Mathematics
UNIT 2
Algebra

Name: _____

Class: _____

Contents

How to use booklets:
You will write in a printed booklet in lessons as directed by your teacher. An online version is available via the QR code or at https://hgsmaths.com/year-9/

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2.1 Expanding and Factorising

Expanding Single Brackets

Worked Example

Expand:

a) $2(x-3)$

b) $-2(x-3)$

Your Turn

Expand:

a) $2(3-x)$

b) $-2(3-x)$

Intelligent Practice

Expand:

1) $3(x+4)$
2) $3(4+x)$
3) $3(4-x)$
4) $3(x-4)$
5) $3(-x-4)$
6) $-3(x+4)$
7) $-3(x-4)$
8) $-3(2x-4)$
9) $-3(4-2x)$
10) $-6(2-x)$
11) $6(x-2)$
12) $6(x-2y)$
13) $6(5x-2y)$
14) $-6(2y-5x)$
15) $-6(2y-5x-7z)$
16) $-6(5x-2y-7z)$
17) $-6(-5x-2y-7z)$
18) $-(-5x-2y-7z)$

- Why are the answers to questions 1 and 2 the same?
- Why are the answers to questions 5 and 6 the same?
- Why are the answers to questions 9 and 10 the same?

Worked Example	Your Turn
Expand: a) $2x(x-3)$ b) $-2x(x-3)$	Expand: a) $2x(3-x)$ b) $-2x(3-x)$

Intelligent Practice

Expand:

1) $x(x+4)$
2) $x(4+x)$
3) $x(4-x)$
4) $x(x-4)$
5) $x(-x-4)$
6) $-x(x+4)$
7) $-x(x-4)$
8) $-x(2x-4)$
9) $-x(4-2x)$
10) $-3x(2-x)$
11) $3x(x-2)$
12) $3x(x-2y)$
13) $3x(5x-2y)$
14) $-3x(2y-5x)$
15) $-3x^2(2y-5x)$
16) $-3y^2(2y-5x)$
17) $-3y^2(2y-5xy)$
18) $-3y^3(2-5x)$

- Why are the answers to questions 1 and 2 the same?
- Why are the answers to questions 5 and 6 the same?
- Why are the answers to questions 17 and 18 the same?

Worked Example	Your Turn
Expand and simplify: a) $2(x-1)+3(x-4)$ b) $2(x-1)-3(x-4)$	Expand and simplify: a) $2(x-1)+5(x-4)$ b) $2(x-1)-5(x-4)$

Intelligent Practice

Expand and simplify:

1) $2(x+1)+3(x+4)$
2) $3(x+4)+2(x+1)$
3) $3(x+1)+2(x+4)$
4) $3(x-1)+2(x+4)$
5) $3(x+1)+2(x-4)$
6) $3(x-1)+2(x-4)$
7) $3(x-1)-2(x-4)$
8) $3(x+1)-2(x+4)$
9) $3(x+1)-2(2x+4)$
10) $3(x+1)-4(x+2)$
11) $3(x+1)-(x+2)$
12) $3(x-1)-(x-2)$
13) $3(x-1)-(5x-2)$
14) $3(x-1)-5x$
15) $5x-3(x-1)$
16) $5-3(x-1)$
17) $5+3(1-x)$
18) $5+3(y-x)$

- Why are the answers to questions 1 and 2 the same?
- Why are the answers to questions 9 and 10 the same?
- Why are the answers to questions 16 and 17 the same?

Worked Example	Your Turn
Expand and simplify: a) $2x(x-1)-3x(x-4)$ b) $2x(x-1)-3(x-4)$	Expand and simplify: a) $2x(x-1)-5x(x-4)$ b) $2x(x-1)-5(x-4)$

Intelligent Practice

Expand and simplify:

1) $2x(x+1)+3x(x+4)$

2) $3x(x+4)+2x(x+1)$

3) $3x(x+1)+2x(x+4)$

4) $3x(x-1)+2x(x+4)$

5) $3x(x+1)+2x(x-4)$

6) $3x(x-1)+2x(x-4)$

7) $3x(x-1)-2x(x-4)$

8) $3x(x+1)-2x(x+4)$

9) $3x(x+1)-2(x+4)$

10) $3x(x+1)-5(x+4)$

11) $3x^2(x+1)-5x(x+4)$

12) $3x^2(x-1)-5x(x-4)$

13) $3x^2(x-1)-(5x^2-4)$

14) $3x^2(x-1)-5x^2$

15) $5x^2-3x^2(x-1)$

16) $5-3x^2(x-1)$

17) $5+3x^2(1-x)$

18) $5+3x^2(y-x)$

- Why are the answers to questions 1 and 2 the same?
- Why are the answers to questions 16 and 17 the same?

