

Year 8
Mathematics
Unit 5 – Student



Name: _____

Class: _____

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1 Data Collection and Sampling

1.1 Qualitative and Quantitative Data

Qualitative Data

<u>Definition</u> Non-numerical data that records qualities of the subject.	<u>Characteristics</u> <ul style="list-style-type: none">• Any data that isn't numbers.• Also called "categorical data" as it sorts the subject of the data into categories.
<u>Examples</u> <ul style="list-style-type: none">• Make of car• Most/least popular names• Political party voted for	<u>Non Examples</u> <ul style="list-style-type: none">• Shoe size• House number• Height• Weight• Reaction time• Number of passengers

Quantitative Data

<u>Definition</u> Numerical data that counts or measures quantities associated with the subject.	<u>Characteristics</u> <ul style="list-style-type: none">• Data that is numbers.• Can be sub-divided into "discrete", which is counted data, and "continuous", which is measured data.
<u>Examples</u> <u>Discrete</u> <ul style="list-style-type: none">• Shoe size• House number• Number of passengers <u>Continuous</u> <ul style="list-style-type: none">• Height• Weight• Reaction time	<u>Non Examples</u> <ul style="list-style-type: none">• Make of car• Most/least popular names• Political party voted for

Fluency Practice

Question 1: Define the term **qualitative data**

Question 2: Give three examples of qualitative data

Question 3: Define the term **quantitative data**

Question 4: Give three examples of quantitative data

Question 5: Emily is doing a survey on the colours of cars.
She is going to count the number of cars of each colour in a car park.
Decide if the following data is qualitative or quantitative

(a) The number of cars

(b) The colour of the cars

Question 6: Eddie carries out a survey about the pet dogs his classmates own.
Decide if the following data is qualitative or quantitative

(a) How many dogs each person owns

(b) The colour of the dogs

(c) The type of dog

(d) The name of each dog

(e) The age of each dog

(f) The mass of each dog

Question 7: Max is writing a report about the Statue of Liberty

(a) List 5 quantitative variables that Max could include in his report

(b) List 5 qualitative variables that Max could include in his report



1.2 Discrete and Continuous Data

Discrete Data: Numerical data that can only take certain values, for example, the number of children in a classroom or a shoe size.

Continuous Data: Numerical data that can take any value within a given range, for example, the masses of 10 babies or the heights of some adults.

Fluency Practice

Question 1: What does the term **discrete data** mean?

Question 2: Write down 3 examples of discrete data

Question 3: What does the term **continuous data** mean?

Question 4: Write down 3 examples of continuous data

Question 5: For each of the following, state if the data would be discrete or continuous:

- | | |
|--|-------------------------------------|
| (a) The number of people in a room | (b) The mass of a book |
| (c) The number of pages in a book | (d) The length of a line |
| (e) The time taken to complete a puzzle | (f) The size of a shoe |
| (g) The number of glasses in a dishwasher | (h) The volume of water in a bottle |
| (i) The number of songs in an album | (j) The weight of an apple |
| (k) The number of people at a football match | |

Question 6: A teacher collects the ages of students in her school.
Is that variable discrete or continuous?

Question 7: Steven keeps a record of the prices of all the cars he sold in one year.
Is that variable discrete or continuous?

Question 1: Is money discrete or continuous? Explain your answer.

Question 2: Is the value of an antique discrete or continuous?

1.3 Primary and Secondary Data

Primary Data

<u>Definition</u> Data obtained through the direct efforts of the user.	<u>Characteristics</u> Data that has either been collected by the person who will use it, or to their exact requirements.
<u>Examples</u> <ul style="list-style-type: none">• Writing and administering a survey.• Paying a marketing company to conduct a focus group for you.• Measuring the reaction time of a set of different chemical reactions.	<u>Non Examples</u> <ul style="list-style-type: none">• Looking up world records on the internet.• Going through the minutes of meetings.• Using the results of someone else's survey.

Secondary Data

<u>Definition</u> Data obtained by means other than as directed by the user.	<u>Characteristics</u> <ul style="list-style-type: none">• Data collected for other purposes, but that is then found and used by someone else.• The end user of the data did not specify what/how it was to be collected.
<u>Examples</u> <ul style="list-style-type: none">• Looking up world records on the internet.• Going through the minutes of meetings.• Using the results of someone else's survey.	<u>Non Examples</u> <ul style="list-style-type: none">• Writing and administering a survey.• Paying a marketing company to conduct a focus group for you.• Measuring the reaction time of a set of different chemical reactions.

Fluency Practice

Question 1: Define the term **primary data**

Question 2: Give three examples of primary data

Question 3: Define the term **secondary data**

Question 4: Give three examples of secondary data

Question 5: For each of the following, state if the data would be primary or secondary:

- (a) Richard wants to know his friends' favourite colour.
He asks his 10 friends their favourite colour.
- (b) Laura wants to know how many cars travel down her street between 9am and 10am.
She stands outside her house and records how many cars drive down her street.
- (c) Hollie wants to know how many people live in her village.
She looks it up on the internet.
- (d) Joseph wants to find out if students like school dinners in his school.
He carries out a survey.
- (e) Kyle collects information from the internet the weather in April over the last 10 years.
- (f) Erin wants to know find out information on the life expectancy of penguins.
She wants a documentary on penguins to find out.
- (g) Rosie wants to find out the mass of an orange.
She weighs 5 oranges.
- (h) Neil wants to find out information about how often people visit the cinema and how much money they spend while there.
Neil asks people to fill out a questionnaire.

1.4 Questionnaires

Questionnaire

I am a student at this school, and I am gathering data on homework. Please answer the questions below, and return this form to me.

1. What is your name? _____

2. Are you male or female?

Yes

No

3. How old are you?

0-5

5-10

10-15

15+

4. How many pets do you have?

1

2

3

5. What is your favourite subject?

Maths

Other

6. How much homework do you do a year?

Less than 500 hours

Between 500 and 800 hours

Between 800 and 1000 hours

More than 1000 hours

7. Do you think you get enough homework?

Far too much

Not nearly enough

8. What do you do in your free time? _____

Questionnaires

Things to remember when designing questionnaires

For some questions, it is important to state a **time period** - e.g. How many times to do you go to the cinema *each month?*

Offer **precise options**; words like “sometimes” or “a lot” aren't always clear.

Don't offer **overlapping options**, otherwise the respondent won't know which box to tick.

Make sure you include enough **options to cover all possibilities**. Boxes such as “Other”, “I don't know” and “more than 10” are good ways of doing this.

Avoid **leading questions**, which try to encourage a particular answer.

Also avoid **personal or embarrassing questions**.

Fluency Practice

α

Alpha Exercise

Zara wants to find out how long it takes her classmates to get to school. She designs the following question.

How long does it usually take you to travel to school?

- 10-15 min 15-20 min 20-25 min

- Do the response boxes cover **all possibilities**?
- Do the response boxes have **overlapping options**?
- Is this a **leading question**?
- Is this question too **personal** or **embarrassing**?
- Improve Zara's question. Include response boxes with your question.

β

Beta Exercise

Rajesh wants to find what colours of car are most liked. He designs the following questionnaire:

Silver cars look elegant. What is your favourite colour for a car?

- Silver Red Blue

- Is this a leading question? Explain your answer.
- What else is wrong with this question?
- Improve Rajesh's question. Include response boxes with your question.

γ

Gamma Exercise

How often do you visit the dentist?

- Never Sometimes A lot

- Write down **two** problems with this question, and **explain** why they are problems.
- Design a better version of this question. You should include response boxes with your question.



Fluency Practice



Explain the mistake

How many hours of TV do you watch?

under 1 hour 1-2 hours over 2 hours

Tyler writes:

This question does not need to specify a time period because the response boxes include time periods.

Tyler is wrong. Explain why.

Exam-style question

Sergey wants to find out how many apps people download each month. He creates the following question:

How many apps do you download?

1-5 5-10 10-15 15+

- (a) Write down **three** things wrong with this question.
- (b) Design a better question to find out the information Sergey wants. You should include response boxes with your question.



Fluency Practice

Here are two versions of a questionnaire about skateboarding.
The aim is to find out about pupils' attitudes to skateboarding at school.

What is different about the questionnaires?
Why do you think these pupils wrote the questionnaires the way they did?

Andy's questionnaire

1. Do you think that keeping healthy is important? Yes/No
2. Do you think that skateboarding is a good form of exercise? Yes/No
3. Do you think that the school should encourage pupils to do things that promote their health? Yes/No
4. Do you agree that there should be a skateboarding area at school? Yes/No

Billie's questionnaire

1. Do you think that keeping healthy is important? Yes/No
2. Do you think that skateboarding can be dangerous? Yes/No
3. Do you think that the school should discourage pupils from injuring themselves on pointless and dangerous activities? Yes/No
4. Do you agree that skateboarding should be banned at school? Yes/No

What makes these questionnaires biased?

Can you write a better version.

Can you make a version that other people agree is unbiased?

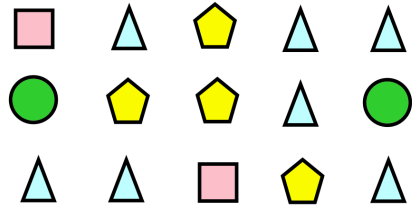
1.5 Tally Charts

A tally chart is a simple way of recording and counting frequencies. Each occurrence is shown by a tally mark and every fifth tally is drawn diagonally to make a “gate” of five. The tallies can then be counted to give the frequency.

Fluency Practice

Question 1: Copy and complete the tally chart

Shape	Tally	Frequency
Circle		
Pentagon		
Square		
Triangle		



Question 2: Dara has recorded how many tries he scored in 25 rugby matches
Copy and complete the tally chart

1	2	0	0	1
0	1	0	2	0
0	3	0	1	0
0	1	2	1	2
0	1	1	1	0

Number of tries	Tally	Frequency
0		
1		
2		
3		

Question 3: Isabelle is creating a tally chart.
Complete the tally chart for her.

Day	Tally	Frequency
Monday		12
Tuesday		
Wednesday		7
Thursday		
Friday		10

Question 4: Jessica rolls a dice 30 times and records the scores.

- Draw a tally chart to show her results
- Which score was the most common?
- Do you think the dice was fair?

6	1	2	3	2	1	5	1	4	1
1	4	1	6	6	5	1	2	3	1
1	3	2	3	2	1	1	6	1	1

Fluency Practice

Question 5: Danielle asked 50 people how they travelled to school.
The tally chart below shows her results.

	Tally	Frequency
Walk		
Bus		
Cycle		
Car		

- Copy and complete the tally chart
- Which method of travel was the most popular?
- Danielle says twice as many people walked than travelled by car. Is Danielle right?

Question 6: Miss Wallace gave the students in a year 6 class a quiz.
The results are shown below.

34	15	31	24	8	11	35
32	27	19	21	39	25	23
14	26	25	26	18	27	30

Score	Tally	Frequency
1 - 10		
11 - 20		
21 - 30		
31 - 40		

- Copy and complete the tally chart
- How many students are in the class?

Question 7: Thomas records the ages of people at a party.

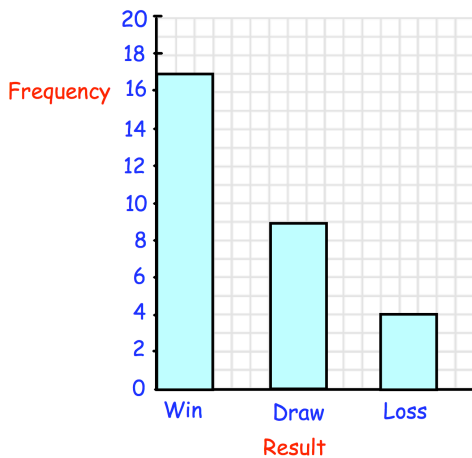
Age	Tally	Frequency
21 - 30		
31 - 40		
41 - 50		
51 - 60		
61 - 70		

- Complete the tally chart
- How many people when to the party?
- How many people were 40 years or younger?
- Thomas says the oldest person was 70. Explain why he might not be correct.

Extension

Question 1: Flip a coin 40 times and record your results in a tally chart.

Question 2: Winston has drawn a bar chart to show his football team's results.



(a) How many matches did the team draw?

(b) Draw a frequency table (tally chart) to show these results.

A win is worth 3 points
 A draw is worth 1 point
 A loss is worth 0 points

(c) How many points did the team receive in total?

Question 3: Each student at a school studies one language.

The tally chart shows the languages that a group of 20 students study.

(a) What percentage of the students study Spanish?

(b) What fraction of the students study French?

A student is selected at random.

(c) What is the probability that the student studies German?

Language	Tally
French	
German	
Latin	
Spanish	

Question 4: Orla organises a charity film showing. The tally chart shows the tickets sold.

An adult ticket costs £6.

The price of a child ticket is half the price of an adult ticket.

Pensioners pay £1 less than adults.

How much money did Orla raise for charity?

Group	Tally
Children	
Adults	
Pensioners	

1.6 Sampling

I wish to find out the proportion of animals in my zoo that have contracted the deadly disease 'Chilcotius'.



Sampling

I could test the entire **population** of animals at the zoo.
(Looking at the entire population is known as a **census**)
But this would be time consuming.



Sampling

I could select just some of the animals to test. This selection is known as a **sample**.

We hope that the proportion of animals with the disease in the sample accurately reflects the whole population.

We therefore instead want to just test some of the animals, and hope that the percentage of these animals with the disease within the sample accurately reflects the whole population of animals, e.g. if 20% have the disease within our selected group, we assume roughly 20% of all animals have the disease.

1.7 Random Sampling

But how do we get our sample?
We could just animals randomly.



“Describe a random sample”

A sample where each thing in the population is equally likely to be chosen.

“You want to take a random sample a student’s favourite TV programmes at school. Describe how you could achieve a random sample.”

- Put all student names into a hat and pick them out to decide who to sample.
- Use a random number generator where each number represents a student.

Worked Example

Describe a method for taking a random sample of 5 students from a class of 30.

Your Turn

Describe a method for taking a random sample of 10 students from a class of 50.

Fluency Practice

<https://corbettmaths.com/wp-content/uploads/2013/02/random-sampling-pdf.pdf>

1.8 Stratified Sampling

Instead of sampling animals completely randomly, we might want to ensure that we sample the same proportion/percentage from each group, so that each type of animal is fairly represented. (With random sampling, it is possible to avoid having any lions!) This is known as a stratified sample.



“Describe a stratified sample”

In stratified sampling, the population is divided into groups, and random samples are taken from each stratum.

(Stratum means group, and literally means ‘layer’)

Worked Example

A stratified sample of 50 is taken from the following information

Year 7	Year 8	Year 9	Year 10
72	108	66	54

How many of each year group should be sampled?

Your Turn

A stratified sample of 20 is taken from the following information

Ireland	Wales	Scotland
8	28	44

How many people of each nationality should be sampled?

Worked Example

A stratified sample of 50 is taken from the following information

Teachers	Teaching Assistants	Admin	Other
94	16	41	29

How many of each type of employee should be sampled?

Your Turn

A stratified sample of 40 is taken from the following information

Type	Milk	Dark	White
Number	600	220	130

How many of each chocolate should be sampled?

Fluency Practice

Question 1: This table shows information about the number of students in years 9, 10 and 11.

Year 9	Year 10	Year 11
100	50	50

The headteacher is going to survey some of the students about the school library.

(a) Explain why it is appropriate to take a stratified sample

The headteacher takes a stratified sample of 40 students.

(b) Work out how many students should be surveyed from each year group.

Question 2: A vet treats 100 pets over 1 week.

Cats	Dogs	Rabbits
30	50	20

A stratified sample of 10 is required.

Find the number of each type of pet that the vet should choose.

Question 3: The table shows information about the inhabitants of a village.

Age	Population Size
0 - 20	70
21 - 40	80
41 - 60	40
Over 60	10

Henry takes a stratified sample of 40.

Work out the number of each age group that Henry should choose.

Question 4: The table shows the holiday destinations of some tourists.

Spain	Portugal	France	Italy
48	66	102	84

A stratified sample of 50 is planned.

Calculate the number of visitors to each country that should be selected.

Fluency Practice

Question 5: The table shows how all 240 students travel to school.

Walk	Car	Bus	Train	Cycle
105	30	70	10	25

A stratified sample of 48 students is to be taken.

Calculate the number using each method that should be surveyed.

Question 6: A business has 80 employees.

Production	44
Dispatch	16
Admin	20

The director wants to survey 10 employees, stratified by job role.

Calculate the number of each type of employee needed for his survey.

Question 7: A factory uses four machines to make parts for microwaves. In one hour, the machines made 120 parts and the table below shows information on the number of parts created by each machine.

A	B	C	D
40	11	44	25

Eve wants to test the quality of the parts.

She wants to choose a stratified sample of 30 parts.

Calculate the number of parts Eve should test from each machine.

Question 8: A cinema sold 1500 tickets last week
A stratified sample of 100 customers is chosen based on the film genre.

Horror	Comedy	Action	Other
210	660	130	500

Calculate the number of customers for each genre chosen.

Question 9: Shaun works in a confectioners. He is asked to test a sample of 200 chocolates stratified by type of chocolate. The table shows the number of each type of chocolate in the shop.

Dark	Milk	White
2500	3600	1500

Calculate the number of each type of chocolate required for his stratified sample.

Fluency Practice

Question 10: The table below shows the age group of the members of a tennis club.

Age Group	Junior	Adult	Senior
Number	320	500	130

A stratified sample of 40 is to be taken.

Calculate the number for each age group in the sample.

Question 11: The table shows information about the ages of people on a train.

Age range	Frequency
under 18	16
18 to 40	41
41 to 60	84
over 60	29

The train conductor gives a questionnaire to some of these passengers.

She takes a sample of exactly 50 passengers stratified by age range.

Work out the number of passengers in each age range that the train conductor should have in her sample.

Question 12: The table below shows information about the vehicles sold by a dealership.

Car	Van	Motorbike	Caravan
5112	1048	2948	750

The manager takes a sample of 150 customers, stratified by type of vehicle sold. Calculate the number of each vehicle type in the sample.

Extension

Question 1: Mr Henderson is going to survey the students in his school.
The table shows the number of students in each year group.

Year	Boys	Girls	Total
8	91	100	191
9	82	95	177
10	84	84	168
11	75	70	145
12	68	71	139
			820

Mr Henderson wants to take a sample of 60 students, stratified by year and by gender.

- What is a stratified sample?
- Work out the number of year 9 girls that should be in the sample.
- Describe a method to randomly select the year 9 girls

Question 2: A cricket club has 300 members.
A stratified sample of members is taken, by age group.

Some information is given in the table.

	Junior	18 - 40	41 - 60	Senior
Number of members	40		115	
Number in sample	8			7

Complete the table.

Question 3: Matthew owns 2400 stamps from several different countries.
He takes a stratified sample, by country.

Some information is given in the table.

Country	France	Spain	Turkey	UK
Number of stamps		320		1120
Number in sample		20	7	

Complete the table.

Question 4: Here is some information about the colour of raffle tickets sold.

Pink	Green	Yellow
145	125	340

A sample of size 20, stratified by colour of raffle ticket is taken.
From the sample of 20, two winning tickets are chosen at random.

Work out the probability that the two tickets are different colours.

1.9 Review and Problem Solving

Fluency Practice

Data is another word for _____

There are two types of data:

_____ and _____

_____ data is to do with **numbers** and **amounts** (*quantity*)

_____ data is to do with **things** or **descriptions** (*quality*)

There are two different types of **quantitative** data:

_____ and _____

_____ data can take any value, along a **sliding scale**.
(eg. **height** or _____)

_____ data can only ever take **specific values**.
(eg. **age** or _____)

Fluency Practice

Situation	Example of data	Type of data
Dave wants to find out how much money his friends earn per hour.	£3.45	Quantitative, Discrete
Amy is measuring how long swimmers stay under water.		
Mark is keeping track of the type of sandwiches bought in a shop.		
Julie needs to find out the age of people being treated for asthma.		
Pete is analysing the size of earthquakes around the world.		
Maggie weighs her puppy every week to see how he is growing.		
Maria keeps track of how much electricity she uses each day.		

