



**Year 9**  
**Mathematics**  
**UNIT 3**



**Name:** \_\_\_\_\_

**Class:** \_\_\_\_\_

# CONTENTS PAGE

[2 Pre-requisite skills Angle rules](#)

[12 Angles in polygons](#)

[45 Rotation and Translation](#)

[69 Prerequisite skills for Pythagoras](#)

[75 Pythagoras' Theorem](#)

Please see unit 3 course on [drfrostmaths.com](http://drfrostmaths.com)

The screenshot shows a website interface for a Year 9 course. At the top, there is a navigation bar with a back arrow, the text 'Courses → Schools → King Edward VI Handsworth Grammar School for Boys →', and 'Year 9'. Below this, a row of buttons lists school classes: '9J1/Ma [RSA]', '9J2/Ma [YLI]', '9J3/Ma [GDH]', '9K1/Ma [BBA]', '9K2/Ma [PAL/RSA]', and '9K3/Ma [PTO/ACA]'. A school crest is visible on the right. The main content area features six numbered sections, each with a list of topics and 'Revision' at the bottom. A hand-drawn black circle highlights the '3. Reasoning with Geometry' section.

Section	Topics
1. Reasoning with Number	Numbers Using percentages Maths and money Revision
2. Algebra	Expanding and Factorising Changing the Subject Functions Revision
3. Reasoning with Geometry	PR Angles in Polygons and construction Angles in Polygons and construction Rotation and translation Pythagoras' theorem Revision
4. Reasoning with Algebra	Straight line graphs Forming and solving equations Testing conjectures Revision
5. Reasoning with Proportion	Enlargement and similarity Ratio and proportion problems Rates Revision
6. Representations	Right-angled Trigonometry Probability Algebraic representation Revision

## PRE-REQUISITES

What you should know from previous years:

K103: Angles on a line.

K104: Angles about a point.

K107: Angles in a triangle.

K108: Angles in a triangle where one side is extended.

K307: Vertically opposite angles.

K301: Angles in an isosceles triangle.

K302: Find an angle in an isosceles triangle using angles on parallel lines.

K305: Alternate angles on parallel lines.

K306: Corresponding angles on parallel lines.

K308: Cointerior (allied) angles on parallel lines.

K309: Angles in a quadrilateral.

# NEED TO KNOW

## POLYGONS

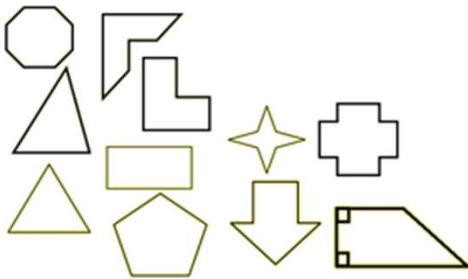
### Definition

Literally translates to “many angles”. Generally recognised as a 2D shape made up of 3 or more connected straight lines.

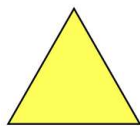
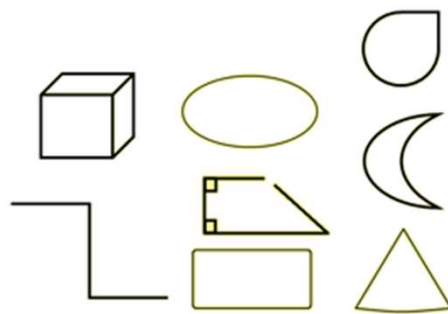
### Characteristics

- Made of connected straight lines (no gaps)
- Flat shape

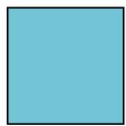
### Examples



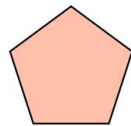
### Non Examples



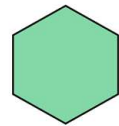
Triangle



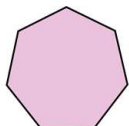
Quadrilateral



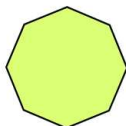
Pentagon



Hexagon



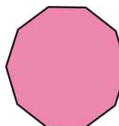
Heptagon



Octagon



Nonagon



Decagon

## REGULAR POLYGONS

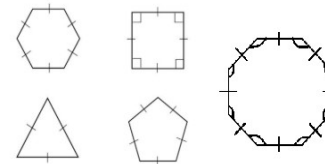
### Definition

A polygon with all sides equal sized and all interior angles equal sized.

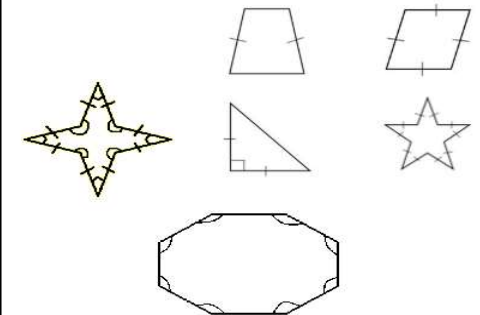
### Characteristics

- All connected straight sides
- All sides equal sized
- All angles equal sized

### Examples

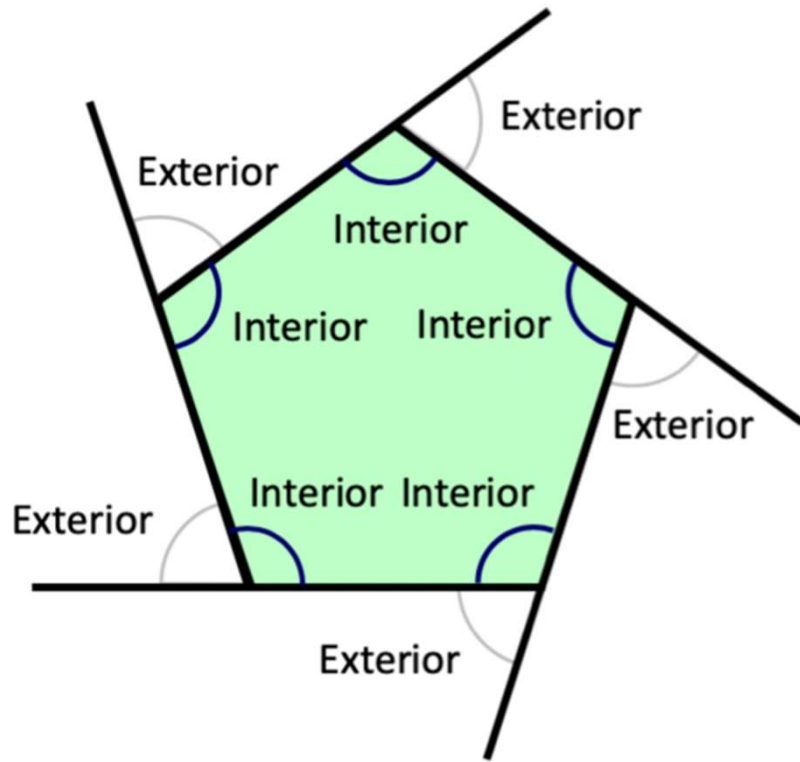


### Non Examples


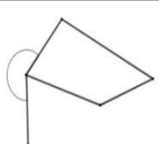
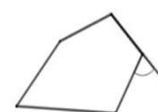
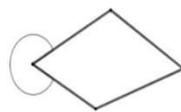

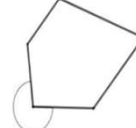




# Interior and Exterior Angles



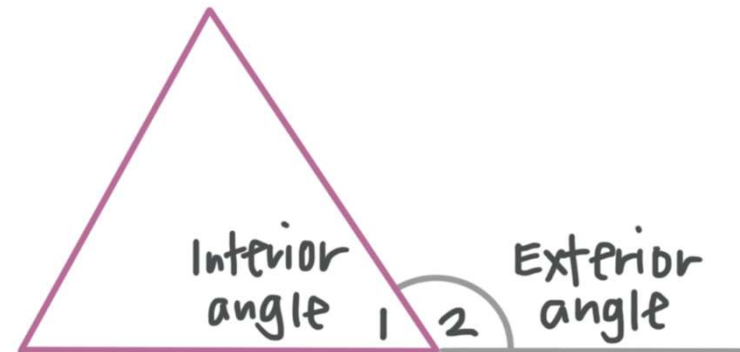
Exterior angles

Examples	Nonexamples
	
	
	

Search:  Topic:

- The interior angles of a polygon are on the inside.
- The exterior angles of a polygon are on the outside.
- The interior and exterior angles form a straight line.

$$\text{Interior Angle} + \text{Exterior Angle} = 180^\circ$$



## Polygons – interior and Exterior Angle Rules

### **ALL POLYGONS**

$$\text{Interior angle} + \text{exterior angle} = 180^\circ$$

$$\text{Sum of interior Angles} = (n - 2) \times 180^\circ$$

$$\text{Sum of exterior Angles} = 360^\circ$$

***n* – number of sides**

### **REGULAR POLYGONS**

$$\text{EACH exterior angle} = \frac{360^\circ}{n}$$

$$\text{EACH interior angle} = 180^\circ - \frac{360^\circ}{n}$$

***n* – number of sides**

Name	Number of angles	Sum of interior angles	Size of one interior angle in a regular polygon	Size of one exterior angle in a regular polygon
	3			
		360°	90°	
Octagon				45°
Hexadecagon		2520°		
Pentadecagon	15		156°	
				72°
		720°	120°	
	12			
		1620°		$\frac{360}{11}$

# Fluency practice

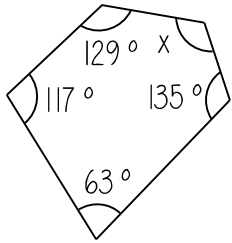
The diagrams are not drawn accurately

1. Find the sum of the interior angles in each polygon

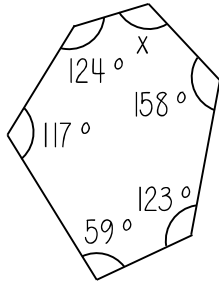
- a. 12 sides    b. 15 sides    c. 18 sides    d. 22 sides    e. 25 sides    f. 30 sides    g. 52 sides    h. 120 sides

2. Find the value of  $x$

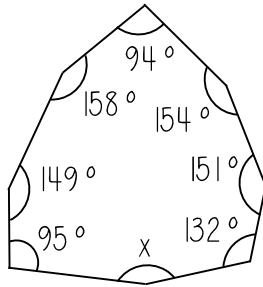
a.