Factorising into Single Brackets

Worked Example

Factorise:

a) $12x + 18$

b) $12x + 18y$

c) $12x^2 + 18$

Your Turn

Factorise:

a) $12x - 20$

b) $12x - 20y$

c) $12x^3 - 20$

Intelligent Practice

Factorise:

1. $30x - 18$
2. $30x - 19$
3. $30x - 20$
4. $30x + 20$
5. $-30x + 20$
6. $35x + 20$
7. $35x + 21$
8. $30x + 18$
9. $27x + 18$
10. $18x + 18$
11. $9x + 18$
12. $9x + 19$
13. $ax + 2a$
14. $ax + ay$
15. $ax + ay + a$
16. $2x + 4y + 6$
17. $2x + 4y - 6z$
18. $2x^2 + 4xy - 6xz$
19. $2x^2y + 4xy - 6xz$
20. $2x^2y + 3y - 5z$

Worked Example	Your Turn
Factorise: a) $12x^2 + 18x$ b) $12x^2 + 18xy$ c) $12x^3 + 18x^2$	Factorise: a) $12x^2 - 20x$ b) $12x^2 - 20xy$ c) $12x^4 - 20x^2$

Worked Example	Your Turn
Factorise: a) $x^4y^2 - x^3y^5$ b) $10x^7y^4 - 25x^3y^2$	Factorise: a) $x^2y^5 - xy^3$ b) $20e^5f^2 - 12e^2f$

Fluency Practice

Factorise:

1) $3a^3b^2c^5 + 12b^3c^2$

2) $8q^3r^4 - 4p^4q^2$

3) $6p^4q^5r^2 - 20pq^2r^5$

4) $16a^4b^2c - 14a^2b^4c^2$

5) $18p^4q^4 + 3q^4r^3$

6) $9pq^4r + 3pq^3r^3$

7) $8a^3b + 18a^3b$

8) $10x^5y^2z^2 - 2x^2y^4$

9) $14x^2y^2z^4 + 4x^5y^2z^2$

10) $11p^4qr^5 - 11p^4q^3r$

Definitions

Expanding and factorising single bracket:

EXAMPLE	NON-EXAMPLE
Expand $3(2x-6) = 6\text{x} - 18$	Expand $3(2x-6) = 6x - 6$
Factorise $2x - 8 = 2(x-4)$	Factorise $2x - 8 = 2(x-8)$
Expand $-2(y-4) = -2y + 8$	Expand $-2(y-4) = -2y - 8$
Factorise $5x^2 - 11x = x(5x-11)$	Factorise $5x^2 - 11 = x(5x-11)$

Fill in the Gaps

Q	*Expanded*	*Factorised*
1	$3p + 15$	
2		$3(p + 6)$
3	$6p + 18$	
4	$6p + 15$	
5		$2(3p + 5)$
6	$6p - 10$	
7		$2p(3p - 5)$
8	$6p^2 - 10$	
9	$6 - 10p^2$	
10		$-2(3 - 5p^2)$
11	$-6 + 12p^2$	
12	$-6 + 11p^2$	

Expanding Double Brackets

Worked Example

Expand and simplify:

a) $(x-3)(x-4)$

b) $(2x-3)(3x-4)$

Your Turn

Expand and simplify:

a) $(x-9)(x-12)$

b) $(2x-9)(3x-12)$

Worked Example	Your Turn
Expand and simplify: a) $(x+3)^2$ b) $(2x-3)^2$	Expand and simplify: a) $(x-3)^2$ b) $(3x-12)^2$

Expand and simplify fully:

