Year 8
Mathematics
Unit 1 – Student

Name: _____

Class: _____

Contents

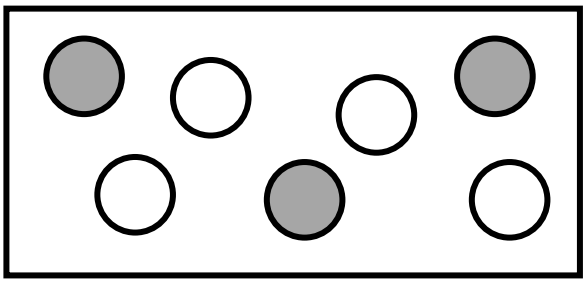
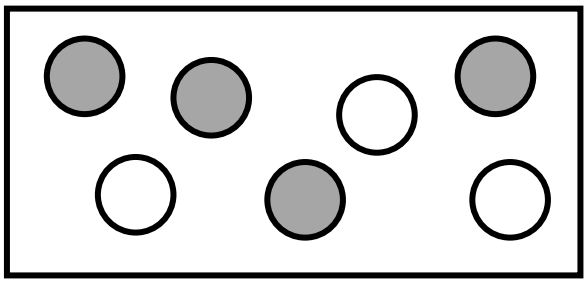
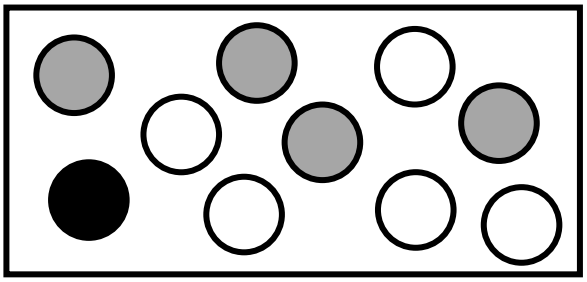
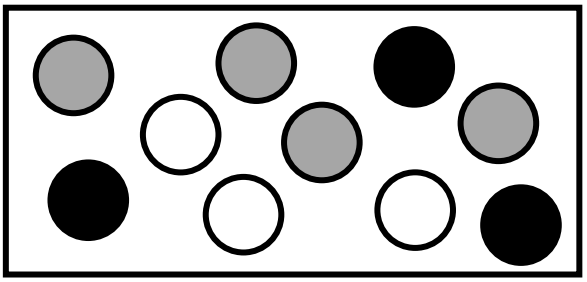
1 Ratio and Proportion

1.1 Writing Ratios

A ratio shows how much of one thing there is compared to another. Ratios are usually written in the form $a : b$.

If you are making orange squash and you mix one part orange to four parts water, then the ratio of orange to water will be $1 : 4$ (1 to 4).

The order in which a ratio is stated is important. Changing the order of the numbers in a ratio changes the proportions.

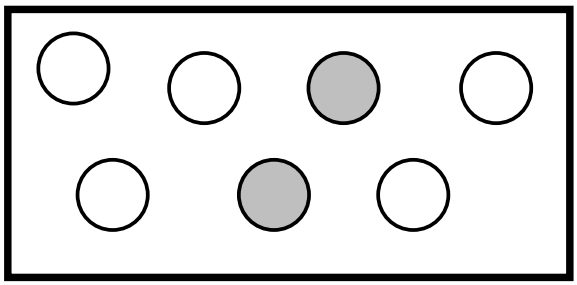
Worked Example	Your Turn
a) Write down the ratio of shaded circles to unshaded circles in the diagram below.	a) Write down the ratio of shaded circles to unshaded circles in the diagram below.
	
b) Write down the ratio of White : Grey : Black in the diagram below.	b) Write down the ratio of White : Grey : Black in the diagram below.
	

Intelligent Practice

For each of the diagrams below:

- **i** Write down the ratio of shaded shapes to unshaded shapes
- **ii** Write down the ratio of unshaded shapes to shaded shapes
- **iii** Write down the fraction of shaded shapes
- **iv** Write down the fraction of unshaded shapes

a



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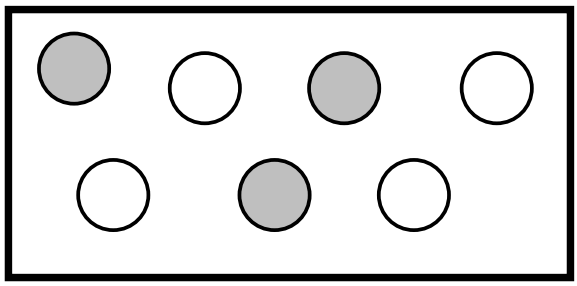

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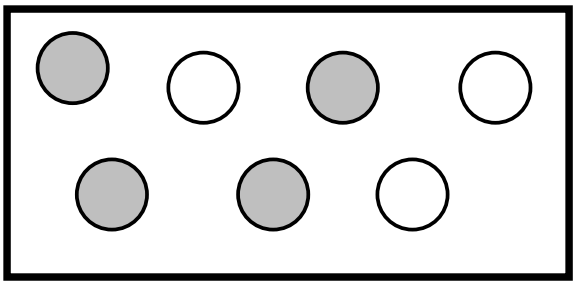

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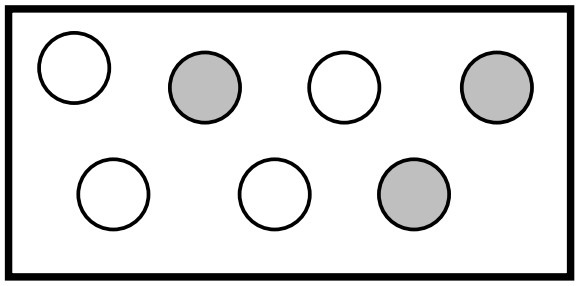

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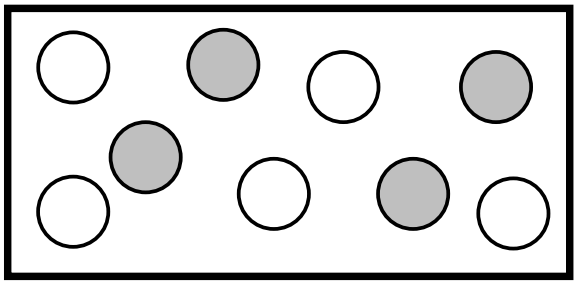

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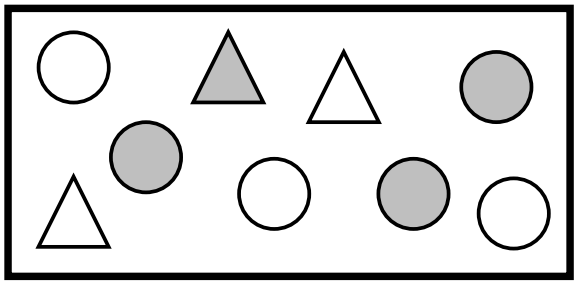

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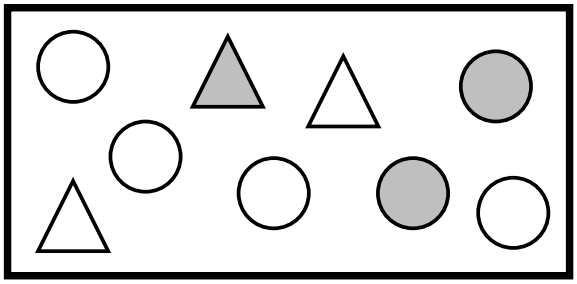

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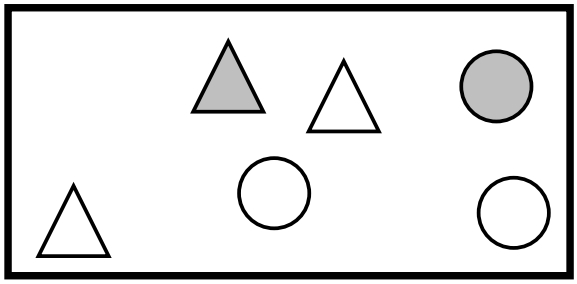


1.2 Ratios to Fractions

Ratios can be used to show fractions and proportions of amounts.

Worked Example	Your Turn
The ratio of red balls to green balls in a bag is $2:3$. What fraction of the balls are red?	The ratio of red balls to green balls in a bag is $2:5$. What fraction of the balls are red?

Fluency Practice

Question 1: The ratio of red apples to green apples in a basket is 1:2

(a) What fraction of the apples in the basket are red?
(b) What fraction of the apples in the basket are green?

Question 2: The ratio of blue pens to black pens in a box is 3:5

(a) What fraction of the pens are black?
(b) What fraction of the pens are blue?

Question 3: A farmer keeps cows and pigs on his farm.
The ratio of cows to pigs on the farm is 2:3

(a) What percentage of the animals on the farm are cows?
(b) What percentage of the animals on the farm are pigs?

Question 4: Olivia has blue, pink and orange counters in a bag.
The ratio of blue to pink to orange counters is 4:5:2

(a) What fraction of the counters are blue?
(b) What fraction of the counters are orange?

Question 5: Chris makes a drink by mixing lemonade and orange juice in the ratio 13:7

(a) What percentage of the drink is lemonade?
(b) What percentage of the drink is orange juice?

Question 6: There are white, red and yellow counters in a box.
The number of white counters, the number of red counters and the number of yellow counters are in the ratio 2:7:16

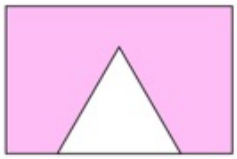
What percentage of the counters in the box are red?

Question 7: Cards in a pack are either orange or purple.
80% of the cards are orange.
Write the ratio of orange cards to purple cards.

Fluency Practice

Question 8: The counters in a bag are red or yellow.
30% of the counters in the bag are red.
Write the ratio of yellow counters to red counters.

Question 9: Chris designs a flag.
20% of the flag is white and the rest is pink.
What is the ratio of white to pink?



Question 10: 14% of the students in a class are left handed.
Write down the ratio of left handed to right handed students.

Question 11: $\frac{1}{3}$ of the beads in a bag are white.

The rest of the beads are grey.

(a) Write down the ratio of white beads to grey beads.
(b) Write down the ratio of grey beads to white beads.

Question 12: An American football team won a sixth of their matches.
They lost the rest.
Work out the ratio matches won : matches lost

Question 13: $\frac{3}{4}$ of the apples in a bag are red.

Write down the ratio of red apples to green apples.

Question 14: $\frac{7}{15}$ of the buses arriving in a town are late.

Write down the ratio of on time buses to late buses.

Intelligent Practice

1) The ratio of red balls to green balls in a bag is $1:3$. What fraction of the balls are red?

2) The ratio of red balls to green balls in a bag is $1:3$. What fraction of the balls are green?

3) The ratio of red balls to green balls in a bag is $3:1$. What fraction of the balls are green?

4) The ratio of red balls to green balls in a bag is $4:1$. What fraction of the balls are green?

5) The ratio of red balls to green balls in a bag is $4:1$. What fraction of the balls are red?

6) The ratio of red balls to green balls in a bag is $8:1$. What fraction of the balls are red?

7) The ratio of red balls to green balls in a bag is $2:1$. What fraction of the balls are red?

8) The ratio of red balls to green balls in a bag is $4:2$. What fraction of the balls are red?

9) The ratio of red balls to green balls in a bag is $4:4$. What fraction of the balls are red?

10) The ratio of red balls to green balls in a bag is $1:1$. What fraction of the balls are red?

11) The ratio of red balls to green balls in a bag is $1:1$. What fraction of the balls are green?

12) The ratio of red balls to green balls in a bag is $1:5$. What fraction of the balls are green?

13) The ratio of red balls to green balls to blue balls in a bag is $1:5:2$. What fraction of the balls are green?

14) The ratio of red balls to green balls to blue balls in a bag is $1:5:2$. What fraction of the balls are red?

15) The ratio of red balls to green balls to blue balls in a bag is $3:5:2$. What fraction of the balls are red?

16) The ratio of red balls to green balls to blue balls in a bag is $9:15:6$. What fraction of the balls are red?

Extension

Question 1: Bethany and Summer are waitresses.
They share the tips in the ratio of the hours they have worked.
Bethany worked from 11am until 5pm.
Summer worked from 1pm until 9pm

What fraction of the tips does Bethany keep?

Question 2: Oscar and Theo collect coins and stamps.
Altogether they have the same number of coins as stamps.

The ratio of coins Oscar has to coins Theo has is 3:7
The ratio of stamps Oscar has to stamps Theo has is 1:4

Show Theo has more stamps than coins.

1.3 Equivalent Ratios

If you are making orange squash and you mix one part orange to four parts water, then the ratio of orange to water will be $1 : 4$ (1 to 4).

If you use 1 litre of orange, you will use 4 litres of water ($1 : 4$).
If you use 2 litres of orange, you will use 8 litres of water ($2 : 8$).
If you use 10 litres of orange, you will use 40 litres of water ($10 : 40$).

These ratios are all equivalent:
$1 : 4 = 2 : 8 = 10 : 40$

Both sides of the ratio can be multiplied or divided by the same number to give an equivalent ratio.

Worked Example	Your Turn
All of the ratios below are equivalent. Complete the gaps below: $1 : 3$ $\_\_ : 6$ $\_\_ : 12$ $24 : \_\_$ $\_\_ : 36$ $\_\_ : 3.6$	All of the ratios below are equivalent. Complete the gaps below: $1 : 4$ $\_\_ : 8$ $\_\_ : 16$ $12 : \_\_$ $\_\_ : 12$ $\_\_ : 1.2$

Worked Example	Your Turn
All of the ratios below are equivalent. Complete the gaps below: $2 : 3$ $\_\_ : 9$ $\_\_ : 18$ $24 : \_\_$ $\_\_ : 54$ $\_\_ : 0.54$	All of the ratios below are equivalent. Complete the gaps below: $2 : 5$ $\_\_ : 15$ $\_\_ : 30$ $24 : \_\_$ $\_\_ : 0.6$ $\_\_ : 4.8$

Worked Example	Your Turn
All of the ratios below are equivalent. Complete the gaps below: $3 : 2 : 4$ $\_\_ : 4 : \_\_$ $\_\_ : 8 : \_\_$ $24 : \_\_ : \_\_$ $2.4 : \_\_ : \_\_$	All of the ratios below are equivalent. Complete the gaps below: $3 : 2 : 5$ $\_\_ : 4 : \_\_$ $\_\_ : 8 : \_\_$ $24 : \_\_ : \_\_$ $2.4 : \_\_ : \_\_$

Equivalent Ratios

pair off the equivalent ratios

(1)

5 : 20

10 : 25

1½ : 2½

9 : 12

3 : 12

6 : 7½

24 : 40

3 : 7½

15 : 20

20 : 25

(2)

10½ : 7

2½ : 1

2 : 1½

½ : 0.3

1½ : 1

12½ : 7½

7½ : 3

10 : 7½

(3)

27 : 72

28 : 63

24 : 84

66 : 121

24 : 64

42 : 77

24 : 54

16 : 56

(4)

96 : 88

75 : 70

81 : 72

98 : 91

132 : 121

108 : 96

70 : 65

90 : 84

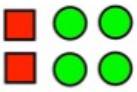
1.4 Simplifying Ratios

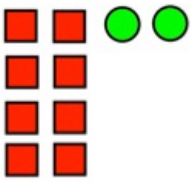
Ratios can be fully simplified just like fractions. To simplify a ratio, divide all of the numbers in the ratio by the same number until they cannot be divided any more.

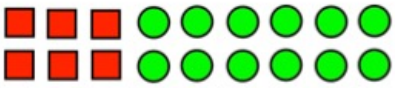
Worked Example	Your Turn
Simplify: a) $25 : 30$ b) $45 : 75$ c) $15 : 20 : 35$ d) $150\text{cm} : 1\text{m}$	Simplify: a) $42 : 35$ b) $24 : 60$ c) $16 : 32 : 72$ d) $450\text{g} : 1.3\text{kg}$

Fluency Practice

Question 1: For each of the following, write down the ratio of red squares to green circles. Give your ratios in their simplest forms.

(a) 

(b) 

(c) 

Question 2: Simplify the following ratios

(a) 4 : 6	(b) 14 : 8	(c) 15 : 10	(d) 6 : 15
(e) 30 : 10	(f) 12 : 16	(g) 6 : 18	(h) 45 : 10
(i) 12 : 28	(j) 24 : 36	(k) 25 : 60	(l) 27 : 63
(m) 48 : 60	(n) 120 : 260	(o) 8000 : 75	(p) 33 : 121
(q) 2.5 : 4.5	(r) 1.5 : 20	(s) 6 : 1.2	(t) 2.25 : 4.95

Question 3: Write the following as ratios in their simplest forms.

(a) £4 to £20	(b) 240cm to 400cm	(c) 50 minutes to 20 minutes
(d) 60kg to 72kg	(e) 12 miles to 30 miles	(f) 15cm to 75cm
(g) 8.5g to 3.5g	(h) £0.50 to £20	(i) 1.02 litres to 0.74 litres

Question 4: Write the following as ratios in their simplest forms.

