KING EDWARD VI HANDSWORTH GRAMMAR SCHOOL FOR BOYS

KING EDWARD VI ACADEMY TRUST BIRMINGHAM

Year 7

2023 Mathematics 2024

Unit 2 Booklet

HGS Maths

Tasks

Dr Frost Course

Name: ____________________________

Class: ____________________________

Contents

1 Powers and Roots

1.1 Squaring

Worked Example	Your Turn
a) Write 5^2 as a multiplication and then work it out	a) Write 8^2 as a multiplication and then work it out
b) Use a calculator to work out 2.11^2	b) Use a calculator to work out 31.7^2

1.2 Square Roots

Worked Example	Your Turn
a) Work out $\sqrt{25}$ b) Use a calculator to work out $\sqrt{4.4521}$	a) Work out $\sqrt{64}$ b) Use a calculator to work out $\sqrt{1004.89}$

1.3 Cubing

Worked Example	Your Turn
a) Write 4^3 as a multiplication and then work it out	a) Write 8^3 as a multiplication and then work it out
b) Use a calculator to work out 2.11^3	b) Use a calculator to work out 31.7^3

1.4 Cube Roots

Worked Example	Your Turn
a) Work out $\sqrt[3]{64}$	a) Work out $\sqrt[3]{512}$
b) Use a calculator to work out $\sqrt[3]{9.393931}$	b) Use a calculator to work out $\sqrt[3]{31855.013}$

1.5 Notation

7^3

7 is the base

3 is the power

3 is the index

3 is the exponent

What we write	What we say
8^2	"Eight squared"
8^3	"Eight cubed"
8^4	"Eight to the power four"
$8^{16\,350\,761.3}$	"Eight to the power sixteen million three hundred and fifty thousand seven hundred and sixty one point three"

1.6 Powers

Worked Example	Your Turn
Write 3^4 as a multiplication and then work it out	Write 2^5 as a multiplication and then work it out

Fill in the Gaps

We say	We write	We work out	Answer
2 to the power of 4	2^4	$2 \times 2 \times 2 \times 2$	
3 to the power of 4		$3 \times 3 \times 3 \times 3$	
	4^4		256
5 to the power of 2			
	6^5		7776
		$8 \times 8 \times 8 \times 8$	
		$9 \times 9 \times 9$	
	3^9		
10 to the power of 2			
2 to the power of 10			

1.7 Roots

Powers recap: fill in the table without a calculator
Do as many as you can in 5 minutes!

$2^1 =$	$3^1 =$	$4^1 =$	$5^1 =$	$6^1 =$
$2^2 =$	$3^2 =$	$4^2 =$	$5^2 =$	$6^2 =$
$2^3 =$	$3^3 =$	$4^3 =$	$5^3 =$	$6^3 =$
$2^4 =$	$3^4 =$	$4^4 =$	$5^4 =$	$6^4 =$
$2^5 =$	$3^5 =$	$4^5 =$	$5^5 =$	$6^5 =$
$2^6 =$				
$2^7 =$	$7^1 =$	$8^1 =$	$9^1 =$	$10^1 =$
$2^8 =$	$7^2 =$	$8^2 =$	$9^2 =$	$10^2 =$
$2^9 =$	$7^3 =$	$8^3 =$	$9^3 =$	$10^3 =$
$2^{10} =$	$7^4 =$	$8^4 =$	$9^4 =$	$10^4 =$

$11^2 =$	$12^2 =$	$13^2 =$	$14^2 =$	$15^2 =$
$16^2 =$	$17^2 =$	$18^2 =$	$19^2 =$	$20^2 =$

Worked Example	Your Turn
Work out $\sqrt[4]{81}$	Work out $\sqrt[5]{32}$

2 Order of Operations

2.1 Commutativity

So far, we have studied three groups of operations.

	Multiplication	**Addition**	**Exponentiation**
Operation	$2 \times 3 = 6$ $2 \cdot 3 = 6$	$2 + 3 = 5$	$2^3 = 8$
Inverse Operation	$6 \div 3 = 2$ $\frac{6}{3} = 2$	$5 - 3 = 2$	$\sqrt[3]{8} = 2$

Commutativity

Which of the operations are commutative?

