Year 7
Mathematics
Unit 1

Name: ___________________________

Class: ___________________________

Contents

1 Factors, Multiples and Primes

1.1 Types of Numbers

In this section you will look at the different types of numbers including:

- Integers
- Square Numbers
- Cube Numbers
- Triangular (or triangle) Numbers

Of course, there are many more types of numbers!
Maybe you could do some research and list the other types of numbers below.

Frayer Model – Integers

Definition	**Characteristics**
Examples	**Non-Examples**

Frayer Model – Square Numbers

Definition	Characteristics
Examples	**Non-Examples**

Frayer Model – Cube Numbers

Definition	**Characteristics**
Examples	**Non-Examples**

Frayer Model – Triangular Numbers

Definition	**Characteristics**
Examples	**Non-Examples**

1.2 Multiples

In this section you will look at multiples. A good way to remember multiples is to think of multipacks:

If cola is sold in multipacks of 6, I can only buy a multiple of 6 bottles.

Frayer Model – Multiples

Definition	**Characteristics**
Examples	**Non-Examples**

Worked Example	Your Turn
Write down the first six multiples of 6	Write down the first six multiples of 8

1.3 Common Multiples

In this section you will look at common multiples.

Worked Example	Your Turn
Find the first three common multiples of 6 and 15	Find the first three common multiples of 6 and 20

1.4 Lowest Common Multiple

In this section you will look at lowest common multiple.
Can you suggest a reason why there is no such thing as highest common multiple?

Frayer Model – Lowest Common Multiple

Definition	**Characteristics**
Examples	**Non-Examples**

Worked Example	Your Turn
Find the LCM of 6 and 15	Find the LCM of 6 and 20

1.5 Divisibility Tests

In this section you will look at divisibility tests:

A divisibility test is a rule for determining whether one whole number is divisible by another.

Divisibility Tests for 2, 5 and 10

Number	Test	Example	Non-Example
2	Number ends in $0, 2, 4, 6$ or 8	1246	3273
5	Number ends in 0 or 5	3825	1011
10	Number ends in 0	4890	3568

Divisibility Tests for 4 and 8

Number	Test	Example	Non-Example
4	Last two digits divisible by 4	7356	9382
8	Last three digits divisible by 8	4512	8148

Divisibility Tests for 3 and 9

Number	Test	Example	Non-Example
3	Sum of digits divisible by 3	1353	4567
9	Sum of digits divisible by 9	1458	3057

Divisibility Test for 7

Number	Test	Example	Non-Example
7	Multiply the last digit by 5 and add it to the remaining part of the number, and see if the result is divisible by 7	9961	3581

This divisibility test was discovered by a 12 year old!
https://www.westminsterunder.org.uk/chikas-test/

Divisibility Test for 11

Number	Test	Example	Non-Example
11	Sum of odd-positioned digits subtract sum of even-positioned digits and see if the result is divisible by 11	2761 8261	5476

Divisibility Tests for 6 and 12

Number	Test	Example	Non-Example
6	Divisible by both 2 and 3	4728	7352
12	Divisible by both 3 and 4	3576	1222

1.6 Factors

In this section you will look at factors. A good way to remember factors is to think of a factory:

A factory is a place where lots of separate parts are put together to make something like a car. All of the separate things that go into the car are factors.

Frayer Model – Factors

Definition	**Characteristics**
Examples	**Non-Examples**

Worked Example	Your Turn
Find all the factors of 44	Find all the factors of 88

1.7 Factors of Larger Numbers

In this section you will look at how to find the factors of larger numbers. The key to this is the divisibility tests you learnt earlier!

Fluency Practice

Use divisibility rule to circle the factors of each number.

1) 3,642

is divisible by

3 4 5 6 12

2) 516

is divisible by

2 3 4 9 10

3) 569,820

is divisible by

2 3 4 5 10

4) 55

is divisible by

2 4 5 7 11

5) 48,704

is divisible by

2 3 4 8 9

6) 9,541

is divisible by

3 7 8 9 12

7) 21,208

is divisible by

2 4 8 10 11

8) 114,786

is divisible by

2 3 5 7 9

9) 248

is divisible by

2 3 4 5 8

10) 758,428

is divisible by

2 3 4 9 10

11) 6,040

is divisible by

2 4 5 8 9

12) 835,752

is divisible by

2 3 4 6 8

13) 16,596

is divisible by

2 3 4 7 12

14) 684,342

is divisible by

2 4 6 8 9

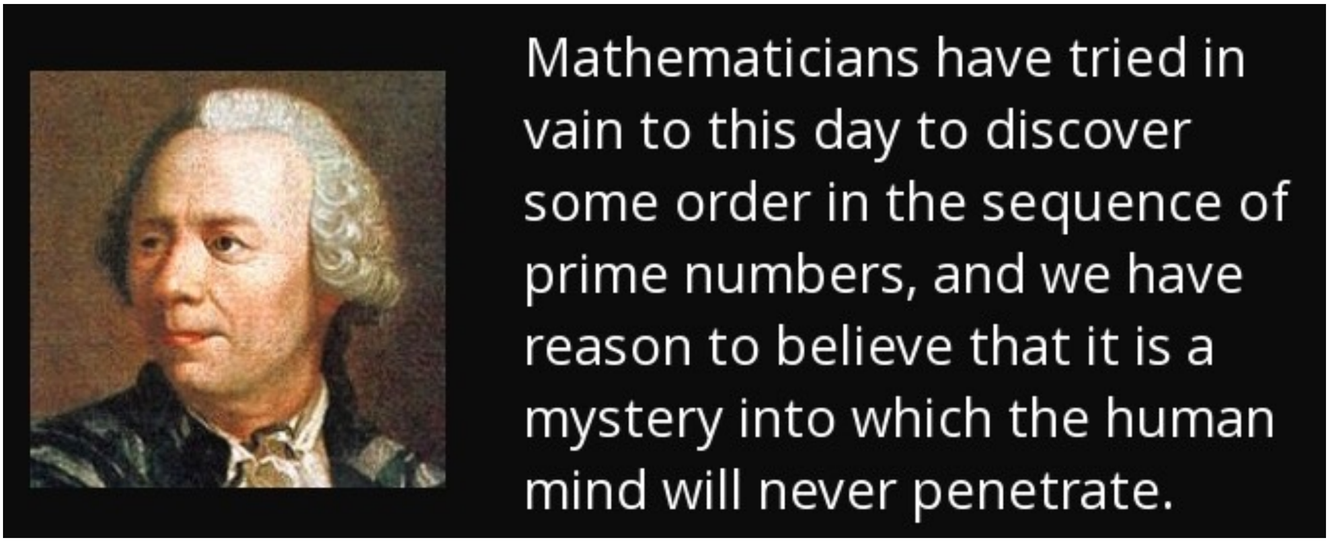
15) 96,415

is divisible by

4 5 10 11 12

1.8 Prime Numbers

In this section you will look at prime numbers. Here is a quote from the Swiss mathematician Leonhard Euler about the prime numbers:



Despite the prime numbers appearing to be ‘random’, they are very useful. Maybe you could research how they are used in the real world.

Frayer Model – Prime Numbers

Definition	**Characteristics**
Examples	**Non-Examples**

1.9 Common Factors

In this section you will look at common factors.

