Year 7
Mathematics
Unit 1 – Student

Name: _____

Class: _____

Contents

1 Negative Numbers

Where do we use negative numbers in real life?

- Temperature
- Bank Balance
- Sea Level
- Elevator/Lift
- Golf
- Time Zones

Who discovered negative numbers?

https://www.youtube.com/watch?v=p62QItqtkQA&t=1361s

Do we say minus 4 or negative 4?

minuere *v.* to lessen, diminish (Latin)

minuet *n.*
1. A slow, graceful dance for two, involving very small steps.
2. A tune for a minuet dance, commonly in triple time, popular in the 18th century.

minus *prep.*
1. prep. Without e.g. I left my house minus my wallet.
2. prep. In mathematics, less or reduced by. Used to find the difference between two quantities.
3. n. A symbol, −, used to denote the operation of subtraction.
4. Often used to mean *negative*.

People might often refer to -4 as "minus 4" but mathematicians prefer to call this "negative 4".

1.1 Ordering Numbers

Numbers get smaller as you go down the number line.

```
↑
— 20
— 19
— 18
— 17
— 16
— 15
— 14
— 13
— 12
— 11
— 10
—  9
—  8
—  7
—  6
—  5
—  4
—  3
—  2
—  1
—  0
— −1
— −2
— −3
— −4
— −5
— −6
— −7
— −8
— −9
— −10
— −11
— −12
— −13
— −14
— −15
— −16
— −17
— −18
— −19
— −20
↓
```
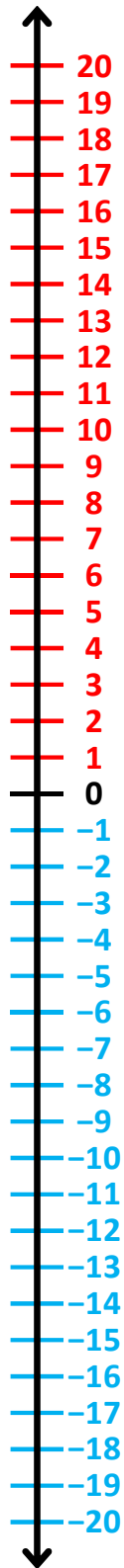
Worked Example	Your Turn
Write the following numbers in ascending order: $-2, -4, -3, 5, 0$	Write the following numbers in ascending order: $-7, -9, -8, 6, 0$

Fluency Practice

Question 2:   Arrange in order from smallest to largest

(a) 3, −5, 1, 0, −2, 4

(b) −1, 8, −5, 2, −9, −4, 3

(c) −1, −7, −2, 5, −6, 1

(d) 10, −7, −3, 5, −9, −2, −12

(e) 21, −3, 16, −19, −15, 23, −30

(f) −25, 35, 15, −5, 25, −45, 20

(g) 129, 101, −11, −111, 92, −91, 133, −29

Question 3:   Arrange these temperatures in order, from lowest to highest

(a)   8°C, 12°C, 9°C, 15°C, 11°C, 7°C, 2°C

(b)   2°C, −5°C, 4°C, 8°C, −3°C, 1°C, −7°C

(c)   5°C, −3°C, 11°C, 9°C, −14°C, 21°C, −1°C
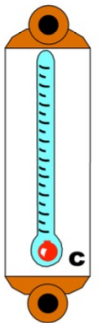
Question 5:

(a)   Arrange the towns in order of temperature, starting with the lowest.

(b)   How much warmer is it in Leek than Newtown?

(c)   Which town has a temperature closest to 0°C
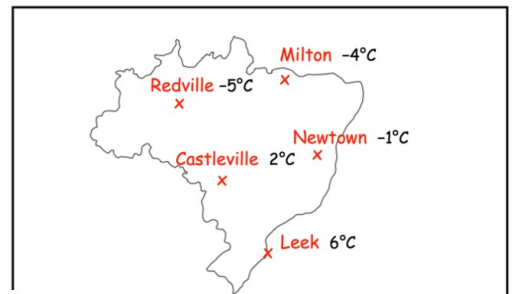
Milton −4°C
Redville −5°C
Newtown −1°C
Castleville 2°C
Leek 6°C

Maths Venns

If you think a region is impossible to fill, convince me why!

Ends in a 9

Bigger than –7
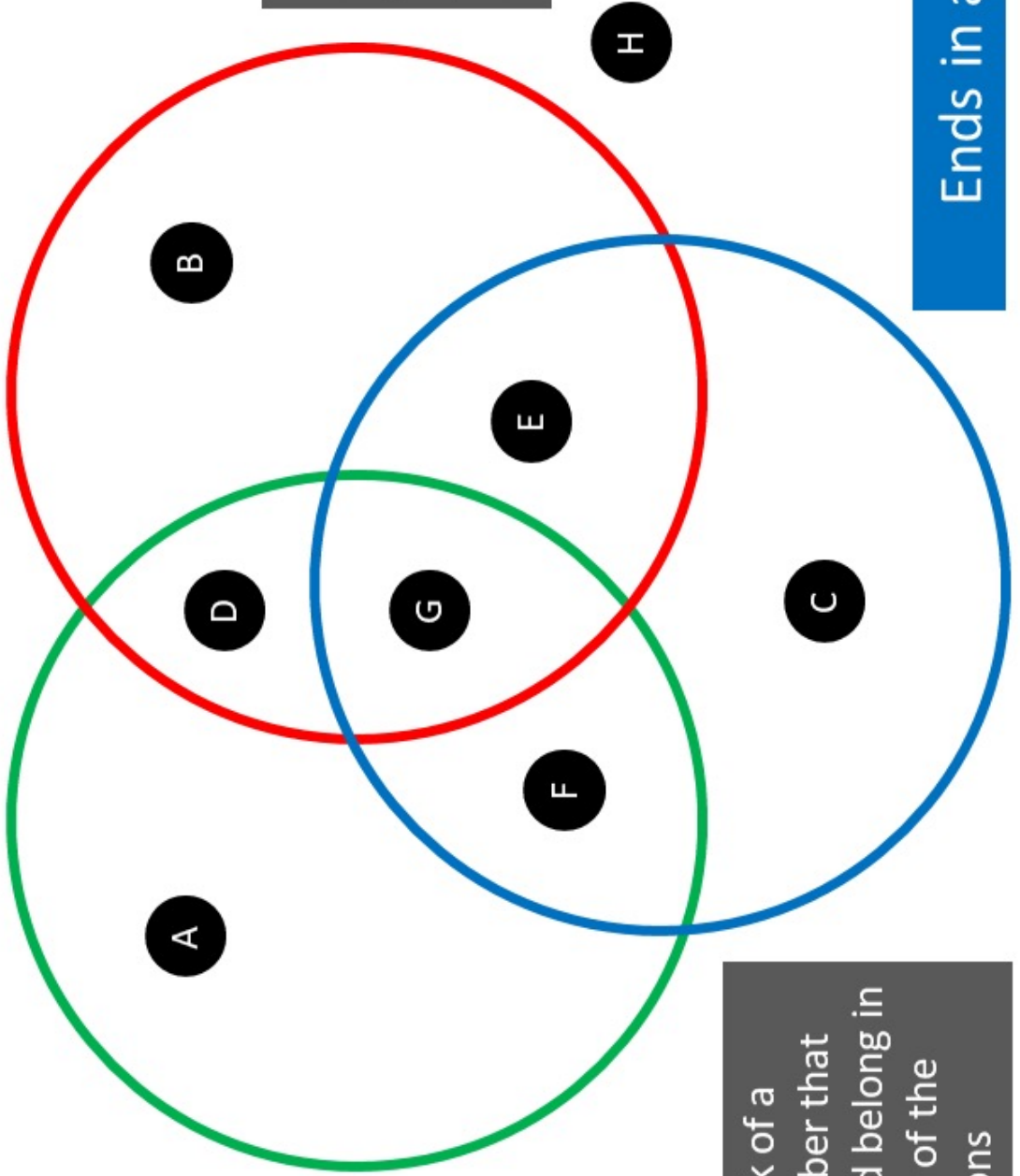
Think of a number that could belong in each of the regions

B

H

E

D

G

C

F

A

Signs Not Next to Each Other

We will first look at how to add and subtract negative numbers when the signs are *not* next to each other.

Worked Example	Your Turn
Calculate:	Calculate:
a) $3 - 4 =$	a) $5 - 7 =$
b) $-3 + 4 =$	b) $-5 + 7 =$
c) $-3 - 4 =$	c) $-5 - 7 =$
d) $-4 + 3 =$	d) $-7 + 5 =$
e) $-4 - 3 =$	e) $-7 - 5 =$

Fluency Practice

Question 1:   Work out the answers to each of the following

(a) 2 – 3   (b) 3 – 5   (c) 4 – 9   (d) 1 – 5

(e) 5 – 7   (f) 6 – 7   (g) 8 – 11   (h) 2 – 10

(i) –2 + 4   (j) –3 + 9   (k) –7 + 10   (l) –6 + 1

(m) –5 + 8   (n) –9 + 7   (o) –20 + 11   (p) –12 + 18

(q) –3 – 2   (r) –4 – 1   (s) –6 – 3   (t) –1 – 5

(u) –7 – 3   (v) –8 – 5   (w) –9 – 12   (x) –15 – 13

Question 2:   Work out the answers to each of the following

(a) 3 + 5 – 4   (b) 2 + 1 – 6   (c) 5 – 8 – 1   (d) 7 – 10 + 1

(e) 8 + 3 – 15   (f) 5 – 6 – 4   (g) 1 – 7 – 4   (h) –3 + 6 + 1

(i) –8 + 2 + 3   (j) –10 + 4 – 6   (k) –9 – 3 – 1   (l) –2 – 7 + 4

(m) –20 + 11 – 6   (n) –5 + 14 – 8   (o) –13 – 4 + 6   (p) –30 – 80 + 40

Signs Next to Each Other

We will now look at how to add and subtract negative numbers when the signs are next to each other.

Adding Negative Numbers Pattern Spotting

$3 + 5 =$ $(-3) + 5 =$

$3 + 4 =$ $(-3) + 4 =$

$3 + 3 =$ $(-3) + 3 =$

$3 + 2 =$ $(-3) + 2 =$

$3 + 1 =$ $(-3) + 1 =$

$3 + 0 =$ $(-3) + 0 =$

$3 + (-1) =$ $(-3) + (-1) =$

$3 + (-2) =$ $(-3) + (-2) =$

$3 + (-3) =$ $(-3) + (-3) =$

$3 + (-4) =$ $(-3) + (-4) =$

$3 + (-5) =$ $(-3) + (-5) =$

…   …

$3 + (-12) =$ $(-3) + (-12) =$

$3 + (-59) =$ $(-3) + (-59) =$

$$+ \ - \ \rightarrow \ -$$

$$- \ + \ \rightarrow \ -$$

Different Signs $\rightarrow$ $-$

Adding Negative Numbers Your Turn

$1 + (-5) =$

$(-5) + 1 =$

$(-1) + 5 =$

$(-1) + (-5) =$

$(-5) + (-1) =$

Subtracting Negative Numbers Pattern Spotting

$3 - 5 =$

$(-3) - 5 =$

$3 - 4 =$

$(-3) - 4 =$

$3 - 3 =$

$(-3) - 3 =$

$3 - 2 =$

$(-3) - 2 =$

$3 - 1 =$

$(-3) - 1 =$

$3 - 0 =$

$(-3) - 0 =$

$3 - (-1) =$

$(-3) - (-1) =$

$3 - (-2) =$

$(-3) - (-2) =$

$3 - (-3) =$

$(-3) - (-3) =$

$3 - (-4) =$

$(-3) - (-4) =$

$3 - (-5) =$

$(-3) - (-5) =$

…

…

$3 - (-12) =$

$(-3) - (-12) =$

$3 - (-59) =$

$(-3) - (-59) =$

Rules

$-\ -\ \rightarrow\ +$

$+\ +\ \rightarrow\ +$

Same Signs $\rightarrow\ +$

Subtracting Negative Numbers Your Turn

$1 - 3 =$

$(-1) - 3 =$

$(-1) - (-3) =$

$(-3) - (-1) =$

$3 - (-1) =$

Worked Example	Your Turn
Calculate:	Calculate:
a) $3 + (-4) =$	a) $5 + (-7) =$
b) $3 - (-4) =$	b) $5 - (-7) =$
c) $4 + (-3) =$	c) $7 + (-5) =$
d) $4 - (-3) =$	d) $7 - (-5) =$
e) $-3 + (-4) =$	e) $-5 + (-7) =$
f) $-3 - (-4) =$	f) $-5 - (-7) =$
g) $-4 + (-3) =$	g) $-7 + (-5) =$
h) $-4 - (-3) =$	h) $-7 - (-5) =$

Fluency Practice

Question 3:   Work out the answers to each of the following

(a) 4 + −1   (b) 6 + −2   (c) 8 + −7   (d) 3 + −5

(e) 1 + −7   (f) 3 + −10   (g) −2 + −1   (h) −1 + −6

(i)   −5 + −5   (j) −4 + −5   (k) −10 + −11   (l) −8 + −4

Question 4:   Work out the answers to each of the following

(a) 6 − +1   (b) 3 − +2   (c) 8 − +4   (d) 2 − +5

(e) 1 − +9   (f) −2 − +5   (g) −10 − +3   (h) −1 − +1

(i) 5 − +11   (j) −2 − +6   (k) −20 − +13   (l) 15 − +25

Question 5:   Work out each of the following

(a) 1 − −2   (b) 3 − −1   (c) 3 − −5   (d) 6 − −4

(e) 9 − −2   (f) −1 − −4   (g) −2 − −1   (h) −8 − −3

(i) −5 − −9   (j) − 6 − −7   (k) −15 − −8   (l) −12 − −30

Question 6:   Work out each of the following

(a) 11 − 15   (b) −9 + 5   (c) −4 − 8   (d) −4 + −3

(e) −9 − +4   (f) 10 − −3   (g) 7 − 20   (h) −2 − −5

(i) 12 + −7   (j) −4 − −1   (k) −9 + −8   (l) 8 − 13

(m) 6 − −11   (n) −7 − +7   (o) −6 − 5   (p) −20 + −3

(q) −9 − −15   (r) −8 + 25   (s) 31 − 50   (t) −30 − −16

(u) −41 − 14   (v) − 5 − +23   (w) −16 + −15   (x) 40 − −40

(y) −18 − −27   (z) −52 + 90

Intelligent Practice

Calculate:

1) $5 + 3 =$

2) $3 + 5 =$

3) $(-3) + 5 =$

4) $5 + (-3) =$

5) $(-5) + (-3) =$

6) $(-5) + 3 =$

7) $(-5) - 3 =$

8) $(-3) - 5 =$

9) $3 - 5 =$

10) $3 - (-5) =$

11) $-3 - (-5) =$

12) $(-5) - (-3) =$

13) $(-5.2) - (-3) =$

14) $(-5.2) + (-3) =$

15) $(-1.2) + (-3) =$

16) $(-1.2) + 3 =$

17) $(-1.2) - (-3) =$

18) $(-1.2) - (-5) =$

19) $1.2 - 5 =$

Think of 2 Numbers

	Sum is Positive	Sum is Zero	Sum is Negative
Positive + Positive			
Negative + Negative			
Positive + Negative			
Negative + Positive			

	Difference is Positive	Difference is Zero	Difference is Negative
Positive - Positive			
Negative - Positive			
Positive - Negative			
Negative - Negative			