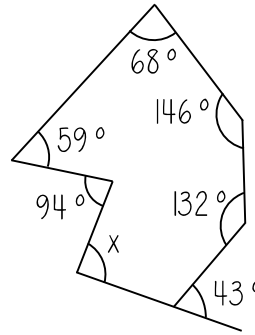
b.



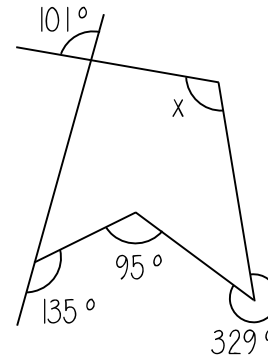
c.



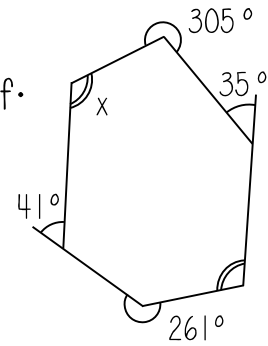
d.



e.



f.

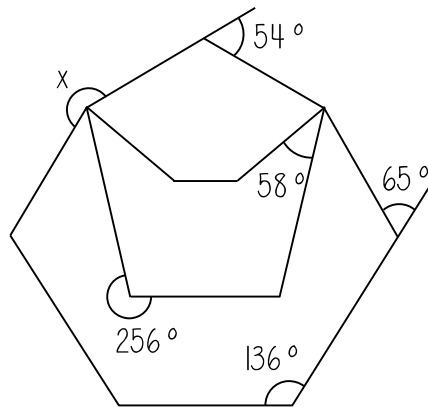


3. Find the number of sides each polygons has, given the sum of the interior angles

- a.  $1800^\circ$     b.  $1980^\circ$     c.  $3060^\circ$     d.  $3240^\circ$     e.  $3780^\circ$     f.  $5940^\circ$     g.  $9720^\circ$     h.  $14220^\circ$

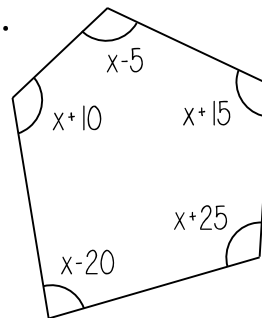
4.

The polygon has one line of symmetry.  
Find the value of  $x$ .

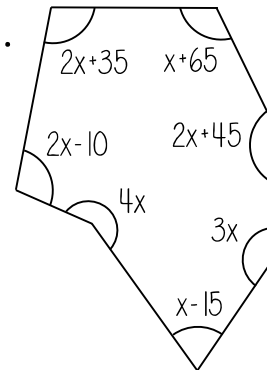


5. Find the value of  $x$  (and  $y$ )

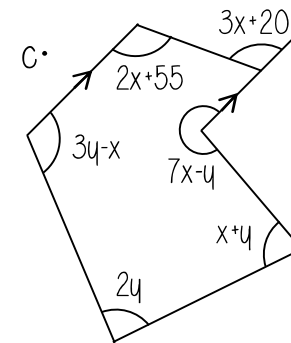
a.



b.



c.



Examples



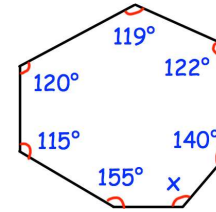
Scan here

Workout

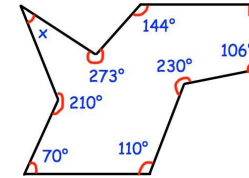
Question 1: Find the missing angle in each irregular polygon

- (a)
- (b)
- (c)
- (d)
- (e)
- (f)
- (g)
- (h)
- (i)
- (j)
- (k)
- (l)

(m)



(n)



Question 2: Work out the sum of the interior angles for polygons with

- (a) 10 sides      (b) 14 sides      (c) 20 sides      (d) 45 sides  
(e) 50 sides      (f) 80 sides      (g) 100 sides      (h) 200 sides

Question 3: Work out the number of sides of polygons with these sum of interior angles

- (a) 1260°      (b) 2880°      (c) 3960°      (d) 5040°  
(e) 12240°      (f) 15840°      (g) 2340°      (h) 89640°

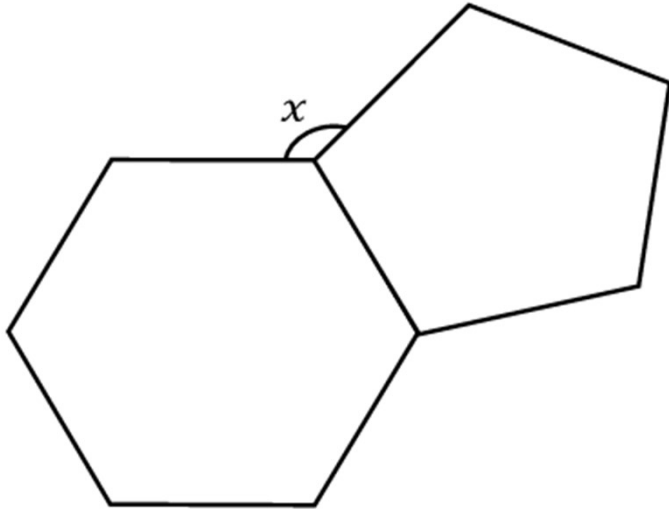
Question 4: Each of the polygons below are regular.  
Calculate the size of each interior angle,  $x$ .

- (a) regular pentagon
- (b) regular hexagon
- (c) regular octagon
- (d) regular nonagon
- (e) regular decagon
- (f) regular dodecagon

# Polygons and angles

## Worked Example

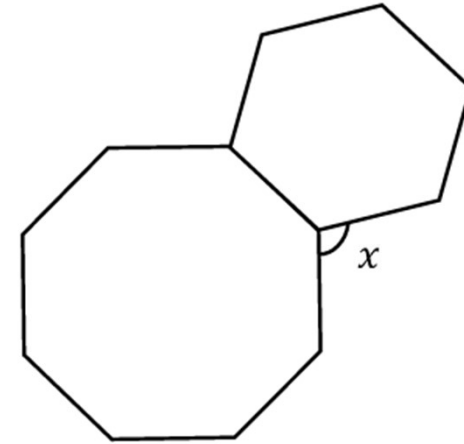
These are regular polygons.  
Find  $x$



## Thinking

## Your Turn

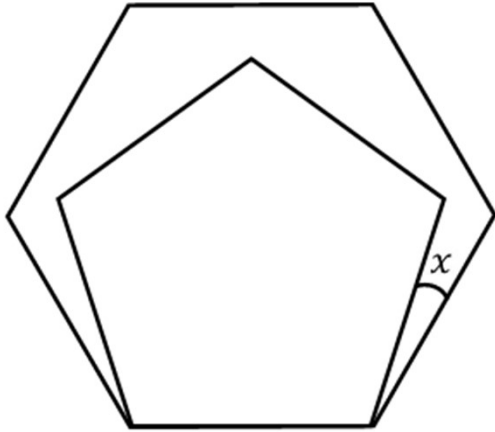
These are regular polygons.  
Find  $x$



# Polygons and angles

## Worked Example

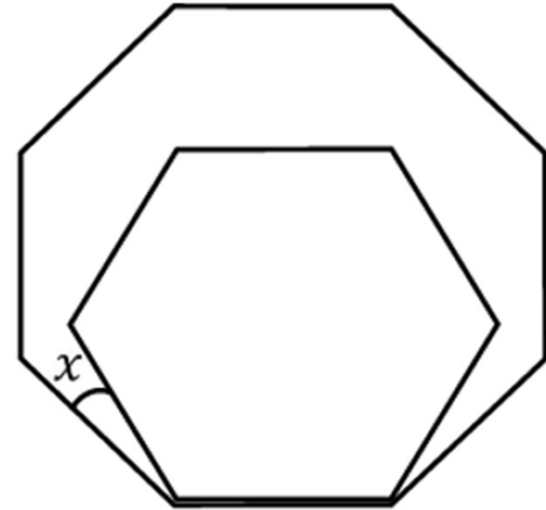
These are regular polygons.  
Find  $x$



## Thinking

## Your Turn

These are regular polygons.  
Find  $x$



## Polygons and angles

### Worked Example

A regular polygon has an exterior angle of  $30^\circ$ . How many sides does it have?

A regular polygon interior angles of size  $135^\circ$ . How many sides does it have?

### Thinking

### Your Turn

A regular polygon has an exterior angle of  $60^\circ$ . How many sides does it have?

A regular polygon has interior angles of size  $120^\circ$ . How many sides does it have?



## Polygons and angles

### Worked Example

In a quadrilateral, the four angles are listed from largest to smallest. Each angle is three times the previous angle.

What are the angles?

### Thinking

### Your Turn

In a quadrilateral, the four angles are listed from largest to smallest. Each angle is one third of the previous angle.

What are the angles?

