KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 8

2023 Mathematics 2024

Unit 8 Booklet

HGS Maths

Tasks

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

1 Factorising to a Single Bracket

1.1 Highest Common Factor

Worked Example	Your Turn
Write the following as a product of factors: a) $3a$ b) $6a$ c) $6a^2$ d) $6a^2\text{b}$	Write the following as a product of factors: a) $2b$ b) $12b$ c) $12b^2$ d) $12a^2b^2$

Worked Example	Your Turn
Find the highest common factor of: a) $3a$ and $5a$ b) 6 and $6a$ c) $3a$ and $6a$ d) $4ab^2$ and $6a^2\mathrm{b}$	Find the highest common factor of: a) $2b$ and $3b$ b) 6 and $12b$ c) $6b$ and $12b^2$ d) $8a^2b$ and $12a^2b^2$

1.2 Factorising to a Single Bracket

Factorising means:
To turn an expression into a **product** of factors.

Year 8 Factorisation

$$2x^2 + 4xz \xrightarrow{\text{Factorise}} 2x(x+2z)$$

Year 9 Factorisation

$$x^2 + 3x + 2 \xrightarrow{\text{Factorise}} (x+1)(x+2)$$

A Level Factorisation

$$2x^3 + 3x^2 - 11x - 6 \xrightarrow{\text{Factorise}} (2x+1)(x-2)(x+3)$$

Factorising is the **reverse of expanding**.
When you have a sum of terms, just **identify the common factor**.
i.e. Find the largest expression each of your terms is divisible by.

Worked Example	Your Turn
a) Factorise $12x + 18$ b) Factorise $12x + 18y$ c) Factorise $12x^2 + 18$	a) Factorise $12x - 20$ b) Factorise $12x - 20y$ c) Factorise $12x^3 - 20$

Worked Example	Your Turn
a) Factorise $12x^2 + 18\text{x}$ b) Factorise $12x^2 + 18xy$ c) Factorise $12x^2y + 18xy$	a) Factorise $12x^2 - 20x$ b) Factorise $12x^2 - 20xy$ c) Factorise $12x^2y - 20xy^2$

Fill in the Gaps

Expanded Expression	HCF of Numbers	HCF of Variables	Factorised Expression
$7x + 14$	7		$7(x + 2)$
$20 + 30a$	10		$10(\square + \square)$
$15b - 5$	5		
$12x + 15$			
$30a - 12b$			
$8cd + de$		d	$d(\square + \square)$
$10a + ab$			
$x^2 - 5x$		x	
$6x^2 + xy$			
$4ab + 8b$	4	b	$4b(\square + \square)$
$10cd - 25de$	5	d	
$4x^2 + 2x$			
$14xy - 21x^2$			
$6x + 3 - 9y$			
$5x^2 - 10xy + 20x$			
$24a^2b + 16abc$			
$\square - 18xyz$			$\square(x - 3z)$
$12x + \square - 16yz$			$4(\square + 2y - \square)$
$35a^2b^2 + \square$			$\square(5a^2b + 2cd)$

1.3 Factorising to a Single Bracket with Index Laws

Worked Example	Your Turn
Factorise: a) $x^4y^2 - x^3y^5$ b) $10x^7y^4 - 25x^3y^2$	Factorise: a) $x^2y^5 - xy^3$ b) $20e^5f^2 - 12e^2f$

1.4 Finish Factorising

Worked Example	Your Turn
Finish factorising: a) $4(10x + 50)$ b) $4(30x + 50)$	Finish factorising: a) $4(5x + 15)$ b) $4(25x + 15)$

2 Solving Linear Equations 2

2.1 Brackets

To solve an equation means that we find the value of the variable(s).

Strategy: To get x on its own on one side of the equation, we gradually need to 'claw away' the things surrounding it.

Note: In algebra, we tend to give our answers as fractions rather than decimals (unless asked). And never recurring decimals. Don't round also (unless asked).