Maths Venns

One number is -2

The answer ends in a 5

The answer is negative
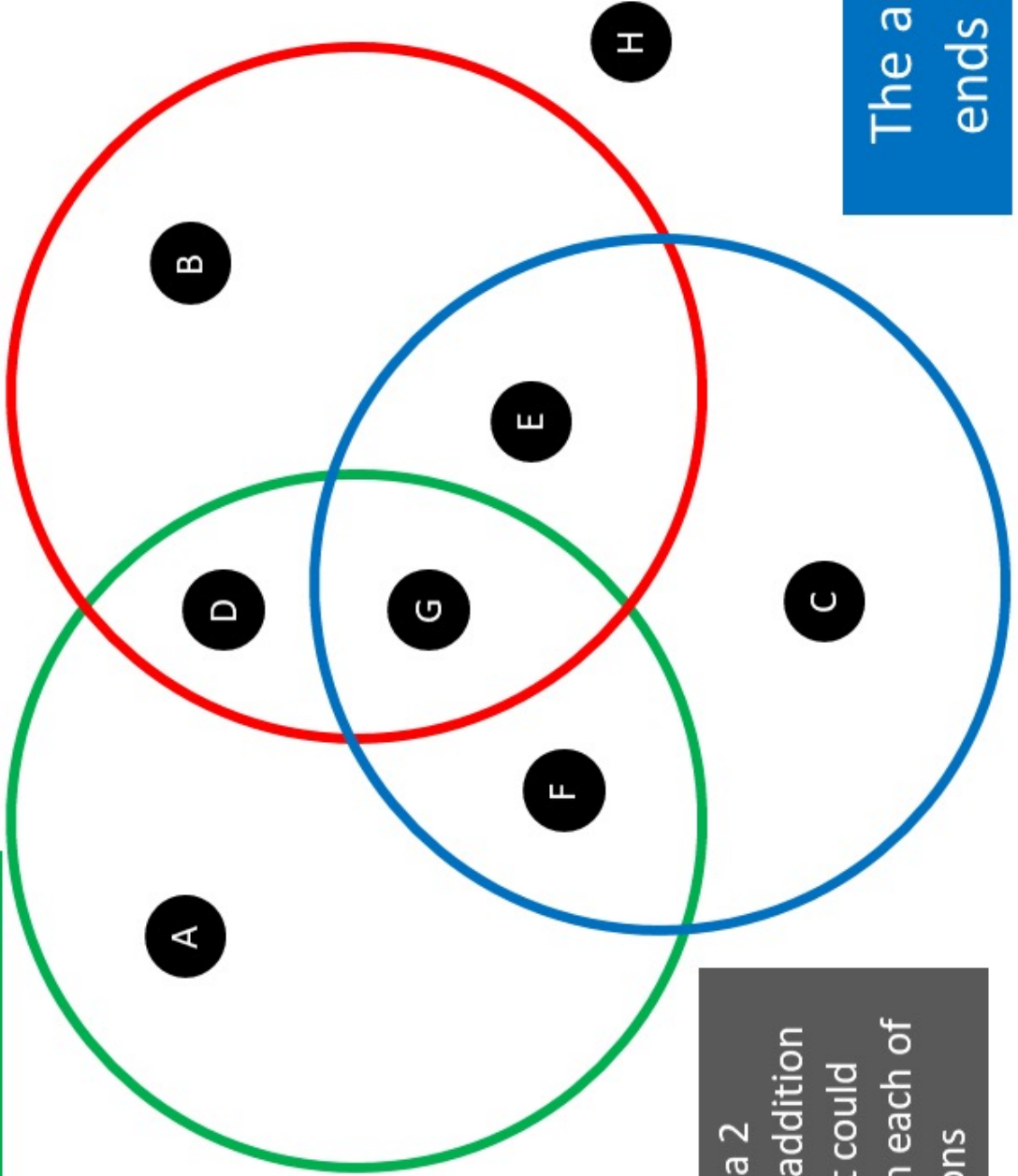
Think of a 2 number addition sum that could belong in each of the regions

Maths Venns

One number is -4

The answer ends in a 2

The answer is positive
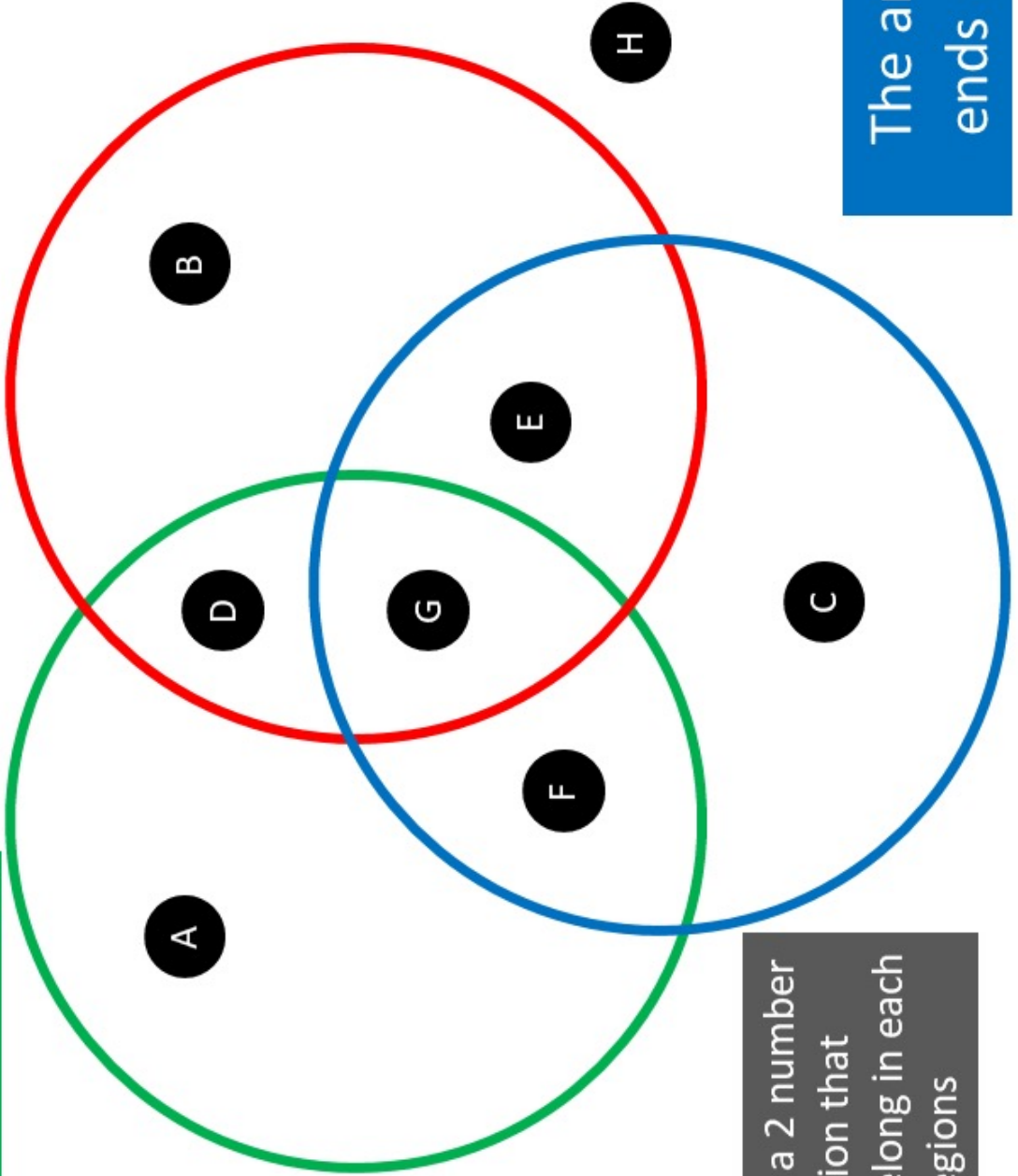
Think of a 2 number subtraction that could belong in each of the regions

H

B

E

D

G

C

F

A

Multiplying Negative Numbers Pattern Spotting

$3 \times 5 =$ $(-3) \times 5 =$

$3 \times 4 =$ $(-3) \times 4 =$

$3 \times 3 =$ $(-3) \times 3 =$

$3 \times 2 =$ $(-3) \times 2 =$

$3 \times 1 =$ $(-3) \times 1 =$

$3 \times 0 =$ $(-3) \times 0 =$

$3 \times (-1) =$ $(-3) \times (-1) =$

$3 \times (-2) =$ $(-3) \times (-2) =$

$3 \times (-3) =$ $(-3) \times (-3) =$

$3 \times (-4) =$ $(-3) \times (-4) =$

$3 \times (-5) =$ $(-3) \times (-5) =$

…   …

$3 \times (-12) =$ $(-3) \times (-12) =$

$3 \times (-59) =$ $(-3) \times (-59) =$

Rules

+ × + → +

+ × − → −

− × + → −

− × − → +

Different Signs → −

Same Signs → +

Multiplying Negative Numbers Your Turn

$4 \times (-5) =$

$(-4) \times 5 =$

$5 \times (-4) =$

$(-5) \times (-4) =$

$(-5) \times (-4) \times (-3) =$

Worked Example	Your Turn
Calculate:	Calculate:
a) $3 \times (-4) =$	a) $5 \times (-7) =$
b) $4 \times (-3) =$	b) $7 \times (-5) =$
c) $(-3) \times 4 =$	c) $(-5) \times 7 =$
d) $(-3) \times (-4) =$	d) $(-5) \times (-7) =$
e) $(-4) \times 3 =$	e) $(-7) \times 5 =$
f) $(-4) \times (-3) =$	f) $(-7) \times (-5) =$

Worked Example	Your Turn
Calculate:	Calculate:
a) $(-4)^2 =$	a) $(-7)^2 =$
b) $(-4)^3 =$	b) $(-7)^3 =$
c) $(-4)^4 =$	c) $(-7)^4 =$

Fluency Practice

Question 1: Answer each of the following multiplications

(a) 2×-3

(b) -4×3

(c) -5×5

(d) -7×-2

(e) -6×-3

(f) 8×-4

(g) -9×3

(h) -5×-8

(i) -9×7

(j) 10×-8

(k) 7×-4

(l) 6×8

(m) -11×3

(n) 4×-15

(o) -12×-12

(p) -5×7

(q) 9×-8

(r) -7×-8

(s) 12×-6

(t) 4×-13

(u) -11×10

(v) -20×-6

(w) 14×7

(x) -18×-13

(y) 25×-7

(z) -16×21

Question 2: Answer each of the following multiplications

(a) $2 \times 3 \times -2$

(b) $-3 \times 2 \times 5$

(c) $-5 \times -6 \times 2$

(d) $10 \times -3 \times -4$

(e) $-9 \times 2 \times -2$

(f) $-4 \times -3 \times -5$

(g) $-8 \times -8 \times -2$

(h) $5 \times -4 \times -7$

Question 3: Work out each of the following

(a) $(-3)^2$

(b) $(-6)^2$

(c) $(-2)^2$

(d) $(-1)^2$

(e) $(-10)^2$

(f) $(-8)^2$

(g) $(-12)^2$

(h) $(-20)^2$

Question 4: Work out each of the following

(a) $(-2)^3$

(b) $(-3)^3$

(c) $(-1)^3$

(d) $(-5)^3$

(e) $(-1)^4$

(f) $(-10)^4$

(g) $(-2)^4$

(h) $(-3)^4$

Maths Venns

One number is -1

The answer ends in a 4

The answer is negative
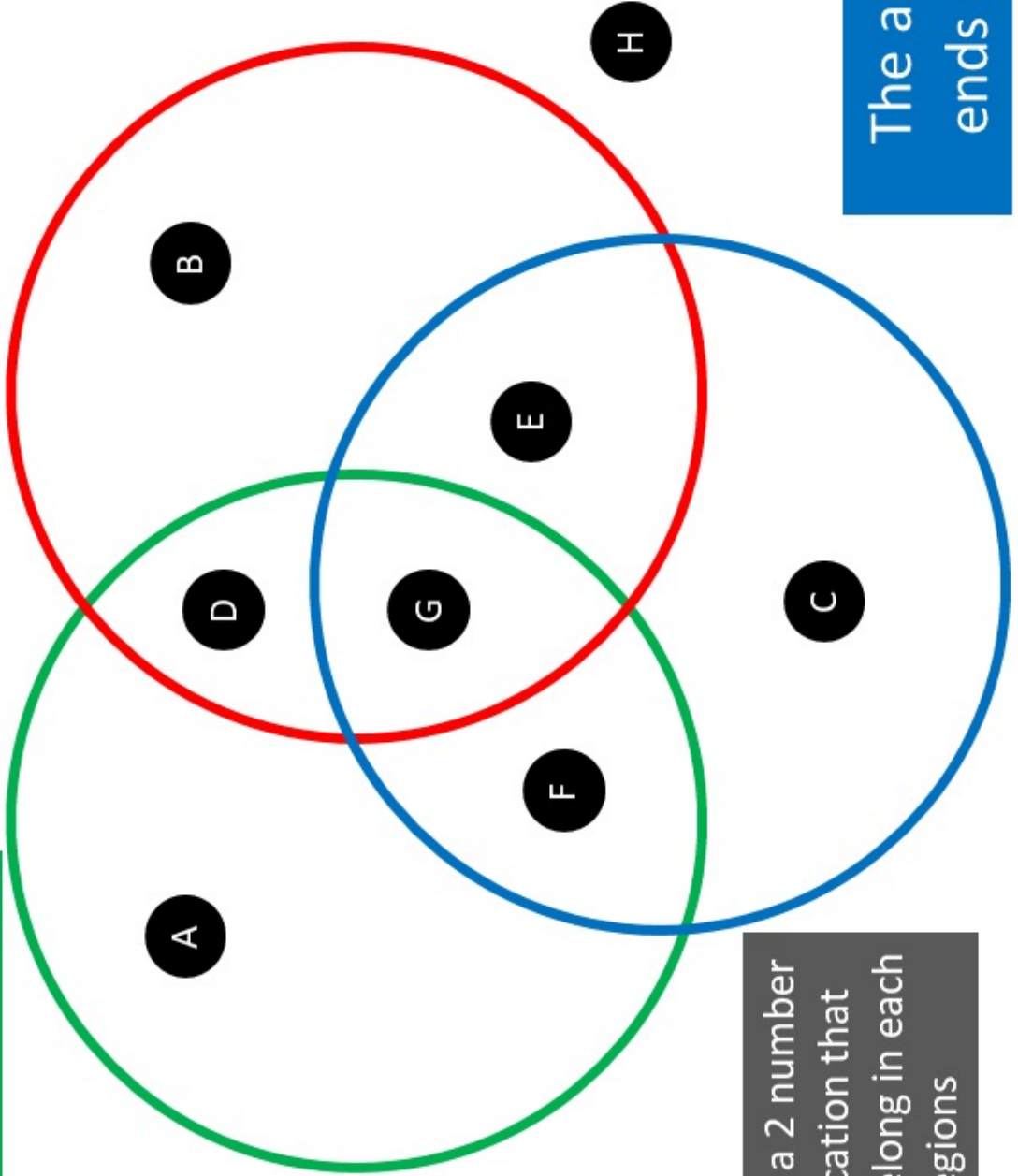
Think of a 2 number multiplication that could belong in each of the regions

H

B

E

D

G

C

F

A

Dividing Negative Numbers Pattern Spotting

$15 \div 3 =$ $15 \div (-3) =$

$12 \div 3 =$ $12 \div (-3) =$

$9 \div 3 =$ $9 \div (-3) =$

$6 \div 3 =$ $6 \div (-3) =$

$3 \div 3 =$ $3 \div (-3) =$

$0 \div 3 =$ $0 \div (-3) =$

$(-3) \div 3 =$ $(-3) \div (-3) =$

$(-6) \div 3 =$ $(-6) \div (-3) =$

$(-9) \div 3 =$ $(-9) \div (-3) =$

$(-12) \div 3 =$ $(-12) \div (-3) =$

$(-15) \div 3 =$ $(-15) \div (-3) =$

...   ...

$(-36) \div 3 =$ $(-36) \div (-3) =$

$(-81) \div 3 =$ $(-81) \div (-3) =$

Rules

$$+ \div + \rightarrow +$$

$$+ \div - \rightarrow -$$

$$- \div + \rightarrow -$$

$$- \div - \rightarrow +$$

Different Signs $\rightarrow$ $-$

Same Signs $\rightarrow$ $+$

Dividing Negative Numbers Your Turn

$12 \div (-2) =$

$(-12) \div (-2) =$

$(-12) \div 2 =$

$(-12) \div (-2) =$

$(-12) \div (-2) \div (-2) =$

Worked Example	Your Turn
Calculate:	Calculate:
a) $12 \div -3 =$	a) $35 \div -5 =$
b) $12 \div -4 =$	b) $35 \div -7 =$
c) $-12 \div -3 =$	c) $-35 \div -5 =$
d) $-12 \div -4 =$	d) $-35 \div -7 =$

Fluency Practice

Question 5:   Answer each of the following divisions

(a) $-10 \div 2$

(b) $-12 \div 3$

(c) $-24 \div 4$

(d) $-42 \div 6$

(e) $9 \div -3$

(f) $21 \div -7$

(g) $-44 \div 11$

(h) $-72 \div 9$

(i) $-10 \div -5$

(j) $-28 \div -4$

(k) $-30 \div -3$

(l) $-48 \div -8$

(m) $-6 \div 6$

(n) $24 \div -3$

(o) $-12 \div -12$

(p) $-132 \div 11$

(q) $72 \div -8$

(r) $-108 \div -9$

(s) $36 \div -9$

(t) $100 \div -4$

(u) $-95 \div 5$

(v) $-49 \div -7$

(w) $144 \div 12$

(x) $-215 \div -5$

(y) $90 \div -15$

(z) $-342 \div 9$

Question 6:   Answer each of the following divisions

(a) -9×-5

(b) $-32 \div 8$

(c) $66 \div -6$

(d) 2×-12

(e) $-24 \div -3$

(f) -12×7

(g) $-54 \div 6$

(h) -16×-2

(i) 8×-6

(j) -7×-6

(k) $40 \div -8$

(l) $56 \div -7$

(m) $-81 \div -9$

(n) -14×-5

(o) 10×-11

(p) $-65 \div 5$

(q) -90×-3

(r) $-170 \div -10$

(s) $1 \div -1$

(t) -1.5×-3

(u) $-17 \div 2$

(v) 2.2×-10

(w) $-93 \div -10$

(x) -6.2×-3

(y) -9×10.5

(z) $52 \div -5$

Intelligent Practice

Calculate:

1) $2 \times 10 =$

2) $10 \times 2 =$

3) $(-10) \times 2 =$

4) $10 \times (-2) =$

5) $(-10) \times (-2) =$

6) $(-10) \div (-2) =$

7) $10 \div (-2) =$

8) $(-10) \div 2 =$

9) $2 \div (-10) =$

10) $2 \div 10 =$

11) $10 \times 2 \times 2 =$

12) $10 \times 2 \times (-2) =$

13) $10 \times (-2) \times (-2) =$

14) $(-10) \times (-2) \times (-2) =$

15) $(-10) \div (-2) \times (-2) =$

16) $10 \div (-2) \times (-2) =$

Maths Venns

One number is −4

The answer is a whole number

The answer is negative
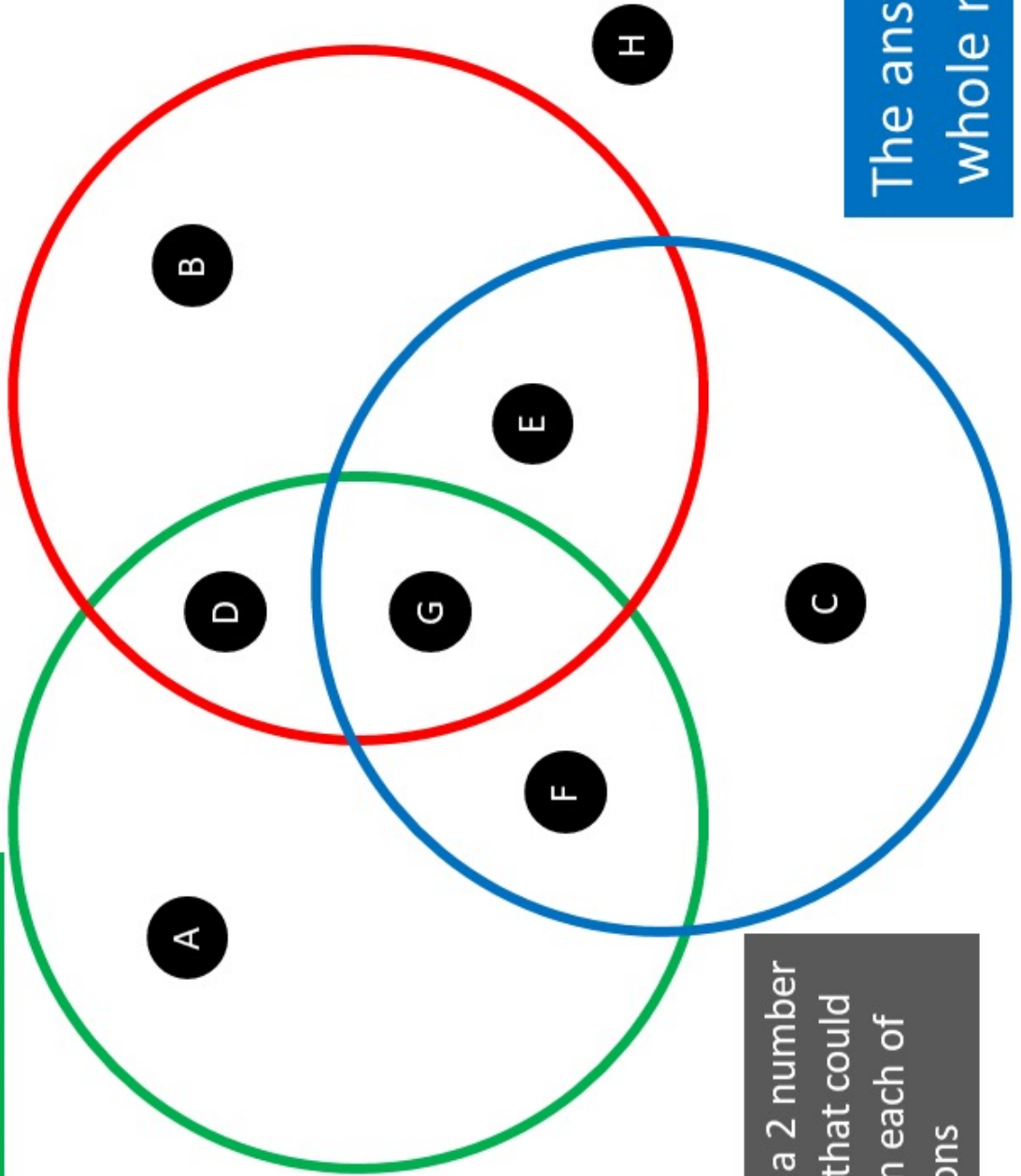
Think of a 2 number division that could belong in each of the regions

B

H

E

D

G

C

F

A

Fill in the Gaps

Q	Number 1	+ or −	Amount		Number 2
1	8	−	3	=	
2	3	−	8	=	
3	3	−		=	−4
4		−	6	=	−4
5	−2	−	6	=	
6	−2	+	6	=	
7	−2	+		=	5
8		+	7	=	4
9	−3	+	−7	=	
10	−3	−		=	−10
11	−3	−	−7	=	
12	−3	−		=	−4
13		−	−1	=	−4
14	−5	+	1	=	
15	−5	+		=	−6
16		+	−1	=	0
17		−	−1	=	0
18	−1	−	−0.5	=	
19		−	−0.5	=	0.5
20		+	−0.5	=	0.5

Directed Numbers Puzzle

Use these clues to work out the numbers in the grid.

Clues

1. The number in the centre equals - 5 - 3

2. The number that goes in the top right box is 5 less than -5

3. The number in the bottom left is –3 × –3

4. The number in the top left is the number in the centre add –3 × –1

5. The number in the middle of the bottom row is 6 less than the number above it.

6. There is a number 6 in one of the middle row boxes.

7. There is a number equal to –4 × –5 next to the box containing –5

8. When you add up the right hand column you get –27

9. The number below –5 is -6 - 10

Directed Target

using the three given numbers try to make the target of **− 10**
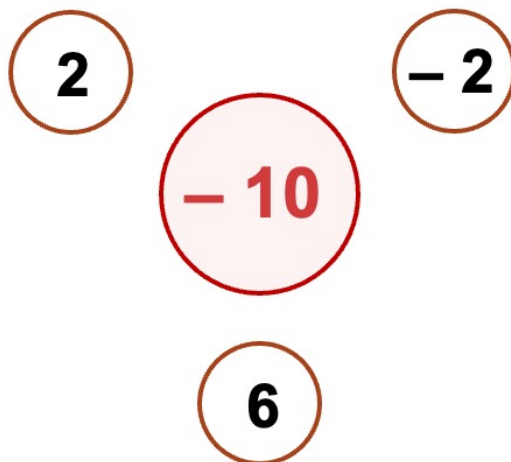
use any of the operations:

$-$, $+$, $\times$

2

− 2

− 10

6

---

can you make the target given?
you must use *all* three numbers and any of: $-$, $+$, $\times$

target: **− 5** using   1 , 3 , − 2

target: **− 5** using   1 , 2 , − 3

target: **− 13** using   2 , 3 , − 5

target: **− 17** using   3 , 5, − 4

target: **− 37** using   5 , 7 , − 6

try to find two (or more) solutions to each of these

what other *negative* integer targets can be reached?

Magic Squares

complete the magic squares so they have the same row, column and diagonal totals

−1		
30	6	
		13

8		
	1	
	2	−6

11		
	4	16
		−3

		−3
	1	
5		8

		−9
20	1	
	11	

	9	
	1	
−1	−7	

explore the relationship for the sum of the four corner numbers with a 1 in the centre

Magic Squares

Puzzle 1

		1
	−3	−8
−7		

Puzzle 2

−3		−19
		6
		−17

Puzzle 3

		2
	−2	
−6		8

Puzzle 4

		−8
	−5	
−2		−1

Puzzle 5

		4
	−1	−2
−6		

Puzzle 6

2		−8
		6
		−10

bidmas

(	)
n^a	$\sqrt{n}$
÷	×
+	−
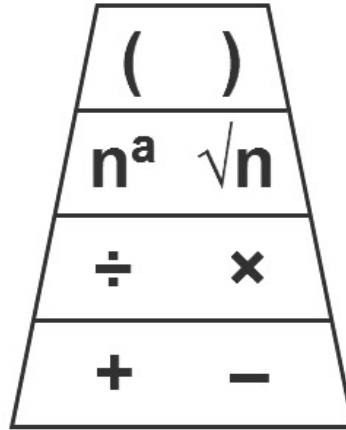
BODMAS / BIDMAS

Remember, it must be used like this:

First do any: **(B**rackets**)**

Followed by any: **I**ndices

Left to right do any: **D÷**vision & **M×**ltiplication

Lastly, left to right: **A**ddi**+**ion & **S**ubtr**−**ction

Why do we need an Order of Operations?

What's the answer to this?

$$4 + 10 \div 2 = ?$$

Did you get **7**?

Do you see how the answer could be **9**?

There are two possible ways to do this problem:

$$4 + 10 \div 2$$

Do the addition first:

$$4 + 10 \div 2$$
$$14 \div 2$$
$$14 \div 2$$
$$7$$

Do the division first:

$$4 + 10 \div 2$$
$$4 + 5$$
$$4 + 5$$
$$9$$

Which is it?

Both answers can't be right or we'd always be arguing about the answers to math problems. The nice thing about math is that there's always just ONE answer!

So, a long time ago, math geeks decided to make a set of rules for what to do first in a math problem.

These rules are called "the order of operations."

2.1 Same Level Operations

We will now look at questions where we have to work from left to right using addition and subtraction or multiplication and division.

Worked Example	Your Turn
Calculate: a) $8 - 3 + 2 =$ b) $8 \times 3 \div 2 =$	Calculate: a) $10 - 4 + 1 =$ b) $9 \times 4 \div 3 =$

Intelligent Practice

Calculate:

1) $5 + 4 + 3 =$

2) $5 - 4 + 3 =$

3) $5 + 4 - 3 =$

4) $5 - 4 - 3 =$

5) $5 - (4 - 3) =$

6) $(5 + 4) - 3 =$

7) $5 + (4 - 3) =$

8) $5 - (4 + 3) =$

9) $(5 - 4) + 3 =$

10) $20 \times 10 \times 2 =$

11) $20 \div 10 \times 2 =$

12) $20 \times 10 \div 2 =$

13) $20 \div 10 \div 2 =$

14) $20 \div (10 \div 2) =$

15) $(20 \times 10) \div 2 =$

16) $20 \times (10 \div 2) =$

17) $20 \div (10 \times 2) =$

18) $(20 \div 10) \times 2 =$

2.2 Addition and Multiplication/Division

We will now look at questions where we have use addition and multiplication/division.

"Pencils come in packs of 10. I have 4 full packs and 7 extra pencils. How many pencils do I have?"

Strategies:
I have 4 packs of 10 pencils plus 7 extra so I do:
$$4 \times 10 + 7 = 47$$

It is like 5 packs of 10 pencils with three missing so I do:
$$5 \times 10 - 3 = 47$$

I have 7 pencils plus 4 packs of 10, so I do:
$$7 + 4 \times 10 = 47$$

In the final example, we do not try to add the 7 and the 4 before multiplying by 10 because it doesn't make sense to do so. The **context** of the question helps them understand this.

Worked Example	Your Turn
Calculate: $2 + 3 \times 4 =$	Calculate: $3 + 2 \times 4 =$

Intelligent Practice

Calculate:

1) $5 \times 2 + 3 =$

2) $3 + 5 \times 2 =$

3) $4 + 5 \times 2 =$

4) $4 + 5 \times 3 =$

5) $4 + (5 \times 3) =$

6) $(4 + 5) \times 3 =$

7) $3 \times (4 + 5) =$

8) $3 \times (4 + 6) =$

9) $3 \times 4 + 6 =$

10) $3 \times 4 + 6 \times 2 =$

11) $6 \times 2 + 3 \times 4 =$

12) $6 \times 2 + (3 \times 4) =$

13) $6 \times (2 + 3) \times 4 =$

14) $6 + (2 + 3) \times 4 =$

15) $(6 + 2 \times 3) \times 4 =$

Worked Example	Your Turn
Calculate: $6 + 8 \div 3 =$	Calculate: $8 + 12 \div 4 =$

Intelligent Practice

Calculate:

1) $(4 + 10) \div 2 =$

2) $4 + 10 \div 2 =$

3) $4 + \dfrac{10}{2} =$

4) $\dfrac{10}{2} + 4 =$

5) $\dfrac{10+4}{2} =$

6) $\dfrac{10}{2} + \dfrac{4}{2} =$

Fluency Practice

Question 1:   Work out

(a) 7 + 2 x 3

(b) 9 + 4 x 2

(c) 10 + 2 x 2

(d) 18 + 4 ÷ 2

(e) 20 – 5 x 2

(f) 8 – 2 x 3

(g) 21 – 9 ÷ 3

(h) 100 – 40 x 2

(i) 16 ÷ 1 – 3

(j) 5 + 5 x 5

(k) 13 – 7 ÷ 1

(l) 7 x 6 – 4

(m) 9 + 3 – 2

(n) 20 – 5 + 6

(o) 21 – 17 + 4

(p) 30 x 4 ÷ 2

(q) (7 + 7) ÷ 2

(r) 35 – (9 + 3)

(s) 40 x (2 + 3)

(t) 60 ÷ (1 + 5)

(u) 15 ÷ (3 + 2)

(v) 9 x (7 + 4)

(w) 90 ÷ (52 – 7)

(x) (8 + 9) x 3

(y) 10 + 5 + 3 x 3

(z) 100 – 6 + 2 x 3

2.3 Powers and Roots

We will now look at questions where we have to calculate powers and roots.

Worked Example	Your Turn
Calculate: $5 + 3 \times 4^2 =$	Calculate: $5 + 4 \times 3^2 =$

Intelligent Practice

Calculate:

1) $4 + 2 \times 3^2 =$

2) $(4 + 2) \times 3^2 =$

3) $4 + (2 \times 3)^2 =$

4) $(4 + 2 \times 3)^2 =$

5) $(4 + 2 - 3)^2 =$

6) $4 + (2 - 3)^2 =$

7) $4 \times (2 - 3)^2 =$

8) $(4 \times 2 - 3)^2 =$

Worked Example	Your Turn
Calculate:	Calculate:
a) $3 \times 9 - \sqrt{16} =$	a) $3 \times 16 - \sqrt{9} =$
b) $\sqrt{25} \times 2 + 14 =$	b) $\sqrt{25 \times 2 + 14} =$

Intelligent Practice

Calculate:

1) $12 + 4 \times \sqrt{9} =$

2) $(12 + 4) \times \sqrt{9} =$

3) $12 + \sqrt{4 \times 9} =$

4) $\sqrt{12 + 4} \times 9 =$

5) $12 + \sqrt{4} \times 9 =$

6) $12 \times \sqrt{4} + 9 =$

7) $12 \times \sqrt{4} - 9 =$

8) $12 \times \left(\sqrt{4} - 9\right) =$

9) $12 - 9 \times \sqrt{4} =$

10) $12^2 - 9 \times \sqrt{4} =$

11) $(12^2 - 9) \times \sqrt{4} =$

12) $12^2 - \sqrt{9 \times 4} =$

Fluency Practice

Question 2:   Work out

(a) $5 - 2^2$

(b) $7 + 3^2$

(c) $9^2 + 1$

(d) $6^2 - 5^2$

(e) $(7 - 2)^2$

(f) $(4 + 3)^2$

(g) $(1 + 2)^3$

(h) $(2 + 8)^3$

(i) $10 - \sqrt{16}$

(j) $\sqrt{(2 + 14)}$

(k) $\sqrt{4} + 3^2$

(l) $2 \times 5 - \sqrt{4}$

Question 3:   Work out

(a) $5 \times 3 + 2 \times 6$

(b) $9 \div 3 + 15 \times 2$

(c) $10 \div 2 - 2 \times 1$

(d) $5 \times (2 + 1) + 4$

(e) $8 + (5 - 1) \times 3$

(f) $50 - (1 + 4) \times 4$

(g) $19 \times 2 + 5^2$

(h) $8^2 + 2 \times 3^2$

(i) $7 \times (8 \div 4)^2$

(j) $11 + 11 - 6^2 \div 2$

2.4 Inserting Brackets

We will now look at questions where we have to insert brackets to make the calculation true.

Worked Example	**Your Turn**
Insert brackets to make the following calculations true: a) $8 + 4 \times 5 - 2 = 20$ b) $8 + 4 \times 5 - 2 = 58$ c) $8 + 4 \times 5 - 2 = 26$ d) $8 + 4 \times 5 - 2 = 36$	Insert brackets to make the following calculations true: a) $7 + 3 \times 5 - 1 = 49$ b) $7 + 3 \times 5 - 1 = 40$ c) $7 + 3 \times 5 - 1 = 19$ d) $7 + 3 \times 5 - 1 = 21$

Fluency Practice

Question 4:   Copy out the following and insert brackets in each to make the correct answer.