Give an example of a piece of data in the first column

You may need to look up some of the information, or ask parents to help you. Make sure you write down sensible estimates for the numbers and use the correct units (eg how electricity usage is measured, or the size of earthquakes).

Describe the type of data in the second column:

Qualitative, Quantitative Continuous, Quantitative Discrete.

Fluency Practice

Use words from the box below to complete these sentences.

<i>continuous</i>	<i>random</i>	<i>experiment</i>	<i>primary</i>	<i>grouped</i>
<i>non-numerical</i>	<i>stratified</i>	<i>numerical</i>	<i>closed</i>	<i>census</i>
<i>open</i>	<i>cost</i>	<i>bias</i>	<i>time</i>	<i>secondary</i>
<i>size</i>	<i>discrete</i>	<i>population</i>	<i>systematic</i>	<i>survey</i>

- Data that is collected directly yourself is called data.
 - Data taken from an existing source, such as a book or website, is called data.
 - The most common methods for collecting data yourself are to perform an or to complete a
 - If a sample of data is collected it must represent the whole picture, called the This means it needs to be an appropriate and must avoid
 - Questions should not be 'leading', but could allow only specified answers (.....) or any answer (.....).
 - Surveying every member of the population is called a
 - Sampling by taking a lottery or picking names out of a hat is called a sample.
 - Sampling every 5th or 10th (or similar) item is called a sample.
 - Taking a sample from each sub-section of the population so each group is represented in proportion is called a sample.
 - As well as the type of data being collected, factors influencing which method to use include and
 - Data can be (numbers) or (words).
 - Data that can only take specific values (e.g. shoe size, number of...) is called data. Data that can take any value in a given range (e.g. length, time) is called data.
 - Once data has been collected, it can be put into classes to make it easier to analyse. This is called data.
-

2 Charts and Quartiles

2.1 Averages and Range Recap

There are three averages:

- Mode – The most common item in a set of data.
- Median – The value at the middle of a numerically ordered list of values.
- Mean – The single value that if all numbers in a list are changed into, maintains the total of the list.

And one measure of spread:

- Range – The difference between the largest and smallest values in a list.

Worked Example

1, 5, 5, 7, 12

Mean =

Median =

Mode =

Range =

Your Turn

0, 4, 4, 6, 11

Mean =

Median =

Mode =

Range =

Intelligent Practice

Data	Mean	Median	Mode	Range
2, 2, 4, 5, 7				
2, 2, 4, 5, 12				
3, 3, 5, 6, 13				
6, 6, 10, 12, 26				
6, 6, 10, 18, 20				
6, 6, 13, 15, 20				
6, 6, 13, 15, 20, 24				
0, 6, 6, 13, 15, 20, 24				

Fluency Practice

Data	Mean	Median	Mode	Range
1, 2, 3, 4, 5				
10, 20, 30, 40, 50				
0.1, 0.2, 0.3, 0.4, 0.5				
-1, -2, -3, -4, -5				
1a, 2a, 3a, 4a, 5a				
£1, £2, £3, £4, £5				
5, 2, 1, 4, 3				
2, 3, 4, 5, 6				
1, 2, 3, 4, 10				
0, 3, 3, 4, 5				
0, 0, 6, 4, 5				
-3, -2, -1, 0, 1, 2, 3				
1, 2, 3, 4, 500				
0, 1, 2, 3, 4, 5				
0, 0, 1, 2, 3, 4, 5				
1, 2, 3, 4, 5, 6				
0.4, 2.1, 0.9, 1.7, 2.9				
$\frac{1}{2}, \frac{1}{5}, \frac{1}{10}$				
4x, 2x, 7x, 3x, 9x				
3a + b, b, 6a				

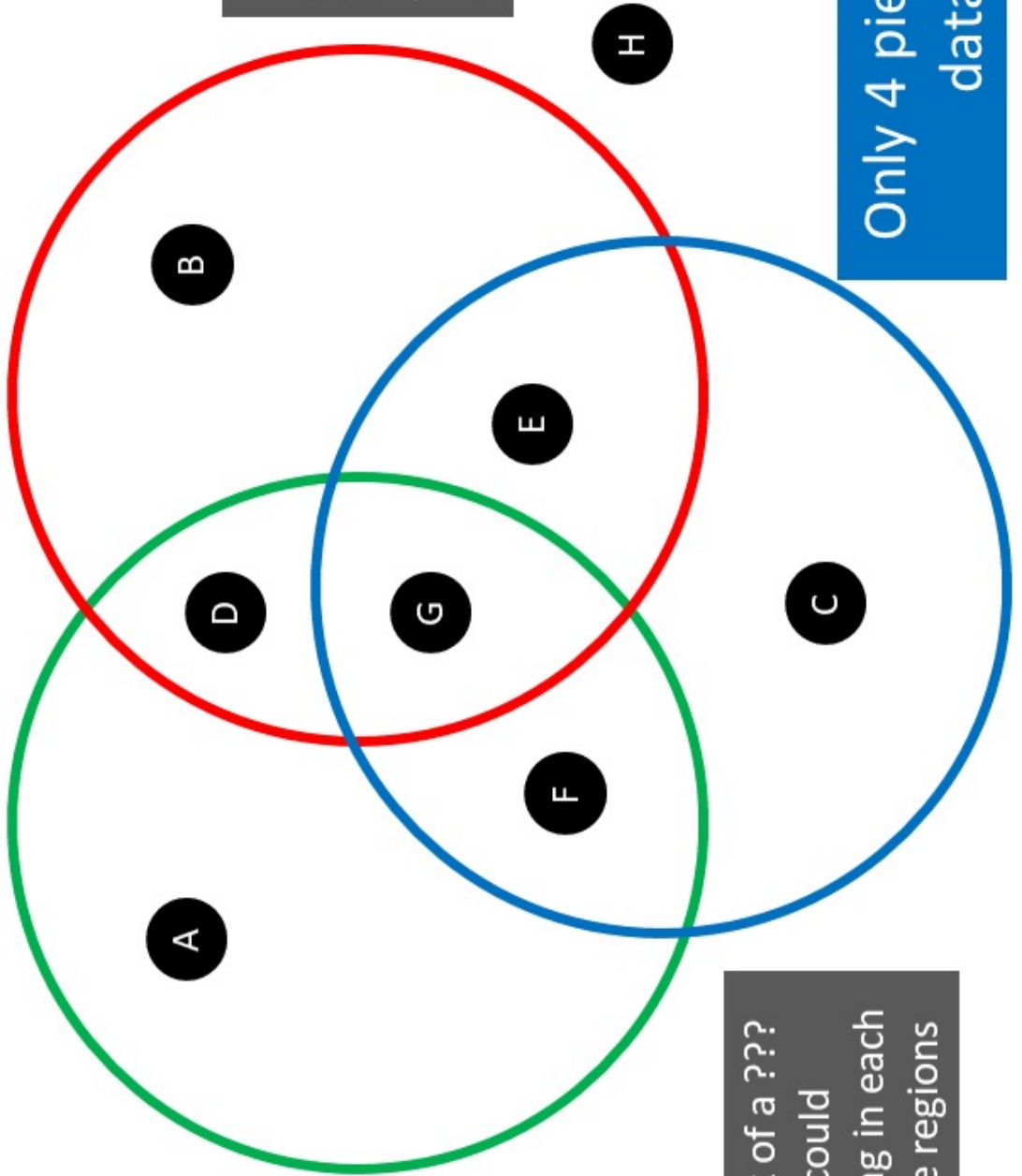
Fill in the Gaps

Data Set						Mode	Median	Range	Mean
2	3	3	3	4		3	3	2	3
2	2	3	4	5					3.1
4	4	6	8	10		4			
4	5	6	7	8					
6	6	6	6	6					
6	6	6	6	7					
-4	-2	-2	0	8					
0.6	0.6	0.8	0.8	1					
2	2	4	5	6	8				
-3	1	5	8	8	11				
8	2	5	9	5	10				
5.3	2.9	2.3	3.5	6.7	1.1				
5	7	7					7	6	
3	6	4							4
						8	8	10	7
						10	7	8	6.5
16	10	13				16	14		
						2	3	13	5.4
						7	4	10	3

Maths Venns

Median = 5

Range > mode



If you think a region is impossible to fill, convince me why!

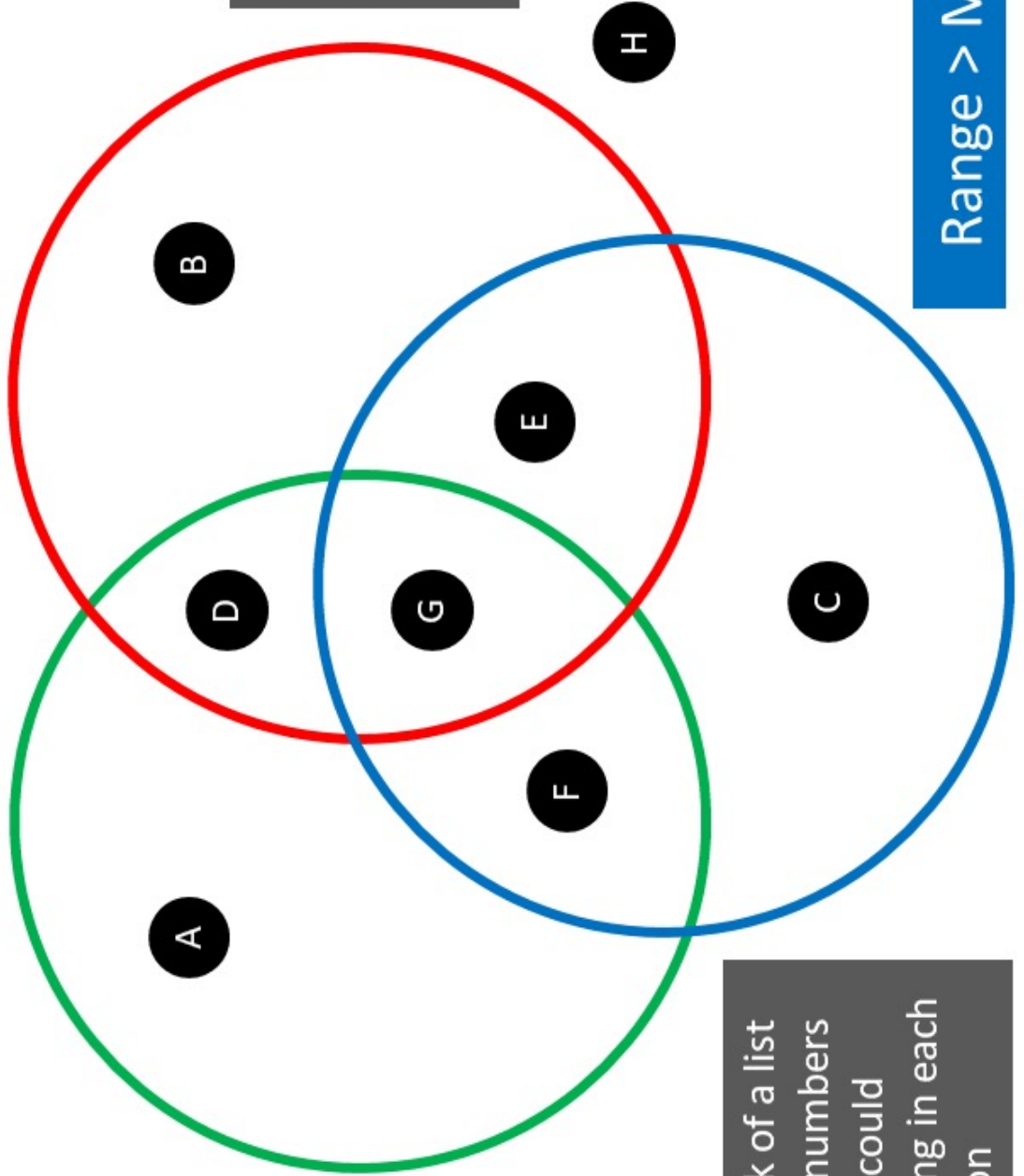
Only 4 pieces of data

Think of a ??? that could belong in each of the regions

Maths Venns

Median = 5

Mode = 4



If you think a region is impossible to fill, convince me why!

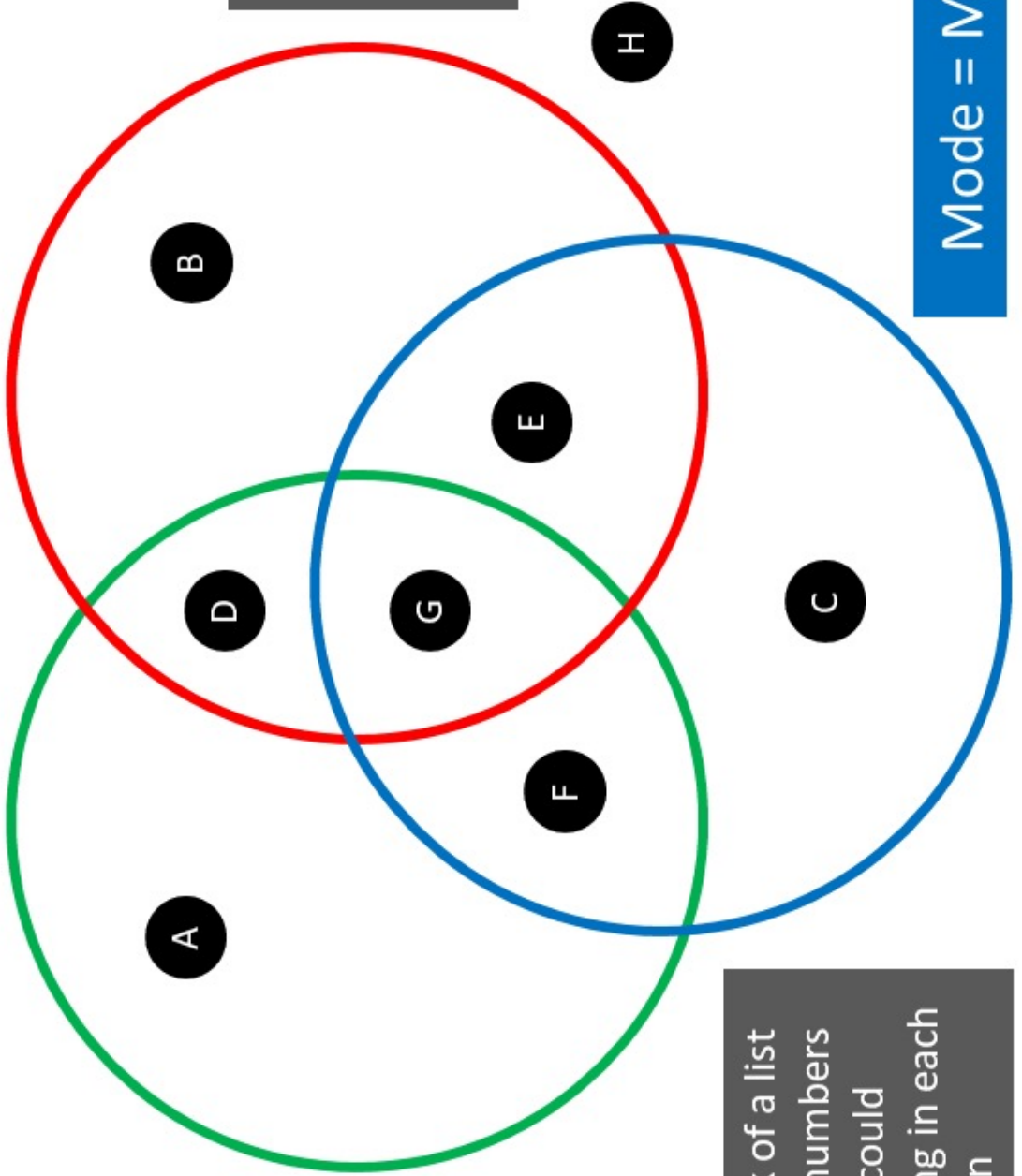
Think of a list of 5 numbers that could belong in each region

Range > Mean

Maths Venns

Median > Range

Mean > Median



If you think a region is impossible to fill, convince me why!