Worked Example	Your Turn
Solve the following equations: a) $4(x+8)=50$ b) $4(2x+8)=50$	Solve the following equations: a) $6(x-8)=50$ b) $6(3x-8)=50$

Worked Example	Your Turn
Solve the following equations: a) $-4(2x+8)=50$ b) $-4(2x-8)=50$	Solve the following equations: a) $-6(3x+8)=50$ b) $-6(3x-8)=50$

Worked Example	Your Turn
Solve the following equations: a) $8(x+3)+3(2x+6)=84$ b) $8(x+3)-3(2x-6)=84$	Solve the following equations: a) $3(x-3)+4(2x-6)=110$ b) $3(x-3)-4(2x-6)=110$

2.2 Both Sides

- Collect the variable terms (i.e. the terms involving x) on one side of the equation, and the 'constants' (i.e. the individual numbers) on the other side.

- Collect the variable terms on the side of the equation where there's more of them (and move constant terms to other side).

Balancing

- We eliminate the variable from the side with the smaller number of the variable.
- We eliminate the variable by applying the inverse to both sides.

Which side do you eliminate the variable from?
How would you balance both sides?

- $3x + 4 = 2x + 6$
- $2x + 4 = 3x + 6$
- $2x - 4 = 3x - 6$
- $4 - 2x = 3x - 6$
- $4 - 2x = 6 - 3x$

Worked Example	Your Turn
Solve the following equations: a) $5x + 7 = 2x + 31$ b) $2x - 23 = 7 - x$	Solve the following equations: a) $5x + 7 = 3x + 23$ b) $2x - 23 = 12 - 3x$

Worked Example	Your Turn
Solve the following equations: a) $17x = 10x + 21$ b) $10x = 17x + 21$	Solve the following equations: a) $10x = 13x - 21$ b) $13x = 10x - 21$

Worked Example	Your Turn
Solve the following equations: a) $3(x+2)=2(x+3)$ b) $3(x+5)-7=2(x+2)$	Solve the following equations: a) $9(x-3)=4(x+7)$ b) $7(x+6)-7=4(x+2)$

Worked Example	Your Turn
Solve the following equation: $3(2w-1)-4=4(w+2)+1$	Solve the following equation: $2(2p-2)-4=2(p+3)-3$

2.3 Variable in the Denominator

Worked Example	Your Turn
Solve the following equation: a) $\frac{3}{x} + 2 = 6$ b) $\frac{3}{x+2} = 6$	Solve the following equation: a) $\frac{15}{x} - 2 = 6$ b) $\frac{15}{x-2} = 6$

Worked Example	Your Turn
Solve the following equation: $\frac{3x+6}{2} = x + 3$	Solve the following equation: $\frac{9x-27}{4} = x + 7$

Worked Example	Your Turn
Solve the following equation: $\frac{3x+6}{x+3} = 2$	Solve the following equation: $\frac{7x-21}{x+7} = 2$

2.4 Cross Multiplication

You can cross multiply to solve equations which are in the form:
$$\frac{a}{b} = \frac{c}{d}$$

Are the following equations ready to be cross multiplied?

- $\frac{2x}{3} = \frac{5}{9}$
- $\frac{2x}{3} + 1 = \frac{5}{9}$
- $\frac{2x}{3} + 1 = 5$
- $\frac{2x+1}{3} = 5$
- $\frac{3}{2x+1} = \frac{5}{x}$

Worked Example	Your Turn
Solve the following equations: a) $\frac{x}{5} = \frac{3}{2}$ b) $\frac{x+1}{5} = \frac{3}{2}$	Solve the following equations: a) $\frac{2x}{5} = \frac{3}{2}$ b) $\frac{2x+1}{5} = \frac{3}{2}$

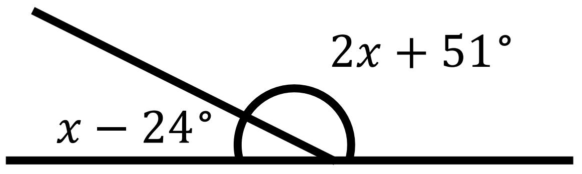
Worked Example	**Your Turn**
Solve the following equations: a) $\frac{3x-4}{5}=\frac{x+4}{3}$ b) $\frac{4}{2-3x}=\frac{5}{6-2x}$	Solve the following equations: a) $\frac{x+4}{7}=\frac{x-4}{3}$ b) $\frac{4}{2+3x}=\frac{5}{6+2x}$

