Year 7
Mathematics
Unit 4 – Student

Name: _____

Class: _____

Contents

1 Number Theory

1.1 Types of Numbers

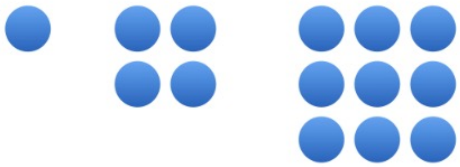
Frayer Model – Integers

Definition	**Characteristics**
A whole number, either positive or negative.	• No decimal or fractional part when simplified.
Examples	**Non Examples**
• 1 • $\frac{1}{1}$ • 11 • -11 • 275 • 275.0 • 36 • $\sqrt{36}$ • $\frac{4}{2}$ • $-\frac{4}{2}$	• 1.1 • 27.5 • $\sqrt{275}$ • $\frac{27}{5}$ • 0.42

Frayer Model – Square Numbers

Square Numbers:

$1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, \ldots$

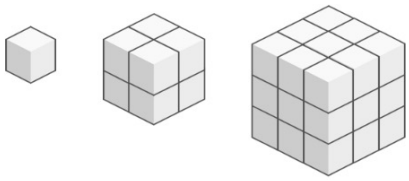


Definition	**Characteristics**
The multiple created when a positive integer is multiplied by the same positive integer.	• The process of creating a square number is called "squaring" and is shown using a power of 2 (^{2})
Examples	**Non Examples**
• $4 = 2^2$ • $9 = 3^2$ • $100 = 10^2$ • $144 = 12^2$ • $1 = 1^2$	• 5 • 1000 • -4 • $\frac{1}{4} = \left(\frac{1}{2}\right)^2$ • $2 \neq 1^2$

Frayer Model – Cube Numbers

Cube Numbers:

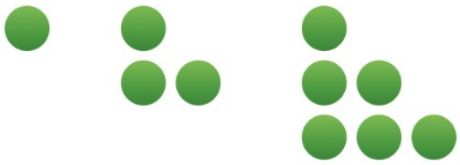
$1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, \ldots$



Definition	**Characteristics**
The multiple created when a positive integer is multiplied by the same positive integer two more times.	• The process of creating a cube number is called "cubing" and is shown using a power of 3 (^{3})
Examples	**Non Examples**
• $8 = 2^3$ • $27 = 3^3$ • $1000 = 10^3$ • $1728 = 12^3$ • $1 = 1^3$	• 5 • 100 • $-8 = (-2)^3$ • $\frac{1}{8} = \left(\frac{1}{2}\right)^3$ • $3 \neq 1^3$

Frayer Model – Triangular Numbers

Triangular (or Triangle) Numbers:
$1, 3, 6, 10, 15, 21, 28, 36, 45, 55, \ldots$



Definition	**Characteristics**
A number created by adding all the whole numbers from 1 to n.	• Each number makes a triangular dot pattern. • Adding two consecutive triangular numbers makes a square number.
Examples	**Non Examples**
• 1 • 3 • 6 • 10 • 15	• 2 • 4 • 5 • 7 • 8

Maths Venns

Triangle Number
Square Number
Cube Number
If you think a region is impossible to fill, convince me why!
Think of a number that could belong in each of the regions
A
B
C
D
E
F
G
H

1.2 Multiples

If cola is sold in multipacks of 6, I can only buy a multiple of 6 bottles.

Worked Example	Your Turn
Write down the first six multiples of 6	Write down the first six multiples of 8

Intelligent Practice

Write down the first six multiples of these numbers:

1) 5

2) 3

3) 4

4) 10

5) 7

6) 9

7) 11

8) 20

9) 100

10) 50

11) 12

12) 35

Fluency Practice

Question 1: Write down the first six multiples of these numbers

(a) 5 (b) 3 (c) 4 (d) 10 (e) 7 (f) 9

(g) 11 (h) 20 (i) 100 (j) 50 (k) 12 (l) 35

Question 2: Below is a list of numbers.

12 15 17 20 22 25 27 30 32 35 39 40

From the list write down any numbers that are multiples of:

(a) 2 (b) 5 (c) 10 (d) 3 (e) 4 (f) 8

Question 3: List all the numbers between 40 and 60 (inclusive) that are multiples of:

(a) 5 (b) 3 (c) 6 (d) 8 (e) 9 (f) 14

Question 4: Below is a list of numbers.

100 101 102 103 104 105 106 107 108 109

From the list write down any numbers that are multiples of:

(a) 2 (b) 3 (c) 5 (d) 10 (e) 4 (f) 15

Question 5: (a) List the first ten multiples of 3.
(b) List the first ten multiples of 4.
(c) Write down any numbers listed that are multiples of both 3 and 4.

Question 6: (a) List the first ten multiples of 5.
(b) List the first ten multiples of 6.
(c) Write down any numbers listed that are multiples of both 5 and 6.

Question 7: (a) List the first ten multiples of 6.
(b) List the first ten multiples of 9.
(c) Write down any numbers listed that are multiples of both 6 and 9.

Question 8: Write down three common multiples of 8 and 12.

Question 9: Write down three common multiples of 4 and 6.

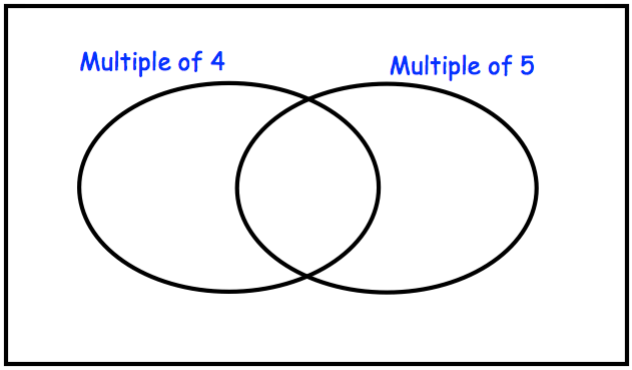
Question 10: Write down three common multiples of 15 and 20.

Extension

Question 1: A light flashes every 8 seconds. How many times will it flash in 3 minutes?

Question 2: Find the smallest number over 200 that is a multiple of 6.

Question 3: Copy the Venn diagram below.
Place these numbers into the Venn diagram: 8, 10, 12, 13, 20, 22, 25, 40, 50



Question 4: Find the first even number that is a multiple of 5 and 7.

Question 5: A crate can hold 12 cans of lemonade.
Thomas has 200 cans of lemonade.
How many crates can be filled?

Question 6: Find a number that is a multiple of 2, 3, 4, 5 and 6.

Frayer Model – Multiple

Definition	**Characteristics**
A multiple is the result of multiplying a positive integer by another positive integer.	• Often given as a list by multiplying the same number by $1, 2, 3, 4$, etc. in turn. • The multiples of a number are the numbers in its time tables. • Will always be greater than or equal to the starting number.
Examples • Multiples of $6 =$ $6, 12, 18, \ldots$ • Multiples of $19 =$ $19, 38, 57, \ldots$	**Non Examples** • 3 is not a multiple of 6 • 2 is not a multiple of 4 or 6 or 8 etc. • 1 is not a multiple of any number except 1

1.3 Divisibility Tests

A divisibility test is a rule for determining whether one whole number is divisible by another. It is a quick way to find factors of large numbers.

Divisibility Tests for 2, 5 and 10

Number	Test	Example	Non Example
2	Ends in 0, 2, 4, 6 or 8	1246	3273
5	Ends in 0 or 5	3825	1011
10	Ends in 0	4890	3568

Divisibility Tests for 4 and 8

Number	Test	Example	Non Example
4	Last two digits divisible by 4	7356	9382
8	Last three digits divisible by 8	4512	8148

Divisibility Tests for 3 and 9

Number	Test	Example	Non Example
3	Sum of digits is divisible by 3	1353	4567
9	Sum of digits is divisible by 9	1458	3057

Divisibility Tests for 7 and 11

Number	Test	Example	Non Example
7	Multiply the last digit by 5 and add it to the remaining number, and see if the result is divisible by 7	9961	3581
11	Sum odd-positioned digits and subtract sum of even-positioned digits and see if the result is divisible by 11	2761 8261	5476

Divisibility Tests for 6 and 12

Number	Test	Example	Non Example
6	Divisible by both 2 and 3	4728	7352
12	Divisible by both 3 and 4	3576	1222

Fluency Practice

Is the number to the left of each row divisible by the number at the top of each column? Check the boxes.

	2	3	4	5	6	7	8	9	10	11	12
497,652											
8,118											
28,755											
640											
874,629											
981,274											
19,746											
6,804											
246,972											
30,630											
176											
78,900											

Extension

How many different solutions can you find, without a calculator, using the digits 1 to 9 once only in this sum?

□□□□□□□□□ × □ = 98765432

The boxes at the end of each row and the foot of each column give the result of multiplying the three numbers in that row or column.

Can you arrange the numbers 1 to 9 in the grid?

			15
			108
			224
144	8	315	

Extension

What is the **largest** multiple you can make using the digits below?
You don't have to use each digit and can use each one at most once

2 3 4 5

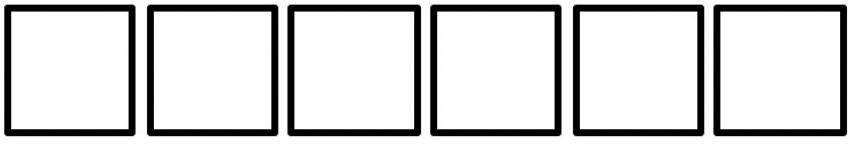
Multiple of 2 **Multiple of 3** **Multiple of 6**

What are the smallest and largest multiples of 4 you can make using **all** the digits below?

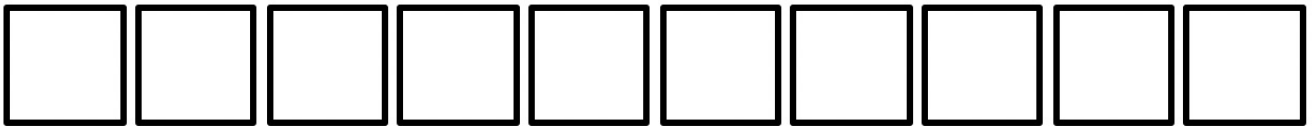
4 5 6 7 8

Smallest _____ **Largest** _____

Using the digits 1 to 6, create a 6 digit number so that the first two digits are divisible by two, the first three digits are divisible by three, etc
How many answers are there?



Do the same using the digits 0 to 9 (one answer!)



1.4 Factors

A factory is a place where lots of separate parts are put together to make something like a car. All of the separate things that go into the car are factors.

Worked Example	Your Turn
Find all the factors of 44	Find all the factors of 88

Intelligent Practice

Find all the factors of:

1) 8
2) 10
3) 7
4) 12
5) 20
6) 22
7) 18
8) 50
9) 15
10) 19
11) 30
12) 100
13) 32
14) 24
15) 42
16) 28
17) 66
18) 70
19) 45
20) 60
21) 25

Count the number of factors for each question.

- Which numbers have 2 factors?
- Which numbers have a odd number of factors?
- Take the factors of 28 (not including 28) add them together. What do you notice?

Fluency Practice

Question 1: List all the factors of these numbers

(a) 8 (b) 10 (c) 7 (d) 12 (e) 20 (f) 22 (g) 18

(h) 50 (i) 15 (j) 19 (k) 30 (l) 100 (m) 32 (n) 24

(o) 42 (p) 28 (q) 66 (r) 70 (s) 45 (t) 60 (u) 25

Question 2: Is 3 a factor of.... ?