				Commutative?
Multiplication	**Multiplication**	$2 \cdot 3 = 6$	$2 \cdot 3 = 3 \cdot 2$	Yes
	Division	$\frac{6}{3} = 2$	$\frac{6}{3} \neq \frac{3}{6}$	No
Addition	**Addition**	$2 + 3 = 5$	$2 + 3 = 3 + 2$	Yes
	Subtraction	$5 - 3 = 2$	$5 - 3 \neq 3 - 5$	No
Exponentiation	**Exponents**	$2^2 = 8$	$2^3 \neq 3^2$	No
	Roots	$\sqrt[3]{8} = 2$	$\sqrt[3]{8} \neq \sqrt[8]{3}$	No

Notice how most operations are not commutative.
That means the order you write and work out matters.
It is only multiplication and addition where you can change the order of the inputs and not affect the output.

Fill in the Gaps

	Calculation	Order Reverse	Commutative?
e.g.	$5 \times 4 = 20$	$4 \times 5 = 20$	Yes
a	$12 \times 3 = 36$	$3 \times 12 =$	
b	$9 \cdot 7 =$		
c	$24 \div 6 = 4$	$6 \div 24 = 0.25$	
d	$\frac{3}{2} =$	$\frac{2}{3} =$	
e	$15 + 19 =$		
f	$20 - 15 = 5$	$15 - 20 = -5$	
g	$6.5 + 1.2 =$		
h	$14 - 8 =$		
i	$5^2 =$	$2^5 =$	
j	$\sqrt[2]{121}$	$\sqrt[121]{2}$	
k	$0.03 - 0.2 =$		
l	$\sqrt[3]{8} =$		
m		$3^4 =$	
n		$123 \cdot 19 =$	

2.2 Moving Numbers Around

What happens when we have more than two numbers in a calculation?

Which of these sums are the same?

$9 + 8 + 25$

$25 + 8 + 9$

$9 + 25 + 8$

$8 + 25 + 9$

What other sums would be the same?

Which of these differences are the same?

$30 - 4 - 10$

$30 - 10 - 4$

$10 - 30 - 4$

$4 - 10 - 30$

Why are the top two the same, but the bottom two different?

Key Words

summand + summand = sum
multiplier × multiplier = product

← same word, you can change the order: commutative

minuend – subtrahend = difference
dividend – divisor = quotient

← different words, you cannot change the order: not commutative

Subtraction

$12 - 3 - 5$

minuend subtrahends

$-3 - 5$

-8

$12 - 5 - 3$

minuend subtrahends

$-5 - 3$

-8

$$12 - 3 - 5 = 12 - 5 - 3$$

but

$$12 - 3 - 5 \neq 5 - 3 - 12$$

and

$$12 - 3 - 5 \neq 3 - 12 - 5$$

We can subtract in any order. What we can’t do is switch a subtrahend with a minuend.

Division

$23 \div 3 \div 2$

dividend divisors

$\div 3 \div 2$

$\div 6$

$23 \div 2 \div 3$

dividend divisors

$\div 2 \div 3$

$\div 6$

$24 \div 3 \div 2 = 24 \div 2 \div 3$

but

$24 \div 3 \div 2 \neq 2 \div 3 \div 24$

and

$24 \div 3 \div 2 \neq 3 \div 24 \div 2$

We can divide in any order. What we can't do is switch a divisor with a dividend.

Moving Numbers Around

When you have a mix of addition and subtraction, remember:

Addition	
$+$	Summands can move anywhere
$-$	Subtrahends can move as long as they are always behind a subtraction sign

When you have a mix of multiplication and division, remember:

Multiplication	
$\times$	Multipliers can move anywhere
$\div$	Divisors can move as long as they are always behind a division sign

Worked Example	Your Turn
Write down as many calculations as you can that are equivalent to these. a) $43 + 189 + 72$ b) $11 \times 17 \times 19$ c) $360 \div 9 \div 8$ d) $34 - 5 - 15.2$	Write down as many calculations as you can that are equivalent to these. a) $12 \times 23 \times 71$ b) $180 \div 10 \div 2$ c) $95 - 17 - 51$ d) $1.2 + 3.6 + 0.4$

Worked Example	Your Turn
Write down as many calculations as you can that are equivalent to these. a) $43 + 189 - 72 - 121 + 18$ b) $11 \div 4 \times 16 \times 3 \div 6$	Write down as many calculations as you can that are equivalent to these. a) $2 + 13 - 5 + 11 - 6$ b) $40 \div 10 \times 3 \div 6 \times 8$

Notation

Using better notation for × and ÷ can help us to see this more clearly.