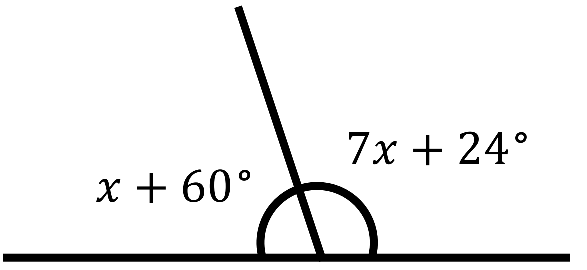
2.5 Forming and Solving Equations

Worked Example	Your Turn
I think of a number. I multiply the number by 6 then subtract 3. The result is 15. What was my original number?	I think of a number. I multiply the number by 4 then subtract 5. The result is 27. What was my original number?

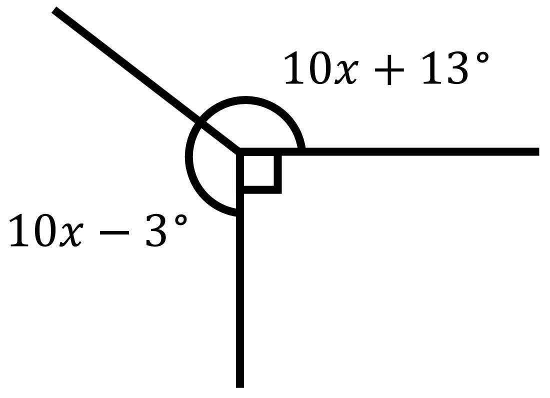
Worked Example	Your Turn
A is x years old. B is 3 years older than A. C is twice as old as A. The sum of the ages of A, B and C is 51. What are their ages?	A is x years old. B is 3 years younger than A. C is three times as old as A. The sum of the ages of A, B and C is 57. What are their ages?

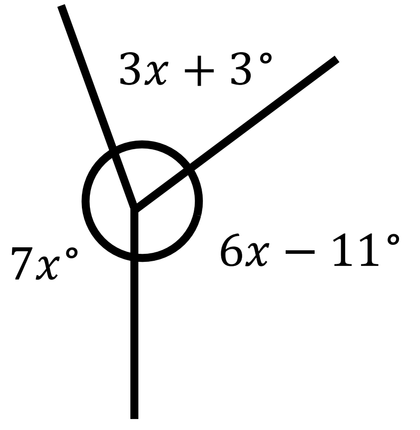
Worked Example	Your Turn
Find x	Find x

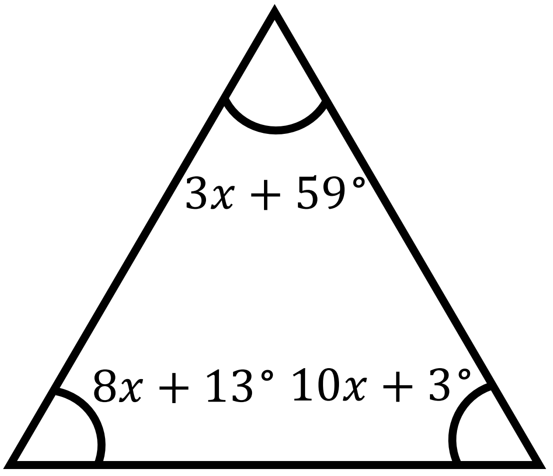
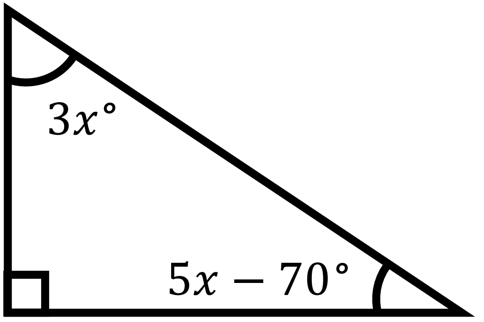




Worked Example	Your Turn
Find x	Find x

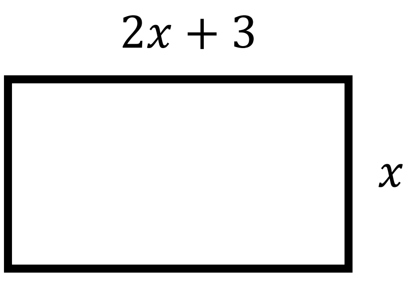




Worked Example	Your Turn
Find x	Find x
	

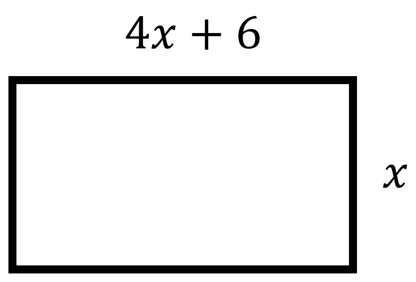
Worked Example

The perimeter of the rectangle is equal to 72 square units. Find x.