(a) 14 (b) 21 (c) 27 (d) 32 (e) 57 (f) 301 (g) 100

Question 3: Is 5 a factor of.... ?

(a) 20 (b) 34 (c) 40 (d) 38 (e) 45 (f) 102 (g) 135

Question 4: List all the factors of these numbers (you may use a calculator)

(a) 84 (b) 140 (c) 200 (d) 240 (e) 145 (f) 192 (g) 244

Question 5: Is 9 a factor of.... ?

(a) 38 (b) 90 (c) 72 (d) 108 (e) 909 (f) 9001 (g) 293

Extension

Question 1: 21 25 30 45
Which number is the odd one out? why?

Question 2: 15 24 28 33
Which number is the odd one out? why?

Question 3: Mary has 26 sweets and is able to share them evenly between her friends.
Mary has more than 1 friend.
Write down how many friends Mary might have.

Question 4: James says that all numbers have an even number of factors.
Is he correct?

Factors of Square Numbers

The number of factors of a square number will always be odd. Can you explain why this is?

Normally factors come in pairs e.g. for 20, we have
$1 \times 20, 2 \times 10, 4 \times 5$

However, in 49, the 7 in 7×7 only counts once, so we will have an odd number of factors.

Factors of Perfect Numbers

A perfect number is a number whose factors (excluding itself) add up to itself.

For example: The factors of 6 (excluding 6) are 1, 2, and 3, and $1 + 2 + 3 = 6$.

Note: All perfect numbers are triangular numbers.

Frayer Model – Factors

Definition	**Characteristics**
A positive integer that will divide exactly into a given positive integer.	• Factors are normally identified in pairs. • The smallest factor of any number is 1. • The largest factor of any number is the number. • Will always be smaller than or equal to the starting number.
Examples	**Non Examples**
• Factors of $8 = 1, 2, 4, 8$ • Factors of $9 = 1, 3, 9$ • Factors of $20 = 1, 2, 4, 5, 10, 20$	• 4 is not a factor of 2 • 2 is not a factor of 1 • 4 is not a factor of 18

Factors of 12

12 has exactly six factors:

1	12
2	6
3	4

Not factors of 12

All other numbers are NOT factors of 12, e.g.:

0.5, 24, -1, 9, -3, $\frac{1}{3}$, 36, 5, -12

1.5 Prime Numbers

Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate.

Definition	**Characteristics**
A positive integer with precisely two distinct factors.	• The factors of the number will be 1 and the number itself. • Nearly all odd.
Examples	**Non Examples**
• 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97	• 1, 4, 6, 9, 15, 25, 27, 36, 39, 50, 63, 72, 81, -2, $\frac{1}{2}$

The largest known prime number is $2^{82,589,933} - 1$, a number which has $24{,}862{,}048$ digits. It takes about 8000 sheets of paper to print it!

Sieve of Eratosthenes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Find the smallest unshaded number.
- Shade in all of the multiples of this number except for the number itself.
- Repeat steps 1 and 2 until you reach a number that has no unshaded multiplies other than itself on the grid.

What types of numbers do you have left?
Why were these numbers left?

1.6 Review and Problem Solving

Factor, Multiple, Both, Neither

For the **Relationship**, choose either: *is a factor of*, *is a multiple of, is neither a factor nor a multiple of*, or *is both a factor and a multiple of*

1st number	Relationship	2nd number
3		9
9		3
9		27
9		49
9		6
6		9
9		9
1		9
9		1
9		2
9		4.5
4.5		9
0		9
9		0

Extension

1. Complete these statements with the most simple examples you can think of

2. Then complete the statements with the most interesting examples you can think of

   _____ is a factor of ____

   _____ is a multiple of _____

   _____ is both a factor and a multiple of ____

   _____ is neither a factor nor a multiple of _____

Sequences of Multiples

consecutive numbers

three consecutive numbers

(1) are multiples of 2 , 3 and 4 (in this order)
what could they be? in general?

(2) are multiples of 3 , 4 and 5 (in this order)
what could they be? in general?

(3) are multiples of 4 , 5 and 6 (in this order)
what could they be? in general?

four consecutive numbers

(4) are multiples of 2 , 3 , 4 and 5 (in this order)
what could they be? in general?

five consecutive numbers

(5) are multiples of 2 , 3 , 4 , 5 and 6 (in this order)
what could they be? in general?

Divisibility Rules

[1] in the number
235, A11B
replace A and B by digits so that the number divides exactly by 3 and by 5

try to establish all the possible answers

[2] in the number
56, A2B
replace A and B by digits so that the number is a multiple of 15

show that there are 7 solutions

[3] replace A and B in the number
22,A2B so that the number is a multiple of 45

show that there are 2 solutions

[4] replace A and B in the number
2A7,69B so that the number is divisible by 3 , 5 and 11

show that there are 2 solutions

[5] replace A and B in the number
6A,9B0 so that 44 is a factor of the number

show that there are 5 solutions

use 1 to 9, once only in the cells so that you obtain row and column products as shown:

(1)

			15
			108
			224
144	8	315	

(2)

			21
			64
			270
36	30	336	

(3)

			80
			63
			72
72	48	105	

(4)

			12
			189
			160
15	144	168	

use 1 to 9, once only in the cells so that you obtain row and column products as shown:

(5)

			48
			105
			72
96	45	84	

(6)

			54
			56
			120
16	210	108	

(7)

			96
			45
			84
32	70	162	

(8) two solutions

			42
			54
			160
35	144	72	

Primes using 0 to 9

make exactly 6 one or two digit prime numbers using the digits 1 to 9 exactly once

how many different ways can this be done?

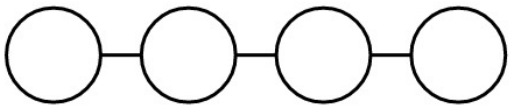
For example: 2, 3, 5, 41, 67, 89

Consecutive Chains

consecutive chains

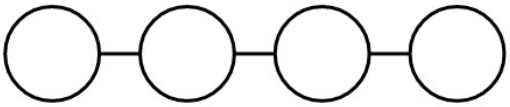
Use the clues to find sets of increasing consecutive numbers. All numbers used are less than 100.

A



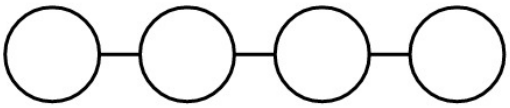
prime — *cube number* — *square number* — *factor of 60*

B



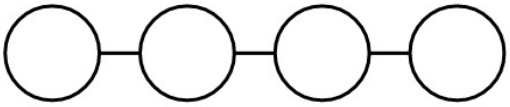
prime — *prime* — *square number* — *prime*

C



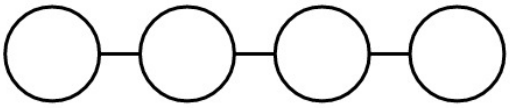
factor of 36 — *prime* — *multiple of 10* — *multiple of 3*

D



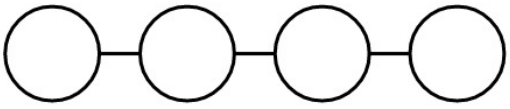
multiple of 1 — *multiple of 4 & 6* — *square number* — *factor of 100*

E



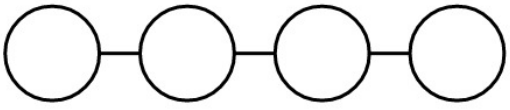
prime — *even* — *multiple of 3 & 5* — *square number*

F



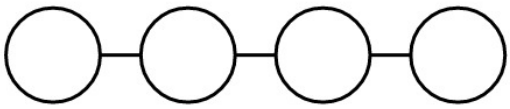
prime — *factor of 48 & 60* — *prime* — *even*

G



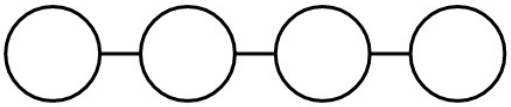
greater than 15 — *divisible by 3* — *even* — *less than 25*

H



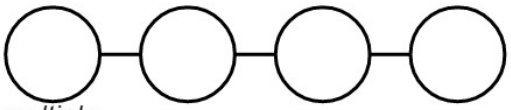
square number — *not prime* — *cube number* — *not prime*

I



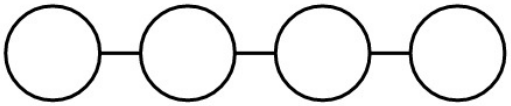
not prime — *not prime* — *not prime* — *square number*

J



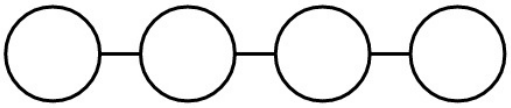
multiple of 2 & 11 — *prime* — *multiple of 5* — *not prime*

K



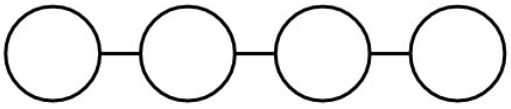
multiple of 6 — *multiple of 5* — *multiple of 4* — *multiple of 3*

L



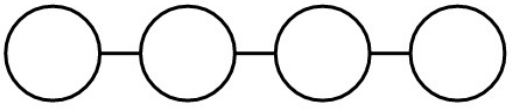
multiple of 7 — *no clue* — *no clue* — *multiple of 9*

M



prime — *no clue* — *no clue* — *prime*

N



multiple of 2 & 7 — *no clue* — *multiple of 3 & 8* — *no clue*

Maths Venns

Multiple of 3
If you think a region is impossible to fill, convince me why!
Factor of 36
H
B
E
D
G
C
F
A
Square Number
Think of a number that could belong in each of the regions

Maths Venns

Factor of 40
Multiple of 5
Multiple of 4
If you think a region is impossible to fill, convince me why!
Think of a number that could belong in each of the regions
A
B
C
D
E
F
G
H

Maths Venns

Triangle Number
Prime Number
Cube Number
If you think a region is impossible to fill, convince me why!
Think of a number that could belong in each of the regions
A
B
C
D
E
F
G
H

1.7 Highest Common Factor

Worked Example	Your Turn
Find the HCF of 6 and 15	Find the HCF of 6 and 20

Intelligent Practice

Find the HCF of:

1) 5 and 10
2) 10 and 5
3) 20 and 5
4) 20 and 10
5) 20 and 30
6) 4 and 30
7) 5 and 30
8) 7 and 30
9) 14 and 30
10) 28 and 30
11) 30 and 30
12) 30 and 48
13) 36 and 48
14) 24, 36 and 48
15) 240, 360 and 480

Can you spot any patterns between questions and answers? Can you explain why they occur?

Fluency Practice

Question 1: (a) List all the factors of 10
(b) List all the factors of 15
(c) Write down all the common factors of 10 and 15.

Question 2: (a) List all the factors of 12
(b) List all the factors of 18
(c) Write down all the common factors of 12 and 18.

Question 3: Write down all the common factors of each of these pairs of numbers.

(a) 6 and 8 (b) 15 and 20 (c) 9 and 15 (d) 7 and 14

(e) 30 and 40 (f) 21 and 27 (g) 18 and 30 (h) 16 and 24

Question 4: (a) List all the factors of 14
(b) List all the factors of 21
(c) Find the highest common factor (HCF) of 14 and 21.

