

# Mathematics

## UNIT 1

### Reasoning with Number



**Name:** \_\_\_\_\_

**Class:** \_\_\_\_\_

# Contents

## **How to use booklets:**

You will write in a printed booklet in lessons as directed by your teacher. An online version is available via the QR code.

You can also access and download this booklet from the Maths section of the school website or your year group team. The PDF version will allow you to click on any links to other resources so we highly recommend you do this.

## **1.1 Number**

[Fraction Arithmetic pg 2](#)

[Prime Factorisation, HCF and LCM pg 8](#)

[Standard Index Form \(SIF\) pg 28](#)

[Types of Number/Surds pg 65](#)

## **1.2 Percentages**

[FDP and Percentage Change pg 83](#)

[Reverse Percentages pg 93](#)

[Repeated Percentage Change pg 106](#)

## **1.3 Maths and Money**

[Compound and Simple Interest pg 111](#)

[Unit Pricing and Exchange Rates pg 114](#)

## **Review**

[Summary 1.2 + 1.3 pg 120](#)

[Mixed Exam Questions pg 124](#)

# 1.1 Number

## Fraction Arithmetic

**Simplifying Fractions** - you can scale a fraction by multiplying or dividing the top AND bottom by the same number

$$\frac{30}{45} \xrightarrow{\div 5} \frac{6}{9} \xrightarrow{\times \frac{1}{3}} \frac{2}{3}$$

**Mixed and improper fractions -**

$$2\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$$

Mixed  $\longrightarrow$  improper

$$\frac{17}{3} \rightarrow 3\frac{5}{3} \rightarrow 5\frac{2}{3}$$

improper  $\longrightarrow$  mixed.

**Multiplying and dividing:**

- ① Must express as improper
  - if dividing  $\swarrow$
  - if multiplying  $\searrow$
- ② Flip 2<sup>nd</sup> fraction, change  $\div$  to  $\times$
- ③ cancel down
- ④ multiply across the top and the bottom

e.g.

$$1\frac{3}{5} \div \frac{6}{7}$$

- ① =  $\frac{8}{5} \div \frac{6}{7}$
- ② =  $\frac{8}{5} \times \frac{7}{6}$
- ③ =  $\frac{48}{5} \times \frac{7}{6}$
- ④ =  $\frac{28}{5}$

**Adding and subtracting:**

FIND COMMON DENOMINATOR (LCM)

$$5\frac{3}{5} - 2\frac{5}{6} = 5 + \frac{3}{5} - 2 - \frac{5}{6}$$

$$= 5 - 2 + \frac{3}{5} - \frac{5}{6}$$

$$\text{lcm}(5,6) = 30 \quad = 3 + \frac{18 - 25}{30}$$

$$= 3 + \frac{-7}{30}$$

$$\begin{array}{r} 1+1+1-\frac{7}{30} \\ \hline 2+\frac{23}{30} \end{array} = 2\frac{23}{30}$$

# Prerequisite Knowledge Check

## Quiz A

Website: <https://diagnosticquestions.com/Quizzes/Go#172778>

PDF: [https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow\\_handsworth\\_bham\\_sc\\_h\\_uk/ETE9FRBMbuVHqIJU655IpTQBZHDIABJePnuT-6q1irhtUQ?e=FNM885](https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow_handsworth_bham_sc_h_uk/ETE9FRBMbuVHqIJU655IpTQBZHDIABJePnuT-6q1irhtUQ?e=FNM885)

## Quiz B

Website: <https://diagnosticquestions.com/Quizzes/Go#172779>

PDF: [https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow\\_handsworth\\_bham\\_sc\\_h\\_uk/EUyyRsXH6gIHtVWGggqYIrYBPaDwEsb4niWOHN2iBADswA?e=3dZelA](https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow_handsworth_bham_sc_h_uk/EUyyRsXH6gIHtVWGggqYIrYBPaDwEsb4niWOHN2iBADswA?e=3dZelA)

## Extension

the fraction

$$\frac{5 \square 2 \square}{1 \square 4 \square 9}$$

uses all the digits 1 to 9

and cancels down to  $\frac{1}{3}$

# Questions

## Exercise 1M

Work out without a calculator and give the answer in its simplest form.

- |                                 |                                 |                                  |                                  |
|---------------------------------|---------------------------------|----------------------------------|----------------------------------|
| 1 $\frac{1}{4} + \frac{3}{8}$   | 2 $\frac{3}{5} + \frac{1}{10}$  | 3 $\frac{2}{3} + \frac{1}{6}$    | 4 $\frac{5}{12} + \frac{1}{4}$   |
| 5 $\frac{7}{8} - \frac{1}{2}$   | 6 $\frac{1}{3} + \frac{1}{2}$   | 7 $\frac{3}{5} - \frac{1}{4}$    | 8 $\frac{4}{7} - \frac{1}{2}$    |
| 9 $\frac{2}{3} + \frac{1}{4}$   | 10 $\frac{2}{5} + \frac{1}{3}$  | 11 $\frac{1}{7} + \frac{1}{2}$   | 12 $\frac{1}{5} - \frac{1}{6}$   |
| 13 $\frac{2}{3} - \frac{5}{12}$ | 14 $\frac{7}{9} - \frac{1}{6}$  | 15 $\frac{4}{5} - \frac{2}{7}$   | 16 $\frac{7}{10} - \frac{1}{3}$  |
| 17 $1\frac{1}{4} - \frac{2}{5}$ | 18 $1\frac{3}{4} - \frac{2}{3}$ | 19 $3\frac{1}{4} + 1\frac{3}{5}$ | 20 $2\frac{5}{6} + 1\frac{1}{4}$ |

Questions 21 to 40 involve either multiplying or dividing.

- |                                       |                                     |                                       |                                      |
|---------------------------------------|-------------------------------------|---------------------------------------|--------------------------------------|
| 21 $\frac{2}{3} \times \frac{1}{5}$   | 22 $\frac{3}{5} \times \frac{3}{4}$ | 23 $\frac{5}{9} \times \frac{3}{4}$   | 24 $1\frac{3}{4} \times \frac{1}{3}$ |
| 25 $\frac{3}{8} \times \frac{4}{5}$   | 26 $\frac{2}{9} \times \frac{6}{7}$ | 27 $\frac{5}{12} \times \frac{3}{10}$ | 28 $\frac{5}{8} \times \frac{6}{15}$ |
| 29 $\frac{5}{6} \div \frac{1}{2}$     | 30 $\frac{7}{8} \div \frac{2}{3}$   | 31 $\frac{5}{9} \div \frac{3}{4}$     | 32 $2\frac{1}{2} \div \frac{1}{3}$   |
| 33 $3\frac{1}{4} \times 2\frac{1}{2}$ | 34 $\frac{5}{8} \div 1\frac{1}{2}$  | 35 $\frac{5}{9} \div \frac{1}{3}$     | 36 $\frac{3}{5} \div \frac{9}{100}$  |
| 37 $\frac{3}{5} \div 2$               | 38 $\frac{4}{7} \div 3$             | 39 $1\frac{1}{4} \div 4$              | 40 $5\frac{1}{2} \div 3$             |
- 41  $(\frac{3}{5} \div \frac{1}{3}) + (1\frac{1}{4} \times \frac{1}{10})$       42  $(\frac{1}{2} + \frac{1}{3} + \frac{1}{9}) \div (\frac{1}{4} - \frac{1}{9})$

## Exercise 1E

Copy each square and fill in the missing numbers or symbols (+, -, ×, ÷). The arrows act as equals signs.

1

	+	2	→	$1\frac{1}{2}$
+		+		
	×		→	2
↓		↓		
	×	8	→	

2

$\frac{1}{4}$	-	$\frac{1}{16}$	→	
		+		
$\frac{1}{8}$	+		→	1
↓		↓		
$\frac{1}{8}$			→	$\frac{5}{16}$

5

$\frac{2}{3}$	×	4	→	
		+		
$\frac{1}{2}$	÷		→	$\frac{1}{16}$
↓		↓		
$\frac{1}{3}$			→	$\frac{5}{6}$

3

	-	$\frac{1}{5}$	→	$\frac{7}{15}$
-		×		
	+		→	$1\frac{1}{2}$
↓		↓		
	+	$\frac{1}{20}$	→	

4

	-	$\frac{1}{5}$	→	$\frac{1}{20}$
×		÷		
2	+		→	
↓		↓		
	×	$\frac{4}{5}$	→	

6

	×	$\frac{1}{3}$	→	$\frac{1}{8}$
		+		
$\frac{1}{4}$			→	$\frac{11}{12}$
↓		↓		
$\frac{5}{8}$	-		→	$\frac{1}{8}$

# Resources

## Tasks

- [https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow\\_handsworth\\_bham\\_sch\\_uk/ERMR4Qpxi5IJtFVWpPE2F74Bc6JX5mVaWwLPR1WLh6Oq2Q?e=oXqEar](https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow_handsworth_bham_sch_uk/ERMR4Qpxi5IJtFVWpPE2F74Bc6JX5mVaWwLPR1WLh6Oq2Q?e=oXqEar)

## Exercises

- Textbook 1.4 pg 18
- <https://corbettmaths.com/wp-content/uploads/2018/11/Fractions-Addition-2-pdf.pdf>
- <https://corbettmaths.com/wp-content/uploads/2018/12/Multiplying-Fractions-pdf.pdf>
- <https://corbettmaths.com/wp-content/uploads/2018/11/Dividing-Fractions-pdf.pdf>

## Extensions

- <https://diagnosticquestions.com/Quizzes/Go#46398>
- Simplify:

$$\frac{3}{12}$$

$$\frac{12x}{6x}$$

$$\frac{3x}{12}$$

$$\frac{12xy}{6x}$$

$$\frac{3}{12x}$$

$$\frac{6x}{12xy}$$

$$\frac{12x}{3}$$

$$\frac{10x}{12xy}$$

$$\frac{12x}{6}$$

$$\frac{10x}{12xy^2}$$

# Extra Notes

# Extra Notes



# Prime Factorisation, HCF and LCM

Every number bigger than one, is either prime or is made up of a product of prime numbers.