(a) $10 \times 2 + 6 = 80$

(b) $5 + 5 \div 5 = 2$

(c) $18 - 6 \div 2 = 6$

(d) $5 + 2 \times 3 + 1 = 13$

(e) $2 \times 7 + 1 \times 3 = 48$

(f) $9 + 3^2 \times 10 \div 2 = 90$

Intelligent Practice

Insert brackets to make the following calculations true:

1) $5 + 4 \times 2 = 13$

2) $5 + 4 \times 3 = 22$

3) $5 + 4 \times 3 = 27$

4) $5 + 4 \times 3^2 = 81$

5) $5 + 4 \times 3^2 = 41$

6) $5 + 4 \times 3^2 = 149$

7) $5 + 4 \times 3^2 - 2 = 147$

8) $5 + 4 \times 3^2 - 2 = 63$

9) $5 - 4 + 3^2 \times 2 = 19$

10) $5 - 4 + 3^2 \times 2 = 20$

order of operating

what are:

(1) $7 - 3 \times 2 =$

(2) $4 + 6 \div 2 =$

(3) $(7 + 8) \div 5 =$

(4) $(2 \times 3^2 - 2^2) \div 7 =$

(5) $3 \times 5 - 6 \div 3 \times 5 =$

(6) $2 \times 5^2 - 2^3 \times 5 =$

(7) $5 \times 3 - 2^2 \times 3 \div 4 - 3 =$

(8) $2(5^2 - 4^2) \div 3 + 2 =$

(9) $1 + 2(3 \times 4 - 5) - 3^2 =$

(10) $2(2 \times 3^2 - 4) - 3 \times 2^3 =$

use the digits:
1 , 1 , 2 , 4 and 6
once only and in
any order
involving brackets
and + , − , × and ÷
(as many times as
you like) to make

39

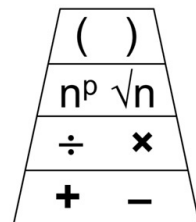try to find three

solutions

$$2 \times 3 \times 2^3 \div (4 + 2) \div 2$$

$$4(4^2 - 1 - 3^2) \div 3 + 1$$

$$(2 \times 4 + 3^2 \times 2^3) \div (2^2 + 1)$$

$$(2 + 3)(2^3 + 4^2 \div (2 \times 4)) \div 2$$

$$(4 + 1)(2 + 1)^2 - 3(2^3 - 2) \div 2$$

some more?

Bracketed

bracketed

some of these need brackets to make them correct
copy them out and place brackets if and where they are needed

(1) $2 \times 3 + 7 = 20$

(2) $2 + 5 \times 6 = 32$

(3) $13 - 2 \times 5 = 55$

(4) $20 - 6 \div 2 = 7$

(5) $7 - 4 - 1 = 4$

(6) $10 - 4 + 2 = 4$

(7) $20 \div 4 \times 5 = 1$

(8) $15 \div 3 + 2 = 7$

(9) $9 - 2 \times 4 = 1$

(10) $7 \times 6 \div 2 = 21$

(11) $18 - 8 - 3 = 13$

(12) $20 \div 10 \div 2 = 1$

(13) $16 \div 8 \div 2 = 4$

(14) $20 - 5 - 2 - 1 = 18$

(15) $36 \div 2 \times 3 + 4 = 10$

(16) $3 + 2 \times 5 - 3 = 7$

(17) $4 \times 5 + 2 \times 3 = 44$

(18) $8 \div 4 + 4 \times 2 = 2$

(19) $15 - 4 - 2 - 1 = 12$

(20) $4 \times 5 + 10 \div 2 = 15$

where do you put brackets with

$$20 \div 5 + 5 \times 2 =$$

to get these answers:

(a) 14

(b) 4

(c) 1

(d) 18

BIDMAS

B i D/M A/S   find the missing numbers – you should use 1 to 9, once only   (i)

1.]

□	+	8	−	□	= 6
−		−		−	
9	÷	□	+	□	= 7
×		+		×	
□	+	6	÷	2	= 4
= 4		= 1		= 1	

2.]

□	+	□	÷	2	= 4
×		−		÷	
□	−	5	+	□	= 7
−		×		+	
9	÷	□	+	7	= 10
= 1		= 9		= 9	

3.]

5	+	□	÷	□	= 8
+		×		×	
□	−	□	×	4	= 4
−		−		−	
3	+	7	−	□	= 1
= 0		= 1		= 1	

4.]

□	÷	□	+	5	= 8
−		÷		+	
7	−	6	÷	□	= 1
+		−		÷	
□	÷	□	÷	□	= 1
= 6		= 2		= 7	

5.]

□	÷	□	×	□	= 4
−		×		÷	
7	−	1	×	□	= 1
+		−		−	
□	+	5	−	□	= −1
= 2		= 1		= 6	

6.]

□	−	□	×	□	= 3
−		÷		÷	
□	+	4	÷	□	= 7
+		×		÷	
7	+	3	−	□	= 2
= 3		= 2		= 4	

BIDMAS

B i D/M A/S   find the missing numbers – you should use 1 to 9, once only (ii)

1.]
```
 □  ×  2  ÷  □  = 1
 ÷     ÷     -
 9  -  □  ÷  □  = 7
 +     -     +
 □  +  □  -  5  = 3
 =4    =3    =3
```

2.]
```
 7  -  □  +  □  = 4
 -     ÷     -
 □  -  4  ÷  □  = 7
 +     ×     ×
 1  +  □  ÷  □  = 3
 =3    =3    =1
```

3.]
```
 4  ÷  □  +  □  = 8
 -     ÷     -
 □  -  □  +  □  = 4
 +     -     ×
 1  +  9  ÷  □  = 4
 =4    =5    =9
```

4.]
```
 7  -  □  ×  □  = 2
 -     ÷     +
 □  ×  □  ÷  □  = 2
 ×     ÷     -
 □  +  □  -  8  = 3
 =1    =3    =7
```

5.]
```
 9  -  □  ×  □  = 1
 -     ×     ÷
 □  +  3  -  □  = 0
 +     ÷     +
 □  -  □  ÷  □  = 1
 =3    =4    =3
```

6.]
```
 □  ×  □  -  □  = 2
 +     ÷     -
 □  -  9  +  □  = 2
 -     -     +
 8  -  □  ÷  □  = 1
 =5    =4    =3
```

Missing Operations

find the missing operations

1.) $4 \,\square\, 3 \,\square\, 2 = 6$

2.) $5 \,\square\, 7 \,\square\, 2 = 19$

3.) $5 \,\square\, 4 \,\square\, 3 = 12$

4.) $3 \,\square\, 4 \,\square\, 5 = 7$

5.) $(8 \,\square\, 4) \,\square\, 2 = 1$

6.) $8 \,\square\, (4 \,\square\, 2) = 1$

7.) $(5 \,\square\, 3) \,\square\, 2 = 4$

8.) $5 \,\square\, (3 \,\square\, 2) = 4$

9.) $(3 \,\square\, 4) \,\square\, 2 = 6$

10.) $3 \,\square\, (4 \,\square\, 2) = 6$

11.) $(7 \,\square\, 7) \,\square\, 3 = 3$

12.) $7 \,\square\, (7 \,\square\, 3) = 3$

13.) $6 \,\square\, 3 \,\square\, 9 = 2$

14.) $6 \,\square\, (9 \,\square\, 3) = 2$

15.) $6 \,\square\, (3 \,\square\, 9) = 2$

16.) $(6 \,\square\, 9) \,\square\, 3 = 2$

17.) $(6 \,\square\, 8) \,\square\, 4 = 3$

18.) $(4 \,\square\, 8) \,\square\, 6 = 3$

19.) $6 \,\square\, 4 \,\square\, 2 = 3$

20.) $6 \,\square\, (2 \,\square\, 3) = 9$

test out any general rules that you think might apply

Missing Number

order of operating

find the missing number

1.) $4 + ? \times 2 = 18$

2.) $5 + ? \div 2 = 8$

3.) $10 - ? \times 2 = 4$

4.) $5 + ? \div 3 = 10$

5.) $4 \times 5 - ? \div 2 = 15$

6.) $3 + ? \times 3 = 15$

7.) $6 + ? \div 2 = 8$

8.) $12 - ? \times 2 = 4$

9.) $7 + ? \div 3 = 10$

10.) $10 \times 6 - ? \div 2 = 20$

11.) $\dfrac{9 + ?}{3 + 2} = 3$

12.) $\dfrac{40 - 28}{10 - ?} = 3$

13.) $\dfrac{30 - 14}{12 - ?} = 4$

14.) $\dfrac{3 \times 8 - ?^2}{10 - 2 \times 3} = 2$

15.) $\dfrac{4 \times 9 - ?^2}{1 + 4 \times 2} = 3$

Directed Number Gaps

directed number gaps (ii)

(1) $\Box \times \Box + \Box = 28$
use $-7, -6$ and 2

(2) $\Box \times \Box + \Box = -15$
use $-5, -3$ and 6

(3) $\Box \times \Box + \Box = -40$
use $10, -5$ and -2

(4) $\Box \times \Box - \Box = -70$
use $-6, 8$ and -5

(5) $\Box \times \Box - \Box = 24$
use $-8, -2$ and -4

(6) $\Box \times \Box - \Box = -63$
use $-6, -7$ and 3

(7) $\Box + \Box \div \Box = \Box$
use $-5, -8, 3$ and -7

(8) $\Box - \Box \div \Box = \Box$
use $-8, -7, 6$ and -2

(9) $\Box + \Box \div \Box = \Box$
use $-3, -3, 9$ and -2

(10) $\Box + \Box \div \Box = \Box$
use $-9, -2, 8$ and -7

(11) $\Box - \Box \div \Box = \Box$
use $-3, 9, 1$ and 6

(12) $\Box + \Box \div \Box = \Box$
use $-4, -5, 3$ and -7

Directed Number Gaps

directed number gaps (i)

(1) ☐ × ☐ + ☐ = −13
use −3 , −2 and 5

(2) ☐ × ☐ + ☐ = −11
use −5 , −2 and 3

(3) ☐ × ☐ + ☐ = −23
use 4 , −3 and −5

(4) ☐ × ☐ + ☐ = 21
use −3 , 6 and −5

(5) ☐ × ☐ + ☐ = 17
use −5 , −3 and −4

(6) ☐ × ☐ + ☐ = −27
use −6 , −7 and 3

(7) ☐ × ☐ − ☐ = 13
use −5 , 2 and −3

(8) ☐ × ☐ − ☐ = −19
use 4 , −3 and 5

(9) ☐ × ☐ − ☐ = 17
use −2 , −5 and −6

(10) ☐ × ☐ − ☐ = −19
use −5 , 3 and −8

(11) ☐ × ☐ − ☐ = 34
use −7 , −4 and −6

(12) ☐ × ☐ − ☐ = −26
use −8 , −6 and 4

3 Algebraic Expressions

Algebra concerns representing missing information.

Put simply, we use letters, known as variables, to represent numbers.

Usually the value of variables are not initially known, but we hope to combine available information to find their value.

For example:
- a might represent someone's age this year.
- θ might represent an unknown angle.

https://www.youtube.com/watch?v=p62QItqtkQA&t=2219s

algebra
/ˈaldʒɪbrə/

Origin

ITALIAN

SPANISH

ARABIC — ARABIC — MEDIEVAL LATIN

jabara → al-jabr
reunite, restore — the reunion of broken parts → algebra
late Middle English

'ilm al-jabr wa'l-muqābala
the science of restoring what is missing and equating like with like

ENGLISH

algorithm

late Middle English: from Italian, Spanish, and medieval Latin, from Arabic *al-jabr* 'the reunion of broken parts', 'bone-setting', from *jabara* 'reunite, restore'. The original sense, 'the surgical treatment of fractures', probably came via Spanish, in which it survives; the mathematical sense comes from the title of a book, *'ilm al-jabr wa'l-muqābala* 'the science of restoring what is missing and equating like with like', by the mathematician al-Ḵwārizmī (see algorithm).

Conventions

- We tend to use single lower case letters for variables, either using the English alphabet or using the Greek alphabet.

- An algebraic x is written using two back to back c's. Do NOT write it as a $\times$ symbol.

- Do NOT include the multiplication sign, for example $3 \times p = 3p$

- Write division as fractions, for example $3 \div p = \dfrac{3}{p}$

- Write numbers first in products, for example $p \times 3 = 3p$

- Write letters in products in alphabetical order, for example $4 \times q \times r \times p = 4pqr$

- $1x$ is written simply as x

3.1 Definitions

- **Variable** is a letter used to represent an unknown number.

- **Coefficient** is the number in front of a variable.

- **Constant** is a number that cannot change its value.

- **Term** is either a constant, a variable or a constant multiplied by a variable.

- **Expression** is terms and operators ($+$ and $-$) grouped together.

Worked Example	Your Turn
Write down the following for the expression: $2x - 4y - 9$ Variables: Coefficient of x: Coefficient of y: Constant: Terms:	Write down the following for the expression: $-2a + 4b + 9$ Variables: Coefficient of a: Coefficient of b: Constant: Terms:

Worked Example	Your Turn
Write down the following for the expression: $2x^2 - 4xy - 9$ Variables: Coefficient of x^2: Coefficient of xy: Constant: Terms:	Write down the following for the expression: $-2ab + 4b^2 + 9$ Variables: Coefficient of ab: Coefficient of b^2: Constant: Terms:

Intelligent Practice

Question	Variables	Coefficients	Constant	Terms
$3x - 9$				
$3x + 4y - 9$				
$3x - 4y - 9$				
$3x - 4y + 9$				
$-3x - 4y + 9$				
$9 - 3x - 4y$				
$9 - 3a - 4b$				
$3a^2 - 4b^2 + 9$				
$3a^2 - 4a + 9$				
$3a^2 - 4a$				
$3a^2 - 4$				
$3ab - 4$				
$3ab - 4a$				
$3ab - 4a - 5b$				
$3a^2b - 4a - 5b$				
$3ab^2 - 4a - 5b$				
$3ab^2 - 4ab - 5b$				
$3ab^2 - 4a^2b - 5b$				
$3ab^2 - 4a^2b - 5ab$				
$3ab^2 - 4a^2b - 5ab - 6$				

Worked Example	Your Turn
Explain what the following mean:	Explain what the following mean:
$7x$	$7a$
xy	ab
xy^2	ab^2
$(xy)^2$	$(ab)^2$

Intelligent Practice

Explain what the following mean:

1) $3x$

2) $3b$

3) ab

4) ab^2

5) a^2b^2

6) a^2b

7) $5a^2$

8) $(5a)^2$

9) $(5ab)^2$

10) $(5ab)^3$

11) $5ab^3$

12) $5a^2b^3$

13) $(5a^2b)^3$

14) $(5a^2b^2)^3$

15) $(5ab^2)^3$

16) $(10ab^2)^3$

3.3 Forming Expressions

Unscramble the words below to make synonyms of the operations.

Addition $+$	Subtraction $-$	Multiplication $\times$	Division $\div$
atlot	enrmai	optcrdu	sitlp
smu	uderec	imset	uleqa cesepi
uspl	cferefiend	pliuetldim	evddidi
omre	esls	tlso of	dshrea
goeettalrh	hgance		raitnocf
xeart	suimn		

Forming Expressions

'Four more than a number'

$n + 4$

This letter does not have to be n. It could be any letter or symbol. People often use x. We could write $\smiley + 4$. We are not going to, though. That would be silly.

Write the following sentences algebraically:

- A number add 6

- A number add 10

- A number subtract 10

- 8 subtract a number

Forming Expressions

'Four lots of a number'

$4n$

We do not tend to use the × symbol in algebra. Instead we write things next to each other to show multiplication.

Write the following sentences algebraically:

- A number multiplied by 6

- A number multiplied by 10

- a multiplied by b

- $4a$ multiplied by b

- $2a$ multiplied by $2b$

Forming Expressions

'A number divided by 5'

$$\frac{n}{5}$$

We tend not to use ÷ in expressions. We use fraction notation (writing a division as a fraction).

