



**KING EDWARD VI  
HANDSWORTH GRAMMAR  
SCHOOL FOR BOYS**



**KING EDWARD VI  
ACADEMY TRUST  
BIRMINGHAM**

**Year 8**

**2023**

**Mathematics**

**2024**

**Unit 7 Tasks – Part 1**

**DO NOT WRITE INSIDE**



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**2024**

**Unit 7 Tasks – Part 2**

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**Mathematics**

**2024**

**Unit 7 Tasks – Part 3**

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# 1 Prime Factorisation

# Fluency Practice

Question 1: Write each of these numbers as the product of their prime factors.

- (a) 10      (b) 12      (c) 20      (d) 18      (e) 16      (f) 30      (g) 100  
(h) 26      (i) 24      (j) 27      (k) 42      (l) 33      (m) 38      (n) 64

Question 2: Write each of these numbers as the product of their prime factors. Give your answers in index form.

- (a) 36      (b) 40      (c) 28      (d) 48      (e) 80      (f) 200      (g) 75  
(h) 32      (i) 105      (j) 81      (k) 52      (l) 242      (m) 108      (n) 500

Question 3: Some numbers have been written as products of their prime factors. Work out each number.

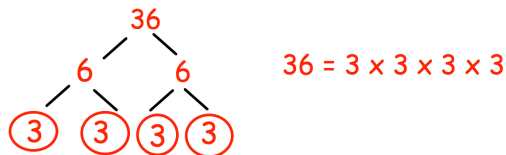
- (a)  $2 \times 7$       (b)  $2 \times 3 \times 5$       (c)  $2 \times 5 \times 11$       (d)  $2 \times 2 \times 2 \times 3$   
(e)  $2^2 \times 5$       (f)  $3 \times 5^2$       (g)  $2^3 \times 3^2$       (h)  $3^2 \times 11$   
(i)  $5^4$       (j)  $2^4 \times 5^2$       (k)  $3^3 \times 13$       (l)  $7 \times 17^2$

Question 4: Write each of these numbers as the product of their prime factors.

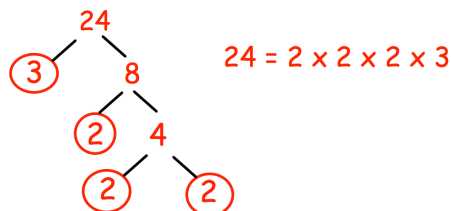
- (a) 9000      (b) 235      (c) 392      (d) 715      (e) 444      (f) 792      (g) 5625

Question 3: Ashley has completed his homework. Can you spot any mistakes?

Express 36 as a product of its prime factors.



Write 24 as the product of its prime factors. Give your answer in index form.



# Fluency Practice

Write the following as a product of prime factors.

1.) 140

8.) 44

2.) 48

9.) 63

3.) 99

10.) 36

4.) 54

11.) 70

5.) 45

12.) 72

6.) 30

13.) 165

7.) 66

14.) 135

15.) 180

<b>S:</b> $2 \times 3 \times 5$	<b>M:</b> $3^2 \times 11$
<b>B:</b> $3 \times 5 \times 11$	<b>M:</b> $2^3 \times 3^2$
<b>I:</b> $2 \times 3 \times 11$	<b>P:</b> $2 \times 3^3$
<b>N:</b> $2^2 \times 3^2$	<b>C:</b> $2^2 \times 5 \times 7$
<b>O:</b> $3^2 \times 5$	<b>T:</b> $2^2 \times 11$
<b>O:</b> $2^4 \times 3$	<b>E:</b> $3^2 \times 7$
<b>E:</b> $3^3 \times 5$	<b>U:</b> $2 \times 5 \times 7$
	<b>R:</b> $2^2 \times 3^2 \times 5$

# Intelligent Practice

		Product of prime factors
1.	18	
2.	90	
3.	180	
4.	60	
5.	360	
6.	240	
7.	24	
8.	12	
9.	144	
10.	1296	

11.	89	
12.	91	
13.	93	
14.	95	
15.	97	

16.	$3^2$	Product of prime factors
17.	$5^2$	
18.	$15^2$	
19.	900	

20.	$2^3$	
21.	$10^3$	
22.	$20^3$	
23.	216 000	

24.	$14^2$	
25.	$42^2$	
26.	$126^2$	
27.	$126^3$	
28.	$126^4$	
29.	$126^n$	
30.	$63^n$	



## Extension

Seven students were asked the following:

**Express 45 as a product of its prime factors**

Here are their solutions:

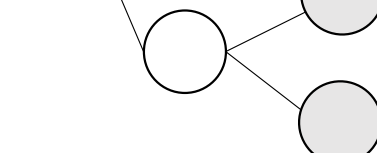
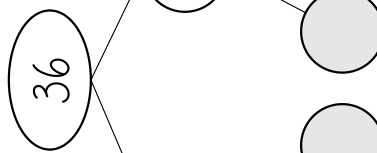
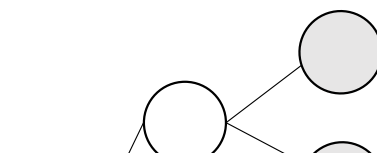
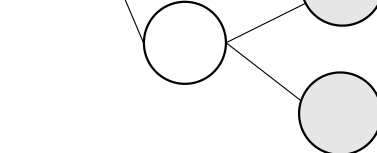
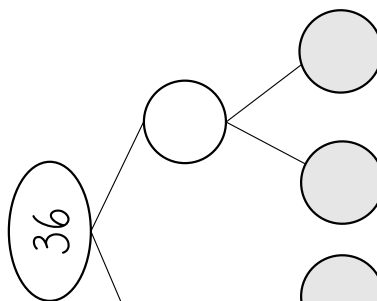
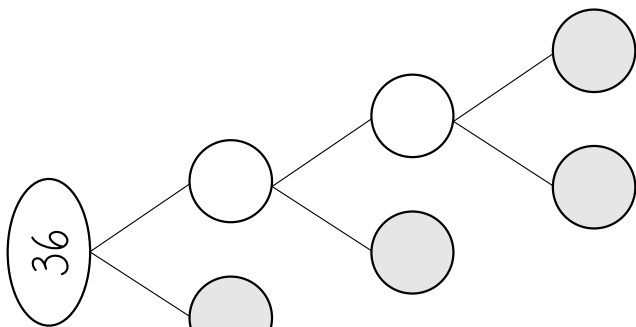
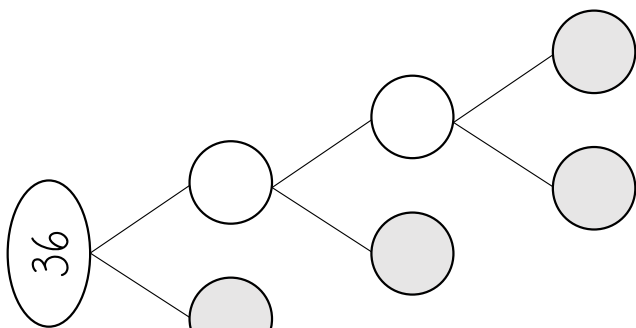
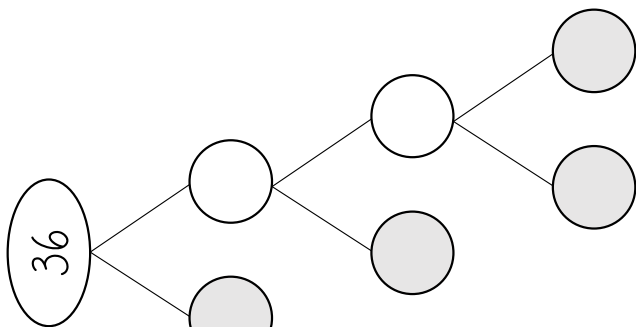
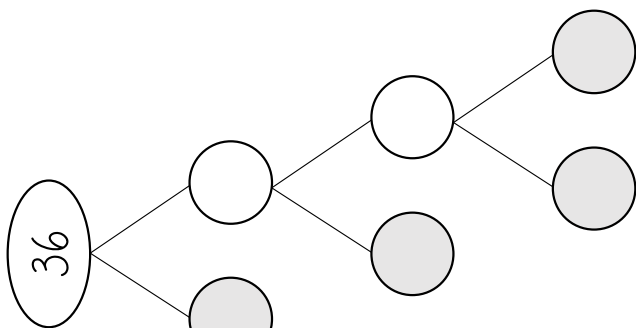
Ashley	$5 \times 9$
Beverly	$3 \times 5 \times 3$
Caitlin	$3 + 3 + 5$
Des	3, 3, 5
Ezra	$3^2 \times 5$
Fatima	3 and 5
Gavin	1, 3, 5, 9, 15, 45

Two answers are correct, which ones?

Can you explain the misconception for each of the others?

# Extension

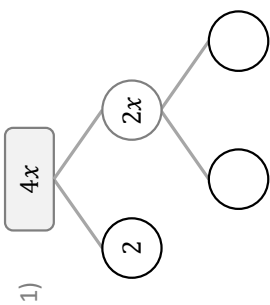
Can you find all the unique factor trees for 36?



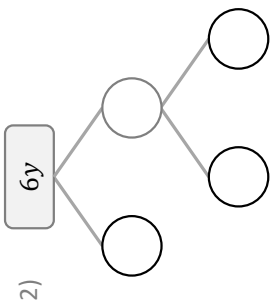
# Extension

**Prime Factorisation... with Algebra!**

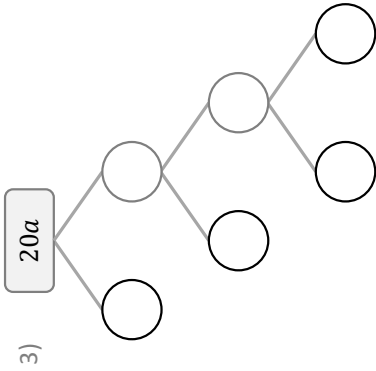
Complete each Prime Factorisation Tree, then write the expression as a product of prime factors.



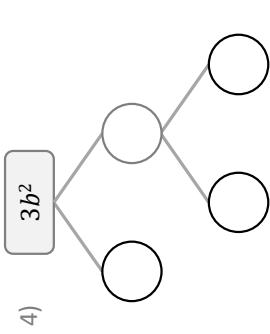
$4x = 2 \times 2 \times x$  \_\_\_\_\_



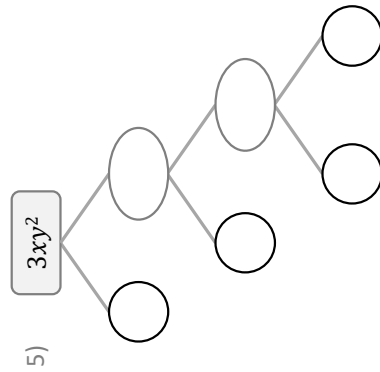
$6y =$  \_\_\_\_\_



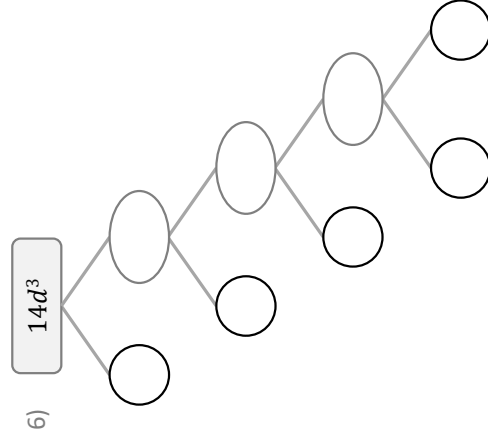
$20a =$  \_\_\_\_\_



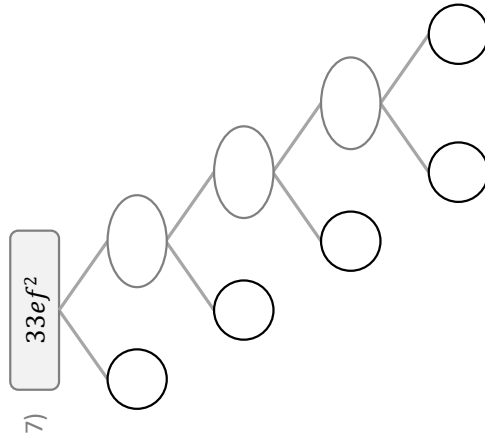
$3b^2 =$  \_\_\_\_\_



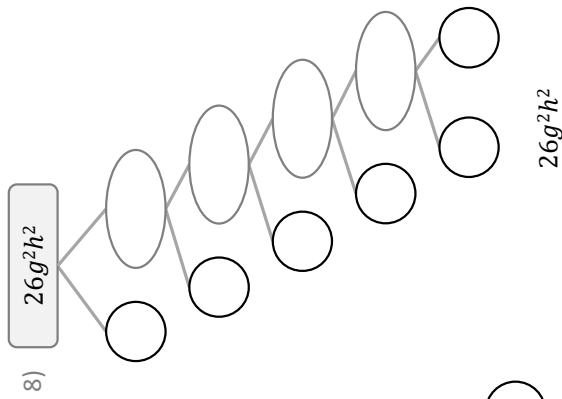
$3xy^2 =$  \_\_\_\_\_



$14d^3 =$  \_\_\_\_\_



$33ef^2 =$  \_\_\_\_\_



$26g^2h^2 =$  \_\_\_\_\_

9)  $49t^2 =$

10)  $34j^4 =$

11)  $121kl^2m =$

12)  $550n^3p^2 =$

# Problem Solving

Draw a Prime Factor Tree for each number. Any prime factors they have in common go in the middle, shaded squares. All other prime factors go to the sides.

<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">□ 21</div> <div style="text-align: center;">■ 30</div> <div style="text-align: center;">□ □</div> </div>	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">□ 66</div> <div style="text-align: center;">■ ■</div> <div style="text-align: center;">150</div> <div style="text-align: center;">□ □</div> </div>
<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">□ □ □</div> <div style="text-align: center;">120</div> <div style="text-align: center;">■ 84</div> <div style="text-align: center;">□</div> </div>	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">□ 45</div> <div style="text-align: center;">■ ■</div> <div style="text-align: center;">126</div> <div style="text-align: center;">□ □</div> </div>
<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">□ □</div> <div style="text-align: center;">110</div> <div style="text-align: center;">■ 42</div> <div style="text-align: center;">□ □</div> </div>	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">□ □</div> <div style="text-align: center;">60</div> <div style="text-align: center;">■ ■</div> <div style="text-align: center;">770</div> <div style="text-align: center;">□ □</div> </div>
<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">□ □</div> <div style="text-align: center;">165</div> <div style="text-align: center;">■ 154</div> <div style="text-align: center;">□ □</div> </div>	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">□ 78</div> <div style="text-align: center;">■ ■</div> <div style="text-align: center;">130</div> <div style="text-align: center;">□</div> </div>
<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">□ □</div> <div style="text-align: center;">175</div> <div style="text-align: center;">■ 182</div> <div style="text-align: center;">□ □</div> </div>	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">□ □</div> <div style="text-align: center;">120</div> <div style="text-align: center;">■ ■ ■</div> <div style="text-align: center;">84</div> <div style="text-align: center;">□</div> </div>

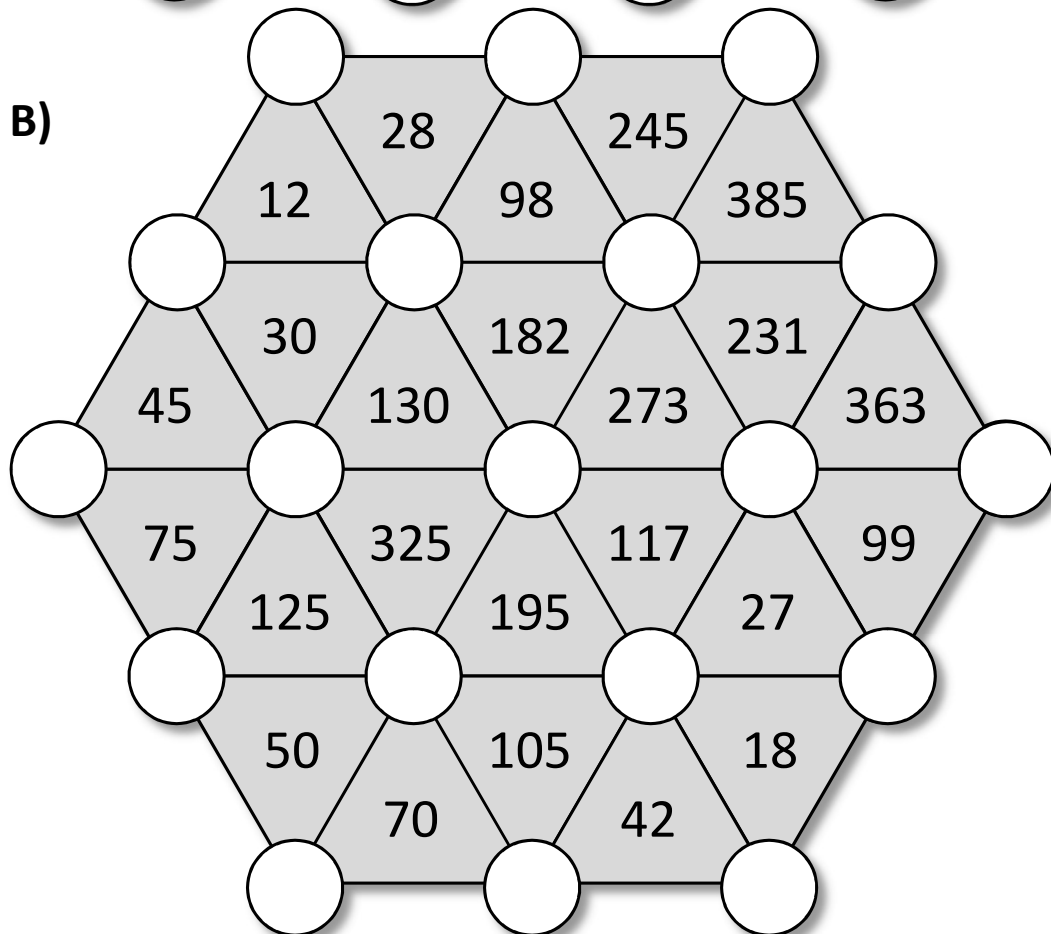
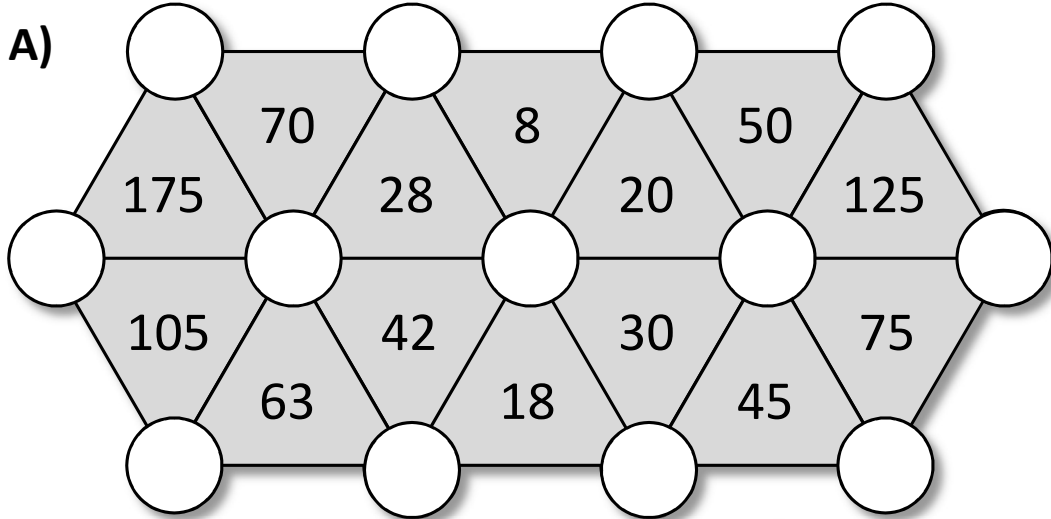
# Problem Solving

1

## Prime Factorisation Grids

Each circle contains the **prime factors** of the number in the triangle.

Can you complete each puzzle?

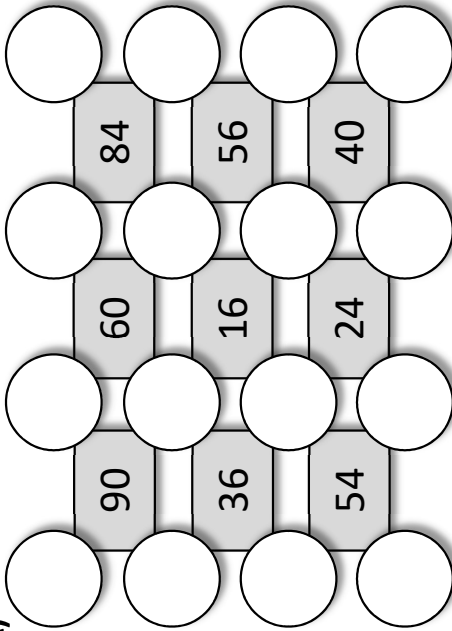


# Problem Solving

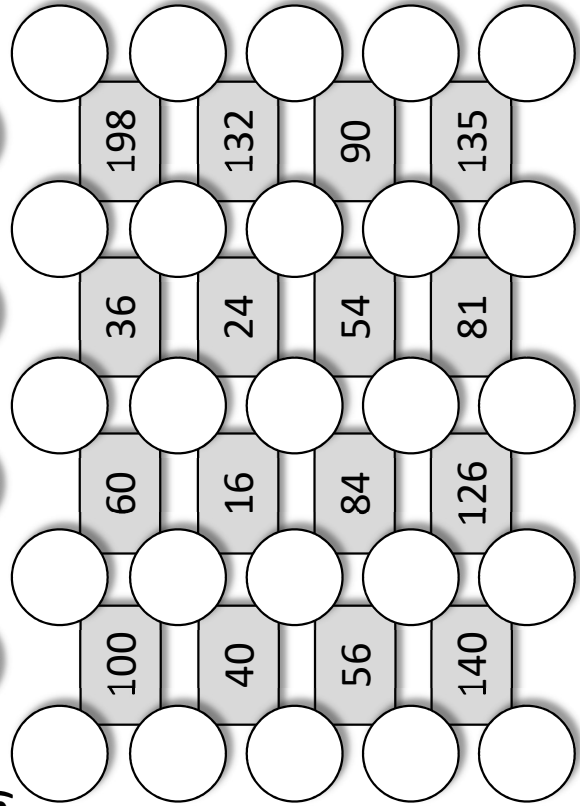
## Prime Factorisation Grids

Each circle contains the **prime factors** of the number in the box.  
Can you complete each puzzle?

A)



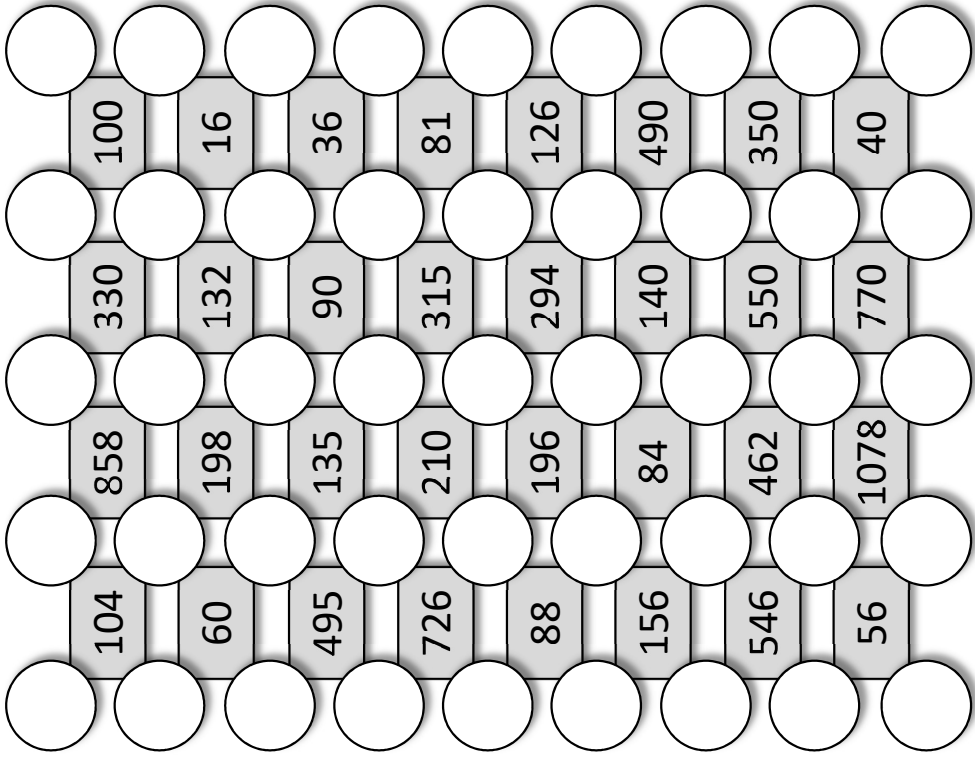
B)



## Prime Factorisation Grid

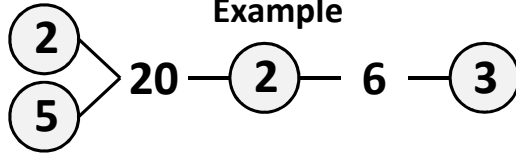
Each circle contains the **prime factors** of the number in the box.  
Can you complete the puzzle?

C)

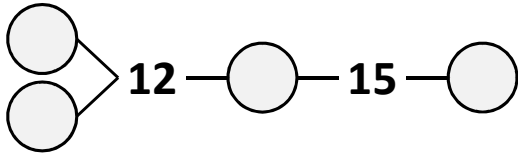


# Problem Solving

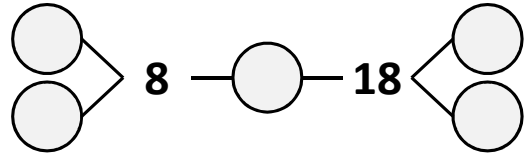
## Prime Factorisation Puzzles



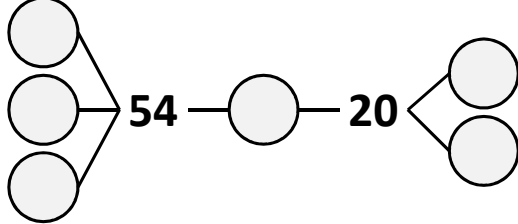
A)



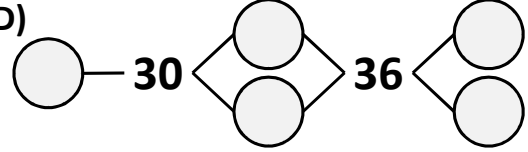
B)



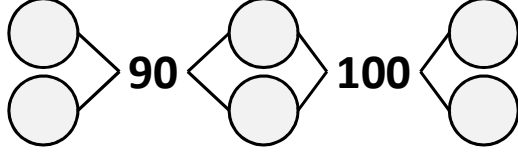
C)



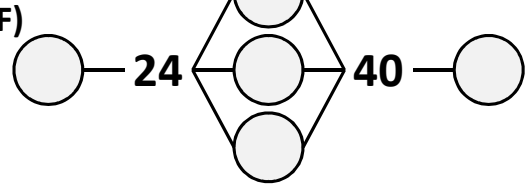
D)



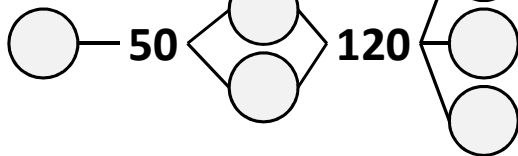
E)



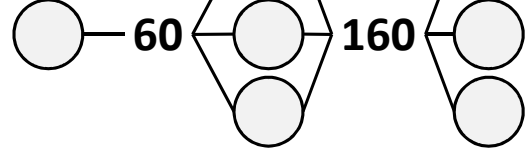
F)



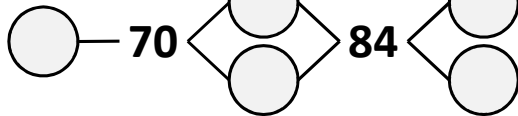
G)



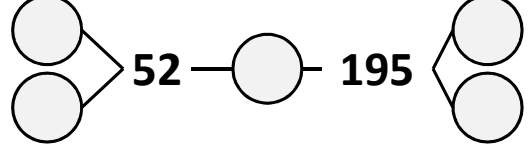
H)



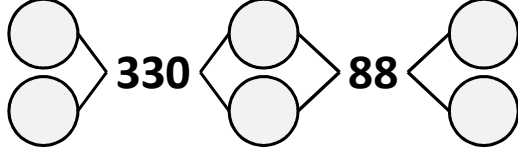
I)



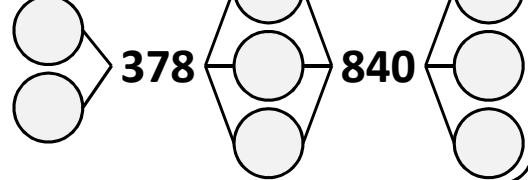
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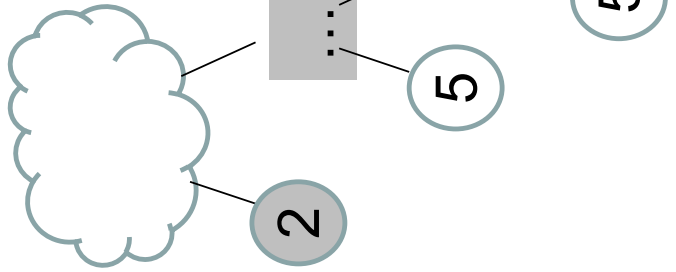
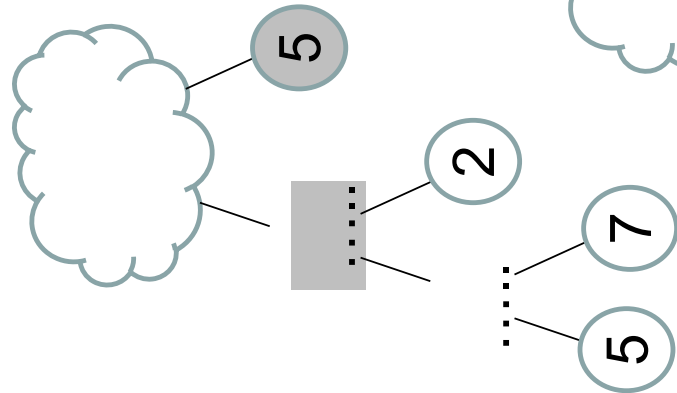
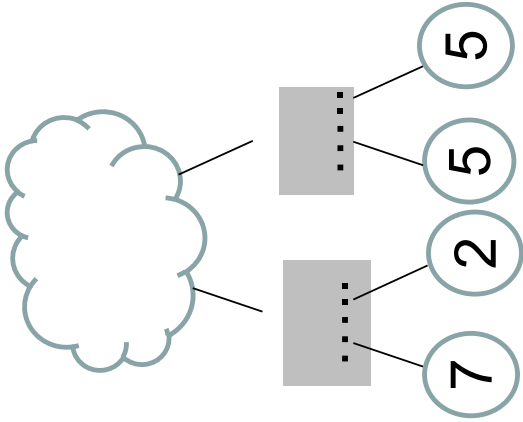
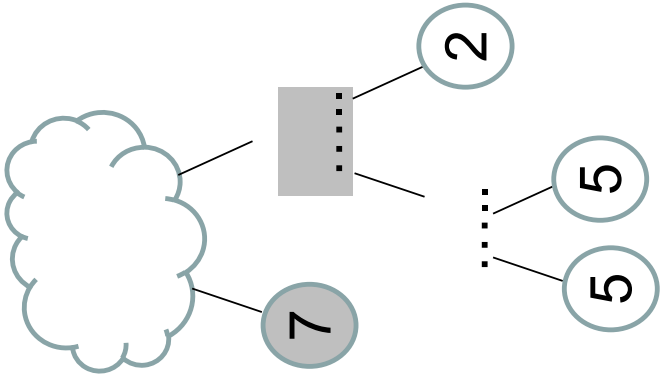
K)



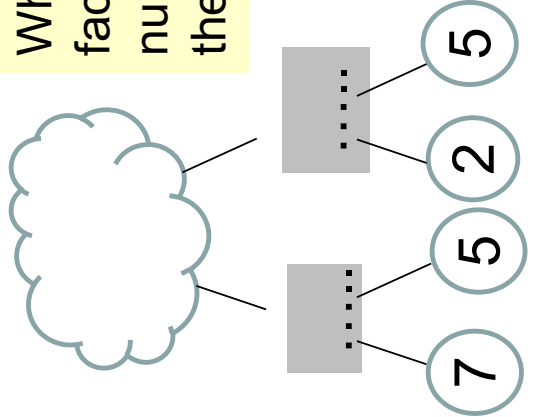
L)



# Extension



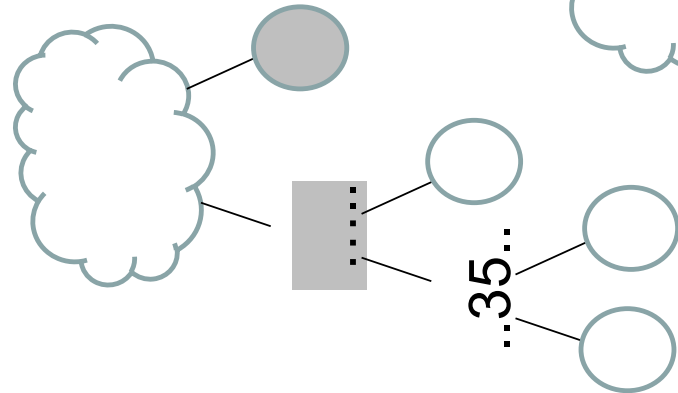
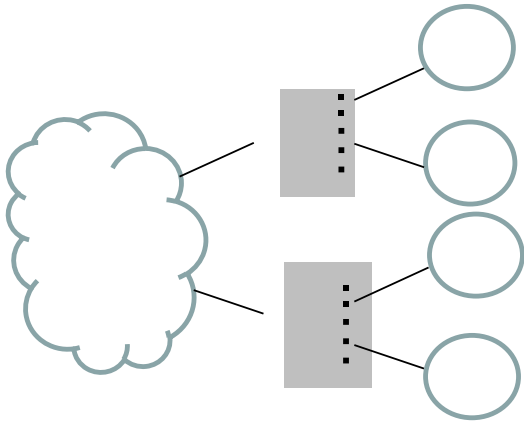
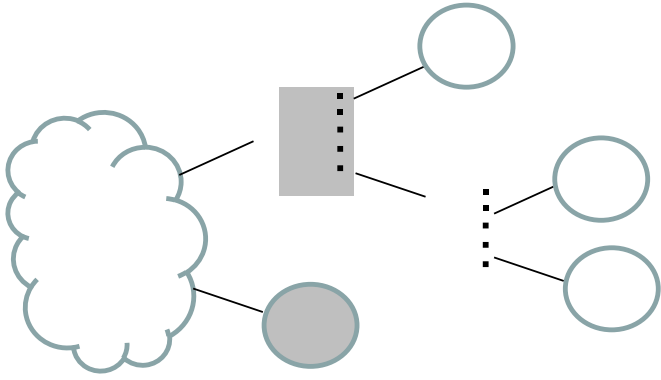
What are the factors of the numbers in the cloud?



