Year 8
Mathematics
Unit 6

Name: ______________________________

Class: ______________________________

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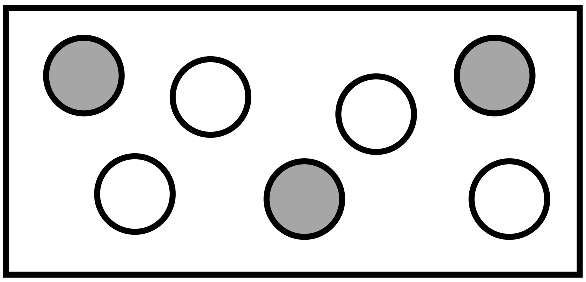
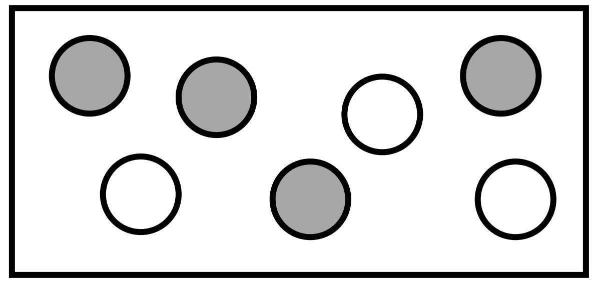
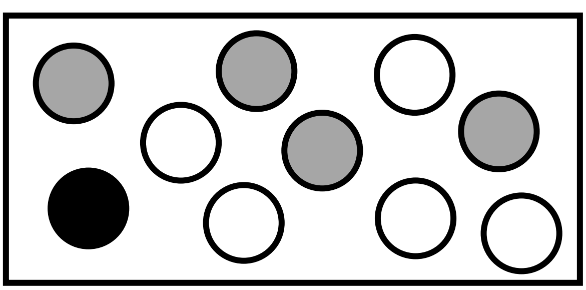
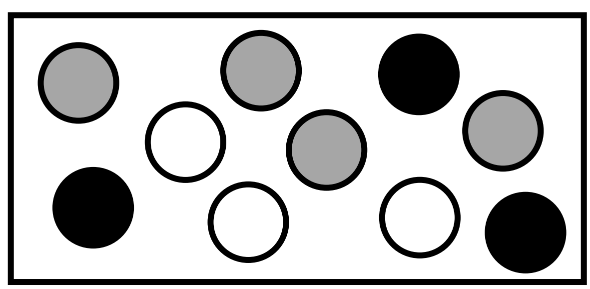
1 Ratio

1.1 Writing Ratios

In this section you will look at how to write ratios. A ratio shows how much of one thing there is compared to another. Ratios are usually written in the form $a : b$.

Note that the order in which a ratio is written is important!

Worked Example	Your Turn
a) Write down the ratio of shaded circles to unshaded circles in the diagram below.	a) Write down the ratio of shaded circles to unshaded circles in the diagram below.
	
b) Write down the ratio of White : Grey : Black in the diagram below.	b) Write down the ratio of White : Grey : Black in the diagram below.
	

1.2 Ratios to Fractions and Percentages

In this section you will look at how to convert ratios to fractions and percentages.

Worked Example	Your Turn
The ratio of blue and red counters in a bag is $4:3$ a) What fraction of the counters are blue? b) What fraction of the counters are red?	The ratio of blue and red counters in a bag is $5:7$ a) What fraction of the counters are blue? b) What fraction of the counters are red?

Worked Example	Your Turn
The ratio of blue, red and yellow counters in a bag is $4 : 3 : 7$ a) What fraction of the counters are blue? b) What fraction of the counters are red? c) What fraction of the counters are yellow?	The ratio of blue, red and yellow counters in a bag is $5 : 7 : 3$ a) What fraction of the counters are blue? b) What fraction of the counters are red? c) What fraction of the counters are yellow?

1.3 Equivalent Ratios

In this section you will look at equivalent ratios.

Note that both sides of a ratio can be multiplied or divided by the same number to give an equivalent ratio.

Worked Example	Your Turn
All the ratios below are equivalent. Complete the gaps below: $1 : 3$ $\_\_ : 6$ $\_\_ : 12$ $24 : \_\_$ $\_\_ : 36$ $\_\_ : 3.6$	All the ratios below are equivalent. Complete the gaps below: $1 : 4$ $\_\_ : 8$ $\_\_ : 16$ $12 : \_\_$ $\_\_ : 12$ $\_\_ : 1.2$

Worked Example	Your Turn
All the ratios below are equivalent. Complete the gaps below: $2 : 3$ $\_\_ : 9$ $\_\_ : 18$ $24 : \_\_$ $\_\_ : 54$ $\_\_ : 0.54$	All the ratios below are equivalent. Complete the gaps below: $2 : 5$ $\_\_ : 15$ $\_\_ : 30$ $24 : \_\_$ $\_\_ : 0.6$ $\_\_ : 4.8$

Worked Example	Your Turn
All the ratios below are equivalent. Complete the gaps below: $3 : 2 : 4$ $\_\_ : 4 : \_\_$ $\_\_ : 8 : \_\_$ $24 : \_\_ : \_\_$ $2.4 : \_\_ : \_\_$	All the ratios below are equivalent. Complete the gaps below: $3 : 2 : 5$ $\_\_ : 4 : \_\_$ $\_\_ : 8 : \_\_$ $24 : \_\_ : \_\_$ $2.4 : \_\_ : \_\_$