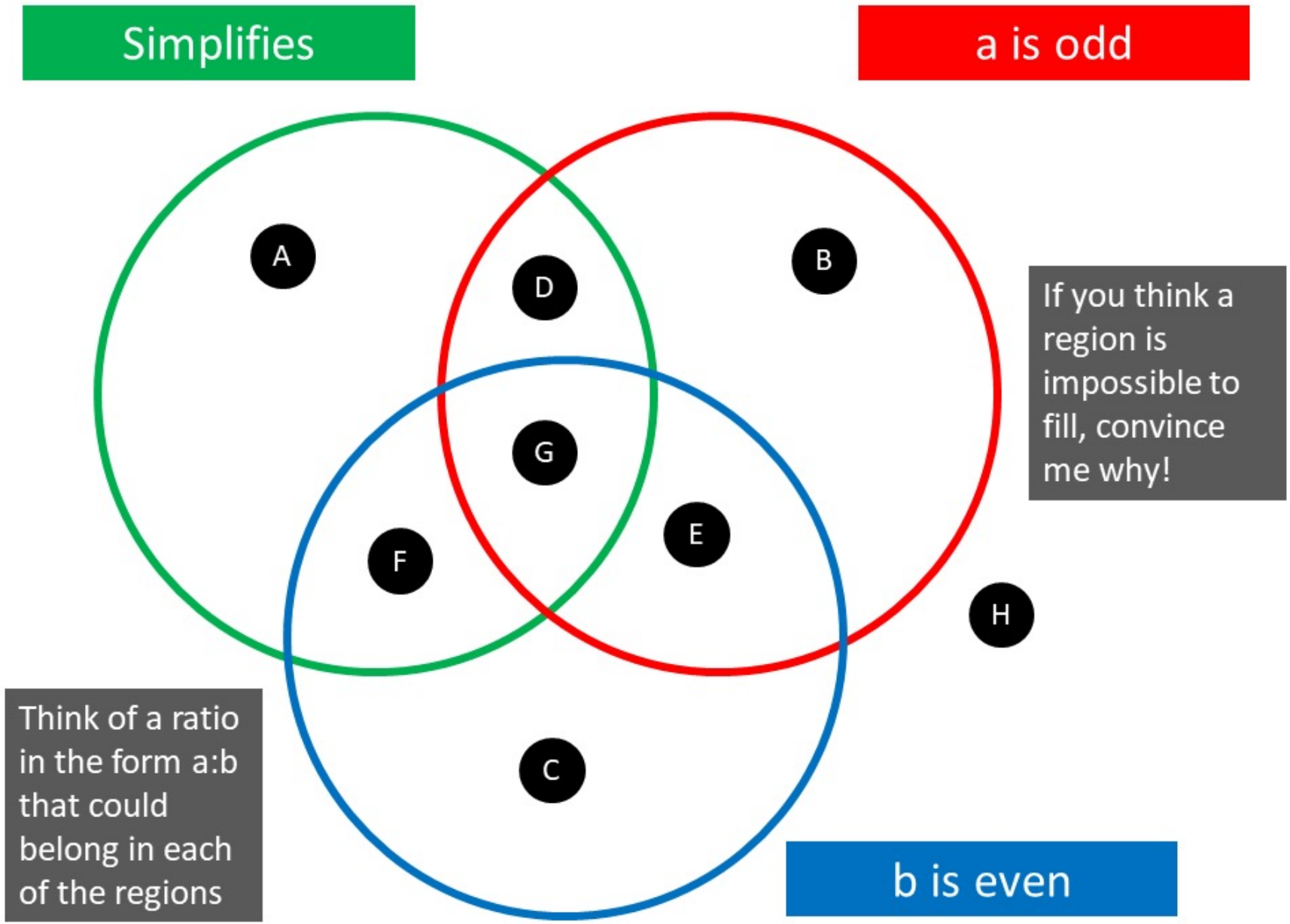
(a) 8 days to 2 weeks	(b) 1 hour to 15 minutes	(c) 2 hours to 1 day
(d) 95p to £3.00	(e) 400m to 1.5km	(f) 15kg to 900g
(g) 4500ml to 2 litres	(h) 8km to 50mm	(i) 90 minutes to 2 days

Intelligent Practice

Simplify:

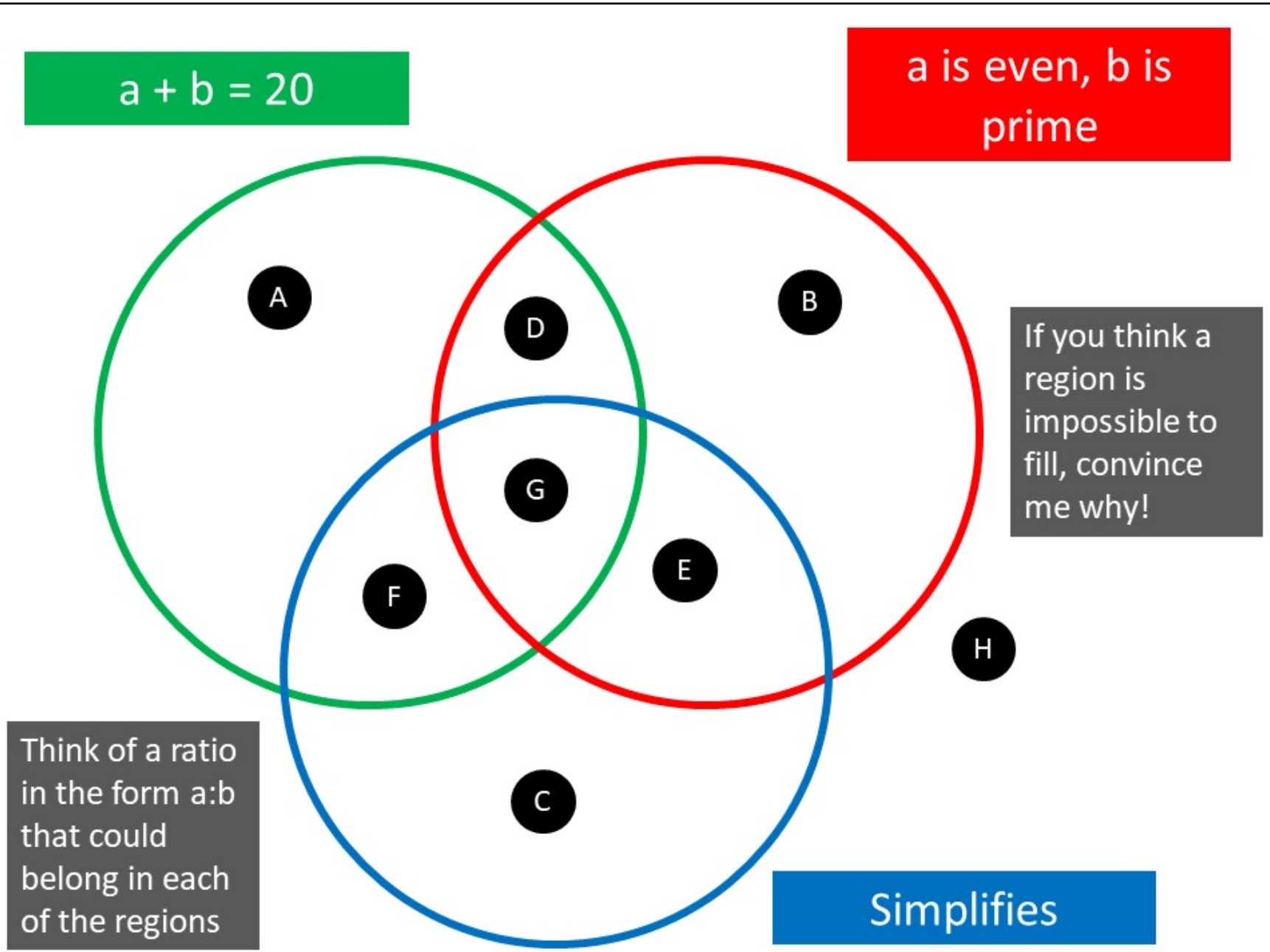
1) $10 : 16$
2) $16 : 10$
3) $8 : 10$
4) $4 : 5$
5) $4.5 : 5$
6) $32 : 24$
7) $32 : 48$
8) $64 : 96$
9) $64 : 96 : 20$
10) $128 : 96 : 40$
11) $\frac{10}{16}$
12) $50\text{p} : £1.50$
13) $\frac{8}{10}$
14) $\frac{16}{20}$
15) $\frac{32}{20}$
16) $50\text{ cm} : 1.5\text{ m}$
17) $60\text{ minutes} : 1\text{ hour}$
18) $\frac{64}{40}$
19) $600\text{ seconds} : 2.5\text{ minutes}$
20) $28 : 42 : 28$
21) $10a : 16a$
22) $10a : 16b$
23) $10a^2 : 16a$
24) $50\text{p} : £2.70$
25) $32ab : 16bc$
26) $32ba : 16cb$
27) $1.5\text{ km} : 400\text{ m}$
28) $30a^2b : 18b^2a$
29) $18a^3b^2 : 24b^4a^2$
30) $18a^3b^2 : 24b^4a^2 : 21ac$

Maths Venns


Simplifies
a is odd
A
D
B
G
E
F
H
C
b is even
If you think a region is impossible to fill, convince me why!
Think of a ratio in the form a:b that could belong in each of the regions

Maths Venns

a is prime
If you think a region is impossible to fill, convince me why!
b is a square number
Simplifies
Think of a ratio in the form a:b that could belong in each of the regions
A
B
C
D
E
F
G
H


a + b = 20
a is even, b is prime
A
D
B
G
E
F
H
C
If you think a region is impossible to fill, convince me why!
Think of a ratio in the form a:b that could belong in each of the regions
Simplifies

Maths Venns

b is odd
If you think a region is impossible to fill, convince me why!
a as a fraction of the whole simplifies
H
B
E
D
G
C
F
A
a is prime
Think of a ratio in the form $a:b$ that could belong in each of the regions

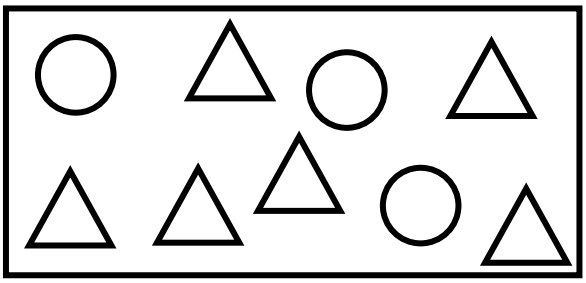
1.5 n:1 and 1:n Ratios

In order that a ratio is written in the form $1 : n$ we must make the left hand side equal to one.

In order that a ratio is written in the form $n : 1$ we must make the right hand side equal to one.

Worked Example

The diagram below shows a number of circles and triangles.

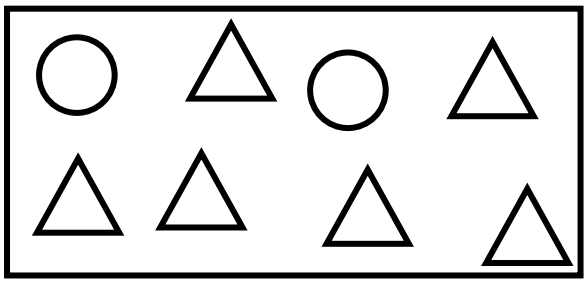


a) Write the ratio of circles to triangles in the ratio $1 : n$.

b) Write the ratio of circles to triangles in the ratio $n : 1$.

Your Turn

The diagram below shows a number of circles and triangles.



a) Write the ratio of circles to triangles in the ratio $1 : n$.

b) Write the ratio of circles to triangles in the ratio $n : 1$.

Worked Example	Your Turn
a) Write the ratio $2 : 5$ in the ratio $1 : n$.	a) Write the ratio $4 : 5$ in the ratio $1 : n$.
b) Write the ratio $2 : 5$ in the ratio $n : 1$.	b) Write the ratio $4 : 5$ in the ratio $n : 1$.

Fluency Practice

Question 5: Express each of the following ratios in the form 1 : n

(a) 2 : 3 (b) 5 : 4 (c) 4 : 10 (d) 10 : 7

(e) 8 : 13 (f) 5 : 81 (g) 100 : 131 (h) 200 : 77

(i) 25 : 29 (j) 21 : 40

Question 6: Express each of the following ratios in the form n : 1

(a) 7 : 2 (b) 9 : 5 (c) 11 : 3 (d) 5 : 8

(e) 3 : 10 (f) 19 : 20 (g) 207 : 50 (h) 38 : 55

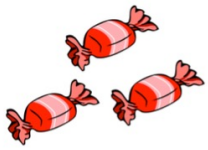
Do not round!

Extension

Question 1: Daisy mixes 50 ml of orange juice with 200 ml of water.
Write down the ratio of orange juice to water.
Give your answer in its simplest form.

Question 2: At a football match, there are 3000 men and 1800 women.
Write down the ratio of male fans to female fans.
Give your answer in its simplest form.

Question 3: Aidan, Bill and Cara share sweets in the ratio of their ages.
Aidan is 12 years old.
Bill is 9 years old.
Cara is 3 years old.
Write down the ratio of their ages.
Give your answer in its simplest form.



Question 4: In a nursery, there are 5 adults and 14 children.
Write the ratio of adults to children in the form 1 : n

Question 5: Ellie is making a cake.
The instructions say that the ratio of sugar to flour should be 1 : 3
Ellie uses 250g of sugar and 650g of flour.
Has Ellie used the correct ratio of sugar to flour?

Question 6: Shannon is revising for her summer exams.
The table below shows the number of minutes Shannon spends revising on each of 5 evenings.
It also shows the number of minutes Shannon spends relaxing on the 5 evenings.

	Monday	Tuesday	Wednesday	Thursday	Friday
Number of minutes revising	88	198	150	133	160
Number of minutes relaxing	20	40	28	25	34

Shannon wants to spend at least 5 minutes revising for every 1 minute of relaxing.
On which days did Shannon spend enough time revising?

Question 7: Four teachers are planning school trips.
The table shows the number of students and the number of teachers planned to go on the trip.

	Karting	Museum	Theme Park	University
Number of students	140	221	342	159
Number of teachers	8	12	19	9

For every 18 students there must be at least 1 teacher.
Which trips have planned to bring enough teachers?

Maths Venns

b:c could be simplified
If you think a region is impossible to fill, convince me why!
a:c could be simplified
H
B
E
D
G
C
F
A
a:b could be simplified
Think of a ratio in the form a:b:c that could belong in each of the regions

1.6 Ratio in Different Forms

Worked Example	Your Turn
$a : b$ $7 : 1$	$a : b$ $8 : 1$
a as a fraction of the whole:	a as a fraction of the whole:
a as a fraction of b:	a as a fraction of b:
In the form $1 : n$	In the form $1 : n$
In the form $n : 1$	In the form $n : 1$

Fill in the Gaps

Ratio $a : b$	a as a fraction of the whole	a as a fraction of b	In the form $1 : n$	In the form $n : 1$
$1 : 3$				
	$\frac{1}{3}$			
		$\frac{2}{5}$		
			$1 : 5$	
$5 : 1$				
	$\frac{5}{7}$			
		$\frac{5}{7}$		
			$1 : 0.7$	
				$1\frac{4}{7} : 1$
				$\frac{7}{11} : 1$
$x : y$				

1.7 Combining Ratios

Sometimes it will be the case that we are given two ratios and asked to combine the ratios to make one ratio. We are only able to combine ratios if one of the components is present in both of the ratios. If this is the case, we can combine the ratios by making the value for the component that appears in both of the ratios the same. We make the value for the component that appears in both of the ratios the lowest common multiple of the current values for that component in the two separate ratios. After we have made the values for the component that appears in both of the ratios the same, we can combine the ratios.

Worked Example	Your Turn
The ratio of $a : b$ is $2 : 3$. The ratio of $b : c$ is $1 : 4$. What is the ratio of $a : c$?	The ratio of $a : b$ is $2 : 5$. The ratio of $b : c$ is $1 : 4$. What is the ratio of $a : c$?

Intelligent Practice

1) The ratio of $a : b$ is $1 : 2$. The ratio of $b : c$ is $2 : 3$. What is the ratio of $a : c$?

2) The ratio of $a : b$ is $1 : 2$. The ratio of $b : c$ is $2 : 4$. What is the ratio of $a : c$?

3) The ratio of $a : b$ is $3 : 2$. The ratio of $b : c$ is $2 : 4$. What is the ratio of $a : c$?

4) The ratio of $a : b$ is $4 : 2$. The ratio of $b : c$ is $2 : 3$. What is the ratio of $a : c$?

5) The ratio of $a : b$ is $4 : 1$. The ratio of $b : c$ is $1 : 3$. What is the ratio of $a : c$?

6) The ratio of $a : b$ is $4 : 6$. The ratio of $b : c$ is $6 : 3$. What is the ratio of $a : c$?

7) The ratio of $a : b$ is $4 : 3$. The ratio of $b : c$ is $6 : 3$. What is the ratio of $a : c$?

8) The ratio of $a : b$ is $4 : 2$. The ratio of $b : c$ is $6 : 3$. What is the ratio of $a : c$?

9) The ratio of $a : b$ is $4 : 1$. The ratio of $b : c$ is $6 : 3$. What is the ratio of $a : c$?

10) The ratio of $a : b$ is $4 : 1$. The ratio of $b : c$ is $5 : 3$. What is the ratio of $a : c$?

11) The ratio of $a : b$ is $4 : 2$. The ratio of $b : c$ is $5 : 3$. What is the ratio of $a : c$?

12) The ratio of $a : b$ is $4 : 3$. The ratio of $b : c$ is $5 : 3$. What is the ratio of $a : c$?

13) The ratio of $a : b$ is $4 : 6$. The ratio of $b : c$ is $10 : 3$. What is the ratio of $a : c$?

14) The ratio of $a : b$ is $4 : 4$. The ratio of $b : c$ is $10 : 3$. What is the ratio of $a : c$?

15) The ratio of $a : b$ is $4 : 2$. The ratio of $b : c$ is $5 : 3$. What is the ratio of $a : c$?

16) The ratio of $a : b$ is $3 : 2$. The ratio of $b : c$ is $5 : 3$. What is the ratio of $a : c$?

17) The ratio of $a : b$ is $3 : 5$. The ratio of $b : c$ is $2 : 3$. What is the ratio of $a : c$?

18) The ratio of $a : b$ is $9 : 5$. The ratio of $b : c$ is $2 : 9$. What is the ratio of $a : c$?

Worked Example

There are red, yellow and blue counters in a bag. Find the ratio Red : Yellow : Blue if

(a) The ratio of Red : Yellow is $1 : 2$ and the ratio of Yellow : Blue is $2 : 3$.

(b) The ratio of Red : Yellow is $1 : 5$ and the ratio of Yellow : Blue is $10 : 7$.

(c) The ratio of Red : Yellow is $1 : 3$ and the ratio of Yellow : Blue is $8 : 5$.

Your Turn

There are red, yellow and blue counters in a bag. Find the ratio Red : Yellow : Blue if

(a) The ratio of Red : Yellow is $1 : 3$ and the ratio of Yellow : Blue is $3 : 4$.

(b) The ratio of Red : Yellow is $2 : 5$ and the ratio of Yellow : Blue is $10 : 3$.

(c) The ratio of Red : Yellow is $2 : 5$ and the ratio of Yellow : Blue is $7 : 1$.

Fluency Practice

Question 1: In a bag there are blue, green and yellow counters.

The ratio of blue counters to green counters is 3:2
The ratio of green counters to yellow counters is 2:5

(a) Write down the ratio of blue to green to yellow counters in the bag.
(b) What percentage of the beads are green?

Question 2: Archie made some cupcakes for a charity coffee morning.

The ratio of chocolate cupcakes to strawberry cupcakes was 3:1
The ratio of strawberry cupcakes to lemon cupcakes was 2:3

(a) Write down the ratio of chocolate to strawberry to lemon cupcakes.
(b) Work out the smallest possible number of cupcakes that Archie could have made.

Question 3: At a safari park, the ratio of lions to tigers is 7:4.
The ratio of elephants to tigers is 1:2

Write down the ratio of lions to tigers to elephants in the safari park.

Question 4: A bag contains three different shaped pieces of card.

The ratio of circles to triangles is 2:3
The ratio of triangles to rectangles is 2:5

Find the ratio of circles to triangles to rectangles.

Question 5: In a school, all students are taught either French, German or Spanish.

The ratio of the number of students taught French to those taught German is 3:4
The ratio of the number of students taught French to taught Spanish is 12:11

Find the ratio of the number of students taught Spanish to taught German.

Question 6: In a box there are white chocolates, milk chocolates and dark chocolates.

The ratio of white chocolates to milk chocolates is 3:5
The ratio of milk chocolates to dark chocolates is 8:1

What fraction of the chocolates are white chocolate?

Intelligent Practice

1) The ratio of red to blue is $2:3$, the ratio of blue to green is $3:5$, what is the ratio of red to green?

2) The ratio of red to blue is $2:3$, the ratio of blue to green is $6:10$, what is the ratio of red to green?

3) The ratio of red to blue is $4:6$, the ratio of blue to green is $6:10$, what is the ratio of red to green?

4) The ratio of red to blue is $8:12$, the ratio of blue to green is $6:10$, what is the ratio of red to green?

5) The ratio of red to blue is $9:12$, the ratio of blue to green is $6:10$, what is the ratio of red to green?

6) The ratio of red to blue is $12:9$, the ratio of blue to green is $6:10$, what is the ratio of red to green?

7) The ratio of red to blue is $12:10$, the ratio of blue to green is $6:10$, what is the ratio of red to green?

8) The ratio of red to blue is $12:10$, the ratio of blue to green is $15:10$, what is the ratio of red to green?

9) The ratio of red to blue is $120:100$, the ratio of blue to green is $150:100$, what is the ratio of red to green?

10) The ratio of red to blue is $\frac{1}{2}:\frac{1}{3}$, the ratio of blue to green is $\frac{1}{3}:\frac{1}{4}$, what is the ratio of red to green?