Question 5: (a) List all the factors of 24
(b) List all the factors of 36
(c) Find the highest common factor (HCF) of 24 and 36.

Question 6: Find the highest common factor (HCF) of each of these pairs of numbers.

(a) 4 and 14 (b) 6 and 9 (c) 9 and 21 (d) 8 and 12

(e) 6 and 15 (f) 10 and 17 (g) 30 and 45 (h) 40 and 60

(i) 28 and 63 (j) 24 and 36 (k) 16 and 28 (l) 18 and 45

(m) 150 and 200 (n) 12 and 54 (o) 90 and 270 (p) 39 and 65

Question 7: Find the highest common factor (HCF) of each of these sets of numbers.

(a) 12, 6 and 15 (b) 27, 33 and 12 (c) 30, 15 and 25 (d) 8, 20 and 12

(e) 10, 25 and 13 (f) 12, 24 and 30 (g) 9, 36 and 45 (h) 100, 125 and 200

Extension

Question 1: Martin says that 6 is a common factor of 42, 36 and 50.
Is he correct?

Question 3: Sam has completed his maths homework.
Can you spot any mistakes?

Find the highest common factor of 18 and 36

Factors of 18: 2, 3, 6, 9
Factors of 36: 2, 3, 4, 6, 9, 12, 18

HCF = 9

Frayer Model – Highest Common Factor

Definition	**Characteristics**
The largest integer which is a factor of two or more given positive integers.	• Will be less than or equal to the smallest of the given numbers. • Often abbreviated to HCF. • All of the given numbers will appear in the times table of the HCF.
Examples	**Non Examples**
• $\text{HCF}\ (8, 12) = 4$ • $\text{HCF}\ (30, 15) = 15$ • $\text{HCF}\ (4, 15) = 1$ • $\text{HCF}\ (4, 6, 10) = 2$	• $\text{HCF}\ (1, 2) \neq 2$ • $\text{HCF}\ (4, 8) \neq 8$ • $\text{HCF}\ (4, 5) \neq 20$

Frayer Model – Co-Prime

Definition	**Characteristics**
Positive integers are co-prime when their HCF is 1.	• At least two numbers. • Neither of the numbers have to be prime. • All groups of prime numbers are co-prime. • If one of the numbers is prime, the others will be co-prime unless they are multiples of the prime.
Examples	**Non Examples**
• 8 and 15 are co-prime • 5 and 31 are co–prime • 24 and 25 are co-prime • 1 and any other positive integer are co-prime	• 2 and 4 are not co-prime • 9 and 15 are not co-prime • 5 and 5 are not co-prime • 5 and 25 are not co-prime • 10 and 25 are not co-prime

1.8 Lowest Common Multiple

Worked Example	Your Turn
Find the LCM of 6 and 15	Find the LCM of 6 and 20

Intelligent Practice

Find the LCM of:

1) 5 and 10
2) 10 and 5
3) 20 and 5
4) 20 and 10
5) 20 and 30
6) 4 and 30
7) 5 and 30
8) 7 and 30
9) 14 and 30
10) 28 and 30
11) 30 and 30
12) 30 and 48
13) 36 and 48
14) 24, 36 and 48
15) 240, 360 and 480

Can you spot any patterns between questions and answers? Can you explain why they occur?

Fluency Practice

Question 1: (a) Write down the first ten multiples of 2.
(b) Write down the first ten multiples of 3.
(c) List the first three common multiples of 2 and 3.

Question 2: (a) Write down the first ten multiples of 4.
(b) Write down the first ten multiples of 5.
(c) List the first three common multiples of 4 and 5.

Question 3: Write down three common multiples of each of these pairs of numbers.

(a) 2 and 5 (b) 3 and 4 (c) 4 and 6 (d) 10 and 15

(e) 20 and 30 (f) 3 and 5 (g) 6 and 9 (h) 6 and 12

Question 4: (a) Write down the first ten multiples of 5.
(b) Write down the first ten multiples of 8.
(c) Find the lowest common multiple (LCM) of 5 and 8.

Question 5: (a) Write down the first ten multiples of 6.
(b) Write down the first ten multiples of 8.
(c) Find the lowest common multiple (LCM) of 6 and 8.

Question 6: Find the lowest common multiple (LCM) of each of these pairs of numbers.

(a) 5 and 6 (b) 2 and 7 (c) 3 and 8 (d) 4 and 10

(e) 9 and 4 (f) 6 and 7 (g) 6 and 8 (h) 9 and 12

(i) 15 and 40 (j) 12 and 20 (k) 13 and 4 (l) 18 and 6

(m) 25 and 35 (n) 22 and 33 (o) 16 and 24 (p) 20 and 28

Question 7: Find the lowest common multiple (LCM) of each of these sets of numbers.

(a) 2, 3 and 5 (b) 3, 4 and 5 (c) 2, 5 and 7 (d) 5, 6 and 9

(e) 10, 12 and 15 (f) 2, 3, 4 and 5 (g) 1, 2, 3, 4, 5 and 6.

Extension

Question 6: Explain why Charlie is wrong

To find the LCM of two numbers, just multiply them together

Question 8: Jennifer says that the lowest common multiple of two consecutive numbers is equal to the product of the two numbers.
By trying four different pairs of consecutive numbers, explore her theory.

Frayer Model – Lowest Common Multiple

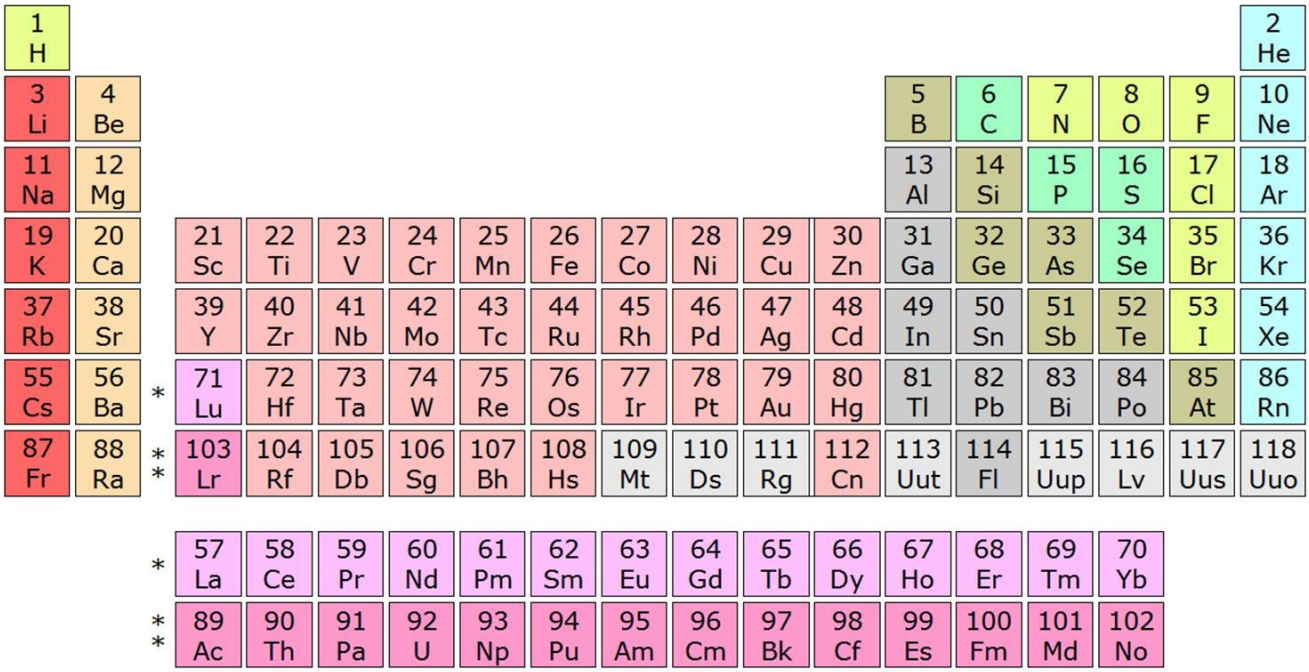
Definition	**Characteristics**
The smallest integer which is a multiple of two or more positive integers.	• Will be greater than or equal to the largest of the numbers. • Often abbreviated to LCM. • Appears in the times table of all given numbers.
Examples	**Non Examples**
• $\text{LCM}\ (8, 12) = 24$ • $\text{LCM}\ (30, 15) = 30$ • $\text{LCM}\ (3, 7) = 21$ • $\text{LCM}\ (4, 5, 6) = 60$	• $\text{LCM}\ (4, 6) \neq 2$ • $\text{LCM}\ (4, 8) \neq 4$ • $\text{LCM}\ (3, 5) \neq 1$

1.9 Prime Factorisation

- In chemistry, the elements on the periodic table make up the world around us.

- In mathematics, prime numbers are our elements.

- Every number bigger than one, is either prime or is made up of a product of prime numbers.



Prime Factors

3 is a prime factor of 36 (True / False)

9 is a prime factor of 36 (True / False)

1 is a prime factor of 36 (True / False)

2 is a prime factor of 36 (True / False)

7 is a prime factor of 36 (True / False)

Intelligent Practice

7 is a prime factor of 12 (True / False)

6 is a prime factor of 12 (True / False)

5 is a prime factor of 12 (True / False)

4 is a prime factor of 12 (True / False)

3 is a prime factor of 12 (True / False)

2 is a prime factor of 12 (True / False)

1 is a prime factor of 12 (True / False)

1 is a prime factor of 27 (True / False)

2 is a prime factor of 27 (True / False)

3 is a prime factor of 27 (True / False)

7 is a prime factor of 27 (True / False)

9 is a prime factor of 27 (True / False)

13 is a prime factor of 27 (True / False)

13 is a prime factor of 26 (True / False)

3 is a prime factor of 26 (True / False)

2 is a prime factor of 26 (True / False)

2 is a prime factor of 25 (True / False)

5 is a prime factor of 25 (True / False)

12.5 is a prime factor of 25 (True / False)

Product of Prime Factors

Product of Prime Factors	Yes / No ?
9×11	
19×11	
19×11^2	
$2 \times 19 \times 11^2$	
$2 \times 19 \times 101^2$	

Intelligent Practice

Product of Prime Factors	Yes / No ?
$5 + 7$	
5×7	
4×7	
3×7	
2×7	
1×7	
$1 \times 7 \times 9$	
$2 \times 7 \times 9$	
$2 \times 7 \times 11$	
$2 \times 7 + 11$	
$2 \times 7 \times 11 \times 21$	
$2 \times 7 \times 11 \times 31$	
$1 \times 2 \times 7 \times 11 \times 31$	
$2 \times 7 \times 7 \times 11 \times 31$	
$2 \times 7^2 \times 11 \times 31$	
$2^2 \times 7^2 \times 11 \times 31$	
$2^3 \times 7^2 \times 11 \times 31$	
$2^3 \times 7^2 \times 11^5 \times 31^4$	
$1^3 \times 7^2 \times 11^5 \times 31^4$	
$2^3 \times 7^2 \times 11^5 \times 41^4$	

Worked Example	Your Turn
Express 24 as a product of prime factors	Express 48 as a product of prime factors

Worked Example	Your Turn
Express 40 as a product of prime factors	Express 80 as a product of prime factors

