Year 8
Mathematics
Unit 6 – Student

Name: _____

Class: _____

Contents

1 Factorising to a Single Bracket

Factorising means:
To turn an expression into a **product** of factors.

Year 8 Factorisation

$2x^2 + 4xz$ → (Factorise) $2x(x+2z)$

Year 9 Factorisation

$x^2 + 3x + 2$ → (Factorise) $(x+1)(x+2)$

A Level Factorisation

$2x^3 + 3x^2 - 11x - 6$ → (Factorise) $(2x+1)(x-2)(x+3)$

Factorising is the **reverse of expanding**.
When you have a sum of terms, just **identify the common factor**.
i.e. Find the largest expression each of your terms is divisible by.

1.1 Factors

What does the factor of a number mean?

Numbers which divide the original number without a remainder.

Write down the factors of each expression

$6x$

$8x$

$3x^2y$

$7pq^2$

$8x^2$

$99x$

Worked Example	Your Turn
Find the highest common factor of: a) $3a$ and $5a$ b) 6 and $6a$ c) $3a$ and $6a$	Find the highest common factor of: a) $2b$ and $3b$ b) 6 and $12b$ c) $6b$ and $12b^2$

Intelligent Practice

Term	Term	Highest Common Factor
$2p$	$3p$	
$2p$	$5p$	
$2p$	$6p$	
$3p$	$9p$	
$3p$	$12p$	
$3pr$	$12pr$	
$6pr$	$12pr$	
$9pr$	$12pr$	
$12pr$	$20pr$	
$12r$	$20pr$	
$12p^2r$	$20pr$	
$12p^2r^2$	$20pr$	
$12p^2r^2$	$20p^3r^2$	

1.2 Does it Factorise?

$6a + 7$	Yes	No
$6a + 7b$	Yes	No
$6a + 9b$	Yes	No
$5a^2 + 9b$	Yes	No
$5a^2 + 9ab$	Yes	No

Fluency Practice

$5x + 15$	Yes	No
$5x - 15$	Yes	No
$5y - 15$	Yes	No
$4y - 15$	Yes	No
$4y - 16$	Yes	No
$4y - 18$	Yes	No
$5y - 18$	Yes	No
$15y - 18$	Yes	No
$14y - 17$	Yes	No

Fluency Practice

$3y + 4$	Yes	No
$3x + 4$	Yes	No
$3x + 4y$	Yes	No
$3xy + 4y$	Yes	No
$3xy + 4x$	Yes	No
$3x^2 + 4xy$	Yes	No
$3x^2 + 4x^2y$	Yes	No
$3x^2 + 4x^3y$	Yes	No
$3x^2 + 4y^3$	Yes	No

1.3 Factorising to a Single Bracket

Worked Example	Your Turn
a) Factorise $12x + 18$ b) Factorise $12x + 18y$ c) Factorise $12x^2 + 18$	a) Factorise $12x - 20$ b) Factorise $12x - 20y$ c) Factorise $12x^3 - 20$

Fluency Practice

Question 1: Factorise the following expressions

(a) $4x + 6$	(b) $15x + 20$	(c) $9y - 12$	(d) $5x + 15$
(e) $6x - 3$	(f) $4x + 8$	(g) $5y - 25$	(h) $8w + 24$
(i) $10y + 15$	(j) $14w + 21$	(k) $20y - 30$	(l) $27x + 18$
(m) $6 - 4x$	(n) $9 + 12y$	(o) $45 + 60x$	(p) $16y - 32$
(q) $22a + 55$	(r) $100 - 40y$	(s) $6x + 9y$	(t) $4w - 2a$
(u) $25y - 35z$	(v) $8x^2 + 20$	(w) $30y^3 - 15$	(x) $42y + 28x - 56c$

Worked Example	Your Turn
a) Factorise $12x^2 + 18\mathrm{x}$ b) Factorise $12x^2 + 18xy$ c) Factorise $12x^2y + 18xy$	a) Factorise $12x^2 - 20x$ b) Factorise $12x^2 - 20xy$ c) Factorise $12x^2y - 20xy^2$

Fluency Practice

Question 2: Factorise the following expressions

(a) $x^2 + 7x$ (b) $x^2 - 3x$ (c) $y^2 + y$ (d) $w^2 + 9w$

(e) $x^2 - 7x$ (f) $4w^2 + 10w$ (g) $6x^2 - 8x$ (h) $9y^2 - 6y$

(i) $10c + c^2$ (j) $5g - g^2$ (k) $14x^2 + 35x$ (l) $40x^2 - 50x$

(m) $12x^2 + 18x$ (n) $24x^2 - 18x$ (o) $45y^2 + 60y$ (p) $7w^2 + 2w$

Question 3: Factorise the following expressions

(a) $x^2 + xy$ (b) $a^2 - ab$ (c) $xy + xz$ (d) $ab + ac - ad$

(e) $6c^2 - 4cd$ (f) $10x^2 + 15xy$ (g) $12ab + 18bc$ (h) $8xy + 4y^2$

(i) $8cdf + 10cde$ (j) $7w^2 + 6w + wy$ (k) $8ab^2 - 10ab$ (l) $4xy^2 + 6xy + 2x^2y$

(m) $6mn - 7m^2n$ (n) $11g^2h + 22h^2$

Question 4: Factorise the following expressions

(a) $x^3 + 2x^2$ (b) $5x^3 - x^2$ (c) $8c^3 + 12c$ (d) $10w^2 - 15w^3$

(e) $32y^3 + 24y^2$ (f) $12x^4 + 15x$ (g) $4a^5 - 12a^2$ (h) $8w^9 + w^7$

Worked Example	Your Turn
Factorise: a) $x^4y^2 - x^3y^5$ b) $10x^7y^4 - 25x^3y^2$	Factorise: a) $x^2y^5 - xy^3$ b) $20e^5f^2 - 12e^2f$

Fluency Practice

Factorise:

1) $3a^3b^2c^5 + 12b^3c^2$

2) $8q^3r^4 - 4p^4q^2$

3) $6p^4q^5r^2 - 20pq^2r^5$

4) $16a^4b^2c - 14a^2b^4c^2$

5) $18p^4q^4 + 3q^4r^3$

6) $9pq^4r + 3pq^3r^3$

7) $8a^3b + 18a^3b$

8) $10x^5y^2z^2 - 2x^2y^4$

9) $14x^2y^2z^4 + 4x^5y^2z^2$

10) $11p^4qr^5 - 11p^4q^3r$

1.4 Finish Factorising

Worked Example	Your Turn
Finishing factorising: a) $4(10x + 50)$ b) $4(30x + 50)$	Finishing factorising: a) $4(5x + 15)$ b) $4(25x + 15)$

