

Year 8
Mathematics
Unit 4 – Student



Name: _____

Class: _____

Contents

- 1 [Compound Measures](#)
 - 1.1 [Speed](#)
 - 1.2 [Density](#)
 - 1.3 [Pressure](#)
 - 1.4 [Review and Problem Solving](#)

- 2 [Inequalities](#)
 - 2.1 [Reading Inequalities](#)
 - 2.2 [Two-Ended Inequalities](#)
 - 2.3 [Inequalities on Number Lines](#)
 - 2.4 [Solving Linear Inequalities](#)
 - 2.5 [Solving Linear Inequalities with Variable on Both Sides](#)
 - 2.6 [Solving Compound Inequalities](#)
 - 2.7 [Review and Problem Solving](#)

- 3 [Probability](#)
 - 3.1 [Probability Scale](#)
 - 3.2 [Probability of Single Events](#)
 - 3.3 [Mutually Exclusive Events](#)
 - 3.4 [Exhaustive Events](#)
 - 3.5 [Expectation](#)
 - 3.6 [Relative Frequency](#)
 - 3.7 [Listing Outcomes](#)
 - 3.8 [Sample Space Diagrams](#)
 - 3.9 [Review and Problem Solving](#)

1 Compound Measures

Compound measures are measures that rely on other measures:

- Speed
- Density
- Pressure

1.1 Speed

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Worked Example

An object travels 40 miles in 2 hours. Calculate its speed in mph?

Your Turn

An object travels 40 miles in 30 minutes. Calculate its speed in mph?

Intelligent Practice

An object travels ___ miles in ___ hours/minutes.
Find its speed in mph.

- | | |
|------------------------|----------------------------|
| 1) 60 miles in 1 hour | 9) 36 miles in 8 hours |
| 2) 60 miles in 2 hours | 10) 36 miles in 1 hour |
| 3) 60 miles in 3 hours | 11) 36 miles in 30 minutes |
| 4) 60 miles in 4 hours | 12) 36 miles in 20 minutes |
| 5) 60 miles in 8 hours | 13) 36 miles in 10 minutes |
| 6) 60 miles in 8 hours | 14) 36 miles in 15 minutes |
| 7) 60 miles in 8 hours | 15) 36 miles in 25 minutes |
| 8) 60 miles in 8 hours | |

Worked Example

An object travels at 40 mph for 2 hours. How far has it travelled in miles?

Your Turn

An object travels at 40 mph for 30 minutes. How far has it travelled in miles?

Intelligent Practice

An object travels at ___ mph in ___ hours/minutes.
Find its distance in miles.

- 1) 60 mph in 1 hour
- 2) 60 mph in 2 hours
- 3) 60 mph in 3 hours
- 4) 60 mph in 4 hours
- 5) 60 mph in 8 hours
- 6) 60 mph in 8 hours
- 7) 60 mph in 8 hours
- 8) 60 mph in 8 hours
- 9) 9.5 mph in 8 hours
- 10) 9.5 mph in 1 hour
- 11) 9.5 mph in 30 minutes
- 12) 9.5 mph in 20 minutes
- 13) 9.5 mph in 10 minutes
- 14) 9.5 mph in 15 minutes
- 15) 9.5 mph in 12 minutes

Worked Example

An object travels 80 miles at 40 mph. How long does the journey take in hours?

Your Turn

An object travels 20 miles at 40 mph. How long does the journey take in hours?

Intelligent Practice

An object travels ___ miles at ___ mph.
Find the time taken in hours.

- | | |
|------------------------|------------------------|
| 1) 60 miles at 60 mph | 9) 24 miles at 7.5 mph |
| 2) 120 miles at 60 mph | 10) 24 miles at 30 mph |
| 3) 180 miles at 60 mph | 11) 24 miles at 60 mph |
| 4) 240 miles at 60 mph | 12) 8 miles at 60 mph |
| 5) 240 miles at 30 mph | 13) 4 miles at 30 mph |
| 6) 240 miles at 15 mph | 14) 8 miles at 45 mph |
| 7) 240 miles at 5 mph | 15) 5 miles at 72 mph |
| 8) 24 miles at 5 mph | |

Fluency Practice

Question 1: Calculate the average speeds for each of the following, without using a calculator.

- (a) A car travels 60 miles in 2 hours
- (b) A lorry travels 120 miles in 3 hours
- (c) A cyclist travels 45 miles in 5 hours
- (d) A jogger travels 30km in 4 hours
- (e) A runner runs 100 metres in 10 seconds
- (f) A car travels 195 miles in 3 hours
- (g) A helicopter travels 425 miles in 5 hours
- (h) A helicopter flies 840 miles in 7 hours
- (i) A dog runs 216 metres in 12 seconds
- (j) An airplane travels 984 miles in 6 hours
- (k) A bird flies 19 miles in 2 hours
- (l) A car travels 600km in 8 hours

Question 2: Calculate the average speeds for each of the following, without using a calculator.

- (a) A car travels 20 miles in 30 minutes
- (b) A lorry travels 32 miles in 30 minutes
- (c) A bird flies 17 kilometres in 30 minutes
- (d) A man jogs 2 kilometres in 15 minutes.
- (e) A helicopter flies 18 miles in 15 minutes
- (f) An F1 car travels 32 miles in 15 minutes.
- (g) A dog runs 3 kilometres in 10 minutes
- (h) A jet travels 23 miles in 6 minutes.
- (i) A car travels 12 miles in 20 minutes
- (j) A car travels 9 miles in 12 minutes
- (k) A motorcycle travels 36 miles in 40 minutes
- (l) A car travels 27 kilometres in 45 minutes.

Question 3: Calculate the average speeds for each of the following.

- (a) A car travels 63 miles in 1 hour 30 minutes
- (b) A man runs 15 miles in 2 hours 30 minutes
- (c) A helicopter flies 238 miles in 3 hours 30 minutes
- (d) A car travels 85.5 miles 2 hours 15 minutes
- (e) An airplane flies 315 kilometres in 1 hour 45 minutes
- (f) A lorry travels 351 miles in 6 hours 45 minutes
- (g) A car drives 154 miles in 2 hours 20 minutes
- (h) A helicopter flies 160 kilometres in 1 hour 40 minutes

Question 4: Calculate the average speeds for each of the following.

- (a) A man jogs 6 miles in 1 hour 12 minutes
- (b) A motorcycle drives 130 miles in 2 hours 36 minutes
- (c) A helicopter flies 152 miles in 1 hour 54 minutes
- (d) A plane travels 1272 kilometres in 5 hours 18 minutes
- (e) A car travels 98 miles in 2 hours 27 minutes
- (f) A rocket travels 750 miles in 3 minutes
- (g) A car travels 6.4 miles in 7 minutes. Give your answer to 2 decimal places.
- (h) A ship sails 105 miles in 4 hours 28 minutes. Give your answer to 2 decimal places.
- (i) A plane travels 400 miles in 1 hour 55 minutes. Give your answer to 2 decimal places.
- (j) A car drives 500 kilometres in 7 hours 13 minutes. Give your answer to 2 decimal places.

Fluency Practice

Question 5: Calculate how far each of the following travels.

- (a) A car travels at a speed of 50mph for 3 hours.
- (b) A plane flies at a speed of 230 kilometres per hour for 2 hours.
- (c) A lorry drives for 4 hours at a speed of 45 miles per hour.
- (d) A man runs at a speed of 8 metres per second for 15 seconds.
- (e) A helicopter flies for 8 hours at a speed of 80 miles per hour.
- (f) A dog runs at a speed of 15 m/s for 20 seconds.
- (g) A car travels at a speed of 48 mph for 3 hours.
- (h) A truck travels at a speed of 29 mph for 5 hours.

Question 6: Calculate the distance travelled by each of the following.

- (a) A car drives at a speed of 60mph for 30 minutes.
- (b) A taxi travels for 30 minutes at a speed of 28 mph.
- (c) A car travels at a speed of 44mph for 15 minutes.
- (d) A lorry drives at a speed of 51mph for 20 minutes.
- (e) An airplane travels at a speed of 441mph for 20 minutes.
- (f) A car drives at a speed of 48mph for 45 minutes.
- (g) A helicopter flies at a speed of 72miles per hour for 10 minutes
- (h) A bird flies for 40 minutes at a speed of 60 kilometres per hour.

Question 7: Work out the distance travelled by each of the following.

- (a) A car drives at a speed of 40mph for 1 hour 30 minutes
- (b) A bird flies at a speed of 32 kilometres per hour for 1 hour 30 minutes
- (c) A lorry travels for 2 hours 30 minutes at a speed of 52 mph
- (d) A F1 race car drives for 1 hour 15 minutes at a speed of 124 mph
- (e) A helicopter flies at a speed of 104 mph for 1 hour 45 minutes
- (f) A car drives at a speed of 58 mph for 3 hours 15 minutes
- (g) A man runs at 6 mph for 1 hour 24 minutes
- (h) A car drives for 2 hours 54 minutes at a speed of 50 mph
- (i) A plane flies at a speed of 306 kilometres per hour for 3 hours 20 minutes
- (j) A hot air balloon flies at a speed of 18 mph for 1 hour 40 minutes
- (k) A bird flies for 4 hours 36 minutes at a speed of 40 kilometres per hour.
- (l) A helicopter travels at 98mph for 5 hours 6 minutes.
- (m) A car travels at 40 mph for 1 hour 7 minutes. Give your answer to 2 decimal places.
- (n) A lorry drives at 65 mph for 2 hours 19 minutes. Give your answer to 2 decimal places.
- (o) A car drives at 70 mph for 44 minutes. Give your answer to 2 decimal places.
- (p) A car drives at 32 mph for 1 minute. Give your answer to 2 decimal places.

Question 8: Work out the distance travelled by each of the following.

- (a) A runner runs at a speed of 8m/s for 2 minutes
- (b) A jog runs at a speed of 4m/s for 10 minutes.
- (c) A car drives at 60mph for 90 seconds.
- (d) A lorry drives at 30 mph for 150 seconds.

Fluency Practice

Question 9: Work out how long each of the journeys take.

- (a) A car drives 120 miles at a speed of 40 mph.
- (b) A lorry drives 250 miles at a speed of 50 mph.
- (c) A bird flies 330 kilometres at a speed of 55 kilometres per hour.
- (d) An object travels 48 miles at speed of 16 mph.
- (e) A man runs 240 metres at a speed of 6m/s
- (f) A dog runs 168 metres at a speed of 12m/s
- (g) A lorry travels 240 miles at a speed of 60mph.
- (h) A helicopter travels 345 miles at a speed of 115 mph.
- (i) A plane travels at a speed of 250 mph and covers a distance of 2250 miles.

Question 10: Calculate how long each journey lasts.

Give each answer in hours and minutes.

- (a) A car travels 100 miles at a speed of 40mph.
- (b) A lorry travels 90 miles at a speed of 60 mph.
- (c) A bus drives at a speed of 48mph and covers a distance of 60 miles.
- (d) A helicopter flies 105 kilometres at a speed of 140 km/h
- (e) A bird covers a distance of 95 miles at a speed of 20 miles per hour.
- (f) A car travels at 50 mph and covers a distance of 110 miles.
- (g) A lorry drives a distance of 452.4 kilometres at a speed of 52 km/h.
- (h) A bird flies 80 miles at a speed of 15 miles per hour
- (i) A ship sails 208 miles a speed of 24 miles per hour
- (j) A jet flies at a speed of 480km/h and covers a distance of 2088 kilometres
- (k) A racing car drives 256 miles at a speed of 120 mph
- (l) A helicopter flies 764 kilometres at a speed of 80 km/h

Extension



1. A bus travels 222 miles in 6 hours.
What was the average speed of the bus?

2. Thomas drives 130 miles at an average speed of 40 mph.
How long does the journey take Thomas?



3. A jumbo jet flies at 484 mph for 4 hours 30 minutes.
How far does the jet travel?

4. Greg and Kevin both travel between two towns that are 90 miles apart.
Greg drives and it takes him 1 hour 30 minutes.
Kevin cycles and it takes him 7 hours 30 minutes.
Work out the difference between their average speeds?

5. Harry catches the train from Belfast to Dublin at 4pm.
The average speed of the train is 70mph and the distance from Belfast to Dublin is 105 miles.
What time does Harry arrive in Dublin?

6. The distance from Sunderland to Wigan is 150 miles.
Mollie leaves Sunderland in her car at 07:50.
Her average speed on the journey is 60mph.
What time does she arrive in Wigan?

7. Jenny drives from Paris to Rochefort, a distance of 483 km
Her average speed on the journey is 84 km/h.
She leaves at 9:50pm.
What time does she arrive in Rochefort?

8. Philip runs at an average speed of 4 m/s.
How long will it take Philip to complete a 10 kilometre race?
Give your answer in minutes and seconds.



9. A car travels for 4 hours at an average speed of 45 mph and then 6 hours at an average speed of 35 mph.

(a) Work out the total distance travelled.

(b) Work out the average speed for the entire journey.

10. David cycles at 20mph for $1\frac{1}{4}$ hours, then at 16mph for 2 hours and then 12mph for 45 minutes.

(a) Work out the total distance travelled.

(b) Work out the average speed for the entire journey.

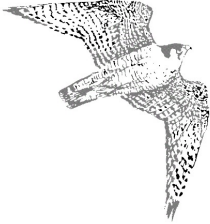
Extension






11. Mr Jenkins catches the 11:45am bus from London to Glasgow.
The distance between the two cities is 407 miles.
The bus travels at an average speed of 55mph.
What time should he arrive in Glasgow?
12. Michael drives 143 miles from town A to town B in 2 hours 36 minutes.
He then drives from town B to town C at the same speed and it takes 21 minutes.
- (a) Work out Michael's average speed from town A to town B.
(b) How far did Michael travel, in total, from town A to town C?
13. The distance from Junction 19 to Junction 20 on a motorway is 14 miles.
Bethany drove the distance in 15 minutes.
Max drove the distance at a speed of 52mph.
Who was faster?
14. The distance from Swindon to a village is 40 miles.
Vicky drives from the village to Swindon at 60 mph.
Charlie drives from the village to Swindon at 50mph.
Work out how much longer the journey takes Charlie.
Give your answer in minutes.
15. Miss Black completes a journey in 3 stages.
In stage 1, she drives at a speed of 40km/h for 45 minutes.
In stage 2, she drives at 60 km/h for 2 hours 9 minutes.
Altogether, over the 3 stages, Miss Black drives 171.6km in 3 hours 15 minutes
What is her average speed, in km/h, in stage 3?
16. The speed limit on a road is 40mph.
A scooter drives 9 miles in 13 minutes.
Is the scooter breaking the speed limit?