We can use factor trees to fully factorise any number into a unique product of prime factors e.g.:

① choose any obvious factor pair

② split every number again unless it's prime, if prime circle

③ write out in order

$280 = 2 \times 2 \times 2 \times 5 \times 7$   
 $= 2^3 \times 5 \times 7$

## HCF – Highest Common Factor      LCM – Lowest Common Multiple

You can find the HCF and LCM of numbers by either:

(i) Listing    or    (ii) Using prime factors, e.g.:

LCM of 8 and 14 :

(i) 8: 8, 16, 24, 32, 40, 48, 56, 64, ...  
 14: 14, 28, 42, 56  
 LCM(8, 14) = 56

(ii)  $8 = 2^3$      $14 = 2^1 \times 7^1$     ← hidden  $\times 1 = 7^0$   
 LCM(8, 14) =  $2^3 \times 7^1$   
 = 56

[For LCM pick largest]

HCF of 36 and 90

(i)  $36 = \begin{cases} 1 \times 36 \\ 2 \times 18 \\ 3 \times 12 \\ 4 \times 9 \end{cases}$      $90 = \begin{cases} 1 \times 90 \\ 2 \times 45 \\ 3 \times 30 \\ 4 \times 15 \\ 5 \times 18 \\ 10 \times 9 \end{cases}$   
 HCF(36, 90) = 18

(ii)  $36 = 2^2 \times 3^2$      $90 = 2^1 \times 3^2 \times 5^1$     ←  $5^0$   
 hcf(36, 90) =  $2^1 \times 3^2 \times 5^0$   
 = 18

[For HCF pick smallest]

## Worked Example

Find the HCF and LCM of:

$$2^2 \times 3^2 \times 5^2 \times 11$$

$$2^3 \times 3 \times 5^2 \times 7$$

## Your Turn

Find the HCF and LCM of:

$$2 \times 3^3 \times 5 \times 7^2$$

$$2^2 \times 3^2 \times 7^2 \times 11$$

# Questions

Find the HCF and LCM of:

1)  $2^2 \times 3^3$   
 $2^3 \times 3^2$

2)  $2^2 \times 3^3$   
 $2^3 \times 3^3$

3)  $2^2 \times 3^3 \times 5$   
 $2^3 \times 3^3$

4)  $2^2 \times 3^3 \times 5$   
 $2^3 \times 3^3 \times 7$

5)  $2^2 \times 3^3 \times 5$   
 $2^3 \times 3^3 \times 5^2 \times 7$

6)  $2^2 \times 3^3 \times 5 \times 7$   
 $2^3 \times 3^3 \times 5^2$

7)  $2^2 \times 3^3 \times 5 \times 7$   
 $2^3 \times 3^3 \times 5^2 \times 11$

8)  $2^2 \times 5$   
 $3^3 \times 11$

9)  $2^2 \times 5$   
 $2^2 \times 5$

# Questions

Using the digits 1 to 9 only once fill in the following boxes to form two 3 digit numbers:


Find the HCF and LCM of the two numbers above.





## Worked Example

HCF	LCM	Number $a$	Number $b$
7	42	14	

## Your Turn

HCF	LCM	Number $a$	Number $b$
11	66	22	

# Questions

HCF	LCM	Number $a$	Number $b$
2	30		10
3	45	9	
		10	15
3	30		15
		12	20
6	60	12	
10	60	60	
1	60	T	15
8	120	24	
4	120	24	
4	120	40	
12	120		60
30	120	30	
1	120	15	
5	120	15	
120	120		



## Worked Example

The HCF of two numbers is 5.  
The LCM of two numbers is a multiple of 12. Write down two possible numbers.

## Your Turn

The HCF of two numbers is 8.  
The LCM of two numbers is a multiple of 5. Write down two possible numbers.



# SSDD Problem

Find the lowest common multiple and the highest common factor for:

120 and 432

I think of two numbers, one is 140.

Their highest common factor is 20.

Their lowest common multiple is 420.

What is my other number?

## Factors and multiples!

The lowest common multiple of two numbers is 420.

Their sum is 144.

Find their highest common factor.

I think of two numbers

One is  $8x^2y^2$ .

Their highest common factor is  $4x^2y$ .

Their lowest common multiple is  $24x^5y^2$

What is my other number?















# Resources

## Exercises

- Textbook ex 13 pg 15
- <https://corbettmaths.com/wp-content/uploads/2013/02/product-of-primes-pdf3.pdf>
- <https://corbettmaths.com/wp-content/uploads/2013/02/lcm-and-hcf-using-product-of-primes-pdf.pdf>

## Extensions

- <https://diagnosticquestions.com/Quizzes/Go#46392>

# Extra Notes

# Extra Notes

## Standard Index Form (SIF)

Standard form is written in the form of  $a \times 10^n$ , where  $a$  is a number bigger than or equal to 1 and less than 10.  $n$  can be any positive or negative whole number.

In Standard Form	Not in Standard Form
$7.3 \times 10^3$	438,000
$1 \times 10^{-3}$	$54 \times 10^7$
$9.36 \times 10^{18}$	$0.6 \times 10^{-4}$
$4 \times 10^1$	$389 \times 10000$
$5.002 \times 10^{-7}$	$6 \times 10^{1.5}$
	0.000372

Why use standard form?

- It allows us to write really small or really big numbers concisely.
- It allows us to easily compare small and big numbers.

# Questions

Decide if the following numbers are in standard form:

$3 \times 10^5$

$3 \div 10^5$

$3 \times 10^6$

$3 + 10^5$

$3 \times 10^{67}$

$3 - 10^5$

$3 \times 10^{6.7}$

$4 \times 10^5$

$3 \times 10^{0.67}$

$40 \times 10^5$

$3 \times 10^{0.7}$

$46 \times 10^5$

$3 \times 10^7$

$4.6 \times 10^5$

$3 \times 10^{-7}$

$0.46 \times 10^5$

$3 \times 10^{-0.7}$

$3.46 \times 10^5$

$3 \times 11^5$

$3.46434561 \times 10^5$

$3 \times 100^5$

$-3.46434561 \times 10^5$

$3 \times 10.5^5$

$3 \times -10^5$

$3 \times (-10)^5$

## Converting Numbers to Standard Form

- For the first number, keep dividing or multiplying by 10 until you get a number between 1 and 9.9.
- For the power of 10, count how many times the decimal place moved leftwards or rightwards.

## Worked Example

Write the following numbers in standard form:

70,000

72,000

720,000

## Your Turn

Write the following numbers in standard form:

60,000

63,000

630,000



# Questions

Write the following numbers in standard form:

1) 200

10) 19,947

2) 2,000

11) 10,000

3) 20,000

12) 10,100

4) 29,000

13) 10,010

5) 29,400

14) 1,001,000,000

6) 29,470

15)  $10.01 \times 10^8$

7) 294,700

16)  $100.1 \times 10^7$

8) 994,700

9) 1,994,700

## Worked Example

Write the following numbers in standard form:

0.05

0.005

0.00572

## Your Turn

Write the following numbers in standard form:

0.06

0.006

0.00683

# Questions

Write the following numbers in standard form:

1) 0.2

10) 0.0010

2) 0.02

11) 0.0090

3) 0.002

12) 0.00000090

4) 0.0023

13) 0.00000099

5) 0.00239

14) 0.00000199

6) 0.002039

15)  $0.199 \times 10^{-5}$

7) 0.0020309

16)  $0.0199 \times 10^{-4}$

8) 0.0010309

9) 0.001



## Converting Numbers from Standard Form

- Recall that the index of the 10 tells us how many times we are multiplying by 10 (or if negative, dividing by 10). Therefore count the number of decimal place jumps, **adding 0's if necessary**.
- Remember that we use negative powers for small numbers (numbers less than 1), positive powers for large numbers (numbers bigger than or equal to 1).