1) $(x+3)(x+4)$
2) $(x+3)(x+5)$
3) $(x+5)(x+3)$
4) $(x+5)(x+5)$
5) $(x+5)(x-5)$
6) $(x+7)(x-5)$
7) $(x-7)(x-5)$
8) $(x-7)(x-7)$
9) $(x-7)^2$
10) $(x-7)(x+7)$
11) $(2x-7)(x+7)$
12) $(2x-7)(x+6)$
13) $(2x-7)(x-6)$
14) $(2x-7)(2x-6)$
15) $(2x-6)(2x-6)$
16) $(2x-6)^2$
17) $(3x-6)(2x-6)$
18) $(3x-6)(2x+6)$
19) $(3x-6)(2x+8)$
20) $(3x-6)(3x+8)$
21) $(3x+6)(3x+8)$
22) $(3x+8)(3x+8)$
23) $(3x+8)^2$
24) $(3x+8)(3x-8)$
25) $(8+3x)(3x-8)$
26) $(8+3x)(8-3x)$
27) $(8-3x)(8-3x)$
28) $(8-3x)^2$
29) $(3x-8)^2$
30) $(3x-8y)^2$

Intelligent Practice

Further Factorising – Grouping

Worked Example

Factorise:

a) $xy - 3y$

b) $x(x+1) - 3(x+1)$

Your Turn

Factorise:

a) $2xy - 3y$

b) $2x(x+1) - 3(x+1)$

Intelligent Practice

Factorise:

1) $x(x+1)+3(x+1)$

2) $x(x+1)+2(x+1)$

3) $x(x+1)+1(x+1)$

4) $3x(x+1)+1(x+1)$

5) $3x(x+1)-1(x+1)$

6) $3x(2x+1)-1(2x+1)$

7) $3x(1+2x)-1(2x+1)$

8) $3x(1-2x)-1(1-2x)$

9) $2x(1-2x)-1(1-2x)$

10) $2x(1-2x)-1(2x-1)$

Worked Example	Your Turn
Factorise: a) $3x(x+1)-5(x+1)$ b) $3x(x+1)^2-5(x+1)$	Factorise: a) $5x(x+1)-3(x+1)$ b) $5x(x+1)^2-3(x+1)$

Intelligent Practice

Factorise:

1) $3x(x+1)+2(x+1)$

2) $3x(x+1)-2(x+1)$

3) $3x(x+1)-1(x+1)$

4) $3x(x+1)-(x+1)$

5) $3x(x+1)-4(x+1)$

6) $3x(x+1)^2-4(x+1)$

7) $3x(x+1)^2-2(x+1)$

8) $3x(x+1)^2-2x(x+1)$

9) $3x(x+1)^2-(2x+1)(x+1)$

10) $3x(x+1)^3-(2x+1)(x+1)^2$

Worked Example	Your Turn
Factorise: $2x^2 + 2x - 3x - 3$	Factorise: $2x^2 - 2x - 3x + 3$

Intelligent Practice

Factorise:

1) $x^2 + 2x + 3x + 6$

2) $x^2 + 3x + 2x + 6$

3) $3x^2 + 3x + 2x + 2$

4) $3x^2 + 3x + 4x + 4$

5) $3x^2 - 3x + 4x - 4$

6) $3x^2 - 3x - 4x + 4$

7) $3x^2 + 3x - 4x - 4$

8) $3x^2 - 4x + 3x - 4$

9) $5x^2 - 4x + 5x - 4$

10) $-5x^2 + 4x - 5x + 4$

Worked Example	Your Turn
Factorise: a) $3x^2 + 3x - 5x - 5$ b) $3x^2 - 5x + 3x - 5$	Factorise: a) $5x^2 + 5x - 3x - 3$ b) $5x^2 - 3x + 5x - 3$

Intelligent Practice

Factorise:

1) $3x^2 + 3x + 2x + 2$

2) $3x^2 + 3x - 2x - 2$

3) $3x^2 + 3x - 1x - 1$

4) $3x^2 + 3x - x - 1$

5) $3x^2 + 3x - 4x - 4$

6) $3x^2 - 4x + 3x - 4$

7) $3x^2 + 4x - 3x - 4$

8) $6x^2 + 8x - 6x - 8$

9) $6x^2 + 9x - 6x - 9$

10) $-6x^2 - 9x + 6x + 9$

Factorising a Monic Quadratic

Definition:
A quadratic is an expression involving a **sum of powers of a variable where the highest power is 2.**

The general form of a quadratic expression is: $ax^2 + bx + c$
where a, b and c are numbers, $a \neq 0$ and x is the variable.