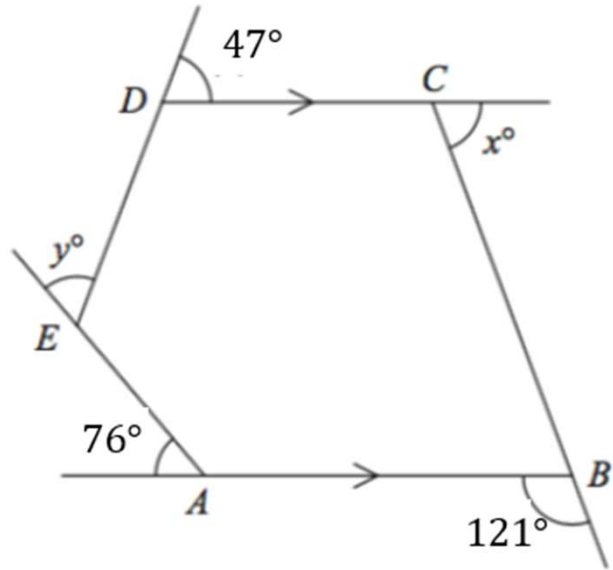
# Polygons and angles

## Worked Example

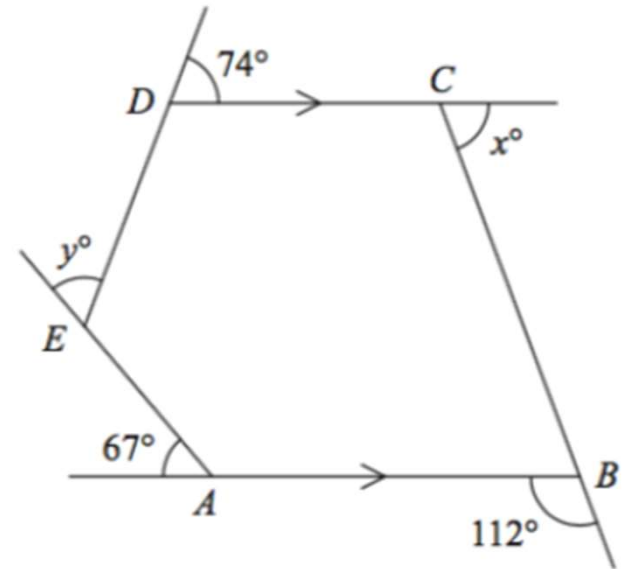
## Thinking

## Your Turn

Calculate  $x$  and  $y$



Calculate  $x$  and  $y$



## Polygons and angles

### Worked Example

In a quadrilateral, the four angles are listed from largest to smallest. Each angle is three times the previous angle.

What are the angles?

### Thinking

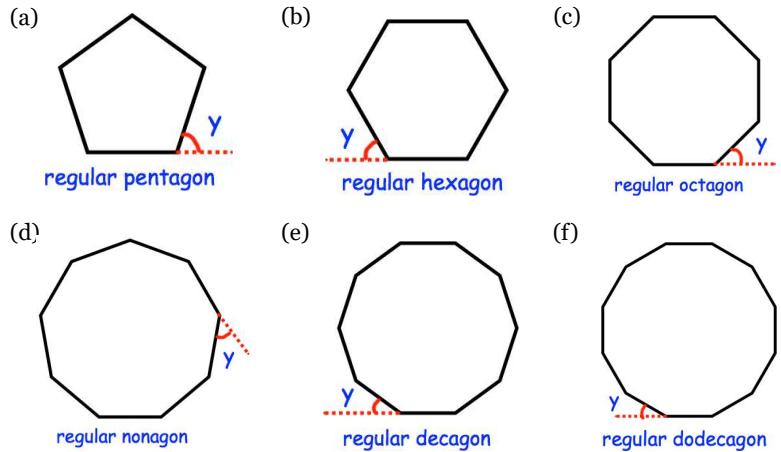
### Your Turn

In a quadrilateral, the four angles are listed from largest to smallest. Each angle is one third of the previous angle.

What are the angles?

Apply

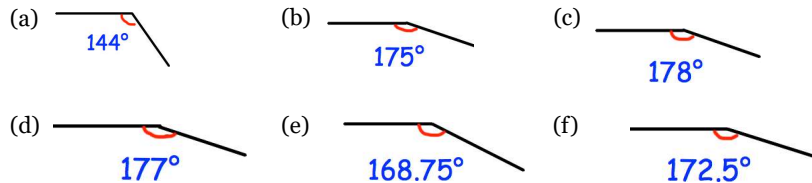
Question 6: Each of the polygons below are regular.  
Calculate the size of each exterior angle,  $y$ .



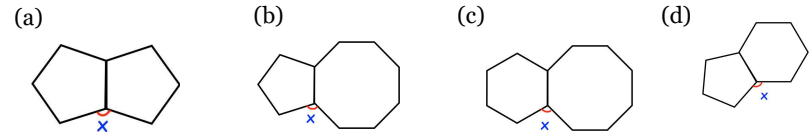
Question 7: Calculate the size of each exterior angle in regular polygons with

- (a) 15 sides      (b) 18 sides      (c) 20 sides      (d) 24 sides  
(e) 30 sides      (f) 36 sides      (g) 40 sides      (h) 45 sides  
(i) 60 sides      (j) 72 sides      (k) 90 sides      (l) 200 sides

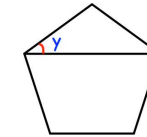
Question 8: Shown below is one interior angle from regular polygons.  
Calculate how many sides the polygons have.



Question 1: In each diagram below, two regular polygons are shown.  
Calculate  $x$ .



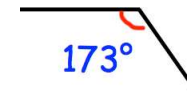
Question 2: Shown is a regular pentagon.  
Find  $y$ .



Question 3: A regular polygon has 18 sides.  
Calculate the size of each interior angle.

Question 4: A regular polygon has 30 sides.  
Calculate the size of each interior angle.

Question 5: Explain why this cannot be an interior angle from regular polygons.

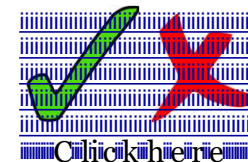


Question 6: A polygon has an interior angle that is  $\sqrt{6}$  times larger than the exterior angle.  
How many sides does it have?

Question 7: Explain why regular hexagons tessellate.

Question 8: Explain why regular pentagons do not tessellate.

Answers



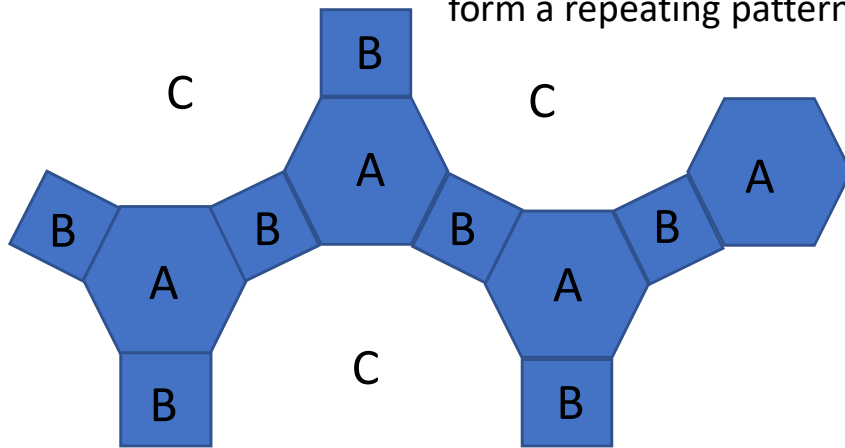
Scan here

# Problem Solving with Interior/Exterior Angles

There are variety of skills that harder questions involving interior/exterior angles might involve:

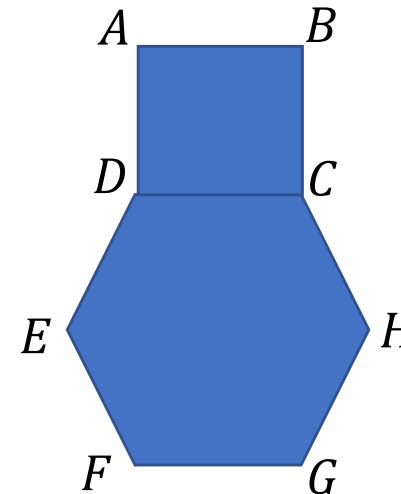
## #1: Tessellation

Shapes 'tessellate' if they fit together, without overlap, to form a repeating pattern.



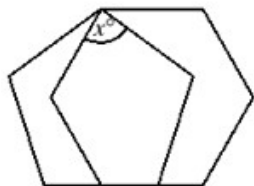
"The above repeating pattern consists of three regular polygons, A (hexagon), B (square) and C. Determine how many sides C has."

## #2: Using isosceles triangles



" $ABCD$  is a square and  $CDEFGH$  is a regular hexagon. Determine the angle  $CBH$ ."

# Problem Solving with Interior/Exterior Angles



[IMC 2006 Q19] The diagram shows a regular pentagon and a regular hexagon which overlap. What is the value of  $x$ ?

Corrections/notes:

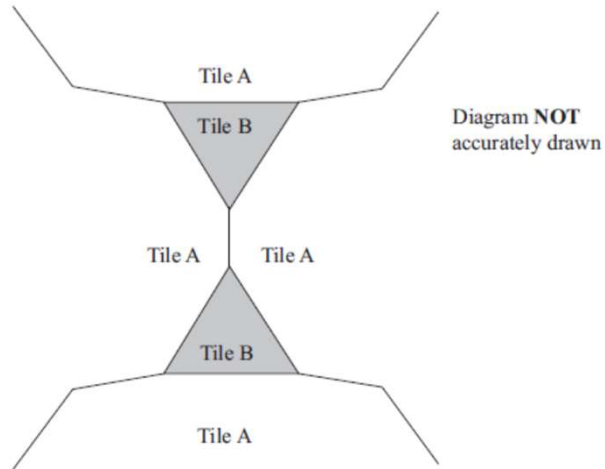
Your attempt

# Problem Solving with Interior/Exterior Angles

[Edexcel GCSE Nov2012-1H Q18] The pattern is made from two types of tiles, tile A and tile B.

Both tile A and tile B are regular polygons.

Work out the number of sides tile A has.

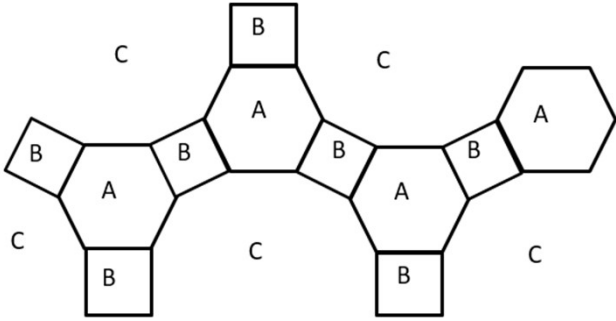


Your attempt

Corrections/notes:

### Worked Example

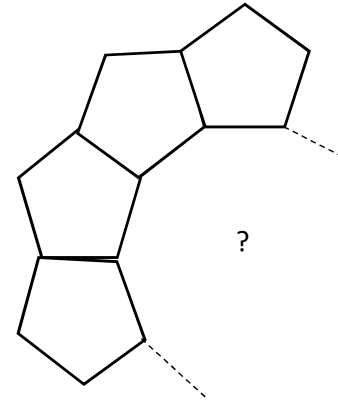
This repeating pattern consists of three regular polygons, A (hexagon), B (square) and C. Determine how many sides C has.