Let's take some questions from the last page.

$11 \div 4 \times 16 \times 3 \div 6$

$= \frac{11 \cdot 16 \cdot 3}{4 \cdot 6}$ ⟵ anything multiplied on top, anything divided on bottom

$= \frac{16 \cdot 3 \cdot 11}{6 \cdot 4}$ ⟵ now we can change the order of things

See how it's clearer that dividing by 6 and 4 is the same as dividing by 24.

$40 \div 10 \times 3 \div 6 \times 8$

$= \frac{40 \cdot 3 \cdot 8}{10 \cdot 6}$ ⟵ anything multiplied on top, anything divided on bottom

$= \frac{3 \cdot 8 \cdot 40}{6 \cdot 10}$ ⟵ now we can change the order of things

See how it's clearer that dividing by 6 and 10 is the same as dividing by 60.

2.3 Mixing the Four Operations

In the last exercise, every question was either from the multiplication group or the addition group.

	Multiplication	**Addition**
Operation	$\times$	$+$
Inverse Operation	$\div$	$-$

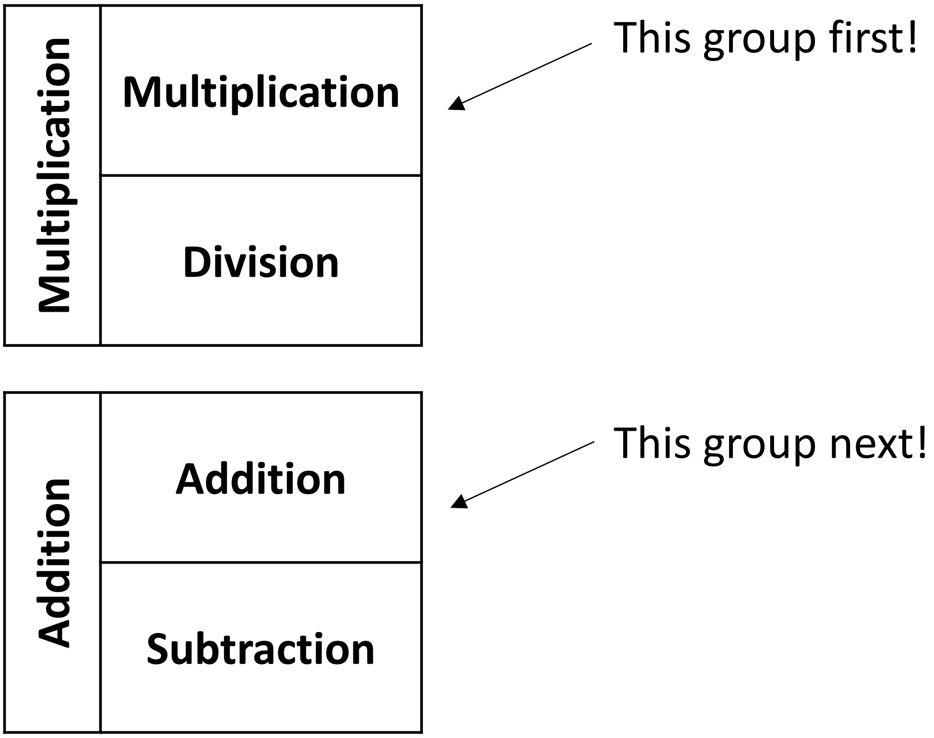
So, what happens if we have a mix of the multiplication and addition groups in the same calculation?

Mixing the Four Operations

If there is a mix of multiplication and addition, work out the multiplication first.

When we say "multiplication" we mean the multiplication group: all multiplication and division.

