Year 7
Mathematics
Unit 2

Name: ______________________________

Class: ______________________________

Contents

1 Powers and Roots

1.1 Squaring

In this section you will look at squaring. You met square numbers briefly in the previous booklet. Do you remember the first 10 square numbers?

Worked Example	Your Turn
a) Write 5^2 as a multiplication and then work it out	a) Write 8^2 as a multiplication and then work it out
b) Use a calculator to work out 2.11^2	b) Use a calculator to work out 31.7^2

1.2 Square Roots

In this section you will look at square roots. The square root is just the inverse of squaring.

Worked Example	Your Turn
a) Work out $\sqrt{25}$	a) Work out $\sqrt{64}$
b) Use a calculator to work out $\sqrt{4.4521}$	b) Use a calculator to work out $\sqrt{1004.89}$

1.3 Cubing

In this section you will look at cubing. You met cube numbers briefly in the previous booklet. Do you remember the first 10 cube numbers?

Worked Example	Your Turn
a) Write 4^3 as a multiplication and then work it out	a) Write 8^3 as a multiplication and then work it out
b) Use a calculator to work out 2.11^3	b) Use a calculator to work out 31.7^3

1.4 Cube Roots

In this section you will look at cube roots. The cube root is just the inverse of cubing.

Worked Example	Your Turn
a) Work out $\sqrt[3]{64}$ b) Use a calculator to work out $\sqrt[3]{9.393931}$	a) Work out $\sqrt[3]{512}$ b) Use a calculator to work out $\sqrt[3]{31855.013}$

1.5 Powers

In this section you will look at powers.

The 'powers of 2'