		Product of prime factors
1.	18	
2.	90	
3.	180	
4.	60	
5.	360	
6.	240	
7.	24	
8.	12	
9.	144	
10.	1296	

11.	89	
12.	91	
13.	93	
14.	95	
15.	97	

		Product of prime factors
16.	3^2	
17.	5^2	
18.	15^2	
19.	900	

20.	2^3	
21.	10^3	
22.	20^3	
23.	216 000	

24.	14^2	
25.	42^2	
26.	126^2	
27.	126^3	
28.	126^4	
29.	126^n	
30.	63^n	

Extension

Seven students were asked the following:

Express 45 as a product of its prime factors

Here are their solutions:

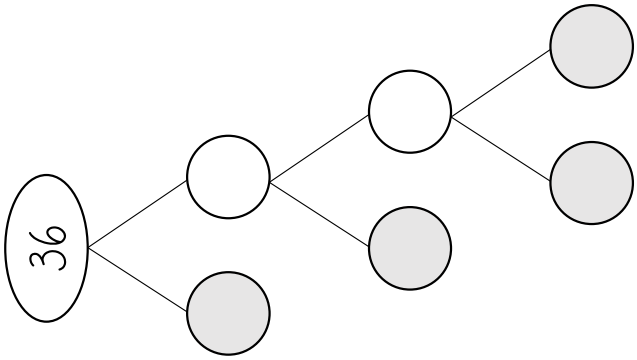
Ashley	5×9
Beverly	$3 \times 5 \times 3$
Caitlin	$3 + 3 + 5$
Des	3, 3, 5
Ezra	$3^2 \times 5$
Fatima	3 and 5
Gavin	1, 3, 5, 9, 15, 45

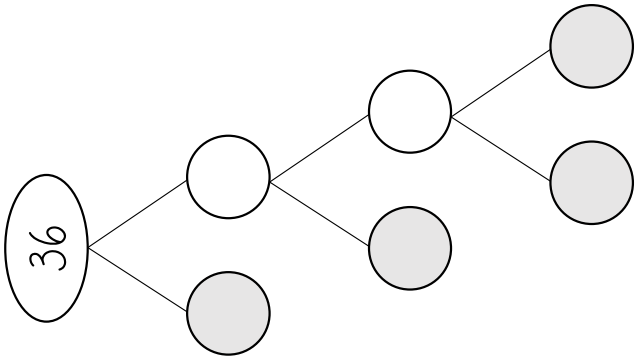
Two answers are correct, which ones?

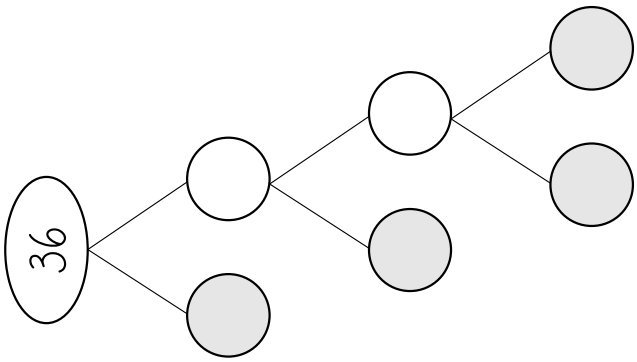
Can you explain the misconception for each of the others?

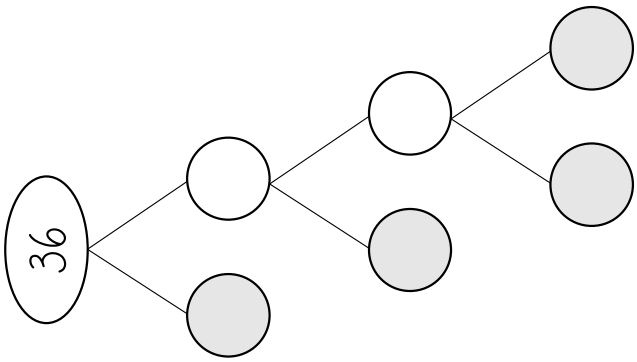
Extension

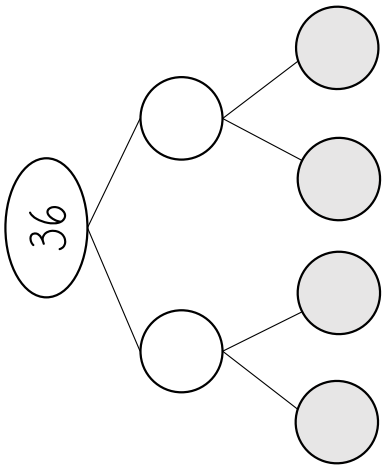
Can you find all the unique factor trees for 36?

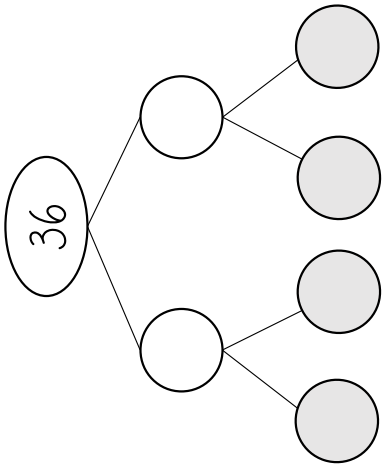




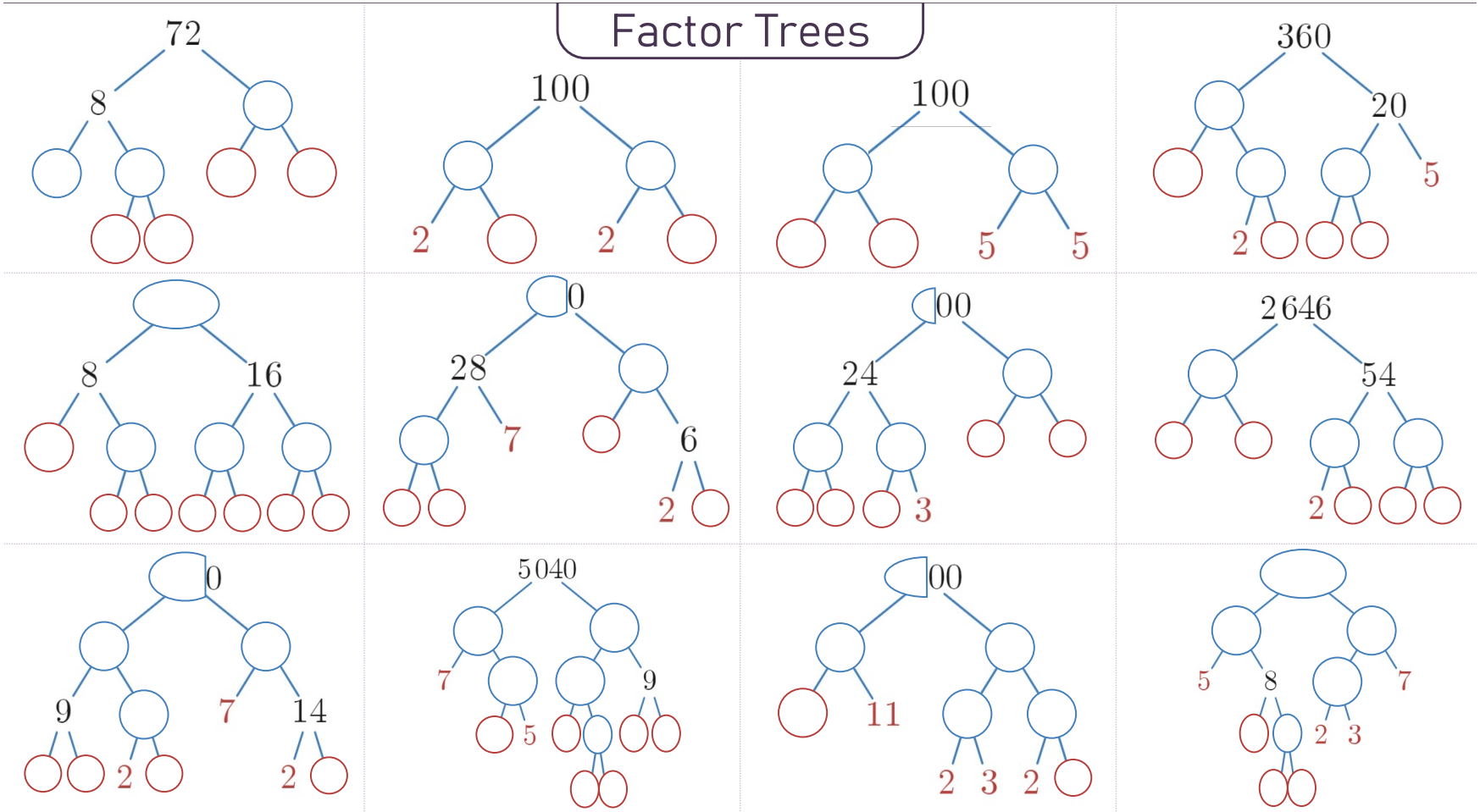








Factor Trees



Worked Example	Your Turn
Express $2^3 \times 3$ as an ordinary number	Express $2^4 \times 3$ as an ordinary number

Fluency Practice

Question 1: Write each of these numbers as the product of their prime factors.

(a) 10 (b) 12 (c) 20 (d) 18 (e) 16 (f) 30 (g) 100

(h) 26 (i) 24 (j) 27 (k) 42 (l) 33 (m) 38 (n) 64

Question 2: Write each of these numbers as the product of their prime factors.
Give your answers in index form.

(a) 36 (b) 40 (c) 28 (d) 48 (e) 80 (f) 200 (g) 75

(h) 32 (i) 105 (j) 81 (k) 52 (l) 242 (m) 108 (n) 500

Question 3: Some numbers have been written as products of their prime factors.
Work out each number.

(a) 2×7 (b) $2 \times 3 \times 5$ (c) $2 \times 5 \times 11$ (d) $2 \times 2 \times 2 \times 3$

(e) $2^2 \times 5$ (f) 3×5^2 (g) $2^3 \times 3^2$ (h) $3^2 \times 11$

(i) 5^4 (j) $2^4 \times 5^2$ (k) $3^3 \times 13$ (l) 7×17^2

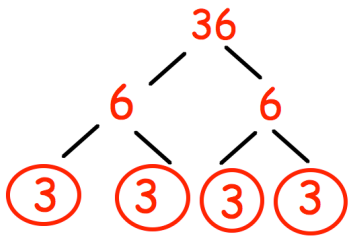
Question 4: Write each of these numbers as the product of their prime factors.

(a) 9000 (b) 235 (c) 392 (d) 715 (e) 444 (f) 792 (g) 5625

Extension

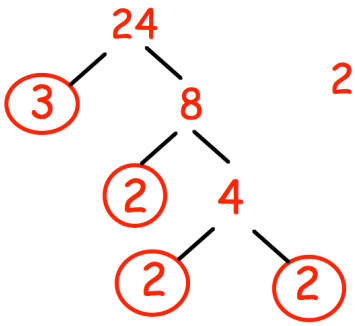
Question 3: Ashley has completed his homework.
Can you spot any mistakes?

Express 36 as a product of its prime factors.



36 = 3 x 3 x 3 x 3

Write 24 as the product of its prime factors.
Give your answer in index form.