11) The ratio of red to blue is $\frac{1}{2}:\frac{1}{3}$, the ratio of blue to green is $\frac{1}{2}:\frac{1}{4}$, what is the ratio of red to green?

12) The ratio of red to blue is $x:y$, the ratio of blue to green is $y:z$, what is the ratio of red to green?

13) The ratio of red to blue is $x:y$, the ratio of blue to green is $3y:4z$, what is the ratio of red to green?

14) The ratio of red to blue is $x:19$, the ratio of blue to green is $17:y$, what is the ratio of red to green?

15) If y and z are prime, what is the ratio of red to blue is $x:y$, the ratio of blue to green is $z:w$, what is the ratio of red to green?

Extension

Question 1: In a drawer, there are white, black and grey socks.
The ratio of white socks to black socks is 3:2
The ratio of white socks to grey socks is 9:4

(a) Write down the ratio of white socks to black socks to grey socks.

Elsie says there is an odd white sock.

(b) Explain why Elsie might be wrong.

Question 2: The ratio of red pens to black pens is 2:9
The ratio of black pens to blue pens is 5:4

Show less than 50% of the pens are black.

Question 3: A quadrilateral, ABCD, is drawn.

The ratio of the size of angle A to angle B is 1:3
The ratio of the size of angle B to angle D is 5:3
The ratio of the size of angle C to angle A is 7:5

Find the difference in size between the largest and smallest angles in quadrilateral ABCD.

Question 4: The ratio of Scott's age to Georgia's age to Fiona's age is 11:6:7
The ratio of Oscar's age to Georgia's age is 3:4

Find the ratio of Fiona's age to Oscar's age.

Question 5: Given $4x = 3y$ and $y : z = 1 : 2$

Find x in terms of z

Question 6: w is 15% of x

y is $\frac{3}{5}$ of x

Find the ratio w:x:y

1.8 One Part Given

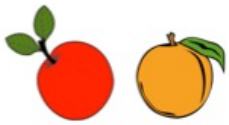
Worked Example	Your Turn
Anju and Kieran share some money in the ratio $5:2$. Anju receives £30. How much does Kieran receive?	Anju and Kieran share some money in the ratio $5:3$. Anju receives £30. How much does Kieran receive?

Fluency Practice

Question 1: A drawer contains white socks and black socks only.
The number of white socks to the number of black socks is in the ratio 1:3
There are 12 white socks.

(a) Work out the number of black socks in the drawer.
(b) Work out the total number of socks in the drawer.

Question 2: James has some apples and oranges.
The ratio of apples and oranges is 2:5
He has 15 oranges.
How many apples does James have?



Question 3: The ratio of lemon sweets to strawberry sweets in a tub is 5:3
There are 120 lemon sweets in the tub.
How many strawberry sweets are in the tub?

Question 4: Rachel has some first class and some second class stamps.
The ratio of the number of first class to the number of second class stamps is 3:4
Rachel has 18 first class stamps.

(a) How many second class stamps does Rachel have?
(b) How many stamps does Rachel have in total?

Question 5: Abby, Neil and Dylan share a sum of money in the ratio 2:4:5
Neil receives £60
Work out how much money Dylan receives.

Question 6: The ratio of the number of girls to the number of boys in a school is 9:10
There are 900 boys in the school.

Work out the total number of students in the school.

Question 7: Flour, sugar and butter are mixed in the ratio 6:2:3
How many grams of flour and sugar are needed to mix with 180g of butter?

Intelligent Practice

1) Claire and John share some money in the ratio $3:2$. Claire receives £18. How much does John receive?

2) Claire and John share some money in the ratio $6:4$. Claire receives £18. How much does John receive?

3) Claire and John share some money in the ratio $3:2$. Claire receives £36. How much does John receive?

4) Sandy and Mark share some sweets in the ratio $1:4$. Mark receives 48 sweets. How many does Sandy receive?

5) Sandy and Mark share some sweets in the ratio $1:9$. Mark receives 18 sweets. How many does Sandy receive?

6) George and Joe share some sweets in the ratio $1:9$. Joe receives 18 sweets. How many sweets are shared in total?

7) George and Joe share some sweets in the ratio $7:9$. Joe receives 18 sweets. How sweets are shared in total?

8) Huda, Nicole and Bethan share some sweets in the ratio $7:8:9$. Huda receives 14 sweets. How many sweets do Nicole and Bethan each receive?

9) A piece of wood is split into three pieces into the ratio $7:8:9$. The smallest piece of wood is 21cm. Work out the length of the other two pieces.

10) A piece of wood is split into three pieces into the ratio $3:6:5$. The smallest piece of wood is 21m. Work out the total length of the wood.

11) Three angles are in the ratio $3:6:5$. The largest angle is $72°$. Could these be the angles of a triangle?

12) Four angles are split in the ratio $3:6:5:2$. The smallest angle is $45°$. Could these four angles meet at a point?

Extension

Question 1: Four angles are in the ratio 2:3:4:11
The largest angle is 198°
Show the four angles will fit together at a point with no gaps.

Question 2: Matthew makes a drink using lemonade and orange juice.
210ml of his drink was lemonade
140ml of his drink was orange juice
Rosie makes more of the drink for a party using the same ratio of lemonade to orange juice.
Rosie uses 6 litres of lemonade.

How much orange juice does Rosie use?

Question 3: There are red and green apples in a crate.
There are 60 green apples in the crate.
The ratio of the number of red apples to green apples is 1:5

Georgia puts in some more red apples into the crate.
The ratio of the number of red apples to green apples is now 2:3

How many red apples does Georgia put into the crate?

Question 4: Olive has 600 fruit trees.
Some of the trees are apple trees.
The rest of the trees are pears trees and plum trees in the ratio 7:5.

There are 175 plum trees.

Work out what fraction of the trees are apple trees.

Question 5: Two numbers are in the ratio 4:3
One of the numbers is 1.8
What are the two possible values for the other number?

Question 6: Jason, Katie and Leonard share some money.
Jason gets $\frac{1}{5}$ of the money.
Katie and Leonard share the rest of the money in the ratio 17:3

Jason gives 45% of his share of the money to his mother.
He has £198 left.

How much more money does Katie receive than Jason?

1.9 Difference Given

Worked Example	Your Turn
Zach and Olivia share some money in the ratio $2:5$. Olivia receives £30 more than Zach. How much do they each receive?	Zach and Olivia share some money in the ratio $2:5$. Olivia receives £15 more than Zach. How much do they each receive?

Fluency Practice

Question 1: A bag contains yellow and blue blocks in the ratio 1:3
There are 8 more blue blocks than yellow blocks.

(a) How many yellow blocks are there?
(b) How many blue blocks are there?

Question 2: The ratio of boys to girls in a class is 2:3
There are 6 more girls than boys in the class.
How many girls are in the class?

Question 3: Thomas and Emma share some money in the ratio 3:5
Emma receives £30 more than Thomas.

(a) How much money does Emma receive?
(b) How much money does Thomas receive?

Question 4: In a survey, the ratio of the number of people who preferred tea to those who preferred coffee was 9:5
36 more people preferred tea to coffee.
How many people were in the survey?

Question 5: The ratio of Mollie's age to Heather's age is 4:9
Heather is 40 years older than Mollie
How old is Mollie?

Intelligent Practice

1) Tony and Luke share some money in the ratio $3:7$. Luke receives £20 more than Tony. How much do they each receive?

2) Tony and Luke share some money in the ratio $3:7$. Luke receives £10 more than Tony. How much do they each receive?

3) Katy and Becky share some money in the ratio $2:1$. Katy receives £10 more than Becky. How much do they each receive?

4) The ratio of boys to girls in a class is $2:1$. There are 8 more boys than girls. How many boys and girls in the class?

5) There are blue, red and yellow counters in a bag in the ratio $3:2:1$. There are 6 more blue counters than red counters. How many counters are there in total?

6) There are blue, red and yellow counters in a bag in the ratio $9:6:3$. There are 6 more blue counters than red counters. How many counters are there in total?

7) There are blue, red and yellow counters in a bag in the ratio $9:6:3$. There are 18 more blue counters than red counters. How many counters are there in total?

8) A flapjack is made of oats, butter and syrup in the ratio $7:2:5$. A recipe requires 180g more syrup than butter. How many grams of oats should I use?

9) A flapjack is made of oats, butter and syrup in the ratio $7:1:5$. A recipe requires 180g more syrup than butter. How many grams of oats should I use?

10) A flapjack is made of oats, butter and syrup in the ratio $9:1:5$. A recipe requires 140g more oats than butter. How many grams of flapjack will the recipe make in total?

Extension

Question 1: A box contains red, purple and green beads in the ratio 4:6:7
There are 1428 more green beads than red beads.
How many green beads are in the box?

1.10 Total Given

Lots of things in everyday life are shared in ratios. Money is shared, liquids are mixed and teams are assigned using ratios.

Worked Example	Your Turn
Divide 30 in the ratio $2:3$	Divide 45 in the ratio $8:1$

Fluency Practice

Question 1:

(a) Share £20 in the ratio 2:3

(b) Share 15cm in the ratio 1:2

(c) Divide £24 in the ratio 1:3

(d) Share 35 sweets in the ratio 4:3

(e) Divide 55g in the ratio 3:2

(f) Divide 54kg in the ratio 1:5

(g) Share £210 in the ratio 2:5

(h) Share 120 hours in the ratio 5:7

(i) Share 350m in the ratio 3:7

(j) Divide 360° in the ratio 1:4

Question 2:

(a) Share £104 in the ratio 3:5

(b) Divide 161 miles in the ratio 6:1

(c) Divide 315ml in the ratio 2:7

(d) Share $650 in the ratio 4:9

(e) Share £800 in the ratio 11:14

(f) Share 1200kg in the ratio 3:37

(g) Divide €510 in the ratio 13:2

(h) Share 1116mm in the ratio 1:8

Question 3:

(a) Share £40 in the ratio 1:3:4

(b) Divide 63ml in the ratio 2:3:4

(c) Share 88p in the ratio 2:4:5

(d) Share 180° in the ratio 2:2:5

(e) Divide $165 in the ratio 1:2:12

(f) Share 720cm in the ratio 3:4:2:9

Question 4:

(a) Share 1km in the ratio 2:3

(b) Divide 2m in the ratio 9:1

(c) Divide 1 day in the ratio 1:2

(d) Share 4 minutes in the ratio 2:3

(e) Share £6 in the ratio 1:4

(f) Share €12 in the ratio 7:17

Question 5: Work out each of the following. You may use a calculator

(a) Share 10ml in the ratio 1:3

(b) Divide 17g in the ratio 2:3

(c) Divide 345ml in the ratio 3:5

(d) Divide £260 in the ratio 5:11

(e) Share 58° in the ratio 2:7

(f) Share 880 seconds in the ratio 2:5:11

Intelligent Practice

1) Share 20 in the ratio $2:3$

2) Share 20 in the ratio $3:2$

3) Share 20 in the ratio $4:1$

4) Share 40 in the ratio $4:1$

5) Share 40 on the ratio $8:2$

6) Share 40 in the ratio $8:12$

7) Share 40 in the ratio $10:6$

8) Share 40 in the ratio $5:3$

9) Share 40 in the ratio $1:4:3$

10) Share 40 in the ratio $5:1:2$

11) Jarvis and Damon share sweets in the ratio $2:3$. Jarvis gets 4 sweets, how many does Damon get?

12) Share 30 in the ratio $1:2$

13) Justine and Brett share sweets in the ratio $7:3$. Brett gets 30 sweets, how many does Justine get?

14) Liam and Noel share some money in the ratio $1:5$. Noel gets £25, how many does Liam get?

15) Share £9 in the ratio $1:5$

16) Mark and Gaz share some money in the ratio $7:4$. Gaz gets £30, how many does Mark get?

17) Crispin and Louise share sweets in the ratio $2:5$. Louise gets 60 more than Crispin. How many does Crispin get?

18) Tommy and Thom share sweets in the ratio $7:9$. Tommy gets 80 less than Thom. How many does Tommy get?

19) Share $180°$ in the ratio $2:3:5$

20) Richard and Tjinder share money in the ratio $4:9$. Tjinder gets £24 more than Richard. How much money did they get altogether?

21) Tim and Mark share some money in the ratio $4:5$. Tim gets £25, how many does Mark get?

22) Share $360°$ in the ratio $7:5:4:2$

Extension

Question 1: Ed has 30 sweets.
The ratio of red sweets to yellow sweets is 2:3
How many red sweets does Ed have?

Question 2: Liam and Nathan share £60 in the ratio 1:3
How much money does each man receive?

Question 3: The ratio of adults to children at a cricket match is 7:3.
There 150 people at the match.
How many children attended the cricket match?

Question 4: Mark is making concrete.
Concrete is made by mixing cement, sand and gravel in the ratio 1:2:3.
Mark wants to make 300kg of concrete.

(a) How much cement does Mark need?
(b) How much sand does Mark need?
(c) How much gravel does Mark need?

Question 5: The angles in a triangle are in the ratio 1:1:4

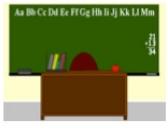
(a) Find the size of each angle
(b) What type of triangle is it?

Question 6: Dorothy has green and blue beads in the ratio 1:4
Dorothy has 80 beads.

(a) How many blue beads does she have?
(b) What fraction of the beads are green?
(c) What percentage of the beads are blue?

Question 7: The ratio of boys to girls in a class is 2:3
Ben says there are 28 students in the class.

(a) Explain why Ben must be wrong
(b) Write down a possible number of students in the class



Question 8: At a football match, the ratio of children to adults is 2:7
There are 2700 people in the crowd.
Each adult ticket is £8
Each child ticket costs £3 less than an adult ticket.

Work out the total money made from ticket sales.

Question 9: In a school, all students study one language, French or Spanish.
The ratio of girls to boys in Year 11 is 4:3
¾ of the boys study French
There are 168 students in Year 11.

How many of the boys study Spanish?

Extension

Question 10: In a school election there were four candidates: Tom, Rebecca, Olly and Wendy.
540 students voted in the election.

5% of the votes were for Tom

$\frac{2}{9}$ of the votes were for Rebecca

The ratio of the number of votes for Olly to the number of votes to Wendy was 1:2

How many votes were for Wendy?

Question 11: A drink is made by mixing orange juice and lemonade in the ratio 1:4
Lemonade costs 80p per litre
Orange juice costs £1.20 per litre
Work out the cost of making 3 litres of the drink.

Question 12: Hannah baked some chocolate, strawberry and vanilla cupcakes.
She baked four times as many chocolate as strawberry cupcakes.
She baked three times as many chocolate as vanilla cupcakes.
Altogether Hannah made 152 cupcakes.

How many cupcakes of each flavour did Hannah make?

Question 13: In a car park the ratio of white cars to black cars is 2:7
The ratio of white cars to red cars is 3:11
Altogether there are 343 white, black and red cars.

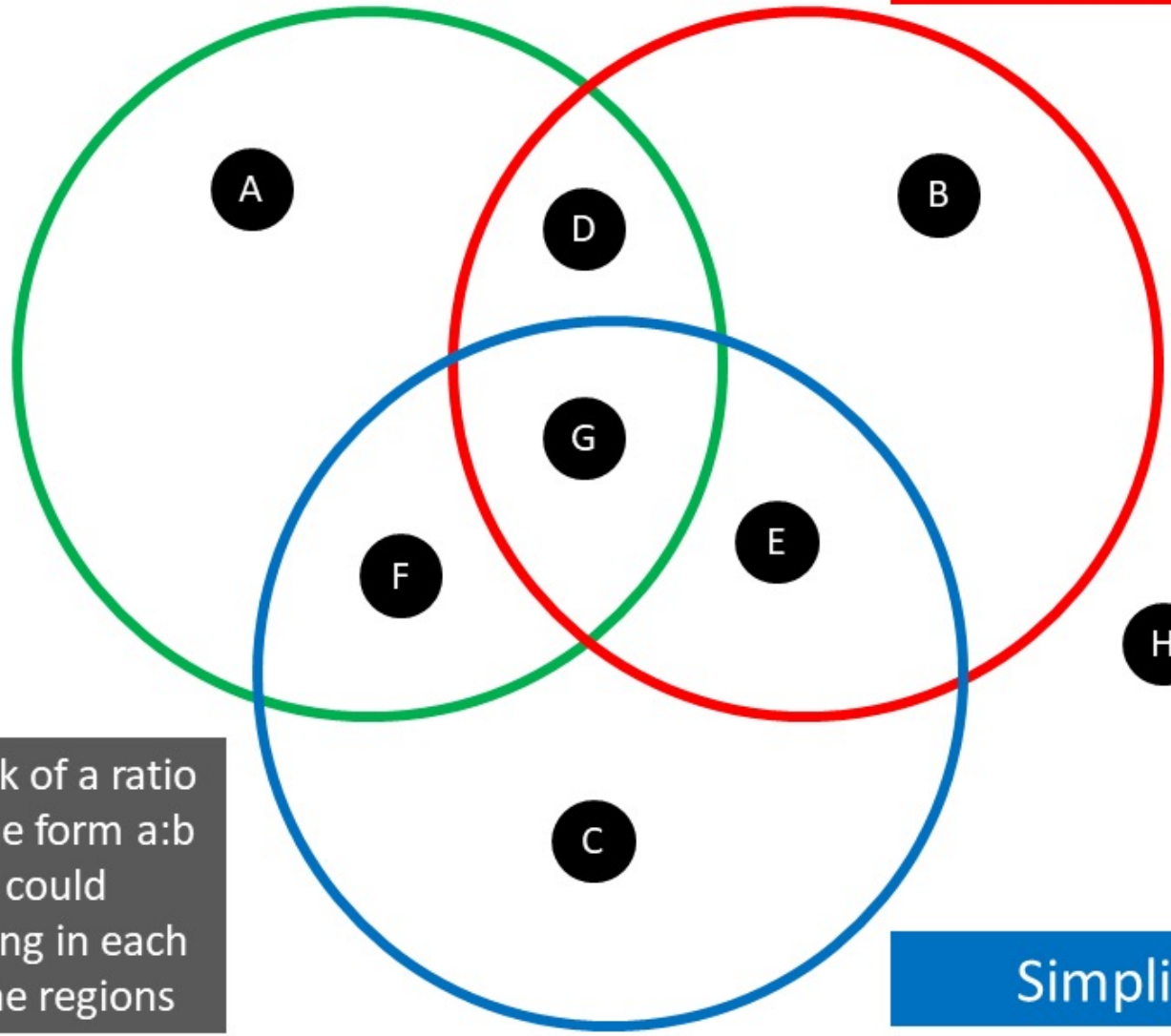
How many black cars are in the car park?

Question 14: At a holiday park, guests either stay in a caravan or in a tent.
In 2017 there were 460 guests.
In 2017 the number of guests was 15% greater than in 2016.
The ratio, in 2016, of people staying in a caravan to staying in a tent was 5:3.

How many guests stayed in caravans in 2016?

a is 3

24 counters can be shared in this ratio



If you think a region is impossible to fill, convince me why!