Write the following sentences algebraically:

- A number divided by 6

- 6 divided by a number

- A number divided by $6 + a$

- $6 - a$ divided by a number

Worked Example	Your Turn
Write an algebraic expression for each of the following: 3 more than a 5 less than a b multiplied by a b multiplied by a then squared	Write an algebraic expression for each of the following: 3 less than a a more than 5 b divided by a b divided by a then squared

Fluency Practice

Question 1:   Write an algebraic expression for each of the following

(a) 4 more than c

(b) 2 lots of a

(c) 3 less than b

(d) m divided by 5

(e) 7 multiplied by s

(f) w subtract 1

(g) e squared

(h) y add 9

(i) m shared between 3

(j) 10 times x

(k) k less than 8

(l) 12 less than g

Question 2:   Write an algebraic expression for each of the following

(a) c add p

(b) f minus m

(c) a times b

(d) p divided by z

(e) b taken away from u

(f) k add n add r

(g) w less than c

(h) l multiplied by m

(i) y multiplied by m multiplied by a

Question 3:   Write an algebraic expression for each of the following

(a) m multiplied by 2 and then add 3

(b) h divided by 4 and then add 7

(c) p squared and then add 10

(d) t add 2 and then multiplied by 5

(e) 9 times e and then add 1

(f) h divided by 3 then add 1

(g) m subtract 6 and then divided by 3

(h) y squared and then multiplied by 4

(i) k multiplied by 4 and then squared

(j) a squared and then multiplied by b

Intelligent Practice

Write an algebraic expression for each of the following:

1) 7 more than x

2) 7 less than x

3) 9 less than x

4) 9 lots of x

5) 19 lots of x

6) x divided by 19

7) x shared between 19

8) x less than 19

9) x less than 3

10) x less than y

11) x more than y

12) x multiplied by y

13) x divided by y

14) x divided by 3

15) x divided by 3 and then add 2

16) x divided by 3 and then subtract 2

17) x lots of 3 and then subtract 2

18) x lots of 3 and then squared

19) x lots of 3 squared

20) x squared and then multiply by 3

Extension

which goes with which?

translation

1) any number add five

2) any number added to six and then times by three

3) any number divided by another

4) any number added to two times another number all multiplied by five

5) any number add ten all squared

6) any number divided by six added to three then all times by five

7) five times any number added to two times another number

8) any number times by three then add six

9) any number times by three then all squared

10) any number add ten then multiply by two

11) one number divided by another number then times by seven

12) any number times by itself then times by three

13) one square number subtract another square number

14) any number add three all divided into another number

15) three times any number then times by two

a) $3(k + 6)$

b) $(d + 10)^2$

c) $5f + 2g$

d) $5\left(\dfrac{e}{6} + 3\right)$

e) $n + 5$

f) $\dfrac{7b}{h}$

g) $h^2 - a^2$

h) $3d^2$

i) $6y$

j) $\dfrac{w}{z}$

k) $(3y)^2$

l) $\dfrac{w}{k + 3}$

m) $5(b + 2c)$

n) $2(k + 10)$

o) $3m + 6$

Forming Expressions in Context

Often you will be asked to take a 'real life' scenario and turn into mathematical code. For instance
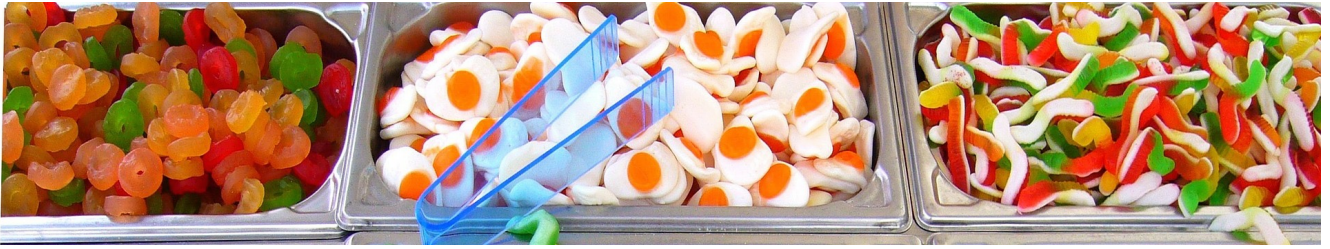
Gummy rings cost 2p per gram, fried eggs cost 3p per gram, gummy snakes cost 4p per gram.

Find an expression for the total cost of x grams of rings, y grams of eggs and z grams of snakes.

$$2x + 3y + 4z$$

Notice that the coefficent of x (the number in front of x) stands for the price of the rings, not the number of them.

Writing Algebraically

	Write the calculation in numbers	Write the expression in algebra
Ex	Becky begins a game with 12 marbles. She wins 3 and loses 5. How many marbles does she have at the end of the game? **12 + 3 − 5 = 10**	Becky begins a game with x marbles. She wins y and loses z. How many marbles does she have at the end of the game? $$x + y - z$$
1	In a class of 30 children, 10 have a sister. How many children don't have a sister?	In a class of a children, b have a sister. How many children don't have a sister?
2	There were 30 questions in a spelling test. Megan got 25 correct. How many did she get wrong?	There were x questions in a spelling test. Megan got y correct. How many did she get wrong?
3	Lucy went on a journey, going 65 miles by train, 4 miles by bus, and 3 miles on foot. How far did she travel?	Lucy went on a journey, going x miles by train, y miles by bus, and z miles on foot. How far did she travel?
4	Mark buys 3 chocolates bars, each costing 25p. How much does he spend on chocolate bars?	Mark buys a chocolates bars, each costing b pence. How much does he spend on chocolate bars, in pence?
5	Nihal is 11 years old. His sister is 5 years older. How old is his sister?	Nihal is x years old. His sister is y years older. How old is his sister?
6	Maddie has 4 cards. Emma has 10 times as many. How many cards does Emma have?	Maddie has a cards. Emma has b times as many. How many cards does Emma have?
7	Aisha went shopping with £5. She spent £2.50 on a toy and £1 on a bag of sweets. How much did she have left?	Aisha went shopping with x pounds. She spent y pounds on a toy and z pounds on a bag of sweets. How much did she have left, in pounds?
8	Apples cost 15p and bananas cost 10p. Jacob buys 2 apples and 3 bananas. How much does he spend?	Apples cost a pence and bananas cost b pence. Jacob buys x apples and y bananas. How much does he spend, in pence?
9	How far will a car go in 4 hours at 30 miles per hour?	How far will a car go in t hours at v miles per hour?
10	Daniel is 12 years old. Anna is twice as old as Daniel. Grace is three years younger than Anna. How old is Grace?	Daniel is d years old. Anna is c times as old as Daniel. Grace is b years younger than Anna. How old is Grace?

Like Terms

Definition	Characteristics
Two or more terms, each with the same variables, to the same power or with the same function applied.	• Any variables must be identical. • If variables are multiplied, the order listed does not matter. • If powers or functions are used, then the same power or function must be used.

Examples	Non Examples
• $2x$ and $3x$ • $2y$ and $3y$ • 5 and -2 • d and $3d$ • $2y^2$ and $3y^2$ • $-2y^2$ and $3y^2$ • $\frac{2}{3}x$ and $\frac{4}{5}x$ • $3a^4$ and $5a^4$ • $\frac{3}{5}a^2b$ and $\frac{1}{5}a^2b$ • $3ab$ and $5ab$ • $3ba$ and $5ab$ • $3\sqrt{x}$ and $5\sqrt{x}$	• $2x$ and $3y$ • $2y$ and $3y^2$ • $5x^2$ and $6x^3$ • $3a$ and $5b$ • $3a^2b$ and $5ab^2$

Like Terms

$3p$	p	Like	Unlike
x^2	$3x^2$	Like	Unlike
x^2	$2x$	Like	Unlike
$-3\sqrt{x}$	$27\sqrt{x}$	Like	Unlike
$7a$	$7b$	Like	Unlike

$3a$	$3a$	Like	Unlike
a	$2a$	Like	Unlike
$2a$	$2A$	Like	Unlike
$-3a$	$2a$	Like	Unlike
$4a$	$4b$	Like	Unlike
$3a$	$3a^2$	Like	Unlike
$2a^2$	$7a^2$	Like	Unlike
$-3a^2$	$7a^2$	Like	Unlike
$2a^2$	$2a^{-2}$	Like	Unlike
2^a	a^2	Like	Unlike
x	$\sqrt{x}$	Like	Unlike

Worked Example	Your Turn
Simplify:	Simplify:
$$a - b + a + b$$	$$a + b - a + b$$
$$3a + 2b - a + b$$	$$3a - 2b + a + b$$

Intelligent Practice

Simplify:

1) $a + a + a + a$

2) $a + b + a + a$

3) $a + b + a + b$

4) $a + b + a - b$

5) $a + b - a - b$

6) $a - b - a - b$

7) $6a - b - a - b$

8) $6a + b - a - b$

9) $6a + 5b - a - b$

10) $6a + 5b - 4a - b$

11) $6a + 5b - 4a - 3b$

12) $6a - 5b - 4a - 3b$

13) $6a - 5b - 4a + 3b$

14) $6a - 5b + 4a + 3b$

15) $-6a - 5b + 4a + 3b$

16) $-6a - 5b - 4a + 3b$

17) $-6a - 5b - 4a - 3b$

18) $-6a - 5 - 4a - 3$

19) $-6ab - 5 - 4ab - 3$

20) $-5 - 6ab - 3 - 4ab$

Extension

$6a + 5b - 4ab + 3 - 3a + 4b - 5ab + 6$

Extension

a) Simplify $a - 2a + 3a - 4a + 5a - 6a + \cdots + 49a - 50a$
b) What happens if the signs switch?
c) Simplify $a - 2a + 3a - 4a + 5a - 6a + \cdots + 99a - 100a$
d) What generalisations can you make for n terms?

Worked Example	Your Turn
Simplify:	Simplify:
$$3a^2 + 2b^2 - a^2 + b^2$$	$$3a^2 - 2b^2 + a^2 + b^2$$
$$3a^2 - 2ab^2 + a^2 + ab^2$$	$$3a^2 + 2ab^2 - a^2 + ab^2$$

Intelligent Practice

Simplify:

1) $a^2 + b^2 + a^2 + b^2$

2) $6a^2 + b^2 + a^2 + b^2$

3) $6a^2 + 5b^2 + a^2 + b^2$

4) $6a^2 + 5b^2 + 4a^2 + b^2$

5) $6a^2 + 5b^2 + 4a^2 + 3b^2$

6) $6a^2 + 5b^2 - 4a^2 + 3b^2$

7) $6a^2 + 5b^2 - 4a^2 - 3b^2$

8) $6a^2 - 5b^2 - 4a^2 - 3b^2$

9) $6a^2 - 5b - 4a^2 - 3b$

10) $6a^2 - 5ab - 4a^2 - 3ab$

11) $6a^2 - 5ab - 4a^2 - 3ba$

12) $6a^2 - 5a^2b - 4a^2 - 3a^2b$

13) $6a^2 - 5a^2b - 4a^2 + 3a^2b$

14) $6a^2 - 5a^2b - 4a^2 + 3ab^2$

15) $6a^2 - 5ab^2 - 4a^2 + 3ab^2$

16) $-6a^2 - 5ab^2 - 4a^2 + 3ab^2$

17) $-6 - 5ab^2 - 4 + 3ab^2$

18) $-6a - 5ab^2 - 4a + 3ab^2$

19) $-6ab - 5ab^2 - 4ab + 3ab^2$

20) $-6a^2b - 5ab^2 - 4a^2b + 3ab^2$

Extension

$6a^2 + 5b^2 - 4a^2b + 3ab^2 - 2a^2b^2 + 1 - a^2 + 2b^2 - 3a^2b + 4ab^2 - 5a^2b^2 + 6$

Fluency Practice

Question 1: Simplify each of the following

(a) $y + y + y + y$ (b) $w + w + w + w + w$ (c) $a + a + a + a + a + a$ (d) $s + s + s$

(e) $n + n$ (f) $g + g + g + g - g$ (g) $y + y + y + y - y - y$ (h) $p + p - p - p$

(i) $3y + 2y$ (j) $4a + 3a$ (k) $9k + 5k$ (l) $7m + m$

(m) $15c + 20c$ (n) $6w - 3w$ (o) $10y + 3y - 5y$ (p) $20t - 14t$

(q) $7x - 3x - x$ (r) $8k - 8k$ (s) $7y - 2y + y$ (t) $5u - 4u$

(u) $y^2 + y^2$ (v) $a^2 + a^2 + a^2$ (w) $c^2 + c^2 + c^2 + c^2 + c^2$ (x) $7y^2 + 3y^2$

(y) $2w^2 + 4w^2 + 8w^2$ (z) $6y^2 - 2y^2 + 3y^2$

Question 2: Simplify the following expressions

(a) $4u - 6u$ (b) $8w - 9w$ (c) $4a + 2a - 9a$ (d) $2y - 9y$

(e) $-3g - 2g$ (f) $-4f + 9f$ (g) $-m - 7m$ (h) $5y^2 - 7y^2$

(i) $6a^2 + 2a^2 - 9a^2$ (j) $ab + ab + ab$

Question 3: Simplify the following expressions

(a) $3a + 2b + 4a + b$ (b) $7y + 5y + 2h + 2h$ (c) $g + 8a + 2a + g$

(d) $7m + 7p + 8m + p + 2p$ (e) $9e + 2 + e + 2$ (f) $4 + 3a + 2a + 8$

(g) $2y + 4 + 3y - 1$ (h) $8 + 3w - w - 3$ (i) $5 - 4s - 2 + 10s$

(j) $3x + 6y + 5x - 2y$ (k) $6m - 2s + 11s + m$ (l) $2a + 3b - 2 + a + 3b + 4$

(m) $3a - 2b + a - 5b$ (n) $2x - 2y - 6x + 5y$ (o) $y - 4m - 3y - 5m$

(p) $7p - 2q - q + 3r + 4r$ (q) $11c + 8d - 6c - 11d$

Question 4: Simplify the following

(a) $3y^2 + 4ab + 7y^2 + ab$ (b) $9x^2 - 2x - 11x^2 + 5x$ (c) $7ac - 3ab + 9ab - 7ac$

Worked Example	Your Turn
Simplify:	Simplify:
$3x \times 5$	$6x \times 2$
$3x \times 5x$	$6x \times 2x$
$3x \times 5y$	$6x \times 2y$
$3x^2y \times 5xy^2$	$6xy^2 \times 2x^2y$

Fluency Practice

Question 1:   Simplify the following expressions.

(a) $3 \times y$ (b) $w \times 3$ (c) $7 \times x$ (d) $a \times 4$

(e) $a \times c$ (f) $f \times g$ (g) $h \times d$ (h) $a \times y \times m$

(i) $t \times t$ (j) $p \times p$ (k) $a \times a \times a$ (l) $m \times m \times m$

(m) $4 \times f \times g$ (n) $3 \times w \times y$ (o) $p \times 5 \times s$ (p) $n \times c \times 7$

(q) $t \times c \times w$ (r) $y \times x \times w$ (s) $5 \times a \times a$ (t) $y \times 3 \times y$

Question 2:   Simplify the following expressions.

(a) $5 \times 3w$ (b) $4y \times 2$ (c) $3 \times 3m$ (d) $10g \times 3$

(e) $4 \times 2 \times y$ (f) $3 \times 2 \times 2p$ (g) $5 \times 2y \times 3$ (h) $9a \times 2 \times 2$

(i) $3a \times c$ (j) $4y \times z$ (k) $5c \times b$ (l) $c \times 6y$

(m) $2a \times 3y$ (n) $6c \times 3t$ (o) $9w \times 3a$ (p) $2y \times 2g$

(q) $2y \times y$ (r) $5w \times w$ (s) $m \times 3m$ (t) $x \times 2x$

(u) $4t \times 2t$ (v) $6y \times 3y$ (w) $9a \times 9a$ (x) $12y \times 10y$

(y) $2a \times 3p \times 5w$ (z) $10y \times 2p \times 3c \times m$

Question 3:   Simplify the following expressions

(a) $a^2 \times a$ (b) $y \times y^2$ (c) $w^2 \times w^2$ (d) $m^2 \times m^3$

(e) $2t^2 \times t$ (f) $4m \times m^2$ (g) $g \times 2g^2$ (h) $p^2 \times 3p^2$

(i) $3p^2 \times 2p$ (j) $2v^2 \times 7v^2$ (k) $9p^2 \times 7p^2$ (l) $5w^2 \times 2w^3$

(m) $7a^3 \times 4a^3$ (n) $6c^4 \times 5c^3$ (o) $aw \times w$ (p) $r \times ry$

(q) $ay \times ay$ (r) $c^2f \times f$ (s) $dg \times d^2$ (t) $3x^2y \times 2x$

(u) $4ab \times 2ab$ (v) $3m^2n^2 \times 4mn$ (w) $2cd^2 \times d^2$ (x) $4a^2bc^2 \times a^3b$

(y) $2ad^2e \times a^3c$ (z) $8m^2n \times 3no^5$

Intelligent Practice

Simplify:

1) $3 \times x$

2) $x \times 3$

3) $x \times y$

4) $x \times x$

5) $2x \times x$

6) $2x \times 3$

7) $3x \times 2$

8) $3x \times 2y$

9) $6x \times 4y$

10) $8x \times 3y$

11) $8x^2 \times 3y$

12) $8x^2 \times 3x$

13) $8x^2 \times 3xy$

14) $8x^3 \times 3y$

15) $8x^3y \times 3xy$

16) $8x^3y \times 3xy^3$

17) $8x^3y \times 3xy^3z$

18) $8xy^3 \times 3xyz^3$

19) $12zy^3 \times 2zyx^3$

20) $12z^2y^3 \times 2z^2y^2x^3$

Worked Example	Your Turn
Simplify:	Simplify:
$15x \div 5$	$12x \div 2$
$15x^2 \div 5x$	$12x^2 \div 2x$
$15xy \div 5y$	$12xy \div 2y$
$15x^3y^3 \div 5xy^2$	$12x^3y^3 \div 2x^2y$

Fluency Practice

Question 1: Simplify the following expressions.

(a) $12x \div 2$ (b) $9y \div 3$ (c) $15a \div 5$ (d) $28c \div 7$

(e) $8m \div 2m$ (f) $10c \div 2c$ (g) $18d \div 3d$ (h) $35m \div 5m$

(i) $5ac \div a$ (j) $6xy \div y$ (k) $7mn \div n$ (l) $20ab \div 2a$

(m) $25xy \div 5y$ (n) $80gh \div 10h$ (o) $27xy \div 3xy$ (p) $32abc \div 8ac$

Question 2: Simplify the following expressions.

(a) $\dfrac{14c}{2}$ (b) $\dfrac{56w}{7}$ (c) $\dfrac{45a}{9a}$ (d) $\dfrac{105y}{5y}$

(e) $\dfrac{mw}{m}$ (f) $\dfrac{8cf}{c}$ (g) $\dfrac{15xy}{3x}$ (h) $\dfrac{70ab}{2a}$

(i) $\dfrac{30ef}{6ef}$ (j) $\dfrac{20cde}{5cde}$ (k) $\dfrac{42ghk}{6gh}$

Question 3: Simplify the following expressions.