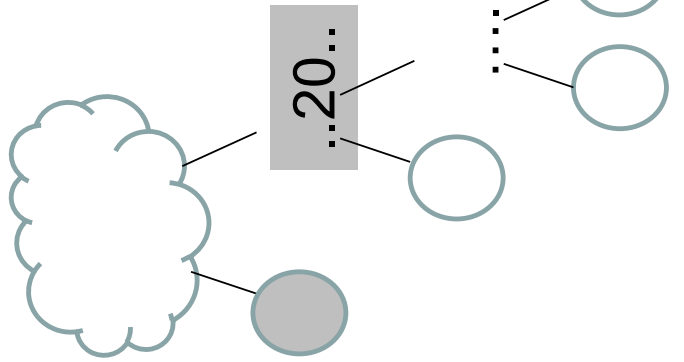
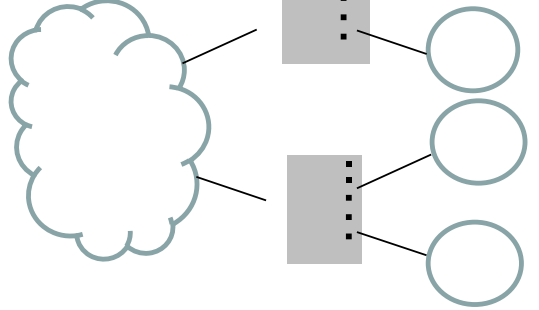
All numbers shaded in grey are unique  
 All numbers circled are Prime  
 All the numbers in the clouds are equal



# Extension



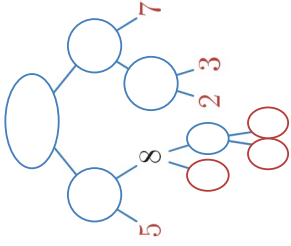
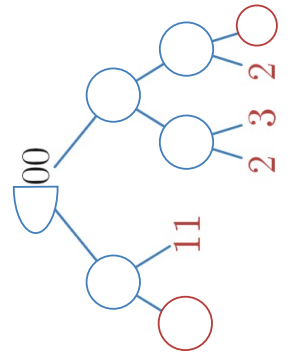
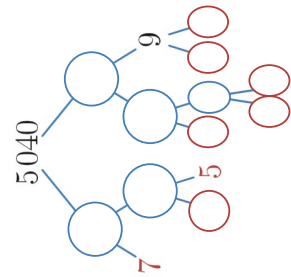
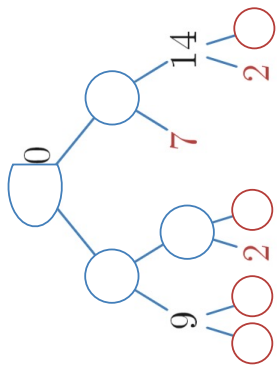
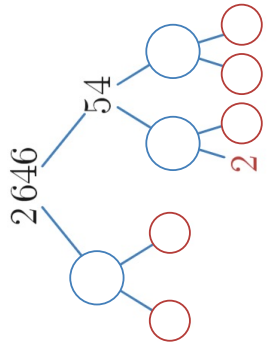
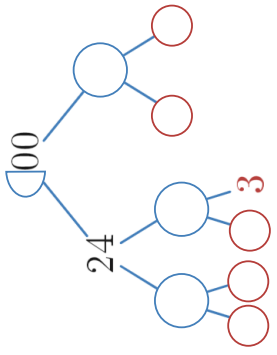
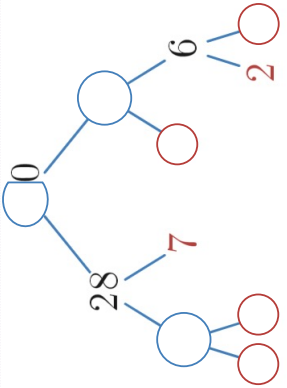
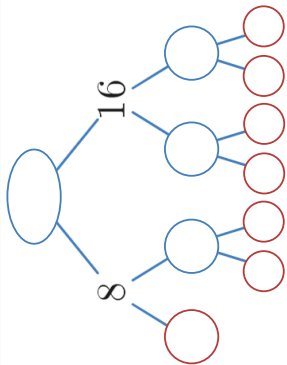
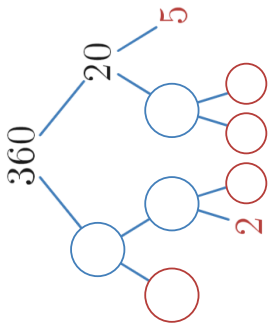
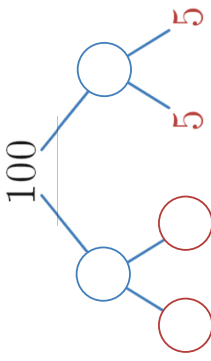
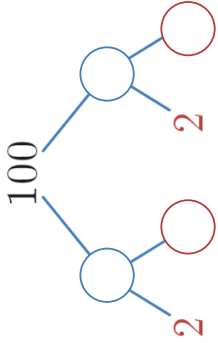
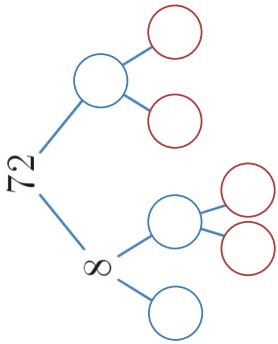
How many factors does the number in the cloud have?



All numbers shaded in grey are unique  
 All numbers circled are Prime  
 All the numbers in the clouds are equal

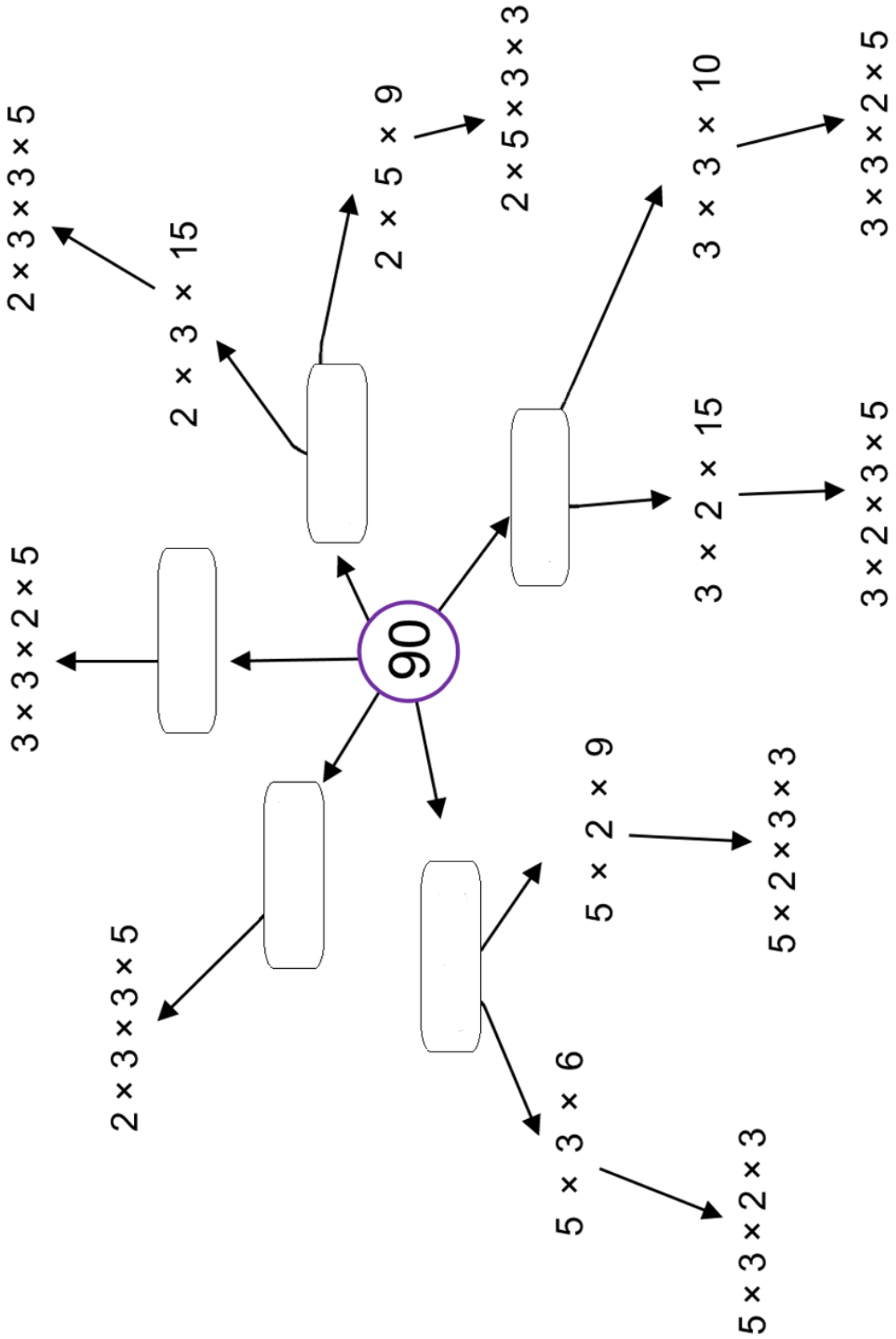
# Extension

## Factor Trees



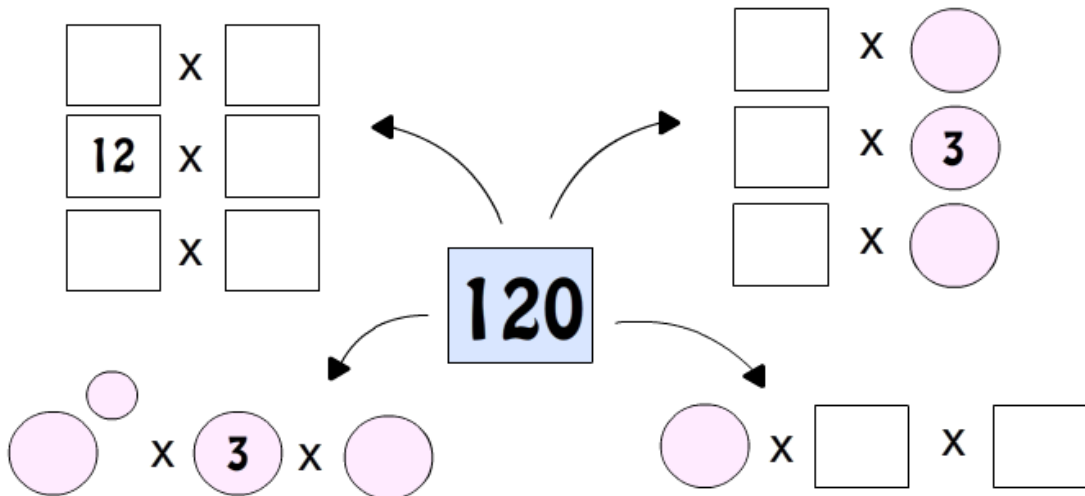
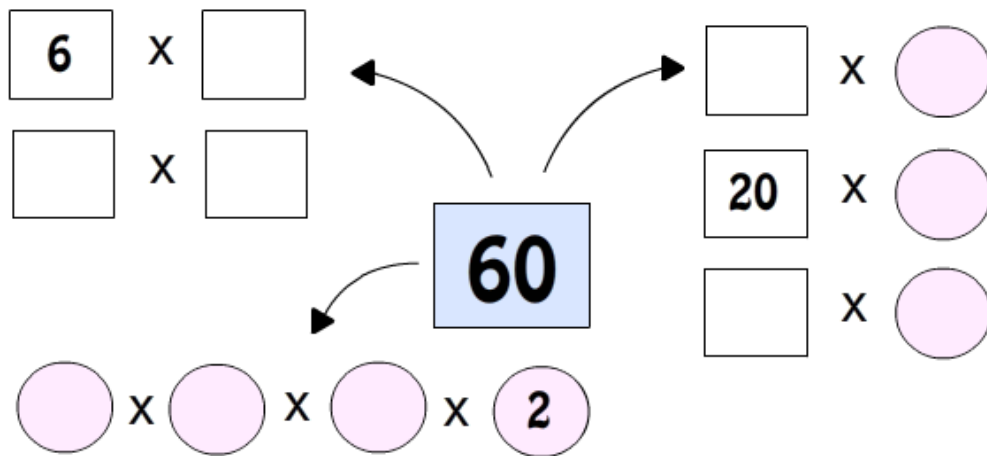
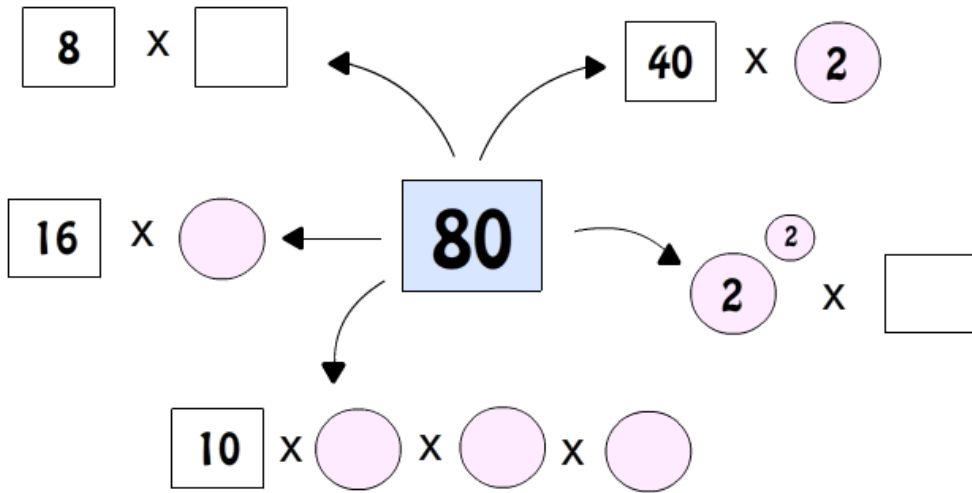


# Extension



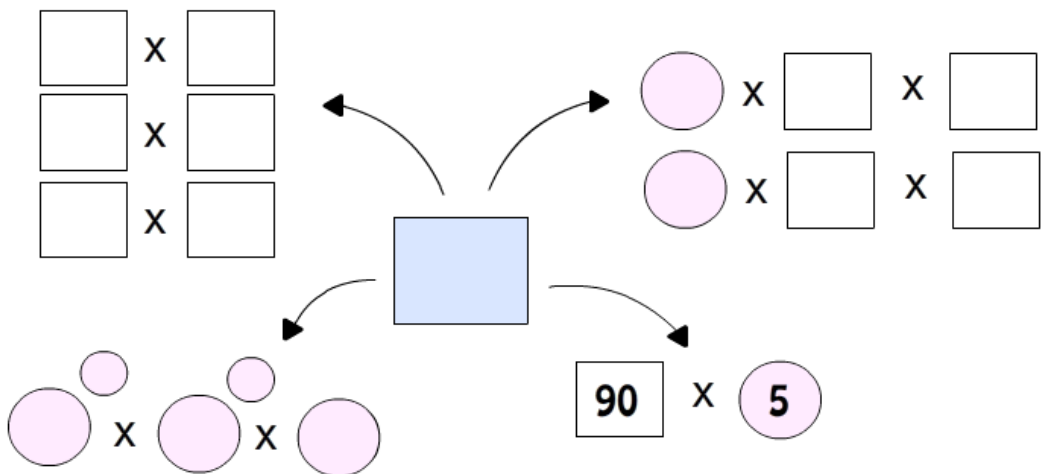
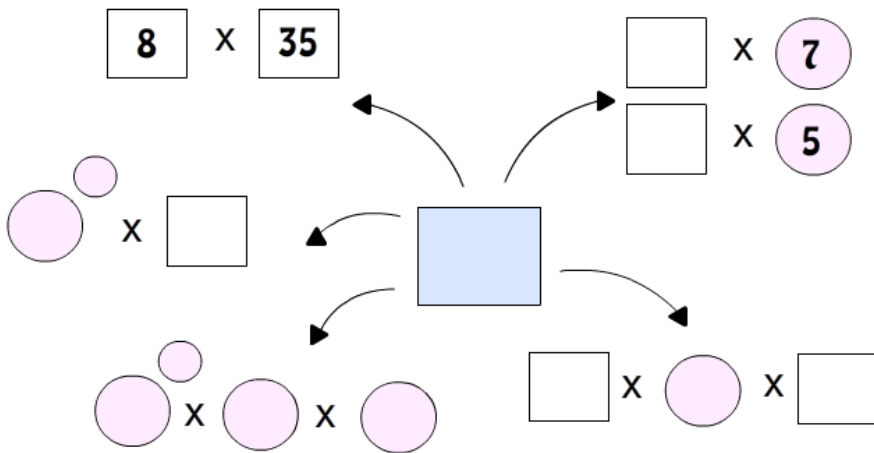
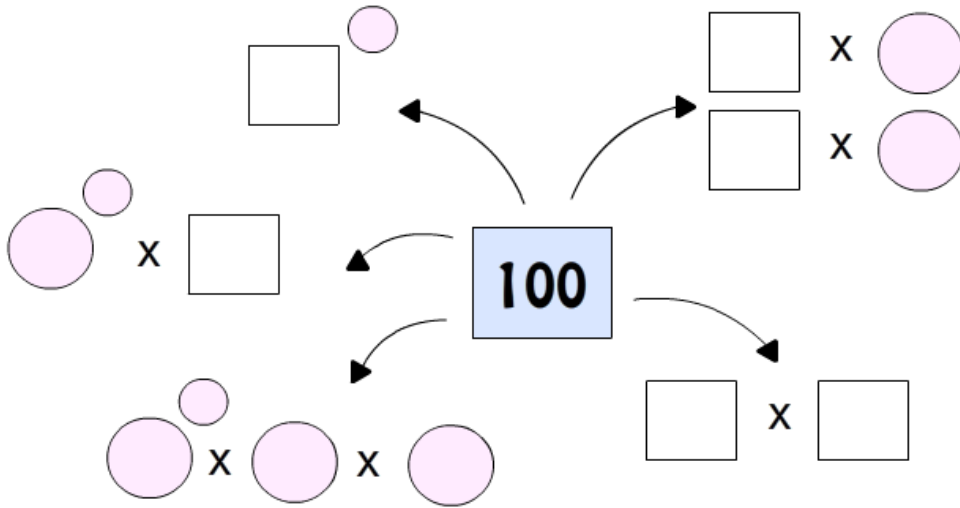
# Problem Solving

Can you fill the circles with prime numbers and the squares with composites (non primes) to make the target numbers in the middle?



# Problem Solving

Can you fill the circles with prime numbers and the squares with composites (non primes) to make the target numbers in the middle?



## Fluency Practice

Question 1: Using the fact that  $12 = 2^2 \times 3$ , write each of the following as the product of prime factors in index form.

(a) 24      (b) 36      (c) 60      (d) 48      (e) 120      (f) 84

Question 2: Using the fact that  $300 = 2^2 \times 3 \times 5^2$ , write each of the following as the product of prime factors in index form.

(a) 600      (b) 150      (c) 900      (d) 3300      (e) 1500      (f) 2400

# Intelligent Practice

$$2 \times 3 = 6.$$

What is 30 as its product of prime factors?

$$2 \times 3^2 \times 5^2 = 450.$$

What is 900 as its product of prime factors?

$$7^2 \times 11 = 539.$$

$$2^2 \times 3 = 12.$$

What is 60 as its product of prime factors?

$$2 \times 3^2 \times 5^2 = 450.$$

What is 4500 as its product of prime factors?

What is 2156 as its product of prime

factors?

$$2 \times 3 \times 5 = 30.$$

What is 150 as its product of prime factors?

$$3^2 \times 5^2 \times 7 = 1575.$$

What is 15750 as its product of prime factors?

$$7^3 \times 11 \times 13 = 49049.$$

What is 490490 as its product of prime

factors?

$$2 \times 3^2 \times 5 = 90.$$

What is 180 as its product of prime factors?

$$7^2 \times 11 = 539.$$

What is 1617 as its product of prime factors?



## Fluency Practice

1)  $X = 63 \times 35$

Write  $X$  as a product of its prime factors.

2)  $P = 154 \times 18^3$

Write  $P$  as a product of its prime factors.

3)  $P = 121 \times 15^5$

Write  $P$  as a product of its prime factors.

4)  $N = 98 \times 90$

Write  $N$  as a product of its prime factors.

5)  $P = 5^2 \times 18^5$

Write  $P$  as a product of its prime factors.

6)  $N = 5^6 \times 10^2$

Write  $N$  as a product of its prime factors.

7)  $N = 9^6 \times 6^3$

Write  $N$  as a product of its prime factors.

8)  $N = 12^3 \times 20^4$

Write  $N$  as a product of its prime factors.

9)  $N = 264 \times 5^4$

Write  $N$  as a product of its prime factors.

10)  $N = 15^6 \times 20^4$

Write  $N$  as a product of its prime factors.

# Intelligent Practice

$A =$	$2^p \times 3^q$
$2A =$	
$3A =$	
$4A =$	
$5A =$	
$6A =$	
$18A =$	
$20A =$	
$30A =$	

$B =$	$2^m \times 5^n$
$2B =$	
$3B =$	
$4B =$	
$5B =$	
$6B =$	
$18B =$	
$20B =$	
$30B =$	

$AB =$	
$2AB =$	
$3AB =$	
$4AB =$	
$5AB =$	
$6AB =$	
$18AB =$	
$20AB =$	
$2.5AB =$	

$\frac{1}{2}A =$	
$\frac{1}{3}A =$	
$\frac{1}{6}A =$	
$\frac{1}{18}A =$	

$\frac{1}{2}B =$	
$\frac{1}{4}B =$	
$\frac{1}{25}B =$	
$\frac{1}{250}B =$	

$\frac{1}{2}AB =$	
$\frac{1}{30}AB =$	
$\frac{2}{3}AB =$	
$\frac{2}{75}AB =$	

$A^2 =$	
$A^3 =$	
$A^6 =$	

$B^2 =$	
$\sqrt{B} =$	
$B^n =$	

$(AB)^2 =$	
$AB^2 =$	
$A + B =$	

# Fluency Practice

Express 360 as a product of primes

Express  $360^5$  as a product of primes

Express  $360^8$  as a product of primes

Fill in each box with a product of primes to make the equality true

$180 =$

$1080 =$

$3600 =$

$108 =$

$360^8 = 360^7 \times 2$

$360^8 = 360^3 \times 2$

$360^5 = 360^3 \times 2$

$360^5 = 36^3 \times 2$

$360^5 = 27^2 \times 2$

$360^6 = 36^6 \times 2$

$360^8 = 36^7 \times 2$

$360^8 = 12^7 \times 2$

$360^6 = 36^3 \times 2$

$360^5 = 1000 \times 2$

# Intelligent Practice

$a$	$b$	$a + b$	$a - b$	$a \times b$	$a \div b$
$2^5 \times 3^8 \times 5^3$	$2^4 \times 3^8 \times 5^3$	$(2 + 1) \times 2^4 \times 3^8 \times 5^3$ $= 2^4 \times 3^9 \times 5^3$	$(2 - 1) \times 2^4 \times 3^8 \times 5^3$ $= 2^4 \times 3^8 \times 5^3$	$2^9 \times 3^{16} \times 5^6$	2
$2^4 \times 3^9 \times 5^3$	$2^4 \times 3^8 \times 5^3$				
	$2^4 \times 3^8 \times 5^3$	$2^5 \times 3^9 \times 5^3$			
	$2^4 \times 3^8 \times 5^3$		$2^4 \times 3^9 \times 5^3$		
	$2^4 \times 3^8 \times 5^3$			$2^9 \times 3^{17} \times 5^6$	
	$2^4 \times 3^8 \times 5^3$				$3^2$

## Extension

Question 4: (a) Write 980 as a product of prime factors.  
Express your answer in index form.

(b) Find the lowest number by which 980 would need to be multiplied by to give a square number.

Question 5: (a) Write 480 as a product of prime factors.  
Express your answer in index form.

(b) Find the lowest number by which 480 would need to be multiplied by to give a square number.

Question 6: (a) Write 2646 as a product of prime factors.  
Express your answer in index form.

(b) Find the lowest number by which 2646 would need to be multiplied by to give a cube number.

## Problem Solving

Find as many different values that could fill the box below,  
so the result of the calculation is a SQUARE number

$$36 \times \boxed{\phantom{000}}$$

What do you notice about the values that work?  
Which of the following would work?

$$36 \times \boxed{288}$$

$$36 \times \boxed{256}$$

$$36 \times \boxed{216}$$

## Extension

Find as many different values that could fill the box below,  
so the result of the calculation is a SQUARE number

$$36 \times \boxed{4} = 2^2 \times 3^2 \times \boxed{2^2} = 2^4 \times 3^2 = 16 \times 9 = 144$$

$$36 \times \boxed{9} = 2^2 \times 3^2 \times \boxed{3^2} = 2^2 \times 3^4 = 4 \times 81 = 324$$

$$36 \times \boxed{16} = 2^2 \times 3^2 \times \boxed{2^4} = 2^6 \times 3^2 = 64 \times 9 = 576$$

$$36 \times \boxed{25} = 2^2 \times 3^2 \times \boxed{5^2} = 2^2 \times 3^2 \times 5^2 = 4 \times 9 \times 25 = 900$$

Use the calculations above to evaluate the following:

$$\sqrt{144}$$

$$\sqrt{324}$$

$$\sqrt{576}$$

$$\sqrt{900}$$

# Problem Solving

$a$  is an integer  
 $a \times a$  is a square  
number

A

$a$  and  $b$  are integers  
 $a \times b$  is a square  
number

E

$a$  is an integer  
 $a \times a \times a$  is a square  
number

G

$a$  and  $b$  are prime  
numbers  $a \times b$  is a  
square number

B

**Determine whether  
each of the statements  
are never TRUE,  
Sometimes TRUE or  
always TRUE**

$a$  and  $b$  are prime  
numbers  $a \times b \times a \times b$   
is a square number

C

$a$  is a prime number  
 $a \times a \times a$  is a square  
number

H

$a$  is a square number  
 $|a \times a \times a$  is a square  
number

I

$a$  and  $b$  are  
integers  $a \times b \times a \times b$   
is a square number

D

$a, b$  and  $c$  are integers  $a \times b \times c$  is a square number

F

$a$  is a cube number  
 $a \times a$  is a square  
number

J



# Fluency Practice

Simplify:

1)  $\frac{30}{42}$

2)  $\frac{70}{105}$

3)  $\frac{154}{182}$

4)  $\frac{60}{616}$

5)  $\frac{375}{875}$

6)  $\frac{385}{455}$

7)  $\frac{833}{931}$

8)  $\frac{2310}{3465}$

9)  $\frac{3773}{4459}$

# Fluency Practice

Find the prime factor decomposition of:

72	120	420	700

Hence fully simplify:

$\frac{72}{120}$	$\frac{120}{420}$
$\frac{72}{420}$	$\frac{120}{700}$
$\frac{72}{700}$	$\frac{420}{700}$

## Fluency Practice

**cancelling down** (without using a calculator)  
these cancel down to much nicer, equivalent fractions

$$(1) \quad \frac{70}{105}$$

$$(4) \quad \frac{5940}{9900}$$

$$(2) \quad \frac{315}{420}$$

$$(5) \quad \frac{297}{5346}$$

$$(3) \quad \frac{1056}{2376}$$

$$(6) \quad \frac{585}{819}$$

# Fluency Practice

Find the square and cube roots:

1)  $\sqrt{400}$

2)  $\sqrt{441}$

3)  $\sqrt{576}$

4)  $\sqrt{676}$

5)  $\sqrt{1024}$

6)  $\sqrt[3]{729}$

7)  $\sqrt[3]{1728}$

8)  $\sqrt[3]{3375}$

9)  $\sqrt[3]{5832}$

# Fluency Practice

How many factors do each of the following have:

- a) 8
- b) 10
- c) 7
- d) 12
- e) 20
- f) 22
- g) 18
- h) 50
- i) 15
- j) 19
- k) 30
- l) 100
- m) 32
- n) 24
- o) 42
- p) 28
- q) 66
- r) 70
- s) 45
- t) 60
- u) 25

# Fluency Practice

List all the factors of 1944.

You may leave your answers as a product of primes


# Fluency Practice

Find the prime factor decomposition of the following numbers, and identify how many factors they have.

You DO NOT have to list all the factors if you can explain how you answered the question without doing so.

8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
30		
32		
48		
64		
120		
150		
1000		
4000		
159000		

## Extension

34 can be written as:  $2 \times 17$

Adil says:

This means 34  
has exactly 4  
factors

Is Adil correct?

List the factors of 34

Can you find other numbers  
with only 4 factors?

63 can be written as:  $7 \times 9$

Balraj says:

This means 63  
has exactly 4  
factors

Is Balraj correct?

List the factors of 63



# Fluency Practice

By drawing factor trees, write the following numbers as a product of their prime factors.

