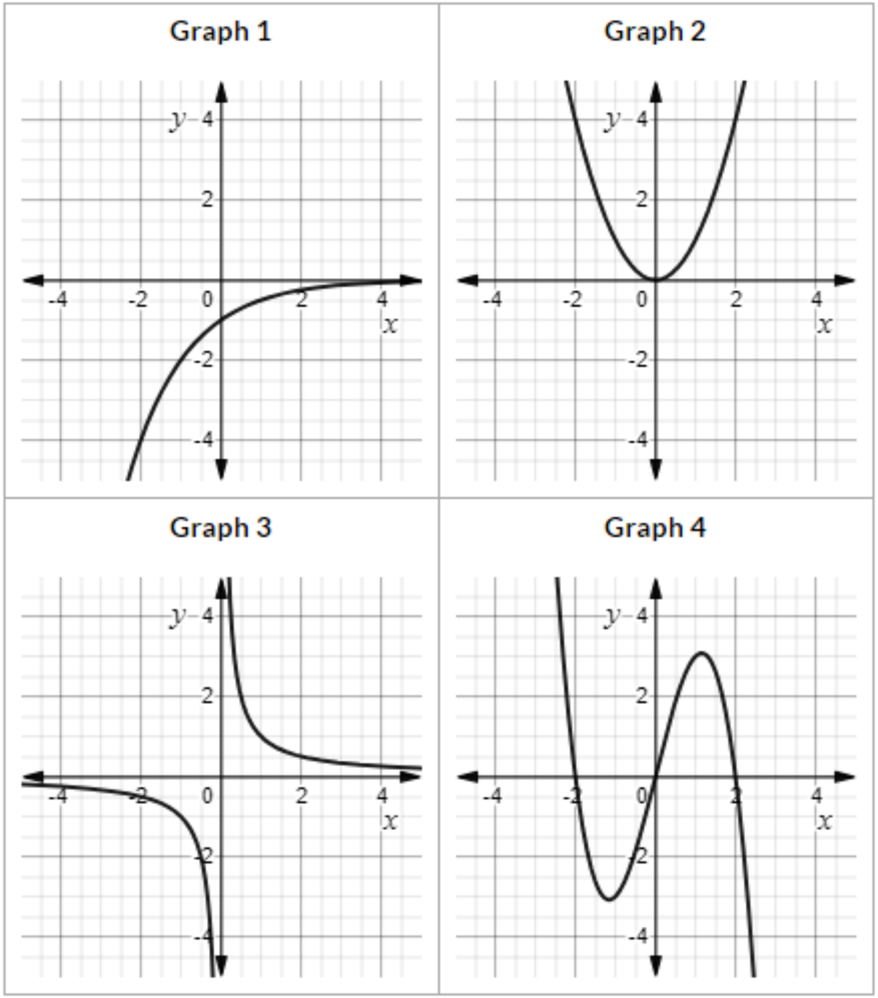
Equation	Graph Number
$y = -x^2$	
$y = x^3$	
$y = \frac{1}{x^2}$	
$y = -0.5^x$	

Submit Answer

Secondary → Algebra → Curved Graphs

426b: Recognise the shape of simple quadratic, cubic, reciprocal and exponential graphs.

Four graphs are sketched below.



Match each equation in the table with a graph above.

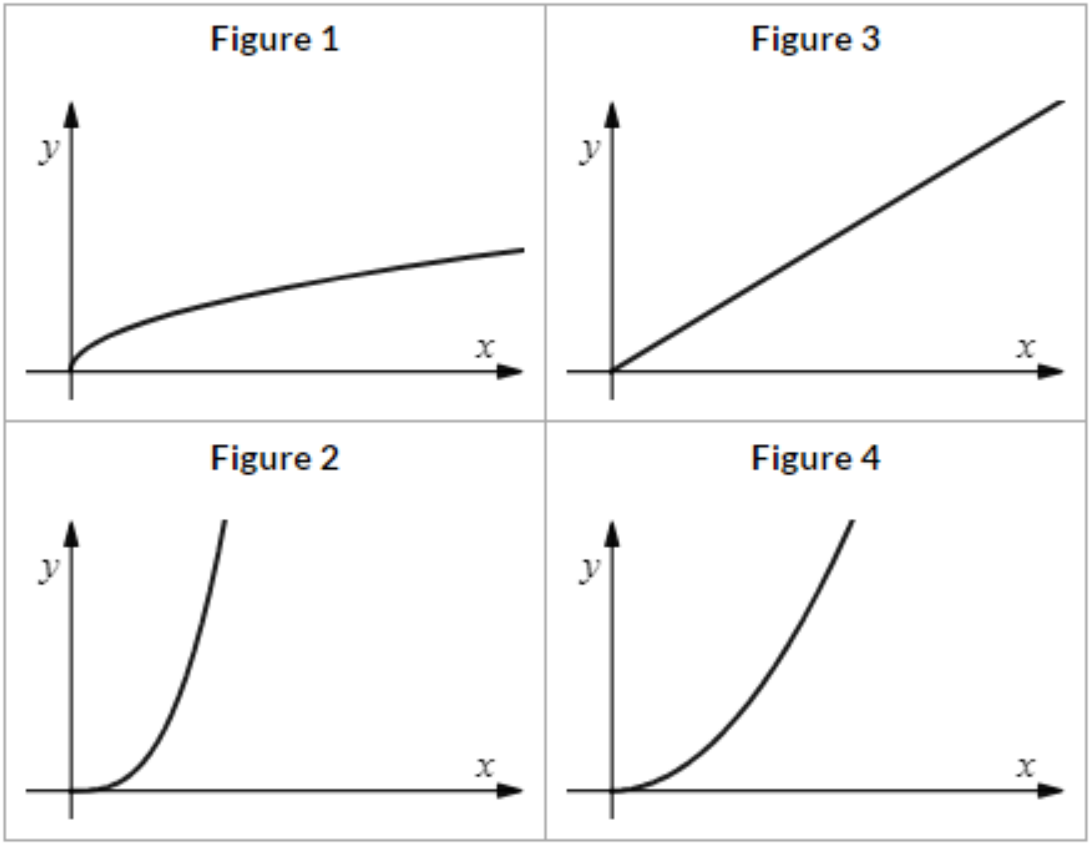
Equation	Graph Number
$y = x^2$	
$y = x(-x+2)(x+2)$	
$y = \dfrac{1}{x}$	
$y = -0.5^x$	

Submit Answer

Secondary → Algebra → Curved Graphs

426c: Recognise graphs for directly proportional and inversely proportional relationships.

Four graphs are sketched below.



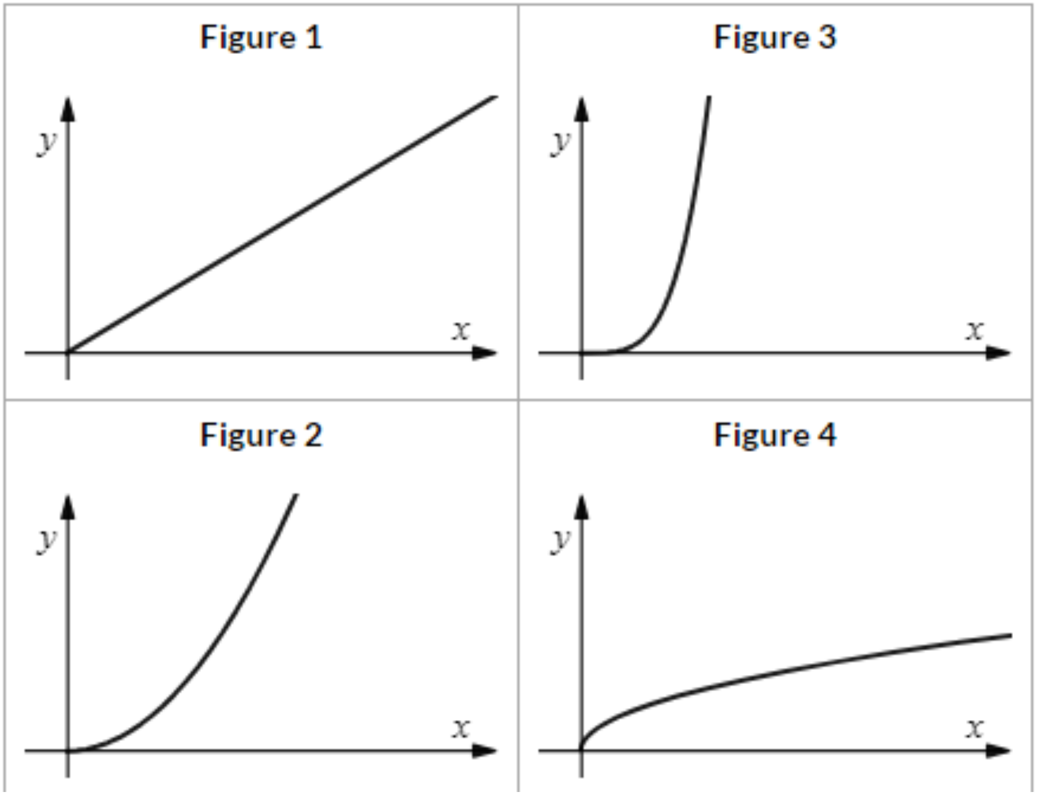
Match each equation in the table with a graph above.

Type of proportionality	Figure Number
$y \propto \sqrt{x}$	
$y \propto x^2$	
$y \propto x$	
$y \propto x^3$	

Secondary → Algebra → Curved Graphs

426c: Recognise graphs for directly proportional and inversely proportional relationships.

Four graphs are sketched below.



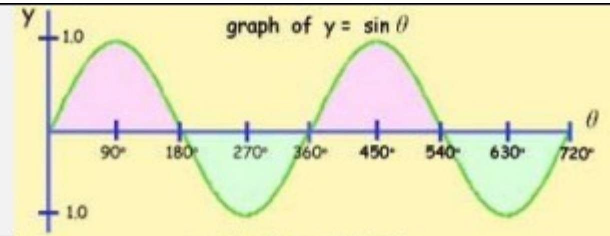
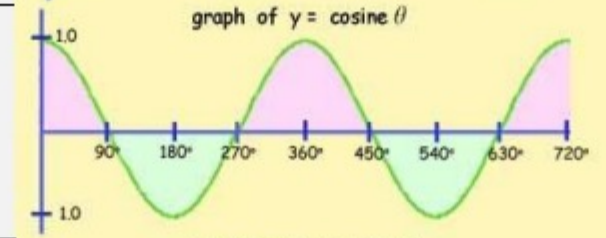
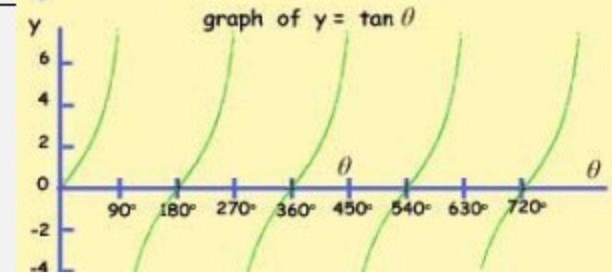
Match each equation in the table with a graph above.