$2^1 = 2$

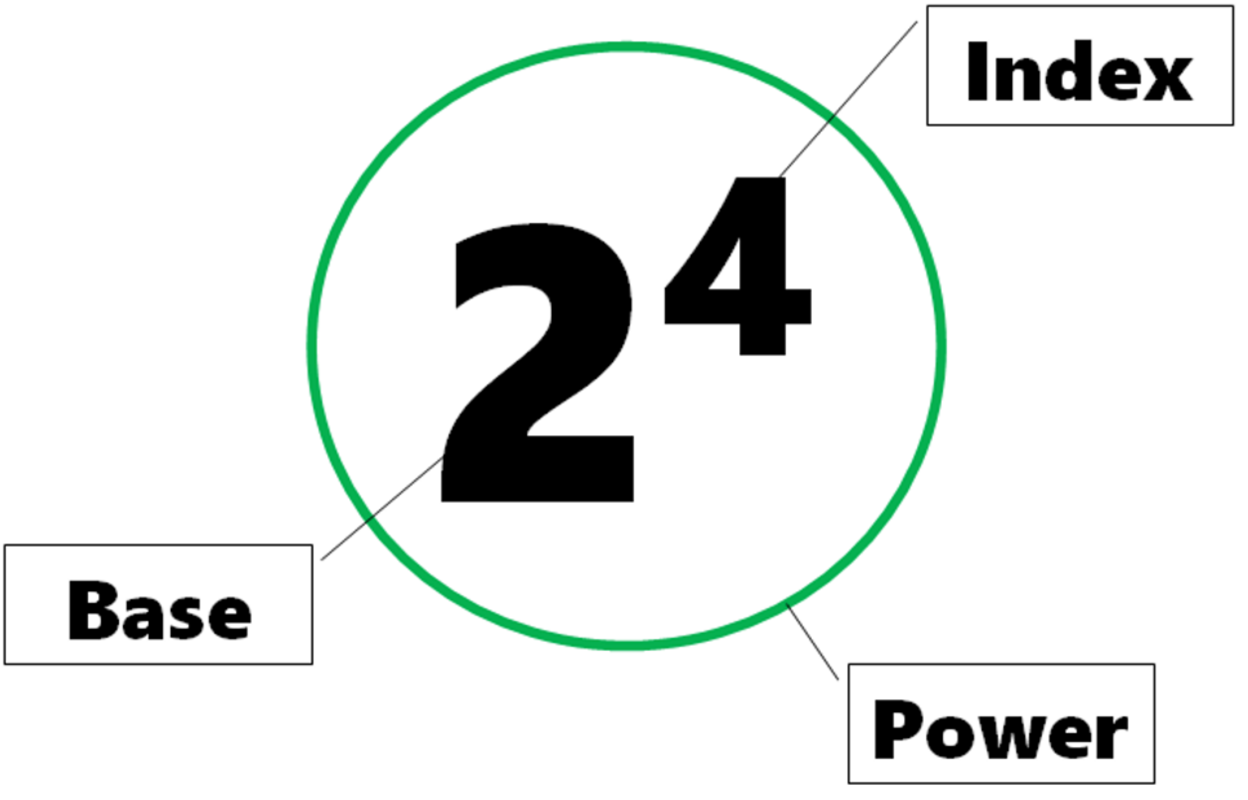
$2^2 = 2 \times 2$

$2^3 = 2 \times 2 \times 2$

$2^4 = 2 \times 2 \times 2 \times 2$

$2^5 = 2 \times 2 \times 2 \times 2 \times 2$

$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$



Worked Example	Your Turn
Write 3^4 as a multiplication and then work it out	Write 2^5 as a multiplication and then work it out

1.6 Roots

In this section you will look at roots.

Worked Example	Your Turn
Work out $\sqrt[4]{81}$	Work out $\sqrt[5]{32}$

2 Order of Operations

Operations

So far, we have studied three groups of operations.

	Multiplication	Addition	Exponentiation
Operation	$2 \times 3 = 6$ $2 \cdot 3 = 6$	$2 + 3 = 5$	$2^3 = 8$
Inverse Operation	$6 \div 3 = 2$ $\frac{6}{3} = 2$	$5 - 3 = 2$	$\sqrt[3]{8} = 2$

2.1 Commutativity

In this section you will look at commutativity.

The commutative law says that we can swap numbers around and still get the same answer.

Commutativity

Which of the operations are commutative?

				Commutative?
Multiplication	**Multiplication**	$2 \cdot 3 = 6$	$2 \cdot 3 = 3 \cdot 2$	Yes
	Division	$\frac{6}{3} = 2$	$\frac{6}{3} \neq \frac{3}{6}$	No
Addition	**Addition**	$2 + 3 = 5$	$2 + 3 = 3 + 2$	Yes
	Subtraction	$5 - 3 = 2$	$5 - 3 \neq 3 - 5$	No
Exponentiation	**Exponents**	$2^2 = 8$	$2^3 \neq 3^2$	No
	Roots	$\sqrt[3]{8} = 2$	$\sqrt[3]{8} \neq \sqrt[8]{3}$	No

Notice how most operations are not commutative.
That means the order you write and work out matters.
It is only multiplication and addition where you can change the order of the inputs and not affect the output.

Fill in the Gaps

	Calculation	Order Reverse	Commutative?
e.g.	$5 \times 4 = 20$	$4 \times 5 = 20$	Yes
a	$12 \times 3 = 36$	$3 \times 12 =$	
b	$9 \cdot 7 =$		
c	$24 \div 6 = 4$	$6 \div 24 = 0.25$	
d	$\frac{3}{2} =$	$\frac{2}{3} =$	
e	$15 + 19 =$		
f	$20 - 15 = 5$	$15 - 20 = -5$	
g	$6.5 + 1.2 =$		
h	$14 - 8 =$		
i	$5^2 =$	$2^5 =$	
j	$\sqrt[2]{121}$	$\sqrt[121]{2}$	
k	$0.03 - 0.2 =$		
l	$\sqrt[3]{8} =$		
m		$3^4 =$	
n		$123 \cdot 19 =$	

2.2 Moving Numbers Around

In this section you will look at what happens when you move numbers around.

What happens when we have more than two numbers in a calculation?

Which of these sums are the same?

$9 + 8 + 25$

$25 + 8 + 9$

$9 + 25 + 8$

$8 + 25 + 9$

What other sums would be the same?

Which of these differences are the same?

$30 - 4 - 10$

$30 - 10 - 4$

$10 - 30 - 4$

$4 - 10 - 30$

Why are the top two the same, but the bottom two different?

