Year 8
Mathematics
Unit 2 – Student

Name: _____

Class: _____

Contents

1 Equations

- An **expression** is a collection of letters and numbers with no equals sign, for example $3x + 1$

- An **equation** contains an equals sign and an unknown letter to be solved, for example $3x + 1 = 10$

- A **formula** is a relationship between two or more letters and it contains an equals sign, for example $A = bh$

- An **identity** is an equation that is always true, no matter what values are substituted, for example $2x + 3x = 5x$

Decide whether the following are an expression, an equation. a formula, or an identity.		
$y = mx + c$	$2a + b$	$3(b + 4) = 12$
$\frac{y}{5} = 6$	abc	$A = \pi r^2$
$xy = yx$	$2x + 4 = 12$	$x + 2y$
$2a - 5b$	$x + x + x$	$a + bc$
$5x - 6 = 46$	$2x + 4$	$a^2 + b^2 = c^2$
$3(a - 5)$	$C = \pi d$	$2x = x - 5$
$s = \frac{d}{t}$	$2x^2 = 64$	$x + y = y + x$

1.1 Forming Expressions

Form the following expressions starting from x:

$4x - 5$	
$5 - 4x$	
$\frac{x}{4} - 5$	
$\frac{x - 5}{4}$	
$4(x - 5)$	

Intelligent Practice

Form the following expressions starting from x:

- $2x + 3$
- $2x - 3$
- $-2x + 3$
- $3 - 2x$
- $-3 - 2x$
- $\frac{x}{2} + 3$
- $\frac{x}{2} - 3$
- $\frac{x-3}{2}$
- $\frac{x+3}{2}$
- $\frac{4x+3}{2}$
- $\frac{4x-3}{2}$
- $2(4x - 3)$
- $\frac{2(4x-3)}{5}$
- $\frac{2(4x-3)}{5} + 6$
- $\frac{8\left(\frac{2\left(\frac{4x}{7}-3\right)}{5}+6\right)}{9}$

1.2 One Side

To solve an equation means that we find the value of the variable(s).

Strategy: To get x on its own on one side of the equation, we gradually need to 'claw away' the things surrounding it.

Note: In algebra, we tend to give our answers as fractions rather than decimals (unless asked). And never recurring decimals. Don't round also (unless asked).

Worked Example	Your Turn
Solve the following equations: $4x + 17 = 43$ $17 + 4x = 43$	Solve the following equations: $6x + 27 = 53$ $27 + 6x = 43$

Worked Example	Your Turn
Solve the following equations: $4x - 17 = 43$ $17 - 4x = 43$	Solve the following equations: $6x - 27 = 53$ $27 - 6x = 53$

Intelligent Practice

Solve the following equations using the balancing method:

1) $2x + 3 = 22$

2) $22 = 2x + 3$

3) $2x - 3 = 22$

4) $4x - 3 = 22$

5) $4x - 7 = 23$

6) $7 - 4x = 23$

7) $7 - 4x = 25$

8) $7 + 4x = 25$

9) $7 + 4x = -25$

10) $-7 + 4x = -25$

11) $-7 - 4x = -25$

12) $-7 - 6x = -25$

13) $-7 - 6x = -28$

14) $-6x - 7 = -28$

15) $6x + 7 = 28$

16) $6x + 8 = 28$

17) $3x + 4 = 14$

18) $3x + 14 = 4$

Maths Venns

All numbers in the equation are between 3 and -3
If you think a region is impossible to fill, convince me why!
Has a solution x = -9
Equation is of the form ..... = 3
A
B
C
D
E
F
G
H
Create an equation that could belong in each of the regions

Worked Example	Your Turn
Solve the following equations: $4(x+8)=50$ $4(2x+8)=50$	Solve the following equations: $6(x-8)=50$ $6(3x-8)=50$

Worked Example	Your Turn
Solve the following equations: $-4(2x + 8) = 50$ $-4(2x - 8) = 50$	Solve the following equations: $-6(3x + 8) = 50$ $-6(3x - 8) = 50$

Intelligent Practice

Solve the following equations using the balancing method:

1) $2(x+8)=41$

2) $2(x+8)=43$

3) $2(x-8)=43$

4) $3(x-8)=43$

5) $3(2x-8)=43$

6) $6(x-4)=43$

7) $6(x-4)=44$

8) $6(x-4)=48$

9) $6(4-x)=48$

10) $6(4-x)=50$

11) $3(4-x)=25$

12) $3(4-2x)=25$

13) $-3(2x-4)=25$

14) $-3(2x-4)=-25$

15) $-5(2x-4)=-25$

16) $-5(-2x-4)=-25$

17) $5(-2x-4)=-25$

18) $6(-2x-4)=-26$

Worked Example	Your Turn
Solve the following equations: $8(x+3)+3(2x+6)=84$ $8(x+3)-3(2x-6)=84$	Solve the following equations: $3(x-3)+4(2x-6)=110$ $3(x-3)-4(2x-6)=110$

Fluency Practice

Solve the following equations using the balancing method:

1) $2(x+3)+4(x+1)=22$

2) $7(x-3)+5(x+2)=37$

3) $3(x-2)+2(x-5)=14$

4) $2(2x-1)-4(3x-1)=18$

5) $5(2x+3)-6(x-1)=29$

6) $4(5x-2)-6(3x-1)=12$

1.3 Both Sides

- Collect the variable terms (i.e. the terms involving x) on one side of the equation, and the 'constants' (i.e. the individual numbers) on the other side.

- Collect the variable terms on the side of the equation where there's more of them (and move constant terms to other side).

Balancing

- We eliminate the variable from the side with the smaller number of the variable.

- We eliminate the variable by applying the inverse to both sides.

Which side do you eliminate the variable from?
How would you balance both sides?

- $3x + 4 = 2x + 6$

- $2x + 4 = 3x + 6$

- $2x - 4 = 3x - 6$

- $4 - 2x = 3x - 6$

- $4 - 2x = 6 - 3x$

Questions

Which side do you eliminate the variable from?
How would you balance both sides?

1) $2x + 5 = x + 8$

2) $x + 8 = 2x + 5$

3) $8 - x = 2x + 5$

4) $8 - x = 2x - 5$

5) $x + 8 = 2x - 5$

6) $2x + 8 = 4x + 5$

7) $2x + 8 = 5 - 4x$

8) $8 - 2x = 5 - 4x$

9) $8 - 2x = -4x + 5$

10) $8 - 2x = 4x + 5$

Worked Example	Your Turn
Solve the following equations: $5x + 7 = 2x + 31$ $2x - 23 = 7 - x$	Solve the following equations: $5x + 7 = 3x + 23$ $2x - 23 = 12 - 3x$

Fluency Practice

Question 1: Solve the following equations

(a) $4x + 1 = 2x + 7$ (b) $5x + 4 = 3x + 16$ (c) $2x + 8 = x + 12$

(d) $7x + 1 = 2x + 46$ (e) $6x - 3 = 2x + 13$ (f) $9x - 10 = 7x + 24$

(g) $2x + 21 = 4x + 5$ (h) $x + 2 = 5x - 2$ (i) $6x - 9 = 4x - 1$

(j) $5x + 2 = 16 - 2x$ (k) $3x - 1 = 23 - x$ (l) $6x + 8 = 38 - 4x$

(m) $80 - x = 8x - 1$ (n) $2x + 7 = 17 - 8x$ (o) $15 - x = 27 - 3x$

(p) $12x - 20 = 15x - 38$ (q) $35x + 10 = 20x + 175$ (r) $14x = 2x + 60$

Question 2: Solve the following equations

(a) $3x + 3 = x + 8$ (b) $9x + 10 = 7x + 39$ (c) $3x + 1 = 7x - 17$

(d) $x + 4 = 13 - x$ (e) $16x + 3 = 6x + 24$ (f) $9x + 12 = 6x + 14$

(g) $7x + 24 = 12x - 12$ (h) $2x + 9 = 48 - 6x$ (i) $34 - 12x = 28x - 36$

Question 3: Solve the following equations

(a) $4x + 15 = x + 3$ (b) $8x + 40 = 3x + 5$ (c) $9x + 7 = 11x + 20$

(d) $7x + 9 = 2x - 16$ (e) $9x - 70 = 2x - 91$ (f) $4 - 5x = 3x + 28$

(g) $10x + 136 = -8 - 2x$ (h) $-6x + 2 = -4x + 10$ (i) $-11x - 4 = -3x + 60$

Intelligent Practice

Solve the following equations using the balancing method:

1) $5x + 7 = 2x + 13$

2) $9x + 5 = 7x + 15$

3) $7x + 3 = 2x + 28$

4) $10x + 19 = 9x + 9$

5) $7x + 33 = 4x + 18$

6) $3x + 85 = 12x + 4$

7) $2x + 21 = 6x + 5$

8) $6x + 11 = 3x - 4$

9) $6x + 19 = 10x - 5$

10) $8x - 7 = 3x - 32$

11) $7 - 2x = 23 - 4x$

12) $2 - 5x = 110 - 14x$

13) $3 - 6x = 7 - 2x$

14) $17 - 18x = 81 - 10x$

15) $2x + 3 = 10 + 2x$

Maths Venns

Equation with a solution of x = 3
If you think a region is impossible to fill, convince me why!
Equation where one side is 3x + 4
Equation with x-terms on both sides
Create an equation that could belong in each of the regions
A
B
C
D
E
F
G
H

Worked Example	Your Turn
Solve the following equations: $3(x+2)=2(x+3)$ $3(x+5)-7=2(x+2)$	Solve the following equations: $9(x-3)=4(x+7)$ $7(x+6)-7=4(x+2)$

Fluency Practice

Question 4: Solve the following equations

(a) $5(x+3) = 3(x+9)$ (b) $8(x-1) = 4(x+3)$ (c) $3(x+13) = 10(x-1)$

(d) $2(4x-3) = 5(2x-5)$ (e) $9(2x-5) = 3(4x+7)$ (f) $2(9-x) = 3(x+16)$

(g) $5(2x+9) + 2(x+11) = 3(3x+4) + 46$

(h) $8(x-2) - 3(1-x) = 9(x+2) + 1$

Intelligent Practice

Solve the following equations using the balancing method:

1) $3(x+3) = 3(2x-4)$

2) $2(3+x) = 3(2x-4)$

3) $-2(3+x) = 3(2x-4)$

4) $-2(3+x) = -3(2x-4)$

5) $-3(3-x) = 3(2x-4)$

6) $3(-3-x) = 6(2x-4)$

7) $3(x-5)-5 = 5(x+6)$

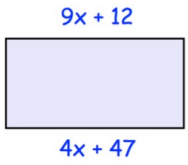
8) $7(x+6)-1 = 4(x+2)$

9) $6(x+2) = 24-3(x-5)$

10) $5(6-x)-3 = 5-3(x-2)$

Extension

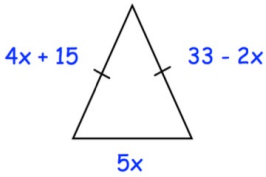
Question 1: Shown is a rectangle



(a) Explain why $9x + 12 = 4x + 47$

(b) Find x

Question 2: Shown is an isosceles triangle



(a) Explain why $4x + 15 = 33 - 2x$

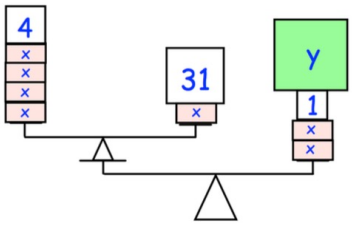
(b) Find x

(c) Find the perimeter of the isosceles triangle

Question 3: Explain why $8x + 3 = 2(4x + 1)$ has no solution.

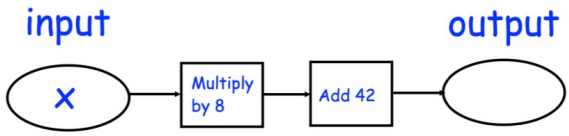
Question 4: (a) Find the value of x

(b) Find the value of y



* The mass of the balances are very small, so may be ignored

Question 5: Shown below is a function machine.
The input and output have the same value.



(a) Write an equation in terms of x.

(b) Solve the equation to find the value of x.

Question 6: Toby has completed his homework.
Can you spot any mistakes?

(a)

Solve $7x - 5 = 5x + 23$

$-5x \quad -5x$

$2x - 5 = 23$

$-5 \quad -5$

$2x = 18$

$\div 2 \quad \div 2$

$x = 9$

(b)

Solve $3x + 11 = 41 - 2x$

$-2x \quad -2x$

$x + 11 = 41$

$-11 \quad -11$

$x = 30$

1.4 Fractions

In general, when solving an equation, we should multiply by the denominator of any fraction, so that we no longer have any fractions lurking about.

Worked Example	Your Turn
Solve the following equations: $\frac{x}{3} + 12 = 49$ $\frac{x + 12}{3} = 49$	Solve the following equations: $\frac{x}{6} - 12 = 49$ $\frac{x - 12}{6} = 49$

Fluency Practice

Question 2: Solve the following equations

(a) $\frac{x}{4} + 1 = 9$ (b) $\frac{x}{2} - 5 = 9$ (c) $\frac{w}{5} + 2 = 3$ (d) $\frac{x}{8} - 7 = 2$

(e) $\frac{m}{3} - 4 = 0$ (f) $\frac{x}{6} + 7 = 2$ (g) $\frac{k}{4} + 5 = -6$ (h) $\frac{x}{6} - 2 = -8$

Question 3: Solve the following equations

(a) $\frac{x+1}{2} = 3$ (b) $\frac{w-4}{3} = 2$ (c) $\frac{x-2}{7} = 6$ (d) $\frac{w+9}{4} = 8$

(e) $\frac{w-25}{3} = -7$ (f) $\frac{x+2}{4} = -1$ (g) $\frac{w+20}{8} = -2$ (h) $\frac{x-9}{4} = -2$

Intelligent Practice

Solve the following equations using the balancing method:

1) $\frac{x}{2} + 4 = 41$

2) $\frac{x+8}{2} = 41$

3) $\frac{x-8}{2} = 41$

4) $\frac{x-8}{3} = 41$

5) $\frac{x}{3} - 8 = 41$

6) $8 - \frac{x}{3} = 41$

7) $8 - \frac{x}{3} = 43$

8) $\frac{8-\mathrm{x}}{3} = 43$

9) $\frac{8-x}{-3} = 43$

10) $\frac{x-8}{3} = 43$

11) $-\frac{x-8}{3} = 43$

12) $\frac{x-8}{3} = -43$

13) $\frac{x-16}{6} = -43$

14) $\frac{x}{6} - 16 = -43$

15) $\frac{-x}{6} - 16 = -43$

16) $\frac{x}{-6} - 16 = -43$

17) $-\frac{x}{6} - 16 = -43$

18) $-\frac{x}{6} - 20 = -50$

Worked Example	Your Turn
Solve the following equations: $\frac{2x}{3} + 12 = 49$ $\frac{2x + 12}{3} = 49$	Solve the following equations: $\frac{5x}{6} - 12 = 49$ $\frac{5x - 12}{6} = 49$

Fluency Practice

Question 4: Solve the following equations

(a) $\frac{2a}{3} = 6$ (b) $\frac{2x}{5} = 4$ (c) $\frac{3x}{10} = 6$ (d) $\frac{7x}{2} = 28$

(e) $\frac{3x}{4} = 12$ (f) $\frac{2x}{9} = -8$ (g) $\frac{3x}{2} = 2$ (h) $\frac{5x}{14} = 3$

Question 5: Solve the following equations

(a) $\frac{3x+5}{2} = 7$ (b) $\frac{5x-12}{3} = 11$ (c) $\frac{4x+2}{6} = 5$ (d) $\frac{10x+3}{4} = 4$

(e) $\frac{5x-8}{2} = 10$ (f) $\frac{8x+4}{5} = 12.8$ (g) $\frac{2x+13}{3} = 1$ (h) $\frac{3x-4}{7} = -4$

(i) $\frac{7x-12}{3} = -25$ (j) $\frac{29-2x}{3} = 5$ (k) $\frac{100-5x}{3} = 30$ (l) $\frac{24-3x}{12} = 5$

Intelligent Practice

Solve the following equations using the balancing method:

1) $\frac{2x}{3} + 4 = 41$

2) $\frac{2x+12}{3} = 41$

3) $\frac{2x-12}{3} = 41$

4) $\frac{2x-12}{5} = 41$

5) $\frac{2x}{5} - 12 = 41$

6) $\frac{2x}{5} - 12 = 42$

7) $12 - \frac{2x}{5} = 42$

8) $\frac{12-2x}{5} = 42$

9) $\frac{12-6x}{5} = 42$

10) $\frac{12-6x}{5} = 43$

11) $\frac{12-6x}{9} = 43$

12) $\frac{4-2x}{3} = 43$

13) $\frac{-4-2x}{3} = 43$

14) $\frac{-4-2x}{-3} = 43$

15) $\frac{4+2x}{3} = 43$

16) $-\frac{4+2x}{3} = -43$

17) $-4 + \frac{2x}{3} = -43$

18) $-43 + \frac{2x}{3} = -4$

Worked Example	Your Turn
Solve the following equation: $\frac{3}{x} + 2 = 6$ $\frac{3}{x+2} = 6$	Solve the following equation: $\frac{15}{x} - 2 = 6$ $\frac{15}{x-2} = 6$

Intelligent Practice

Solve the following equations using the balancing method:

1) $\frac{2}{x} - 6 = 9$

2) $\frac{2}{x} - 7 = 9$

3) $\frac{3}{x} - 7 = 9$

4) $\frac{3}{x} + 7 = 9$

5) $7 + \frac{3}{x} = 9$

6) $7 - \frac{3}{x} = 9$

7) $\frac{3}{x-1} = 9$

8) $\frac{3}{2x-1} = 9$

9) $\frac{3}{2(x-1)} = 9$

10) $\frac{2}{3(x-1)} = 9$

Worked Example	Your Turn
Solve the following equations: $\frac{3x+6}{2} = x+3$ $\frac{3x+6}{x+3} = 2$	Solve the following equations: $\frac{9x-27}{4} = x+7$ $\frac{7x-21}{x+7} = 2$

Fluency Practice

Question 6: Solve the equations below

(a) $\frac{2x+1}{3} = x-2$

(b) $\frac{5x-3}{2} = 2x+4$

(c) $\frac{x+17}{5} = x+1$

(d) $4x-9 = \frac{2x+3}{3}$

(e) $\frac{9-x}{2} = x+3$

(f) $\frac{15-2x}{3} = 2x-3$

Question 7: Solve the equations below

(a) $\frac{2x-1}{x+3} = 9$

(b) $\frac{x+11}{2x-5} = 2$

(c) $\frac{x+1}{x+4} = 3$

(d) $\frac{5x-24}{x-4} = 3$

(e) $\frac{x+7}{x-3} = -4$

(f) $\frac{3x+8}{9-x} = -2$

1.5 Cross Multiplication

You can cross multiply to solve equations which are in the form:
$\frac{a}{b} = \frac{c}{d}$

Are the following equations ready to be cross multiplied?

- $\frac{2x}{3} = \frac{5}{9}$

- $\frac{2x}{3} + 1 = \frac{5}{9}$

- $\frac{2x}{3} + 1 = 5$

- $\frac{2x+1}{3} = 5$

- $\frac{3}{2x+1} = \frac{5}{x}$

Questions

Are the following equations ready to be cross multiplied?

1) $\frac{x}{5} + 2 = \frac{4}{5}$

2) $\frac{2x}{5} + 2 = \frac{4}{5}$

3) $\frac{x}{5} = \frac{4}{5}$

4) $\frac{2x}{5} = \frac{4}{5}$

5) $\frac{x}{5} = 4$

6) $\frac{x}{5} + 2 = 4$

7) $\frac{x+2}{5} = 4$

8) $\frac{2x}{5} + 1 = \frac{4}{5}$

9) $\frac{5}{2x} + 1 = \frac{4}{5}$

10) $\frac{5}{2x+1} = \frac{4}{5}$

11) $\frac{5}{2x} + 1 = \frac{4}{5x}$

12) $\frac{5}{2x+1} = \frac{4}{5x}$

Worked Example	Your Turn
Solve the following equations: $\frac{x}{5} = \frac{3}{2}$ $\frac{x+1}{5} = \frac{3}{2}$	Solve the following equations: $\frac{2x}{5} = \frac{3}{2}$ $\frac{2x+1}{5} = \frac{3}{2}$

Fluency Practice

Question 1: Solve each equation below

(a) $\frac{x}{7} = \frac{3}{2}$ (b) $\frac{a}{6} = \frac{3}{10}$ (c) $\frac{c}{20} = \frac{7}{8}$ (d) $\frac{w}{13} = \frac{3}{4}$

(e) $\frac{7}{10} = \frac{c}{8}$ (f) $\frac{4}{5} = \frac{d}{9}$ (g) $\frac{2}{11} = \frac{5}{h}$ (h) $\frac{14}{p} = \frac{3}{20}$

(i) $\frac{2a}{5} = \frac{3}{2}$ (j) $\frac{5x}{11} = \frac{9}{2}$ (k) $\frac{4a}{9} = \frac{6}{5}$ (l) $\frac{7}{4} = \frac{2y}{3}$

(m) $\frac{25}{4y} = \frac{1}{10}$ (n) $\frac{35}{8} = \frac{5w}{12}$ (o) $\frac{10}{17} = \frac{9}{10w}$ (p) $\frac{1}{3y} = \frac{2}{5}$

Intelligent Practice

Solve the following equations using cross multiplication:

1) $\frac{x}{7} = \frac{3}{2}$

2) $\frac{2x}{7} = \frac{3}{2}$

3) $\frac{2x}{7} = \frac{3}{14}$

4) $\frac{2x}{14} = \frac{3}{14}$

5) $\frac{2x}{14} = \frac{3}{7}$

6) $\frac{14}{2x} = \frac{7}{3}$

7) $\frac{14}{2x+1} = \frac{7}{3}$

8) $\frac{14}{2x-1} = \frac{7}{3}$

9) $\frac{7}{2x-1} = \frac{7}{3}$

10) $\frac{2x-1}{7} = \frac{7}{3}$

11) $\frac{2x-1}{6} = \frac{7}{3}$

12) $\frac{1-2x}{6} = \frac{7}{3}$

Worked Example	Your Turn
Solve the following equations: $\frac{3x-4}{5}=\frac{x+4}{3}$ $\frac{4}{2-3x}=\frac{5}{6-2x}$	Solve the following equations: $\frac{x+4}{7}=\frac{x-4}{3}$ $\frac{4}{2+3x}=\frac{5}{6+2x}$

Fluency Practice

Question 2: Solve the following equations

(a) $\frac{x+1}{5} = \frac{3}{2}$ (b) $\frac{x+3}{4} = \frac{2}{3}$ (c) $\frac{2x+1}{5} = \frac{7}{2}$

(d) $\frac{3x-1}{5} = \frac{x+11}{2}$ (e) $\frac{x+3}{4} = \frac{2x-1}{7}$ (f) $\frac{3x-4}{3} = \frac{2x-1}{4}$

(g) $\frac{2}{3x-5} = \frac{1}{x+8}$ (h) $\frac{x+2}{2x+3} = \frac{1}{5}$ (i) $\frac{9}{4} = \frac{2x+1}{x-1}$

Question 3: Solve the following equations

(a) $\frac{2}{x+2} = \frac{x+1}{3}$ (b) $\frac{x+1}{3} = \frac{1}{x-1}$ (c) $\frac{4}{x+5} = \frac{x-2}{2}$

(d) $\frac{x-3}{2} = \frac{2}{2x+1}$ (e) $\frac{5}{2x+3} = \frac{3x+1}{2}$ (f) $\frac{2}{7x-1} = \frac{x+3}{12}$

Fluency Practice

Solve the following equations using the balancing method:

1) $\frac{3x-5}{5} = \frac{2x+4}{7}$

2) $\frac{4x-1}{3} = \frac{3x-2}{2}$

3) $\frac{2(x+3)}{5} = \frac{3x-2}{2}$

4) $\frac{14}{2x-1} = \frac{7}{3x-2}$

5) $\frac{6}{2x-1} = \frac{7}{3x-2}$

6) $\frac{6}{1-2x} = \frac{12}{7x-14}$

7) $\frac{3}{7x-1} = \frac{5}{13-14x}$

Extension

section (i)

(1) $20 - x = \frac{1}{3}x$

(2) $\frac{1}{2}x = 12 - x$

(3) $\frac{1}{2}x = \frac{1}{3}x + 5$

(4) $7(\frac{1}{2}x + 6) = 14x$

(5) $4(\frac{1}{3}x + 8) = 12x$

(6) $\frac{1}{2}x = 3(2x - 11)$

(7) $4(3x - 35) = \frac{1}{3}x$

(8) $\frac{1}{3}x + 2 = \frac{1}{2}x + 1$

section (ii)

(1) $\frac{1}{3}x + 5 = \frac{1}{2}x + 3$

(2) $\frac{1}{4}x + 3 = \frac{1}{2}x + 1$

(3) $\frac{1}{3}x - 2 = \frac{1}{5}x + 2$

(4) $\frac{1}{3}x - 18 = \frac{1}{4}x - 13$

(5) $20 - \frac{1}{3}x = 15 - \frac{1}{2}x$

(6) $\frac{1}{2}x - 11 = \frac{1}{11}x + 7$

(7) $\frac{1}{6}x + 4 = \frac{1}{3}x - 2$

(8) $\frac{1}{4}x - 7 = \frac{1}{5}x - 5$

section (iii)

(1) $\frac{3(2x + 3)}{5} = 3(x - 3)$

(2) $\frac{3x + 2}{4} = ½x + 3$

(3) $5(\frac{1}{3}x + 1) = 3(\frac{1}{2}x + 2) + 1$

(4) $\frac{1}{2}(\frac{1}{6}x - 1) = \frac{1}{5}(\frac{1}{3}x - 1)$

(5) $\frac{1}{2}(\frac{1}{5}x - 1) = \frac{1}{5}(\frac{1}{3}x + 2)$

(6) $\frac{2x - 1}{5} + 3 = \frac{5x - 2}{3} + 1$

1.6 Forming and Solving Equations

Worked Example	Your Turn
I think of a number. I multiply the number by 6 then subtract 3. The result is 15. What was my original number?	I think of a number. I multiply the number by 4 then subtract 5. The result is 27. What was my original number?

Worked Example	Your Turn
A is x years old. B is 3 years older than A. C is twice as old as A. The sum of the ages of A, B and C is 51. What are their ages?	A is x years old. B is 3 years younger than A. C is three times as old as A. The sum of the ages of A, B and C is 57. What are their ages?

Fluency Practice

Question 1: I think of a number.
I multiply the number by 3 and then add 5.
The answer is 29.

(a) Form an equation in terms of x.
(b) Solve the equation to find the original number.