Type of proportionality	Figure Number
$y \propto x^4$	
$y \propto x$	
$y \propto x^2$	
$y \propto \sqrt{x}$	

Submit Answer

Trigonometric graphs

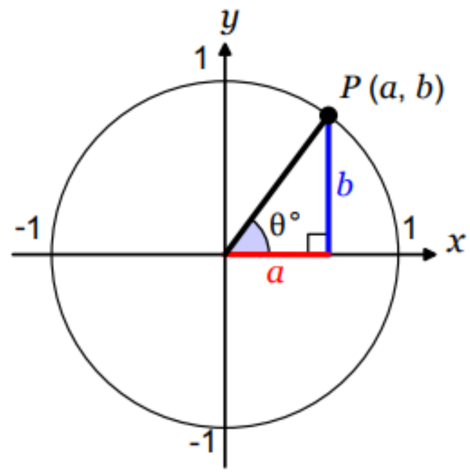
Recap of Exact Trigonometric Values								
Angle (θ Degrees)	0°	30°	45°	60°	90°	180°	270°	360°
$\sin\theta$								
$\cos\theta$								
$\tan\theta$								

8. $y = \sin x$	Key Coordinates: $(\mathbf{0},\mathbf{0}),(\mathbf{90},\mathbf{1}),(\mathbf{180},\mathbf{0}),(\mathbf{270},-\mathbf{1}),(\mathbf{360},\mathbf{0}$ y is never more than 1 or less than -1. Pattern repeats every 360°.	
9. $y = \cos x$	Key Coordinates: $(\mathbf{0},\mathbf{1}),(\mathbf{90},\mathbf{0}),(\mathbf{180},-\mathbf{1}),(\mathbf{270},\mathbf{0}),(\mathbf{360},\mathbf{1}$ y is never more than 1 or less than -1. Pattern repeats every 360°.	
10. $y = \tan x$	Key Coordinates: $(\mathbf{0},\mathbf{0}),(\mathbf{45},\mathbf{1}),(\mathbf{135},-\mathbf{1}),(\mathbf{180},\mathbf{0}),$ $(\mathbf{225},\mathbf{1}),(\mathbf{315},-\mathbf{1}),(\mathbf{360},\mathbf{0})$ **Asymptotes** at $\boldsymbol{x} = \mathbf{90}$ and $\boldsymbol{x} = \mathbf{270}$ Pattern repeats every 360°.	

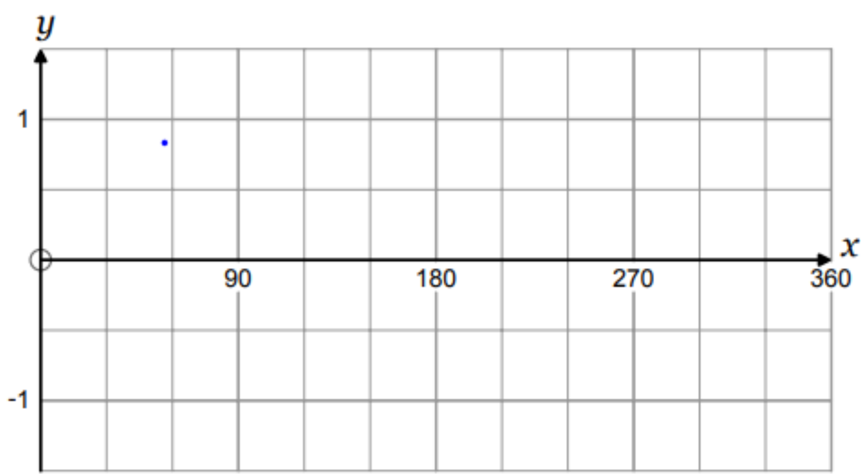
trigonometric graphs

The unit circle is centered on the origin and has a radius of 1.

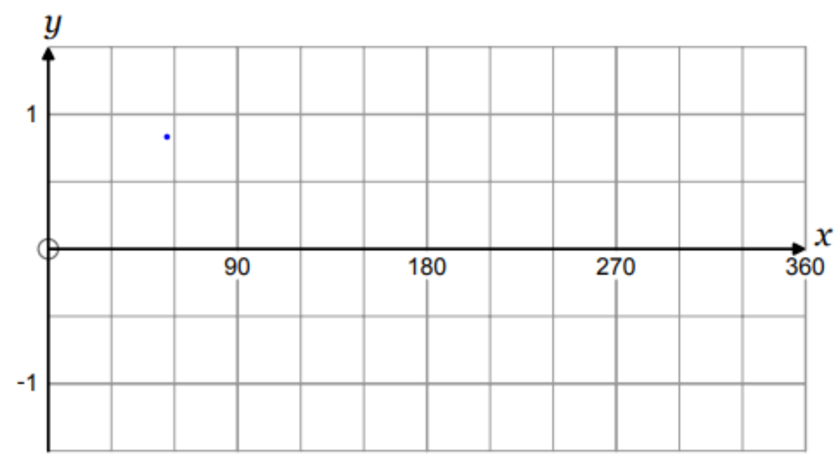
1. Work out a and b in terms of θ.



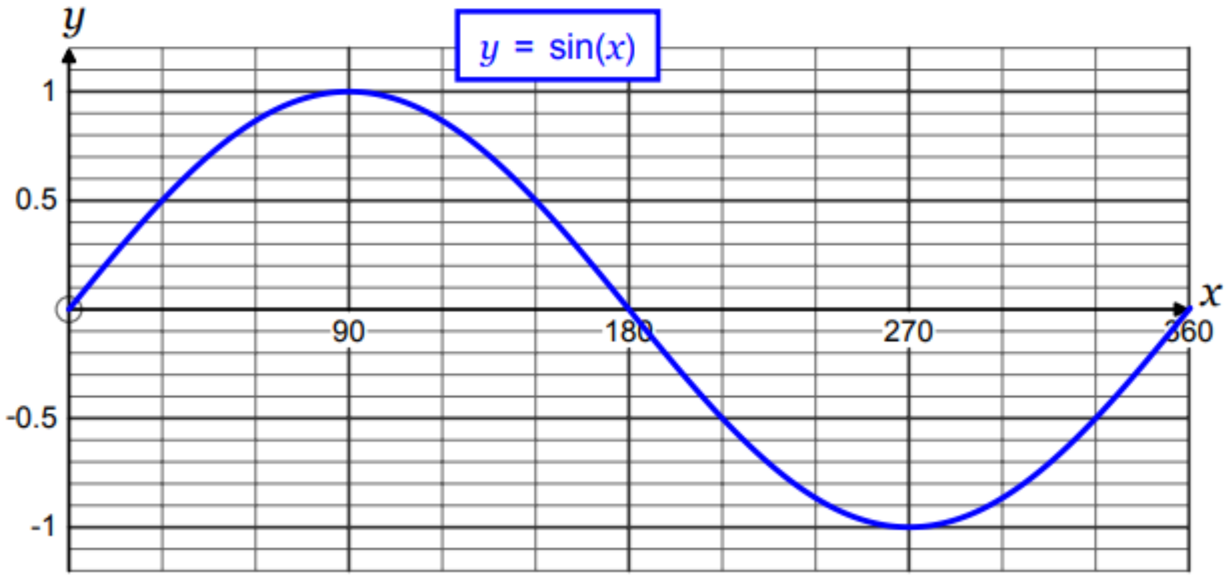
2. Sketch the graph of $y = \sin(x)$ for $0 \leq x \leq 360°$.



3. Sketch the graph of $y = \cos(x)$ for $0 \leq x \leq 360°$.



the sine graph



1. Solve for $0 \leq x \leq 360°$. Give your answers to 1 decimal place.

   a) $\sin(x) = 0.7$
   b) $\sin(x) = 0.4$
   c) $\sin(x) = -0.3$
   d) $\sin(x) = -0.8$

2. Solve for $0 \leq x \leq 360°$. Give your answers to 1 decimal place where necessary.

   a) $\sin(x) = 0.55$
   b) $\sin(x) = -0.9$
   c) $\sin(x) = -0.5$
   d) $\sin(x) = 1$
   e) $\sin(x) = \frac{\sqrt{3}}{2}$
   f) $\sin(x) = 0$
   g) $2\sin(x) = 1$
   h) $\sin(x) = 0.95$
   i) $\sin(x) = -\frac{1}{3}$

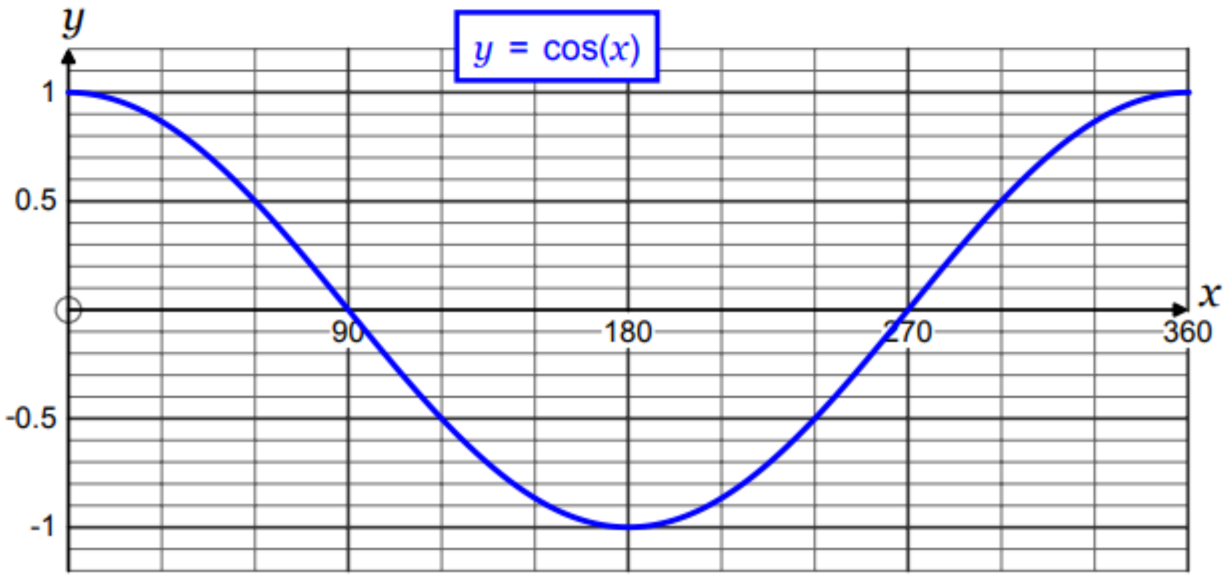
3. a) Given $\sin(40°) = 0.643$, complete: $\sin(140°) =$ ______

   b) Given $\sin(25°) = 0.423$, complete: $\sin(205°) =$ ______

   c) Given $\sin(165°) = 0.259$, complete: $\sin(15°) =$ ______

   d) Given $\sin(315°) = -0.707$, complete: $\sin(45°) =$ ______

the cosine graph



1. Solve for $0 \leq x \leq 360°$. Give your answers to 1 decimal place.

   a) $\cos(x) = 0.75$ b) $\cos(x) = 0.2$

   c) $\cos(x) = -0.6$ d) $\cos(x) = -0.35$

2. Solve for $0 \leq x \leq 360°$. Give your answers to 1 decimal place where necessary.

   a) $\cos(x) = 0.45$ b) $\cos(x) = -0.08$ c) $\cos(x) = 0.6$

   d) $\cos(x) = -1$ e) $\cos(x) = \frac{1}{2}$ f) $\cos(x) = 0$

   g) $4\cos(x) = 1$ h) $\cos(x) = -0.65$ i) $\cos(x) = \frac{7}{8}$

3. a) Given $\cos(25°) = 0.906$, complete: $\cos(335°) =$ ______

   b) Given $\cos(80°) = 0.174$, complete: $\cos(100°) =$ ______

   c) Given $\cos(160°) = -0.940$, complete: $\cos(20°) =$ ______

   d) Given $\cos(235°) = -0.574$, complete: $\cos(125°) =$ ______

Sort It Out...

Trigonometric Graphs

Sort these properties of trigonometric graphs into each of the categories – $y = \sin x$, $y = \cos x$ or $y = \tan x$. Some properties may apply to more than one graph, and some may apply to none.