Extension

speed 1

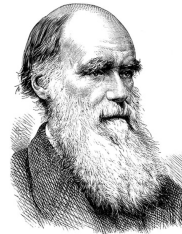


- (1) a peregrine falcon flies 14 miles horizontally in 15 minutes
what is the average speed in miles per hour?
- (2) when it is diving the peregrine falcon goes much faster, at 4.5 miles in 1 minute
how fast is this in miles per hour?
- (3) a cheetah runs at an average speed of 72 mph for 20 minutes
how far did it travel? 
- (4) a mako shark takes 10 minutes to travel 10 miles
what is the average speed? 
- (5) a sea horse travels at 0.01 mph
how long (in days and hours) will it take to travel a mile? 
- (6) a sailfish, the fastest fish, travels 22.73 miles in 20 minutes
how fast does it travel in mph? 
- (7) a snail works hard to travel 0.01 miles in 1 minute
how fast does it go in mph? 
- (8) sound travels at nearly 1200 mph
how far does it travel in 5 minutes?
- (9) a jet aircraft travels 36.5 miles every minute
how fast is this in mph?
- (10) a car runs at an average speed of 42 mph on a fairly built up motorway
how far will it go at this speed in 1 hour 10 minutes?

speed 2



Shrewsbury in Shropshire



- (1) how far is Stoke on Trent away if an escaped horse, travelling at an average speed of 13 mph takes 3 hours to get there?
- (2) how far is it to London if it takes 3 hours at an average speed of 54 mph?
- (3) a runner goes to Chester, 40 miles away, at an average speed of 8 mph
how long does it take them?
- (4) how long does it take a cyclist riding at an average speed of 12mph to travel to Aberystwyth, 72 miles away?
- (5) how long does it take to travel to Telford, 15 miles away at an average speed of 45 mph?
- (6) how long does it take to get to Hereford, 54 miles away at an average speed of 36 mph?
- (7) how long does it take to get to Wolverhampton, 32 miles away,
travelling at an average speed of 48 miles per hour?
- (8) what is the average speed of a truck that goes to Birmingham, 48 miles away, in 1hr 30 mins?
- (9) what is the average speed of a lorry which travels to Gloucester, 80 miles away, in 2.5 hours?
- (10) a very old car takes 3 hours 18 mins to get to Liverpool, 66 miles away
what is the average speed of the car?
how long would it take at this speed to get to Manchester, 74 miles away?

Extension

speed 3



speed = distance ÷ time

1.6 km/h = 1 mph
1 km/h = 0.62 mph
1 km/min = 37.2 mph
50 km/h = 31 mph

m/sec × 60 = m/min
m/min × 60 = m/hr
m/hr ÷ 1000 = km/hr
km/hr × 0.62 = mph

use a calculator to work out the speeds, in mph, of the world records for:

- (1) women's 2000m of 325.36 seconds
- (2) men's 100m of 9.58 seconds
- (3) men's 100 metres freestyle swimming competition of 46.91 seconds
- (4) women's 100 metre Breaststroke competition of 64.45 seconds
- (5) men's 20,000m walk of 4645.6 seconds
- (6) women's 100 metres hurdles of 12.21 seconds
- (7) men's 200 metres butterfly of 111.51 seconds
- (8) men's 10,000 metres of 26 minutes and 17.53 seconds
- (9) men's 50km walk of 3 hours, 34 mins and 14 seconds
- (10) women's 20km walk of 1 hour, 2 mins and 36 seconds

speed review



- (1) a duck flies 7 miles in 10 minutes, what is the average speed in miles per hour?
- (2) an Indian bird: the spine-tailed swift, travels 3.3 miles in 2 minutes, how fast is this in miles per hour?
- (3) a turbo-boosted Wart Hog runs at an average speed of 36 mph for 5 minutes, how far did it travel?
- (4) an Elk takes 7.5 minutes to bounce along for 10 miles, what is the average speed in miles per hour?
- (5) a three-toed sloth whizzes up a tall tree going 0.0075 miles in 3 minutes
what is the average speed in mph?
- (6) a pig, wanting to avoid crowds of onlookers, goes at an average speed of 10.8 mph
it goes 3.6 miles, how long does it take?
- (7) a garden snail nips out for a lettuce from Waitrose, travelling at 0.001 miles in 2 minutes
how fast does it go in mph?
- (8) sound travels at nearly 1200 mph, how long does it take to travel 30 miles?
- (9) a jet aircraft travels 23.4 miles every minute, how fast is this in mph?
- (10) a car runs at an average speed of 54 mph on a fairly clear motorway
how far will it go at this speed in 1 hour 10 minutes?

1.2 Density

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Mass} = \text{Density} \times \text{Volume}$$

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

Worked Example

The density of magnesium is 1.75 g/cm^3 . Find the mass of 20 cm^3 .

Your Turn

The density of magnesium is 1.75 g/cm^3 . Find the mass of 10 cm^3 .

Intelligent Practice

The density of magnesium is 1.75 g/cm^3 .

Find the mass of the following volumes of magnesium.

- 1) 1 cm^3
- 2) 10 cm^3
- 3) 100 cm^3
- 4) 20 cm^3
- 5) 200 cm^3
- 6) 0.2 cm^3

The density of gold is 19.3 g/cm^3 .

Find the mass of the following volumes of gold.

- 7) 1 cm^3
- 8) 100 cm^3
- 9) 2 cm^3
- 10) 20 cm^3
- 11) 200 cm^3
- 12) 0.2 cm^3

Worked Example

The density of magnesium is 1.75 g/cm^3 . Find the volume of 20 g. Round your answer to 2 decimal places.

Your Turn

The density of magnesium is 1.75 g/cm^3 . Find the volume of 10 g. Round your answer to 2 decimal places.

Intelligent Practice

The density of magnesium is 1.75 g/cm^3 .

Find the volume of the following masses of magnesium.

- 1) 1 g
- 2) 10 g
- 3) 100 g
- 4) 20 g
- 5) 200 g
- 6) 0.2 g

The density of gold is 19.3 g/cm^3 .

Find the volume of the following masses of gold.

- 7) 1 g
- 8) 100 g
- 9) 2 g
- 10) 20 g
- 11) 200 g
- 12) 0.2 g

Worked Example

Work out the density of copper.
150 g of a copper block has a volume of 17 cm^3 . Round your answer to 2 decimal places.

Your Turn

Work out the density of gold.
97 g of gold has a volume of 5 cm^3 . Round your answer to 2 decimal places.

Intelligent Practice

Calculate the density of the following materials given their mass and their volume.

- 1) Aluminum: 45 cm³ has a mass of 120 g
- 2) Brass: 117 cm³ has a mass of 1000 g
- 3) Bronze: 60 cm³ has a mass of 500 g
- 4) Copper: 17 cm³ has a mass of 150 g
- 5) Lead: 18 cm³ has a mass of 200 g
- 6) Magnesium: 57 cm³ has a mass of 100 g
- 7) Mercury: 9 cm³ has a mass of 120 g
- 8) Nylon: 118 cm³ has a mass of 200 g
- 9) Rubber: 67 cm³ has a mass of 80 g
- 10) Silver: 4.8 cm³ has a mass of 50 g
- 11) Zinc: 7 cm³ has a mass of 50 g

Fluency Practice

Question 1: Work out the density of each of the following.
State the units of each answer.

- (a) A piece of wood has a mass of 7g and a volume of 10cm^3
- (b) A rod of aluminium has a mass of 575.4g and a volume of 210cm^3
- (c) A piece of nickel has a mass of 3.48kg and a volume of 400cm^3
- (d) An iron statue with volume of 0.05m^3 and a mass of 394kg
- (e) 2.1m^3 of oil with a mass of 1775kg

Question 2: Work out the mass of each of the following.
State the units of each answer.

- (a) A statue with a volume of 120cm^3 made from ceramic which has a density of 2g/cm^3 .
- (b) A rod with a volume of 50cm^3 made from copper which has a density of 8.9g/cm^3 .
- (c) A block with a volume of 1.8m^3 made from silver which has a density of 10490kg/m^3
- (d) A statue with a volume of 3m^3 made from zinc which has a density of 7.14g/cm^3
- (e) 2800cm^3 of butter which has a density of 911kg/m^3

Question 3: Work out the volume of each of the following.
State the units of each answer.

- (a) A 50g piece of wood which has a density of 0.4g/cm^3
- (b) A 770g block made of brass which has a density of 8.67g/cm^3
- (c) A 4kg sheet of glass which has a density of 2.42g/cm^3
- (d) 80kg of rye which has a density of 720kg/m^3
- (e) 5 tonnes of gold which has a density of 19300kg/m^3

Extension

Question 1: A cube of ice has side length of 5cm.
The mass of the cube of ice is 114.5g.

Find the density of ice.
Give your answer in g/cm^3

Question 2: Shown is a solid cylinder made from carbon.
The density of carbon is 1.95g/cm^3



Find the mass of the cylinder.

Question 3: The mass of 4m^3 of silver is 41960kg.
The density of gold is 19300kg/m^3 .

Calculate the difference in mass between 5m^3 of silver and 5m^3 of gold.

Question 4: Beverley is building a toy boat.
If wood has a density under 1g/cm^3 , it will float.
She has a choice of three different pieces of wood.

Piece 1: volume = 400cm^3 and mass = 450g.

Piece 2: volume = 0.02m^3 and mass = 8kg

Piece 3: volume = 1000cm^3 and mass = 1.03kg

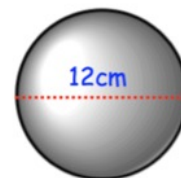
Which piece of wood is the most suitable?

Question 5: Material A has a density of 4.5g/cm^3 .
Material B has a density of 14g/cm^3 .

5kg of Material A and 200g of Material B form Material C.

Work out the density of Material C.

Question 6: A solid sphere has a diameter of 12cm.
The sphere is made from glass.
The density of the glass is 3.02g/cm^3



Find the mass of the glass sphere.

Question 7: An object has a mass of 420kg, correct to two significant figures.
The density of the material it is made from is 5.4g/cm^3 , correct to one decimal place.

Work out the smallest possible volume of the object.
Give your answer to three significant figures.

1.3 Pressure

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$\text{Force} = \text{Pressure} \times \text{Area}$$

$$\text{Area} = \frac{\text{Force}}{\text{Pressure}}$$

Worked Example

An object with an area of 5 m^2 exerts a force of 10 N . Find the pressure.

Your Turn

An object with an area of 2 m^2 exerts a force of 10 N . Find the pressure.

Intelligent Practice

An object with an area of ____ m² exerts a force of ____ N.
Find the pressure.

- 1) 1 m² with 60N
- 2) 2 m² with 60N
- 3) 3 m² with 60N
- 4) 4 m² with 60N
- 5) 5 m² with 60N
- 6) 6 m² with 60N
- 7) 3 m² with 20N
- 8) 3 m² with 30N
- 9) 3 m² with 40N
- 10) 3 m² with 50N
- 11) 3 m² with 70N
- 12) 3 m² with 80N

Worked Example

An object with a cross-sectional area of 2 m^2 exerts a pressure of 40 N/m^2 . Find the force.

Your Turn

An object with a cross-sectional area of 2 m^2 exerts a pressure of 10 N/m^2 . Find the force.

Intelligent Practice

An object with a cross-sectional area of ___ m^2 exerts a pressure of ___ N/m^2 . Find the force.

- 1) 1 m^2 with a pressure of 20 N/m^2
- 2) 2 m^2 with a pressure of 20 N/m^2
- 3) 3 m^2 with a pressure of 20 N/m^2
- 4) 4 m^2 with a pressure of 20 N/m^2
- 5) 5 m^2 with a pressure of 20 N/m^2
- 6) 6 m^2 with a pressure of 20 N/m^2
- 7) 2 m^2 with a pressure of 40 N/m^2
- 8) 2 m^2 with a pressure of 50 N/m^2
- 9) 2 m^2 with a pressure of 60 N/m^2
- 10) 2 m^2 with a pressure of 70 N/m^2
- 11) 2 m^2 with a pressure of 80 N/m^2
- 12) 2 m^2 with a pressure of 90 N/m^2

Fluency Practice

Question 1: Work out the pressure for each of the following.
Give suitable units for each answer.

- (a) A box is placed on a table and exerts a force of 250N on an area of 20cm^2
- (b) An object is placed on the ground and exerts a force of 3000N on an area of 4m^2
- (c) An object is placed on the ground and exerts a force of 54N on an area of 0.5cm^2
- (d) A box is placed on a table and exerts a force of 124 newtons on an area of 10.5cm^2
- (e) An object is placed on the ground and exerts a force of 25958N on an area of 1.4m^2

Question 2: Work out the force for each of the following.
In each case a box has been placed on the floor.

- (a) The area of contact is 16cm^2 and the pressure exerted is $10\text{N}/\text{cm}^2$
- (b) The area of contact is 1.5m^2 and the pressure exerted is $5000\text{N}/\text{m}^2$
- (c) The area of contact is 660cm^2 and the pressure exerted is $8.2\text{N}/\text{cm}^2$
- (d) The area of contact is 0.2m^2 and the pressure exerted is $1.2\text{N}/\text{cm}^2$
- (e) The area of contact is 500cm^2 and the pressure exerted is $450000\text{N}/\text{m}^2$

Question 3: Work out the area of contact for each of the following.
In each case an object has been placed on the floor.
Give suitable units for each answer.

- (a) The object exerts a force of 420N on the floor and the pressure on the floor is $20\text{N}/\text{cm}^2$
- (b) The object exerts a force of 8590N on the floor and the pressure on the floor is $900\text{N}/\text{m}^2$
- (c) The object exerts a force of 30N on the floor and the pressure on the floor is $600\text{N}/\text{m}^2$
- (d) The object exerts a force of 3945N on the floor and the pressure on the floor is $200\text{N}/\text{cm}^2$

Extension

Question 1: Find the pressure exerted by a force of 180 newtons on an area of 50cm^2 .
Give your answer in newtons/m^2

Question 2: A cylinder is placed on a table.
The cylinder has a weight of 400N and has a diameter of 10cm.

Work out the pressure on the table in newtons/cm^2

Question 3: Two cubes are placed on a table.
One cube has a side length of 4cm and the other cube has a cube length of 10cm.

The weight of the smaller cube is 50N and the weight of the large cube is 250N

Which cube exerts a greater pressure on the table?

Question 4: A microwave is placed on a worktop.

The area of the microwave in contact with the table is 600cm^2 .
The pressure of the microwave is 2450 Newtons/m^2 .

Work out the force exerted by the microwave on the worktop.

Question 5: The pressure of a tyre is 32 pounds per square inch.

Given 1 pound = 0.4536 kilograms
 1 inch = 2.54 centimetres

Work out the pressure in grams per square centimetre.

1.4 Review and Problem Solving

Fill in the Gaps

Distance	Time	Speed	Units of Speed
120 km	4 hours		km/h
55 m	5 seconds		m/s
8000 m	2 hours		km/h
450 km	180 minutes		km/h
	20 seconds	10	m/s
	3 hours	25	km/h
900 cm	3 seconds		m/s
132 m		12	m/s
640 km		80	km/h
	120 minutes	65	km/h
30 m	1 minute		m/s
1750 cm		2.5	m/s
	150 minutes	88	km/h
	1.5 minutes	8.5	m/s
20000 m	30 minutes	40	

2 Inequalities

Where in real life might we use phrases like “at least”, “more than”, “less than” and “at most”?

Real-life scenario	How we could represent mathematically
“You can have at most 20 people at your party.”	$x \leq 20$ (where x is number of people)
“I was chased by at least 10 zombies!”	$z \geq 10$ (where z is number of zombies)
I’ll visit next in less than a month.”	$d < 31$ (where d is number of days)
“My cat’s IQ is between 120 and 140.”	$120 \leq x \leq 140$ (where x is my cat’s IQ)

Definition

Relationship between two expressions that are not exactly equal.

Characteristics

Expressions can be connected with the following signs:

- $>$ Greater than
- \geq Greater than or equal to
- $<$ Less than
- \leq Less than or equal to
- \neq Not equal to

Examples

- $5 > -2$
- $x \leq 12$
- $-3 < y \leq 5$
- $x < -1, x \geq 8$
- $a \neq b$
- $2x - 7 < x + 6$

Non Examples

- $x = 5$
- $4x = 2x + 5$
- $-5 > -1$

Why do we need Inequalities?

Inequalities are needed in mathematics when we need to represent a **range of values**.

$x > 4$ has infinite solutions e.g. $x = 4.01, 5, 2000, \dots$

A 'range' of values often involves infinitely possible many values. So we need inequalities to be able to represent them, as it's not possible to list all the values.