(a) $h^2 \div h$ (b) $x^3 \div x$ (c) $7y^2 \div y$ (d) $40m^2 \div 2m$

(e) $16c^2 \div 4c$ (f) $20g^2 \div g^2$ (g) $45x^3 \div x$ (h) $30t^3 \div 3t$

(i) $9h^3 \div 3h^2$ (j) $10x^3 \div 5x^3$ (k) $24m^2 \div 3$

Question 4: Simplify the following expressions.

(a) $\dfrac{g^2}{g}$ (b) $\dfrac{w^3}{w}$ (c) $\dfrac{3a^2}{a}$ (d) $\dfrac{24e^2}{3e}$

(e) $\dfrac{35c^3}{7c^2}$ (f) $\dfrac{52c^3}{13c}$ (g) $\dfrac{100w^3}{10w^3}$

Question 5: Simplify the following expressions

(a) $a^2b^2 \div ab$ (b) $xy^2 \div x$ (c) $4ab^3 \div 2ab^2$ (d) $25c^2d^2 \div 5cd$

(e) $16x^4y^3 \div 4x^2y^2$ (f) $10c^3de^2 \div 2cde$ (g) $15abc^4 \div bc^3$ (h) $24d^3e^9f \div 8d^3f$

Question 6: Simplify the following expressions.

(a) $\dfrac{a^3c^3}{ac}$ (b) $\dfrac{10a^4c^3}{2ac^2}$ (c) $\dfrac{9abc^3}{3ac^2}$ (d) $\dfrac{45a^5b^8c^4}{3a^3b^4c}$

Intelligent Practice

Simplify:

1) $6x \div 2$

2) $6x \div 3$

3) $6x \div 3x$

4) $\dfrac{6x}{3x}$

5) $\dfrac{6xy}{3x}$

6) $\dfrac{6xy}{3y}$

7) $\dfrac{6x^2}{3y}$

8) $\dfrac{6x^2}{3y^2}$

9) $\dfrac{6x^2}{3x^2}$

10) $\dfrac{6x^2y^2}{3x^2}$

11) $\dfrac{6x^3y^2}{3x^2y}$

12) $\dfrac{12x^3y^2}{6x^2y}$

13) $\dfrac{12x^2y}{6x^3y^2}$

14) $\dfrac{12x^2y}{4x^3y^2}$

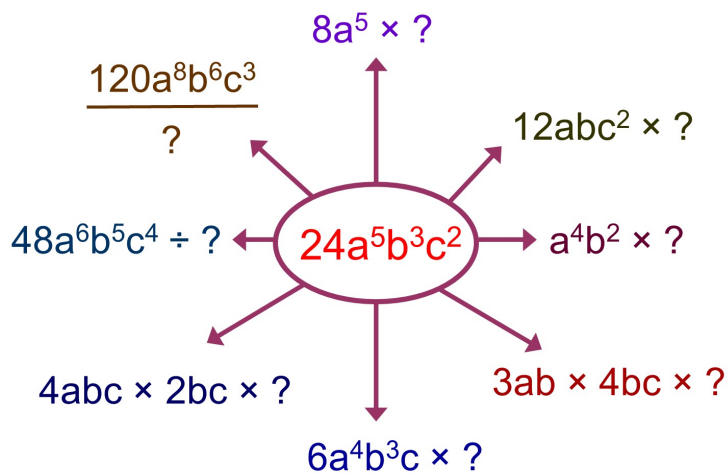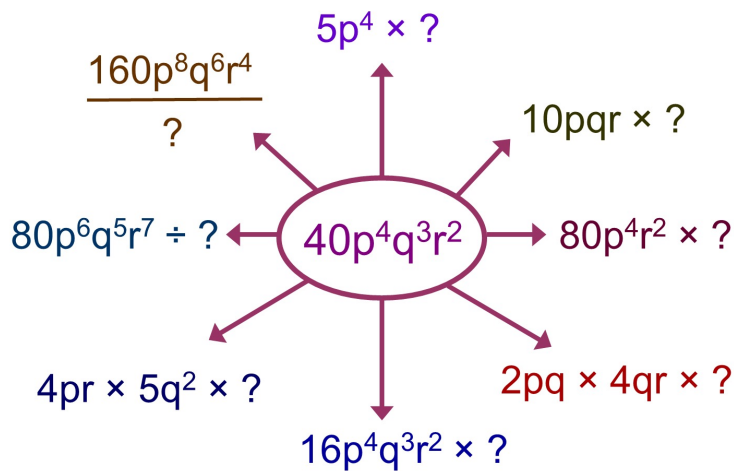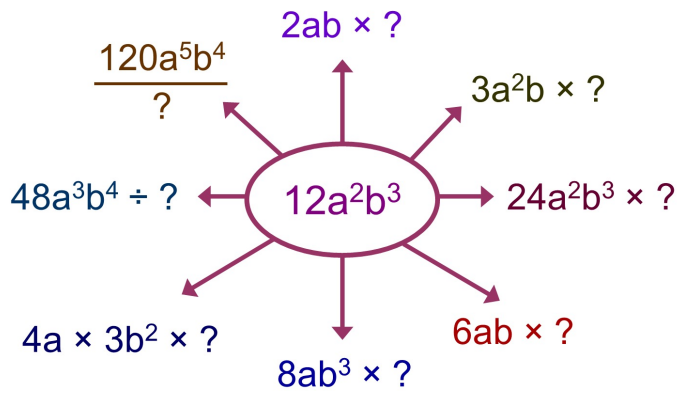15) $\dfrac{12x^2y}{4x^6y^4}$

16) $\dfrac{4x^2y}{12x^6y^4}$

17) $\dfrac{4x^2y}{8x^6y^4}$

18) $\dfrac{4x^2y^2z}{8x^6y^4}$

19) $\dfrac{4x^2y^2z}{8x^6y^4z^2}$

20) $\dfrac{4z^2y^2x}{8x^6y^4z^2}$

Diagram 1 — center: $12a^2b^3$

$2ab \times ?$

$3a^2b \times ?$

$\dfrac{120a^5b^4}{?}$

$48a^3b^4 \div ?$

$24a^2b^3 \times ?$

$4a \times 3b^2 \times ?$

$6ab \times ?$

$8ab^3 \times ?$

Diagram 2 — center: $40p^4q^3r^2$

$5p^4 \times ?$

$10pqr \times ?$

$\dfrac{160p^8q^6r^4}{?}$

$80p^6q^5r^7 \div ?$

$80p^4r^2 \times ?$

$4pr \times 5q^2 \times ?$

$2pq \times 4qr \times ?$

$16p^4q^3r^2 \times ?$

Diagram 3 — center: $24a^5b^3c^2$

$8a^5 \times ?$

$12abc^2 \times ?$

$\dfrac{120a^8b^6c^3}{?}$

$48a^6b^5c^4 \div ?$

$a^4b^2 \times ?$

$4abc \times 2bc \times ?$

$3ab \times 4bc \times ?$

$6a^4b^3c \times ?$

3.7 Algebraic Order of Operations

Example

Simplify $3a^2 + a \times a$

Multiplication first:	$3a^2 + a \times a$	$= 3a^2 + a^2$
Addition second:	$3a^2 + a^2$	$= \mathbf{4a^2}$

Simplify these expressions

1.	$5b + 3b \times 2$	2.	$5b + 2 \times 3b$
3.	$3b \times 2 + 5b$	4.	$3b \times 2 + 5$
5.	$5 + 3b \times 2$	6.	$5 + 2 \times 3b$
7.	$5 + 2b \times 3b$	8.	$5b + 2b \times 3b$
9.	$5b^2 + 2b \times 3b$	10.	$2b \times 3b - b^2$
11.	$5b^2 + 2b \times 3b - b^2$	12.	$2b \times 3b - b^2 + 5b^2$
13.	$2b \times 3b - b \times 5b$	14.	$2b \times 3b + b \times 5b$
15.	$5b^2 + 2 \times 3b - b^2$	16.	$5b^2 + 2 \times 3b - b$
17.	$5b^2 + 3b \times b - 2b^2$	18.	$2b \times 5b + 3b \times b$
19.	$5b^2 + a \times a$	20.	$2a + 5b^2 - a$
21.	$b \times 2b - 5b^2 + 3a \times b - ab$	22.	$b + 6b^2 \div 3b$
23.	$8b \times b + 6b^2 \div 2b$	24.	$\dfrac{10a^2 + 2a \times a}{3a \times 2a}$
25.	$\dfrac{6a + 7 \times 2a}{8a \times 2 + 20a^2 \div 5a}$	26.	$5a^2 \div a - \sqrt{8a^2 + 4a \times 2a}$

Algebraic Order of Operations

Where required, insert brackets to make these identities true.

Example	$(3a + 2a) \times 4a \equiv 20a^2$ ✓
1.	$5a + 4a \times 2a \equiv 18a^2$
2.	$3 \times a + 4a \times 2a \equiv 30a^2$
3.	$3 \times 2a + 4a \div 2a \equiv 9$
4.	$8 \times 2a + 8a^2 \div 2a \equiv 20a$
5.	$4a \times 2a + 5a^2 \equiv 8a^2 + 20a^3$
6.	$3a \times b + b \equiv 6ab$
7.	$4a + b \times 3a \equiv 12a^2 + 3ab$
8.	$2 \times 2b \times 2b + 3 \times b - b \equiv 16b^2$

Spot the mistake

Identify the errors in these solutions. Can you see how they arrived at their answers? Which one is correct?

4a + 2a x 7 – 4

= 6a x 3

= 18a

4a + 2a x 7 – 4

= 4a + 14a – 4

= 18a – 4

= 14a

4a + 2a x 7 – 4

= 6a x 7 – 4

= 42a – 4

4a + 2a x 7 – 4

= 4a + 14a – 4

= 18a – 4

4a + 2a x 7 – 4

= 4a + 2a x 3

= 4a + 6a

= 10a

$x^3 \times x^2$
$= (x \times x \times x) \times (x \times x)$
$= x^5$

$x^3 \times x^3$
$= (x \times x \times x) \times (x \times x \times x)$
$= x^6$

$x^3 \times x^4$
$= x^7$

$x^3 \times x^n$
$= x^{3+n}$

$x^m \times x^n$
$= x^{m+n}$

Worked Example	Your Turn
Simplify:	Simplify:
$x^7 \times x^8$	$x^9 \times x^2$
$3x^4 \times 2x^5$	$4x^3 \times 5x^7$

Fluency Practice

Task 2

Simplify the following in your jotter:

1. $a^2 \times a^4$ 2. $x \times x^3$ 3. $x^2 \times x^6$ 4. $x^6 \times x^{-2}$

5. $a^{12} \times a^{10}$ 6. $f^{23} \times f^{-10}$ 7. $x^7 \times x^8$ 8. $y^{-6} \times y^7$

9. $b^7 \times b^5 \times b^9$ 10. $x^2 \times x \times x^7$ 11. $x^4 \times x^5 \times x^6$ 12. $x^2 \times x^4 \times x^6 \times x^8$

13. $a^2 \times a^4 \times a^{-1}$ 14. $a^3 \times a^{-2} \times a$ 15. $x^{-2} \times x^{-4} \times x^6 \times x^8$

Task 3

Find as many pairs of values for m and n such that the statement below holds true:

$$c^m \times c^n = c^8$$

Task 4

Simplify the following in your jotter:

1. $2a^2 \times a^3$ 2. $2a^4 \times 3a$ 3. $2x^6 \times 3x^4$

4. $5x^4 \times 6x^2$ 5. $\frac{2}{3}x^3 \times 12x^4$ 6. $2a^7 \times 6a^2 \times \frac{1}{4}a$

7. $4a^3 \times 3a^2 \times 5a$ 8. $-3b^5 \times 8b^4$ 9. $-3c^4 \times -4c^9$

10. $2a^4 \times 3a^{-1}$ 11. $4a^5 \times 8a^{-3}$ 12. $4r^{-6} \times 5r^7$

13. $5t^7 \times 2t^{-4} \times 3t$ 14. $3s^2 \times 4s^4 \times -2s^6$ 15. $\frac{2}{3}x^4 \times -12x^3 \times \frac{1}{4}x^{-4}$

Task 5

If $x^2 \times x^y \times x^{-3} = x^{-4} \times x^2 \times x^{2y}$ then:

A $y = 0$ **B** $y = -1$ **C** $y = 1$ **D** $y = \frac{1}{2}$

Question 1:   Write as a single power of m.

(a) $m^2 \times m^3$ (b) $m^3 \times m^3$ (c) $m^6 \times m^2$ (d) $m^7 \times m^3$ (e) $m^6 \times m^8$ (f) $m^2 \times m$ (g) $m \times m^3$

(h) $m^7 \times m^8$ (i) $m^9 \times m^2$ (j) $m \times m^8$ (k) $m^6 \times m^5$ (l) $m^2 \times m^2 \times m^2 \times m^2$

3.9 Index Laws – Dividing

$x^5 \div x$

$= \dfrac{x^5}{x}$

$= \dfrac{x \times x \times x \times x}{x}$

$= x^4$

$x^5 \div x^2$

$= \dfrac{x^5}{x^2}$

$= \dfrac{x \times x \times x \times x}{x \times x}$

$= x^3$

$x^5 \div x^3$

$= \dfrac{x^5}{x^3}$

$= x^2$

$x^5 \div x^n$

$= \dfrac{x^5}{x^n}$

$= x^{5-n}$

$x^m \div x^n$

$= \dfrac{x^m}{x^n}$

$= x^{m-n}$

Worked Example	Your Turn
Simplify:	Simplify:
$y^{12} \div y^4$	$p^{14} \div p^9$
$12y^{11} \div 6y^7$	$56y^4 \div 8y^2$
$\dfrac{5y^{11}}{12y^7}$	$\dfrac{8y^4}{56y^2}$

Simplify

1. $\dfrac{x^6}{x}$

2. $\dfrac{x^{13}}{x^2}$

3. $c^{12} \div c^4$

4. $x^8 \div x^3$

5. $\dfrac{x^{10}}{x^3}$

6. $a^{12} \div a^2$

7. $a^{12} \div a^{-2}$

8. $\dfrac{t^{20}}{t^3}$

9. $\dfrac{t^{20}}{t^{-3}}$

10. $\dfrac{t^2}{t^{-5}}$

11. $\dfrac{t^{-2}}{t^{-5}}$

12. $b^{10} \div b^{-6}$

Simplify

1. $6x^5 \div 3x^2$

2. $3x^5 \div 6x^2$

3. $6x^5 \div 3x^{-2}$

4. $3x^5 \div 6x^{-2}$

5. $\dfrac{20x^6}{4x^5}$

6. $\dfrac{4x^6}{20x^5}$

7. $\dfrac{36x^7}{3x^4}$

8. $\dfrac{3x^7}{36x^4}$

9. $\dfrac{36x^7}{3x^{-4}}$

10. $\dfrac{3x^7}{36x^{-4}}$

11. $\dfrac{1.3x^7}{1.3x^4}$

12. $\dfrac{3}{4}x^5 \div \dfrac{3}{4}x^{-2}$

13. $5.5x^{-1} \div 1.1x^{-5}$

14. $\dfrac{1.1x^{-1}}{5.5x^{-5}}$

15. $\dfrac{2}{3}b^{13} \div \dfrac{1}{3}b^3$

Fluency Practice

Question 2:   Write as a single power of n.

(a) $n^5 \div n^2$ (b) $n^8 \div n^3$ (c) $n^9 \div n^2$ (d) $n^7 \div n^5$ (e) $n^3 \div n$ (f) $n^8 \div n$ (g) $n^7 \div n^4$

(h) $n^9 \div n^3$ (i) $n^4 \div n^8$ (j) $n \div n^3$ (k) $n^{45} \div n^5$ (l) $n^3 \div n^3$

Question 3:   Write as a single power of a.

(a) $\dfrac{a^5}{a^2}$ (b) $\dfrac{a^9}{a^3}$ (c) $\dfrac{a^{10}}{a^2}$ (d) $\dfrac{a^7}{a}$

(e) $\dfrac{a^{14}}{a^7}$ (f) $\dfrac{a^4}{a^4}$ (g) $\dfrac{a^3}{a^4}$ (h) $\dfrac{a^5}{a^9}$

Worked Example	Your Turn
Simplify: $$\frac{15x^9 \times 2x^3}{10x^4}$$	Simplify: $$\frac{24x^{10}}{13x^5 \times 4x^2}$$

Fluency Practice

Task 1

Simplify the following:

1. $\dfrac{x^3 \times x^4}{x}$

2. $\dfrac{x^2 \times x^6}{x^3}$

3. $\dfrac{x^7}{x^4} \times x$

4. $\dfrac{a^{-2} \times a^4}{a}$

5. $\dfrac{3s^2 \times 2s^4}{s^3}$

6. $\dfrac{5t^4 \times 4t^3}{2t^2}$

7. $\dfrac{8s^9 \times 4s^0}{2s^4 \times 3s^{-3}}$

8. $\dfrac{2a^8}{8a^3 \times 3a^4}$

9. $\dfrac{15x^{-4}}{3x^{-3} \times 2x^{-1}}$

10. $\dfrac{16s^6 \times 2s^5}{4s^{15} \times 3s^{-4}}$

11. $\dfrac{4a^{\frac{1}{2}} \times 5a^{\frac{7}{2}}}{10a^0}$

12. $\dfrac{6s^{\frac{2}{3}} \times 3s^{\frac{4}{3}}}{3s^{-2} \times 3s^4}$

Task 2

Fill in the missing exponents:

$$a^5 b \times b^6 c \times \frac{ac^7}{b^9} = a^{\square} b^{\square} c^{\square}$$

Task 3

A particle travels $3ab^2$ metres in $12a^2c$ seconds.