Example	Non-Example
$3x^2 - 9$	$3x - 9$
$x - 3x^2 - 9$	$3x^3 - 9$
$5y^2 + 7y$	$5y^2 + 7x$
$3a^2 - 9a + 2$	$-9a + 2b$

Monic means you have a single x^2, i.e. $a = 1$ in the general form $ax^2 + bx + c$

Monic Quadratic	Non-Monic Quadratic
$x^2 - 9$	$3x^2 - 9$
$x^2 + x$	$x - 3x^2 - 9$
$6 + 8y - y^2$	$5y^2 + 7y$
$15 - a^2$	$3a^2 - 9a + 2$

Sum and Product

	Sum is Positive	**Sum is Negative**
Product is Positive	$\_\_ \times \_\_ = 14$ $\_\_ + \_\_ = 9$	$\_\_ \times \_\_ = 14$ $\_\_ + \_\_ = -9$
Product is Negative	$\_\_ \times \_\_ = -14$ $\_\_ + \_\_ = 5$	$\_\_ \times \_\_ = -14$ $\_\_ + \_\_ = -5$

	Sum is Positive	**Sum is Negative**
Product is Positive	Positive and Positive	Negative and Negative
Product is Negative	Positive and Negative where the size of the positive is greater than the size of the negative	Positive and Negative where the size of the negative is greater than the size of the positive

Intelligent Practice

$\square \times \square = 18$ $\square + \square = 11$	$\square \times \square = 20$ $\square + \square = 9$	$\square \times \square = 180$ $\square + \square = -36$
$\square \times \square = -18$ $\square + \square = 7$	$\square \times \square = -20$ $\square + \square = -1$	$\square \times \square = -252$ $\square + \square = -36$
$\square \times \square = -18$ $\square + \square = -7$	$\square \times \square = -20$ $\square + \square = 1$	$\square \times \square = 252$ $\square + \square = 48$
$\square \times \square = 18$ $\square + \square = -11$	$\square \times \square = 20$ $\square + \square = -9$	$\square \times \square = 432$ $\square + \square = 48$
$\square \times \square = 18$ $\square + \square = -9$	$\square \times \square = -36$ $\square + \square = -9$	$\square \times \square = 576$ $\square + \square = 48$
$\square \times \square = -18$ $\square + \square = 3$	$\square \times \square = -36$ $\square + \square = 0$	$\square \times \square = -576$ $\square + \square = 0$

Worked Example	Your Turn
Factorise:	Factorise:
$x^2 + 10x + 24$	$x^2 + 9x + 20$
$x^2 - 10x + 24$	$x^2 - 9x + 20$
$x^2 + 10x - 24$	$x^2 + 9x - 10$
$x^2 - 10x - 24$	$x^2 - 9x - 10$

Factorising a Non-Monic Quadratic

- Multiply the coefficient of x^2 and the constant.
- Find two numbers that multiply to give this value and add to give the coefficient of x.
- Write the quadratic with the x-term split into two x-terms using these numbers.
- Factorise the pair of terms.
- Factorise again, taking the bracket as the common factor.
- Check by expanding.

Worked Example	Your Turn
Factorise: $3x^2 + 10x + 8$	Factorise: $3x^2 - 10x + 8$

Worked Example	Your Turn
Factorise: $3x^2 + 2x - 8$	Factorise: $3x^2 - 2x - 8$

Worked Example	Your Turn
Factorise: $6x^2 + 20x + 16$	Factorise: $6x^2 - 2x - 4$

Intelligent Practice

Factorise:

1) $3x^2 + 5x + 2$

2) $3x^2 + 8x + 4$

3) $3x^2 + 11x + 6$

4) $3x^2 + 7x - 6$

5) $3x^2 + 4x - 4$

6) $3x^2 + 1x - 2$

7) $3x^2 + x - 2$

8) $3x^2 - x - 2$

9) $3x^2 - 4x - 4$

10) $3x^2 - 7x - 6$

11) $3x^2 - 11x + 6$

12) $3x^2 - 8x + 4$

13) $3x^2 - 5x + 2$

14) $2(3x^2 - 5x + 2)$

15) $6x^2 - 10x + 4$

16) $x^2 - 10x + 24$

17) $x^2 - 10x - 24$

18) $2\mathrm{x}^2 - 20\mathrm{x} - 48$

19) $2x^2 - 29x - 48$

20) $-2x^2 + 29x + 48$

Difference of Two Squares

- The coefficient of the variable is a square number.
- The power of the variable must be even.
- One term will be negative AND the other term will be positive.
- The constant must be a value that can be square rooted.