Question 2: I think of a number.
I multiply the number by 5 and then subtract 2.
The answer is 58.

(a) Form an equation in terms of x.
(b) Solve the equation to find the original number.

Question 3: I think of a number.
I divide the number by 2 and then add 1.
The answer is 7.

(a) Form an equation in terms of x.
(b) Solve the equation to find the original number.

Question 4: Gregory is x years old.
Daisy is 2 years older than Gregory
The sum of their ages is 40.

(a) Form an equation in terms of x
(b) Solve the equation and work out Gregory's and Daisy's ages.

Question 5: Robert is x years old.
Hannah is 7 years younger than Robert
The sum of their ages is 61.

(a) Form an equation in terms of x
(b) Solve the equation and work out Robert's and Hannah's ages.

Question 6: Michael is x years old.
Jenny is twice as old as Michael
The sum of their ages is 57.

(a) Form an equation in terms of x
(b) Solve the equation and work out Michael's and Jenny's ages.

Fluency Practice

Question 7: Fiona is x years old.
Thomas is 3 years older than Fiona.
Cara is twice as old as Fiona.
The sum of their ages is 51.

(a) Form an equation in terms of x
(b) Solve the equation and work out Fiona's, Thomas's and Cara's ages.

Question 8: Alan is x years old.
Barry is ten years younger than Alan.
Kevin is double Alan's age.
The sum of their ages is 54.

(a) Form an equation in terms of x
(b) Solve the equation and work out Alan's, Barry's and Kevin's ages.

Question 9: Rebecca is x years old.
Mary is 8 years older than Rebecca.
Jill is three times older than Mary.
The sum of their ages is 67.

(a) Form an equation in terms of x
(b) Solve the equation and work out Rebecca's, Mary's and Jill's ages.

Question 10: Andy has x pence.
Kelly has 7 pence more than Andy.
Georgia has 9 pence less than Andy.
The total amount of money they have is £1.48

(a) Form an equation in terms of x
(b) Solve the equation and work out how much money each has.

Question 11: Billy has x pounds.
Liam has twice as much money as Billy.
Nicola has three times as much money as Liam.
The total amount of money they have is £180

(a) Form an equation in terms of x
(b) Solve the equation and work out how much money each has.

Fluency Practice

Question 12: Farmer Jones has x sheep
Farmer Smith has 100 more sheep than Farmer Jones.
Farmer White has twice as many sheep as Farmer Jones.
In total there are 2500 sheep.

(a) Form an equation in terms of x
(b) Solve the equation and work out how many sheep each farmer has.

Question 13: The cost of a TV is £x
The cost of a DVD player is £45 less than a TV.
The total cost of the TV and DVD player is £235

(a) Form an equation in terms of x
(b) Find the cost of a TV

Question 14: The sum of three consecutive numbers is 51.

(a) Form an equation in terms of x
(b) Solve the equation and work out each number.

Hint:
Three consecutive numbers can be written as $n, n+1, n+2$

Question 15: The sum of five consecutive numbers is 110.

(a) Form an equation in terms of x
(b) Solve the equation and work out each number.

Question 16: A rectangular field is 7 metres longer than wide.
The perimeter of the field is 106m.

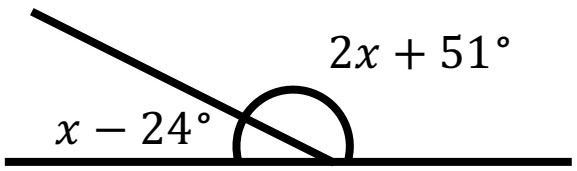
(a) Find the dimensions of the field.
(b) Find the area of the field.

Question 17: A rectangular field is 20 metres longer than wide.
The perimeter of the field is 280m.

(a) Find the dimensions of the field.
(b) Find the area of the field.

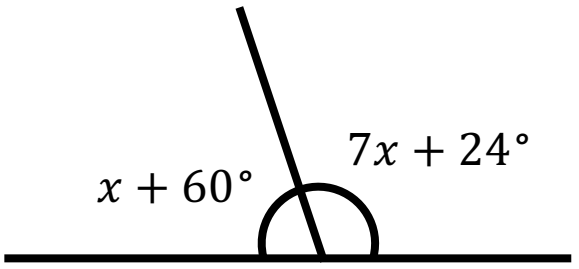
Worked Example

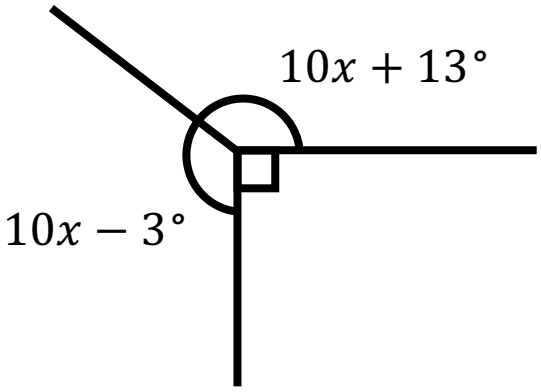
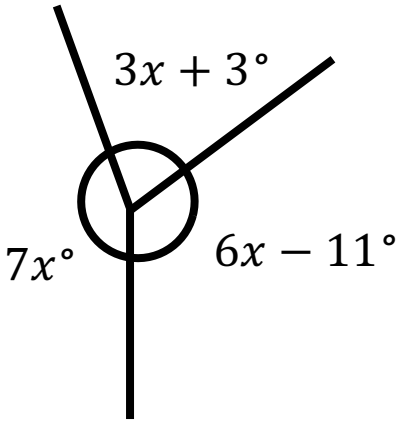
Find x



Your Turn

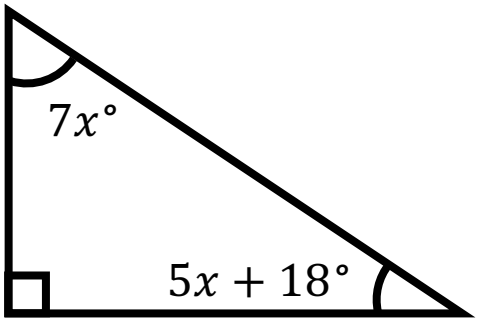
Find x



Worked Example	Your Turn
Find x 	Find x 

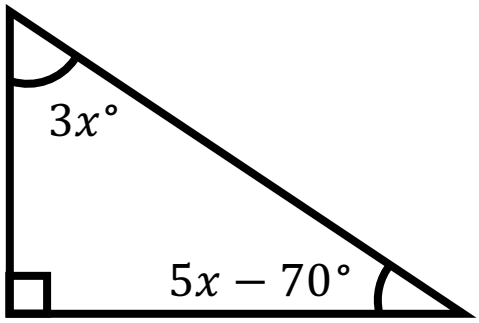
Worked Example

Find x



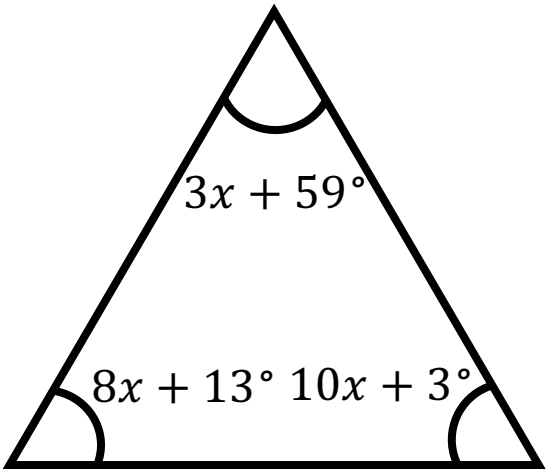
Your Turn

Find x



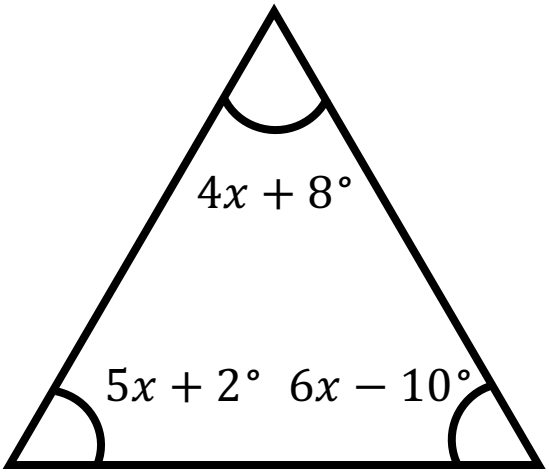
Worked Example

Find x



Your Turn

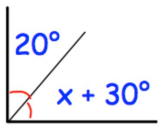
Find x



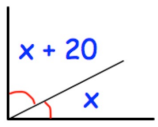
Fluency Practice

Question 4: Calculate x in each of these diagrams.

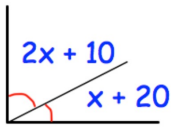
(a)



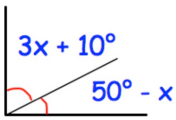
(b)



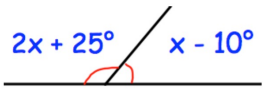
(c)



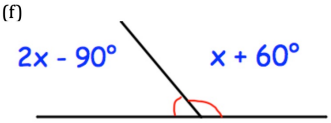
(d)



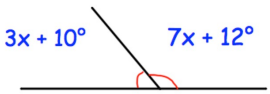
(e)



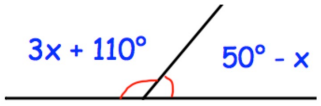
(f)



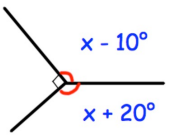
(g)



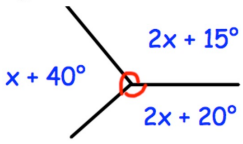
(h)



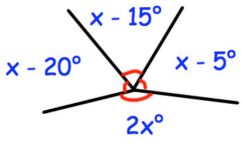
(i)



(j)

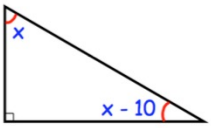


(k)

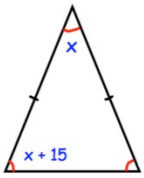


Question 5: Calculate x in each of these diagrams.

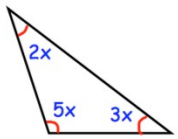
(a)



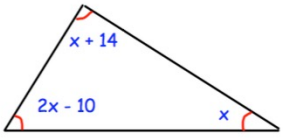
(b)



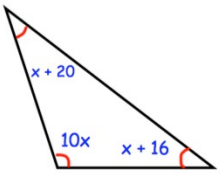
(c)



(d)

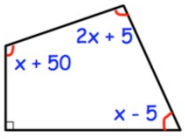


(e)

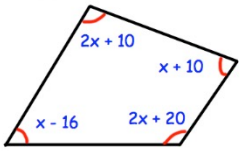


Question 6: Calculate x in each of these diagrams.

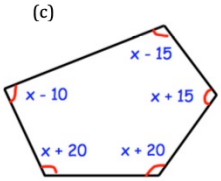
(a)



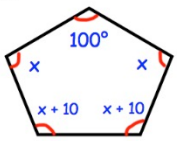
(b)



(c)



(d)

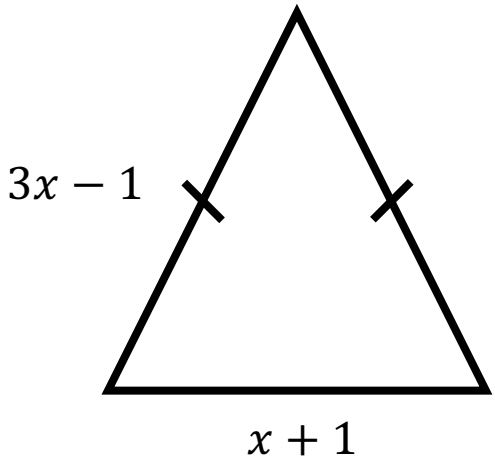


Hint:
Angles in a pentagon add up to 540°

Worked Example	Your Turn
The perimeter of the rectangle is equal to 72 square units. Find x. $2x + 3$  x	The perimeter of the rectangle is equal to 72 square units. Find x. $4x + 6$  x

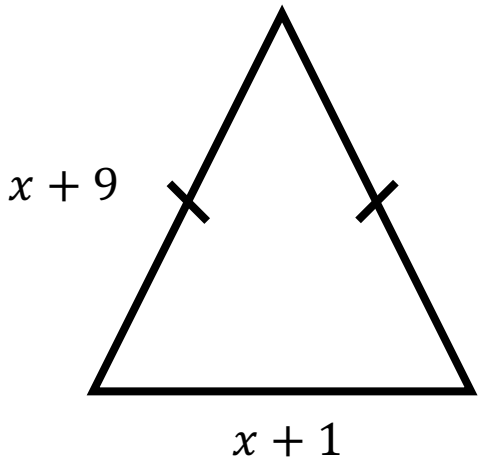
Worked Example

The perimeter of the isosceles triangle is equal to 34 square units. Find x.



Your Turn

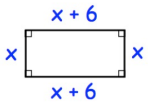
The perimeter of the isosceles triangle is equal to 34 square units. Find x.



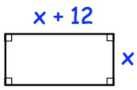
Fluency Practice

Question 1: For each rectangle below, find x. Then use x to find the length and width of each. The expressions for lengths and widths are in centimetres.

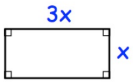
(a) Perimeter = 24cm



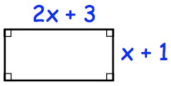
(b) Perimeter = 56cm



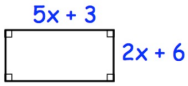
(c) Perimeter = 88cm



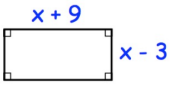
(d) Perimeter = 38cm



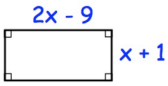
(e) Perimeter = 158cm



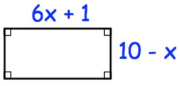
(f) Perimeter = 60cm



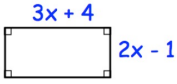
(g) Perimeter = 2m



(h) Perimeter = 82cm

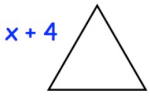


(i) Perimeter = 40cm

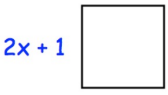


Question 2: Each shape below is regular. For each, find x and the length of each side.

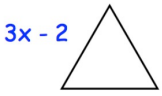
(a) Perimeter = 27cm



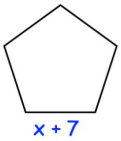
(b) Perimeter = 52cm



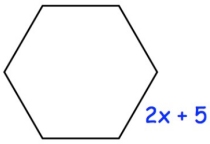
(c) Perimeter = 93cm



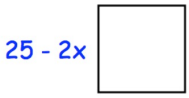
(d) Perimeter = 75cm



(e) Perimeter = 102cm

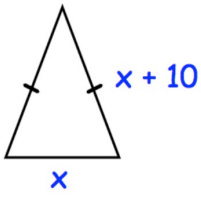


(f) Perimeter = 64cm

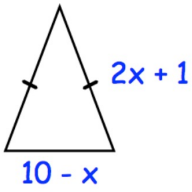


Question 3: For each shape below, find x.

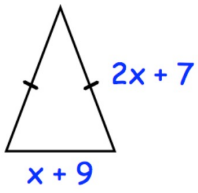
(a) Perimeter = 38cm



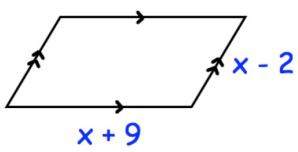
(b) Perimeter = 30cm



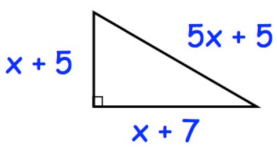
(c) Perimeter = 85.5cm



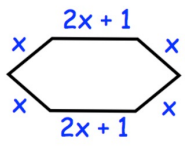
(d) Perimeter = 46cm



(e) Perimeter = 24cm

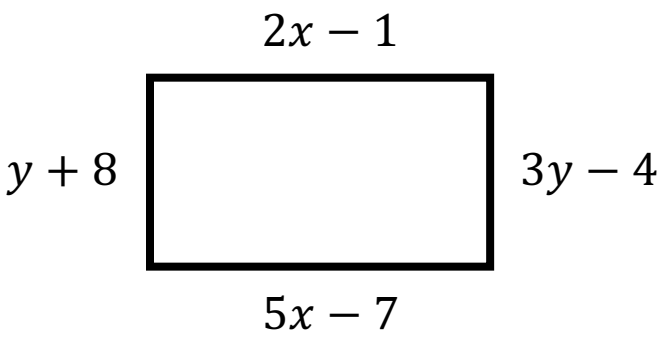


(f) Perimeter = 162cm



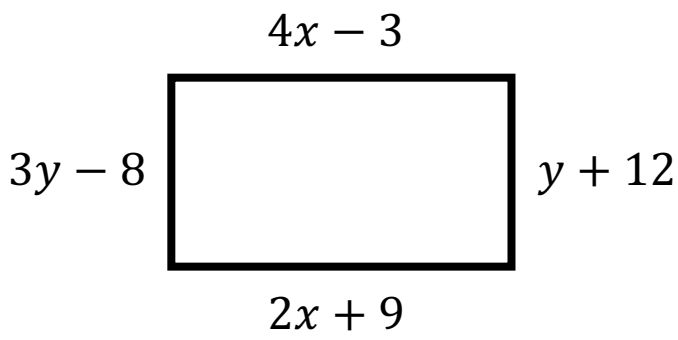
Worked Example

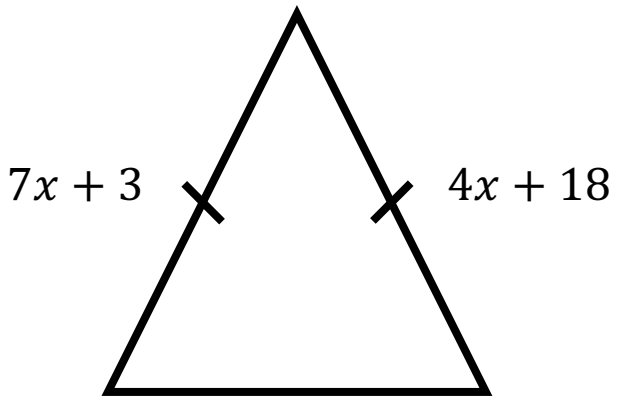
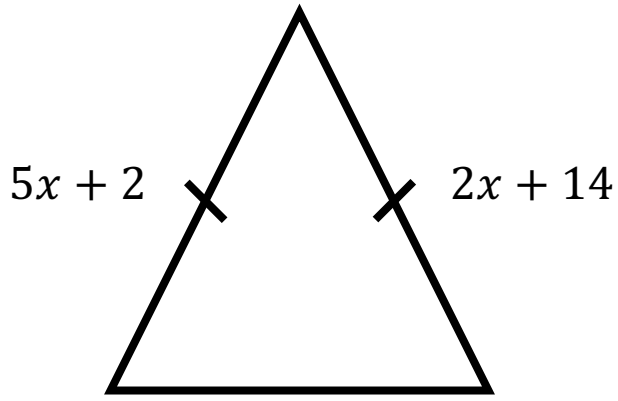
Find x and y



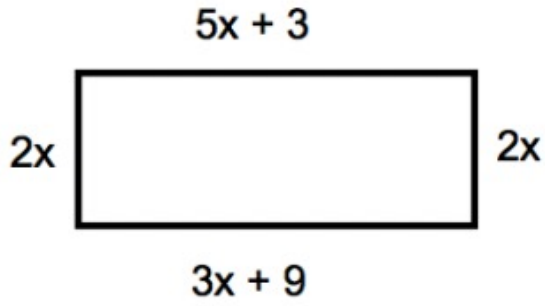
Your Turn

Find x and y



Worked Example	Your Turn
Find x 	Find x 

Question



The diagram shows a rectangle. The sides are measured in centimetres.

(a) Explain why $5x + 3 = 3x + 9$

.....

.....

(1)

(b) Solve $5x + 3 = 3x + 9$

x =.....cm

(2)

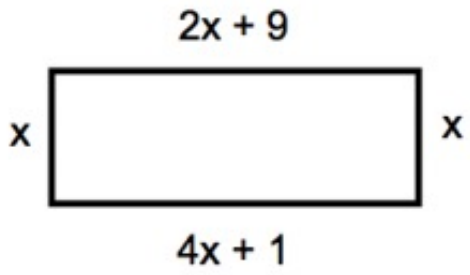
(c) Calculate the perimeter of the rectangle.

.....cm

(2)

Question

A rectangle is shown below.



(a) Explain why $4x + 1 = 2x + 9$

.....

.....

(1)

(b) Find the size of x.

x = ...cm

(2)

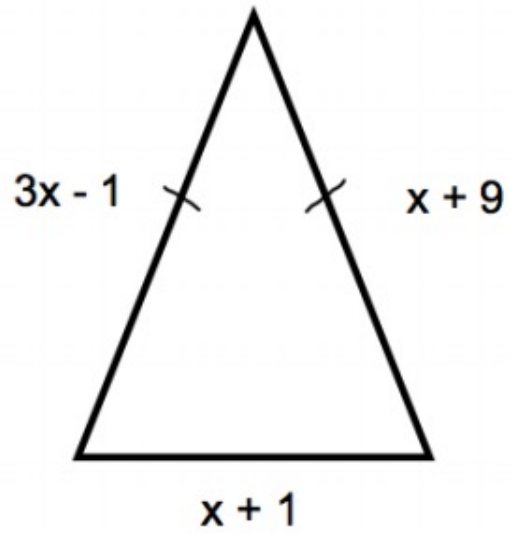
(c) Work out the area of the rectangle.

.....cm^2

(2)

Question

Shown below is an isosceles triangle. Each side is measured in centimetres.



(a) Explain why $3x - 1 = x + 9$

.....

.....

(1)

(b) Solve the equation above.

x = ...cm

(2)

(c) Calculate the perimeter of the triangle.

.....cm

(2)

Extension

Question 1: A rectangle is 5cm longer than it is wide.
The perimeter of the rectangle is 86cm.
How long is the rectangle?

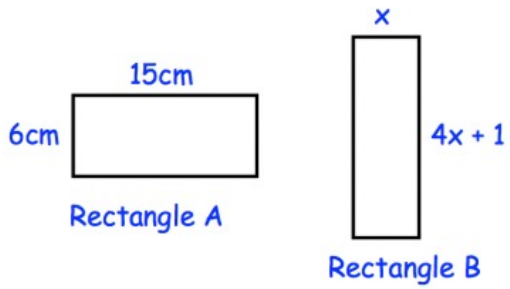
Question 2: A rectangle is 8cm longer than it is wide.
The perimeter of the rectangle is 72cm.
How wide is the rectangle?

Question 3: Each side of an isosceles triangle is 4cm longer than the base.
The perimeter of the triangle is 44cm.
How long is the base of the triangle?

Question 4: Three angles form a straight line.
The two smaller angles are equal.
The largest angle is three times larger than each of the smaller angles.
Work out the size of each of the three angles.

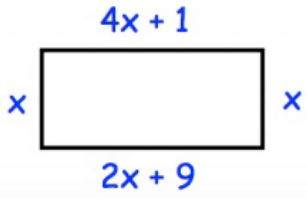
Question 5: Simon draws a right angled triangle.
One angle is 90°.
Of the other two angles, one is 22° larger than the other.
Work out the size of the smallest angle.

Question 6: Below both rectangles have the same perimeter.
Find the area of rectangle B.

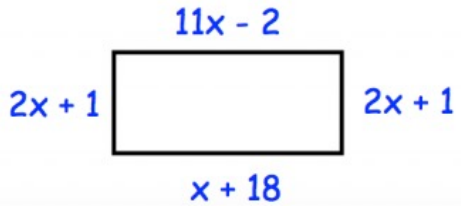


Question 7: Find the perimeters of the rectangles below.
The expressions for the lengths and widths are in centimetres.

(a)



(b)



1.7 Review and Problem Solving

Fluency Practice

Solve the equations and leave your answers as simplified fractions or as decimals.

Section A

1) $\frac{2x+5}{3}=11$

2) $\frac{8-3x}{2}=5$

3) $\frac{5-9x}{6}=-2$

4) $\frac{7x+6}{3}-9=-12$

5) $8x+\frac{1-4x}{8}=7$

6) $\frac{5}{x}=-6$

7) $\frac{11}{4x}+9=3$

8) $5-\frac{3x}{4}=8x$

9) $2+\frac{4x}{3}-7=1$

10) $4-\frac{3x}{2}=3x+5$

11) $6-\frac{2}{x}=10$

12) $4-\frac{2x}{9}+x=-1$

Section B

1) $4(2x-3)=8(2x+5)$

2) $3(4x-5)=5(2x-5)$

3) $8(6x+2)=5(x-2)$

4) $2(3x-4)=7(11-2x)$

5) $7(5-x)=-4(x-11)$

6) $-4(x-8)=-6(4+3x)$

7) $7(4-3x)=2(8x-9)+6$

8) $-6(3-4x)+2x=8(x+11)$

9) $3(2x-6)=3-4(3-x)$

10) $9(2x-1)-3x=3(12+x)$

11) $4x-(2x-8)=5(1+2x)$

12) $10-6(8x-2)=9x-(3+4x)$

Section C

1) $\frac{5x-2}{3}=\frac{4x+1}{2}$

2) $\frac{7x-8}{5}=\frac{2x+5}{4}$

3) $\frac{-8x-1}{2}=\frac{5-3x}{6}$

4) $\frac{5(x+11)}{3}=\frac{3(1+x)}{2}$

5) $\frac{3(2+5x)}{4}=\frac{2(6x-3)}{5}$

6) $\frac{2(3x-5)}{3}=\frac{-4(x-2)}{7}$

7) $\frac{1}{2}(2x-6)=\frac{1}{4}(8-12x)$

8) $\frac{1}{2}(5x+7)=\frac{3}{4}(3x-1)$

9) $\frac{5}{3x+1}=12$

10) $\frac{x+2}{x+3}=4$

11) $\frac{2x-9}{3x-2}=-3$

12) $\frac{2}{3x+10}=\frac{1}{x-1}$

13) $\frac{2}{7x+3}=\frac{9}{2x-5}$

14) $\frac{8}{6x+12}=-\frac{11}{7x-10}$

forming equations

1. two numbers add up to (sum to) 46
the smaller is 4 less than the larger
find the numbers

2. the sum of two numbers is 66
the larger is twice the smaller
find the numbers

3. the sum of three numbers is 42
one number is 9 more than another
two numbers are the same
find the numbers

4. the sum of two numbers is 57
the larger is 3 less than twice the smaller
find the numbers

5. the perimeter of a rectangle is 40m
the length is 1 less than twice the width
find the size of the rectangle

6. two of the same planks overlap by 22 cm
with this overlap the overall length is 58cm
what is the length of each plank?