### Thinking

### Your Turn

The diagram shows 4 congruent regular pentagons that form the sides of an  $n$ -sided regular polygon. Determine the value of  $n$ .





## Worked Example

[Edexcel GCSE June 2016-2H Q12] The diagram shows a regular pentagon.  $AB$  and  $CD$  are two of the lines of symmetry of the pentagon. Work out the size of the angle marked  $x$ .

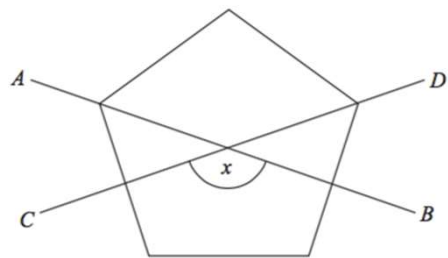
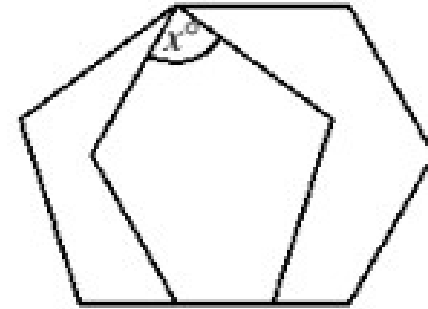


Diagram **NOT** accurately drawn

## Thinking

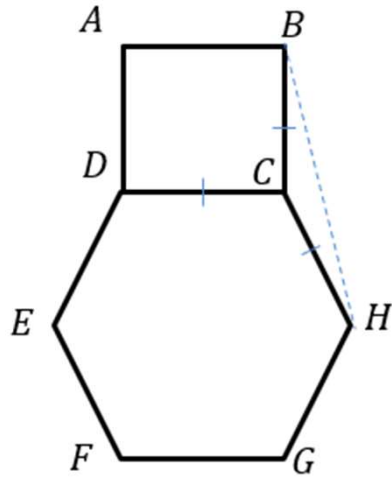
## Your Turn

The diagram shows a regular pentagon and a regular hexagon which overlap. What is the value of  $x$ ?



### Worked Example

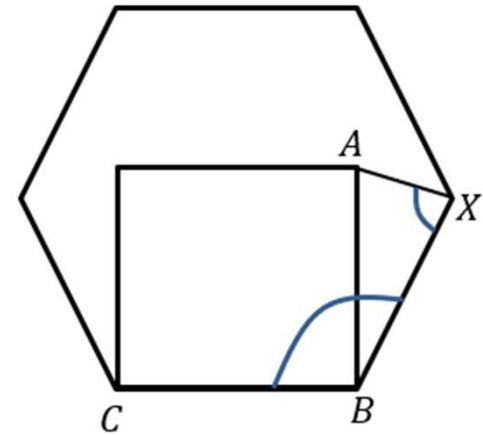
$ABCD$  is a square and  $CDEFGH$  is a regular hexagon. Determine the angle  $CBH$ .



### Thinking

### Your Turn

The diagram shows a square inside a regular hexagon. What is the size of the marked angle at  $X$ ?

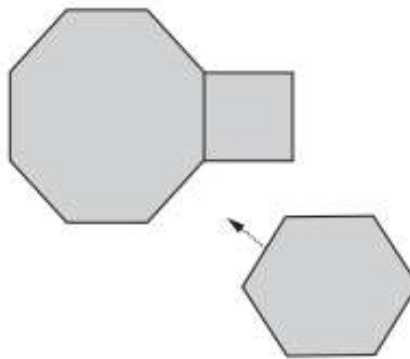


# Exercise 3

1

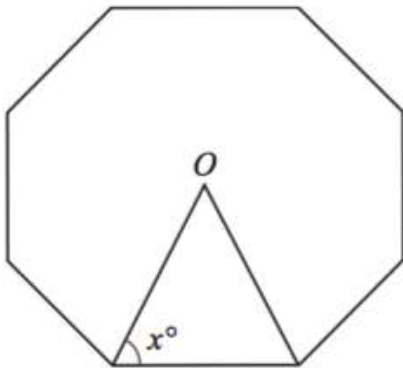
[KS3 SATs 2004 L6-L8 Paper 2 Q19 Edited]

A pupil has three tiles. One is a regular octagon, one is a regular hexagon, and one is a square. The side length of each tile is the same. The pupil says the hexagon will fit exactly like this. Is the pupil correct?



?

2



[Edexcel IGCSE Nov2009-3H Q3a]

The diagram shows a regular octagon, with centre O. Work out the value of  $x$ .

?

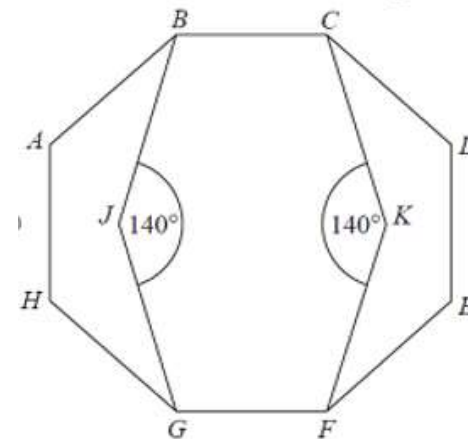
3

[Edexcel IGCSE Nov-2010-4H Q13]

The size of each interior angle of a regular polygon is 11 times the size of each exterior angle. Work out the number of sides the polygon has.

?

4

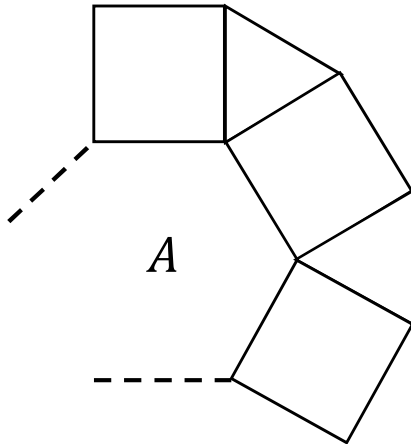


[Edexcel GCSE Nov2014-1H Q17] ABCDEFGH is a regular octagon. BCKFGJ is a hexagon. JK is a line of symmetry of the hexagon. Angle  $BJG =$  angle  $CKF = 140^\circ$ . Work out the size of angle KFE.

?

# Exercise 3

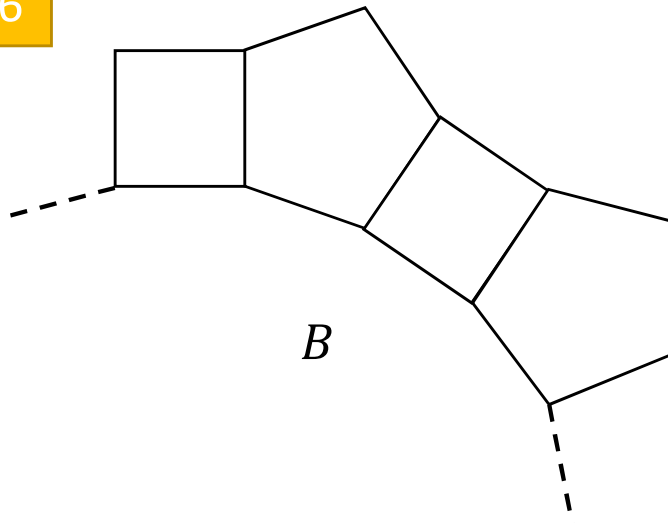
5



A regular polygon  $A$  is surrounded by squares and equilateral triangles in an alternating pattern, as shown. Show that  $A$  is a hexagon.

?

6

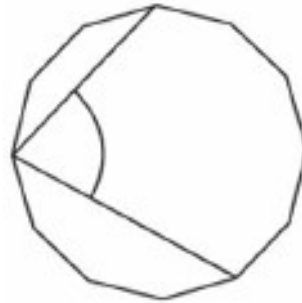


A regular polygon  $B$  with  $n$  sides is surrounded by squares and regular pentagons in an alternating pattern, as shown. Determine the value of  $n$ .

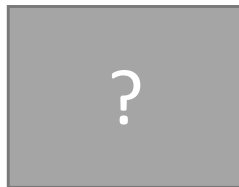
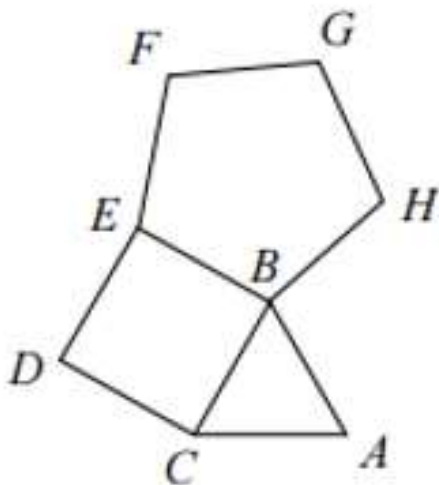
?

# Exercise 3

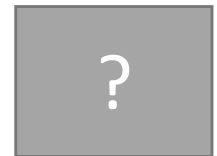
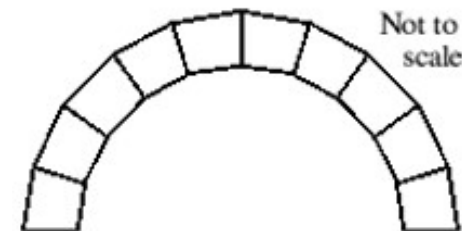
- 7 [IMC 2003 Q22] The diagram shows a regular dodecagon (a polygon with twelve equal sides and equal angles). What is the size of the marked angle?