Calculate the particles average speed in metres per second.

3.10 Index Laws – Powers of Powers

$(y^3)^1 = y^3$

$(y^3)^2 = y^3 \times y^3 = y \times y \times y \times y \times y \times y = y^6$

$(y^3)^3 = y^3 \times y^3 \times y^3 = y \times y \times y \times y \times y \times y \times y \times y \times y = y^9$

$(y^3)^4 = y^{12}$

$(y^3)^5 = y^{15}$

$(y^3)^n = y^{3n}$

$(y^m)^n = y^{mn}$

Worked Example	Your Turn
Simplify:	Simplify:
$(c^4)^2$	$(c^4)^3$
$-(c^4)^2$	$-(c^4)^3$
$(-c^4)^2$	$(-c^4)^3$

Fluency Practice

Task 1

Simplify:

1. $(b^5)^3$
2. $-(b^5)^3$
3. $(-b^5)^3$
4. $(k^2)^9$
5. $-(k^2)^9$

6. $(-k^2)^9$
7. $(p^7)^{10}$
8. $-(p^7)^{10}$
9. $(-p^7)^{10}$
10. $(h^3)^6$

11. $-(h^3)^6$
12. $(-h^3)^6$
13. $(x^2)^5$
14. $-(x^2)^5$
15. $(-x^2)^5$

16. $(j^{-4})^{-7}$
17. $-(j^{-4})^{-7}$
18. $(-j^{-4})^{-7}$
19. $(m^{-6})^{-2}$
20. $-(m^{-6})^{-2}$

21. $(-m^{-6})^{-2}$
22. $(g^{-9})^{-4}$
23. $-(g^{-9})^{-4}$
24. $(-g^{-9})^{-4}$
25. $(a^{-10})^{-10}$

26. $-(a^{-10})^{-10}$
27. $(-a^{-10})^{-10}$
28. $(c^{-15})^{-3}$
29. $-(c^{-15})^{-3}$
30. $(-c^{-15})^{-3}$

31. $(f^{-4})^{-9}$
32. $-(f^{-4})^{-9}$
33. $(-f^{-4})^{-9}$

Worked Example	Your Turn
Simplify: $$(3c^4)^2$$	Simplify: $$(3c^{-4})^2$$

Fluency Practice

Task 1

Simplify:

1. $(2x^3)^2$
2. $(2x^{-3})^2$
3. $(2x^3)^{-2}$
4. $(2x^{-3})^{-2}$

5. $(-2x^3)^2$
6. $(-2x^{-3})^2$
7. $(-2x^3)^{-2}$
8. $(-2x^{-3})^{-2}$

9. $(5b^6)^2$
10. $(5b^{-6})^2$
11. $(5b^6)^{-2}$
12. $(5b^{-6})^{-2}$

13. $(-5b^6)^2$
14. $(-5b^{-6})^2$
15. $(-5b^6)^{-2}$
16. $(-5b^{-6})^{-2}$

17. $(10c^9)^3$
18. $(10c^{-9})^3$
19. $(10c^9)^{-3}$
20. $(10c^{-9})^{-3}$

21. $(-10c^9)^3$
22. $(-10c^{-9})^3$
23. $(-10c^9)^{-3}$
24. $(-10c^{-9})^{-3}$

25. $(5f^6)^2$
26. $(5f^{-6})^2$
27. $(5f^6)^{-2}$
28. $(5f^{-6})^{-2}$

29. $(-5f^6)^2$
30. $(-5f^{-6})^2$
31. $(-5f^6)^{-2}$
32. $(-5f^{-6})^{-2}$

33. $(10k^9)^3$
34. $(10k^{-9})^3$
35. $(10k^9)^{-3}$
36. $(10k^{-9})^{-3}$

37. $(-10k^9)^3$
38. $(-10k^{-9})^3$
39. $(-10k^9)^{-3}$
40. $(-10k^{-9})^{-3}$

41. $(-15h^9k^7)^3$
42. $(3y^6)^2(x^5y^2z)$
43. $(4h^3)^2(-2g^3h)^3$
44. $(14a^4b^6)^2(a^6c^3)^7$

Task 2

Simplify:

1. $(y^4d^6)^8$
2. $(-c^5h^6)^4$
3. $(u^4v^3)^2$
4. $(x^2y^2)^2$

5. $(a^6c^3)^7$
6. $(xy)^2(x^2y^2)^2$
7. $(k^9)^5(k^3)^2$
8. $(3x^2y^3)^2$

9. $(2k)^3(4k^3)^3$
10. $(2y^2c^{-3})^4$
11. $(5dc^5)^3$
12. $(4r^3)^2(r^2)^5$

13. $(2r^{-3})^2(4r)^{-3}(r^3)^4$
14. $(2h^3)^{-3}(3h)^3$
15. $(3z^2)^{-2}(4z^{-2})^{-3}$

Question 4:   Write as a single power of y.

(a) $(y^5)^2$ (b) $(y^3)^2$ (c) $(y^4)^3$ (d) $(y^5)^4$ (e) $(y^3)^6$ (f) $(y^7)^3$ (g) $(y^6)^6$

(h) $(y^9)^2$ (i) $(y^4)^8$ (j) $(y^3)^{-5}$ (k) $(y^{-5})^2$

Question 6:   Write as a single power of x.

(a) $(2x^3)^2$ (b) $(5x^6)^2$ (c) $(5x^5)^3$ (d) $(2x^3)^4$ (e) $(7x^5)^2$ (f) $(4x^7)^3$ (g) $(2x^6)^6$

(h) $(10x^9)^3$ (i) $(3x^4)^4$

Worked Example	Your Turn
Simplify:	Simplify:
$y^{11} \times y^5$	$x^5 \times x^{-2}$
$6y^3 \times 2y^5$	$7x^5 \times 8x^{-3}$
$y^5 \div y^2$	$y^5 \div y^4$
$8y^3 \div 2y$	$15y^3 \div 3y$
$(y^3)^7$	$(y^7)^8$
$(3y^4)^2$	$(5y^4)^3$

Intelligent Practice

Simplify:

1) $y^{13} \times y^4$

2) $6y^{13} \times 5y^4$

3) $y^{13} \div y^4$

4) $40y^{13} \div 8y^4$

5) $(y^{13})^4$

6) $(3y^{13})^4$

7) $7y^4 \div y^2$

8) $7y^4 \times y^2$

9) $(7y^4)^2$

10) $2x^7 \times 5x^4$

11) $12y^5 \times 5x^4$

12) $12y^5 \div 6y^4$

13) $12y^5 \div 12y^{-4}$

14) $(12y^5)^2$

15) $(12y^{-3})^2$

16) $12y^{-3} \div 4y^2$

17) $12y^{-3} \div 4y^{-2}$

18) $12y^{-3} \times 4y^{-2}$

Worked Example	Your Turn
Simplify:	Simplify:
$\dfrac{a^6 \times a^4}{a^2}$	$\dfrac{a^6 \times a^{-4}}{a^2}$
$(4a^6b^3)^2$	$(2a^6b^3)^4$
$\dfrac{8a^5b^3}{4ab^7}$	$\dfrac{12a^2b^3}{4ab^7}$

Intelligent Practice

Simplify:

1) $\dfrac{a^3 \times a^5}{a^6}$

2) $\dfrac{a^6}{a^3 \times a^5}$

3) $\dfrac{x^6}{a^3 \times a^5}$

4) $\dfrac{12x^6}{2a^3 \times 3a^5}$

5) $\dfrac{12x^6}{2x^3 \times 3x^5}$

6) $2x^3 \times 3x^5$

7) $2x^3y^2 \times 3x^5y^2$

8) $12x^6y^2 \times 3x^5y^2$

9) $12x^6y^2 \div 3x^5y^2$

10) $\dfrac{12x^6y^2}{3x^5y^2}$

11) $\dfrac{12x^6y^8}{3x^5y^2}$

3.12 Substitution

When substituting into our expressions, we simply replace our unknown with whatever we're asked to substitute in.

Worked Example	Your Turn
If $a = 4$, find	If $b = 6$, find
$3 + a$	$2 + b$
$3a$	$2b$
$\dfrac{3a}{4}$	$\dfrac{2b}{3}$
a^2	b^2
$3a^2$	$2b^2$

Worked Example	Your Turn
If $a = -4$, find	If $b = -6$, find
$3 + a$	$2 + b$
$3a$	$2b$
$\dfrac{3a}{4}$	$\dfrac{2b}{3}$
a^2	b^2
$3a^2$	$2b^2$

Fluency Practice

Question 1: If $a = 7$ $b = 10$ $c = 3$ $d = 8$ and $e = 15$
Find the value of each expression.

(a) $a + 5$ (b) $b - 4$ (c) $c + d$ (d) $e - d$

(e) $2a$ (f) $4b$ (g) $3e$ (h) $5c$

(i) $\dfrac{b}{2}$ (j) $\dfrac{e}{5}$ (k) $\dfrac{d}{4}$ (l) $\dfrac{a}{2}$

(m) a^2 (n) b^2 (o) c^2 (p) d^2

(q) $2a + 1$ (r) $3b - 7$ (s) $9c + 11$ (t) $4e - 45$

(u) $2a + 3c$ (v) $4d - b$ (w) $5a + 2d$ (x) $e - 4c$

(y) $30 - 4a$ (z) $15 - 3c$

Question 2: If $f = 5$ $g = 6$ $h = 4$ and $i = 2$
Find the value of each expression.

(a) fg (b) hi (c) fgh (d) i^3

(e) $\sqrt{h}$ (f) $3f + 2g$ (g) $5h + 7i$ (h) $9h - 7i$

Question 3: If $a = -2$ $b = 5$ $c = -6$ $d = 10$ and $e = 9$
Find the value of each expression.

(a) $a + 4$ (b) $b - 8$ (c) $c + e$ (d) $a - d$

(e) $d - c$ (f) $2c$ (g) $7a$ (h) $-7b$

(i) $2d + 3c$ (j) $6e + 3a$ (k) $5a + 7$ (l) $20 + 4a$

(m) ac (n) $40 - d$ (o) $2e - a$ (p) $bd + a$

(q) $\dfrac{a}{2}$ (r) $\dfrac{d}{4}$ (s) $\sqrt{e}$ (t) c^2

Question 4: If $a = 1.5$ $b = 4$ $c = 6$ $d = 0.5$ and $e = -3$
Find the value of each expression.

(a) $4(a + d)$ (b) $5(c + b)$ (c) $3(10 - e)$ (d) abc

(e) e^3 (f) d^2 (g) $5b^2$ (h) $8e^2 + 3$

(i) $\dfrac{b + 2}{3}$ (j) $\dfrac{2c - e}{4}$ (k) $\dfrac{10d + 4b}{7}$

Question 5: $P = 2L + 2W$, work out P if $L = 8$ and $W = 3$.

Question 6: $C = 15h + 30$, work out C if $h = 6$.

# Intelligent Practice

Given that $a = 2, \ b = -3, \ c = 4, \ \text{d} = -5$, find:

1) $b + a$

2) $a + b$

3) $a - b$

4) $b - a$

5) $7b - 6a$

6) $6a - 7b$

7) $7a - 7b$

8) $7(a - b)$

9) $7(d - c)$

10) $7a - 7b + 7d - 7c$

11) ab

12) $ab + c$

13) $ac + b$

14) $bc + a$

15) $\dfrac{bc+a}{d}$

16) $\dfrac{7c+6a}{d}$

17) $\dfrac{7c+6a}{d+b}$

18) $\dfrac{7d+6b}{c+a}$

19) $\dfrac{7c+6b-1}{c+a}$

20) $\dfrac{c+a}{7c+6b-1}$

21) a^2

22) b^2

23) d^2

24) cd^2

25) $7d^2$

26) $7d^2 + b^2$

27) $7d^2 + 6b^2$

28) $7d^2 - 6b^2$

29) $\sqrt{7d^2 - 6b^2}$

30) $\dfrac{\sqrt{7d^2-6b^2}}{ac-b}$

Maths Venns

Value of expression is a square number

If you think a region is impossible to fill, convince me why!

Value of expression ends in a 1

Value of expression is an integer
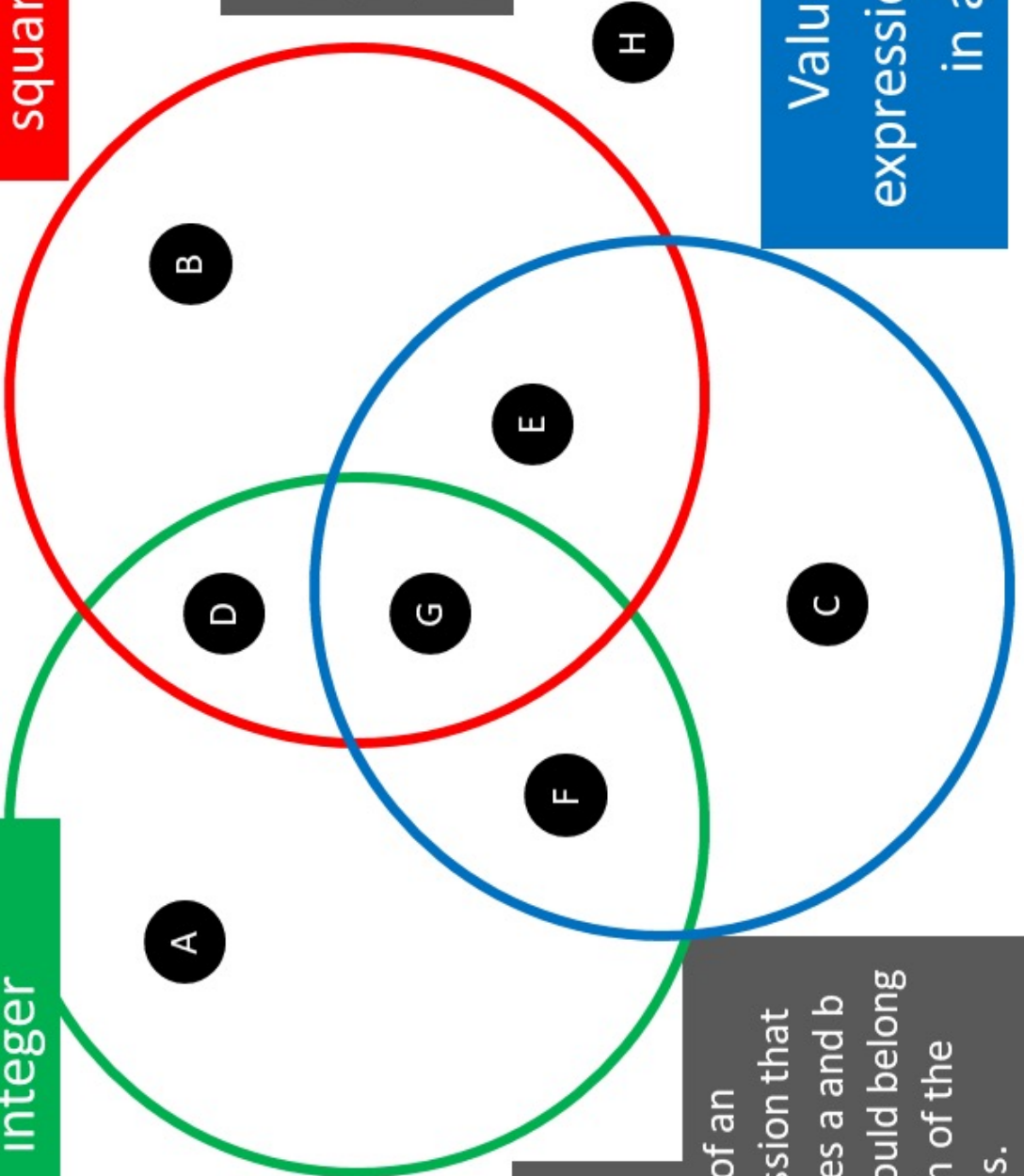
a = 3

b = -2

Think of an expression that includes a and b that could belong in each of the regions.

Frayer Model – Formula

Definition	Characteristics
A relationship between variables, expressed algebraically.	- Is a type of equation. - Must have at least two variables. - Often refers to particular things.