7. start with a number, double this
add the next-door along number to this double (the next consecutive number along) and you get 101
what is the start number?

8. the mean of three numbers is 12
the range is 6
there is a mode
find the numbers

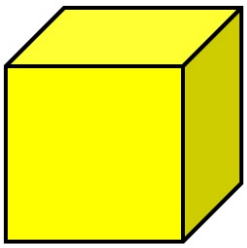
9. of the three angles in a triangle, one is double the smallest and the other is 40° more than the smallest
what are the three angles?

10. Julie is twice as old as Andrew and Carlo is 5 years older than Julie
altogether their ages come to 40 years
what are their three ages?

2 Volume and Surface Area

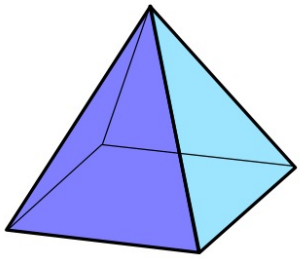
2.1 3D Shapes

Cube

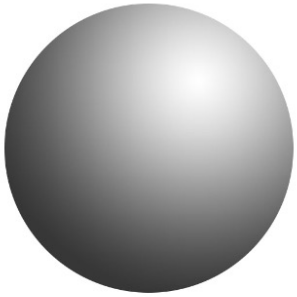


Cuboid

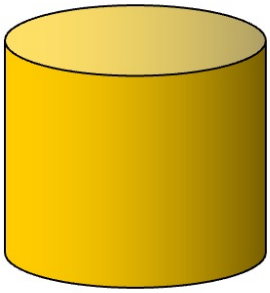
Square-based pyramid



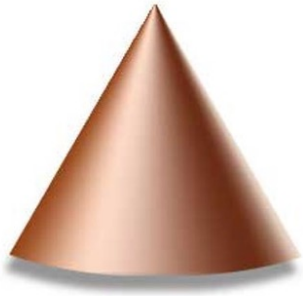
Sphere



Cylinder



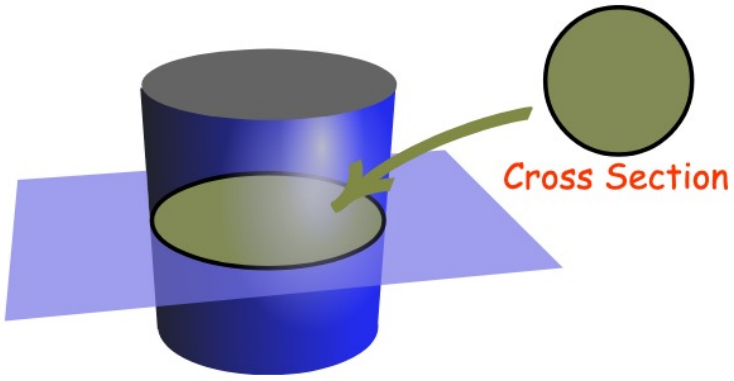
Cone



Prisms

Cross-Section

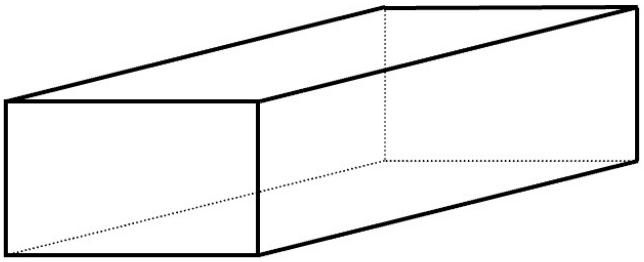
It is the shape made when a solid is cut through parallel to the base.



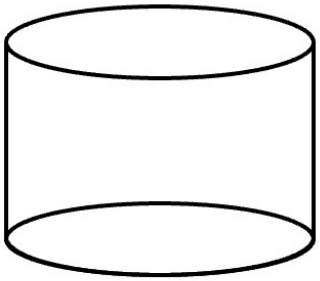
Prisms

A prism is a 3D shape which has the same cross-section along its length.

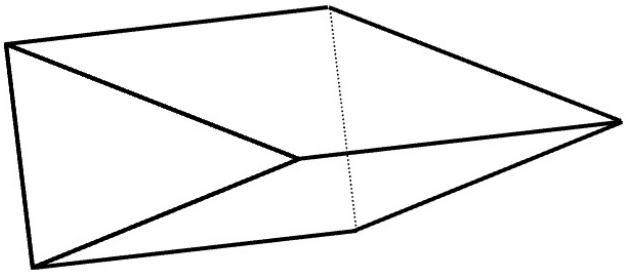
Rectangular prism or cuboid



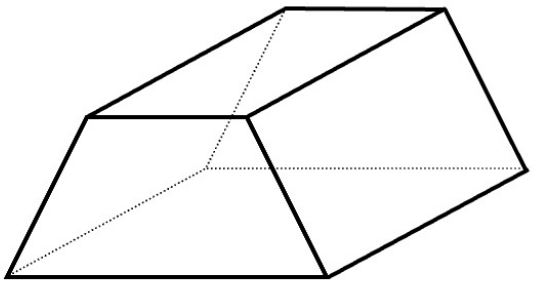
Circular prism or cylinder



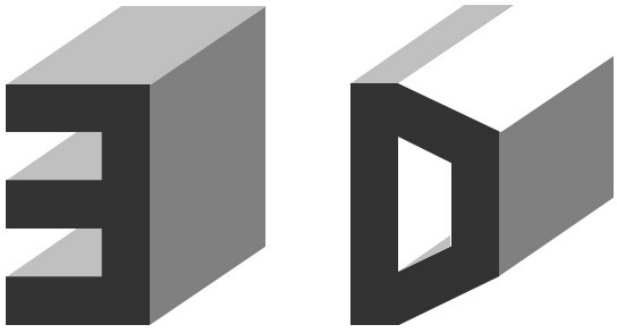
Triangular prism



Trapezoidal prism



2.2 Drawing Prisms and Other 3D Solids



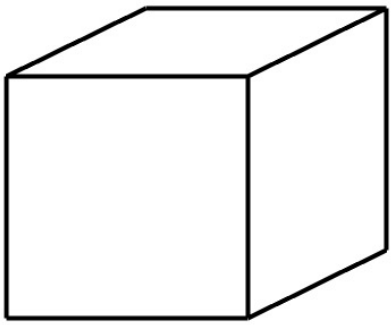
To draw prisms in 3D:

Draw the cross-section, using horizontal & vertical lines where possible

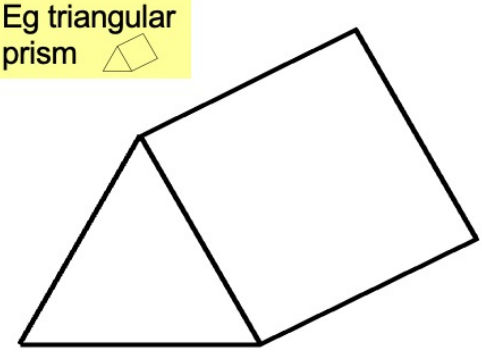
Draw depth lines from each possible corner, making sure they are **equal in length and parallel**

Use horizontal & vertical lines to connect the depth lines

Eg cube

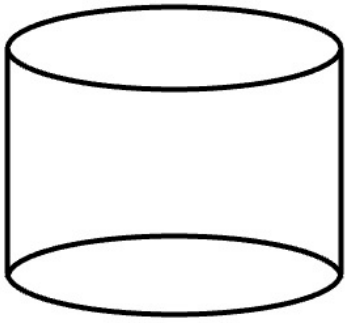


Eg triangular prism



Other 3D solids have unique ways of drawing them

Cylinder

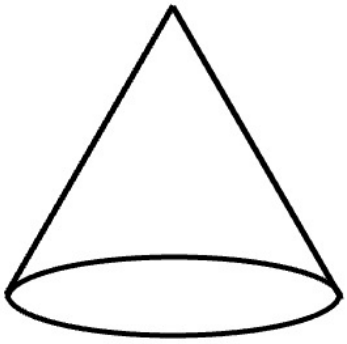


Draw an oval

Add two vertical lines

Draw another oval

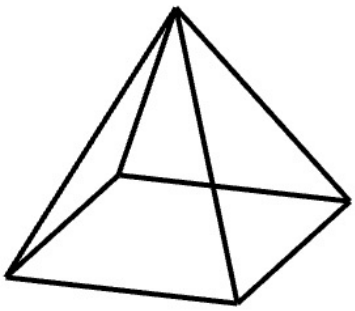
Cone



Draw an oval

Add two sloping lines

Square-based pyramid



Draw a rhombus on an angle

Add four sloping lines

Make 3D sketches of the following solids:

Cube	Cuboid	Square-based pyramid
Triangular prism	Cylinder	Cone

Fluency Practice

Question 1: Draw the following 3D shapes

(a) A cube

(b) A cuboid

(c) A sphere

(d) A cylinder

(e) A triangular prism

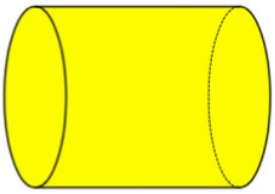
(f) A cone

(g) A square-based pyramid

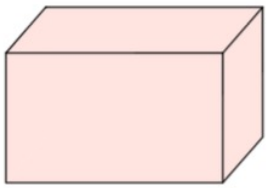
(h) A tetrahedron/triangular-based pyramid

Question 2: Name each of the 3D shapes below

(a)

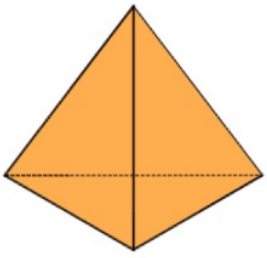


(b)

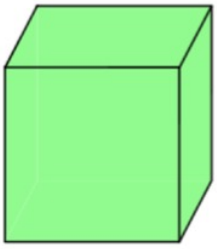


(c)

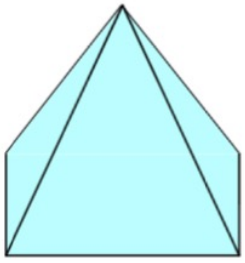
(d)



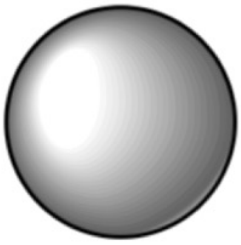
(e)



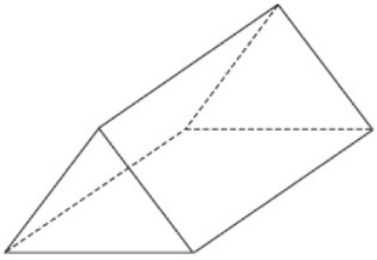
(f)



(g)



(h)



Extension

Question 1: Below is a picture of some objects.
Write down any 3D shapes you see and what they are in the picture.

Question 2: Can you spot any mistakes below?

The names of five solid shapes are given.

triangular prism sphere cube cuboid cylinder

Three of them are drawn below.

A B C

Complete these statements.

Shape A is called a .....Cube.....

Shape B is called a .....Cylinder.....

Shape C is called a .....Sphere.....

2.3 Edges, Faces and Vertices

Faces

A face is a flat or curved surface on a 3D shape.

Edges

An edge is where two faces meet.

Vertices

A vertex is a corner where edges meet. The plural is vertices.

Fluency Practice

Gamma Exercise

Copy and complete the following table. You may construct the shapes from nets or use diagrams to help you.

Shape	Vertices	Edges	Faces
Cube			
Triangle-based pyramid (tetrahedron)			
Square-based pyramid			
Pentagonal-based pyramid			
Hexagonal-based pyramid			
Triangular prism			
Cuboid			
Pentagonal prism			
Hexagonal prism			

Challenge

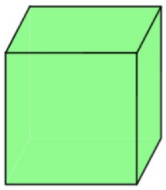
Refer back to the table from the Gamma Exercise. For each shape, work out what you get when you calculate $V - E + F$, where V is the number of vertices, E is the number of edges and F is the number of faces. What do you notice?

Shape	Vertices	Edges	Faces
Cube			
Triangle-based pyramid (tetrahedron)			
Square-based pyramid			
Pentagonal-based pyramid			
Hexagonal-based pyramid			
Triangular prism			
Cuboid			
Pentagonal prism			
Hexagonal prism			

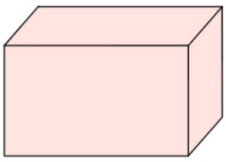
Fluency Practice

Question 1: For each 3D shape below, write down how many edges, faces and vertices it has.

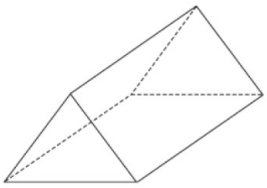
(a)



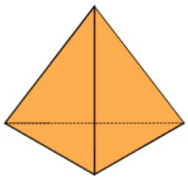
(b)



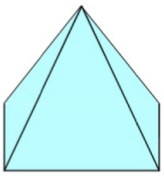
(c)



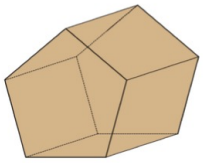
(d)



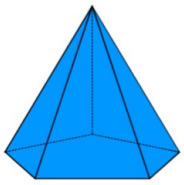
(e)



(f)



(g)



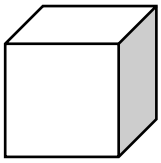
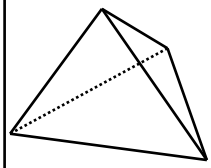
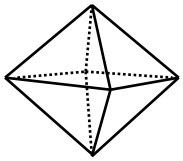
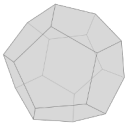
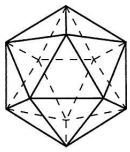
Extension

Question 1: Can you spot any mistakes in the question below?

	Faces	Edges	Vertices
Cube	12	6	8
Square-based Pyramid	5	5	5
Triangular Prism	9	9	6

2.4 Platonic Solids

A Platonic Solid is a 3D solid where all faces are the same regular polygon.

Name	Each Face	Diagram	Faces	Edges	Vertices
Cube	Square		6	12	8
Tetrahedron	Triangle		4	6	4
Octahedron	Triangle		8	12	6
Dodecahedron	Pentagon		12	30	20
Icosahedron	Triangle		20	30	12

2.5 Euler's Polyhedral Formula

For each, calculate $v - e + f$. What do you notice?

The value is always 2!

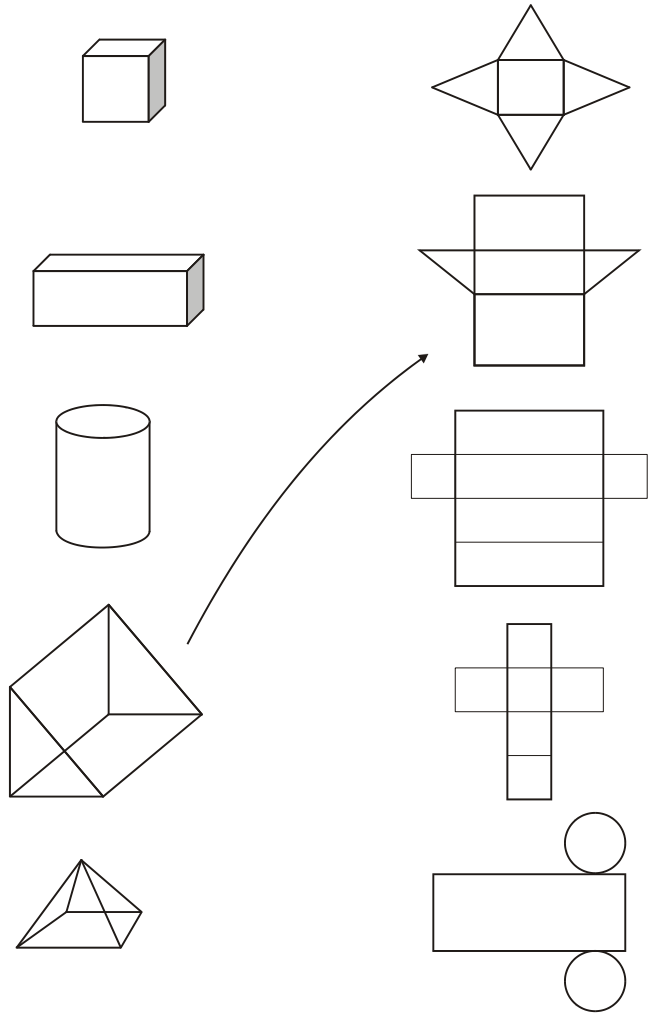
Euler's Polyhedral Formula:
$\boldsymbol{v - e + f = 2}$

Note: A polyhedron is a 3D shape with flat faces and straight edges.

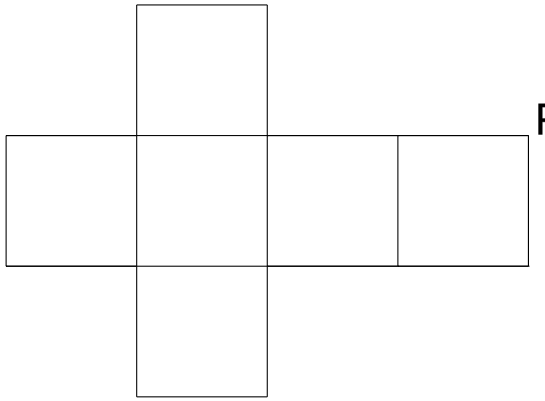
2.6 Nets

A net shows what a 3D solid could look like if ‘unfolded’ and laid out flat

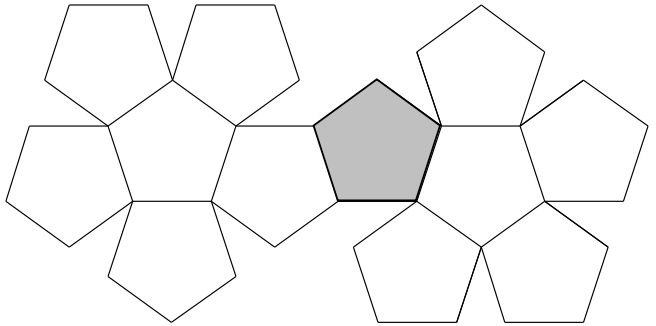
1. Match the 3D solids with their net



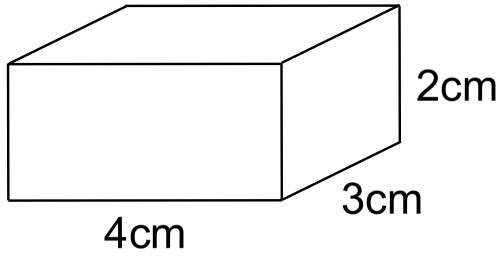
2. The net is folded to make a cube.
Two other vertices meet at *P.*
Mark each of these vertices with the letter *P*.

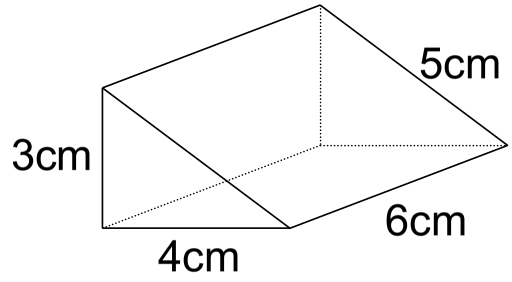


3. The net shown is folded to make a dodecahedron. Label the face which is opposite the shaded one



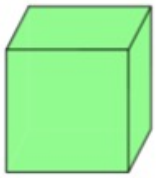
4. Using the grid provided with 1 square = 1 cm, draw an accurate net of these solids

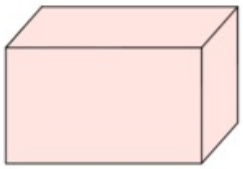


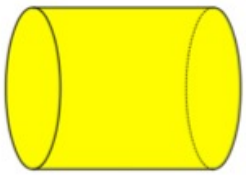


Fluency Practice

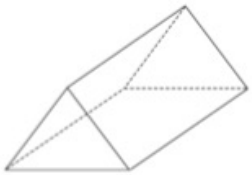
Question 1: Draw the nets for these 3D shapes

(a) 

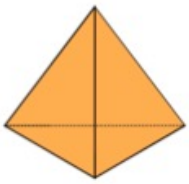
(b) 

(c) 

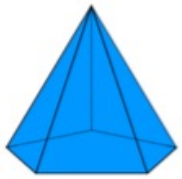
(d) 

(e) 

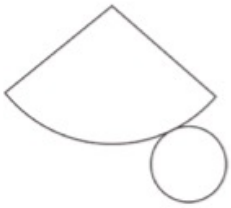
(f) 

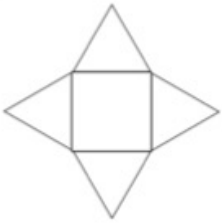
(g) 

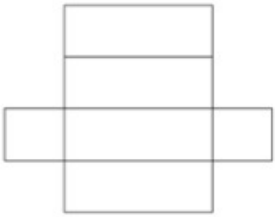
(h) 

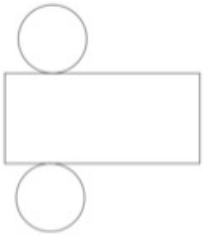
(i) 

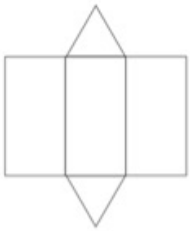
Question 2: Below are nets for various 3D shapes. Name the 3D shapes.

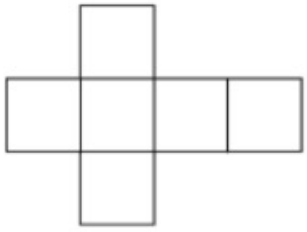
(a) 

(b) 

(c) 

(d) 

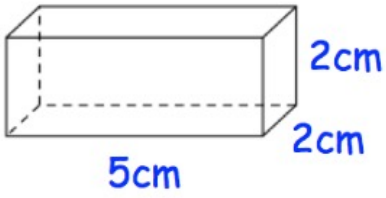
(e) 

(f) 

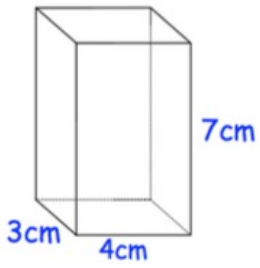
Fluency Practice

Question 3: Draw accurate nets for these 3D shapes on squared paper.

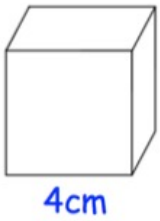
(a)



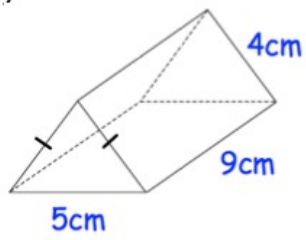
(b)



(c)

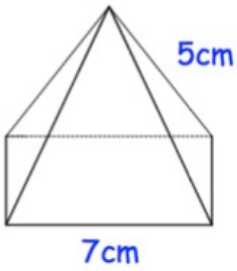


(d)

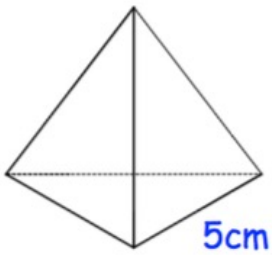


Question 4: Shown below is a square-based pyramid and a tetrahedron.
Draw accurate nets for these 3D shapes on squared paper.

(a)

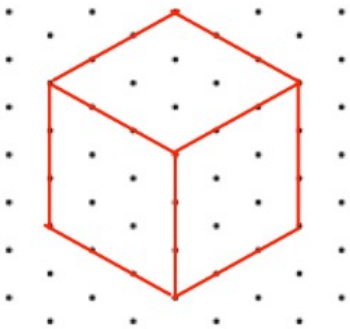


(b)

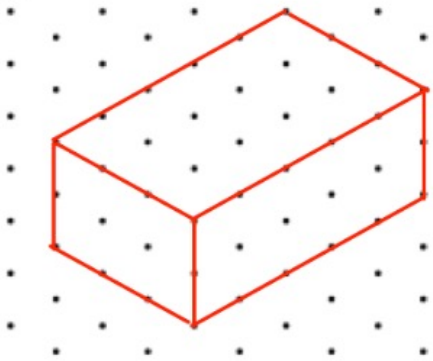


Question 5: The solids on 1cm isometric grids. Draw their nets on squared paper.

(a)

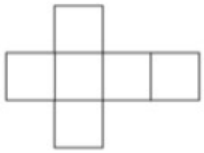


(b)

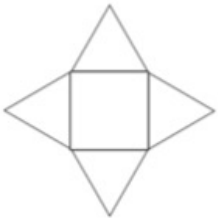


Extension

Question 1: Shown below is a net for a cube. Draw all the other possible nets for a cube.

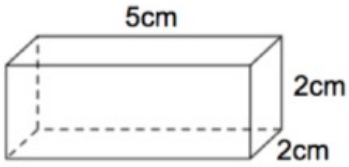


Question 2: Shown below is a net for a square-based pyramid. Draw all other possible nets for a square-based pyramid.



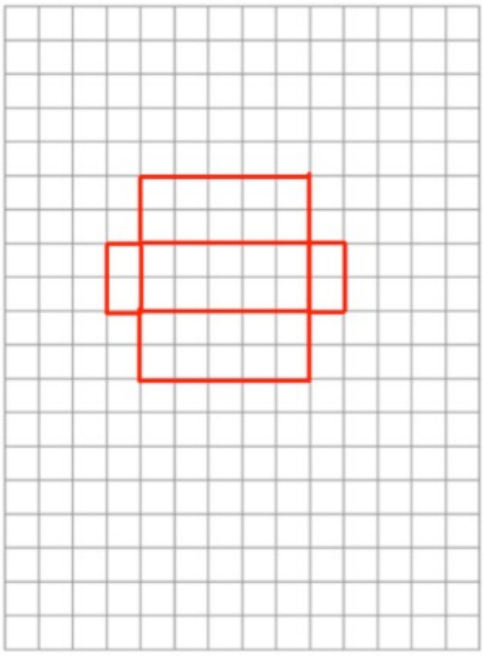
Question 3: Can you spot any mistakes below?

Shown below is a cuboid.