Think of a ratio in the form a:b that could belong in each of the regions

Simplifies

Maths Venns

18 counters can be shared in this ratio
If you think a region is impossible to fill, convince me why!
In the form 1 : n
Equivalent to $\frac{1}{2} : \frac{2}{5}$
Think of a ratio in the form a:b that could belong in each of the regions
A
B
C
D
E
F
G
H

1.11 Review and Problem Solving

Worked Example	Your Turn
Jenny and Ben share £12 in the ratio $2:1$ Jenny's amount:	Jenny and Ben share £12 in the ratio $3:1$ Jenny's amount:
Ben's amount:	Ben's amount:
Jenny gets ____ more	Jenny gets ____ more
Jenny gets $\frac{?}{?}$ of the whole	Jenny gets $\frac{?}{?}$ of the whole

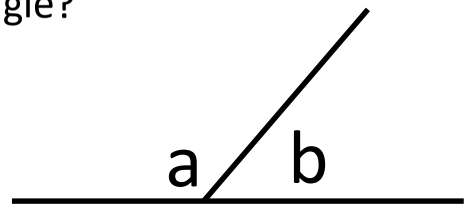
Fill in the Gaps

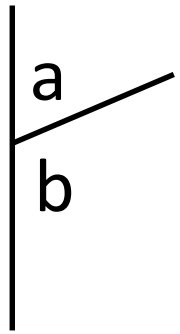
Ratio Jenny : Ben	Amount to share	Jenny's amount	Ben's amount	Jenny gets ? more/less	Jenny's amount as a fraction of the whole
3 : 2	£30				
3 : 2		£24			
		£24		£8 less	
	£40		£32		
3 : 4				£5 less	
	£33	£9			
__ : 8	£33			The same	
8 : __			£8		$\frac{?}{?} = \frac{2}{3}$
				£12 less	$\frac{?}{?} = \frac{2}{7}$
__ : 7	£28	£15.75			

Fill in the Blanks

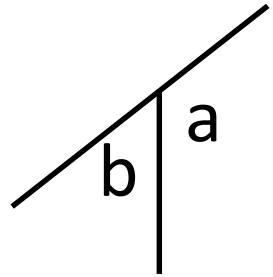
Amount	Ratio	Number of Parts	Amount per Part	First Share	Second Share
£50	4 : 1	5	£10	£40	£10
£100	3 : 2	5	£20		
£100	3 : 7	10			
£100	1 : 4				
£60	2 : 1				
£60	5 : 1				
£60	5 : 7				
£72	7 : 5				
£48	3 : 5				
	:		£5	£25	£15
	:	7		£100	£75
£20	:	10			£6
£90	:	9		£20	
£64	5 :		£8		
	: 1			£35	£7
	3 :	8		£7.50	

The ratio of a to b is 7:3.
What is the size of the largest angle?

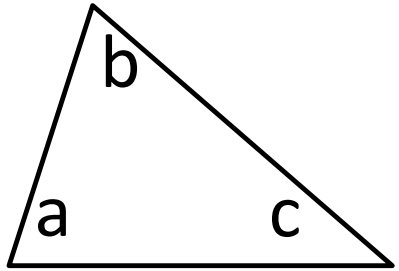




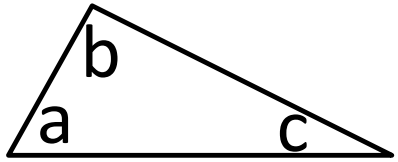
The ratio of a to b is 4:11.
What is the size of the smallest angle?



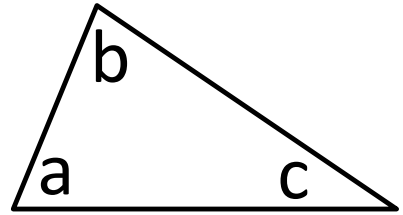
The ratio of a to b is 5:4.
What is the difference in the size of the angles.



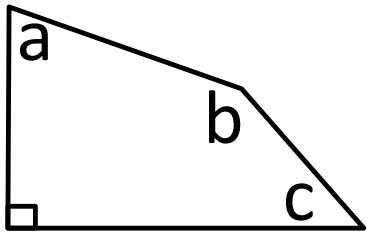
The ratio of a to b to c is 5:3:4.
Calculate the size of each angle



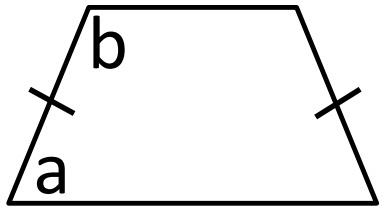
The ratio of a to b is 3:5. If angle $b = 85°$ what is the size of angle C?



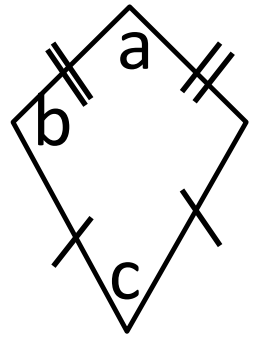
The ratio of a to b is 3:5. The difference between angle a and b is $36°$. What is the size of angle C?



The ratio of a to b to c is 2:5:3.
Calculate the size of each angle



The ratio of a:b is 3:5, what is the difference between the angles.



The ratio of a:b:c is 6:5:4.

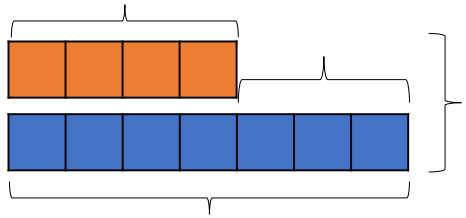
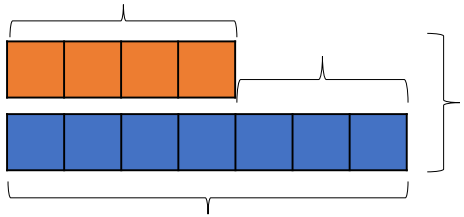
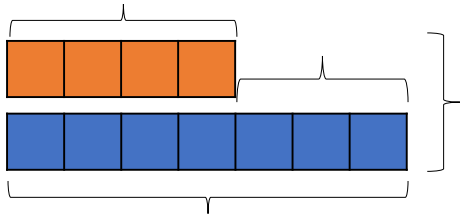
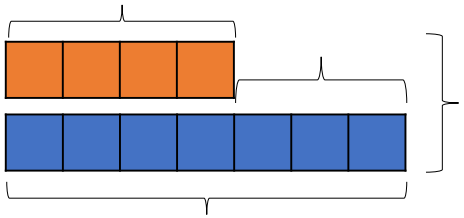
What is the size of the smallest angle?

Review

Anna and Bob shared some money in the ratio $3 : 5$.

a) If the total shared was £120, what else can you work out?
b) If Anna received £120, what else can you work out?
c) If Bob received £120, what else can you work out?
d) If Bob received £120 more than Anna, what else can you work out?

Sharing in a Ratio Decisions

Question	Type	Model	Working Out	Answer
Amy and Bob share £231 in the ratio 4 : 7. How much does Amy get?	Total Given			
Amy and Bob share some money in the ratio 4 : 7. Bob gets £231. How much does Amy get?	One Amount Given			
Amy and Bob share some money in the ratio 4 : 7. Bob gets £231 more than Amy. How much does Amy get?	Difference Given			
Amy and Bob share some money in the ratio 4 : 7. Amy gets £231. How much does is shared in total?				

Sharing in a Ratio Decisions

Question	Type	Model	Working Out	Answer
Amy and Bob share £84 in the ratio 4 : 3. How much does Amy get?				
Amy and Bob share some money in the ratio 4 : 3. Bob gets £84. How much does Amy get?				
Amy and Bob share some money in the ratio 4 : 3. Bob gets £84 more than Amy. How much does Amy get?				
Amy and Bob share some money in the ratio 4 : 3. Amy gets £84. How much does is shared in total?				

Sharing in a Ratio Decisions

Question	Type	Model	Working Out	Answer
The ratio of adults to children at an event are $3:5$. If there are 240 more adults than children how many people are there in total?				
The ratio of adults to children at an event are $3:5$. If there are 240 adults then how many people are there in total?				
The ratio of adults to children at an event are $3:5$. If there are 240 people in total then how many children are there?				
The ratio of adults to children at an event are $3:5$. If there are 240 children then how many adults are there in total?				

Sharing in a Ratio Decisions

Question	Type	Model	Working Out	Answer
Flour, sugar and butter are mixed in the ratio $6:2:3$. How many grams of flour and sugar are needed to mix with 1320g of butter?				
Flour, sugar and butter are mixed in the ratio $6:2:3$. How many grams of flour, sugar and butter are needed to make 1320g of the mixture?				
Flour, sugar and butter are mixed in the ratio $6:2:3$. You make the mixture with 1320g more flour than sugar. How many grams of flour, sugar and butter are used?				
Flour, sugar and butter are mixed in the ratio $6:2:3$.	One Amount Given			Total amount of mixture is 2420g

1.12 Scale Drawings

Scale drawing allows us to draw large objects on a smaller scale while keeping them accurate.

All scale drawings must have a scale written on them. Scales are usually expressed as ratios.

The ratio 1cm : 100cm means that for every 1cm on the scale drawing the length will be 100cm in real life.

Worked Example	Your Turn
A map is scaled down so that 5 centimetres on the map represents a real distance of 1 kilometre. On the map, the distance between two points is 41 cm. Work out the real distance between these two points.	A map is scaled down so that 4 centimetres on the map represents a real distance of 1 kilometre. On the map, the distance between two points is 14 cm. Work out the real distance between these two points.

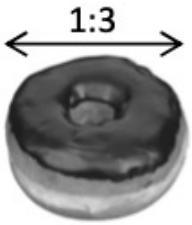
Worked Example	Your Turn
A map is scaled down so that 5 centimetres on the map represents a real distance of 1 kilometre. On the map, the distance between two points is 41 cm. Work out the scale of the map in the form $1 : n$.	A map is scaled down so that 4 centimetres on the map represents a real distance of 1 kilometre. On the map, the distance between two points is 14 cm. Work out the scale of the map in the form $1 : n$.

Scale Drawings

Scale Drawings

These are scale images of real objects.
Each object is at a different scale.
What is the marked length of the real object?

1:2



1:3

1:7

1:40

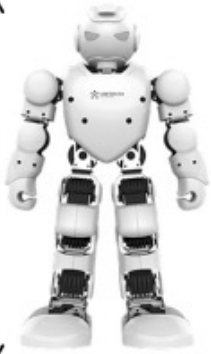
1:500

1:400

1:100

1:35

1:47



1:14

1:125

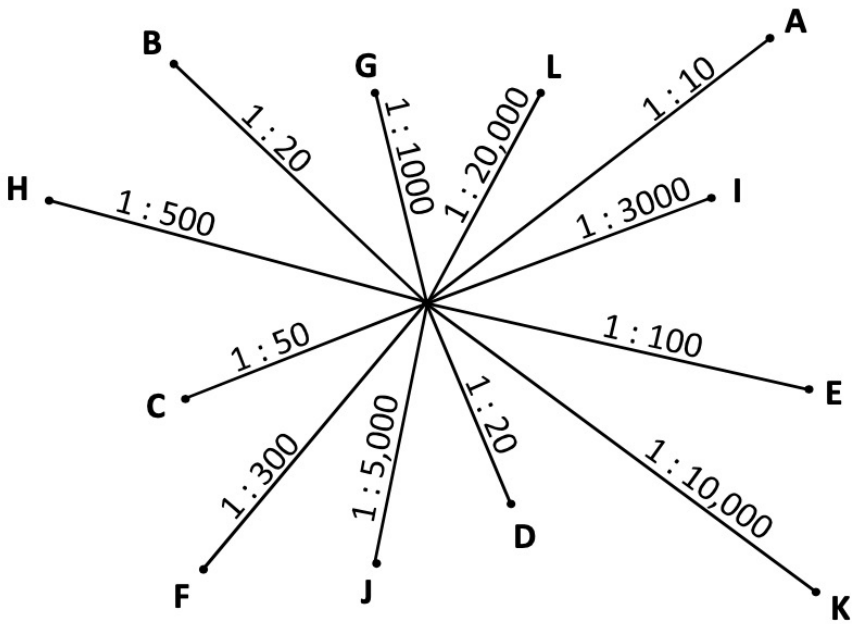
1: 175 million

Scale Stars

Scale Stars 1

Each line is drawn at a **different scale**.

Measure the lines and calculate how long they would be in **real life**.



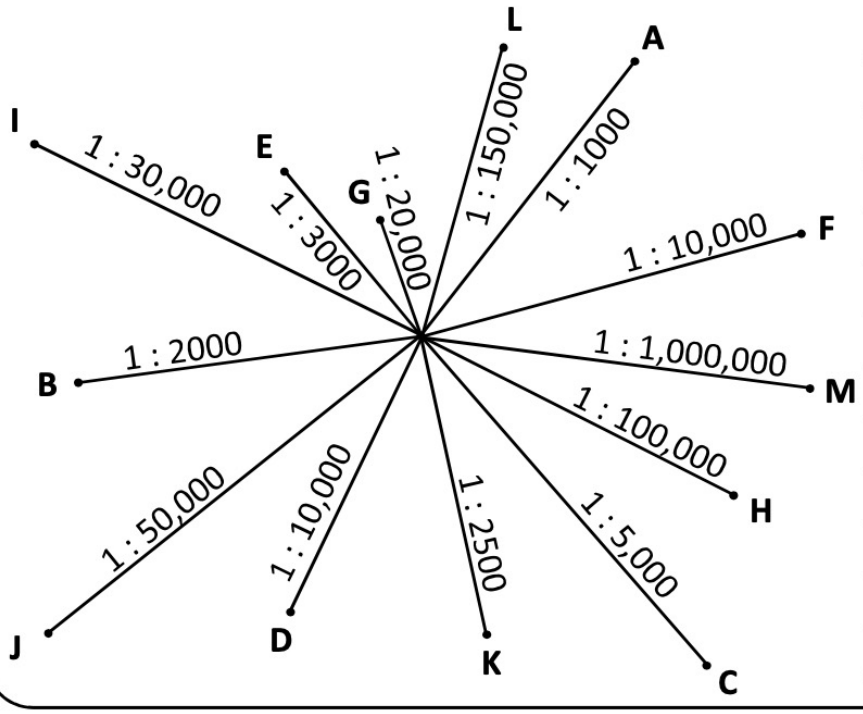
Answers

→ A _____ cm
→ B _____
→ C _____
→ D _____
→ E _____
→ F _____
→ G _____
→ H _____
→ I _____
→ J _____
→ K _____
→ L _____

Scale Stars 2

Each line is drawn at a **different scale**.

Measure the lines and calculate how long they would be in **real life**.



Answers

→ A _____
→ B _____
→ C _____
→ D _____
→ E _____
→ F _____
→ G _____
→ H _____
→ I _____
→ J _____
→ K _____
→ L _____
→ M _____

1.13 Direct Proportion

Two things are directly proportional if they in the same ratio.

Can you suggest variables that might be directly proportional?

- Speed and distance travelled (if you double your speed, you double the distance travelled).
- Total cost and quantity purchased.
- Length of steel rod and weight.

Worked Example	Your Turn
(a) I buy 8 pens for £1.84. How much is 1 pen? (b) If 8 pens cost £1.12, how much are 12 pens?	(a) I buy 6 pens for £1.62. How much is 1 pen? (b) If 6 pens cost £1.14, how much are 8 pens?

Fluency Practice

Question 1: Keith buys 6 pencils for 90p

(a) How much does one pencil cost?
(b) How much would five pencils cost?
(c) How much would eleven pencils cost?

Question 2: Jack and Harry are waiters in a restaurant.
They are both paid the same amount of money for each hour that they work.
Jack worked 6 hours and is paid £48
Harry worked 8 hours.
How much money is Harry paid?

Question 3: A car travels 120 miles in 3 hours at a steady speed.

(a) How far does the car travel in 1 hour?
(b) How far does the car travel in 8 hours?

Question 4: A plumber charges £140 for a 4 hour job.

How much does the plumber charge for a 3 hour job?

Question 5: Seven candles cost £45.29

How much would 25 candles cost?

Question 6: £50 is worth €56

(a) How many euros is £1 worth?
(b) How many euros is £220 worth?

Question 7: If 24 marbles have a mass of 60g, what would the mass of 60 marbles be?

Question 8: Rebecca is making Chilli Con Carne.
Here is a list of ingredients to serve 6 people.

serves 6
1.2kg mince
420g tomatoes
3 chillies
600g kidney beans

Rebecca wants to make enough Chilli Con Carne for 4 people.

How much of each ingredient does Rebecca need?

Question 9: Oscar is making fish pie.
Here is a list of ingredients for 5 people.

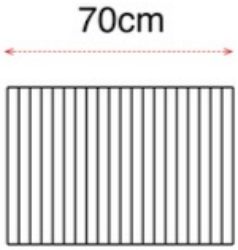
serves 5
500g cod
400g haddock
600ml milk
120g butter
40g flour
1kg potatoes

Oscar wants to make enough fish pie for 6 people.

How much of each ingredient should Oscar use?

Extension

Question 1: On a map, 4cm represents 60 miles.
The distance between two towns is 37.5 miles.
On the map, what is the distance between the two towns?



Question 2: Nathan has 20 identical books on a shelf.
The books take up 70cm of space on the shelf.
Nathan removes seven books.

How much space do the remaining books take up?

Question 3: A car uses 8.4 litres of petrol for a 112 mile journey.
When the tank is full, the car holds 54 litres of petrol.

How far should the car be able to travel on a full tank of petrol?

Question 4: A 345ml tin of paint costs £4.80
A 250ml tin of paint costs £3.35
Which tin is better value for money?

Worked Example	Your Turn
4 rulers cost £32. (a) How much does 1 ruler cost? (b) How much do 5 rulers cost?	8 rulers cost £32. (a) How much does 1 ruler cost? (b) How much do 5 rulers cost?

Intelligent Practice

1) 2 pens cost £30.
How much does 1 pen cost?

2) 2 rulers cost £30.
How much does 1 ruler cost?

3) 4 rulers cost £30.
How much does 1 ruler cost?

4) 4 rulers cost £60.
How much does 1 ruler cost?

5) 6 rulers cost £90.
How much does 1 ruler cost?

6) 60 rulers cost £90.
How much does 1 ruler cost?

7) 60 rulers cost £9.
How much does 1 ruler cost?

8) 60 rulers cost £9.
How much do 2 rulers cost?

9) 60 rulers cost £9.
How much do 20 rulers cost?

10) 40 rulers cost £9.
How much do 20 rulers cost?

11) 20 rulers cost £9.
How much do 40 rulers cost?

12) 5 rulers cost £2.25.
How much do 40 rulers cost?

13) 15 rulers cost £2.25.
How much do 40 rulers cost?

14) 15 rulers cost £2.25.
How much do 7 rulers cost?

15) 15 rulers cost £2.25.
How much do 45 rulers cost?

16) n rulers cost £2.25.
How much do $3n$ rulers cost?

17) $3n$ rulers cost £2.25.
How much do n rulers cost?

18) x rulers cost £y.
How much do n rulers cost?

1.14 Best Buys

Proportion calculations can be used to decide which items in a shop offer the best value. Many items sold in supermarkets show a price per item and a price per 100g or per kg. This lets people compare products and get the best value for money.

Proportion calculations must be used to compare the cost of items. This makes either their cost or their size the same so comparisons can be made. It is generally easier to make the size of the items the same but how this is done can vary depending on the numbers that are used.

Worked Example	Your Turn
A pack of 4 pens in shop A costs £1.32. A pack of 5 pens in shop B costs £1.60. Which pack is better value for money?	A pack of 5 pens in shop A costs £1.20. A pack of 4 pens in shop B costs £1. Which pack is better value for money?

Fluency Practice

Question 1: For each pair, decide which is better value for money.