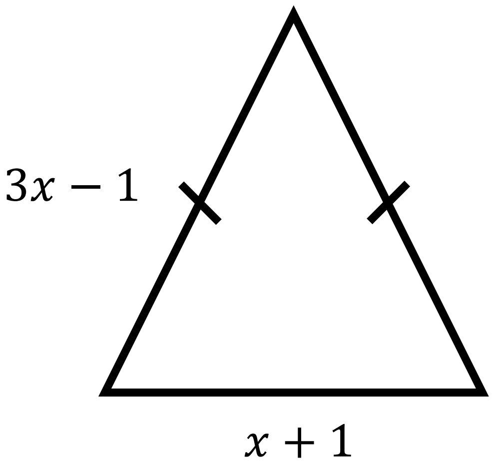
Your Turn

The perimeter of the rectangle is equal to 72 square units. Find x.



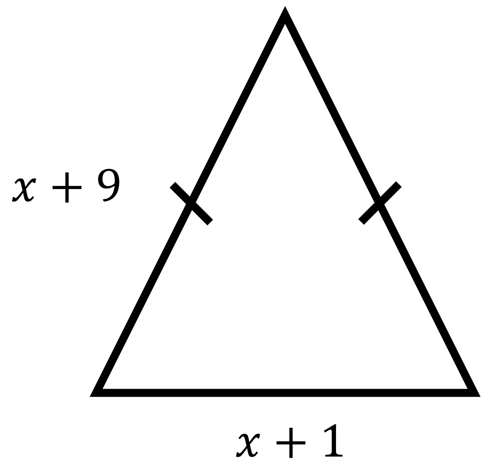
Worked Example

The perimeter of the isosceles triangle is equal to 34 square units. Find x.



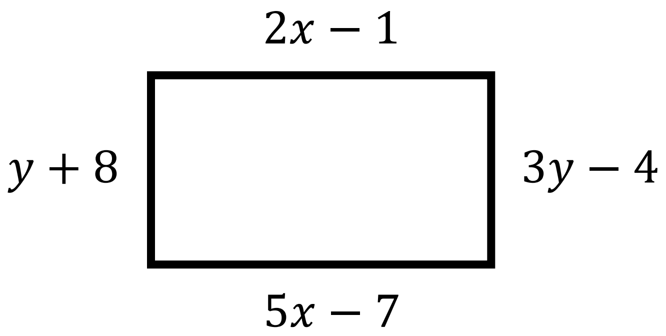
Your Turn

The perimeter of the isosceles triangle is equal to 34 square units. Find x.