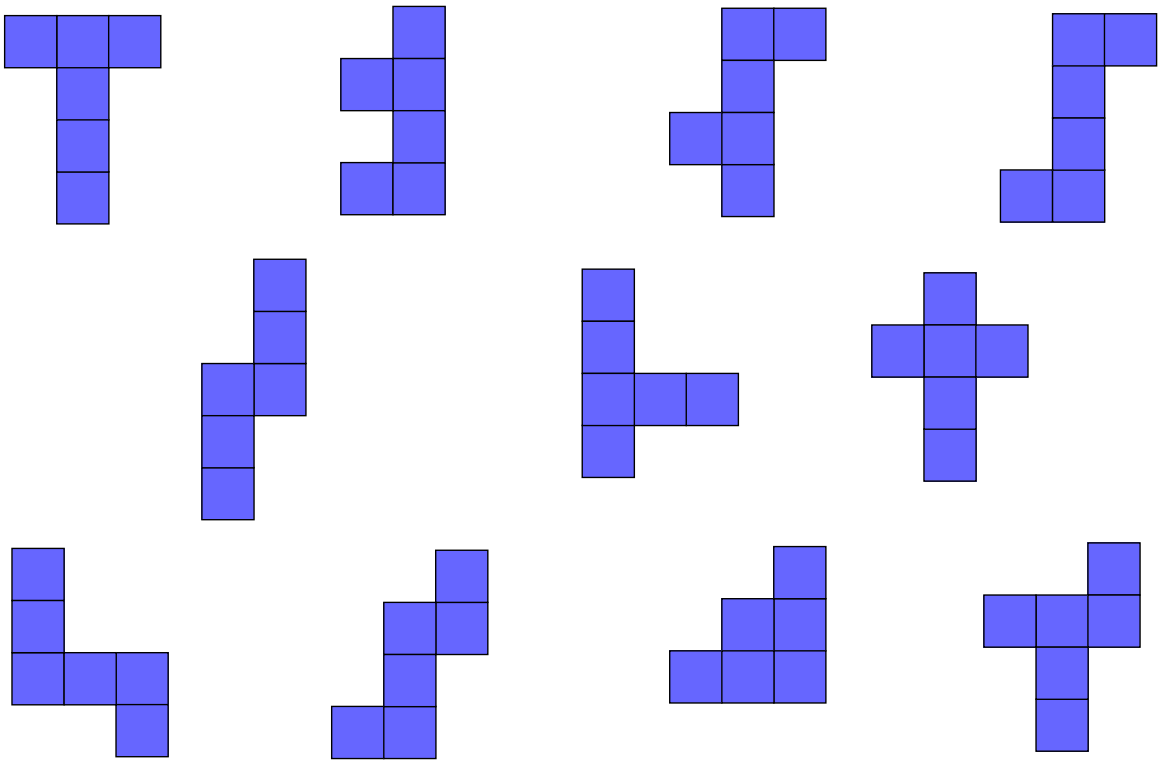
Draw a net for the cuboid.

Each square represents 1cm²



Extension

Which four of these are not real nets of a cube?



Maths Venns

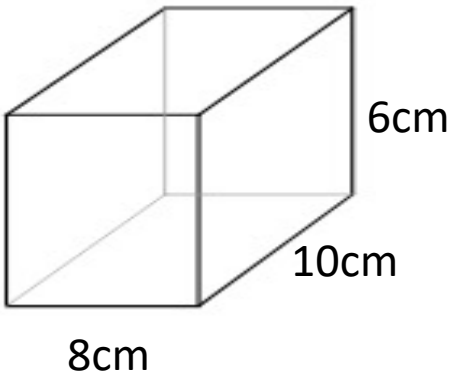
Has at least 1 curved surface
If you think a region is impossible to fill, convince me why!
More than 1 plane of symmetry
Is a prism
Draw a 3D shape that could belong in each of the regions
A
B
C
D
E
F
G
H

2.7 Volume of a Cuboid

Volume is the amount of space an object takes up.

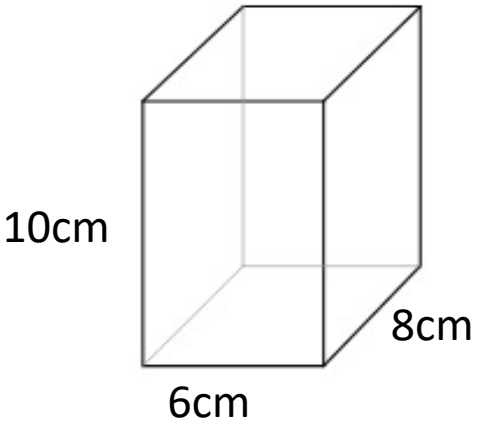
Worked Example

Calculate the volume:



Your Turn

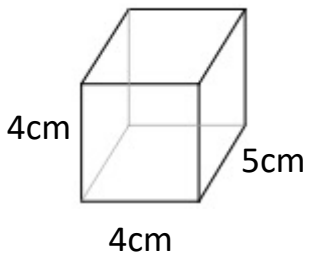
Calculate the volume:



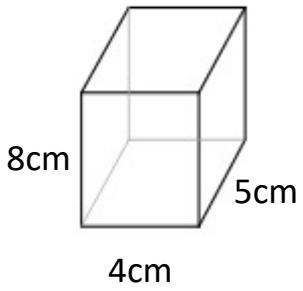
Intelligent Practice

Calculate the volume.

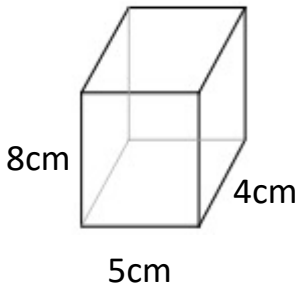
Question 1



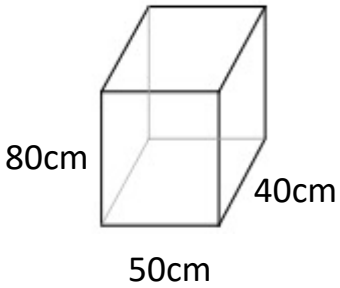
Question 2



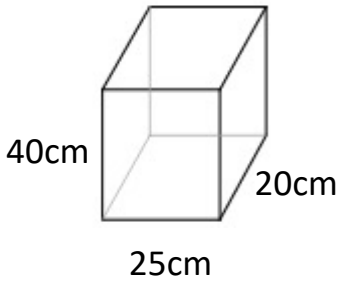
Question 3



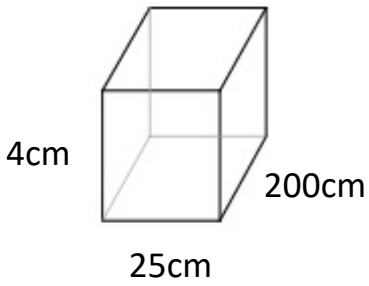
Question 4



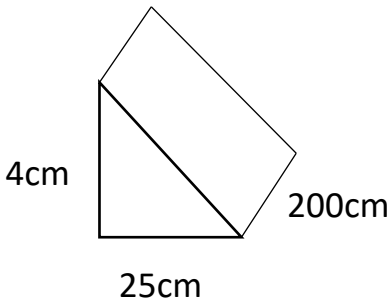
Question 5



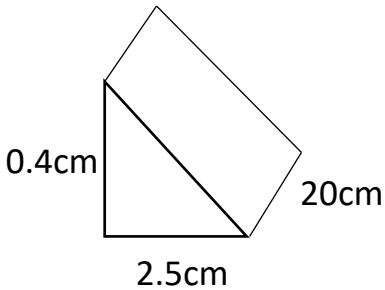
Question 6



Question 7



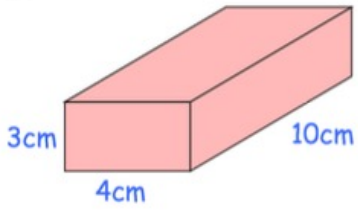
Question 8



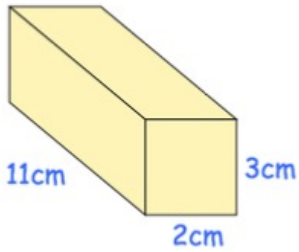
Fluency Practice

Question 1: Work out the volume of each cuboid.
Include suitable units.

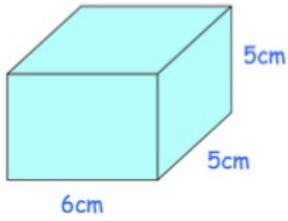
(a)



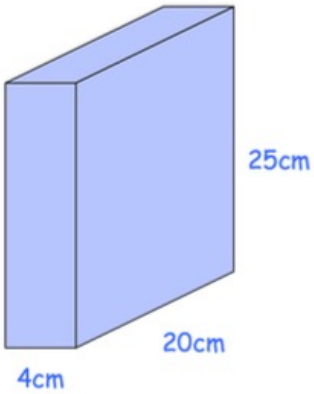
(b)



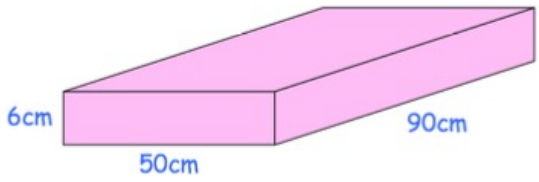
(c)



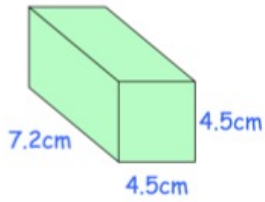
(d)



(e)

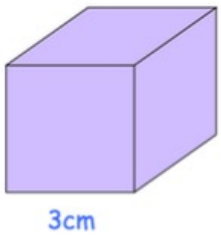


(f)

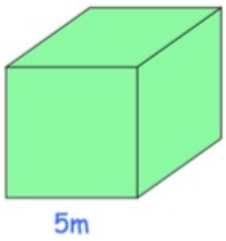


Question 2: Work out the volume of each cube.
Include suitable units.

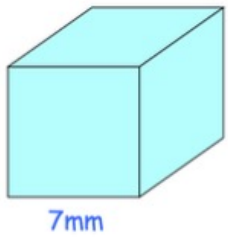
(a)



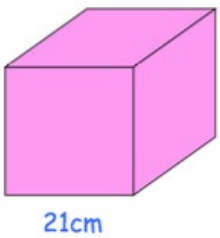
(b)



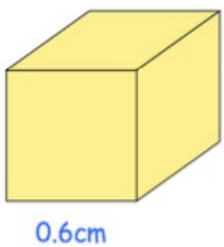
(c)



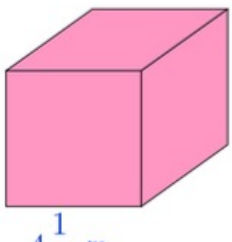
(d)



(e)



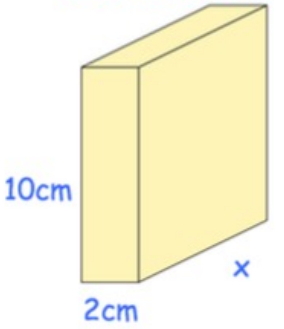
(f)



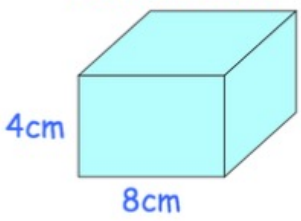
Fluency Practice

Question 3: Find the length of each cuboid.

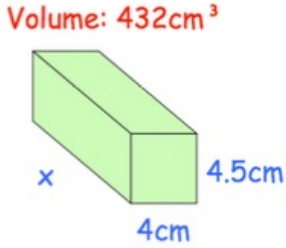
(a)



(b)

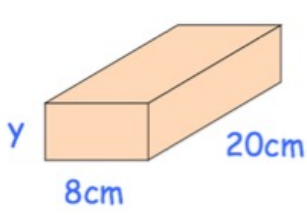


(c)

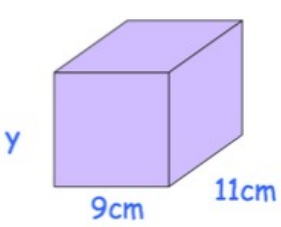


Question 4: For each cuboid below, find the missing measurement, y.

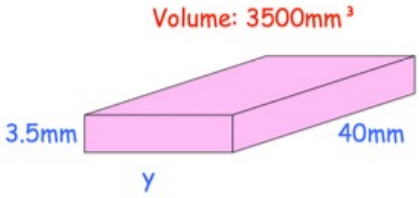
(a)



(b)

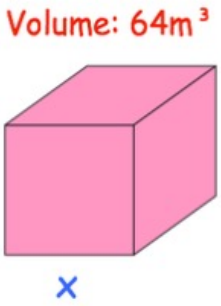


(c)

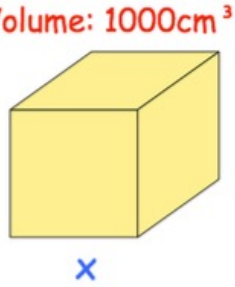


Question 5: The volume of each cube is given.
Find the length of each side, x.

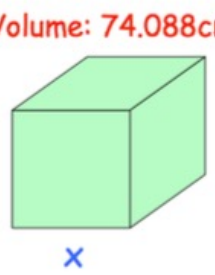
(a)



(b)



(c)



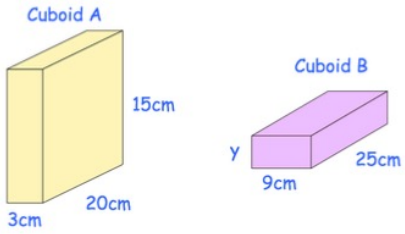
Extension

Question 1: Find the volume of a water tank that is 80cm long, 40cm wide and 20cm high.

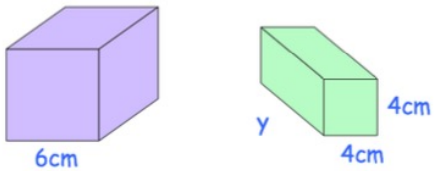
Question 2: A wooden beam measures 4 inches wide by 4 inches high by 60 inches long.
Work out the volume of the wooden beam.

Question 3: The cube on the TV show "The Cube" is a cube with each side measuring 4m.
Work out the volume of the cube.

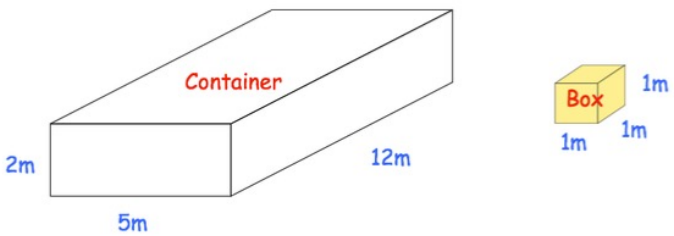
Question 4: Both cuboids below have the same volume.
Find the height of cuboid B.



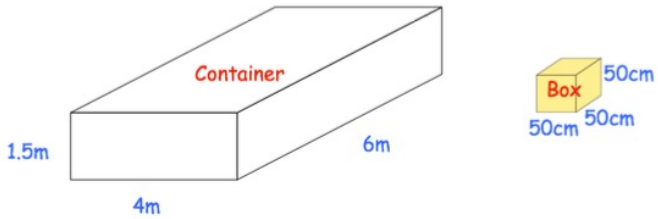
Question 5: The volume of the cube is twice the volume of the cuboid.
Find the length of the cuboid.



Question 6: The cuboid container below is used to store boxes.
Each box is a cube with side length 1m.
How many boxes can be stored in the container?



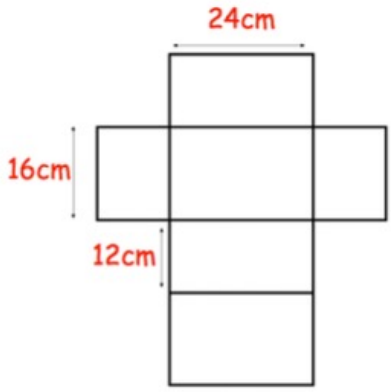
Question 7: The cuboid container below is used to store boxes.
Each box is a cube with side length 50cm.
How many boxes can be stored in the container?



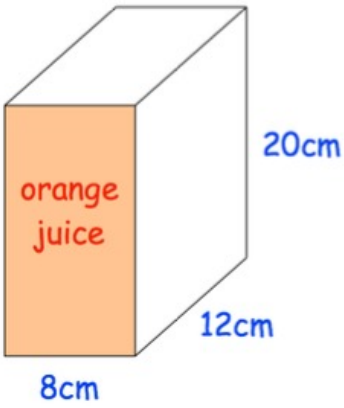
Question 8: An empty swimming pool is going to be filled with water.
The swimming pool is a cuboid, that is 25 metres long, 10 metres wide and 2 metres deep.
It is being filled at a rate of 800 litres per minute
Given $1m^3 = 1000$ Litres, how long it will take to fill the swimming pool?
Give your answer in hours and minutes.

Extension

Question 9: Shown is a net of a cuboid.
Calculate the volume of the cuboid.



Question 10: A carton of orange juice is shown below.
The carton is in the shape of a cuboid.

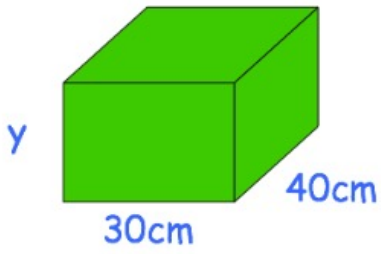


The depth of the orange juice is 6cm.

The carton is turned so that it stands the shaded (orange) face.

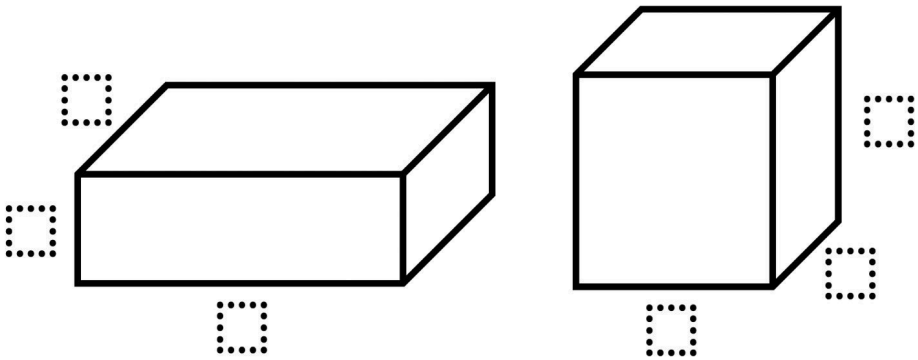
Work out the depth of the orange juice now.

Question 11: Peter is making green paint by mixing blue and yellow paint in a cuboid container, shown below.
The container has a width of 30cm and length of 40cm and is full.
He mixes blue paint and yellow paint in the ratio 2:3.
Peter uses 8.4 litres of blue paint.
Calculate the height of the container.

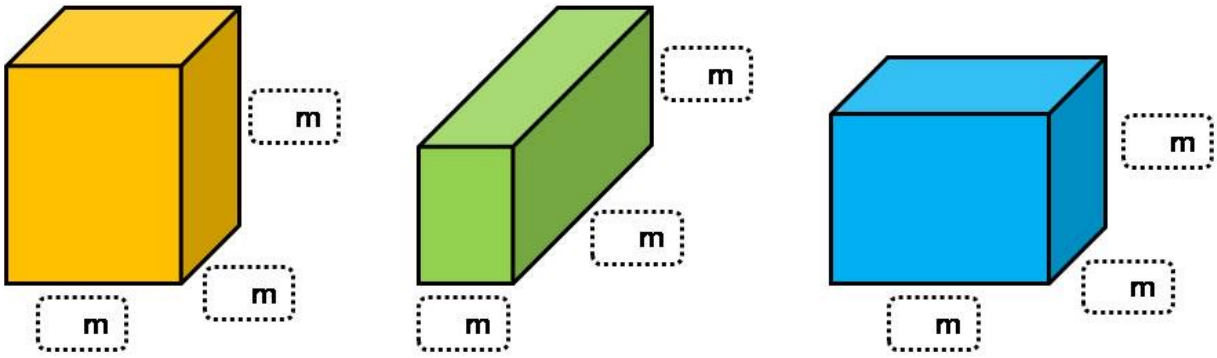


Puzzles

Using digits 0 to 9, without repetition, fill the blanks to create two rectangular prisms with the same volume.

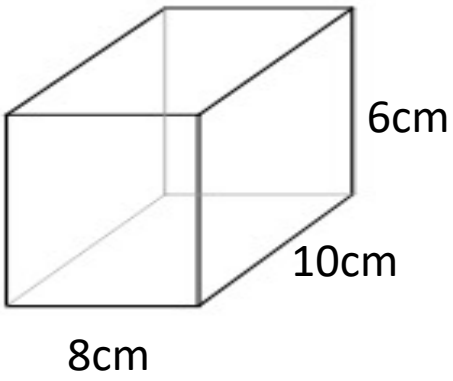
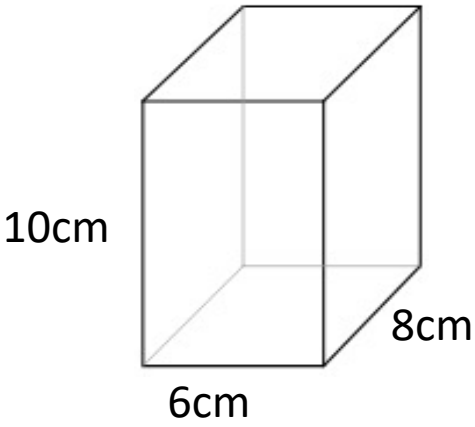


Using the numbers 1 to 9, without repetition, can you fill the blanks so that the volumes of these cuboids are equal? If not equal, how close can you get?



2.8 Surface Area of a Cuboid

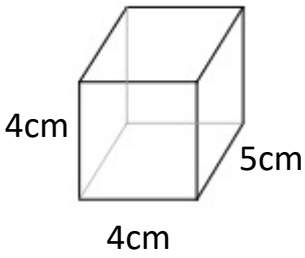
Surface Area is the total area across the surface.

Worked Example	Your Turn
Calculate the total surface area:  	Calculate the total surface area: 

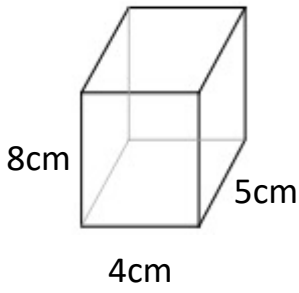
Intelligent Practice

Calculate the total surface area.

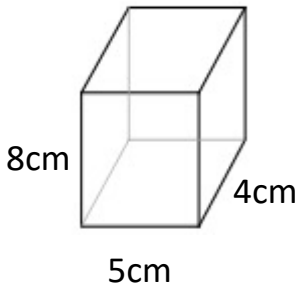
Question 1



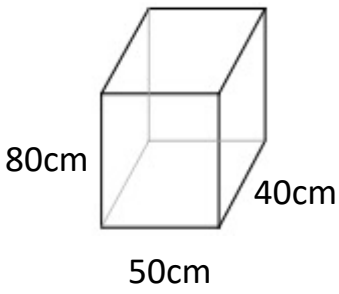
Question 2



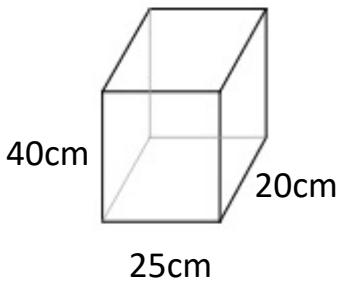
Question 3



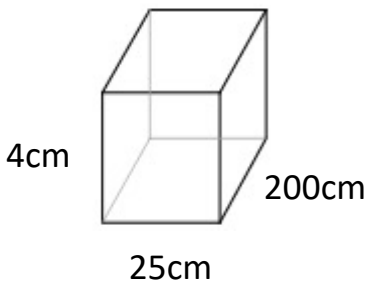
Question 4



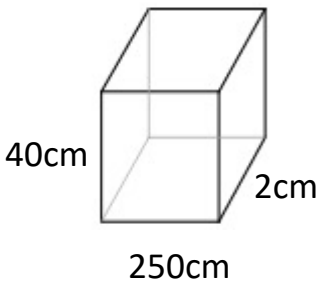
Question 5



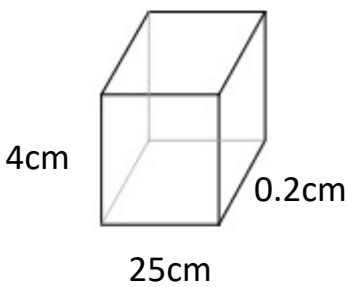
Question 6



Question 7



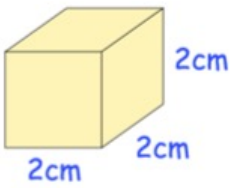
Question 8



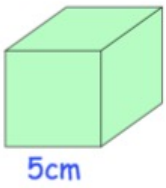
Fluency Practice

Question 1: Work out the surface area of each of the following cubes.

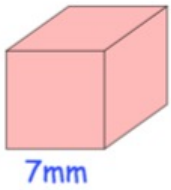
(a)



(b)

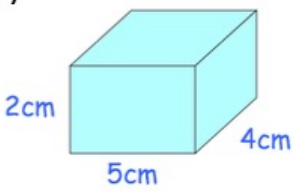


(c)

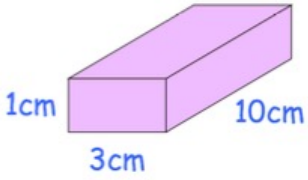


Question 2: Work out the surface area of each of the following cuboids.

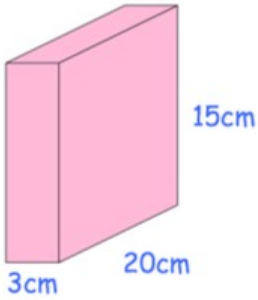
(a)



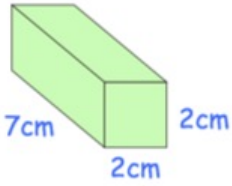
(b)



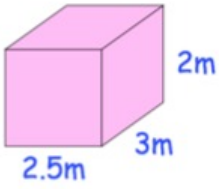
(c)



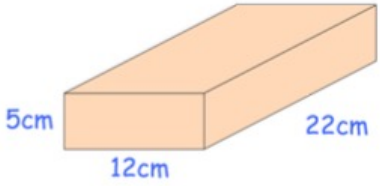
(d)



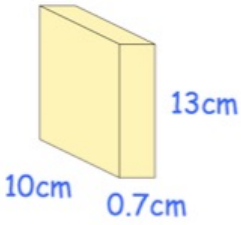
(e)



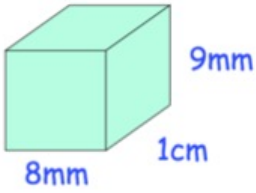
(f)



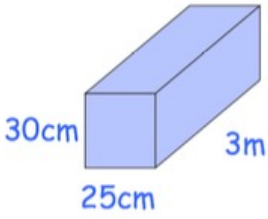
(g)



(h)



(i)

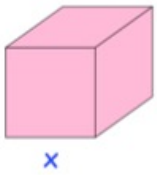


Question 3: Calculate the surface area of a cube with side length 12cm

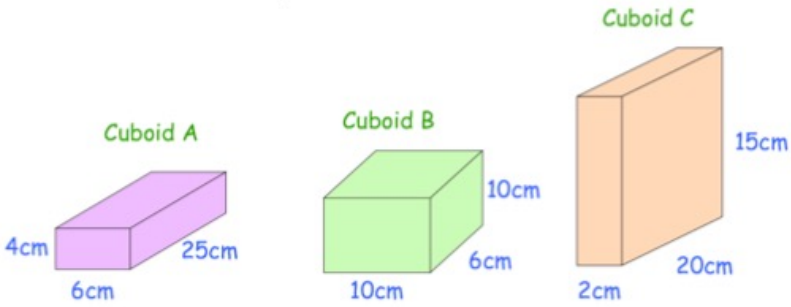
Question 4: Calculate the surface area of a cube with side length ½ cm

Extension

Question 1: A cube has a surface area of $54cm^2$
Find the side length, x, of the cube



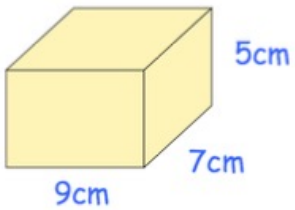
Question 2: A company is designing a new box to hold coffee.
They have 3 designs, cuboids A, B and C.
All 3 designs have the same volume of $600cm^3$
The company want to choose the design with the smallest surface area.
Which cuboid should they choose?