Fluency Practice

pair off the equivalent ratios

(1)

5 : 20

10 : 25

1½ : 2½

3 : 12

9 : 12

6 : 7½

24 : 40

3 : 7½

15 : 20

20 : 25

(2)

10½ : 7

2½ : 1

2 : 1½

½ : 0.3

1½ : 1

12½ : 7½

7½ : 3

10 : 7½

(3)

27 : 72

28 : 63

24 : 84

66 : 121

24 : 64

42 : 77

24 : 54

16 : 56

(4)

96 : 88

75 : 70

81 : 72

98 : 91

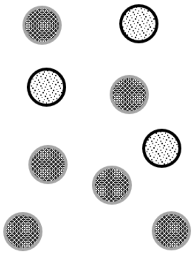
132 : 121

108 : 96

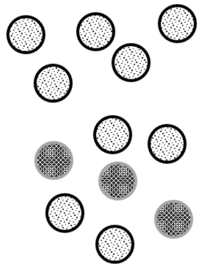
70 : 65

90 : 84

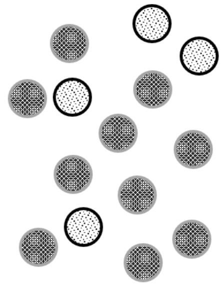
Fluency Practice



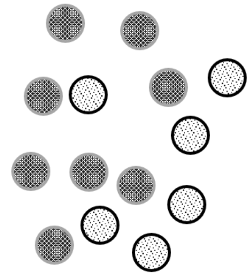
ratio ● : ○ ?
or ?



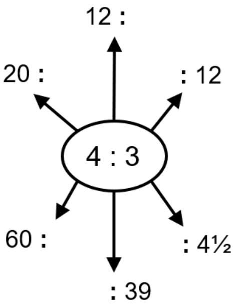
ratio ● : ○ ?
or ?

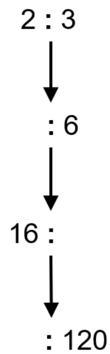


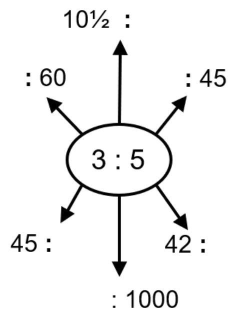
ratio ● : ○ ?
or ?

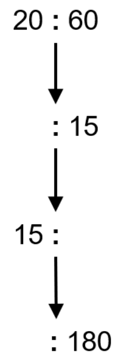


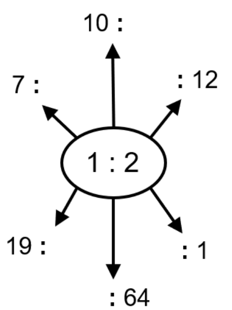
ratio ● : ○ ?
or ?