- |        |         |
|--------|---------|
| (a) 15 | (b) 22  |
| (c) 28 | (d) 24  |
| (e) 32 | (f) 42  |
| (g) 50 | (h) 54  |
| (i) 60 | (j) 75  |
| (k) 80 | (l) 100 |

As a product of its primes, what number is given by:

- (a)  $2 \times 5 \times 11$
- (b)  $3 \times 3 \times 5$
- (c)  $2 \times 5 \times 7$
- (d)  $2 \times 2 \times 3 \times 3 \times 5$

For each of these numbers, draw a factor tree and write as a product of its prime factors.

- |        |        |
|--------|--------|
| (a) 4  | (b) 9  |
| (c) 16 | (d) 25 |
| (e) 36 | (f) 81 |

What do you notice?

As a product of its prime factors,  $120 = 2 \times 2 \times 2 \times 3 \times 5$ . How could you use this information to find all the factors of 120, making sure you do not miss any factor pairs?

# Prime Factor Decomposition

1. Write as a product as prime factors. Give your answer in index form.

a. 18                      b. 50                      c. 24                      d. 72                      e. 80                      f. 96

g. 150                      h. 126                      i. 200                      j. 550                      k. 729                      l. 1050

2. Given the prime factor decomposition, find the number

a.  $2 \times 3 \times 5 \times 7$                       b.  $2^4 \times 5^2$                       c.  $3^2 \times 5^2 \times 11$                       d.  $2^4 \times 3^2 \times 5$                       e.  $2 \times 5^2 \times 7^2 \times 11$

3. Given the prime factor decomposition, find the prime factor decomposition of the new number

a. If  $V = 2^a \times 3^b$ , find  $3V$                       b. If  $W = 2^a \times 5^b$ , find  $10W$                       c. If  $X = 3^a \times 5^b \times 7$ , find  $X^2$                       d. If  $Y = 3^a \times 5^b$ , find  $2Y^2$

4. Explain, using prime factors, why...

a. ... 144 is a square number                      b. ... 216 is a cube number                      c. ... 64 can be both a square and cube number

5. Decide whether the following are square numbers, cube numbers or neither

a.  $2^2 \times 3^2 \times 5^2$                       b.  $2^2 \times 3^2 \times 5^3$                       c.  $2^4 \times 3^4 \times 5^4$                       d.  $2^5 \times 3^5 \times 5^5$

e.  $2^6 \times 3^6 \times 5^6$                       f.  $2^2 \times 3^6 \times 5^4$                       g.  $2^9 \times 3^6 \times 5^6$                       h.  $2^6 \times 3^3 \times 5^3$

6. Find the lowest number to multiply ...

a. 980 by to make a square number                      b. 480 by to make a square number                      c. 60 by to make a cube number

# Prime Factorisation

## Prime Factorisation

1) Complete the boxes below.

18

108

300

90

99

175

2) Use the boxes to find the prime factorisation of each answer.

$99 \times 175$

$18 \times 108$

$18 \times 300$

$18 \times 9$

$90 \times 300$

$108 \div 18$

$90 \div 18 \times 175$

$300 \times 90 \div 108$

$175^2$

3) General rules.

$A = 2^p \times 3^q$	
$2A$	
$3A$	
$4A$	
$6A$	
$\frac{1}{2}A$	
$\frac{1}{6}A$	

$B = 3^p \times 5^q$	
$9B$	
$B^2$	
$B^3$	
$3B^2$	
$\frac{1}{5}B^2$	
$0.6B^3$	

# Fluency Practice

Prime Factorisation			
<p><b>1. Factual recall</b> a) List the factors of 12.</p> <p>b) List the prime numbers between 10 and 20.</p>	<p><b>2. Carry out a routine procedure</b> Find the prime factorisation of the following numbers. Leave you answer in index form.</p> <p>a) 36</p> <p>b) 270</p> <p>c) 189</p>	<p><b>3. Classify some mathematical object</b> By finding the prime factorisations, which number is the odd one out?</p> <p>a) 45</p> <p>b) 225</p> <p>c) 1125</p>	<p><b>4. Interpret a situation or answer</b> This year a grandmother, her daughter and her granddaughter noticed that the sum of their ages is 100 years. Each of their ages is a power of 2. How old is the granddaughter?</p>
<p><b>5. Prove, show, justify</b> Look at these equations:</p> <div style="border: 1px solid black; padding: 2px; display: inline-block; margin: 5px;"> <math>48 = 3 \times 2^a</math> </div> <div style="border: 1px solid black; padding: 2px; display: inline-block; margin: 5px;"> <math>56 = 7 \times 2^b</math> </div> <p>Show that <math>a + b</math> is a prime number:</p>	<p><b>6. Extend a concept</b> The <b>primorial</b> of a number is the product of all the prime numbers less than or equal to that number. For example, the <b>primorial</b> of 6 is <math>2 \times 3 \times 5 = 30</math>. How many different whole numbers have a <b>primorial</b> of 210?</p>	<p><b>7. Construct an instance</b> Create an <b>even</b> number which does not have a repeating prime factor.</p>	<p><b>8. Criticise a fallacy</b> A student tried to find the prime factorisation of 108. Below are their workings. Find and amend any mistakes.</p> <div style="text-align: center;"> </div>

# Factors

**4 has exactly 3 factors: 1, 2 and 4.**

**5 has exactly 2 factors: 1 and 5.**

1. How many numbers are there between 1 and 100 that have exactly 2 factors?
2. There are 10 numbers between 1 and 100 that have an odd number of factors. What are they?
3. What is the smallest number to have exactly 5 different factors?
4. What is the largest number less than 100 to have exactly 3 factors?
5. Emily says that bigger numbers always have more factors. Is this true?
6. There is at least one number between 1 and 100 that has 12 different factors. What is it?
7. How would you describe a “factor” to someone?
8. Is -1 a factor of 1?
9. How many factors does -10 have?
10. If you divide 2 by 0.5 you get 4. Is 0.5 a factor of 2?
11. Jenny says that numbers in the 5 times table always have less factors than the numbers in the 6 times table. Is she correct?

**Numbers with exactly 2 factors are called prime numbers.**

12. How many prime numbers are there between 1 and 100?
13. Why isn't 1 a prime number?
14. Do you think -1 should be a prime number?
15. True or false: There is no number whose factors are all prime numbers

**You can write 12 as a product of prime factors:  $2 \times 2 \times 3 = 12$**

16. Can you write 36 as a product of prime factors?
17. Can you write 50 as a product of prime factors?
18. Can you think of a quick way of writing 225 as a product of prime factors?
19. Which numbers between 1 and 100 cannot be written as a product of prime factors?
20. What is the smallest number with 5 different prime factors?

**You can write 36 as a product of prime factors using indices for shorthand, like this:**

$$36 = 2^2 \times 3^2$$

21. Can you write 100 as a product of prime factors using indices?
22. Can you write 250 as a product of prime factors using indices?
23. Can you write 1000 as a product of prime factors using indices?
24. What number would be written :  $2^3 \times 5 \times 7^3$  ?

# Using Prime Factorisation

$$X = 2^2 \times 5^3 \times 7$$

$$Y = 2^3 \times 5^3$$

## Questions about X

- A) How many zeroes are at the end of X?
- B) How do we know that X is not a square number?
- C) What should we multiply X by, to make it a square number?
- D) Is X a multiple of 14?
- E) Is 25 a factor of X?
- F) What is the largest factor of X that is also less than X?

## Questions about Y

- A) How many zeroes are at the end of Y?
- B) What is the first digit of Y?
- C) How do we know that Y is not a square number?
- D) What should we multiply Y by, to make it a square number?
- E) What is the cube root of Y?
- F) How many factors does Y have?

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## Square & Cube Numbers & Roots

1. Calculate  $\sqrt{2^2 \times 3^2}$

2. Calculate  $\sqrt{2^4 \times 5^2}$

3. Calculate  $\sqrt{3^6}$

4. Calculate  $\sqrt[3]{3^9}$

5. Calculate  $\sqrt[3]{2^6 \times 3^3}$

6. Calculate  $\sqrt[3]{2^2 \times 3^2 \times 6}$

7. Calculate  $\sqrt{1225}$

8. Which of the following are square numbers?

A)  $2^3 \times 3^2$                       B)  $3^2 \times 5^2 \times 7^2$

C)  $2^5$                                 D)  $2^3 \times 5^3$

9. Which of the following are odd numbers?

A)  $3^3 \times 5^2$                       B)  $2^3 \times 5^2 \times 7^2$

C)  $5^2 \times 10$                       D)  $19 \times 120$

10. What should we multiply  $2 \times 5^2 \times 7^2$  by to make it a square number?

11. What should we multiply  $2 \times 5^2 \times 7^2$  to make it a multiple of 100?

# Prime Puzzles

$2 \times 3 \times 13$ is a multiple of 6	$3 \times 15 \times 23$ is a multiple of 9	$2^2 \times 15$ is a multiple of 10	$2^{10}$ is a multiple of 10	$16 \times 30$ is a multiple of 12
$2^2 \times 5^2$ is a multiple of 10	$3 \times 15 \times 23$ is a multiple of 9	$15 \times 23$ is a multiple of 20	$3^2 \times 5^2$ is a multiple of 2	$8^2 \times 5$ is a multiple of 10
$25 \times 8$ is a multiple of 10	$2^2 \times 2^5$ is a multiple of 5	$2 \times 5 \times 18$ is a multiple of 15	$32 \times 30$ is a multiple of 6	$2 \times 15 \times 31$ is an even number
$3^2 \times 5^5$ is a multiple of 6	$21 \times 80$ is a multiple of 28	$2 \times 8 \times 7$ is an even number	$3^2 \times 2^5$ is a multiple of 8	$25 \times 40$ is a multiple of 15
$2^2 \times 5^5$ is a multiple of 3	$27 \times 50$ is a multiple of 45	$18 \times 60$ is a multiple of 27	$5^2 \times 7^5$ is a multiple of 21	$2^{10}$ is a multiple of 16

**Using prime numbers: true or false?**  
Tick the cards that are false...

# Fluency Practice

1.) Express 256 as a product of prime factors.

Use your answer to complete the following:

..... 2	..... 4	..... 8	..... X 8
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## Fluency Practice

2.) By writing 450 as a product of prime factors, show that it isn't a Square number

What is the smallest number you can multiply 450 by to make it a square number?

What is the square root of the number?

## Prime factorisation



1. Given that  $196 = 2^2 \times 7^2$ , write 1960 as a product of prime numbers.
2. Bailey's comet is visible from earth every 196 years. Cayley's comet is visible from earth every 70 years. They were both seen in the year 1170. When can they next be seen in the same year?
3. What is the largest square number that is a factor of 1960?
4. Find the smallest integer  $n$  such that  $1960n$  is a cube number.

## The Product Product

$$45 \times (4 \times 5) = 900$$

$$24 \times (2 \times 4) = 192$$

which two-digit numbers when multiplied by the product of the two digits will give:

- 1) 408
- 2) 336
- 3) 378
- 4) 522
- 5) 1533
- 6) prove that 420 cannot be reached

# Prime Factor Decomposition Logical Puzzle

All numbers have 4 prime factors, and they must fit in around the large number in the small boxes.

	24		132		88		308	
	40		16		56		196	
	60		36		84		140	
	90		135		210		350	
	330		90		735		490	
	462		54		126		364	
	189		81		198		858	

# Exactly Four Factors

exactly 4 factors

a number has 4 factors, one of which is 9, what is it?

a number is one less than a square number; it has 4 factors, one of which is 5; what could it be?

a number has 4 factors, two of which add up to 10; what could it be? how many numbers could it be?

can you find some pairs of consecutive numbers, both of which have 4 factors?

can you find a trio of consecutive even numbers all three having 4 factors?

can you find some trios of consecutive numbers all three of which have 4 factors?

can you find a trio of consecutive odd numbers all of which have 4 factors?

what types of number have exactly 4 factors?

describe the two distinct families of numbers that have 4 factors

can you explain *why* the families of numbers have 4 factors?

# Find the Number

## factors of numbers

- (1) give five (or more) numbers  
try to give the lowest number  
in each case
- (a) with 2 and 3 as factors
  - (b) with 6 and 8 as factors
  - (c) with 9 and 11 as factors
  - (d) with 5 and 15 as factors
  - (e) with 10 and 12 as factors
  - (f) with 12 and 15 as factors
- (2) find the numbers from these clues:
- (a) it has exactly 4 factors, one of which is 9
  - (b) it has exactly 3 factors, one of which is 5
  - (c) smallest number with 6 factors, one of which is 6
  - (d) it has exactly 4 factors, two of which add to 10
  - (e) smallest number with 6 factors, one of which is 10
  - (f) it has 6 factors, one of which is 15
- (3) find the numbers from these clues:
- (a) it has 4 factors, one of which is 7  
and it is one less than a square number
  - (b) it has 5 factors, one of which is 9
  - (c) it has 6 factors, one of which is 10
  - (d) it has 7 factors, one of which is 16  
and it is less than 100
  - (e) it has 8 factors, two of which are 10 and 15
  - (f) it has 8 factors, two of which are 21 and 35
- (4) how many factors do these numbers have?
- (a) 80
  - (b) 72
  - (c) 2000
  - (d) 9625
- (5) what five numbers less than 100 have exactly 12 factors?

# Factors of Numbers Number of Factors

factors of numbers and numbers of factors  
find the numbers less than 100 that have

- (1) exactly 4 factors and
- a. one of the factors is 58
  - b. one of the factors is 57
  - c. one of the factors is 11 (four answers)
  - d. one of the factors is 91
  - e. two of the factors sum to 10 (three answers)
  - f. two of the factors sum to 8 and two sum to 16 (two answers)
- (2) exactly 6 factors and
- a. one of the factors is 6 (two answers)
  - b. one of the factors is 10 (two answers)
  - c. one of the factors is 14 (two answers)
  - d. one of the factors is 15 (two answers)
  - e. one of the factors is 25 (two answers)
  - f. are in the 90s decade (three answers)
- (3) exactly 3 factors (four answers)
- a. exactly 3 factors (four answers)
  - b. exactly 4 factors, one of which is 7 and it is one less than a square number
  - c. exactly 5 factors (two answers)
  - d. exactly 6 factors, one of which is 21
  - e. exactly 7 factors
  - f. exactly 8 factors, two of which are 10 and 15
- (4) exactly 9 factors (less than 50)
- a. exactly 9 factors (less than 50)
  - b. exactly 10 factors (two answers, one less than 50)
  - c. exactly 12 factors (5 answers, all bigger than 50)

# More-Same-Less – Prime Factorisation

Instructions: Find the prime factorised form of the number in the middle box. Next choose a different number which fits in each box and then write it in prime factorised form. Try to make your questions and answers as similar as possible to the middle box.

**Number of prime factors (include repeats)**

	Less	Same	More
Less			
Same			
More			

Value of the number

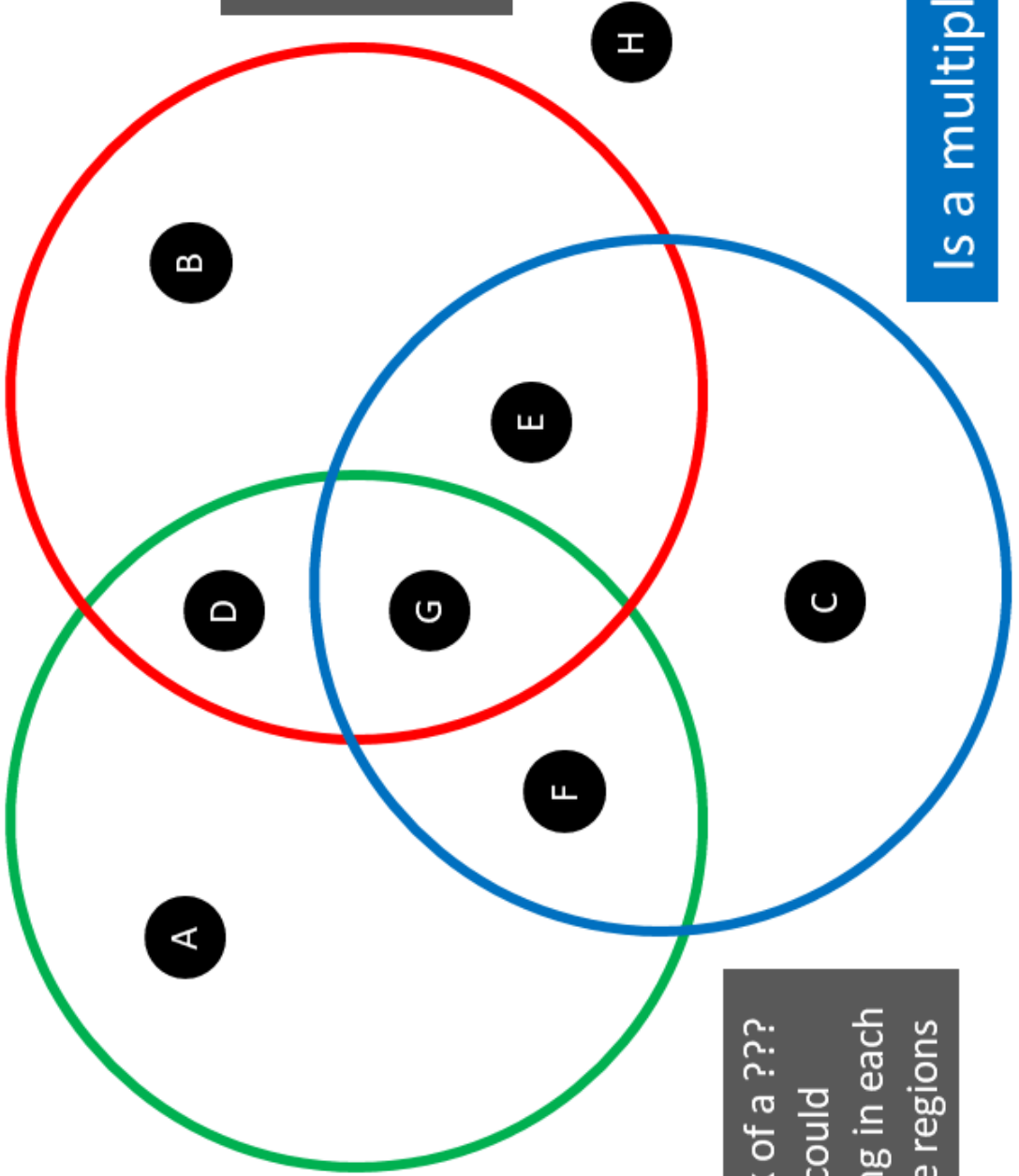


# Maths Venns

Is a product of at least 3 primes

Has 3 as a factor

Is a multiple of 4



If you think a region is impossible to fill, convince me why!

Think of a ??? that could belong in each of the regions

## 2 Probability

# Intelligent Practice

Put these probabilities on their own probability line and state how likely they are:

1)  $\frac{7}{10}$

2)  $\frac{3}{10}$

3)  $\frac{5}{10}$

4)  $\frac{5}{8}$

5)  $\frac{8}{8}$

6)  $\frac{8}{10}$

7)  $\frac{0}{10}$

8)  $\frac{0}{7}$

9)  $\frac{5}{7}$

10)  $\frac{3}{7}$

# Fluency Practice

Question 1: Which phrase from the box best describes the likelihood of each of these events?  
You may use each phrase more than one.

Impossible   Unlikely   Even Chance   Likely   Certain

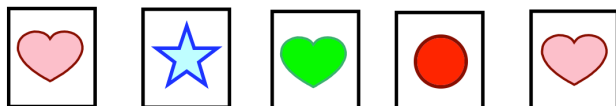
- (a) Rolling a 9 on an ordinary six sided dice.
- (b) A newborn baby being a boy.
- (c) A day picked at random ending with the letter y
- (d) Getting a tail when a coin is flipped.
- (e) It snowing in London in May.
- (f) Rolling a number greater than 1 on an ordinary six sided dice.

Question 2: Which word from the box best describes the likelihood of each of these events?

Impossible   Unlikely   Even   Likely   Certain

- (a) You throw a coin and get a Heads.
- (b) You take a green counter from a bag that only contains black counters.
- (c) May 18th 2018 is the day after May 17th 2017.

Question 3: Here are some cards



Impossible   Unlikely   Even   Likely   Certain

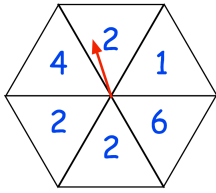
A card is picked at random.

Which word from the box best describes the likelihood of each of these events?

- (a) The card has a blue star on it.
- (b) The card has a heart on it.
- (c) The card has a shape on it that is symmetrical.

# Fluency Practice

Question 4: A fair spinner has six equal sections.



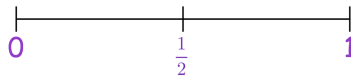
Impossible   Unlikely   Even   Likely   Certain

Which word from the box best describes the likelihood of each of these events?

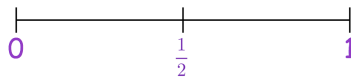
- (a) The arrow landing on an even number
- (b) The arrow landing on 4.
- (c) The number landing on 2.

Question 5: Francesca rolls an ordinary 6-sided dice.

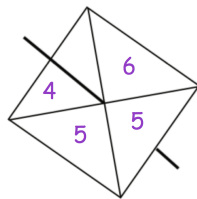
- (a) Mark with a cross the probability that Francesca gets an 8.



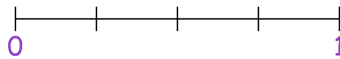
- (b) Mark with a cross the probability that Francesca gets an odd number.



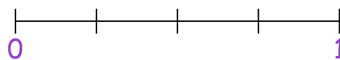
Question 6: A fair 4-sided spinner is spun once.



- (a) On the probability scale, mark with a letter A, the probability that the spinner will land on the number 4.

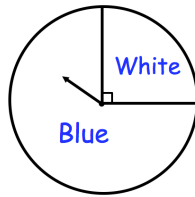


- (b) On the probability scale, mark with a letter B, the probability that the spinner will land on the number 5.

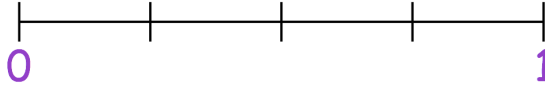


# Fluency Practice

Question 7: The diagram shows a fair spinner.



(a) Which colour is the arrow least likely to land on?



(b) Mark the probability scale with an arrow to show the probability of landing on white. Label the arrow, W.

(c) Mark the probability scale with an arrow to show the probability of landing on blue. Label the arrow, B.

Question 8: A fair six sided dice is rolled once.

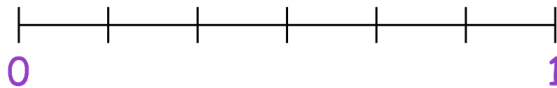


Mark the probability of each of the following events onto the probability scale.

A: The dice lands on an even number.

B: The dice lands on the number 5

C: The dice lands on a number less than 5.

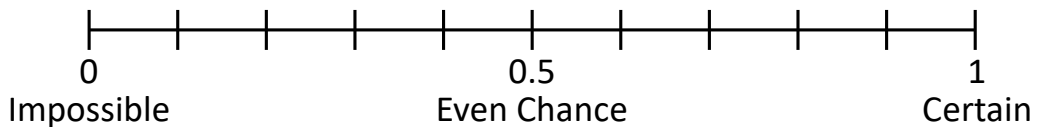


# Fluency Practice

## The Probability Scale

1) How likely are these events? Mark the letter on the probability scale.

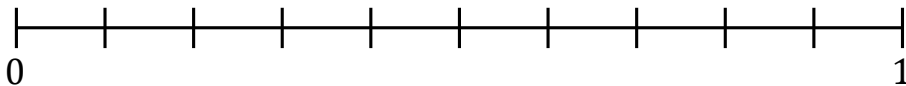
- A) It will rain tomorrow.                      D) I will pass my next English test.  
B) There will be school next week.            E) It will snow at Christmas.  
C) I will do homework today.                  F) A flipped coin lands on tails.



2) The probability of each event is given.

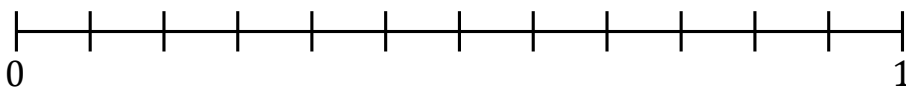
Mark each letter on the probability scale.

- A) Jack will cycle home = 99%                      D) United will win =  $\frac{1}{5}$   
B) Jenny will have potato for dinner =  $\frac{7}{10}$             E) See a pigeon today = 55%  
C) It will be sunny tomorrow = 0.4                  F) Thunder tomorrow = 0.05



3) Mark the letter of each event on the probability scale.

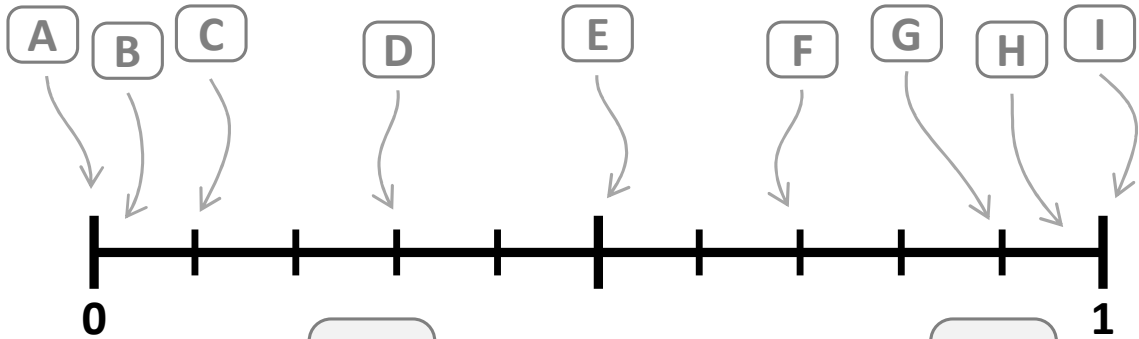
- A) Rain tomorrow =  $\frac{7}{12}$                       D) Roll a 6 on a dice =  $\frac{1}{6}$   
B) Test next lesson =  $\frac{3}{4}$                       E) Roll an even number on a dice.  
C) Pasta next lunch =  $\frac{1}{3}$                       F) Not roll a 6 on a dice.



# Fluency Practice

## Probability Vocabulary

Match each expression with a letter on the probability scale.



Impossible	<input type="text"/>	Even Chance	<input type="text"/>
Likely	<input type="text"/>	Certain	<input type="text"/>
A Tiny Chance	<input type="text"/>	No Chance	<input type="text"/>
Unlikely	<input type="text"/>	Very Unlikely	<input type="text"/>
Definite	<input type="text"/>	Very Likely	<input type="text"/>
Expected	<input type="text"/>	1-in-a-Million	<input type="text"/>
Extremely Likely	<input type="text"/>	Inconceivable	<input type="text"/>
Fifty-Fifty	<input type="text"/>	Probable	<input type="text"/>
Guaranteed	<input type="text"/>	High Risk	<input type="text"/>

What is the problem with using language to describe probabilities?

*More Likely*

*Most Likely*

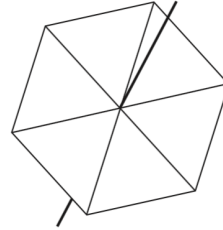
*Less Probable*

How would you label these 2 expressions on the probability scale?



# Extension

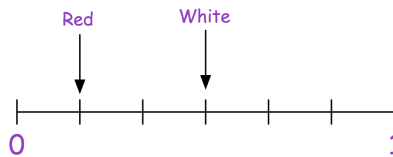
Question 1: Curtis has a fair 6-sided spinner.  
 The spinner has numbers less than 7 on it.  
 The number 5 is the least likely number that the spinner will land on.  
 There is an even chance that the spinner will land on a 3.  
 It is impossible that the spinner will land on an even number.  
 Write the numbers on the spinner.



Question 2: Reggie has a bag holding red, white and green counters.  
 Altogether there are 6 counters in the bag.

The probability scale shows the probability that a counter picked at random will be white.

It also shows the probability that a counter picked at random will be white.



Show on the probability scale the probability that a counter picked at random will be green.

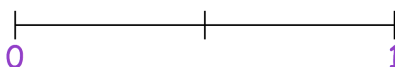
Question 3: A school offers students 3 lunchtime clubs each week: hockey, golf and cricket.

- (a) Which clubs does Helen attend?
- (b) Which of the children attend the cricket club?
- (c) Which of the club do the least of the 5 children attend?
- (d) Which child attends the most clubs?

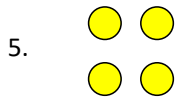
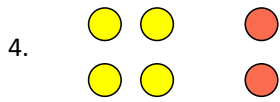
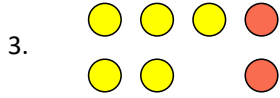
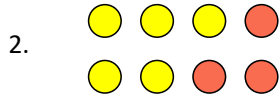
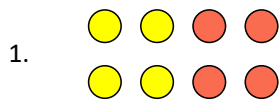
	Hockey	Golf	Cricket
Helen	✓		✓
Leah			✓
Emily	✓	✓	✓
Mia	✓	✓	
Sally	✓		

Mr White picks one of the 5 children at random

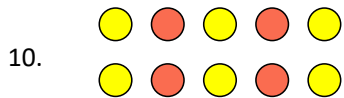
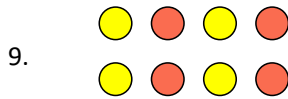
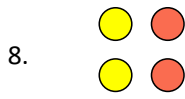
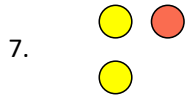
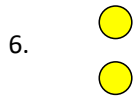
- (e) On the probability scale, mark with a cross the probability that he will pick a child that attends the hockey club.



# Intelligent Practice



Increase / Decrease / Same?	P(yellow) =



Increase / Decrease / Same?	P(yellow) =

## Intelligent Practice

A bag of sweets contains only 15 red sweets, 10 yellow sweets and 5 green sweets.

- 1) What is the probability of choosing a red sweet?
- 2) What is the probability of choosing a yellow sweet?
- 3) What is the probability of choosing a green sweet?
- 4) What is the probability of choosing a purple sweet?
- 5) What is the probability of choosing a banana?
- 6) What is the probability of choosing a red or yellow?
- 7) What is the probability of choosing a yellow or red?
- 8) What is the probability of choosing a red, yellow, green?
- 9) What is the probability of choosing a red, yellow, green, or purple?

# Intelligent Practice

A fair dice has faces 1, 1, 2, 3, 4 and 5.

- 1) What is the probability of choosing 1?
- 2) What is the probability of choosing 2?
- 3) What is the probability of choosing 3?
- 4) What is the probability of choosing 4?
- 5) What is the probability of choosing 5?
- 6) What is the probability of choosing 1 or 2?
- 7) What is the probability of choosing 1 or 3?
- 8) What is the probability of choosing 1, 2, or 3?
- 9) What is the probability of choosing 1, 2, 3, or 5?

# Fluency Practice

Match the statements to the probabilities on the right

The chance of picking an Ace from a normal pack of playing cards

The chance of landing on an even number when a normal, six sided die is thrown.

The chance of picking a yellow counter out of a bag containing 3 yellows, 4 blues and 2 reds

The chance of getting a heads when flipping a coin

The chance of picking a blue pen out of a pencil case holding 3 blue pens and 1 black pen

The school bus is late 3 days out of 5. What is the chance that the bus will be on time?

The chance of rolling a 3 on a normal, 6 sided die.

The chance of picking a club card from a normal pack of 52 playing cards

The chance of picking a blue ball from a bag containing 1 blue ball and 12 red balls.

The chance of getting either a head or a tail when flipping a normal coin

The chance of rolling a normal 6 sided die and getting neither an odd number nor an even number.