2.1 Reading Inequalities

Notice the symbol is taller on the side which is larger.

$$x \{ > \} 7$$

Inequality	What It Means
$x > 7$	" x is greater than 7" This doesn't include 7 Examples: 7.2, 10
$x \geq 7$	" x is greater than or equal to 7" or " x is at least 7" This does include 7 Examples: 7, 8, 100.5
$x < 10$	" x is less than 10" Examples: -3 , 4, 9.2
$x \leq 8$	" x is less than or equal to 8" or " x is at most 8" Examples: 8, -3 , 4, 9.2

Worked Example

Write an inequality in between the two numbers:

4 5

Your Turn

Write an inequality in between the two numbers:

4.1 4.05

Fluency Practice

Write an inequality or equality in between the two numbers:

1) 9 5

10) $\frac{1}{4}$ 0.26

2) 3 3.5

11) $\frac{1}{4}$ $\frac{3}{8}$

3) 3.55 3.5

12) 0.1 0.1001

4) 3.09 3.091

13) -3 -4

5) 4.44 4.04

14) -3.2 -3.3

6) 0.5 $\frac{1}{2}$

15) -11 -10.9

7) 0.89 0.98

16) 0.33 $\frac{1}{3}$

8) 0.99 1.01

9) 3.101 3.099

True or False

Are the following inequalities true or false?

- $3 < 4$

In words: "3 is less than 4". This is true: 3 is a smaller value than 4.

- $-5 > 1$

In words: "-5 is greater than 1". This is not true: -5 is not the larger value.

- $5 \leq 5$

In words: "5 is less than or equal to 5". This is true: the left can either be less than **or equal to** the right. 5 is equal to 5!

Fluency Practice

Decide if the following statements are true or false for the values given.

1) $n = 7$

a) $n > 8$

b) $n < 8$

c) $n \geq 8$

d) $n < 3$

e) $n \leq 7$

2) $n = 0.5$

a) $n > 0$

b) $n < 0.55$

c) $n \geq 0.05$

d) $n < -1$

e) $n \leq 1$

3) $n = -3$

a) $n > -4$

b) $n < -2$

c) $n \geq 0$

d) $n < -3.5$

e) $n \leq -2.9$

4) $n = \frac{1}{3}$

a) $n > \frac{2}{6}$

b) $n < \frac{1}{4}$

c) $n \geq \frac{5}{12}$

d) $n < \frac{5}{12}$

e) $n \leq \frac{1}{2}$

Fluency Practice

Question 1: Write out the following with either an $<$ or $>$ symbol

(a) $8 \square 6$

(b) $2 \square 3$

(c) $7 \square 10$

(d) $5 \square 0$

(e) $4 \square -1$

(f) $-4 \square 6$

(g) $9 \square 9.4$

(h) $0 \square -1$

Question 2: Write down an inequality for each of the following

(a) x is greater than 8

(b) x is less than 3

(c) x is less than or equal to 1

(d) x is greater than or equal to 0

(e) x is less than 7

(f) x is greater than or equal to -2

(g) x is less than or equal to -10

(h) x is greater than 5

Question 3: Write down the meaning of these inequalities

(a) $x > 6$

(b) $x < 2$

(c) $x \geq 1$

(d) $x \leq 4$

(e) $x \geq 0$

(f) $x \leq -4$

(g) $x < -2$

(h) $x > 20$

(i) $x < y$

(j) $a \geq b$

(k) $c > 5$

(l) $y \leq 100$

2.2 Two-Ended Inequalities

$$-1 \leq x < 3$$

What does this mean in words?

“ x is greater or equal to -1 , and less than 3 ”

Or we could more simply say:

“ x is between -1 and 3 , inclusive of -1 ”

Worked Example

Write this as a compound inequality:

x is greater than 3, but less than 7.

Your Turn

Write this as a compound inequality:

x is greater than or equal to 1 and less than 7.

Fluency Practice

Write these as compound inequalities:

- 1) x is between -1 [included] and 3 [excluded]
- 2) x is between -5 [excluded] but less than 1 [excluded]
- 3) x between -3 [included] and 5 [included]
- 4) x is between 1 [excluded] and 3 [included]
- 5) x between -5 [included] and 10 [excluded]
- 6) x is between -3 [included] and 3 [not included]
- 7) x is between 3.4 [included] and 4.5 [included]
- 8) x is between 3.5 [included] and 4.5 [not included]

Worked Example

Write all the integers that satisfy
 $4 < n \leq 12$

Your Turn

Write all the integers that satisfy
 $3.5 \leq n \leq 8.9$

Fluency Practice

Find the integers that satisfy the following:

1) $5 \leq n \leq 10$

2) $5 \leq n < 7$

3) $0 < n \leq 5$

4) $10 < n \leq 15$

5) $-5 \leq n \leq -2$

6) $-5 < n < -2$

7) $-3 \leq n < 3$

8) $-2 < n \leq \frac{1}{2}$

9) $\frac{1}{2} \leq n < 3$

10) $\frac{1}{2} \leq n < \frac{3}{2}$

11) $\frac{7}{5} \leq n \leq \frac{13}{5}$

12) $-\frac{7}{5} < n < \frac{11}{5}$

13) $3.5 \leq n \leq 4.5$

14) $-3.5 \leq n \leq 4.5$

15) $-9.1 < n \leq 1.1$

16) $-3.5 < n < -1.5$

Extension

Which group would each value of x belong in?

$$x = 10$$

$$x = 0$$

$$x = 6.7103$$

$$x = -5.5$$

$$x = 12$$

$$x = 0.0001$$

Group	Inequality
<i>A</i>	$-12 \leq x < -6$
<i>B</i>	$-6 \leq x < 0$
<i>C</i>	$0 \leq x < 6$
<i>D</i>	$6 \leq x < 12$
<i>E</i>	$12 \leq x < 18$

Fluency Practice

Question 6: Write down an inequality for each of the following

- (a) x is greater than 2, but less than 5
- (b) x is greater than 0, but less than 4
- (c) x is greater than 1, but less than or equal to 7
- (d) x is greater than -5 , but less than or equal to 2
- (e) x is greater than or equal to -8 , but less than 3
- (f) x is greater than or equal to 10, but less than 20
- (g) x is greater than or equal to 3, but less than or equal to 6
- (h) x is greater than or equal to 8, but less than or equal to 11

Question 7: Write down the meaning of these inequalities

- (a) $3 < x < 5$
- (b) $2 < x < 9$
- (c) $19 \leq x < 20$
- (d) $5 \leq x \leq 10$
- (e) $0 < x \leq 4$
- (f) $-4 \leq x < 1$
- (g) $-8 \leq x \leq -6$
- (h) $100 < x < 200$

Question 8: List all the integers (whole number) that satisfies each inequality

- (a) $2 < x < 6$
- (b) $5 < x < 10$
- (c) $4 \leq x < 8$
- (d) $12 \leq x \leq 15$
- (e) $-2 < x \leq 3$
- (f) $-5 \leq x < 1$
- (g) $-10 \leq x \leq -5$
- (h) $-4 < x < 4$

2.3 Inequalities on Number Lines

$$x = 4$$

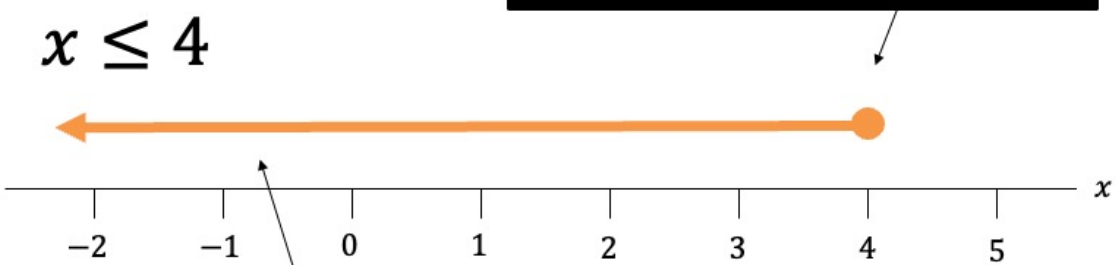
We can use a filled circle on a number line to indicate we want to include the value.



But what about:

$$x \leq 4$$

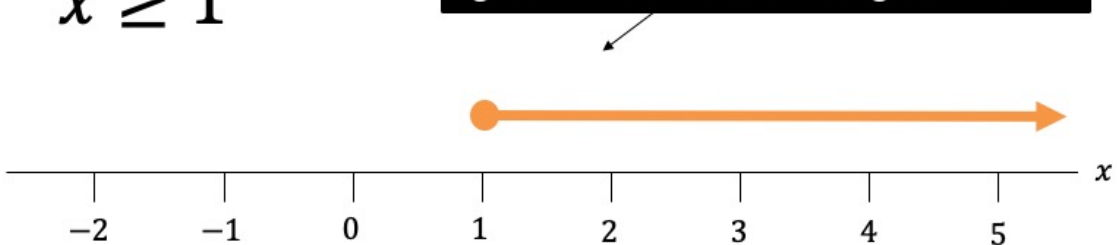
We again use a filled circle to indicate that we want to include 4.



But we also have an arrow pointing left to say we also want any value less than 4.

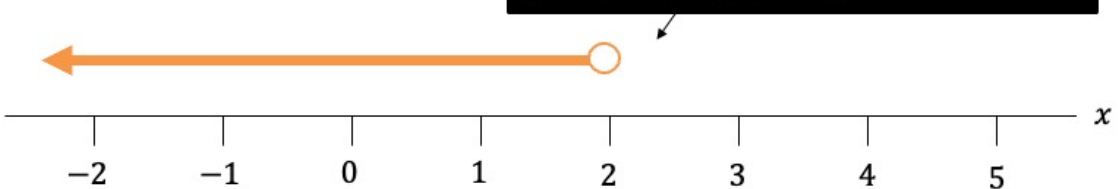
$$x \geq 1$$

We again want to include 1, but our arrow is right this time to indicate values greater than 1.



$$x < 2$$

We again have an arrow left to indicate "less than 2", but this time we **DON'T** want to include 2 itself. We use an unfilled circle to indicate that 2 is excluded.



Worked Example

a) Plot $x < 3$ on a number line

b) Plot $x \geq 14$ on a number line

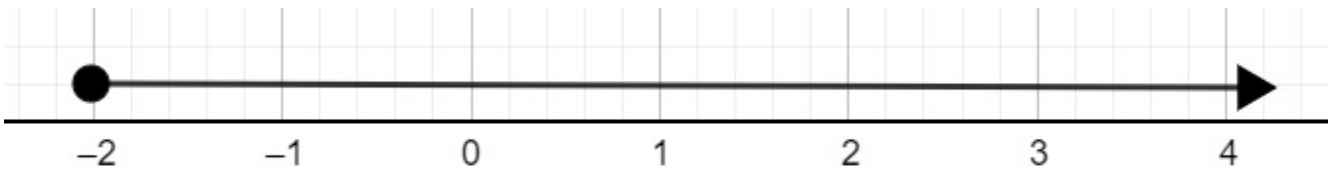
Your Turn

a) Plot $x > 14$ on a number line

b) Plot $x \leq -2$ on a number line

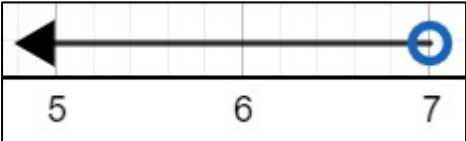
Discussion

Which diagram shows $x \geq -2$?



Fluency Practice

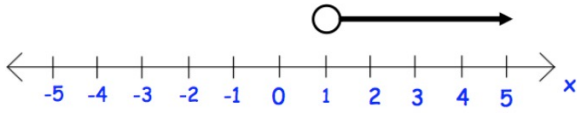
For each sentence, write an inequality then draw a number line representation.

1) x is less than 7	$x < 7$	 A number line with tick marks from 5 to 7. An open circle is drawn at 7, and an arrow points to the left from this circle. The numbers 5, 6, and 7 are labeled below the line.
2) x is less than or equal to 7		
3) x is more than 4		
4) x is more than 10		
5) x is more than 3.5		
6) x is more than or equal to 7.5		
7) x is less than or equal to 0		
8) x is more than or equal to 3.5		

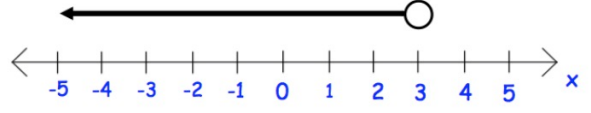
Fluency Practice

Question 4: Write down the inequalities shown below

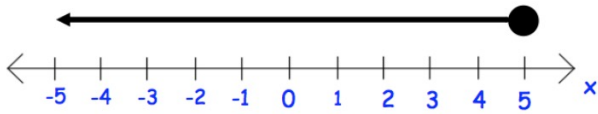
(a)



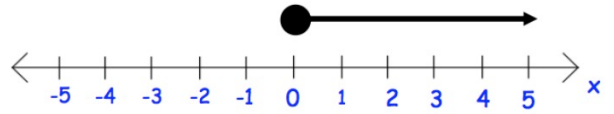
(b)



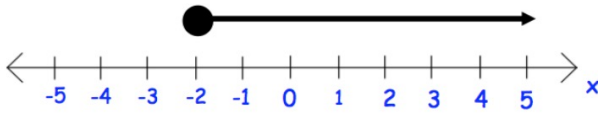
(c)



(d)



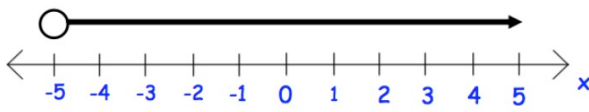
(e)



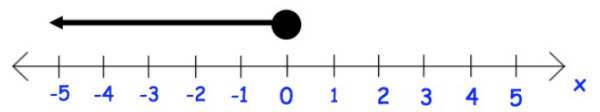
(f)



(g)



(h)



Question 5: Show these inequalities on a number line.

(a) $x > 2$

(b) $x < 4$

(c) $x \geq 3$

(d) $x \leq 5$

(e) $x \geq 0$

(f) $x \leq -1$

(g) $x < -4$

(h) $x > -5$

(i) $x \geq -6$

(j) $x > 0$

(k) $x < -2$

(l) $x > -1$

Worked Example

- a) Plot $2 < x < 3$ on a number line
- b) Plot $x < 3$ or $x > 7$ on a number line

Your Turn

- a) Plot $2 \leq x \leq 3$ on a number line
- b) Plot $x \leq -3$ or $x > 5$ on a number line

Fluency Practice

Plot the following on a number line:

1) $5 \leq n \leq 10$

2) $5 \leq n < 7$

3) $0 < n \leq 5$

4) $10 < n \leq 15$

5) $-5 \leq n \leq -2$

6) $-5 < n < -2$

7) $-3 \leq n < 3$

8) $-2 < n \leq \frac{1}{2}$

9) $\frac{1}{2} \leq n < 3$

10) $\frac{1}{2} \leq n < \frac{3}{2}$

11) $\frac{7}{5} \leq n \leq \frac{13}{5}$

12) $-\frac{7}{5} < n < \frac{11}{5}$

13) $3.5 \leq n \leq 4.5$

14) $-3.5 \leq n \leq 4.5$

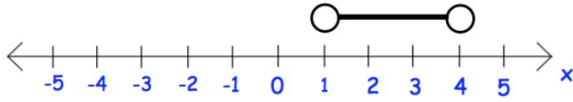
15) $-9.1 < n \leq 1.1$

16) $-3.5 < n < -1.5$

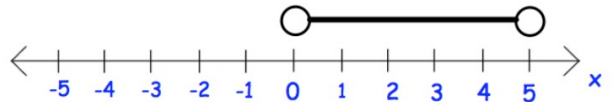
Fluency Practice

Question 9: Write down the inequalities shown below

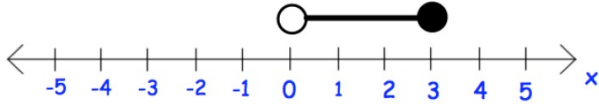
(a)



(b)



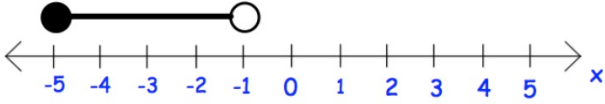
(c)



(d)



(e)



(f)



Extension

Question 1: The cost, c , of a TV is less than £300. Write this as an inequality.