24 = 2 x 2 x 2 x 3

Fill in the Gaps

Number	Prime Factor Decomposition	Index Form
6		
	$2 \times 2 \times 3$	
48		
240		
		$2^4 \times 3^2 \times 5$
	$2 \times 2 \times 2 \times 3 \times 3$	
216		
		$2^2 \times 3^2$
	$2 \times 2 \times 3 \times 3 \times 5 \times 5$	
		$2 \times 3 \times 5$
420		
12 600		

1.10 Number of Factors

To get the number of factors of a number in prime factorised form, add one to each power and times the powers together.

Worked Example	Your Turn
a) How many factors does 36 have? b) How many factors does 37 have? c) How many factors does 38 have?	a) How many factors does 72 have? b) How many factors does 73 have? c) How many factors does 74 have?

Fluency Practice

How many factors do each of the following have:

a) 8

b) 10

c) 7

d) 12

e) 20

f) 22

g) 18

h) 50

i) 15

j) 19

k) 30

l) 100

m) 32

n) 24

o) 42

p) 28

q) 66

r) 70

s) 45

t) 60

u) 25

Extension

Smallest Number	Prime Factor Form	Factors	Number of Factors
			1
			2
			3
			4
			5
			6
			7
			8
			9
			10
			11
60	$2^2 \times 3 \times 5$	1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60	12

1.11 Review and Problem Solving

Exactly Four Factors

exactly 4 factors

a number has 4 factors, one of which is 9, what is it?

a number is one less than a square number; it has 4 factors, one of which is 5; what could it be?

a number has 4 factors, two of which add up to 10; what could it be? how many numbers could it be?

can you find some pairs of consecutive numbers, both of which have 4 factors?

can you find a trio of consecutive even numbers all three having 4 factors?

can you find some trios of consecutive numbers all three of which have 4 factors?

can you find a trio of consecutive odd numbers all of which have 4 factors?

what types of number have exactly 4 factors?

describe the two distinct families of numbers that have 4 factors

can you explain *why* the families of numbers have 4 factors?

factors of numbers

(1) give five (or more) numbers

try to give the lowest number in each case

(a) with 2 and 3 as factors

(b) with 6 and 8 as factors

(c) with 9 and 11 as factors

(d) with 5 and 15 as factors

(e) with 10 and 12 as factors

(f) with 12 and 15 as factors

(2) find the numbers from these clues:

(a) it has exactly 4 factors, one of which is 9

(b) it has exactly 3 factors, one of which is 5

(c) smallest number with 6 factors, one of which is 6

(d) it has exactly 4 factors, two of which add to 10

(e) smallest number with 6 factors, one of which is 10

(f) it has 6 factors, one of which is 15

(3) find the numbers from these clues:

(a) it has 4 factors, one of which is 7 and it is one less than a square number

(b) it has 5 factors, one of which is 9

(c) it has 6 factors, one of which is 10

(d) it has 7 factors, one of which is 16 and it is less than100

(e) it has 8 factors, two of which are 10 and 15

(f) it has 8 factors, two of which are 21 and 35

(4) how many factors do these numbers have?

(a) 80

(b) 72

(c) 2000

(d) 9625

(5) what five numbers less than 100 have exactly 12 factors?

factors of numbers and numbers of factors

find the numbers less than 100 that have

(1) exactly 4 factors and

a. one of the factors is 58

b. one of the factors is 57

c. one of the factors is 11 (four answers)

d. one of the factors is 91

e. two of the factors sum to 10 (three answers)

f. two of the factors sum to 8 and two sum to 16 (two answers)

(2) exactly 6 factors and

a. one of the factors is 6 (two answers)

b. one of the factors is 10 (two answers)

c. one of the factors is 14 (two answers)

d. one of the factors is 15 (two answers)

e. one of the factors is 25 (two answers)

f. are in the 90s decade (three answers)

(3)

a. exactly 3 factors (four answers)

b. exactly 4 factors, one of which is 7 and it is one less than a square number

c. exactly 5 factors (two answers)

d. exactly 6 factors, one of which is 21

e. exactly 7 factors

f. exactly 8 factors, two of which are 10 and 15

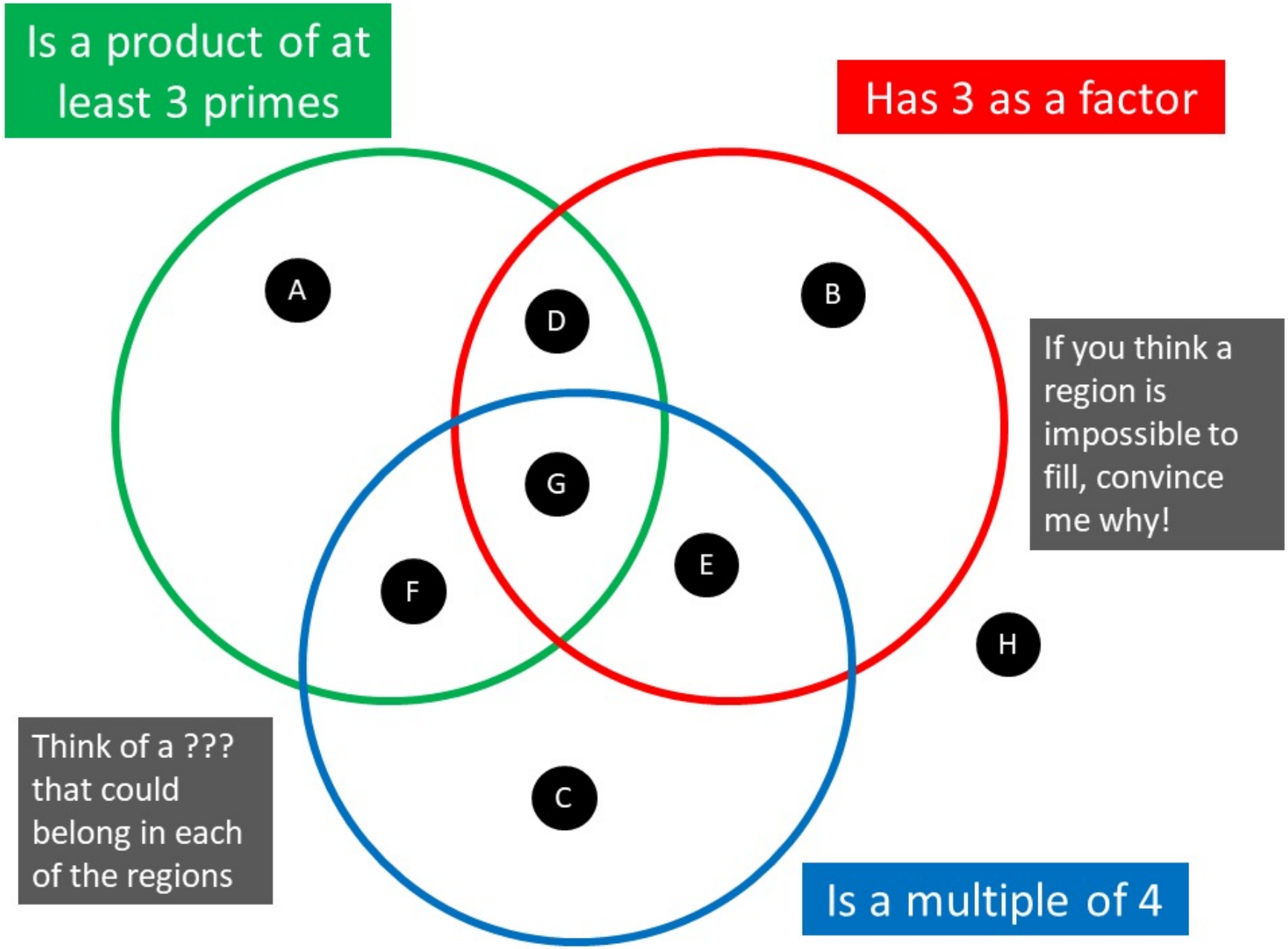
(4)

a. exactly 9 factors (less than 50)

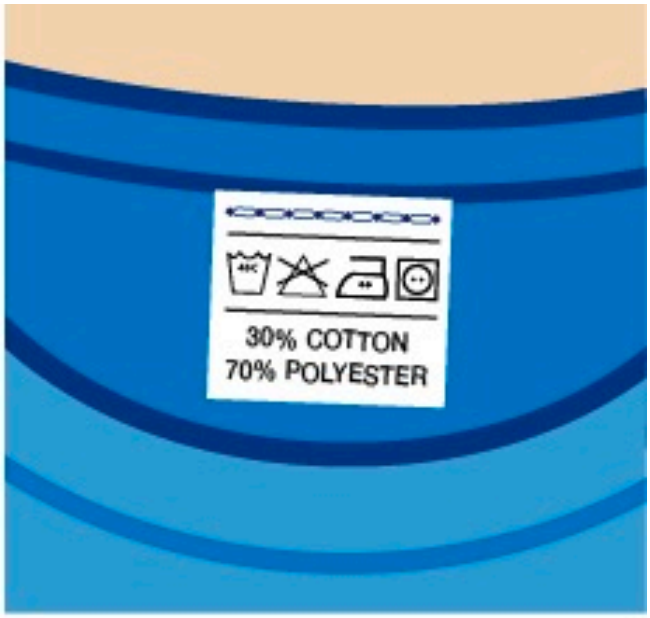
b. exactly 10 factors (two answers, one less than 50)

c. exactly 12 factors (5 answers, all bigger than 50)

Maths Venns


Is a product of at least 3 primes
Has 3 as a factor
Is a multiple of 4
A
B
C
D
E
F
G
H
If you think a region is impossible to fill, convince me why!
Think of a ??? that could belong in each of the regions

2 Percentages



Frayer Model – Percentages

Definition	**Characteristics**
A measure of the proportion of one whole, expressed as parts per 100.	• Shown by the symbol $\%$. • Equivalent to fractions with a denominator of 100.
Examples	**Non Examples**
• Find 20% of… • Increase 37kg by 2.6% • A sale takes 33% off… • Over its life, the height of a human increases by an average of 225%	• 0.3 • 0.2356 • 1.4 • $\frac{5}{8}$ • $\frac{1}{4}$

2.1 Expressing as Percentages

Worked Example	Your Turn
a) Write 15 as a percentage of 30	a) Write 30 as a percentage of 60
b) Write 10 as a percentage of 25	b) Write 20 as a percentage of 50
c) Write 15 as a percentage of 150	c) Write 3 as a percentage of 30
d) Write 10 as a percentage of 80	d) Write 5 as a percentage of 40

Intelligent Practice

1) Write 5 as a percentage of 20

2) Write 50 as a percentage of 200

3) Write 150 as a percentage of 200

4) Write 15 as a percentage of 20

5) Write 5 as a percentage of 40

6) Write 50 as a percentage of 400

7) Write 150 as a percentage of 400

8) Write 15 as a percentage of 40

9) Write 150 as a percentage of 40

10) Write 40 as a percentage of 150

Fluency Practice

Question 1:

(a) Write £5 as a percentage of £10

(b) Write 5cm as a percentage of 20cm

(c) Write 7 days as a percentage of 10 days

(d) Write 27 as a percentage of 50

(e) Write 3g as a percentage of 20g

(f) Write 4m as a percentage of 5m

(g) Write 164 as a percentage of 200

(h) Write 130ml as a percentage of 1000ml

Question 2:

(a) Write 6 out of 8 marks as a percentage

(b) Write 10kg as a percentage of 40kg

(c) Write 22 as a percentage of 40

(d) Write $15 as a percentage of $75

(e) Write £21 as a percentage of £30

(f) Write €18 as a percentage of €40

(g) Write 20p as a percentage of £1

(h) Write 60cm as a percentage of 2m

Question 3:

(a) Write 3 as a percentage of 8

(b) Write 13 out of 200 as a percentage

(c) Write 7cm as a percentage of 40cm

(d) Write $5 as a percentage of $16

(e) Write 19 marks out of 32 as a percentage

(f) Write 20 out of 30 as a percentage

Extension

Question 1: Kristina receives £5 from her Grandmother.
She gives £1 to her sister.
What percentage of the £5 did she give to her sister?