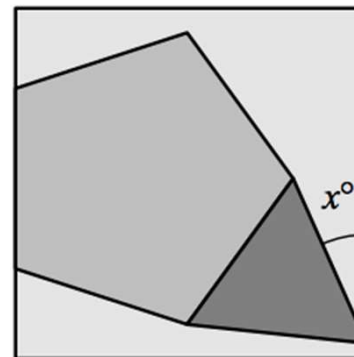
- 8 [JMO 2014 B1] The figure shows an equilateral triangle ABC, a square BCDE, and a regular pentagon BEFGH. What is the difference between the sizes of  $\angle ADE$  and  $\angle AHE$ ?



- 9 [IMC 2005 Q14] Ten stones, of identical shape and size, are used to make an arch, as shown in the diagram. Each stone has a cross-section in the shape of a trapezium with three equal sides. What is the size of the smallest angles of the trapezium?



10



- [IMC 2018 Q18] The diagram shows a regular pentagon and an equilateral triangle placed inside a square. What is the value of  $x$ ?



# Exercise 3

11

Find all regular polygons which tessellate (when restricted only to one type of polygon).

?

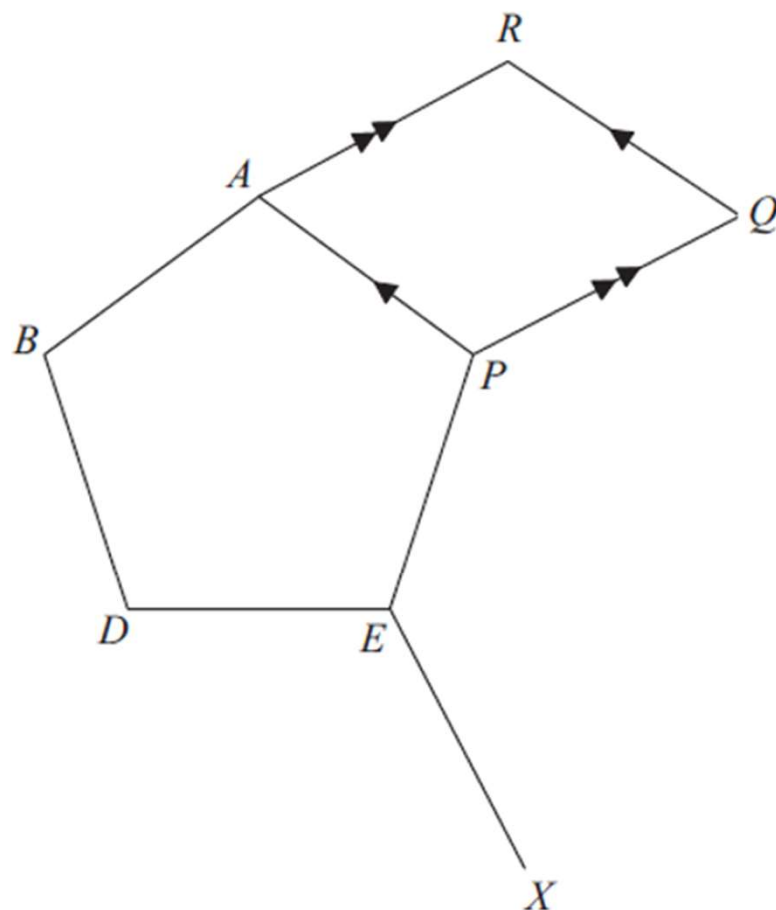


By thinking about interior angles, prove that the regular polygons you identified above are the only regular polygons which tessellate.

?

# Exam Questions

Edexcel (Linked Pair Pilot).



$ABDEP$  is a regular pentagon.

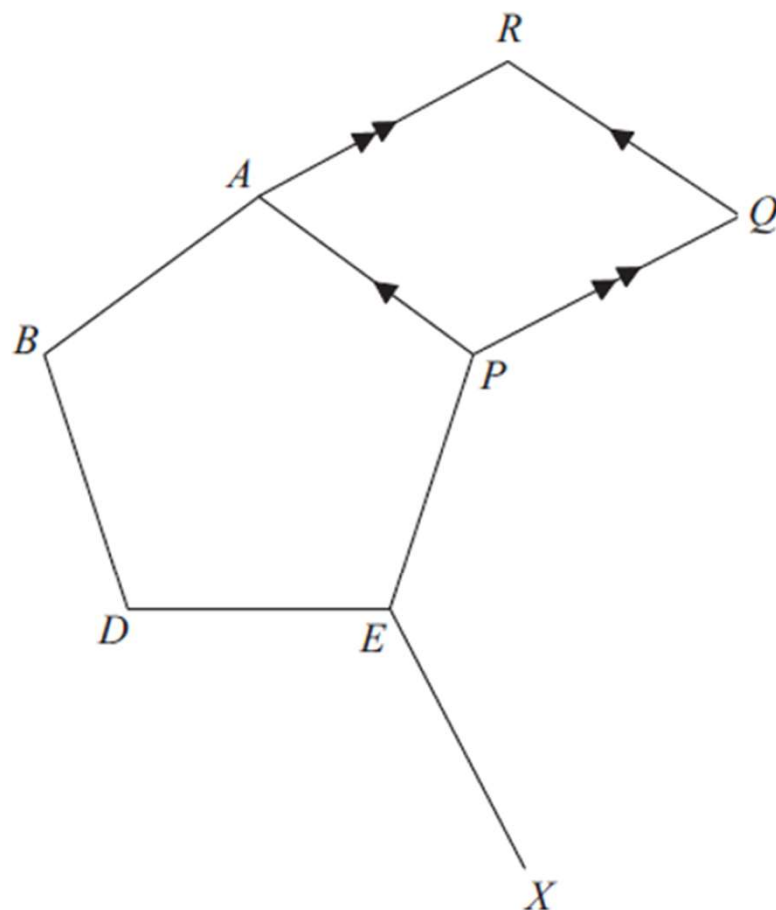
$QPEX$  is part of a regular octagon.

$PARQ$  is a parallelogram.

Calculate the size of angle  $PAR$ .

# Exam Questions

Edexcel (Linked Pair Pilot).



$ABDEP$  is a regular pentagon.

$QPEX$  is part of a regular octagon.

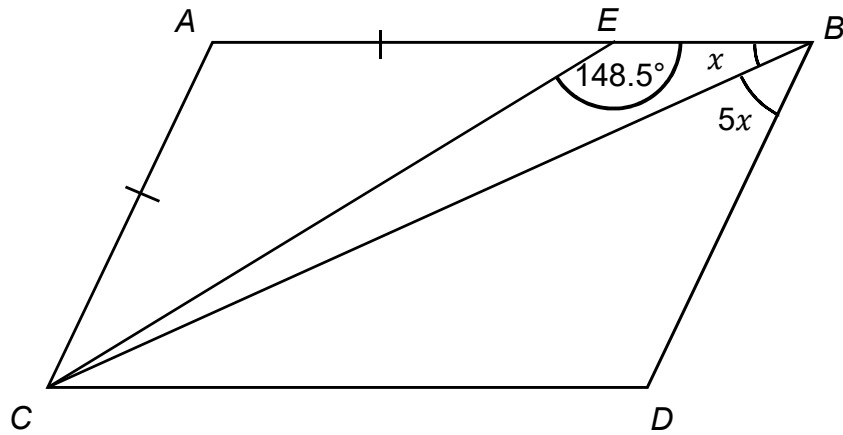
$PARQ$  is a parallelogram.

Calculate the size of angle  $PAR$ .





- 1  $ABCD$  is a parallelogram.  
 $AE = AC$



Not drawn accurately

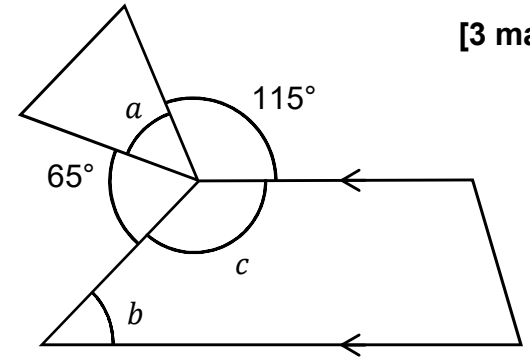
Work out the size of angle  $x$ .

[4 marks]

- 1 The diagram shows a triangle and a trapezium.

[3 marks]

Prove that  $a = b$




---



---



---



---

## Rotations

**A transformation that turns all points through a given angle, in a given direction, around a given centre.**

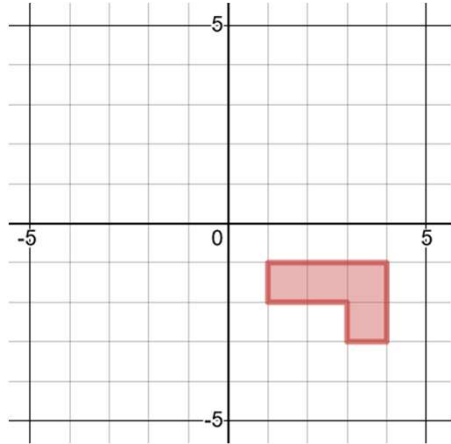
- Shapes turn around a centre point.
- Produces a congruent shape.