Example	Non-Example
$x^2 - 9$	$x^2 - 8$
$9 - x^2$	$9 - 3x^2$
$4y^2 - 36$	$5y^2 - 36$

$$a^2 - b^2 = (\mathrm{a} + \mathrm{b})(\mathrm{a} - \mathrm{b})$$

Worked Example	Your Turn
Factorise: $x^2 - 4$ $4 - x^2$ $2x^2 - 8$ $16x^2 - 25$	Factorise: $x^2 - 9$ $9 - x^2$ $3x^2 - 27$ $16x^2 - 9$

Intelligent Practice

Factorise:

1) $x^2 - 16$

2) $x^2 - 25$

3) $x^2 - 36$

4) $-36 + x^2$

5) $36 - x^2$

6) $4 - x^2$

7) $36 - 9x^2$

8) $9x^2 - 36$

9) $100 - 9x^2$

10) $100 - x^2$

11) $200 - 2x^2$

12) $300 - 3x^2$

13) $300 - 27x^2$

14) $300 - 12x^2$

15) $12x^2 - 300$

16) $12x^2 - 3y^2$

17) $3x^2 - 3y^2$

18) $9x^2 - 9y^2$

19) $9x^4 - 9y^2$

20) $9x^4 - 9y^6$

2.2 Functions Part 1

Prerequisite Work – Solving Equations

1. Solve each of these equations.

(a)	$3x + 6 = 48$	(b)	$5x - 6 = 39$	(c)	$2x - 6 = 22$
(d)	$6x - 7 = 41$	(e)	$8x - 3 = 29$	(f)	$6x + 12 = 20$
(g)	$4x + 18 = 2$	(h)	$5x + 10 = 5$	(i)	$3x + 6 = 1$
(j)	$5(x + 2) = 45$	(k)	$3(x - 2) = 12$	(l)	$2(x + 7) = 10$
(m)	$3(2x - 1) = 57$	(n)	$3(2x + 7) = 27$	(o)	$5(5x + 1) = 20$
(p)	$4(2x + 3) = -8$	(q)	$5(3x - 1) = -2$	(r)	$2(8x + 5) = -2$
(s)	$6x - 8 = -26$	(t)	$4(x + 15) = 60$	(u)	$5x - 8 = -10$
(v)	$\frac{x}{4} - 1 = 8$	(w)	$\frac{x}{3} + 2 = 7$	(x)	$\frac{2x}{5} + 1 = 3$

2. Solve these equations.

(a)	$2x + 6 = x + 3$	(b)	$4x - 8 = 5x - 2$
(c)	$6x + 7 = 2x + 20$	(d)	$x + 6 = 2x - 8$
(e)	$3x + 7 = 2x + 11$	(f)	$10x + 2 = 8x + 22$
(g)	$6 - x = 5$	(h)	$2 - x = 5$
(i)	$3 - x = -10$	(j)	$14 - 3x = 5$
(k)	$10 - 2x = 2$	(l)	$4 - 3x = 2$
(m)	$x + 2 = 8 - x$	(n)	$x + 4 = 10 - 2x$
(o)	$x + 4 = 9 - 2x$	(p)	$8 - x = 12 - 2x$
(q)	$22 - 4x = 18 - 2x$	(r)	$3 - 6x = 2 - 4x$
(s)	$3(x + 2) = 5(x - 2)$	(t)	$4 = 8 - \frac{x}{3}$
(u)	$3 - \frac{x}{4} = -5$	(v)	$4(x - 2) = 3(x + 2)$
(w)	$5 = 18 - \frac{x}{3}$	(x)	$2 - \frac{x}{4} = 1 - \frac{x}{6}$

Functions – The Basics

What is a function? *A function is like a number machine in that you INPUT a value and perform some operations to OUTPUT a value.*

Notation:

To denote the above we use function notation 'f(x)', i.e. for the above

$$f(x) = 2x - 1$$

So here x represents the input. We know that an input of 3 become 5, so we denote as:

$$f(3) = 5$$

Uses:

- Substitution: Evaluate $g(2)$ if $g(x) = \frac{x+6}{2}$
- Solving: Solve $h(a) = 18$ if $h(x) = 3(x + 2)$

Worked Example	Your Turn
If $f(x) = \frac{x}{3} + 2$, evaluate: a) $f(3)$ b) $f(-6)$	If $g(x) = \frac{x}{4} - 5$, evaluate: a) $g(24)$ b) $g(4)$

Worked Example	Your Turn
If $f(x) = x^2 + 3$, evaluate: a) $f(4)$ b) $f(-2)$	If $g(x) = x^2 - 4$, evaluate: *a)* $g(5)$ b) $g(-2)$

Worked Example	Your Turn
If $f(x) = \frac{x}{3} + 2$, find x when $f(x) = 8$	If $g(x) = \frac{x}{4} - 5$, find x when $g(x) = -2$

Worked Example	Your Turn
If $f(x) = x^2 + 3$, find x when $f(x) = 19$	If $g(x) = x^2 - 4$, find x when $g(x) = 21$

Fluency Practice

Question 1: Given $f(x) = 3x + 5$

Work out the values of

(a) $f(2)$ (b) $f(8)$ (c) $f(0)$ (d) $f(-2)$

Question 2: Given $g(x) = \frac{2x + 9}{4}$

Work out the values of

(a) $g(6)$ (b) $g(-1)$ (c) $g(0)$ (d) $g(-10)$

Question 3: Given $h(x) = x^2 - 5$

Work out the values of

(a) $h(7)$ (b) $h(-1)$ (c) $h(-3)$ (d) $h(15)$

Question 4: The function f is such that $f(x) = 3x - 8$

Solve $f(x) = 7$

Question 5: The function g is such that $g(x) = 19 - 4x$

Solve $g(x) = 31$

Question 6: The function h is such that $h(x) = \frac{5x - 1}{2}$

Solve $h(x) = 32$

Worked Example	Your Turn
If $f(x) = x^2 - 2$, evaluate: a) $f(x-2)$ b) $f(2x)$	If $g(x) = x^2 + 3$, evaluate: a) $g(x-3)$ b) $g(3x)$

Worked Example	Your Turn
If $f(x) = 3x^2 - 2$, evaluate: a) $f(x-2)$ b) $f(2x)$	If $g(x) = 5x^2 + 3$, evaluate: a) $g(x-3)$ b) $g(3x)$

Worked Example	Your Turn
If $f(x) = 3x^2 - 5x - 2$, evaluate $f(x - 2)$	If $g(x) = 5x^2 - 2x + 3$, evaluate $g(x - 3)$

Fluency Practice

1. The function f is such that $f(x) = 5x + 2$

   Find (a) f(3) (b) f(7) (c) f(−4)

   (d) f(−2) (e) f(−0.5) (f) f(0.3)

2. The function f is such that $f(x) = x^2 - 4$

   Find (a) f(4) (b) f(6) (c) f(−2)

   (d) f(−6) (e) f(−0.2) (f) f(0.9)

3. The function g is such that $g(x) = x^3 - 3x^2 - 2x + 1$

   Find (a) g(0) (b) g(1) (c) g(2)

   (d) g(−1) (e) g(−0.4) (f) g(1.5)

4. The function f is such that $f(x) = \sqrt{2x + 5}$

   Find (a) f(0) (b) f(1) (c) f(2)

   (d) f(−1) (e) f(−0.7) (f) f(1.5)

5. $f(x) = 3x^2 - 2x - 8$

   Express $f(x + 2)$ in the form $ax^2 + bx$

6. The functions f and g are such that

   $f(x) = 3x - 5$ and $g(x) = 4x + 1$

   (a) Find (i) f(−1) (ii) g(2)

   (b) Find the value of x for which $f(x) = g(x)$.