Question 2: To go on a rollercoaster, a person's height, h , must be over 140cm. Write this as an inequality.

Question 3: The value of a house, v , is £100,000 or more. Write this as an inequality.



Question 4: There are 20 students in a class. The number of students present on a particular day is 20 or less. Write this as an inequality.

Question 5: Write down any integers (whole numbers) that satisfies **both** $x > 4$ and $x \leq 8$

Question 6: Write down any integers (whole numbers) that satisfies **both** $2 < x \leq 9$ and $x > 5$

2.4 Solving Linear Inequalities

Inequalities behave in a similar way to equations: whatever we do to one side of the equation, we have to do the same to the other.

'Solving an inequality' means to get x on its own on one side of the equation. This is so that the range is then clear.

When you divide or multiply both sides of an inequality by a negative number, **reverse the direction of the inequality**.

Why?

Consider the inequality $2 < 4$

This is clearly true as 2 is less than 4

But, if we multiple/divide by both sides by -1 , we get $-2 < -4$, which is false.

However, if we reverse the inequality sign, we get $-2 > -4$, which is true as -2 is more than -4 .

But it is probably easiest to avoid needing to divide by a negative number in the first place...

Worked Example

Solve:

a) $2x - 8 < 16$

b) $2(4 - x) < 16$

Your Turn

Solve:

a) $3x - 9 > 27$

b) $3(3 - x) > 27$

Intelligent Practice

Solve the following inequalities:

1) $5x - 40 \leq 80$

1) $-2x + 5 < -35$

2) $5x - 40 < 40$

2) $5 - 2x < -35$

3) $40 - 5x \geq 40$

3) $-5 - 2x \leq -35$

4) $5(8 - x) < -40$

4) $-7 - 2x \leq -35$

5) $5(8 - 2x) > -40$

5) $-7 - 4x > -35$

6) $-5(8 - 2x) > -40$

6) $-7 - 7x > -35$

Worked Example

Solve:

$$10(x + 3) + 3(2x + 6) < 144$$

Your Turn

Solve:

$$5(x + 3) + 2(2x - 6) \leq 111$$

Worked Example

Solve:

$$7(x + 3) - 3(2x - 6) = 84$$

Your Turn

Solve:

$$5(x - 3) - 2(2x - 6) \geq 111$$

Fluency Practice

Solve the following inequalities:

1) $4(x + 3) + 8(x + 1) < 44$

2) $7(x - 3) + 5(x + 2) \leq 37$

3) $3(x - 2) + 2(x - 5) > 24$

4) $2(2x - 1) - 4(3x - 1) > 26$

5) $5(2x + 3) - 6(x - 1) < 29$

6) $2(5x - 2) - 3(3x - 1) \geq 6$

Fluency Practice

Question 1: Solve each of the inequalities below

(a) $x + 4 > 9$ (b) $x - 3 < 2$ (c) $2x \leq 14$ (d) $8x < 16$

(e) $5x \geq 15$ (f) $\frac{x}{3} > 4$ (g) $\frac{x}{5} \leq 2$ (h) $x + 6 \geq 4$

Question 2: Solve each of the inequalities below

(a) $2x + 1 \leq 9$ (b) $3x - 5 > 16$ (c) $4x + 8 < 32$ (d) $5x - 2 \geq 68$

(e) $\frac{x}{2} + 1 \leq 5$ (f) $\frac{x}{9} - 6 > 4$ (g) $\frac{x + 3}{2} \geq 5$ (h) $\frac{x - 5}{4} > 2$

Question 3: Solve each inequality below and represent the solution on a number line.

(a) $4x + 7 < 11$ (b) $3x - 2 \geq 10$ (c) $\frac{x}{2} - 3 > 0$ (d) $\frac{x + 18}{4} \leq 5$

Question 4: Solve each of the inequalities below

(a) $5(x - 3) \geq 40$ (b) $6(x + 2) < 42$ (c) $2(5x + 1) \leq 36$

(d) $4(x - 2) < 18$ (e) $2(2x - 9) \geq 22$ (f) $3(2x + 7) \leq 9$

2.5 Solving Inequalities with Variable on Both Sides

If the variable appears on both sides of the equation, again we can solve in a similar way to how solve equations.

Often the best strategy is to first get all the variable terms (e.g. x) on the side of the equation where there is more of them.

Worked Example

Solve:

a) $9x + 4 < 2x + 60$

b) $3x - 23 \leq 7 - 2x$

Your Turn

Solve:

a) $5x + 7 > 2x + 22$

b) $2x - 23 \geq 9 - 2x$

Intelligent Practice

Solve the following inequalities:

1) $5x + 3 < 3x + 13$

10) $8x - 39 \geq 6 - 7x$

2) $5x + 2 \leq 3x + 44$

11) $39 - 8x \geq 6 - 7x$

3) $11x + 2 \geq 5x + 44$

12) $39 - 10x \geq 6 - 7x$

4) $11x + 44 \geq 5x + 2$

13) $6 - 10x \leq 39 - 7x$

5) $11x + 39 > 5x + 21$

14) $6 - 18x \leq 39 - 7x$

6) $8x + 39 > 5x + 21$

7) $8x + 39 < 2x + 21$

8) $8x - 39 < 21 - 2x$

9) $8x - 39 \leq 21 - 17x$

Extension

Why is it not possible to solve the following? Explain your answer.

$$3x + 3 < 15 + 3x$$

Worked Example

Solve:

a) $3(x + 2) < 2(x + 3)$

b) $3(x + 8) > 3(2 - x)$

Your Turn

Solve:

a) $7(x - 3) \leq 2(x + 7)$

b) $3(x - 5) \geq 5(5 - x)$

Intelligent Practice

Solve the following inequalities:

1) $3(x - 5) \leq 3(2x + 1)$

2) $3(x - 5) < -3(2x + 1)$

3) $-3(x + 5) \geq -3(2x + 1)$

4) $-3(x - 5) < -3(2x + 1)$

5) $-3(x - 5) > -3(2x - 1)$

6) $-3(2x - 1) > -3(x - 5)$

Extension

Explain your thinking process to solve the inequality $\frac{x}{4} - 2 < 3(2x - 7)$.

Fluency Practice

Question 5: Solve each of the inequalities below

(a) $4x + 3 > 2x + 11$

(b) $x + 1 \geq 3x - 18$

(c) $13x - 12 < 3x + 13$

(d) $7x - 5 \geq 3x + 11$

Fluency Practice

Question 6: Find the largest integer that satisfies each inequality below.

(a) $x + 3 < 9$

(b) $2x + 5 < 12$

(c) $7x + 10 \leq 31$

(d) $3x - 5 \leq 9$

(e) $\frac{x}{4} + 3 \leq 8$

(f) $4x + 14 \leq 8$

Question 7: Find the smallest integer that satisfies each inequality below.

(a) $2x - 5 \geq 12$

(b) $4x > 9$

(c) $\frac{x + 9}{3} \geq 7$

(d) $7x + 1 > 60$

(e) $10x - 16 \geq 76$

(f) $9x + 4 > 7x + 15$

Exam Questions

1. (a) (i) Solve the inequality

$$5x - 7 < 28$$

- (ii) On the number line, represent the solution set to part (i).
(3)
-

2. (a) Solve $5x + 12 < 17$ (2)

- (b) Solve the inequality $3(2y + 1) > 10$ (2)
-

3. (i) Solve the inequality $7x - 3 > 18$ (2)

x is a whole number such that $7x - 3 > 18$

- (ii) Write down the smallest value of x . (2)

2.6 Solving Compound Inequalities

We can even solve compound inequities, we just have to remember to do the same thing to every part of the equation.

Worked Example

Solve:

a) $-1 < 2x + 3 < 9$

b) $-1 \leq 2x + 6 < 9$

Your Turn

Solve:

a) $-9 < 2x + 3 < 1$

b) $-9 \leq 2x + 6 \leq 1$

Intelligent Practice

Solve:

1) $4 < x + 1 < 10$

10) $-20 < 4x - 2 \leq 8$

2) $4 < x - 1 < 10$

11) $-20 < 4x \leq 8$

3) $4 < 2x - 1 \leq 10$

12) $-20 < \frac{1}{4}x \leq 8$

4) $-4 \leq 2x - 1 \leq 10$

13) $-20 < \frac{1}{2}x \leq 8$

5) $-10 \leq 2x - 1 \leq -4$

14) $-20 < \frac{1}{2}x - 8 \leq 8$

6) $-10 \leq 4x - 2 \leq -4$

15) $-20 < \frac{x-8}{2} \leq 8$

7) $-10 \leq 4x - 2 \leq 4$

16) $-20 < \frac{8-x}{2} < 8$

8) $-10 \leq 4x - 2 \leq 8$

9) $-20 \leq 4x - 2 \leq 8$

Fluency Practice

Question 8: Solve each of the inequalities below

(a) $6 < x + 3 < 10$

(b) $4 \leq 2x \leq 7$

(c) $1 \leq 3x < 9$

(d) $4 < \frac{x}{5} < 6$

(e) $9 \leq 2x + 3 \leq 25$

(f) $-3 \leq \frac{x}{4} - 1 < 0$

Question 9: Find the integers that satisfy each of the inequalities below

(a) $5 < x < 9$

(b) $-3 < x \leq 1$

(c) $4 \leq 2x \leq 8$

(d) $16 \leq 5x + 1 < 31$

(e) $0 \leq \frac{x-6}{2} < 2$

(f) $-9 < \frac{x}{4} - 1 < -8$

Extension

Question 1: Lauren goes shopping and has £50 to spend.
She bought a T-shirt and 3 pairs of leggings.
The T-shirt cost £23.
Each pair of leggings cost £x

- (a) Form an inequality in terms of x.
- (b) Solve the inequality to find the possible price of the leggings.

Question 2: Farmer Taylor is placing a fence around his field.
He has 300 metres of fencing but this is not enough.

- (a) Form an inequality in terms of x.
- (b) Solve the inequality to find the possible width of the field.

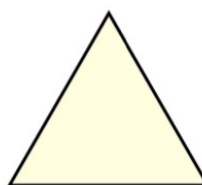
2x + 5 metres



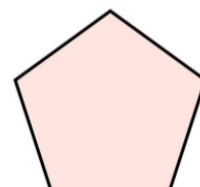
x metres

Question 3: The perimeter of the regular pentagon is larger than the perimeter of the equilateral triangle.

- (a) Form an inequality in terms of x
- (b) Solve the inequality to find the possible range of values for x.



x + 6



x + 2

Question 4: Find the range of values of x that satisfies **both**


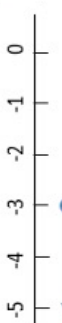

$$3(x + 2) \leq 30 \quad \text{and} \quad 4x + 3 > 21$$

Question 5: y is a prime number and also satisfies $7 < 2y - 3 \leq 25$

List the possible values of y.

2.7 Review and Problem Solving

Fill in the Gaps

Q	Inequality	Represent on a number line	Integer solutions
1	$x > 3$		
2			$x = 3, 4, 5, \dots$
3			$x = -3, -4, -5, \dots$
4	$-3 \leq x$		
5	$x - 1 > 2$		
6			
7	$x + 5 \leq 2$		
8			
9			$x = 4, 5, 6, \dots$ or $x = -1, -2, -3, \dots$
10	$< x \leq$		$x = -2, -1, 0, 1, 2, 3$
11	$x \geq 1$ and $x < 3$		
12	$3x > 9$		

3 Probability

Could it be a Probability?

0.35674

Yes / No

1.35674

Yes / No

1

Yes / No

$\frac{1}{3}$

Yes / No

$-\frac{1}{3}$

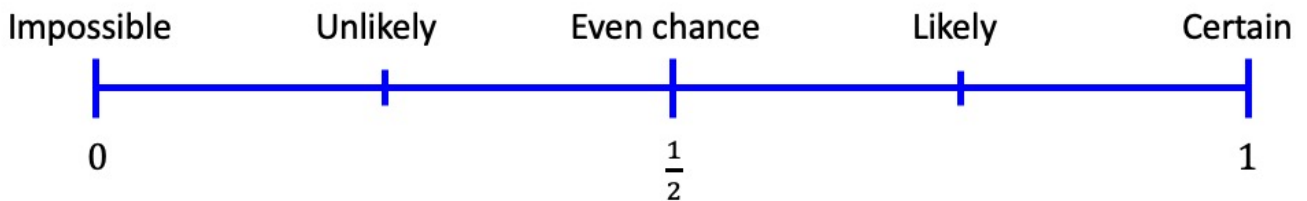
Yes / No

Intelligent Practice

0.3	Yes / No	1	Yes / No
-0.3	Yes / No	2	Yes / No
1.3	Yes / No	-1	Yes / No
0.000003	Yes / No	$\frac{2}{3}$	Yes / No
0.43045783	Yes / No	$1\frac{2}{3}$	Yes / No
1.43045783	Yes / No	$-\frac{2}{3}$	Yes / No
-0.43045783	Yes / No	$\frac{3}{2}$	Yes / No
$0.\dot{4}$	Yes / No	$\frac{43}{51}$	Yes / No
0	Yes / No		

3.1 Probability Scale

- Probability is a numerical measure of how likely or unlikely an event is to occur.
- Probabilities are usually written as fractions, but can be written in any form equivalent to that fraction, e.g. $\frac{3}{4} = 0.75 = 75\%$
- Probabilities can be anywhere between 0 (impossible) and 1 (certain):



Worked Example

Describe using impossible, unlikely, even chance, likely or certain the probability that:

- a) You will walk to Mars.
- b) The day after Monday is Tuesday.
- c) You roll a three on a fair die.
- d) You flip a tails on a fair coin.

Your Turn

Describe using impossible, unlikely, even chance, likely or certain the probability that:

- a) You roll an even number on a fair die.
- b) The day after Monday is Wednesday.
- c) You roll a number between 1 and 6 on a fair die.
- d) You will go to space in your life.

Fluency Practice

Question 1: Which phrase from the box best describes the likelihood of each of these events?
You may use each phrase more than one.

Impossible Unlikely Even Chance Likely Certain

- (a) Rolling a 9 on an ordinary six sided dice.
- (b) A newborn baby being a boy.
- (c) A day picked at random ending with the letter y
- (d) Getting a tail when a coin is flipped.
- (e) It snowing in London in May.
- (f) Rolling a number greater than 1 on an ordinary six sided dice.

Question 2: Which word from the box best describes the likelihood of each of these events?

Impossible Unlikely Even Likely Certain

- (a) You throw a coin and get a Heads.
- (b) You take a green counter from a bag that only contains black counters.
- (c) May 18th 2018 is the day after May 17th 2017.

Question 3: Here are some cards



Impossible Unlikely Even Likely Certain

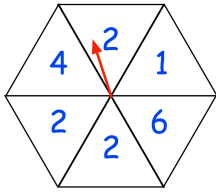
A card is picked at random.

Which word from the box best describes the likelihood of each of these events?

- (a) The card has a blue star on it.
- (b) The card has a heart on it.
- (c) The card has a shape on it that is symmetrical.

Fluency Practice

Question 4: A fair spinner has six equal sections.