When we say "addition" we mean the addition group: all addition and subtraction.



When we solve calculations, we must look for the multiplication group first.

Think of + and − as <u>*separators*</u> between the multiplication groups.

Worked Example	Your Turn
Find the value of these calculations. a) $40 \div 8 - 2 \times 5 + 2 \times 11$ b) $20 \div 2 \times 3 + 4 \div 8 \times 2$	Find the value of these calculations. a) $25 \div 5 - 4 \times 10 + 3 \times 20$ b) $50 \div 10 \times 3 + 3 \div 9 \times 6$

Worked Example	Your Turn
Find the value of this calculation. $16 \div 4 \times 5 - 2 \times 50 \div 4 + 3 \times 5 \times 2$	Find the value of this calculation. $25 \div 5 \times 2 - 4 \times 10 \div 2 + 3 \times 2 \times 4$

2.4 Exponentiation

At the start of this unit, we looked at a third group of operations, which we called exponentiation.

It included exponents and their inverses, roots, which we learnt about in detail in the last unit.

	Exponentiation
Operation	
Inverse Operation	$\sqrt{}$

Where does exponentiation fit into the order of calculating?

Exponentiation

We have seen that multiplication should be worked out before addition.

Remember how a power comes from repeated multiplication:

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$

This means powers should be worked out before other multiplication.

In fact, all exponentiation should be worked out before other multiplication.

Exponentiation	**Exponents**
	Roots
Multiplication	**Multiplication**
	Division
Addition	**Addition**
	Subtraction

Worked Example	Your Turn
Find the value of these calculations. a) $2^3 + 5$ b) $15 - \sqrt{81}$ c) $10 \cdot 4^2$	Find the value of these calculations. a) $5^2 - 11$ b) $5 + \sqrt[3]{64}$ c) $2 \cdot \sqrt{49}$ d) $2 \cdot 6^2$

Worked Example	Your Turn
Find the value of these calculations. a) $3\times2^3 + 5\times3$ b) $\frac{66}{2} - \frac{\sqrt{36}}{2} + 2^4$	Find the value of these calculations. a) $\frac{32}{4^2} + 9^2 \cdot 2$ b) $\frac{\sqrt{144}}{2} + 8^2 - 2 \cdot 5^2$

2.5 Brackets

Breaking the Order

Work out the value of

$10 + 2 \times 3$

Of course, the answer is 16, because we multiply before adding.

But what if we *want* to add first?
How can we show that we want to add *before* multiplying?

We have a clever way of showing that we want to break the normal order.

$(10 + 2) \times 3$

By putting brackets ( ) around the addition, we mean
"break the order, do this first!"

Work out the new value.

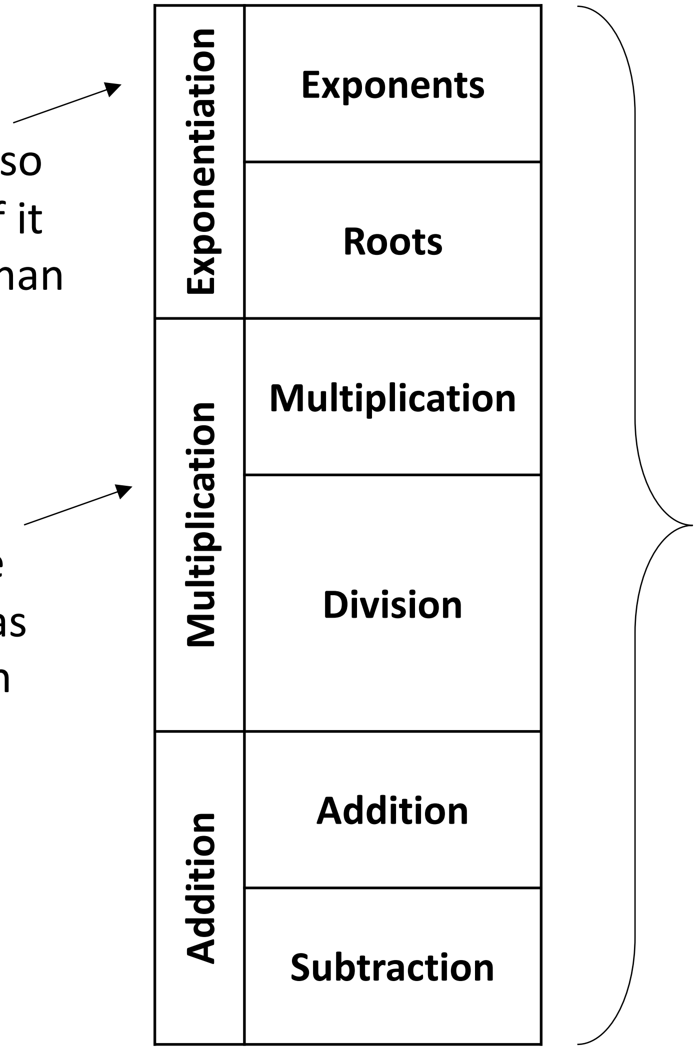
Worked Example	Your Turn
Find the value of these calculations. a) $3 \cdot 9 + 5$ b) $3 \cdot (9 + 5)$ c) $15 - 9 \div 3$ d) $(15 - 9) \div 3$	Find the value of these calculations. a) $12 \div 4 + 2$ b) $12 \div (4 + 2)$ c) $5 + 7 \cdot 2$ d) $(5 + 7) \cdot 2$