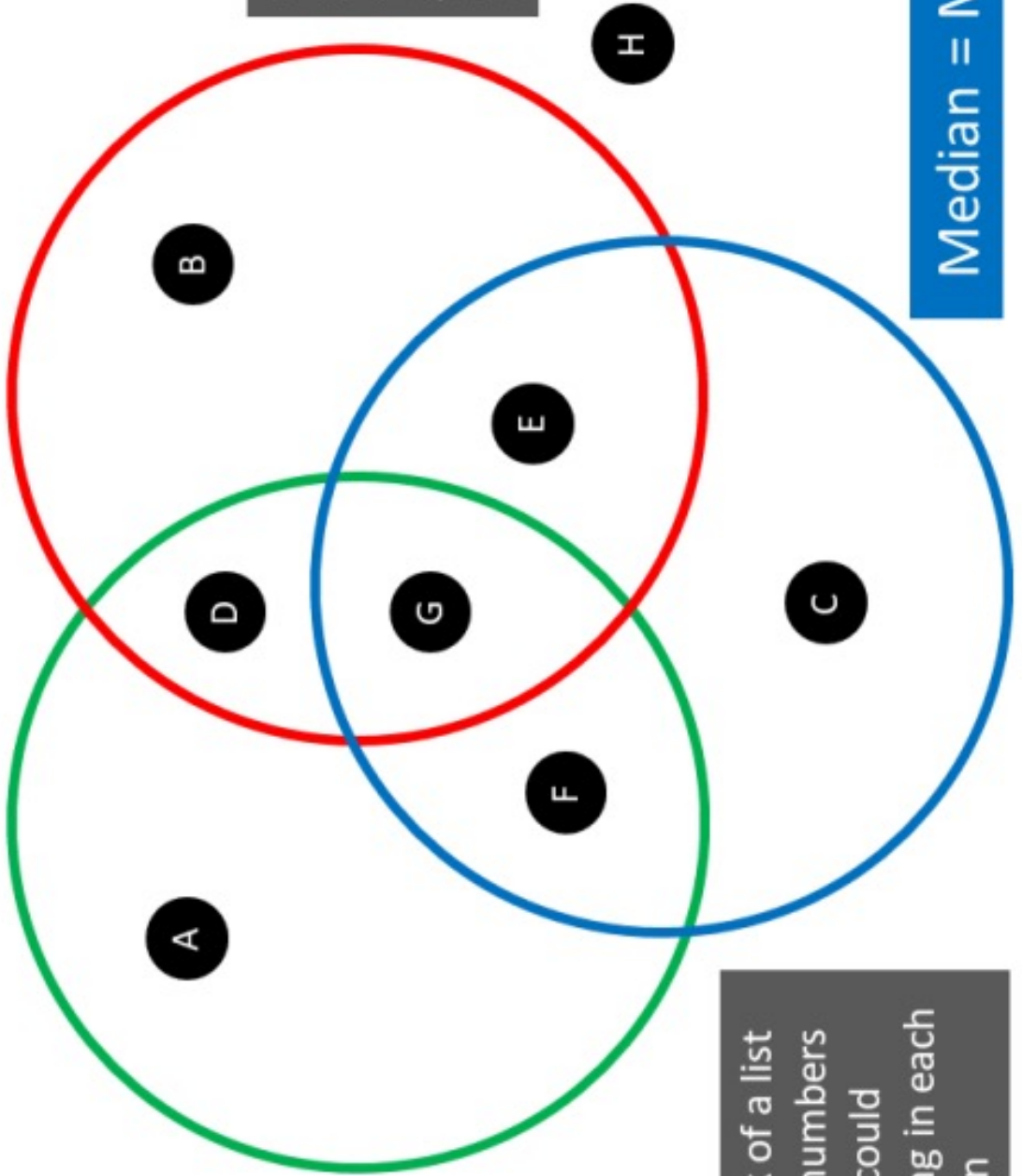
Mode = Mean

Think of a list of 5 numbers that could belong in each region

Maths Venns

Range < Mean

No mode



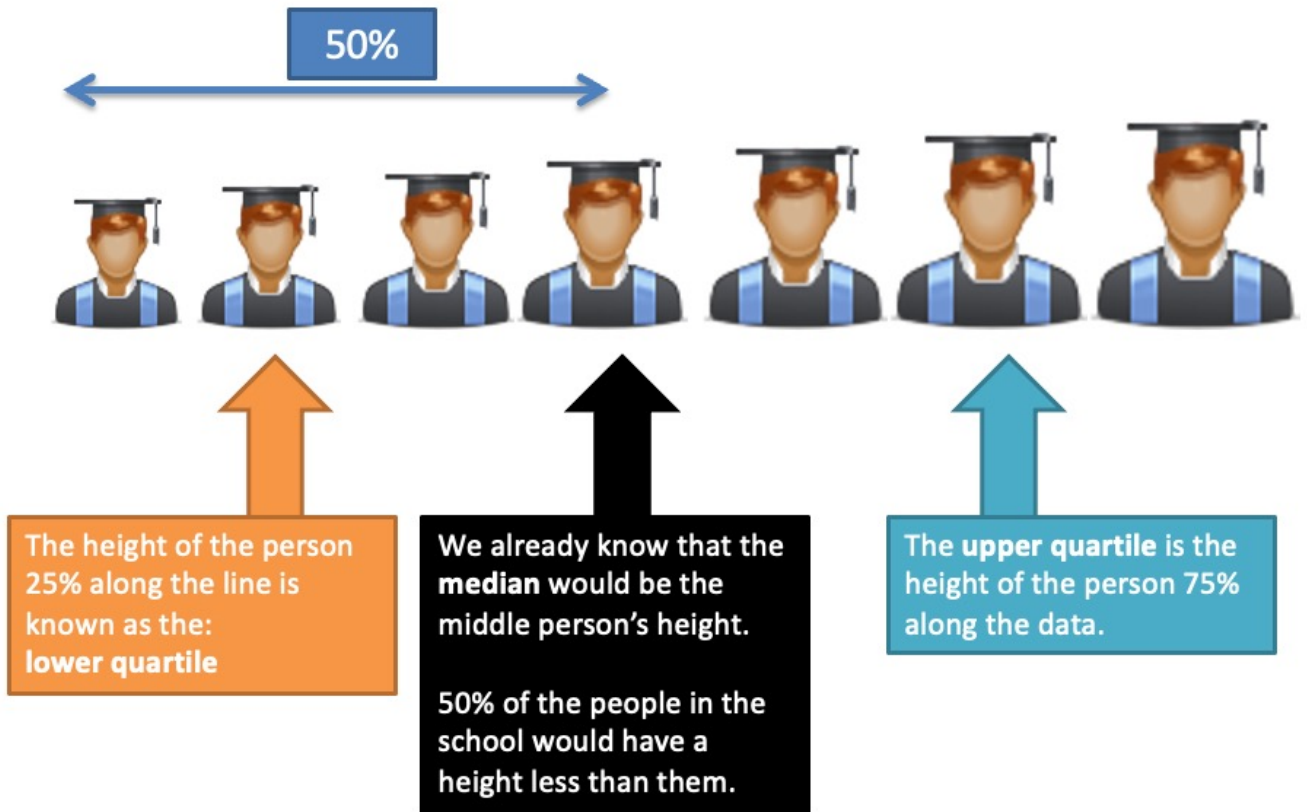
If you think a region is impossible to fill, convince me why!

Median = Mean

Think of a list of 5 numbers that could belong in each region

2.2 Quartiles

Suppose that we line up everyone in the school according to height.



Rule for lower quartile:

- Even number of items: find median of bottom half.
- Odd number of items: throw away middle item, find median of remaining half.

Worked Example

Find the lower quartile, upper quartile and interquartile range for 3, 4, 4, 4, 6, 7, 7, 8, 10, 11

Your Turn

Find the lower quartile, upper quartile and interquartile range for 4, 5, 5, 5, 7, 8, 8, 9, 11, 12

Worked Example

Find the lower quartile, upper quartile and interquartile range for 25, 25, 27, 28, 31, 31, 32, 35

Your Turn

Find the lower quartile, upper quartile and interquartile range for 26, 26, 28, 29, 32, 32, 33, 36

Worked Example

Find the lower quartile, upper quartile and interquartile range for 2.3, 2.4, 2.7, 2.8, 2.9, 3.0, 3.0

Your Turn

Find the lower quartile, upper quartile and interquartile range for 2.4, 2.5, 2.8, 2.9, 3.0, 3.1, 3.1

Worked Example

Find the lower quartile, upper quartile and interquartile range for 2, 6, 8, 6, 5, 4, 2, 2, 3

Your Turn

Find the lower quartile, upper quartile and interquartile range for 12, 6, 18, 6, 15, 4, 12, 2, 13

Fluency Practice

Find the lower quartile, upper quartile and interquartile range for each of these sets of data:

a) 4, 5, 7, 8, 8, 8, 9, 10, 10

b) 8, 9, 11, 12, 12, 12, 14, 14, 15, 17

c) 7.1, 7.2, 7.4, 7.4, 7.5

d) 19, 29, 31, 21, 28, 27, 24

e) 0.6, 0.23, 0.2, 0.7, 0.14, 0.1, 0.68

f) 20, 31, 25, 45, 46, 20, 34, 31

Extension

For each statement, decide if its possible and suggest 7 values that fit the description

The interquartile range is equal to the range



The interquartile range is equal to the median



The interquartile range is larger than the range



The upper quartile is equal to the median



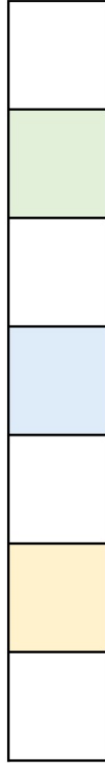
The median is 0 and the interquartile range is 5



The range is negative



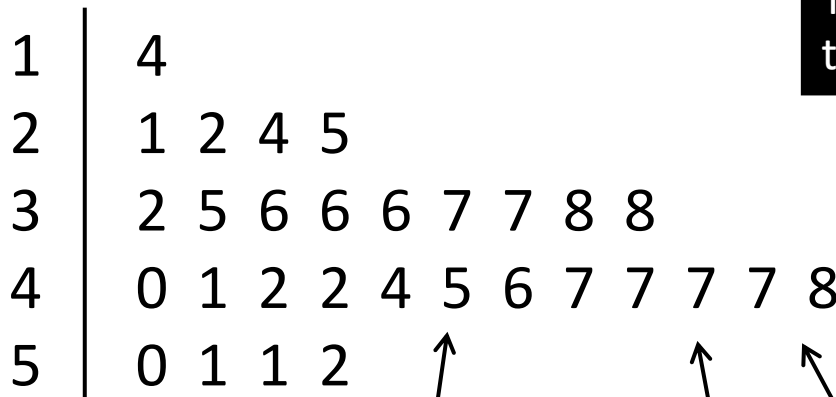
The upper quartile is 1 and the range is 5



2.3 Stem and Leaf Diagrams Recap

A stem and leaf diagram is a simple but effective way of showing data. It puts the data into order, puts it into classes (groups) and we can quickly see patterns. As the data is in order it is also useful for finding averages and the range.

Suppose this “stem and leaf diagram” represents the lengths of beetles.



The key tells us how two digits combine.

Key:
2 | 1 means 2.1cm

Value represented = 4.5cm

These numbers (known as the 'stems') represent the first digit of the number.

The 'leaves' must be in order.

These numbers (the 'leaves') represent the second.

Worked Example

Draw an ordered stem and leaf diagram for this data:

12 21 13 31 53
47 29 21 18 46
21 53 45

- a) Work out the mode
- b) Write down the median
- c) Work out the mean
- d) Work out the range

Your Turn

Draw an ordered stem and leaf diagram for this data:

55 23 48 29 41
47 36 35 40 35
44 34 35

- a) Work out the mode
- b) Write down the median
- c) Work out the mean
- d) Work out the range

Worked Example

Draw an ordered stem and leaf diagram for this data:

12 21 13 31 53
47 29 21 18 46
21 53 45 21

- a) Work out the mode
- b) Write down the median
- c) Work out the mean
- d) Work out the range

Your Turn

Draw an ordered stem and leaf diagram for this data:

42 35 56 39 40
51 47 38 42 55
42 48 49 41

- a) Work out the mode
- b) Write down the median
- c) Work out the mean
- d) Work out the range

Fluency Practice

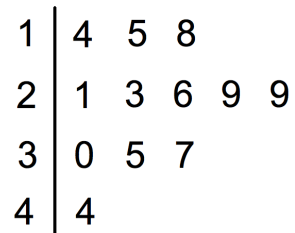
Question 1: Draw ordered stem and leaf diagrams for the following sets of data. Remember to include a suitable key.

- (a) 35, 50, 38, 44, 53, 41, 39, 45, 48, 55
- (b) 18, 42, 5, 28, 33, 9, 15, 38, 32, 9, 11, 24, 40, 29, 24
- (c) 153, 144, 148, 140, 149, 145, 144, 142, 158, 135, 140, 139, 160
- (d) 3.4kg, 1.9kg, 2.8kg, 3.1kg, 5.1kg, 3.9kg, 4.8kg, 4.5kg, 2.2kg, 3.7kg,

Question 2: The stem and leaf diagram below shows the ages of a group of people.

- (a) How many people are there in the group?
- (b) How old is the youngest member of the group?
- (c) How old is the oldest member of the group?
- (d) How many people are under 20?
- (e) How many people are over 25?

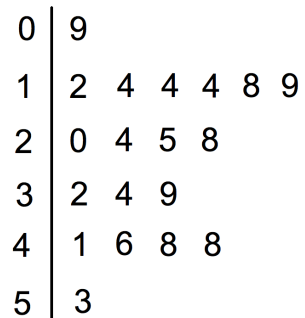
Key: 1|4 means 14 years old



Question 3: The stem and leaf diagram below shows heights of Mrs Smith's flowers.

- (a) How many flowers does Mrs Smith have?
- (b) What is the height of the shortest flower?
- (c) What is the height of the tallest flower?
- (d) How many flowers have a height of 14cm?
- (e) How many flowers have a height greater than 40cm?
- (f) What fraction of the flowers have a height under 20cm?

Key: 0|9 means 9cm



Fluency Practice

Question 4: Fiona recorded the times it took 11 students to run 200 metres.
The times are measured in seconds and are:

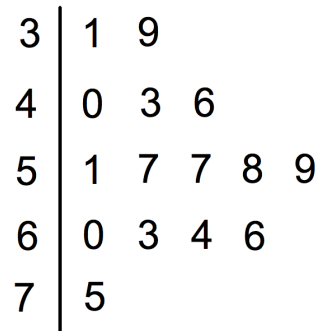
27 38 42 35 43 49
50 37 38 41 48

- (a) Draw an ordered stem and leaf diagram to show this information.
- (b) Work out the median time.
- (c) Work out the range of the times
- (d) How many students finished the race in under 40 seconds?

Question 5: The following stem and leaf diagram shows times taken for 15 people to complete a jigsaw.

Key: 3|1 means 31 minutes

- (a) Write down the modal time taken.
- (b) Write down the median time taken.
- (c) Write down the range of times taken.
- (d) What fraction of the people took over one hour?



Worked Example

Draw an ordered stem and leaf diagram for this data:

12 21 13 31 53
47 29 21 18 46
21 53 45

- a) Work out the lower quartile
- b) Work out the upper quartile
- c) Work out the interquartile range

Your Turn

Draw an ordered stem and leaf diagram for this data:

55 23 48 29 41
47 36 35 40 35
44 34 35

- a) Work out the lower quartile
- b) Work out the upper quartile
- c) Work out the interquartile range

Worked Example

Draw an ordered stem and leaf diagram for this data:

12 21 13 31 53
47 29 21 18 46
21 53 45 21

- a) Work out the lower quartile
- b) Work out the upper quartile
- c) Work out the interquartile range

Your Turn

Draw an ordered stem and leaf diagram for this data:

42 35 56 39 40
51 47 38 42 55
42 48 49 41

- a) Work out the lower quartile
- b) Work out the upper quartile
- c) Work out the interquartile range

Fluency Practice

The test results of 12 students are shown below.

48, 47, 53, 55, 55, 69, 45,
45, 51, 50, 65, 55, 43

Record them in a stem and leaf diagram.

First, put the results into an unordered stem and leaf:

4	8 7 5 5 3
5	3 5 5 1 0 5
6	9 5

Then, order the results

4	3 5 5 7 8
5	0 1 3 5 5 5
6	5 9

Don't forget a key!

Key
4|3 means 43 marks.

Draw ordered stem and leaf diagrams for these sets of data. Remember to include a suitable key. Then, write down the median, upper quartile, lower quartile, and interquartile range

<p>Example 28, 29, 31, 39, 42, 44, 48, 48, 50, 52, 56,</p> <table style="margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding: 5px;">2</td> <td style="padding: 5px;">8 9</td> <td style="padding: 5px;">Key</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">3</td> <td style="padding: 5px;">1 9</td> <td style="padding: 5px;">2 8 means 28</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">4</td> <td style="padding: 5px;">2 4 8 8</td> <td style="padding: 5px;">Median: 44</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">5</td> <td style="padding: 5px;">0 2 6</td> <td style="padding: 5px;">LQ: 31</td> </tr> <tr> <td></td> <td></td> <td style="padding: 5px;">UQ: 50</td> </tr> <tr> <td></td> <td></td> <td style="padding: 5px;">IQR:</td> </tr> </table>	2	8 9	Key	3	1 9	2 8 means 28	4	2 4 8 8	Median: 44	5	0 2 6	LQ: 31			UQ: 50			IQR:	<p>a) 45, 48, 51, 54, 61, 62, 66, 68, 70, 74, 75, 77, 78, 82, 93</p> <table style="margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding: 5px;">4</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">Key</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">5</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">4 5 means</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">6</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">Median:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">7</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">LQ:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">8</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">UQ:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">9</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">IQR:</td> </tr> </table>	4		Key	5		4 5 means	6		Median:	7		LQ:	8		UQ:	9		IQR:	<p>b) 26, 28, 33, 39, 40, 46, 47, 51, 55, 60, 64</p> <table style="margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding: 5px;">2</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">Key</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">3</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">Median:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">4</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">LQ:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">5</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">UQ:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">6</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">IQR:</td> </tr> </table>	2		Key	3		Median:	4		LQ:	5		UQ:	6		IQR:
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		UQ: 50																																																			
		IQR:																																																			
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6		IQR:																																																			
<p>c) 12, 24, 21, 16, 8, 9, 3, 31, 18, 27, 35, 41, 26, 12, 17, 6, 5, 19, 29</p> <table style="margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding: 5px;">0</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">Key</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">1</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">Median:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">2</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">LQ:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">3</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">UQ:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">4</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">IQR:</td> </tr> </table>	0		Key	1		Median:	2		LQ:	3		UQ:	4		IQR:	<p>d) 47, 51, 63, 39, 42, 57, 36, 37, 49, 32, 60, 54, 56, 45, 52</p> <table style="margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">Key</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">Median:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">LQ:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">UQ:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">IQR:</td> </tr> </table>			Key			Median:			LQ:			UQ:			IQR:	<p>e) 153, 147, 160, 146, 162, 158, 159, 149, 152, 150, 163</p> <table style="margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">Key</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">Median:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">LQ:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">UQ:</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">IQR:</td> </tr> </table>			Key			Median:			LQ:			UQ:			IQR:						
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The back-to-back stem and leaf diagram below shows the test scores of pupils in two classes.

Class A		Class B
8 3 3	5	5 7 7 7
6 5 4 3 1	6	2 6 8 8
8 7 4 0 0	7	6
1	8	2 8 8
	9	9
	10	0

Key
6|2 means
62 marks

Work out the range in each class.

Class A: _____ Class B: _____

Work out the median in each class.

Class A: _____ Class B: _____

State the mode in each class.

Class A: _____ Class B: _____

Extension

Question 1: The stem and leaf diagram shows the weights of 10 books that are placed on a book shelf.

Key: 0|3 means 0.3kg

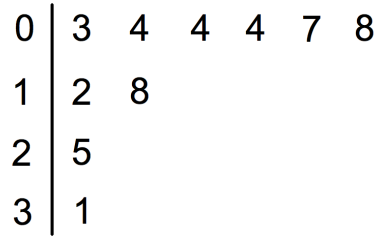
(a) Write down the modal weight.

(b) Find the median weight.

(c) Find the range of the weights.

The bookshelf can hold 12kg.

(d) Will the bookshelf be able to support the 10 books?



Question 2: The stem and leaf diagram shows the heights of 14 students on a school trip to a theme park.

(a) Find the median height.

Key: 13|5 means 135cm

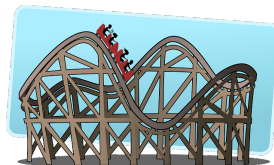
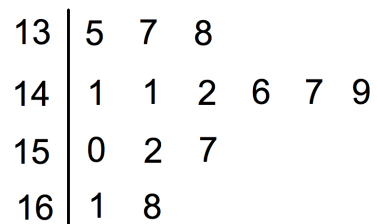
(b) Work out the range of the heights

A "fast pass" allows one of the students to go on a ride at the theme park without queueing.

One of the students is picked at random to win a "fast pass" for a ride.

To go on the ride, the student must be at least 140cm tall.

(c) Write down the probability that the student who wins the "fast pass" cannot go on the ride.



Question 3: This dual stem and leaf diagram shows the results for the students in Mr Turner's class.

(a) How many boys are there in the class?

(b) How many girls are there in the class?

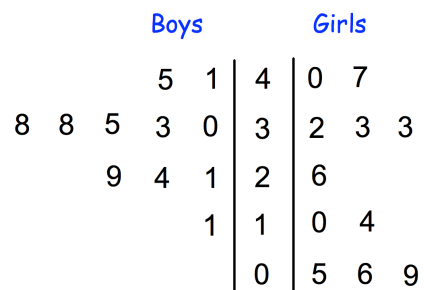
(c) What was the highest mark in the class?

(d) Find the range of the boys' results

(e) Find the median of the girls' results

(f) Find the modal mark for the whole class.

(g) Compare the boy's and girls' results.



Boys Key: 9|2 means 29 marks

Girls Key: 2|6 means 26 marks

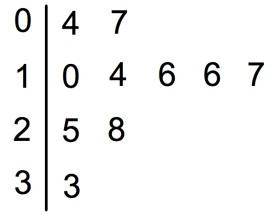
Extension

Question 4: The stem and leaf diagram shows the time taken 10 students to answer a puzzle

(a) Work out the range

Key: 0|4 means 4 seconds

(b) Work out the median



Another student answers the puzzle in 38 seconds.

(c) Tick the box to show how this will effect the range

The range will decrease

The range will stay the same

The range will increase

(d) Tick the box to show how this will effect the median

The median will decrease

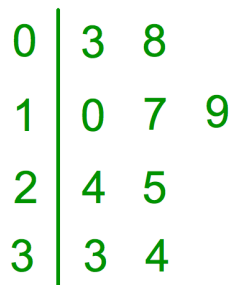
The median will stay the same

The median will increase

Question 5: Amelia has been asked to draw a stem and leaf diagram.
Can you spot any mistakes?