The chance of picking a red ball from a bag containing 3 yellow and 5 red balls.

$$\frac{5}{8}$$

$$\frac{3}{4}$$

$$\frac{3}{9}$$

$$\frac{3}{6}$$

$$\frac{1}{2}$$

$$\frac{2}{5}$$

$$\frac{1}{4}$$

$$\frac{1}{13}$$

$$0$$

$$1$$

$$\frac{1}{6}$$

$$\frac{4}{52}$$

# Fluency Practice

A fair dice is rolled once. What is the probability that the dice lands on:

- (a) 1
- (b) 4 or more
- (c) a prime number
- (d) a factor of 6
- (e) 7
- (f) not 5

A fair spinner has 8 equal sections, numbered 1 to 8. If the spinner is spun once, what is the probability that it lands on:

- (a) an even number
- (b) a number less than 4
- (c) 1 or 2
- (d) a number less than 10
- (e) not a prime number

A bag contains 3 red balls, 6 blue balls and 5 yellow balls. A ball is picked at random. What is the probability that:

- (a) the ball is red
- (b) the ball is blue or yellow
- (c) the ball is not blue
- (d) the ball is white

A letter is chosen at random from the word {S T A T I S T I C S}. What is the probability that the letter is:

- (a) an S
- (b) a C or a T
- (c) a vowel
- (d) not a T

At brunch, Tomek has a choice of toast, croissant or pain au chocolat. If  $P(\text{toast}) = 0.25$  and  $P(\text{croissant}) = 0.35$ , what is the probability that Tomek chooses pain au chocolat?

Bag A contains 5 red balls and 7 white balls. Bag B contains 3 red balls and 5 white balls. From which bag do you have the highest probability of choosing a white ball at random?

# Fluency Practice

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

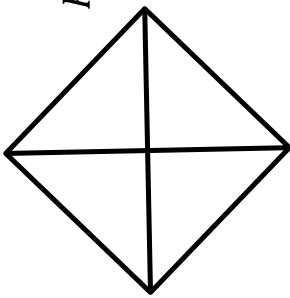
Use the number square above to help you work out these probabilities.  
Write your answers as fractions in their simplest forms.

A number is chosen at random between 1 and 100 (inclusive).  
Work out the probability that the number is ...

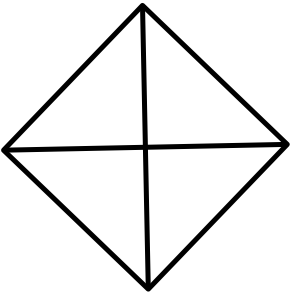
- |  |                             |
|--|-----------------------------|
| <b>1</b> a multiple of 10              | <b>11</b> a factor of 24    |
| <b>2</b> a multiple of 5               | <b>12</b> a factor of 11    |
| <b>3</b> a multiple of 2               | <b>13</b> a multiple of 11  |
| <b>4</b> greater than 90               | <b>14</b> a square number   |
| <b>5</b> less than 5                   | <b>15</b> an odd number     |
| <b>6</b> greater than 5                | <b>16</b> a multiple of 13  |
| <b>7</b> between 20 and 30 (inclusive) | <b>17</b> a power of 2      |
| <b>8</b> a two-digit number            | <b>18</b> a multiple of 3   |
| <b>9</b> a three-digit number          | <b>19</b> a triangle number |
| <b>10</b> a factor of 50               | <b>20</b> a prime number    |

# Problem Solving

Design a spinner by placing numbers in the gaps, so that the probabilities are correct.

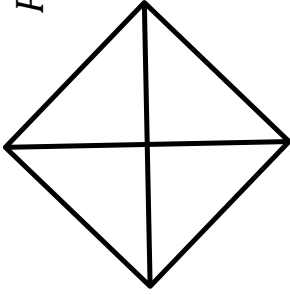


$$P(2) = \frac{1}{4}$$

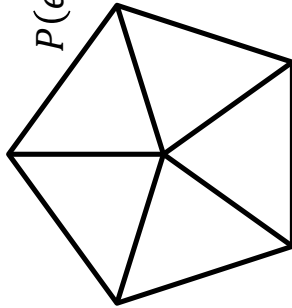


$$P(3) = \frac{1}{4}$$

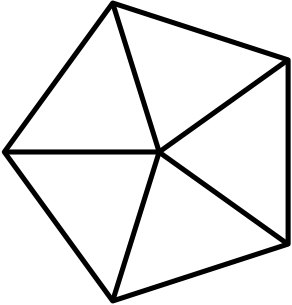
$$P(1) = \frac{1}{2}$$



$$P(\text{odd}) = 1$$



$$P(\text{even}) = \frac{3}{5}$$

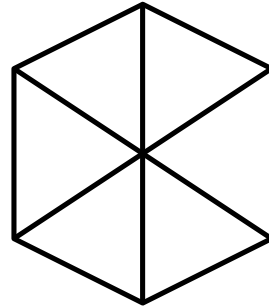
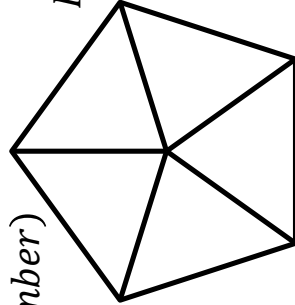


$$P(\text{square number}) = \frac{4}{5}$$

$$P(\text{square number}) = \frac{4}{5}$$

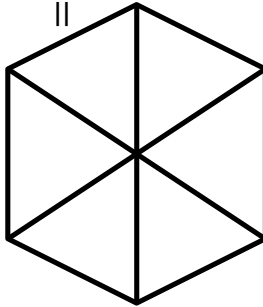
$$P(\text{prime}) = \frac{2}{5}$$

$$P(\text{Even}) = \frac{4}{5}$$



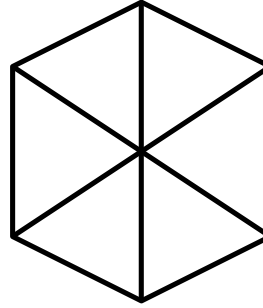
$$P(5) = \frac{1}{2}$$

$$P(4) = \frac{1}{3}$$



$$P(\text{prime}) = \frac{2}{3}$$

$$P(\text{square number}) = \frac{1}{6}$$



$$P(\text{cube number}) = \frac{1}{2}$$

$$P(\text{square number}) = \frac{1}{2}$$

$$P(\text{prime number}) = \frac{1}{3}$$



## Fluency Practice

Are the following pairs of statements mutually exclusive?

- 1) Winning a football match AND losing the same football match.
- 2) Getting the bus AND missing the same bus.
- 3) Watching a comedy DVD AND watching a romance DVD.
- 4) Rolling a 2 on a dice AND rolling a number less than 3.
- 5) Choosing a spade AND choosing a king.

# Intelligent Practice

Aiden is a boxer. The table shows the probability that Aiden will win, lose or draw.

**Work out the probability that Aiden will draw**

Win	Lose	Draw
$\frac{1}{4}$	$\frac{1}{4}$	

**Work out the probability that Aiden will draw**

Win	Lose	Draw
0.2	0.2	

**Work out the probability that Aiden will draw or win**

Win	Lose	Draw
$\frac{1}{4}$	$\frac{1}{4}$	

**Work out the probability that Aiden will draw**

Win	Lose	Draw
0.3	0.03	

**Work out the probability that Aiden will draw**

Win	Lose	Draw
$\frac{1}{4}$	$\frac{2}{8}$	

**Work out the probability that Aiden will draw or lose**

Win	Lose	Draw
0.7		0.07

# Intelligent Practice

## Castle FC play football matches every Saturday.

In their last 10 matches, Castle FC have drawn 5 matches, lost 2 and won the rest.

Complete the probability table for Castle FC

Win	Lose	Draw

## Castle FC play football matches every Saturday.

In their last 50 matches, Castle FC have drawn 10 matches, lost 5 and won the rest.

Complete the probability table for Castle FC as decimals

Win	Lose	Draw

# Intelligent Practice

Aiden is a boxer. The table shows the probability that Aiden will win, lose or draw.

**Work out the probability that Aiden will win**

Win	Lose
$x$	$x$

**Work out the probability that Aiden's match will be postponed**

Win	Lose	Draw	Postponed
$2x$	$2x$	$2x$	$4x$

**Work out the probability that Aiden's match will be postponed**

Win	Lose	Draw	Postponed
$\frac{3}{4}x$	$2x$	$2x$	$\frac{1}{4}x$

# Fluency Practice

1) Here are the probabilities of some events ( $h$ ) happening, write down the probabilities of the events not happening ( $h'$ ):

- |                          |                                    |                           |                                    |
|--------------------------|------------------------------------|---------------------------|------------------------------------|
| a) $P(h) = \frac{3}{10}$ | $P(h') = \underline{\hspace{2cm}}$ | e) $P(h) = 98\%$          | $P(h') = \underline{\hspace{2cm}}$ |
| b) $P(h) = \frac{1}{4}$  | $P(h') = \underline{\hspace{2cm}}$ | f) $P(h) = 55.5\%$        | $P(h') = \underline{\hspace{2cm}}$ |
| c) $P(h) = 0.21$         | $P(h') = \underline{\hspace{2cm}}$ | g) $P(h) = \frac{2}{5}$   | $P(h') = \underline{\hspace{2cm}}$ |
| d) $P(h) = 25\%$         | $P(h') = \underline{\hspace{2cm}}$ | h) $P(h) = \frac{12}{15}$ | $P(h') = \underline{\hspace{2cm}}$ |

2) There are some blue, red, green and purple balls in a bag. Find the probability of a purple ball being pulled out if these are the probabilities of the other colours:

a)

Blue	Red	Green	Purple
0.2	0.4	0.3	

b)

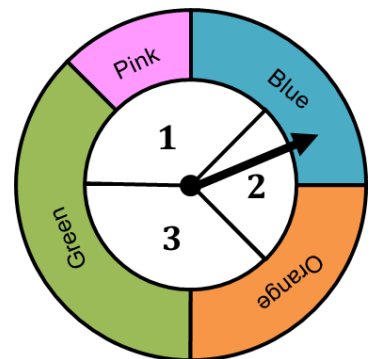
Blue	Red	Green	Purple
35%	21%	40%	

c)

Blue	Red	Green	Purple
$\frac{3}{20}$	$\frac{4}{20}$	$\frac{6}{20}$	

3) A spinner consists of an outer ring of coloured sectors and an inner circle of numbered sectors, as shown.

- a) The probability of getting 2 is  $\frac{2}{8}$ . The probability of getting 1 or 3 are equal. What is the probability of getting 3?  $P(3) = \underline{\hspace{2cm}}$
- b) The probability of getting blue is  $\frac{2}{8}$ . The probability of getting orange is  $\frac{2}{8}$ . The probability of getting green is  $\frac{3}{8}$ . What is the probability of getting pink?  $P(\text{pink}) = \underline{\hspace{2cm}}$
- c) Which of these pairs of events are mutually exclusive?
- i. Getting 3 AND getting 2
  - ii. Getting 3 AND getting green
  - iii. Getting 3 AND getting blue
  - iv. Getting blue AND getting pink



## Intelligent Practice


- 1) The relative frequency of a teacher throwing a pen in the bin is 0.5. A teacher throws a pen 50 times. How many will be successful?
- 2) The relative frequency of a teacher throwing a pen in the bin is 0.1. A teacher throws a pen 50 times. How many will be successful?
- 3) The relative frequency of a teacher throwing a pen in the bin is 1. A teacher throws a pen 50 times. How many will be successful?
- 4) The relative frequency of a teacher throwing a pen in the bin is 0.9. A teacher throws a pen 100 times. How many will be successful?
- 5) The relative frequency of a teacher throwing a pen in the bin is 0.10. A teacher throws a pen 100 times. How many will be successful?
- 6) The relative frequency of a teacher throwing a pen in the bin is 0.15. A teacher throws a pen 100 times. How many will be successful?
- 7) The relative frequency of a teacher throwing a pen in the bin is 0.015. A teacher throws a pen 100 times. How many will be successful?

## Intelligent Practice

- 1) If I throw a fair coin 10 times, how many times would you expect it to land on heads?
- 2) If I throw a fair coin 20 times, how many times would you expect it to land on heads?
- 3) If I throw a fair coin 60 times, how many times would you expect it to land on heads?
- 4) If I throw a fair coin 600 times, how many times would you expect it to land on heads?
- 5) If I roll a fair dice 600 times, how many times would you expect it to land on the number 1?
- 6) If I roll a fair dice 300 times, how many times would you expect it to land on the number 1?
- 7) If I roll a fair dice 150 times, how many times would you expect it to land on the number 1?
- 8) If I roll a fair dice 750 times, how many times would you expect it to land on the number 1?
- 9) If I roll a fair dice 1500 times, how many times would you expect it to land on the number 1?
- 10) If I throw a fair coin 1500 times, how many times would you expect it to land on heads?

# Fluency Practice

**No Low!** 10p to play


 Roll a dice,  
if it's 5 or 6, you win 20p!

	Win	Lose
Tally		
Total		

Probability of winning =  $\frac{\quad}{\quad}$  ← Total wins  
 ← Total games

Profit = income – pay outs = \_\_\_\_\_

**Twice Dice** 20p to play


 Roll a dice twice,  
if **both** numbers are even,  
you win 40p!

	Win	Lose
Tally		
Total		

Probability of winning =  $\frac{\quad}{\quad}$  ← Total wins  
 ← Total games

Profit = income – pay outs = \_\_\_\_\_

**Flippin' 'Eck** 10p to play


 Flip a coin,  
if it's tails, you win 20p!

	Win	Lose
Tally		
Total		

Probability of winning =  $\frac{\quad}{\quad}$  ← Total wins  
 ← Total games

Profit = income – pay outs = \_\_\_\_\_

**Tricky Tails** 10p to play


 Flip a coin twice,  
if **either** result is tails,  
you win 20p!

	Win	Lose
Tally		
Total		

Probability of winning =  $\frac{\quad}{\quad}$  ← Total wins  
 ← Total games

Profit = income – pay outs = \_\_\_\_\_

**No Low!** 10p to play


 Roll a dice,  
if it's 5 or 6, you win 20p!

	Win	Lose
Tally		
Total		

Probability of winning =  $\frac{\quad}{\quad}$  ← Total wins  
 ← Total games

Profit = income – pay outs = \_\_\_\_\_

**Twice Dice** 20p to play


 Roll a dice twice,  
if **both** numbers are even,  
you win 40p!

	Win	Lose
Tally		
Total		

Probability of winning =  $\frac{\quad}{\quad}$  ← Total wins  
 ← Total games

Profit = income – pay outs = \_\_\_\_\_

**Flippin' 'Eck** 10p to play


 Flip a coin,  
if it's tails, you win 20p!

	Win	Lose
Tally		
Total		

Probability of winning =  $\frac{\quad}{\quad}$  ← Total wins  
 ← Total games

Profit = income – pay outs = \_\_\_\_\_

**Tricky Tails** 10p to play

 Flip a coin twice,  
if **either** result is tails,  
you win 20p!

	Win	Lose
Tally		
Total		

Probability of winning =  $\frac{\quad}{\quad}$  ← Total wins  
 ← Total games

Profit = income – pay outs = \_\_\_\_\_



# Fluency Practice

1) Simon records the colour of cars going past his house for an hour.

Colour	Frequency
Blue	5
Red	4
Yellow	1
White	7
Black	3

- a) What is the probability the next car will be  
 i) blue \_\_\_\_\_ ii) red \_\_\_\_\_ iii) Not black \_\_\_\_\_
- b) How many Red cars would you expect if  
 i) 100 cars went past \_\_\_\_\_ ii) 60 cars went past \_\_\_\_\_

2) Sammy throws a drawing pin 200 times and records how it lands.

Pin up	160
Pin down	40

- a) What is the probability the pin will land  
 i) pin up? \_\_\_\_\_ ii) pin down \_\_\_\_\_
- b) How many pin ups would you expect if the pin was thrown  
 i) 80 times \_\_\_\_\_ ii) 320 times \_\_\_\_\_ iii) 400 times \_\_\_\_\_

3) A group of children are asked to write for their favourite food, and child is picked at random.

Favourite Food	Number of people
Chinese	20
Pizza	16
Mexican	18

- a) What is the probability the person liked Chinese?  
 i) \_\_\_\_\_ ii) Didn't like Mexican best. \_\_\_\_\_
- b) How many people would you expect to like pizza if  
 i) 100 people were asked \_\_\_\_\_  
 ii) 250 people were asked \_\_\_\_\_  
 iii) 1000 people were asked? \_\_\_\_\_  
 iv) 460 people were asked? \_\_\_\_\_

# Fluency Practice

(a) Mikel throws a biased coin 180 times and it lands on heads 120 times. What is the experimental probability that the coin lands on heads?

(b) Billie spins a four-sided spinner 200 times and it lands on four 45 times. What is the relative frequency that the spinner will land on a four?

(c) The probability that Julie pulls a red ball from a bag is 0.15. If there are 80 balls in the bag, how many of them would you expect to be red?

(d) In a class 6 out of 30 students wear glasses. If there are 450 students in the school, how many of them would you expect to wear glasses in the whole school?

(e) Samir records the colours of 50 cars passing school, and 14 are black. Samir then records the colours of 400 cars. Work out an estimate for the number of cars he would expect to be black.

(f) Jim has a choice of cereal, toast or fruit for breakfast.  $P(\text{cereal})=0.3$  and  $P(\text{toast})=0.25$ . Over 300 days, how many times would you expect Jim to have fruit for breakfast?

(g) A spinner can land on red, blue or yellow. The probability that it lands on red is 0.1 and the probability that it lands on yellow is 0.25. If Hadiyah spins the spinner 400 times, how many times would she expect it to land on blue?

(h) Tom throws a fair coin 30 times. Explain why Tom might not get exactly 15 heads and 15 tails.

(i) Mabel throws a coin 1000 times. It lands on heads 492 times. State with reason whether you think the coin is fair.

# Fluency Practice

1. The table shows the number of times a coin landed on heads and tails.

a) Estimate the probability that this coin lands on heads.

Heads	Tails
3	7

b) Is the coin fair? Explain your answer.

2. The table shows the number of vehicles that park in an underground car park during a single day.

a) What is the relative frequency of cars?

Cars	Vans	Other
60	21	3

b) How many vans would you expect to park in this car park during a week?

3. Andrew and Jenny play a game of chance. The number of times they win and lose the game are recorded in the table.

a) Use the data to work out the most reliable estimate of the probability of winning.

	Number of Wins	Number of Losses
Andrew	18	45
Jenny	48	192

b) If you played the game 100 times, how many times would you expect to win?

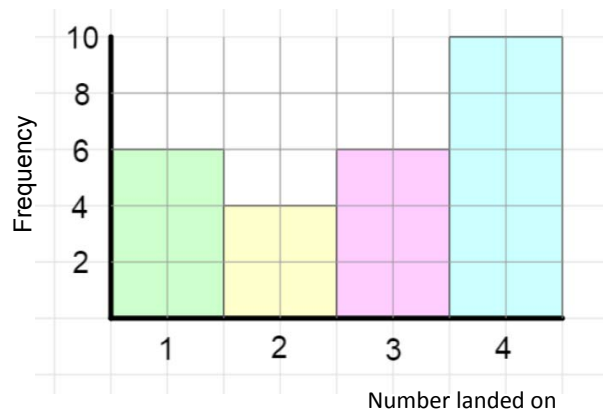
4. The probability of picking a red marble from a bag is  $\frac{1}{5}$

If you pick a marble 800 times, how many times would you expect get a red marble?

5. The chart shows the number of times a spinner landed on the numbers 1,2,3 and 4.

a) Estimate the probability that the spinner will land on a 4.

b) If we spin this spinner 150 times, how many times would you expect it to land on number 4?



# Fluency Practice

Question 1: An ordinary coin is thrown 50 times.  
Barry says “I am going to get heads 25 times and tails 25 times.”  
Explain why he could be wrong.

Question 2: A coin is thrown 30 times.  
The coin lands on tails 20 times.  
What is the relative frequency of the coin landing on tails?

Question 3: A dice is rolled 50 times.  
It lands on six 37 times.  
(a) Write down the relative frequency of the dice landing on a six

Robyn says “the dice is biased towards landing on a six.”

(b) Do you think the dice is biased? Explain your answer.

Question 4: Jessica wants to test if a coin is biased.  
She throws the coin 24 times.

T T H H T H H H T H T T  
T H H T T H H T H H H T

(a) Complete the relative frequency table.

	Heads	Tails
Relative frequency		

(b) Do you think the coin is biased? Explain your answer.

Question 5: A biased dice is rolled 30 times.



3 4 1 3 6 2 6 6 6 5 6 3 6 4 6  
1 6 3 4 6 6 2 6 3 6 6 3 6 3 6

(a) Complete the relative frequency table

Number	1	2	3	4	5	6
Relative Frequency						

(b) Do you think the dice is biased? Explain your answer.

# Fluency Practice

Question 6: Esme takes the bus to university 40 times during a term.  
The relative frequency of the bus being late is 0.3.  
How many times was the bus late?

Question 7: Katie rolls a dice 100 times.  
The table shows the results

Number	1	2	3	4	5	6
Frequency	22	9	14	31	19	5

Work out the relative frequency of throwing:

- (a) An even number      (b) A square number      (c) A prime number  
(d) A cube number      (e) A multiple of 3      (f) A factor of 18

Question 8: A spinner lands of white, black, red or orange.  
The relative frequencies after 300 spins are shown in the table below.

Colour	White	Black	Red	Orange
Relative Frequency	0.25	0.4	0.2	0.15

- (a) How many times did the spinner land on white?  
(b) How many times did the spinner land on red?  
(c) How many more times did the spinner land on black than orange?

Question 7: Martin and Laura want to estimate how many green jelly beans are in a tub of 600 jelly beans.  
A trial consists of taking a jelly bean at random, noting the colour and replacing the jelly bean into the tub.

	Number of trials	Number of green jelly beans chosen
Martin	30	4
Laura	150	12

- (a) Write down the relative frequency of Martin taking a green jelly bean.  
(b) Write down the relative frequency of Laura taking a green jelly bean.  
(c) Whose experiment gives the more reliable estimate of the number of green jelly beans in the tub? Give a reason for your answer.  
(d) How many green jelly beans do you expect to be in tub altogether?

# Extension

Question 1: Leo plants and grows 50 flowers.  
The table shows information about the colours.

Colour	Red	Yellow	White
Frequency	16		28
Relative Frequency	0.32		

(a) Copy and complete the table.

Altogether, Leo grows 125 flowers.

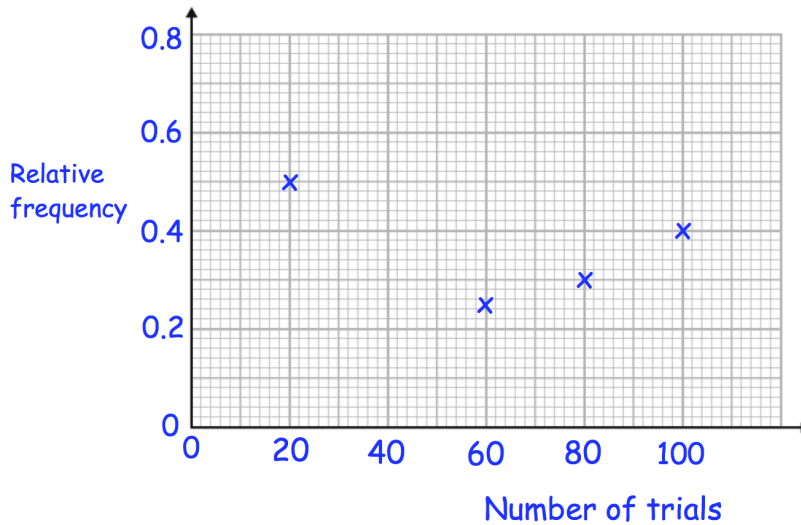
(b) How many flowers would you expect to be yellow?

Question 2: Four students have each written a 2000 word essay.  
The spellings are checked for all four students.  
The relative frequencies of a spelling mistake for the 4 students are:

Student	Alan	Beryl	Connie	Diego
Relative Frequency	0.032	0.01	0.009	0.017

Work out the mean number of spelling mistakes made by the four students.

Question 3: There are 70 sweets in a jar.  
In a trial, a sweet is chosen at random and then replaced.  
The graph shows the relative frequency of a blue sweet.



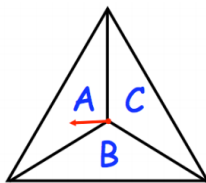
In the first forty trials, twelve blue sweets were chosen.

- Plot this result on the graph.
- How many blue sweets were chose after 60 trials.
- Which is the best estimate, from the graph, of the probability of choosing a blue sweet?
- Use your answer to estimate the number of blue sweets in the jar

# Extension

Question 4: A coin lands on heads 300 times.  
The relative frequency of heads is 0.6  
Work out the number of times the coin was flipped.

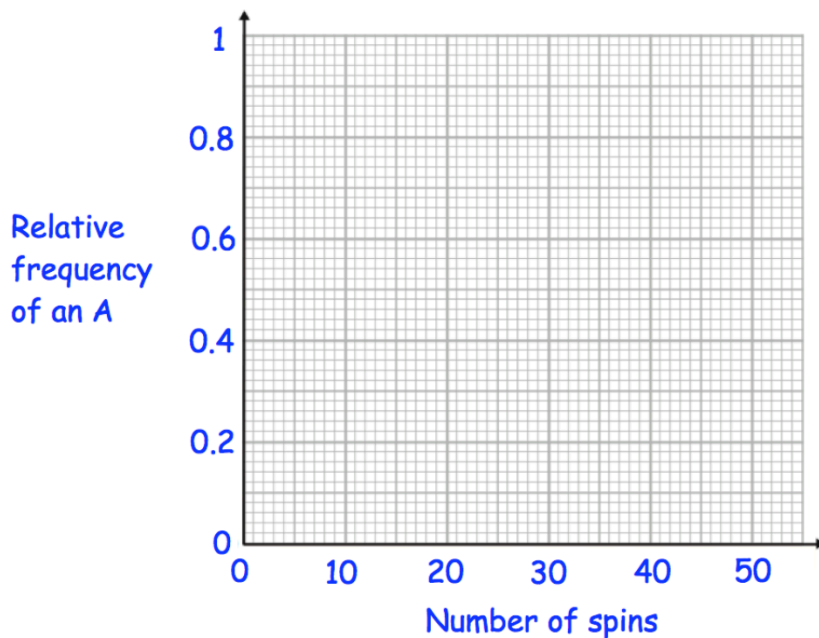
Question 5: A three sided spinner is labelled A, B and C.



The spinner is spun and the frequency of the letter A is recorded every 10 spins.  
The table below shows this information.

Spins	10	20	30	40
Frequency of an A	6	14	18	26

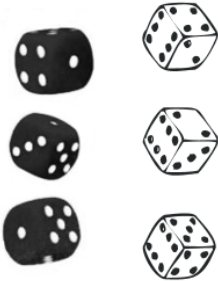
(a) Plot the relative frequencies on the graph below.



(b) Jacob says the relative frequency after 50 spins is 0.8.  
Explain why Jacob must be wrong.

# Problem Solving

## probability: estimated frequency

- 1.) a bag contains marbles: 3 red and 5 blue  
a marble is chosen repeatedly (with  
replacement), 320 times  
how many of each would you expect to get?
- 2.) the probability of picking an ace is  $\frac{4}{52}$   
how many aces would you expect to obtain if  
a card is picked, with replacement, 260 times?
- 3.) Albert has a biased coin  
the probability of landing on a tail is 0.3  
if he flips the coin 500 times, how many times  
would he expect to get a head?
- 4.) around 12% of people are left handed  
how many left handed people would you  
expect to find at a Shrewsbury football game  
that 6000 people attend?
- 5.) a bag contains red and pink counters  
there are 60 pink counters in the bag  
the probability of picking a pink counter is 0.2  
how many red counters are in the bag?
- 6.) in France about  $\frac{1}{25}$  of people  
have red hair  
how many students would you  
expect to have red hair in a school  
in France of 1250 students?
- 7.) a spinner has regions numbered 1 to 10  
(inclusive)  
in 1000 spins how often would you expect to  
get a number that is even or a multiple of 5?
- 8.) when three dice are  
rolled the probability of  
all three numbers being  
different is  $\frac{5}{9}$   
what is the probability of  
three being the same?  
  
in 3600 throws how often would you  
expect to get:  
(i) all three numbers different?  
(ii) all three numbers the same?  
(iii) exactly two (a pair) the same?



# Fluency Practice

1. Different coloured counters are placed in a bag. The probabilities of each counter is given.

<b>Colour</b>	Red	Blue	Green	Purple
<b>Probability</b>	$2x$	$0.5 - x$	$3x - 0.05$	0.15

- a) Find the probability of selecting a green counter
- b) You are told there are 18 purple counters in the bag. Find how many blue, green and red counters there are?

2. Different coloured Lego bricks are placed in a bag. The probabilities of getting each Lego brick is given.

<b>Colour</b>	Yellow	Green	Brown	Pink
<b>Probability</b>	$4x$	$5x + 0.02$	$3x$	0.26

- a) Find the probability of selecting a brown Lego brick
- b) Given that there are 156 pink Lego bricks, how many bricks are there in total?

3. Different coloured sponges are placed in a bag. The probabilities of getting each sponge is given. You are three times more likely to get a red sponge than a blue sponge

<b>Colour</b>	Blue	Red	Pink	Yellow
<b>Probability</b>	$3x + 0.08$		0.12	$2x$

- a) Find the probability of selecting a yellow sponge
- b) Given that there are 24 pink sponges, how many sponges are there in total?

4. A number of beetles are dipped in coloured paint and put into a jar. The probability of picking beetle with a certain colour is below. You are half as likely to get a blue beetle as you are a green beetle. You are 25% more likely to get gold beetle than a green one.

<b>Colour</b>	Green	Blue	Gold	Silver
<b>Probability</b>	$8x + 0.04$			0.01

- a) Find the probability of getting a green beetle.
- b) Given that the probability of getting a Silver beetle is 0.01, what is the minimum amount of beetles in the bag?

5. Mick's pantry is flooded and all the tins have their labels washed away. They are indistinguishable from one another. He has tins of beans, tomatoes, cat food and peaches. The probability of picking a tin of each type is given below.

<b>Tin contents</b>	Beans	Tomatoes	Cat Food	Peaches
<b>Probability</b>	$\frac{1}{3}$	$x + \frac{1}{10}$	$\frac{4}{15} - x$	$3x$

- a) Find the probability of selecting tomatoes **or** peaches.
  - b) You are told that you have 4 more tins of tomatoes than cat food. How many tins do you have in total?
  - c) What is the minimum number of tins? Give a reason for your answer.
6. Bess, Cress, Jess and Tess are having a pumpkin carving competition. Tess has a one in five chance of winning. Cress has 5% less than double the chance Bess has of winning. Whilst Jess has the same chance of winning as Bess or Cress. What is the probability of Bess winning the competition?

# Fluency Practice

## Probabilities & Equations

- 1) 120 students were surveyed about their hair length.
- 70 students had **long** hair.  
20 students had **medium-length** hair.  
 $4x + 18$  students had **short** hair.

Use this information to form an equation and solve to find  $x$ .

- 2) 200 students were surveyed about their pets.
- 60 students **only** had a dog.  
70 students **only** had a cat.  
 $2x + 25$  students had a dog **and** a cat.  
 $x$  students had **no** dog **or** cat.

What is the probability a student has a dog **and** a cat?

- 3) Some students were surveyed about their lunch.
- 60 of the students ate **healthy** food.  
 $8x - 42$  of the students ate **unhealthy** food.

The probability of picking a student who ate **healthy** food is  $\frac{2}{7}$

- a) How many students were there in total?  
b) Use this information to find  $x$ .

- 4) The table shows how people get to work.

The probability they took the train is  $\frac{3}{8}$ .

Work out the probability a person picked at random walks.

Walk	Train	Car
$3x + 5$	12	$4x + 1$

- 5) The probability a train is **late** is  $10x - 15$   
The probability a train is **early** is  $3x - 3$   
The probability a train is **on time** is  $34 - x$

Express **numerically** the probability a train is **not** late.  
(**Hint:** Give the total a value.)

# Fluency Practice

Question 1: Emily flips a coin twice.  
One of the possible outcomes is a tail and a tail (TT)  
List all the possible outcomes.

Question 2: Benjamin rolls an ordinary six-sided dice once and flips a coin.  
List all the possible outcomes.



Question 3: A rugby team plays two matches.  
They can win (W), draw (D) or lose (L) each match.  
List all the possible outcomes.