## Worked Example

Write the following as an ordinary number:

$$3.1 \times 10^6$$

## Your Turn

Write the following as an ordinary number:

$$3.2 \times 10^7$$

# Questions

Write the following as ordinary numbers:

1)  $3.2 \times 10^3$

2)  $3.3 \times 10^3$

3)  $3.37 \times 10^3$

4)  $3.37 \times 10^6$

5)  $3.378 \times 10^6$

6)  $1.3378 \times 10^6$

7)  $1.3378 \times 10^9$

8)  $1.03378 \times 10^9$

9)  $9 \times 10^{11}$

10)  $9.003378 \times 10^{11}$





## Worked Example

Write the following as an ordinary number:

$$4.1 \times 10^{-6}$$

## Your Turn

Write the following as an ordinary number:

$$4.2 \times 10^{-7}$$

# Questions

Write the following as ordinary numbers:

1)  $4.2 \times 10^{-3}$

2)  $4.3 \times 10^{-3}$

3)  $4.37 \times 10^{-3}$

4)  $4.37 \times 10^{-6}$

5)  $4.378 \times 10^{-6}$

6)  $4.2378 \times 10^{-6}$

7)  $4.2378 \times 10^{-9}$

8)  $4.02378 \times 10^{-9}$

9)  $4 \times 10^{-11}$

10)  $4.002378 \times 10^{-11}$



# Multiplying and Dividing in Standard Form

To multiply  $(a \times 10^n) \times (b \times 10^m)$  :

All the four things are being multiplied, so we can multiply in any order!

- Multiply  $a \times b$
- Multiply  $10^m \times 10^n$  (add the powers)
- Make sure the answer is in standard form

Division works in the same way.

## Worked Example

Work out:

$$(3 \times 10^5) \times (2 \times 10^4)$$

$$(3 \times 10^5) \times (4 \times 10^{-4})$$

## Your Turn

Work out:

$$(3 \times 10^5) \times (4 \times 10^4)$$

$$(3 \times 10^{-5}) \times (2 \times 10^4)$$

# Questions

Work out:

1)  $(4 \times 10^5) \times (2 \times 10^4)$

2)  $(2 \times 10^4) \times (4 \times 10^5)$

3)  $(4 \times 10^5) \times (4 \times 10^4)$

4)  $(2 \times 10^5) \times (8 \times 10^4)$

5)  $(8 \times 10^5) \times (2 \times 10^4)$

6)  $(8.1 \times 10^5) \times (2 \times 10^4)$

7)  $(8.01 \times 10^5) \times (2 \times 10^4)$

8)  $(2 \times 10^5) \times (8.01 \times 10^4)$

9)  $(2 \times 10^5) \times (8.01 \times 10^{-4})$

10)  $(2 \times 10^{-5}) \times (8.01 \times 10^4)$

11)  $(2 \times 10^{-5}) \times (8.01 \times 10^{-4})$



## Worked Example

Work out:

$$(4 \times 10^9) \div (2 \times 10^3)$$

$$(8 \times 10^5) \div (2 \times 10^{-4})$$

## Your Turn

Work out:

$$(2 \times 10^9) \div (4 \times 10^3)$$

$$(2 \times 10^5) \div (8 \times 10^{-4})$$



# Questions

Work out:

1)  $(9 \times 10^6) \div (3 \times 10^2)$

2)  $(6 \times 10^6) \div (3 \times 10^2)$

3)  $(3 \times 10^6) \div (6 \times 10^2)$

4)  $(3 \times 10^2) \div (6 \times 10^6)$

5)  $(3 \times 10^{-2}) \div (6 \times 10^6)$

6)  $(3 \times 10^6) \div (6 \times 10^{-2})$

7)  $(3 \times 10^6) \div (1.5 \times 10^2)$

8)  $(1.5 \times 10^6) \div (3 \times 10^2)$

9)  $(1.5 \times 10^{-6}) \div (3 \times 10^2)$

10)  $(1.5 \times 10^6) \div (6 \times 10^{-2})$

11)  $(1.5 \times 10^{-6}) \div (6 \times 10^{-2})$

12)  $(6 \times 10^{-6}) \div (1.5 \times 10^{-2})$





# Calculator

Use the  $\times 10^x$  button on your calculator to make calculations involving standard form. While you can explicitly write  $3 \times 10^7$  using the  $x^y$  button, it is faster to use the specialised standard form key.

Check the following using your calculator:

$$(2.41 \times 10^{19}) \times (7.1 \times 10^{23}) = \mathbf{1.7111 \times 10^{43}}$$





# Adding and Subtracting in Standard Form

If the powers are not the same, either:

- Convert both numbers to normal numbers first, then add or subtract, then convert back to standard form.
- Or better, change the number with the smaller power of 10 so it matches the power of the larger one.

## Worked Example

Work out:

$$(3 \times 10^4) + (4 \times 10^4)$$

$$(3 \times 10^4) + (8 \times 10^4)$$

$$(3 \times 10^5) + (8 \times 10^4)$$

## Your Turn

Work out:

$$(3 \times 10^7) + (2 \times 10^7)$$

$$(3 \times 10^7) + (9 \times 10^7)$$

$$(3 \times 10^8) + (9 \times 10^7)$$



# Questions

Work out:

1)  $(5 \times 10^4) + (4 \times 10^4)$

2)  $(15 \times 10^4) + (4 \times 10^4)$

3)  $(150 \times 10^4) + (4 \times 10^4)$

4)  $(5 \times 10^3) + (4 \times 10^4)$

5)  $(5 \times 10^2) + (4 \times 10^4)$

6)  $(5 \times 10^4) + (4 \times 10^3)$

7)  $(30 \times 10^3) + (4 \times 10^4)$

8)  $(3 \times 10^4) + (40 \times 10^3)$

9)  $(30 \times 10^3) + (40 \times 10^3)$

10)  $(0.3 \times 10^2) + (4 \times 10^4)$

11)  $(35 \times 10^{-2}) + (4.5 \times 10^4)$



## Worked Example

Work out:

$$(7 \times 10^4) - (4 \times 10^4)$$

$$(7 \times 10^4) - (0.4 \times 10^4)$$

$$(7 \times 10^5) - (0.4 \times 10^4)$$

## Your Turn

Work out:

$$(6 \times 10^7) - (2 \times 10^7)$$

$$(6 \times 10^7) - (0.2 \times 10^7)$$

$$(6 \times 10^7) - (0.2 \times 10^8)$$

# Questions

Work out:

1)  $(5 \times 10^4) - (4 \times 10^4)$

2)  $(15 \times 10^4) - (5.1 \times 10^4)$

3)  $(15 \times 10^4) - (3 \times 10^4)$

4)  $(5 \times 10^{-4}) - (4 \times 10^{-4})$

5)  $(5 \times 10^5) - (4 \times 10^4)$

6)  $(50 \times 10^4) - (4 \times 10^5)$

7)  $(30 \times 10^{-3}) - (4 \times 10^{-4})$

8)  $136,000 - (40 \times 10^3)$

9)  $(0.045 \times 10^4) - (35 \times 10^{-2})$

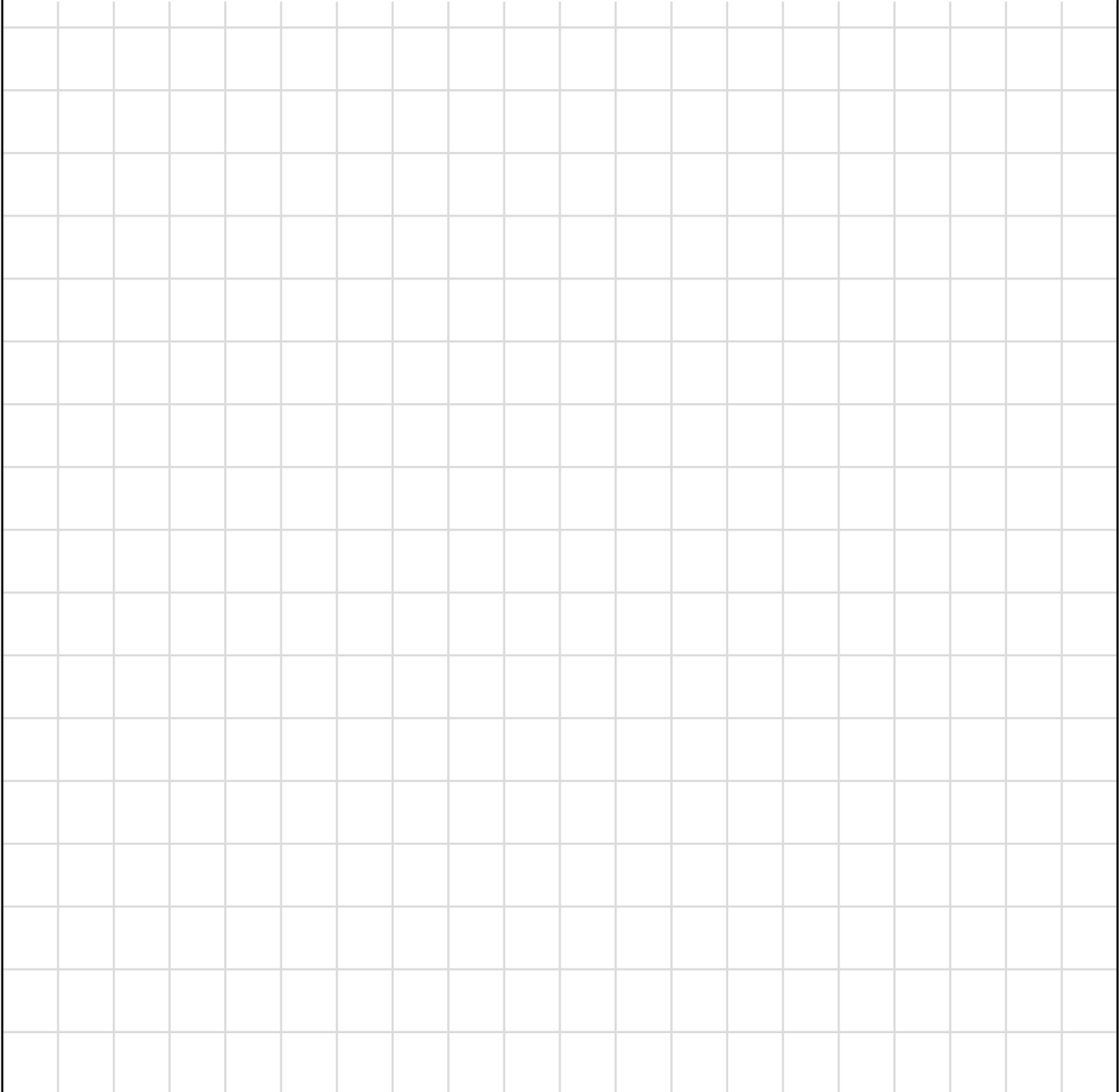
10)  $(1,360 \times 10^{-2}) - (0.004 \times 10^3)$



## Goal Free Problem

Rob is learning about the planets. Rob makes a model of the sun. He also makes a model of the planet Jupiter. Rob is going to hang the two models in the school hall. Rob wants a distance of 16 m between the two models. The real distance between the planet Jupiter and the sun is  $8 \times 10^8$  km.