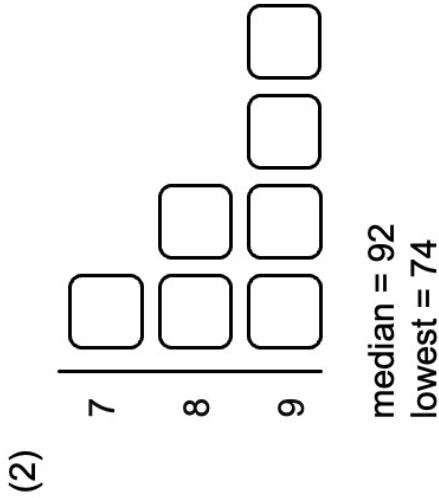
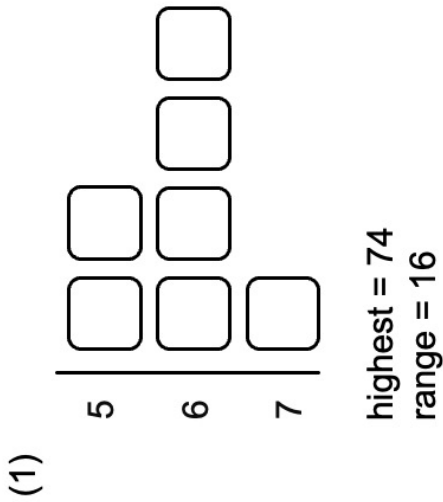
Draw an ordered stem and leaf for:

24 19 28 8 17 33 34 10 3 25

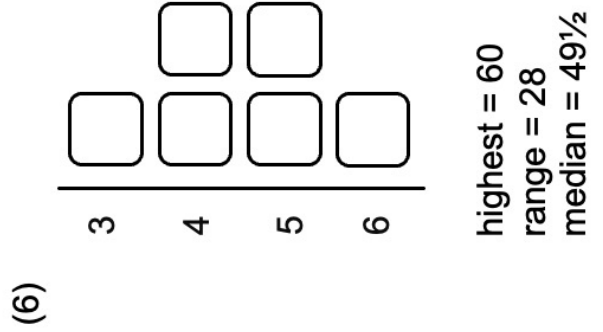
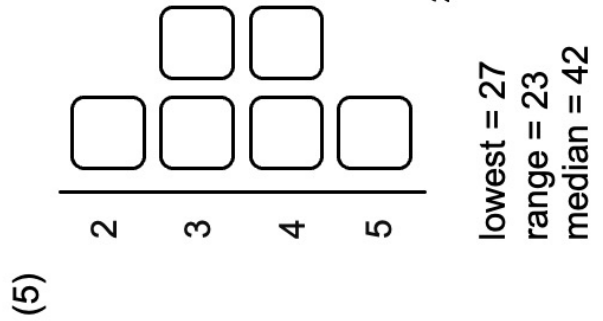
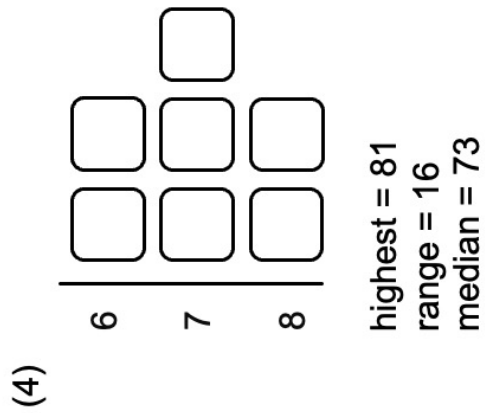
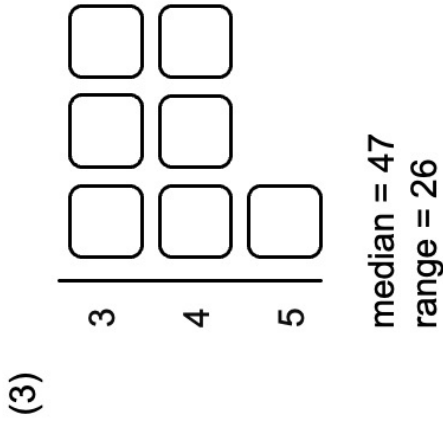


Stem and Leaf Diagram Problems

the digits from 0 to 9 inclusive are used in an ordered stem and leaf diagram:

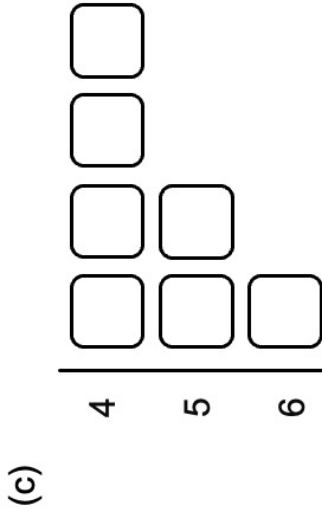
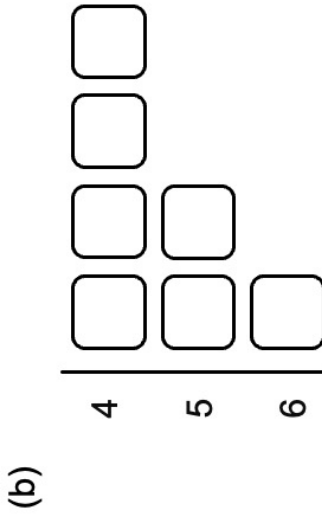
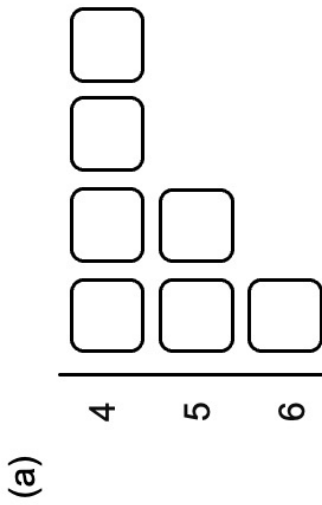


find where the other digits go



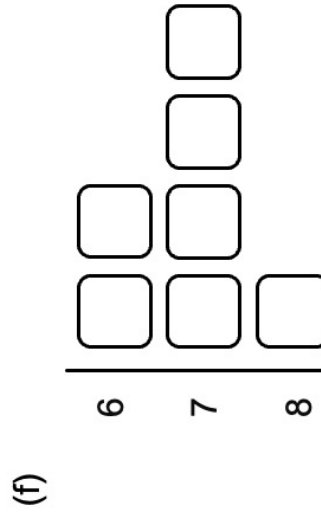
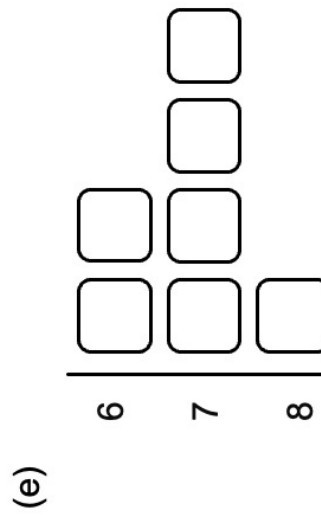
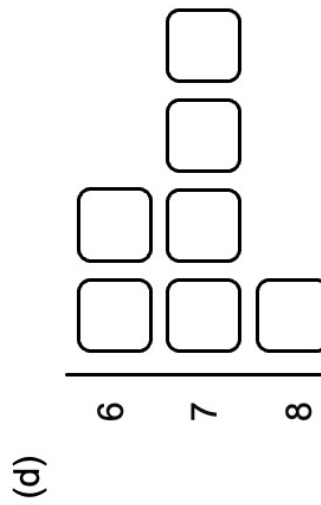
Stem and Leaf Diagram Problems

the digits from 0 to 9 inclusive are used in an ordered stem and leaf diagram:



create your own examples

what are the means of your data sets for (a), (b) and (c)?



what are the means of your data sets for (d), (e) and (f)?

2.4 Ungrouped Frequency Tables

25 packets of sweets were opened. The numbers of sweets in the packets were:

11, 8, 9, 12, 10, 10, 9, 8, 9, 13, 9, 11, 10, 10, 12, 12, 10, 10, 10, 11, 12, 8, 9, 8, 9

Construct a frequency table to show this data:

Number of sweets	Frequency

Fluency Practice

Favourite subject	Frequency
Maths	7
English	7
Science	
History	3
Geography	3
	24

In a survey, 24 students say what their favourite subject is. The results are shown in the frequency table.

- How many students said Geography was their favourite subject?
- Work out how many students said Science was their favourite subject. Add this information to the table.

Items purchased	Frequency
1	
2	3
3	5
4	4
5	3
6	3
7	1
	25

The number of items purchased one day by 25 customers in a shop is shown in the table.

- Six customers bought exactly one item. Add this information to the table.
- How many customers bought exactly three items?
- How many customers bought five or more items?

Score	Frequency
0	0
1	1
2	0
3	4
4	14
5	36
6	48
7	
8	
9	17
10	8
	200

200 students sat a test. The results are shown in the table.

- 42 students scored 7 marks. Add this information to the table.
- How many students scored 8 marks?
- How many students scored *less than* 5 marks?

Fluency Practice

The frequency table shows the number of crisps found in 20 packets of crisps. Billy and Nathan want to work out the total number of crisps in all 20 packets.

Number of crisps	Frequency
14	6
15	7
16	5
17	1
18	1
	20

Billy says:

The total number of crisps is the total of the frequencies, so there were 20 crisps altogether.

Nathan disagrees:

Billy just worked out that there were 20 packets of crisps. The correct answer is $14 + 15 + 16 + 17 + 18 = \underline{80}$ crisps.

Billy is obviously wrong, but so is Nathan's total. Explain why the total number of crisps in the packets is not 80.

Score	Frequency
0	0
1	0
2	0
3	
4	6
5	4
6	0
7	3
8	5
9	2
10	1
	23

Twenty-three students sat a test. Their scores are shown in the table.

A score of 9 or 10 gets an A grade. A score of 7 or 8 gets a B grade, and a score of 5 or 6 gets a C grade.

- How many students scored 3 marks?
- How many students got an A grade?
- How many students got at least a B grade?

2.5 Mode of Ungrouped Data

Worked Example

Determine the modal score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Determine the modal score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Fluency Practice

Items purchased	Frequency
1	6
2	3
3	5
4	4
5	3
6	3
7	1
	25

The number of items purchased one day by 25 customers in a shop is shown in the table. What is the modal number of items purchased?

Score	Frequency
0	0
1	1
2	0
3	4
4	14
5	36
6	48
7	42
8	30
9	17
10	8
	200

200 students sat a test. The results are shown in the table.

What was the modal score achieved by the students?

Animal	Frequency
Cat	7
Dog	6
Hamster	3
Guinea pig	2
Mouse	1
Rabbit	1
Snake	1

Shaun asks everyone in his class how many pets they have, and what type of animal they are. He records the number of pets in the table shown.

- (a) What is the mode?
- (b) There are 28 students in Shaun's class. Explain why the frequencies **don't** add up to 28.

Fluency Practice

The frequency table shows the number of crisps found in 20 packs of crisps.

Kieron looks at the table and says:

The modal number of crisps is 18 because 18 is the biggest number.

Lauren looks at the table and says:

The modal number of crisps is 7 because 7 is the highest frequency.

Kieron and Lauren are wrong. What is the modal number of crisps in a packet?

This table shows the number of goals scored by a hockey team in their first 9 matches of a tournament. In their tenth and final match of the tournament, the team scores 3 goals. What is the modal number of goals scored by the team in the tournament?

Number of goals scored	Frequency
0	3
1	4
2	1
3	0
4	1



Fluency Practice

1)

Colour	Frequency
Red	7
Blue	7
Green	8
Pink	10

Mode = 7
True / False

4)

Colour	Frequency
Red	10
Blue	8
Green	7
Pink	7

Mode = Red
True / False

2)

Colour	Frequency
Red	7
Blue	7
Green	8
Pink	10

Mode = Red and Blue
True / False

5)

Colour	Frequency
Red	10
Blue	10
Green	8
Pink	7

Mode = Red and Blue
True / False

3)

Colour	Frequency
Red	7
Blue	7
Green	8
Pink	10

Mode = Pink
True / False

6)

Colour	Frequency
Red	10
Blue	10
Green	10
Pink	10

Mode = Red, Blue, Green and Pink
True / False

7)

Age	Frequency
2	10
3	10
4	10
5	10

Mode = 10
True / False

10)

Age	Frequency
2	10
3	10
4	3
5	8

Mode = 10
True / False

8)

Age	Frequency
2	10
3	10
4	10
5	10

Mode = no mode
True / False

11)

Age	Frequency
2	10
3	10
4	3
5	8

Mode = 10 and 3
True / False

9)

Age	Frequency
2	10
3	7
4	3
5	8

Mode = 2
True / False

12)

Age	Frequency
2	10
3	10
4	3
5	8

Mode = 2 and 3
True / False

Fluency Practice

13)

Age	Frequency
2	8
3	8
4	10
5	8

Mode = 8

True / False

15)

Age	Frequency
2	2
3	5
4	6
5	8

Mode = 2

True / False

14)

Age	Frequency
2	8
3	8
4	10
5	8

Mode = 4

True / False

16)

Age	Frequency
2	2
3	5
4	6
5	8

Mode = 5

True / False

2.6 Range of Ungrouped Data

Worked Example

Determine the range of the scores:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Determine the range of the scores:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Fluency Practice

Cars sold	Frequency
0	1
1	9
2	3
3	5
4	4
5	3
6	3
7	2
	30

Information about the number of cars sold by a dealer each day in June is shown in the table. What is the range in number of cars sold?

Score	Frequency
0	0
1	1
2	0
3	4
4	14
5	36
6	48
7	42
8	30
9	17
10	8
	200

200 students sat a test. The results are shown in the table.

What was the range of scores achieved by the students?

Fluency Practice

The frequency table shows the number of crisps found in 20 packs of crisps.

Number of crisps	Frequency
14	6
15	7
16	5
17	1
18	1
	20

Micah looks at the table and says:

The range in number of crisps is 19 because 20 is the highest and 1 is the lowest.

Niamh disagrees with Micah and says:

The range is 6 because 7 is the highest and 1 is the lowest. You can't use 20 because that's just a total.

Both Micah and Niamh haven't found the correct range in number of crisps. What is the correct range?

This table shows the number of goals scored by a hockey team in their first 9 matches of a tournament. In their tenth and final match of the tournament, the team scores 3 goals. What is the range in number of goals scored by the team in the tournament?

Number of goals scored	Frequency
0	3
1	4
2	1
3	0
4	1



2.7 Median of Ungrouped Data

Fluency Practice

Number
of pieces
of data:

Position
of the
median:

(a) 4, 10, 11, 12, 12, 15, 20



(b) 4, 10, 11, 12, 12, 15



(c) 10, 11, 12, 12, 15



(d) 10, 11, 12, 12



(e) 1, 3, 6, 8, 9, 12



Number of pieces of data:	Position of the median:
7	
11	
10	
41	
24	
	8
	3.5
	40
	21.5

Fluency Practice

Number of pets	Frequency	Which pieces of data are in this category?
0	3	1 st 2 nd 3 rd
1	2	4 th 5 th
2	4	

Number of pets	Frequency	Which pieces of data are in this category?
0	5	
1	1	
2	3	

Number of pets	Frequency	Which pieces of data are in this category?
0	2	
1	1	
2	5	

Number of pets	Frequency	Which pieces of data are in this category?
0	1	
1	3	
2	3	

Fluency Practice

Number of pets	Frequency	Which pieces of data are in this category?
0		1 st 2 nd
1		3 rd
2		4 th 5 th 6 th 7 th 8 th
3		9 th 10 th
4		11 th 12 th 13 th

Number of pets	Frequency	Which pieces of data are in this category?
0		1 st
1		2 nd 3 rd 4 th
2		5 th 6 th 7 th 8 th
3		9 th 10 th
4		11 th 12 th

Number of pets	Frequency	Which pieces of data are in this category?
0	21	1 st to 21 st
1	15	22 nd to...
2	18	
3	25	
4	32	

Number of pets	Frequency	Which pieces of data are in this category?
0	10	
1	12	
2	15	
3	20	
4	5	

Fluency Practice

Number of pets	Frequency	Which pieces of data are in this category?
0	8	
1	9	
2	13	
3	12	
4	9	

Number of pets	Frequency	Which pieces of data are in this category?
0		1 st to 13 th
1		14 th to 29 th
2		30 th to 59 th
3		60 th to 80 th
4		81 st to 92 nd

Worked Example

Calculate the median score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Calculate the median score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Worked Example

Calculate the median score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	7

Your Turn

Calculate the median score:

Score	Frequency
0	9
1	6
2	2
3	4
4	4
5	8

Fluency Practice

Question 1: Work out the median from each of the frequency tables.

(a)

Age	Frequency
18	2
19	3
20	13
21	1

(b)

Shoe Size	Frequency
5	2
6	11
7	5
8	4
9	1

(c)

Number of TVs	Frequency
0	3
1	15
2	9
3	11
4	1

(d)

Days absent	Frequency
0	31
1	8
2	3
3	4
4	1
5	3

(e)

Age	Frequency
5	12
6	20
7	23
8	65

(f)

Goals Scored	Frequency
0	2
1	4
2	5
3	8
4	0
5	1

Extension

Question 1: There are 30 students in a class.
Miss Williamson knows that the median shoe size is 5.
Fill in the frequency table with two possible values.

Shoe Size	Frequency
4	4
5	
6	
7	10

Question 2: The frequency table shows the piano grade of 17 students in a class.

Grade	Frequency
2	3
3	3
4	4
5	3
6	2
7	2

3 new students, who are all Grade 6, join the class.
The teacher says the median piano grade will increase.
Is she correct?

2.8 Mean of Ungrouped Data

Worked Example

Calculate the mean score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Calculate the mean score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Fluency Practice

Question 1: Work out the mean for each of these frequency tables.
You may not use a calculator

(a)

Age	Frequency
5	2
6	2
7	5
8	1

(b)

Number of phones	Frequency
0	1
1	3
2	2
3	0
4	4
5	0

(c)

Number of pets	Frequency
0	13
1	28
2	50
3	9

(d)

Money Withdrawn	Frequency
£10	16
£20	19
£30	4
£40	3
£50	6
£60	2

(e)

Number of bedrooms	Frequency
1	34
2	275
3	512
4	179

(f)

Level	Frequency
3	1
4	9
5	7
6	2
7	1

Question 2: Work out the mean for each of these frequency tables.
You may use a calculator

(a)

Age	Frequency
16	28
17	7
18	3
19	2

(b)

Grade	Frequency
3	16
4	27
5	45
6	49
7	50
8	13

(c)

Siblings	Frequency
0	71
1	25
2	14

(d)

Pocket Money	Frequency
£1	5
£2	34
£3	86
£4	19
£5	3
£6	3

(e)

Star rating	Frequency
0	9
1	12
2	17
3	19
4	21
5	8

(f)

Times visited	Frequency
0	131
1	873
2	599
3	205

Extension

Question 1: A teacher asked his class how long they spent revising for a test, to the nearest hour. By calculating the mean, compare the amount of time the boys and girls spent revising

Boys

Hours	Frequency
0	0
1	2
2	3
3	4
4	5
5	1

Girls

Hours	Frequency
0	2
1	7
2	2
3	2
4	1
5	1

Question 2: Aidan plays 50 games in an arcade. The table shows how many tickets he won in each game.