(a) 1 ticket for £8 or 3 tickets for £20

(b) 1 sandwich for £2.50 or 2 sandwiches for £5.20

(c) 2 pizzas for £12 or 4 pizzas for £28

(d) 3 doughnuts for 60p or 6 doughnuts for £1

(e) 6 eggs for 96p or 12 eggs for £1.80

(f) 1 litre of milk for 67p or 2 litres of milk for £1.35

(g) 100g of ham for £1.20 or 300g of ham for £3.50

(h) 5kg of potatoes for £2.50 or 20kg of potatoes for £10.50

(i) 500ml of lemonade for 89p or 1 litre of lemonade for £1.70

Question 2: For each pair, decide which is better value for money.

(a) 2 croissants for 48p or 3 croissants for 75p

(b) 3 cupcakes for £1.05 or 5 cupcakes for £1.70

(c) 4 pens for £3.50 or 6 pens for £5

(d) 10 chocolate bars for £4.80 or 15 chocolate bars for £6.90

(e) 6 chicken wings for £3.50 or 9 chicken wings for £5.30

(f) 400g of porridge for £1.52 or 500g of porridge for £1.86

(g) 500ml of lemonade for 94p or 750ml of lemonade for £1.44

(h) 200 minutes of calls for £7 or 350 minutes of calls for £12.50

(i) 600g of honey for £4.25 or 1kg of honey for £6.99

Question 3: For each pair, decide which is better value for money.
You may use a calculator.

(a) 250 sheets of paper for £1.25 or 400 sheets of paper for £2.08

(b) 350g of coffee for £2.45 or 540g of coffee for £3.60

(c) 0.8kg of carrots for £1 or 1.3kg of carrots for £1.70

(d) 345ml of paint for £4.80 or 250ml of paint for £3.35

(e) 0.9 grammes of gold for $38.20 or 6.5 grammes for gold for $270

(f) A taxi journey of 8.7 miles for £17 or A taxi journey of 3.3 miles for £7

Extension

Question 1: Mr McClean wants to hire a taxi.
He rings three different taxi companies and asks them for their prices.

A1 Taxis: A 5 mile journey costs £15
Crazy Cabs: A 4 mile journey costs £13
Value Cars: A 10 mile journey costs £28

Order the taxi companies from best to least value for money.

Question 2: Bethany wants to buy 9 chairs.
Which shop is best value for money?

Question 3: Baked beans are sold in different sizes:
415g tin for 75p.
3 x 200g pack for £1.69.
1kg fridge pack for £2.39.
Which is best value for money?

Question 4: Flower pots normally cost £4 each.

Two shops have special offers.

Gardenbase
20% off

Lawn Factory
Buy 5 get 2 free

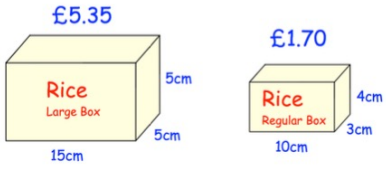
Laura wants to buy 30 flower pots.
Which shop should Laura buy them from?

Question 5: A cereal bar is sold in packs of 4, 6 or 8.

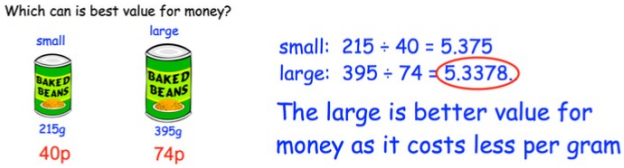
The 4 pack of cereal bars costs £1.80 and it is the least value for money.
The 8 pack of cereal bars cost £3.52 and it is the best value for money.

Work out (a) the lowest price of the 6 pack of cereal bar
(b) the highest price of the 6 pack of cereal bar

Question 6: A shop sells two different boxes of rice.
Work out which box is best value for money.



Question 7: Phil has completed his maths homework.
Can you spot any mistakes?



1.15 Recipes

Worked Example	Your Turn
A recipe for making 12 muffins requires ingredients including: 300g flour 150g sugar ml milk (a) How much sugar is needed to make 4 muffins? (b) How much flour is needed to make 18 muffins?	A recipe for making 10 muffins requires ingredients including: 240g flour g sugar ml milk (a) How much flour is needed to make 2 muffins? (b) How much milk is needed to make 25 muffins?

Fluency Practice

Question 1: Jake is making scones.
Here is a list of ingredients to make 8 scones.

8 Scones

200g flour
30g caster sugar
50g butter
140ml milk
1 egg

How much of each ingredient would be needed to make:

(a) 16 scones? (b) 4 scones? (c) 24 scones?

(d) 40 scones? (e) 80 scones? (f) 2 scones?

Question 2: Chloe is making ice cream.
She is using the recipe below.

serves 4

300ml double cream
320ml milk
120g caster sugar
1 vanilla pod
4 egg yolks

How much of each ingredient would Chloe need to make enough for:

(a) 8 people? (b) 2 people? (c) 1 person?

(d) 3 people? (e) 6 people? (f) 10 people?

Question 3: Rupert is making a fish pie.
He is using the recipe below.

serves 5

500g cod
400g haddock
600ml milk
120g butter
40g flour
1kg potatoes

How much of each ingredient would Rupert need to make enough for:

(a) 15 people? (b) 1 person? (c) 2 people?

(d) 4 people? (e) 8 people? (f) 11 people?

Extension

Question 1: Tia uses this recipes to make hot cross buns.
Tia is going to use this recipe to make 9 hot cross buns.

(a) How much of each ingredient does Tia need?

Grace uses the same recipe.
She uses 500ml of milk.

(b) How many hot cross buns is Grace making?

makes 12
480g flour
60g caster sugar
200ml milk
1 egg
50g butter
100g currant

Question 2: Timothy is making a Rice Krispie cakes.
A recipe uses 240g of chocolate and 160g of Rice Krispies to make 24 cakes.

(a) Write down the ratio of chocolate to Rice Krispies in its simplest form.

(b) How much Rice Krispies should Timothy use to make 30 cakes?

Question 3: Sarah is making shortbread biscuits.
She has:

600g of butter
300g of caster sugar
1kg of plain flour
500g of cornflour

She has found this list of ingredients for making 8 shortbread biscuits

makes 8

Butter	150g
Caster Sugar	60g
Plain Flour	200g
Cornflour	50g

Sarah wants to make as many shortbread biscuits as possible.
Work out how many shortbread biscuits Sarah can make.

Question 4: Rebecca has a recipe for Chilli Con Carne

She only has 400g of kidney beans

How much of the other ingredients should she use?

serves 6
1.2kg mince
420g tomatoes
3 chillies
600g kidney beans

Extension

Question 5: Heather is making chocolate biscuits.
She has:

2kg of flour
1kg of butter
340g of icing sugar
200g of chocolate

Here is the list of ingredients for making 20 biscuits.

makes 20

100g flour
120g butter
80g icing sugar
25g chocolate

Heather wants to make as many biscuits as she can.
Work out how many biscuits Heather can make.

Question 6: David has a full 800ml bottle of car shampoo.
He is going to mix some of the car shampoo with water.

The bottle has this guidance

Car Shampoo - 800ml

Mix $\frac{1}{4}$ of the car shampoo
with 1.8 litres of water

David is going to use 360ml of water.
How much car shampoo should David use?

Question 7: James is making concrete using cement, sand and gravel in the ratio 1 : 2 : 3
James has:

63kg cement
112kg sand
210kg gravel

What is the maximum amount of concrete that James can make?

Worked Example	Your Turn
5 People 100g egg 50g pork 10g sugar 20g carrots How much ingredients do I need for 10 People?	10 People 100g egg 50g pork 10g sugar 20g carrots How much ingredients do I need for 15 People?

Worked Example	Your Turn
5 People 100g egg 50g pork 10g sugar 20g carrots How much ingredients do I need for 7 People?	10 People 100g egg 50g pork 10g sugar 20g carrots How much ingredients do I need for 7 People?

Intelligent Practice

1) How much ingredients do I need for 8 People?

5 People
120g egg
60g pork
10g sugar
40g carrots

2) How much ingredients do I need for 8 People?

6 People
120g egg
60g pork
12g sugar
48g carrots

3) How much ingredients do I need for 8 People?

7 People
140g egg
63g pork
21g sugar
42g carrots

4) How much ingredients do I need for 28 People?

14 People
140g egg
60g pork
10g sugar
40g carrots

5) How much ingredients do I need for 21 People?

14 People
140g egg
60g pork
10g sugar
40g carrots

1.16 Exchange Rates

The currency of the United Kingdom is the British pound, or pound sterling. When we refer to foreign currency, we mean the money that a different country uses such as baht in Thailand or rupees in India.

Not all currencies have the same value. We use exchange rates to convert from one currency to another.

Exchange rates are published in newspapers and online where the pound is matched against various currencies.

Worked Example	Your Turn
The exchange rate for Pound to Yen in June 2020 was $1 : 132$ How much would I get in return for exchanging: (a) £10 (b) ¥660	The exchange rate for Pound to Yen in June 2020 was $1 : 132$ How much would I get in return for exchanging: (a) £20 (b) ¥6600

Fluency Practice

Question 1: Given £1 = 5 złoty convert each of the following into Polish złoty

(a) £4 (b) £9 (c) £20 (d) £35 (e) £70 (f) £410 (g) £88

Question 2: Given £1 = 5 złoty convert each of the following into UK pounds

(a) 15 zł (b) 35 zł (c) 250 zł (d) 180 zł (e) 715 zł (f) 900 zł (g) 95 zł

Question 3: Given £1 = 25 Mexican Peso convert each of the following into Pesos

(a) £4 (b) £20 (c) £25 (d) £40 (e) £37 (f) £66 (g) £360

Question 4: Given £1 = 25 Mexican Peso convert each of the following into UK pounds

(a) $75 (b) $250 (c) $825 (d) $4000 (e) $9200 (f) $38000 (g) $1275

Question 5: Given £1 = $1.50 convert each of the following into US dollars.

(a) £3 (b) £5 (c) £7 (d) £20 (e) £40 (f) £50 (g) £100

Question 6: Given £1 = $1.50 convert each of the following into UK pounds

(a) $3 (b) $6 (c) $15 (d) $45 (e) $300 (f) $12 (g) $33

Question 7: Given £1 = ₺4.25 convert each of the following into Turkish lira.

(a) £9 (b) £15 (c) £9.60 (d) £73 (e) £853 (f) £9500 (g) 80p

Question 8: Given £1 = ₺4.25 convert each of the following into UK pounds.

(a) ₺29.75 (b) ₺76.50 (c) ₺110.50 (d) ₺2550 (e) ₺5100 (f) ₺0.85 (g) ₺4.59

Question 9: Given £1 = €1.28 convert each of the following into euros.

(a) £6 (b) £4.50 (c) £13 (d) £58 (e) £190 (f) £5730 (g) £809

Question 10: Given £1 = €1.28 convert each of the following into pounds.

(a) €64 (b) €153.60 (c) €1152 (d) €0.32 (e) €44.80 (f) €140.80 (g) €2.24

Extension

Question 1: Nicola went to Italy. She changed £800 into euros (€).
The exchange rate was £1 = €1.40
Change £800 into euros.

Question 2: A new TV in Tokyo costs ¥53380
The exchange rate is £1 = ¥157
How much is the TV in pounds?

Question 3: In Lisbon, a watch costs €80.
In Liverpool, the same watch costs £65.
The exchange rate is £1 = €1.25

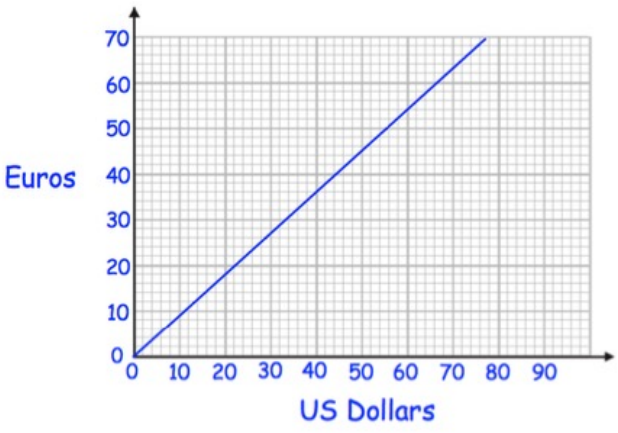
Work out the difference in cost.

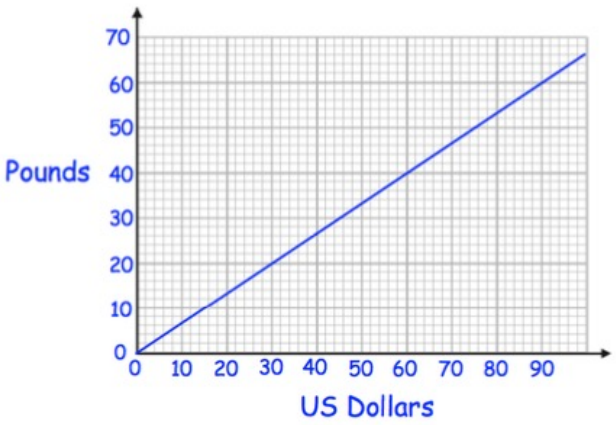
Question 4: Below are the prices of the same car in different countries.
£1 = €1.18 £1 = ¥140 £1 = $1.25

USA	Ireland	England	Japan
$20000	€17500	£15000	¥3000000

In which country is the car the best value?

Question 8: Convert £160 into Euros





Extension

Question 5: Martina wants to convert £300 into Euros.
The Post Office only has €20 notes.
The exchange rate is £1 = €1.17

(a) How many €20 notes will Martina receive?
(b) How much will it cost Martina?

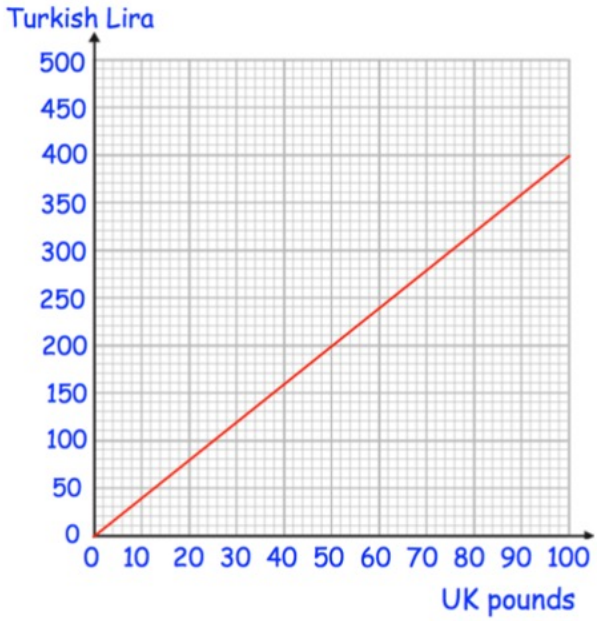
Question 6: Shown is a conversion graph to convert between GB pounds and Turkish lira.

(a) Convert £90 into Turkish lira.

(b) Convert 100 lira into pounds.

(c) Convert £250 into Turkish lira.

(d) Convert 800 lira into pounds.

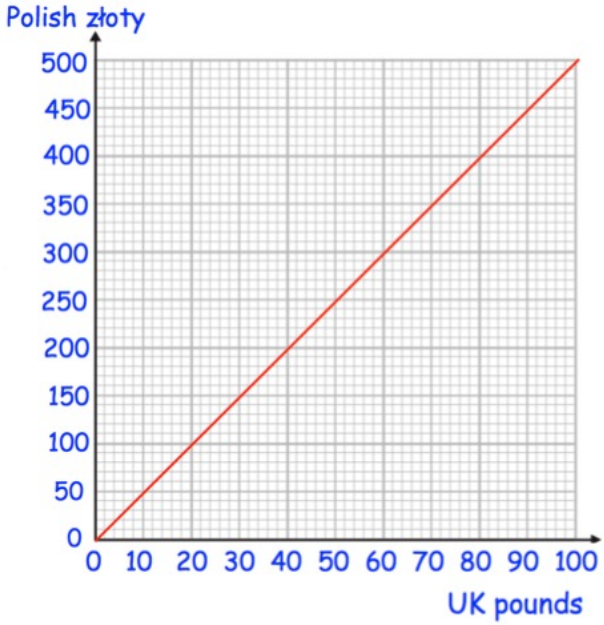


Question 7: Below is a conversion graph to convert between GB pounds and Polish złoty.

Jack has £400 and 1200 złoty
His hotel bill is 2000 złoty

He pays the bill with 1200 złoty and some of the pounds.

Work out how many pounds he has left.



Worked Example	Your Turn
$\$12 = £9$ Work out the value of $\$5$ in £	$\$5 = £9$ Work out the value of $\$12$ in £

Intelligent Practice

1) Given $\$8 = £1$
   Work out the value of $\$2$ in £

2) Given $\$2 = £1$
   Work out the value of $\$8$ in £

3) Given $\$3 = £1$
   Work out the value of $\$8$ in £

4) Given $\$6 = £4$
   Work out the value of $\$8$ in £

5) Given $\$12 = £8$
   Work out the value of $\$4$ in £

6) Given $\$1.20 = £1$
   Work out the value of $\$4$ in £

7) Given $\$2.40 = £4$
   Work out the value of $\$4$ in £

8) Given $\$0.60 = £1$
   Work out the value of $\$4$ in £

9) Given $\$0.61 = £1.01$
   Work out the value of $\$4.01$ in £

10) Given $\$6.1 = £1.01$
    Work out the value of $\$4.01$ in £

11) Given $£101 = \$6.1$
    Work out the value of $£4.01$ in $

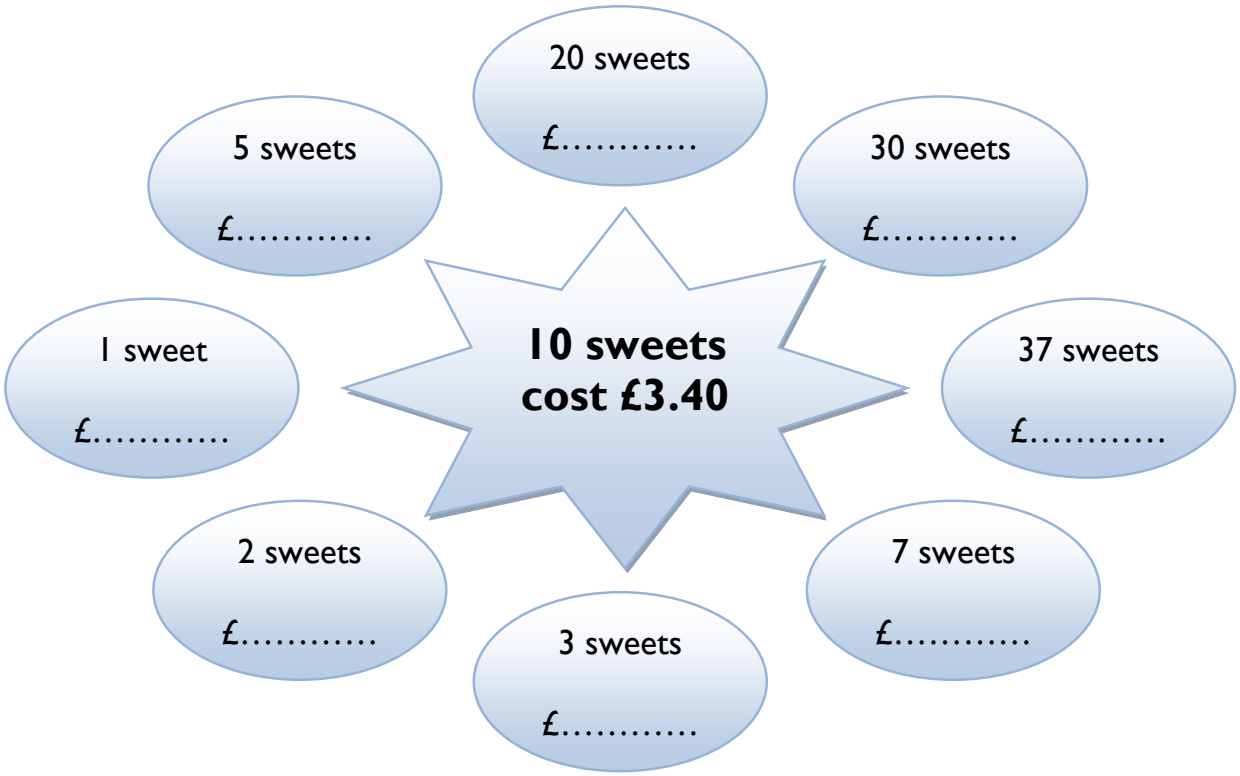
12) Given $\$x = £8$
    Work out the value of $\$7$ in £