Question 3: A cube has a volume of $1000cm^3$.
Work out the surface area of the cube.

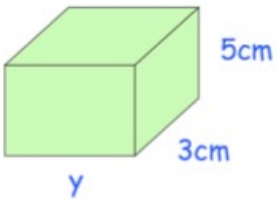
Question 4: Jamie is trying to work out the surface area of the cuboid below.
Can you spot any mistakes?

$$9 \times 5 = 45$$
$$7 \times 5 = 35$$
$$9 \times 7 = 63$$
$$45 + 35 + 63 = 143cm^3$$

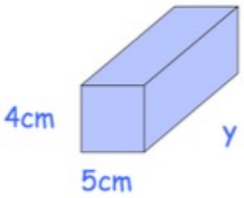


Question 5: Find y for each of the cuboids below

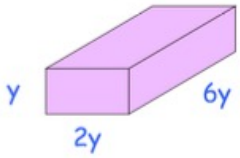
(a) Surface area = $158cm^2$



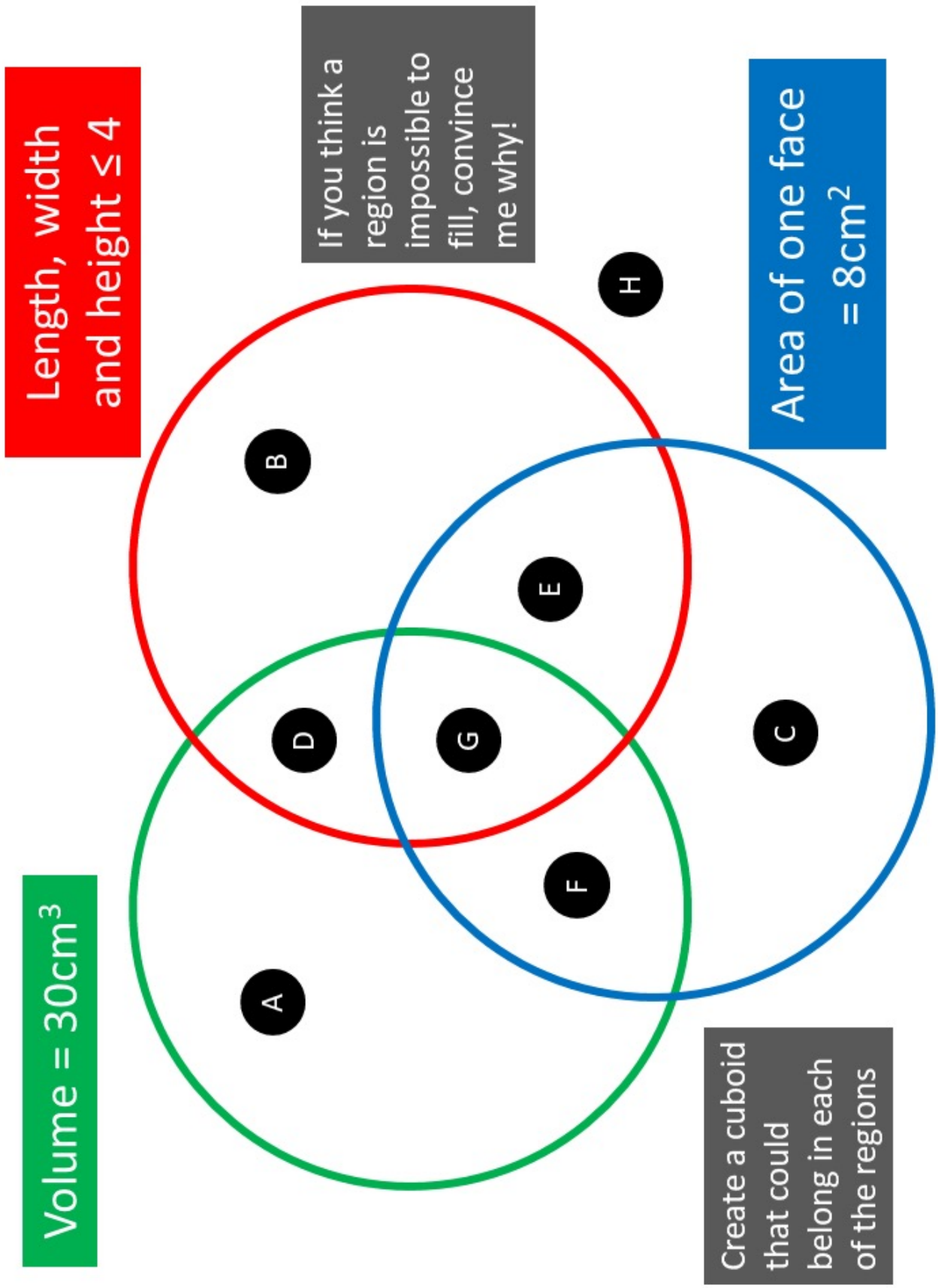
(b) Surface area = $346cm^2$



(c) Surface area = $90cm^2$



Maths Venns


Length, width and height ≤ 4
If you think a region is impossible to fill, convince me why!
Area of one face = 8cm²
Volume = 30cm³
Create a cuboid that could belong in each of the regions
A
B
C
D
E
F
G
H

Maths Venns

Length, width and height ≤ 4
If you think a region is impossible to fill, convince me why!
All faces have an area ≥ 10cm²
Volume = 70cm³
Create a cuboid that could belong in each of the regions
A
B
C
D
E
F
G
H

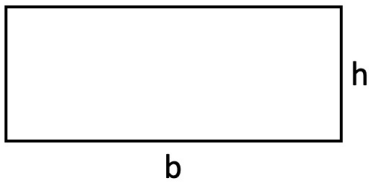
Maths Venns

Surface Area
> 60cm²
If you think a region is impossible to fill, convince me why!
Height = 6cm
Volume < 50cm³
A
B
C
D
E
F
G
H
State the dimensions of a cuboid that could belong in each of the regions

2.9 Area Recap

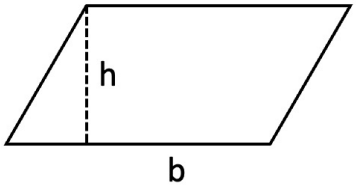
- Area of a rectangle = base x height

$$A = b \times h$$



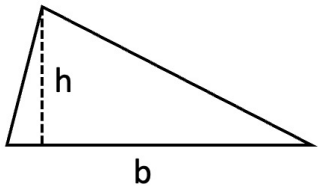
- Area of a parallelogram = base x perpendicular height

$$A = b \times h$$



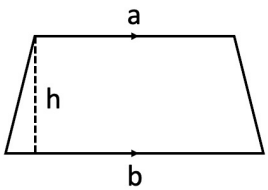
- Area of a triangle = $\frac{\text{base x perpendicular height}}{2}$

$$A = \frac{b \times h}{2}$$



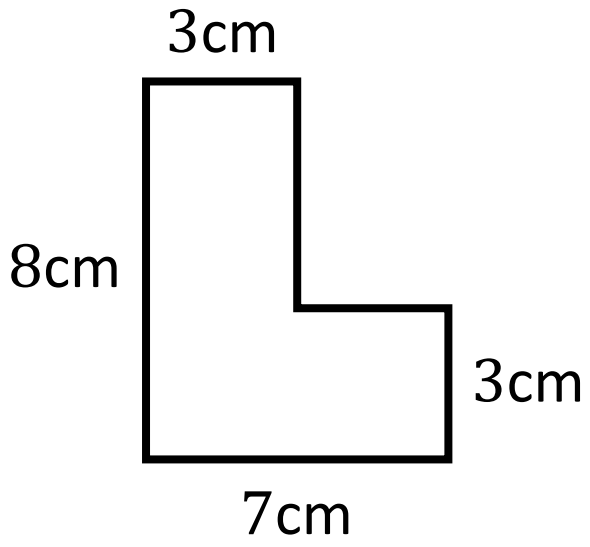
- Area of a trapezium = $\frac{\text{sum of parallel sides}}{2}$ x perpendicular height

$$A = \frac{a+b}{2} \times h$$



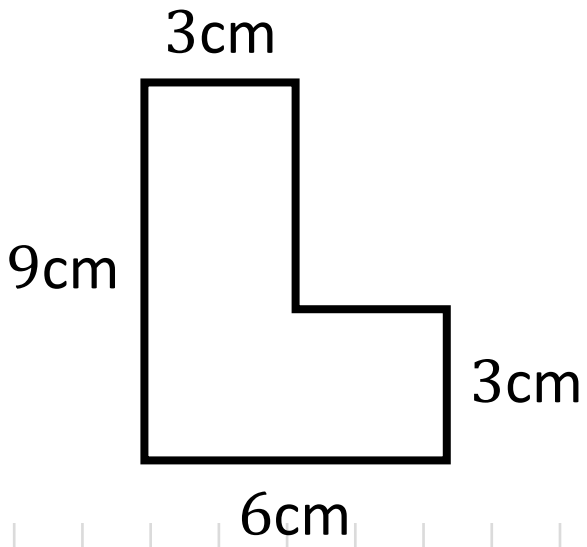
Worked Example

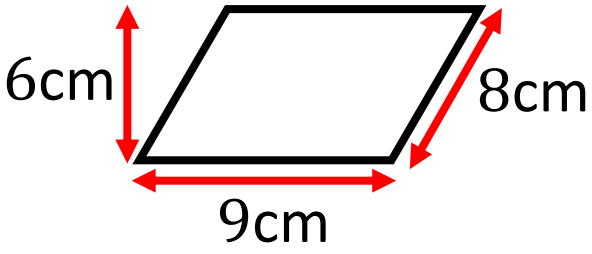
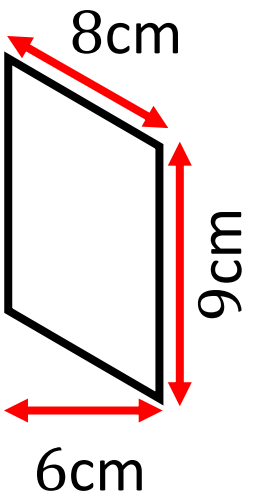
Calculate the area of the shape below:

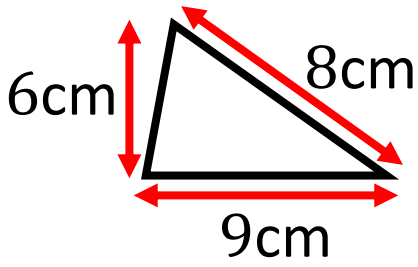
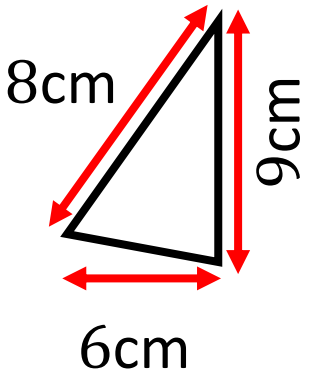


Your Turn

Calculate the area of the shape below:

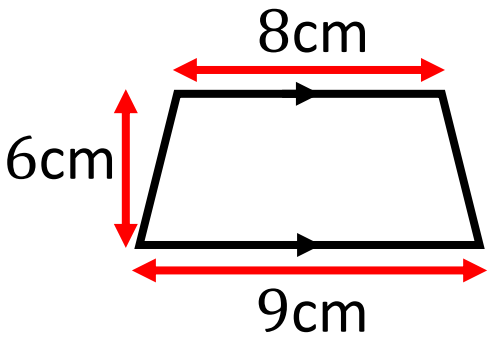


Worked Example	Your Turn
Calculate the area of the parallelogram below:  	Calculate the area of the parallelogram below: 

Worked Example	Your Turn
Calculate the area of the triangle below:  	Calculate the area of the triangle below: 

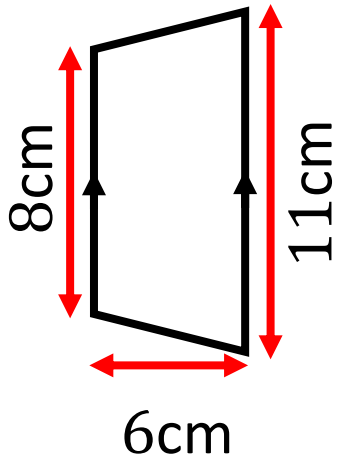
Worked Example

Calculate the area of the trapezium below:



Your Turn

Calculate the area of the trapezium below:



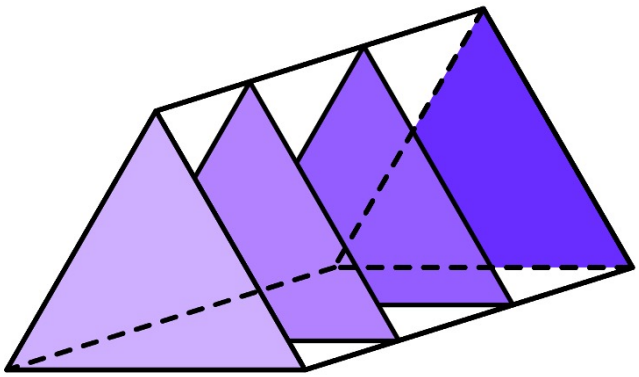
Fluency Practice

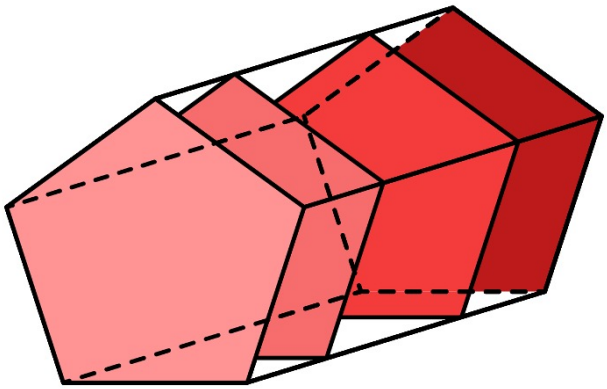
Area Practice

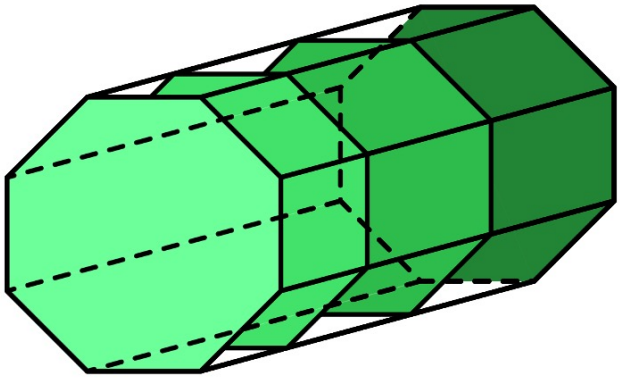
2.10 Volume of a Prism

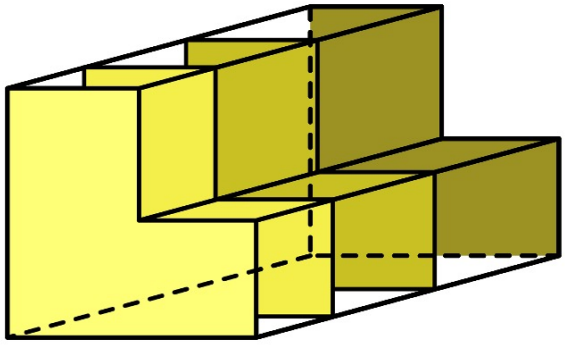
Volume is the amount of space an object takes up.

A prism is a 3D shape that has a constant cross-section along its length.



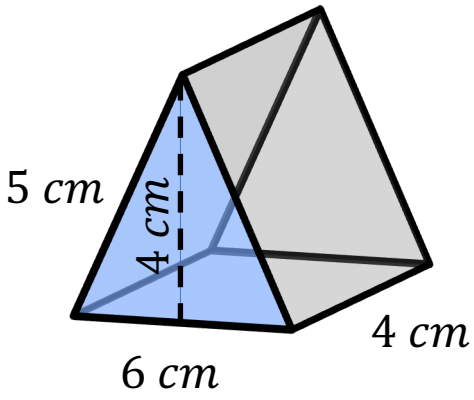
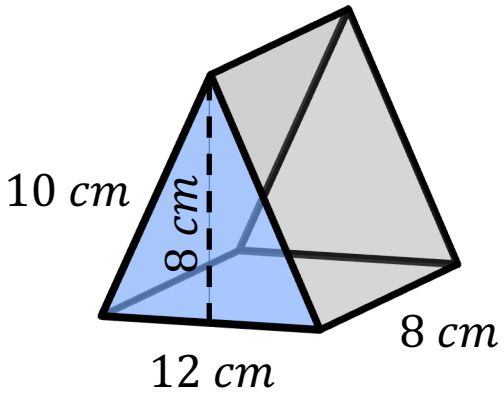


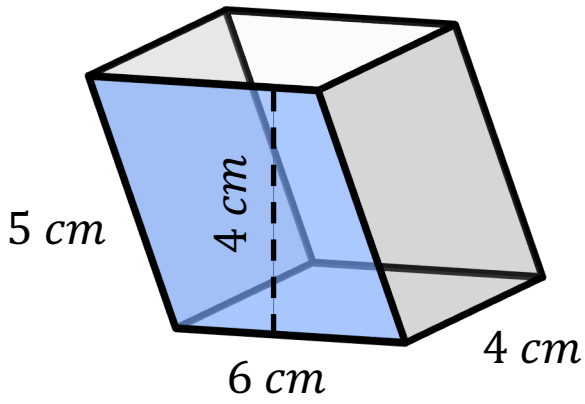
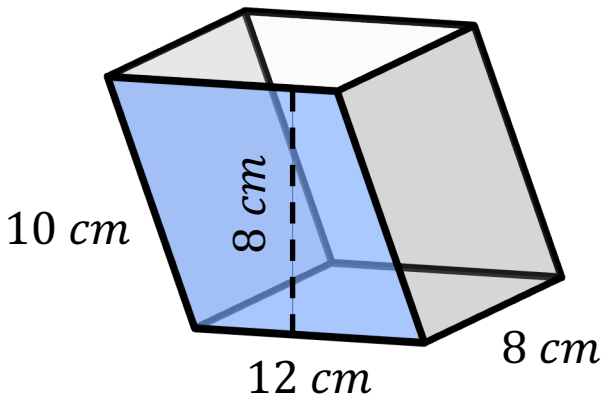




Before finding the volume of a prism, you first have to identify its cross-section and its length.

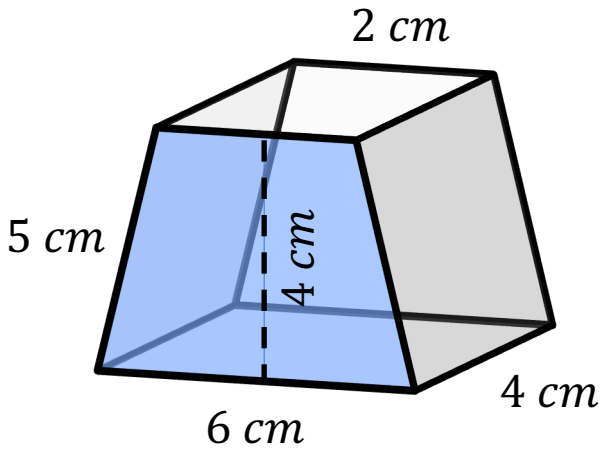
Volume of a prism = Area of cross-section × length

Worked Example	Your Turn
Calculate the volume of the triangular prism:  	Calculate the volume of the triangular prism: 

Worked Example	Your Turn
Calculate the volume of the parallelepiped:  	Calculate the volume of the parallelepiped: 

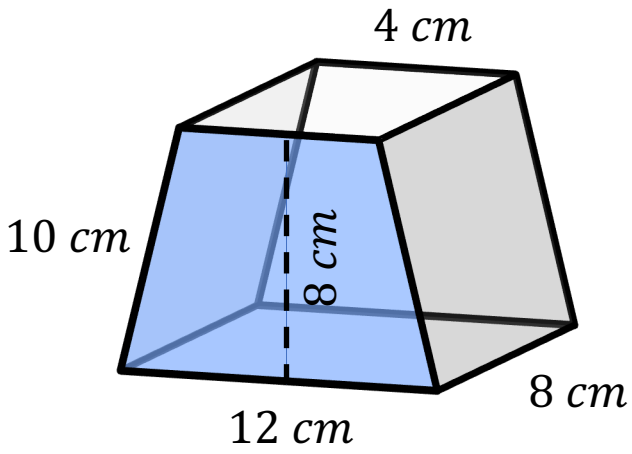
Worked Example

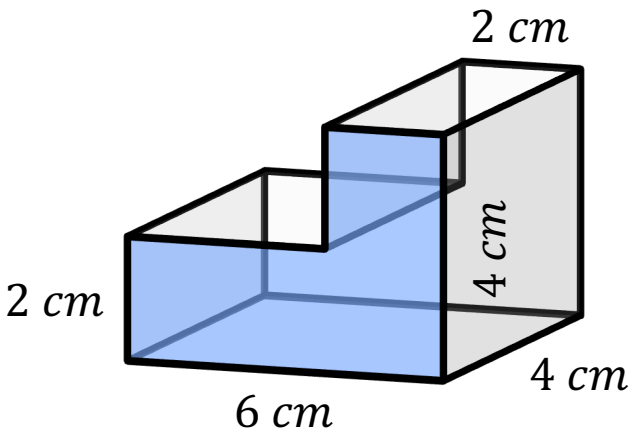
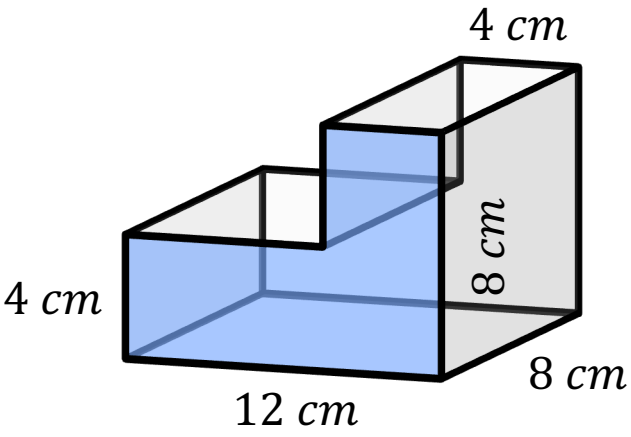
Calculate the volume of the trapezium prism:



Your Turn

Calculate the volume of the trapezium prism:

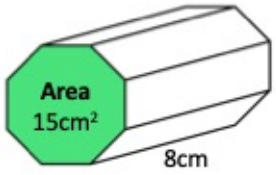


Worked Example	Your Turn
Calculate the volume of the L-shaped prism:  	Calculate the volume of the L-shaped prism: 

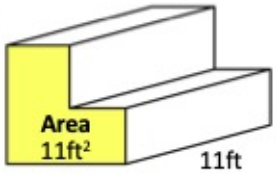
Fluency Practice

Q1 Work out the volume of each of the following prisms.

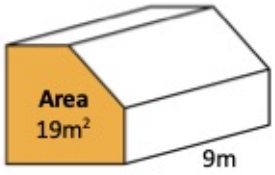
[a]



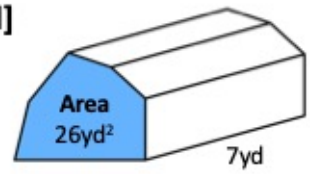
[b]



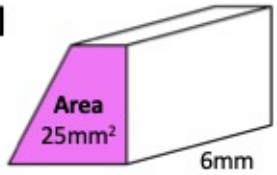
[c]



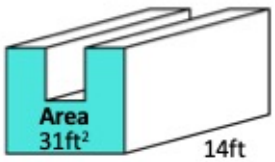
[d]



[e]

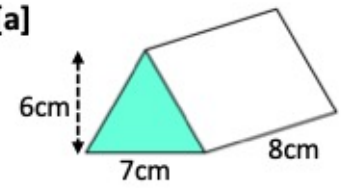


[f]

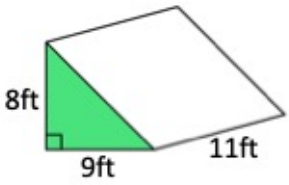


Q2 Calculate the volume of the following triangular prisms.

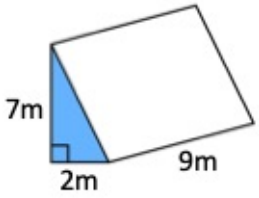
[a]



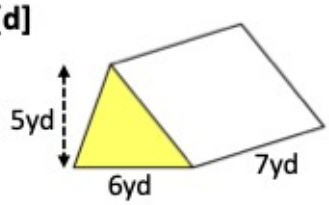
[b]



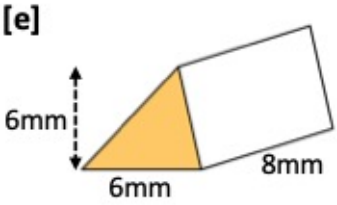
[c]



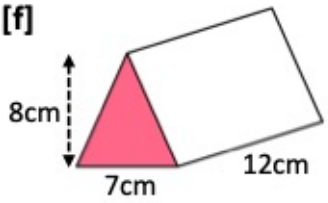
[d]



[e]

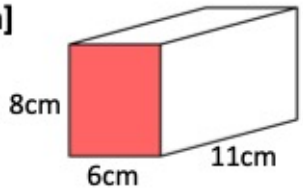


[f]

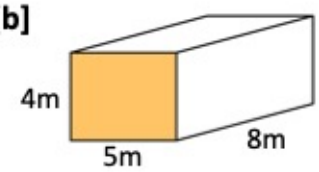


Q3 Calculate the volume of the following triangular prisms.