- (a) Work out the missing frequency
- (b) Work out the total number of tickets won
- (c) Work out the mean number of tickets won per game.

Tickets won	Frequency
0	4
1	3
2	5
3	
4	11
5	6
6	10
7	2
8	3

Aidan wants to exchange his ticket for a prize that costs 800 tickets.

- (d) How many more games do you expect Aidan would have to play?

Question 3: Max rolls a dice 80 times. The table shows the results.

- (a) Find the value of x
- (b) Work out the mean score

Number	Frequency
1	4
2	6
3	$x + 5$
4	x
5	$2x$
6	5

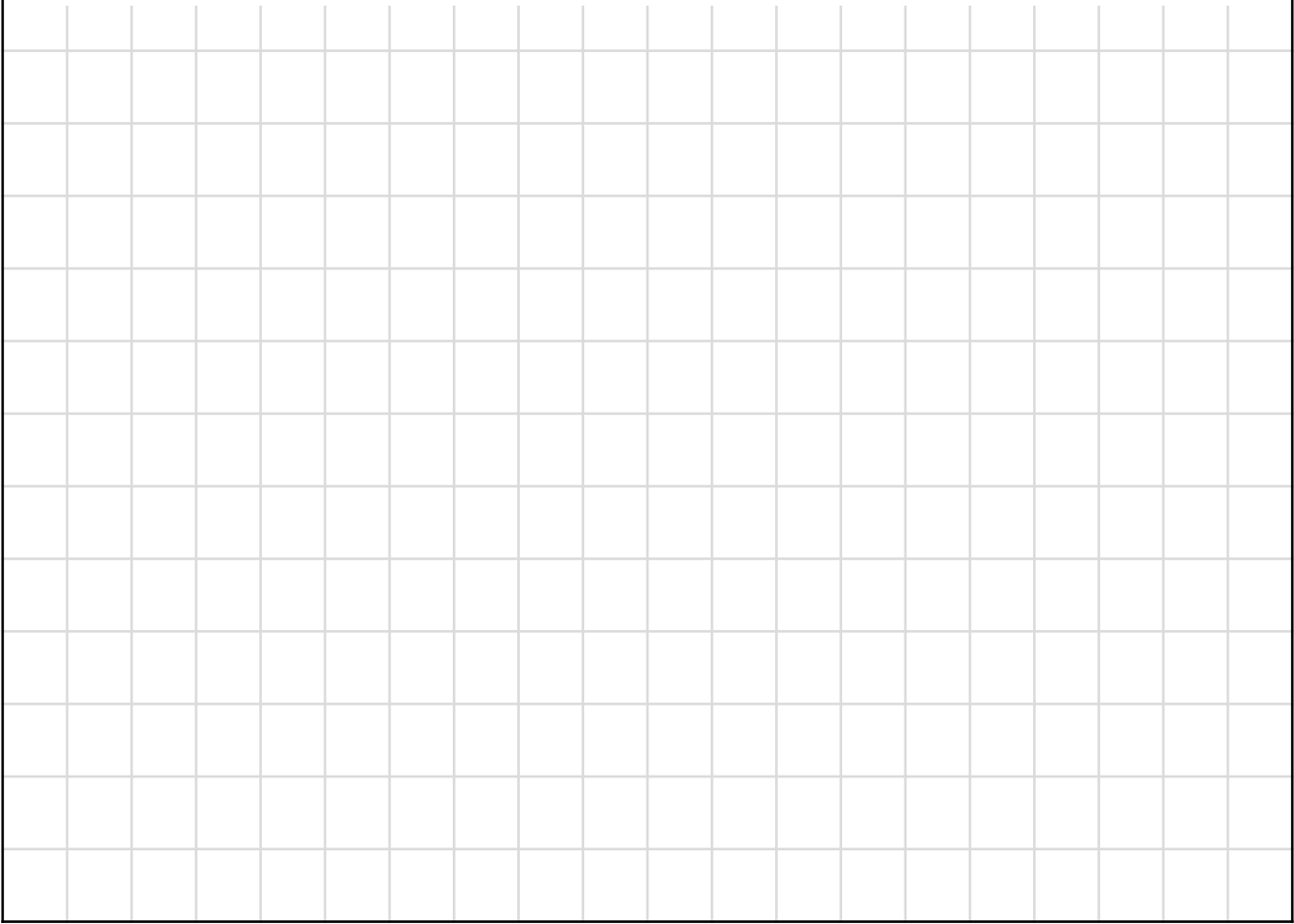
2.9 Review and Problem Solving

Worked Example

The table gives information about the numbers of badges gained by the girls in a Guide group.

- a) Write down the mode.
- b) Find the range.
- c) Work out the median
- d) Calculate the mean.

Number of badges	Frequency
0	2
1	8
2	4
3	3
4	5
5	3



Fluency Practice

Vicky counts the number of birds in her garden at 5 pm on each of 20 days. She records the information in a frequency table.

Number of birds	Frequency
0	3
1	2
2	3
3	4
4	5
5	3

Marta asked some students how many cans of drink they each drank yesterday. The table shows her results.

Number of cans	Frequency
0	6
1	9
2	7
3	3
4	2
5	1

Jim asked each person in his class how many cars their family have. The frequency table shows the results.

Number of cars	Frequency
0	2
1	12
2	8
3	6
4	2

Chris works in a cafe. At noon one day, he records the number of customers sitting at each table in the cafe. Here are his results.

Number of customers sitting at a table	Number of tables
0	4
1	5
2	10
3	7
4	3
5	1

Extension

9

Age	Frequency
5	3
6	
7	
8	3
Total	20

Mean = 6.5

Mode =

Median =

Range =

11

Age	Frequency
5	
6	
7	6
8	
Total	20

Mean =

Mode = 5 and 6

Median =

Range =

10

Age	Frequency
5	3
6	2
7	
8	
Total	20

Mean =

Mode =

Median = 7.5

Range =

12

Age	Frequency
5	
6	
7	
8	
Total	20

Mean = 6.75

Mode =

Median =

Range = 1

2.10 Grouped Frequency Tables

80 people take part in a survey. Their ages are shown in the frequency table. How many respondents are in their thirties?

Age range	Frequency
$20 \leq \text{age} < 30$	8
$30 \leq \text{age} < 40$	
$40 \leq \text{age} < 50$	12
$50 \leq \text{age} < 60$	16
$60 \leq \text{age} < 70$	11
$70 \leq \text{age} < 80$	10
$80 \leq \text{age} < 90$	9
	80

Fluency Practice

Height, h cm	Frequency
$120 \leq h < 130$	1
$130 \leq h < 140$	4
$140 \leq h < 150$	
$150 \leq h < 160$	16
$160 \leq h < 170$	20
$170 \leq h < 180$	23
$180 \leq h < 190$	8
	80

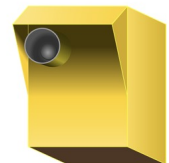
The heights of 80 people are measured. The results are shown in the table.

- Eight people have a height that is at least 140 cm but less than 150 cm. Add this information to the table.
- How many people measured were at least 180 cm tall?
- How many people measure were under 120 cm tall?

Speed, s mph	Frequency
$20 < s \leq 25$	5
$25 < s \leq 30$	8
$30 < s \leq 35$	7
$35 < s \leq 40$	4
$40 < s \leq 45$	1
	25

The speeds of 25 drivers in a 30 mph zone were measured and recorded in the table shown.

- How many drivers were not exceeding the speed limit?
- How many drivers exceeded the speed limit by over 10 mph?



Bill total, £ x	Frequency
$0 < x \leq 5$	19
$5 < x \leq 10$	33
$10 < x \leq 15$	27
$15 < x \leq 20$	17
$20 < x \leq 25$	4
	100

The table shows information about the shopping bills of 100 customers at a shop one day.

How many of the customers spent more than £10?



2.11 Mode of Grouped Data

Worked Example

Determine the modal class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 20$	3
$20 < x \leq 40$	2
$40 < x \leq 46$	6
$46 < x \leq 50$	7

Your Turn

Determine the modal class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	15
$10 < x \leq 20$	6
$20 < x \leq 40$	4
$40 < x \leq 46$	12
$46 < x \leq 50$	8

Fluency Practice

80 people take part in a survey. Their ages are shown in the frequency table. What is the **modal class**?

Age range	Frequency
$20 \leq \text{age} < 30$	8
$30 \leq \text{age} < 40$	14
$40 \leq \text{age} < 50$	12
$50 \leq \text{age} < 60$	16
$60 \leq \text{age} < 70$	11
$70 \leq \text{age} < 80$	10
$80 \leq \text{age} < 90$	9
	80

Height, h cm	Frequency
$120 \leq h < 130$	1
$130 \leq h < 140$	4
$140 \leq h < 150$	8
$150 \leq h < 160$	16
$160 \leq h < 170$	20
$170 \leq h < 180$	23
$180 \leq h < 190$	8
	80

The heights of 80 people are measured. The results are shown in the table.

What is the modal class?

Bill total, £ x	Frequency
$0 < x \leq 5$	5
$5 < x \leq 10$	8
$10 < x \leq 15$	7
$15 < x \leq 20$	4
$20 < x \leq 25$	1
	25

The table shows information about the shopping bills of 25 customers at a shop one day. What is the modal class?



2.12 Range of Grouped Data

Worked Example

Determine the upper and lower bounds for the range:

Mass, x (kg)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 20$	3
$20 < x \leq 40$	2
$40 < x \leq 46$	6
$46 < x \leq 50$	7

Your Turn

Determine the upper and lower bounds for the range:

Mass, x (kg)	Frequency
$10 < x \leq 20$	5
$20 < x \leq 30$	3
$30 < x \leq 50$	2
$50 < x \leq 56$	6
$56 < x \leq 60$	7

Fluency Practice

Scientists measured the lengths of 80 turtles on a beach. Their lengths are shown in the frequency table. Find upper and lower bounds for the range of lengths.

Length, cm	Frequency
$20 \leq \text{length} < 30$	8
$30 \leq \text{length} < 40$	14
$40 \leq \text{length} < 50$	12
$50 \leq \text{length} < 60$	16
$60 \leq \text{length} < 70$	11
$70 \leq \text{length} < 80$	10
$80 \leq \text{length} < 90$	9
	80

Height, h cm	Frequency
$120 \leq h < 130$	1
$130 \leq h < 140$	4
$140 \leq h < 150$	8
$150 \leq h < 160$	16
$160 \leq h < 170$	20
$170 \leq h < 180$	23
$180 \leq h < 190$	8
	80

The heights of 80 people are measured. The results are shown in the table.

Find upper and lower bounds for the range of heights.

Time spent in the shop	Frequency
$0 < x \leq 5$	5
$5 < x \leq 10$	8
$10 < x \leq 15$	7
$15 < x \leq 20$	4
$20 < x \leq 25$	1
	25

The table shows information about the time spent by 25 people in a shop one day. Find the upper bound and lower bound for the range of times spent in the shop.



2.13 Median Class of Grouped Data

Worked Example

Determine the median class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 20$	3
$20 < x \leq 40$	2
$40 < x \leq 46$	6
$46 < x \leq 50$	7

Your Turn

Determine the median class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	15
$10 < x \leq 20$	6
$20 < x \leq 40$	4
$40 < x \leq 46$	12
$46 < x \leq 50$	8

Fluency Practice

Question 2: Work out which class interval contains the median for each table below.

(a)

Time taken	Frequency
$0 < t \leq 5$	5
$5 < t \leq 10$	14
$10 < t \leq 15$	10
$15 < t \leq 20$	1

(b)

Lifetime (months)	Frequency
$0 < t \leq 12$	1
$12 < t \leq 24$	9
$24 < t \leq 36$	13
$36 < t \leq 48$	56
$48 < t \leq 60$	21

(c)

Mass (Kg)	Frequency
$50 < m \leq 60$	41
$60 < m \leq 70$	39
$70 < m \leq 80$	28
$80 < m \leq 90$	6
$90 < m \leq 100$	2

(d)

Mass (Kg)	Frequency
$0 < m \leq 100$	123
$100 < m \leq 200$	290
$200 < m \leq 300$	2009
$300 < m \leq 400$	1817
$400 < m \leq 500$	584
$500 < m \leq 600$	177

2.14 Midpoint of Two Numbers

Worked Example

Numbers	Midpoint
40 and 60	

Your Turn

Numbers	Midpoint
40 and 70	

Intelligent Practice

Numbers	Midpoint
11. 142 and 194	
12. 14.2 and 19.4	
13. 7.1 and 9.7	
14. 7 and 9.6	
15. -9.6 and -7	
16. -9.9 and -7	
17. -9.9 and -6.9	
18. -6.9 and 9.9	
19. $-6\frac{3}{4}$ and $9\frac{3}{4}$	
20. $-6\frac{3}{5}$ and $9\frac{3}{4}$	

Numbers	Midpoint
1. 8 and 10	
2. 7 and 11	
3. 2 and 16	
4. 22 and 36	
5. 22 and 46	
6. 22 and 47	
7. 22 and 48	
8. 21 and 48	
9. 21 and 47	
10. 42 and 94	

2.15 Estimated Mean of Grouped Data

Worked Example

Calculate an estimate for the mean:

Mass, x (kg)	Frequency
$0 < x \leq 8$	3
$8 < x \leq 16$	6
$16 < x \leq 24$	7
$24 < x \leq 32$	4

Your Turn

Calculate an estimate for the mean:

Mass, x (kg)	Frequency
$0 < x \leq 8$	3
$8 < x \leq 16$	0
$16 < x \leq 24$	7
$24 < x \leq 32$	4

Fluency Practice

Question 1: Work out an estimate of the mean for each of these frequency tables.

(a)

Length	Frequency	Midpoint	
$0 < L \leq 10$	6		
$10 < L \leq 20$	7		
$20 < L \leq 30$	5		
$30 < L \leq 40$	1		
$40 < L \leq 50$	1		

(b)

Cost	Frequency	Midpoint	
$0 < c \leq 4$	2		
$4 < c \leq 8$	3		
$8 < c \leq 12$	5		
$12 < c \leq 16$	12		
$16 < c \leq 20$	3		

(c)

Length	Frequency	Midpoint	
$0 < t \leq 5$	11		
$5 < t \leq 10$	37		
$10 < t \leq 15$	43		
$15 < t \leq 20$	9		

(d)

Mass	Frequency	Midpoint	
$50 < m \leq 55$	3		
$55 < m \leq 60$	5		
$60 < m \leq 65$	10		
$65 < m \leq 70$	12		
$70 < m \leq 75$	10		

Question 2: Work out an estimate of the mean for each of these frequency tables.

(a)

Duration (years)	Frequency
$0 \leq d < 10$	9
$10 \leq d < 20$	13
$20 \leq d < 30$	16
$30 \leq d < 40$	2

(b)

Length (cm)	Frequency
$0 \leq L < 30$	8
$30 \leq L < 60$	43
$60 \leq L < 90$	25
$90 \leq L < 120$	4

(c)

Mass	Frequency
$20 < m \leq 25$	12
$25 < m \leq 30$	24
$30 < m \leq 35$	17
$35 < m \leq 40$	15
$40 < m \leq 45$	4

(d)

Height	Frequency
$120 < h \leq 130$	51
$130 < h \leq 140$	120
$140 < h \leq 150$	66
$150 < h \leq 160$	59
$160 < h \leq 170$	4

Intelligent Practice

Mass, x (kg)	Frequency
$0 < x \leq 10$	1
$10 < x \leq 20$	2
$20 < x \leq 30$	4
$30 < x \leq 40$	3

1.

Mass, x (kg)	Frequency
$0 < x \leq 10$	2
$10 < x \leq 20$	4
$20 < x \leq 30$	8
$30 < x \leq 40$	6

2.

Mass, x (kg)	Frequency
$0 < x \leq 20$	2
$20 < x \leq 40$	4
$40 < x \leq 60$	8
$60 < x \leq 80$	6

3.

Mass, x (kg)	Frequency
$0 < x \leq 20$	6
$20 < x \leq 40$	8
$40 < x \leq 60$	4
$60 < x \leq 80$	2

4.

Mass, x (kg)	Frequency
$0 < x \leq 20$	6
$20 < x \leq 40$	8
$40 < x \leq 60$	4
$60 < x \leq 80$	20

5.

Mass, x (kg)	Frequency
$0 < x \leq 20$	6
$20 < x \leq 40$	8
$40 < x \leq 60$	20
$60 < x \leq 80$	20

6.

Mass, x (kg)	Frequency
$0 < x \leq 20$	6
$20 < x \leq 40$	8
$40 < x \leq 60$	0
$60 < x \leq 80$	20

7.

Mass, x (kg)	Frequency
$1 < x \leq 21$	6
$21 < x \leq 41$	8
$41 < x \leq 61$	0
$61 < x \leq 81$	20

8.

Mass, x (kg)	Frequency
$1 < x \leq 21$	6
$21 < x \leq 41$	8
$41 < x \leq 61$	0
$61 < x \leq 66$	5
$66 < x \leq 81$	15

9.

Mass, x (kg)	Frequency
$1 < x \leq 21$	6
$21 < x \leq 41$	8
$41 < x \leq 81$	20

10.

Extension

Question 1: Sally is raising money for charity for a fun run.
The table below has been given to her from the website.

Sally says the average donation is £10.
By calculating the estimated mean, decide if you agree with Sally.

Donation	Frequency
$0 < d \leq 5$	44
$5 < d \leq 10$	35
$10 < d \leq 20$	16
$20 < d \leq 50$	3
$50 < d \leq 100$	2

Question 2: Nathan delivers pizzas.
The table below shows information about his delivery times.
The pizza company has a promotion that if the delivery time is over 30 minutes, the customer gets their meal for free

(a) Calculate an estimate for the mean delivery time

(b) What percentage of deliveries took over 30 minutes?

Nathan's manager thinks that the promotion should be changed to 40 minutes

Delivery Time	Frequency
$0 < t \leq 10$	3
$10 < t \leq 20$	10
$20 < t \leq 30$	14
$30 < t \leq 40$	19
$40 < t \leq 50$	4

(c) Do you agree? Explain your answer.

Question 3: The manager of a small company is calculating the mean salary for his workers.
He has calculated this to be £568,500 per year.
Can you spot any mistakes?

Salary	Frequency	Midpoint	fx
$0 < s \leq 15000$	2	7500	15000
$15000 < s \leq 30000$	15	22500	337500
$30000 < s \leq 45000$	6	37500	2250000
$45000 < s \leq 60000$	2	52500	105000
$60000 < s \leq 100000$	2	67500	135000

2842500

$$\text{Mean salary} = 2842500 \div 5 = \text{£}568500$$

2.16 Review and Problem Solving

Worked Example

Bob asked each of 40 friends how many minutes they took to get to work. The table shows some information about his results.

- Write down the modal class.
- Work out the upper and lower bounds for the range.
- Work out the class in which the median lies.
- Calculate an estimate for the mean.

Time taken (m minutes)	Frequency
$0 < m \leq 10$	3
$10 < m \leq 20$	8
$20 < m \leq 30$	11
$30 < m \leq 40$	9
$40 < m \leq 50$	9



Fluency Practice

The table gives some information about the lengths of time, in hours, that some adults watched TV last week.

Length of time (t hours)	Frequency
$0 \leq t < 10$	8
$10 \leq t < 15$	15
$15 \leq t < 20$	11
$20 \leq t < 30$	10
$30 \leq t < 50$	6

The table shows some information about the prices of 64 second-hand cars that are for sale.

Price (£ x)	Frequency
$0 < x \leq 2000$	8
$2000 < x \leq 4000$	14
$4000 < x \leq 6000$	28
$6000 < x \leq 8000$	10
$8000 < x \leq 10000$	4

The table shows some information about the times, in minutes, 60 people took to get to work.