Intelligent Practice

Where possible, finish factorising:

1) $2(3x + 9)$

2) $2(6x + 9)$

3) $2(6x + 10)$

4) $2(6x + 11)$

5) $2(6x + 12)$

6) $2(6x - 12)$

7) $4(12 - 6x)$

8) $4(12 - 6x + 2y)$

9) $4(12 - 6x + 3y)$

10) $4(12 - 6x + 4y)$

11) $4(12 - 6x + 4y)$

12) $4(12 - 6x + 5y)$

13) $4(12x - 6x^2 + 5xy)$

14) $4(12x - 6x^2 + 4xy)$

15) $4(12x^2 - 6x^2 + 4x^2y)$

1.5 Review and Problem Solving

Intelligent Practice

	A		B		C
1)	$6x + 20$	1)	$30x^2 - 9xy$	1)	$3x + 3y + 3z$
2)	$20 + 6x$	2)	$30x^2y - 9xy$	2)	$3x + 6y + 3z$
3)	$6x - 20$	3)	$30x^2y^2 - 9xy$	3)	$3x + 6y + 15z$
4)	$12x - 30$	4)	$50 - 45g$	4)	$21x^2y^2 - 35xy^3$
5)	$12x - 60$	5)	$30x^2y^2 - 9x^2y$	5)	$3x + 6xy + 15z$
6)	$6x - 30$	6)	$30x^4y^2 - 9x^2y$	6)	$3x + 6xy + 15xz$
7)	$30x - 6$	7)	$30x^4y^2 + 9x^2y$	7)	$3x^2 + 6xy + 15xz$
8)	$30x - 9$	8)	$x^4y^2 + x^2y$	8)	$x^2 + xy + xz$
9)	$30x - 9x$	9)	$x^4y^2 - x^2y$	9)	$45f + 81$
10)	$30x^2 - 9x$	10)	$21v - 35$	10)	$7w + 9xy + 15xz$

Fill in the Gaps

Expanded Expression	HCF of Numbers	HCF of Variables	Factorised Expression
$7x + 14$	7		$7(x + 2)$
$20 + 30a$	10		$10(\square + \square)$
$15b - 5$	5		
$12x + 15$			
$30a - 12b$			
$8cd + de$		d	$d(\square + \square)$
$10a + ab$			
$x^2 - 5x$		x	
$6x^2 + xy$			
$4ab + 8b$	4	b	$4b(\square + \square)$
$10cd - 25de$	5	d	
$4x^2 + 2x$			
$14xy - 21x^2$			
$6x + 3 - 9y$			
$5x^2 - 10xy + 20x$			
$24a^2b + 16abc$			
$\square - 18xyz$			$\square (x - 3z)$
$12x + \square - 16yz$			$4(\square + 2y - \square)$
$35a^2b^2 + \square$			$\square (5a^2b + 2cd)$

Fill in the Gaps

- $2(x+5) = \square x + \square$
- $\square(x+6) = 3x + \square$
- $\square(x-5) = \square x - 25$
- $4(\square x + \square) = 28x + 12$
- $x(x + \square) = \square^2 + 4x$
- $\square(\square - 5) = 6x^2 - 10x$
- $\square(\square - \square) = 6x - 21$
- $\square(\square + \square) = 20x^2 + 24x$

Extension

Question 1: Explain why 8x + 3y cannot be factorised.

Question 2: James has factorised an expression correctly.
His answer is 2(7y – 3).
What was the expression that he factorised?

Question 3: Alexandra is trying to factorise fully 15y + 30.
Rebecca says the answer is 3(5y + 10)
Victoria says the answer is 5(3y + 6)
Alexandra says both Rebecca and Victoria are incorrect, why?

Question 4: Can you spot any mistakes?

Factorise

$w^2 - 5w$

$w(w + 5)$

.....

(1)

Question 5: Can you spot any mistakes?

Factorise completely

$24x^2 + 20x$

$4(6x^2 + 5x)$

.....

(2)

Question 6: Can you spot any mistakes?

Factorise completely

$20a^2c + 30ac$

$5ac(4a^2 + 6)$

.....

(2)

Extension

Factorise fully the following expressions:

(a) $6x - 8$.

(b) $10z^2 + 5z$.

(c) $5x^2 - 20xy$.

(d) $36x^3y + 30xy^2$.

(e) $8ab^2c - 16a^2b^2c$.

(f) $6d^3e - d^4e$.

(g) $\pi r^3 - 2\pi r^2 h$.

(h) $120l^3h - 100h^2l^4$.

(i) $y(2x + 4y) - 6y$.

(j) $8xy(x^2 + y^2) + 2x(yx^3 + y^3)$.

(k) $30d^3e^4 - 15d^2e^3$.

(l) $14b^3c + 28b^3c^2 + 7a^2b^3c$.

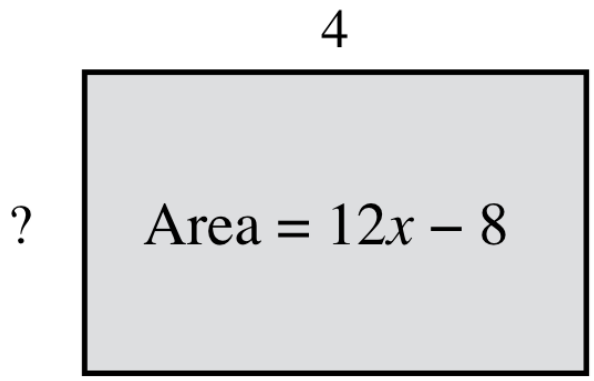
(m) $6x^5y^2 - 8y^3x^3 - 10x^8y$.

(n) $5a^{20}b^{17} + 20a^{30}b^{14} - 15a^{20}b^{15}$.