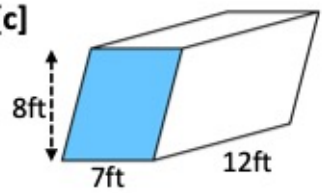
[a]



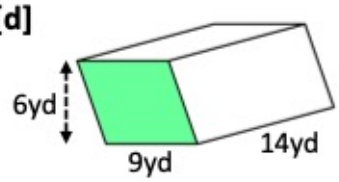
[b]



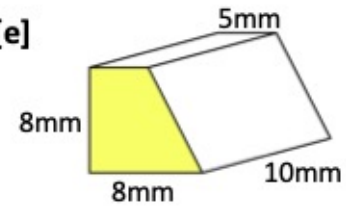
[c]



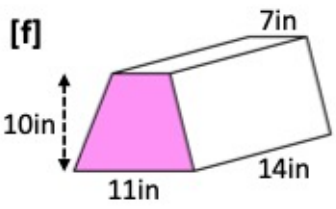
[d]



[e]



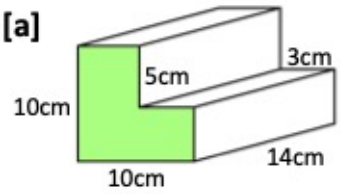
[f]



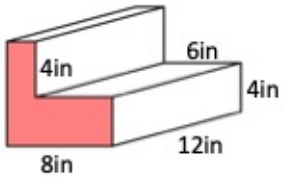
Fluency Practice

Q1 Calculate the volume of the following L-shape prisms.

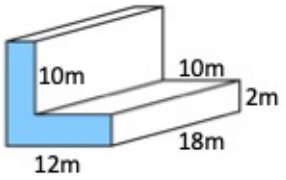
[a]



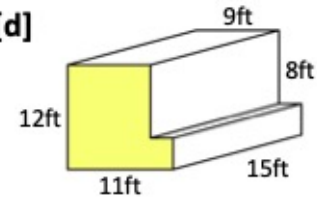
[b]



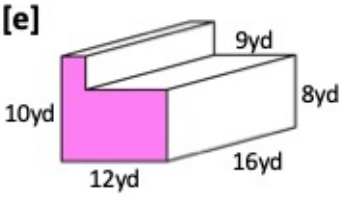
[c]



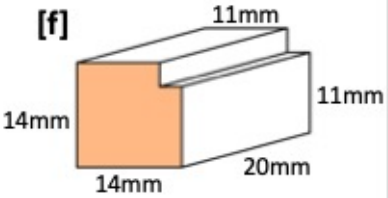
[d]



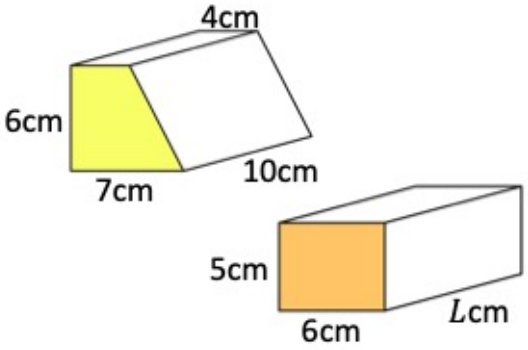
[e]



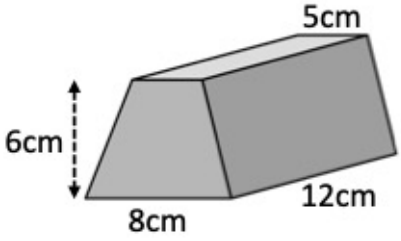
[f]



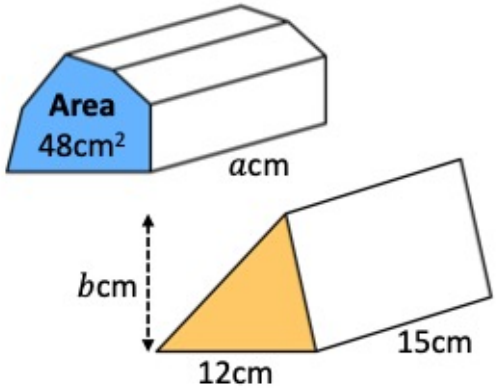
Q2 Both of the following prisms have the same volume. Find the missing length.



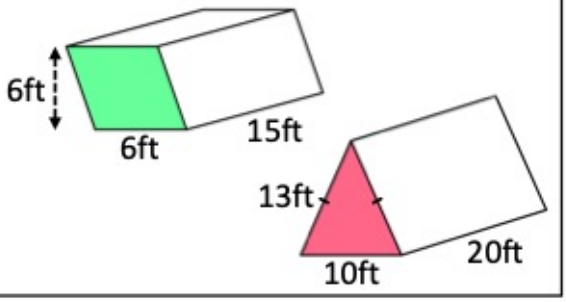
Q3 25 kilograms of metal is melted and moulded into prisms. The density of the metal is $10g/cm^3$. How many prisms can be made?



Q4 Each prism has a volume of $720cm^3$. Find the values of a and b.



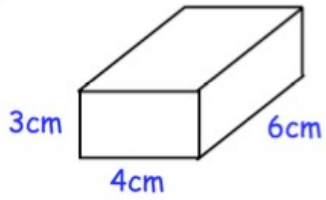
Q5 Find the ratio of the volume of the triangular prism to the volume of the parallelepiped. Give your answer in its simplest form.



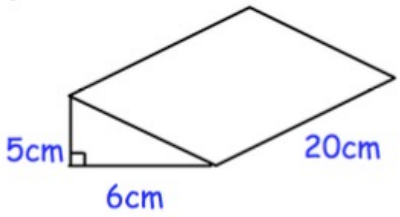
Fluency Practice

Question 1: Calculate the volume of each prism below

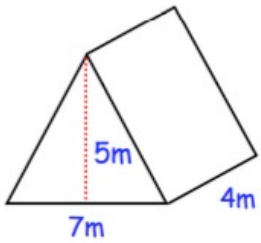
(a)



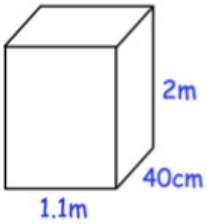
(b)



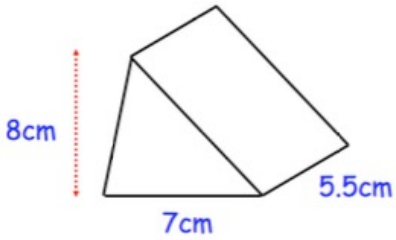
(c)



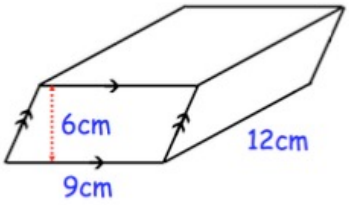
(d)



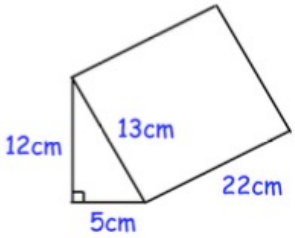
(e)



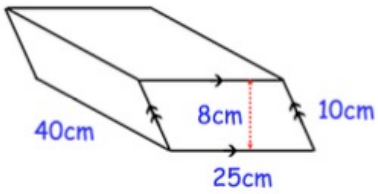
(f)



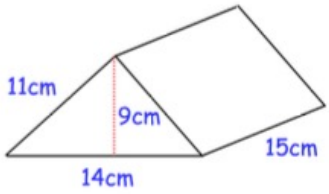
(g)



(h)

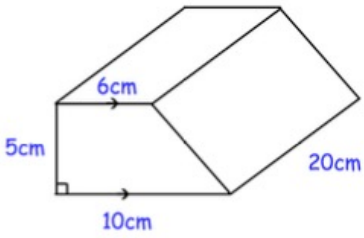


(i)

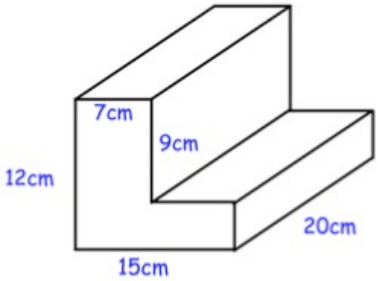


Question 2: Calculate the volume of each prism below

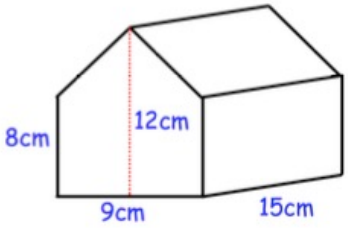
(a)



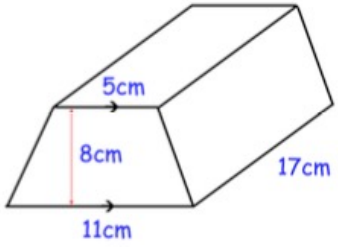
(b)



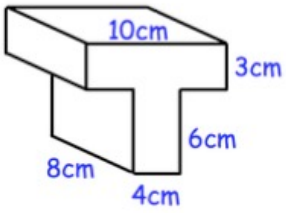
(c)



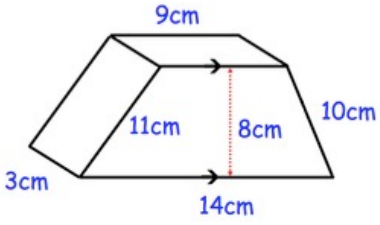
(d)



(e)

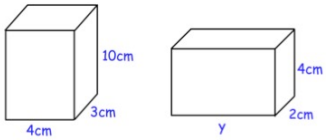


(f)

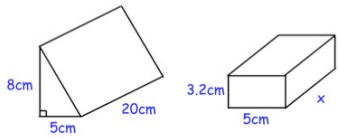


Extension

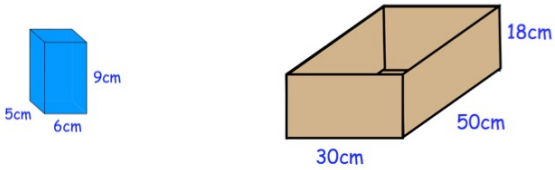
Question 1: Cillian makes two cuboids out of clay.
Both cuboids have the same volume.
Find y



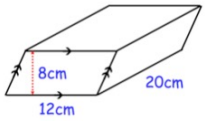
Question 2: The cuboid and the triangular prism have the same volume.
Find x.



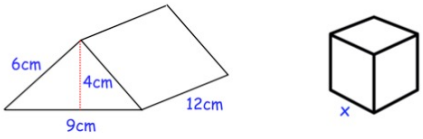
Question 3: Boxes of coffee are placed into a crate.
Each box of coffee is a cuboid and the crate is also a cuboid.
How many boxes of coffee fit into the crate?



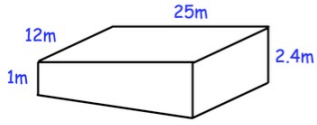
Question 4: James wants to fill an empty flowerpot with soil.
He has 2 litres of soil.
Given 1 litre = $1000cm^3$
Does James have enough soil to fill the flowerpot?



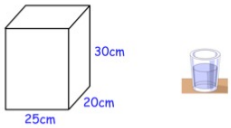
Question 5: The solid triangular prism shown below is made from metal.
The prism is melted down and the metal is used to create a solid cube.
Find the side length of the cube.



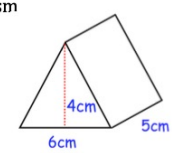
Question 6: The swimming pool in a leisure centre is shown below.
The length of the swimming pool is 25m and the width is 12m.
The depth of the shallow end is 1m and the depth of the deep end is 2.4m.
Given $1m^3$ = 1000 litres
Work out how much water, in litres, the swimming pool holds.



Question 7: A fish tank is ¼ full of water.
Harry pours a 400ml glass of water into the fish tank.
Given 1ml = $1cm^3$
What will the depth of the water of the fish tank then be?



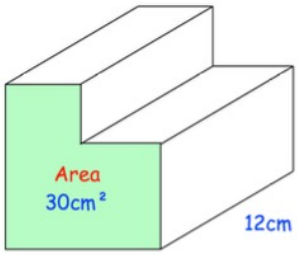
Question 8: A solid glass paperweight is in the shape of a triangular prism
The density of the glass is $2.4g/cm^3$
Work out the mass of the paperweight.



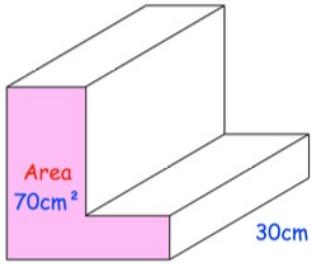
Fluency Practice

Question 1: Work out the volume of each of the prisms below.
The area of the front of each prism has been given.

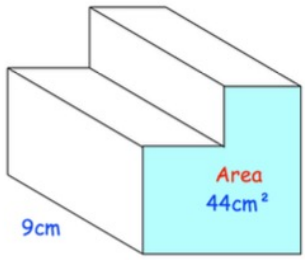
(a)



(b)

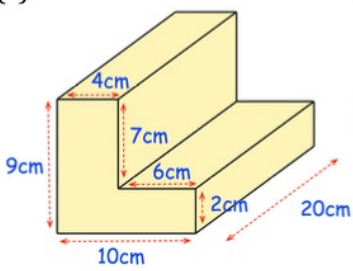


(c)

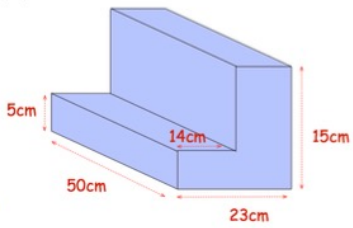


Question 2: Work out the volume of each of the prisms below.

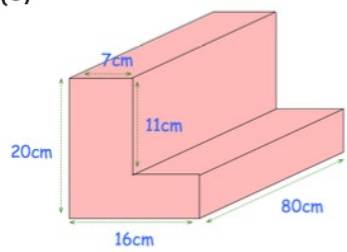
(a)



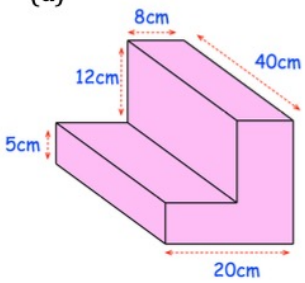
(b)



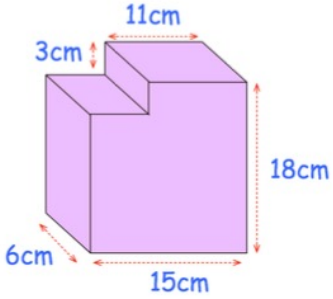
(c)



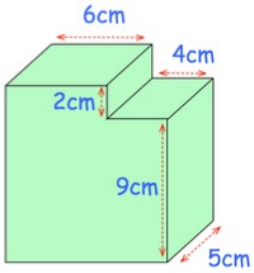
(d)



(e)

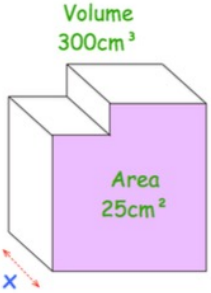


(f)

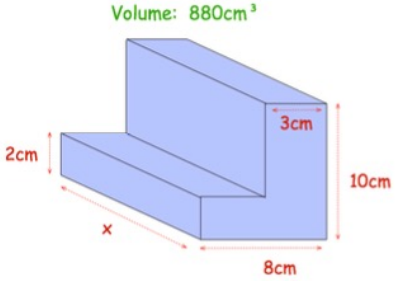


Question 3: Work out the length of each of these prisms.

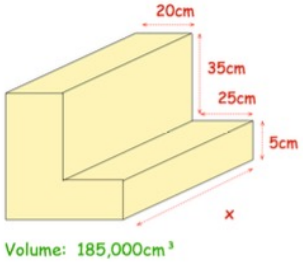
(a)



(b)

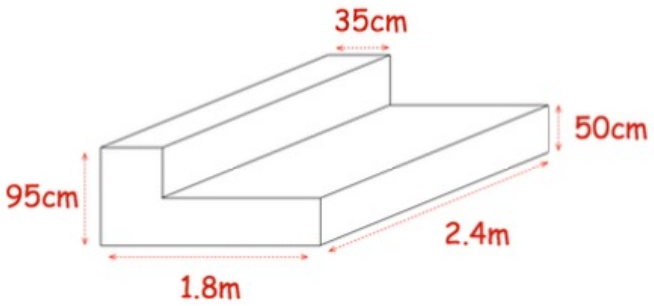


(c)

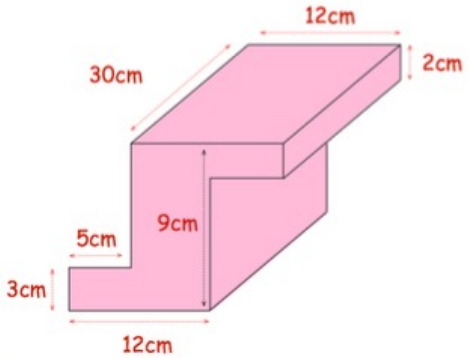


Extension

Question 1: Shown below is a prism.
Find the volume of the prism.
Give your answer in m^3.

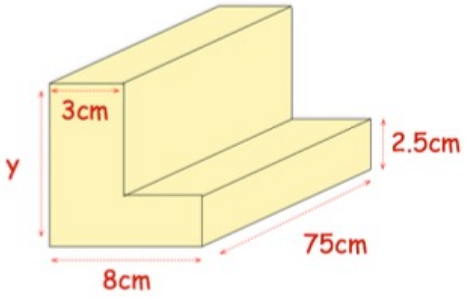


Question 2: Find the volume of the prism.



Volume: $2580cm^3$

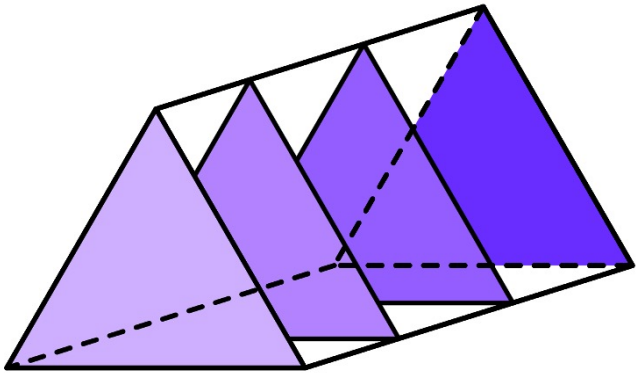
Question 3: Find y

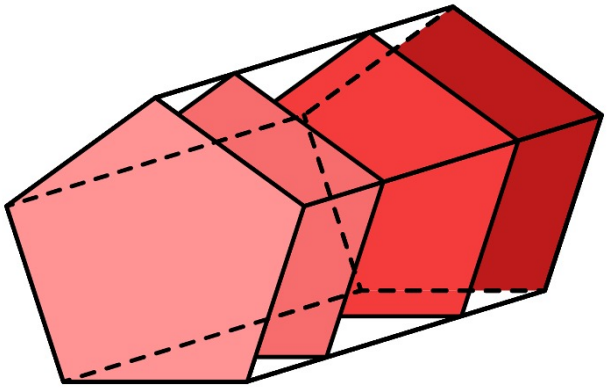


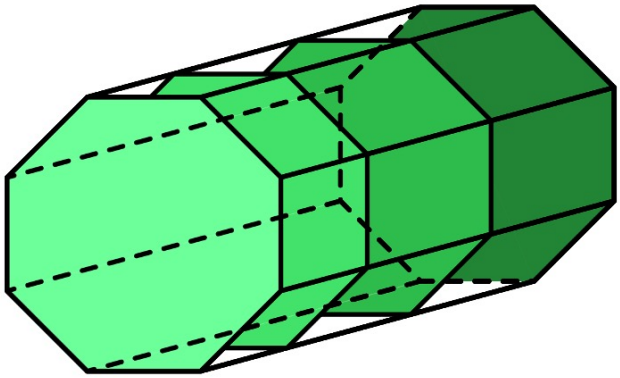
2.11 Surface Area of a Prism

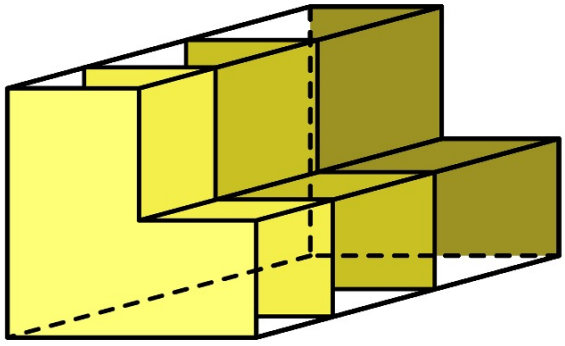
Surface Area is the total area across the surface.

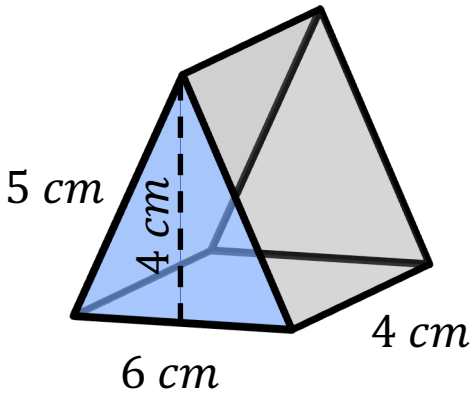
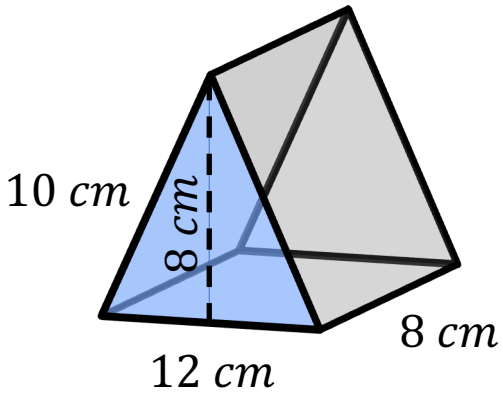
A prism is a 3D shape that has a constant cross-section along its length.

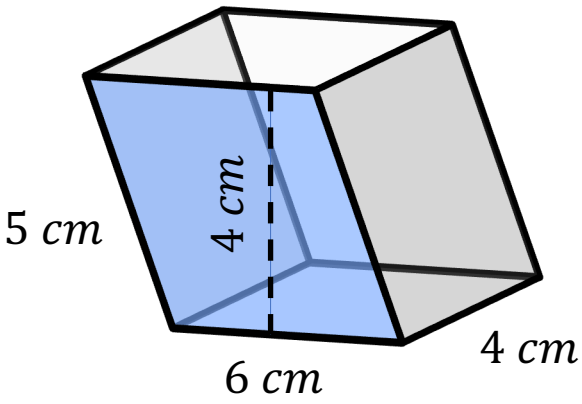
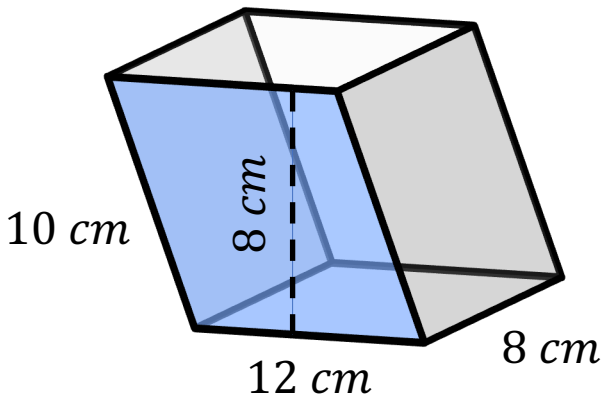


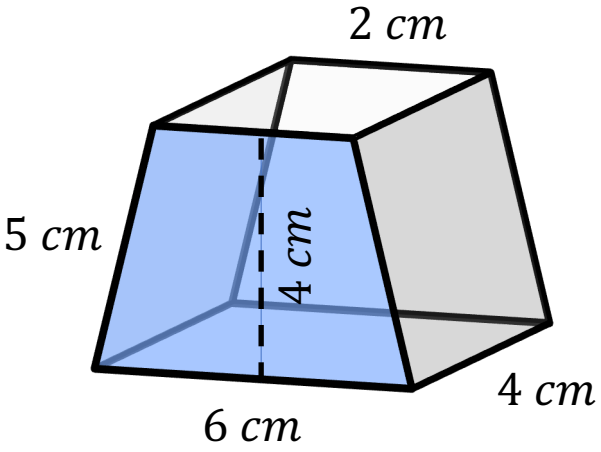
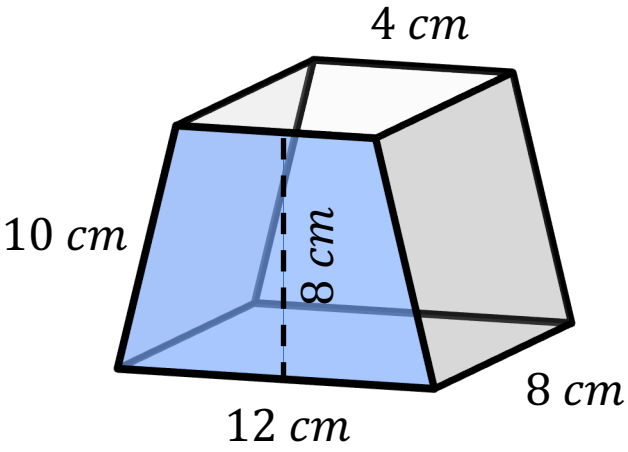


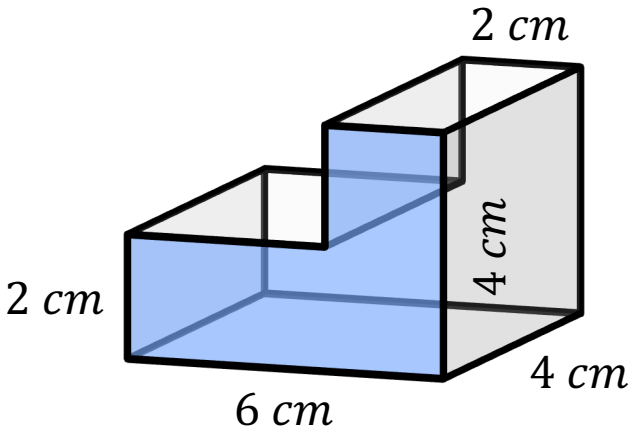
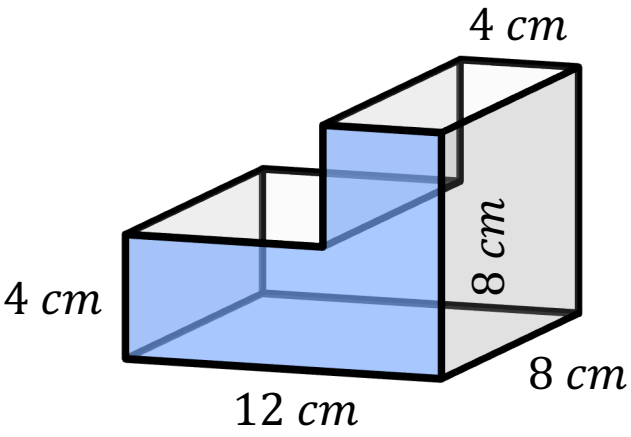




Worked Example	Your Turn
Calculate the surface area of the triangular prism:  	Calculate the surface area of the triangular prism: 

Worked Example	Your Turn
Calculate the surface area of the parallelepiped:  	Calculate the surface area of the parallelepiped: 

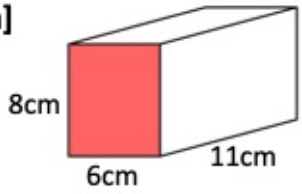
Worked Example	Your Turn
Calculate the surface area of the trapezium prism:  	Calculate the surface area of the trapezium prism: 

Worked Example	Your Turn
Calculate the surface area of the L-shaped prism:  	Calculate the surface area of the L-shaped prism: 

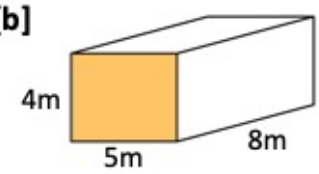
Fluency Practice

Q1 Calculate the surface area of the following cuboids.