Question 4: There are five students in a group: Alison, Beth, Conor, David and Eddie.  
Miss Jenkins chooses two students at random from the group to give a presentation.  
List all the possible outcomes.

Question 5: Here are four cards.  
Each card has a number on it.

7

2

6

4

- (a) Write down all the 2-digit numbers that can be made using the cards  
(b) Write down all the 3-digit numbers that can be made using the cards

Question 6: Marco visits a restaurant with his friends.  
Shown is the menu.  
Marco chooses one starter, one main and one dessert.  
List all possible outcomes.

Starter	Main	Dessert
Soup	Curry	Ice Cream
Fish	Pizza	Danish
	Burger	

# Extension

Question 1: Andrew has attempted his maths homework.  
Can you spot any mistakes?

**Q1** Orla has four types of vegetable.

Peas  
Carrots  
Turnip  
Spinach

Orla is going to choose 2 different types of vegetable.

Write down all the possible combinations of vegetable she can choose.

PC, PT, PS      CP, CT, CS  
-----  
TP, TC, TS      SP, SC, ST  
-----

Question 2: Here are four cards.  
Each card has a number on it.



Write down all the 3-digit even numbers that can be made using the cards

Question 3: In a restaurant, there are 5 possible pizza toppings:  
Chicken, Pineapple, Olives, Mushrooms and Beef.

Freddie picks two different toppings on his pizza

(a) List all possible outcomes

Freddie picks the toppings at random

(b) Write down the probability that the pizza contains meat

Question 4: There are two bags.

Bag 1 contains a red counter and a pink counter.

Bag 2 contains a blue counter, a yellow counter and a white counter.

Sam picks a counter at random from bag 1 and notes its colour

He then places this counter into bag 2.

Sam then picks a counter at random from bag 2.



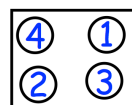
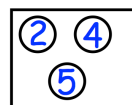
Write down the probability that Sam picks two counters that are the same colour

# Extension

Question 5: Heather has made up a game for a school fête to raise money for charity. There are two boxes of counters. Each counter has a number on it.

Box 1

Box 2



The person playing the game will select one counter at random from box 1.

They will then select one counter at random from box 2.

(a) Write down all the possible combinations of counters picked.

The person playing the game wins when the numbers multiply to give an odd number.

During the fête the game is played 300 times.

The game costs 80p to play.

Each prize costs £2

(b) Work out how much money Heather should raise for charity.

Question 6: Ali is having a meal with his friends. He will either have:

- one starter and one main
- or
- one main and one dessert

Shown is the menu

Starter		Main		Dessert	
Soup	£3.20	Chicken	£6.25	Trifle	£2.50
Prawns	£3.55	Beef	£8.00	Brownie	£2.15
Melon	£2.45	Pork	£6.75	Eton Mess	£3.50
Duck	£3.95			Ice Cream	£1.95

Ali has £10.

List all the possible combinations that Ali **cannot** afford.

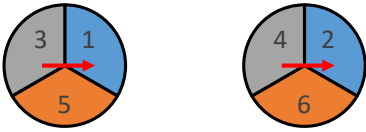
# Fluency Practice

Draw a sample space diagram for each question:

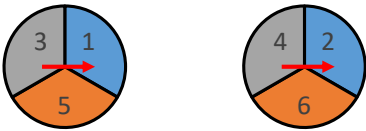
- 1) Kayleigh throws a four-sided dice numbered 2, 4, 6 and 8 and a four-sided dice at the same time and multiplies the scores.
- 2) Joel throws a four-sided dice numbered 1, 3, 5 and 7 and a four-sided dice numbered 1, 3, 5 and 7 at the same time and adds up the scores.
- 3) Noel throws a four-sided dice numbered 1, 3, 5 and 7 and a four-sided dice numbered 2, 4, 6 and 8 at the same time and adds up the scores.
- 4) Lisa throws a four-sided dice and a four-sided dice at the same time and find the difference between the scores.
- 5) Noel throws a four-sided dice numbered 2, 4, 6 and 8 and a four-sided dice numbered -1, -2, -3 and -4 at the same time and multiplies the scores.
- 6) Paul throws a four-sided dice numbered -1, -2, -3 and -4 and a four-sided dice numbered 1, 3, 5 and 7 at the same time and find the difference between the scores.
- 7) Ethan throws a six-sided dice and a spinner with faces labelled R, G, B and Y at the same time.
- 8) Kayleigh throws a four-sided dice numbered 1, 3, 5 and 7 and a spinner with faces labelled R, G, B and Y at the same time.
- 9) Ethan throws a spinner with faces labelled R, G, B and Y and a six-sided dice at the same time.
- 10) Lisa throws a four-sided dice numbered -1, -2, -3 and -4 and a four-sided dice numbered -1, -2, -3 and -4 at the same time and find the difference between the scores.

# Intelligent Practice

I spin these two spinners then add the numbers together to get a score. Work out the probability that I get a score of 4.



I spin these two spinners then add the numbers together to get a score. Work out the probability that I get a score of 7.



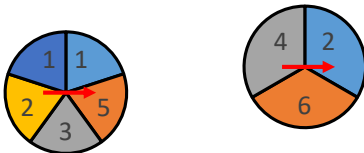
I spin these two spinners then add the numbers together to get a score. Work out the probability that I get a score of 7.



I spin these two spinners then find the product of the numbers to get a score. Work out the probability that I get a score of 4.



I spin these two spinners. What is the probability that I get the same number on both spinners?



I spin the same spinner twice. What is the probability that I get the same number on both spins?



I spin these two spinners then add the numbers together to get a score. Work out the probability that I get a score of 7.



I spin these two spinners then find the difference between the numbers to get a score. Work out the probability that I get a score of 7.



I spin these two spinners then find the difference between the numbers to get a score. Work out the probability that I get a score of 4.



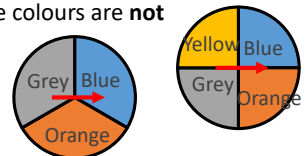
I spin the same spinner twice. What is the probability that I get the same colour on both spins? What is the probability that the colours are **not** the same?



I spin the same spinner twice. What is the probability that I get the same colour on both spins? What is the probability that the colours are **not** the same?



I spin these two spinners. What is the probability that I get the same colour on both spinners? What is the probability that the colours are **not** the same?



# Fluency Practice

At a café, children can choose from either fish fingers, chicken nuggets or pizza for their main course, and ice cream, fruit or jelly for their dessert. List all the possible combinations of meals. What is the probability that a child chooses chicken nuggets followed by jelly?

A door code is made up of three digits. The first digit can be 1, 2 or 3. The second digit can be 4 or 5, and the third digit can be 6 or 7. List all the possible door codes. What is the probability that the three-digit door code is a multiple of three?

Lucy has two four-sided fair spinners, each number 1 to 4. She spins both spinners, the add their scores together.

- (a) Complete the sample space.  
(b) What is the probability of the total being a multiple of 3?  
(c) What is the probability of the total being greater than 5?

	1	2	3	4
1				
2				
3				
4				

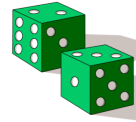
Tariq has two five-sided fair spinners. The first spinner is numbered 1, 2, 3, 4 and 5 and the second spinner is numbered 2, 3, 5, 7 and 11. He spins each spinner once and finds the difference between their scores.

- (a) Complete the sample space.  
(b) Find the probability that the difference between the scores is zero.  
(c) Find the probability that the difference between the scores is greater than four.

	1	2	3	4	5
2					
3					
5					
7					
11					



# Fluency Practice



Question 1: Two fair six sided dice are rolled.  
 The numbers on the two dice are **added** together to give a score.  
 The table shows all possible scores.

- (a) Which score is the most likely?
- (b) Which scores are the least likely?
- (c) Write down the probability of scoring a
- (i) 3            (ii) 5            (iii) 6            (iv) 7
- (d) Write down the probability of scoring a number
- (i) over 10    (ii) under 7    (iii) 4 or less    (iv) 6 or more
- (e) Write down the probability of scoring
- (i) an odd number            (ii) a square number            (iii) a prime number

		Dice 1					
+		1	2	3	4	5	6
1		2	3	4	5	6	7
2		3	4	5	6	7	8
3		4	5	6	7	8	9
4		5	6	7	8	9	10
5		6	7	8	9	10	11
6		7	8	9	10	11	12

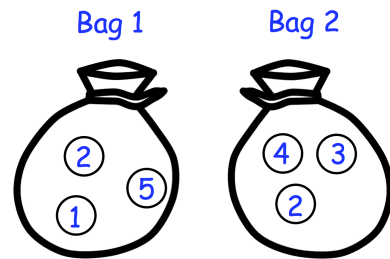
Question 2: Two fair six sided dice are rolled.  
 The numbers on the two dice are **multiplied** together to give a score.

- (a) Complete the table to show all possible scores.
- (b) Write down the probability
- (i) 10            (ii) 9            (iii) 12            (iv) 8
- (c) Write down the probability of scoring
- (i) an even number            (ii) an odd number
- (iii) a number less than 20

		Dice 1					
x		1	2	3	4	5	6
1							
2							
3							
4							
5							
6							

# Fluency Practice

Question 3: Two bags, 1 and 2, each contain three counters.  
 In bag 1, the counters are labelled 1, 2 and 5.  
 In bag 2, the counters are labelled 2, 3 and 4.



A counter is drawn at random from bag 1 and a counter is drawn from bag 2.

The two numbers are multiplied together to give a score

- Complete the table to show all possible scores
- Find the probability of scoring a 6
- Find the probability of scoring a multiple of 4
- Find the probability of scoring an odd number

		Bag 1		
		1	2	5
Bag 2	x			
	2			
	3			
4				

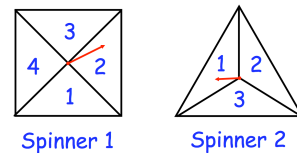
Question 4: Rose is playing a game with a fair six sided dice and a fair coin.  
 She rolls the dice and flips the coin.

If the coin lands on heads, her score is **one less** than the number on the dice.  
 If the coin lands on tails, her score is **two more** than the number on the dice

- Complete the table to show all possible scores
- Find the probability of scoring a 0
- Find the probability of scoring a 5
- Find the probability of scoring a number less than 4
- Find the probability of scoring a square number

		Dice					
		1	2	3	4	5	6
Coin	Heads						
	Tails						

Question 5: Two fair spinners are spun.  
 Spinner 1 has four equation sections labelled 1, 2, 3 and 4.  
 Spinner 2 has three equal sections labelled 1, 2 and 3.  
 Each spinner is spun once.  
 The score is the **difference** between the numbers



- Complete the table to show all possible scores
- Find the probability of scoring a 1
- Find the probability of scoring a 2 or more

		Spinner 1			
		1	2	3	4
Spinner 2	1	0	1	2	
	2	1			
	3	2			

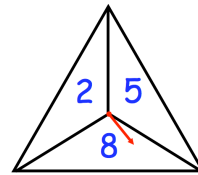
# Extension

Question 1: Adam uses two fair spinners in a game.  
He spins both spinners and **adds** the two numbers together.

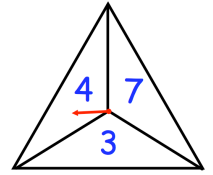
(a) Draw a table to show all possible scores

Adam thinks that the probability of an even score is  $\frac{1}{2}$

(b) Explain why Adam is incorrect

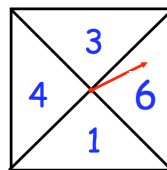


Spinner 1



Spinner 2

Question 2: Jessica is playing a game with a fair four sections spinner and a fair coin.  
She spins the spinner and flips the coin.



Spinner



Coin

If the coin lands on heads, Jessica applies **rule 1** to the number on the spinner  
If the coin lands on tails, Jessica applies **rule 2** to the number on the spinner

The table below shows some information about the scores that Jessica can get.

		Spinner			
		1	3	4	6
Coin	Heads	1	9		36
	Tails	3	7	9	13

(a) What could **rule 1** be?

(b) What could **rule 2** be?

(c) Complete the table

(d) Find the probability that Jessica scores a number less than 15

# Extension

Question 3: Katie has organised a game to raise money for charity at a local fair.  
The person playing rolls two fair six sided dice  
The numbers on the dice are **multiplied** together.

Each person pays £1 to play.  
If they score a square number, they win £3  
The game is played 180 times.

How much money does Katie raise for charity?

Question 4: Two bags, 1 and 2, each contain equal size counters.

Bag 1 contains a pink counter, yellow counter and white counter  
Bag 2 contains counters labelled 1, 4, 5 and 7.

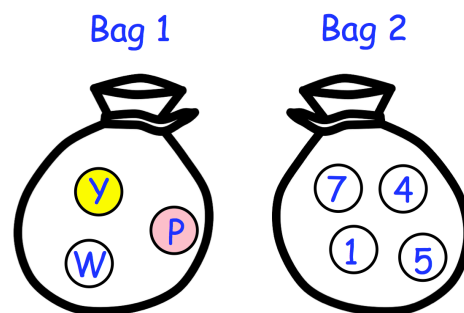
A counter is drawn at random from bag 1 and  
a counter is drawn at random from bag 2.

If the counter from bag 1 is pink, the number on the  
counter from bag 2 is **increased** by 1

If the counter from bag 1 is yellow, the number on the  
counter from bag 2 is **decreased** by 5

If the counter from bag 1 is white, the number is **halved**.

Find the probability of scoring a number below 3



Question 5: A fair dice has six faces numbered 1, 1, 1, 2, 3 and 4.  
The dice is rolled twice and the number shown is recorded each time.  
Find the probability that the sum of the two numbers recorded is at least 4.

Question 6: A fair six sided dice is rolled twice.  
Find the probability that the number obtained on the first roll is at least 3  
greater than the second roll

# Problem Solving

Each puzzle is a sample space diagram representing a two spinners being spun. The spinners are both fair, so each outcome has the same probability. The rule to calculate the score for each spin is different in each puzzle. Complete the sample space diagrams.

1.

	1	
+		2
2		5

2.

	2	3	
×		10	-6
4		12	
	5		18

3.

	1	2	4	6
+				

4.

	1	3	5	2
×				

**Rule:** Add the results of the two spinners to get your score.

**Rule:** Multiply the results to get your score.

**Rule:** Add the results to get your score.

**Rule:** Multiply the results to get your score.

5.

	-1	2
×		
-1		
	-	

The probability of getting a negative score is  $\frac{10}{16}$ . What could the missing numbers be?

6.

	-1	-3	-5	-7
+				
4				

The probability of getting a negative score is  $\frac{10}{16}$ . What could the missing numbers be?

7.

	3	7
÷		
5		
	6	

The probability of getting a recurring decimal is 0.5. What could the missing numbers be?

**Rule:** Multiply the results to get your score.

**Rule:** Add the results to get your score.

**Rule:** Divide the smaller number by larger.

## Problem Solving

a number is selected from 1, 2, 3, 4

another number is selected from 1, 4, 9, 16

A wins if the product of the two numbers is less than 16

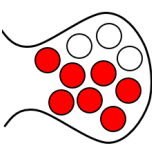
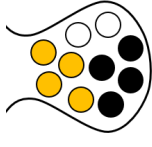
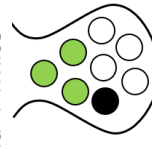
B wins if the product of the two numbers is 16 or more

is it fair or not?

x	1	2	3	4
1				
4				
9				
16				

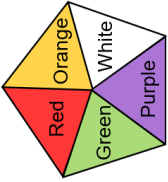
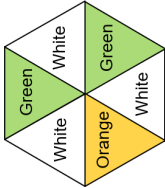
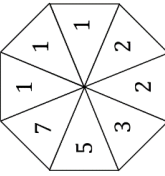
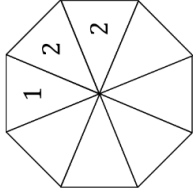
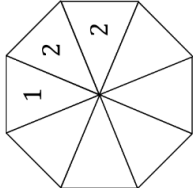
sample space

# Theoretical Probability with Counters

Theoretical Probability with Counters			
(a)	(b)	(c)	(d)
<p>A bag contains 7 red counters and 3 white counters. A counter is chosen at random.</p> 	<p>What is the probability that a white counter is chosen?</p>	<p>What is the probability that the counter chosen is not white?</p>	<p>How many white counters would need to be added to the bag to make the probability of choosing a white counter equal to <math>\frac{1}{2}</math>?</p>
<p>A bag contains 2 white counters, 4 orange counters and 4 black counters. A counter is chosen at random.</p> 	<p>What is the probability that a black counter is chosen?</p>	<p>What is the probability that the counter chosen is not white?</p>	<p>How many black counters would need to be added to the bag to make the probability of choosing a black counter equal to <math>\frac{1}{2}</math>?</p>
<p>A bag contains 1 black counter, 3 green counters and 4 white counters. A counter is chosen at random.</p> 	<p>What is the probability that a white counter is chosen?</p>	<p>What is the probability that the counter chosen is not black?</p>	<p>How many white counters would need to be added to the bag to make the probability of choosing a white counter equal to <math>\frac{2}{3}</math>?</p>
<p><b>Bag A</b> contains 7 blue counters and 5 red counters. The rest of the counters are white. <b>Bag B</b> contains 3 blue counters, 2 white counters and 5 red counters.</p>	<p>The probability of choosing a blue counter from bag A is 0.35. What is the total number of counters in bag A?</p>	<p>Ali takes a counter at random from bag A. Ben takes a counter at random from bag B. Who has the greater probability of taking a blue counter?</p>	<p>How many red counters does Ben need to add to bag A to make the probability of choosing a red counter from bag A the same as from bag B?</p>

# Theoretical Probability with Spinners

## Theoretical Probability with Spinners

<p>The fair five-sided spinner shown is spun once.</p> 	<p><b>(a)</b> What is the probability of the spinner landing on green?</p>	<p><b>(b)</b> What is the probability of the spinner landing on purple or white?</p>	<p><b>(c)</b> What is the probability of the spinner landing on black?</p>	<p><b>(d)</b> Sania spins the spinner 50 times. How many times would she expect it to land on orange?</p>
<p>The fair six-sided spinner shown is spun once.</p> 	<p><b>(e)</b> What is the probability of the spinner landing on white?</p>	<p><b>(f)</b> What is the probability that the spinner does not land on orange?</p>	<p><b>(g)</b> Which is more likely – the spinner landing on white or the spinner landing on green?</p>	<p><b>(h)</b> Lola spins the spinner 120 times. How many times would she expect it to land on white?</p>
<p>The fair eight-sided spinner shown is spun once.</p> 	<p><b>(i)</b> What is the probability of the spinner landing on a number less than 10?</p>	<p><b>(j)</b> What is the probability of the spinner landing on an odd number?</p>	<p><b>(k)</b> What is the probability of the spinner not landing on a prime number?</p>	<p><b>(l)</b> Aidan spins the spinner 80 times. How many times would he expect it to land on a 2 or 3?</p>
<p><b>(m)</b> Here is a fair eight-sided spinner. Complete the spinner so that:</p> <ul style="list-style-type: none"> <li>The probability of landing on a 1 is the same as the probability of landing on a 2</li> <li>The probability of landing on a 4 is <math>\frac{1}{8}</math></li> <li>The total of all the numbers on the spinner is 16.</li> </ul> 		<p><b>(n)</b> Here is a fair eight-sided spinner. Complete the spinner so that:</p> <ul style="list-style-type: none"> <li>The probability of landing on an odd number is 0.5</li> <li>The probability of spinning a 3 is the same as the probability of spinning a 4</li> <li>All the numbers on the spinner are less than 8</li> <li>The total of all the numbers is 24.</li> </ul> 		


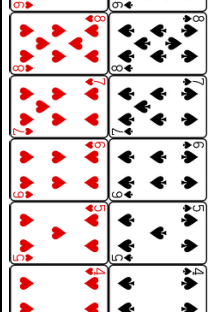


# Theoretical Probability with Dice

Theoretical Probability with Dice	
<b>(a)</b>	A fair, six-sided dice numbered 1 to 6 is rolled once. What is the probability of the dice landing on a 5?
<b>(b)</b>	A fair four-sided dice numbered 1 to 4 is rolled once. What is the probability of the dice landing on a 3?
<b>(c)</b>	A fair ten-sided dice numbered 1 to 10 is rolled once. What is the probability of the dice landing on a 7?
<b>(d)</b>	A fair ten-sided dice numbered 1 to 10 is rolled once. What is the probability of the dice landing on a 5 or 6?
<b>(e)</b>	A fair, six-sided dice numbered 1 to 6 is rolled once. What is the probability of the dice landing on an odd number?
<b>(f)</b>	A fair four-sided dice numbered 1 to 4 is rolled once. What is the probability of rolling a number less than 3?
<b>(g)</b>	A fair ten-sided dice numbered 1 to 10 is rolled once. What is the probability of rolling a number less than 7?
<b>(h)</b>	A fair, six-sided dice numbered 1 to 6 is rolled once. What is the probability of the dice landing on a number that is not six?
<b>(i)</b>	A fair, six-sided dice numbered 1 to 6 is rolled once. What is the probability of the dice landing on a prime number?
<b>(j)</b>	A fair four-sided dice numbered 3, 4, 5 and 6 is rolled once. What is the probability of the dice landing on a multiple of 3?
<b>(k)</b>	A fair six-sided dice numbered 1, 1, 1, 2, 2 and 3 is rolled once. What is the probability of rolling a 1?
<b>(l)</b>	A fair six-sided dice numbered 1, 1, 1, 2, 2 and 3 is rolled once. What is the probability of rolling a number that is not a 2?
<b>(m)</b>	A fair ten-sided dice numbered 1 to 10 is rolled once. What is the probability of rolling a square number?
<b>(n)</b>	A fair ten-sided dice numbered 1 to 10 is rolled once. What is the probability of rolling a number that is a factor of 12?
<b>(o)</b>	The probability of rolling a 6 on a biased dice is 0.3. What is the probability of not rolling a 6?
<b>(p)</b>	If a fair six-sided dice is rolled 60 times, how many times would you expect it to land on a 3?

# Theoretical Probability with Playing Cards

## Theoretical Probability with Playing Cards

	<p><b>(a)</b></p> <p>A playing card is chosen at random. What is the probability that it is a red card?</p>	<p><b>(b)</b></p> <p>A playing card is chosen at random. What is the probability that it is a king (K)?</p>	<p><b>(c)</b></p> <p>A playing card is chosen at random. What is the probability that it is the ace (A) of hearts?</p>	<p><b>(d)</b></p> <p>A playing card is chosen at random. What is the probability that it is a spade card?</p>
	<p><b>(e)</b></p> <p>A playing card is chosen at random. What is the probability that it is not a diamond card?</p>	<p><b>(f)</b></p> <p>A playing card is chosen at random. What is the probability that it is a 2, 3 or 4?</p>	<p><b>(g)</b></p> <p>A playing card is chosen at random. What is the probability that it is a queen (Q) or a king (K)?</p>	<p><b>(h)</b></p> <p>A playing card is chosen at random. What is the probability that it is a red card with a prime number on it?</p>
<p><b>(i)</b></p> <p>A playing card is chosen at random. What is the probability that it is a red non-picture card?</p>		<p><b>(j)</b></p> <p>Bruce chooses a card at random, looks at it and then replaces it in the deck. He repeats this 520 times. How many times would Bruce expect to see an ace?</p>		
<p><b>(k)</b></p> <p>Nadia chooses a card at random, looks at it and then replaces it in the deck. She repeats this 260 times. How many times would Nadia expect to see a red jack (J) or red queen (Q)?</p>				

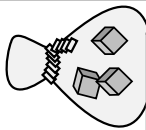
# Theoretical and Experimental Probability

<b>(a)</b> A bag contains 6 red sweets, 5 orange sweets and 3 yellow sweets. Find the probability of choosing an orange sweet at random from the bag.	<b>(b)</b> A fair six-sided spinner is numbered 1 to 6. The spinner is spun once. Find the probability that the spinner lands on a multiple of 3.	<b>(c)</b> There are 10 balls in a bag. 7 of the balls are red and the rest are yellow. When a ball is picked from the bag at random, what is the probability that it is blue?	<b>(d)</b> There are 5 white counters, 8 black counters and 7 grey counters in a bag. A counter is chosen at random. What is the probability that it is not white?																									
<b>(e)</b> A purse contains 20 coins. They are either 10p or 5p coins. The probability of choosing a 5p coin at random is 0.4. How many 10p coins are in the purse?	<b>(f)</b> Zack rolls a biased dice. The probability that it lands on each of the numbers 1 to 4 is shown in the table. The dice is twice as likely to land on a 5 as it is to land on a 6. Complete the table. <table border="1" data-bbox="611 607 729 1425"> <tbody> <tr> <td>Number</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>Probability</td> <td>0.2</td> <td>0.05</td> <td>0.1</td> <td>0.2</td> <td></td> <td></td> </tr> </tbody> </table>	Number	1	2	3	4	5	6	Probability	0.2	0.05	0.1	0.2			<b>(g)</b> The probability that a biased spinner lands on a 2 is 0.3. Jemima spins the spinner 150 times. Work out an estimate for the number of times the spinner will land on a 2.												
Number	1	2	3	4	5	6																						
Probability	0.2	0.05	0.1	0.2																								
<b>(i)</b> Leon has a fair four-sided spinner containing the numbers 1, 3, 5 and 7. He spins it twice and adds the two numbers together to get a total. (a) Complete the sample space. (b) Calculate the probability of Leon getting a total of 10 or more.																												
<b>(k)</b> A bag contains 12 red counters and 6 blue counters. Some more blue counters are added to the bag, so that the probability of choosing a blue counter is now $\frac{3}{7}$ . How many blue counters have been added to the bag?																												
<table border="1" data-bbox="861 1011 1153 1301"> <tbody> <tr> <td></td> <td><b>1</b></td> <td><b>3</b></td> <td><b>5</b></td> <td><b>7</b></td> </tr> <tr> <td><b>1</b></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td><b>3</b></td> <td></td> <td>6</td> <td></td> <td></td> </tr> <tr> <td><b>5</b></td> <td></td> <td></td> <td></td> <td>12</td> </tr> <tr> <td><b>7</b></td> <td></td> <td></td> <td></td> <td></td> </tr> </tbody> </table>					<b>1</b>	<b>3</b>	<b>5</b>	<b>7</b>	<b>1</b>					<b>3</b>		6			<b>5</b>				12	<b>7</b>				
	<b>1</b>	<b>3</b>	<b>5</b>	<b>7</b>																								
<b>1</b>																												
<b>3</b>		6																										
<b>5</b>				12																								
<b>7</b>																												

# Exhaustive Events


## Exhaustive Events For each scenario, complete the table.

- A) A bag contains 100 cubes in three colours. There are the same amount of blue & green cubes.



Colour	Red	Blue	Green
Frequency	40		

- D) A bag contains 85 cubes in three colours. The ratio of black cubes to yellow cubes 3 : 5



Colour	Purple	Black	Yellow
Frequency	45		


- G) A bag contains 120 cubes in three colours. The table shows algebraic expressions for each frequency. Find the actual amount of each colour.

Colour	Red	Green	Silver
Expression	$2x$	$x$	$3x$
Frequency			

- J) A bag contains 80 cubes in three colours. The table shows algebraic expressions for each frequency. Find the actual amount of each colour.

Colour	Black	Blue	Grey	Green
Expression	$3x$	$2x$	$2x + 21$	$x - 5$
Frequency				

- B) A bag contains some cubes in three colours. The probability of picking a white cube is twice the probability of picking a yellow cube.



Colour	Black	White	Yellow
Probability	0.4		

- E) A bag contains some cubes in three colours. The probability of NOT picking a gold cube is 0.84  
The ratio of grey cubes to gold cubes 7 : 2

Colour	Orange	Grey	Gold
Probability			


- H) A bag contains some cubes in three colours. It has 5 times as many pick cubes as red cubes, 4 times as many gold cubes as red cubes, and 2 times as many grey cubes as pink cubes.

Colour	Gold	Red	Pink	Grey
Probability				

- K) A bag contains some cubes in three colours. The probability of picking a blue cube is 0.2  
How many cubes are there in total?  
black cubes to yellow cubes = 2 : 3

Colour	Blue	Black	Yellow
Frequency	15		

- C) A bag contains some cubes in three colours. The probability of NOT picking a pink cube is 0.7



Colour	Grey	Red	Pink
Probability	0.15		

- F) A bag contains some cubes in three colours.  $P(\text{pick a pink cube or a brown cube}) = 0.3$   
pink cubes to brown cubes = 1:5

Colour	Yellow	Pink	Brown
Probability			

- I) A bag contains some cubes in three colours. white cubes to gold cubes = 1 : 6  
gold cubes to silver cubes = 2 : 1

Colour	White	Gold	Silver
Probability			

- L) The probability of picking a blue cube is 0.15  
The probability of NOT picking a pink cube is  $\frac{7}{15}$   
 $P(\text{pick a Black cube}) = ?$

Colour	Pink	Blue	Black	Red
Frequency		9	$2x + 7$	$3x - 8$

# Exhaustive Probabilities and Frequency

①

## Exhaustive Probabilities & Frequency

A scientist studies penguins in a colony. They are either healthy or unhealthy. If one of the penguins is picked at random, what is the probability it is healthy? (Complete the table)

	Healthy	Unhealthy
<b>Frequency</b>	35	15
<b>Probability</b>		



②

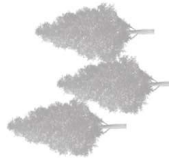
An online business records information for 80 customers. If they contact one of the customers randomly, what is the probability they are a child (under 18)?

Age	0-17	18-30	31+
<b>Frequency</b>		24	40
<b>Probability</b>			



③

A scientist selects trees in a forest to study. They are rated on how healthy they are. If one of these trees is picked at random, what is the probability it is rated as 'Good'?



Health	Unhealthy	OK	Good	Excellent
<b>Frequency</b>	40			20
<b>Probability</b>		0.4		0.1

④

Students are asked how they travel to school. If a student is picked at random, what is the probability they cycle to school?



Transport	Walk	Cycle	Bus	Car
<b>Frequency</b>	27		45	
<b>Probability</b>	0.3			0.1

Dr Kay records the eye colour of her patients.

35% of her patients had blue eyes.

If a patient is picked at random, what is the probability they have brown eyes?

⑤

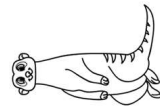


Eye-Colour	Blue	Green	Brown	Other
<b>Frequency</b>	56		24	16
<b>Probability</b>		0.4		

⑥

$\frac{3}{5}$  of the meerkats in a zoo are (adult or juvenile) males.

If a meerkat is picked at random, what is the probability they are an adult meerkat?



⑦

	Adult		Juvenile	
	Male	Female	Male	Female
<b>Frequency</b>		36	6	12
<b>Probability</b>				



Jake records the colour of cars parked at the mall.

If one of the cars is picked at random, the probability it is green is three-times the probability it is red. How many green cars did Jake find?

Car-Colour	Black	Silver	Red	Green	Other
<b>Frequency</b>	63				
<b>Probability</b>	0.35	0.2			0.25

⑧



A teacher records the science test results of 300 students in Year 7.

The ratio of students who scored above 80 to students who scored less than 21 was 1 : 3

If a student is picked at random, what is the probability they scored more than 40 marks?

Score	0-20	21-40	41-60	61-80	81-100
<b>Frequency</b>		78	93	81	
<b>Probability</b>					

# Relative Frequency

## Relative Frequency

1) Sarah conducts an experiment & spins the spinner 20 times.

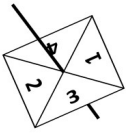
4 2 3 4 2 2 1 3 2 3  
3 4 2 4 1 4 2 2 4 2

Score	1	2	3	4
Frequency				
Relative Frequency				

a) Complete the table with frequencies & relative frequencies.

b) What is the **theoretical** probability of the spinner landing on four?

c) Do you think the spinner is **biased**? Explain why.



2) A dice is rolled 30 times. It lands on four 12 times.

a) What is the relative frequency of the dice landing on four?

b) Do you think the dice is biased? Explain why.

3) Toby spins the spinner 50 times and records his results.

Complete his table.