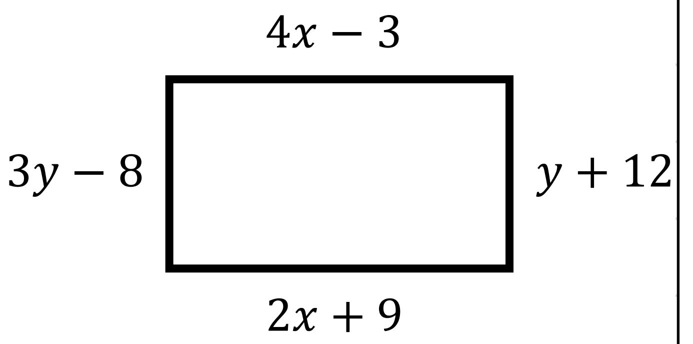
Worked Example

Find x and y

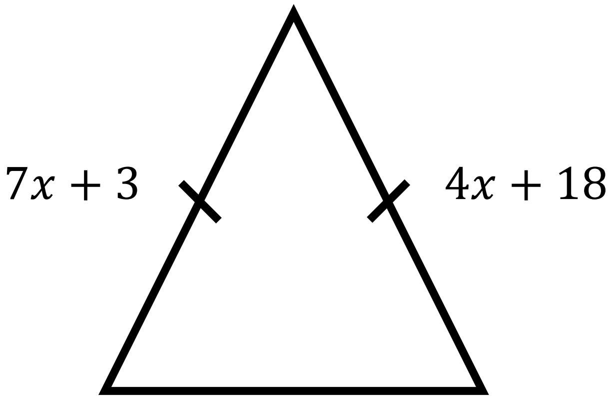


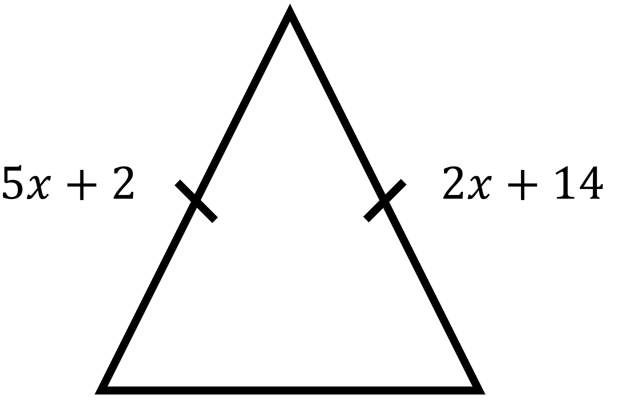
Your Turn

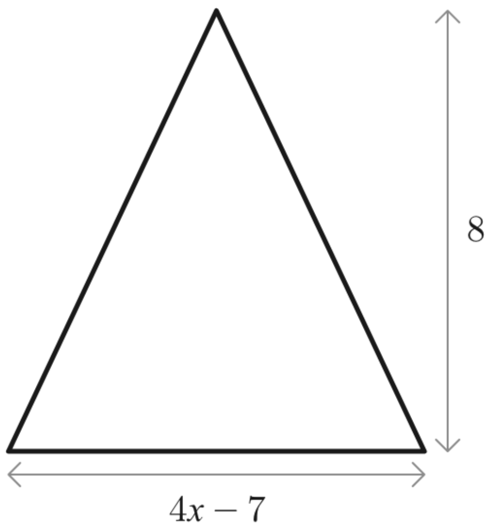
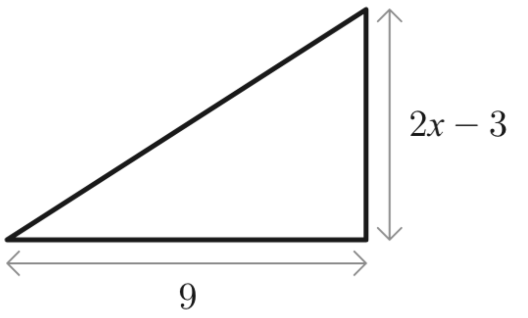
Find x and y

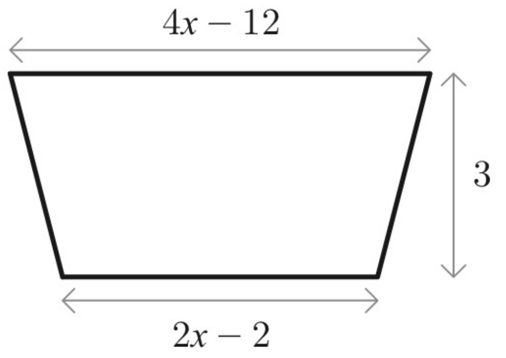
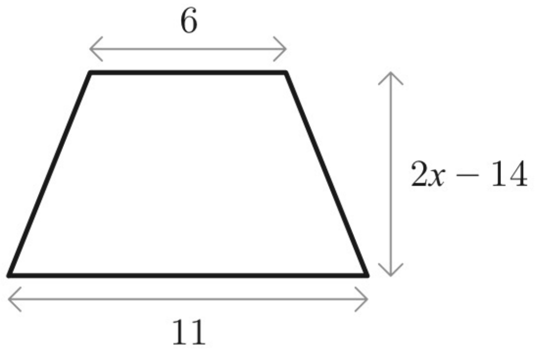


Worked Example	Your Turn
Find x	Find x



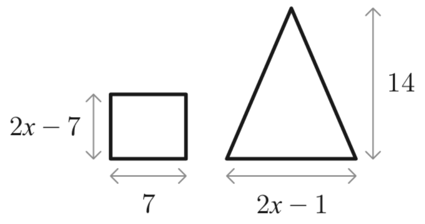


Worked Example	Your Turn
A triangle is shown in the diagram below.  All the measurements are in centimetres. The area of the triangle is 28 cm^2. Find the value of x.	The diagram below shows a triangle.  All the measurements are in centimetres. The area of the triangle is 9 cm^2. Find the value of x.