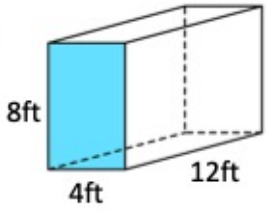
[a]



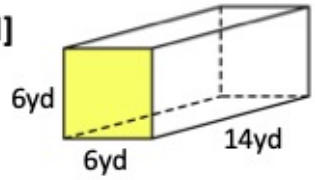
[b]



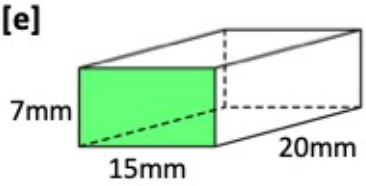
[c]



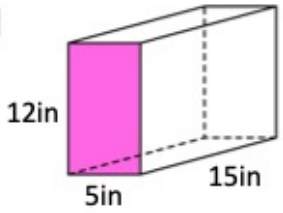
[d]



[e]

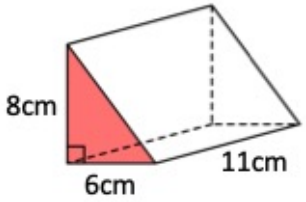


[f]

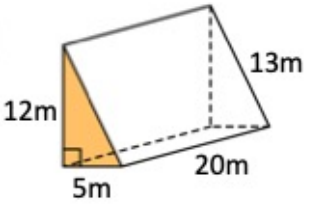


Q2 Calculate the surface area of the following prisms.

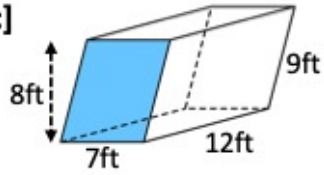
[a]



[b]

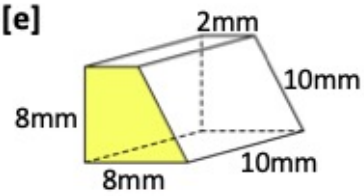


[c]

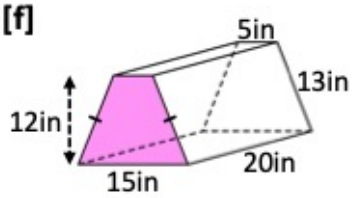


[d]

[e]



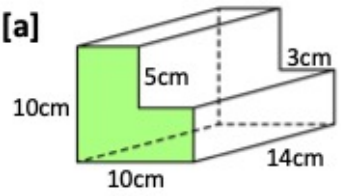
[f]



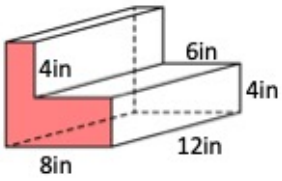
Fluency Practice

Q3 Calculate the volume of the following L-shape prisms.

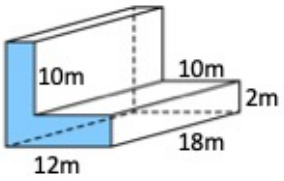
[a]



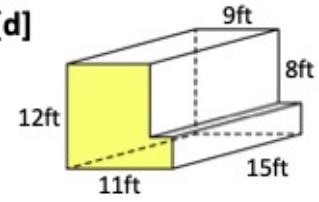
[b]



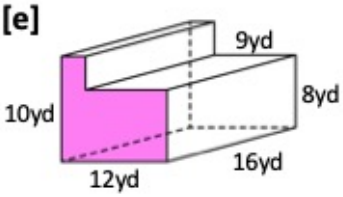
[c]



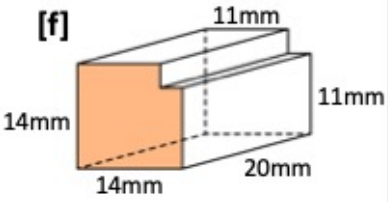
[d]



[e]

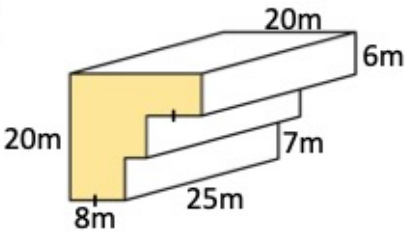


[f]

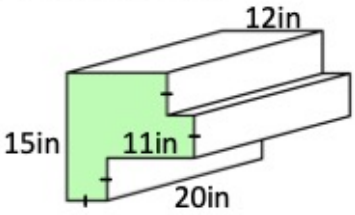


Q4 Calculate the surface area of the following prisms.

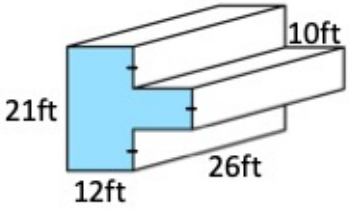
[a]



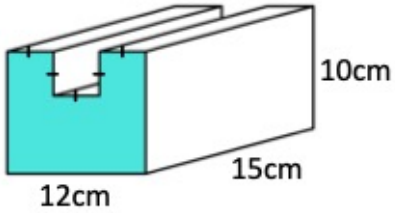
[b]



[c]

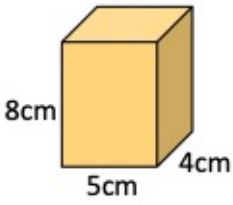


[d]

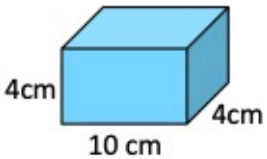


Q5 A company is designing a new box to hold cereal. They have two designs, cuboids A and B, with the same volume. The company wants to choose the smallest surface area. Which cuboid should they choose?

Cuboid A



Cuboid B



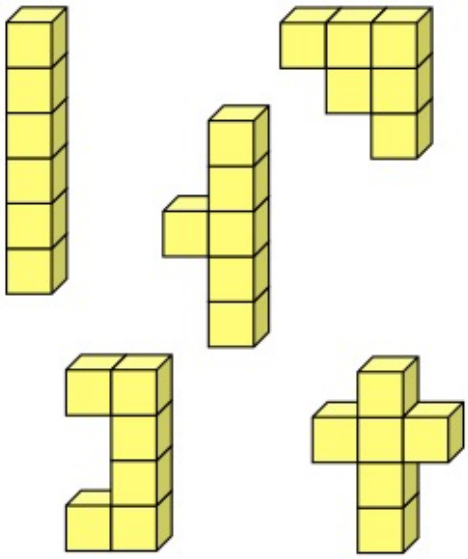
Q6 A cube has a surface area of 96cm^2. Find the side length of the cube.

Q7 A cube has a surface area of 294cm^2. Find the side length, x, of the cube.

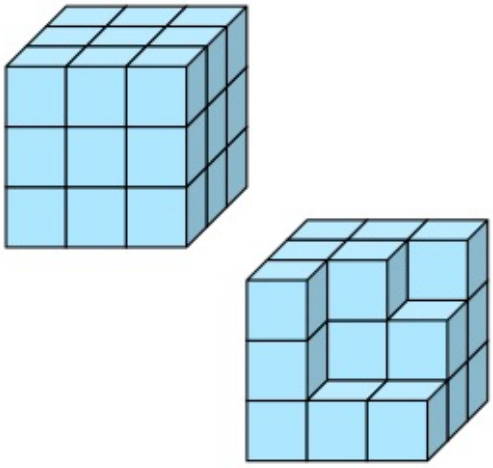
Q8 A cube has a surface area of 150cm^2. Find the volume of the cube.

Fluency Practice

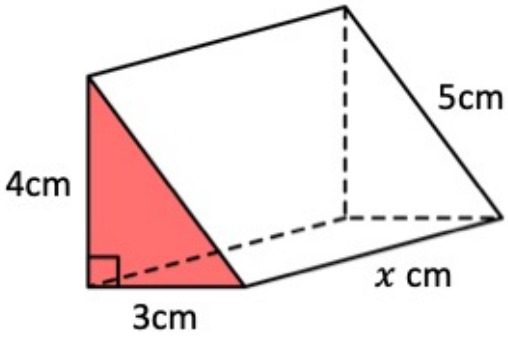
Q9 Each of the four shapes shown below is made from 6 unit cubes. Which has the smallest surface area?



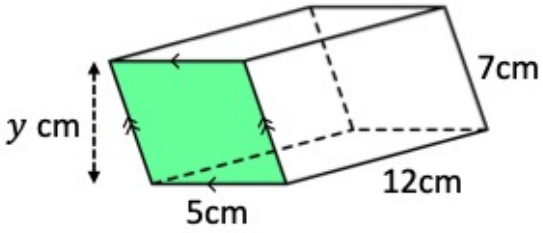
Q10 It takes 15 litres of paint to cover the surface are of the cube on the left. How much paint would it take to cover the surface area of the shape on the right?

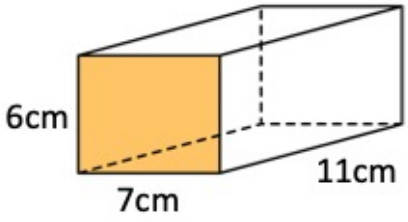


Q11 Given that the surface are of the triangular prism below is 96cm^2, find x.



Q12 Given that the surface area of the following prisms are equal, find y.

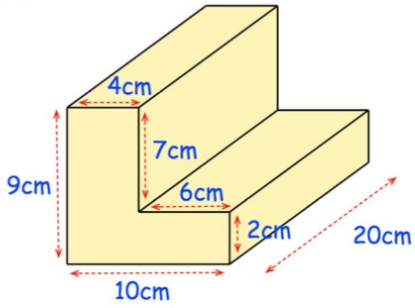




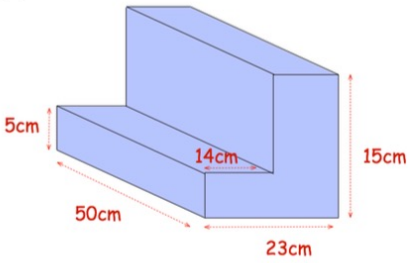
Fluency Practice

Question 1: Work out the surface area of each of the prisms below.

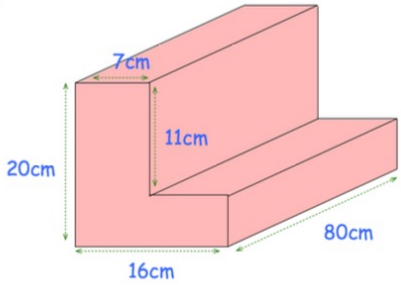
(a)



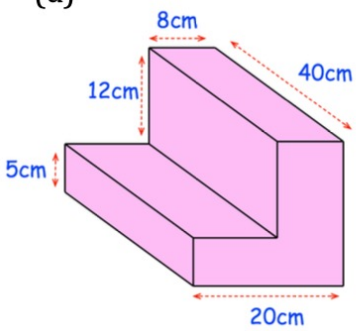
(b)



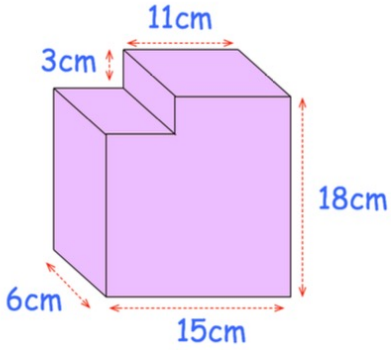
(c)



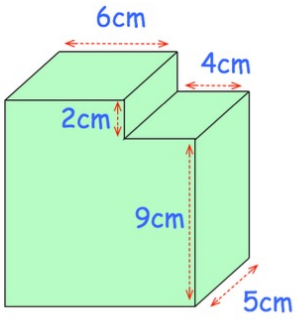
(d)



(e)

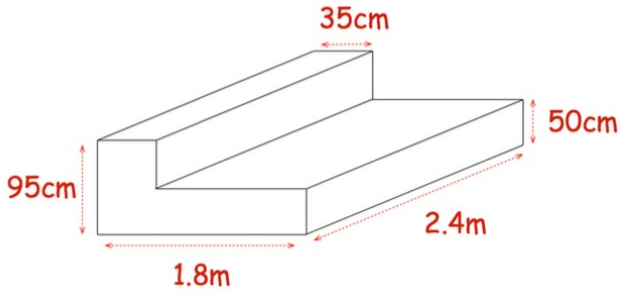


(f)



Extension

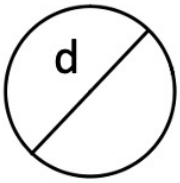
Question 1: Shown below is a prism.
Find the surface area of the prism.
Give your answer in m^2.



2.12 Circles Recap

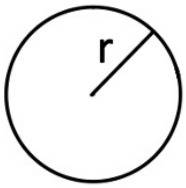
- Circumference = π x diameter

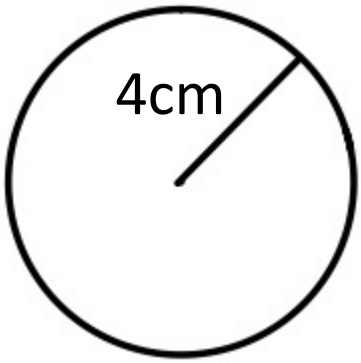
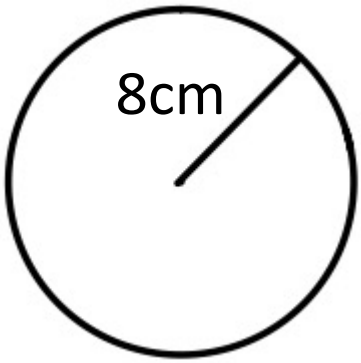
$$C = \pi \times d$$

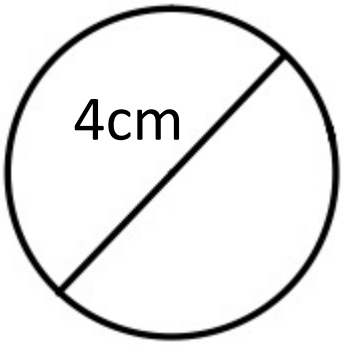
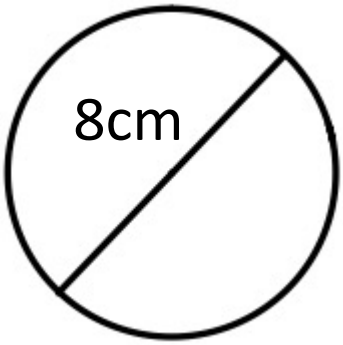


- Area = π x radius^2

$$A = \pi \times r^2$$



Worked Example	Your Turn
Calculate the circumference of the circle below:  	Calculate the circumference of the circle below: 

Worked Example	Your Turn
Calculate the area of the circle below:  	Calculate the area of the circle below: 

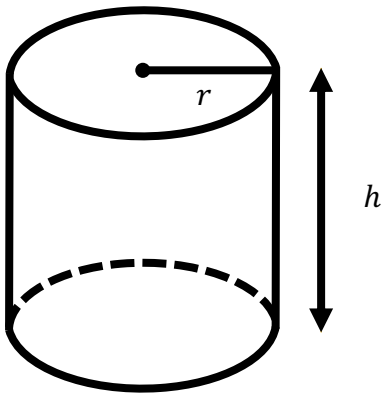
Fluency Practice

Circumference and Area

2.13 Volume of a Cylinder

Volume is the amount of space an object takes up.

A cylinder is just a prism with a circular cross-section.

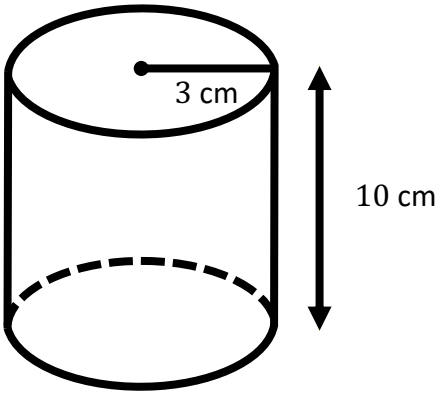


$$\textbf{Volume of Cylinder} = \textbf{Area of Circle} \times \textbf{Height}$$

$$\textbf{Volume of Cylinder} = \pi r^2 h$$

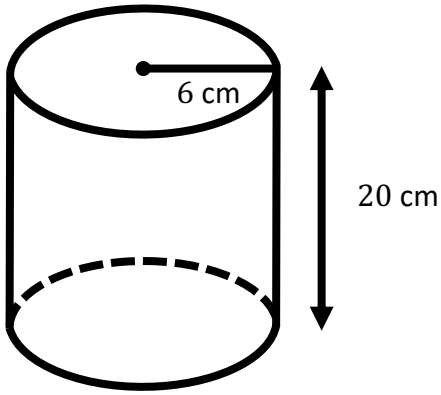
Worked Example

Calculate the volume of the following cylinder. Give your answer in terms of π and to 1 decimal place.



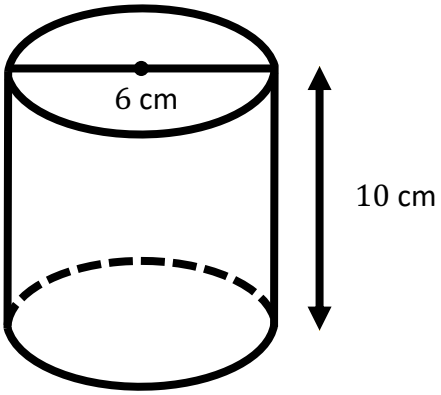
Your Turn

Calculate the volume of the following cylinder. Give your answer in terms of π and to 1 decimal place.



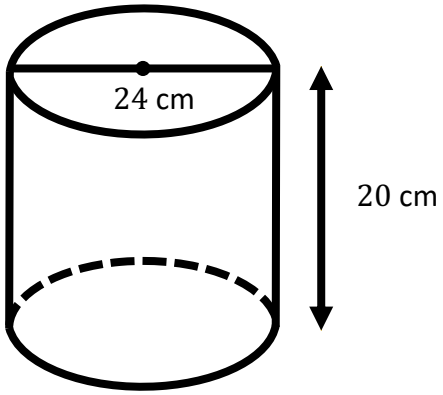
Worked Example

Calculate the volume of the following cylinder. Give your answer in terms of π and to 1 decimal place.



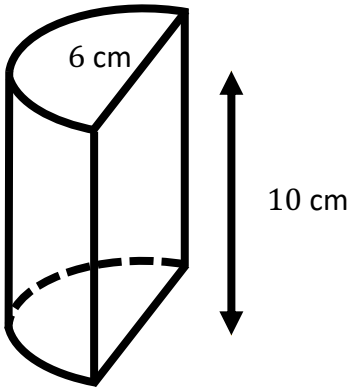
Your Turn

Calculate the volume of the following cylinder. Give your answer in terms of π and to 1 decimal place.



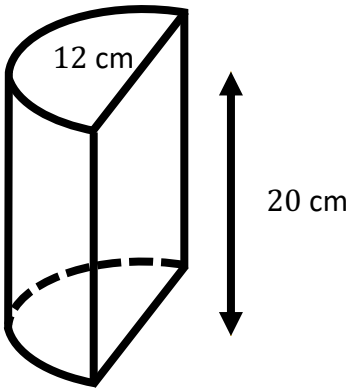
Worked Example

Calculate the volume of the following half cylinder. Give your answer in terms of π and to 1 decimal place.



Your Turn

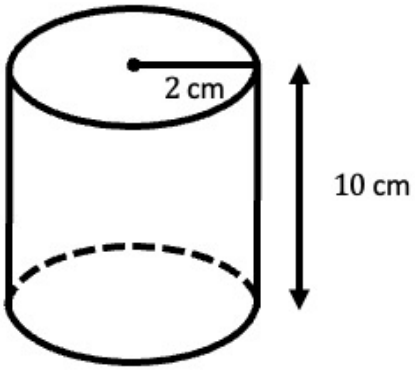
Calculate the volume of the following half cylinder. Give your answer in terms of π and to 1 decimal place.



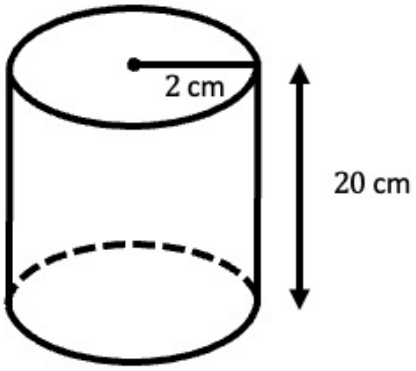
Intelligent Practice

Calculate the volume. Give your answers in terms of π and to 1 decimal place.

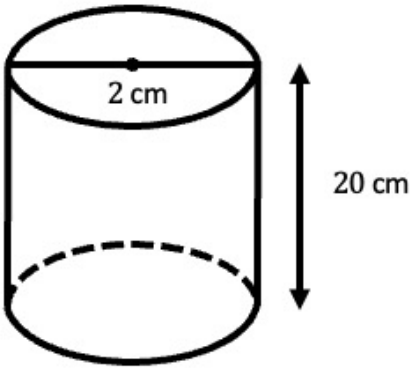
Question 1



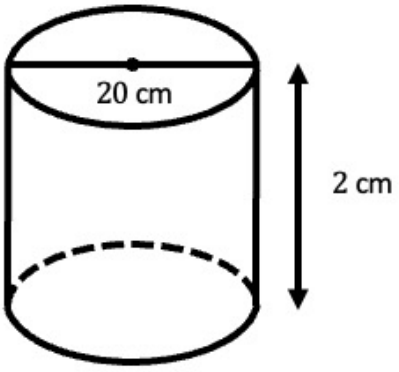
Question 2



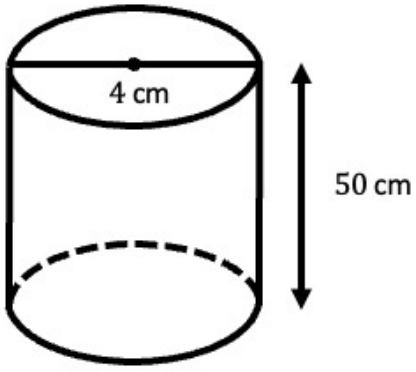
Question 3



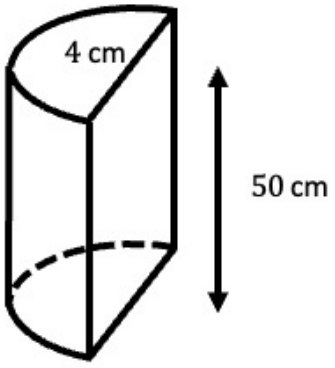
Question 4



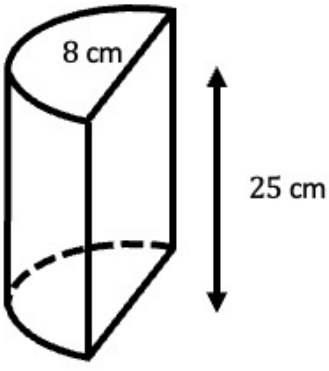
Question 5



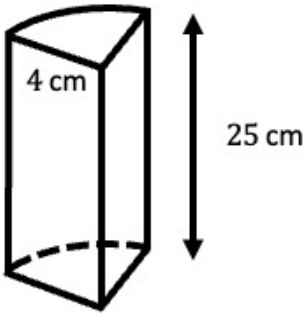
Question 6



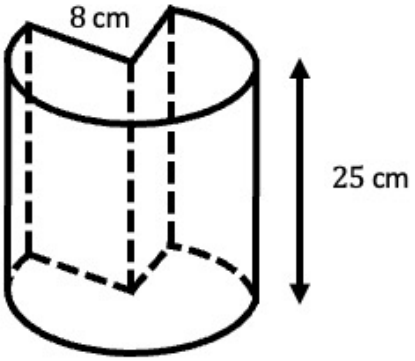
Question 7



Question 8



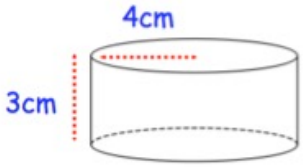
Question 9



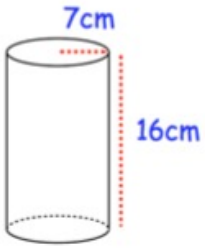
Fluency Practice

Question 3: Calculate the volume of each cylinder below

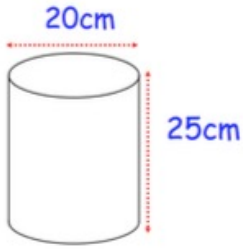
(a)



(b)



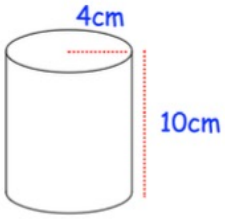
(c)



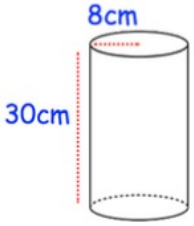
Fluency Practice

Question 1: Work out the volume of each cylinder.
Give each answer to one decimal place.