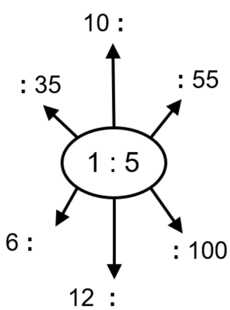




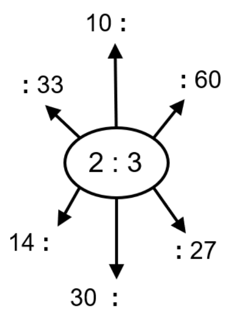




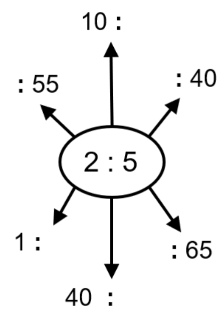
ratio £1 for me, £2 for you



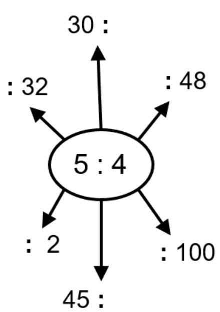
ratio £1 for me, £5 for you

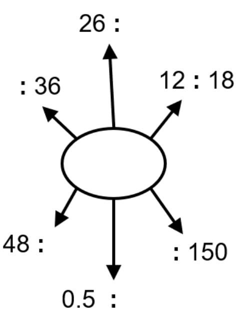


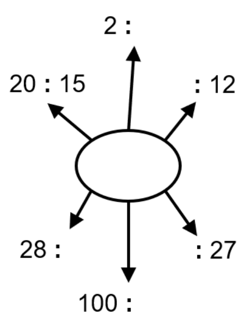
ratio £2 for me, £3 for you

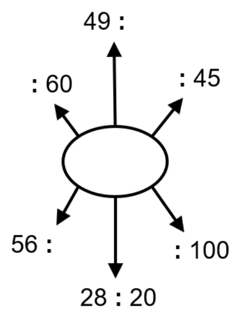


ratio £2 for me, £5 for you









1.4 Simplifying Ratios

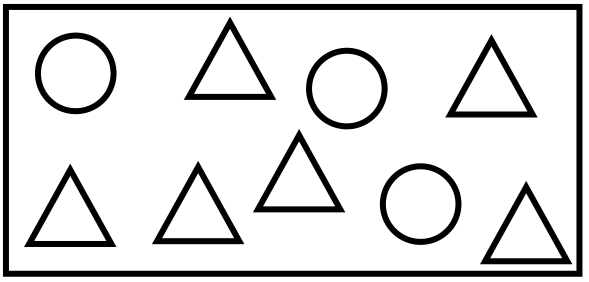
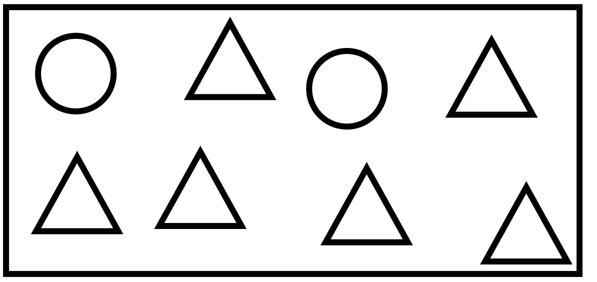
In this section you will look at simplifying ratios.

Note that to simplify a ratio, divide all the numbers in the ratio by the same number until they cannot be divided any more.

Worked Example	Your Turn
Simplify: a) $25 : 30$ b) $45 : 75$ c) $15 : 20 : 35$ d) $150\text{cm} : 1\text{m}$	Simplify: a) $42 : 35$ b) $24 : 60$ c) $16 : 32 : 72$ d) $450\text{g} : 1.3\text{kg}$

1.5 n:1 and 1:n Ratios

In this section you will look at simplifying ratios into the form $n : 1$ and $1 : n$

Worked Example	Your Turn
The diagram below shows a number of circles and triangles.	The diagram below shows a number of circles and triangles.
	
a) Write the ratio of circles to triangles in the ratio $1 : n$	a) Write the ratio of circles to triangles in the ratio $1 : n$
b) Write the ratio of circles to triangles in the ratio $n : 1$	b) Write the ratio of circles to triangles in the ratio $n : 1$

Worked Example	Your Turn
a) Write the ratio $2 : 5$ in the ratio $1 : n$ b) Write the ratio $2 : 5$ in the ratio $n : 1$	a) Write the ratio $4 : 5$ in the ratio $1 : n$ b) Write the ratio $4 : 5$ in the ratio $n : 1$

1.6 Ratio in Different Forms

In this section you will look at how to write ratios in different forms.

Worked Example	Your Turn
$a : b$ $7 : 1$	$a : b$ $8 : 1$
a as a fraction of the whole	a as a fraction of the whole
a as a fraction of b	a as a fraction of b
In the form $1 : n$	In the form $1 : n$
In the form $n : 1$	In the form $n : 1$

Fill in the Gaps

Ratio $a:b$	a as a fraction of the whole	a as a fraction of b	In the form $1:n$	In the form $n:1$
$1:3$				
	$\frac{1}{3}$			
		$\frac{2}{5}$		
			$1:5$	
$5:1$				
	$\frac{5}{7}$			
		$\frac{5}{7}$		
			$1:0.7$	
				$1\frac{4}{7}:1$
				$\frac{7}{11}:1$
$x:y$				

1.7 Scale Drawings

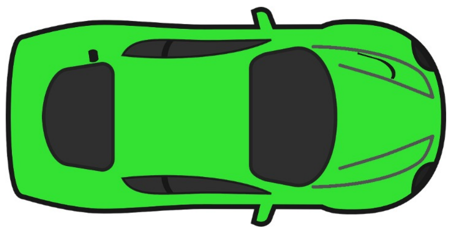
In this section you will look at how to work with scale drawings. Scale drawings allow us to draw large objects on a smaller scale while keeping them accurate. All scale drawings must have a scale written on them. Scales are usually expressed as ratios. The ratio $1 : 100$ means that for every 1cm on the scale drawing the length will be 100cm in real life.

Worked Example

Each of these drawings are at different scales: how big is each real-life measurement?

a)

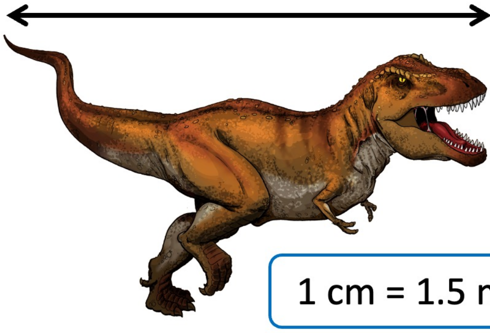
3 cm



1 cm = 1 metre

b)

10 cm



1 cm = 1.5 metres

Your Turn

Each of these drawings are at different scales: how big is each real-life measurement?

a)

4 cm

1 cm = 3 metres

b)