Work out what you can from this information.



# Resources

## Exercises

- Student Textbook 1.6 pg 14
- <https://corbettmaths.com/wp-content/uploads/2019/10/Standard-Form-Textbook.pdf>

# Extra Notes



# Extra Notes

# Types of Number

TYPE	Definition	Examples
Integer	Whole number	-5, 0, 1, $\sqrt{4}$ , $10^6$
Rational	Can be written as a fraction	0, 0.32, $0.\dot{3}$ , $0.\dot{2}1\dot{7}$
Irrational	Can't be written as a fraction	$\sqrt{2}$ , $3\sqrt{2}$ , $\pi$ , $\frac{\pi}{3}$
Negative	Less than zero	-21, $-\pi$ , $-\sqrt{2}$
Multiple	In a number's times table	Of 3: 3, 6, 15, 42
Factor	Divides into a number	Of 10, 1, 2, 5, 10
Prime	Only factors are 1 and itself	2, 3, 5, 7, 11, ( <i>not 1!</i> )

Which of the equations have rational solutions?

$2x = 4$

$4x = 2$

$x^2 = 4$

$x^2 = 2$

$2x = 5$

$2x = 4.5$

$2x = \pi$

$2x^2 = 4$

Investigate whether the statements are always, sometimes or never true.  
Give examples and/or counterexamples to justify your answers.

- Integers are rational numbers
- Rational numbers are integers
- Rational numbers are real numbers
- Terminating decimals are rational numbers
- Recurring decimals are rational numbers
- The square roots of negative numbers are real, but not rational

Which of these cards have integer values?

$\sqrt{25}$

$\sqrt{50}$

$\sqrt{125}$

$\sqrt[3]{125}$

$\sqrt{9 \times 4}$

$\sqrt{2 \times 32}$

$\sqrt{12 \times 4}$

$\sqrt[3]{9 \times 3}$

# Surds

When the root (square root, cube root or higher root) of a number cannot be obtained exactly, the root is called a surd. A surd cannot be written as a fraction but can be written as a decimal, that goes on forever, without repeating (recurring) or ending (terminating). Hence, surds are irrational numbers.

Surds	Not Surds
$\sqrt{8}$	8
$\sqrt{10}$	-12.05
$\sqrt{91}$	0.62
$\sqrt[3]{7}$	$\frac{3}{7}$
$\sqrt[3]{16}$	$7\frac{1}{2}$
$\sqrt[4]{73}$	$\sqrt{16}$
$2\sqrt{2}$	$\sqrt{25}$
$2 + \sqrt{5}$	$\sqrt[3]{8}$
$3 - \sqrt{5}$	$\sqrt{2.25}$
$\frac{\sqrt{17}}{5}$	$\frac{\sqrt{100}}{\sqrt{4}}$

## Is this a surd?

$\sqrt{1}$

$\frac{1}{(\sqrt{5})^2}$

$\sqrt{4}$

$\sqrt{9}$

$\frac{\sqrt{1}}{\sqrt{4}}$

$\sqrt{36}$

$\sqrt{6}$

$\sqrt{\frac{1}{4}}$

$\sqrt{24}$

$\sqrt{3}$

$\sqrt{\frac{2}{8}}$

$2\sqrt{3}$

$3\sqrt{3}$

$\sqrt{\frac{2}{9}}$

$3\sqrt{4}$

$\sqrt{5}$

$\sqrt{\frac{4}{9}}$

$\sqrt{5^2}$

$\frac{2}{\sqrt{9}}$

$\frac{\sqrt{7}}{2}$

## Simplifying Surds

To simplify  $\sqrt{x}$  :

Write down a list of all the square numbers up to  $x$ .

1. Find the biggest square number which is a factor of  $x$ . Write  $x$  as a product using this square number.
2. Now write  $\sqrt{x}$  as this product.
3. Use the fact  $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$  to split the product.
4. Simplify your answer.

## Worked Example

Simplify:

$$\sqrt{60}$$

$$\sqrt{120}$$

## Your Turn

Simplify:

$$\sqrt{50}$$

$$\sqrt{200}$$

# Questions

Simplify:

1)  $\sqrt{8}$

10)  $\sqrt{90}$

2)  $\sqrt{12}$

11)  $\sqrt{160}$

3)  $\sqrt{32}$

12)  $\sqrt{1600}$

4)  $\sqrt{64}$

13)  $\sqrt{200}$

5)  $\sqrt{128}$

14)  $\sqrt{250}$

6)  $\sqrt{192}$

15)  $\sqrt{175}$

7)  $\sqrt{320}$

16)  $\sqrt{350}$

8)  $\sqrt{80}$

17)  $\sqrt{351}$

9)  $\sqrt{40}$

18)  $\sqrt{353}$

## Worked Example

Simplify:

$$2\sqrt{20}$$

$$4\sqrt{40}$$

## Your Turn

Simplify:

$$3\sqrt{20}$$

$$4\sqrt{50}$$



# Questions

Simplify:

1)  $\sqrt{8}$

2)  $2\sqrt{8}$

3)  $3\sqrt{8}$

4)  $\sqrt{72}$

5)  $5\sqrt{72}$

6)  $5\sqrt{144}$

7)  $5\sqrt{1440}$

8)  $5\sqrt{720}$

9)  $5\sqrt{360}$

10)  $5\sqrt{36}$

11)  $6\sqrt{36}$

12)  $7\sqrt{49}$

13)  $7\sqrt{98}$

14)  $7\sqrt{147}$

15)  $14\sqrt{147}$

16)  $7\sqrt{294}$

17)  $7\sqrt{295}$

18)  $7\sqrt{2950}$

## Worked Example

Write the following as a single root:

$$2\sqrt{15}$$

$$2\sqrt{30}$$

## Your Turn

Write the following as a single root:

$$5\sqrt{2}$$

$$10\sqrt{2}$$

# Questions

Write the following as a single root:

1)  $2\sqrt{2}$

2)  $3\sqrt{2}$

3)  $2\sqrt{3}$

4)  $4\sqrt{6}$

5)  $5\sqrt{7}$

6)  $5\sqrt{8}$

7)  $5\sqrt{9}$

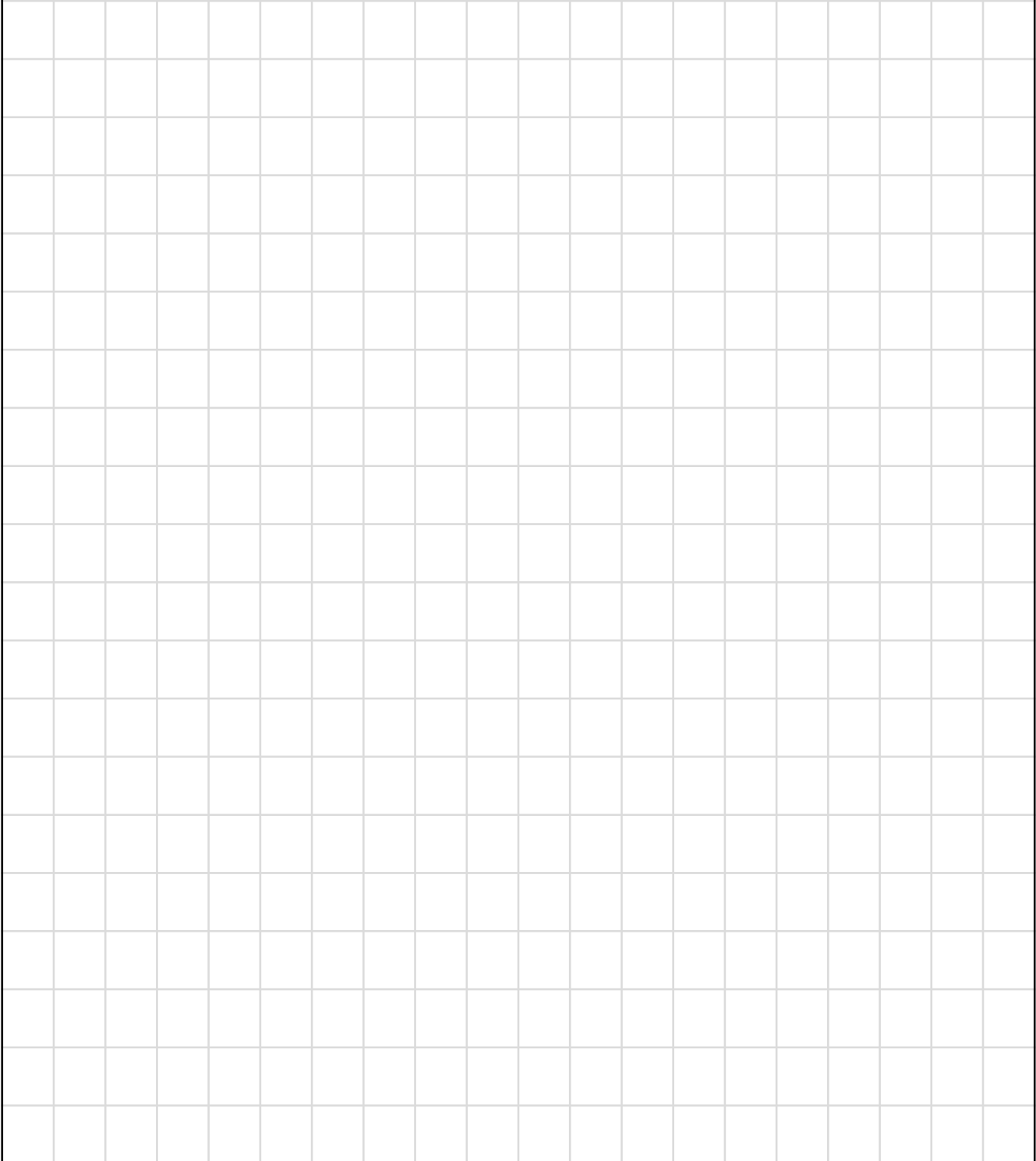
8) 15

9)  $15^2$

## Exam Question

Write  $\sqrt{8}$  in the form  $m\sqrt{2}$ , where  $m$  is an integer.

2 marks













# Resources

## Exercises

- Pearson Student Textbook 1.7 pg 17
- <https://corbettmaths.com/wp-content/uploads/2013/02/surds-pdf1.pdf>

# Extra Notes

# Extra Notes

# 1.2 Percentages

## Percentages Prerequisite Knowledge Checks

### Fraction, Decimal, Percentages Equivalence

Website: <https://diagnosticquestions.com/Quizzes/Go#172798>

PDF: [https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow\\_handsworth\\_bham\\_sc\\_h\\_uk/ETdXXmMFFaxNp1GGLgS-U7cB9aiKnVaYcxCGDmqCHKevRQ?e=dfzhKO](https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow_handsworth_bham_sc_h_uk/ETdXXmMFFaxNp1GGLgS-U7cB9aiKnVaYcxCGDmqCHKevRQ?e=dfzhKO)

### Percentages of Amounts

Website: <https://diagnosticquestions.com/Quizzes/Go#172791>

PDF: [https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow\\_handsworth\\_bham\\_sc\\_h\\_uk/EXfKs\\_NsqFIPsxmj\\_zMTsBIBIDc\\_KADKgSwmWOXSMussqQ?e=JJB1Gb](https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow_handsworth_bham_sc_h_uk/EXfKs_NsqFIPsxmj_zMTsBIBIDc_KADKgSwmWOXSMussqQ?e=JJB1Gb)

## Worked Example

## Your Turn

To find 20% ...

To find 30% ...

To increase by 20% ...

To increase by 30% ...

To decrease by 20% ...

To decrease by 30% ...

## Worked Example

## Your Turn

To find 12.5% ...

To find 0.5% ...

To increase by 12.5% ...

To increase by 0.5% ...

To decrease by 12.5% ...

To decrease by 0.5% ...

Percentage	To find	To increase by	To decrease by
40%			
50%			
60%			
6%			
7%			
8%			
18%			
28%			
48%			
88%			
98%			
108%			
118%			
218%			

Percentage	To find	To increase by	To decrease by
21.8%			
2.18%			
2.08%			
0.08%			
	$\times 0.15$		
		$\times 1.25$	
			$\times 0.76$
			$\times 0.66$
	$\times 0.66$		
		$\times 2.66$	
		$\times 1.06$	
			$\times 0.994$
	$\times 0.606$		



## Worked Example

Original Amount: 40  
Percentage: 24%

As a fraction:

Multiplier:

Percentage of...:

Increased by...:

Decreased by...:

## Your Turn

Original Amount: 40  
Percentage: 72%

As a fraction:

Multiplier:

Percentage of...:

Increased by...:

Decreased by...:

## Worked Example

Original Amount: £120  
Percentage: 36%

As a fraction:

Multiplier:

Percentage of...:

Increased by...:

Decreased by...:

## Your Turn

Original Amount: £350  
Percentage: 55%

As a fraction:

Multiplier:

Percentage of...:

Increased by...:

Decreased by...:

	Original Amount	Percentage	As a fraction	Multiplier	Percentage of...	Increased by...	Decreased by....
1.	60	20%					
2.	60		$\frac{3}{10}$				
3.	60			0.25			
4.		25%			7.5		
5.			$\frac{1}{40}$			30.75	29.25
6.	30				6.75		
7.				0.225	67.5		

Original Amount	Percentage	As a fraction	Multiplier	Percentage of...	Increased by...	Decreased by...
8. 300		$\frac{41}{200}$				
9. 60					72.3	47.7
10.		$\frac{41}{40}$		61.5		
11. 60			1.125			
12. 6				0.675		
13. 6					24.675	
14. 6						-31.35

# Resources

## Exercises

- Pearson Higher Student Book 4.4 pg 105 (Mixed Problems incl. FDP in section 4.5)
- <https://corbettmaths.com/wp-content/uploads/2013/02/multipliers-pdf.pdf>
- <https://corbettmaths.com/wp-content/uploads/2013/02/percentage-of-an-amount-calculator-pdf.pdf>
- CIMT: 11.1 – 11.5 of: [https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow\\_bham\\_sch\\_uk/EWINuTeVHq1GtzwP04Slwq4BE2V87zmUx-uTenVuPvKRIA?e=jHapCm](https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow_bham_sch_uk/EWINuTeVHq1GtzwP04Slwq4BE2V87zmUx-uTenVuPvKRIA?e=jHapCm) (answers: <https://www.cimt.org.uk/projects/mepres/allgcse/bs11pba.pdf> )

Calculate the percentage increase from £2500 to £3100.

Jack used to get paid £1770 per month. Now he gets £25,000 per year. What is the percentage increase of his wage?

A rectangular field has been made smaller. The old dimensions were 50m by 28m. The new dimensions are 44m by 25m. Calculate the percentage change of the size of this field.

Sam and Luke share a flat. They split the bill into a ratio of 6:4.

Sam used to pay £300 per month. Luke's new monthly payment is £248.

Work out the percentage increase of the old price to the new.

## Extension



$x\%$  of  $x\%$  of  $y$  is 9. Write  $x$  in terms of  $y$ .

$x =$  



[JMC 2012 Q24] After playing 500 games, my success rate in Spider Solitaire is 49%. Assuming I win every game from now on, how many extra games do I need to play in order that my success rate increases to 50%?

  extra games



[JMO 2006 A5]

A balloon seller starts the day with a certain number of balloons. He then sells one third of his balloons to boys, 20% to girls, and three times the difference between these amounts to adults. At the end of the day, he has eight balloons left. How many balloons did the seller have at the start?





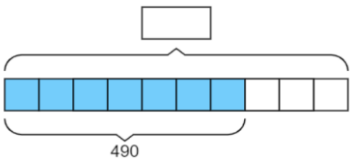
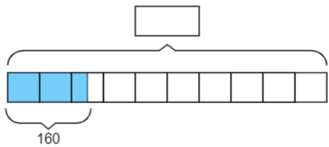
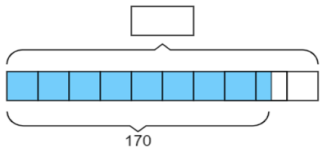
[SMC 2007 Q9] In a sale, a shopkeeper reduced the advertised selling price of a dress by 20%. This resulted in a profit of 4% over the cost price of the dress. What percentage profit would the shopkeeper have made if the dress had then been sold at the original selling price?

  %

# Reverse Percentages – Non Calculator

Question	Diagram	Working	Answer
90% of a number is 180. What is the number?		$180 \div 9 = 20$ $20 \times 10 = 200$	200
90% of a number is 450. What is the number?		$450 \div 9 = \underline{\quad}$ $\underline{\quad} \times 10 = \underline{\quad}$	
90% of a number is 72. What is the number?			
80% of a number is 720. What is the number?			
80% of a number is 120. What is the number?			
60% of a number is 120. What is the number?			
20% of a number is 120. What is the number?			

# Reverse Percentages – Non Calculator

Question	Diagram	Working	Answer
70% of a number is 490. What is the number?			
70% of a number is 245. What is the number?			
50% of a number is 160. What is the number?			
25% of a number is 160. What is the number?			
85% of a number is 170. What is the number?			
____ % of a number is 120. What is the number?			150
____ % of a number is 120. What is the number?			400



# Reverse Percentages – Calculator

## Reverse Percentages

Increasingly  
Difficult  
Exercises

Find 100% when...