Score	1	2	3	4	5
Frequency	12	14	8		
Relative Frequency			0.2	0.16	

4) Sam rolls a biased dice 200 times.

He calculates that the relative frequency of scoring a six is 0.2

How many times did Sam roll a six during his experiment?

5) Mack records the meals bought at lunch. 60 students buy a sandwich.

He calculated the relative frequency of a student buying

a sandwich was 0.8 How many meals did Mack record?

6) Dan records his wins & losses playing online & compares his results with Alice's results.

Alice says she's the better player.

Do you agree?



	Dan		Alice	
Result	Win	Loss	Win	Loss
Frequency	13	7	66	54
Relative Frequency				

Over the next 40 games...

...how many games does Dan expect to win?

...how many games does Alice expect to win?

7) Ash records the speed that cars are driving on the road by his home that has a 30 mph speed limit.

Speed (mph)	1-10	11-20	21-30	30+
Frequency	30		63	24
Relative Frequency	0.2	0.22		

a) What is the probability the next car is breaking the law?

b) What is the probability the next car is driving legally?

c) 300 cars passed by his home over the next weekend.

Estimate how many were driving faster than 20 mph.

8) A factory records faults in the three types of robots in makes.

Robot	Type-A	Type-B	Type-C
Fault Relative Frequency		0.16	0.12

Type-A robots have 50% more faults compared to Type-B robots.

3 times the amount of Type-C robots are manufactured compared to

Type-A robots & compared to Type-B robots.

What is the probability that the factory produces a faulty robot?

# Worded Probability

## WORDDED: PROBABILITY

- 1) A bag contains buttons in 3 colours. There are 10 black, 8 red and 7 white buttons. If a button is picked randomly, what is the probability it is black? Express this as a simplified fraction, decimal, and a percentage.
- 2) At set of 10 cards are numbered 1 to 10. What is the probability...
  - ... the card with 6 on it is picked?
  - ... a card with a value of 12 is piked?
  - ... a card with a 1 digit is piked?
  - ... a card with a value greater than 7 is chosen?
- 3) A lake contains approximately 600 fish. George catches a fish, tags it, and then releases it back into the lake. What is the approximate probability George will now catch the tagged fish again? How many more fish does George need to catch and tag to have a 0.25 chance of catching a tagged fish?
- 4) Before a running race, Kimmy has a 0.62 chance of winning the race. What is the percentage probability she doesn't win the race?
- 5) There are 9 packets of salted crisps, 7 packets of cheese & onion crisps and 13 packets of cheese crisps. If one packet is chosen at random, what is the probability it is **not** a packet of salted crisps?
- 6) A bag contains red and yellow cubes in the ratio 3 : 4. What is the probability a red cube is picked at random from the bag?
- 7) At set of 20 cards are numbered 1 to 20. If the 8 is randomly picked and removed, then the 13 is, then the card with 16 is picked and removed, now what is the probability of randomly picking an even number?
- 8) A spinner has 6 outcomes: A, B, C, D, E, F. The probability of the spinner landing on F is twice the probability of landing on each of the other outcomes. What is the probability the spinner lands on B?
- 9) A bag contains green and blue spheres. The probability of picking a green sphere is 0.55. If there are more than 100 spheres in the bag, what is the least amount of green spheres there can be?

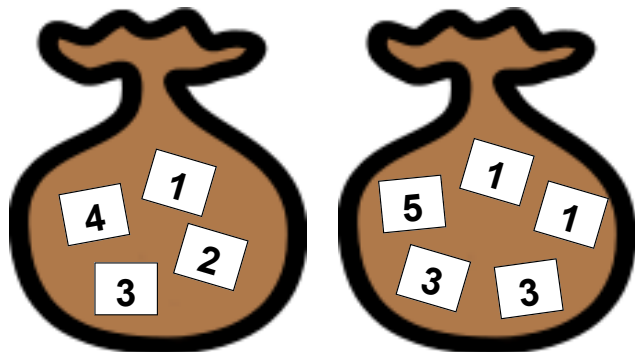
# Listing Outcomes

1. Jemma shakes the two bags shown below and picks a numbered card from each.

She multiplies the numbers on her cards together.

a) Complete the sample space diagram to show the different scores she could make:

	1	1	3	3	5
1					
2					
3					
4					



b) What is the probability that Jemma gets a score of 9?



2. Peter is choosing his dinner from the menu.

There are 12 different ways he could choose his meal, if he has a starter, main meal and desert. List them all.

He tells the chef to choose a meal for him at random. What is the probability that:

- He gets soup for starter and ice cream for desert?
- He gets beef for his main course and apple tart for desert?

3. Carl rolls two dice. He subtracts the numbers on the dice to get a score. For example, if he rolls a 2 and then a 6, he scores 4. What is the probability that Carl scores:

- 0?
- 7?

4. Andrew picks a card from a normal pack of playing cards. He records whether he gets a heart, club, diamond or spade. At the same time, he throws a normal six sided die and records the number it lands on.

If he gets a heart and rolls a 6, he wins a prize.

- What is the chance he wins?
- What is the chance he loses?



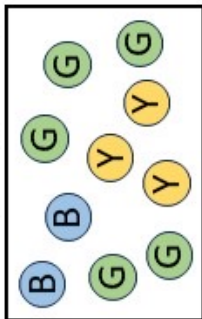


# Fluency Practice

<p><b>A1</b> Cameron throws a fair coin. He gets a Head. Cameron's sister then throws the same coin. What is the probability that she will get a Head?</p>	<p><b>A2</b> Damien throws a coin 30 times. Explain why he may not get exactly 15 Heads and 15 Tails.</p>	<p><b>A3</b> Lucas throws a bias coin 180 times. It lands on tails 120 times. Lucas throws the coin once more. Work out an estimate for the probability that it will show <b>heads</b>.</p>	<p><b>A4</b> Serena throws a fair coin three times and gets two heads and a tail. Serena's then throws the same coin once more. What is the probability that the coin will land on heads?</p>
<p><b>B1</b> Every morning Joanne eats one of cereal, toast or croissants.  <math>P(\text{cereal}) = 0.45</math>  <math>P(\text{croissants}) = 0.3</math>            Find <math>P(\text{toast})</math></p>	<p><b>B2</b> Rosie throws a coin 1000 times. She gets heads 490 times. State, with a reason, whether the coin is fair.</p>	<p><b>B3</b> In a class of 30 students, 6 of the students are left handed and 9 of the students wear glasses. Anthony says 'the probability that a student is left-handed or wears glasses is 0.5'. State, with a reason, whether Anthony is right.</p>	<p><b>B4</b> Millie takes a bead at random from a bag. The probability that she will take a red bead is 0.3. There are 120 beads in the bag. How many red beads are there in the bag?</p>
<p><b>C1</b> Felix throws a dice 600 times. He scores six 200 times. Is the dice fair? Explain your answer.</p>	<p><b>C2</b> Amy spins a spinner once. <math>P(\text{she scores 4}) = 0.3</math> If Amy were to spin the spinner 200 times, work out an estimate for the number of times that she would score 4</p>	<p><b>C3</b> A bag contains some red beads, black beads and yellow beads. Sarah takes a bead at random from the bag.  <math>P(\text{red}) = 0.3</math>  <math>P(\text{black}) = P(\text{yellow})</math>            Find <math>P(\text{yellow})</math></p>	<p><b>C4</b> A bag contains 10 coloured counters. James is going to take at random, a counter from the bag. He states "The probability that I will take a red counter is 0.25". Explain why James is wrong.</p>

# Single Event Probabilities

**1** A box contains different coloured counters, blue, green and yellow.



Complete the probability table for picking a random counter.

Colour	Blue	Green	Yellow
Probability			

**5** A barista measured the number of customers who ordered tea or coffee and the time of day they ordered. Some of the results are recorded in the table.

	Tea	Coffee	Total
Morning		41	
Evening	32		
Total		46	90

- a) Complete the two-way table  
 b) A customer is picked at random  
 Find:  
 i. P(Coffee)  
 ii. P(Tea in the morning)

**2** Tom has 12 cards with the numbers 1 to 12 on them, he selects a card at random.

- Find:  
 a) P(5)  
 b) P(Multiple of 4)  
 c) P(Even Number)  
 d) P(Factor of 12)  
 e) P(Prime Number)  
 f) P(Square Number)

**3** A bag contains blue, green and red counters in the ratio 2:5:7.

- Find:  
 a) P(Red)  
 b) P(Red or Green)  
 c) P(Not Green)

**4** There are only blue, green and pink marbles in a bag.

There are twice as many green marbles as blue marbles and four times as many pink marbles as green marbles. A marble is picked at random.

Complete the probability table for picking a random marble.

Colour	Blue	Green	Pink
Probability			

**6** A scrabble bag contains vowels and consonants.

There are 16 vowels in the scrabble bag.  
 The probability of picking vowel is  $\frac{2}{3}$ .  
 How many consonants are in the bag?

**7** A bag contains 1p, 2p and 5p coins.

There are 3 times as many 2p coins as 5p coins.  
 There are 7 more 1p coins than 5p coins.  
 There are 15 2p coins.  
 A random coin is picked out of the bag.  
 Find P(2p coin).

**8** A bag of mixed berries contains four different types of berries.

The ratio of strawberries to black berries is 3:5.  
 There are an equal number of raspberries and blue berries.  
 The probability of picking a blue berry out is  $\frac{1}{6}$ .  
 Find the least amount of each berry that could be in the bag.

# Relative Frequency and Expected Outcomes

**1** The relative frequency of flipping a tails on a biased coin is 0.18. If the coin is flipped 50 times, how many heads can we expect?

**2** The following spinner is spun 48 times. Complete the table for the expected outcomes of each colour.



Colour	B	G	Y
Expected Outcome			

**3** Ben rolls a die 50 times, here are his results:

Number	Frequency	Relative Frequency
1	12	
2	8	
3	1	
4	15	
5	5	
6	6	

**a** Ben states: "The die must be biased because I only rolled one 3 in 50 rolls" Is Ben correct? Explain.

**b** By calculating the relative frequencies, complete the table.

**c** If Ben rolled the dice 80 times how many even numbers can he expect to roll?

**4** Lewis and Gracie roll a biased 4 sided die. Lewis rolls 20 times, Gracie rolls 50 times. Lewis calculates his relative frequency of rolling a 4 is 0.15, Gracie calculates hers to be 0.2.

**a** Who has the better estimate for the probability of rolling a 4? Explain.

**b** Using all their data, calculate the amount of 4s expected from 200 rolls.

**5** A student rolls a biased die. Complete the following table.

Number	Frequency	Relative Frequency
1		
2		0.25
3	2	
4		0.2
5	4	0.1
6	12	

**6** Tom and Sally flip a biased coin. Tom says that they flipped 36 more heads than tails. Sally says that they flipped heads 4 times as much as tails.

**a** Calculate the relative frequency of flipping a head.

**b** How many flips did they complete?

**7** Neda has played a game numerous times, her relative frequency of winning is 0.6. Neda plays an additional game and wins, her relative frequency after the win is now 0.625.

How many games has Neda played including the recent win?

# Single Event Probabilities

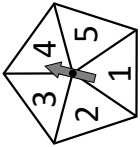
## Single Event Probabilities

①

Put these probabilities in order, from least likely to occur to most likely to occur.

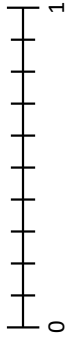
$\frac{2}{5}$     33%    0.52     $\frac{3}{10}$

②



This is a fair spinner.

On the probability scale, mark & label the probability the spinner lands on 3.



⑤ Jim has 15 cards numbered 1 to 15.

Jim picks a card at random, what is the percentage probability that the card has a prime number on it?

⑦



A bag contains some cubes in 3 colours.

Red	Yellow	Green
0.4		

What is the probability it does **not** have 1 digit?

Complete the table with probabilities.

⑨

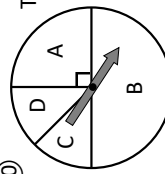
A student is chosen at random.

Yr 7	Yr 8	Yr 9	Yr 10
0.1	$x$	$2x$	

The probability of picking a student in Year 8 is 50% greater than the probability of picking a student in Year 7.

What is the probability of picking a student in Year 10?

⑩

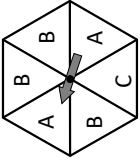


This fair spinner will be spun many times.

A	B	C	D
		40	

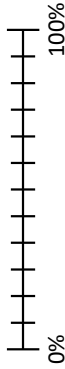
Complete the table showing total expected outcomes.

③



This is a fair spinner.

Mark & label the probability the spinner does **not** land on A.



⑧

A fair dice is rolled 150 times.

How many times do we expect it to land on a 6?

How many times do we expect it to land on a number greater than 4?

⑫

A bag contains 80 disks in 3 colours.

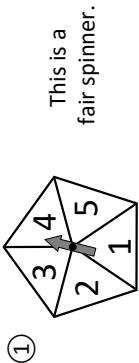
Pink	White	Green
0.15		

The ratio of white to green disks is 9 : 8

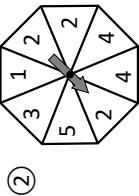
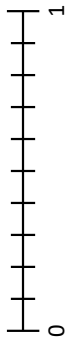
How many pink disks do we need to add to the bag so that there is an equal probability of picking a pink or a green disk?

# Single and Multiple Event Probabilities

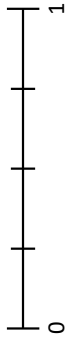
## Single & Multiple Event Probabilities



On the probability scale, mark & label the probability the spinner lands on an odd number.



Mark & label: P(it lands on a 2 or a 4)



③ A factor of 28 is chosen at random. What is the probability it is prime?

What is the probability it does **not** have 1 digit?

④ A fair 6-sided dice is rolled 210 times. How many times do we expect it to land on a 1?

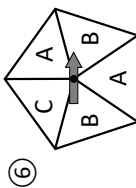
How many times do we expect it to land on a number lower than 3?

⑤ A bag contains disks of 4 colours. The table shows the probability of picking each colour.

Red	Blue	Green	Brown
0.1	$x$	$x$	$x$

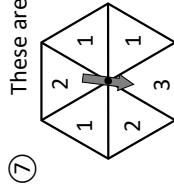
The probability of picking a blue disk is 20% greater than the probability of picking a red disk.

If there are 300 disks in the bag, how many brown disks are there?



Outcome	A	B	C
Frequency	10	80	

If we spin the spinner 350 times, how many times should we expect to get A or C??



If both spinners are spun, what is...

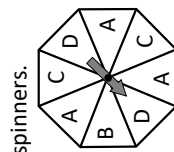
P(1 & A)?

P(Odd & C)?

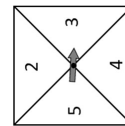
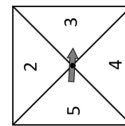
⑧ A fair 8-sided dice is rolled twice and the scores are added. P(total of 16) =



P(total less than 4) =



⑨ These two fair spinners are spun and their scores are **multiplied**.



1<sup>st</sup> Spinner

	2	3	4	5
2				
3				
4				
5				

What is...

P(a square number)?

P(greater than 10)?

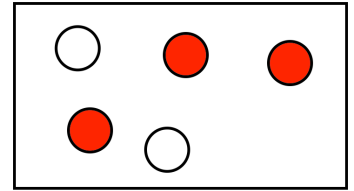
⑩ Dan has 2 sets of cards. The first set are the factors of 8. The second set are the first 3 prime numbers. Dan takes a card from each set and sums the two numbers. Dan runs this trial 100 times. How many times should he expect to score more than 4?



# Fluency Practice

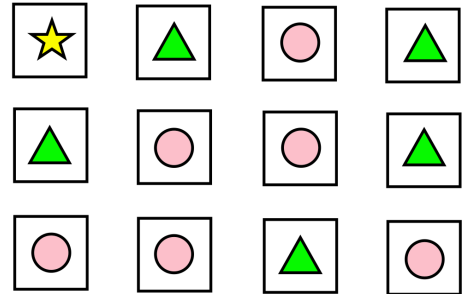
Question 1: Theo has 3 red sweets and 2 white sweets.  
He picks a sweet at random.

- (a) Write down the probability that Theo picks a red sweet.  
(b) Write down the probability that Theo picks a white sweet.



Question 2: Leah has 12 cards, each with a shape on it.  
She takes a card at random.

- (a) What is the probability that Leah takes a card with a star on it?  
(b) What is the probability that Leah takes a card with a triangle on it?  
(c) What is the probability that Leah takes a card with a circle on it?



Question 3: Ralph has 9 cards, each with a number on it.



He picks a card at random.  
Write down the probability that the chosen card is

- (a) the number 8                      (b) an even number                      (c) a number less than 7  
(d) a multiple of 4                      (e) a square number                      (f) a prime number

Question 4: There are 12 red roses, 5 yellow roses and 3 white roses in a vase.  
Felix takes a rose, at random, from the vase.

- (a) Write down the probability that he takes a white rose.  
(b) Write down the probability that he takes a red **or** a white rose.  
(c) Write down the probability that Felix takes a rose that is **not** red.

# Fluency Practice

Question 5: Leon throws a biased coin.  
The probability of getting tails is 0.4  
Work out the probability of getting heads.



Question 6: Edith plants a daffodil bulb.  
The probability that the bulb will grow is 0.8  
What is the probability that the bulb will **not** grow?

Question 7: Wycombe Wanderers play a match of football.  
The probability that they win the match is 0.28  
The probability that they draw the match is 0.55  
Work out the probability that they lose the match.

Question 8: Evelyn has 80 pens in a drawer.  
15 pens are black and the other pens are blue.

Evelyn picks a pen at random from the drawer.

- (a) What is the probability that Evelyn picks a black pen?
- (b) What is the probability that Evelyn picks a blue pen?



Question 9: There are 20 counters in a bag.

2 of the counters are white.  
1 of the counters is pink.  
4 of the counters are black.  
The rest of the counters are purple.

Carter takes a counter at random from the bag.

Show that the probability that the counter is white or purple is  $\frac{3}{4}$

Question 10: There are only pink, yellow, green and blue counters in a bag.  
The table shows the probability that a counter taken at random from the bag will be pink, green or blue.

Colour	Pink	Yellow	Green	Blue
Probability	0.5		0.1	0.2

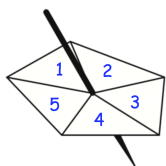
- (a) Work out the probability that the counter taken is yellow

There are 40 counters in the bag.

- (b) Work out the number of blue counters in the bag.

# Fluency Practice

- Question 11: Darcy has a biased spinner.  
A spinner has sections labelled 1, 2, 3, 4 and 5.  
The table below shows information about some of the probabilities



Number	1	2	3	4	5
Probability	$x$	0.15	0.05	0.2	0.35

Work out the value of  $x$ .

- Question 12: Frederick organises a raffle for his school fayre.  
The top prize is a ride in a hot air balloon, which will be won by 1 ticket.  
Altogether Frederick sells 700 raffle tickets.  
Miss Robinson buys 5 tickets for the raffle.



Work out the probability that Miss Robinson does **not** win.

- Question 13: There are 20 chocolates in a box.  
Some of the chocolates contain nuts and the rest do not.  
The probability that a chocolate containing nuts is picked at random from the box is 0.6  
How many of the chocolates in the box contain nuts?

- Question 14: A bag contains 600 coloured counters.  
The counters are yellow, brown or orange.  
There are 117 yellow counters in the bag.  
The probability that a brown counter is chosen from the bag is 0.35

Calculate the number of orange counters in the bag.



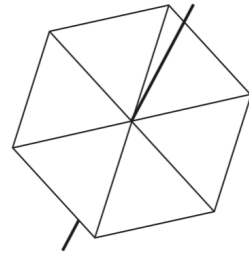
# Extension

Question 1: Megan has a fair 6 sided spinner.  
The spinner has the letters A, B and C on it.

The probability that the spinner will land on an A is  $\frac{1}{2}$

The probability that the spinner will land on a C is  $\frac{1}{3}$

Write the letters on the spinner.



Question 2: Elliott has eight numbered cards.



One of the cards is chosen at random.

Elliott says:

The probability of a 8 is  $\frac{1}{4}$

The range of the numbers is 5.

The probability of a number greater than 10 is 0.

The probability of a 7 is  $\frac{1}{2}$

Fill in the six missing numbers.

Question 3: The two-way table gives information about 90 people who sat their driving test.

(a) Complete the two-way table

A person is picked at random.

	Under 20 driving lessons	20 or over driving lessons	total
Pass		21	30
Fail	45		
total			90

(b) Write down the probability that the person failed their driving test.

(c) Write down the probability that the person had under 20 driving lessons.

Somebody who passed their driving test is picked at random.

(d) Work out the probability that this person had under 20 driving lessons.

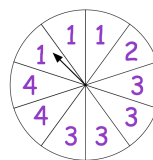
Question 4: Isaac has made two fair spinners.  
Spinner 1 has 10 equal sized sections.  
Spinner 2 has 4 equal sized sections.

Isaac says

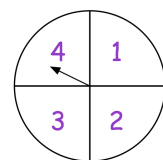
“It is more likely to get a 4 on spinner 1 than spinner 2 because there are two number 4s on spinner 1 and only one number 4 on spinner 2.”

Explain why Isaac is incorrect.

Spinner 1



Spinner 2



# Extension

Question 5: The table shows the shoe size of 23 students.

Shoe Size	Frequency
5	2
6	11
7	5
8	4
9	1

A student is picked at random.

- (a) Work out the probability that the student has a shoe size of 8.  
(b) Work out the probability that the student has a shoe size of 7 or smaller.

Question 6: A football team can win, draw or lose a match.  
The table shows the probabilities of each result.

Result	Win	Draw	Lose
Probability		0.05	0.3

Each win is worth 3 points.  
Each draw is worth 1 point.  
Each loss is worth 0 points.  
The football team plays 40 games in a season.

Work out how many points the football team should receive in one season.

Question 7: Beatrice has a biased four sided spinner.  
The table shows the probabilities that the spinner will land on a 2 or 3.

Number	1	2	3	4
Probability		0.1	0.3	

The probability that the spinner will land on 1 is three times the probability that the spinner will land on 4.

Work out the probability that the spinner will land on 1.

Question 8: Finn has some sweets in a bag.  
5 of the sweets are lemon flavoured.  
7 of the sweets are strawberry flavoured.  
The rest of the sweets are mint flavoured.

The probability that Finn takes a mint flavoured sweet is  $\frac{2}{5}$

How many mint flavoured sweets are in the bag?

# Extension

Question 9: Gracie has more than 5 coins.  
The total value of the coins is 50p.

Gracie is going to pick one of the coins at random.

The probability that Gracie picks a **1p** coin is  $\frac{1}{5}$

List all the coins that Gracie has.

Question 10: A box contains lego blocks of the same size.  
Each block is white, blue, green or red.

Colour	White	Blue	Green	Red
Probability	0.25	0.45		0.2

The table shows the probabilities that a block picked at random is white, blue or red.

(a) Work out the probability of a green block

There are 60 red lego blocks.

(b) How many white lego blocks are there?

Question 11: A bag contains good and bad apples.  
 $n$  of the apples are good.  
The other 5 apples are bad.

(a) Write down an expression, in terms of  $n$ , for the number of apples in the bag altogether.

Maryam will take at random, an apple from the bag.

(b) Write down an expression, in terms of  $n$ , for the probability that Maryam will take a good apple.

(c) Write down an expression, in terms of  $n$ , for the probability that Maryam will take a bad apple.

Question 12: There are only red, black and green pens in a box.

There are three times as many red pens as green pens.  
There are four as many black pens than red pens.

Work out the probability of a black pen being selected.


# Probability with Words

## probability with words

- (1) give some words where the probability of picking a vowel is  $\frac{1}{2}$   
try to find words with different lengths
- (2) try to find some words where the probability of picking a letter 'a' is  $\frac{1}{3}$   
try to find words with different lengths
- (3) give some words where the probability of picking a letter 'e' is  $\frac{1}{3}$   
try to find words with different lengths
- (4) try to find words where the probability of picking a vowel is greater than  $\frac{1}{2}$   
try to find words with different lengths
- (5) give some words where the probability of picking a consonant is  $\frac{3}{4}$   
try to find words with different lengths
- (6) give some words where the probability of picking a consonant is equal to or close to 1  
how close can you get for a five letter word?

# Probability with Words

## probability with words

- (1)  what is the probability of picking an 'E' out of the letters in the word S N E E Z E ?
- (2) put these words in order for the probability of picking a 'T' from them:  
BETTERMENT  
DAUNT  
TWIST
- (3) put these words in order for the probability of picking a 'S' from them:  
BOSSSES  
ASSESSSES  
SUSS
- (4) put these words in order for the probability of picking a 'R' from them:  
ERRORS  
RARE  
REFERRER
- (5) put these words in order for the probability of picking a vowel (a, e, i, o, u) from them:  
AVENUE  
QUEUEING  
AREA
- (6) what do these words have in common (in terms of probabilities)?  
BETTER  
TEA  
BEVERAGES
- (7) what do these words have in common (in terms of probabilities)?  
ADDITION  
SIDE  
DODECAHEDRON
- (8) what do these words have in common (in terms of probabilities)?  
DAMAGE  
READ  
UNIDENTIFIED  
SOLITUDE

# Dice and Cards

probability questions on dice

## one dice



what is the probability of throwing:

- (a) a prime number ?
- (b) a factor of 15 ?
- (c) a multiple of 2 ?
- (d) a square number ?
- (e) a factor of 60 ?

if the dice is rolled **720** times  
how many times would you expect to obtain

- (f) a factor of 12 ?
- (g) a multiple of 3 ?
- (h) a factor of 25 ?

if the dice is rolled **100** times  
how many times would you expect to obtain

- (i) a factor of 8 ?
- (j) a multiple of 3 ?
- (k) a factor of 18 ?

## two dice

two fair dice  
each is numbered 1 to 6  
the probability of throwing a **double 6** :

(both dice showing number 6) is  $\frac{1}{36}$

- (a) what is the probability of **not** throwing double 6?
- (b) I throw the two dice and get double 6  
then I throw them again  
what is the probability that I will throw **double 6** *this*  
time?



I start again and throw the two dice  
(c) what is the probability of throwing **double 3**?



- (d) what is the probability of throwing *any* double?

# Dice and Cards

## three dice

four students threw 3 fair dice they recorded how many times the numbers on the dice were the same:



(a) write the name of the student whose data is **most likely** to give the best estimate of the probability of getting each result

name	throws	results		
		different	2 the same	3 the same
Meg	40	26	12	2
Sue	140	81	56	3
Zia	20	10	10	0
Ali	100	54	42	4

(b) use *all* of their results to estimate the probability of obtaining two numbers the same

theoretical results

(c) what is the theoretical probability of obtaining **three numbers the same** ?

the theoretical probability of throwing **all different** numbers is  $\frac{5}{9}$

(d) write down **how many times** you would theoretically expect to get each result

for 5400 throws of the three dice

(i) all different =

(ii) all the same =

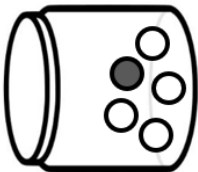
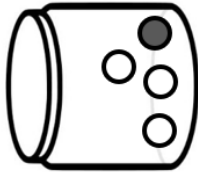
(iii) two the same =

# Comparing Probabilities

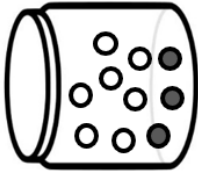
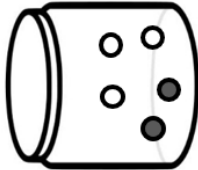
out of which jar do you stand more of a chance of picking a dark bead? (i)

you know what is in the jars but you cannot see into them

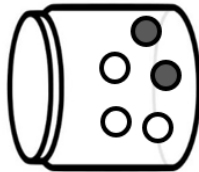
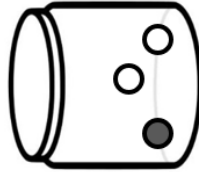
(1)



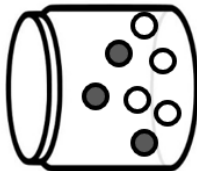
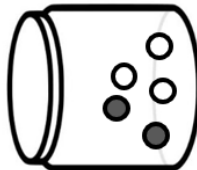
(4)



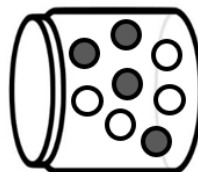
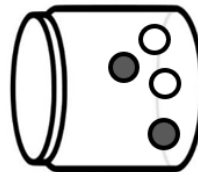
(2)



(5)



(3)



(6)

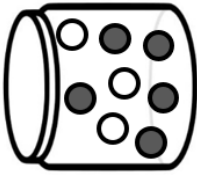
make up your own two jars  
so that the probabilities are  
close but not the same



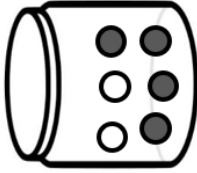
# Comparing Probabilities

out of which jar do you stand more of a chance of picking a dark bead? (ii)

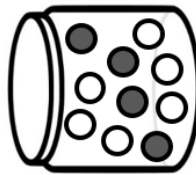
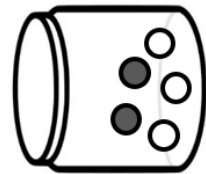
you know what is in the jars but you cannot see into them



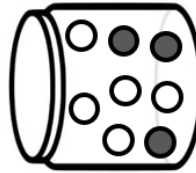
(1)



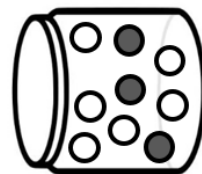
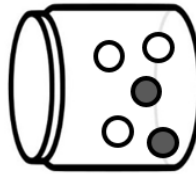
(4)



(5)



(2)



(3)

make up your own two jars  
so that the probabilities are  
both close to  $\frac{1}{2}$   
but are not the same

# Comparing Probabilities

greater chance

(1)

decide which jar you are more likely to pick a red counter out of by writing the probabilities as percentages:

A : 7 red, 3 blue

B : 13 red, 7 blue

(2)

decide which jar you are more likely to pick a red counter out of by writing the probabilities as percentages:

C : 7 red, 3 blue

D : 17 red, 8 blue

(3)

decide which jar you are more likely to pick a red counter out of by writing the probabilities as percentages:

E : 7 red, 3 blue

F : 18 red, 7 blue

(4)

decide which jar you are more likely to pick a red counter out of by writing the probabilities as percentages:

G : 13 red, 7 blue

H : 16 red, 9 blue

# Probability with Words

a c a c i a

if the letters of the name of this shrub  
are jumbled up in a bag  
what are the probabilities of picking:

$$P(a) = \quad P(\text{not } a) =$$

$$P(c) = \quad P(\text{not } c) =$$

$$P(i) = \quad P(\text{not } i) =$$

$$P(e) = \quad P(\text{not } e) =$$

a e r

these three letters  
are jumbled up and  
then placed in a  
row

three letters are  
jumbled up and then  
placed in a row

what is the  
probability of  
getting a common  
word?

the probability of  
getting a common  
word is  $\frac{1}{3}$

what could the letters  
be?

# Probability How Many?

probability **how many?** counters (i)

- 1.) a bag contains red counters, blue counters, and pink counters
- the probability of picking a pink counter is  $\frac{1}{2}$
  - there are **3** red counters
  - there are **4** blue counters
- how many pink counters are in the bag?
- 2.) a bag contains red counters, blue counters, and pink counters
- the probability of picking a blue counter is  $\frac{1}{3}$
  - there are **4** red counters
  - there are **8** pink counters
- how many blue counters are in the bag?
- 3.) a bag contains red counters, blue counters, and pink counters
- the probability of picking a pink counter is  $\frac{3}{4}$
  - there is **1** red counter
  - there are **2** blue counters
- how many pink counters are in the bag?
- 4.) a bag contains red counters, blue counters, and pink counters
- the probability of picking a red counter is  $\frac{3}{6}$
  - there is **10** blue counters
  - there are **5** pink counters
- how many red counters are in the bag?
- 5.) a bag contains red counters, blue counters, and pink counters
- the probability of picking a red counter is  $\frac{1}{2}$
  - the probability of picking a blue counter is  $\frac{1}{3}$
  - there are **8 more** red than blue counters
- how many counters are in the bag?
- 6.) a box contains green counters, white counters, and orange counters
- the probability of picking a green counter is  $\frac{3}{8}$
  - the probability of picking a white counter is  $\frac{1}{3}$
  - there are **35** orange counters
- how many counters are in the box?
- 7.) a container has some blue, red, green and grey counters put into it
- the probability of picking a blue counter is **0.15**
  - the probability of picking a red counter is **0.3**
  - the probability of picking a green counter is **0.35**
- what is the smallest possible number of grey counters in the container?
- 8.) a bag contains *some* red counters, **twice** as many blue than red counters, and **three times** as many green than red counters
- what is the probability of choosing each colour?

