## Year 8

## Mathematics Unit 9



Name:

Class:

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### 1.1 Direct Proportion

In this section you will look at direct proportion.

At a steady speed, a car uses 30 litres of petrol to travel 90 km . At the same speed, what distance could be travelled on 10 litres?

At a steady speed, a car uses 10 litres of petrol to travel 40 km . At the same speed, what distance could be travelled on 40 litres?

### 1.2 Recipes

In this section you will look at recipes.

## Worked Example

This is a list of ingredients for making a cake for 8 people.

Ingredients for 8 people:
70 g flour
120 g fruits
150 g rolled oats
100 ml water
70 g butter

Work out the amount of each ingredient needed to make a cake for 20 people.

This is a list of ingredients for making a cake for 6 people.

Ingredients for 6 people:
100 g flour
190 g chocolate
7 eggs
180 g fruits

Work out the amount of each ingredient needed to make a cake for 15 people.

### 1.3 Best Buys

In this section you will look at best buys.
Proportion calculations can be used to decide which items in a shop offer the best value. Many items sold in supermarkets show a price per item and a price per 100 g or per kg . This lets people compare products and get the best value for money.

Plants are sold in three different sizes of tray.

A small tray of 20 plants costs $£ 4.20$.
A medium tray of 40 plants costs $£ 7.20$.
A large tray of 70 plants costs $£ 13.30$.

Which size tray of plants is the best value for money?

Plants are sold in three different sizes of tray.

A small tray of 20 plants costs $£ 4.00$.
A medium tray of 40 plants costs $£ 10.80$.
A large tray of 90 plants costs $£ 9.00$.

Which size tray of plants is the best value for money?

### 1.4 Exchange Rates

In this section you will look at exchange rates.
The currency of the United Kingdom is the British pound, or pound sterling. When we refer to foreign currency, we mean the money that a different country uses such as baht in Thailand or rupees in India.

Not all currencies have the same value. We use exchange rates to convert from one currency to another.

Exchange rates are published in newspapers and online where the pound is matched against various currencies.
a) Phil goes on holidays. Phil changes $£ 640$ to euros.
The exchange rate is $£ 1=$ 1.14 euros. How many euros should Phil get?
b) Dave hired a car in Germany. The cost of hiring the car was 429 euros. The exchange rate is $£ 1=1.1$ euros. Work out the cost of hiring the car in pounds.
a) Alice hired a car in Greece. The cost of hiring the car was $£ 700$. The exchange rate is $£ 1=1.1$ euros. Work out the cost of hiring the car in euros.
b) Nina goes on holidays. Nina changes 147.60 euros to pounds. The exchange rate is $£ 1=1.23$ euros. How many pounds should Nina get?

### 1.5 Inverse Proportion

In this section you will look at inverse proportion.

In a school, 8 classrooms are required if each class
has 27 pupils. How many classrooms would be required if the class size has reduced to 18 ?

In a school, 4 classrooms are required if each class
has 21 pupils. How many classrooms would be required if the class size has reduced to 14 ?

## 2 Averages

### 2.1 Range

In this section you will look at working out the range from listed data.

The range is the difference between the largest and smallest values in a list.

Note: The range is not an average but a measure of spread.

## Advantages:

- Easy to calculate.


## Disadvantages:

- Affected by outliers (extreme values).

Find the range of:
$3,5,9,13,18$

Find the range of:
1,3, 7, 11, 16

### 2.2 Mode

In this section you will look at working out the mode from listed data.

The mode is the most common item in a set of data.

## Advantages:

- Is not affected by outliers (extreme values).
- Can be used with words.
- Always a possible value (e.g., an integer if the data must be integers).


## Disadvantages:

- May be more than one mode or no mode.
- Does not include all the data.

Find the mode of:
a) $5,3,2,9,13,3$
b) $9,13,5,2,3,18$

Find the mode of:
a) $3,2,19,14,10,2$
b) $10,19,5,3,14,4$

### 2.3 Median

In this section you will look at working out the median from listed data.

The median is the value at the middle of a numerically ordered list of values.

- Position of the median found by $\frac{n+1}{2}$ where $n$ is the number of values, for a list.


## Advantages:

- Unaffected by outliers (extreme values).


## Disadvantages:

- Does not include all the data.
- Cannot be used with words.