Time (x minutes)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 30$	11
$30 < x \leq 50$	23
$50 < x \leq 80$	13
$80 < x \leq 100$	8

The table shows information about the ages of 90 employees in a factory.

Age (a years)	Frequency
$15 < a \leq 25$	12
$25 < a \leq 35$	27
$35 < a \leq 45$	18
$45 < a \leq 55$	23
$55 < a \leq 65$	10

Tomato Plant Heights

tomato plant heights

the heights of 50 tomato plants were measured in metres, to the nearest cm:



1.52	2.05	2.39	2.14	1.84	1.65	1.91	2.34	1.04	2.95
1.72	2.28	2.32	2.00	2.11	1.66	1.74	1.97	2.21	1.43
2.08	1.76	2.68	1.91	2.07	1.85	2.19	2.14	1.99	1.57
2.06	2.45	1.82	1.11	2.68	1.86	2.19	1.56	2.78	1.23
2.83	2.01	2.44	2.04	2.63	1.90	2.21	1.37	2.57	2.54

how could you organise the data into groups to get an idea of the shape of the data?

10 groups could have these intervals

$1.0 < h \leq 1.2$
 $1.2 < h \leq 1.4$
 $1.4 < h \leq 1.6$
 $1.6 < h \leq 1.8$
 $1.8 < h \leq 2.0$
 $2.0 < h \leq 2.2$
 $2.2 < h \leq 2.4$
 $2.4 < h \leq 2.6$
 $2.6 < h \leq 2.8$
 $2.8 < h \leq 3.0$

7 groups could have these intervals

$1.0 < h \leq 1.3$
 $1.3 < h \leq 1.6$
 $1.6 < h \leq 1.9$
 $1.9 < h \leq 2.2$
 $2.2 < h \leq 2.5$
 $2.5 < h \leq 2.8$
 $2.8 < h \leq 3.1$

5 groups could have these intervals

$1.0 < h \leq 1.4$
 $1.4 < h \leq 1.8$
 $1.8 < h \leq 2.2$
 $2.2 < h \leq 2.6$
 $2.6 < h \leq 3.0$

work out an estimate for the mean by grouping the data into

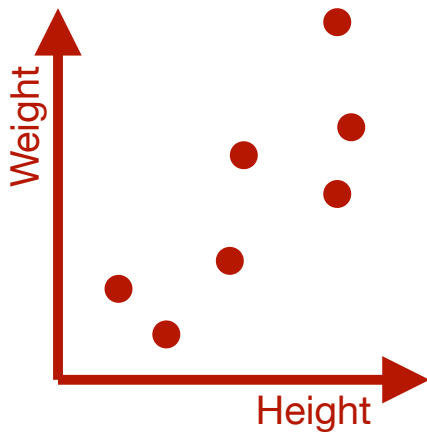
(i) 10 groups (ii) 7 groups and (iii) 5 groups

compare the estimate of the mean with the actual mean: $101.59 \div 50 = 2.0318$

comment on the accuracy of the estimates

3 Scatter Graphs

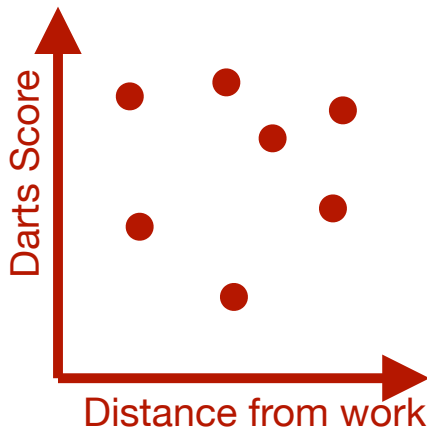
Scatter Graphs can show a relationship between two variables.



...such as people's height and weight.



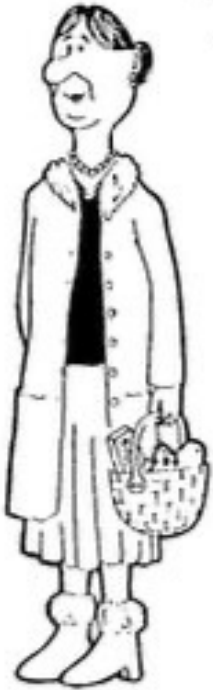
...or the number of staff working in KFC and the wait time for food.



...or the distance people live from work and their best score in darts.

Fluency Practice

Who is represented by each point on the scattergraph, below?



Alice



Brenda



Cathy



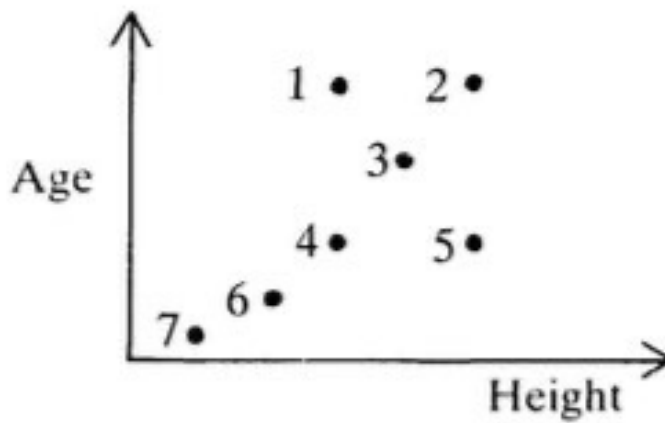
Dennis



Errol



Freda Gavin

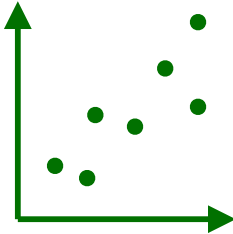


3.1 Correlation

If the two variables have a relationship we call it **correlation**.

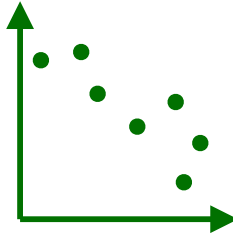
There are different types of **correlation**:

Positive correlation:



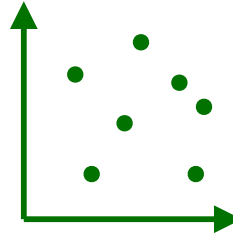
As one value goes up, so does the other.

Negative correlation:



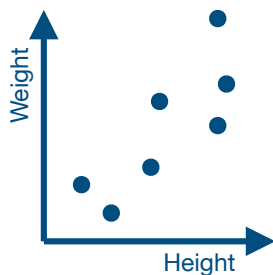
As one value goes up, the other goes down.

No correlation:



There is no obvious relationship.

Sometimes you might be asked to explain the correlation **in context**.

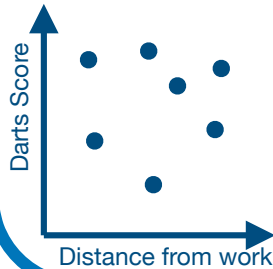


This means describing what is actually happening. eg:

“Taller people are usually heavier.”



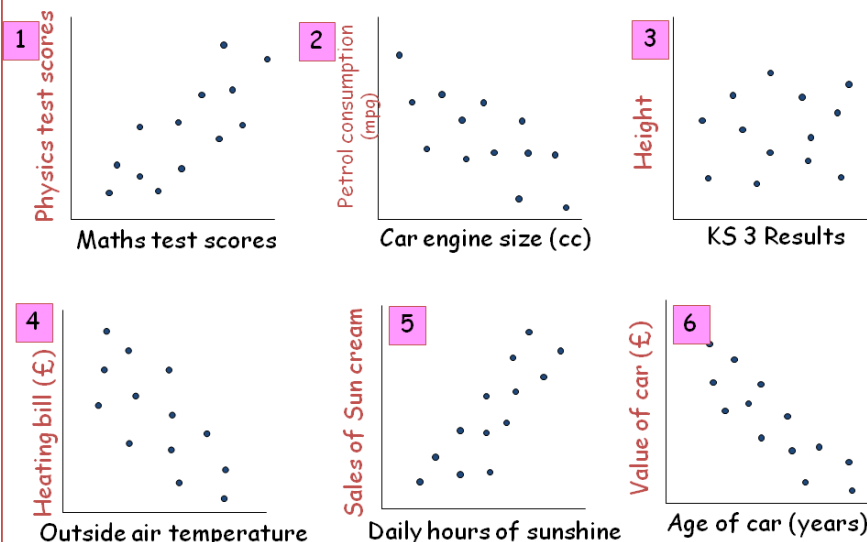
“When there are more staff working, you wait less.”



“There is no relationship between how far people live from work and their darts ability.”

Fluency Practice

State the type of **correlation** for the scatter graphs below and write a sentence describing the relationship in each case.



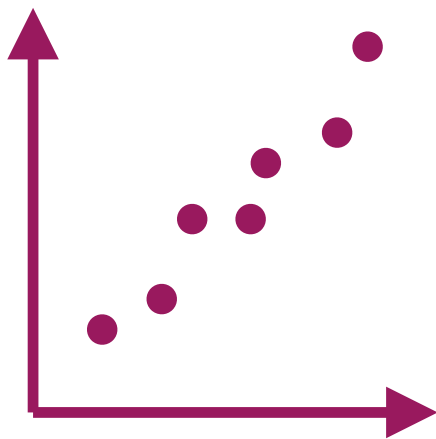
Complete the sentences using **positive/negative/no** and then **increase/decrease/not affected**.

1. There is a correlation between Physics and Maths test scores. As the Maths test results increase the Physics test results
2. There is a correlation between car engine size and petrol consumption.. As the car engine size increases the petrol consumption
3. There is correlation between KS3 results and height. As the KS3 results increase the height of the person is
4. There is a correlation between outside air temperature and the heating bill. As the air temperature increases the heating bill
5. There is a correlation between the daily hours of sunshine and sales of sun cream. As the hours of sunshine increase sales of sun cream
6. There is a correlation between the age of a car and its value. As the car gets older its value

3.2 Correlation Strength

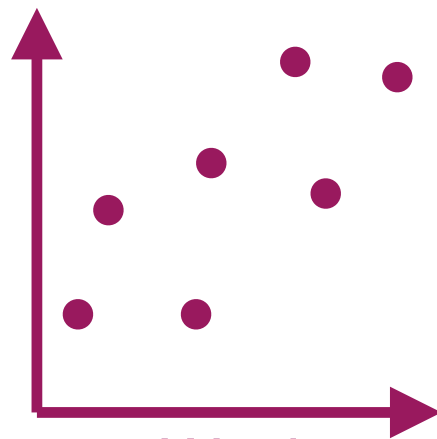
Correlation can be strong or weak.

If the correlation is strong, all the points will closely follow a straight line.



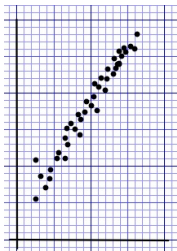
Strong
correlation

If the correlation is weak, the points will follow the line more loosely.

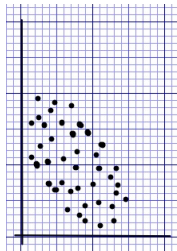


Weak
correlation

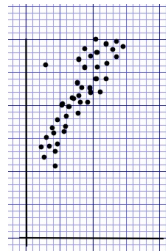
Fluency Practice



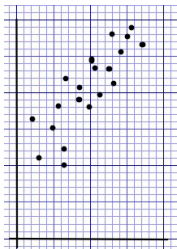
Strong
Moderate
Weak



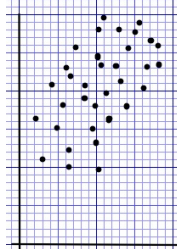
Strong
Moderate
Weak



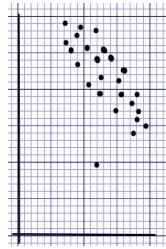
Strong
Moderate
Weak



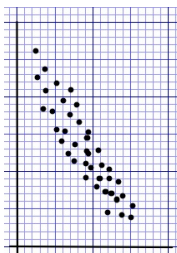
Strong
Moderate
Weak



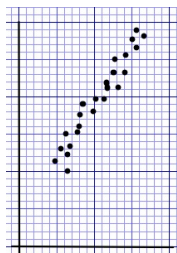
Strong
Moderate
Weak



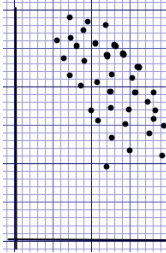
Strong
Moderate
Weak



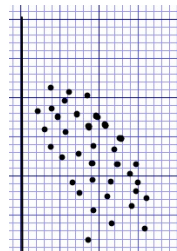
Strong
Moderate
Weak



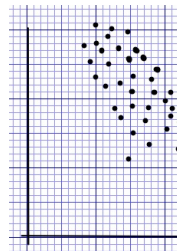
Strong
Moderate
Weak



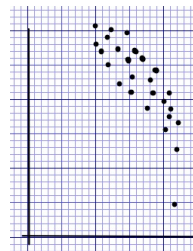
Strong
Moderate
Weak



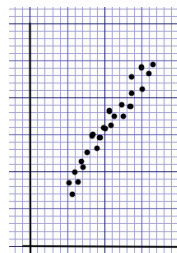
Strong
Moderate
Weak



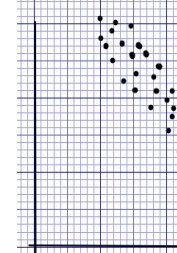
Strong
Moderate
Weak



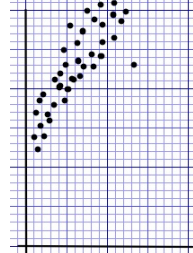
Strong
Moderate
Weak



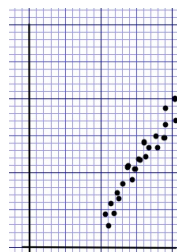
Strong
Moderate
Weak



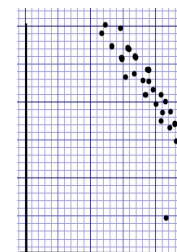
Strong
Moderate
Weak



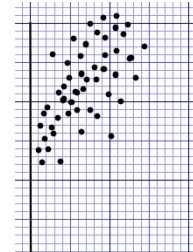
Strong
Moderate
Weak



Strong
Moderate
Weak



Strong
Moderate
Weak

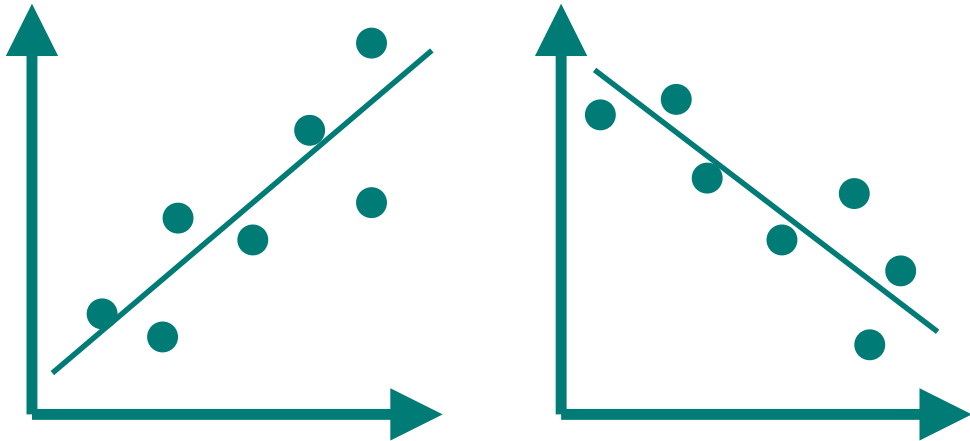


Strong
Moderate
Weak

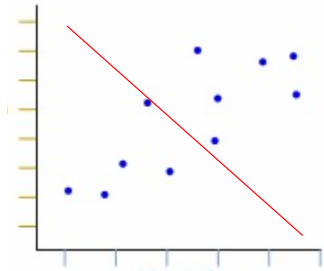
3.3 Line of Best Fit

We can show the correlation more clearly by drawing a **Line of Best Fit**.

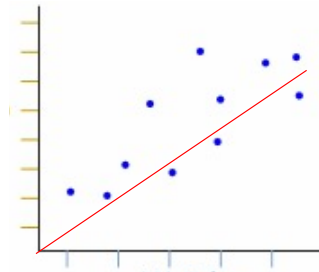
This should pass through the middle of all the points (but does not have to touch any of the points).



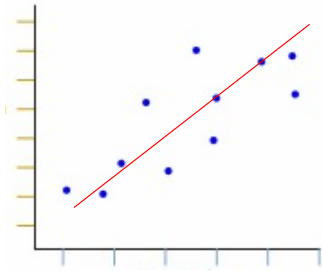
Fluency Practice



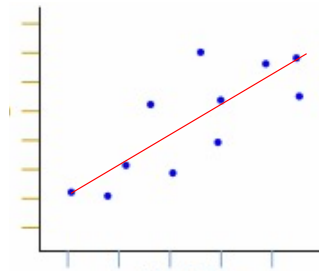
Yes / No



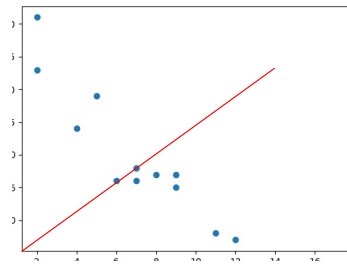
Yes / No



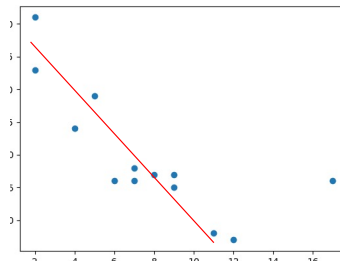
Yes / No



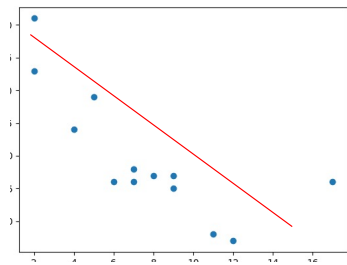
Yes / No



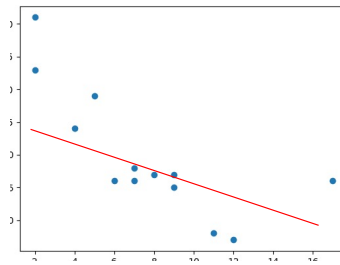
Yes / No



Yes / No



Yes / No



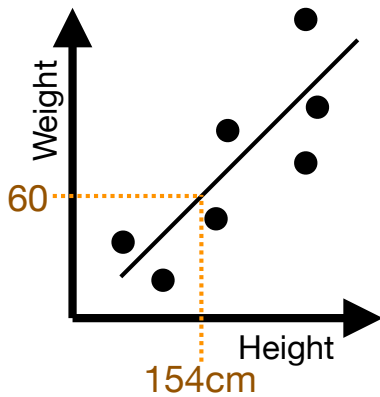
Yes / No

3.4 Drawing and Interpreting Scatter Graphs

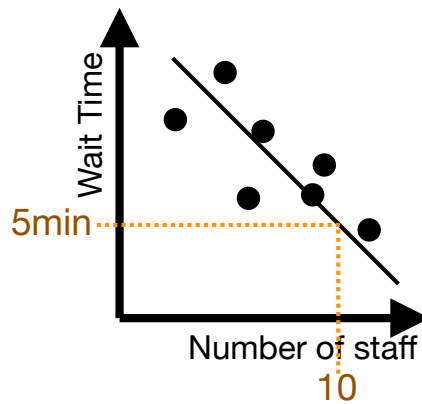
We can use the **Line of Best Fit** to make predictions of other results.

For example, we can estimate:

...someone's height if we know their weight is 60kg.