1. $3x + 12$
2. $x^2 + x$
3. $3x^2 + 12x$
4. $x^3 - x$
5. $x^3 + x^2$
6. $7x^3 + 21x^2$
7. $x^3y - x^2$
8. $x^3y + x^2y^2$
9. $2x^3y + 10x^2y^2$
10. $2x^3y^5 - 10x^2y^2$
11. $-2x^3y^5 - 20x^2y^2$
12. $2hx^3y^5 + 10x^2y^2$
13. $15h^4x^3y^5 - 20h^7x^2y^2$
14. $15\pi h^4x^3y^5z + 20\pi h^7x^2y^2$

Extension

The length of this rectangle is 4 units.
Its area is $(12x - 8)$ square units.
Find the width of the rectangle.



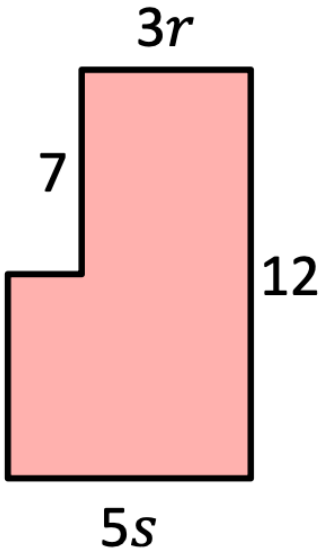
Q5 Simplify and then factorise the following expressions.

[a] $5a - 8b + 13a - 4b$

[b] $7r - 8s + 20s + 8r$

[c] $11m - 4t + 14m - 6t$

Q6 Find and factorise the expression for the area of the following compound shape.



2 Sets and Venns

2.1 Sets

A set is a collection of numbers, or letters, or symbols, or objects, etc., which are related in some way.

The items in a set are called '**members**' or '**elements**'

Curly brackets (often called 'braces') are usually used when listing or describing sets – this helps to distinguish sets from lists of unrelated items.

The elements within a set are usually described in words or listed

Examples:

Description in words	**List of elements**
{even numbers less than 11}	{2, 4, 6, 8, 10}
{the first five prime numbers}	{2, 3, 5, 7, 11}
{multiples of three between 10 and 20}	{12, 15, 18}
{factors of 27 which are even}	{ }

More examples of sets:

Description in words	**List of elements**
{quadrilaterals with four equal length sides}	{square, rhombus}
{vowels}	{a, e, i, o, u}
{letters in the word 'banana'}	{a, b, n}
{yellow fruit}	{grapefruit, banana, lemon, …}

Notes:

Elements are only ever included once – as shown with {letters in the word 'banana'} = {a, b, n}

{yellow fruits} is an imprecise description and the list of elements contains only examples.

Worked Example

List the following sets:

a) {factors of 15}

b) {the first four square numbers}

c) {letters in the word LONDON}

d) {possible outcomes when an ordinary coin is thrown}

Fluency Practice

A1 List {vowels}	**A2** List {the first six consonants}	**A3** List {vowels in the word 'NUMBER'}	**A4** List {consonants in the word 'MATHS'}
B1 List {vowels in the word 'ALGEBRA'}	**B2** List {consonants in the word 'SETS'}	**B3** List {letters in the word 'ISOSCELES'}	**B4** List {vowels in 'SQUARE ROOT'}
C1 List {days of the week}	**C2** List {seasons in the year}	**C3** List {colours in the rainbow}	**C4** List {countries in the United Kingdom}
D1 List {first three months of the year}	**D2** List {months of the year with four letters}	**D3** List {months of the year beginning with 'A'}	**D4** List {days of the week which contain an 'E'}
E1 Describe the following set: {spring, summer}	**E2** Describe the following set: {square, rhombus}	**E3** Describe the following set: {north, east, south, west}	**E4** Describe the following set: {orange, yellow, indigo, violet}

2.2 Multiple Sets

When we have more than one set, capital letters are usually used to represent them.

Examples:

Description in words	**List of elements**
A = {prime numbers between 10 and 20}	A = {11, 13, 17, 19}
B = {factors of 24}	B = {1, 2, 3, 4, 6, 8, 12, 24}
C = {vowels}	C = {a, e, i, o, u}

Note that it is often convenient to use letters that are in some way connected to the description of the set.

e.g. P = {prime numbers between 10 and 20}, F = {factors of 24} and V = {vowels}

Universal Set

The Universal set is the set of all elements under consideration.

Elements that can be in other sets are restricted to those within the Universal set. For example, if the Universal set was {integers less than 10}, then {prime numbers} would be limited to {2, 3, 5, 7}.

Likewise if the Universal set was {even numbers}, then {factors of 18} would be {2, 6, 18}

Notation

In Britain the special symbol '$\mathcal{E}$' is used to represent the Universal set but in some countries, such as America, the letter 'U' is used.

Thus we could write

$\mathcal{E}$ = {integers less than 10} or $\mathcal{E}$ = {prime numbers}

Worked Example

a) U = {odd numbers less than 15}
A = {prime numbers}
B = {multiples of 3}
List:
i) A
ii) B

b) U = {first 10 letters of the alphabet}
X = {vowels}
Y = {letters in the word ‘ENGLISH’}
List:
i) X
ii) Y

c) U = {factors of 24}
P = {prime numbers}
E = {even numbers}
O = {odd numbers}
List:
i) P
ii) E
iii) O

A1 List {the first six multiples of 3}	**A2** List {prime numbers less than 10}	**A3** List {all the factors of 12}	**A4** List {even numbers between 3 and 11}
B1 Describe the set: {1, 2, 3, 4, 5}	**B2** Describe the set: {1, 3, 5, 7, 9}	**B3** Describe the set: {1, 2, 3, 6, 9, 18}	**B4** Describe the set: {11, 13, 17, 19}
C1 A = {positive integers less than 5} List set A	**C2** B = {negative integers more than 6} List set B	**C3** C = {integers between 4 and 9} List set C	**C4** D = {integers between –3 and 4} List set D
D1 M = {the first five multiples of 6} List set M	**D2** F = {all the factors of 20} List set F	**D3** P = {the first six prime numbers} List set P	**D4** S = {square numbers less than 20} List set S
E1 A = {factors of 20} B = {1, 2, 5, 10, 20} Are the sets A and B the same?	**E2** C = {first five multiples of 7} D = {7, 14, 21, 27, 35} Are the sets C and D the same?	**E3** E = {prime numbers less than 20} F = {the first nine prime numbers} Are the sets E and F the same?	**E4** G = {numbers on a dice} H = {positive integers less than 7} Are the sets G and H the same?