1	Passes through $(0, 0)$	**2**	Graph repeats itself every $360°$	**3**	Has a maximum y-value of 1
4	Symmetrical about the y-axis	**5**	Passes through $(0, 1)$	**6**	Symmetrical about the x-axis
7	Has rotational symmetry order 2 about origin	**8**	Has an asymptote at $x = 90°$	**9**	Has a minimum y-value of -1
10	Passes through $(0, -1)$	**11**	Symmetrical about the line $x = 180°$	**12**	Passes through $(360, 0)$
13	Has an asymptote at $x = 180°$	**14**	Graphs repeats itself every $180°$	**15**	Has rotational symmetry order 2 about $(90, 0)$

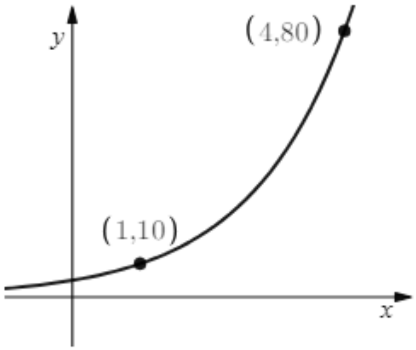
A	$y = \sin x$

C	$y = \tan x$

B	$y = \cos x$

D	*None of them*

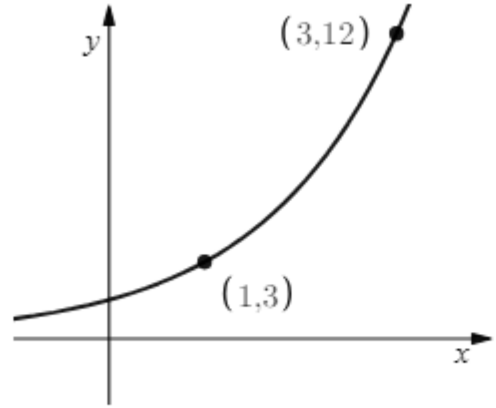
Worked Example	Your Turn
425a: Determine the values of p and q in the exponential function $y = pq^x$	



The sketch graph shows a curve with equation $y = pq^x$

The curve passes through the points $(1, 10)$ and $(4, 80)$.

Calculate the value of p and the value of q.



The sketch graph shows a curve with equation $y = pq^x$

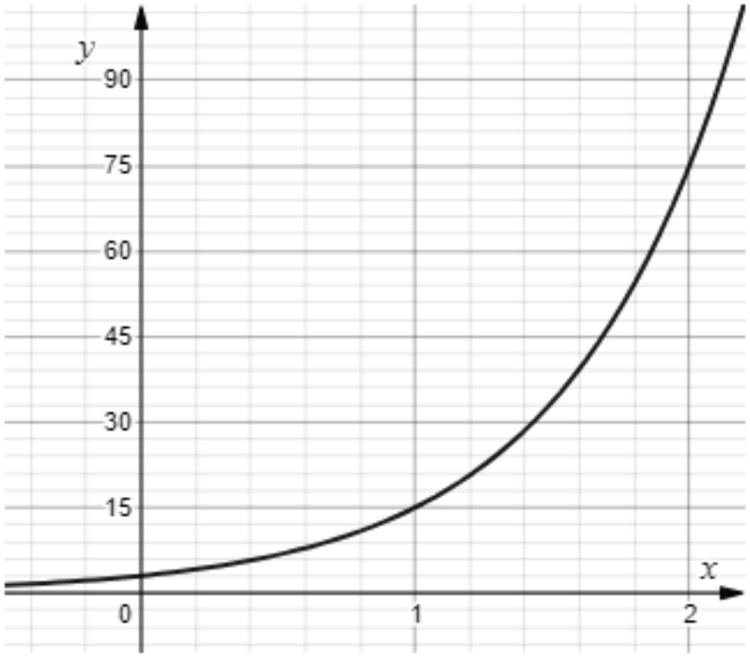
The curve passes through the points $(1, 3)$ and $(3, 12)$.

Calculate the value of p and the value of q.

Worked Example	Your Turn

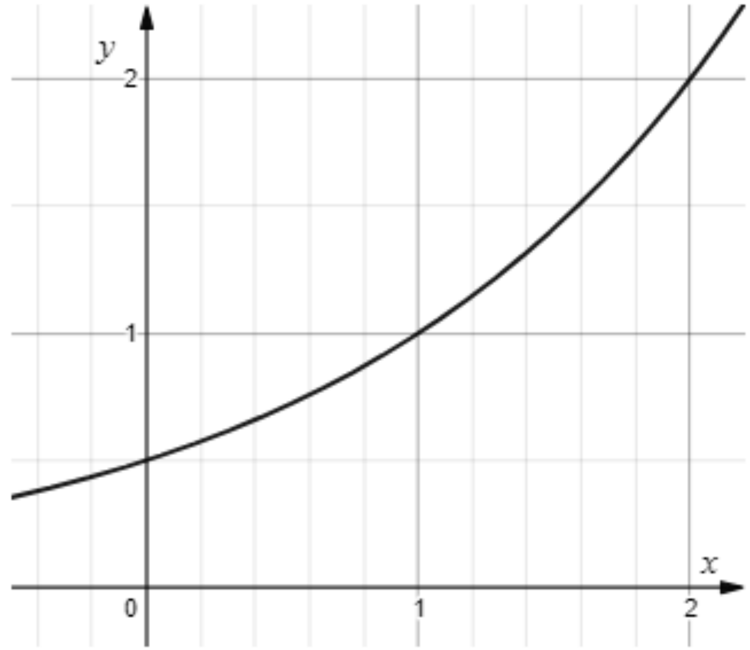
425b: Use a graph to determine the function $y = pq^x$

The graph below shows a curve with equation $y = pq^x$



Calculate the value of p and q.

The graph below shows a curve with equation $y = pq^x$



Calculate the value of p and q.

Worked Example	Your Turn

425c: Determine an exponential model $y = pq^x$ from two data points to make a prediction.

Initially, a capacitor held a charge of 2000 mC.

It begins discharging, and after 3 seconds, it holds 900 mC.

The charge of the capacitor is given by the formula

$$Q = ar^t$$

where Q is the charge of the capacitor in mC, t seconds after it began discharging.

Calculate the charge of the capacitor 5 seconds after it began discharging, giving your answer to 3 significant figures.

425c: Determine an exponential model $y = pq^x$ from two data points to make a prediction.

At the start of 2010, a savings account contained $\$80\,000$.

At the start of 2015, the value of the savings account was $\$190\,000$.

The value of the account is given by the formula

$$V = ar^t$$

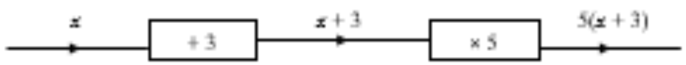
where V is the amount in the account, t years after the start of 2010.

Calculate the value of the account at the start of 2017.

Composite Functions

A **composite function** is a function consisting of two or more functions.

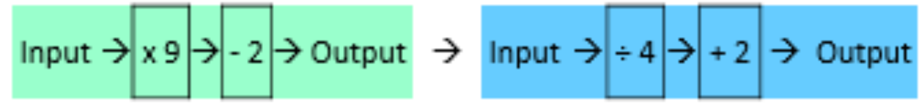
The term composition is used when one operation is performed after another operation.
For instance:



This function can be written as $f(x) = 5(x + 3)$

Composite Functions

Here are two number machines.



What is the output of the second machine, when the input of the first machine is 2.

Here are two functions:

$f(x) = 9x - 2$

$g(x) = \frac{x}{4} + 2$

Calculate the value of $gf(2)$

Worked Example	Your Turn
If $f(x) = 3x + 4$, $g(x) = 2x - 5$ a) $fg(6) =$ b) $gf(7) =$	If $f(x) = 4x - 3$, $g(x) = 5x + 2$ a) $fg(8) =$ b) $gf(8) =$

Worked Example	Your Turn
If $f(x) = 3x^2$, $g(x) = x - 4$ a) $fg(5) =$ b) $gf(6) =$	If $f(x) = 5x^2$, $g(x) = x + 3$ a) $fg(7) =$ b) $gf(7) =$

Worked Example	Your Turn
If $f(x) = 5x^2$, $g(x) = 2x + 3$ a) $fg(2) =$ b) $gf(3) =$	If $f(x) = 4x^2$, $g(x) = 3x + 2$ a) $fg(4) =$ b) $gf(4) =$

Worked Example	Your Turn
If $f(x) = x + 3$, $g(x) = \frac{1}{x-2}$ a) $fg(5) =$ b) $gf(5) =$	If $f(x) = x - 5$, $g(x) = \frac{1}{x+4}$ a) $fg(8) =$ b) $gf(8) =$

Question 8: The functions $f(x)$ and $g(x)$ are given by the following:

$$f(x) = x + 5$$
$$g(x) = 3x - 1$$

Calculate the value of:

(a) $fg(1)$ (b) $fg(-5)$ (c) $gf(4)$ (d) $gf(0)$

(e) $ff(2)$ (f) $ff(-4)$ (g) $gg(10)$ (h) $gg(-2)$

Question 9: The functions $f(x)$, $g(x)$ and $h(x)$ are given by the following:

$$f(x) = x^2 + 7$$
$$g(x) = 3x - 8$$
$$h(x) = \frac{x}{4}$$

Calculate the value of:

(a) $fg(3)$ (b) $hf(5)$ (c) $gh(20)$ (d) $gf(-2)$

(e) $fh(12)$ (f) $ff(1)$ (g) $gg(4)$ (h) $hh(40)$

Question 10: The functions $f(x)$, $g(x)$ and $h(x)$ are given by the following:

$$f(x) = \frac{32}{x^2} \qquad g(x) = 2x^3 \qquad h(x) = \frac{12 - 2x}{5}$$

Calculate the value of:

(a) $fg(1)$ (b) $gf(4)$ (c) $gh(-19)$ (d) $hf(2)$

(e) $ff(2)$ (f) $ggg(1)$ (g) $hgf(8)$ (h) $hgh(6)$

Corbett maths

Functions
Videos 369, 370 on Corbettmaths

Question 11: The functions $f(x)$ and $g(x)$ are given by the following:

$$f(x) = 2x + 1$$
$$g(x) = x - 5$$

Find:

(a) $fg(x)$ (b) $gf(x)$ (c) $ff(x)$ (d) $gg(x)$

Question 12: The functions $f(x)$, $g(x)$ and $h(x)$ are given by the following:

$$f(x) = 4x - 3 \qquad g(x) = 2x + 6 \qquad h(x) = x^2$$

Find

(a) $fg(x)$ (b) $gf(x)$ (c) $hf(x)$ (d) $fh(x)$

(e) $hg(x)$ (f) $gh(x)$ (g) $fgh(x)$ (h) $hgf(x)$

Question 13: Find $f^{-1}(x)$ for each of the following:

(a) $f(x) = 2x$ (b) $f(x) = x - 6$ (c) $f(x) = \frac{x}{3}$

(d) $f(x) = 5x + 1$ (e) $f(x) = \frac{2x}{7}$ (f) $f(x) = \frac{x - 2}{6}$

Question 14: Given $h(x) = \frac{x}{4}$

(a) Find $h^{-1}(x)$

(b) Calculate the value of $h^{-1}(1.5)$

Question 15: Given $f(x) = 2x - 3$

(a) Find $f^{-1}(x)$

(b) Calculate the value of $f^{-1}(7)$

Functions
Videos 369, 370 on Corbettmaths

Question 16: Given $g(x) = \frac{3x+1}{2}$

(a) Find $g^{-1}(x)$

(b) Calculate the value of $g^{-1}(11)$

Question 17: Given $f(x) = \frac{4x}{9} - 8$

(a) Find $f^{-1}(x)$

(b) Calculate the value of $f^{-1}(-10)$

Apply

Question 1: Given $f(x) = 5x + 7$ and $g(x) = 3x - 18$

Find the value of a such that $f(a) = g(a)$

Question 2: Given $f(x) = x^2 + 9$ and $g(x) = x + 21$

Find the values of a such that $f(a) = g(a)$

Question 3: Given $f(x) = \frac{x+1}{3}$ and $g(x) = \frac{2}{x+2}$

Find the values of a such that $f(a) = g(a)$

Question 4: Given $f(x) = x^2 + 4x - 1$

Express the following in the form $ax^2 + bx + c$

(a) $f(x+2)$ (b) $f(x-1)$ (c) $f(2x)$

(d) $f(3x)$ (e) $f(2x-1)$ (f) $f(4x+3)$

Functions
Videos 369, 370 on Corbettmaths

Question 5: The function f is such that $f(x) = kx + 7$

The function g is such that $g(x) = 3x - 2$

Given that $gf(1) = 34$

Work out the value of k

Question 6: The function g is such that $f(x) = \frac{kx+2}{4}$

The function h is such that $g(x) = 2x + 5$

Given that $fg(4) = -9.25$

Work out the value of k

Question 7: For all values of x

$$f(x) = x^2 + 5$$

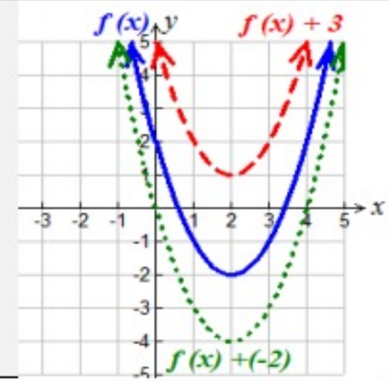
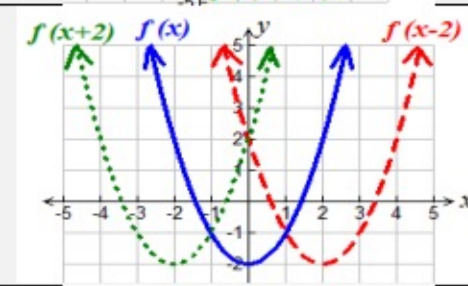
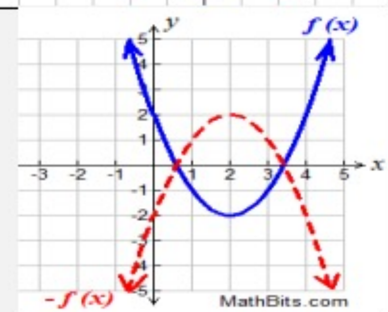
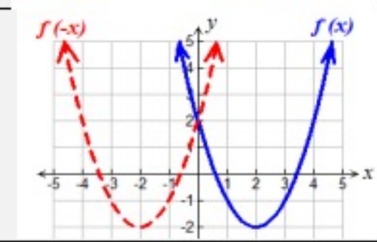
$$g(x) = x - 4$$

Solve $fg(x) = gf(x)$

Question 8: $f(x) = x^2 + 3x + 8$

Show that $f(x+1) - f(x) = 2x + 4$

Graph transformation Rules you must learn

11. $f(x)+a$	**Vertical translation** up a units. $\begin{pmatrix}0\\a\end{pmatrix}$	
12. $f(x+a)$	**Horizontal translation** left a units. $\begin{pmatrix}-a\\0\end{pmatrix}$	
13. $-f(x)$	**Reflection** over the **x-axis**.	
14. $f(-x)$	**Reflection** over the **y-axis**.	

Worked Example	Your Turn

448c: Understand the effect on a point under the transformation $y = f(x + a)$

The curve with equation $y = f(x)$ has the minimum point $P(-9, 2)$.

Find the image of P on the curve with equation $y = f(x - 3)$

The point $P(-7, -1)$ lies on the curve with equation $y = f(x)$.

Find the image of P on the curve with equation $y = f(x + 2)$

448d: Understand the effect on a point under the transformation $y = f(x) + a$

The point $P(1, -6)$ lies on the curve with equation $y = f(x)$.

Find the image of P on the curve with equation $y = f(x) - 2$

The point $P(-4, 1)$ lies on the curve with equation $y = f(x)$.

Find the image of P on the curve with equation $y = f(x) + 3$

Worked Example	Your Turn

Worked Example

448g: Determine the equation in $f(x)$ after a translation of a given graph to $f(x) + a$ and $f(x + a)$

The graph of $y = f(x)$ is shown in Figure 1.

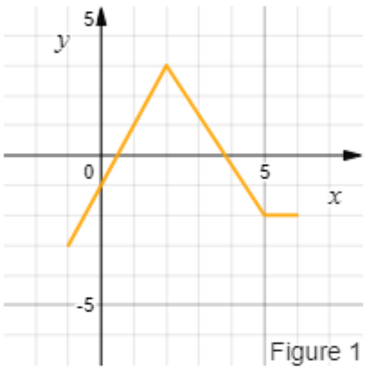


Figure 1

The graph of $y = f(x) + a$ is shown in Figure 2.

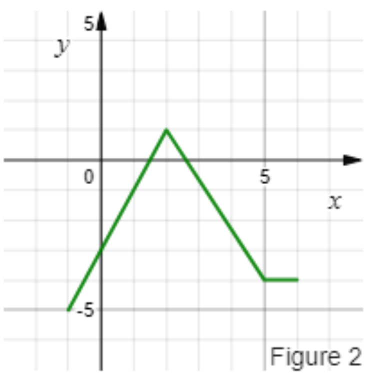


Figure 2

Determine the value of a.

Your Turn

The graph of $y = f(x)$ is shown in Figure 1.

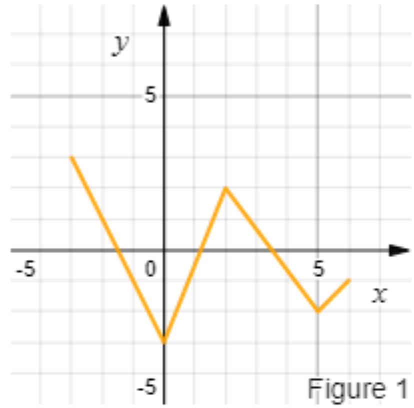


Figure 1

The graph of $y = f(x) + a$ is shown in Figure 2.

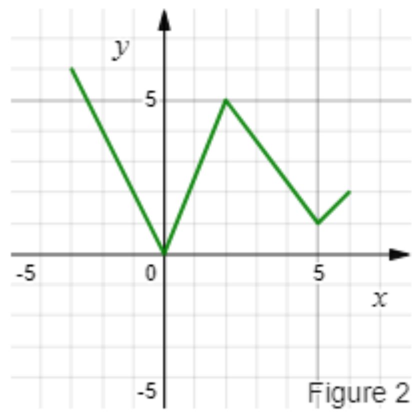


Figure 2

Determine the value of a.

Worked Example	Your Turn

448h: Determine the new equation of a function after a translation by $\begin{pmatrix} a \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ a \end{pmatrix}$

The curve $y = 2\tan(4x)$ is translated by 1 unit in the positive y- direction.

State the equation of the new curve after this transformation.

448h: Determine the new equation of a function after a translation by $\begin{pmatrix} a \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ a \end{pmatrix}$

The curve $y = \frac{2}{2x-1}$ is translated by $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$.

State the equation of the new curve after this transformation.

The curve $y = 5\sqrt{2x-1}$ is translated by $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$.

State the equation of the new curve after this transformation.

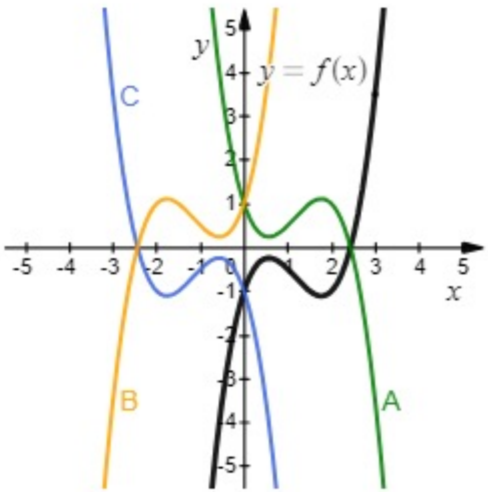
The curve $y = \frac{2}{3x+1}$ is translated by 1 unit in the negative y- direction.

State the equation of the new curve after this transformation.

Worked Example	Your Turn

449a: Sketch a graph of $y = -f(x)$ given the graph of $y = f(x)$

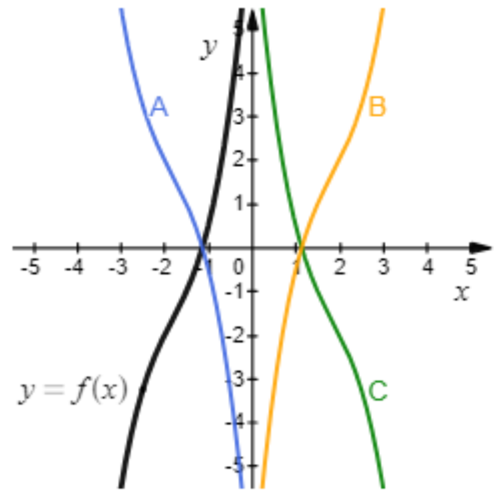
The graph of $y = f(x)$ is drawn in black on the grid below.



Select the graph which represents the transformation $y = -f(x)$.

- ○ A
- ○ B
- ○ C

The graph of $y = f(x)$ is drawn in black on the grid belov



Select the graph which represents the transformation $y = -f(x)$.

- ○ A
- ○ B
- ○ C

Worked Example	Your Turn

449c: Understand the effect on a point under the transformation $y = -f(x)$

The point $P(6, 0)$ lies on the curve with equation $y = f(x)$.

Find the image of P on the curve with equation $y = -f(x)$

449d: Understand the effect on a point under the transformation $y = f(-x)$

The curve with equation $y = f(x)$ has the maximum point $P(2, -5)$.

Find the image of P on the curve with equation $y = f(-x)$

The point $P(5, 4)$ lies on the curve with equation $y = f(x)$
.

Find the image of P on the curve with equation $y = -f(x)$

The curve with equation $y = f(x)$ has the maximum point $P(5, 1)$.

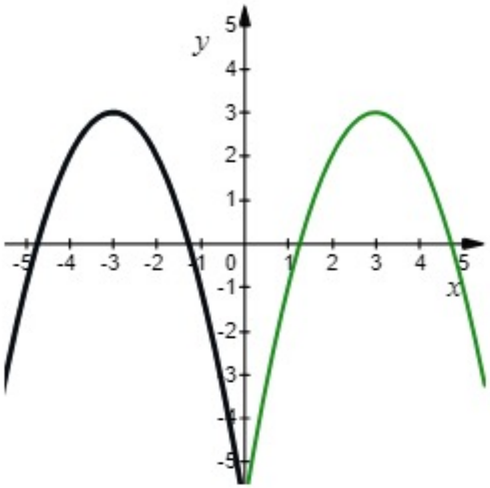
Find the image of P on the curve with equation $y = f(-x)$

Worked Example

449f: Determine the equation in $f(x)$ notation after a reflection of a given graph in one axis only.

The graph of $y = f(x)$ is drawn in black on the grid below.

The graph drawn in green is a transformation of $y = f(x)$.



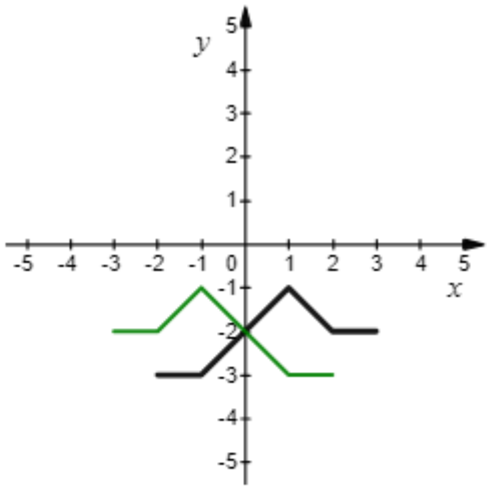
Determine which of the following functions represents the transformation?