Find the median of:
a) $5,3,2,9,13$
b) $9,13,5,2,5,18$

Find the median of:
a) $3,2,19,14,10$
b) $10,19,5,3,14,4$

### 2.4 Mean

In this section you will look at working out the mean from listed data.

Then mean can be found by sharing the total of all the numbers in the list equally between them.

## Advantages:

- Includes all the data.


## Disadvantages:

- Affected by outliers (extreme values).
- Cannot be used with words.
- Not always a possible value.

Find the mean of:
$2,4,5,6,13$

Find the mean of:
$2,4,5,6,13,30$

### 2.5 Using Totals

In this section you will look at using the total given the mean.
Total $=$ Mean $\times$ Number of Items

## Worked Example

Find the missing number:
$5,1,10$, ?
Mean $=6$

Find the missing number:
$6,2,11$, ?
Mean $=6$

Four numbers have a mean of 10. Three of the numbers are $8,15,7$. What is the fourth number?

Five numbers have a mean of 10. Four of the numbers are $8,15,7,8$. What is the fifth number?

## Worked Example

The mean height of 14 players is 172 cm . A player with a height of 197 cm leaves the team. What is the new mean height of the team?

The mean height of 14 players is 127 cm . A player with a height of 142 cm leaves the team. What is the new mean height of the team?

## Worked Example

The mean score after six tests is 5. One more test is taken. After this test the mean score is 6 . What was the score on the final test?

The mean score after five tests is 6 . One more test is taken. After this test the mean score is 7. What was the score on the final test?

### 2.6 Combined Mean

In this section you will look at working out the combined mean given multiple sets of listed data.

A group of students take a test. The group consists of 24 boys and 16 girls. The mean mark for the boys is 36 . The mean mark for the girls is 33 . Calculate the mean mark for the whole group.

A group of students take a test. The group consists of 12 boys and 8 girls. The mean mark for the boys is 18 . The mean mark for the girls is 16.5 . Calculate the mean mark for the whole group.

## Worked Example

A group of men, women and children take a test. The mean score for women is 31.2. The mean score for children is 18.4. The mean score for all 80 people is 22.4 . Work out the mean score for men.

A group of men, women and children take a test. The mean score for women is 15.6. The mean score for children is 9.2. The mean score for all 40 people is 11.2 . Work out the mean score for men.

### 2.7 Determining List of Numbers

In this section you will look at determining list of numbers given information on the range, mode, median and mean.

## Worked Example

Write a list of five numbers with:

Mean $=4$

Median $=4$

Mode $=4$

Range $=4$

Write a list of five numbers with:

Mean $=5$

Median $=5$

Mode $=5$

Range $=5$

Write a list of four numbers with:

Mean $=4$

Median $=4$

Mode $=4$

Range $=4$

Write a list of four numbers with:

Mean $=5$

Median $=5$

Mode $=5$

Range $=5$

### 2.8 Comparing Data

In this section you will look at comparing data.
You compare two sets of data using a measure of central tendency (usually the mean) and a measure of spread (usually the range).

## Worked Example

Zayd plants two different types of tomato plant. He record the number of tomatoes that he picks from each plant every day for 10 days. His records are shown below:

Plant A: $4 \begin{array}{llllllllll}6 & 7 & 3 & 5 & 2 & 1 & 3 & 6 & 5\end{array}$

Plant $B: \begin{array}{llllllllll}5 & 6 & 7 & 6 & 8 & 9 & 6 & 7 & 8 & 9\end{array}$

Compare the two plants and recommend which type he should buy next year.

### 2.9 Deciding which Average to Use

In this section you will look at deciding which average is the best to use.

## Advantages and Disadvantages

Choosing an Appropriate Average to Use
Can you think of an example of a data set for each point?


## Worked Example

Charlie keeps a record of the number of carrier bags that he is given when he does his weekly shopping. The data he collects over 10 weeks is listed below:
$9, \quad 8, \quad 5, \quad 9, \quad 12, \quad 8, \quad 7, \quad 6, \quad 5, \quad 9$
a) Calculate: (i) the mean (ii) the median (iii) the mode
b) Explain why the mean is not very useful in this context.
c) Which value might be used by an environmental group who thinks that supermarkets cause pollution by giving out too many carrier bags?
d) Which value might be used by a shopper who thinks that the supermarket doesn't give him enough carrier bags for his shopping?