Worked Example	Your Turn
Find the common factors of 6 and 15	Find the common factors of 6 and 20

1.10 Highest Common Factor

In this section you will look at highest common factor.
Can you suggest a reason why there is no such thing as lowest common factor?

Frayer Model – Highest Common Factor

Definition	**Characteristics**
Examples	**Non-Examples**

Worked Example	Your Turn
Find the HCF of 6 and 15	Find the HCF of 6 and 20

1.11 HCF and LCM Worded Problems

In this section you will look at HCF and LCM worded problems.

Worked Example	Your Turn
Two strings of different lengths, 15 cm and 24 cm are to be cut into equal lengths. What is the greatest possible length of each piece?	Two strings of different lengths, 18 cm and 30 cm are to be cut into equal lengths. What is the greatest possible length of each piece?

Worked Example	Your Turn
Two lighthouses flash their lights every 15 s and 24 s respectively. They both flash at the same time. After how many seconds will they next both flash at the same time.	Two lighthouses flash their lights every 18 s and 30 s respectively. They both flash at the same time. After how many seconds will they next both flash at the same time.

2 Sets and Venn Diagrams

2.1 Sets

In this section you will look at sets. However, a lot of the notation for sets will be covered in year 11.

A set is a collection of numbers, or letters, or symbols, or objects, etc., which are related in some way.

The items in a set are called '**members**' or '**elements**'

Curly brackets (often called 'braces') are usually used when listing or describing sets – this helps to distinguish sets from lists of unrelated items.

The elements within a set are usually described in words or listed

Examples:

Description in words	**List of elements**
{even numbers less than 11}	{2, 4, 6, 8, 10}
{the first five prime numbers}	{2, 3, 5, 7, 11}
{multiples of three between 10 and 20}	{12, 15, 18}
{factors of 27 which are even}	{ }

More examples of sets:

Description in words	**List of elements**
{quadrilaterals with four equal length sides}	{square, rhombus}
{vowels}	{a, e, i, o, u}
{letters in the word 'banana'}	{a, b, n}
{yellow fruit}	{grapefruit, banana, lemon, …}

Notes:

Elements are only ever included once – as shown with {letters in the word 'banana'} = {a, b, n}

{yellow fruits} is an imprecise description and the list of elements contains only examples.

Worked Example

List the following sets:

a) {factors of 15}

b) {the first four square numbers}

c) {letters in the word LONDON}

d) {possible outcomes when an ordinary coin is thrown}

Your Turn

List the following sets:

a) {the first four multiples of 15}

b) {the first four cube numbers}

c) {letters in the word BIRMINGHAM}

d) {possible outcomes when an ordinary dice is thrown}

Fluency Practice

A1 List {vowels}	**A2** List {the first six consonants}	**A3** List {vowels in the word 'NUMBER'}	**A4** List {consonants in the word 'MATHS'}
B1 List {vowels in the word 'ALGEBRA'}	**B2** List {consonants in the word 'SETS'}	**B3** List {letters in the word 'ISOSCELES'}	**B4** List {vowels in 'SQUARE ROOT'}
C1 List {days of the week}	**C2** List {seasons in the year}	**C3** List {colours in the rainbow}	**C4** List {countries in the United Kingdom}
D1 List {first three months of the year}	**D2** List {months of the year with four letters}	**D3** List {months of the year beginning with 'A'}	**D4** List {days of the week which contain an 'E'}
E1 Describe the following set: {spring, summer}	**E2** Describe the following set: {square, rhombus}	**E3** Describe the following set: {north, east, south, west}	**E4** Describe the following set: {orange, yellow, indigo, violet}

2.2 Multiple Sets and The Universal Set

In this section you will look at multiple sets and the universal set.

When we have more than one set, capital letters are usually used to represent them.

Examples:

Description in words	**List of elements**
A = {prime numbers between 10 and 20}	A = {11, 13, 17, 19}
B = {factors of 24}	B = {1, 2, 3, 4, 6, 8, 12, 24}
C = {vowels}	C = {a, e, i, o, u}

Note that it is often convenient to use letters that are in some way connected to the description of the set.

e.g. P = {prime numbers between 10 and 20}, F = {factors of 24} and V = {vowels}

The Universal set is the set of all elements under consideration.

Elements that can be in other sets are restricted to those within the Universal set. For example, if the Universal set was {integers less than 10}, then {prime numbers} would be limited to {2, 3, 5, 7}.

Likewise if the Universal set was {even numbers}, then {factors of 18} would be {2, 6, 18}

Notation

In Britain the special symbol '$\mathcal{E}$' is used to represent the Universal set but in some countries, such as America, the letter 'U' is used.

Thus we could write

$\mathcal{E}$ = {integers less than 10} or $\mathcal{E}$ = {prime numbers}

Worked Example

a) U = {odd numbers less than 15}
A = {prime numbers}
B = {multiples of 3}
List:

i) A

ii) B

b) U = {first 10 letters of the alphabet}
X = {vowels}
Y = {letters in the word 'ENGLISH'}
List:

i) X

ii) Y

c) U = {factors of 24}
P = {prime numbers}
E = {even numbers}
O = {odd numbers}
List:

i) P

ii) E

iii) O

Your Turn

a) U = {even numbers less than 15}
A = {prime numbers}
B = {multiples of 3}
List:

i) A

ii) B

b) U = {first 10 letters of the alphabet}
X = {vowels}
Y = {letters in the word ‘FRENCH’}
List:

i) X

ii) Y

c) U = {factors of 30}
P = {prime numbers}
E = {even numbers}
O = {odd numbers}
List:

i) P

ii) E

iii) O

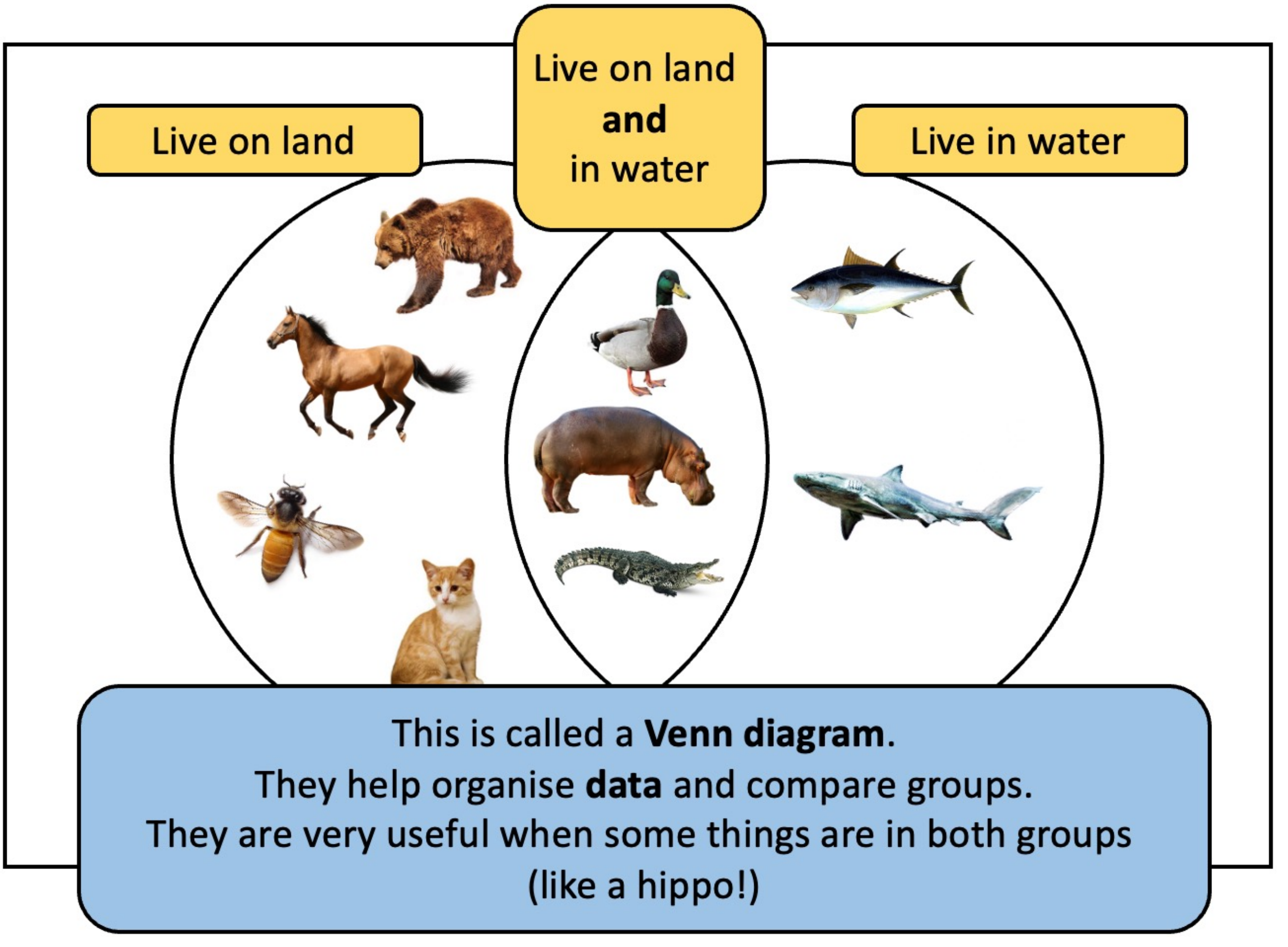
Fluency Practice

A1 List {the first six multiples of 3}	**A2** List {prime numbers less than 10}	**A3** List {all the factors of 12}	**A4** List {even numbers between 3 and 11}
B1 Describe the set: {1, 2, 3, 4, 5}	**B2** Describe the set: {1, 3, 5, 7, 9}	**B3** Describe the set: {1, 2, 3, 6, 9, 18}	**B4** Describe the set: {11, 13, 17, 19}
C1 A = {positive integers less than 5} List set A	**C2** B = {negative integers more than 6} List set B	**C3** C = {integers between 4 and 9} List set C	**C4** D = {integers between –3 and 4} List set D
D1 M = {the first five multiples of 6} List set M	**D2** F = {all the factors of 20} List set F	**D3** P = {the first six prime numbers} List set P	**D4** S = {square numbers less than 20} List set S
E1 A = {factors of 20} B = {1, 2, 5, 10, 20} Are the sets A and B the same?	**E2** C = {first five multiples of 7} D = {7, 14, 21, 27, 35} Are the sets C and D the same?	**E3** E = {prime numbers less than 20} F = {the first nine prime numbers} Are the sets E and F the same?	**E4** G = {numbers on a dice} H = {positive integers less than 7} Are the sets G and H the same?

2.3 Venn Diagrams with Two Circles

In this section you will look at Venn diagrams (with two circles), named after the English mathematician John Venn.

Venn Diagrams



This is called a **Venn diagram**.
They help organise **data** and compare groups.
They are very useful when some things are in both groups (like a hippo!)

A Venn diagram ('created' by John Venn)
is a pictorial view of the relationships between sets.

A rectangle is drawn to represent the Universal set, and one or more ovals to represent the other sets.

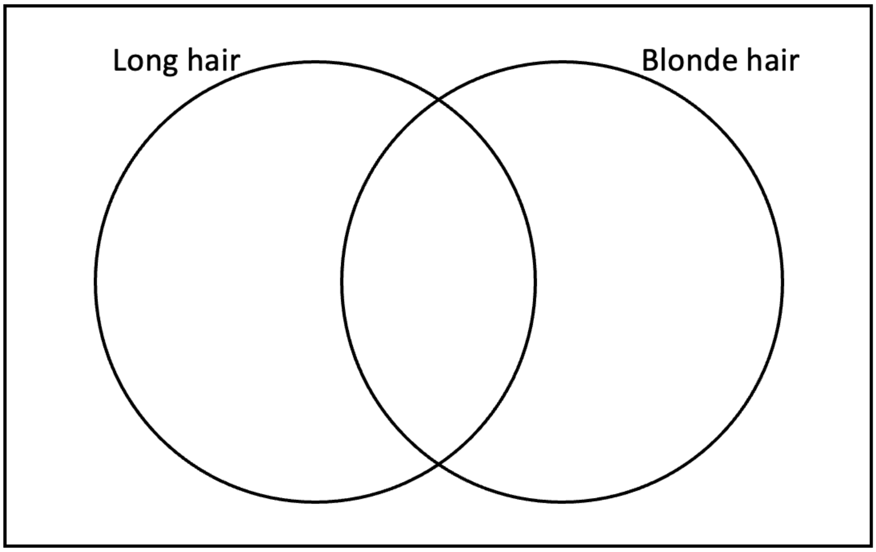
Worked Example

Complete the Venn Diagram:

4 students were picked from Year 7

ξ =

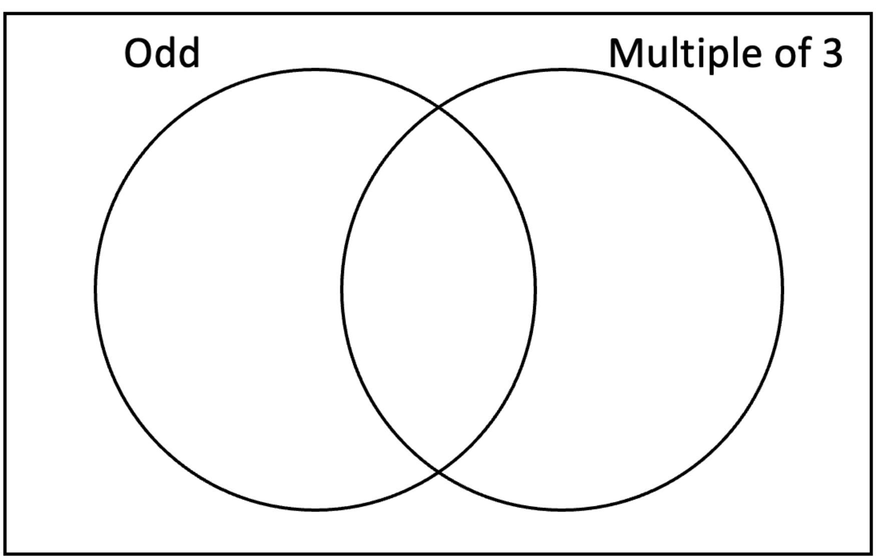
ξ



We want to sort the numbers 1 to 10.