- ○ $f(x) - 1$
- ○ $-f(x)$
- ○ $f(-x)$
- ○ $f(x-2)$
- ○ $f(x-1)$

Your Turn

The graph of $y = f(x)$ is drawn in black on the grid below.

The graph drawn in green is a transformation of $y = f(x)$.



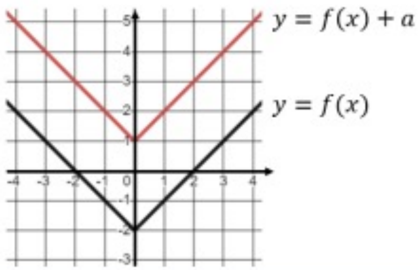
Determine which of the following functions represents the transformation?

- ○ $f(x-1)$
- ○ $-4f(x)$
- ○ $-f(x)$
- ○ $f(-2x)$
- ○ $f(-x)$

Describing Transformations of Graphs

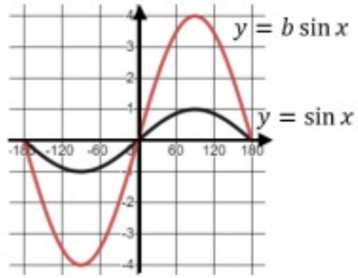
(a)

The graphs of $y = f(x)$ and $y = f(x) + a$ are shown below. Find the value of a.



(b)

The graphs of $y = \sin x$ and $y = b\sin(x)$ are shown below. Find the value of b.



(c)

The graph of $y = f(x)$ is transformed to give the equation $y = -f(x)$. Describe the transformation in words.

(d)

The graph of $y = f(x)$ is transformed to give the equation $y = f(x-4)$. Describe the transformation in words.

(e)

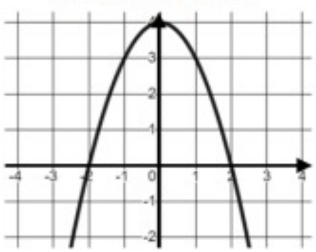
The graph of $y = f(x)$ is transformed to give the equation $y = f(2x)$. Describe the transformation in words.

(f)

The graph of $y = f(x)$ is transformed to give the equation $y = f(-x)$. Describe the transformation in words.

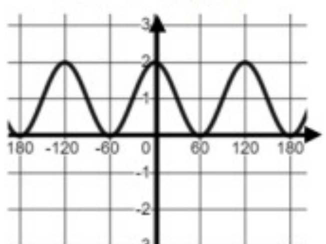
(g)

The graph of $y = x^2$ has been transformed to give the graph shown below. Write down the equation of the transformed graph.



(h)

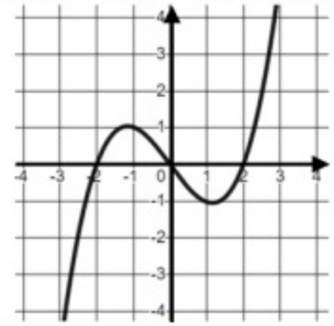
The graph of $y = \cos x$ has been transformed to give the graph shown below. Write down the equation of the transformed graph.



Drawing Transformations of Graphs

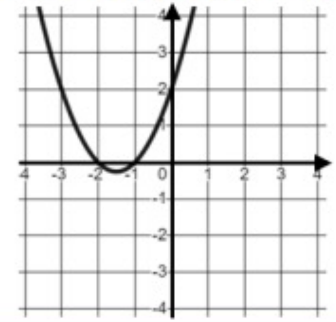
(a)

Here is a graph of $y = f(x)$. On the same axes, draw the graph of $y = f(x) - 2$.



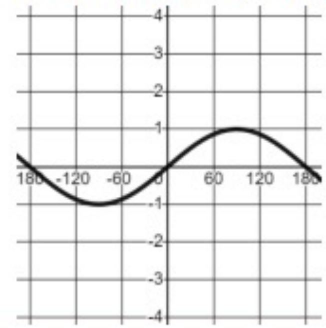
(b)

Here is a graph of $y = f(x)$. On the same axes, draw the graph of $y = -f(x)$



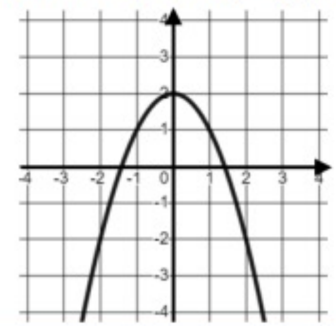
(c)

Here is a graph of $y = f(x)$. On the same axes, draw the graph of $y = 3f(x)$



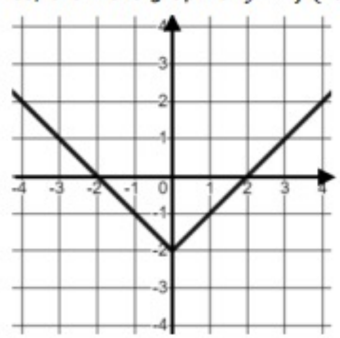
(d)

Here is a graph of $y = f(x)$. On the same axes, draw the graph of $y = f(x+1)$



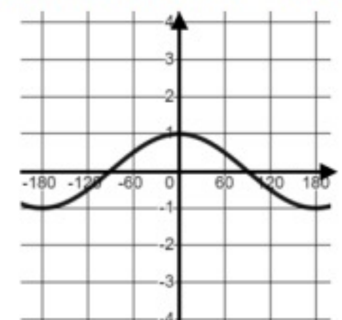
(e)

Here is a graph of $y = f(x)$. On the same axes, draw the graph of $y = f(2x)$



(f)

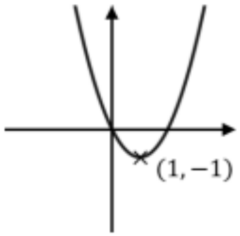
Here is a graph of $y = f(x)$. On the same axes, draw the graph of $y = 2f(x) - 1$



Transformations of Points on Graphs

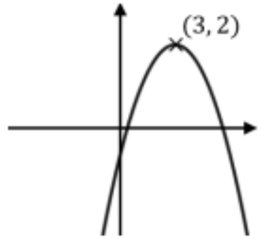
(a)

The curve $y = f(x)$ shown below has a minimum point with coordinates $(1, -1)$. Write down the coordinates of the minimum point of the curve $y = f(x) + 3$



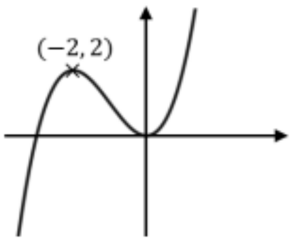
(b)

The point $P\ (3, 2)$ lies on the curve with equation $y = f(x)$ shown below. Write down the coordinates of the point P on the transformed curve $y = -f(x)$



(c)

The curve $y = f(x)$ shown below has a maximum point with coordinates $(-2, 2)$.



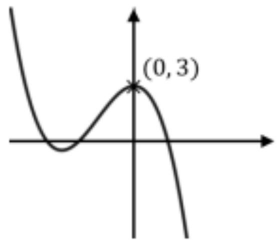
Write down the coordinates of the maximum point of the transformed curve

(i) $y = f(2x)$

(ii) $y = f(x + 5)$

(d)

The curve $y = f(x)$ shown below has a maximum point with coordinates $(0, 3)$.



Write down the coordinates of the maximum point of the transformed curve

(i) $y = \frac{1}{2}f(x)$

(ii) $y = f(-x)$

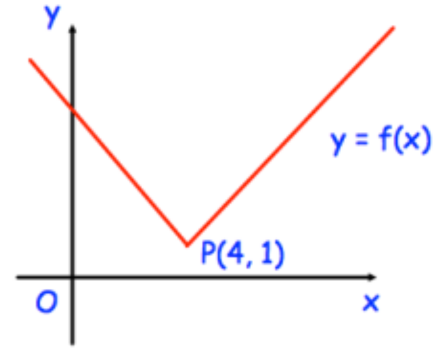
(e)

The curve A with equation $y = f(x)$ is transformed to give the curve B with equation $y = f(-x) + 2$. The point $(1, 1)$ lies on the curve A. What point does this map to on the transformed curve B?

(f)

The curve C with equation $y = f(x)$ is transformed to give the curve D with equation $y = -f(x + 1) - 2$. The point $(3, -2)$ lies on the curve C. What point does this map to on the transformed curve D?

1. Here is the graph of y = f(x)
The point P(4, 1) is a point on the graph.



What are the coordinates of the new position of P when the graph y = f(x) is transformed to the graph of

(a) y = −f(x)

(.............. , ...)
(1)

(b) y = f(x) + 4

(.............. , ...)
(1)

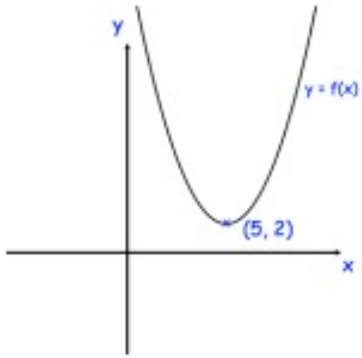
(c) y = f(−x)

(.............. , ...)
(1)

(d) y = f(x + 5)

(.............. , ...)
(1)

2.



Shown is the curve with equation y = f(x)
The coordinates of the minimum point of the curve are (5, 2).

Write down the coordinates of the minimum point of the curve with equation

(a) y = f(x) − 4

(................ , ...)
(1)

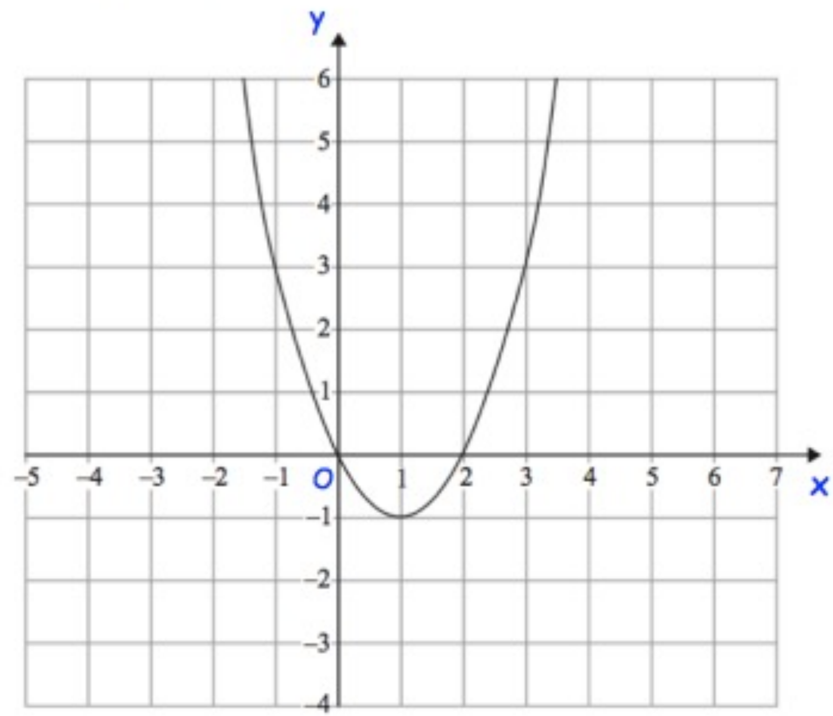
(b) y = f(x − 2)

(................ , ...)
(1)

(c) y = f(−x)

(................ , ...)
(1)

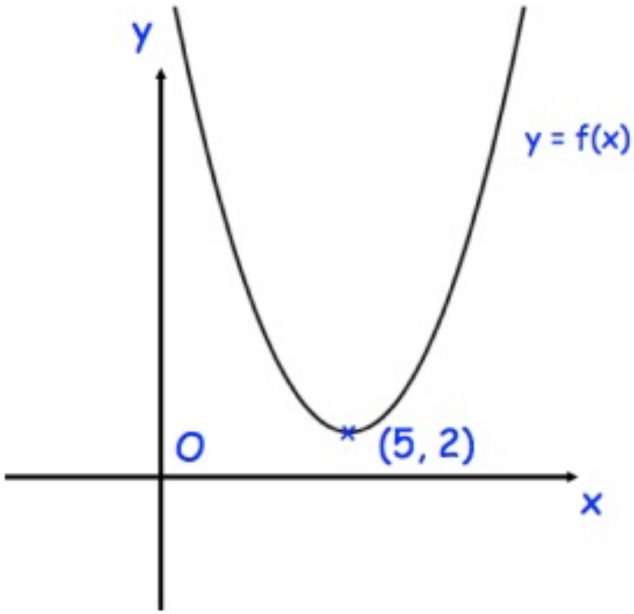
3. The graph of y = f(x) is shown below.