Fluency Practice

Describe these sets in words.

(a) {4, 8, 12, 16, 20, 24, 28}

(b) {1, 4, 9, 16, 25}

(c) {Europe, Asia, Africa,…}

(d) {1, 2, 3, 4, 6, 12}

List the elements of the sets:

(a) Multiples of 7 less than 30

(b) Months of the year

(c) Factors of 25

$A = \{factors\ of\ 6\}$

$B = \{prime\ numbers\ less\ than\ 20\}$

$C = \{integers\ from\ 1\ to\ 10\}$

(a) List the elements of A

(b) List the elements of B

(c) List the elements of C

2.3 Venns

Starter





How could we categorise (group) these animals?




List 5 ways in your book.

Starter



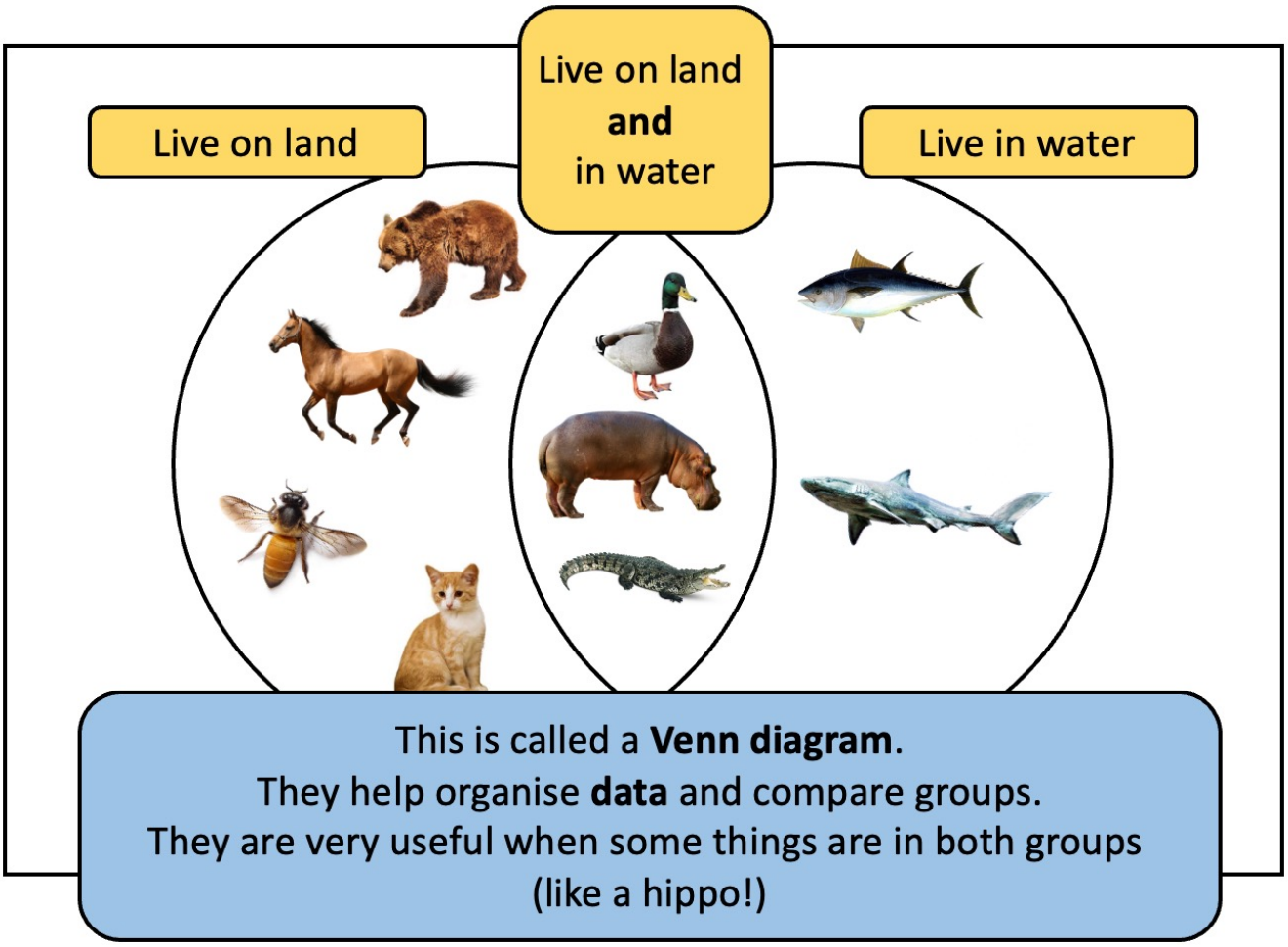

Live on land


Live in water

Which group would these animals go into?



2.4 Venn Diagrams

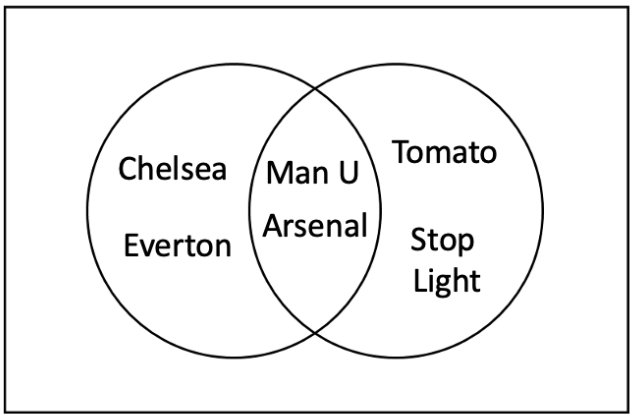


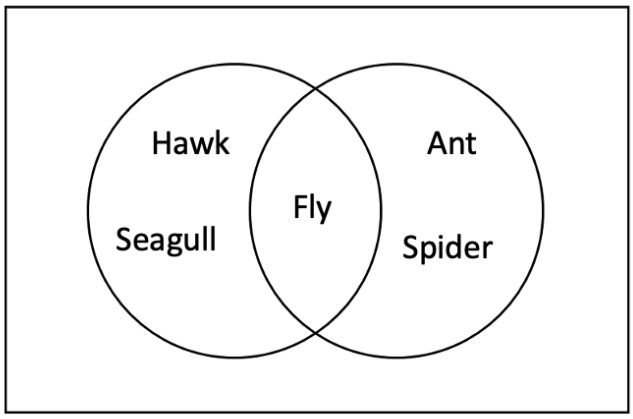
A Venn diagram ('created' by John Venn)
is a pictorial view of the relationships between sets.