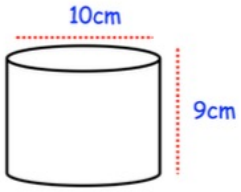
(a)



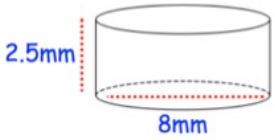
(b)



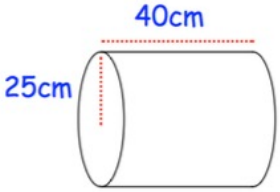
(c)



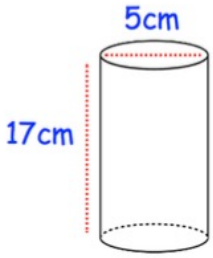
(d)



(e)

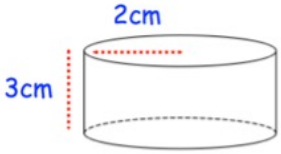


(f)

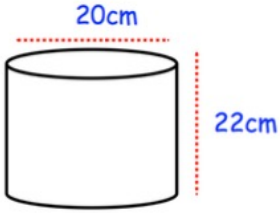


Question 2: Work out the volume of each cylinder.
Give each answer in terms of **π**.

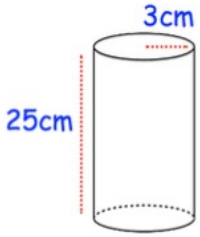
(a)



(b)

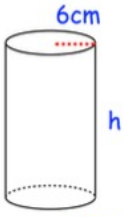


(c)



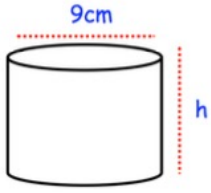
Question 3: Work out the height of each cylinder.
Give each answer to one decimal place.

(a)



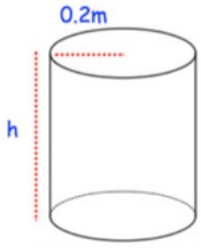
Volume = 1600cm³

(b)



Volume = 800cm³

(c)

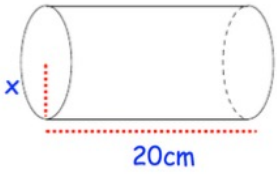


Volume = 0.11m³

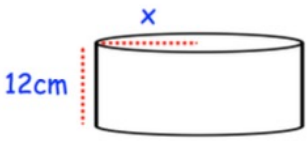
Question 4: Work out the value of x.
Give each answer to one decimal place.

(a)

Volume = 725cm³

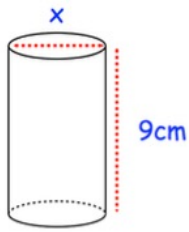


(b)



Volume = 9000cm³

(c)



Volume = 170cm³

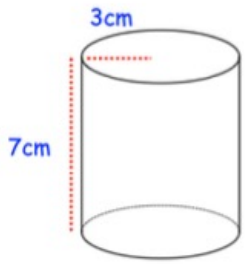
Extension

Question 1: A cylindrical oil drum has a diameter of 48cm and a height of 92cm.
Calculate the volume of the oil drum.

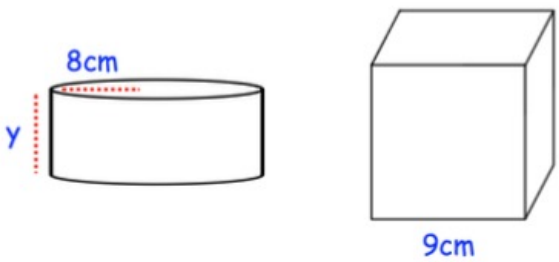
Question 2: A cylinder has a radius of 2m and a height of 5cm.
Work out the volume of the cylinder in terms of **π**.

Question 3: Timothy is filling cups with orange juice.
Each cup is a cylinder with radius 3cm and height 7cm.
Timothy has 2 litres of orange juice.
1 litre = $1000cm^3$

How many cups can be filled?

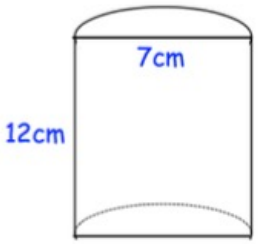


Question 4: Shown below is a cylinder and a cube.
The volume of the cylinder is equal to the volume of the cube.
Find y.

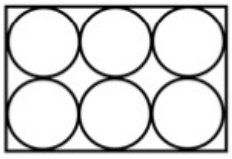


Extension

Question 5: Calculate the volume of this shape.

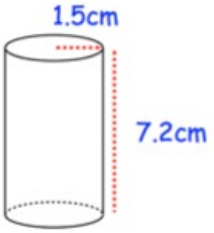


Question 6: 6 cylinders are placed in a crate as shown below.
The radius of each cylinder is 4cm and the height of each cylinder is 14cm.
The crate also has a height of 14cm.



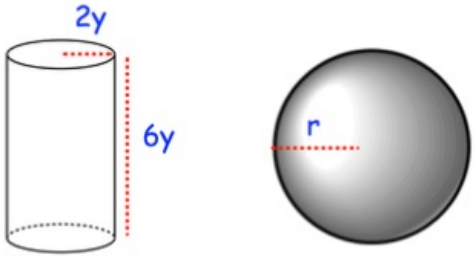
What percentage of space in the crate is empty?

Question 7: A solid glass cylinder has a radius of 1.5cm and a height of 7.2cm
The density of the glass is 2.61g/cm^3
Work out the mass of the cylinder.



Question 8: The diagram shows a solid cylinder.
The cylinder has radius of 2y and height of 6y.
The cylinder is melted down and made into a sphere of radius r.

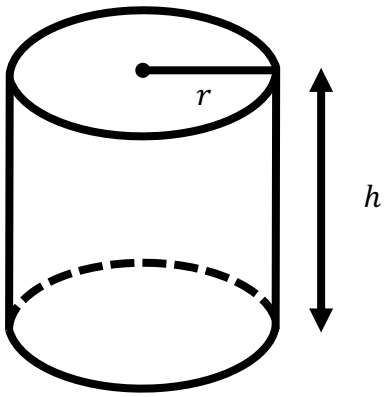
Express r in terms of y.



2.14 Surface Area of a Cylinder

Surface Area is the total area across the surface.

A cylinder is just a prism with a circular cross-section.

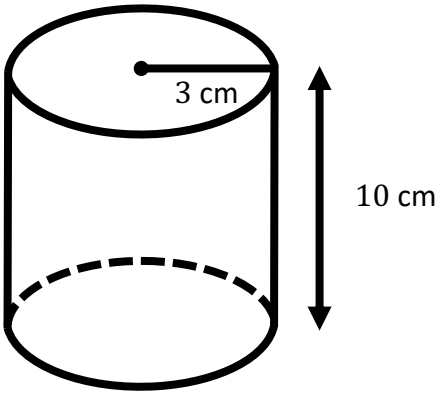


Curved Surface Area of Cylinder $= \boldsymbol{2\pi rh}$

Total Surface Area of Cylinder $= \boldsymbol{2\pi rh + 2\pi r^2}$

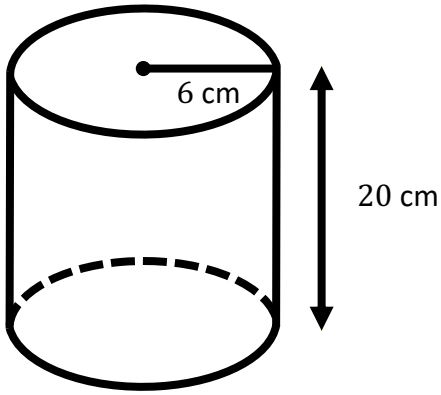
Worked Example

Calculate the curved surface area of the following cylinder. Give your answer in terms of π and to 1 decimal place.



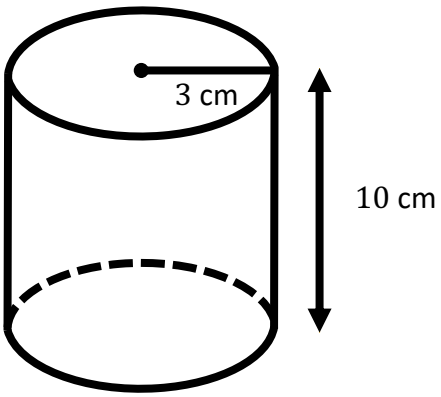
Your Turn

Calculate the curved surface area of the following cylinder. Give your answer in terms of π and to 1 decimal place.



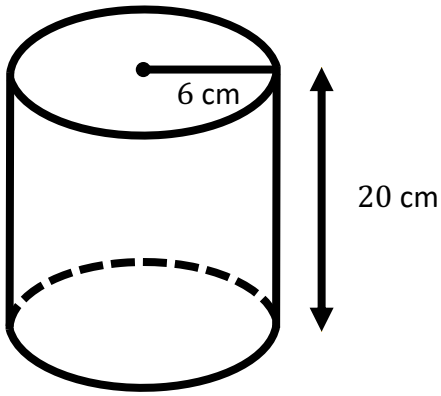
Worked Example

Calculate the total surface area of the following cylinder. Give your answer in terms of π and to 1 decimal place.



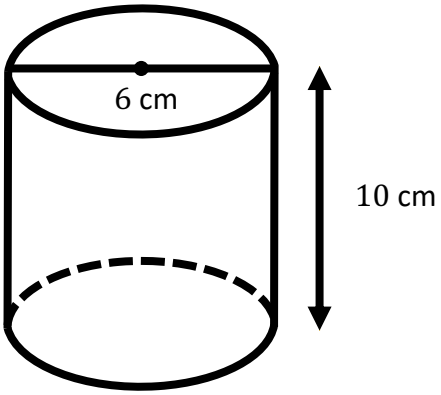
Your Turn

Calculate the total surface area of the following cylinder. Give your answer in terms of π and to 1 decimal place.



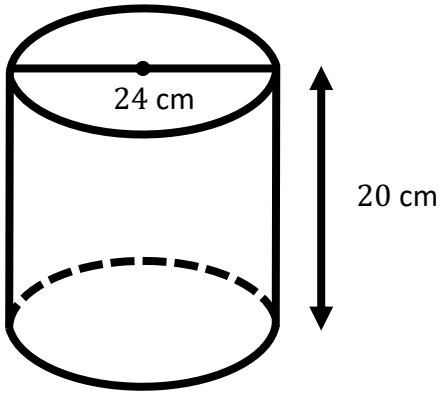
Worked Example

Calculate the total surface area of the following cylinder. Give your answer in terms of π and to 1 decimal place.



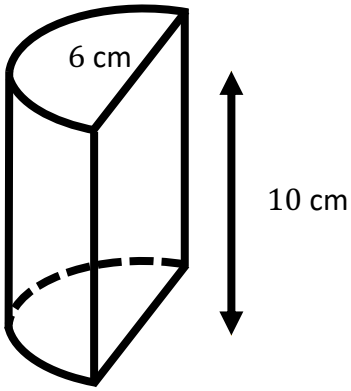
Your Turn

Calculate the total surface area of the following cylinder. Give your answer in terms of π and to 1 decimal place.



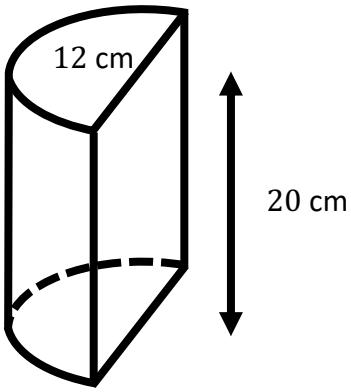
Worked Example

Calculate the curved surface area of the following half cylinder. Give your answer in terms of π and to 1 decimal place.



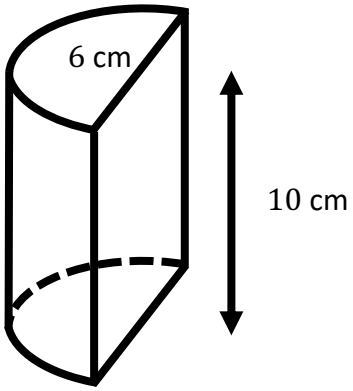
Your Turn

Calculate the curved surface area of the following half cylinder. Give your answer in terms of π and to 1 decimal place.



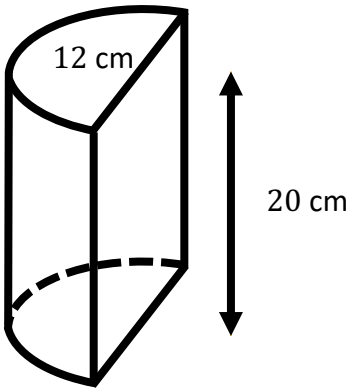
Worked Example

Calculate the total surface area of the following half cylinder. Give your answer in terms of π and to 1 decimal place.



Your Turn

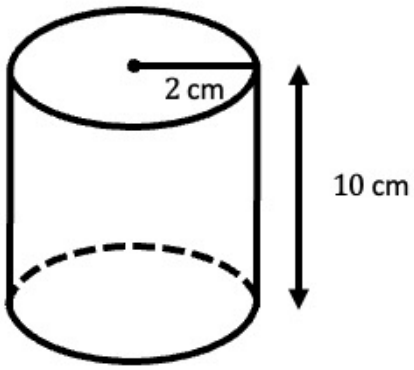
Calculate the total surface area of the following half cylinder. Give your answer in terms of π and to 1 decimal place.



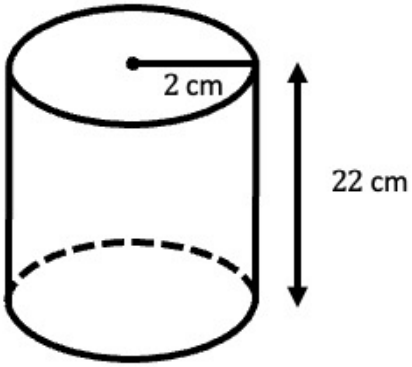
Intelligent Practice

Calculate the total surface area. Give your answers in terms of π and to 1 decimal place.

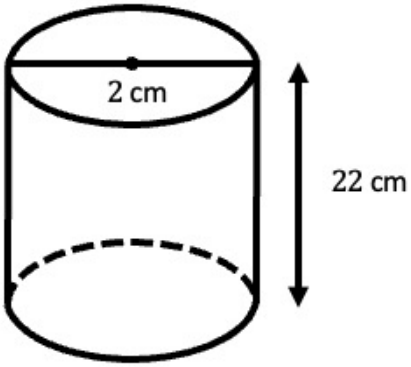
Question 1



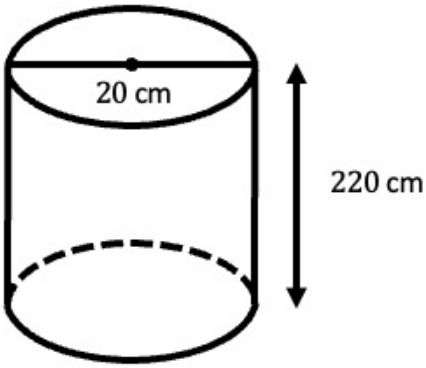
Question 2



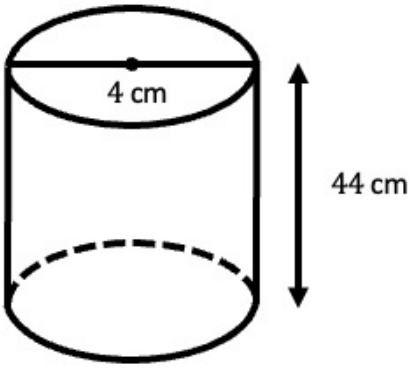
Question 3



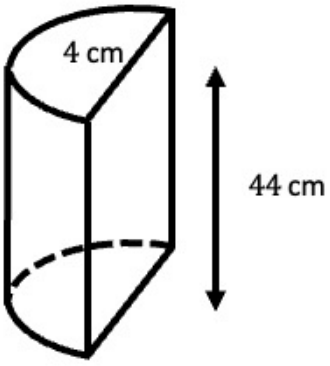
Question 4



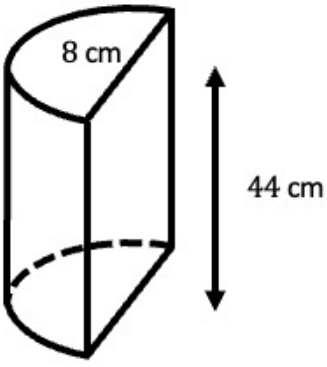
Question 5



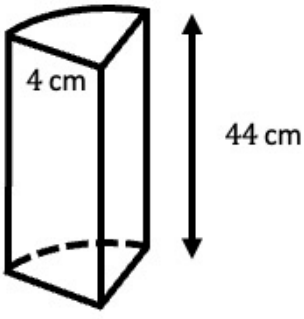
Question 6



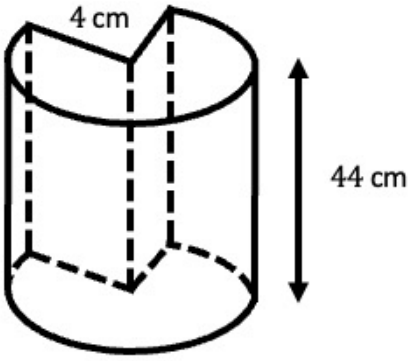
Question 7



Question 8



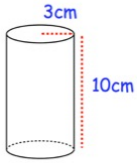
Question 9



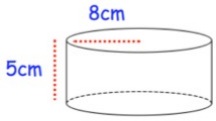
Fluency Practice

Question 1: Work out the surface area of each of the following cylinders.
Give each answer to 2 decimal places.

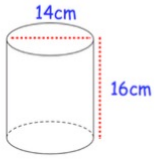
(a)



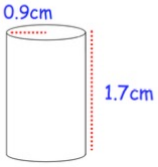
(b)



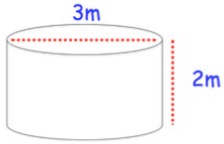
(c)



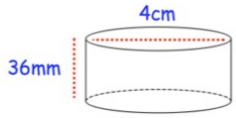
(d)



(e)



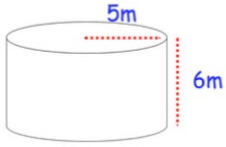
(f)



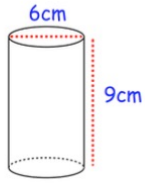
Question 2: Work out the surface area of each of the following cylinders.
Leave your answers in terms of **π**

(a)

(b)

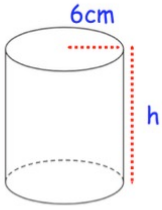


(c)



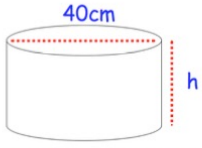
Question 3: Work out the height of each cylinder below

(a)



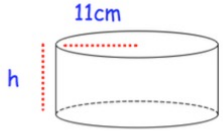
Surface area = 980.18cm²

(b)



Surface area = 4715cm²

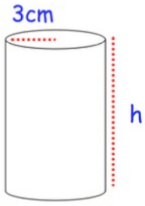
(c)



Surface area = 850cm²

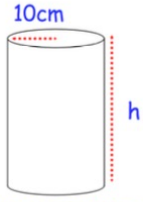
Question 4: Work out the height of each cylinder below

(a)



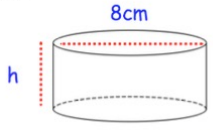
Surface area = 84π cm²

(b)



Surface area = 900π cm²

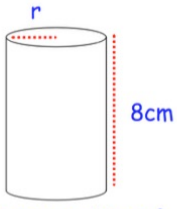
(c)



Surface area = 56π cm²

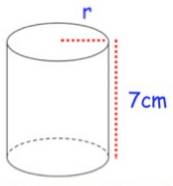
Question 5: Work out the radius of each cylinder below

(a)



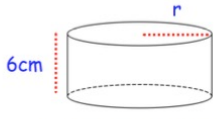
Surface area = 18π cm²

(b)



Surface area = 36π cm²

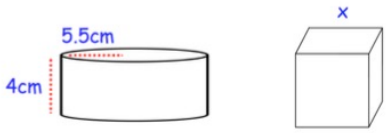
(c)



Surface area = 1040π cm²

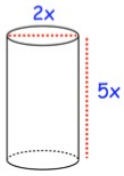
Extension

Question 1: The cylinder and cube below have the same surface area.
Find the side length of the cube, x.



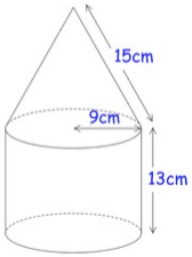
Question 2: A can of baked beans has a paper label wrapped around the outside.
The can has a height of 11cm and radius of 4.5cm.
The label covers the entire height of the can.
The label has a 1cm overlap vertically so that it can be stuck together
Calculate the area of the label.

Question 3: The cylinder below has a surface area of 972π cm^2.
Find x.

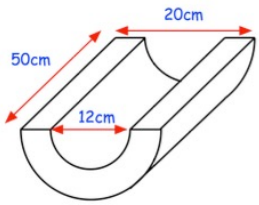


Question 4: A cylinder has a height of 18cm and volume of 1715cm^3.
Work out the surface area of the cylinder.

Question 5: A cylinder and a cone are joined together to make a solid.
The cylinder has a radius of 9cm and height of 13cm.
The cone has a slant height of 15cm.
Find the total surface area of the solid.

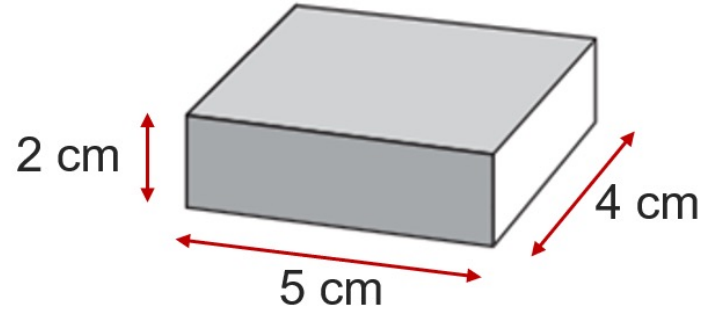


Question 6: Work out the surface area of the shape below.

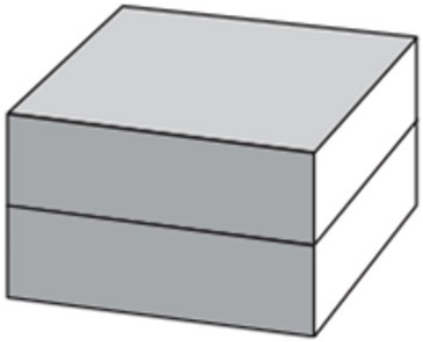


2.15 Review and Problem Solving

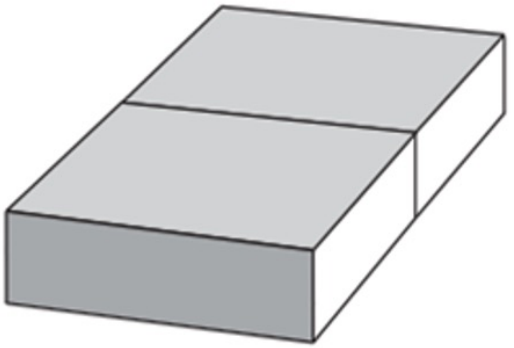
two boxes stuck together



(1)

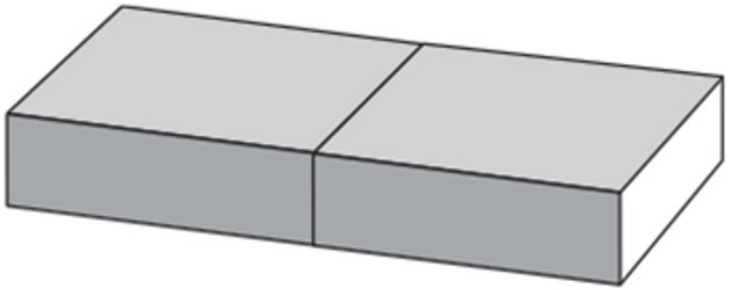


(2)



what are the outside surface areas of these boxes?

(3)



Kellogg's Corn Flakes Investigation

Which size is the most eco-friendly?

Which size gives the best value, and by how much?

Kellogg's Corn Flakes come in various sizes of box. The main ones are listed below. Use the measurements given for each type to calculate the volume and surface area.

Box Size	Weight (g)	Cost	Height	Width	Depth	Volume	Surface Area
Small	250	£1.39	***25cm***	***19cm***	***5.5cm***		
Medium	500	£1.98	***29.5cm***	***23cm***	***7cm***		
Large	750	£2.68	***35cm***	***24.5cm***	***9cm***		

Remember:

- To calculate volume, *multiply all 3 dimensions together*.
- To calculate surface area, *find the area of all 6 faces and add them together.*

1. How much cardboard per gram of cornflakes does the most efficient box use?

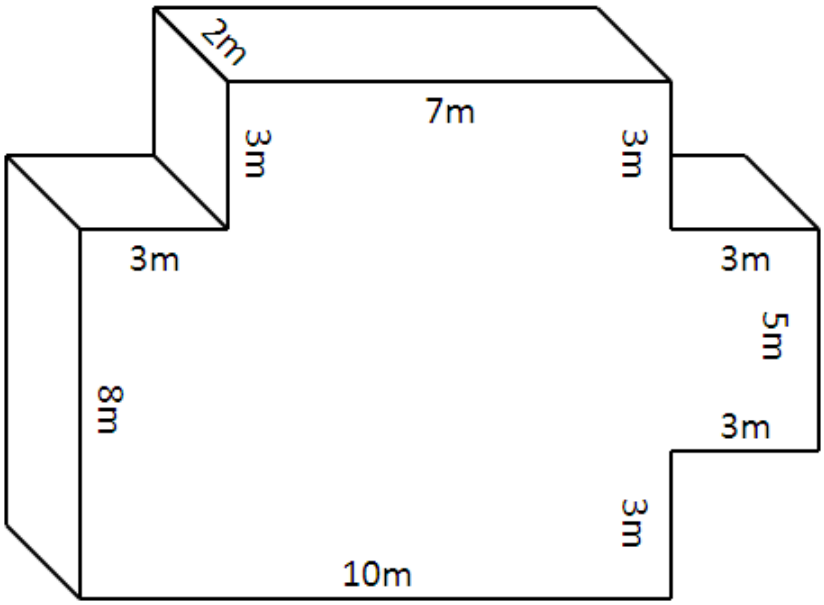
2. Which box gives the best value (most cornflakes for your money)?

3. If you planned to make a 1500g box of cereal by doubling *one* of the dimensions of the 750g box, which dimension would you double to minimise the surface area, and what would the resulting surface area of the box be?

Extension: If you could choose the dimensions of your 1500g box, how would you minimise surface area? If you could choose *any* shape whatsoever, what would be best?

Fuel Tank

An underground fuel storage unit is designed as shown:



Task 1
Diesel costs around £1200 per cubic metre. Calculate how much it would cost to fill this storage unit completely.

Task 2
The tank is to be constructed from sheets of stainless steel at a cost of £25 per square metre. Draw a set of scale drawings for each plate required, and calculate the total cost of the stainless steel.

Task 3
A welder is employed to put the tank together. He charges £30 per metre for welding stainless steel seams. Calculate the total cost of welding the tank together.