The Order of Operations

We now know the order in which we calculate.

This group is repeated multiplication, so we can think of it as “stronger” than multiplication.

This group is repeated addition, so we can think of it as “stronger” than addition.



Brackets break the order, so we must always look at them first.

The Order of Operations

Brackets break the order.

(Addition	Addition
	Subtraction)
Exponentiation	Exponents
	Roots
Multiplication	Multiplication
	Division

(Multiplication	Multiplication
	Division)
Exponentiation	Exponents
	Roots
Addition	Addition
	Subtraction

Multiplying a Bracket

We have seen questions that look like this: $2 \times (5 + 3)$
which can also be written easily like this: $2 \cdot (5 + 3)$

We normally don't even write the dot: $2(5 + 3)$

these all mean the same thing

There is no symbol between the 2 and the bracket.

No symbol still means 'multiply'.

We know that multiplication is commutative, so these two calculations have the same answer:

$2 \cdot (5 + 3)$ and $(5 + 3) \cdot 2$

When we remove the multiplication symbol, we always write the number in front of the bracket.

$2(5 + 3)$ not $(5 + 3)2$

The main reason for this is to avoid confusion with exponents: $(5 + 3)^2$

Worked Example	Your Turn
Find the value of these calculations. a) $3(9+5)$ b) $(15-9)3$	Find the value of these calculations. a) $12(4+2)$ b) $(5+7)2$

Hidden Brackets

Many calculations use hidden brackets.

Here is a division calculation: $(6+4) \div 2$

We can write it like this: $\frac{(6+4)}{2}$

We don't need to write the brackets: $\frac{6+4}{2}$

The long bar under the $6+4$ tells us there's a hidden bracket.

Be careful, though. It's different to this $\frac{6}{2}+4$ and this $6+\frac{4}{2}$

Many calculations use hidden brackets.

Here is a root calculation: $\sqrt{(4+5)}$

We don't need to write the brackets: $\sqrt{4+5}$

The long bar over the $4+5$ tells us there's a hidden bracket.

Be careful, though. It's different to this $\sqrt{4}+5$.

Many calculations use hidden brackets.

Here is an exponent calculation: $2^{(3+1)}$

We don't need to write the brackets: 2^{3+1}

The addition is clearly in the exponent, which tells us there's a hidden bracket.

Be careful, though. It's different to this 2^3+1.