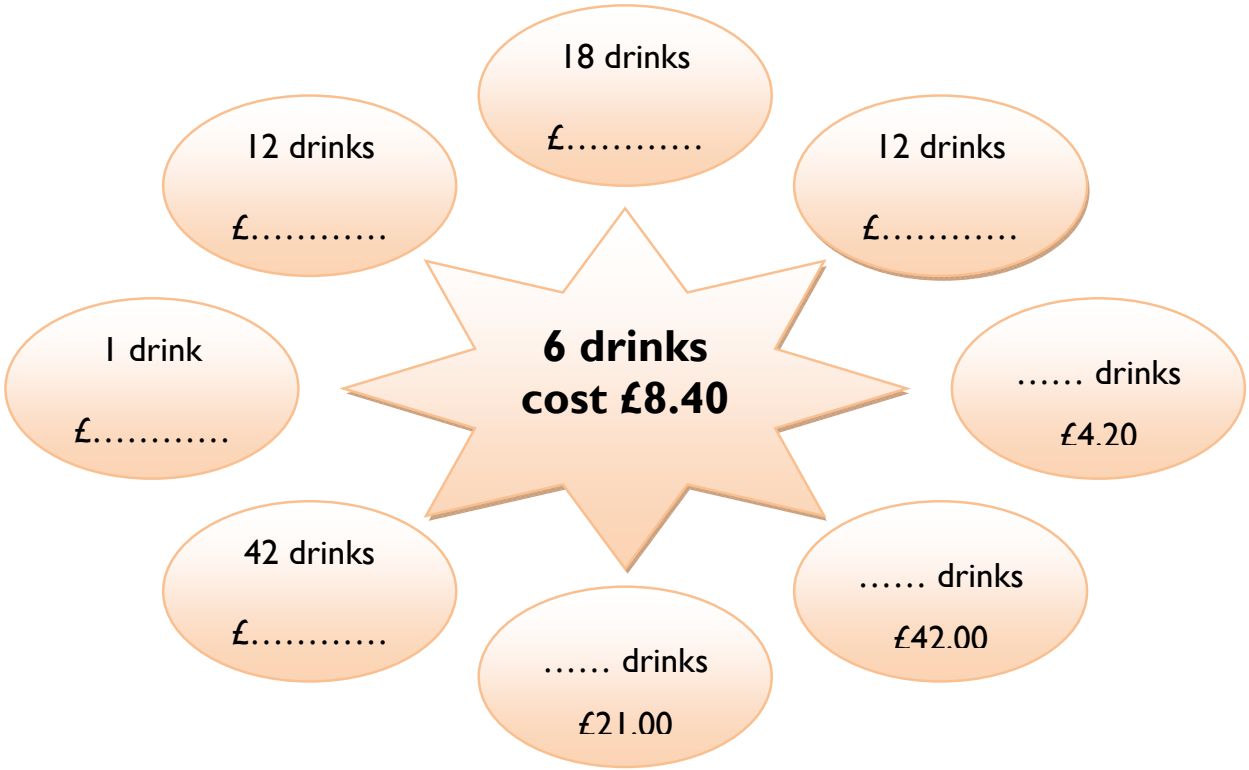
13) Given $\$8 = £x$
    Work out the value of $\$7$ in £

14) Given $\$8 = £7$
    Work out the value of $\$x$ in £

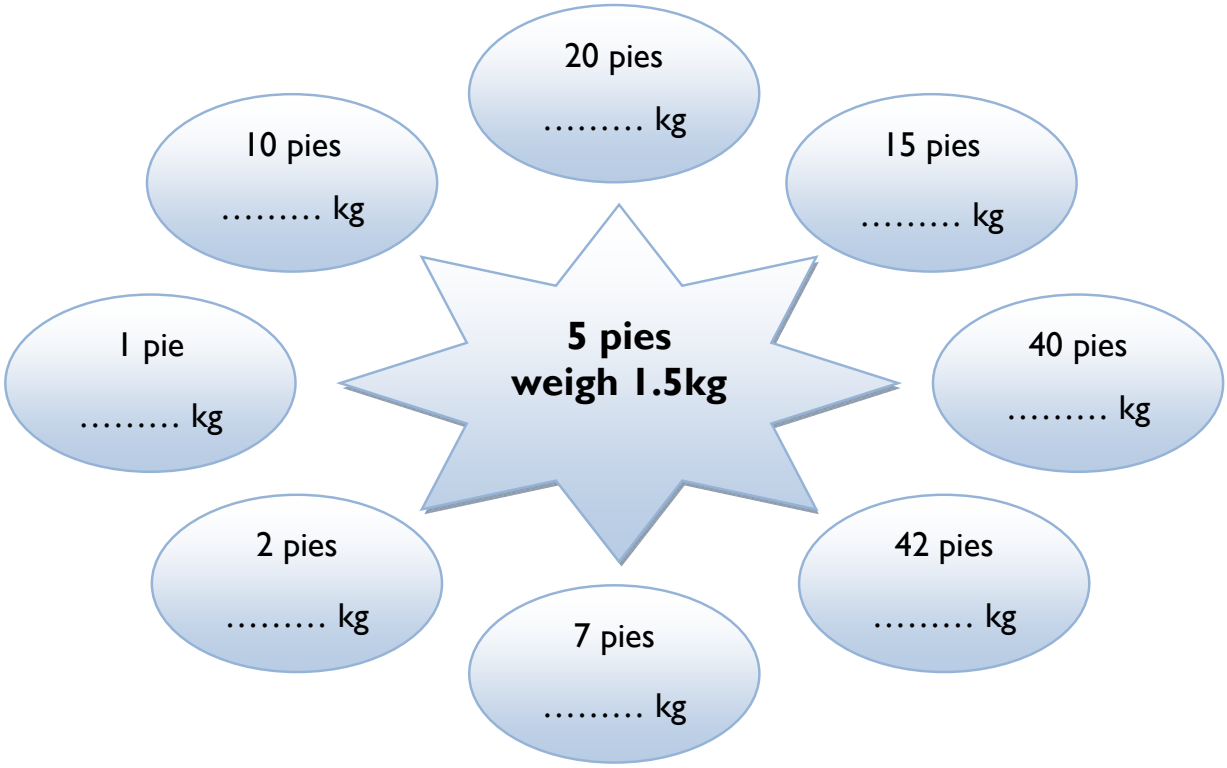
1.17 Review and Problem Solving

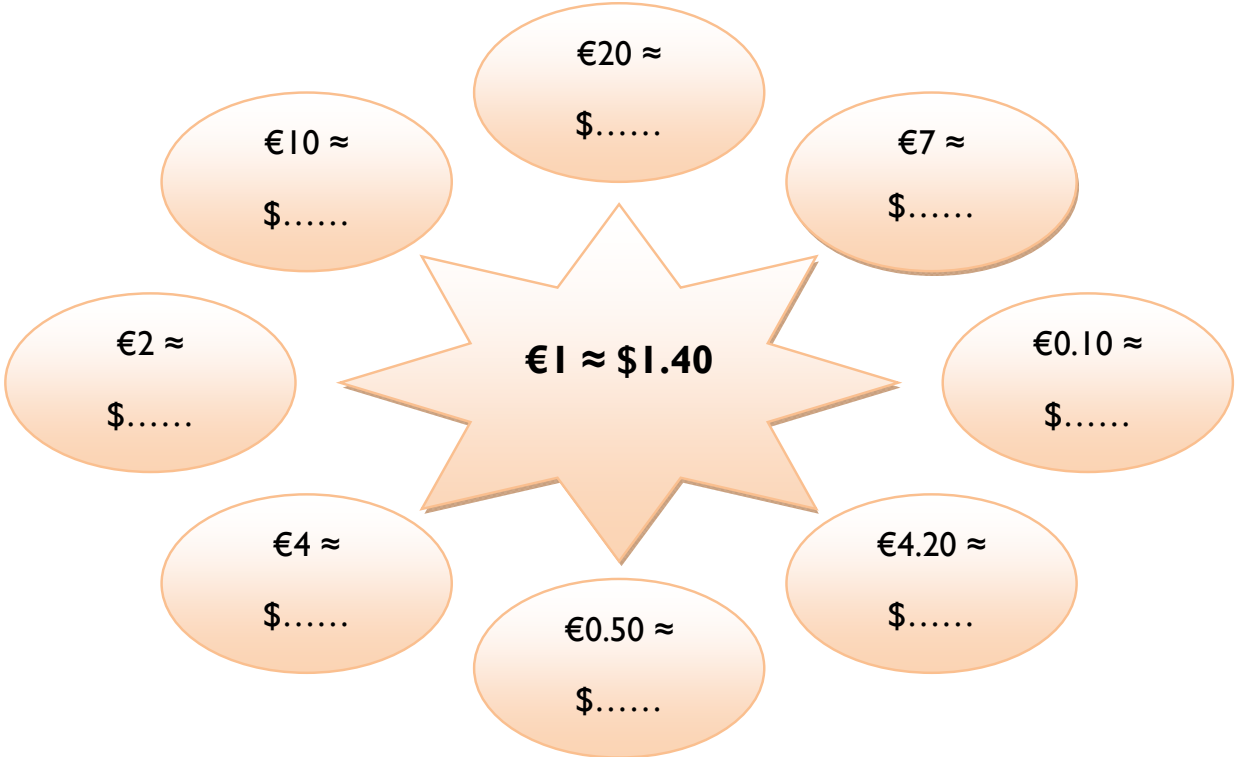
Proportion Bursts





Proportion Bursts





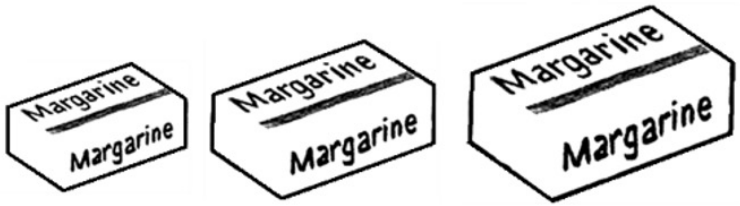
ratio

which is the best buy?

(1)

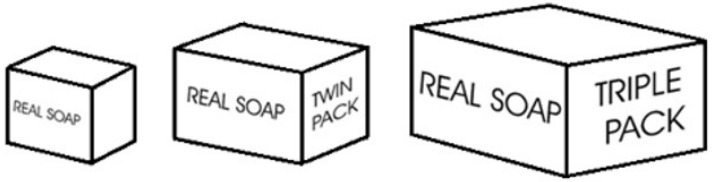
28p 52p 81p

(2)



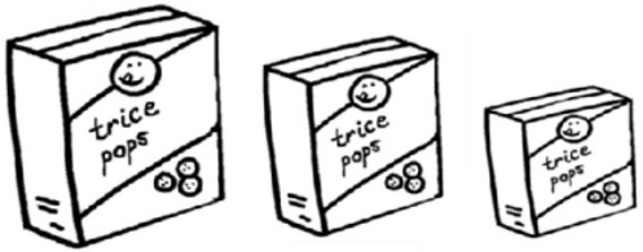
500g £1.25 1kg £2.48 2kg £4.98

(3)



83p £1.68 £2.50

(4)



500g £2.21 300g £1.32 200g 87p

Best Buys

Best Buys

£2.00	£3.55	£5.30
80 tea bags	150 tea bags	220 tea bags

what do each of these calculations tell you?

200 ÷ 80

355 ÷ 150

150 ÷ 3.55

530 ÷ 220

2 ÷ 8

5.30 ÷ 22

220 ÷ 5.30

80 ÷ 2.00

3.55 ÷ 15

pancakes

250 ml milk
1 egg
140 g flour
5g butter/fat

this recipe is enough for **8** pancakes

(i) how much milk is needed for **32** pancakes?
(ii) how much flour is needed for **20** pancakes?

apple crumble

90 g sugar
60 g butter/fat
80 g flour
6 apples

this recipe is enough for **4** people

how much of each ingredient is needed for **10** people?

trifle

120 g jelly
8 sponge fingers
420 ml custard
180 g tinned fruit

this recipe is enough for **4** people

(i) how much custard is needed for **6** people?
(ii) how much tinned fruit is needed for **7** people?

scones

225 g self raising flour
55 g butter
25 g caster sugar
150 ml milk

this recipe is enough for **10** scones

(i) how much flour is needed for **8** scones?
(ii) how much of each ingredient is needed for **6** scones?

onion soup

onion soup recipe for **8** people:

10 onions
2 pints of water
4 vegetable stock cubes
2 dessertspoons butter
½ pint cream

(1) for **4** people
(a) how much water is needed?
(b) how many stock cubes are needed?
(c) how many onions?

(2) for **12** people
(a) how much water is needed?
(b) how many stock cubes are needed?
(c) how many onions?

(3) for **20** people
(a) how many onions are needed?
(b) how many stock cubes are needed?
(c) how much water is needed?

(4) how much cream is needed for
(a) 4 people?
(b) 12 people?
(c) 20 people?

Money Exchange

Jesse decides to sell the 198 dollars he had left over from a holiday. Comparing rates in a few places, he opts for the highest rate of 1.8 (dollars to the pound) rather than the lowest, of 1.5. Too late, he realises his mistake. How much did he lose out?

2 Algebra Recap

- We tend to use single lower case letters for variables, either using the English alphabet or using the Greek alphabet.
- An algebraic x is written using two back to back c's. Do NOT write it as a $\times$ symbol.
- Do NOT include the multiplication sign, for example $3 \times p = 3p$
- Write division as fractions, for example $3 \div \mathrm{p} = \frac{3}{p}$
- Write numbers first in products, for example $p \times 3 = 3p$
- Write letters in products in alphabetical order, for example $4 \times q \times r \times p = 4pqr$
- $1x$ is written simply as x

2.1 Definitions

- Variable is a letter used to represent an unknown number.
- Coefficient is the number in front of a variable.
- Constant is a number that cannot change its value.
- Term is either a constant, a variable or a constant multiplied by a variable.
- Expression is terms and operators ($+$ and $-$) grouped together.

Worked Example	Your Turn
Write down the following for the expression: $2x - 4y - 9$ Variables: Coefficient of x: Coefficient of y: Constant: Terms:	Write down the following for the expression: $-2a + 4b + 9$ Variables: Coefficient of a: Coefficient of b: Constant: Terms:

Worked Example	Your Turn
Write down the following for the expression: $2x^2 - 4xy - 9$ Variables: Coefficient of x^2: Coefficient of xy: Constant: Terms:	Write down the following for the expression: $-2ab + 4b^2 + 9$ Variables: Coefficient of ab: Coefficient of b^2: Constant: Terms:

Intelligent Practice

Question	Variables	Coefficients	Constant	Terms
$3x - 9$				
$3x + 4y - 9$				
$3x - 4y - 9$				
$3x - 4y + 9$				
$-3x - 4y + 9$				
$9 - 3x - 4y$				
$9 - 3a - 4b$				
$3a^2 - 4b^2 + 9$				
$3a^2 - 4a + 9$				
$3a^2 - 4a$				
$3a^2 - 4$				
$3ab - 4$				
$3ab - 4a$				
$3ab - 4a - 5b$				
$3a^2b - 4a - 5b$				
$3ab^2 - 4a - 5b$				
$3ab^2 - 4ab - 5b$				
$3ab^2 - 4a^2b - 5b$				
$3ab^2 - 4a^2b - 5ab$				
$3ab^2 - 4a^2b - 5ab - 6$				

2.2 Forming Expressions

Unscramble the words below to make synonyms of the operations.

Addition +	**Subtraction −**	**Multiplication ×**	**Division ÷**
atlot	enrmai	optcrdu	sitlp
smu	uderec	imset	uleqa cesepi
uspl	cferefiend	pliuetldim	evddidi
omre	esls	tlso of	dshrea
goeettalrh	hgance		raitnocf
xeart	suimn		

Forming Expressions

‘Four more than a number’

$n + 4$

This letter does not have to be n. It could be any letter or symbol. People often use x. We could write ☺ $+ 4$. We are not going to, though. That would be silly.

Write the following sentences algebraically:

- A number add 6

- A number add 10

- A number subtract 10

- 8 subtract a number

Forming Expressions

‘Four lots of a number’

$4n$

We do not tend to use the $\times$ symbol in algebra. Instead we write things next to each other to show multiplication.

Write the following sentences algebraically:

- A number multiplied by 6

- A number multiplied by 10

- a multiplied by b

- $4a$ multiplied by b

- $2a$ multiplied by $2b$

Forming Expressions

'A number divided by 5'

$$\frac{n}{5}$$

We tend not to use ÷ in expressions. We use fraction notation (writing a division as a fraction).