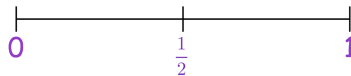
Impossible Unlikely Even Likely Certain

Which word from the box best describes the likelihood of each of these events?

- (a) The arrow landing on an even number
- (b) The arrow landing on 4.
- (c) The number landing on 2.

Question 5: Francesca rolls an ordinary 6-sided dice.

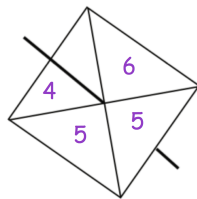
- (a) Mark with a cross the probability that Francesca gets an 8.



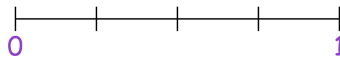
- (b) Mark with a cross the probability that Francesca gets an odd number.



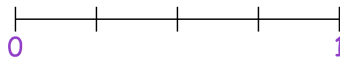
Question 6: A fair 4-sided spinner is spun once.



- (a) On the probability scale, mark with a letter A, the probability that the spinner will land on the number 4.

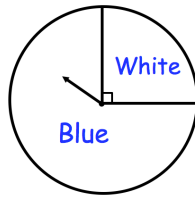


- (b) On the probability scale, mark with a letter B, the probability that the spinner will land on the number 5.

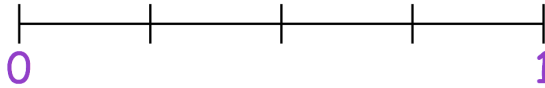


Fluency Practice

Question 7: The diagram shows a fair spinner.



(a) Which colour is the arrow least likely to land on?



(b) Mark the probability scale with an arrow to show the probability of landing on white. Label the arrow, W.

(c) Mark the probability scale with an arrow to show the probability of landing on blue. Label the arrow, B.

Question 8: A fair six sided dice is rolled once.

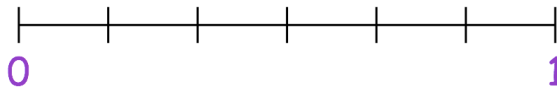


Mark the probability of each of the following events onto the probability scale.

A: The dice lands on an even number.

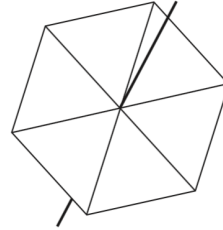
B: The dice lands on the number 5

C: The dice lands on a number less than 5.



Extension

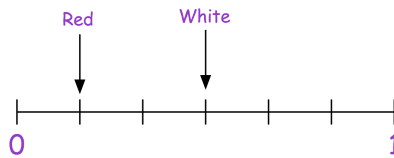
Question 1: Curtis has a fair 6-sided spinner.
 The spinner has numbers less than 7 on it.
 The number 5 is the least likely number that the spinner will land on.
 There is an even chance that the spinner will land on a 3.
 It is impossible that the spinner will land on an even number.
 Write the numbers on the spinner.



Question 2: Reggie has a bag holding red, white and green counters.
 Altogether there are 6 counters in the bag.

The probability scale shows the probability that a counter picked at random will be white.

It also shows the probability that a counter picked at random will be white.



Show on the probability scale the probability that a counter picked at random will be green.

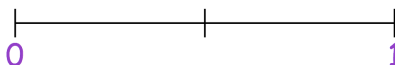
Question 3: A school offers students 3 lunchtime clubs each week: hockey, golf and cricket.

- Which clubs does Helen attend?
- Which of the children attend the cricket club?
- Which of the club do the least of the 5 children attend?
- Which child attends the most clubs?

	Hockey	Golf	Cricket
Helen	✓		✓
Leah			✓
Emily	✓	✓	✓
Mia	✓	✓	
Sally	✓		

Mr White picks one of the 5 children at random

- On the probability scale, mark with a cross the probability that he will pick a child that attends the hockey club.



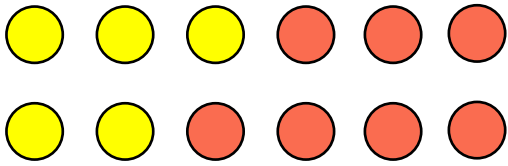
3.2 Probability of Single Events

The probability of an event occurring is defined as:

$$\text{Probability} = \frac{\text{Number of desired outcomes}}{\text{Number of possible outcomes}}$$

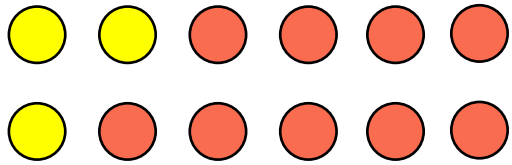
- How many cards in a standard deck of cards?
- How many weeks in a year?
- How many sides on a standard die?
- What are the names of the two sides of a coin?
- Name of each suit in a deck of cards.
- How many aces in pack of cards?
- How many cards in each suit in a pack of cards?
- How many picture cards in a deck of cards?

Worked Example



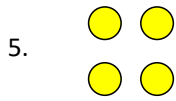
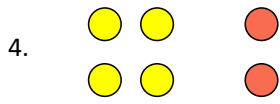
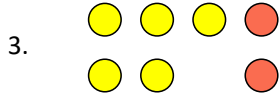
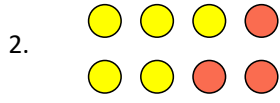
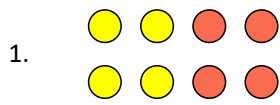
$P(\text{yellow}) =$

Your Turn

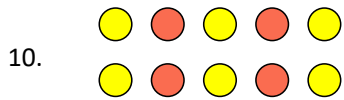
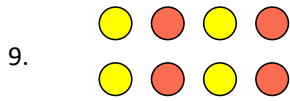
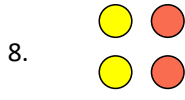
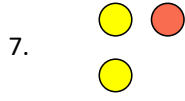
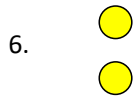


$P(\text{yellow}) =$

Intelligent Practice



Increase / Decrease / Same?	P(yellow) =



Increase / Decrease / Same?	P(yellow) =

Worked Example

A bag of sweets contains only 4 red sweets, 2 yellow sweets and 4 green sweets.

- a) What is the probability of choosing a red sweet?
- b) What is the probability of choosing a red or yellow sweet?
- c) What is the probability of choosing a mint?

Your Turn

A bag of sweets contains only 8 red sweets, 4 yellow sweets and 8 green sweets.

- a) What is the probability of choosing a red sweet?
- b) What is the probability of choosing a red or yellow sweet?
- c) What is the probability of choosing a mint?

Intelligent Practice

A bag of sweets contains only 15 red sweets, 10 yellow sweets and 5 green sweets.

- 1) What is the probability of choosing a red sweet?
- 2) What is the probability of choosing a yellow sweet?
- 3) What is the probability of choosing a green sweet?
- 4) What is the probability of choosing a purple sweet?
- 5) What is the probability of choosing a banana?
- 6) What is the probability of choosing a red or yellow?
- 7) What is the probability of choosing a yellow or red?
- 8) What is the probability of choosing a red, yellow, green?
- 9) What is the probability of choosing a red, yellow, green, or purple?

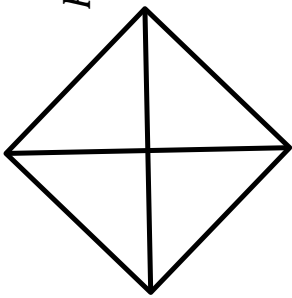
Intelligent Practice

A fair dice has faces 1, 1, 2, 3, 4 and 5.

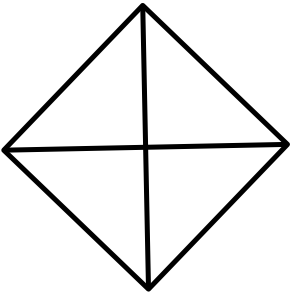
- 1) What is the probability of choosing 1?
- 2) What is the probability of choosing 2?
- 3) What is the probability of choosing 3?
- 4) What is the probability of choosing 4?
- 5) What is the probability of choosing 5?
- 6) What is the probability of choosing 1 or 2?
- 7) What is the probability of choosing 1 or 3?
- 8) What is the probability of choosing 1, 2, or 3?
- 9) What is the probability of choosing 1, 2, 3, or 5?

Extension

Design a spinner by placing numbers in the gaps, so that the probabilities are correct.

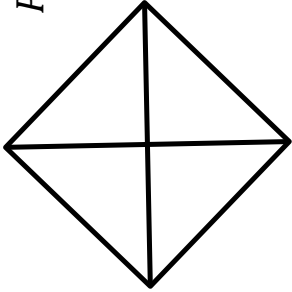


$$P(2) = \frac{1}{4}$$

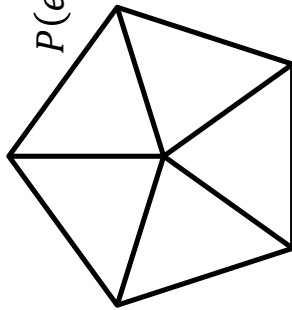


$$P(3) = \frac{1}{4}$$

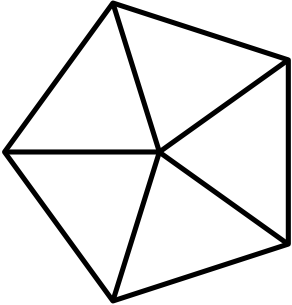
$$P(1) = \frac{1}{2}$$



$$P(\text{odd}) = 1$$



$$P(\text{even}) = \frac{3}{5}$$

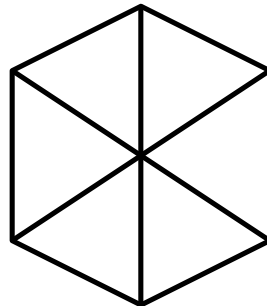
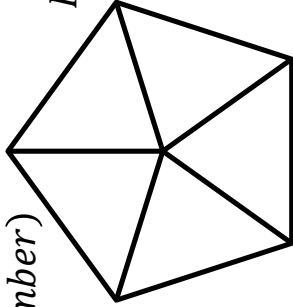


$$P(\text{square number}) = \frac{4}{5}$$

$$P(\text{square number}) = \frac{4}{5}$$

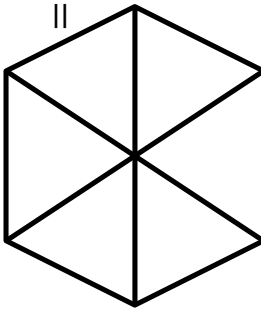
$$P(\text{prime}) = \frac{2}{5}$$

$$P(\text{Even}) = \frac{4}{5}$$



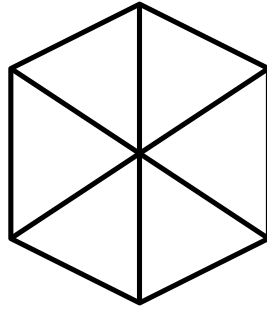
$$P(5) = \frac{1}{2}$$

$$P(4) = \frac{1}{3}$$



$$P(\text{square number}) = \frac{1}{6}$$

$$P(\text{prime}) = \frac{2}{3}$$



$$P(\text{cube number}) = \frac{1}{2}$$

$$P(\text{square number}) = \frac{1}{2}$$

$$P(\text{prime number}) = \frac{1}{3}$$

Extension

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Use the number square above to help you work out these probabilities.
Write your answers as fractions in their simplest forms.

A number is chosen at random between 1 and 100 (inclusive).
Work out the probability that the number is ...

- | | |
|--|-----------------------------|
| 1 a multiple of 10 | 11 a factor of 24 |
| 2 a multiple of 5 | 12 a factor of 11 |
| 3 a multiple of 2 | 13 a multiple of 11 |
| 4 greater than 90 | 14 a square number |
| 5 less than 5 | 15 an odd number |
| 6 greater than 5 | 16 a multiple of 13 |
| 7 between 20 and 30 (inclusive) | 17 a power of 2 |
| 8 a two-digit number | 18 a multiple of 3 |
| 9 a three-digit number | 19 a triangle number |
| 10 a factor of 50 | 20 a prime number |

3.3 Mutually Exclusive Events

Mutually exclusive means “cannot happen at the same time”.

Examples

- Turning left or turning right (you cannot turn left and right at the same time).
- Going to Liverpool at 9am tomorrow, or going to Manchester at 9am tomorrow (you cannot be in two places at once).

Non-Examples

- Turning left and scratching your head can happen at the same time.
- Kings and hearts, because you can have a king of hearts.

Fluency Practice

Are the following pairs of statements mutually exclusive?

- 1) Winning a football match AND losing the same football match.
- 2) Getting the bus AND missing the same bus.
- 3) Watching a comedy DVD AND watching a romance DVD.
- 4) Rolling a 2 on a dice AND rolling a number less than 3.
- 5) Choosing a spade AND choosing a king.

3.4 Exhaustive Events

The probabilities of all possible outcomes add up to 1.

Worked Example

Castle FC play football matches every Saturday.

The table shows the probability that Castle FC will win or lose.

- a) Work out the probability that Castle FC will lose

Win	Lose
$\frac{3}{4}$	

- b) Work out the probability that Castle FC will lose

Win	Lose
0.75	

Your Turn

Castle FC play football matches every Saturday.

The table shows the probability that Castle FC will win or lose.

- a) Work out the probability that Castle FC will lose

Win	Lose
$\frac{6}{8}$	

- b) Work out the probability that Castle FC will win

Win	Lose
	0.75

Intelligent Practice

Aiden is a boxer. The table shows the probability that Aiden will win, lose or draw.

Work out the probability that Aiden will draw

Win	Lose	Draw
$\frac{1}{4}$	$\frac{1}{4}$	

Work out the probability that Aiden will draw

Win	Lose	Draw
0.2	0.2	

Work out the probability that Aiden will draw or win

Win	Lose	Draw
$\frac{1}{4}$	$\frac{1}{4}$	

Work out the probability that Aiden will draw

Win	Lose	Draw
0.3	0.03	

Work out the probability that Aiden will draw

Win	Lose	Draw
$\frac{1}{4}$	$\frac{2}{8}$	

Work out the probability that Aiden will draw or lose

Win	Lose	Draw
0.7		0.07

Castle FC play football matches every Saturday.

In their last 10 matches, Castle FC have drawn 5 matches, lost 2 and won the rest.

Complete the probability table for Castle FC

Win	Lose	Draw

Castle FC play football matches every Saturday.

In their last 50 matches, Castle FC have drawn 10 matches, lost 5 and won the rest.

Complete the probability table for Castle FC as decimals

Win	Lose	Draw

Fluency Practice

1) Here are the probabilities of some events (h) happening, write down the probabilities of the events not happening (h'):

- | | | | |
|--------------------------|------------------------------------|---------------------------|------------------------------------|
| a) $P(h) = \frac{3}{10}$ | $P(h') = \underline{\hspace{2cm}}$ | e) $P(h) = 98\%$ | $P(h') = \underline{\hspace{2cm}}$ |
| b) $P(h) = \frac{1}{4}$ | $P(h') = \underline{\hspace{2cm}}$ | f) $P(h) = 55.5\%$ | $P(h') = \underline{\hspace{2cm}}$ |
| c) $P(h) = 0.21$ | $P(h') = \underline{\hspace{2cm}}$ | g) $P(h) = \frac{2}{5}$ | $P(h') = \underline{\hspace{2cm}}$ |
| d) $P(h) = 25\%$ | $P(h') = \underline{\hspace{2cm}}$ | h) $P(h) = \frac{12}{15}$ | $P(h') = \underline{\hspace{2cm}}$ |

2) There are some blue, red, green and purple balls in a bag. Find the probability of a purple ball being pulled out if these are the probabilities of the other colours:

a)

Blue	Red	Green	Purple
0.2	0.4	0.3	

b)

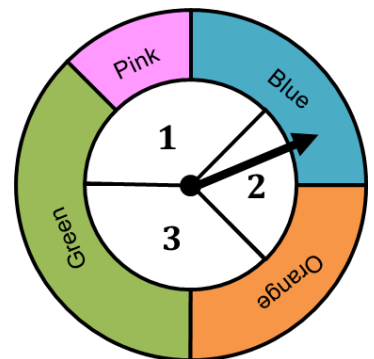
Blue	Red	Green	Purple
35%	21%	40%	

c)

Blue	Red	Green	Purple
$\frac{3}{20}$	$\frac{4}{20}$	$\frac{6}{20}$	

3) A spinner consists of an outer ring of coloured sectors and an inner circle of numbered sectors, as shown.