Subtraction

$12 - 3 - 5$

minuend subtrahends

$-3 - 5$

-8

$12 - 5 - 3$

minuend subtrahends

$-5 - 3$

-8

We can subtract in any order. What we can't do is switch a subtrahend with a minuend.

Division

$23 \div 3 \div 2$

dividend divisors

$\div 3 \div 2$

$\div 6$

$23 \div 2 \div 3$

dividend divisors

$\div 2 \div 3$

$\div 6$

We can divide in any order. What we can’t do is switch a divisor with a dividend.

Moving Numbers Around

When you have a mix of addition and subtraction, remember:

Addition	
$+$	Summands can move anywhere
$-$	Subtrahends can move as long as they are always behind a subtraction sign

When you have a mix of multiplication and division, remember:

Multiplication	
$\times$	Multipliers can move anywhere
$\div$	Divisors can move as long as they are always behind a division sign

2.3 Mixing the Four Operations

In this section you will look at what happens when you mix the four operations.

In the last exercise, every question was either from the multiplication group or the addition group.

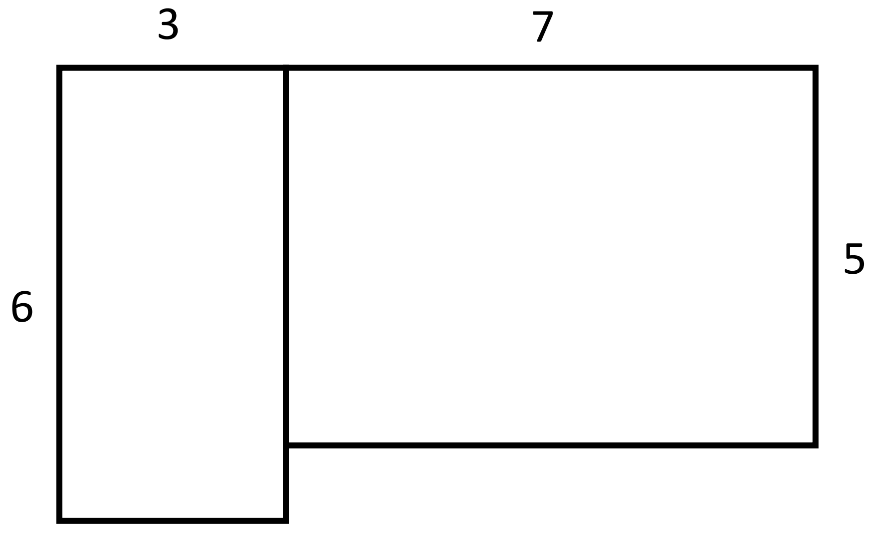
	Multiplication	**Addition**
Operation	$\times$	$+$
Inverse Operation	$\div$	$-$

So, what happens if we have a mix of the multiplication and addition groups in the same calculation?

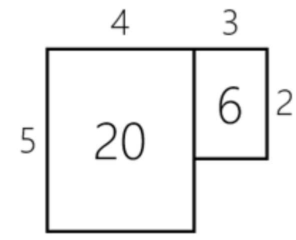
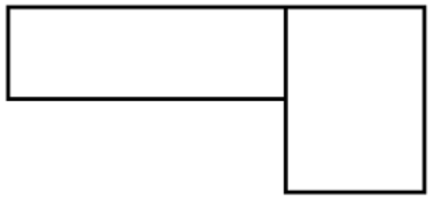
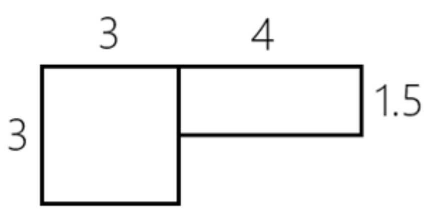
Mixing the Four Operations

Let's look at some area models.

What is the total area of this shape?