ξ = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

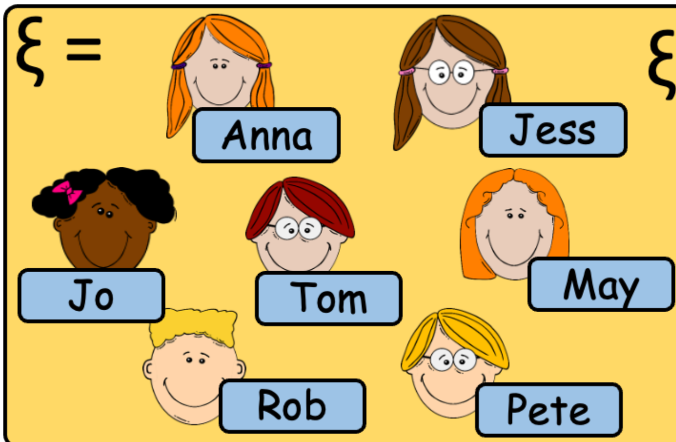
ξ



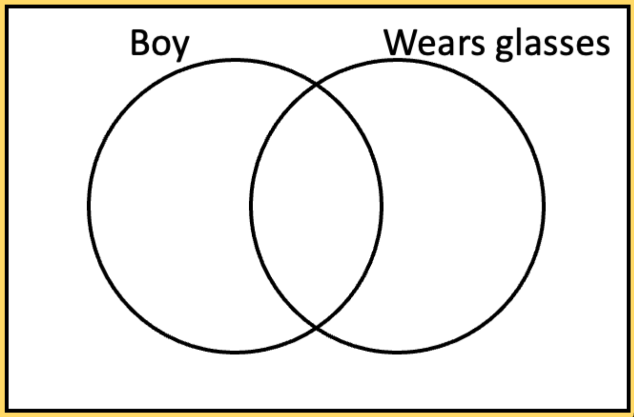
Your Turn

Complete the Venn Diagram:

ξ =

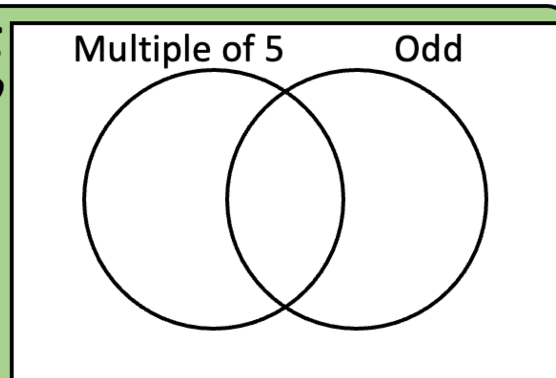


ξ



ξ = 3, 4, 5, 7, 10, 12, 13, 15, 20, 24, 25

ξ



Fluency Practice

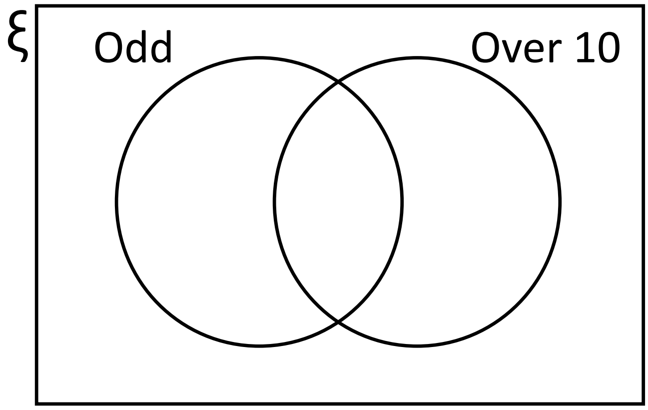
Venn Diagrams

1

Complete each Venn Diagram

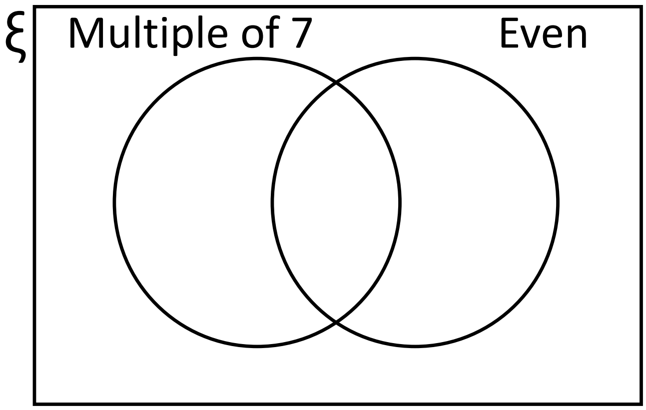
A) ξ =

1	2	3
7	8	9
11	12	13
16	17	18

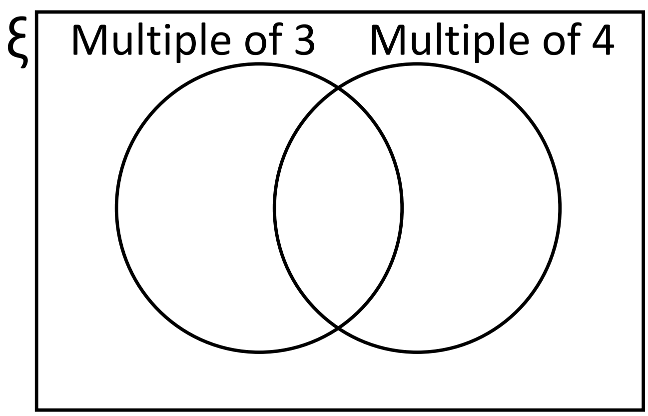


B) ξ =

34	14	15	28
21	70	20	13
1	25	7	16
6	35	18	41



C) ξ = Numbers from 1 to 25



Fluency Practice

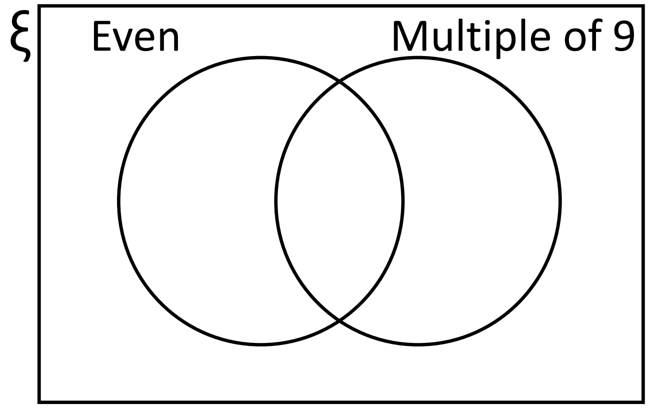
Venn Diagrams

2

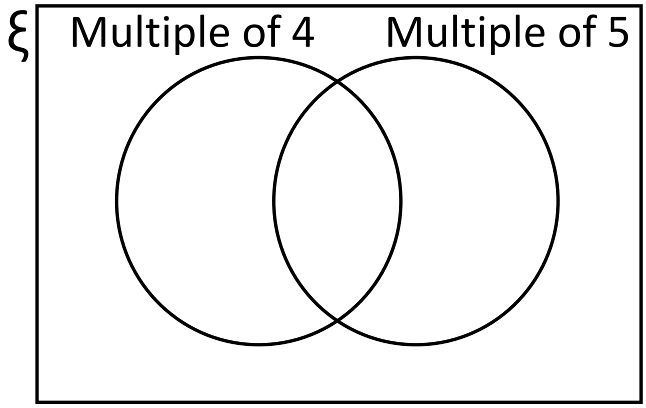
Complete each Venn Diagram

A) ξ =

44	90	45	88
54	27	6	26
91	16	71	9
18	13	24	33



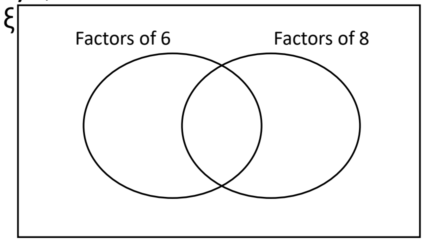
B) ξ = Even numbers from 10 to 40



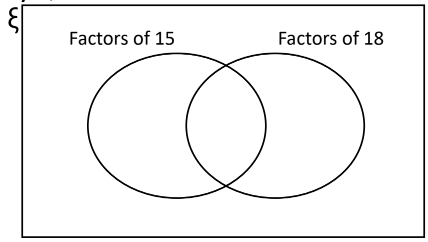
Fluency Practice

Venn Diagrams for Factors

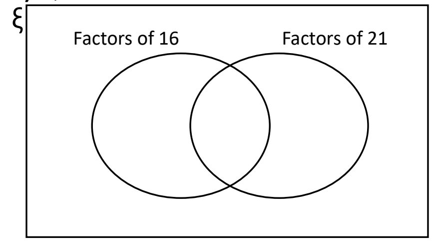
A) ξ = Factors of 24



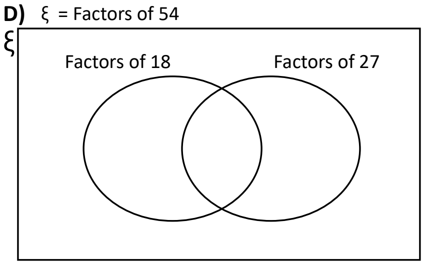
B) ξ = Factors of 36



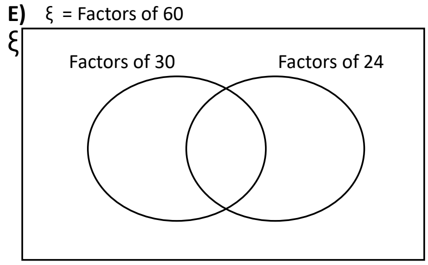
C) ξ = Factors of 28



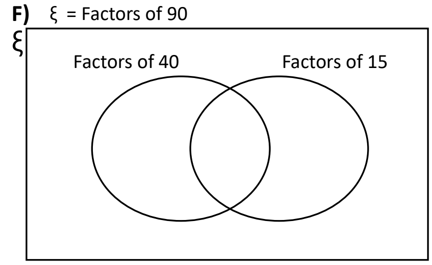
D) ξ = Factors of 54



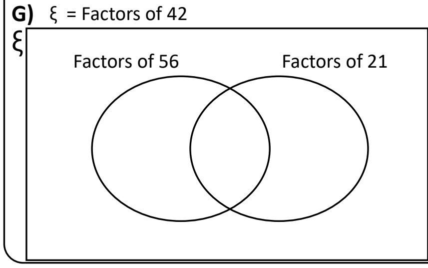
E) ξ = Factors of 60



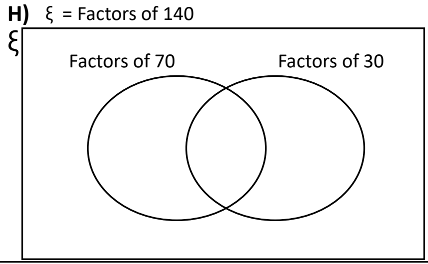
F) ξ = Factors of 90



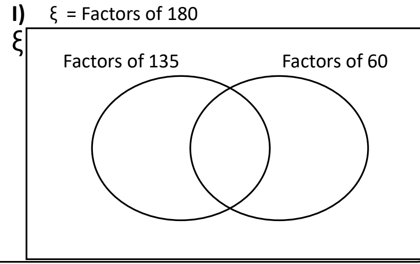
G) ξ = Factors of 42



H) ξ = Factors of 140



I) ξ = Factors of 180



Worked Example	Your Turn
Represent as a Venn diagram: $\xi = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $A = \{0, 1, 3, 5, 8\}$ $B = \{2, 5, 8, 9\}$	Represent as a Venn diagram: $\xi = \{2, 3, 4, 5, 7, 11, 13, 17, 19\}$ $A = \{2, 3, 5, 11, 13\}$ $B = \{5, 7, 13, 17, 19\}$

Worked Example	Your Turn
Represent as a Venn diagram: ξ =Positive integers between 1 and 10 inclusive $\mathrm{A} = \{$Prime numbers$\}$ $\mathrm{B} = \{$Even numbers$\}$	Represent as a Venn diagram: ξ = Integers between 0 and 5 inclusive $\mathrm{A} = \{$Prime numbers$\}$ $\mathrm{B} = \{$Odd numbers$\}$

Worked Example	Your Turn
ξ = $\{Days\ of\ the\ week\}$ A = $\{Tuesday, Thursday\}$ B = $\{Days\ starting\ with\ S\ or\ T\}$ Draw a Venn diagram to represent this information.	ξ = $\{Months\ of\ the\ year\}$ A = $\{Months\ starting\ with\ A\}$ B = $\{Months\ with\ six\ letters\}$ Draw a Venn diagram to represent this information.

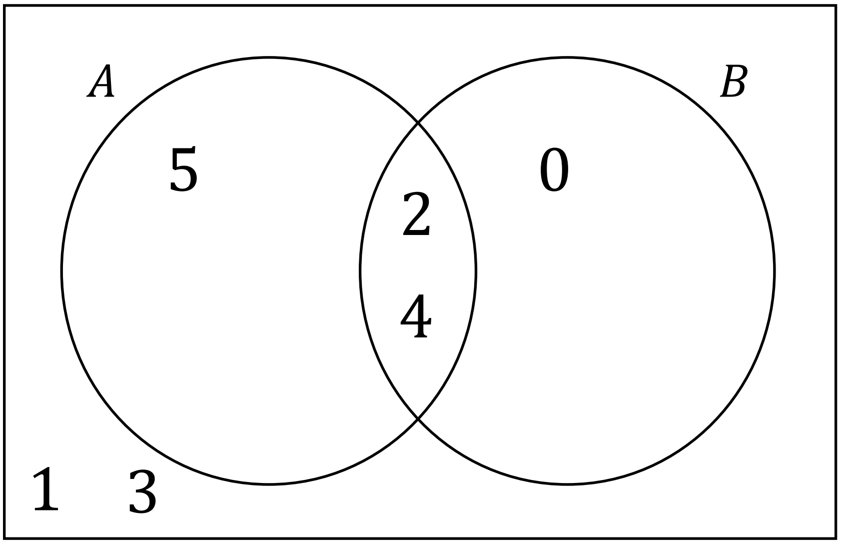
Worked Example

From the Venn diagram below, write in roster notation:

$\xi =$

$A =$

$B =$



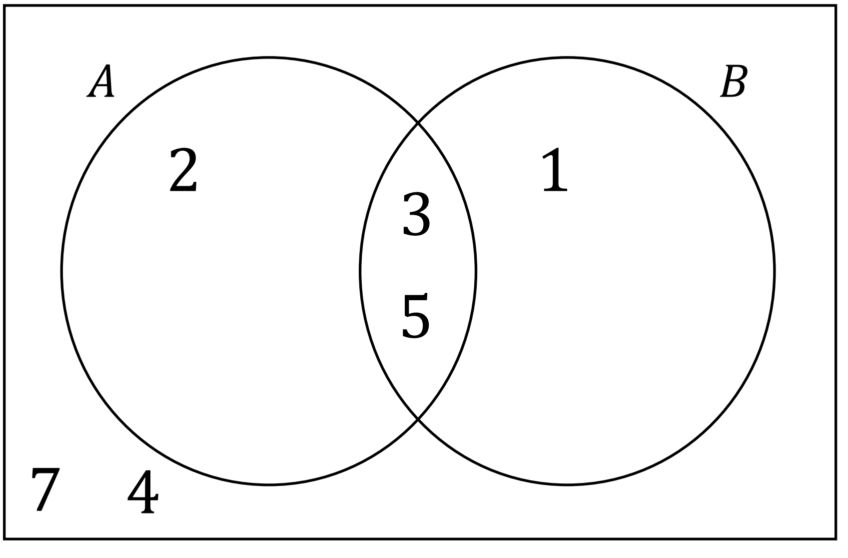
Your Turn

From the Venn diagram below, write in roster notation:

$\xi =$

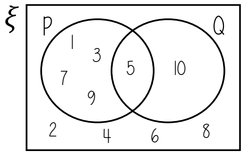
$A =$

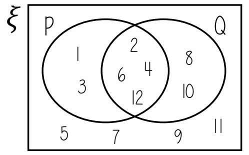
$B =$

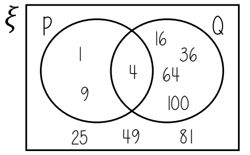


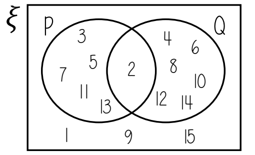
Fluency Practice

1. For each Venn diagram, describe the sets: ξ, P and Q

a.


b.


c.


d.


2. Given the sets, can you place the members into a Venn diagram

a.
ξ = {10, 11, 12, 13, 14, 15, 16}
P = {12, 14, 16}
Q = {10, 11, 12, 16}

b.
ξ = {integers from 15 to 21, inclusive}
X = {15, 18, 21}
Y = {16, 18, 20}

c.
ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
E = {even numbers}
F = {factors of 10}

d.
ξ = {a, b, c, d, e, f, g, h, i, j}
A = {a, e, i}
B = {a, c, e, g, i}

e.
ξ = {integers from 1 to 12, inclusive}
M = {multiples of 2}
N = {numbers less than or equal to 5}

2.4 Venn Diagrams with Three Circles

In this section you will look at Venn diagrams (with three circles), named after the English mathematician John Venn.

Worked Example

Represent in a Venn diagram:

$\xi = \{Integers\ between\ 1\ and\ 10\ inclusive\}$

$A = \{odd\ numbers\}$

$B = \{numbers\ greater\ than\ 4\}$

$C = \{numbers\ less\ than\ 3\}$

Your Turn

Represent in a Venn diagram:

$\xi = \{Integers\ between\ 1\ and\ 20\ inclusive\}$

$A = \{prime\ numbers\}$

$B = \{square\ numbers\}$

$C = \{even\ numbers\}$

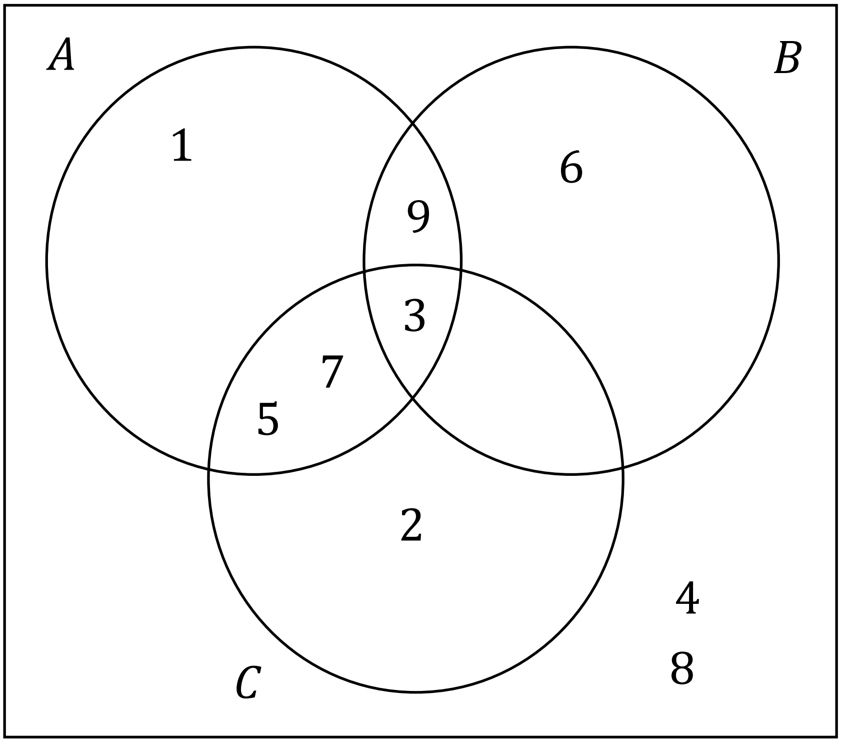
Worked Example

From the Venn diagram below, write in roster notation:

$\xi =$

$A =$

$B =$



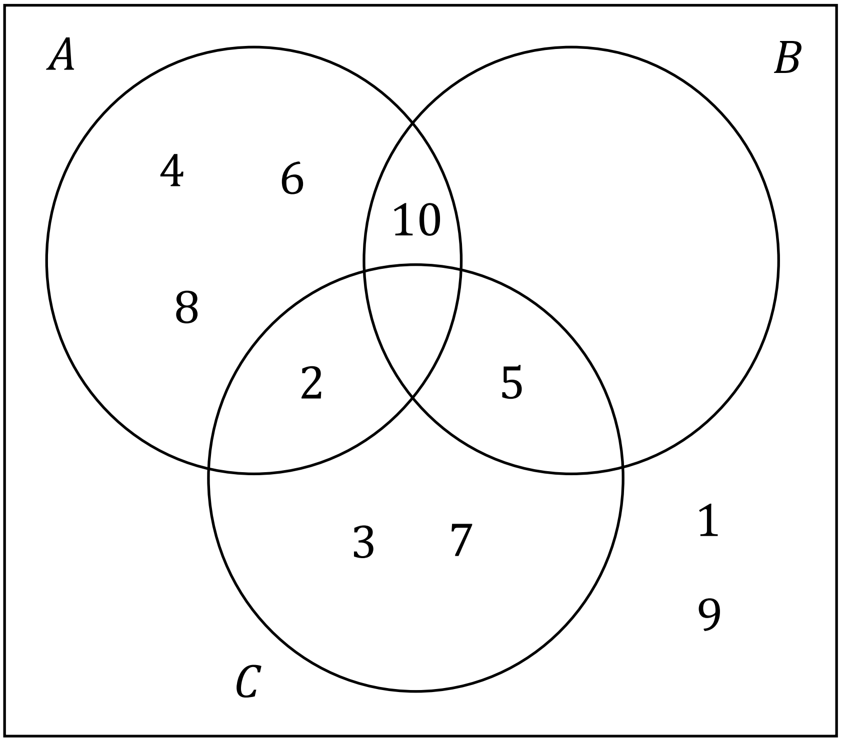
Your Turn

From the Venn diagram below, write in roster notation:

$\xi =$

$A =$

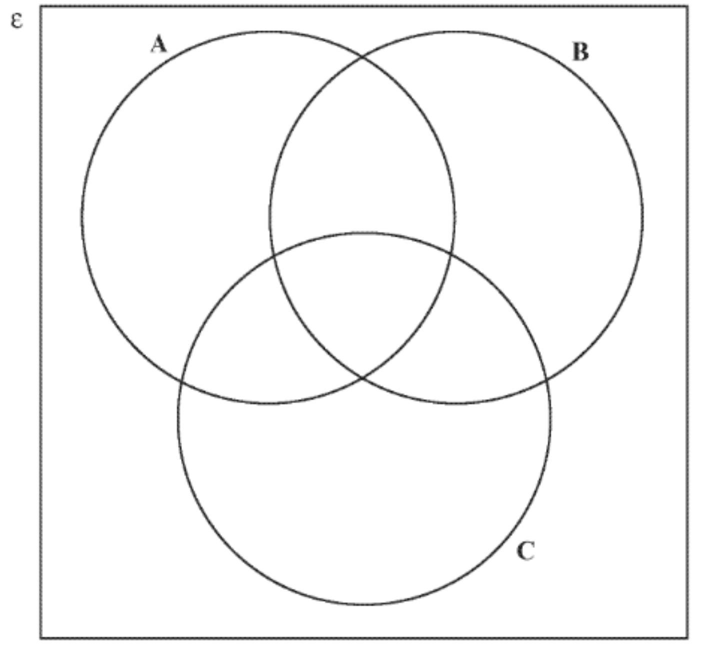
$B =$



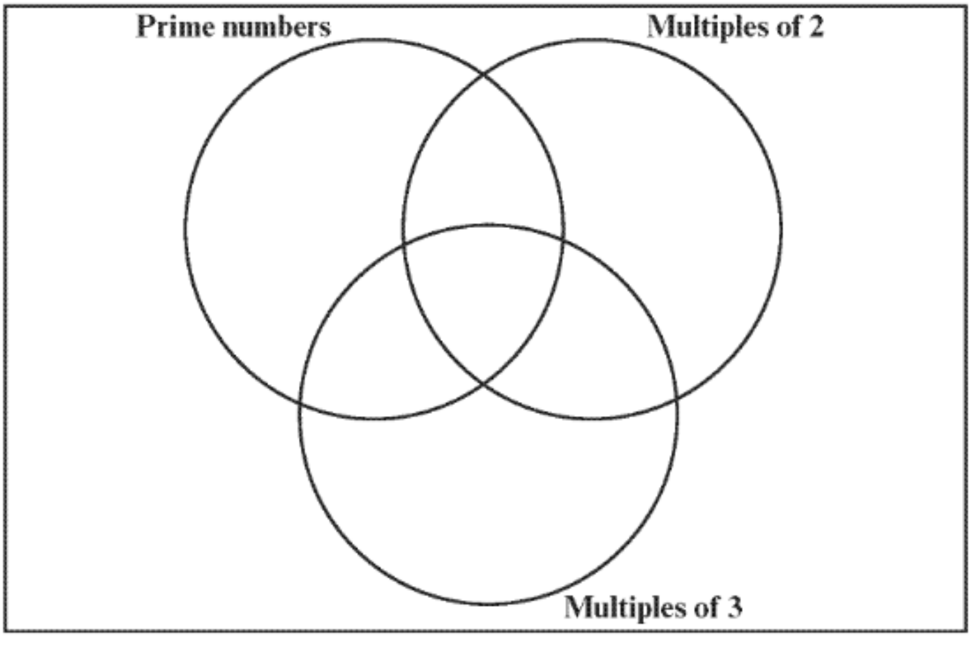
Fluency Practice

Given the following information, complete the Venn diagram shown below.

1.
- ε = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
- **A** is the set of factors of 24
- **B** is the set of multiples of 3
- **C** is the set of common factors of 30 and 70



2. (i) Place each of the whole numbers 42, 43, 44, 45, 46, 47, 48, 49, 50 in the correct positions in the Venn diagram.

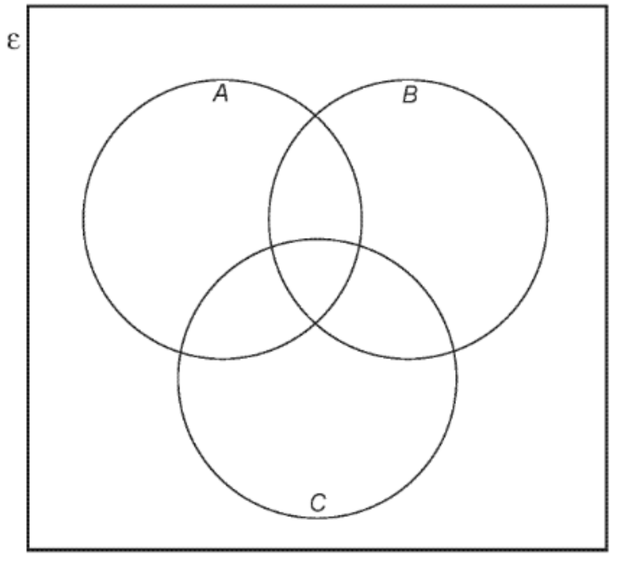


3. The universal set, ε = {22, 23, 24, 25, 26, 27, 28, 29, 30}.

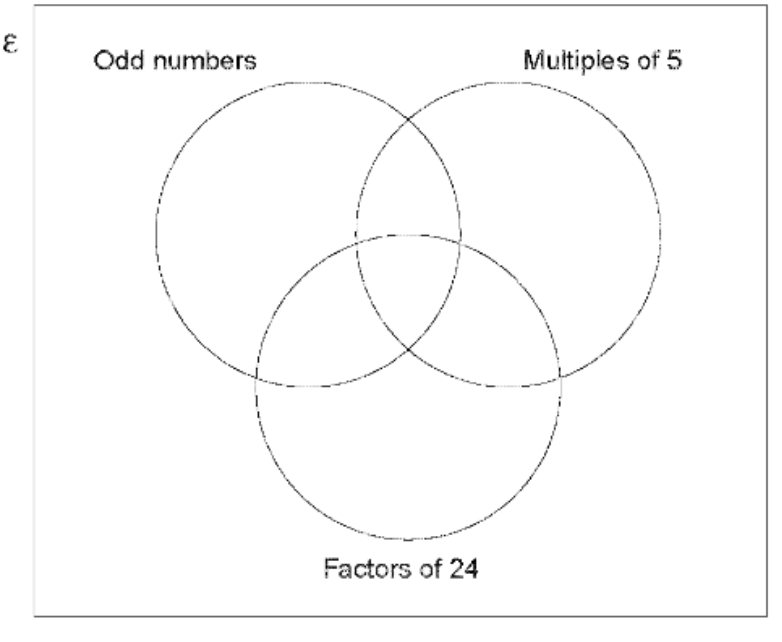
Within this universal set ε,

- set A is the multiples of 2
- set B is the multiples of 4
- set C is the multiples of 5

(a) Complete the Venn diagram. [3]



4. Place the whole numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 in the correct positions in the Venn diagram. [3]



3 Negative Numbers

3.1 Adding and Subtracting Negative Numbers

In this section you will look at adding and subtracting negative numbers.