Question 2: For every 50 fans at an ice hockey match between Belfast and Cardiff, 20 of the fans support Cardiff.
(a) Work out 20 as a percentage of 50.

1000 fans attend the match between Belfast and Cardiff.
(b) How many Cardiff fans attend the match?

Question 3: Danny scored 13 out of 20 in a quiz.

(a) Work out the percentage of questions Danny answered correctly.
(b) Work out the percentage of questions Danny answered incorrectly.

Question 4: Jake brings 400 cupcakes to a school fête.
He sells 350 of the cupcakes.
Jake says that he has sold over 85% of the cupcakes.

Is Jake correct?

Question 5: A cereal bar weighs 24g.
The cereal bar contains 3.8g of protein.
Work out what percentage of the cereal bar is protein.

Question 6: Hannah scored 60 out of 90 in a French test.
She scored 50 out of 80 in a drama test.
Hannah scored 85 out of 130 in an art test.
She scored 13 out of 20 in a maths test.
Arrange the subject in order from the highest percentage to lowest percentage.

Question 7: Bryan and Ryan are buying a car that costs £15000.
Bryan pays a deposit of £2000
Ryan pays a deposit that is 40% more than Bryan's deposit.
Work out the percentage of total cost that is left to pay.

Question 8: 370 students attend a primary school.
Mrs Jones says that at least 95% of students attended the school every day.

Mon	Tues	Wed	Thurs	Fri
360	355	352	347	357

Is Mrs Jones correct?

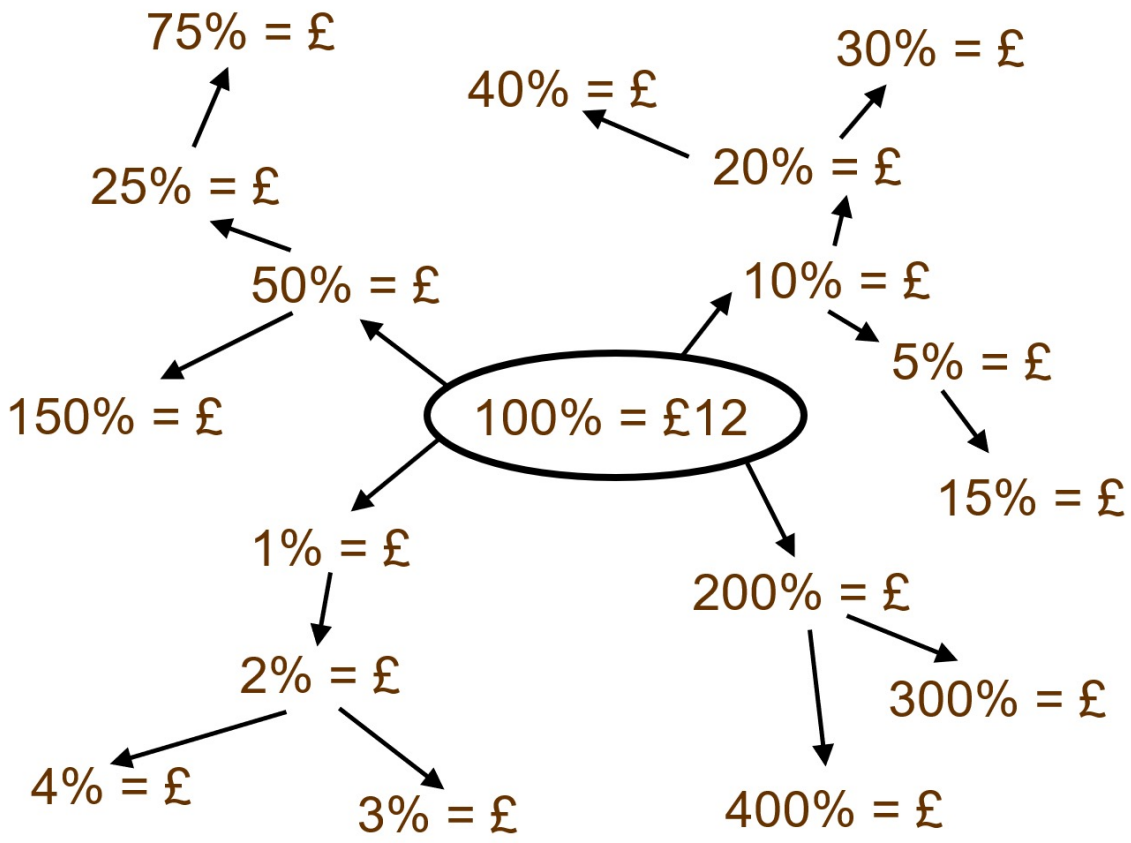
Question 9: The population of a town is 4.52×10^4
The number of people that own a goldfish is 1.34×10^3
Calculate the percentage of the population that own a goldfish.

Maths Venns

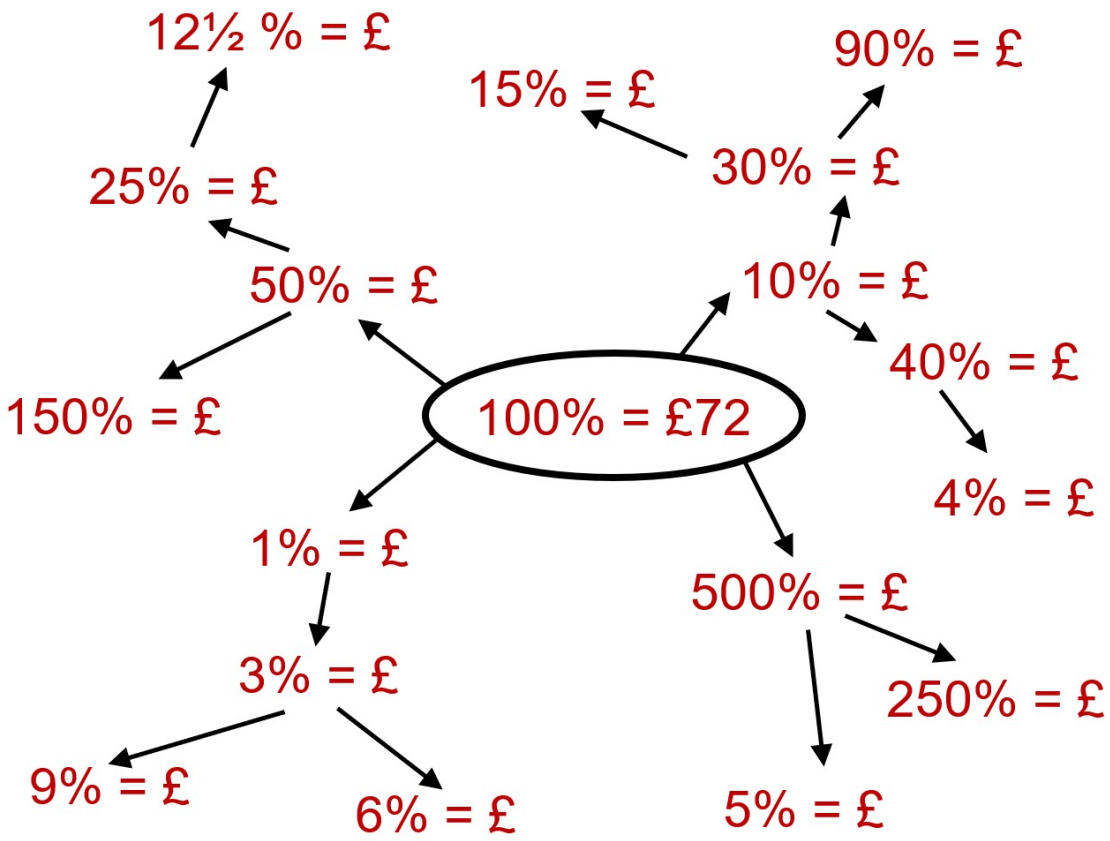
? As a percentage of ? Is between 35 and 50 percent
If you think a region is impossible to fill, convince me why!
? As a percentage of ? Is less than 70%
? As a percentage of ? Is between 30 and 40 percent
Think of a number that could belong in each of the regions
A
B
C
D
E
F
G
H