Write the following sentences algebraically:

- A number divided by 6

- 6 divided by a number

- A number divided by $6 + a$

- $6 - a$ divided by a number

Worked Example	Your Turn
Write an algebraic expression for each of the following:	Write an algebraic expression for each of the following:
3 more than a	3 less than a
5 less than a	a more than 5
b multiplied by a	b divided by a
b multiplied by a then squared	b divided by a then squared

Fluency Practice

Question 1: Write an algebraic expression for each of the following

(a) 4 more than c (b) 2 lots of a (c) 3 less than b (d) m divided by 5

(e) 7 multiplied by s (f) w subtract 1 (g) e squared (h) y add 9

(i) m shared between 3 (j) 10 times x (k) k less than 8 (l) 12 less than g

Question 2: Write an algebraic expression for each of the following

(a) c add p (b) f minus m (c) a times b (d) p divided by z

(e) b taken away from u (f) k add n add r (g) w less than c (h) l multiplied by m

(i) y multiplied by m multiplied by a

Question 3: Write an algebraic expression for each of the following

(a) m multiplied by 2 and then add 3 (b) h divided by 4 and then add 7

(c) p squared and then add 10 (d) t add 2 and then multiplied by 5

(e) 9 times e and then add 1 (f) h divided by 3 then add 1

(g) m subtract 6 and then divided by 3 (h) y squared and then multiplied by 4

(i) k multiplied by 4 and then squared (j) a squared and then multiplied by b

Intelligent Practice

Write an algebraic expression for each of the following:

1) 7 more than x
2) 7 less than x
3) 9 less than x
4) 9 lots of x
5) 19 lots of x
6) x divided by 19
7) x shared between 19
8) x less than 19
9) x less than 3
10) x less than y
11) x more than y
12) x multiplied by y
13) x divided by y
14) x divided by 3
15) x divided by 3 and then add 2
16) x divided by 3 and then subtract 2
17) x lots of 3 and then subtract 2
18) x lots of 3 and then squared
19) x lots of 3 squared
20) x squared and then multiply by 3

translation

1) any number add five
2) any number added to six and then times by three
3) any number divided by another
4) any number added to two times another number all multiplied by five
5) any number add ten all squared
6) any number divided by six added to three then all times by five
7) five times any number added to two times another number
8) any number times by three then add six
9) any number times by three then all squared
10) any number add ten then multiply by two
11) one number divided by another number then times by seven
12) any number times by itself then times by three
13) one square number subtract another square number
14) any number add three all divided into another number
15) three times any number then times by two

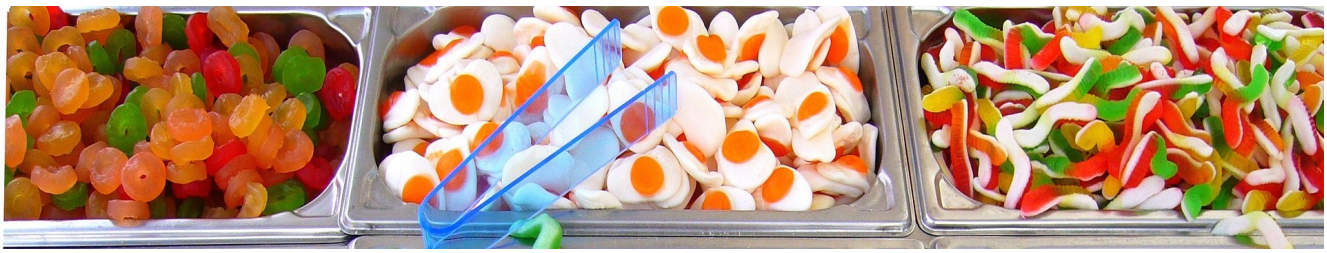
which goes with which?

a) $3(k + 6)$
b) $(d + 10)^2$
c) $5f + 2g$
d) $5\left(\frac{e}{6} + 3\right)$
e) $n + 5$
f) $\frac{7b}{h}$
g) $h^2 - a^2$
h) $3d^2$
i) $6y$
j) $\frac{w}{z}$
k) $(3y)^2$
l) $\frac{w}{k + 3}$
m) $5(b + 2c)$
n) $2(k + 10)$
o) $3m + 6$

Extension

Forming Expressions in Context

Often you will be asked to take a ‘real life’ scenario and turn into mathematical code. For instance



Gummy rings cost 2p per gram, fried eggs cost 3p per gram, gummy snakes cost 4p per gram.

Find an expression for the total cost of x grams of rings, y grams of eggs and z grams of snakes.

$$2x + 3y + 4z$$

Notice that the coefficent of x (the number in front of x) stands for the price of the rings, not the number of them.

Writing Algebraically

	Write the calculation in numbers	Write the expression in algebra
Ex	Becky begins a game with 12 marbles. She wins 3 and loses 5. How many marbles does she have at the end of the game? **12 + 3 − 5 = 10**	Becky begins a game with x marbles. She wins y and loses z. How many marbles does she have at the end of the game? $x + y - z$
1	In a class of 30 children, 10 have a sister. How many children don't have a sister?	In a class of a children, b have a sister. How many children don't have a sister?
2	There were 30 questions in a spelling test. Megan got 25 correct. How many did she get wrong?	There were x questions in a spelling test. Megan got y correct. How many did she get wrong?
3	Lucy went on a journey, going 65 miles by train, 4 miles by bus, and 3 miles on foot. How far did she travel?	Lucy went on a journey, going x miles by train, y miles by bus, and z miles on foot. How far did she travel?
4	Mark buys 3 chocolates bars, each costing 25p. How much does he spend on chocolate bars?	Mark buys a chocolates bars, each costing b pence. How much does he spend on chocolate bars, in pence?
5	Nihal is 11 years old. His sister is 5 years older. How old is his sister?	Nihal is x years old. His sister is y years older. How old is his sister?
6	Maddie has 4 cards. Emma has 10 times as many. How many cards does Emma have?	Maddie has a cards. Emma has b times as many. How many cards does Emma have?
7	Aisha went shopping with £5. She spent £2.50 on a toy and £1 on a bag of sweets. How much did she have left?	Aisha went shopping with x pounds. She spent y pounds on a toy and z pounds on a bag of sweets. How much did she have left, in pounds?
8	Apples cost 15p and bananas cost 10p. Jacob buys 2 apples and 3 bananas. How much does he spend?	Apples cost a pence and bananas cost b pence. Jacob buys x apples and y bananas. How much does he spend, in pence?
9	How far will a car go in 4 hours at 30 miles per hour?	How far will a car go in t hours at v miles per hour?
10	Daniel is 12 years old. Anna is twice as old as Daniel. Grace is three years younger than Anna. How old is Grace?	Daniel is d years old. Anna is c times as old as Daniel. Grace is b years younger than Anna. How old is Grace?

2.3 Collecting Like Terms

Like Terms

Definition

Two or more terms, each with the same variables, to the same power or with the same function applied.

Characteristics

- Any variables must be identical.
- If variables are multiplied, the order listed does not matter.
- If powers or functions are used, then the same power or function must be used.

Examples

- $2x$ and $3x$
- $2y$ and $3y$
- 5 and -2
- d and $3d$
- $2y^2$ and $3y^2$
- $-2y^2$ and $3y^2$
- $\frac{2}{3}x$ and $\frac{4}{5}x$
- $3a^4$ and $5a^4$
- $\frac{3}{5}a^2b$ and $\frac{1}{5}a^2b$
- $3ab$ and $5ab$
- $3ba$ and $5ab$
- $3\sqrt{x}$ and $5\sqrt{x}$

Non Examples

- $2x$ and $3y$
- $2y$ and $3y^2$
- $5x^2$ and $6x^3$
- $3a$ and $5b$
- $3a^2b$ and $5ab^2$

Like Terms

$3p$	p	Like	Unlike
x^2	$3x^2$	Like	Unlike
x^2	$2x$	Like	Unlike
$-3\sqrt{x}$	$27\sqrt{x}$	Like	Unlike
$7a$	$7b$	Like	Unlike

$3a$	$3a$	Like	Unlike
a	$2a$	Like	Unlike
$2a$	$2A$	Like	Unlike
$-3a$	$2a$	Like	Unlike
$4a$	$4b$	Like	Unlike
$3a$	$3a^2$	Like	Unlike
$2a^2$	$7a^2$	Like	Unlike
$-3a^2$	$7a^2$	Like	Unlike
$2a^2$	$2a^{-2}$	Like	Unlike
2^a	a^2	Like	Unlike
x	$\sqrt{x}$	Like	Unlike

Worked Example	Your Turn
Simplify: $a - b + a + b$ $3a + 2b - a + b$	Simplify: $a + b - a + b$ $3a - 2b + a + b$

Intelligent Practice

Simplify:

1) $a + a + a + a$

2) $a + b + a + a$

3) $a + b + a + b$

4) $a + b + a - b$

5) $a + b - a - b$

6) $a - b - a - b$

7) $6a - b - a - b$

8) $6a + b - a - b$

9) $6a + 5b - a - b$

10) $6a + 5b - 4a - b$

11) $6a + 5b - 4a - 3b$

12) $6a - 5b - 4a - 3b$

13) $6a - 5b - 4a + 3b$

14) $6a - 5b + 4a + 3b$

15) $-6a - 5b + 4a + 3b$

16) $-6a - 5b - 4a + 3b$

17) $-6a - 5b - 4a - 3b$

18) $-6a - 5 - 4a - 3$

19) $-6ab - 5 - 4ab - 3$

20) $-5 - 6ab - 3 - 4ab$

Extension

$6a + 5b - 4ab + 3 - 3a + 4b - 5ab + 6$

Extension

a) Simplify $a - 2a + 3a - 4a + 5a - 6a + \cdots + 49a - 50a$
b) What happens if the signs switch?
c) Simplify $a - 2a + 3a - 4a + 5a - 6a + \cdots + 99a - 100a$
d) What generalisations can you make for n terms?

Worked Example	Your Turn
Simplify:	Simplify:
$3a^2 + 2b^2 - a^2 + b^2$	$3a^2 - 2b^2 + a^2 + b^2$
$3a^2 - 2ab^2 + a^2 + ab^2$	$3a^2 + 2ab^2 - a^2 + ab^2$

Intelligent Practice

Simplify:

1) $a^2 + b^2 + a^2 + b^2$

2) $6a^2 + b^2 + a^2 + b^2$

3) $6a^2 + 5b^2 + a^2 + b^2$

4) $6a^2 + 5b^2 + 4a^2 + b^2$

5) $6a^2 + 5b^2 + 4a^2 + 3b^2$

6) $6a^2 + 5b^2 - 4a^2 + 3b^2$

7) $6a^2 + 5b^2 - 4a^2 - 3b^2$

8) $6a^2 - 5b^2 - 4a^2 - 3b^2$

9) $6a^2 - 5b - 4a^2 - 3b$

10) $6a^2 - 5ab - 4a^2 - 3ab$

11) $6a^2 - 5ab - 4a^2 - 3ba$

12) $6a^2 - 5a^2b - 4a^2 - 3a^2b$

13) $6a^2 - 5a^2b - 4a^2 + 3a^2b$

14) $6a^2 - 5a^2b - 4a^2 + 3ab^2$

15) $6a^2 - 5ab^2 - 4a^2 + 3ab^2$

16) $-6a^2 - 5ab^2 - 4a^2 + 3ab^2$

17) $-6 - 5ab^2 - 4 + 3ab^2$

18) $-6a - 5ab^2 - 4a + 3ab^2$

19) $-6ab - 5ab^2 - 4ab + 3ab^2$

20) $-6a^2b - 5ab^2 - 4a^2b + 3ab^2$

<u>Extension</u>

$6a^2 + 5b^2 - 4a^2b + 3ab^2 - 2a^2b^2 + 1 - a^2 + 2b^2 - 3a^2b + 4ab^2 - 5a^2b^2 + 6$

Fluency Practice

Question 1: Simplify each of the following

(a) $y + y + y + y$ (b) $w + w + w + w + w$ (c) $a + a + a + a + a + a$ (d) $s + s + s$

(e) $n + n$ (f) $g + g + g + g - g$ (g) $y + y + y + y - y - y$ (h) $p + p - p - p$

(i) $3y + 2y$ (j) $4a + 3a$ (k) $9k + 5k$ (l) $7m + m$

(m) $15c + 20c$ (n) $6w - 3w$ (o) $10y + 3y - 5y$ (p) $20t - 14t$

(q) $7x - 3x - x$ (r) $8k - 8k$ (s) $7y - 2y + y$ (t) $5u - 4u$

(u) $y^2 + y^2$ (v) $a^2 + a^2 + a^2$ (w) $c^2 + c^2 + c^2 + c^2 + c^2$ (x) $7y^2 + 3y^2$

(y) $2w^2 + 4w^2 + 8w^2$ (z) $6y^2 - 2y^2 + 3y^2$

Question 2: Simplify the following expressions

(a) $4u - 6u$ (b) $8w - 9w$ (c) $4a + 2a - 9a$ (d) $2y - 9y$

(e) $-3g - 2g$ (f) $-4f + 9f$ (g) $-m - 7m$ (h) $5y^2 - 7y^2$

(i) $6a^2 + 2a^2 - 9a^2$ (j) $ab + ab + ab$

Question 3: Simplify the following expressions

(a) $3a + 2b + 4a + b$ (b) $7y + 5y + 2h + 2h$ (c) $g + 8a + 2a + g$

(d) $7m + 7p + 8m + p + 2p$ (e) $9e + 2 + e + 2$ (f) $4 + 3a + 2a + 8$

(g) $2y + 4 + 3y - 1$ (h) $8 + 3w - w - 3$ (i) $5 - 4s - 2 + 10s$

(j) $3x + 6y + 5x - 2y$ (k) $6m - 2s + 11s + m$ (l) $2a + 3b - 2 + a + 3b + 4$

(m) $3a - 2b + a - 5b$ (n) $2x - 2y - 6x + 5y$ (o) $y - 4m - 3y - 5m$

(p) $7p - 2q - q + 3r + 4r$ (q) $11c + 8d - 6c - 11d$

Question 4: Simplify the following

(a) $3y^2 + 4ab + 7y^2 + ab$ (b) $9x^2 - 2x - 11x^2 + 5x$ (c) $7ac - 3ab + 9ab - 7ac$

2.4 Multiplying Terms

Worked Example	Your Turn
Simplify:	Simplify:
$3x \times 5$	$6x \times 2$
$3x \times 5x$	$6x \times 2x$
$3x \times 5y$	$6x \times 2y$
$3x^2y \times 5xy^2$	$6xy^2 \times 2x^2y$

Fluency Practice

Question 1: Simplify the following expressions.

(a) $3 \times y$ (b) $w \times 3$ (c) $7 \times x$ (d) $a \times 4$

(e) $a \times c$ (f) $f \times g$ (g) $h \times d$ (h) $a \times y \times m$

(i) $t \times t$ (j) $p \times p$ (k) $a \times a \times a$ (l) $m \times m \times m$

(m) $4 \times f \times g$ (n) $3 \times w \times y$ (o) $p \times 5 \times s$ (p) $n \times c \times 7$

(q) $t \times c \times w$ (r) $y \times x \times w$ (s) $5 \times a \times a$ (t) $y \times 3 \times y$

Question 2: Simplify the following expressions.

(a) $5 \times 3w$ (b) $4y \times 2$ (c) $3 \times 3m$ (d) $10g \times 3$

(e) $4 \times 2 \times y$ (f) $3 \times 2 \times 2p$ (g) $5 \times 2y \times 3$ (h) $9a \times 2 \times 2$

(i) $3a \times c$ (j) $4y \times z$ (k) $5c \times b$ (l) $c \times 6y$

(m) $2a \times 3y$ (n) $6c \times 3t$ (o) $9w \times 3a$ (p) $2y \times 2g$

(q) $2y \times y$ (r) $5w \times w$ (s) $m \times 3m$ (t) $x \times 2x$

(u) $4t \times 2t$ (v) $6y \times 3y$ (w) $9a \times 9a$ (x) $12y \times 10y$

(y) $2a \times 3p \times 5w$ (z) $10y \times 2p \times 3c \times m$

Question 3: Simplify the following expressions

(a) $a^2 \times a$ (b) $y \times y^2$ (c) $w^2 \times w^2$ (d) $m^2 \times m^3$

(e) $2t^2 \times t$ (f) $4m \times m^2$ (g) $g \times 2g^2$ (h) $p^2 \times 3p^2$

(i) $3p^2 \times 2p$ (j) $2v^2 \times 7v^2$ (k) $9p^2 \times 7p^2$ (l) $5w^2 \times 2w^3$

(m) $7a^3 \times 4a^3$ (n) $6c^4 \times 5c^3$ (o) $aw \times w$ (p) $r \times ry$

(q) $ay \times ay$ (r) $c^2f \times f$ (s) $dg \times d^2$ (t) $3x^2y \times 2x$

(u) $4ab \times 2ab$ (v) $3m^2n^2 \times 4mn$ (w) $2cd^2 \times d^2$ (x) $4a^2bc^2 \times a^3b$

(y) $2ad^2e \times a^3c$ (z) $8m^2n \times 3no^5$

Intelligent Practice

Simplify:

1) $3 \times x$
2) $x \times 3$
3) $x \times y$
4) $x \times x$
5) $2x \times x$
6) $2x \times 3$
7) $3x \times 2$
8) $3x \times 2y$
9) $6x \times 4y$
10) $8x \times 3y$
11) $8x^2 \times 3y$
12) $8x^2 \times 3x$
13) $8x^2 \times 3xy$
14) $8x^3 \times 3y$
15) $8x^3y \times 3xy$
16) $8x^3y \times 3xy^3$
17) $8x^3y \times 3xy^3z$
18) $8xy^3 \times 3xyz^3$
19) $12zy^3 \times 2zyx^3$
20) $12z^2y^3 \times 2z^2y^2x^3$

2.5 Dividing Terms

Worked Example	Your Turn
Simplify:	Simplify:
$15x \div 5$	$12x \div 2$
$15x^2 \div 5x$	$12x^2 \div 2x$
$15xy \div 5y$	$12xy \div 2y$
$15x^3y^3 \div 5xy^2$	$12x^3y^3 \div 2x^2y$

Fluency Practice

Question 1: Simplify the following expressions.

(a) $12x \div 2$ (b) $9y \div 3$ (c) $15a \div 5$ (d) $28c \div 7$

(e) $8m \div 2m$ (f) $10c \div 2c$ (g) $18d \div 3d$ (h) $35m \div 5m$

(i) $5ac \div a$ (j) $6xy \div y$ (k) $7mn \div n$ (l) $20ab \div 2a$

(m) $25xy \div 5y$ (n) $80gh \div 10h$ (o) $27xy \div 3xy$ (p) $32abc \div 8ac$

Question 2: Simplify the following expressions.

(a) $\frac{14c}{2}$ (b) $\frac{56w}{7}$ (c) $\frac{45a}{9a}$ (d) $\frac{105y}{5y}$

(e) $\frac{mw}{m}$ (f) $\frac{8cf}{c}$ (g) $\frac{15xy}{3x}$ (h) $\frac{70ab}{2a}$

(i) $\frac{30ef}{6ef}$ (j) $\frac{20cde}{5cde}$ (k) $\frac{42ghk}{6gh}$

Question 3: Simplify the following expressions.

(a) $h^2 \div h$ (b) $x^3 \div x$ (c) $7y^2 \div y$ (d) $40m^2 \div 2m$

(e) $16c^2 \div 4c$ (f) $20g^2 \div g^2$ (g) $45x^3 \div x$ (h) $30t^3 \div 3t$

(i) $9h^3 \div 3h^2$ (j) $10x^3 \div 5x^3$ (k) $24m^2 \div 3$

Question 4: Simplify the following expressions.

(a) $\frac{g^2}{g}$ (b) $\frac{w^3}{w}$ (c) $\frac{3a^2}{a}$ (d) $\frac{24e^2}{3e}$

(e) $\frac{35c^3}{7c^2}$ (f) $\frac{52c^3}{13c}$ (g) $\frac{100w^3}{10w^3}$

Question 5: Simplify the following expressions

(a) $a^2b^2 \div ab$ (b) $xy^2 \div x$ (c) $4ab^3 \div 2ab^2$ (d) $25c^2d^2 \div 5cd$

(e) $16x^4y^3 \div 4x^2y^2$ (f) $10c^3de^2 \div 2cde$ (g) $15abc^4 \div bc^3$ (h) $24d^3e^9f \div 8d^3f$

Question 6: Simplify the following expressions.