- a) The probability of getting 2 is $\frac{2}{8}$. The probability of getting 1 or 3 are equal. What is the probability of getting 3? $P(3) = \underline{\hspace{2cm}}$
- b) The probability of getting blue is $\frac{2}{8}$. The probability of getting orange is $\frac{2}{8}$. The probability of getting green is $\frac{3}{8}$. What is the probability of getting pink? $P(\text{pink}) = \underline{\hspace{2cm}}$
- c) Which of these pairs of events are mutually exclusive?
- i. Getting 3 AND getting 2
 - ii. Getting 3 AND getting green
 - iii. Getting 3 AND getting blue
 - iv. Getting blue AND getting pink



3.5 Expectation

Expectation is the long-run average you would get if a test was repeated many times.

If an event has probability p , the expectation in n trials is $n \times p$.

Expectation is used as an estimate for how many times an event will occur.

Worked Example

The relative frequency of a teacher throwing a pen in the bin is 0.5. A teacher throws a pen 100 times. How many throws will be successful?

Your Turn

The relative frequency of a teacher throwing a pen in the bin is 0.5. A teacher throws a pen 1000 times. How many throws will be successful?

Intelligent Practice

- 1) The relative frequency of a teacher throwing a pen in the bin is 0.5. A teacher throws a pen 50 times. How many will be successful?
- 2) The relative frequency of a teacher throwing a pen in the bin is 0.1. A teacher throws a pen 50 times. How many will be successful?
- 3) The relative frequency of a teacher throwing a pen in the bin is 1. A teacher throws a pen 50 times. How many will be successful?
- 4) The relative frequency of a teacher throwing a pen in the bin is 0.9. A teacher throws a pen 100 times. How many will be successful?
- 5) The relative frequency of a teacher throwing a pen in the bin is 0.10. A teacher throws a pen 100 times. How many will be successful?
- 6) The relative frequency of a teacher throwing a pen in the bin is 0.15. A teacher throws a pen 100 times. How many will be successful?
- 7) The relative frequency of a teacher throwing a pen in the bin is 0.015. A teacher throws a pen 100 times. How many will be successful?

Worked Example

If I roll a fair dice 12 times, how many times would you expect it to land on the number 1?

Your Turn

If I roll a fair dice 60 times, how many times would you expect it to land on the number 1?

Intelligent Practice

- 1) If I throw a fair coin 10 times, how many times would you expect it to land on heads?
- 2) If I throw a fair coin 20 times, how many times would you expect it to land on heads?
- 3) If I throw a fair coin 60 times, how many times would you expect it to land on heads?
- 4) If I throw a fair coin 600 times, how many times would you expect it to land on heads?
- 5) If I roll a fair dice 600 times, how many times would you expect it to land on the number 1?
- 6) If I roll a fair dice 300 times, how many times would you expect it to land on the number 1?
- 7) If I roll a fair dice 150 times, how many times would you expect it to land on the number 1?
- 8) If I roll a fair dice 750 times, how many times would you expect it to land on the number 1?
- 9) If I roll a fair dice 1500 times, how many times would you expect it to land on the number 1?
- 10) If I throw a fair coin 1500 times, how many times would you expect it to land on heads?

3.6 Relative Frequency

In most events, it is difficult to accurately predict the probability of an event happening.

When there is no theory behind the probability of an event happening, we use **relative frequency** to calculate probabilities.

Because it is often calculated after performing experiments, it is often called **experimental probability**.

The more trials there are, the more accurate that experimental probability becomes.

Worked Example

A coin is flipped 30 times. The results are:

H T H H H H H H T T T H H T T

T T T H H H T T H H T H T H H

- a) What are the relative frequencies for heads and tails?
- b) The coin is flipped 300 more times. Estimate how many times the coin will land on tails.

Your Turn

A coin is flipped 20 times. The results are:

T H T T T T T H H H

H H T H T H H H H H

- a) What are the relative frequencies for heads and tails?
- b) The coin is flipped 100 more times. Estimate how many times the coin will land on tails.

Fluency Practice

1) Simon records the colour of cars going past his house for an hour.

Colour	Frequency
Blue	5
Red	4
Yellow	1
White	7
Black	3

- a) What is the probability the next car will be
 i) blue _____ ii) red _____ iii) Not black _____
- b) How many Red cars would you expect if
 i) 100 cars went past _____ ii) 60 cars went past _____
- 2) Sammy throws a drawing pin 200 times and records how it lands.

Pin up	160
Pin down	40

- a) What is the probability the pin will land
 i) pin up? _____ ii) pin down _____
- b) How many pin ups would you expect if the pin was thrown
 i) 80 times _____ ii) 320 times _____ iii) 400 times _____

3) A group of children are asked to write for their favourite food, and child is picked at random.

Favourite Food	Number of people
Chinese	20
Pizza	16
Mexican	18

- a) What is the probability the person liked Chinese?
 i) _____ ii) Didn't like Mexican best. _____
- b) How many people would you expect to like pizza if
 i) 100 people were asked _____
 ii) 250 people were asked _____
 iii) 1000 people were asked? _____
 iv) 460 people were asked? _____

Fluency Practice

Question 1: An ordinary coin is thrown 50 times.
Barry says “I am going to get heads 25 times and tails 25 times.”
Explain why he could be wrong.

Question 2: A coin is thrown 30 times.
The coin lands on tails 20 times.
What is the relative frequency of the coin landing on tails?

Question 3: A dice is rolled 50 times.
It lands on six 37 times.
(a) Write down the relative frequency of the dice landing on a six

Robyn says “the dice is biased towards landing on a six.”
(b) Do you think the dice is biased? Explain your answer.

Question 4: Jessica wants to test if a coin is biased.
She throws the coin 24 times.

T T H H T H H H T H T T
T H H T T H H T H H H T

(a) Complete the relative frequency table.

	Heads	Tails
Relative frequency		

(b) Do you think the coin is biased? Explain your answer.

Question 5: A biased dice is rolled 30 times.



3 4 1 3 6 2 6 6 6 5 6 3 6 4 6
1 6 3 4 6 6 2 6 3 6 6 3 6 3 6

(a) Complete the relative frequency table

Number	1	2	3	4	5	6
Relative Frequency						

(b) Do you think the dice is biased? Explain your answer.

Fluency Practice

Question 6: Esme takes the bus to university 40 times during a term.
The relative frequency of the bus being late is 0.3.
How many times was the bus late?

Question 7: Katie rolls a dice 100 times.
The table shows the results

Number	1	2	3	4	5	6
Frequency	22	9	14	31	19	5

Work out the relative frequency of throwing:

- (a) An even number (b) A square number (c) A prime number
(d) A cube number (e) A multiple of 3 (f) A factor of 18

Question 8: A spinner lands of white, black, red or orange.
The relative frequencies after 300 spins are shown in the table below.

Colour	White	Black	Red	Orange
Relative Frequency	0.25	0.4	0.2	0.15

- (a) How many times did the spinner land on white?
(b) How many times did the spinner land on red?
(c) How many more times did the spinner land on black than orange?

Question 7: Martin and Laura want to estimate how many green jelly beans are in a tub of 600 jelly beans.
A trial consists of taking a jelly bean at random, noting the colour and replacing the jelly bean into the tub.

	Number of trials	Number of green jelly beans chosen
Martin	30	4
Laura	150	12

- (a) Write down the relative frequency of Martin taking a green jelly bean.
(b) Write down the relative frequency of Laura taking a green jelly bean.
(c) Whose experiment gives the more reliable estimate of the number of green jelly beans in the tub? Give a reason for your answer.
(d) How many green jelly beans do you expect to be in tub altogether?

Extension

Question 1: Leo plants and grows 50 flowers.
The table shows information about the colours.

Colour	Red	Yellow	White
Frequency	16		28
Relative Frequency	0.32		

(a) Copy and complete the table.

Altogether, Leo grows 125 flowers.

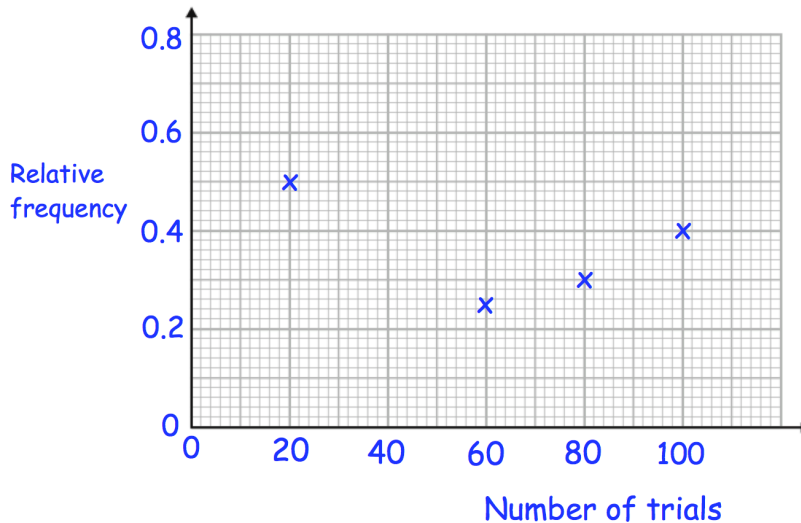
(b) How many flowers would you expect to be yellow?

Question 2: Four students have each written a 2000 word essay.
The spellings are checked for all four students.
The relative frequencies of a spelling mistake for the 4 students are:

Student	Alan	Beryl	Connie	Diego
Relative Frequency	0.032	0.01	0.009	0.017

Work out the mean number of spelling mistakes made by the four students.

Question 3: There are 70 sweets in a jar.
In a trial, a sweet is chosen at random and then replaced.
The graph shows the relative frequency of a blue sweet.



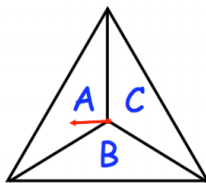
In the first forty trials, twelve blue sweets were chosen.

- Plot this result on the graph.
- How many blue sweets were chose after 60 trials.
- Which is the best estimate, from the graph, of the probability of choosing a blue sweet?
- Use your answer to estimate the number of blue sweets in the jar

Extension

Question 4: A coin lands on heads 300 times.
The relative frequency of heads is 0.6
Work out the number of times the coin was flipped.

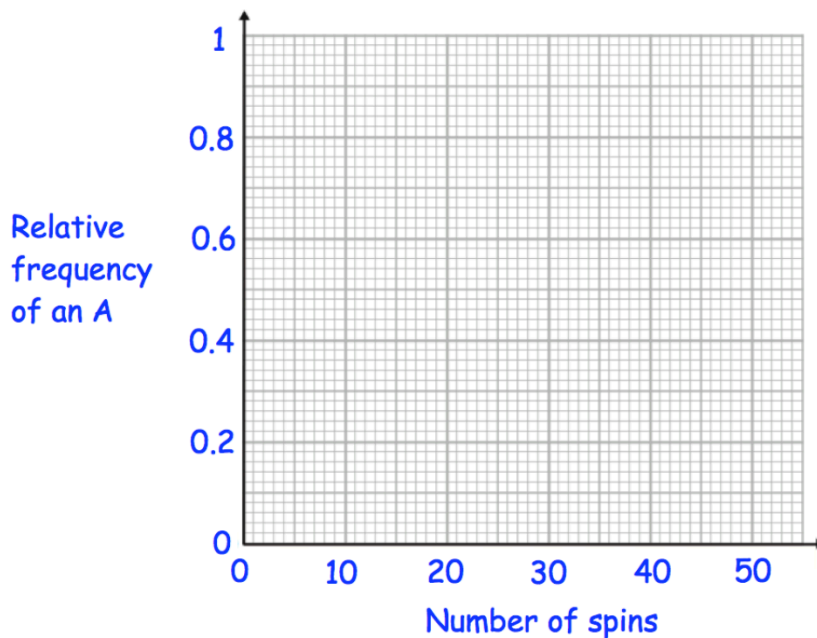
Question 5: A three sided spinner is labelled A, B and C.



The spinner is spun and the frequency of the letter A is recorded every 10 spins.
The table below shows this information.

Spins	10	20	30	40
Frequency of an A	6	14	18	26

(a) Plot the relative frequencies on the graph below.



(b) Jacob says the relative frequency after 50 spins is 0.8.
Explain why Jacob must be wrong.

Misconceptions

If you toss a fair coin and get heads 5 times in a row, you are more likely to get tails the next time.

The probability is the same each time – previous results are irrelevant.

In a football match, you can either win, lose or draw. So the probability of winning is $\frac{1}{3}$.

Winning may not have the same probability as losing.

You are less likely to win with lottery numbers 1, 2, 3, 4, 5, 6 than if you pick numbers at random.

Every number has the same chance and so does every combination.

If you toss a coin 50 times and get heads 40 times, the coin must be biased.

It might be biased, as you would only expect 25 heads, but it is still possible to get 40 out of 50 heads with a fair coin.

If you roll two dice and add the results, the probability of getting 9 is $\frac{1}{11}$ as there are 11 possibilities (2 – 12).

There are more ways to get some totals than others.

When tossing a coin, you are just as likely to get 5 heads in a row as 10 in a row – it's just chance.

$$P(5 \text{ heads in a row}) = \frac{1}{32}$$

$$P(10 \text{ heads in a row}) = \frac{1}{1024}$$

3.7 Listing Outcomes

Worked Example

List all the ways of arranging the letters in the word:

CAT

Your Turn

List all the ways of arranging the letters in the word:

DOG

Worked Example

I flip a coin and then roll a six-sided die. List the possible outcomes.

Your Turn

I flip a coin and then roll a 4-sided die. List the possible outcomes.

Worked Example

The first five positive integers are 1, 2, 3, 4, 5. I choose two numbers from this list. Write down all possible combinations of two numbers I can choose.

Your Turn

The four square numbers are 1, 4, 9, 16. I choose two numbers from this list. Write down all possible combinations of two numbers I can choose.

Fluency Practice

Question 1: Emily flips a coin twice.
One of the possible outcomes is a tail and a tail (TT)
List all the possible outcomes.

Question 2: Benjamin rolls an ordinary six-sided dice once and flips a coin.
List all the possible outcomes.



Question 3: A rugby team plays two matches.
They can win (W), draw (D) or lose (L) each match.
List all the possible outcomes.

Question 4: There are five students in a group: Alison, Beth, Conor, David and Eddie.
Miss Jenkins chooses two students at random from the group to give a presentation.
List all the possible outcomes.

Question 5: Here are four cards.
Each card has a number on it.

7

2

6

4

- (a) Write down all the 2-digit numbers that can be made using the cards
(b) Write down all the 3-digit numbers that can be made using the cards

Question 6: Marco visits a restaurant with his friends.
Shown is the menu.
Marco chooses one starter, one main and one dessert.
List all possible outcomes.

Starter	Main	Dessert
Soup	Curry	Ice Cream
Fish	Pizza	Danish
	Burger	

Extension

Question 1: Andrew has attempted his maths homework.
Can you spot any mistakes?