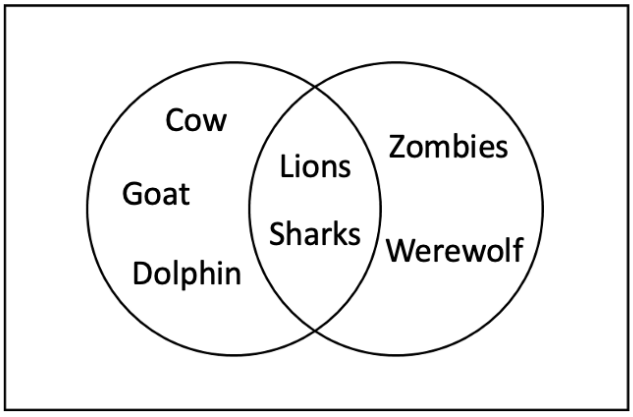
A rectangle is drawn to represent the Universal set, and one or more ovals to represent the other sets.

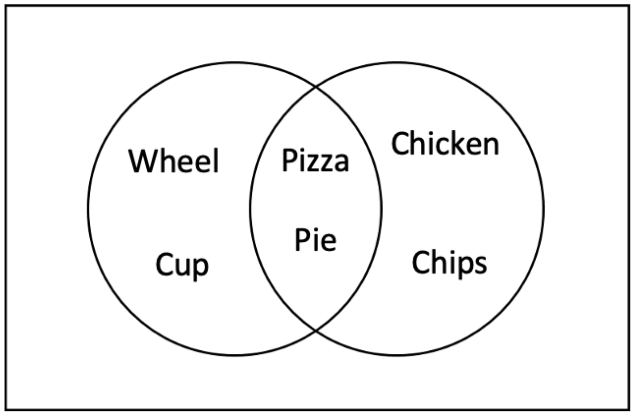
Venn Diagrams

What are the different groups in these Venn diagrams?









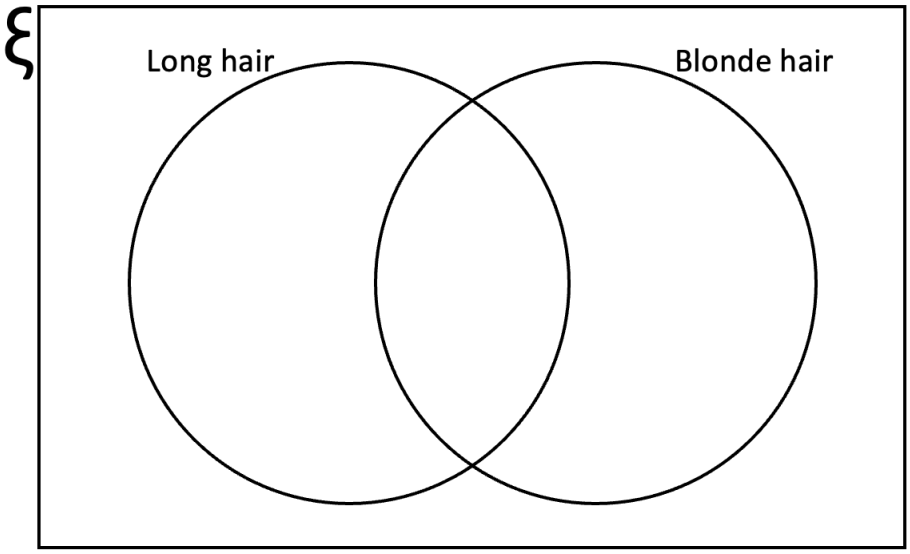
Worked Example

Complete the Venn Diagram:

4 students were picked from Year 7

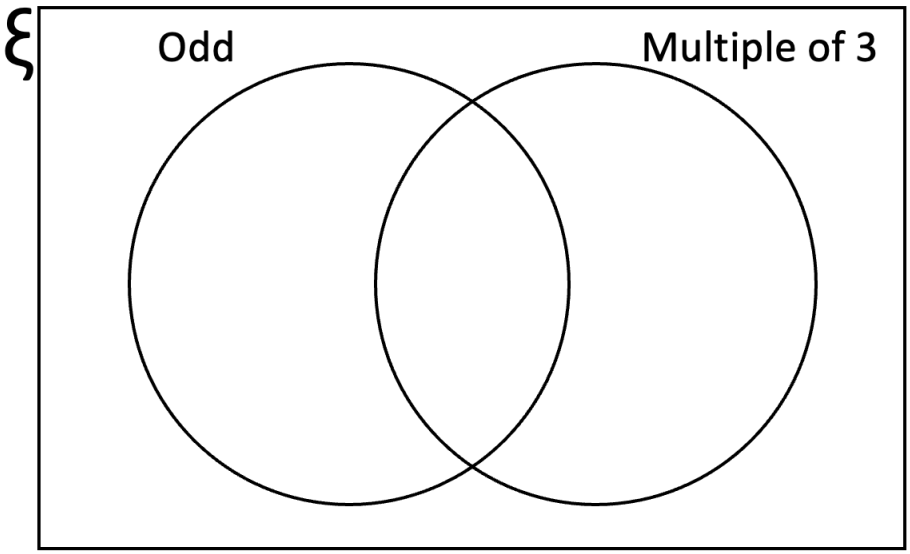
ξ =

Anna Tom Mary Jack Beth



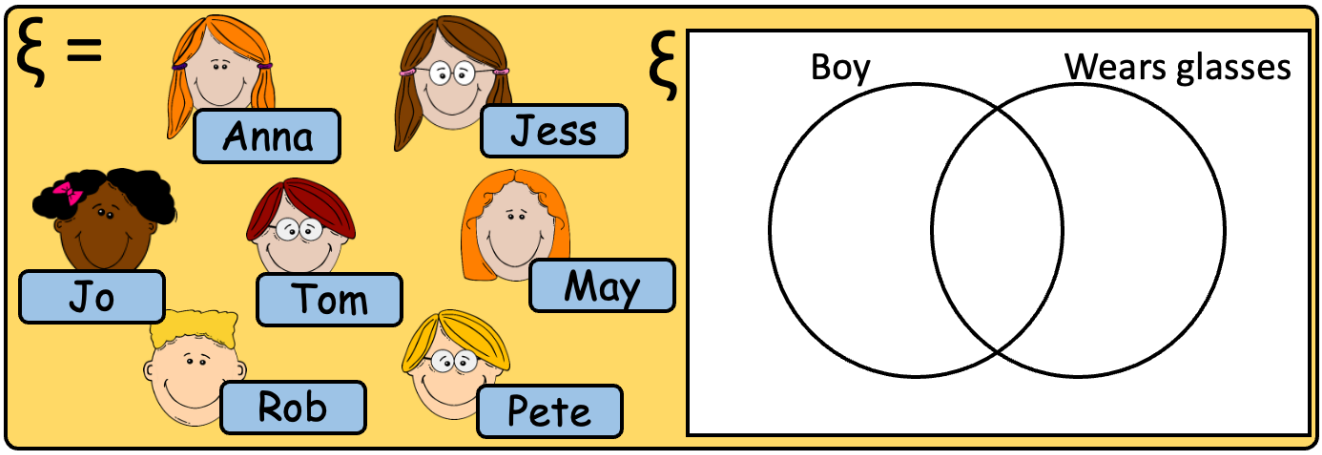
We want to sort the numbers 1 to 10.

ξ = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

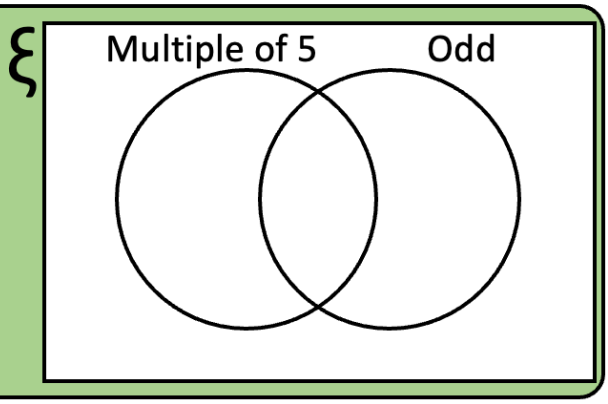


Your Turn

Complete the Venn Diagram:



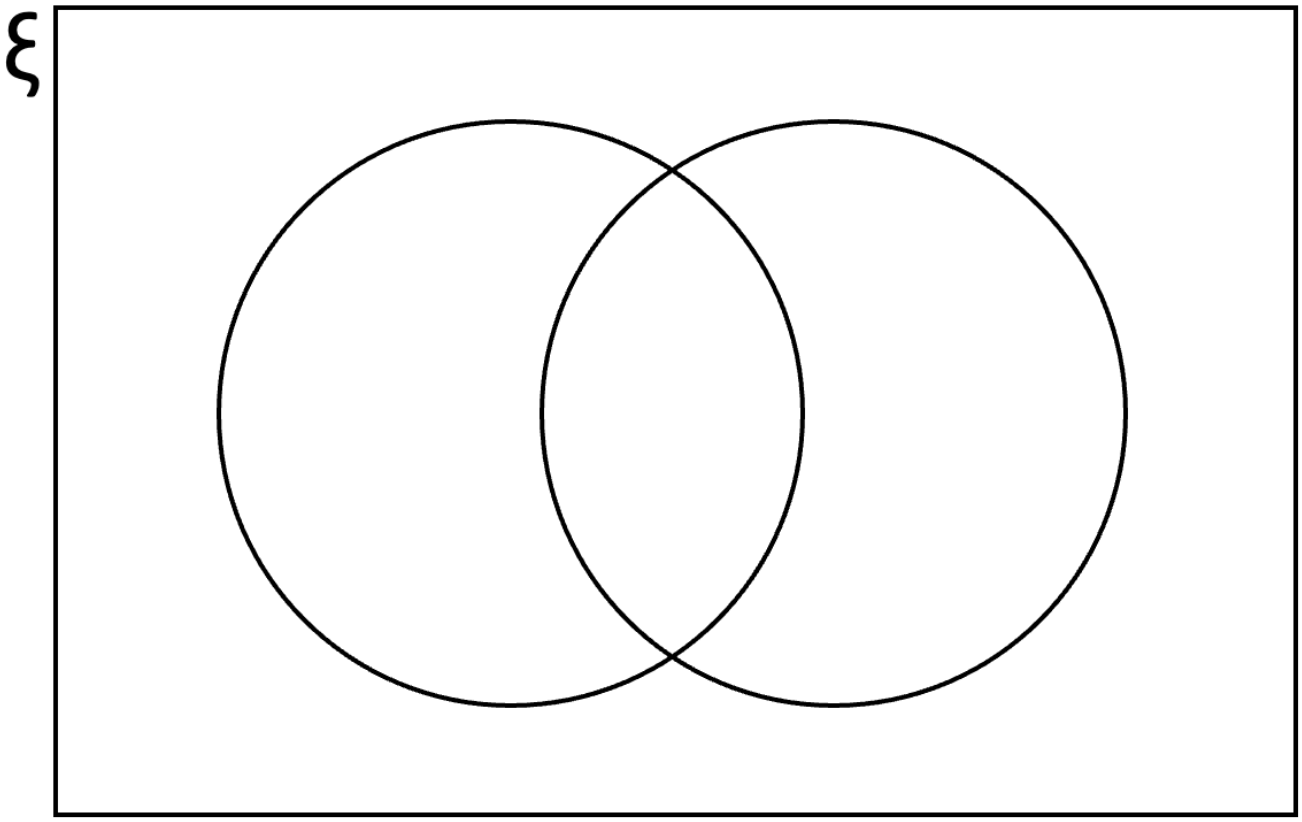
ξ = 3, 4, 5, 7, 10, 12, 13, 15, 20, 24, 25