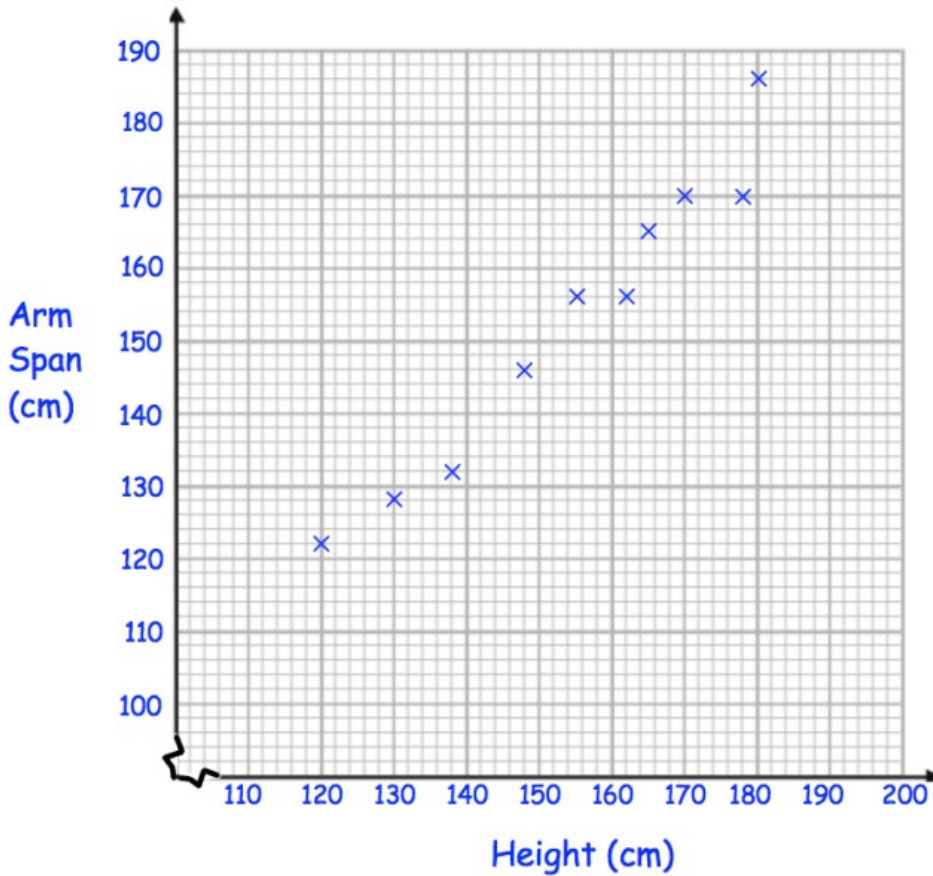


...or the wait time in KFC if we know they have 10 staff on today.



Worked Example

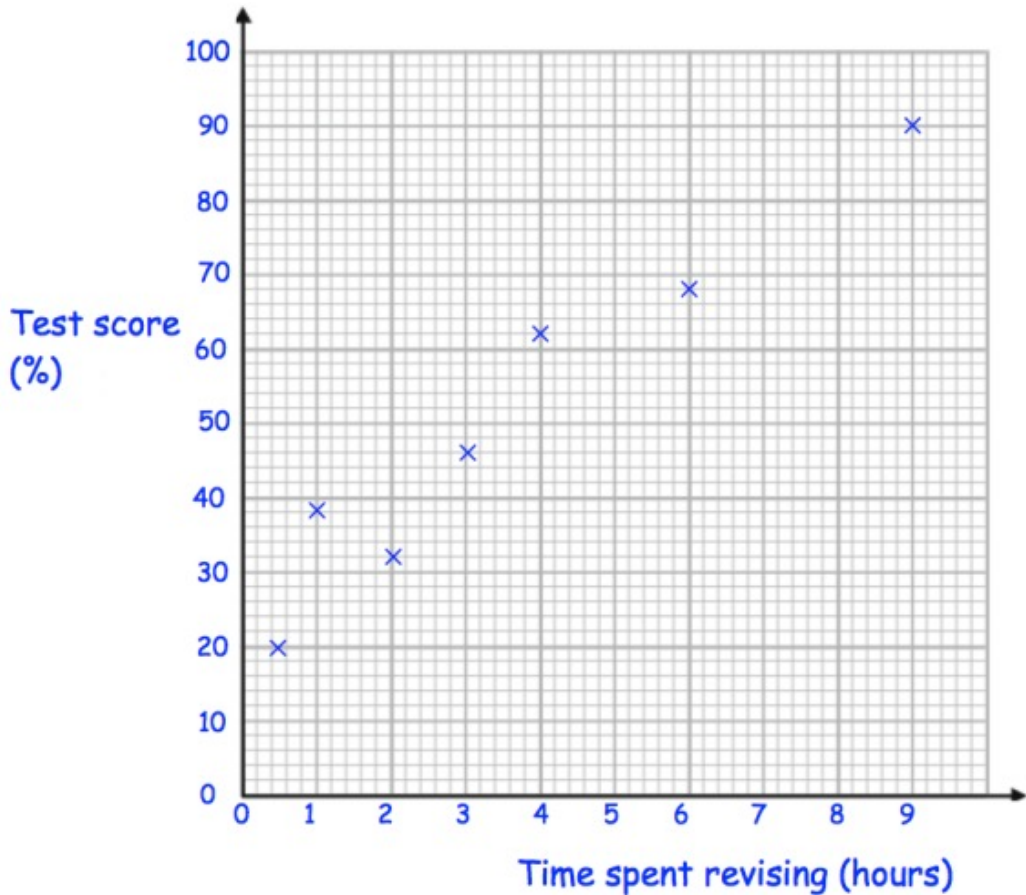
The scatter graph shows the height and arm span of ten students.



- Describe the correlation.
- Another student is 174 *cm* tall and has an arm span of 180 *cm*. Plot this on the graph.
- Another student is 142 *cm* tall. Estimate the arm span of this student.

Your Turn

The scatter graph shows the time spent revising and the test score of seven students.

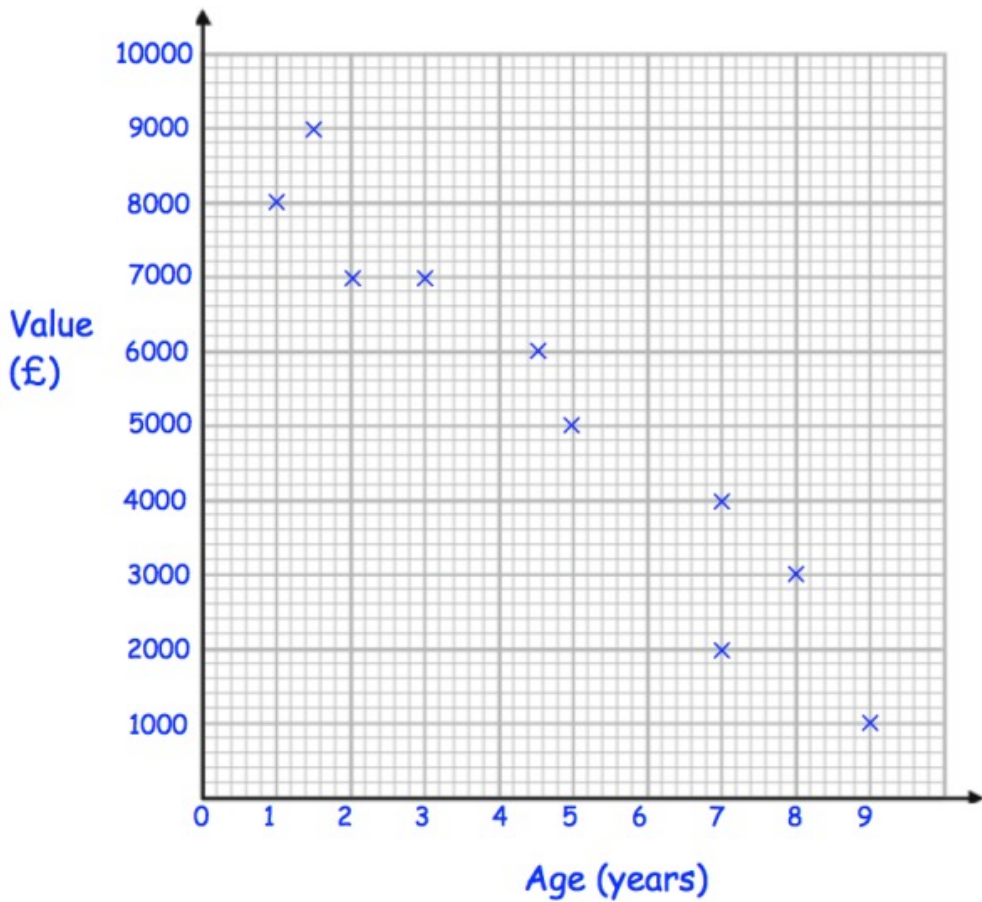


- Describe the correlation.
- Another student revised for 8 hours and scored 98%. Plot this on the graph.
- Another student achieved a test score of 80%. Estimate the time they spent revising.



Worked Example

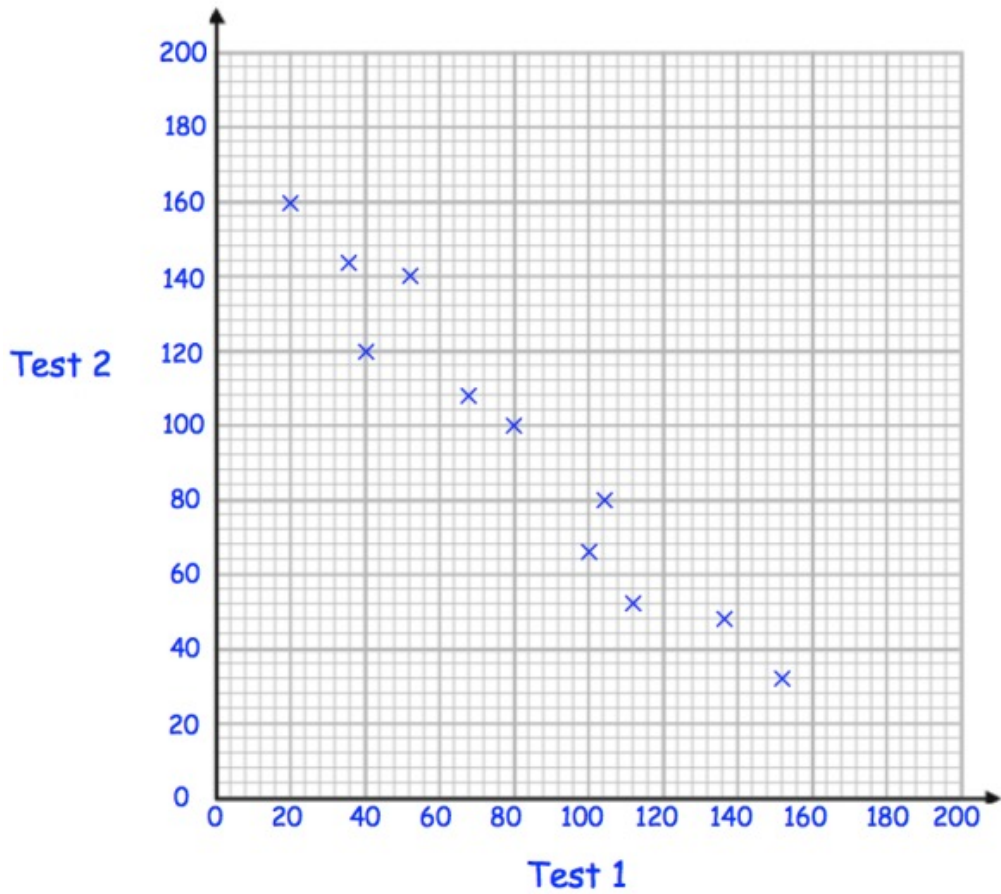
The scatter graph shows the value of cars and their age.



- Describe the correlation.
- Another car is 6 years old and worth £1500. Plot this on the graph.
- Another car is 4 years old. Estimate its value.

Your Turn

The scatter graph shows the scores on two tests



- Describe the correlation.
- Another student scores 80 on test 1 and 120 on test 2. Plot this on the graph.
- Another student scored 20 on test 2. Estimate the student's score on test 1.

Fluency Practice

Question 1: Plot the following information as scatter graphs

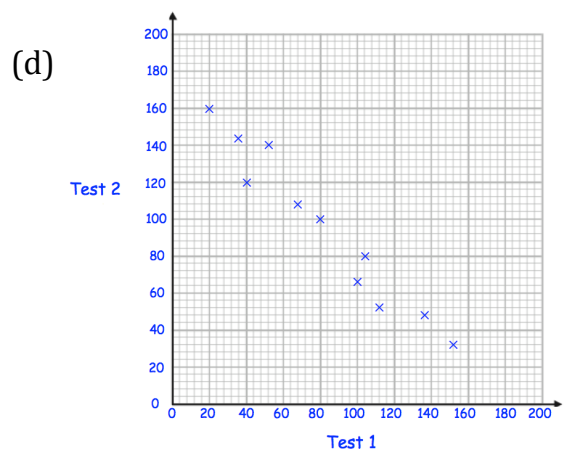
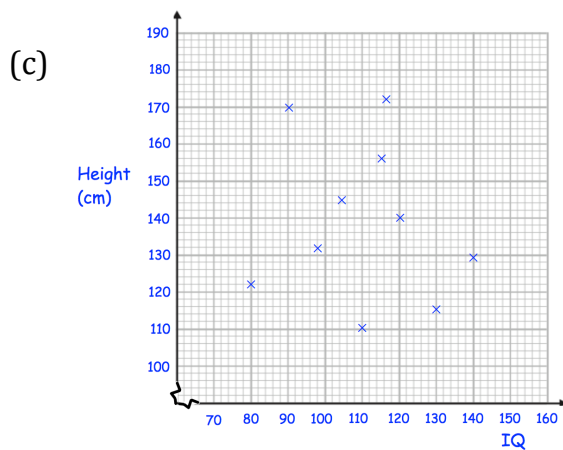
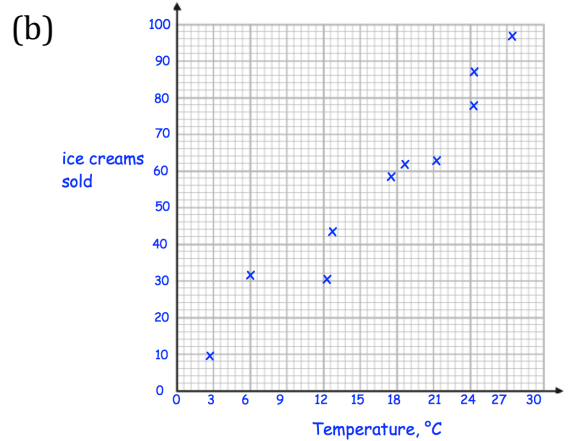
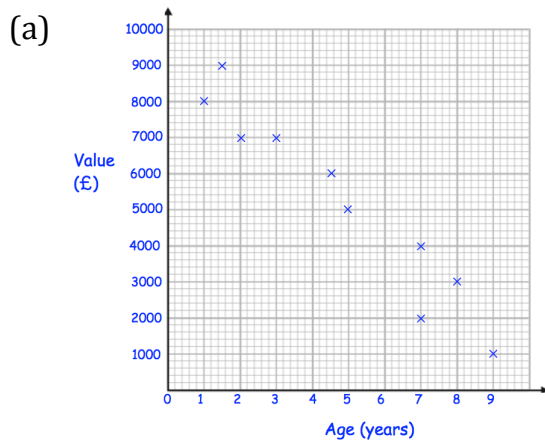
(a)	Maths score	9	13	6	18	11	4	15	10
	Physics score	10	13	5	20	8	5	12	14

(b)	Age, years	4	7	2	4	1	9	3	6
	Cost, £	6000	3000	7500	5000	8000	1500	6000	4000

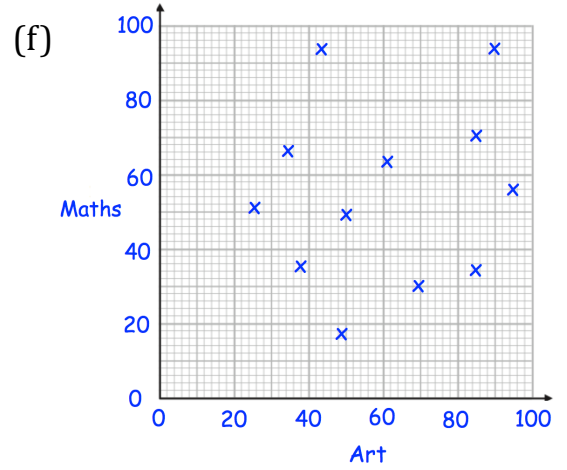
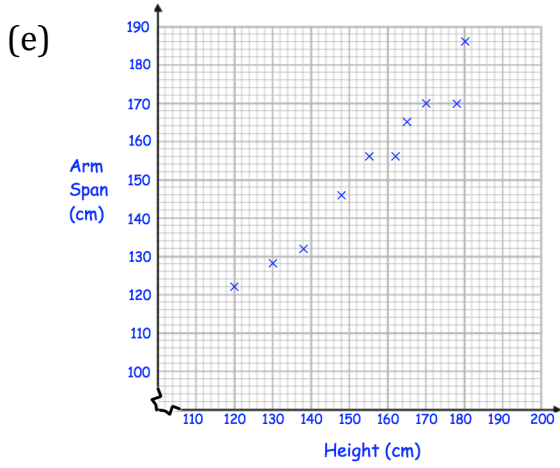
(c)	Height, cm	157	160	148	160	177	156	166	170
	Weight, kg	53	60	44	53	54	60	54	70

(d)	Distance, miles	2.5	0.8	1.2	4.1	2.8	3.3	3.7	1.5
	Cost	£3.20	£1.40	£1.80	£4.40	£3.00	£3.60	£4.80	£2.40

Question 2: What type of correlation does each scatter graph show below



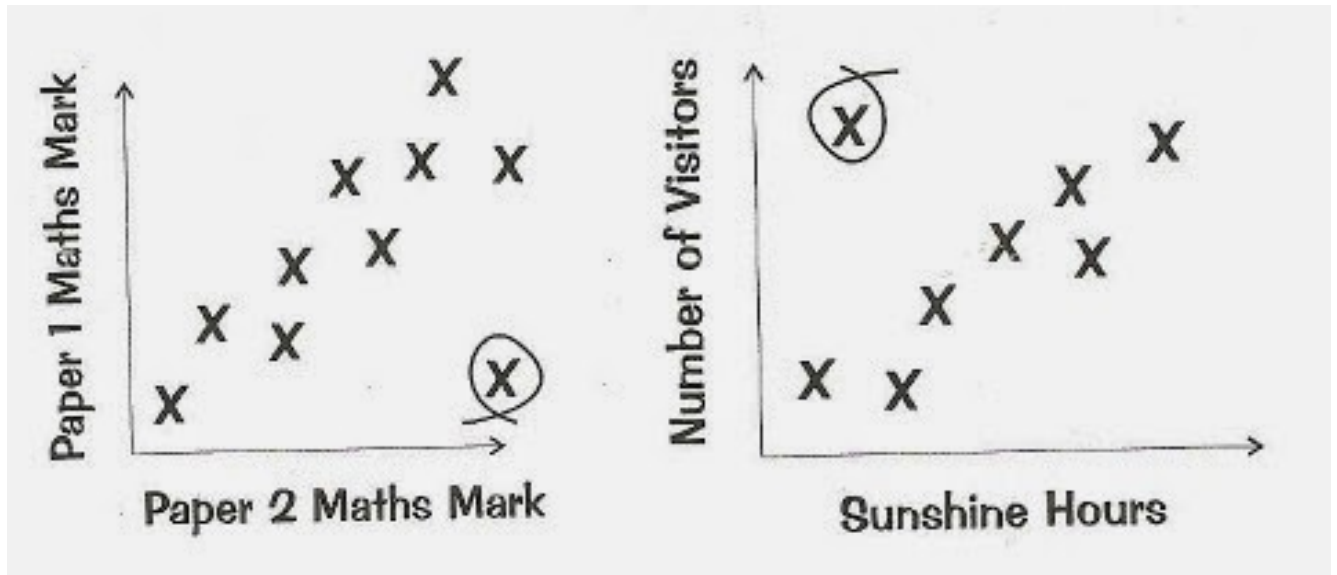
Fluency Practice



Question 3: Describe the relationships shown in each scatter graph in Question 2.