2.3 Changing the Subject of a Formula

Changing the Subject – Without Factorising

Worked Example	Your Turn
Make x the subject of the following formulae: $y = mx + c$	Make x the subject of the following formulae: $y = abx + c$

Intelligent Practice

Make x the subject for each of the following formulae:

1) $y = ax + b$

2) $y = b^2 + ax$

3) $y = ax + bcd$

4) $y = d^2ef + ax$

5) $y = ax - b$

6) $y = -\sqrt{b} + ax$

7) $y = ax - bcd$

8) $y = -\sqrt{bcd} + ax$

Worked Example	Your Turn
Make x the subject of the following formulae: a) $y = \frac{x}{m} + c$ b) $y = -\frac{x}{ef} + c^2$	Make x the subject of the following formulae: a) $y = \frac{x}{ab} + c$ b) $y = -\frac{x}{cd} + e^2$

Intelligent Practice

Make x the subject for each of the following formulae:

1) $y = \frac{x}{a} + c$

2) $y = -\frac{x}{a} + c$

3) $y = \frac{x}{a^2} - c$

4) $y = -\frac{x}{a^2} - c$

5) $y = \frac{x}{bc} + \sqrt{qrs}$

6) $y = -\frac{x}{bc} - \sqrt{qrs}$

Worked Example	Your Turn
Make x the subject of the following formulae: $y = p(x + q)$	Make x the subject of the following formulae: $y = p(x - q)$

Fluency Practice

Make x the subject of these equations.

There are two ways to rearrange each equation so there are two sets of answer boxes. You only need to fill in your answer once. Use the boxes where your answer fits best.

$a(x + b) = c$ $\quad x = \frac{\square}{\square} - \square$ or $x = \frac{\square}{\square}$

$2(x + a) = b$ $\quad x = \frac{\square}{\square} - \square$ or $x = \frac{\square}{\square}$

$a(x - 6) = b$ $\quad x = \frac{\square}{\square} + \square$ or $x = \frac{\square}{\square}$

$c(7 + x) = 2$ $\quad x = \frac{\square}{\square} - \square$ or $x = \frac{\square}{\square}$

Worked Example	Your Turn
Make a the subject of the following formulae: a) $2a^2 = c$ b) $2\sqrt{a} = c$	Make a the subject of the following formulae: a) $2(a+b)^2 = c$ b) $2\sqrt{a-b} = c$

Intelligent Practice

Make a the subject for each of the following formulae:

1) $2a^2 + b = c$

2) $2a^2 - b = c$

3) $6a^2 - b = c$

4) $2(a - 2b)^2 + b = c$

5) $2(a + 2b)^2 - b = c$

6) $6(a + 2b)^2 - b = c$

1) $2\sqrt{a} + b = c$

2) $2\sqrt{a} - b = c$

3) $6\sqrt{a} - b = c$

4) $2\sqrt{a - 2b} + b = c$

5) $2\sqrt{a + 2b} - b = c$

6) $6\sqrt{a + 2b} - b = c$

Changing the Subject – With Factorising

Worked Example	Your Turn
Make x the subject of the following formula: $ax + ay = cx + by$	Make x the subject of the following formula: $yx + wz = 3xz + 3yz$

Intelligent Practice

Make x the subject for each of the following formulae:

1) $5x + b = 2x + a$

2) $5x + bd = 2x + ac$

3) $bx + 5 = ax + 2$

4) $bx + 5d = ax + 2c$

5) $5(x + b) = 2(x + a)$

6) $5(x + bd) = 2$

7) $b(x + 5) = a(x + 2)$

8) $b(x + 5d) = a(x + 2c)$

Worked Example	Your Turn
Make x the subject of the following formula: $w = \frac{x + a}{x - a}$	Make x the subject of the following formula: $w = \frac{x + 2y}{x - y}$

Intelligent Practice

Make x the subject for each of the following formulae:

1) $w = \frac{5x+5b}{x+a}$

2) $w = \frac{5x+5b}{4x+4a}$

3) $w = \frac{5x+5b}{3x-3a}$

Worked Example	Your Turn
Make a the subject of the following formula: $ax + ay = 3$	Make a the subject of the following formula: $ak + am = 5$

Worked Example	Your Turn
Make a the subject of the following formula: $ax + 2y = 5y + am$	Make a the subject of the following formula: $ab + 3y = 7y + ak$