To fully describe a rotation you need to give four pieces of information:

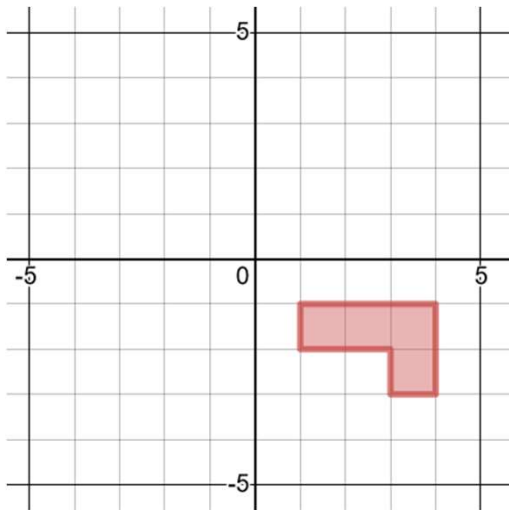
1. Type of Transformation: Rotation
2. Angle (in degrees):  $90^\circ$ ,  $180^\circ$ ,  $270^\circ$
3. Direction: Clockwise or Anticlockwise
4. Centre of Rotation: Coordinate  $(x, y)$

## Worked Example

Rotate  $90^\circ$  clockwise about the origin

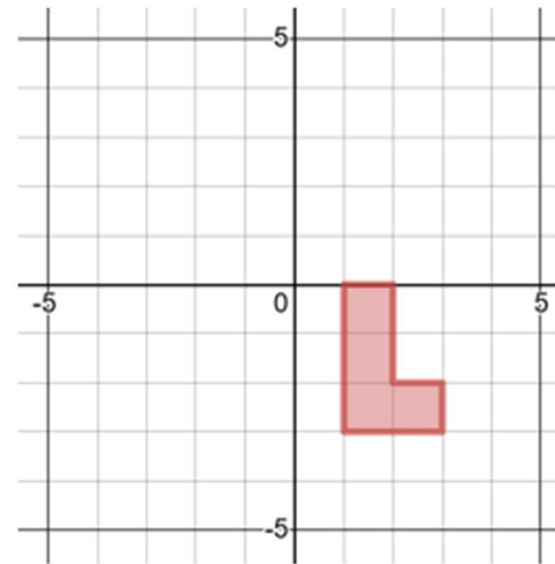


Rotate  $90^\circ$  anticlockwise about the origin

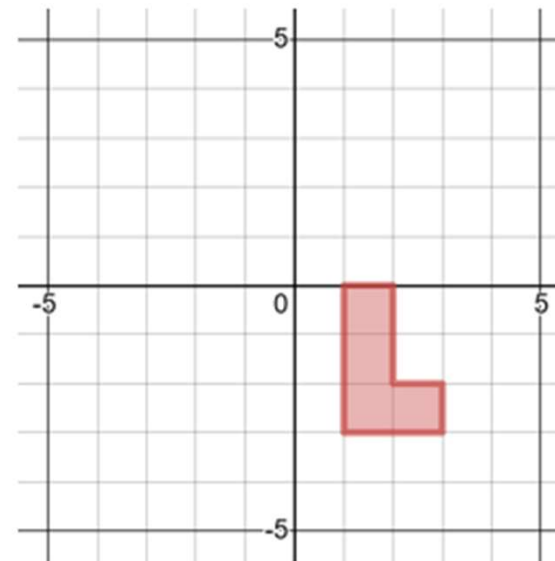


## Your Turn

Rotate  $90^\circ$  clockwise about the origin

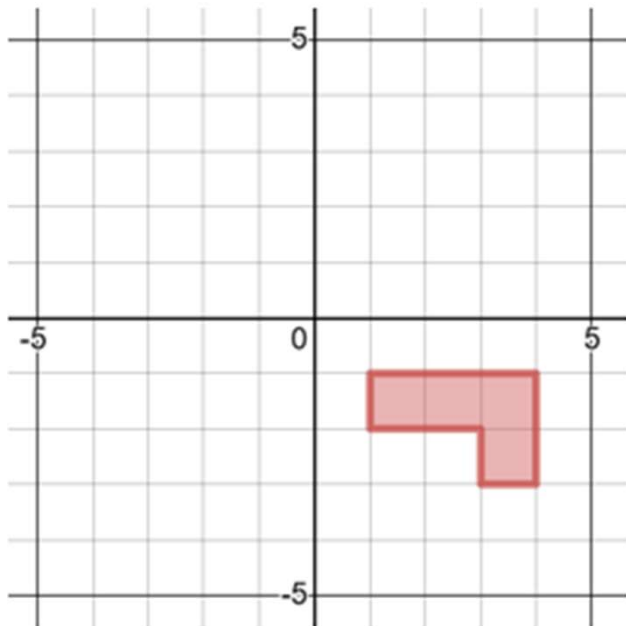


Rotate  $90^\circ$  anticlockwise about the origin

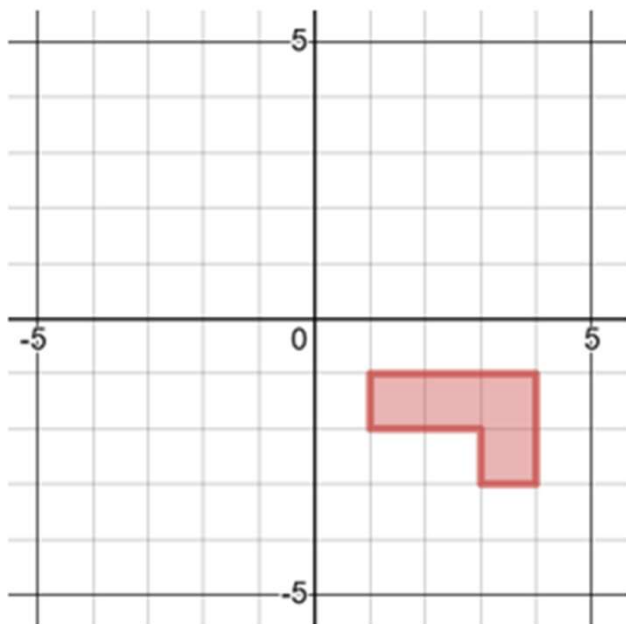


## Worked Example

Rotate  $90^\circ$  clockwise about  $(1, -1)$

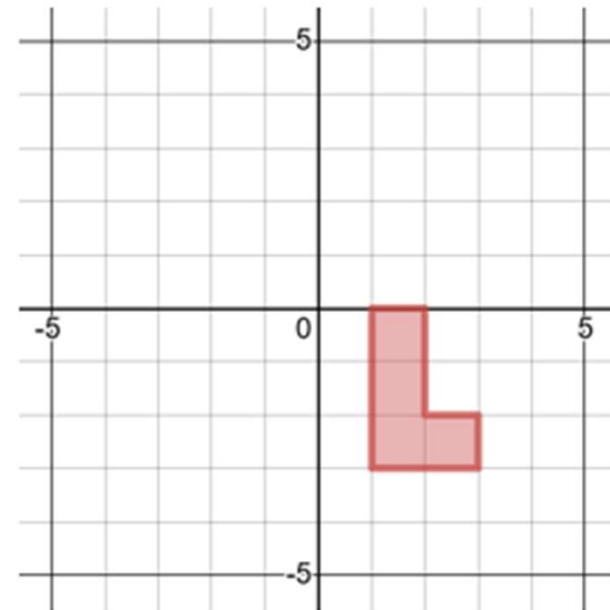


Rotate  $90^\circ$  anticlockwise about  $(1, -1)$

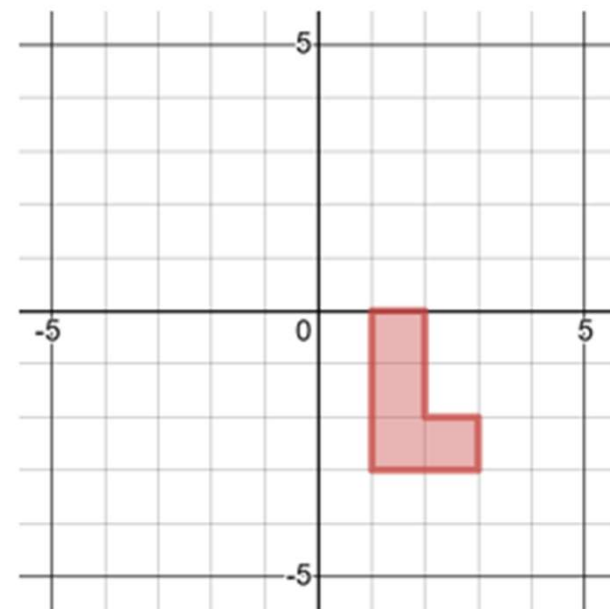


## Your Turn

Rotate  $90^\circ$  clockwise about  $(2, 1)$

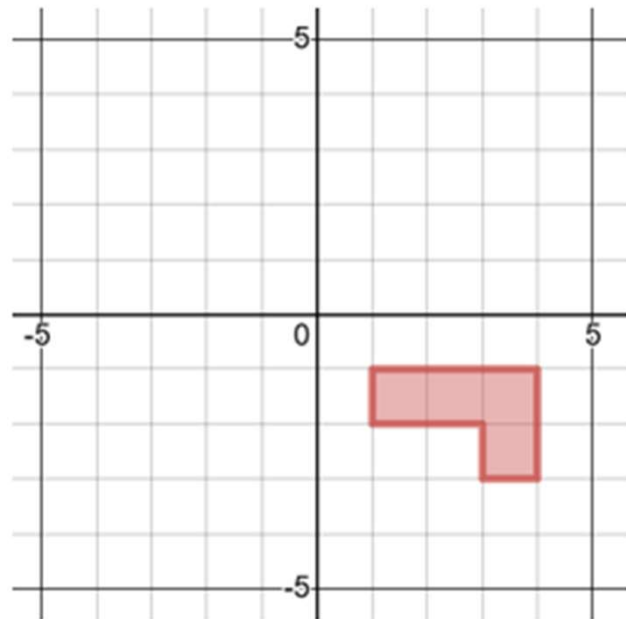


Rotate  $90^\circ$  anticlockwise about  $(-1, 1)$

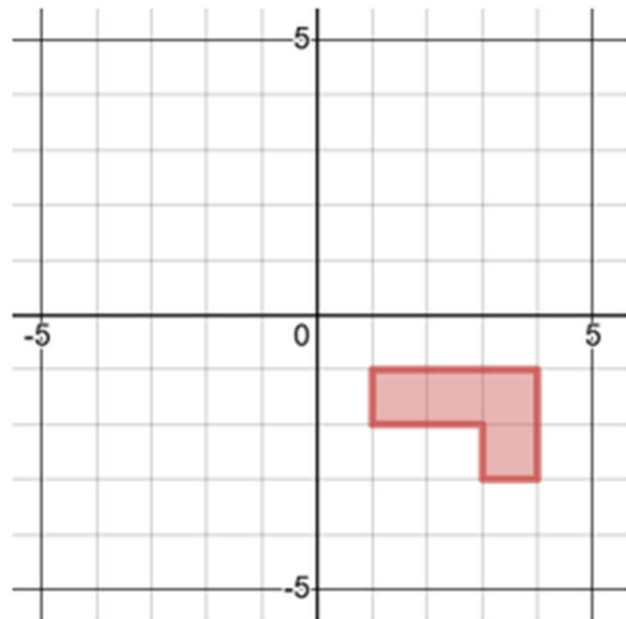


## Worked Example

Rotate  $180^\circ$  about the origin

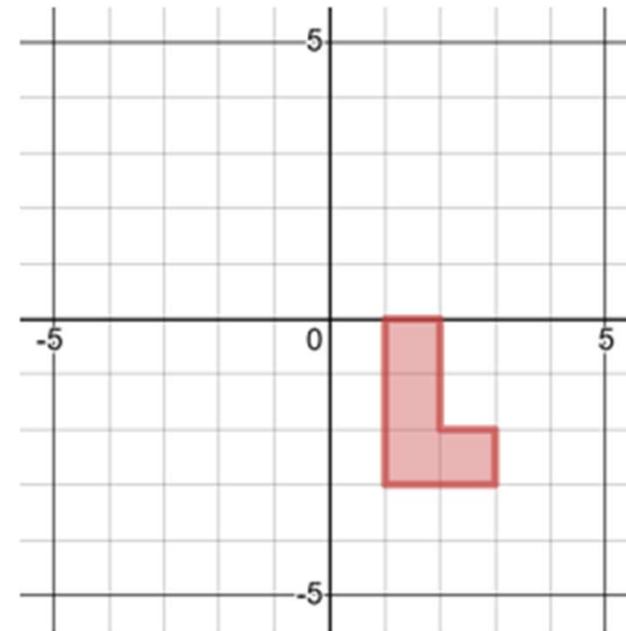


Rotate  $180^\circ$  about  $(1, -1)$

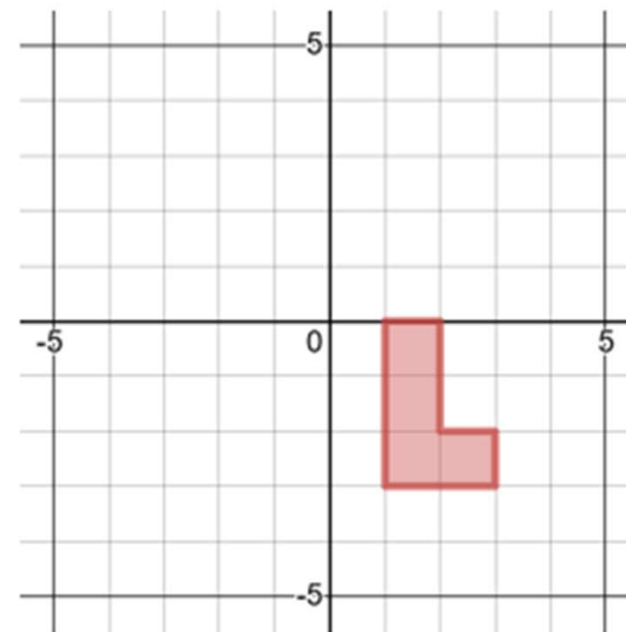


## Your Turn

Rotate  $180^\circ$  about the origin