# Probability How Many?

- probability **how many?**    counters (ii)    what is the **smallest** number of counters in the container?
- 1.) a box has 3 colours of counters it:  
**aqua, amber and aero**  
a counter is picked out of the box at random  
the probability of picking aqua is  $\frac{3}{4}$   
the probability of picking amber is  $\frac{1}{6}$
- 2.) a bag has 3 colours of counters it:  
**bisque, brown and beige**  
a counter is picked out of the bag at random  
the probability of picking bisque is  $\frac{5}{8}$   
the probability of picking brown is  $\frac{1}{6}$
- 3.) an urn has 3 colours of counters it:  
**cyan, capri and cerise**  
a counter is picked out of the urn at random  
the probability of picking cyan is  $\frac{2}{15}$   
the probability of picking capri is  $\frac{1}{6}$
- 4.) a bin has 3 colours of counters it:  
**dirt, desert and drab**  
a counter is picked out of the bin at random  
the probability of picking dirt is  $\frac{7}{10}$   
the probability of picking desert is  $\frac{1}{6}$
- 5.) a tub has 3 colours of counters it:  
**fawn, flax and flame**  
a counter is picked out of the tub at random  
the probability of picking fawn is  $\frac{3}{8}$   
the probability of picking flax is  $\frac{7}{12}$
- 6.) a jar has 3 colours of counters it:  
**gold, garnet and grape**  
a counter is picked out of the jar at random  
the probability of picking gold is  $\frac{1}{9}$   
the probability of picking garnet is  $\frac{11}{15}$

# Probability How Many?

probability **how many?** counters (iii)

(1)

there are only **red** and **pink** counters in a bag  
there are **12 red** counters

a counter is picked at random

how many **pink** counters should there be in the  
bag for the probability of choosing a **pink**  
counter to be:

- (a)  $1/2$
- (b)  $1/13$
- (c)  $1/5$
- (d)  $1/7$
- (e)  $1/3$  ?

(2)

there are only **red, blue** and **pink** counters in a bag  
there are **6 red** counters and **6 pink** counters

a counter is picked at random

how many **blue** counters should there be in the bag  
for the probability of choosing a **blue**  
counter to be:

- (a)  $1/5$
- (b)  $1/4$
- (c)  $1/7$
- (d)  $1/3$  ?

(3)

there are only **3 white** and **1 pink** counter in a bag

some more pink counters are put into the bag

a counter is picked without looking  
the probability that the counter is pink is now  $\frac{5}{6}$

how many more pink counters were  
put into the bag?

explore for other fractions where the numerator is  
one less than the denominator

(4)

there are only **4 white** and **2 red** counters in a bag

some more white counters are put into the bag  
and double the number of red counters are put  
into the bag

a counter is picked without looking  
the probability that the counter is white is now  $\frac{3}{7}$

how many more of each colour counters were  
put into the bag?

# Probability How Many?

probability **how many?** counters (iv)

(5)

there are only red, blue and pink counters in a bag there are **15** pink counters

the probability of a **blue** counter is  $\frac{3}{14}$

the probability of a **red** counter is twice the probability of a **blue** counter

how many **red** counters and **blue** counters are in this bag?

(7)

there are only red, blue and pink counters in a bag

the probability of a **blue** counter is twice the probability of a **red** counter

the probability of a **pink** counter is twice the probability of a **blue** counter

what are the probabilities for each of the colours?

(6)

there are only red, blue and pink counters in a bag there are **6** blue counters

the probability of a **red** counter is  $\frac{1}{3}$

the probability of a **pink** counter is twice the probability of a **blue** counter

how many **red** counters and **pink** counters are in this bag?

(8)

in a bag there are only red, blue and pink counters

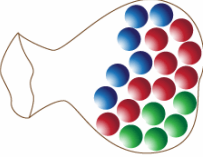
the number of **red** counters is one more than the number of **pinks**

it is twice as likely to be **blue** as **red**

show that the probability of picking a **red** counter must be greater than  $\frac{1}{4}$

# More-Same-Less – Probability

Instructions: Complete the remaining boxes by making the minimum change possible to the centre box. If there are boxes that cannot be filled in, say why.

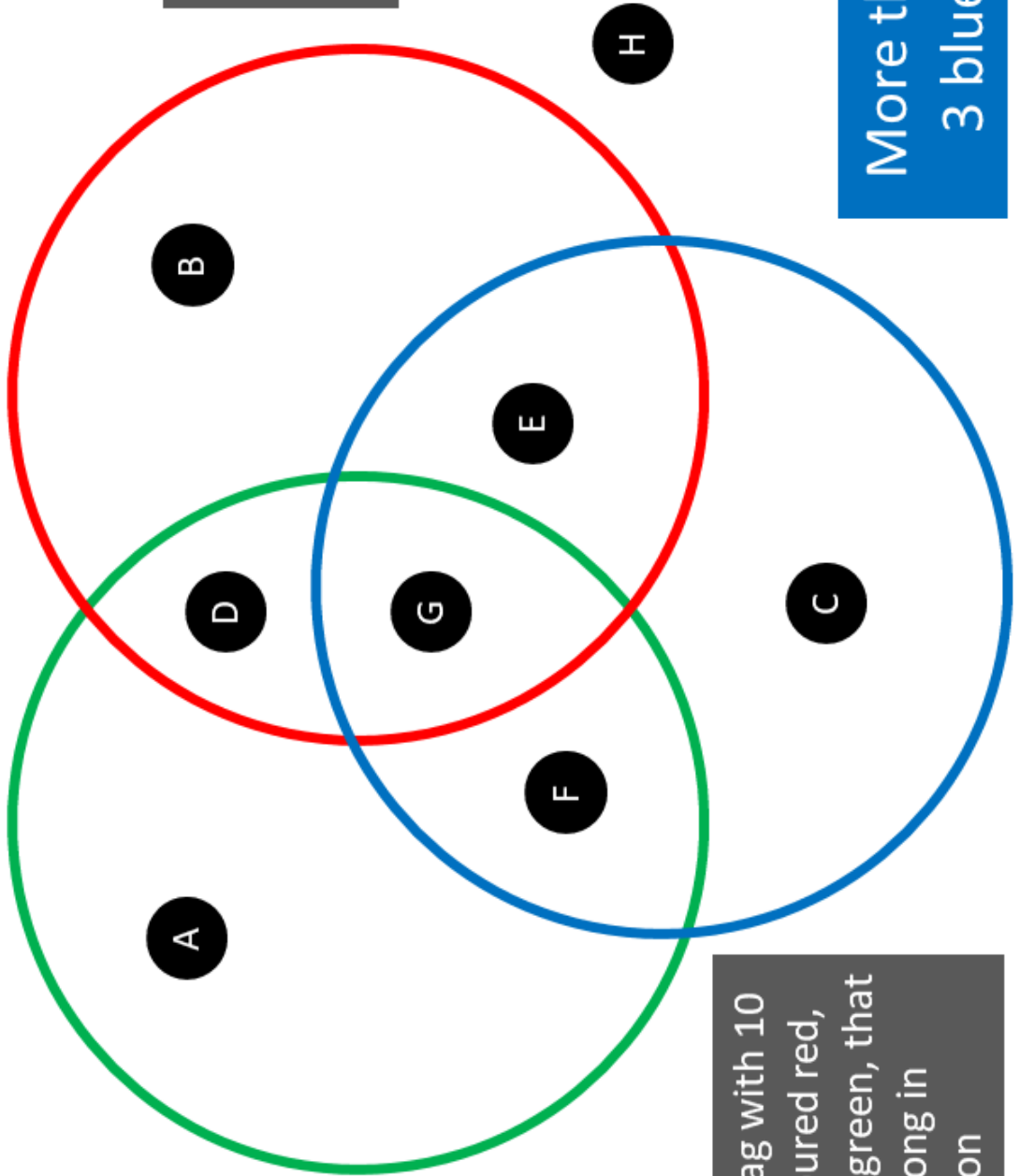
		Probability of Blue		
		Less	Same	More
Probability of Green	More			
	Same			
	Less			



# Maths Venns

$P(\text{red}) > P(\text{green})$

$P(\text{blue}) = P(\text{red})$



If you think a region is impossible to fill, convince me why!

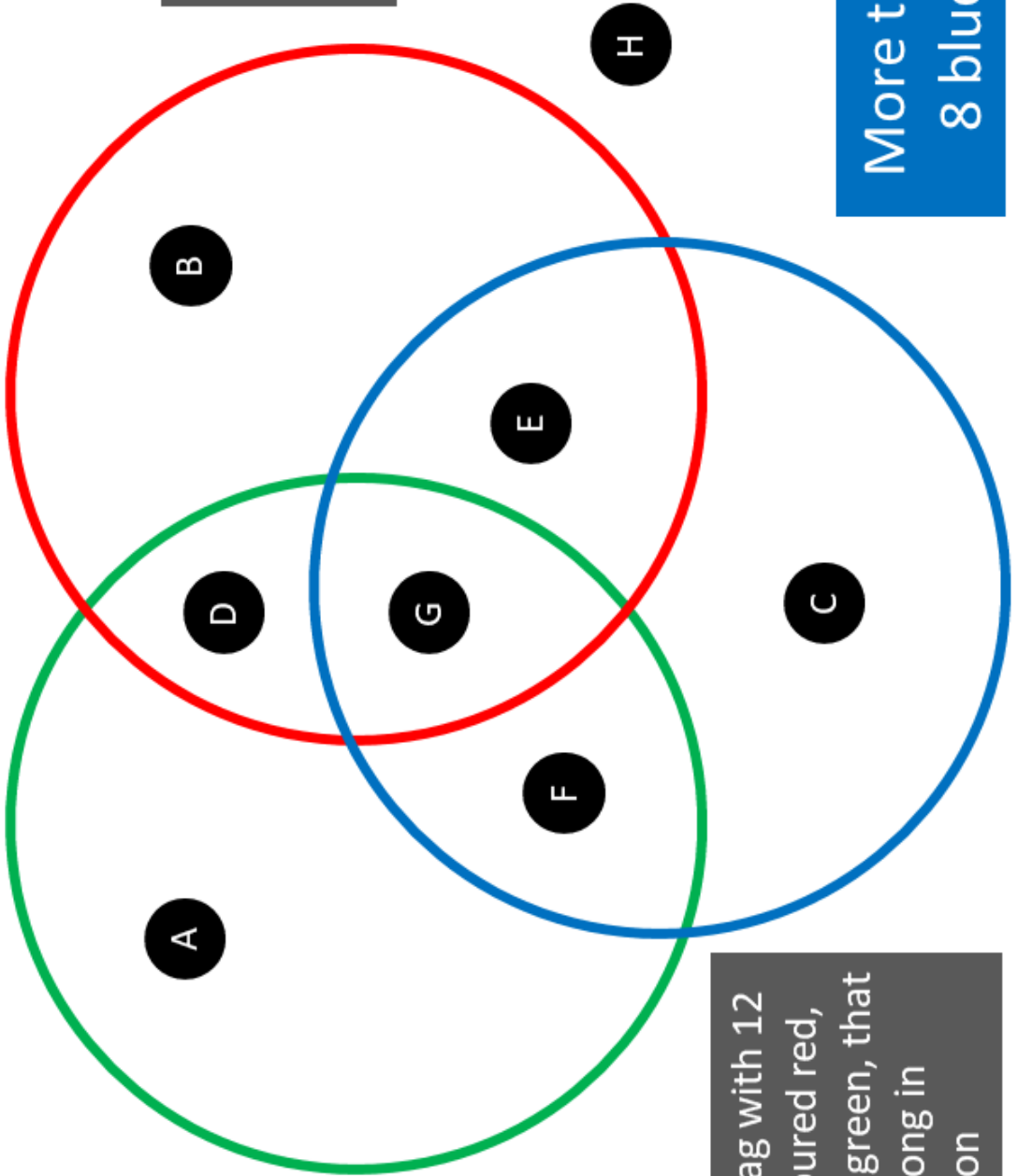
Draw a bag with 10 balls coloured red, blue and green, that could belong in each region

More than 3 blues

# Maths Venns

$$P(\text{red}) < \frac{1}{4}$$

$$P(\text{green}) > \frac{1}{2}$$



If you think a region is impossible to fill, convince me why!

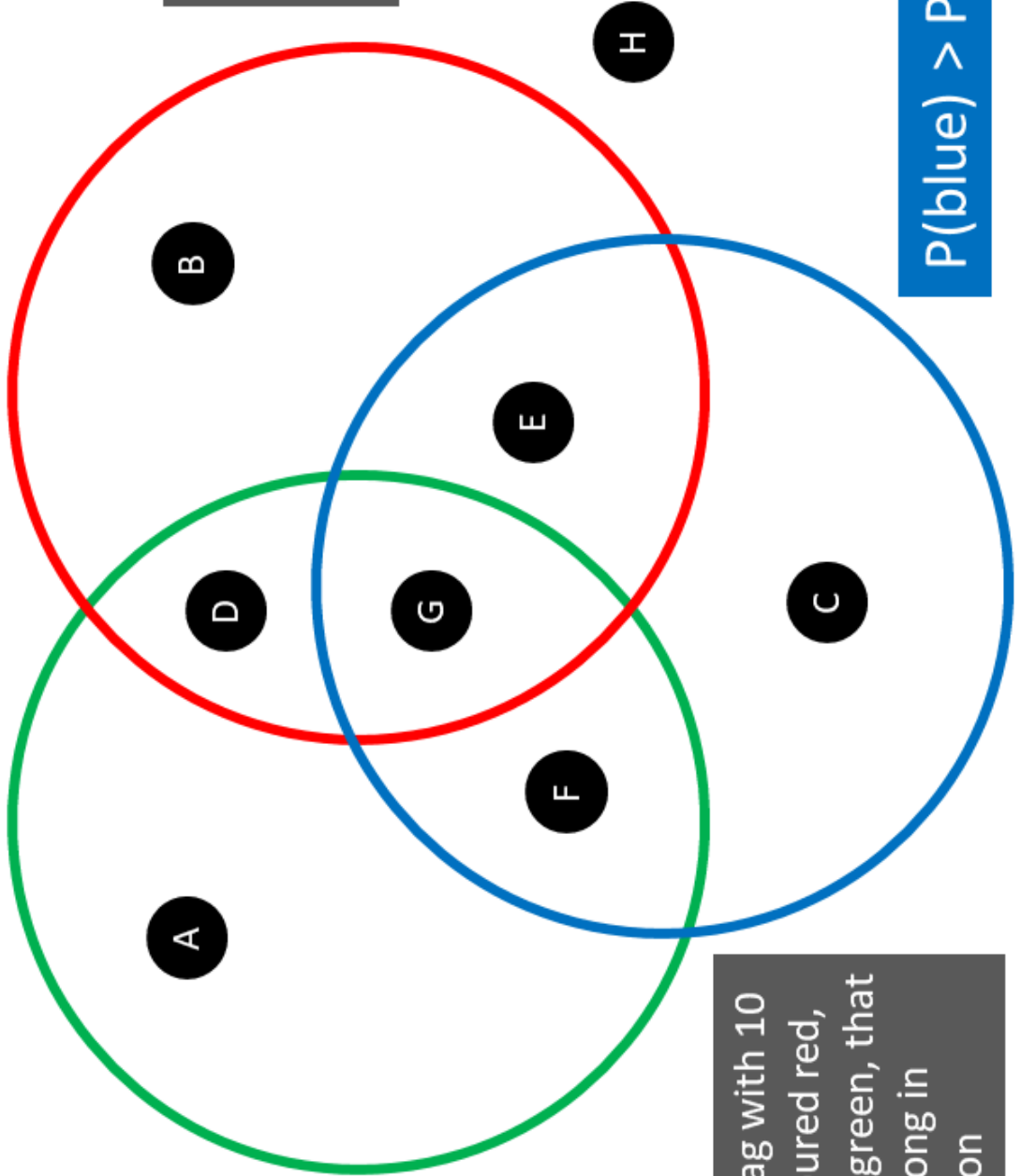
More than 8 blues

Draw a bag with 12 balls coloured red, blue and green, that could belong in each region

# Maths Venns

$P(\text{green}) < P(\text{red})$

$P(\text{red}) < P(\text{blue})$



If you think a region is impossible to fill, convince me why!

Draw a bag with 10 balls coloured red, blue and green, that could belong in each region

$P(\text{blue}) > P(\text{green})$

## 3 Expanding Single Brackets

# Intelligent Practice

Use the distributive property to calculate:

1)  $7 \times (80 + 4)$

2)  $7 \times (80 + 5)$

3)  $8 \times (80 + 5)$

4)  $8 \times (90 + 5)$

5)  $(90 + 5) \times 8$

6)  $(70 + 5) \times 8$

7)  $(70 + 5) \times 16$

8)  $(70 + 5) \times y$

9)  $(70 + y) \times 5$

10)  $(y + 70) \times 5$

# Fluency Practice

Question 1: Expand the following brackets

- |                  |                   |                  |                       |
|------------------|-------------------|------------------|-----------------------|
| (a) $5(y + 3)$   | (b) $4(a + 2)$    | (c) $8(w + 10)$  | (d) $3(x - 7)$        |
| (e) $9(s - 1)$   | (f) $2(8 - t)$    | (g) $7(4 + h)$   | (h) $10(a + 2b + 3c)$ |
| (i) $4(3y + 2)$  | (j) $5(2p - 1)$   | (k) $3(7a + 2)$  | (l) $9(2x - 5)$       |
| (m) $5(4 + 3t)$  | (n) $7(9 - 2c)$   | (o) $8(3w + 1)$  | (p) $9(1 - 4p)$       |
| (q) $11(2k - 5)$ | (r) $20(6a + 5c)$ | (s) $3(15w - 7)$ | (t) $3(9 - 2a)$       |

Question 2: Expand the following brackets

- |                 |                  |                  |                   |
|-----------------|------------------|------------------|-------------------|
| (a) $-2(w + 5)$ | (b) $-3(c + 7)$  | (c) $-8(c + 7)$  | (d) $-10(y - 2)$  |
| (e) $-7(g - 3)$ | (f) $-4(2w + 3)$ | (g) $-9(3w - 5)$ | (h) $-9(5x - 1)$  |
| (i) $-5(6 - c)$ | (j) $-6(4 + 3m)$ | (k) $-2(1 + 9c)$ | (l) $-5(8a - 7w)$ |

Question 3: Expand the following brackets

- |                  |                  |                  |                   |
|------------------|------------------|------------------|-------------------|
| (a) $a(c + 2)$   | (b) $c(d - 3)$   | (c) $a(b + c)$   | (d) $w(8 - y)$    |
| (e) $c(5 + a)$   | (f) $w(a - 9)$   | (g) $y(s + t)$   | (h) $2a(c - 3)$   |
| (i) $5x(y + 8)$  | (j) $3a(2c + 9)$ | (k) $6g(2c - 1)$ | (l) $9k(2 + d)$   |
| (m) $5(2f + 9w)$ | (n) $3y(5p + 2)$ | (o) $2s(t + 1)$  | (p) $-4a(8x - 3)$ |

# Intelligent Practice

Expand:

10)  $-6(2 - x)$

1)  $3(x + 4)$

11)  $6(x - 2)$

2)  $3(4 + x)$

12)  $6(x - 2y)$

3)  $3(4 - x)$

13)  $6(5x - 2y)$

4)  $3(x - 4)$

14)  $-6(2y - 5x)$

5)  $3(-x - 4)$

15)  $-6(2y - 5x - 7z)$

6)  $-3(x + 4)$

16)  $-6(5x - 2y - 7z)$

7)  $-3(x - 4)$

17)  $-6(-5x - 2y - 7z)$

8)  $-3(2x - 4)$

18)  $-w(-5x - 2y - 7z)$

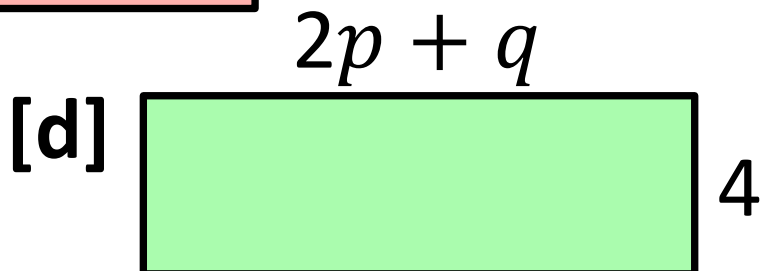
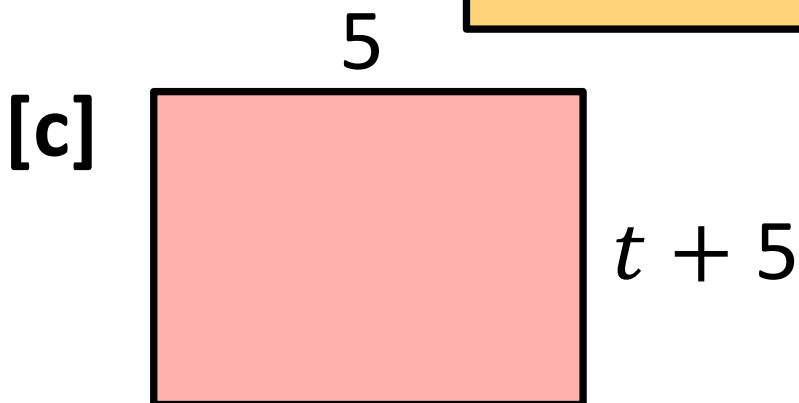
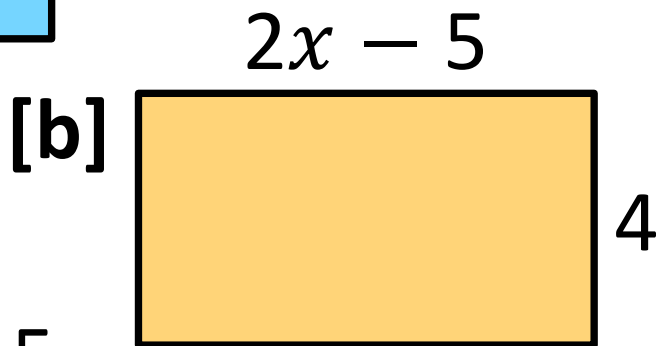
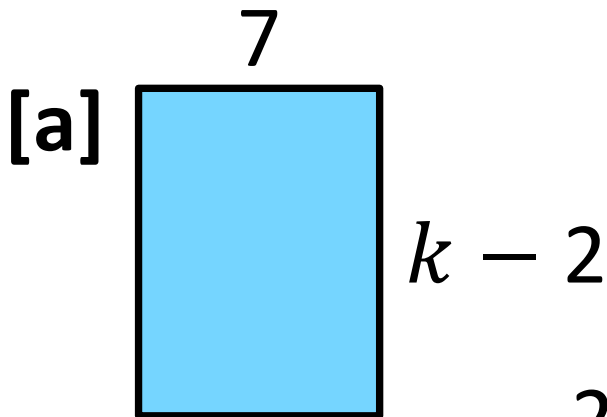
9)  $-3(4 - 2x)$

19)  $-(-5x - 2y - 7z)$

- Why are the answers to questions 1 and 2 the same?
- Why are the answers to questions 5 and 6 the same?
- Why are the answers to questions 9 and 10 the same?

## Extension

Write an expanded expression for the area of each of the following rectangles.





## Fluency Practice

Question 4: Expand the following brackets

(a)  $a(a + 2)$

(b)  $y(y - 5)$

(c)  $w(a + w)$

(d)  $c(9 - c)$

(e)  $p(2p + 5)$

(f)  $2w(3w - 1)$

(g)  $9y(2y + 3)$

(h)  $4c(2a + 5c)$

(i)  $2u(3 - u)$

(j)  $m(m^2 + 3)$

(k)  $y(y^2 - 7)$

(l)  $g^2(g - 8)$

(m)  $2w(w^2 + 6)$

(n)  $4a(2a^2 - 3)$

(o)  $5c(3c^2 - a)$

(p)  $8w(3w^2 + 3y)$

(q)  $x^2(x^2 + 4)$

(r)  $3w^2(7 + 2w^2)$

## Intelligent Practice

Expand:

10)  $-3x(2 - x)$

1)  $x(x + 4)$

11)  $3x(x - 2)$

2)  $x(4 + x)$

12)  $3x(x - 2y)$

3)  $x(4 - x)$

13)  $3x(5x - 2y)$

4)  $x(x - 4)$

14)  $-3x(2y - 5x)$

5)  $x(-x - 4)$

15)  $-3x^2(2y - 5x)$

6)  $-x(x + 4)$

16)  $-3y^2(2y - 5x)$

7)  $-x(x - 4)$

17)  $-3y^2(2y - 5xy)$

8)  $-x(2x - 4)$

18)  $-3y^3(2 - 5x)$

9)  $-x(4 - 2x)$

19)  $-3y^3(2y^2 - 5x^2)$

- Why are the answers to questions 1 and 2 the same?
- Why are the answers to questions 5 and 6 the same?
- Why are the answers to questions 17 and 18 the same?

## Fluency Practice

Expand and simplify:

1)  $5a^3b^5c(9a^5b^2 - 4a^5b)$

2)  $7x^4y^2(2x^2y - 5x^2y^2)$

3)  $a^2b^5c^2(4a^3b^3 - 7a^3b^3c)$

4)  $9x^4y^5(7y^3 + 6x^5y)$

5)  $2a^2b^3(5a^4b^4 - 2ab^4)$

6)  $3ab^4(5a - 4a^5)$

7)  $2a^5b^2(5b + 8a^2b^5)$

8)  $2abc^2(9a^4b^2c^2 + 4a^3b^2)$

9)  $7x^3y^3z^2(9x^5y^4 - 4y^2)$

10)  $3x^3y^4(5xy^2z - 6x^4y^4)$

## Fluency Practice

Expand and simplify:

1)  $2 + 8(5y - 7)$

2)  $10(9x + 4) - 5$

3)  $8 + 9(10y + 3)$

4)  $8(5x + 8) + 2$

5)  $2(3z - 4) + 7$

6)  $4x + 6 + 3(2x + 5)$

7)  $3(9x + 10) + 6x + 5$

8)  $4z - 6 + 2(5z + 9)$

9)  $-8 + 7(6z + 1)$

10)  $8(7y + 1) + 8y - 3$

## Fluency Practice

Question 5: Expand and simplify

(a)  $5(y + 3) + 2(y + 7)$

(b)  $6(2w + 5) + 9(w + 2)$

(c)  $3(y - 2) + 4(2y + 5)$

(d)  $7(2g + 3) - 5(g + 2)$

(e)  $6(x - 2) - 4(x - 8)$

(f)  $2(3y - 8) - 5(2y - 1)$

(g)  $8(5 + 2m) + 3(5 - 3m)$

(h)  $4(w + 7) - 2(2w + 1)$

(i)  $9(1 + 2y) + 3(3 - y)$

## Intelligent Practice

Expand and simplify:

- |                           |                           |
|---------------------------|---------------------------|
| 1) $2(x + 1) + 3(x + 4)$  | 10) $3(x + 1) - 4(x + 2)$ |
| 2) $3(x + 4) + 2(x + 1)$  | 11) $3(x + 1) - (x + 2)$  |
| 3) $3(x + 1) + 2(x + 4)$  | 12) $3(x - 1) - (x - 2)$  |
| 4) $3(x - 1) + 2(x + 4)$  | 13) $3(x - 1) - (5x - 2)$ |
| 5) $3(x + 1) + 2(x - 4)$  | 14) $3(x - 1) - 5x$       |
| 6) $3(x - 1) + 2(x - 4)$  | 15) $5x - 3(x - 1)$       |
| 7) $3(x - 1) - 2(x - 4)$  | 16) $5 - 3(x - 1)$        |
| 8) $3(x + 1) - 2(x + 4)$  | 17) $5 + 3(1 - x)$        |
| 9) $3(x + 1) - 2(2x + 4)$ | 18) $5 + 3(y - x)$        |
|                           | 19) $5 - 3(y - x)$        |

- Why are the answers to questions 1 and 2 the same?
- Why are the answers to questions 9 and 10 the same?
- Why are the answers to questions 16 and 17 the same?

# Fluency Practice

Expand and simplify

- (a)  $2(x + 4) + 5(x + 3)$
- (b)  $3(x + 5) + 2(x + 1)$
- (c)  $5(x + 7) + 3(x + 2)$
- (d)  $6(x + 1) + 4(x + 3)$
- (e)  $2(2x + 3) + 4(3x + 5)$

Expand and simplify

- (a)  $3(x + 9) + 6(x - 2)$
- (b)  $5(x - 2) + 3(x + 4)$
- (c)  $2(x + 8) + 4(x - 1)$
- (d)  $6(x + 3) + 2(x - 4)$
- (e)  $3(2x + 5) + 2(x - 3)$

Expand and simplify

- (a)  $5(x + 5) - 2(x + 3)$
- (b)  $6(x - 1) - 3(x + 2)$
- (c)  $4(x + 7) - 2(x + 5)$
- (d)  $3(x - 1) - 2(x + 4)$
- (e)  $5(2x + 3) - 4(x + 2)$

Expand and simplify

- (a)  $3(x + 10) - 2(x - 4)$
- (b)  $5(x + 4) - 4(x - 1)$
- (c)  $7(x + 5) - 3(x - 2)$
- (d)  $6(x - 3) - 2(x - 4)$
- (e)  $3(2x - 7) - 2(x - 2)$

When two sets of single brackets are added the result is  $7x - 5$ . What could the two sets of brackets have been?

## Problem Solving

Pick 2 of the expressions below, and add them up...

$$6(x + 2) \quad 5(2x + 1) \quad 5(4x + 1) \quad 2(2x + 1)$$

$$5(3x + 1) \quad 3(x + 9) \quad 3(x + 7) \quad 3(3x + 4)$$

$$7(x + 4) \quad 5(x + 1) \quad 2(6x + 1) \quad 8(x + 4)$$

$$8(x + 5) \quad 4(3x + 2) \quad 2(8x + 3) \quad 9(2x + 3)$$

...and try and get one of the answers below!

$$11x + 17$$

$$7x + 23$$

$$15x + 35$$

$$19x + 30$$

$$28x + 32$$

$$16x + 72$$

$$25x + 18$$

$$35x + 10$$



## Fluency Practice

Question 6: Expand and simplify

(a)  $w(w + 5) + w(w + 7)$

(b)  $2g(4g + 3) + g(g - 7)$

(c)  $n(n - 4) - n(5 - n)$

(d)  $2e(4e + 3) - 3e(e - 5)$

(e)  $a(3 + c) + c(a + 2)$

(f)  $m(a + 7) - a(4 - 3m)$

(g)  $8c(8 - 3a) + 3(4 - c)$

(h)  $5y(3y + z) - 2y(4y - 3z)$

(i)  $4c(3c - c^2) - 2c^2(4 - 5c)$

## Intelligent Practice

Expand and simplify:

10)  $3x(x + 1) - 5(x + 4)$

1)  $2x(x + 1) + 3x(x + 4)$

11)  $3x^2(x + 1) - 5x(x + 4)$

2)  $3x(x + 4) + 2x(x + 1)$

12)  $3x^2(x - 1) - 5x(x - 4)$

3)  $3x(x + 1) + 2x(x + 4)$

13)  $3x^2(x - 1) - (5x^2 - 4)$

4)  $3x(x - 1) + 2x(x + 4)$

14)  $3x^2(x - 1) - 5x$

5)  $3x(x + 1) + 2x(x - 4)$

15)  $5x^2 - 3x^2(x - 1)$

6)  $3x(x - 1) + 2x(x - 4)$

16)  $5 - 3x^2(x - 1)$

7)  $3x(x - 1) - 2x(x - 4)$

17)  $5 + 3x^2(1 - x)$

8)  $3x(x + 1) - 2x(x + 4)$

18)  $5 + 3x^2(y - x)$

9)  $3x(x + 1) - 2(x + 4)$

19)  $5 - 3x^2(y - x)$

- Why are the answers to questions 1 and 2 the same?
- Why are the answers to questions 16 and 17 the same?