- |                         |                                 |                                 |
|-------------------------|---------------------------------|---------------------------------|
| a)<br>15 represents 50% | b)<br>12 represents 10%         | c)<br>8 represents 20%          |
| d)<br>6 represents 5%   | e)<br>15% of an amount<br>is 36 | f)<br>35% of an amount<br>is 21 |

## Reverse Percentages – Calculator

g)

A coat is reduced by 15% to £68

h)

A top is reduced by 6% to £47

i)

A sofa is reduced by 17% to £1,162

j)

John gets a raise of 10% to £7.48/hr

k)

A house depreciates by 0.3% to £249,250

l)

Population increases by 0.04% to 718,262

## Worked Example

(CALCULATOR)

## Your Turn

Calculate the original amount:

Percentage change: 10% decrease

New value: £360

Calculate the original amount:

Percentage change: 25% decrease

New value: £150

Percentage change: 10% increase

New value: £440

Percentage change: 25% increase

New value: £250

# Questions

% change: 10% decrease

New value: £36

% change: 20% decrease

New value: £32

% change: 10% decrease

New value: £18

% change: 10% decrease

New value: £180

% change: 5% decrease

New value: £190

% change: 5% decrease

New value: £19

# Questions

% change: 10% increase

New value: £44

% change: 10% increase

New value: £88

% change: 20% increase

New value: £960

% change: 5% increase

New value: £84

% change: 1% increase

New value: £808

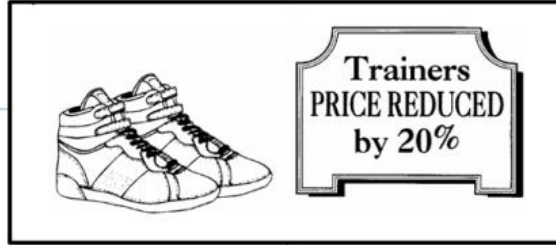
% change: 5% increase

New value: £840

# SSDD

How much would you pay for a pair of trainers priced at £56.50?

I paid £48.80 for my trainers. What was the original price?



Another shop is offering  $\frac{1}{4}$  off the price. Which is the better offer? Convince me.

You have £500 to spend. How many pairs of trainers, costing £96 before the reduction can you buy?

A company announces a 2.41% pay rise. Arthur is currently paid £23.74 per hour. What is his new salary?

A company announces a 2.41% pay rise. Merlin's new salary is £23.74 per hour. What was his salary before the increase?

A company announces a 2.41% pay rise. Lancelot's salary has increased by £23.74. How much was he paid before?

A company announces a 2.41% pay rise. Percival was paid £23.74 per hour. His new salary is £25 per hour. Did he get the same pay rise as everyone else?

# Resources

## Exercises

- <https://corbettmaths.com/wp-content/uploads/2019/10/Reverse-Percentages.pdf>
- <https://corbettmaths.com/wp-content/uploads/2013/02/reverse-percentages-pdf.pdf>
- Textbook pg 55 ex 8: [https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow\\_handsworth\\_bham\\_sch\\_uk/Ec7Dr337dVRDuSVzVuIdypUBeEosDbxsT\\_4oRnWklWtmWw?e=LMBQ1s](https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow_handsworth_bham_sch_uk/Ec7Dr337dVRDuSVzVuIdypUBeEosDbxsT_4oRnWklWtmWw?e=LMBQ1s)
- CIMT pg 1666

## Extensions

- <https://diagnosticquestions.com/Quizzes/Go#46411>  
PDF: [https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow\\_handsworth\\_bham\\_sch\\_uk/Ef1ELua1g\\_pHouR\\_vCJbyKUBuemI5k0N9IsoXwZot2PC6Q?e=qEROoj](https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow_handsworth_bham_sch_uk/Ef1ELua1g_pHouR_vCJbyKUBuemI5k0N9IsoXwZot2PC6Q?e=qEROoj)

# Extra Notes



# Extra Notes



# Repeated Change

1 <sup>st</sup> percentage change	1 <sup>st</sup> percentage multiplier	2 <sup>nd</sup> percentage change	2 <sup>nd</sup> percentage multiplier	Overall percentage change	Overall percentage multiplier
30% increase	× 1.3	15% increase	× 1.15	49.5% increase	× 1.495
15% increase		30% increase			
20% increase		25% increase			
5% increase		40% increase			
7.5% increase			× 1.375		
	× 1.06		× 1.39		× 1.68
	× 1.2				
		50% increase		68% increase	
10% decrease		10% decrease			
20% decrease		20% decrease			
30% decrease		30% decrease			
30% decrease		30% increase			
30% increase		10% decrease			
	× 0.85				× 1.19
			× 0.92	35.6% decrease	

# Questions

- 1) What is £320 after:
  - a) An increase of 21%
  - b) An increase of 21 % followed by an increase of 18%
  - c) An increase of 21 % followed by an increase of 23.2%
  - d) An increase of 21 % followed by an increase of 21%
  - e) An increase of 21 % followed by an **decrease** of 21%
  
- 2) What is 2.5 Kg after:
  - a) An increase of 35%
  - b) An increase of 35 % followed by an increase of 62%
  - c) An increase of 35 % followed by an increase of 6.2%
  - d) An increase of 35 % followed by an increase of 35%
  - e) An increase of 35 % followed by an **decrease** of 62%

## Extension

- 1) If you increase a number of 10% then decrease by 10%, what is the overall percentage change?
  
- 2) If you increase by  $x$  % twice what is the overall percentage increase?
  
- 3) What is you do the above but with decreasing?

# Questions

## Section 2: Reasoning and Problem Solving

- 1) Tarquin says “the order of percent change matters” do you agree? Explain your answer.
- 2) The sum of the percentage increases on the first five row is 45%. If you increase an amount by two percentages which add 45%, what is the biggest possible amount the original value it can be increase by?
- 3) Two percentages sum to 45%, one of the is a decrease and one is an increase. What is the biggest possible amount the original value can be increase by.
- 4) Rows 9, 10 and 11 all have a pattern cane you explain what is going on here?

## Section 3: Worded Problems

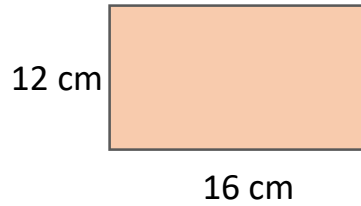
- 1) A shop decreases all its prices by 15%. A week later the shop increases all prices by 17.5%. What is the overall change?
- 2) Amy is given a 7% pay rise. The next year Amy is given another 4.5% pay rise. Her manager says that Amy’s pay has increased by 11.5% overall. Explain why Amy’s manager is wrong.
- 3) Paul is building a wardrobe. The wood costs 30% more than he had estimated. He needs 40% more than he had estimated. How much more than his original estimate does the material for the wardrobe cost?
- 4) Create two word problems involving repeated percentage change of your own. Make one very difficult.

# Questions

## Section 4: Shapes

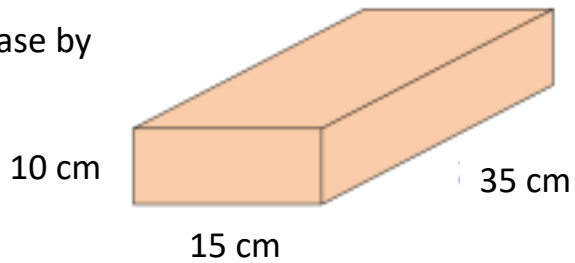
The sides lengths of this shape increase by 25%, what happens to the:

- a) Perimeter
- b) Area



The sides lengths of this shape increase by 20%, what happens to the:

- a) Surface area
- b) Volume



# Compound Interest

Question	Starting amount	Compound interest rate	Years	Calculation	End amount (from calculator, round to 2 dp)
<b>Example</b>	<b>£800</b>	<b>37%</b>	<b>2</b>	$800 \times 1.37^2$	<b>£1501.52</b>
A	£900	18%	5		
B	£4000	25%	6		
C	£350	9%	8		
D	£148	10%	10		
E	£100,000	2%	10		
F	£500	2.5%	50		

2. Challenge:

- In Question A in the table above, how much interest was earned altogether?
- If the compound interest rate is 4%, the money is invested for 2 years, and the end amount is £2163.20, what was the starting amount?



# Simple vs Compound Interest

# Worked Example

(CALCULATOR)

# Your Turn

## Example 1

You deposit £400 in to a bank account paying 5% simple interest per year. How much interest would you have earned after 3 years?

## Example 2

You deposit £345 in to a bank account paying 7% simple interest per year. How much interest would you earn after 5 years?

## Example 3

You take out a loan of £800 and the bank charges you 15% compound interest per year. If you don't pay off any of the loan in 4 years, how much would you owe the bank?

## Example 4

You invest £4000 in a fund which earns an 11% compound return per year. How much would the fund be worth after 10 years?