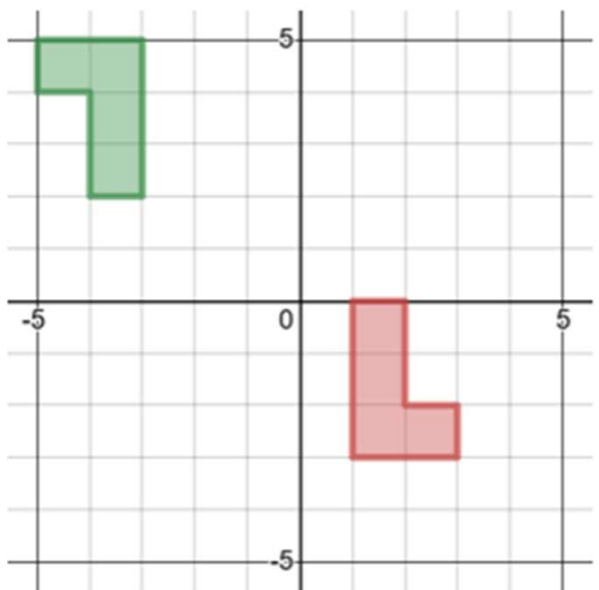
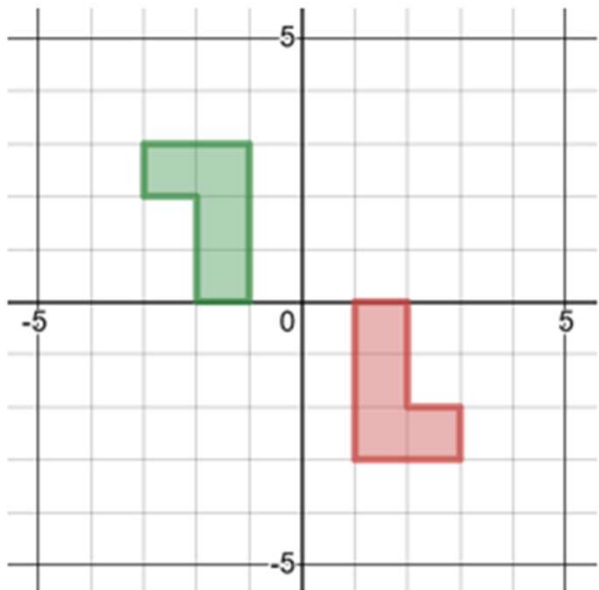


Rotate  $180^\circ$  about  $(-1, 1)$



### Worked Example

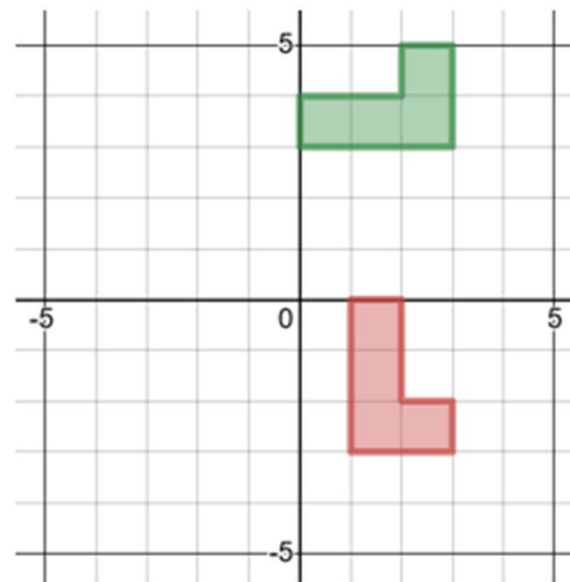
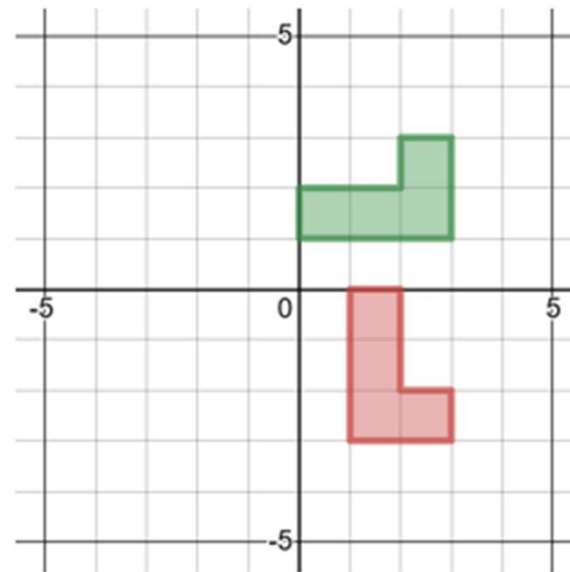
Describe the single transformation of the red object onto the green image



### Thinking

### Your Turn

Describe the single transformation of the red object onto the green image



# Rotations

Video 275 on [www.corbettmaths.com](http://www.corbettmaths.com)



Scan here

Question 1: Rotate each of the shapes below as instructed, using P as the centre of rotation.

<p>(a)</p> <p>rotate 90° clockwise about P</p>	<p>(b)</p> <p>rotate 90° anticlockwise about P</p>	<p>(c)</p> <p>rotate 90° clockwise about P</p>
<p>(d)</p> <p>rotate 180° about P</p>	<p>(e)</p> <p>rotate 90° anticlockwise about P</p>	<p>(f)</p> <p>rotate 180° about P</p>
<p>(g)</p> <p>rotate 90° clockwise about P</p>	<p>(h)</p> <p>rotate 270° clockwise about P</p>	<p>(i)</p> <p>rotate 270° anticlockwise about P</p>



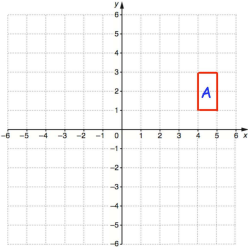
# Rotations

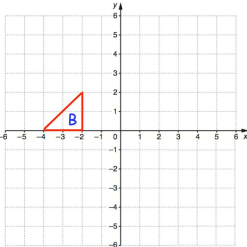
Video 275 on [www.corbettmaths.com](http://www.corbettmaths.com)

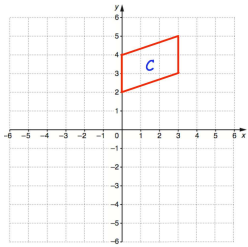
Question 2: Rotate each of the shapes below as instructed, using the origin, (0,0), as the centre of rotation.