Worked Example	Your Turn
A is shown in the diagram below.	The diagram below shows a trapezium.
	
All the measurements are in centimetres.	All the measurements are in centimetres.
The area of the trapezium is $42\ \mathrm{cm}^2$.	The area of the trapezium is $34\ \mathrm{cm}^2$.
Find the value of x.	Find the value of x.

Worked Example

The diagram shows a rectangle and a triangle.



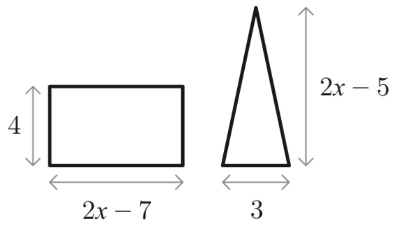
All the measurements are in centimetres.

The area of the rectangle is half the area of the triangle.

Work out the value of x.

Your Turn

The diagram shows a rectangle and a triangle.



All the measurements are in centimetres.

The area of the rectangle is twice the area of the triangle.

Work out the value of x.

3 Sequences

3.1 Finding the Next Term

A **sequence** is simply an ordered list of items (possibly infinitely long), usually with some kind of pattern.

Each item in a sequence is called a **term**.

Worked Example	Your Turn
A sequence starts with: $24, 29, 34, 39 \ldots$ Work out the next 3 terms.	A sequence starts with: $41, 36, 31, 26 \ldots$ Work out the next 3 terms.

Worked Example	Your Turn
A sequence starts with: $2048, 512, 128, 32 \ldots$ Work out the next 3 terms.	A sequence starts with: $7, 42, 252, 1512 \ldots$ Work out the next 3 terms.

Worked Example	Your Turn
A sequence starts with: $5, 9, 14, 23, 37 \ldots$ Work out the next 3 terms.	A sequence starts with: $6, 10, 16, 26, 42 \ldots$ Work out the next 3 terms.

3.2 Constant Differences

Worked Example	Your Turn
What is the constant difference in the sequence? The 10^{th} term is 52 and the 18^{th} term is 76	What is the constant difference in the sequence? The 10^{th} term is 52 and the 22^{nd} term is 76

Worked Example	Your Turn
What is the constant difference in the sequence? The 10^{th} term is 76 and the 18^{th} term is 52	What is the constant difference in the sequence? The 10^{th} term is 76 and the 22^{nd} term is 52

3.3 Term to Term Rule

Some sequences we can generate by stating a rule to say how to generate the next term given the previous term(s).

$\mathbf{3, 7, 11, 15, 19} \ldots$

What is the rule, in words, for this sequence?
We add 4 each time.

The problem is that this also describes many other sequences. Can you think of another sequence that adds 4 every time?

We need to both state our **rule** and our **starting term**.

A better rule for this sequence would be:
Start with 3, add 4 each time.

Fill in the Gaps

First Five Terms of Sequence					Term-to-Term Rule
6	10	14			
5	3	1			
3		5			
1	3	9			
1.5	1.7		2.1		
	7	2	-3		
80	40	20			
	1		$1\frac{1}{2}$		
8					$add\ 3$
2					$add\ 7$
	4				$subtract\ 2$
		2.5			$add\ 0.5$
			5		$subtract\ 2.5$
	2				$multiply\ by\ 2$
100					$divide\ by\ 10$
-4					$subtract\ 3$

3.4 Types of Sequences

Arithmetic/Linear: The terms' first difference is constant.
e.g., $1, 3, 5, 7, \ldots$

Geometric: The terms found by multiplying by the same number each time.
e.g., $2, 4, 8, 16, \ldots$

Quadratic: The terms' second difference is constant.
e.g., $2, 5, 10, 17, \ldots$

Fibonacci-Type: The terms found by adding the previous two terms together.
e.g., $1, 3, 4, 7, 11, \ldots$

Frayer Model – Linear Sequences

Definition	**Characteristics**
Examples	**Non-Examples**

3.5 Position to Term Rule

It is sometimes more helpful to be able to generate a term of a formula based on its position in the sequence.