Fluency Practice

	Calculation	Area model	Value
e.g.	5 · 4 + 3 · 2		26
a	2 · 6 + 4 · 3		
b	4 · 2 + 3 · 5 + 2 · 7		
c			
d	5 · 3 + 3 · ____		33
e	2 · 9 + ____ · ____ + 4 · 3		45

2. Make up some more complex area models of your own and write down the calculations that would find their area.

Mixing the Four Operations

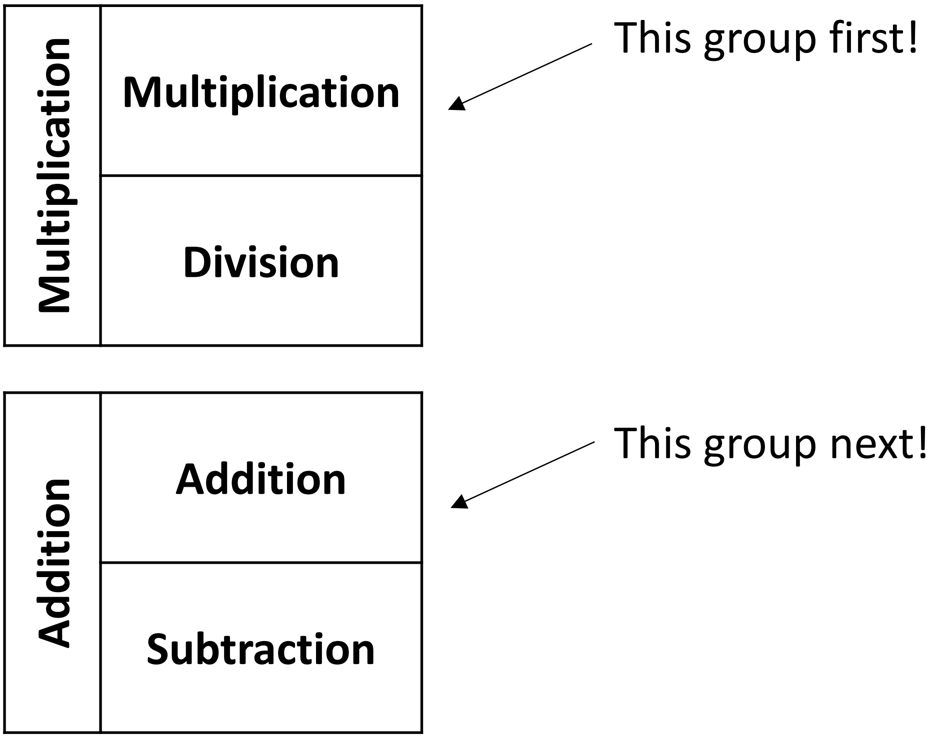
We can see from the area models that we work out the products first, then we add them.

We can extend that to a general principle.

If there is a mix of multiplication and addition, work out the multiplication first.

When we say “multiplication” we mean the multiplication group: all multiplication and division.

When we say “addition” we mean the addition group: all addition and subtraction.



2.4 Exponentiation

In this section you will look at exponentiation.

At the start of this unit, we looked at a third group of operations, which we called exponentiation.

It included exponents and their inverses, roots, which we learnt about in detail in the last unit.

	Exponentiation
Operation	
Inverse Operation	$\sqrt{}$

Where does exponentiation fit into the order of calculating?

Exponentiation

We have seen that multiplication should be worked out before addition.

Remember how a power comes from repeated multiplication:

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$

This means powers should be worked out before other multiplication.

In fact, all exponentiation should be worked out before other multiplication.

Exponentiation	**Exponents**
	Roots
Multiplication	**Multiplication**
	Division
Addition	**Addition**
	Subtraction

2.5 Brackets

In this section you will look at brackets.

The Order of Operations

We now know the order in which we calculate.

This group is repeated multiplication, so we can think of it as “stronger” than multiplication.

This group is repeated addition, so we can think of it as “stronger” than addition.

Exponentiation	**Exponents**
	Roots
Multiplication	**Multiplication**
	Division
Addition	**Addition**
	Subtraction

Brackets break the order, so we must always look at them first.

The Order of Operations

Brackets break the order.