1 cm = 0.5 cm

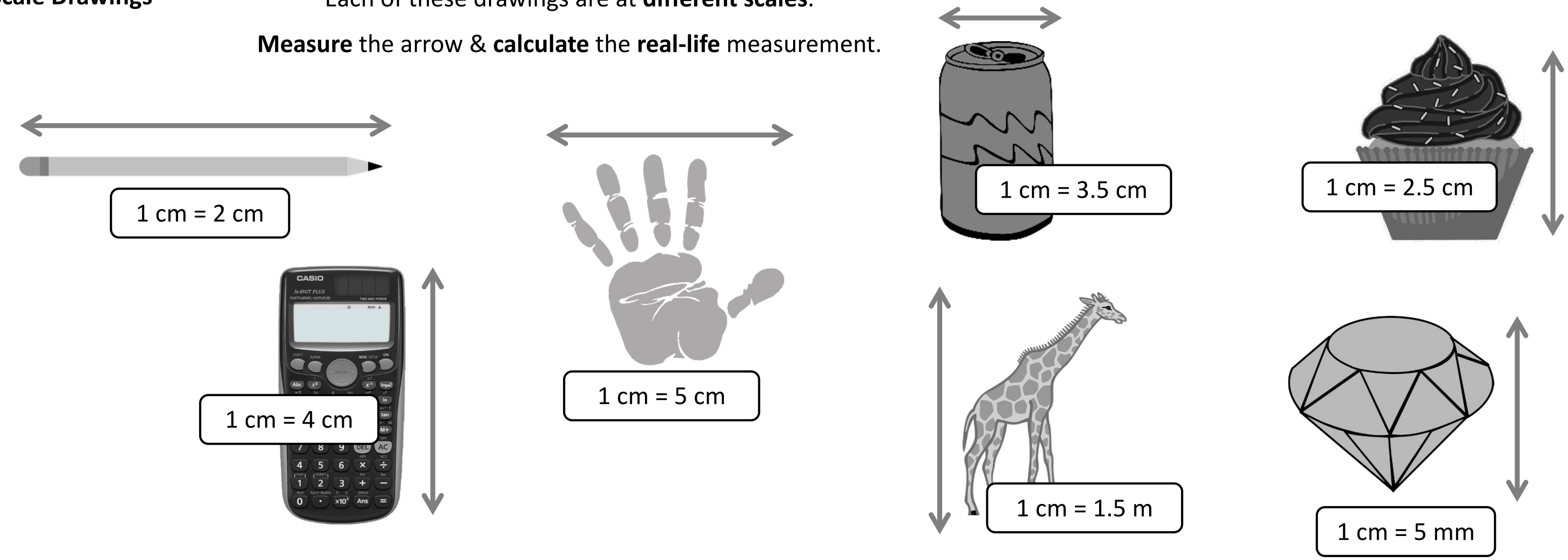
6 cm

Fluency Practice

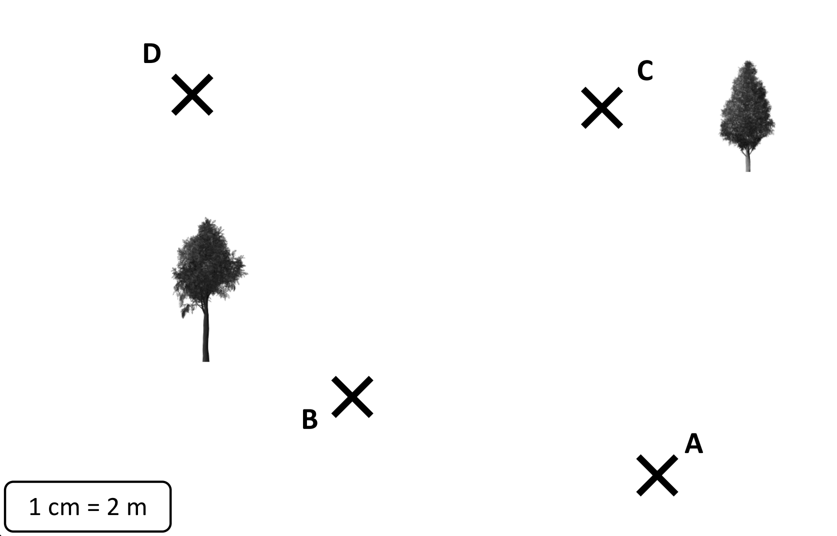
Scale Drawings

Each of these drawings are at **different scales**.

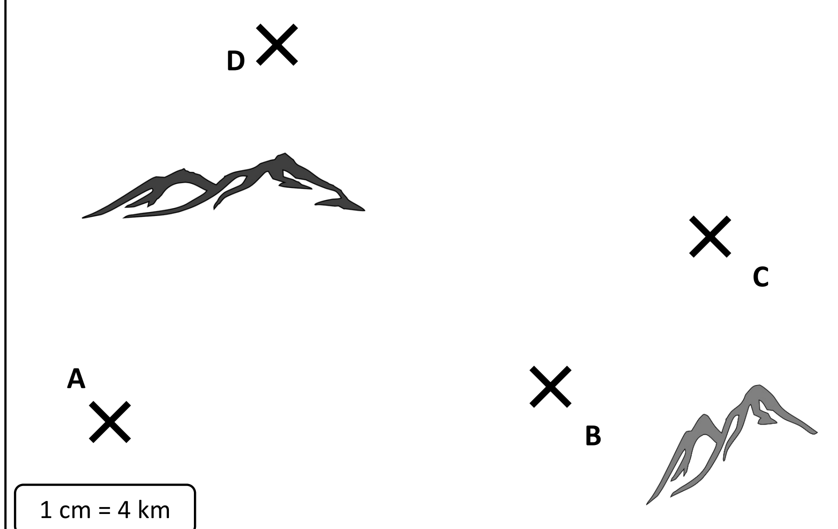
Measure the arrow & **calculate** the **real-life** measurement.