Make a the subject of the following formulae

1. $ab + ac = 3$
2. $3a + ay = x$
3. $a(3 + y) + 2a = x$
4. $2a = x + ak$
5. $ay + k = 3a$
6. $ay - x = ab$
7. $ay + b = 3b + ax$
8. $m(a + y) = x(a + b)$
9. $\frac{y-a}{x+a} = b$
10. $a - n = \frac{a+2}{n}$
11. $\sqrt{\frac{a+x}{a-x}} = 2$
12. $\sqrt{\frac{m(a+n)}{a}} = p$

Make a the subject of the following formulae:

1) $2a = b$

2) $\frac{a}{2} = b$

3) $a + 2 = b$

4) $a - 2 = b$

5) $\frac{a}{c} = b$

6) $\frac{a}{c} + 2 = b$

7) $ac = b$

8) $ac - 2 = b$

9) $ac - d = b$

10) $\frac{a+2}{c} = b$

11) $\frac{2a}{c} = b$

12) $\frac{d(a+2)}{c} = b$

13) $2a = b + a$

14) $2a + c = b + a$

15) $2a + c = b - a$

16) $ae + c = b + a$

17) $ae + c = b + af$

18) $ae + c = b - af$

2.4 Functions Part 2

Inverse Functions

RULES FOR FINDING THE INVERSE $f^{-1}(x)$:

Step 1: Write out the function as $y = \ldots$

Step 2: Swap the x and y

Step 3: Make y the subject

Step 4: Instead of $y=$ write $f^{-1}(x) =$

Worked Example	Your Turn
Find the inverse function: $f(x) = 3x - 5$	Find the inverse function: $g(x) = 4x + 2$

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{2x + 3}{4}$	Find the inverse function: $g(x) = \frac{4x - 3}{2}$

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{2}{x-3}$	Find the inverse function: $g(x) = \frac{3}{x+2}$

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{2}{3-4x}$	Find the inverse function: $g(x) = \frac{4}{5-3x}$

Worked Example	Your Turn
Find the inverse function: $f(x) = 3\sqrt{x} - 2$	Find the inverse function: $g(x) = 4\sqrt{x} + 5$

Fluency Practice

Question 13: Find $f^{-1}(x)$ for each of the following:

(a) $f(x) = 2x$ (b) $f(x) = x - 6$ (c) $f(x) = \frac{x}{3}$

(d) $f(x) = 5x + 1$ (e) $f(x) = \frac{2x}{7}$ (f) $f(x) = \frac{x - 2}{6}$

Question 14: Given $h(x) = \frac{x}{4}$

(a) Find $h^{-1}(x)$

(b) Calculate the value of $h^{-1}(1.5)$

Question 15: Given $f(x) = 2x - 3$

(a) Find $f^{-1}(x)$

(b) Calculate the value of $f^{-1}(7)$

Question 16: Given $g(x) = \frac{3x + 1}{2}$

(a) Find $g^{-1}(x)$

(b) Calculate the value of $g^{-1}(11)$

Question 17: Given $f(x) = \frac{4x}{9} - 8$

(a) Find $f^{-1}(x)$

(b) Calculate the value of $f^{-1}(-10)$

Fluency Practice

1. Find the inverse function, $f^{-1}(x)$, of the following functions:

   (a) $f(x) = 3x - 1$

   (b) $f(x) = 2x + 3$

   (c) $f(x) = 1 - 2x$

   (d) $f(x) = x^2 + 5$

   (e) $f(x) = 6(4x - 1)$

   (f) $f(x) = 4 - x$

   (g) $f(x) = 3x^2 - 2$

   (h) $f(x) = 2(1 - x)$

   (i) $f(x) = \dfrac{2}{x+1}$

   (j) $f(x) = \dfrac{x+1}{x-2}$

2. The function f is such that $f(x) = 7x - 3$

   (a) Find $f^{-1}(x)$.

   (b) Solve the equation $f^{-1}(x) = f(x)$.

3. The function f is such that $f(x) = \dfrac{8}{x+2}$

   (a) Find $f^{-1}(x)$.

   (b) Solve the equation $f^{-1}(x) = f(x)$.

4. The function f is such that $f(x) = \dfrac{1}{x+4}$, $x \neq -4$.

   Evaluate $f^{-1}(3)$. [Hint: First find $f^{-1}(x)$ and then substitute for $x = -3$]