On the grid, sketch the graph of y = f(x − 1)

(2)

4. This is a sketch of the curve with the equation $y = f(x)$.
The only minimum point of the curve is at the point (5, 2).



Write down the coordinates of the minimum point of the curve with equation

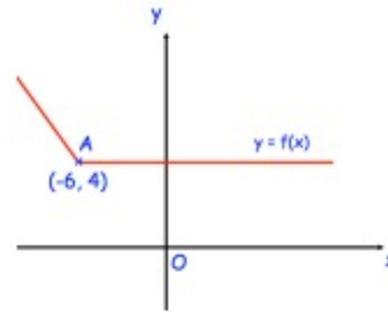
(a) $y = f(x) + 3$

(.................. , ...)
(1)

(b) $y = f(x + 1) - 2$

(.................. , ...)
(2)

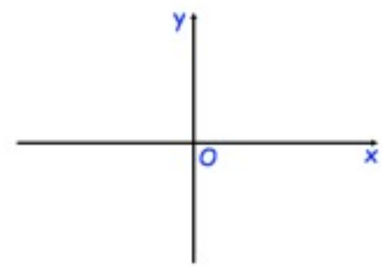
5. The diagram below shows the graph of $y = f(x)$



The point A(−6, 4) lies on the graph.

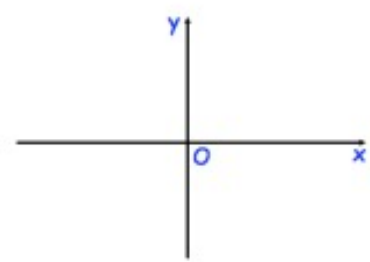
Sketch the graphs with the equations below, clearly giving the point corresponding to A.

(a) $y = -f(x)$



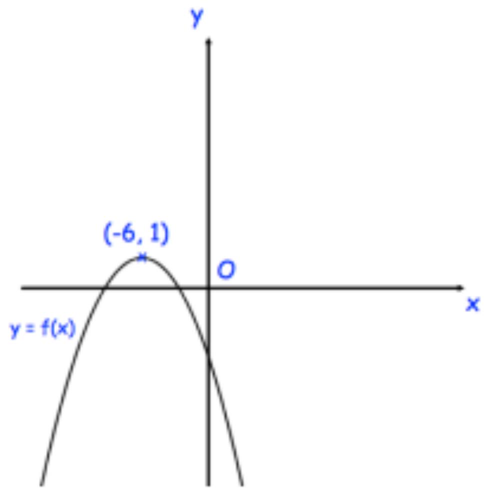
(2)

(b) $y = f(x - 3)$



(2)

6. This is a sketch of the curve with equation y = f(x)



The vertex of the curve is at the point (–6, 1)

Write down the coordinates of the vertex of the curve with equation

(a) $y = f(x + 3)$

(................ , ...)
(1)

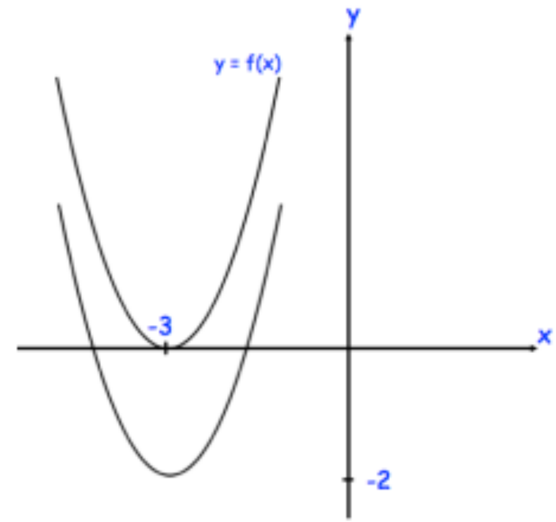
(b) $y = f(-x)$

(................ , ...)
(1)

(c) $y = f(x) - 4$

(................ , ...)
(1)

7.

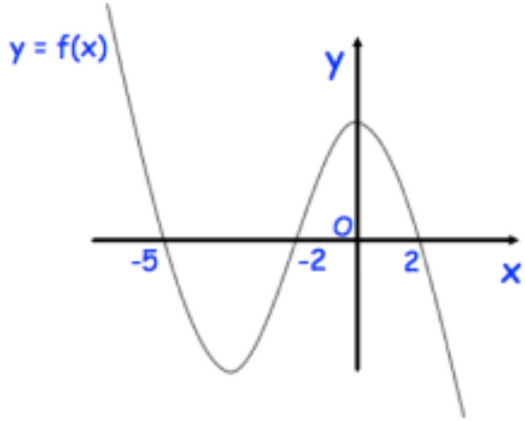


The curve with equation y = f(x) is translated so that the point at (–3, 0) is mapped onto the point (–3, –2).

Find an equation of the translated curve.

....
(2)

8.



The graph of y = f(x) cuts the x axis when x = –5, –2 and 2

Write down the coordinates of the points where these graphs cut the x axis.

(a) y = f(–x)

....

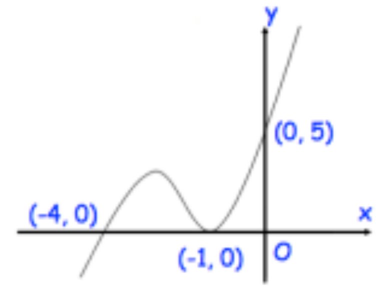
(2)

(b) y = f(x + 2)

....

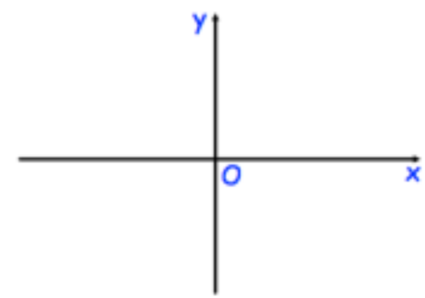
(2)

9. Shown below is the curve with equation y = f(x).
The curve passes through the points (–4, 0), (–1, 0) and (0, 5)



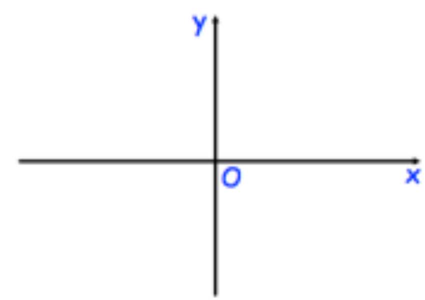
Sketch the curve with equation:

(a) y = f(x – 1)



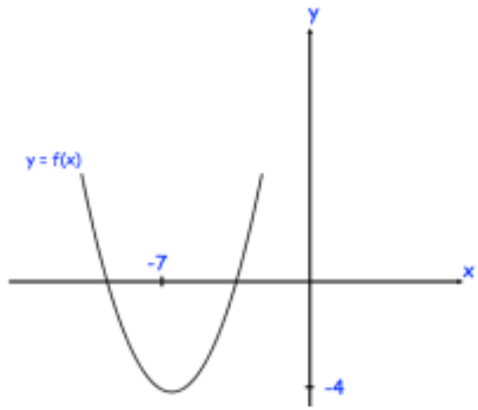
(2)

(b) y = –f(x)



(2)

10. Shown below is a sketch of a curve with equation y = f(x).
The curve has a minimum point at (-7, -4).

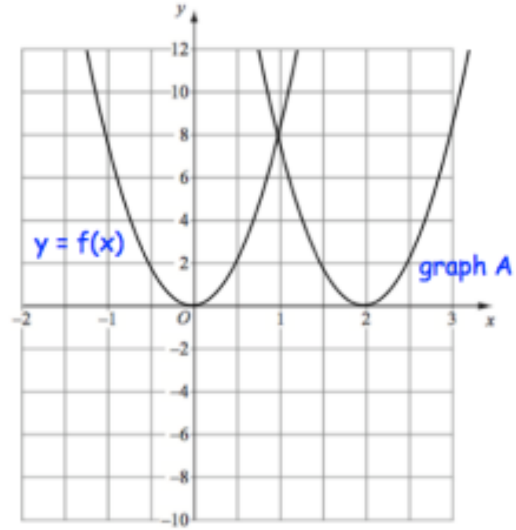


The graph of y = f(x) + a has a minimum point at (−7, 0), where a is a constant.

Write down the value of a.

....
(1)

11. The graph of y = f(x) is shown on the grid.

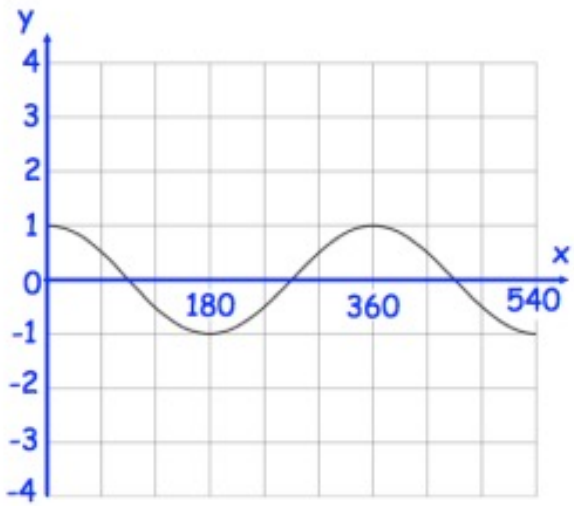


The graph A is a translation of the graph y = f(x)

Write down the equation of graph A.

....
(2)

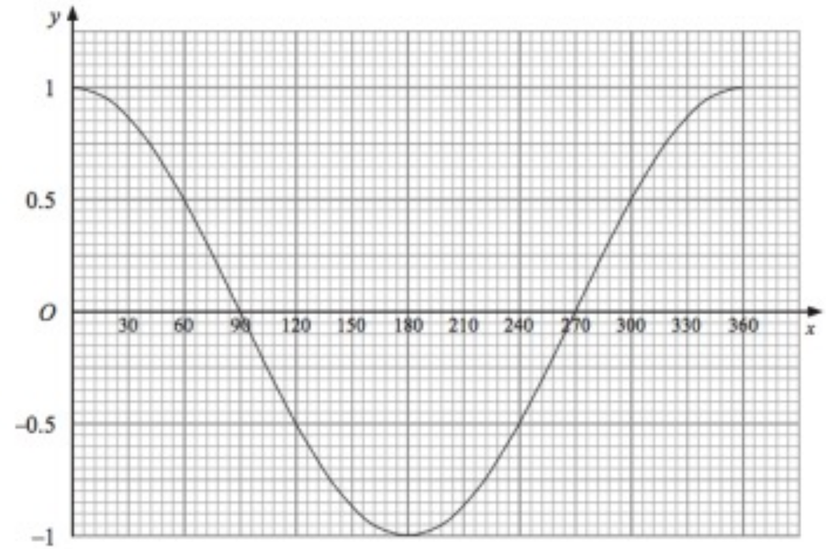
12. Shown below is the graph of y = cos x



On the grid, sketch the graph of y = 3 + cos x for values of x from 0° to 540°

(2)

13. Shown below is the graph of y = cos x



On the grid, sketch the graph of y = cos(x – 90°) for values of x from 0° to 360°

(2)

14. Describe the transformation that maps the curve with equation $y = \sin(x)$ onto the curve with equation

(a) $y = -\sin(x)$

....