## 3 Coordinates

### 3.1 Plotting Coordinates

In this section you will look at plotting coordinates.
Coordinates are pair of numbers written in the form $(x, y)$ where $x$ is the amount moved horizontally, and $y$ the amount moved vertically from the origin on a graph. The two values are referred to, in order, as the $x$-coordinate and the $y$-coordinate.

Plot the coordinates:
$(2,5)$
$(2,-5)$
$(-2,5)$
$(-2,-5)$


Plot the coordinates:
$(3,4)$
$(3,-4)$
$(-3,4)$
$(-3,-4)$


### 3.2 Reading Coordinates

In this section you will look at reading coordinates.
Coordinates are pair of numbers written in the form $(x, y)$ where $x$ is the amount moved horizontally, and $y$ the amount moved vertically from the origin on a graph. The two values are referred to, in order, as the $x$-coordinate and the $y$-coordinate.
3.3 Coordinates with Shapes

In this section you will look at coordinates with shapes.

## Fluency Practice

The shapes for each question are congruent. Calculate the coordinates of points $\mathbf{A}$ to $\mathbf{F}$.


The shapes for each question are congruent. Calculate the coordinates of points $\mathbf{G}$ to $\mathbf{L}$.


## Fluency Practice

Coordinates \& Shape
Find the labelled coordinates, or coordinate. For each diagram, the shapes are congruent.





## Fluency Practice

What are the coordinates of the points $\mathbf{A}$ to $\mathbf{F}$ ?

## 


(




## 4 Charts

### 4.1 Bar Charts

In this section you will look at bar charts.
A bar chart is a chart that uses the heights of rectangles to represent the frequency of qualitative data (data that can only be written in words and not numbers).

- Each rectangle should be of equal width.
- The rectangles should be separated by gaps of equal width.
- The frequency axis should start from 0.
- Each axis should have a title.

The bar chart shows the colour of cars in a car park:


The bar chart shows the hair colour of students in a class:


- What is the most common hair colour?
- How many students had black hair?
- How many more students had red hair than blonde hair?
- How many students are in the class?
- What fraction of the students have brown hair?

Spot the mistakes in the bar chart:

| Football Team | Frequency |
| :---: | :---: |
| Arsenal | 3 |
| Chelsea | 5 |
| Liverpool | 4 |
| Man City | 8 |
| Man United | 8 |

Bar Chart showing the favourite


Spot the mistakes in the bar chart:

| Colour | Frequency |
| :---: | :---: |
| Blue | 14 |
| Red | 9 |
| Silver | 17 |
| White | 12 |
| Green | 4 |

Colours of cars in the school car park


Worked Example
Your Turn
Draw a bar chart for the data:

| Sport | Frequency |
| :---: | :---: |
| Cricket | 4 |
| Football | 3 |
| Hockey | 6 |
| Rugby | 1 |

Draw a bar chart for the data:

| Colour | Frequency |
| :--- | :---: |
| Blue | 15 |
| Green | 8 |
| Red | 21 |
| Yellow | 3 |

### 4.2 Pictograms

In this section you will look at pictograms.
A pictogram is a chart that uses a number of icons to represent the frequency of qualitative data

- Only one labelled axis, which shows the categories.
- The size of each icon should be roughly the same.
- The icons should line up.
- A key is required to show the frequency.
- The same symbol needs to be used for all categories.


## Worked Example

Your Turn
The pictogram shows the type of books a person read last year.

Key represents 8 books

| Romance |  |
| :--- | :--- |
| Crime |  |
| Horror |  |
| Factual |  |

How many books were:
Romance

The pictogram shows the number of hours of sunshine in a day across various cities
= 2 hours of sunshine


How many hours of sunshine were there in:

Dublin

Belfast

Glasgow


### 4.3 Pie Charts

In this section you will look at pie charts.
A pie chart is a chart that uses sectors of a circle to represent the relative frequency of different categories, values or groups.

Pie charts should be used when we interested in proportions/percentage/fractions of some total and are less concerned with the frequencies.

- The fraction of $\frac{\text { frequency of category }}{\text { total of frequencies }}$ is equivalent to the fraction $\frac{\text { sector angle }}{360^{\circ}}$.
- Each sector should be labelled, or a key used.
- It is conventional to draw the first slice of a pie from 12 o'clock and the slices are then arranged clockwise.

Pie charts have been around since William Playfair created his Statistical Breviary of 1801. They were later popularised by Florence Nightingale.