We could use it to say find the 300^{th} term of a sequence without having to write all the terms out!

We use $\boldsymbol{n}$ to mean the **position in the sequence**. So, if we want the 3^{rd} term, $n = 3$.

The **position to term rule** is also called the $\boldsymbol{n}^{\textbf{th}}$ **term rule**.

This year, we will only look at how to work out the position to term rule for linear sequences. You will learn how to find the position to term rule for geometric and quadratic sequences in year 11.

Worked Example	Your Turn
Find the n^{th} term rule:	Find the n^{th} term rule:
$8, 15, 22, 29, 36, \ldots$	$11, 18, 25, 32, 39, \ldots$
$-6, 1, 8, 15, 22, \ldots$	$-3, 4, 11, 18, 25, \ldots$
$36, 29, 22, 15, 8, \ldots$	$39, 32, 25, 18, 11, \ldots$

Worked Example	Your Turn
Find the n^{th} term rule: $\frac{1}{2}, \frac{7}{10}, \frac{9}{10}, 1\frac{1}{10}, \ldots$	Find the n^{th} term rule: $\frac{1}{3}, \frac{7}{9}, 1\frac{2}{9}, 1\frac{2}{3}, \ldots$

Worked Example	Your Turn
Find the n^{th} term rule: $\frac{5}{12}, \frac{7}{19}, \frac{9}{26}, \frac{11}{33}, \ldots$	Find the n^{th} term rule: $\frac{6}{13}, \frac{8}{20}, \frac{10}{27}, \frac{12}{34}, \ldots$

3.6 Generating Linear Sequences

To generate a term of a linear sequence, substitute n (the position number) into the n^{th} term rule.

Worked Example	Your Turn
Generate the first 5 terms of a) $5n + 3$ b) $-3 - 5n$	Generate the first 5 terms of a) $6n - 3$ b) $3 - 6n$

Worked Example	Your Turn
1) The nth term of a sequence is $5(-6n+3)$ Work out the 50th term of the sequence.	1) The nth term of a sequence is $4(-3n-6)$ Work out the 50th term of the sequence.
2) The nth term of a sequence is $4n^2+6n-3$ Work out the 50th term of the sequence.	2) The nth term of a sequence is $2n^2-4n+1$ Work out the 50th term of the sequence.

3.7 Linear Sequences

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Fill in the Gaps

First Five Terms	Term-to-Term Rule	10th Term	30th Term	nth Term	Sum of the First 5 Terms
$3, 5, 7, 9, 11$				$2n + 1$	
$7, 10, 13, 16, 19$					
$8, \square, \square, \square, \square$	$add\ 5$				
$40, 36, 32, 28, 24$				$44 - 4n$	
$25, 22, 19, 16, 13$					
$\square, \square, 7, \square, \square$	$subtract\ 2$				
$\square, \square, \square, 27, 31$					
$\square, \square, \square, \square, \square$		35		$3n + \square$	
$\square, \square, \square, \square, \square$			20	$\square - 6n$	
$\square, \square, \square, \square, \square$	$add\ 1$				35

Fill in the Gaps

	First 5 terms					Term-to-term rule	0^{th} term	10^{th} term	Sum of first 5 terms	nth term rule
a.	5	8								
b.	5		9							
c.	5			8						
d.	5					Add 6				
e.	5						-2			
f.	5	3								
g.	10					Subtract 3				
h.	10						15			
i.	10							19		
j.		10			22					
k.			10				16			
l.				10	22					
m.						Add 3	4			
n.						Add 3		33		
o.						Add 3			30	
p.							6	26		
q.							26	6		
r.					21			36		
s.							2		25	
t.								40	60	

Fill in the Gaps

Work out the missing terms in each sequence, and then the n^{th} term.
All sequences are decreasing arithmetic sequences.