Addition	Addition
	Subtraction
Exponentiation	Exponents
	Roots
Multiplication	Multiplication
	Division

Multiplication	Multiplication
	Division
Exponentiation	Exponents
	Roots
Addition	Addition
	Subtraction

Multiplying a Bracket

We have seen questions that look like this: $2 \times (5+3)$
which can also be written easily like this: $2 \cdot (5+3)$

We normally don't even write the dot: $2(5+3)$

these all mean the same thing

There is no symbol between the 2 and the bracket.

No symbol still means 'multiply'.

Multiplying a Bracket

We know that multiplication is commutative, so these two calculations have the same answer:

$2 \cdot (5 + 3)$ and $(5 + 3) \cdot 2$

When we remove the multiplication symbol, we always write the number in front of the bracket.

$2(5 + 3)$ not $(5 + 3)2$

The main reason for this is to avoid confusion with exponents: $(5 + 3)^2$

Hidden Brackets

Many calculations use hidden brackets.

Here is a division calculation: $(6 + 4) \div 2$

We can write it like this: $\frac{(6+4)}{2}$

We don't need to write the brackets: $\frac{6+4}{2}$

these all mean the same thing

The long bar under the $6 + 4$ tells us there's a hidden bracket.

Be careful, though. It's different to this $\frac{6}{2} + 4$ and this $6 + \frac{4}{2}$

Hidden Brackets

Many calculations use hidden brackets.

Here is a root calculation: $\sqrt{(4+5)}$

We don't need to write the brackets: $\sqrt{4+5}$

} these all mean the same thing

The long bar over the $4+5$ tells us there's a hidden bracket.

Be careful, though. It's different to this $\sqrt{4}+5$.

Hidden Brackets

Many calculations use hidden brackets.

Here is an exponent calculation: $2^{(3+1)}$

We don't need to write the brackets: 2^{3+1}

} these all mean the same thing

The addition is clearly in the exponent, which tells us there's a hidden bracket.

Be careful, though. It's different to this $2^3 + 1$.

3 Introduction to Algebra

Introduction

Algebra concerns representing missing information.

Put simply, we use letters, known as variables, to represent numbers.

Usually, the value of variables are not initially known, but we hope to combine available information to find their value.

For example:
- a might represent someone's age this year.
- θ might represent an unknown angle (θ is a Greek letter).

3.1 Forming Expressions

In this section you will look at forming algebraic expressions.

Forming Expressions

'Four more than a number'

$n + 4$

This letter does not have to be n. It could be any letter or symbol. People often use x. We could write ☺ $+ 4$. We are not going to, though. That would be silly.

Write the following sentences algebraically:

- A number add 6
- A number add 10
- A number subtract 10
- 8 subtract a number

Forming Expressions

'Four lots of a number'

$4n$

We do not tend to use the $\times$ symbol in algebra. Instead we write things next to each other to show multiplication.

Write the following sentences algebraically:

- A number multiplied by 6
- A number multiplied by 10
- a multiplied by b
- $4a$ multiplied by b
- $2a$ multiplied by $2b$

Forming Expressions

'A number divided by 5'

$$\frac{n}{5}$$

We tend not to use ÷ in expressions. We use fraction notation (writing a division as a fraction).

Write the following sentences algebraically:

- A number divided by 6
- 6 divided by a number
- A number divided by $6 + a$
- $6 - a$ divided by a number

Worked Example	Your Turn
Write an algebraic expression for each of the following:	Write an algebraic expression for each of the following:
3 more than a	3 less than a
5 less than a	a more than 5
b multiplied by a	b divided by a
b multiplied by a then squared	b divided by a then squared

3.2 Forming Expressions in Context

In this section you will look at forming algebraic expressions in context.

Often you will be asked to take a 'real life' scenario and turn into mathematical code. For instance

Gummy rings cost 2p per gram, fried eggs cost 3p per gram, gummy snakes cost 4p per gram.

Find an expression for the total cost of x grams of rings, y grams of eggs and z grams of snakes.

Notice that the coefficient of x (the number in front of x) stands for the price of the rings, not the number of them.