Signs Not Next to Each Other

You will first look at how to add and subtract negative numbers when the signs are *not* next to each other.

Worked Example	Your Turn
Calculate: a) $3 - 4 =$ b) $-3 + 4 =$ c) $-3 - 4 =$ d) $-4 + 3 =$ e) $-4 - 3 =$	Calculate: a) $5 - 7 =$ b) $-5 + 7 =$ c) $-5 - 7 =$ d) $-7 + 5 =$ e) $-7 - 5 =$

Signs Next to Each Other

You will now look at how to add and subtract negative numbers when the signs are next to each other.

Adding Negative Numbers Pattern Spotting

$3 + 5 =$	$(-3) + 5 =$
$3 + 4 =$	$(-3) + 4 =$
$3 + 3 =$	$(-3) + 3 =$
$3 + 2 =$	$(-3) + 2 =$
$3 + 1 =$	$(-3) + 1 =$
$3 + 0 =$	$(-3) + 0 =$
$3 + (-1) =$	$(-3) + (-1) =$
$3 + (-2) =$	$(-3) + (-2) =$
$3 + (-3) =$	$(-3) + (-3) =$
$3 + (-4) =$	$(-3) + (-4) =$
$3 + (-5) =$	$(-3) + (-5) =$
…	…
$3 + (-12) =$	$(-3) + (-12) =$
$3 + (-59) =$	$(-3) + (-59) =$

Rules

Worked Example	Your Turn
Calculate: a) $3 + (-4) =$ b) $4 + (-3) =$ c) $(-3) + (-4) =$ d) $(-4) + (-3) =$	Calculate: a) $5 + (-7) =$ b) $7 + (-5) =$ c) $(-5) + (-7) =$ d) $(-7) + (-5) =$

Subtracting Negative Numbers Pattern Spotting

$3 - 5 =$	$(-3) - 5 =$
$3 - 4 =$	$(-3) - 4 =$
$3 - 3 =$	$(-3) - 3 =$
$3 - 2 =$	$(-3) - 2 =$
$3 - 1 =$	$(-3) - 1 =$
$3 - 0 =$	$(-3) - 0 =$
$3 - (-1) =$	$(-3) - (-1) =$
$3 - (-2) =$	$(-3) - (-2) =$
$3 - (-3) =$	$(-3) - (-3) =$
$3 - (-4) =$	$(-3) - (-4) =$
$3 - (-5) =$	$(-3) - (-5) =$
…	…
$3 - (-12) =$	$(-3) - (-12) =$
$3 - (-59) =$	$(-3) - (-59) =$

Rules

Worked Example	Your Turn
Calculate:	Calculate:
a) $3-(-4)=$	a) $5-(-7)=$
b) $4-(-3)=$	b) $7-(-5)=$
c) $(-3)-(-4)=$	c) $(-5)-(-7)=$
d) $(-4)-(-3)=$	d) $(-7)-(-5)=$

3.2 Multiplying Negative Numbers

In this section you will look at multiplying negative numbers.

Multiplying Negative Numbers Pattern Spotting

$3 \times 5 =$	$(-3) \times 5 =$
$3 \times 4 =$	$(-3) \times 4 =$
$3 \times 3 =$	$(-3) \times 3 =$
$3 \times 2 =$	$(-3) \times 2 =$
$3 \times 1 =$	$(-3) \times 1 =$
$3 \times 0 =$	$(-3) \times 0 =$
$3 \times (-1) =$	$(-3) \times (-1) =$
$3 \times (-2) =$	$(-3) \times (-2) =$
$3 \times (-3) =$	$(-3) \times (-3) =$
$3 \times (-4) =$	$(-3) \times (-4) =$
$3 \times (-5) =$	$(-3) \times (-5) =$
…	…
$3 \times (-12) =$	$(-3) \times (-12) =$
$3 \times (-59) =$	$(-3) \times (-59) =$

Rules

Worked Example	Your Turn
Calculate: a) $3 \times (-4) =$ b) $(-3) \times 4 =$	Calculate: a) $5 \times (-7) =$ b) $(-5) \times 7 =$

Worked Example	Your Turn
Calculate: a) $(-3) \times (-4) =$ b) $(-4) \times (-3) =$	Calculate: a) $(-5) \times (-7) =$ b) $(-7) \times (-5) =$

3.3 Dividing Negative Numbers

In this section you will look at dividing negative numbers.

Dividing Negative Numbers Pattern Spotting

$15 \div 3 =$	$15 \div (-3) =$
$12 \div 3 =$	$12 \div (-3) =$
$9 \div 3 =$	$9 \div (-3) =$
$6 \div 3 =$	$6 \div (-3) =$
$3 \div 3 =$	$3 \div (-3) =$
$0 \div 3 =$	$0 \div (-3) =$
$(-3) \div 3 =$	$(-3) \div (-3) =$
$(-6) \div 3 =$	$(-6) \div (-3) =$
$(-9) \div 3 =$	$(-9) \div (-3) =$
$(-12) \div 3 =$	$(-12) \div (-3) =$
$(-15) \div 3 =$	$(-15) \div (-3) =$
…	…
$(-36) \div 3 =$	$(-36) \div (-3) =$
$(-81) \div 3 =$	$(-81) \div (-3) =$

Rules

Worked Example	Your Turn
Calculate: a) $12 \div (-3) =$ b) $12 \div (-4) =$	Calculate: a) $35 \div (-5) =$ b) $35 \div (-7) =$

Worked Example	Your Turn
Calculate: a) $(-12) \div (-3) =$ b) $(-12) \div (-4) =$	Calculate: a) $(-35) \div (-5) =$ b) $(-35) \div (-7) =$

3.4 Real Life Applications

In this section you will look at the real life applications of negative numbers.

Worked Example	Your Turn
The temperature in Wolverhampton on Tuesday is -15°C. On Wednesday, the temperature decreases by 5°C. Find the temperature in Wolverhampton on Wednesday.	The temperature in Lichfield on Saturday is -3°C. On Sunday, the temperature decreases by 6°C. Find the temperature in Lichfield on Sunday.

Worked Example	Your Turn
The temperature in Derby is $-3°C$. The temperature in Birmingham is $9°C$. What is the difference between the temperature in Derby and the temperature in Birmingham?	The temperature in Birmingham is $8°C$. The temperature in Newcastle upon Tyne is $-5°C$. What is the difference between the temperature in Birmingham and the temperature in Newcastle upon Tyne?