....

(2)

(b) $y = 1 + \sin(x)$

....

....

(2)

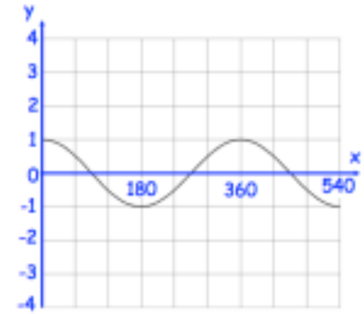
(c) $y = \sin(x - 30°)$

....

....

(2)

15. Shown below is the graph of $y = \cos x$



On the grid, sketch the graph of $y = 2 - \cos(x)$ for values of x from 0° to 540°

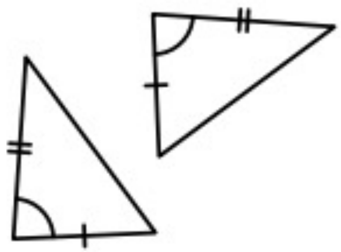
(2)

Congruency

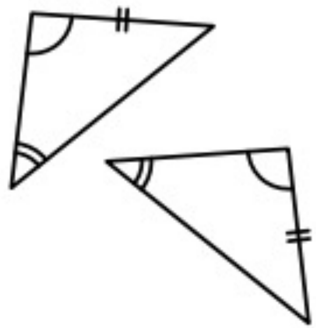
learn by heart

Congruent shapes are identical - they have the same shape and same size. They could fit on top of each other. To prove that two triangles are congruent, we need to know that they meet one of these conditions:

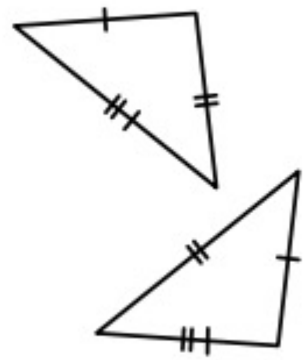
SAS: 2 corresponding sides and the angle inbetween them are the same.



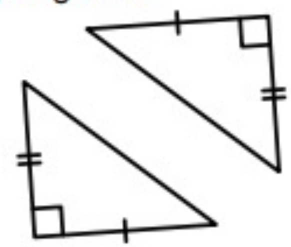
AAS: 2 angles and a corresponding side are the same



SSS: 3 sides the same



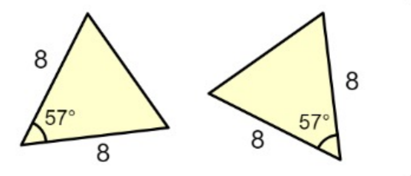
RSS: 2 corresponding sides being equal in a right triangle means all 3 are the same using Pythagoras.



Making the right decision:

For each pair of triangles, decide whether...

- they are congruent, giving a reason (SSS, ...)
- they are not congruent
- there is not enough information to decide

A


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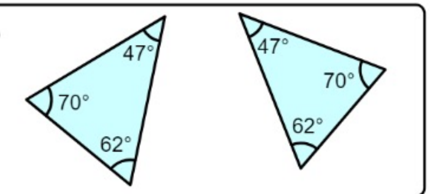

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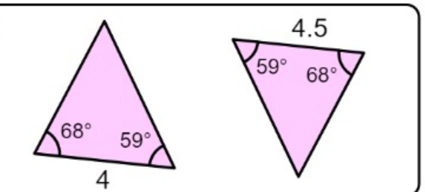

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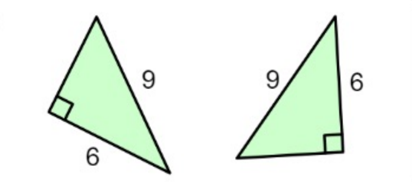

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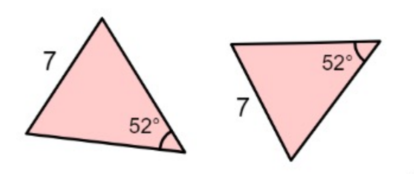

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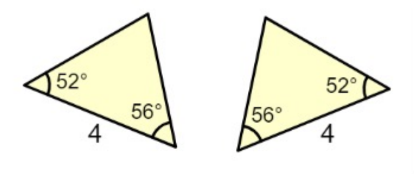

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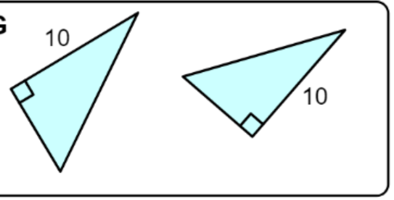

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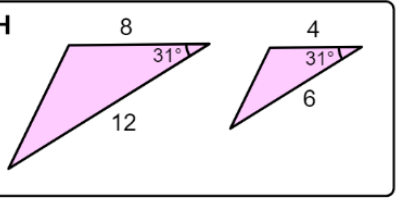

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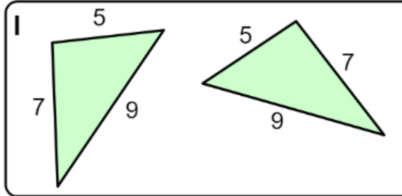

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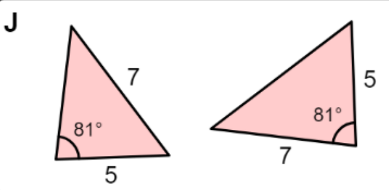

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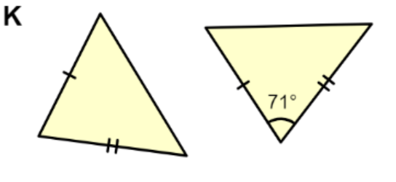

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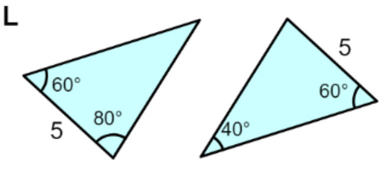

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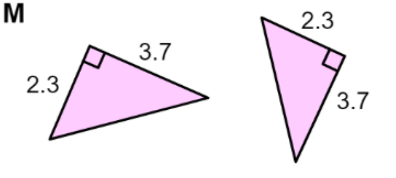

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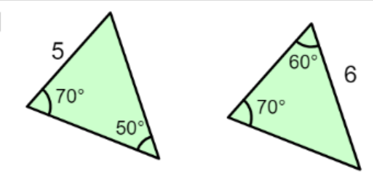

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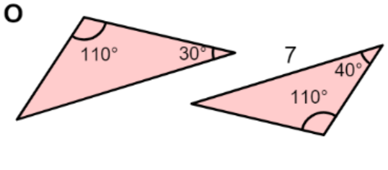

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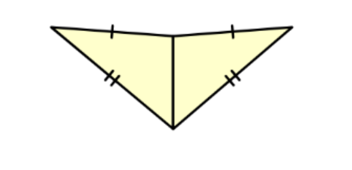

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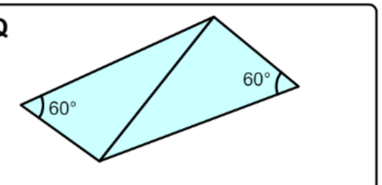

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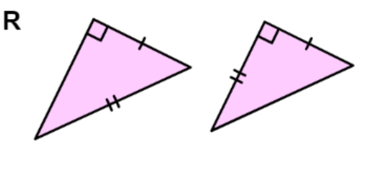
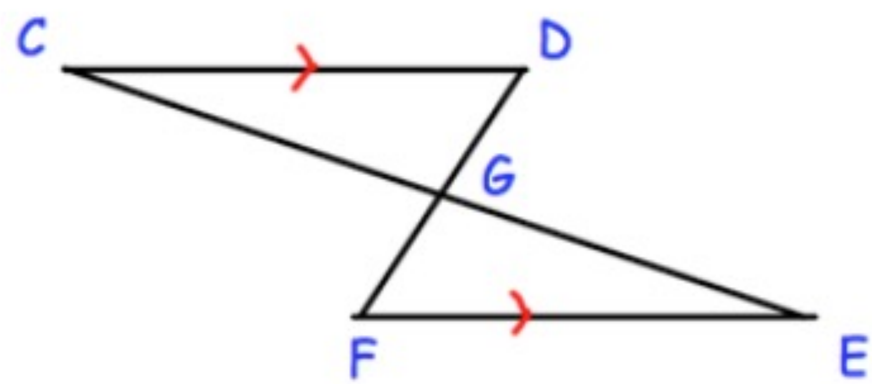
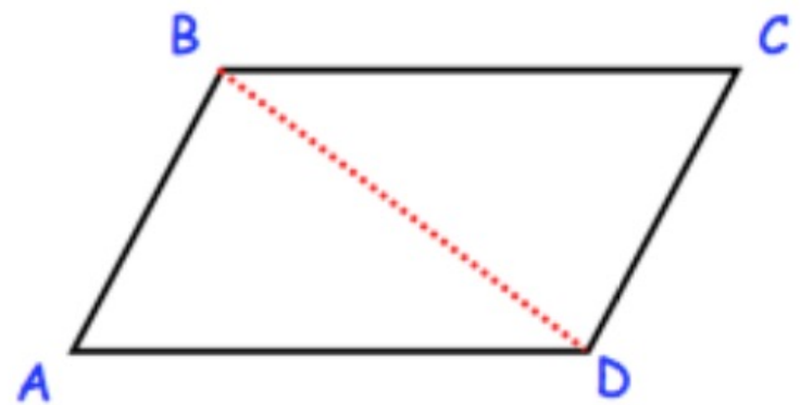


Worked example	Your turn
CD and FE are parallel, and CD = FE. Prove that triangles CDG and EFG are congruent.	ABCD is a parallelogram. Prove that triangles ABD and BCD are congruent.
	

Congruent Triangles

Video 67 on www.corbettmaths.com

Examples

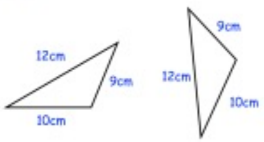
Workout

Click here

Scan here

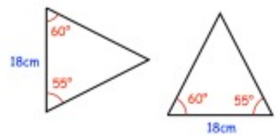
Question 1: The following pairs of triangles are congruent, state the condition that shows they are congruent.

(a)

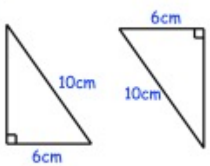


(b)

(c)



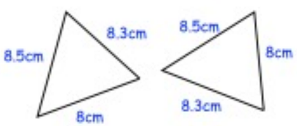
(d)



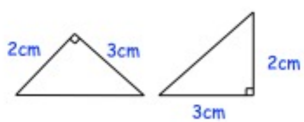
(e)

(f)

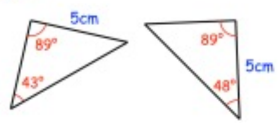
(g)



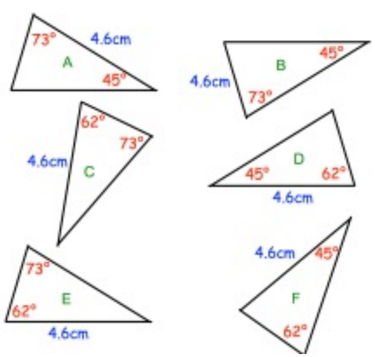
(h)



(i)



Question 2: Shown are six triangles.
Which triangles are congruent?