Worked Example	Your Turn
Adam is x years old. Lucy is 15 years older than Adam. Write down an expression, in terms of x, for Lucy's age.	Albert is z years old. Laura is 3 times as old as Albert. Write down an expression, in terms of z, for Laura's age.

Worked Example	Your Turn
a) Ahmed is z years old. Libby is 3 times as old as Ahmed. John is 19 years older than Libby. Write down an expression, in terms of z, for John's age. b) Alfred has x stickers. Lottie has 11 less stickers than Alfred. John has 5 times as many stickers as Lottie. Write down an expression, in terms of x, for the number of stickers John has.	a) Adam has y cards. Latika has twice as many cards as Adam. Jack has 10 less cards than Latika. Write down an expression, in terms of y, for the number of cards Jack has. b) Alfie is z years old. Lottie is 15 years younger than Alfie. John is 3 times as old as Lottie. Write down an expression, in terms of z, for John's age.

3.3 Conventions and Definitions

In this section you will look at commonly used conventions and definitions in Algebra.

The conventions include:

- We tend to use single lowercase letters for variables, either using the English alphabet or using the Greek alphabet.
- An algebraic x is written using two back-to-back c's. Do NOT write it as a $\times$ symbol.
- Do NOT include the multiplication sign, for example $3 \times p = 3p$
- Write division as fractions, for example $3 \div \mathrm{p} = \frac{3}{p}$
- Write numbers first in products, for example $p \times 3 = 3p$
- Write letters in products in alphabetical order, for example $4 \times q \times r \times p = 4pqr$
- $1x$ is written simply as x

Definitions

The definitions include:

- **Variable** is a letter used to represent an unknown number.

- **Coefficient** is the number in front of a variable.

- **Constant** is a number that cannot change its value.

- **Term** is either a constant, a variable or a constant multiplied by a variable.

- **Expression** is terms and operators ($+$ and $-$) grouped together.

Worked Example	Your Turn
Write down the following for the expression: $2x - 4y - 9$ Variables: Coefficient of x: Coefficient of y: Constant: Terms:	Write down the following for the expression: $-2a + 4b + 9$ Variables: Coefficient of a: Coefficient of b: Constant: Terms:

Worked Example	Your Turn
Write down the following for the expression: $2x^2 - 4xy - 9$ Variables: Coefficient of x^2: Coefficient of xy: Constant: Terms:	Write down the following for the expression: $-2ab + 4b^2 + 9$ Variables: Coefficient of ab: Coefficient of b^2: Constant: Terms:

Intelligent Practice

Question	Variables	Coefficients	Constant	Terms
$3x - 9$				
$3x + 4y - 9$				
$3x - 4y - 9$				
$3x - 4y + 9$				
$-3x - 4y + 9$				
$9 - 3x - 4y$				
$9 - 3a - 4b$				
$3a^2 - 4b^2 + 9$				
$3a^2 - 4a + 9$				
$3a^2 - 4a$				
$3a^2 - 4$				
$3ab - 4$				
$3ab - 4a$				
$3ab - 4a - 5b$				
$3a^2b - 4a - 5b$				
$3ab^2 - 4a - 5b$				
$3ab^2 - 4ab - 5b$				
$3ab^2 - 4a^2b - 5b$				
$3ab^2 - 4a^2b - 5ab$				
$3ab^2 - 4a^2b - 5ab - 6$				

3.4 Collecting Like Terms without Powers

In this section you will look at adding and subtracting algebraic terms without powers.

Frayer Model – Like Terms

Definition	**Characteristics**
Examples	**Non-Examples**

Fluency Practice

$3p$	p	Like	Unlike
x^2	$3x^2$	Like	Unlike
x^2	$2x$	Like	Unlike
$-3\sqrt{x}$	$27\sqrt{x}$	Like	Unlike
$7a$	$7b$	Like	Unlike

$3a$	$3a$	Like	Unlike
a	$2a$	Like	Unlike
$2a$	$2A$	Like	Unlike
$-3a$	$2a$	Like	Unlike
$4a$	$4b$	Like	Unlike
$3a$	$3a^2$	Like	Unlike
$2a^2$	$7a^2$	Like	Unlike
$-3a^2$	$7a^2$	Like	Unlike
$2a^2$	$2a^{-2}$	Like	Unlike
2^a	a^2	Like	Unlike
x	$\sqrt{x}$	Like	Unlike
1	2	Like	Unlike