2.2 Percentages of Amounts

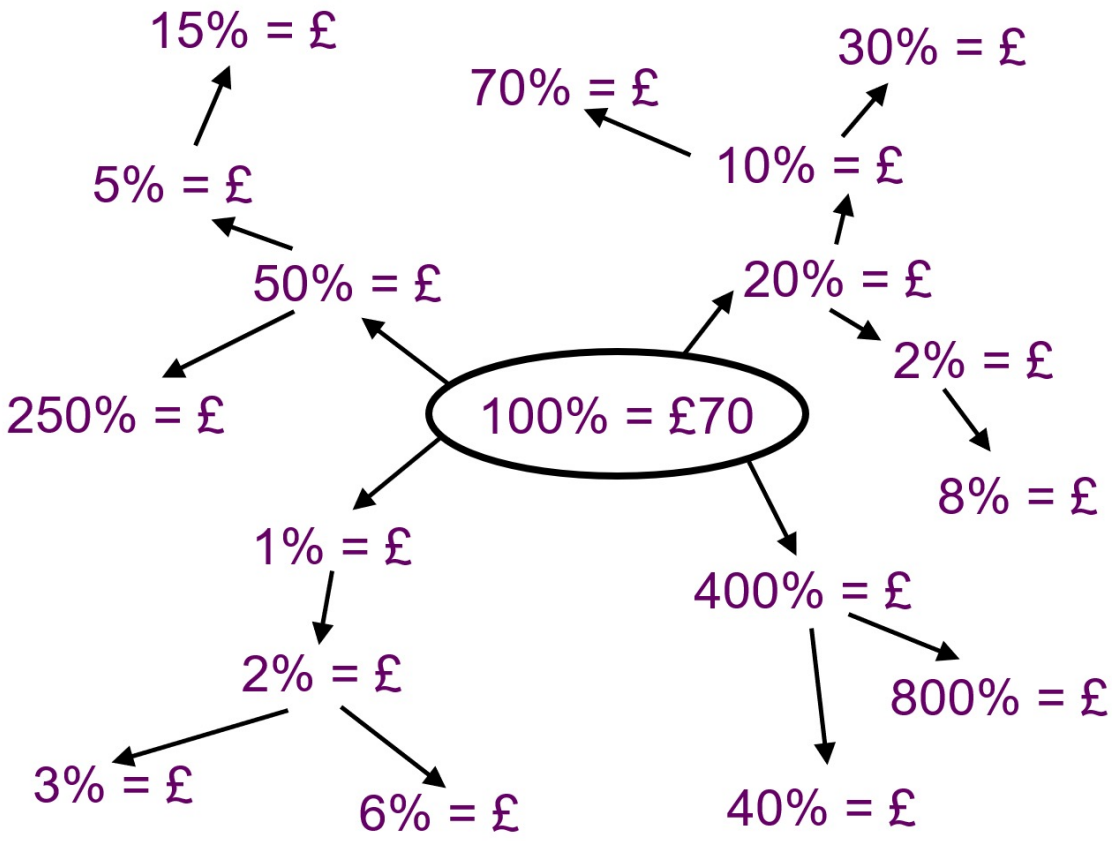
Worked Example



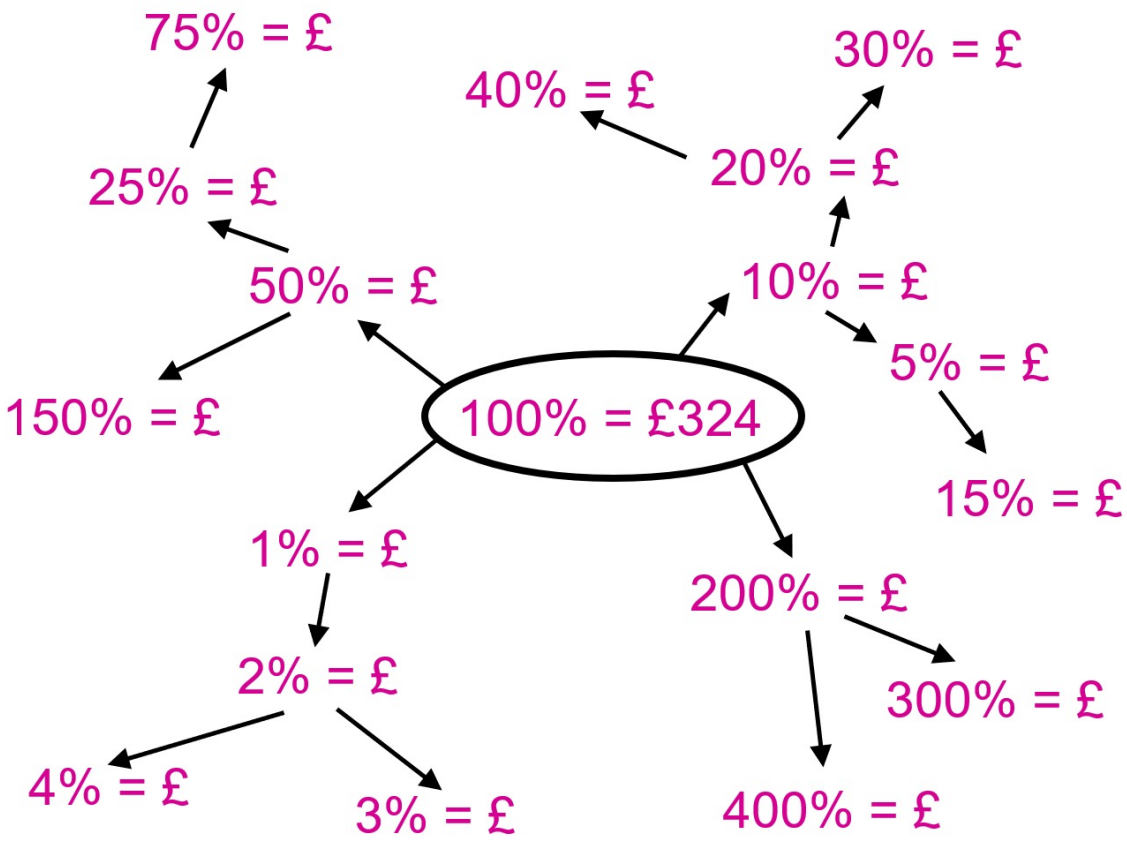
Your Turn



Your Turn



Your Turn



percentages 'of' (try to work out the answer mentally, or just with a few jottings)

(1) 50% of £24
(2) 50% of £72
(3) 25% of £120
(4) 25% of £480
(5) 75% of £120
(6) 10% of £90
(7) 10% of £9
(8) 10% of £19
(9) 20% of £55
(10) 20% of £110
(11) 20% of £165
(12) 40% of £5
(13) 40% of £55
(14) 40% of £155
(15) 80% of £50
(16) 80% of £250
(17) 80% of £3000
(18) 28% of £50
(19) 25% of £56
(20) 35% of £40
(21) 56% of £25
(22) 75% of £48
(23) 75% of £240
(24) 90% of £60
(25) 90% of £15
(26) 70% of £70
(27) 70% of £7
(28) 70% of £35
(29) 16% of £25
(30) 18% of £50
(31) 20% of £45
(32) 45% of £20
(33) 5% of £64
(34) 15% of £64
(35) 8% of £125
(36) 16% of £125
(37) 116% of £125
(38) 3% of £123
(39) 103% of £123
(40) 9% of £320
(41) 19% of £320
(42) 99% of £320

Fluency Practice

Question 1: Work out the following

(a) 10% of 70m (b) 25% of 16 seconds (c) 10% of 400kg (d) 50% of 26g

(e) 75% of 40ml (f) 1% of £300 (g) 25% of 36 days (h) 50% of 9 days

(i) 75% of 24p (j) 25% of £18 (k) 1% of $6300 (l) 10% of £7

(m) 1% of 60m (n) 75% of 8 miles (o) 1% of 80kg (p) 50% of 1.6km

Question 2: Work out the following

(a) 20% of 30km (b) 5% of £60 (c) 2% of 600m (d) 30% of 70p

(e) 3% of $9000 (f) 40% of 75 seconds (g) 15% of 90 hours (h) 5% of 14kg

(i) 60% of 30km (j) 30% of £40 (k) 70% of 900cm (l) 20% of 13cm

(m) 11% of 420m (n) 26% of 4000m (o) 55% of £8 (p) 15% of 340kg

Question 3: Work out the following

(a) 35% of £800 (b) 6% of 160g (c) 23% of 330cm (d) 52% of 910m

(e) 61% of 1400 (f) 7% of 640GB (g) 45% of 350g (h) 80% of 450 people

(i) 90% of 1250ml (j) 76% of £80,000 (k) 85% of 90 hours (l) 12% of £6

(m) 6% of £20 (n) 11% of 6m (o) 28% of 3km (p) 71% of 4 tonnes

Question 4: Calculate the following

(a) 30% of 245m (b) 5% of 84g (c) 30% of £254 (d) 35% of 82 seconds

(e) 15% of 688kg (f) 45% of 3mm (g) 18% of 25miles (h) 65% of 108ml

(i) 98% of 6m (j) 55% of 18 points (k) 20% of 1.8kg (l) 19% of 705ml

(m) 27% of 84g (n) 63% of 38 seconds (o) 86% of 5km (p) 92% of 80 litres

Extension

Question 1: A primary school has 212 students.
50% of the students are boys.
How many of the students are boys?

Question 2: There are 800 fans at a rugby match between Armagh and Malone.
30% of the fans support Malone.
How many fans support Malone?

Question 3: Hannah is paid £280.
She spends 30% on her rent, 25% on food and bills and saves the rest.

(a) How much does Hannah spend on rent?
(b) How much does Hannah spend on food and bills?
(c) How much does Hannah save?

Question 4: There are 220 students in Year 7.
15% cycle to school.
60% are driven to school.
The rest walk to school.

(a) How many students cycle to school?
(b) How many students are driven to school?
(c) How many students walk to school?

Question 5: Fredrick is an estate agent in New York and earns 5% commission on every property sold. How much will he receive is he sells a flat for $830,000?

Question 6: A cake weighs 750g.
40% of the cake is sugar.
Work out how many grams of sugar are in the cake.

Question 7: There are 600 members of a running club.
45% of these members are male.
Work out 45% of 600.

Question 8: Martin gives 40% of £75 to his sister.
How much money does Martin give to his sister?

Extension

Question 9: Emma is paid £24,000 each year.
She is given a pay rise of 12%.
Work out 12% of £24,000.

Question 10: Mrs Jones donates 4% of her salary each year to charity.
She is paid £32,400.
Work out how much money she donates to charity.

Question 11: 13% of the people on an island are left handed.
The population of the island is 0.7million.
Work out how many people are left handed.

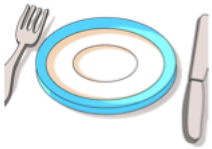
Question 12: Frank organised a raffle.
He sells 300 tickets for £5 each.
The prizes cost £400.
He gives 55% of the profit to Charity A and 45% of the profit to Charity B.
Work out how much each charity receives.

Question 13: Michael is going to buy a car.
The car costs £2400.
He pays a deposit of 20%.
Michael pays the rest of the money over 20 monthly payments.
Work out the cost of each monthly payment.

Question 14: An adult ticket for a museum is £15.00
A child ticket costs 60% of the price of an adult ticket.
Mrs Jenkins and her three children go to the museum.
Mrs Jenkins pays with three £20 notes.
How much change will she receive?

Question 15: Frances and her family go for a meal while on holiday in Florida.
They are told it is normal to tip 15%.

The meal costs $128
Frances tips $16, is this enough?



Maths Venns

...% of 30 is less than 12
If you think a region is impossible to fill, convince me why!
...% of 60 is more than 16
...% of 150 is between 50 and 80
Think of a percentage that could belong in each of the regions
A
B
C
D
E
F
G
H

2.3 Percentage Increase

Worked Example	Your Turn
Increase 40 by 20%	Increase 90 by 20%

Intelligent Practice

1) Increase 30 by 10%
2) Increase 30 by 20%
3) Increase 60 by 20%
4) Increase 60 by 10%
5) Increase 74 by 10%
6) Increase 74 by 50%
7) Increase 84 by 50%
8) Increase 84 by 10%
9) Increase 84 by 5%
10) Increase 84 by 100%

1) Increase 44 by 5%
2) Increase 44 by 10%
3) Increase 44 by 20%
4) Increase 44 by 50%
5) Increase 44 by 60%
6) Increase 88 by 60%
7) Increase 88 by 30%
8) Increase 88 by 15%
9) Increase 88 by 10%
10) Increase 88 by 110%

Fluency Practice

Question 1 

(a) Increase 20 by 50%

(b) Increase 60p by 10%

(c) Increase 12g by 25%

(d) Increase 400 litres by 20%

(e) Increase 32ml by 75%

(f) Increase 70m by 40%

(g) Increase 9000 by 5%

(h) Increase £7 by 20%

(i) Increase 9kg by 100%

2.4 Percentage Decrease

Worked Example	Your Turn
Decrease 40 by 20%	Decrease 90 by 20%

Intelligent Practice

1) Decrease 30 by 10%
2) Decrease 30 by 20%
3) Decrease 60 by 20%
4) Decrease 60 by 10%
5) Decrease 74 by 10%
6) Decrease 74 by 50%
7) Decrease 104 by 50%
8) Decrease 104 by 10%
9) Decrease 104 by 5%
10) Decrease 104 by 100%

1) Decrease 68 by 5%
2) Decrease 68 by 10%
3) Decrease 68 by 20%
4) Decrease 48 by 50%
5) Decrease 48 by 60%
6) Decrease 96 by 60%
7) Decrease 96 by 30%
8) Decrease 96 by 15%
9) Decrease 96 by 10%
10) Decrease 96 by 110%

Fluency Practice

Question 2 

(a) Decrease 40 by 10%

(b) Decrease 30 hours by 50%

(c) Decrease 8kg by 25%

(d) Decrease 55cm by 40%

(e) Decrease 64 by 75%

(f) Decrease £3 by 10%

(g) Decrease 1400 by 30%

(h) Decrease 500g by 3%

(i) Decrease 6kg by 5%

Extension

Question 1: Last year, there were 20 students in a class.
This year, there are 30% more students.
How many students are in the class this year?