Q1 Orla has four types of vegetable.

Peas
Carrots
Turnip
Spinach

Orla is going to choose 2 different types of vegetable.

Write down all the possible combinations of vegetable she can choose.

PC, PT, PS CP, CT, CS

TP, TC, TS SP, SC, ST

Question 2: Here are four cards.
Each card has a number on it.



Write down all the 3-digit even numbers that can be made using the cards

Question 3: In a restaurant, there are 5 possible pizza toppings:
Chicken, Pineapple, Olives, Mushrooms and Beef.

Freddie picks two different toppings on his pizza

(a) List all possible outcomes

Freddie picks the toppings at random

(b) Write down the probability that the pizza contains meat

Question 4: There are two bags.

Bag 1 contains a red counter and a pink counter.

Bag 2 contains a blue counter, a yellow counter and a white counter.

Sam picks a counter at random from bag 1 and notes its colour

He then places this counter into bag 2.

Sam then picks a counter at random from bag 2.



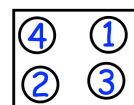
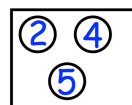
Write down the probability that Sam picks two counters that are the same colour

Extension

Question 5: Heather has made up a game for a school fête to raise money for charity. There are two boxes of counters. Each counter has a number on it.

Box 1

Box 2



The person playing the game will select one counter at random from box 1.

They will then select one counter at random from box 2.

(a) Write down all the possible combinations of counters picked.

The person playing the game wins when the numbers multiply to give an odd number.

During the fête the game is played 300 times.

The game costs 80p to play.

Each prize costs £2

(b) Work out how much money Heather should raise for charity.

Question 6: Ali is having a meal with his friends. He will either have:

- one starter and one main
- or
- one main and one dessert

Shown is the menu

Starter		Main		Dessert	
Soup	£3.20	Chicken	£6.25	Trifle	£2.50
Prawns	£3.55	Beef	£8.00	Brownie	£2.15
Melon	£2.45	Pork	£6.75	Eton Mess	£3.50
Duck	£3.95			Ice Cream	£1.95

Ali has £10.

List all the possible combinations that Ali **cannot** afford.













3.8 Sample Space Diagrams

Snail Race

<https://www.transum.org/software/SW/SnailRace/>

Snail Race

Teachers' Version

1	<input type="text"/>		
2	<input type="text"/>		
3	<input type="text"/>		
4	<input type="text"/>		
5	<input type="text"/>		
6	<input type="text"/>		
7	<input type="text"/>		
8	<input type="text"/>		
9	<input type="text"/>		
10	<input type="text"/>		
11	<input type="text"/>		
12	<input type="text"/>		

Roll Dice



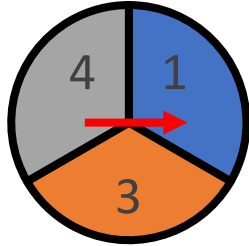
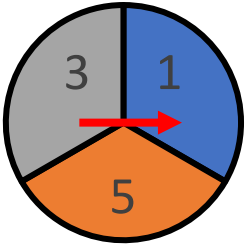
Start Again

Autorace

Worked Example

I spin these two spinners then add the numbers together to get a score.

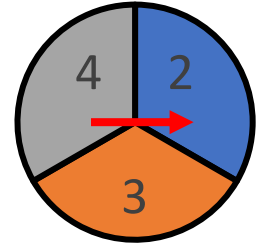
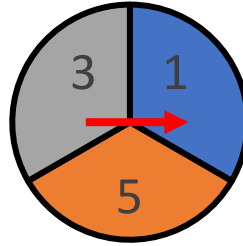
Work out the probability that I get a score of 4.



Your Turn

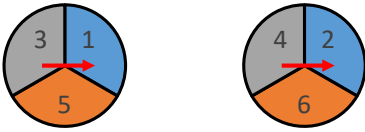
I spin these two spinners then add the numbers together to get a score.

Work out the probability that I get a score of 4.



Intelligent Practice

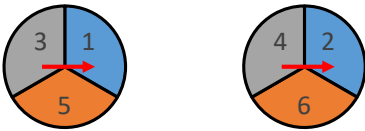
I spin these two spinners then add the numbers together to get a score. Work out the probability that I get a score of 4.



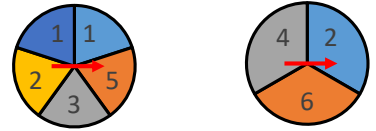
I spin these two spinners then add the numbers together to get a score. Work out the probability that I get a score of 7.



I spin these two spinners then add the numbers together to get a score. Work out the probability that I get a score of 7.



I spin these two spinners then find the difference between the numbers to get a score. Work out the probability that I get a score of 7.



I spin these two spinners then add the numbers together to get a score. Work out the probability that I get a score of 7.



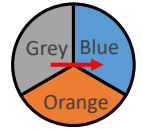
I spin these two spinners then find the difference between the numbers to get a score. Work out the probability that I get a score of 4.



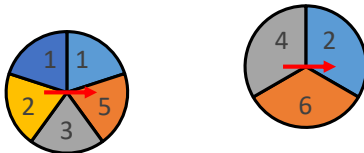
I spin these two spinners then find the product of the numbers to get a score. Work out the probability that I get a score of 4.



I spin the same spinner twice. What is the probability that I get the same colour on both spins?
What is the probability that the colours are **not** the same?



I spin these two spinners. What is the probability that I get the same number on both spinners?



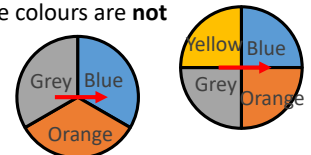
I spin the same spinner twice. What is the probability that I get the same colour on both spins?
What is the probability that the colours are **not** the same?



I spin the same spinner twice. What is the probability that I get the same number on both spins?



I spin these two spinners. What is the probability that I get the same colour on both spinners?
What is the probability that the colours are **not** the same?



Worked Example

Bag A contains four counters, labelled 2, 3, 5 and 7. Bag B contains five counters, labelled 1, 4, 9, 16 and 25. A counter is taken from each bag at random and the numbers are added together. Draw a sample space to show all possible scores.

Your Turn

Bag A contains four counters, labelled 3, 5, 7 and 9. Bag B contains five counters, labelled 1, 8, 27 and 64. A counter is taken from each bag at random and the numbers are added together. Draw a sample space to show all possible scores.

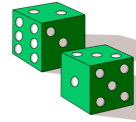
Worked Example

Two four-sided dice are rolled. The numbers on the two dice are multiplied together. Draw a sample space of the all the possible products.

Your Turn

Two six-sided dice are rolled. The numbers on the two dice are multiplied together. Draw a sample space of the all the possible products.

Fluency Practice



Question 1: Two fair six sided dice are rolled.
 The numbers on the two dice are **added** together to give a score.
 The table shows all possible scores.

- (a) Which score is the most likely?
- (b) Which scores are the least likely?
- (c) Write down the probability of scoring a
- (i) 3 (ii) 5 (iii) 6 (iv) 7
- (d) Write down the probability of scoring a number
- (i) over 10 (ii) under 7 (iii) 4 or less (iv) 6 or more
- (e) Write down the probability of scoring
- (i) an odd number (ii) a square number (iii) a prime number

		Dice 1					
+		1	2	3	4	5	6
1		2	3	4	5	6	7
2		3	4	5	6	7	8
3		4	5	6	7	8	9
4		5	6	7	8	9	10
5		6	7	8	9	10	11
6		7	8	9	10	11	12

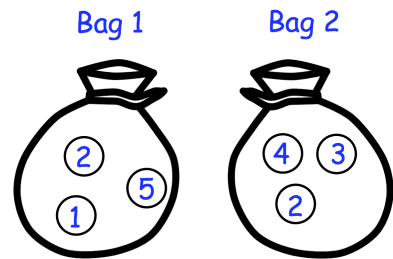
Question 2: Two fair six sided dice are rolled.
 The numbers on the two dice are **multiplied** together to give a score.

- (a) Complete the table to show all possible scores.
- (b) Write down the probability
- (i) 10 (ii) 9 (iii) 12 (iv) 8
- (c) Write down the probability of scoring
- (i) an even number (ii) an odd number
- (iii) a number less than 20

		Dice 1					
x		1	2	3	4	5	6
1							
2							
3							
4							
5							
6							

Fluency Practice

Question 3: Two bags, 1 and 2, each contain three counters.
 In bag 1, the counters are labelled 1, 2 and 5.
 In bag 2, the counters are labelled 2, 3 and 4.



A counter is drawn at random from bag 1 and a counter is drawn from bag 2.

The two numbers are multiplied together to give a score

- Complete the table to show all possible scores
- Find the probability of scoring a 6
- Find the probability of scoring a multiple of 4
- Find the probability of scoring an odd number

		Bag 1		
		1	2	5
Bag 2	x			
	2			
	3			
4				

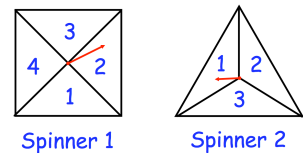
Question 4: Rose is playing a game with a fair six sided dice and a fair coin.
 She rolls the dice and flips the coin.

If the coin lands on heads, her score is **one less** than the number on the dice.
 If the coin lands on tails, her score is **two more** than the number on the dice

- Complete the table to show all possible scores
- Find the probability of scoring a 0
- Find the probability of scoring a 5
- Find the probability of scoring a number less than 4
- Find the probability of scoring a square number

		Dice					
		1	2	3	4	5	6
Coin	Heads						
	Tails						

Question 5: Two fair spinners are spun.
 Spinner 1 has four equation sections labelled 1, 2, 3 and 4.
 Spinner 2 has three equal sections labelled 1, 2 and 3.
 Each spinner is spun once.
 The score is the **difference** between the numbers



- Complete the table to show all possible scores
- Find the probability of scoring a 1
- Find the probability of scoring a 2 or more

		Spinner 1			
		1	2	3	4
Spinner 2	1	0	1	2	
	2	1			
	3	2			

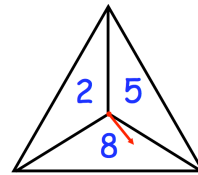
Extension

Question 1: Adam uses two fair spinners in a game.
He spins both spinners and **adds** the two numbers together.

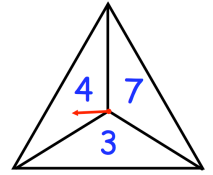
(a) Draw a table to show all possible scores

Adam thinks that the probability of an even score is $\frac{1}{2}$

(b) Explain why Adam is incorrect

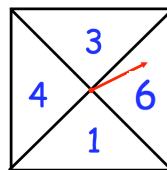


Spinner 1



Spinner 2

Question 2: Jessica is playing a game with a fair four sections spinner and a fair coin.
She spins the spinner and flips the coin.



Spinner



Coin

If the coin lands on heads, Jessica applies **rule 1** to the number on the spinner
If the coin lands on tails, Jessica applies **rule 2** to the number on the spinner

The table below shows some information about the scores that Jessica can get.

		Spinner			
		1	3	4	6
Coin	Heads	1	9		36
	Tails	3	7	9	13

(a) What could **rule 1** be?

(b) What could **rule 2** be?

(c) Complete the table

(d) Find the probability that Jessica scores a number less than 15

Extension

Question 3: Katie has organised a game to raise money for charity at a local fair.
The person playing rolls two fair six sided dice
The numbers on the dice are **multiplied** together.

Each person pays £1 to play.

If they score a square number, they win £3

The game is played 180 times.

How much money does Katie raise for charity?

Question 4: Two bags, 1 and 2, each contain equal size counters.

Bag 1 contains a pink counter, yellow counter and white counter

Bag 2 contains counters labelled 1, 4, 5 and 7.

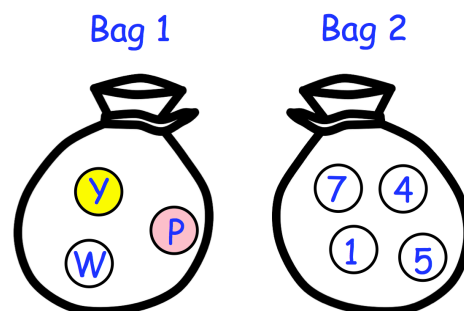
A counter is drawn at random from bag 1 and
a counter is drawn at random from bag 2.

If the counter from bag 1 is pink, the number on the
counter from bag 2 is **increased** by 1

If the counter from bag 1 is yellow, the number on the
counter from bag 2 is **decreased** by 5

If the counter from bag 1 is white, the number is **halved**.

Find the probability of scoring a number below 3



Question 5: A fair dice has six faces numbered 1, 1, 1, 2, 3 and 4.

The dice is rolled twice and the number shown is recorded each time.

Find the probability that the sum of the two numbers recorded is at least 4.

Question 6: A fair six sided dice is rolled twice.

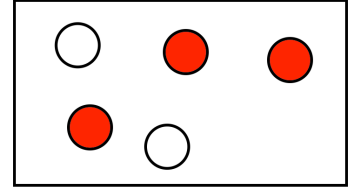
Find the probability that the number obtained on the first roll is at least 3
greater than the second roll

3.9 Review and Problem Solving

Fluency Practice

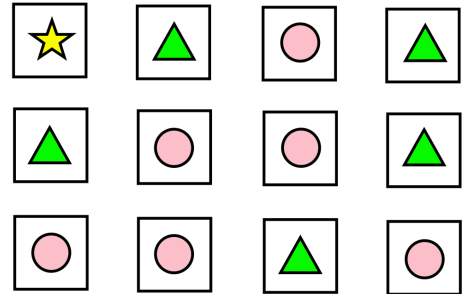
Question 1: Theo has 3 red sweets and 2 white sweets.
He picks a sweet at random.

- (a) Write down the probability that Theo picks a red sweet.
(b) Write down the probability that Theo picks a white sweet.



Question 2: Leah has 12 cards, each with a shape on it.
She takes a card at random.

- (a) What is the probability that Leah takes a card with a star on it?
(b) What is the probability that Leah takes a card with a triangle on it?
(c) What is the probability that Leah takes a card with a circle on it?



Question 3: Ralph has 9 cards, each with a number on it.



He picks a card at random.
Write down the probability that the chosen card is

- (a) the number 8 (b) an even number (c) a number less than 7
(d) a multiple of 4 (e) a square number (f) a prime number

Question 4: There are 12 red roses, 5 yellow roses and 3 white roses in a vase.
Felix takes a rose, at random, from the vase.

- (a) Write down the probability that he takes a white rose.
(b) Write down the probability that he takes a red **or** a white rose.
(c) Write down the probability that Felix takes a rose that is **not** red.

Fluency Practice

Question 5: Leon throws a biased coin.
The probability of getting tails is 0.4
Work out the probability of getting heads.



Question 6: Edith plants a daffodil bulb.
The probability that the bulb will grow is 0.8
What is the probability that the bulb will **not** grow?

Question 7: Wycombe Wanderers play a match of football.
The probability that they win the match is 0.28
The probability that they draw the match is 0.55
Work out the probability that they lose the match.

Question 8: Evelyn has 80 pens in a drawer.
15 pens are black and the other pens are blue.

Evelyn picks a pen at random from the drawer.

- (a) What is the probability that Evelyn picks a black pen?
- (b) What is the probability that Evelyn picks a blue pen?



Question 9: There are 20 counters in a bag.

- 2 of the counters are white.
- 1 of the counters is pink.
- 4 of the counters are black.
- The rest of the counters are purple.

Carter takes a counter at random from the bag.

Show that the probability that the counter is white or purple is $\frac{3}{4}$

Question 10: There are only pink, yellow, green and blue counters in a bag.
The table shows the probability that a counter taken at random from the bag will be pink, green or blue.