Examples
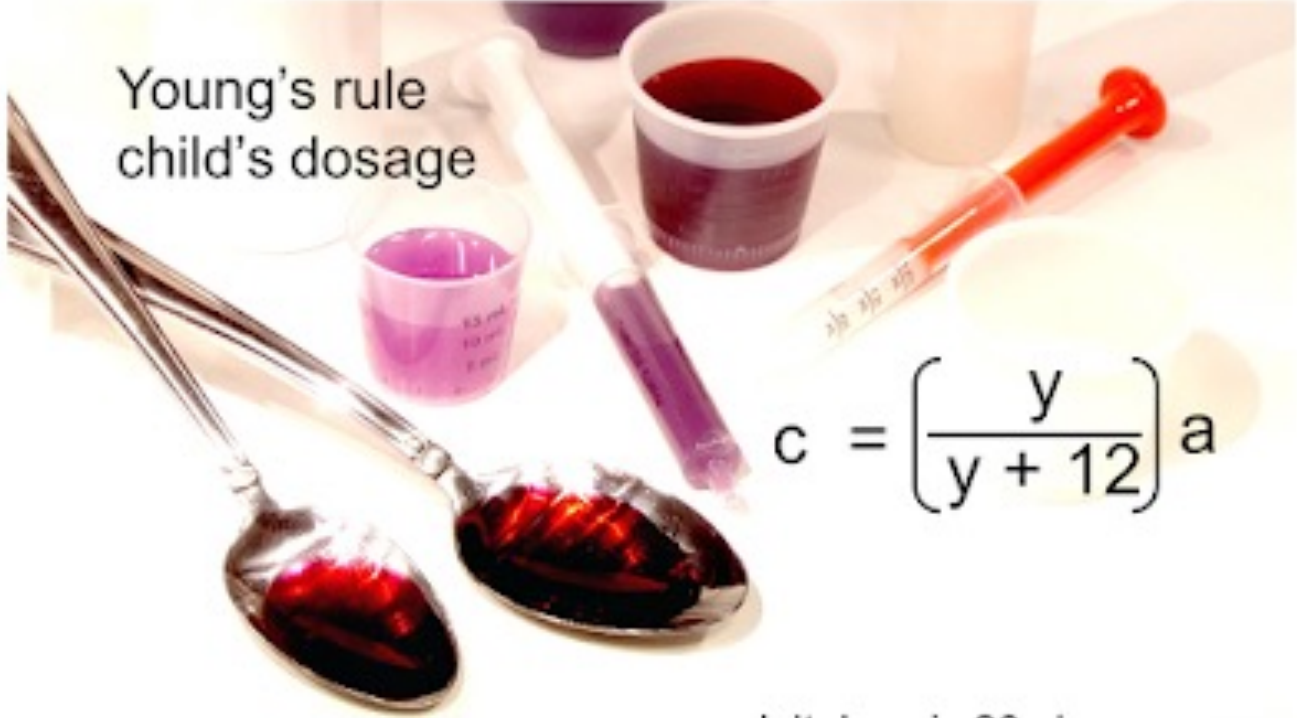
- $6b + 11c + 15d = A$
- $z = 6x + 11y$
- $z = 6x + 11$
- $y = 2x + 7$
- $z = 2x + 7y$
- $a = 6b + 11$
- $z^2 = 2x^2 + 7y$
- $15 = 6x + 11y$
- $y = 2x + 7y$

Non Examples

- $2x + 7$
- $x = 2x + 7$
- $z > 2x + 7y$
- $6b + 11 = 15b$
- $6v + 11$
- $6b + 11 \neq 15a$

Substituting in Formulas

Young's rule
child's dosage

$$c = \left(\frac{y}{y + 12}\right) a$$

c is the amount in ml
y is the age in years
a is the adult dose, in ml

an adult dose is 20ml
how much should be given to
(i) a 4 year old child?
(ii) an 8 year old child?

Substituting in Formulas

substituting in formulas

(1)   cooking a turkey in an oven
      t = time in minutes
      w = weight in pounds (lb)

      in a normal oven: **t = 15w + 40**
      in a convection oven: **t = 7½ w + 30**

      (a) how much quicker is it to cook a small, 8lb turkey in a convection oven?
      (b) how much quicker is it to cook an average, 12lb turkey in a convection oven?
      (c) what is the weight of a turkey that takes 4 hours and 55 minutes to cook in a normal oven?

(2)   a boat can be hired for parties
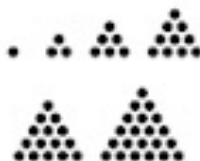   c = cost (£)
      n = number of people
      c = 14n + 25
      (a) work out c if n = 10
      (b) work out c if n = 20
      (c) work out the number of people if the cost of a party was £375

(3)   the number of dots in a triangle
      t = total dots
      n = dots in the bottom row
      t = ½ n(n + 1)
      (a) work out t if n = 10
      (b) work out t if n = 100
      (c) work out the number in the bottom row if the total is 120

(4)   Hero of Alexandria is credited with a formula
      for working out the area of any triangle when
      you know the three sides: a, b and c
      s, the semi-perimeter, = ½ $(a + b + c)$
      $$A = \sqrt{s(s - a)(s - b)(s - c)}$$
      (a) work out A if $a = 5$, $b = 5$ and $c = 6$
      (b) work out A if $a = 5$, $b = 5$ and $c = 8$
      (c) work out A if $a = 7$, $b = 15$ and $c = 20$
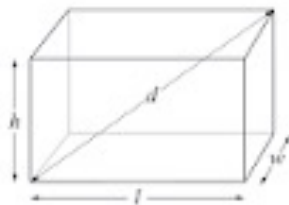   note: usually A will not be an integer

      Hero
      lived in Alexandria
      around 10 – 70 CE

(5)   length of the longest diagonal of a cuboid (box)
      $d^2 = w^2 + l^2 + h^2$
      (a) work out d if $w = 2$, $l = 6$ and $h = 3$
      (b) work out l if $w = 1$, $h = 4$ and $d = 9$
      (c) work out l if $w = 4$, $h = 4$ and $d = 9$
      note: usually d will not be an integer
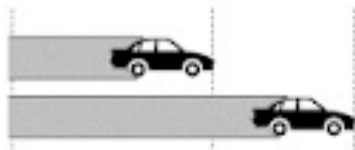
(6)   the stopping distance at various speeds
      v = speed in miles per hour
      d = distance to stop in feet
      $d = v + (v^2)/20$
      (a) work out d if $v = 30$
      (b) work out d if $v = 60$
      (c) work out v if $d = 175$

True or False?

Cut out the identities and sort them into two groups: True or False.

$a + a = a^2$	$a^3 = a \times a \times a$	$2a \times a = 2a^2$
$a + a = 2a$	$3b - b = 2b$	$3a = 2a + a$
$a \times a = a^2$	$3a = 2a \times a$	$a + b = ab$
$a \times a = 2a$	$3b - 2 = b$	$3a \times 4b = 7ab$
$a \times b = ab$	$a \div 2 = \dfrac{a}{2}$	$3a + 4b = 7ab$
$2a + 1 = 3a$	$b \times 3 \times a = 3ab$	$3a \times 4b = 12ab$
$2a + b = 2ab$	$2 \times a \times a \times b = 2ab^2$	$a + b + 2a = 3a + b$
$2a \times b = 2ab$	$3b - 3b = b$	$3 \times a \times a = 6a$

Powers of y Eliminator

Simplify the 31 calculations below, crossing out the corresponding squares in the grid. When you have finished, the remaining squares will reveal a message.

y^6	C	y^{13}	Y	y^{100}	A	y^7	0	$3y^{12}$	R	y^{75}	B
$2y^4$	T	y^{20}	Y	1	A	y^{22}	U	y^{29}	H	y^{11}	S
y^{15}	I	y^4	E	y^{18}	A	y^9	P	y^{33}	T	y^{19}	U
y^{21}	R	y^{16}	Q	$2y^2$	C	y^{28}	E	y^5	X	y^{26}	I
y^{64}	N	y^3	R	y^{24}	K	y^2	Y	y^{32}	B	y^{23}	H
y	D	y^8	M	y^{17}	I	$2y^8$	A	y^{30}	G	y^{27}	G
y^{10}	H	y^{12}	J	$4y^3$	W	y^{14}	F	y^{36}	T	y^{31}	S

1. $y^3 \times y^2$

2. $y^5 \times y^6$

3. $y^7 \times y^2$

4. $y^3 \times y^{16}$

5. $(y^2)^2$

6. $(y^3)^5$

7. $(y^4)^5$

8. $(y^7)^2$

9. $(y^5)^{15}$

10. $y^8 \div y^2$

11. $y^{16} \div y^4$

12. $y^7 \div y^6$

13. $y^{16} \div y^8$

14. $y^{15} \times y^9$

15. $(y^{16})^2$

16. $y^8 \times y^9 \times y^{12}$

17. $y^0 \times y^0$

18. $y \times y^{10} \times y^{20}$

19. $y^0 \times y^2$

20. $y \times (y^7)^9$

21. $y^2 + y^2$

22. $y^{28} \div y^2$

23. $y^4(y^{28} \div y^2)$

24. $y^3 + 3y^3$

25. $y^3(y^{25} \div y^5)$

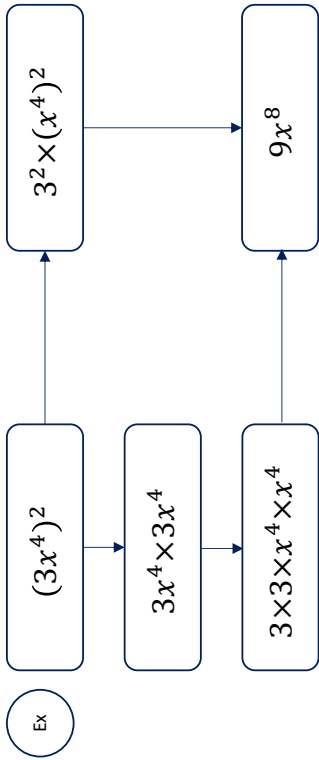26. $y \times y^{29} \times y^3$

27. $2(y^2)^4$

28. $3(y^4)^3$

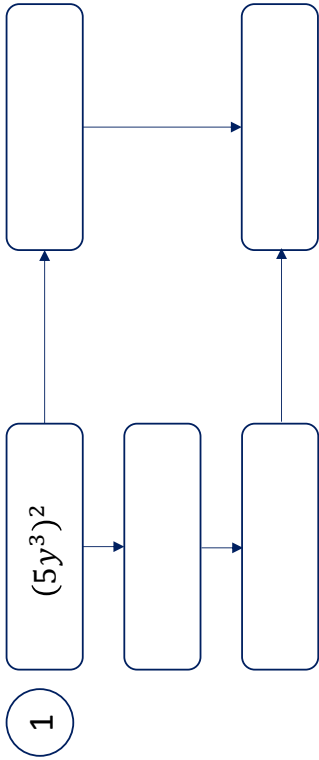29. $y^0 \times (y^{10})^{10}$

30. $y^0(y^{32} \div y^{16})$

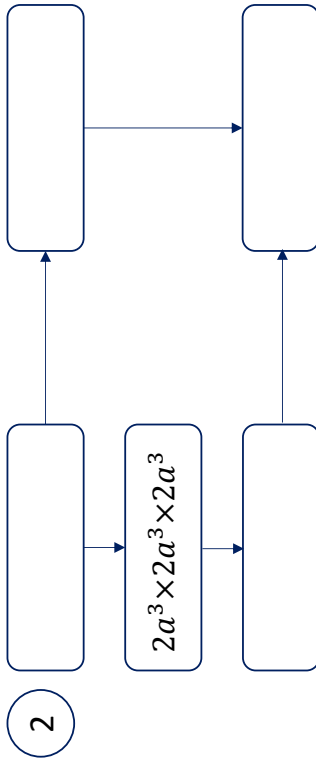31. $y^4 + (y^2)^2$

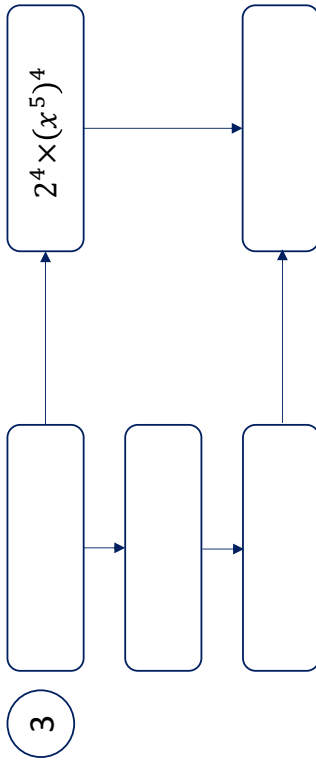# Raising a Power to a Power

Task 1 – Raising a power to a power

Ex

$(3x^4)^2$ → $3^2 \times (x^4)^2$ → [ ]

$3x^4 \times 3x^4$ → $3 \times 3 \times x^4 \times x^4$ → $9x^8$

$(3x^4)^2 = 9x^8$

1

$(5y^3)^2$ → [ ] → [ ] → [ ]

[ ] → [ ]

$(5y^3)^2 = $ ___

2

[ ] → [ ] → [ ]

[ ] → $2a^3 \times 2a^3 \times 2a^3$ → [ ]

___ = ___

3

$2^4 \times (x^5)^4$ → [ ] → [ ] → [ ]

[ ] → [ ]

___ = ___

Extension – Can you create question/flowchart of your own?

Raising a Power to a Power

Ex

$(7x^3y)^2$ → $7x^3y \times 7x^3y$ → $7 \times 7 \times x^3 \times x^3 \times y \times y$ → $49x^6y^2$

$7^2 \times (x^3)^2 \times y^7$ → $49x^6y^2$

$(7x^3y)^2 = 49x^6y^2$

4

$(10a^2b^5)^2$ → ☐ → ☐ → ☐ → ☐

$(10a^2b^5)^2 = $ _____

5

☐ → $3pq^2 \times 3pq^2 \times 3pq^2$ → ☐ → ☐

_____ = _____

6

☐ → ☐ → ☐ → ☐

$2^5 \times (x^2)^5 \times y^5$ → ☐

_____ = _____
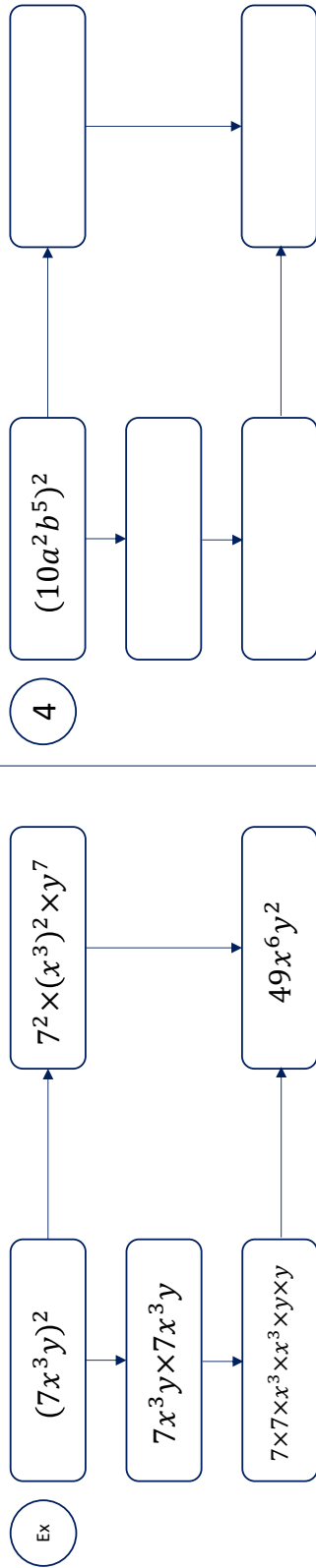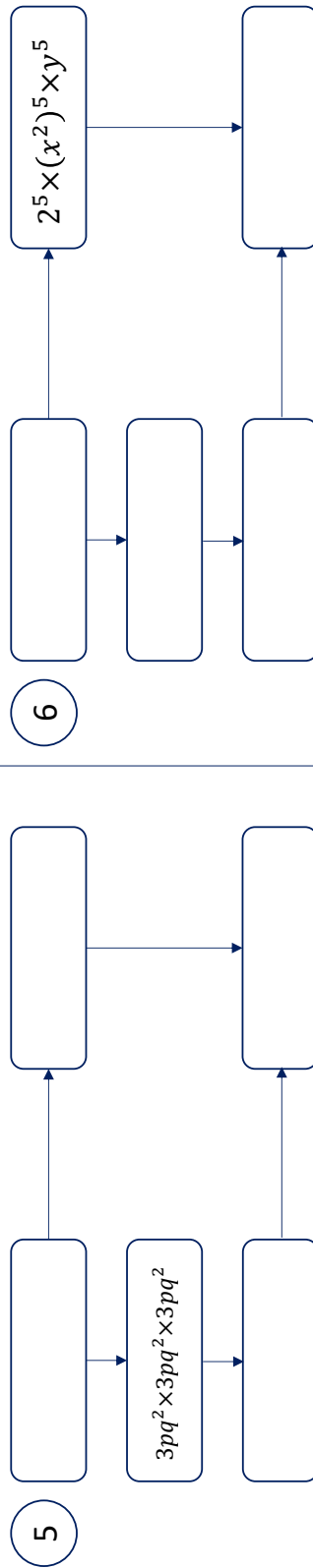
Extension – Can you create question/flowchart of your own?

Substitute

what values of 'a' and 'b' make the three expressions equal?

1) $3a + 5$ $4(b + 1)$ $8a - 5b$

2) $a + 3b + 2$ $2a + b$ $a + b + 10$

3) $5b - 3a$ $2b + 5a$ $11a - b + 6$

what values of 'a' and 'b' make the three expressions equal?

1) $a^2 + b^2$ $2(4b - a)$ $2(ab + 2)$

2) $2a^2 + b + 4$ $9a + \frac{1}{2}b$ $ab + 2$

3) $b^2 - a^2$ $3(b + a)$ $7(b - a)$