Mini-Map 1 Find the distance from **A** to **B**, from **B** to **C** & from **C** to **D**.



Mini-Map 2 Find the distance from **A** to **B**, from **B** to **C** & from **C** to **D**.



Worked Example	Your Turn
a) If a map is in the scale $1:500$ what real-life distance does 1 cm represent?	a) If a map is in the scale $1:2000$ what real-life distance does 1 cm represent?
b) If a map is in the scale $1:20{,}000$ what real-life distance does 1 cm represent?	b) If a map is in the scale $1:250{,}000$ what real-life distance does 1 cm represent?

Fluency Practice

Real-Life Maps

A **straight-line distance** is the shortest distance between 2 points.

It ignores roads, rivers & mountains.

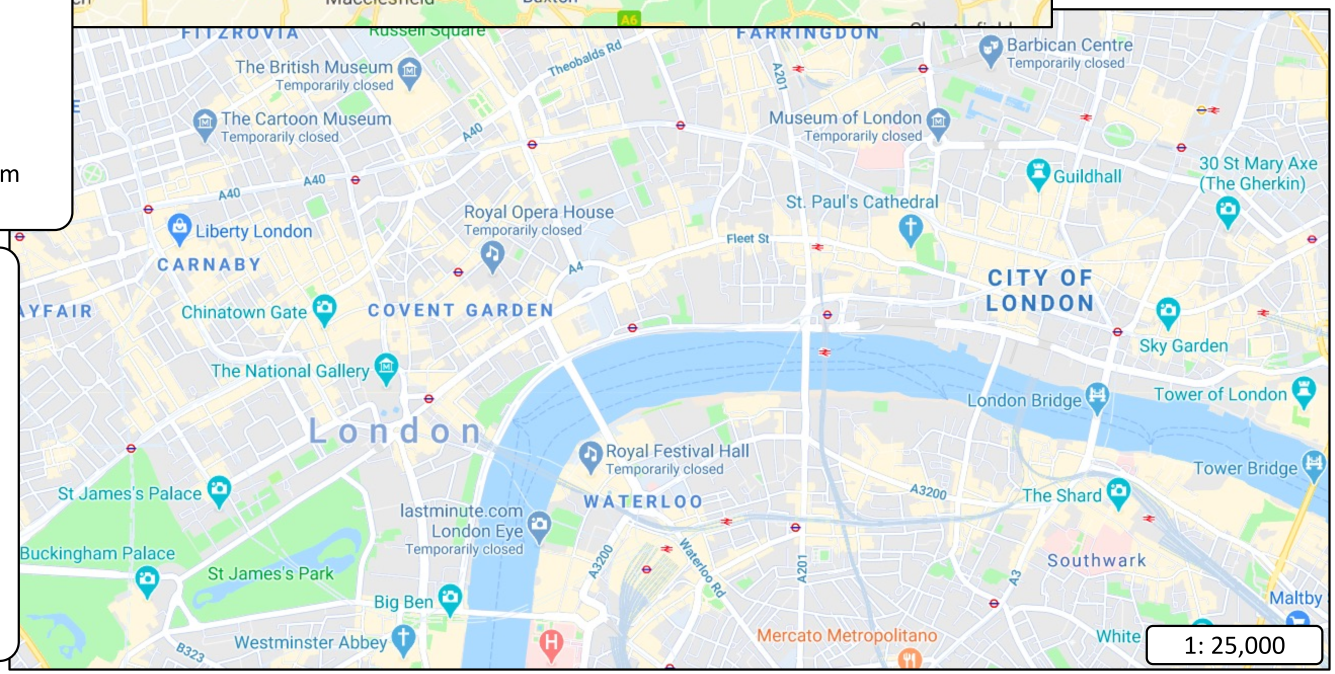
"As the crow flies."

To the nearest kilometre, find the **straight-line distance** of:

Manchester to Liverpool
Manchester to Wigan
Manchester to Sheffield

Bolton to Stockport
Sheffield to Liverpool

Estimate the distance, **by road**, from Manchester to Edale.



To the nearest 0.1 km, find the **straight-line distance** from:

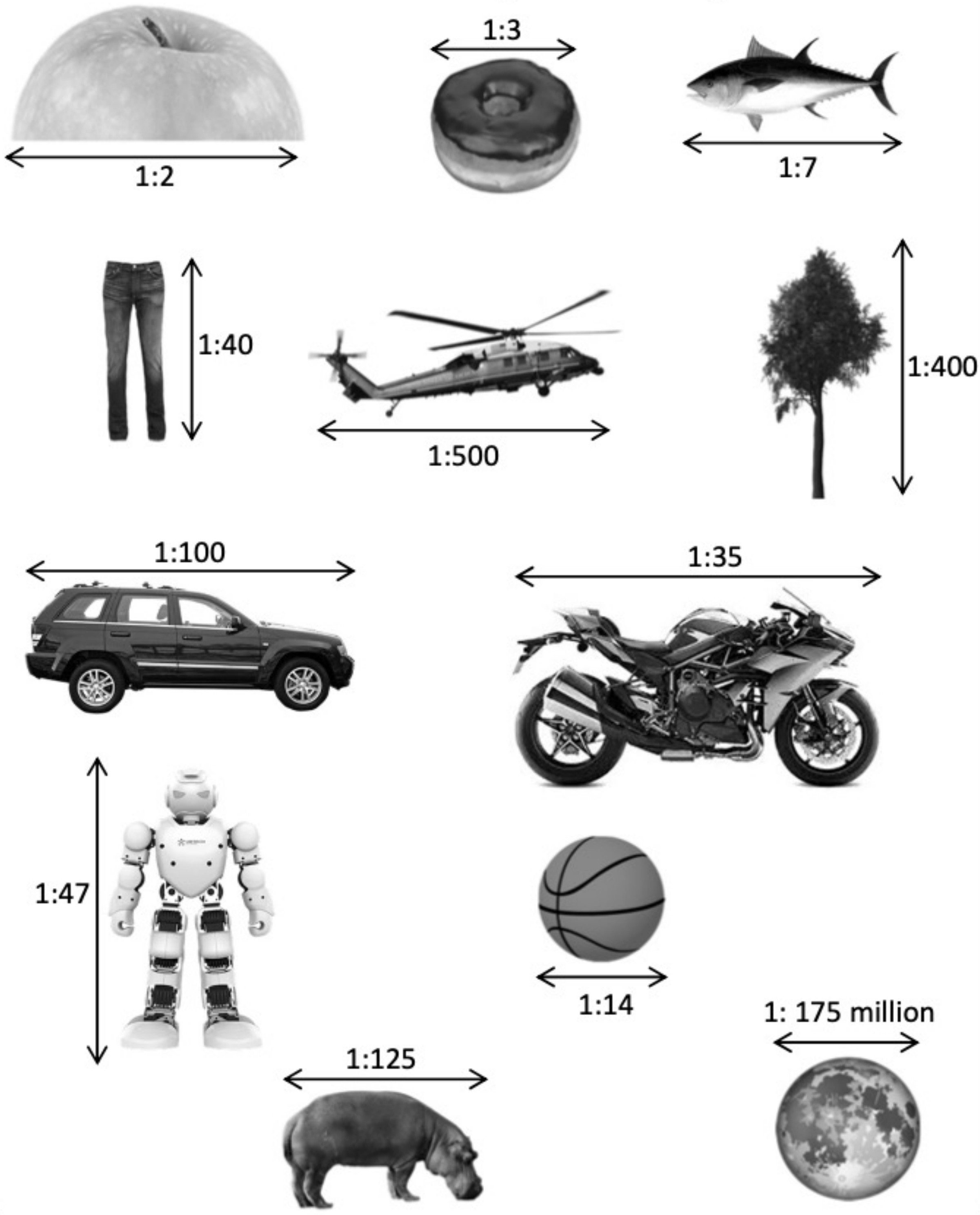
St. Paul's Cathedral to Big Ben
then to Buckingham Palace
then to The Royal Opera House
then to The Shard
then to Tower Bridge
then to Carnaby.

Estimate the length of the Thames on the map.

Fluency Practice

Scale Drawings

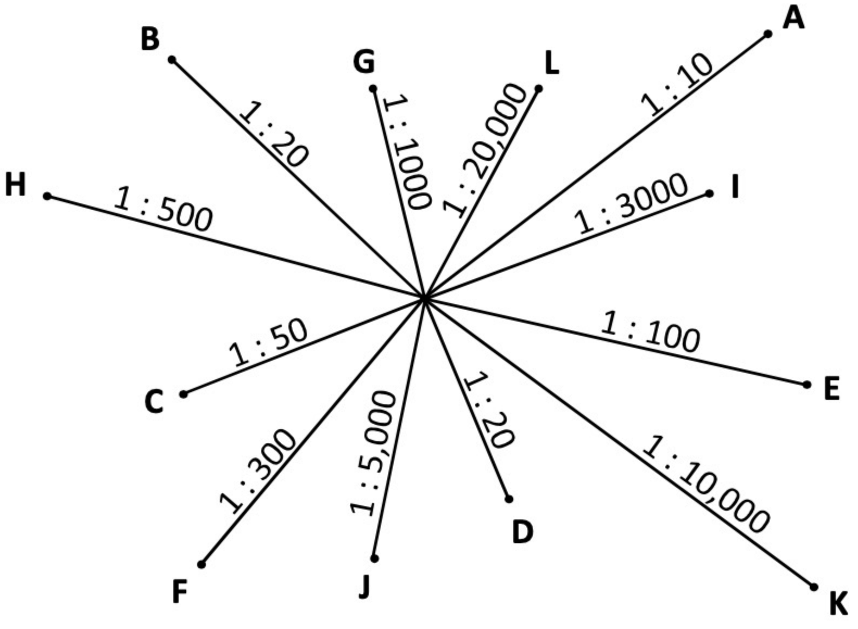
These are scale images of real objects.
Each object is at a different scale.
What is the marked length of the real object?



Fluency Practice

Scale Stars Each line is drawn at a **different scale**. 1

Measure the lines and calculate how long they would be in **real life**.