	1st term	2nd term	3rd term	4th term	5th term	6th term	7th term	8th term	9th term	10th term	n^{th} term
Q1		8		4							
Q2	15		9		3				-9		
Q3			11			-4					
Q4	23						11				
Q5		44				16					
Q6						82		74			
Q7	14						-4				
Q8	-1			-7							
Q9		-5				-21		-29			
Q10			-12						-42		

Fill in the Gaps

Work out the missing terms in each sequence, and then the n^{th} term.
All sequences are decreasing arithmetic sequences.

	1st term	2nd term	3rd term	4th term	5th term	6th term	7th term	8th term	9th term	10th term	n^{th} term
Q11		1.8		1.2							
Q12	2.9				0.5						
Q13					3.8					0.3	
Q14				9.8			2.3				
Q15					4.1		1.5				
Q16		2.7				2.5					
Q17		3.1			-4.4						
Q18	-4				-6.4						
Q19	-0.4						-8.8				
Q20				-7.2			-13.5				

Fill in the Gaps

Finding the nth term of a Linear Sequence

1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	common difference	nth term
		11					21						
					27				43				
		4									31		
	11									65			
		14									−4		
	4						−38						
			−25										

5th	7th	nth term
39	53	

4th	7th	nth term
12	30	

6th	10th	nth term
9	11	

8th	15th	nth term
−20	−48	

12th	20th	nth term
−54	−102	

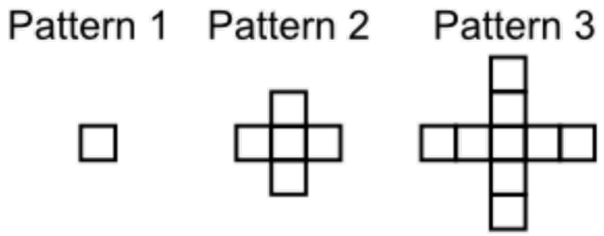
10th	15th	nth term
17	15.5	

Fill in the Gaps

Q	First 4 terms	nth term rule	term to term rule	1st term	10th term	29th term
1	$5, 9, 13, 17, \ldots$					
2		$4n + 3$				
3	$8, 13, 18, 23, \ldots$					
4		$5n - 3$				
5			$+6$	2		
6			$+6$		52	
7				-1	26	
8					28	66
9		$8 - 2n$				
10	$7, 6, 5, 4, \ldots$					
11				7	-20	
12					-20	-67.5

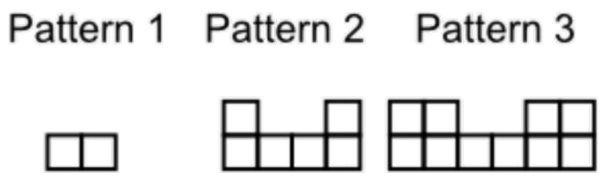
3.8 Patterns

Worked Example



a) Draw the next pattern.
b) How many squares are in the n^{th} pattern?
c) How many squares in the 50^{th} pattern?
d) Which pattern will use 145 squares?

Your Turn



a) Draw the next pattern.

b) How many squares are in the n^{th} pattern?

c) How many squares in the 50^{th} pattern?

d) Which pattern will use 154 squares?

3.9 Fibonacci-Type Sequences

Worked Example	Your Turn
Find the next three terms in these Fibonacci-type sequences:	Find the next three terms in these Fibonacci-type sequences:
$2, 7, 9, 16, \ldots$	$3, 11, 14, 25, \ldots$
$\frac{2}{3}, \frac{5}{6}, \frac{3}{2}, \frac{7}{3}, \ldots$	$\frac{3}{4}, \frac{5}{6}, \frac{19}{12}, \frac{29}{12}, \ldots$
$3a + 4b, a + 7b, 4a + 11b, \ldots$	$3a - 4b, 2a - 5b, 5a - 9b, \ldots$

3.10 Is a Term in the Sequence?

Worked Example	Your Turn
Is 100 in the sequence $16, 20, 24, 28, 32, \ldots?$	Is 100 in the sequence $26, 30, 34, 38, 42, \ldots?$

Worked Example	Your Turn
Is -100 in the sequence $42, 38, 34, 30, 26\ \ldots?$	Is -100 in the sequence $32, 28, 24, 20, 16, \ldots?$