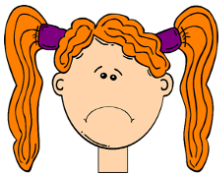
Activity

Classmates in a Venn Diagram

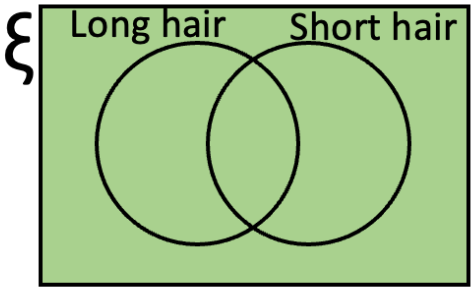
Possible Categories: blonde hair, curly hair, straight hair, short hair, female, male, has a sister, has a brother, wears glasses, right-handed, left-handed, pet dog, pet cat, plays football, loves maths…

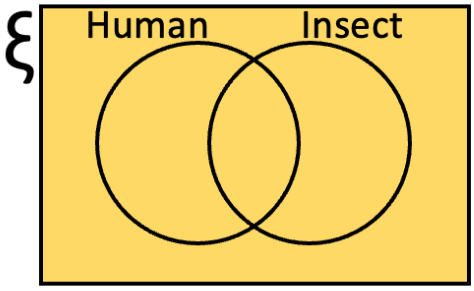


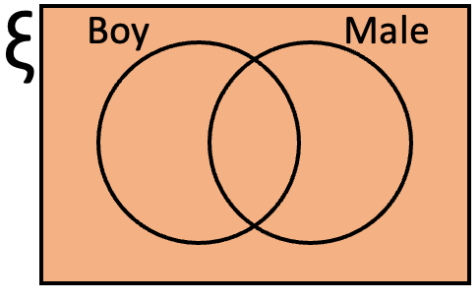
Mistakes

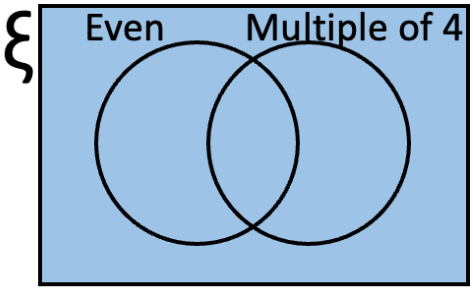


What's wrong with these Venn Diagrams?









Fluency Practice

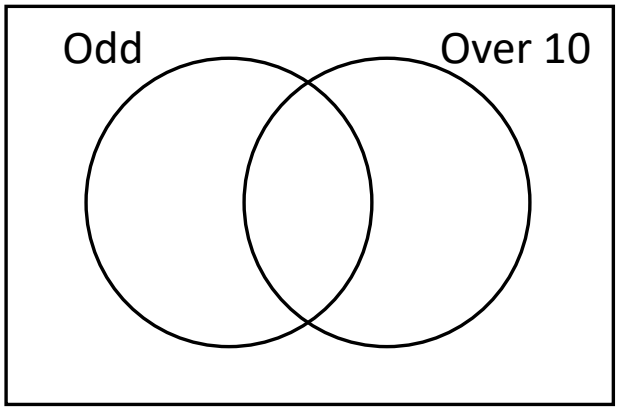
Venn Diagrams

1

Complete each Venn Diagram

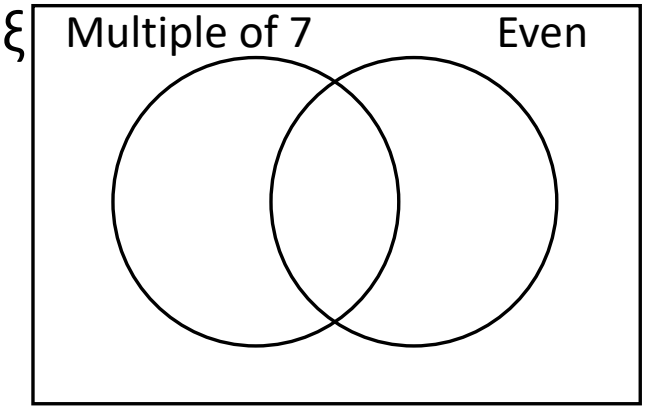
A) ξ =

1	2	3
7	8	9
11	12	13
16	17	18



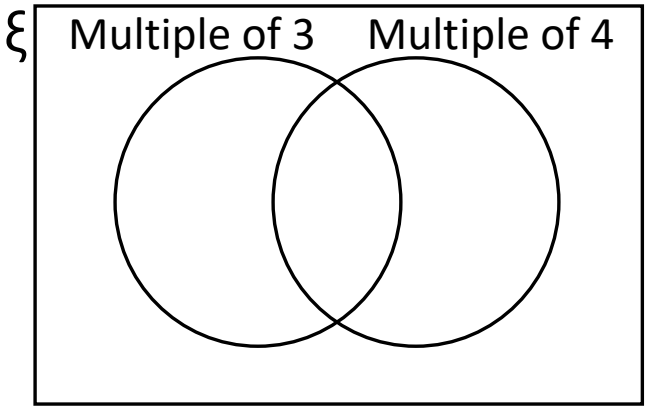
B) ξ =

34	14	15	28
21	70	20	13
1	25	7	16
6	35	18	41



C) ξ = Numbers from 1 to 25

What fraction of the numbers are not a multiple of 3 or 4?



Fluency Practice

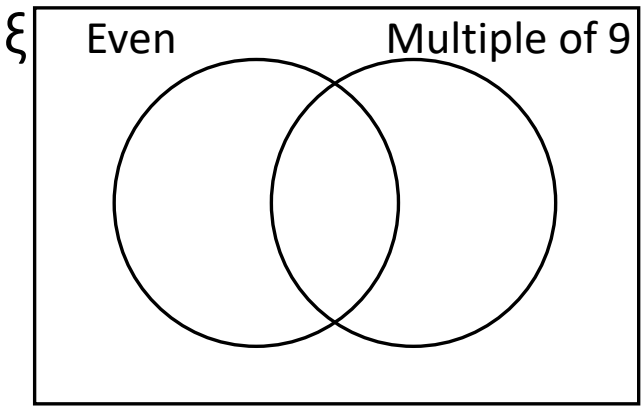
Venn Diagrams

2

Complete each Venn Diagram

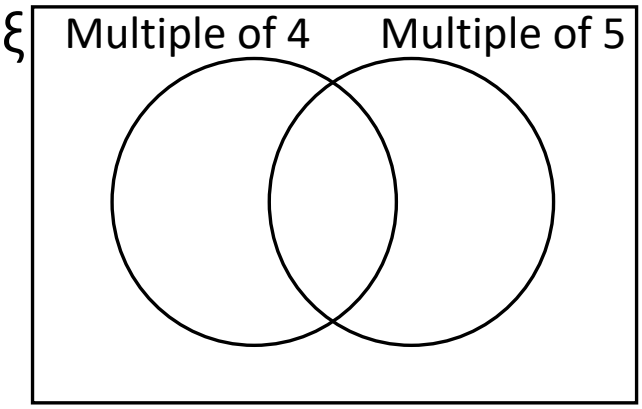
A) ξ =

44	90	45	88
54	27	6	26
91	16	71	9
18	13	24	33



B) ξ = Even numbers from 10 to 40

What fraction of the numbers are multiple of 4 and 5?