Worked Example	Your Turn
Simplify: a) $5y - 3y - y$ b) $4q - 3q - 3q - 4q - 3q$	Simplify: a) $3x + x + 3x$ b) $4z + 5z - 5z - 3z - 2z$

Worked Example	Your Turn
Simplify: a) $6p + 9p + 4q + 7p$ b) $-7x + 5y - y - 6x$	Simplify: a) $3q + q + 6p + 4p$ b) $-p - 7p + 7p + 6q$

3.5 Collecting Like Terms with Powers

In this section you will look at adding and subtracting algebraic terms with powers.

Worked Example	Your Turn
Simplify: a) $y^4 + y^2 + 3y^4 - 4q^4$ b) $5p^4 - 2x^4 - x^4 + 4x^4$	Simplify: a) $3z^4 + 3z^4 + 3z^3 + p^4$ b) $p^2 - 4z^3 - 5z^2 + 3z^3$

3.6 Algebraic Notation

In this section you will look at algebraic notation.

Worked Example	Your Turn
Explain what the following mean: $7x$ xy xy^2 $(\mathrm{xy})^2$	Explain what the following mean: $7a$ ab ab^2 $(ab)^2$

3.7 Multiplying Terms without Powers

In this section you will look at multiplying algebraic terms without powers.

Worked Example	Your Turn
Simplify: a) $5p \times q$ b) $2p \times 8y$ c) $8z \times 7z$	Simplify: a) $p \times 5x$ b) $4x \times 4y$ c) $3z \times 2z$

3.8 Multiplying Terms with Powers

In this section you will look at multiplying algebraic terms with powers.

Worked Example	Your Turn
Simplify: a) $8x^4y^7 \times x^2y^4$ b) $7x^8y^3 \times 4x^8y^6$	Simplify: a) $x^8y \times 8x^5y^2$ b) $8x^2y^4 \times 6x^2y^6$

3.9 Dividing Terms without Powers

In this section you will look at dividing algebraic terms without powers.

Worked Example	**Your Turn**
Simplify: a) $\frac{3x}{x}$ b) $\frac{3xy}{y}$	Simplify: a) $\frac{7y}{y}$ b) $\frac{7xy}{x}$

3.10 Dividing Terms with Powers

In this section you will look at dividing algebraic terms with powers.

Worked Example	Your Turn
Simplify: a) $\frac{x^6y^6}{x^4y^4}$ b) $\frac{10x^8y^5}{5xy^3}$	Simplify: a) $\frac{x^6y^8}{x^2y^3}$ b) $\frac{9x^5y^5}{3x^3y}$

3.11 Algebraic Order of Operations

In this section you will look at algebraic order of operations.

3.12 Substitution

In this section you will look at substituting values into algebraic expressions.

Worked Example	Your Turn
a) Calculate $\frac{14}{y} + y^2$ when $y = 7$	a) Calculate $y^2 + 3y$ when $y = 2$
b) Work out $\frac{4z+1}{4}$ when $z = 6$	b) Work out $\frac{2z-1}{4}$ when $z = 1$

Worked Example	Your Turn
a) Evaluate $p^2 + 4q$ when $p = 6$ and $q = 7$	a) Evaluate $x^2 - 2y$ when $x = 10$ and $y = 1$
b) Work out $(2p + q)^2$ when $p = 8$ and $q = 10$	b) Work out $(4x + 3y)^2$ when $x = 1$ and $y = 3$

Worked Example	Your Turn
a) Evaluate $a^2 + \frac{-12}{b}$ when $a = -3$ and $b = -6$	a) Evaluate $\frac{-36}{x} + y^2$ when $x = -8$ and $y = -9$
b) Work out $p^2 - 2q$ when $p = -2$ and $q = -6$	b) Work out $p^2 - 4q$ when $\mathrm{p} = -8$ and $q = -2$

3.13 Substitution into Formulae

In this section you will look at substituting values into formulae.