(a) $\frac{a^3c^3}{ac}$ (b) $\frac{10a^4c^3}{2ac^2}$ (c) $\frac{9abc^3}{3ac^2}$ (d) $\frac{45a^5b^8c^4}{3a^3b^4c}$

Intelligent Practice

Simplify:

1) $6x \div 2$

2) $6x \div 3$

3) $6x \div 3x$

4) $\frac{6x}{3x}$

5) $\frac{6xy}{3x}$

6) $\frac{6xy}{3y}$

7) $\frac{6x^2}{3y}$

8) $\frac{6x^2}{3y^2}$

9) $\frac{6x^2}{3x^2}$

10) $\frac{6x^2y^2}{3x^2}$

11) $\frac{6x^3y^2}{3x^2y}$

12) $\frac{12x^3y^2}{6x^2y}$

13) $\frac{12x^2y}{6x^3y^2}$

14) $\frac{12x^2y}{4x^3y^2}$

15) $\frac{12x^2y}{4x^6y^4}$

16) $\frac{4x^2y}{12x^6y^4}$

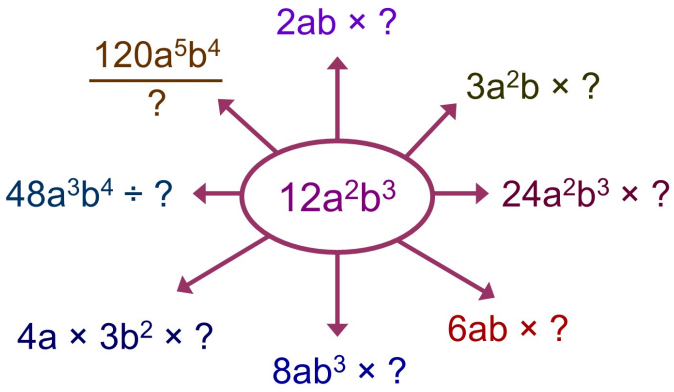
17) $\frac{4x^2y}{8x^6y^4}$

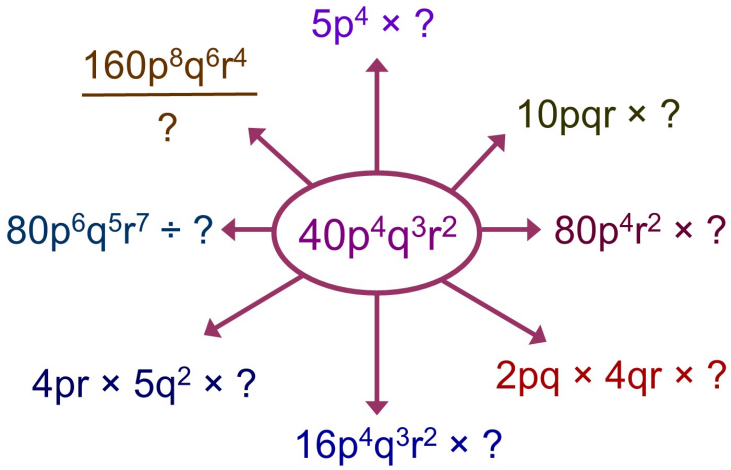
18) $\frac{4x^2y^2z}{8x^6y^4}$

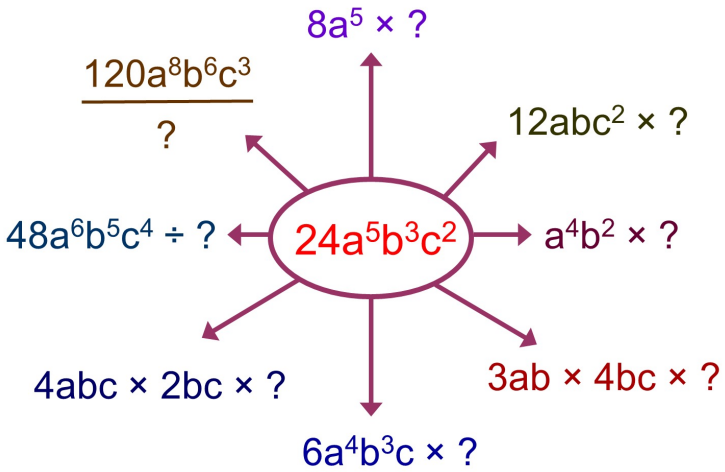
19) $\frac{4x^2y^2z}{8x^6y^4z^2}$

20) $\frac{4z^2y^2x}{8x^6y^4z^2}$

Extension


2ab × ?
120a⁵b⁴ / ?
3a²b × ?
48a³b⁴ ÷ ?
12a²b³
24a²b³ × ?
4a × 3b² × ?
6ab × ?
8ab³ × ?


5p⁴ × ?
160p⁸q⁶r⁴ / ?
10pqr × ?
80p⁶q⁵r⁷ ÷ ?
40p⁴q³r²
80p⁴r² × ?
4pr × 5q² × ?
2pq × 4qr × ?
16p⁴q³r² × ?


8a⁵ × ?
120a⁸b⁶c³ / ?
12abc² × ?
48a⁶b⁵c⁴ ÷ ?
24a⁵b³c²
a⁴b² × ?
4abc × 2bc × ?
3ab × 4bc × ?
6a⁴b³c × ?

2.6 Algebraic Order of Operations

Example

Simplify $\boldsymbol{3a^2 + a \times a}$

Multiplication first: $3a^2 + a \times a = 3a^2 + a^2$

Addition second: $3a^2 + a^2 = \boldsymbol{4a^2}$

Simplify these expressions

1.	$5b + 3b \times 2$	2.	$5b + 2 \times 3b$
3.	$3b \times 2 + 5b$	4.	$3b \times 2 + 5$
5.	$5 + 3b \times 2$	6.	$5 + 2 \times 3b$
7.	$5 + 2b \times 3b$	8.	$5b + 2b \times 3b$
9.	$5b^2 + 2b \times 3b$	10.	$2b \times 3b - b^2$
11.	$5b^2 + 2b \times 3b - b^2$	12.	$2b \times 3b - b^2 + 5b^2$
13.	$2b \times 3b - b \times 5b$	14.	$2b \times 3b + b \times 5b$
15.	$5b^2 + 2 \times 3b - b^2$	16.	$5b^2 + 2 \times 3b - b$
17.	$5b^2 + 3b \times b - 2b^2$	18.	$2b \times 5b + 3b \times b$
19.	$5b^2 + a \times a$	20.	$2a + 5b^2 - a$
21.	$b \times 2b - 5b^2 + 3a \times b - ab$	22.	$b + 6b^2 \div 3b$
23.	$8b \times b + 6b^2 \div 2b$	24.	$\dfrac{10a^2 + 2a \times a}{3a \times 2a}$
25.	$\dfrac{6a + 7 \times 2a}{8a \times 2 + 20a^2 \div 5a}$	26.	$5a^2 \div a - \sqrt{8a^2 + 4a \times 2a}$

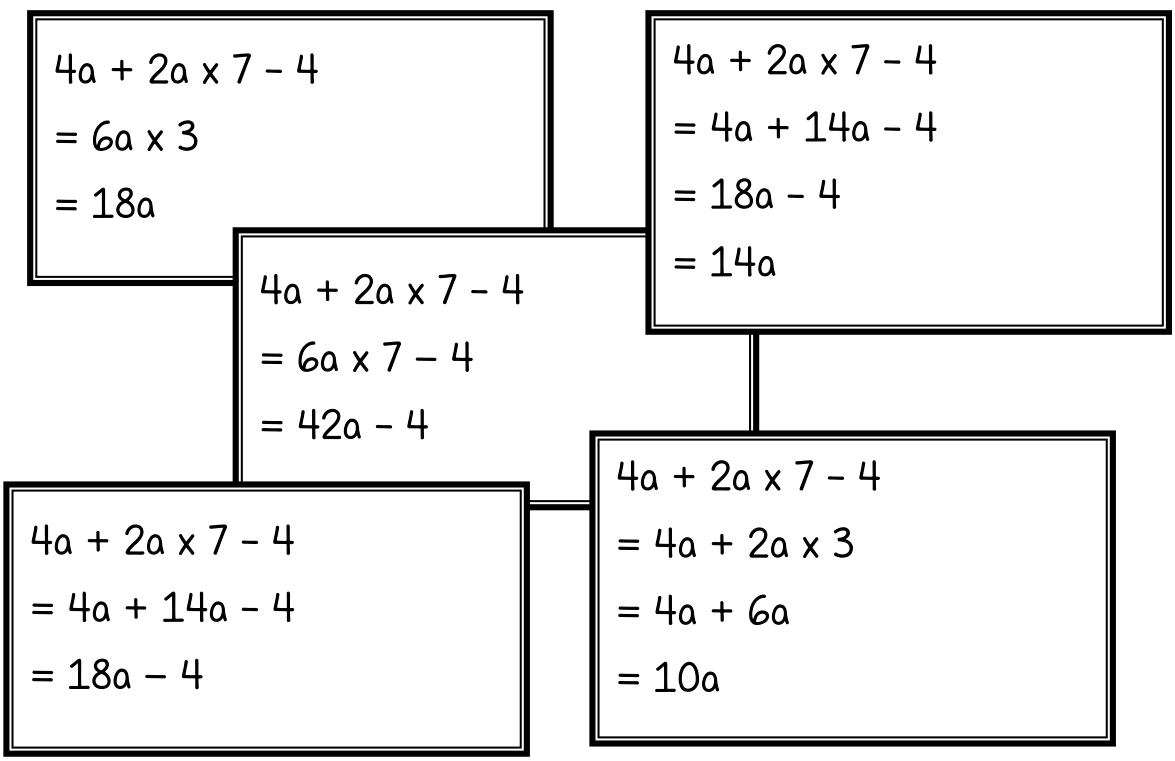
Algebraic Order of Operations

Where required, insert brackets to make these identities true.

Example	$(3a + 2a) \times 4a \equiv 20a^2$ ✓
1.	$5a + 4a \times 2a \equiv 18a^2$
2.	$3 \times a + 4a \times 2a \equiv 30a^2$
3.	$3 \times 2a + 4a \div 2a \equiv 9$
4.	$8 \times 2a + 8a^2 \div 2a \equiv 20a$
5.	$4a \times 2a + 5a^2 \equiv 8a^2 + 20a^3$
6.	$3a \times b + b \equiv 6ab$
7.	$4a + b \times 3a \equiv 12a^2 + 3ab$
8.	$2 \times 2b \times 2b + 3 \times b - b \equiv 16b^2$

Spot the mistake

Identify the errors in these solutions. Can you see how they arrived at their answers? Which one is correct?



2.7 Index Laws

$x^3 \times x^2$
$= (x \times x \times x) \times (x \times x)$
$= x^5$

$x^3 \times x^3$
$= (x \times x \times x) \times (x \times x \times x)$
$= x^6$

$x^3 \times x^4$
$= x^7$

$x^3 \times x^n$
$= x^{3+n}$

$x^m \times x^n$
$= x^{m+n}$

$x^5 \div x$
$= \frac{x^5}{x}$
$= \frac{x \times x \times x \times x}{x}$
$= x^4$

$x^5 \div x^2$
$= \frac{x^5}{x^2}$
$= \frac{x \times x \times x \times x}{x \times x}$
$= x^3$

$x^5 \div x^3$
$= \frac{x^5}{x^3}$
$= x^2$

$x^5 \div x^n$
$= \frac{x^5}{x^n}$
$= x^{5-n}$

$x^m \div x^n$
$= \frac{x^m}{x^n}$
$= x^{m-n}$

$(y^3)^1 = y^3$

$(y^3)^2 = y^3 \times y^3 = y \times y \times y \times y \times y \times y = y^6$

$(y^3)^3 = y^3 \times y^3 \times y^3 = y \times y \times y \times y \times y \times y \times y \times y \times y = y^9$

$(y^3)^4 = y^{12}$

$(y^3)^5 = y^{15}$

$(y^3)^n = y^{3n}$

$(y^m)^n = y^{mn}$

Worked Example	Your Turn
Simplify:	Simplify:
$y^{11} \times y^5$	$x^5 \times x^{-2}$
$6y^3 \times 2y^5$	$7x^5 \times 8x^{-3}$
$y^5 \div y^2$	$y^5 \div y^4$
$8y^3 \div 2y$	$15y^3 \div 3y$
$(y^3)^7$	$(y^7)^8$
$(3y^4)^2$	$(5y^4)^3$

Intelligent Practice

Simplify:

1) $y^{13} \times y^4$

2) $6y^{13} \times 5y^4$

3) $y^{13} \div y^4$

4) $40y^{13} \div 8y^4$

5) $(y^{13})^4$

6) $(3y^{13})^4$

7) $7y^4 \div y^2$

8) $7y^4 \times y^2$

9) $(7y^4)^2$

10) $2x^7 \times 5x^4$

11) $12y^5 \times 5x^4$

12) $12y^5 \div 6y^4$

13) $12y^5 \div 12y^{-4}$

14) $(12y^5)^2$

15) $(12y^{-3})^2$

16) $12y^{-3} \div 4y^2$

17) $12y^{-3} \div 4y^{-2}$

18) $12y^{-3} \times 4y^{-2}$

Worked Example	Your Turn
Simplify: $\frac{a^6 \times a^4}{a^2}$	Simplify: $\frac{a^6 \times a^{-4}}{a^2}$
$(4a^6b^3)^2$	$(2a^6b^3)^4$
$\frac{8a^5b^3}{4ab^7}$	$\frac{12a^2b^3}{4ab^7}$

Intelligent Practice

Simplify:

1) $\frac{a^3 \times a^5}{a^6}$

2) $\frac{a^6}{a^3 \times a^5}$

3) $\frac{x^6}{a^3 \times a^5}$

4) $\frac{12x^6}{2a^3 \times 3a^5}$

5) $\frac{12x^6}{2x^3 \times 3x^5}$

6) $2x^3 \times 3x^5$

7) $2x^3y^2 \times 3x^5y^2$

8) $12x^6y^2 \times 3x^5y^2$

9) $12x^6y^2 \div 3x^5y^2$

10) $\frac{12x^6y^2}{3x^5y^2}$

11) $\frac{12x^6y^8}{3x^5y^2}$

2.8 Review and Problem Solving

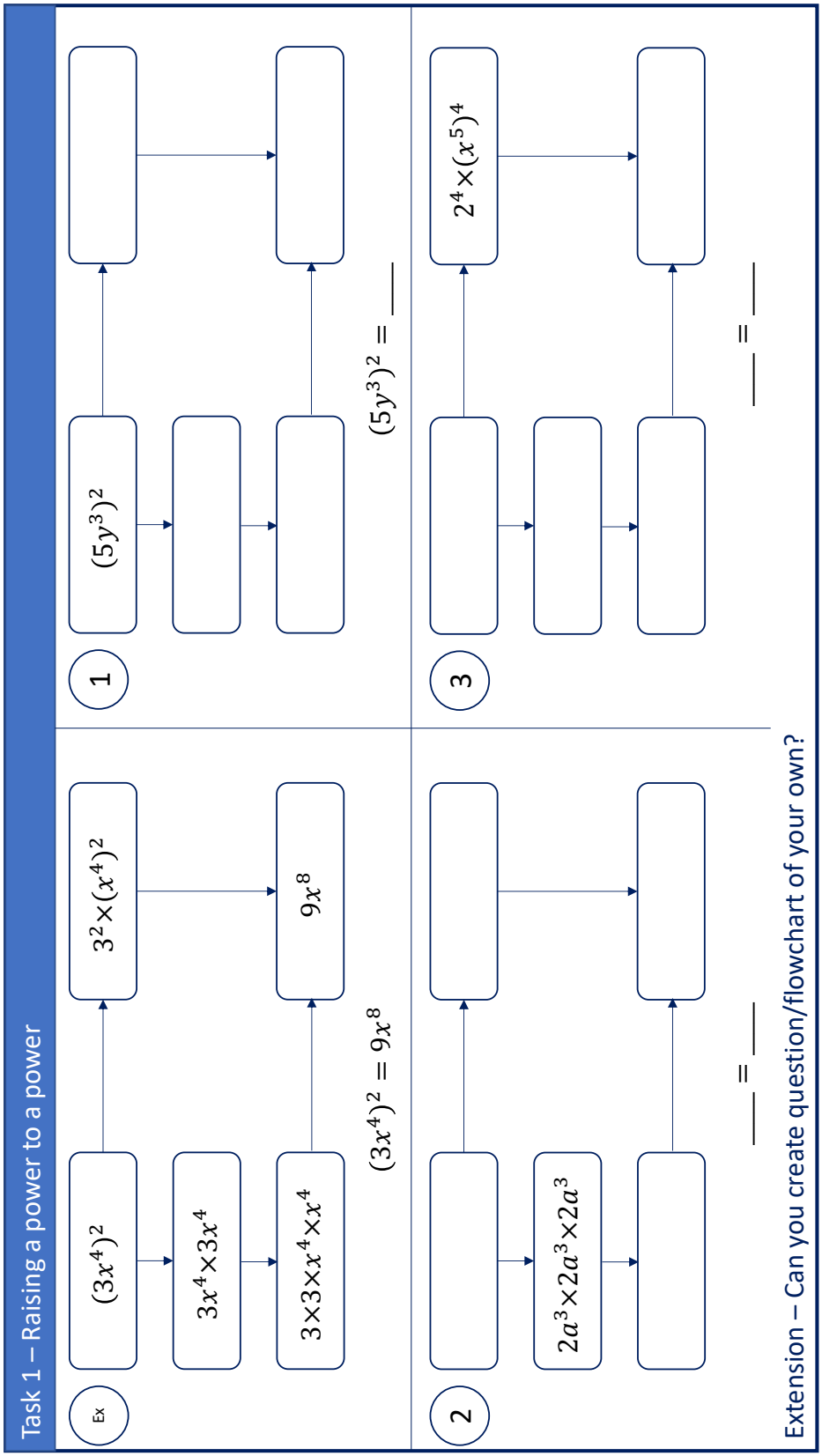
Powers of y Eliminator

Simplify the 31 calculations below, crossing out the corresponding squares in the grid. When you have finished, the remaining squares will reveal a message.

y^6	C	y^{13}	Y	y^{100}	A	y^7	0	$3y^{12}$	R	y^{75}	B
$2y^4$	T	y^{20}	Y	1	A	y^{22}	U	y^{29}	H	y^{11}	S
y^{15}	I	y^4	E	y^{18}	A	y^9	P	y^{33}	T	y^{19}	U
y^{21}	R	y^{16}	Q	$2y^2$	C	y^{28}	E	y^5	X	y^{26}	I
y^{64}	N	y^3	R	y^{24}	K	y^2	Y	y^{32}	B	y^{23}	H
y	D	y^8	M	y^{17}	I	$2y^8$	A	y^{30}	G	y^{27}	G
y^{10}	H	y^{12}	J	$4y^3$	W	y^{14}	F	y^{36}	T	y^{31}	S

1. $y^3 \times y^2$
2. $y^5 \times y^6$
3. $y^7 \times y^2$
4. $y^3 \times y^{16}$
5. $(y^2)^2$
6. $(y^3)^5$
7. $(y^4)^5$
8. $(y^7)^2$
9. $(y^5)^{15}$
10. $y^8 \div y^2$
11. $y^{16} \div y^4$
12. $y^7 \div y^6$
13. $y^{16} \div y^8$
14. $y^{15} \times y^9$
15. $(y^{16})^2$
16. $y^8 \times y^9 \times y^{12}$
17. $y^0 \times y^0$
18. $y \times y^{10} \times y^{20}$
19. $y^0 \times y^2$
20. $y \times (y^7)^9$
21. $y^2 + y^2$
22. $y^{28} \div y^2$
23. $y^4(y^{28} \div y^2)$
24. $y^3 + 3y^3$
25. $y^3(y^{25} \div y^5)$
26. $y \times y^{29} \times y^3$
27. $2(y^2)^4$
28. $3(y^4)^3$
29. $y^0 \times (y^{10})^{10}$
30. $y^0(y^{32} \div y^{16})$
31. $y^4 + (y^2)^2$

Raising a Power to a Power



Task 2 – Raising a power to a power

Ex

$(7x^3y)^2$ → $7^2\times(x^3)^2\times y^7$

$7x^3y\times7x^3y$

$7\times7\times x^3\times x^3\times y\times y$ → $49x^6y^2$

$(7x^3y)^2 = 49x^6y^2$

4

$(10a^2b^5)^2$ → ____

____

____ → ____

$(10a^2b^5)^2 =$ ____

5

____ → ____

$3pq^2\times3pq^2\times3pq^2$

____ → ____

____ = ____

6

____ → $2^5\times(x^2)^5\times y^5$

____

____ → ____

____ = ____

Extension – Can you create question/flowchart of your own?