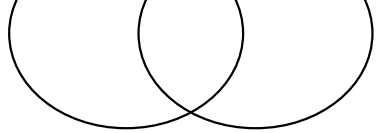


Venn Diagrams for Factors

A) ξ = Factors of 24

ξ

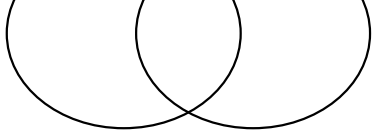
Factors of 6 | Factors of 8



B) ξ = Factors of 36

ξ

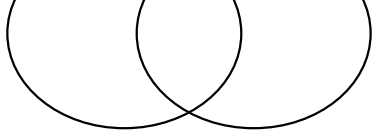
Factors of 15 | Factors of 18



C) ξ = Factors of 28

ξ

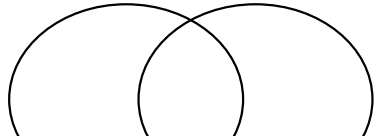
Factors of 16 | Factors of 21



D) ξ = Factors of 54

ξ

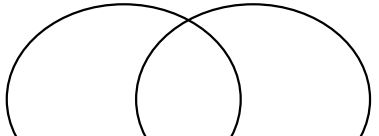
Factors of 18 | Factors of 27



E) ξ = Factors of 60

ξ

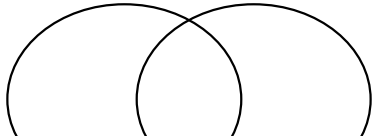
Factors of 30 | Factors of 24



F) ξ = Factors of 90

ξ

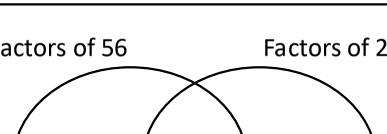
Factors of 40 | Factors of 15



G) ξ = Factors of 42

ξ

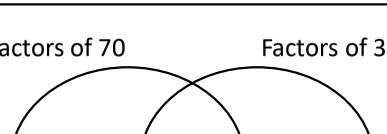
Factors of 56 | Factors of 21



H) ξ = Factors of 140

ξ

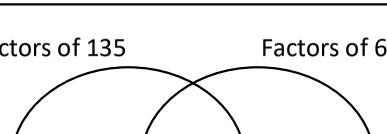
Factors of 70 | Factors of 30



I) ξ = Factors of 180

ξ

Factors of 135 | Factors of 60



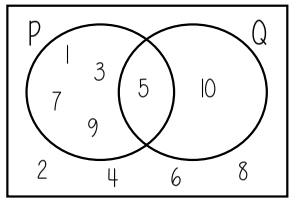
2.5 Review and Problem Solving

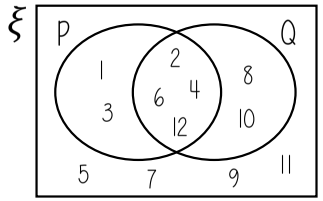
Worked Example	Your Turn
Represent as a Venn diagram: $\xi = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $A = \{0, 1, 3, 5, 8\}$ $B = \{2, 5, 8, 9\}$	Represent as a Venn diagram: $\xi = \{2, 3, 4, 5, 7, 11, 13, 17, 19\}$ $A = \{2, 3, 5, 11, 13\}$ $B = \{5, 7, 13, 17, 19\}$

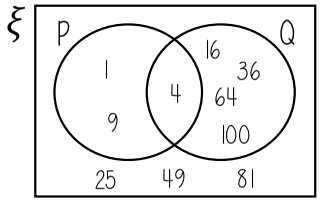
Worked Example	Your Turn
Represent as a Venn diagram: ξ =Positive integers between 1 and 10 inclusive $A = \{\text{Prime numbers}\}$ $B = \{\text{Even numbers}\}$	Represent as a Venn diagram: ξ = Integers between 0 and 5 inclusive $A = \{\text{Prime numbers}\}$ $B = \{\text{Odd numbers}\}$

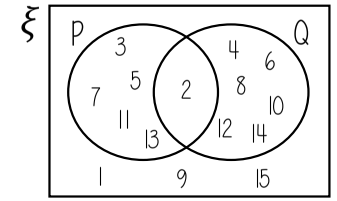
Worked Example	Your Turn
$\xi = \{Days\ of\ the\ week\}$ $A = \{Tuesday, Thursday\}$ B = $\{Days\ starting\ with\ S\ or\ T\}$ Draw a Venn diagram to represent this information.	$\xi = \{Months\ of\ the\ year\}$ $A = \{Months\ starting\ with\ A\}$ B = $\{Months\ with\ six\ letters\}$ Draw a Venn diagram to represent this information.

1. For each Venn diagram, describe the sets: ξ, P and Q

a. 

b. 

c. 

d. 

2. Given the sets, can you place the members into a Venn diagram

a.
ξ = {10, 11, 12, 13, 14, 15, 16}
P = {12, 14, 16}
Q = {10, 11, 12, 16}

b.
ξ = {integers from 15 to 21, inclusive}
X = {15, 18, 21}
Y = {16, 18, 20}

c.
ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
E = {even numbers}
F = {factors of 10}

d.
ξ = {a, b, c, d, e, f, g, h, i, j}
A = {a, e, i}
B = {a, c, e, g, i}

e.
ξ = {integers from 1 to 12, inclusive}
M = {multiples of 2}
N = {numbers less than or equal to 5}

f.
ξ = {C, F, G, H, I, N, S, T, X}
L = {letters with lines of symmetry}
R = {letter with rotational symmetry of order 2}

Template