3.5 Outliers

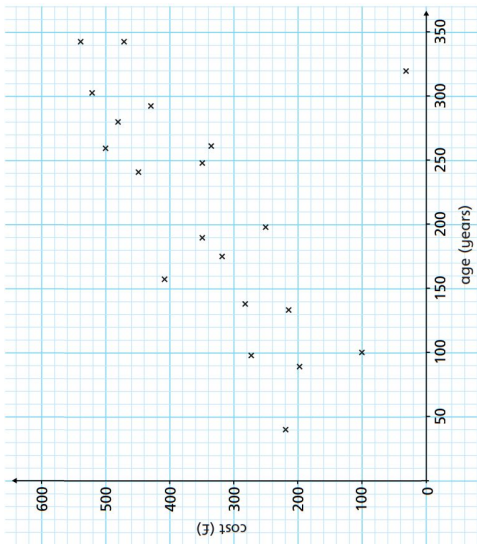
Scatter plots often have a pattern. We call a data point an **outlier** if it doesn't fit the pattern.



Fluency Practice

1. The scatter graph shows the cost and age of some rare books.

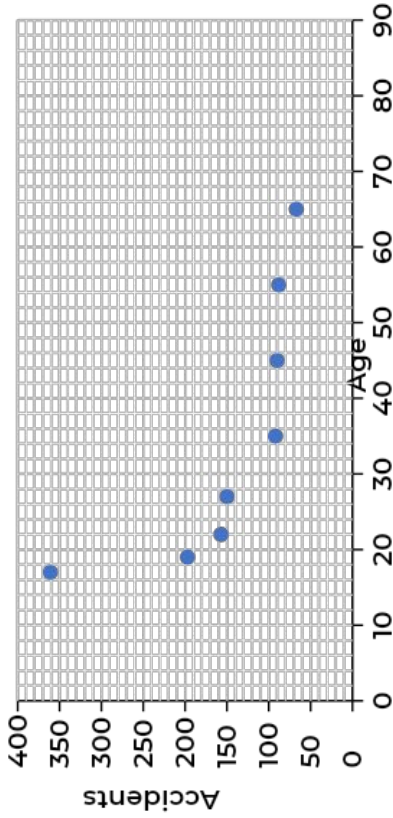
a) Circle the outlier.



b) Describe the correlation.

2. The scatter graph shows the number of accidents per million miles for drivers of different ages.

a) Circle the outlier.



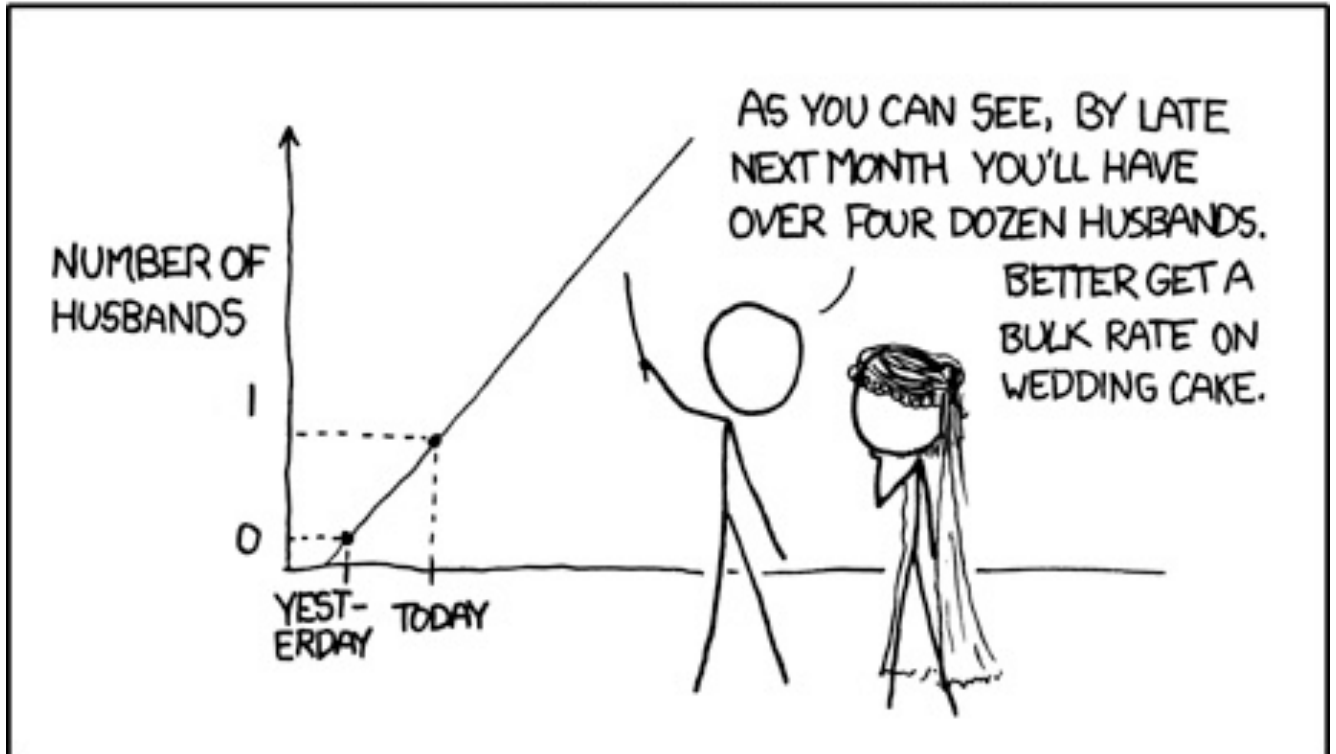
b) Describe the correlation.

3.6 Interpolation vs Extrapolation

When we use our line of best fit to estimate a value **inside** the range of our data, this is known as **interpolation**.

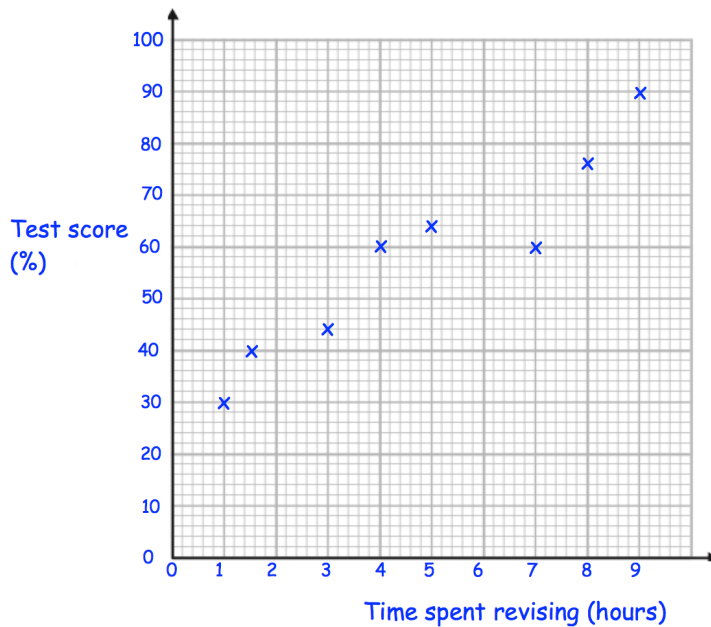
When we use our line of best fit to estimate a value **outside** the range of our data, this is known as **extrapolation**.

MY HOBBY: EXTRAPOLATING



Fluency Practice

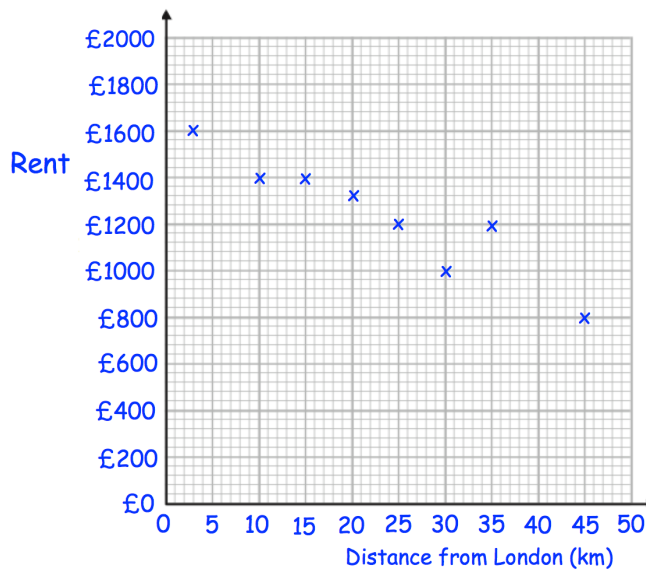
Question 1: The scatter graph below shows information about the number of hours spent revising for a test and the test result for a group of 8 students.



- Daisy spent 7 hours revising for the test. What is Daisy's test score?
- Harry's test score was 30%. How many hours did Harry spend revising?
- Draw a line of best fit.
- Another student spent 6 hours revising for the test. Find an estimate of their test score.
- Explain why it might not be sensible to use the scatter graph to estimate the score for a student that spent 15 hours revising.

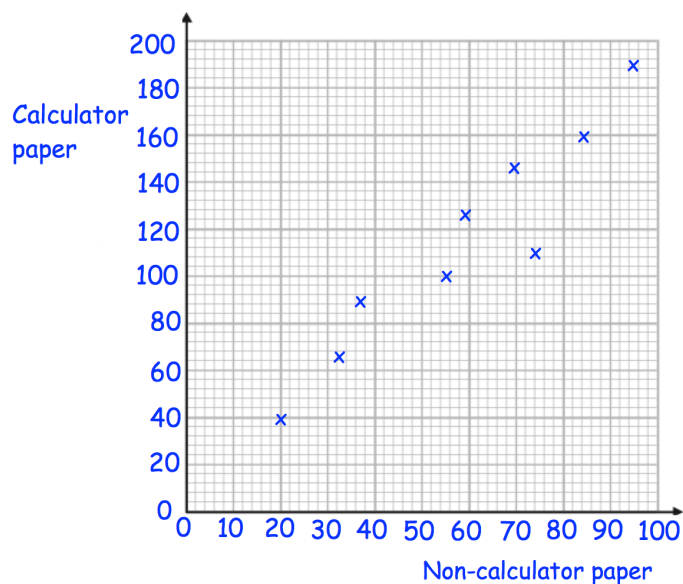
Fluency Practice

Question 2: The scatter graph shows information about the cost of renting apartments and their distance from London.



- Describe the relationship shown in the scatter graph.
- Draw a line of best fit on the diagram.
- Estimate the cost of renting an apartment 40km from London.
- Victor has £1100 to spend on rent. Estimate how close he could live to London.
- Explain why it might not be sensible to use the scatter graph to estimate the price of rent for a property that is 250km from London.

Question 3: The students in a class sit a non-calculator and a calculator maths paper.

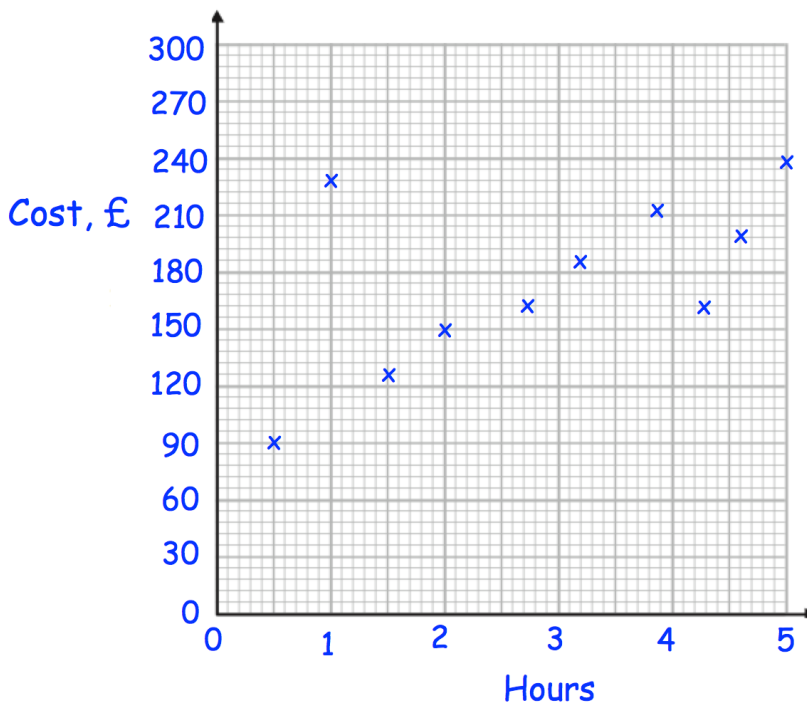


Fluency Practice

- (a) What type of correlation does the scatter graph show?
- (b) Draw a line of best fit.
- (c) Philip was absent for the calculator paper, but he scored 80 in the non-calculator paper. Use your line of best fit to predict his calculator paper score.
- (d) Neil was absent for the non-calculator paper, but he scored 60 in the calculator paper. Use your line of best fit to predict his non-calculator paper score.

Question 4: Mr Hughes is a plumber.

The scatter graph shows the cost and the length of his last 10 jobs.



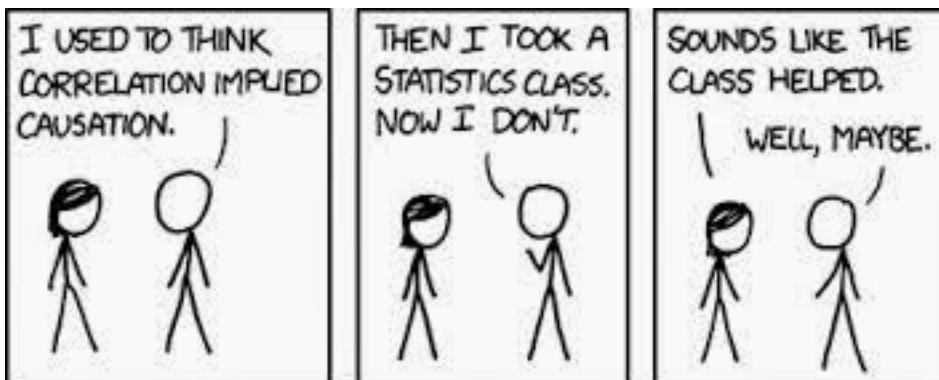
- (a) Draw a line of best fit
- (b) For one job Mr Hughes needed to replace an expensive part that he fitted quickly. How long did that job last?
- (c) Estimate the cost of a job lasting 3.5 hours.
- (d) A job costs £120, estimate the length of the job.

3.7 Correlation vs Causation

Are these sensible correlations?

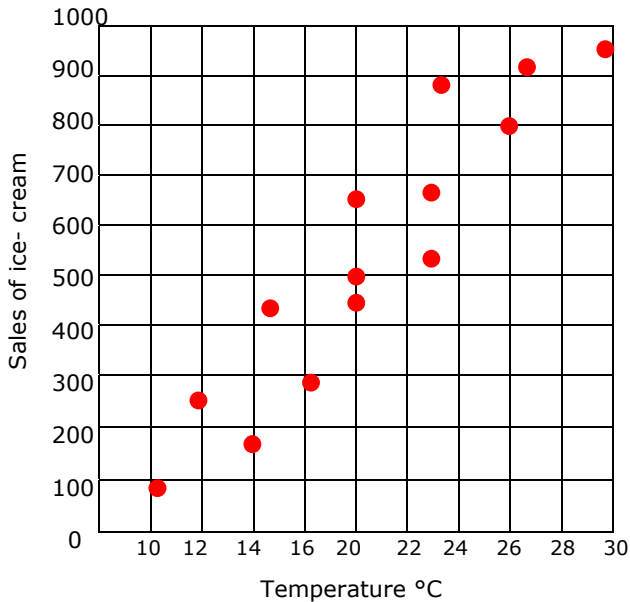
- The more firemen fighting a fire, the bigger the fire is going to be. Therefore, firemen cause fire.
- Sleeping with your shoes on is strongly correlated with waking up with a headache. Therefore, sleeping with your shoes on causes headache.
- People are taller today than 500 years ago. Health and diet have improved over the last 500 years. Therefore, better health and diet have led to people becoming taller.
- As the number of pirates has decreased, global warming has increased. Therefore, global warming is caused by a lack of pirates.
- During summer, people get hayfever. There is also lots of pollen in the air. Therefore, pollen causes hayfever.

Just because we have a correlation does not mean it is true! It does not mean one thing causes the other. For that, we need scientific evidence! Sometimes it is pretty difficult to get.



Fluency Practice

The following graph depicts the relationship between the sales of ice cream and the temperature according to the weather recorded each day.



1. What type of correlation does this data represent?
2. What does the correlation show about the relationship between temperature and sales of ice cream?
3. Can it be claimed that the hotter it gets, the more ice cream is sold?
Explain why or why not.

Fluency Practice

Which of the following statements have link between correlation and causation?

4. Is there any relationship between student's scores on an examination and students cumulative grade- point average (GPA) upon graduation?

5. The price of oil related to the demand and supply graph of oil over the years in the United States.

6. Manufacturing time per unit for a new aircraft tends to decrease each time the total number of units double.

7. A company is working on determining the relationship between workers salary and absentee rate.

8. The Deterrick Waste Management company is interested in seeing if a relationship between the age of a truck and the cost to repair the truck are related. They do find that the older truck, the more costly the repair bills.

9. The relationship between the money spent on research and development and chemical firm annual profits. The firm spends \$ 8 million for R & D in 1984 and expects to earn \$ 36 million in profits that year. This has been the trend over 6 years.

10. Richard Specker is a sales manager for large retailer is measuring his radio advertising campaign featuring major appliances (washers, dryers and dishwashers). Over the last 7 years it has been found that with varying amounts of radio time there happens to be varying amounts of appliances sold that week.

3.8 Review and Problem Solving

Fluency Practice

1. A town recorded the hours of sunshine and rainfall it got over 11 summers.

Sunshine (hrs)	Rainfall (mm)
150	40
80	130
110	80
160	40
20	160
60	120
130	90
90	110
70	120
50	170
120	70

Draw a scatter graph to display the information.

Add a line of best fit to the graph and describe the correlation.

Use your line of best fit to estimate how much sunshine you would expect if there were 100mm of rainfall in a summer.

2. Ten students sat Maths and Science tests. Their scores are shown below.

Draw a scatter graph to display the information.

Add a line of best fit to the graph and describe the correlation.

Use your line of best fit to estimate what you would expect a student who got 75% in their Science test to get in their Maths test.

Maths result %	Science result %
90	80
100	80
30	40
40	50
70	60
70	80
50	50
30	40
90	100
80	70

3. A football coach recorded how shots on target each member of his team made and how many goals each team member scored in a season.

Shots on target	Goals scored
12	8
0	0
4	3
7	6
8	7
5	5
10	8
11	9
15	10
3	3
6	4

Draw a scatter graph to display the information.

Add a line of best fit to the graph and describe the correlation.

Use your line of best fit to estimate how many goals are likely to be made by a player who had 9 attempts at goal.