Turkifh Empire


| Worked Example | Your Turn |
| :---: | :---: |
| A group of 720 people were asked which rugby team they support. <br> How many supported: <br> Ireland <br> England <br> Wales <br> Scotland | There are 1440 counters in a bag. Each is white, red or black. <br> How many counters are: <br> White <br> Red <br> Black |

The table shows the number of ice creams sold in a day. In total 120 were sold. Draw a pie chart for the data.

| Flavour | Number sold |
| :---: | :---: |
| Vanilla | 20 |
| Chocolate | 40 |
| Strawberry | 24 |
| Honeycomb | 24 |
| Mint | 12 |



The table shows the holiday destinations of 60 people. Draw a pie chart for the data.

| Destination | Number of people |
| :---: | :---: |
| Italy | 15 |
| Portugal | 10 |
| Spain | 12 |
| France | 23 |



Draw a pie chart for the data.

| Flavour | Percentage |
| :---: | :---: |
| Vanilla | $20 \%$ |
| Chocolate | $10 \%$ |
| Strawberry | $50 \%$ |
| Honeycomb | $5 \%$ |
| Lemon | $15 \%$ |



Draw a pie chart for the data.

| Drink | Percentage |
| :---: | :---: |
| Cola | $10 \%$ |
| Water | $50 \%$ |
| Lemonade | $40 \%$ |



### 4.4 Stem and Leaf Diagrams

In this section you will look at stem and leaf diagrams.
A stem and leaf diagram is a simple but effective way of showing data. It puts the data into order, puts it into classes (groups) and we can quickly see patterns. As the data is in order it is also useful for finding averages and the range.

Suppose this "stem and leaf diagram" represents the lengths of beetles.


These numbers (the 'leaves') represent the second.

| Worked Example | Your Turn |
| :---: | :---: |
| Draw an ordered stem and leaf diagram for this data: <br> $\begin{array}{lllll}12 & 21 & 13 & 31 & 53\end{array}$ <br> $\begin{array}{lllll}47 & 29 & 21 & 18 & 46\end{array}$ <br> $21 \quad 5345$ | Draw an ordered stem and leaf diagram for this data: |
| Work out the mode | Work out the mode |
| Write down the median | Write down the median |
| Work out the mean (1dp) | Work out the mean (1dp) |
| Work out the range | Work out the range |


| Worked Example | Your Turn |
| :---: | :---: |
| Draw an ordered stem and leaf diagram for this data: | Draw an ordered stem and leaf diagram for this data: |
| Work out the mode | Work out the mode |
| Write down the median | Write down the median |
| Work out the mean (1dp) | Work out the mean (1dp) |
| Work out the range | Work out the range |

### 4.5 Scatter Diagrams

In this section you will look at scatter diagrams

## Scatter Graphs can show a

 relationship between two variables.
...such as
people's height and weight.
...or the number of staff working in KFC and the wait time for food.
...or the distance people live from work and their best score in darts.

## Correlation

If the two variables have a relationship we call it correlation.

There are different types of correlation:


Sometimes you might be asked to explain the correlation in context.

"When there are more staff working, you wait less."
"There is no relationship between how far people live from work and their darts ability."

## Fluency Practice



Compete the sentences using positive/negative/no and then increase/decrease/not affected.

1. There is a $\qquad$ correlation between Physics and Maths test scores. As the Maths test results increase the Physics test results $\qquad$
2. There is a $\qquad$ correlation between car engine size and petrol consumption.. As the car engine size increases the petrol consumption
$\qquad$
3. There is $\qquad$ correlation between KS3 results and height. As the KS3 results increase the height of the person is $\qquad$
4. There is a $\qquad$ correlation between outside air temperature and the heating bill. As the air temperature increases the heating bill
$\qquad$
5. There is a $\qquad$ correlation between the daily hours of sunshine and sales of sun cream. As the hours of sunshine increase sales of sun cream $\qquad$
6. There is a $\qquad$ correlation between the age of a car and its value. As the car gets older its value $\qquad$

## Correlation Strength

## Correlation can be strong or weak.

If the correlation is strong, all the points will closely follow a straight line.


Strong correlation

If the correlation is weak, the points will follow the line more loosely.

correlation

## Fluency Practice




Strong
Moderate
Weak


## Line of Best Fit

## We can show the correlation more clearly by drawing a Line of Best Fit.

This should pass through the middle of all the points (but does not have to touch any of the points).