Necessary or Unnecessary

Are the brackets necessary?
If you remove them and the answer stays the same, then they are not necessary.

a) $2 + (3 + 5)$

b) $(5 \times 2) \times 3$

c) $5 \times (2 + 3)$

d) $\frac{(2+3)}{5}$

e) $2^{(5+3)}$

f) $\sqrt{(5 + 3 + 1)}$

g) $\sqrt{3(5 - 2)}$

h) $5 - (3 - 2)$

Worked Example	Your Turn
Find the value of these calculations. a) $\sqrt{25}\times\frac{4+14}{2}$ b) $\sqrt{25\times 4}+\frac{14}{2}$	Find the value of these calculations. a) $\sqrt{4}\times\frac{9+15}{3}$ b) $\sqrt{4\times 9}+\frac{15}{3}$

Worked Example	Your Turn
Find the value of these calculations. a) $3(9+5)$ b) $(15-9)3$	Find the value of these calculations. a) $12(4+2)$ b) $(5+7)2$

Worked Example	Your Turn
Insert brackets to make the following calculations true:	Insert brackets to make the following calculations true:
a) $8 + 4 \times 5 - 2 = 20$	a) $7 + 3 \times 5 - 1 = 49$
b) $8 + 4 \times 5 - 2 = 58$	b) $7 + 3 \times 5 - 1 = 40$
c) $8 + 4 \times 5 - 2 = 26$	c) $7 + 3 \times 5 - 1 = 19$
d) $8 + 4 \times 5 - 2 = 36$	d) $7 + 3 \times 5 - 1 = 21$

3 Introduction to Algebra

3.1 Forming Expressions

Worked Example	Your Turn
Write an algebraic expression for each of the following:	Write an algebraic expression for each of the following:
3 more than a	3 less than a
5 less than a	a more than 5
b multiplied by a	b divided by a
b multiplied by a then squared	b divided by a then squared

Worked Example	Your Turn
Adam is x years old. Lucy is 15 years older than Adam. Write down an expression, in terms of x, for Lucy's age.	Albert is z years old. Laura is 3 times as old as Albert. Write down an expression, in terms of z, for Laura's age.

Worked Example	Your Turn
a) Ahmed is z years old. Libby is 3 times as old as Ahmed. John is 19 years older than Libby. Write down an expression, in terms of z, for John's age. b) Alfred has x stickers. Lottie has 11 less stickers than Alfred. John has 5 times as many stickers as Lottie. Write down an expression, in terms of x, for the number of stickers John has.	a) Adam has y cards. Latika has twice as many cards as Adam. Jack has 10 less cards than Latika. Write down an expression, in terms of y, for the number of cards Jack has. b) Alfie is z years old. Lottie is 15 years younger than Alfie. John is 3 times as old as Lottie. Write down an expression, in terms of z, for John's age.

3.2 Conventions and Definitions

The conventions include:

- We tend to use single lowercase letters for variables, either using the English alphabet or using the Greek alphabet.
- An algebraic x is written using two back-to-back c's. Do NOT write it as a $\times$ symbol.
- Do NOT include the multiplication sign, for example $3 \times p = 3p$
- Write division as fractions, for example $3 \div \mathrm{p} = \frac{3}{p}$
- Write numbers first in products, for example $p \times 3 = 3p$
- Write letters in products in alphabetical order, for example $4 \times q \times r \times p = 4pqr$
- $1x$ is written simply as x

The definitions include:

- **Variable** is a letter used to represent an unknown number.
- **Coefficient** is the number in front of a variable.
- **Constant** is a number that cannot change its value.
- **Term** is either a constant, a variable or a constant multiplied by a variable.
- **Expression** is terms and operators ($+$ and $-$) grouped together.

Worked Example	Your Turn
Write down the following for the expression: $2x - 4y - 9$ Variables: Coefficient of x: Coefficient of y: Constant: Terms:	Write down the following for the expression: $-2a + 4b + 9$ Variables: Coefficient of a: Coefficient of b: Constant: Terms:

Worked Example	Your Turn
Write down the following for the expression: $2x^2 - 4xy - 9$ Variables: Coefficient of x^2: Coefficient of xy: Constant: Terms:	Write down the following for the expression: $-2ab + 4b^2 + 9$ Variables: Coefficient of ab: Coefficient of b^2: Constant: Terms:

3.3 Collecting Like Terms without Powers

Frayer Model – Like Terms

Definition	**Characteristics**
Examples	**Non-Examples**

Fluency Practice

$3p$	p	Like	Unlike
x^2	$3x^2$	Like	Unlike
x^2	$2x$	Like	Unlike
$-3\sqrt{x}$	$27\sqrt{x}$	Like	Unlike
$7a$	$7b$	Like	Unlike