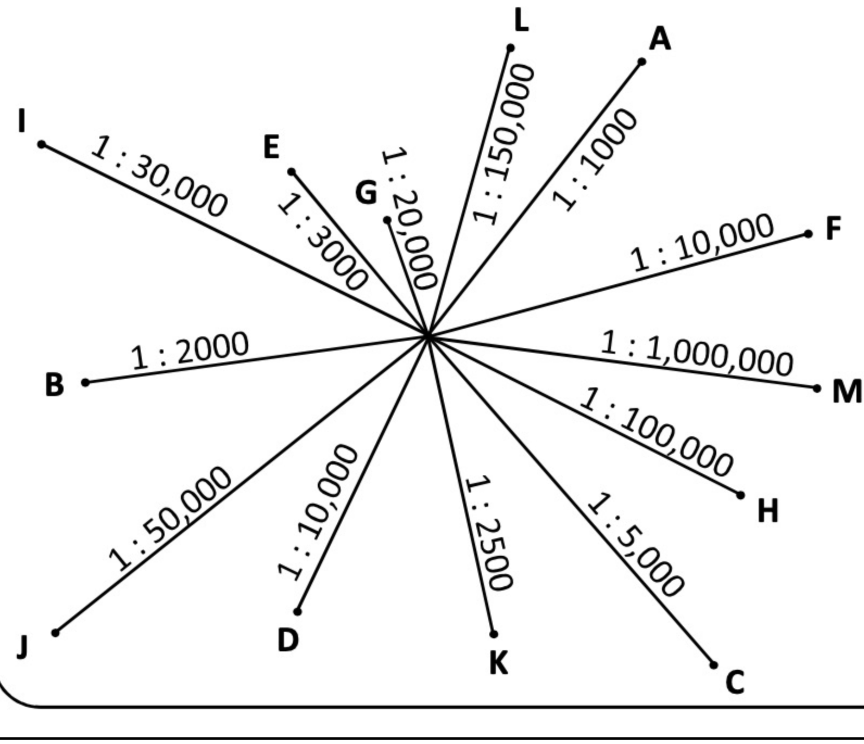


Answers

→ A ________ cm
→ B ________
→ C ________
→ D ________
→ E ________
→ F ________
→ G ________
→ H ________
→ I ________
→ J ________
→ K ________
→ L ________

Scale Stars Each line is drawn at a **different scale**. 2

Measure the lines and calculate how long they would be in **real life**.



Answers

→ A ________
→ B ________
→ C ________
→ D ________
→ E ________
→ F ________
→ G ________
→ H ________
→ I ________
→ J ________
→ K ________
→ L ________
→ M ________

1.8 Combining Ratios

In this section you will look at how to combine two ratios into one ratio.

Note that we are only able to combine ratios if one of the components is present in both of the ratios. If this is the case, we can combine the ratios by making the value for the component that appears in both of the ratios the same. We make the value for the component that appears in both of the ratios the lowest common multiple of the current values for that component in the two separate ratios. After we have made the values for the component that appears in both of the ratios the same, we can combine the ratios.

Worked Example	Your Turn
The ratio of $a : b$ is $2 : 3$ The ratio of $b : c$ is $1 : 4$ What is the ratio of $a : c$?	The ratio of $a : b$ is $2 : 5$ The ratio of $b : c$ is $1 : 4$ What is the ratio of $a : c$?

Worked Example	Your Turn
There are red, yellow and blue counters in a bag. Find the ratio Red : Yellow : Blue if (a) The ratio of Red : Yellow is $1 : 2$ and the ratio of Yellow : Blue is $2 : 3$ (b) The ratio of Red : Yellow is $1 : 5$ and the ratio of Yellow : Blue is $10 : 7$ (c) The ratio of Red : Yellow is $1 : 3$ and the ratio of Yellow : Blue is $8 : 5$	There are red, yellow and blue counters in a bag. Find the ratio Red : Yellow : Blue if (a) The ratio of Red : Yellow is $1 : 3$ and the ratio of Yellow : Blue is $3 : 4$ (b) The ratio of Red : Yellow is $2 : 5$ and the ratio of Yellow : Blue is $10 : 3$ (c) The ratio of Red : Yellow is $2 : 5$ and the ratio of Yellow : Blue is $7 : 1$

1.9 One Quantity Given

In this section you will look at how to find quantities when one quantity is given.

Worked Example	Your Turn
Anju and Kieran share some money in the ratio $5 : 2$. Anju receives £30. How much does Kieran receive?	Anju and Kieran share some money in the ratio $5 : 3$. Anju receives £30. How much does Kieran receive?

1.10 Difference Given

In this section you will look at how to find quantities when the difference is given.

Worked Example	Your Turn
Zach and Olivia share some money in the ratio $2 : 5$. Olivia receives £30 more than Zach. How much do they each receive?	Zach and Olivia share some money in the ratio $2 : 5$. Olivia receives £15 more than Zach. How much do they each receive?

1.11 Total Given

In this section you will look at how to find quantities when the total is given.

Worked Example	Your Turn
Divide 30 in the ratio $2 : 3$	Divide 45 in the ratio $8 : 1$

2 Algebra Recap

2.1 Collecting Like Terms

In this section you will look at collecting like terms.