Colour	Pink	Yellow	Green	Blue
Probability	0.5		0.1	0.2

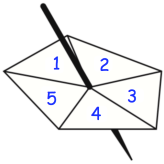
- (a) Work out the probability that the counter taken is yellow

There are 40 counters in the bag.

- (b) Work out the number of blue counters in the bag.

Fluency Practice

- Question 11: Darcy has a biased spinner.
A spinner has sections labelled 1, 2, 3, 4 and 5.
The table below shows information about some of the probabilities



Number	1	2	3	4	5
Probability	x	0.15	0.05	0.2	0.35

Work out the value of x .

- Question 12: Frederick organises a raffle for his school fayre.
The top prize is a ride in a hot air balloon, which will be won by 1 ticket.
Altogether Frederick sells 700 raffle tickets.
Miss Robinson buys 5 tickets for the raffle.



Work out the probability that Miss Robinson does **not** win.

- Question 13: There are 20 chocolates in a box.
Some of the chocolates contain nuts and the rest do not.
The probability that a chocolate containing nuts is picked at random from the box is 0.6
How many of the chocolates in the box contain nuts?

- Question 14: A bag contains 600 coloured counters.
The counters are yellow, brown or orange.
There are 117 yellow counters in the bag.
The probability that a brown counter is chosen from the bag is 0.35

Calculate the number of orange counters in the bag.

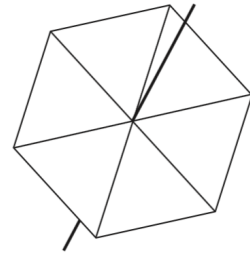
Extension

Question 1: Megan has a fair 6 sided spinner.
The spinner has the letters A, B and C on it.

The probability that the spinner will land on an A is $\frac{1}{2}$

The probability that the spinner will land on a C is $\frac{1}{3}$

Write the letters on the spinner.



Question 2: Elliott has eight numbered cards.



One of the cards is chosen at random.

Elliott says:

The probability of a 8 is $\frac{1}{4}$

The range of the numbers is 5.

The probability of a number greater than 10 is 0.

The probability of a 7 is $\frac{1}{2}$

Fill in the six missing numbers.

Question 3: The two-way table gives information about 90 people who sat their driving test.

(a) Complete the two-way table

A person is picked at random.

(b) Write down the probability that the person failed their driving test.

(c) Write down the probability that the person had under 20 driving lessons.

	Under 20 driving lessons	20 or over driving lessons	total
Pass		21	30
Fail	45		
total			90

Somebody who passed their driving test is picked at random.

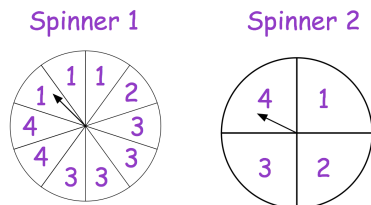
(d) Work out the probability that this person had under 20 driving lessons.

Question 4: Isaac has made two fair spinners.
Spinner 1 has 10 equal sized sections.
Spinner 2 has 4 equal sized sections.

Isaac says

"It is more likely to get a 4 on spinner 1 than spinner 2 because there are two number 4s on spinner 1 and only one number 4 on spinner 2."

Explain why Isaac is incorrect.



Extension

Question 5: The table shows the shoe size of 23 students.

Shoe Size	Frequency
5	2
6	11
7	5
8	4
9	1

A student is picked at random.

- (a) Work out the probability that the student has a shoe size of 8.
(b) Work out the probability that the student has a shoe size of 7 or smaller.

Question 6: A football team can win, draw or lose a match.
The table shows the probabilities of each result.

Result	Win	Draw	Lose
Probability		0.05	0.3

Each win is worth 3 points.
Each draw is worth 1 point.
Each loss is worth 0 points.
The football team plays 40 games in a season.

Work out how many points the football team should receive in one season.

Question 7: Beatrice has a biased four sided spinner.
The table shows the probabilities that the spinner will land on a 2 or 3.

Number	1	2	3	4
Probability		0.1	0.3	

The probability that the spinner will land on 1 is three times the probability that the spinner will land on 4.

Work out the probability that the spinner will land on 1.

Question 8: Finn has some sweets in a bag.
5 of the sweets are lemon flavoured.
7 of the sweets are strawberry flavoured.
The rest of the sweets are mint flavoured.

The probability that Finn takes a mint flavoured sweet is $\frac{2}{5}$

How many mint flavoured sweets are in the bag?

Extension

Question 9: Gracie has more than 5 coins.
The total value of the coins is 50p.

Gracie is going to pick one of the coins at random.

The probability that Gracie picks a **1p** coin is $\frac{1}{5}$

List all the coins that Gracie has.

Question 10: A box contains lego blocks of the same size.
Each block is white, blue, green or red.

Colour	White	Blue	Green	Red
Probability	0.25	0.45		0.2

The table shows the probabilities that a block picked at random is white, blue or red.

(a) Work out the probability of a green block

There are 60 red lego blocks.

(b) How many white lego blocks are there?

Question 11: A bag contains good and bad apples.
 n of the apples are good.
The other 5 apples are bad.

(a) Write down an expression, in terms of n , for the number of apples in the bag altogether.

Maryam will take at random, an apple from the bag.

(b) Write down an expression, in terms of n , for the probability that Maryam will take a good apple.

(c) Write down an expression, in terms of n , for the probability that Maryam will take a bad apple.

Question 12: There are only red, black and green pens in a box.

There are three times as many red pens as green pens.
There are four as many black pens than red pens.

Work out the probability of a black pen being selected.

Dice and Cards

probability questions on dice

one dice



what is the probability of throwing:

- (a) a prime number ?
- (b) a factor of 15 ?
- (c) a multiple of 2 ?
- (d) a square number ?
- (e) a factor of 60 ?

if the dice is rolled **720** times
how many times would you expect to obtain

- (f) a factor of 12 ?
- (g) a multiple of 3 ?
- (h) a factor of 25 ?

if the dice is rolled **100** times
how many times would you expect to obtain

- (i) a factor of 8 ?
- (j) a multiple of 3 ?
- (k) a factor of 18 ?

two dice

two fair dice
each is numbered 1 to 6
the probability of throwing a **double 6** :

(both dice showing number 6) is $\frac{1}{36}$

- (a) what is the probability of **not** throwing double 6?
- (b) I throw the two dice and get double 6
then I throw them again
what is the probability that I will throw **double 6** *this*
time?



I start again and throw the two dice
(c) what is the probability of throwing **double 3**?



- (d) what is the probability of throwing *any* double?

Dice and Cards

three dice

four students threw 3 fair dice they recorded how many times the numbers on the dice were the same:



(a) write the name of the student whose data is **most likely** to give the best estimate of the probability of getting each result

name	throws	results		
		different	2 the same	3 the same
Meg	40	26	12	2
Sue	140	81	56	3
Zia	20	10	10	0
Ali	100	54	42	4

(b) use *all* of their results to estimate the probability of obtaining two numbers the same

theoretical results

(c) what is the theoretical probability of obtaining **three numbers the same** ?

the theoretical probability of throwing **all different** numbers is $\frac{5}{9}$

(d) write down **how many times** you would theoretically expect to get each result

for 5400 throws of the three dice

(i) all different =

(ii) all the same =

(iii) two the same =

Probability with Words

a c a c i a

if the letters of the name of this shrub
are jumbled up in a bag
what are the probabilities of picking:

$$P(a) = \quad P(\text{not } a) =$$

$$P(c) = \quad P(\text{not } c) =$$

$$P(i) = \quad P(\text{not } i) =$$

$$P(e) = \quad P(\text{not } e) =$$

a e r

these three letters
are jumbled up and
then placed in a
row

three letters are
jumbled up and then
placed in a row

what is the
probability of
getting a common
word?

the probability of
getting a common
word is $\frac{1}{3}$

what could the letters
be?

Probability with Words

probability with words

- (1) give some words where the probability of picking a vowel is $\frac{1}{2}$
try to find words with different lengths
- (2) try to find some words where the probability of picking a letter 'a' is $\frac{1}{3}$
try to find words with different lengths
- (3) give some words where the probability of picking a letter 'e' is $\frac{1}{3}$
try to find words with different lengths
- (4) try to find words where the probability of picking a vowel is greater than $\frac{1}{2}$
try to find words with different lengths
- (5) give some words where the probability of picking a consonant is $\frac{3}{4}$
try to find words with different lengths
- (6) give some words where the probability of picking a consonant is equal to or close to 1
how close can you get for a five letter word?

Probability with Words

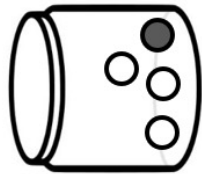
probability with words

- (1) what is the probability of picking an 'E' out of the letters in the word S N E E Z E ?
- (2) put these words in order for the probability of picking a 'T' from them:
BETTERMENT
DAUNT
TWIST
- (3) put these words in order for the probability of picking a 'S' from them:
BOSSSES
ASSESSSES
SUSS
- (4) put these words in order for the probability of picking a 'R' from them:
ERRORS
RARE
REFERRER
- (5) put these words in order for the probability of picking a vowel (a, e, i, o, u) from them:
AVENUE
QUEUEING
AREA
- (6) what do these words have in common (in terms of probabilities)?
BETTER
TEA
BEVERAGES
- (7) what do these words have in common (in terms of probabilities)?
ADDITION
SIDE
DODECAHEDRON
- (8) what do these words have in common (in terms of probabilities)?
DAMAGE
READ
UNIDENTIFIED
SOLITUDE

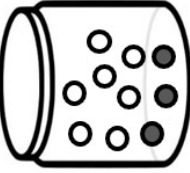
Comparing Probabilities

out of which jar do you stand more of a chance of picking a dark bead? (i)

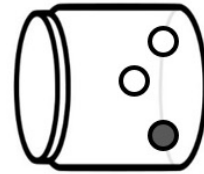
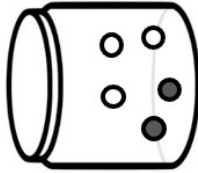
you know what is in the jars but you cannot see into them



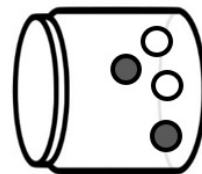
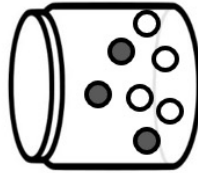
(1)



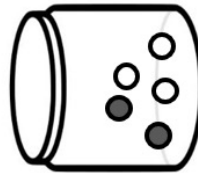
(4)



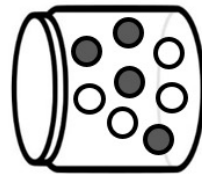
(2)



(3)



(5)



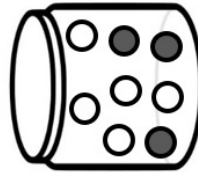
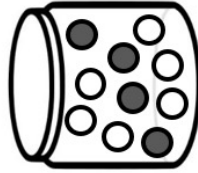
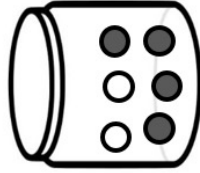
(6)

make up your own two jars
so that the probabilities are
close but not the same

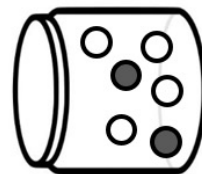
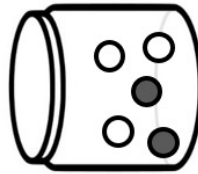
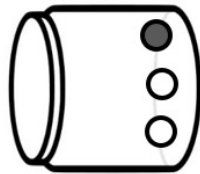
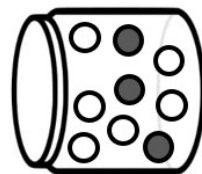
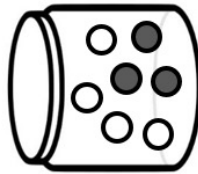
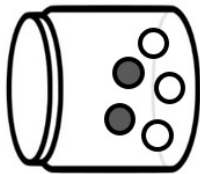
Comparing Probabilities

out of which jar do you stand more of a chance of picking a dark bead? (ii)

you know what is in the jars but you cannot see into them



make up your own two jars
so that the probabilities are
both close to $\frac{1}{2}$
but are not the same



(1)

(2)

(3)

(4)

(5)

(6)

Comparing Probabilities

greater chance

(1)

decide which jar you are more likely to pick a red counter out of by writing the probabilities as percentages:

A : 7 red, 3 blue

B : 13 red, 7 blue

(2)

decide which jar you are more likely to pick a red counter out of by writing the probabilities as percentages:

C : 7 red, 3 blue

D : 17 red, 8 blue

(3)

decide which jar you are more likely to pick a red counter out of by writing the probabilities as percentages:

E : 7 red, 3 blue

F : 18 red, 7 blue

(4)

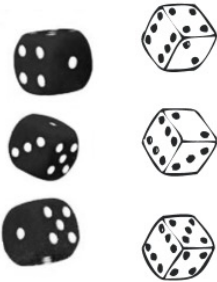
decide which jar you are more likely to pick a red counter out of by writing the probabilities as percentages:

G : 13 red, 7 blue

H : 16 red, 9 blue

Estimated Frequency

probability: estimated frequency

- 1.) a bag contains marbles: 3 red and 5 blue
a marble is chosen repeatedly (with
replacement), 320 times
how many of each would you expect to get?
- 2.) the probability of picking an ace is $\frac{4}{52}$
how many aces would you expect to obtain if
a card is picked, with replacement, 260 times?
- 3.) Albert has a biased coin
the probability of landing on a tail is 0.3
if he flips the coin 500 times, how many times
would he expect to get a head?
- 4.) around 12% of people are left handed
how many left handed people would you
expect to find at a Shrewsbury football game
that 6000 people attend?
- 5.) a bag contains red and pink counters
there are 60 pink counters in the bag
the probability of picking a pink counter is 0.2
how many red counters are in the bag?
- 6.) in France about $\frac{1}{25}$ of people
have red hair
how many students would you
expect to have red hair in a school
in France of 1250 students?
- 7.) a spinner has regions numbered 1 to 10
(inclusive)
in 1000 spins how often would you expect to
get a number that is even or a multiple of 5?
- 8.) when three dice are
rolled the probability of
all three numbers being
different is $\frac{5}{9}$
what is the probability of
three being the same?

in 3600 throws how often would you
expect to get:
(i) all three numbers different?
(ii) all three numbers the same?
(iii) exactly two (a pair) the same?

Sample Space Diagram Puzzle

Each puzzle is a sample space diagram representing a two spinners being spun. The spinners are both fair, so each outcome has the same probability. The rule to calculate the score for each spin is different in each puzzle. Complete the sample space diagrams.

1.

	1		
+		2	
2			5

Rule: Add the results of the two spinners to get your score.

2.

	2	3		
×			10	-6
4			12	
5				
				18

Rule: Multiply the results to get your score.

3.

	1			
+				
2				
4				
6				

Rule: Add the results to get your score.

4.

	1	3	5	2
×				

Rule: Multiply the results to get your score.

The probability of getting an even score is 50%. What could the numbers on the second spinner be?

5.

	-1	2		
×				
-1				

Rule: Multiply the results to get your score.

The probability of getting a negative score is $\frac{10}{16}$. What could the missing numbers be?

6.

	-1	-3	-5	-7
+				
4				

Rule: Add the results to get your score.

The probability of getting a negative score is $\frac{10}{16}$. What could the missing numbers be?

7.

	3	7		
÷				
5				
6				

Rule: Divide the smaller number by larger.

The probability of getting a recurring decimal is 0.5. What could the missing numbers be?