Congruent Triangles

Video 67 on www.corbettmaths.com

Question 3: In triangle ABC, AB = 7cm, ∠BAC = 50° and ∠ABC = 35°
In triangle DEF, EF = 7cm, ∠DEF = 35° and ∠DFE = 50°
Are triangles ABC and DEF congruent? If they are, state the condition.

Question 4: In triangle GHI, GH = 7cm, HI = 4cm and GI = 5cm.
In triangle JKL, JK = 7cm, KL = 4.5cm and JL = 5cm.
Are triangles GHI and JKL congruent? If they are, state the condition.

Question 5: In triangle MNO, ∠MNO = 50°, ∠NOM = 60° and ∠OMN = 70°
In triangle PQR, ∠PQR = 50°, ∠QRP = 60° and ∠RPQ = 70°
Are triangles MNO and PQR congruent? If they are, state the condition.

Question 6: In triangle STU, SU = 13cm, ∠TSU = 20° and ∠TUS = 30°
In triangle VWX, WX = 13cm, ∠WXV = 30° and ∠XVW = 20°
Are triangles STU and VWX congruent? If they are, state the condition.

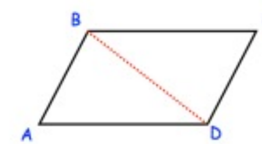
Apply

Question 1: Hannah and Chris each draw a triangle with one side of 3cm, one angle of 35° and one angle of 80°.
Hannah says their triangles **must** be congruent.
Is Hannah correct?

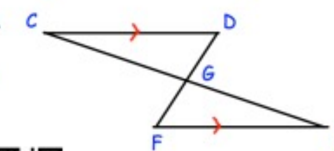
Question 2: Paul and Greg each draw a triangle with one side of 3cm, one side of 9cm and one side of 10cm.
Greg says their triangles **must** be congruent.
Is Greg correct?

Question 3: Carl and Michael each draw a triangle with one angle of 58°, one angle of 68° and one angle of 54°.
Carl says their triangles **must** be congruent.
Is Carl correct?

Question 4: ABCD is a parallelogram.
Prove that triangles ABD and BCD are congruent.



Question 5: In the diagram, the lines CE and DF intersect at G.
CD and FE are parallel and CD = FE.
Prove that triangles CDG and EFG are congruent.

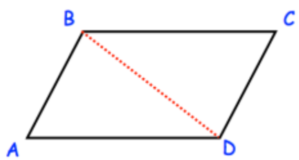


Answers

Click here

Scan here

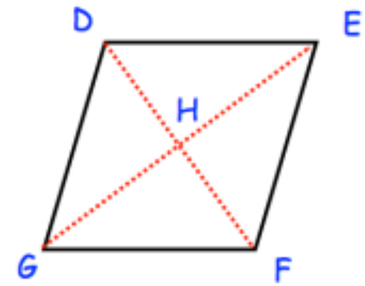
5. ABCD is a parallelogram.



Prove that triangles ABD and BCD are congruent.

(4)

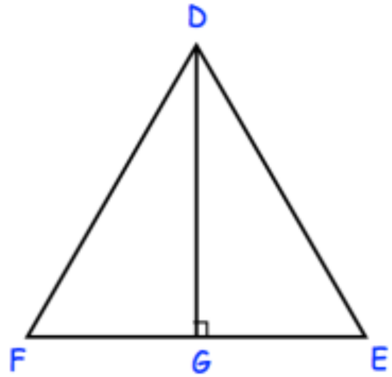
7. The diagram shows a rhombus DEFG.
The diagonals intersect at H.



Prove triangles DGH and EFH are congruent.

(4)

9. DEF is an equilateral triangle.

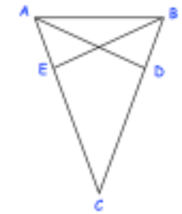


G lies on EF.
DG is perpendicular to FE.

Prove DFG is congruent to DEG.

(3)

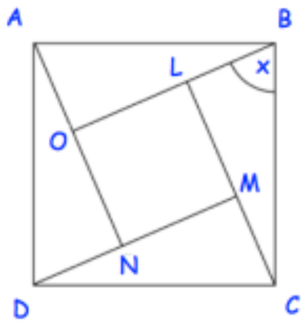
10. ABC is an isosceles triangle in which AC = BC.
D and E are points on BC and AC such that CE = CD.



Prove triangles ACD and BCE are congruent.

(4)

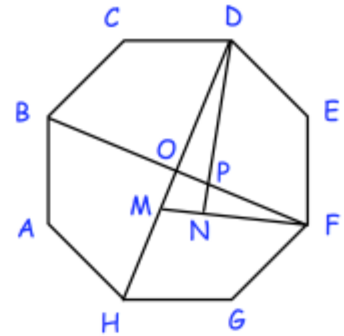
11. ABCD and LMNO are squares.
Angle CBL = x



Prove that triangles ABO and CBL are congruent.

(4)

21. ABCDEFGH is a regular octagon.
M is a point on the line DH.
N is a point on the line FM.
The lines DN and FM are perpendicular.



Prove that triangles FHM and DFP are congruent.

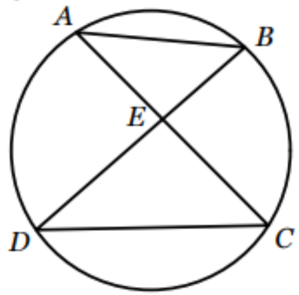
(4)

Circle theorems and Proof

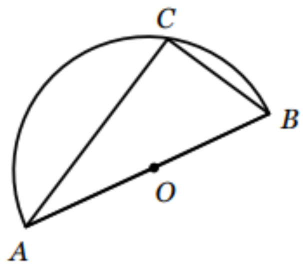
circle theorems and proof

Give reasons for each stage of your working.

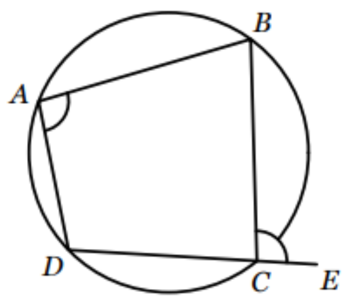
1. Prove that triangle AEB and triangle DEC are similar.



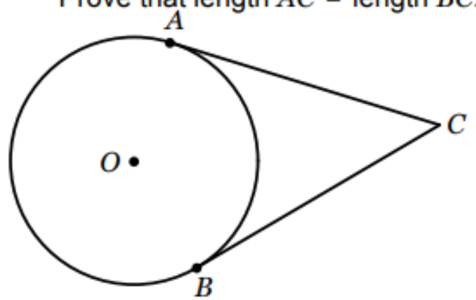
2. Point C lies on the arc of the semicircle. Prove that angle ACB is 90°.



3. Prove that angle DAB is equal to angle BCE.

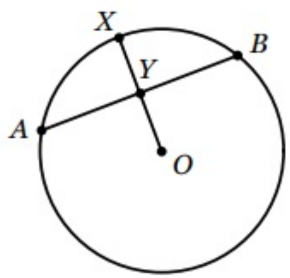


4. AC and BC are tangents to the circle. Prove that length AC = length BC.

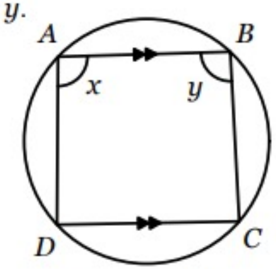


Circle theorems and Proof

5. OX is perpendicular to the chord AB. Prove that Y is the midpoint of AB.

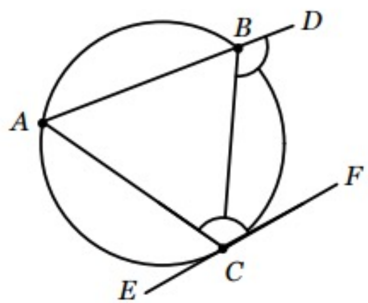


6. a) Prove that $x = y$.

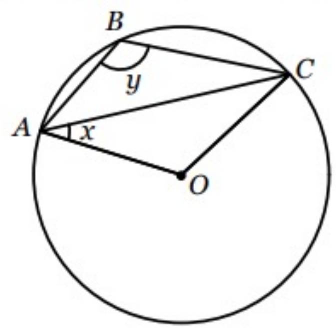


b) Jake says that x and y could both measure 80°. Is he right?

7. Prove that angle CBD = angle ACF.

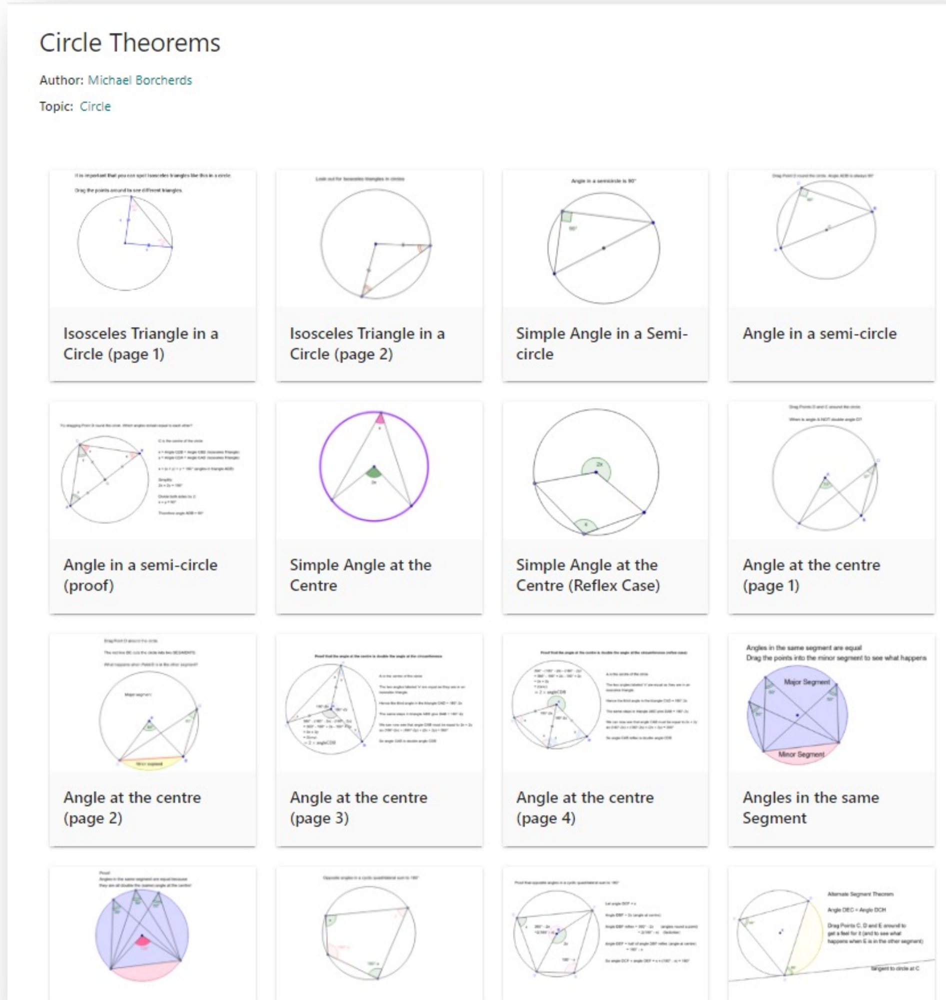


8. Prove that $y = x + 90$.



Proving each Circle Theorem

See: Circle Theorems – GeoGebra (geogebra.org/m/PFf7ehXE)



Proof of angle at the centre twice angle at circumference

Proof of angle in a semi-circle is a right angle

Proof of angles in the same segment are equal

Proof of opposite angles in a cyclic quadrilateral add up to 180 deg.

Proof of Alternate segment theorem