Recall that like terms are two or more terms, each with the same variables, to the same power or with the same function applied.

Like Terms

$3p$	p	Like	Unlike
x^2	$3x^2$	Like	Unlike
x^2	$2x$	Like	Unlike
$-3\sqrt{x}$	$27\sqrt{x}$	Like	Unlike
$7a$	$7b$	Like	Unlike

$3a$	$3a$	Like	Unlike
a	$2a$	Like	Unlike
$2a$	$2A$	Like	Unlike
$-3a$	$2a$	Like	Unlike
$4a$	$4b$	Like	Unlike
$3a$	$3a^2$	Like	Unlike
$2a^2$	$7a^2$	Like	Unlike
$-3a^2$	$7a^2$	Like	Unlike
$2a^2$	$2a^{-2}$	Like	Unlike
2^a	a^2	Like	Unlike
x	$\sqrt{x}$	Like	Unlike
1	2	Like	Unlike

2.2 Multiplying Terms

In this section you will look at multiplying terms.

2.3 Dividing Terms

In this section you will look at dividing terms.

2.4 Substitution

In this section you will look at substitution.

3 Index Laws

Notation

Index/Exponent

$$2^4$$

Base

Power

We say 'two to the power of four'.

3.1 Multiplying

In this section you will look at how to use the index law for multiplying.

Complete the following:

$3^4 \times 3 =$

$3^4 \times 3^2 =$

$3^4 \times 3^3 =$

$3^4 \times 3^n =$

$3^m \times 3^n =$

Worked Example	Your Turn
Simplify a) $9^5 \times 9^2$ b) $9^5 \times 9^{-2}$	Simplify a) $8^6 \times 8^3$ b) $8^6 \times 8^{-3}$

Multiplying

In this section you will look at how to use the index law for multiplying.

Complete the following:

$x^3 \times x^2 =$

$x^3 \times x^3 =$

$x^3 \times x^4 =$

$x^3 \times x^n =$

$x^m \times x^n =$

Worked Example	Your Turn
Simplify a) $x^7 \times x^8$ b) $3x^4 \times 2x^5$	Simplify a) $x^9 \times x^2$ b) $4x^3 \times 5x^7$

3.2 Dividing

In this section you will look at how to use the index law for dividing.

Complete the following:

$2^4 \div 2 =$

$2^4 \div 2^2 =$

$2^4 \div 2^3 =$

$2^4 \div 2^n =$

$2^m \div 2^n =$

Worked Example	Your Turn
Simplify a) $9^5 \div 9^2$ b) $9^5 \div 9^{-2}$	Simplify a) $8^{12} \div 8^3$ b) $8^{12} \div 8^{-3}$

Dividing

In this section you will look at how to use the index law for dividing.

Complete the following:

$x^5 \div x =$

$x^5 \div x^2 =$

$x^5 \div x^3 =$

$x^5 \div x^n =$

$x^m \div x^n =$

Worked Example	Your Turn
Simplify a) $y^{12} \div y^4$ b) $12y^{11} \div 6y^7$ c) $\frac{5y^{11}}{12y^7}$	Simplify a) $p^{14} \div p^9$ b) $56y^4 \div 8y^2$ c) $\frac{8y^4}{56y^2}$

3.3 The Power Zero

In this section you will look at how to use the index law for the power zero.

Complete the following:

$2^4 =$

$2^3 =$

$2^2 =$

$2^1 =$

$2^0 =$

Worked Example	Your Turn
Simplify: a) $4x^0$ b) $x^4 \times x^0$ c) $\frac{x^9}{x^0}$ d) $x^0 \div x^{-2}$	Simplify: a) $8x^0$ b) $x^0 \times x^8$ c) $\frac{x^0}{x^{18}}$ d) $x^{-4} \div x^0$

3.4 Combined

In this section you will look at how to use the index laws combined.

Worked Example	Your Turn
Simplify $\dfrac{15x^9 \times 2x^3}{10x^4}$	Simplify $\dfrac{24x^{10}}{13x^5 \times 4x^2}$

3.5 Powers of Powers

In this section you will look at how to use the index law for powers of powers.

Complete the following:

$(2^2)^1 =$

$(2^2)^2 =$

$(2^2)^3 =$

$(2^2)^4 =$

$(2^2)^5 =$

$(2^2)^n =$

$(2^m)^n =$

Worked Example	Your Turn
Simplify $(2^4)^3$	Simplify$(3^4)^9$

Powers of Powers

In this section you will look at how to use the index law for powers of powers.

Complete the following:

$(y^3)^1 =$

$(y^3)^2 =$

$(y^3)^3 =$

$(y^3)^4 =$

$(y^3)^5 =$

$(y^3)^n =$

$(y^m)^n =$

Worked Example	Your Turn
Simplify a) $(c^4)^2$ b) $-(c^4)^2$ c) $(-c^4)^2$	Simplify a) $(c^4)^3$ b) $-(c^4)^3$ c) $(-c^4)^3$

Worked Example	Your Turn
Simplify a) $(3c^4)^2$ b) $(-3c^4)^2$	Simplify a) $(5c^{-4})^2$ b) $(-5c^{-4})^2$