Question 2: A TV normally costs £520.
In a sale, all prices are reduced by 10%
Calculate the sale price of the TV

Question 3: Over the past 10 years, the population of a town has increased by 25%
The population of the town 10 years ago was 18000
What is the population of the town now?

Question 4: A standard bag of flour contains 600g of flour.
A special edition bag contains 35% more flour.
How much flour is in the special edition bag?

Question 5: Richard owns a coffee shop.
In February, 4500 hot chocolates were sold.
The number of hot chocolates sold in March was 3% less.
How many hot chocolates are sold in March?

Always, Sometimes or Never True

Decrease by 50% followed by an increase of 50% takes you back to the original.

2.5 Percentage Change

Worked Example	Your Turn
Calculate the percentage change: a) Original value: £400 New value: £360 b) Original value: £400 New value: £440	Calculate the percentage change: a) Original value: £200 New value: £150 b) Original value: £200 New value: £250

Intelligent Practice

1) Original value: £20
   New value: £18

2) Original value: £20
   New value: £16

3) Original value: £20
   New value: £10

4) Original value: £200
   New value: £100

5) Original value: £100
   New value: £200

6) Original value: £125
   New value: £225

7) Original value: £88
   New value: £66

8) Original value: £88
   New value: £22

9) Original value: £880
   New value: £220

10) Original value: £88
    New value: £220

11) Original value: £176
    New value: £440

12) Original value: £440
    New value: £176

Fluency Practice

Question 1: In January, a puppy weighed 4kg.
Three months later, the same puppy weighed 5kg.
What was the percentage increase in the puppy's weight?

Question 2: The number of TVs sold increased from 50 to 60.
Work out the percentage increase.

Question 3: Peter's weight decreases from 80kg to 72kg
Calculate the percentage decrease in Peter's weight.

Question 4: A car is travelling at 40 kilometres per hour.
The car increases its speed to 56 kilometres per hour.
Calculate the percentage increase in the speed of the car.

Question 5: Keira buys a coffee table for £120 and sells it for £204.
Work out her percentage profit.

Question 6: Daisy bought a car for £20,000.
She sold the car for £15,000.
Work out the percentage loss.

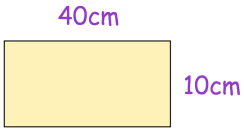
Question 7: The population of an island in 2017 was 30,000.
In 2018, the population was 31,500.
Calculate the percentage increase.

Question 8: Rebecca bought a dress for £80.
She later sold it for £116.
Find the percentage profit.

Question 9: In a sale the price of a football shirt decreases from £50 to £37
Work out the percentage decrease in price.

Extension

Question 2: ABCD is a rectangle with length 40cm and width 10cm.
The length of the rectangle is decreased by 40%.
The width of the rectangle is decreased by 20%
Find the percentage decrease in the area of the rectangle.



Maths Venns

Pair of numbers with more than 40% change
If you think a region is impossible to fill, convince me why!
Integers
Pair of numbers with less than 30% change
Think of a pair of numbers that could belong in each of the regions
A
B
C
D
E
F
G
H

2.6 Reverse Percentages

practice makes perfect: from one percentage to another

1) 25% is £17 50% is ?	2) 50% is £350 10% is ?	3) 5% is £15 20% is ?	4) 75% is £900 25% is ?	5) 10% is £60 25% is ?	6) 5% is £45 ? % is £900
7) 20% is £12 50% is ?	8) 15% is £90 ? % is £60	9) 10% is £60 ? % is £150	10) 25% is £45 10% is ?	11) 40% is £600 ? % is £450	12) 48% is £600 ? % is £1000
13) 45% is £360 ? % is £800	14) 33⅓% is £20 30% is ?	15) 80% is £48 ? % is £15	16) 40% is £36 ? % is £4.50	17) 90% is £150 ? % is £120	18) 40% is £90 ? % is £67.50
19) 35% is £112 25% is ?	20) 66⅔% is £120 15% is ?	21) 80% is £128 ? % is £40	22) 72% is £120 ? % is £55	23) 70% is £840 ? % is £660	24) 15% is £1200 ? % is £2880

Fluency Practice

Worked Example	Your Turn
Calculate the original amount: a) Percentage change: 10% decrease New value: £360 b) Percentage change: 10% increase New value: £440	Calculate the original amount: a) Percentage change: 25% decrease New value: £150 b) Percentage change: 25% increase New value: £250

Intelligent Practice

1) % change: 10% decrease
   New value: £36

2) % change: 20% decrease
   New value: £32

3) % change: 10% decrease
   New value: £18

4) % change: 10% decrease
   New value: £180

5) % change: 5% decrease
   New value: £190

6) % change: 5% decrease
   New value: £19

7) % change: 10% increase
   New value: £44

8) % change: 10% increase
   New value: £88

9) % change: 20% increase
   New value: £960

10) % change: 5% increase
    New value: £84

11) % change: 1% increase
    New value: £808

12) % change: 5% increase
    New value: £840

Fluency Practice

Question 1: 20% of all the children in a class are left handed.

4 children are left handed.
How many children are there in the class altogether?

Question 2: 30% of the members of a tennis club are pensioners.

36 members are pensioners.

(a) How many members are there in total?
(b) How many members are not pensioners?

Question 3: A group of people sit their driving theory test and 24 people passed.

80% of the people passed the driving theory test.
How many people sat the test altogether?

Question 6: Heather invested money into a savers bank account.

Each year the money in the account earns 10% interest.
After one year, the total amount of money in the account was £2200
How much did Heather invest?

Question 8: The population of an island has decreased by 40% over 50 years.

The population in 2018 was 360
What was the population in 1968?

Question 9: Sinead buys a watch.

20% VAT is added to the price of the watch.
Sinead then has to pay a total of £60
What is the price of the watch with no VAT added?

2.7 Review and Problem Solving

% How Close Can You Get

how close can you get?

12%					23%			
7	out of		is	%	9	out of		is	%
	out of	49	is	%		out of	50	is	%
20	out of		is	%	30	out of		is	%
	out of	70	is	%		out of	81	is	%
15	out of		is	%	45	out of		is	%
	out of	102	is	%		out of	102	is	%
3½	out of		is	%	7½	out of		is	%

% How Close Can You Get

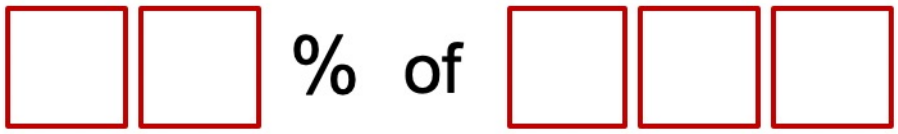
how close can you get?

55%				77%		
11	out of		is	12	out of		is	%
	out of	60	is		out of	40	is	%
20	out of		is	30	out of		is	%
	out of	180	is		out of	75	is	%
27	out of		is	24	out of		is	%
	out of	80	is		out of	120	is	%
2¾	out of		is	3.1	out of		is	%

Percent Of

close to ….

use any of the digits: 1 , 2 , 3 , 4, 5 , 6 , 7 , 8
but you can't use a digit more than once in:



try to get as close as you can to:

(a) 400
(b) 650
(c) 100
(d) 500
(e) 300

Percentage Change

Use these 12 numbers, once each, in the gaps below.

10, 20, 25, 35, 40, 50, 60, 70, 75, 80, 90, 100

£ _____ increased by _____ % = £ _____

£ _____ increased by _____ % = £ _____

£ _____ decreased by _____ % = £ _____

£ _____ decreased by _____ % = £ _____

Worked Example	Your Turn
Original Amount: 40 Percentage: 24% As a fraction Percentage of… Increased by… Decreased by…	Original Amount: 40 Percentage: 72% As a fraction Percentage of… Increased by… Decreased by…

Fill in the Gaps

Original Amount	Percentage	As a fraction	Percentage of…	Increased by…	Decreased by….
60	20%				
60		$\frac{3}{10}$			
60		$\frac{1}{4}$			
	25%		7.5		
		$\frac{1}{40}$		30.75	29.25
30			6.75		
		$\frac{9}{40}$	67.5		
300		$\frac{41}{200}$			
60				72.3	47.7
		$\frac{41}{40}$	61.5		
60		$\frac{9}{8}$			
6			0.675		
6				24.675	
6					-31.35

Fill in the Gaps

Q	Original amount	Percentage change	Increase / decrease	New amount	Change
1	£50	20%	Increase		
2	£60	20%	Increase		
3	£72	20%	Decrease		
4	£72			£54	
5		50%	Increase	£54	
6		50%	Decrease	£54	
7	£54				+ £54
8				£108	− £27
9	£96			£108	
10	£96	1.25%	Increase		
11		25%	Increase	£98.40	
12	£98.40				− £19.68
13	£98.40	100%	Increase		
14	£196.80	100%	Decrease		

Worded Questions

Section A: Percentage Change

1. The population of a village increased from 234 to 275 during one year. Find the percentage increase.

2. When a beaker of sand is dried in a hot oven its mass reduces from 1.2kg to 870g. Find the percentage reduction in its mass.

3. A battery was tested and found to power a camera for 12 hours before it needed recharging. An improved version of the battery powered the camera for an extra 30 minutes. Find the percentage increase in the life of the batteries.

4. The average cost of a local telephone call dropped by 8p to 27p. Find the percentage reduction in the average cost of a local call.

Section B: Increasing and Decreasing

1. In a sale, all the prices are reduced by 30%. Calculate the sale price of the following items:
   a. a bike that cost £250
   b. a pair of gloves that cost £3.20

2. In 2004, 180 parents applied to a school for a place for their child. The following year saw an increase of 35% in the number of applications. Find the number of applications in 2005.

3. Following the opening of a new supermarket nearby, the number of customers using a small store decreased by 21%. If 2,400 customers used to use the store each week, find the number of customers after the store opened.

4. A car costs £9,999.90 before VAT (value added tax). Work out the cost including VAT if it is charged at 20%.

5. Sally's investment of £450 has gone up by 30%, while Susie's investment of £650 has gone down by 10%. Who now has the larger amount of money, Sally or Susie?

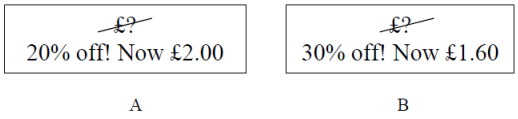
6. A train company increases its rail fares by 4% one year and by 6.5% the following year. Find the percentage increase in cost over the two years.

Section C: Reverse Percentage Problems

1. A jacket is reduced by 12% to £66 in a sale. Find the original price.

2. A baby's weight increases by 8% over a month from birth to 4.05kg, what wasthe weight at birth?

3. Which product has the greatest original price? Show your working.



A B

4. The air pressure increases by 1.2% to 1,214.4 mbar. What was the original air pressure?

5. A dress in a sale is reduced by 7% to £60.45. What is the original price?

6. A stereo system is sold for £1,998 and an 11% profit is made. Find the original cost of the stereo.

7. A shop sells a television to a man and makes a 15% profit. The man sells it to another man for £414 at a loss of 10%. Find the original price of the television.