$3a$	$3a$	Like	Unlike
a	$2a$	Like	Unlike
$2a$	$2A$	Like	Unlike
$-3a$	$2a$	Like	Unlike
$4a$	$4b$	Like	Unlike
$3a$	$3a^2$	Like	Unlike
$2a^2$	$7a^2$	Like	Unlike
$-3a^2$	$7a^2$	Like	Unlike
$2a^2$	$2a^{-2}$	Like	Unlike
2^a	a^2	Like	Unlike
x	$\sqrt{x}$	Like	Unlike
1	2	Like	Unlike

Frayer Model – Expression

Definition	Characteristics
Examples	**Non-Examples**

Worked Example	Your Turn
Simplify: a) $5y - 3y - y$ b) $4q - 3q - 3q - 4q - 3q$	Simplify: a) $3x + x + 3x$ b) $4z + 5z - 5z - 3z - 2z$

Worked Example	Your Turn
Simplify: a) $6p + 9p + 4q + 7p$ b) $-7x + 5y - y - 6x$	Simplify: a) $3q + q + 6p + 4p$ b) $-p - 7p + 7p + 6q$

3.4 Collecting Like Terms with Powers

Worked Example	Your Turn
Simplify: a) $y^4 + y^2 + 3y^4 - 4q^4$ b) $5p^4 - 2x^4 - x^4 + 4x^4$	Simplify: a) $3z^4 + 3z^4 + 3z^3 + p^4$ b) $p^2 - 4z^3 - 5z^2 + 3z^3$

3.5 Algebraic Notation

Worked Example	Your Turn
Explain what the following mean: $7x$ xy xy^2 $(\mathrm{xy})^2$	Explain what the following mean: $7a$ ab ab^2 $(ab)^2$

3.6 Multiplying Terms without Powers

Worked Example	Your Turn
Simplify: a) $5p \times q$ b) $2p \times 8y$ c) $8z \times 7z$	Simplify: a) $p \times 5x$ b) $4x \times 4y$ c) $3z \times 2z$

3.7 Multiplying Terms with Powers

Worked Example	Your Turn
Simplify: a) $8x^4y^7 \times x^2y^4$ b) $7x^8y^3 \times 4x^8y^6$	Simplify: a) $x^8y \times 8x^5y^2$ b) $8x^2y^4 \times 6x^2y^6$

3.8 Dividing Terms without Powers

Worked Example	Your Turn
Simplify: a) $\frac{3x}{x}$ b) $\frac{3xy}{y}$	Simplify: a) $\frac{7y}{y}$ b) $\frac{7xy}{x}$

3.9 Dividing Terms with Powers

Worked Example	Your Turn
Simplify: a) $\frac{x^6y^6}{x^4y^4}$ b) $\frac{10x^8y^5}{5xy^3}$	Simplify: a) $\frac{x^6y^8}{x^2y^3}$ b) $\frac{9x^5y^5}{3x^3y}$

3.10 Substitution

Worked Example	Your Turn
a) Calculate $\frac{14}{y} + y^2$ when $y = 7$ b) Work out $\frac{4z+1}{4}$ when $z = 6$	a) Calculate $y^2 + 3y$ when $y = 2$ b) Work out $\frac{2z-1}{4}$ when $z = 1$

Worked Example	Your Turn
a) Evaluate $p^2 + 4q$ when $p = 6$ and $q = 7$	a) Evaluate $x^2 - 2y$ when $x = 10$ and $y = 1$
b) Work out $(2p + q)^2$ when $p = 8$ and $q = 10$	b) Work out $(4x + 3y)^2$ when $x = 1$ and $y = 3$

Worked Example	Your Turn
a) Evaluate $a^2 + \frac{-12}{b}$ when $a = -3$ and $b = -6$	a) Evaluate $\frac{-36}{x} + y^2$ when $x = -8$ and $y = -9$
b) Work out $p^2 - 2q$ when $p = -2$ and $q = -6$	b) Work out $p^2 - 4q$ when $\mathrm{p} = -8$ and $q = -2$