## Fluency Practice



Yes / No



Yes / No



Yes / No


## Drawing and Interpreting Scatter Graphs

## We can use the Line of Best Fit to make predictions of other results.

For example, we can estimate:
...someone's height if we know their weight is 60 kg .

...or the wait time in KFC if we know they have 10 staff on today.


## Worked Example

The scatter graph shows the height and arm span of ten students.

a) Describe the correlation.
b) Another student is 174 cm tall and has an arm span of 180 cm . Plot this on the graph.
c) Another student is 142 cm tall. Estimate the arm span of this student.

## Your Turn

The scatter graph shows the value of cars and their age.

a) Describe the correlation.
b) Another car is 6 years old and worth $£ 1500$. Plot this on the graph.
c) Another car is 4 years old. Estimate its value.

## Fluency Practice

Question 1: Plot the following information as scatter graphs
(a)

| Maths score | 9 | 13 | 6 | 18 | 11 | 4 | 15 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Physics score | 10 | 13 | 5 | 20 | 8 | 5 | 12 | 14 |

(b)

| Age, years | 4 | 7 | 2 | 4 | 1 | 9 | 3 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cost, $£$ | 6000 | 3000 | 7500 | 5000 | 8000 | 1500 | 6000 | 4000 |

(c)

| Height, cm | 157 | 160 | 148 | 160 | 177 | 156 | 166 | 170 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Weight, kg | 53 | 60 | 44 | 53 | 54 | 60 | 54 | 70 |

(d)

| Distance, miles | 2.5 | 0.8 | 1.2 | 4.1 | 2.8 | 3.3 | 3.7 | 1.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cost | $£ 3.20$ | $£ 1.40$ | $£ 1.80$ | $£ 4.40$ | $£ 3.00$ | $£ 3.60$ | $£ 4.80$ | $£ 2.40$ |

Question 2: What type of correlation does each scatter graph show below
(a)

(b)

(c)

(d)


## Fluency Practice

(e)

(f)


Question 3: Describe the relationships shown in each scatter graph in Question 2.

## Outliers

Scatter plots often have a pattern. We call a data point an outlier if it doesn't fit the pattern.

## Fluency Practice

1. The scatter graph shows the cost and age of some rare books.
a)Circle the outlier.

b) Describe the correlation.
2. The scatter graph shows the number of accidents per million miles for drivers of different ages.
a)Circle the outlier.

b) Describe the correlation.

## Interpolation vs Extrapolation

When we use our line of best fit to estimate a value inside the range of our data, this is known as interpolation.

When we use our line of best fit to estimate a value outside the range of our data, this is known as extrapolation.

## MY HOBBY: EXTRAPOLATING



## Fluency Practice

Question 1: The scatter graph below shows information about the number of hours spent revising for a test and the test result for a group of 8 students.

(a) Daisy spent 7 hours revising for the test. What is Daisy's test score?
(b) Harry's test score was 30\%. How many hours did Harry spend revising?
(c) Draw a line of best fit.
(d) Another student spent 6 hours revising for the test.

Find an estimate of their test score.
(e) Explain why it might not be sensible to use the scatter graph to estimate the score for a student that spent 15 hours revising.

## Fluency Practice

Question 2: The scatter graph shows information about the cost of renting apartments and their distance from London.

(a) Describe the relationship shown in the scatter graph.
(b) Draw a line of best fit on the diagram.
(c) Estimate the cost of renting an apartment 40km from London.
(d) Victor has $£ 1100$ to spend on rent. Estimate how close he could live to London.
(e) Explain why it might not be sensible to use the scatter graph to estimate the price of rent for a property that is 250 km from London.

Question 3: The students in a class sit a non-calculator and a calculator maths paper.


## Fluency Practice

(a) What type of correlation does the scatter graph show?
(b) Draw a line of best fit.
(c) Philip was absent for the calculator paper, but he scored 80 in the non-calculator paper. Use your line of best fit to predict his calculator paper score.
(d) Neil was absent for the non-calculator paper, but he scored 60 in the calculator paper. Use your line of best fit to predict his non-calculator paper score.

Question 4: Mr Hughes is a plumber.
The scatter graph shows the cost and the length of his last 10 jobs.

(a) Draw a line of best fit
(b) For one job Mr Hughes needed to replace an expensive part that he fitted quickly. How long did that job last?
(c) Estimate the cost of a job lasting 3.5 hours.
(d) A job costs $£ 120$, estimate the length of the job.