# Best Buys

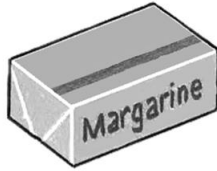
which is the best buy? (better value)



£1.25  
500g



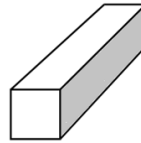
£2.48  
1kg



£4.98  
2kg

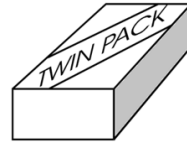
which is the best buy? (better value)

28p



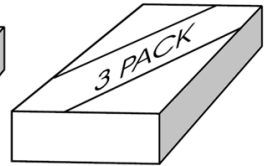
1 bar

52p



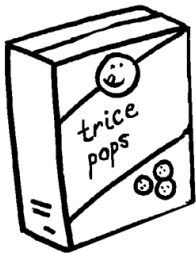
2 bars

81p

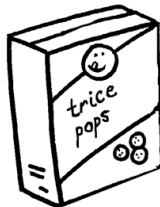


3 bars

which is the best buy? (better value)



500g  
£2.21



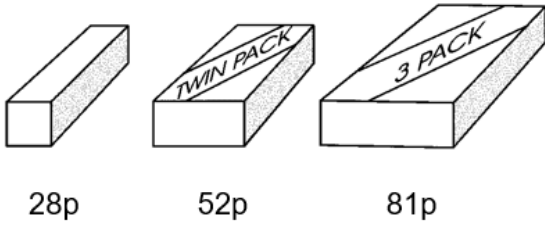
300g  
£1.32



200g  
87p

# Which is the Best Buy?

(1)

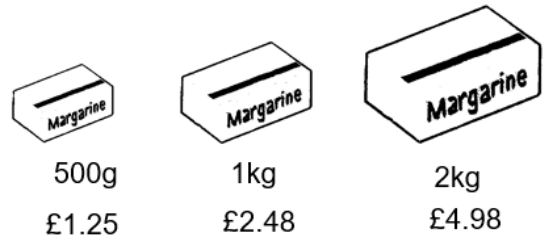


28p

52p

81p

(2)



500g

£1.25

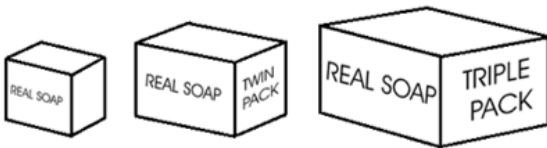
1kg

£2.48

2kg

£4.98

(3)

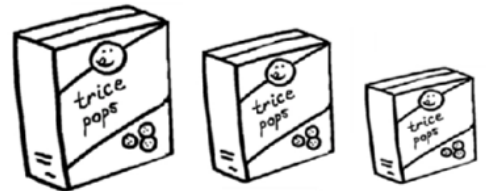


83p

£1.68

£2.50

(4)



500g

£2.21

300g

£1.32

200g

87p

# Which is the Best Buy?



£2.00  
80 tea bags



£3.55  
150 tea bags



£5.30  
220 tea bags

# Resources

## Exercises

- Unitary Method <https://corbettmaths.com/wp-content/uploads/2018/11/Unitary-Method-pdf.pdf>
- Compound Interest: <https://corbettmaths.com/wp-content/uploads/2019/09/Compound-Interest-pdf-1.pdf> and <https://corbettmaths.com/wp-content/uploads/2013/02/compound-interest-pdf1.pdf>
- Textbook pg 56 ex9
- CIMT pg 1662

## Extensions

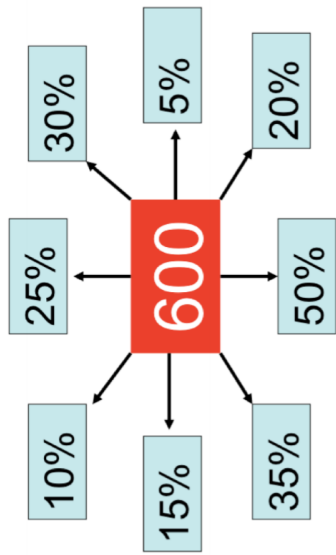
- [https://handsworthgrammar-my.sharepoint.com/:f:/g/personal/gdhillow\\_handsworth\\_bham\\_sch\\_uk/EIRWAVN8sgVFnbjKUAgEhcUBCOOr552yy4epOZSicAlbqHg?e=l3VqeB](https://handsworthgrammar-my.sharepoint.com/:f:/g/personal/gdhillow_handsworth_bham_sch_uk/EIRWAVN8sgVFnbjKUAgEhcUBCOOr552yy4epOZSicAlbqHg?e=l3VqeB)

# Extra Notes

# Extra Notes



### Finding percentages mentally



### Expressing change as a percentage

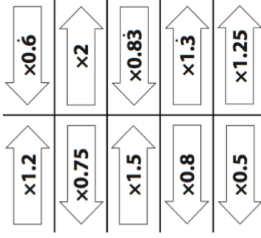
$$\frac{\text{NEW} - \text{OLD}}{\text{OLD}} \times 100$$

Laura used to get £2.50 pocket money. She now gets £3.00. What was the percentage increase?

\*your answer is a percentage

### Using multipliers for increases/decreases

- 100% is your starting point. As a decimal  $100\% = 1$
- A 20% increase means you now have 120%. As a decimal  $120\% = 1.20$ . This is the multiplier.
- A 2.5% decrease means you now have 97.5%. As a decimal  $97.5\% = 0.975$ . This is the multiplier.



### Great offers!

Write each of these as a percentage decrease

**50% off Buy One. Get One Free. 3 for 2**

**1/3 off 100% extra free Half price**

# %

### Compare the amount of working out:

Faraz pays £4000 into a saving account with a 5% interest rate. How much interest does he get after the first year?

- Basic method:
- Multiplier:

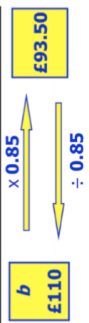
### Reverse percentages

The value of a painting went up by 25% and is now valued at £400.

What did it cost before?



An item costs £93.50 after a 15% discount. What did it cost before the discount?



- 1 The entry price to Paulton Towers is increased by 5% this year to £34.65. How much was entry last year? £
- 2 A restaurant adds on 8% service charge to a bill to make a total bill of £59.40. How much is the bill without the charge? £
- 3 The price of a pair of shoes is cut by 25% in a sale to £12.75. What was it before? £

### Converting between Fractions, Decimals and Percentages

Fraction	Decimal	Percentage
$\frac{3}{10}$		%
$\frac{1}{4}$	0.4	%
$\frac{3}{4}$	0.05	25%
		%
		%
		12.5%
	0.3	%

\*When ordering amounts given in different forms, convert them all into decimals

### Compound percentage increase

A man invests £2,000 in a bank for 5 years earning 8% interest per annum.

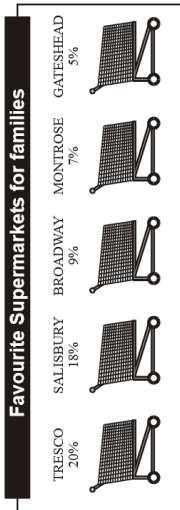
How much money will he have at the end of 5 years?

Starting amount £2000  
Interest is 8%, so multiplier is 1.08  
It's for 5 years, so you need  $1.08^5$   
So, to work it out you do:

$$2000 \times 1.08^5 = \text{£}2938.66$$

- 1 £8000 invested at 6% for 5 years £
- 2 £3800 invested at 8% for 7 years £
- 3 £3550 invested at 5% for 9 years £

In a survey, some families were asked to name their favourite supermarket. Some of the results are shown in the diagram.



200 families took part in the survey.

Work out the number of families whose favourite supermarket was Tesco.

.....

(Total 2 marks)

Loft insulation reduces annual heating costs by 20%.

After he insulated his loft, Curtley's annual heating cost was £520.

Work out Curtley's annual heating cost would have been, if he had not insulated his loft.

£ .....

(Total 3 marks)

5. A hotel has 56 guests.  
35 of the guests are male.

(a) Work out 35 out of 56 as a percentage.

..... %  
(2)

40% of the 35 male guests wear glasses.

(b) Write the number of male guests who wear glasses as a fraction of the 56 guests.  
Give your answer in its simplest form.

.....  
(4)

(Total 6 marks)

6. In a sale the normal price of a book is reduced by 10%.  
The sale price of the book is £4.86  
Calculate the normal price of the book.

£.....  
(Total 3 marks)

Ben bought a car for £12 000.



Each year the value of the car depreciated by 10%.

Work out the value of the car two years after he bought it.

£ .....

(Total 3 marks)

The table gives information about an estate agent's charges for selling a house.

Value of the house	Estate agent's charges
Up to £60 000	2% of the value of the house
Over £60 000	2% of the first £60 000 plus 1% of the remaining value of the house

The estate agent sold a house for £80 000.

Work out the total charge

£.....  
(Total 4 marks)

## Percentage Mixed Problems

<p><b>A1</b> A bank pays 2.5% interest on its current account. Write 2.5% as a decimal.</p>	<p><b>A2</b> Rosie took a science test and scored 41 marks out of 45. Express 41 out of 45 as a percentage.</p>
<p><b>B1</b> Ayesha plays hockey. Last year Ayesha scored 8 goals. This year Ayesha scored 13 goals. Calculate the percentage increase in for the number of goals scored.</p>	<p><b>B2</b> Between 2001 and 2011, the population of a town increased by 8% In 2001 the population was 34 342. Calculate the population in 2011.</p>
<p><b>C1</b> Rohan invested £3000 for 4 years in a savings account. He was paid 2.5% per annum compound interest. How much did Rohan have in his savings account after 4 years?</p>	<p><b>C2</b> Susanna invested £2000 for 3 years at 4% interest per annum compound interest. Work out the amount of interest Susanna had earned after 3 years.</p>
<p><b>D1</b> In a sale, normal prices were reduced by 25%. The sale price of a computer was £442. Work out the normal price of the computer.</p>	<p><b>D2</b> In a sale, all prices are reduced by 15%. The sale price of a shirt is £22.40. Work out the original price of the shirt.</p>

## Percentage Mixed Problems

**A3**

A school has 80 staff.  
15% of the staff wear glasses.  
Calculate the number of staff that wear glasses.