## Extension

Question 1: Can you spot any mistakes in the questions below.

Expand  $3(2y - 1)$

$$6y - 1$$

Multiply out  $x(x + 3)$

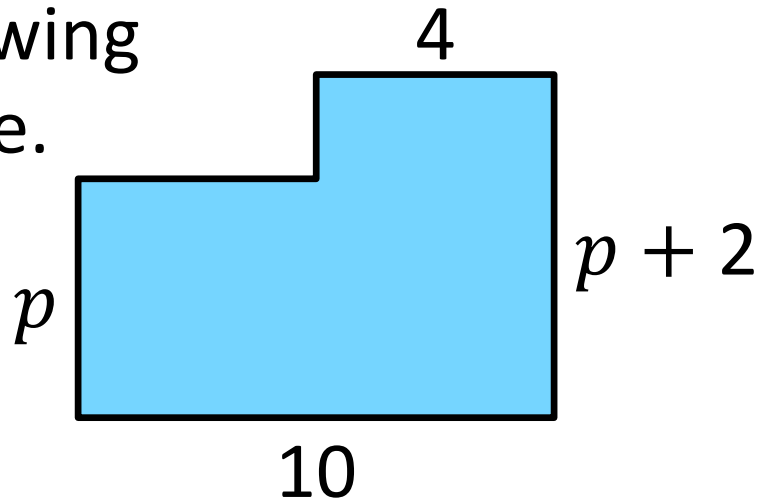
$$2x + 3x = 5x$$

Expand and simplify  $6(w + 3) - 2(w - 5)$

$$\begin{aligned}6w + 18 - 2w - 10 \\ = 4w + 8\end{aligned}$$

## Extension

Find an expanded expression for the area of the following compound shape.



# Fluency Practice

Expand

(a)  $4(x - 3)$       (b)  $2(3 + 4y)$

(c)  $x(x + 4)$       (d)  $x(7 - x)$

(e)  $2x(x + 9)$       (f)  $x(y + 3x)$

(g)  $-2(4 + x)$       (h)  $-(x - 6)$

(i)  $-3x(6 - x)$       (j)  $-y(x + y)$

(k)  $x^2(3x + y)$       (l)  $2y^2(y - x)$

Expand and simplify

(a)  $2(x + 4) + 5(x + 7)$

(b)  $3(a + 2) + 4(a - 1)$

(c)  $4(p - 5) + 6(p - 1)$

(d)  $2(x + 8) - 3(x + 2)$

(e)  $5(x - 2) - 2(x - 9)$

(f)  $3(2x + 1) - 4(x + 5)$

(g)  $2(3x + 1) - (2x - 3)$

(h)  $2(p - 4) + 3(2p - 1)$

Expand and simplify

(a)  $x(x^2 - 2y) - 3x^2(x + 2y)$

(b)  $a(a + 2b + 3c) + 3c(a - 2b + 3c)$

(c)  $a(b - c + d) - a(b - c + d)$

(d)  $6 + 2(x + 7)$

(e)  $6 + 2(3 - x)$

(f)  $6 - (2x + 3)$

(a) A rectangle has a width  $x$  cm and a length  $x + 5$  cm. Write a simplified expression for the area of the rectangle.

(b) A triangle has a base of  $4x$  cm and a height of  $(3x - 5)$  cm. Find a simplified expression for the area of the triangle.

# Fluency Practice

Expanding Single Brackets			
<b>(a)</b>  Expand $2(x + 5)$	<b>(b)</b>  Expand $3(6 - x)$	<b>(c)</b>  Expand $x(x - 3)$	<b>(d)</b>  Expand $a(5 + a)$
<b>(e)</b>  Expand $2a(b - 7)$	<b>(f)</b>  Expand $-2(x + 6)$	<b>(g)</b>  Expand $5(2x - y)$	<b>(h)</b>  Expand $4x(2 + x)$
<b>(i)</b>  Expand $-3(x^2 + 4)$	<b>(j)</b>  Expand $6a(a + 2b)$	<b>(k)</b>  Expand $2x(x^2 + 3y)$	<b>(l)</b>  Expand $ab(8 - a)$
<b>(m)</b>  Expand $-x(3 + x)$	<b>(n)</b>  Expand $-2(5 - x)$	<b>(o)</b>  Expand $3x^2y(2x - 6y)$	<b>(p)</b>  Expand and simplify $4(x + 2) + 3(x + 6)$
<b>(q)</b>  Expand and simplify $4(8 + x) + 3(x - 1)$	<b>(r)</b>  Expand and simplify $6(1 + 2x) - 2(x + 5)$	<b>(s)</b>  Expand and simplify $7(3x + 2) - 4(x - 2)$	<b>(t)</b>  Expand and simplify $6x(x + 4) - x(7 - 2x)$

# Fluency Practice

2 Work these multiplications.

a  $x(x - 2)$

h  $3(5x + 4)$

o  $3x(x + 7)$

b  $x(x - 4)$

i  $4(3x - 2)$

p  $4x(x - 3)$

c  $x(x + 9)$

j  $8(2x - 5)$

q  $8x(x - 5)$

d  $x(x + 12)$

k  $x(4x - 3)$

r  $8x(2x - 5)$

e  $3(x + 6)$

l  $x(3x - 7)$

s  $3x(3x + 4)$

f  $4(x + 5)$

m  $x(5x + 6)$

t  $2x(5x - 3)$

g  $2(2x + 3)$

n  $5x(x + 6)$

u  $4x(2x + 1)$

3 Work these harder multiplications. Take care with the powers of  $x$ .

a  $x(x^2 + 3x + 4)$

i  $4(3x^2 + 4x - 7)$

b  $x(x^2 - 5x + 2)$

j  $3x(4x^2 - 6x + 1)$

c  $x(x^2 + 6x - 7)$

k  $2x(3x^2 + 6x + 4)$

d  $x(2x^2 - 4x + 3)$

l  $4x(2x^2 + x + 3)$

e  $x(3x^2 + 4x - 5)$

m  $3x(4x^2 - x - 1)$

f  $2(x^2 - 3x - 4)$

n  $8x(3x^2 - 6x + 1)$

g  $3(x^2 + 6x - 5)$

o  $8(5x^2 - x + 2)$

h  $6(2x^2 - 2x + 3)$

p  $3x^2(3x^2 + 2x - 7)$

4 Expand the brackets and simplify these expressions.

a  $3(2x + 1) + 2(4x + 3)$

e  $5(x + 2) + 6(2x - 3)$

b  $5(3x + 4) + 4(3x - 4)$

f  $4(x - 3) + 3(2x + 5)$

c  $4(2x + 3) + 2(x - 2)$

g  $5(2x - 4) + 2(x + 8)$

d  $2(3x + 4) + 3(x - 4)$

h  $3(x - 4) + 2(2x - 3)$

# Fluency Practice

i  $8(x - 1) + 3(4x - 3)$

j  $2(4x + 1) - 3(2x - 2)$

k  $4(2x - 3) - 2(3x - 7)$

l  $5(3x + 1) - 3(2x + 2)$

m  $6(2x + 3) - 5(x + 3)$

n  $x(x + 3) + x(x + 4)$

o  $x(x + 6) + x(x - 2)$

p  $x(x - 4) + x(x + 7)$

q  $x(x - 3) + x(x - 5)$

r  $x(x + 6) + x(x - 8)$

s  $x(2x + 3) + x(3x + 4)$

t  $x(4x + 5) + x(2x - 3)$

u  $x(x + 8) + x(3x - 5)$

v  $5x(x + 3) - 3x(x - 2)$

w  $6x(x - 2) - 4x(x + 1)$

x  $3x(2x + 3) + 2x(4x - 3)$

y  $6x(4x + 2) - 3x(7x + 4)$

z  $4x(3x + 9) - 6x(2x + 6)$

5 Expand the brackets and simplify the expressions. These are harder.

a  $x(x^2 + 4x + 2) + x(x^2 + 5x + 7)$

b  $x(x^2 - 5x + 3) + x(x^2 + 8x - 5)$

c  $x(x^2 - 4x - 6) - x(x^2 - 6x - 9)$

d  $x(x^2 + 6) - x(x^2 - 4x + 2)$

e  $x(x^2 - 3) + x(x^2 - 4x + 5)$

f  $x(x^2 + x - 4) - x(x^2 - 2)$

g  $x(2x^2 + 3x + 1) + x(3x^2 - 3x + 2)$

h  $2x(x^2 + 3x + 8) + 3x(x^2 + x - 4)$

i  $5x(2x^2 - 6x + 2) - 2x(4x^2 - 12x + 5)$

j  $3x(4x^2 - 2x + 1) + 5(x^2 + 3x - 2)$

k  $2x(7x^2 - 3) + 3(x^2 + 2x - 1)$

l  $4x(2x^2 - 3x + 1) - 2(x^3 - 4) + 9x$

m  $5x(x^2 + 2x - 3) + x(x - 4) - 6x$

n  $4(2x^2 + 7x - 4) - 2x(4x + 3) + 16$

o  $x^2(3x - 2) - 3x(x^2 - 2x) - 4x^2$

p  $x^2(2 - 5x) + x(5x^3 - 2x + 4)$



# Fluency Practice

6 Work these multiplications, simplifying where possible.

a  $x(y + 3)$

b  $2x(y + 4)$

c  $3x(4y + 5)$

d  $7x(2y - 3)$

e  $5x(3y - 2z)$

f  $2x(5y + 4z)$

g  $x(x + 2y)$

h  $3x(x - 4y)$

i  $5x(x - 3y + 2z)$

j  $2x(y - 1) + 3x(y + 2)$

k  $4x(y + 5) + 2x(y - 8)$

l  $3x(5y - 4) - 5x(2y - 1)$

m  $2x(2y + 3z) - 3x(y + 4z)$

n  $5x(3y - 2z) + 2x(y + 6z)$

o  $x(x - 2y) - x(x - 5y)$

p  $2x(3x + y) + 3x(2x - 2y)$

q  $6x(3x - 2y) - 9x(2x - y)$

r  $2x(x - y) + x(x + 2y)$

s  $3x(2x - y) + 2x(x + 3y)$

t  $x(6x + 2y + 3) + 2x(x - 3y + 2)$

u  $5x(x - y + 1) - 4x(x - 2y + 3)$

v  $3x(x - 3y + 1) - 2x(x + 2y - 3)$

w  $y(x + 7y - 2) + 3y(x - 2y + 1)$

x  $2y(4x + 3y - 5) - y(x - y + 5)$

# Simplifying Expressions

## Simplifying Expressions

### Collecting Like Terms

- a)  $2x + 5 + 4x$   
 b)  $6 + 4x + 3 - 2x$   
 c)  $4 + 2x + x + 3 - 5x$   
 d)  $5x + 3 - x - 5 + (2x)$   
 e)  $7 - 3x + 5 + 6x - (-2x)$   
 f)  $3x^2 + 4x + 2x^2 - x$   
 g)  $5x + x^2 - 3 - 2x - 3x^2 + 3$   
 h)  $-3x + 1 + 2x^2 - 4x + x^2$   
 i)  $(2x + 5x) - (3x - x)$   
 j)  $4x^2 - (x + 4) + 3x + x^2 + 2$   
 k)  $x + x^3 - 2x^2 + 2x^3 - 4x$   
 l)  $4 - x^3 + x^2 - 2x^3 - (5 - 4x^3)$

### Multiplying

- a)  $2x \times 3x$   
 b)  $3x \times 5x^2$   
 c)  $3x \times x^2 \times 3x$   
 d)  $3x^2 \times 2x^3$   
 e)  $3x^2 \div x$   
 f)  $x^6 \div x^2$   
 g)  $\frac{3x^5}{x}$   
 h)  $\frac{4x^5}{2x^2}$   
 i)  $\frac{6x^3}{3x} \times x^2$   
 j)  $2x^3 \times \frac{8x^6}{2x^2}$   
 k)  $(4x)^2$   
 l)  $(2yx^2)^3$   
 m)  $\left(\frac{1}{2}y^4x\right)^3$

### Mixed Arithmetic

- a)  $2x + 2 \times 4x$   
 b)  $3 \times x + 5 \times 2x$   
 c)  $4 + 8x - 3 \times 2x + 3x + 2$   
 d)  $5(2x + 3) + 2$   
 e)  $3(3x^2 + 5)$   
 f)  $2x(2x - 1) + x^2$   
 g)  $3x(2x^2 - 7)$   
 h)  $5x(4 - 2x) + (-3x)$   
 i)  $2(3x - 2) + 2(x + 3)$   
 j)  $5x(x + 4) - 2(x - 3)$   
 k)  $\frac{16x^3}{2x} + 5x(x - 2)$   
 l)  $\left(\frac{9x^3}{3x^2}\right)^2 - 2x(3x - 1) - 3x$

## Simplifying Expressions

Simplify these four expressions.

(a)  $(3x + 4y) + 2(x + 2y)$

(b)  $4(2x + 5y) - 3(x + 4y)$

(c)  $3(2x + 3y) - (x - y)$

(d)  $3(x + 3y) + (2x - y)$

Which one is the odd one out?

If you finish, try to make up some more that fit the pattern.

# The answer is $5x + 8y$ : What's the question?

$5x + 8y$  is the answer – your job is to make up the questions!

The only brackets that you are allowed to use are:

$$(x + y) \quad (x + 2y) \quad (x - 2y) \quad (x + 4y) \quad \text{and} \quad (2x + 3y)$$

Pick any **two** of these brackets and combine them with numbers and + or – to make an expression.

For example, you could pick

- the brackets  $(x + 2y)$  and  $(x + 4y)$
- and the numbers 3 and  $-2$

and make

$$3(x + 2y) - 2(x + 4y)$$

... but unfortunately that **doesn't** make  $5x + 8y$ .

Can you find a way to make  $5x + 8y$  using **two** different brackets?

Can you find a way to make  $5x + 8y$  using **more than two** different brackets?

Can you find a way to make  $5x + 8y$  using **all five** brackets?

# Expanding and Simplifying

simplification

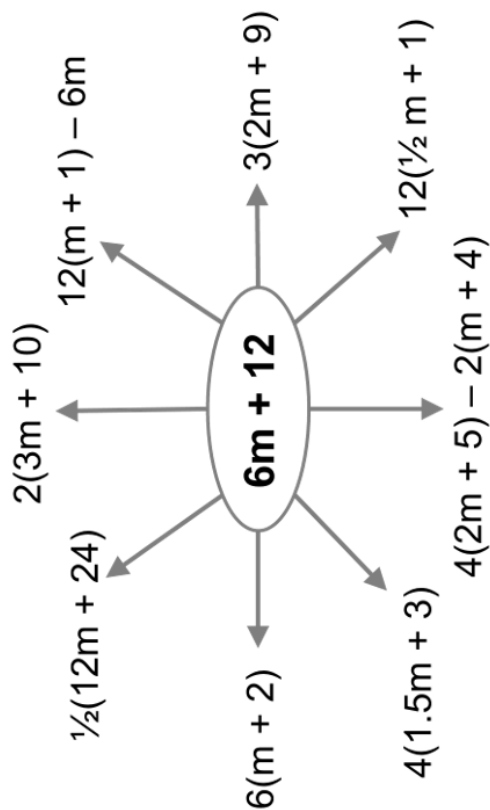
a) expand the brackets and then simplify the expressions

- 1)  $5(b + 5) + 7(b + 2) + 2(b + 1)$
- 2)  $3(2a + 1) + 6(a + 3)$
- 3)  $3(2m + 15) + 10(m + 1) + 4(5m + 2)$
- 4)  $5(2n + 3) + 3(10n + 3) + \frac{1}{2}(6n + 20)$
- 5)  $4(2t + 3) + 5(t + 6) + \frac{1}{2}(4t + 18)$
- 6)  $15(3k + 1) + 2(17k + 1) + \frac{1}{2}(4k + 2)$
- 7)  $6(5d + 8) + 3(4d - 5) + d + 1$
- 8)  $8(p + 5) + 6(7p - 3) + 2(p + 1.5)$
- 9)  $2(4h + 21) + 5(4h - 1) - 5(h + 1)$

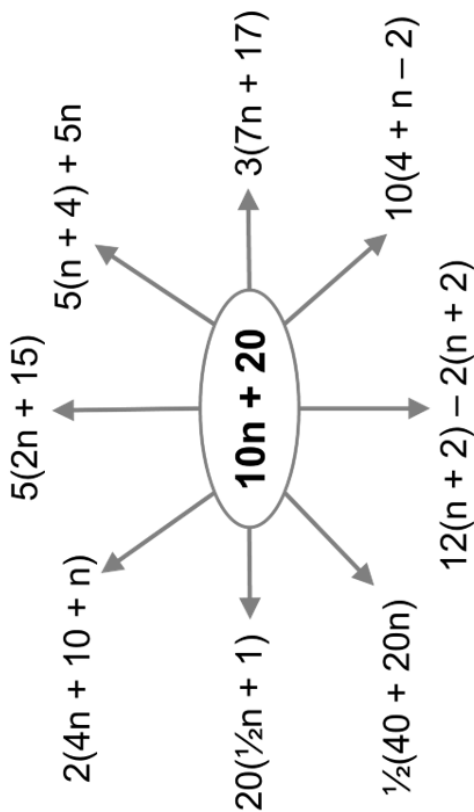
b) six expressions:

- 1)  $2(9b - 13a)$   
which two sum to  $4(3b - 2a)$ ?
- 2)  $3(2b - 11a)$   
which two add to  $7(4a - 3b)$ ?
- 3)  $4(3a - 5b)$   
which three sum to zero?
- 4)  $5(2a - 3b)$   
which three add to  $4(a - 2b)$ ?
- 5)  $6(3a - b)$
- 6)  $7(3a + 2b)$

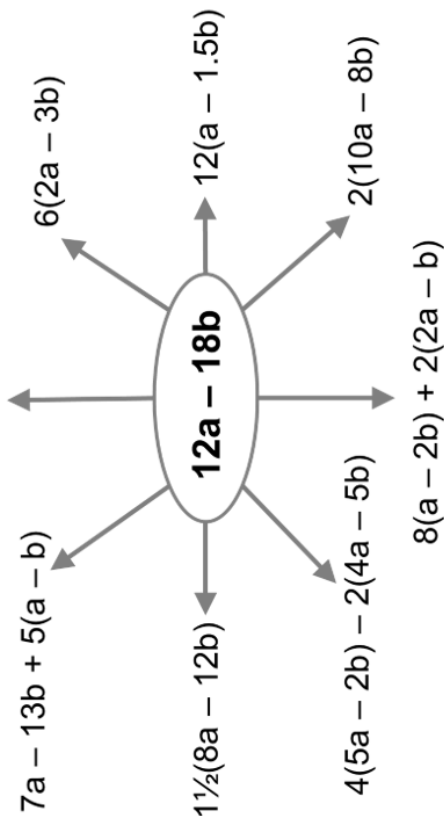
# One Incorrect Simplification



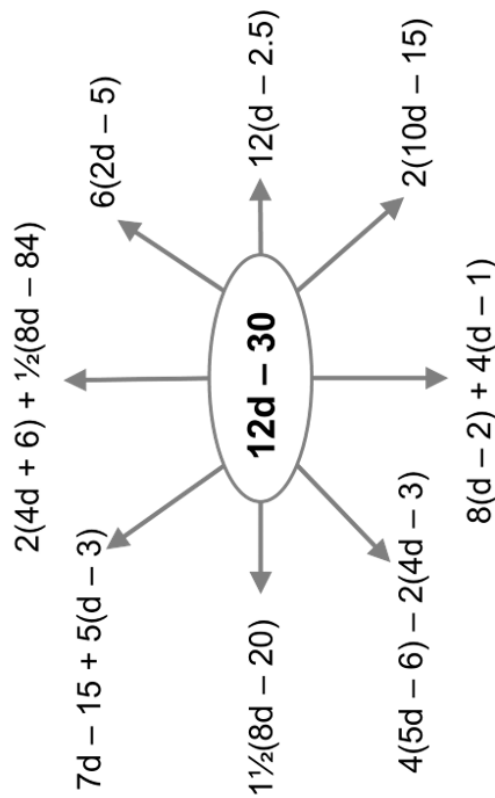
which expressions are **not** the same as  $6m + 12$  ?



which expressions are **not** the same as  $10n + 20$  ?



which expressions are **not** the same as  $12a - 18b$  ?



which expressions are **not** the same as  $12d - 30$  ?

# Find the Gaps

**3p - 1**

find the missing expression or numbers:

(1)  $7(p - 3) - 4(\square) = 3p - 1$

(2)  $3(5p + 9) - 4(\square) = 3p - 1$

(3)  $8(3p + 1) - 3(\square) = 3p - 1$

(4)  $3(5p - 3) - 4(\square) = 3p - 1$

(5)  $11(3p - 5) - 6(\square) = 3p - 1$

(6)  $5(\square + 1) - 6(\bigcirc + 1) = 3p - 1$

(7)  $14(\square + 1) - 5(\bigcirc + ?) = 3p - 1$

(8)  $\square(p - 2) + 5(\bigcirc - 3p) = 3p - 1$

(9)  $\square(2p - 9) - \bigcirc(3p - 16) = 3p - 1$

(10)  $\square(5p - 7) - \bigcirc(2p - 3) = 3p - 1$

## Find the Gaps

**complete the expression sums**

$$(1) \quad p + 3 + p + 5 - \boxed{\phantom{0000}} = p$$

$$(2) \quad 2p - 3 + p - 4 + \boxed{\phantom{0000}} = p$$

$$(3) \quad p + 3 + 2p - 8 + \boxed{\phantom{0000}} = p$$

$$(4) \quad 3p - 2 + 4(2 - p) + 2 \boxed{\phantom{0000}} = p$$

$$(5) \quad 4(p - 1) + 2(3p - 1) + 3 \boxed{\phantom{0000}} = p$$

$$(6) \quad p + 1 + 2p - 2 - \boxed{\phantom{0000}} = p$$

$$(7) \quad 4(p - 1) + 2(p - 3) - 5 \boxed{\phantom{0000}} = p$$

$$(8) \quad 7(p - 1) - 4(3p - 1) + 3 \boxed{\phantom{0000}} = p$$

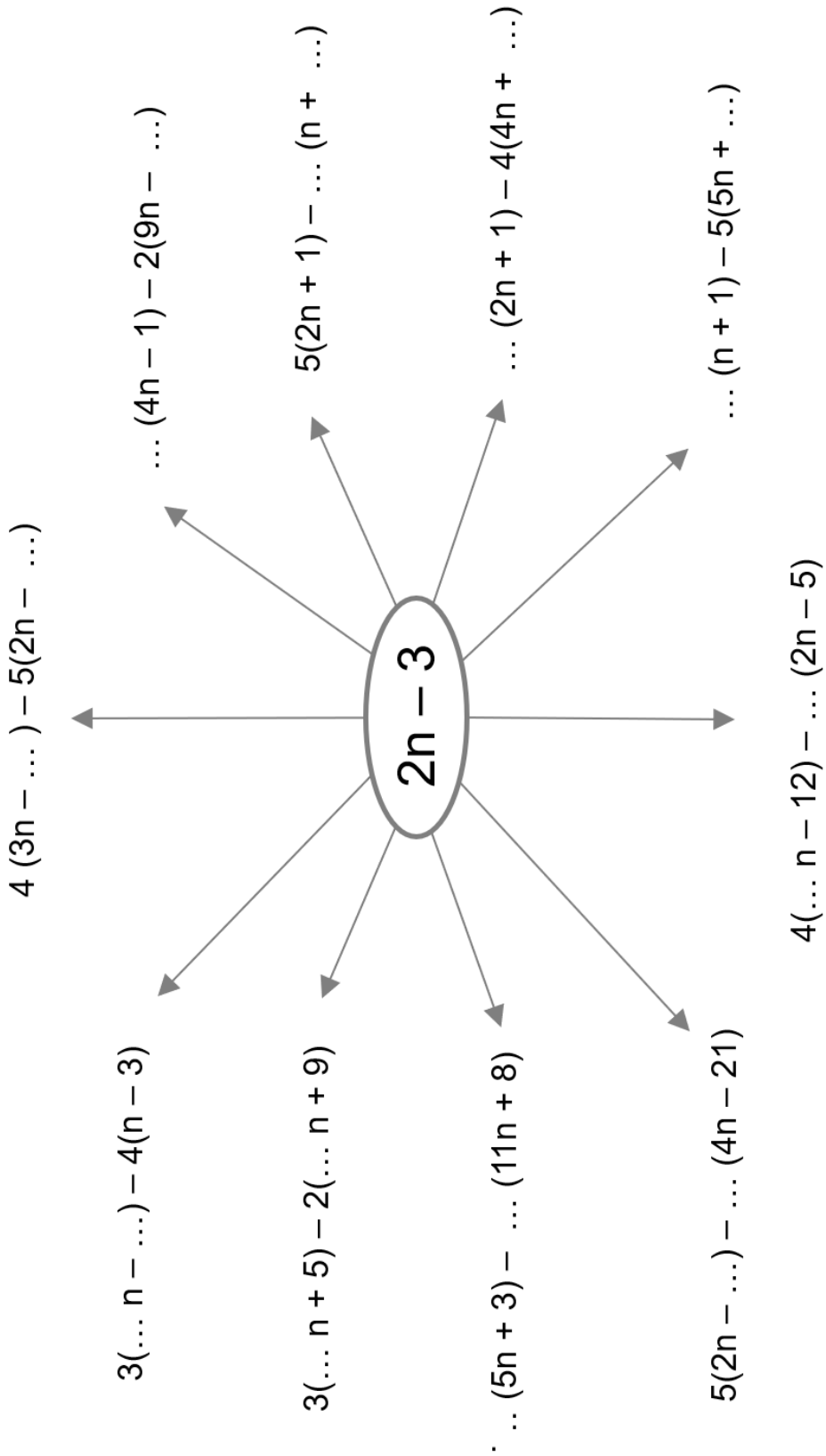
$$(9) \quad 2(3 - p) + 6(2p - 3) - 3 \boxed{\phantom{0000}} = p$$



# Equivalent Things

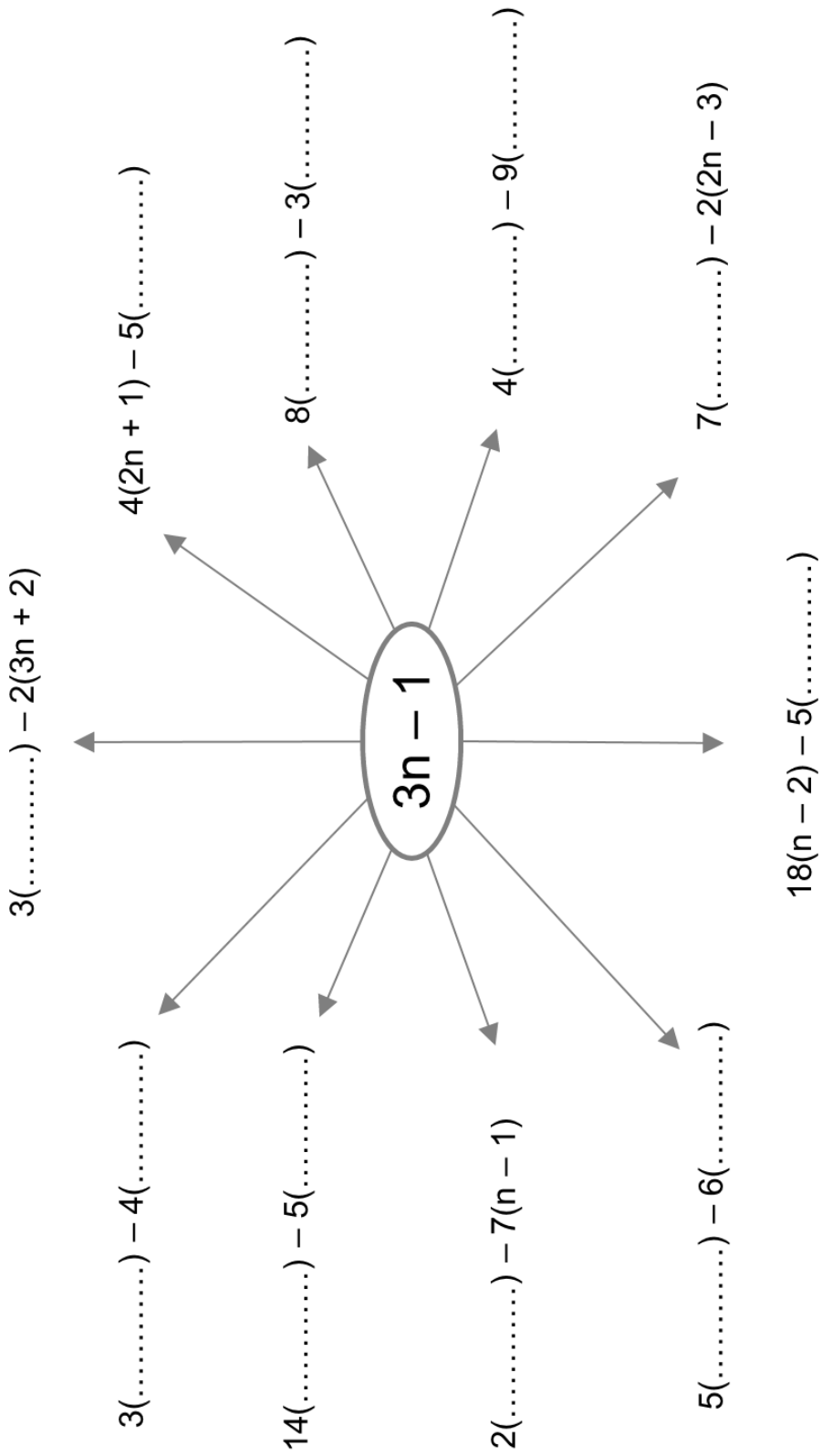
what numbers go in the missing (blank) spaces so that the expression simplifies to  $2n - 3$  ?

e.g.  $7(6n - 5) - 8(5n - 4) \equiv 2n - 3$



# Equivalent Things

what expressions go in the missing (blank) spaces to create  $3n - 1$  ?



# Expressions Sums

complete the expression sums

$$(1) \quad p + 3 + p + 5 - \boxed{\phantom{00}} = p$$

$$(2) \quad 2p - 3 + p - 4 + \boxed{\phantom{00}} = p$$

$$(3) \quad p + 3 + 2p - 8 + \boxed{\phantom{00}} = p$$

$$(4) \quad 3p - 2 + 4(2 - p) + 2\boxed{\phantom{00}} = p$$

$$(5) \quad 4(p - 1) + 2(3p - 1) + 3\boxed{\phantom{00}} = p$$

$$(6) \quad p + 1 + 2p - 2 - \boxed{\phantom{00}} = p$$

$$(7) \quad 4(p - 1) + 2(p - 3) - 5\boxed{\phantom{00}} = p$$

$$(8) \quad 7(p - 1) - 4(3p - 1) + 3\boxed{\phantom{00}} = p$$

$$(9) \quad 2(3 - p) + 6(2p - 3) - 3\boxed{\phantom{00}} = p$$

$$(10) \quad 5p - 6 - 3(p - 1) - \frac{\boxed{\phantom{00}}}{2} = p$$

$$(11) \quad 2(5 - p) + 3(2p - 1) - \frac{\boxed{\phantom{00}}}{3} = p$$

$$(12) \quad 4(1 - 3p) - 3(2 - p) + \frac{8(5p + 1)}{\boxed{\phantom{00}}} = p$$

$$(13) \quad 3(p - 2) - 4(2p - 1) + \frac{3p + 1}{\boxed{\phantom{00}}} = p$$

$$(14) \quad 5(2 - p) - 2(7 - 4p) - \frac{2}{3}\boxed{\phantom{00}} = p$$

$$(15) \quad 2(3 - 4p) - 3(4 - 5p) - \frac{3}{4}\boxed{\phantom{00}} = p$$