**A4**

56% of students in a school are girls.  
There are 420 girls in the school.  
Work out the total number of students in the school.

**B3**

In a sale, normal prices were reduced by 20%. The normal price of a camera was £180.  
Work out the sale price of the camera.

**B4**

Justin bought some clothes.  
The clothes should have cost £84.00 but he got a discount of 15%.  
Work out how much money Justin saved.

**C3**

Anya bought a car for £12 500.  
The car depreciates at a rate of 12% per year.  
Work out the value of the car after five years.

**C4**

The price of shoes was increased by 15%. However, customers were given a 20% discount if they bought two pairs at the same time.  
Work out the cost of two pairs of shoes that originally cost £68 each.

**D3**

The price of a new TV is £540, which includes 20% VAT.  
Find the cost of the TV excluding VAT.

**D4**

Natasha invested some money at 4% per annum compound interest.  
At the end of two years, the value of her investment was £3380.  
Find the amount of money that Natasha invested.

[https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow\\_handsworth\\_bham\\_sch\\_uk/EQAXUROzN7NNqhNKIoKmpmMBLCORtE-82enL6iAPbiMgjQ?e=ma4iYb](https://handsworthgrammar-my.sharepoint.com/:b:/g/personal/gdhillow_handsworth_bham_sch_uk/EQAXUROzN7NNqhNKIoKmpmMBLCORtE-82enL6iAPbiMgjQ?e=ma4iYb)

# Exam Questions



## Simple, Compound Interest, Depreciation, Growth & Decay (H)

A collection of 9-1 Maths GCSE Sample and Specimen questions from AQA, OCR, Pearson-Edexcel and WJEC Eduqas.

Name:	
Total Marks:	

1. (a) During an experiment, a scientist notices that the number of bacteria halves every second.

There were  $2.3 \times 10^{30}$  bacteria at the start of the experiment.

Calculate how many bacteria were left after 5 seconds.

Give your answer in standard form correct to two significant figures.

[3]

- (b) In a different experiment the number of bacteria is reduced by a quarter each second. On this occasion the number of bacteria initially was  $x$ .

Write a formula to calculate the number of bacteria,  $r$ , remaining after  $t$  seconds.

[3]

2. The value of a car £ $V$  is given by

$$V = 20\,000 \times 0.9^t$$

where  $t$  is the age of the car in complete years.

- (a) Write down the value of  $V$  when  $t = 0$ .

(a) £ ..... [1]

(b) What is the value of  $V$  when  $t = 3$ ?

(b) £ ..... [2]

(c) After how many complete years will the car's value drop below £10 000?

(c) ..... [2]

3. Katy invests £2000 in a savings account for 3 years.

The account pays compound interest at an annual rate of  
 2.5% for the first year  
 $x\%$  for the second year  
 $x\%$  for the third year

There is a total amount of £2124.46 in the savings account at the end of 3 years.  
 Work out the rate of interest in the second year.

..... [4]

4. Louis and Robert are investigating the growth in the population of a type of bacteria. They have two flasks A and B.

At the start of day 1, there are 1000 bacteria in flask A.  
 The population of bacteria grows exponentially at the rate of 50% per day.

(a) Show that the population of bacteria in flask A at the start of each day forms a geometric progression.

[2]

The population of bacteria in flask A at the start of the 10th day is  $k$  times the population of bacteria in flask A at the start of the 6th day.

(b) Find the value of  $k$ .

..... [2]



At the start of day 1 there are 1000 bacteria in flask B.  
 The population of bacteria in flask B grows exponentially at the rate of 30% per day.

(c) Sketch a graph to compare the size of the population of bacteria in flask A and in flask B.

[1]

5. Here are the interest rates for two accounts.

Account A	Account B
Interest: 3% per year compound interest.	Interest: 4% for the first year, 3% for the second year and 2% for the third year.
No withdrawals until the end of three years.	Withdrawals allowed at any time.

Derrick has £10 000 he wants to invest.

a) Calculate which account would give him most money if he invests his money for 3 years.

Give the difference in the interest to the nearest penny.

a) Account ..... by ..... p [5]

(b) Explain why he might not want to use Account A.

[1]

6. The population,  $P$ , of an island  $t$  years after January 1st 2016 is given by this formula.

$$P = 4200 \times 1.04^t$$

a) What was the population of the island on January 1st 2016?

(a) ..... [1]

b) Explain how you know that the population is increasing.

[1]

c) What is the annual percentage increase in the population?

c) ..... % [1]

d) Work out the population of the island on January 1st 2021.

d) ..... [2]

7. Toby invested £7500 for 2 years in a savings account.

He was paid 4% per annum compound interest.

How much money did Toby have in his savings account at the end of 2 years?

£ ..... [2]

8. Ibrar bought a house for £145 000

The value of the house depreciated by 4% in the first year.

The value of the house depreciated by 2.5% in the second year.

Ibrar says,

*"4 + 2.5 = 6.5 so in two years the value of my house depreciated by 6.5%"*

a) Is Ibrar right?

You must give a reason for your answer.

[2]

The value of Ibrar's house increases by  $x\%$  in the third year.

At the end of the third year the value of Ibrar's house is £140 000

b) Work out the value of  $x$ .

Give your answer correct to 3 significant figures.

[3]

9. Ian invested an amount of money at 3% per annum compound interest.

At the end of 2 years the value of the investment was £2652.25



a) Work out the amount of money Ian invested.

£..... [3]

Noah has an amount of money to invest for five years.

**Saver Account**

4% per annum  
compound interest.

**Investment Account**

21% interest paid at the  
end of 5 years.

Noah wants to get the most interest possible.

b) Which account is best?

You must show how you got your answer.

[2]

10. A virus on a computer is causing errors.

An antivirus program is run to remove these errors.

An estimate for the number of errors at the end of  $t$  hours is  $10^6 \times 2^{-t}$

a) Work out an estimate for the number of errors on the computer at the end of 8 hours.

..... [2]

b) Explain whether the number of errors on this computer ever reaches zero.

[1]

11. The population of a city increased by 5.2% for the year 2014

At the beginning of 2015 the population of the city was 1560000

Lin assumes that the population will continue to increase at a constant rate of 5.2% each year.

a) Use Lin's assumption to estimate the population of the city at the beginning of 2017  
Give your answer correct to 3 significant figures.

..... [3]

b) (i) Use Lin's assumption to work out the year in which the population of the city will reach 2000000

.....

ii) If Lin's assumption about the rate of increase of the population is too low, how might this affect your answer to (b)(i)?

[3]

12 The number of slugs in a garden  $t$  days from now is  $p_t$  where

$$p_0 = 100$$

$$p_{t+1} = 1.06p_t$$

Work out the number of slugs in the garden 3 days from now.

[3]

13. The number of bees in a beehive at the start of year  $n$  is  $P_n$ .

The number of bees in the beehive at the start of the following year is given by

$$P_{n+1} = 1.05 (P_n - 250)$$

At the start of 2015 there were 9500 bees in the beehive.

How many bees will there be in the beehive at the start of 2018?

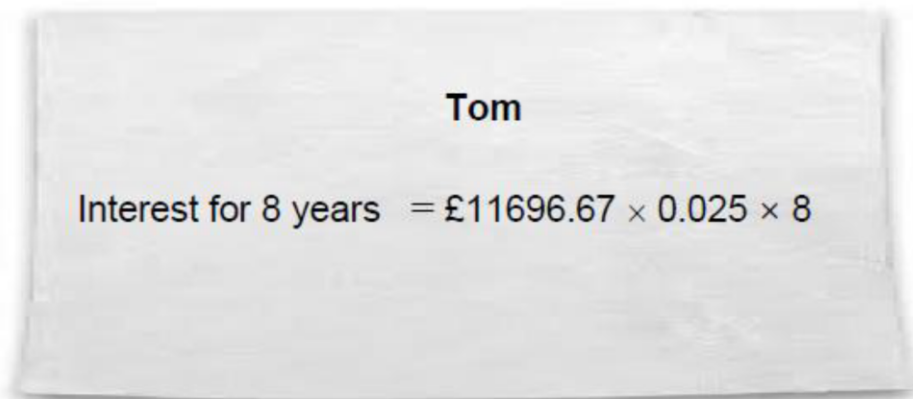
[3]

14. An amount of money was invested for 8 years.

It earned compound interest at 2.5% per year.

After 8 years the total value of the investment was £11 696.67

a) Tom is trying to work out the total interest earned.



State what is wrong with Tom's method.

[1]

b) Work out the total interest earned.

[3]

15. On 1st January 2012 Beth invested some money in a bank account.

The account pays 2.5% compound interest per year.

On 1st January 2013 Beth withdrew £1000 from the account.

On 1st January 2014 she had £17 466 in the account.

Work out how much money Beth originally invested in the account.

[4]