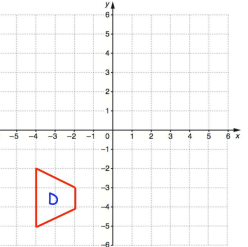
<p>(a)</p> <p>rotate 180° about (0, 0)</p>	<p>(b)</p> <p>rotate 90° clockwise about (0, 0)</p>	<p>(c)</p> <p>rotate 90° anticlockwise about (0, 0)</p>
<p>(d)</p> <p>rotate 90° clockwise about (0, 0)</p>	<p>(e)</p> <p>rotate 90° anticlockwise about (0, 0)</p>	<p>(f)</p> <p>rotate 180° about (0, 0)</p>
<p>(g)</p> <p>rotate 90° anticlockwise about (0, 0)</p>	<p>(h)</p> <p>rotate 180° about (0, 0)</p>	<p>(i)</p> <p>rotate 90° clockwise about (0, 0)</p>

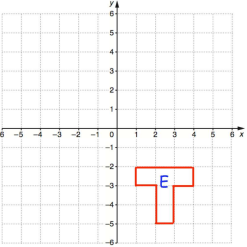
Question 3: Rotate each of the shapes below as instructed.

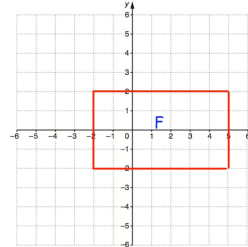
(a)  rotate 90° anticlockwise about (0, 1)

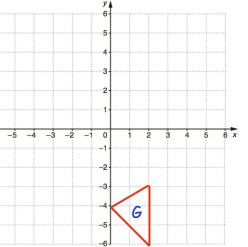
(b)  rotate 90° clockwise about (-1, -2)

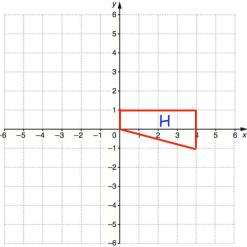
(c)  rotate 180° about (1, 1)

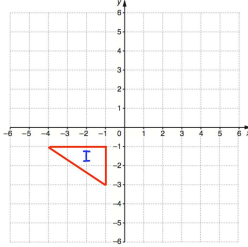
(d)  rotate 90° anticlockwise about (-4, 0)

(e)  rotate 180° about (-1, 0)

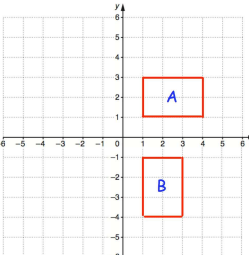
(f)  rotate 90° clockwise about (-1, 2)

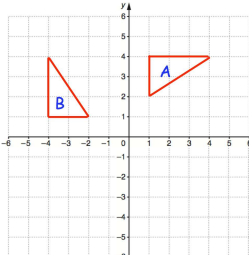
(g)  rotate 90° clockwise about (5, 0)

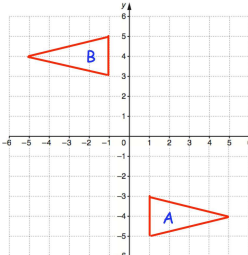
(h)  rotate 90° anticlockwise about (3, 0)

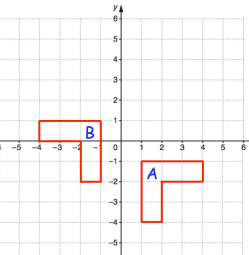
(i)  rotate 180° about (1, 1)

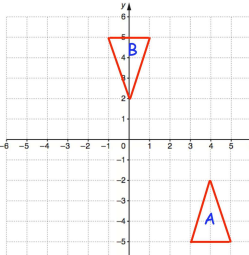
Question 4: Describe fully the single transformation that takes shape A to shape B.

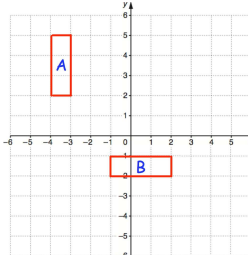
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

Answers

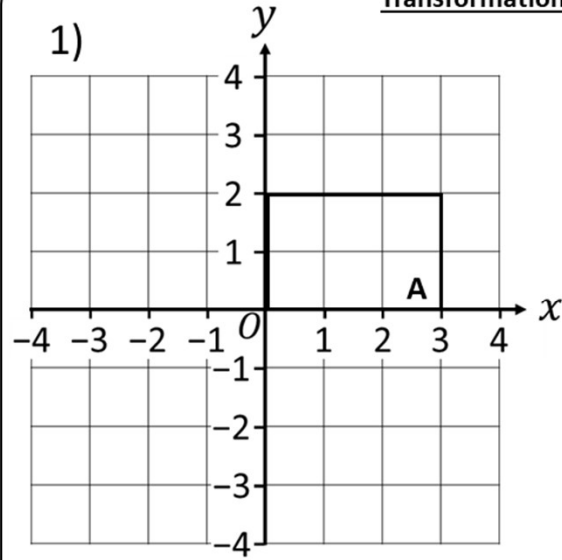


Scan here

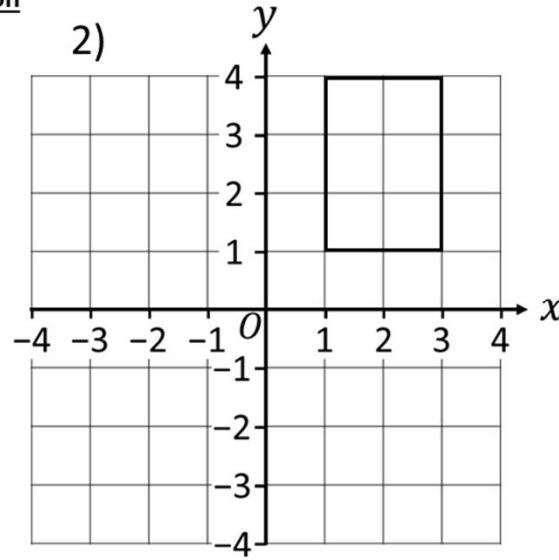


Transformation: Rotation

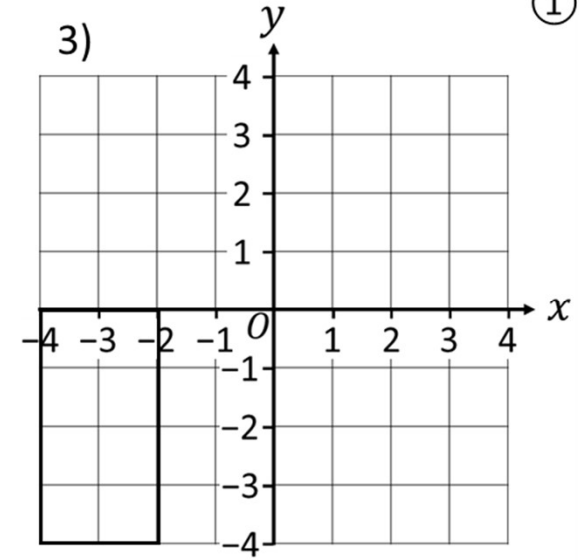
①



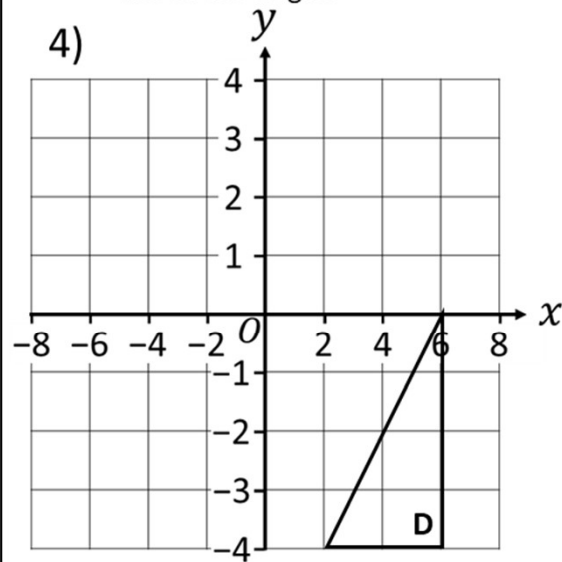
Rotate Shape **A**  $90^\circ$  clockwise about the origin.



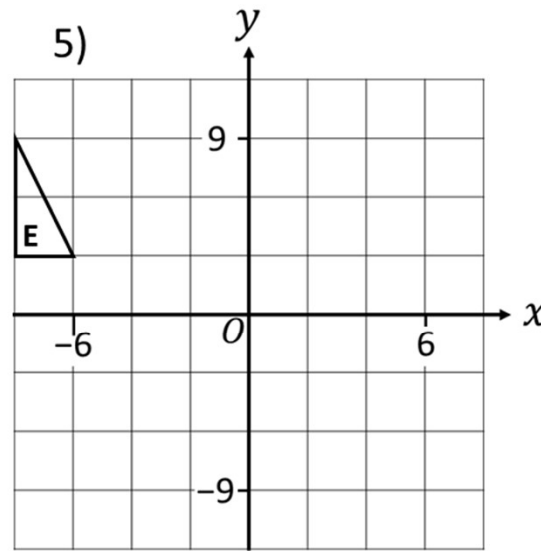
Rotate Shape **B**  $180^\circ$  about  $(0,1)$ .



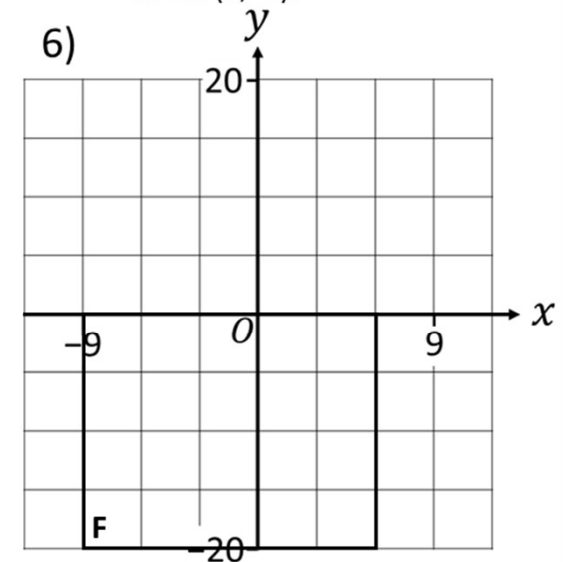
Rotate Shape **C**  $90^\circ$  clockwise about  $(0,-1)$ .



Rotate Shape **D**  $90^\circ$  clockwise about  $(6,2)$ .



Rotate Shape **A**  $90^\circ$  clockwise about  $(-4,-6)$ .

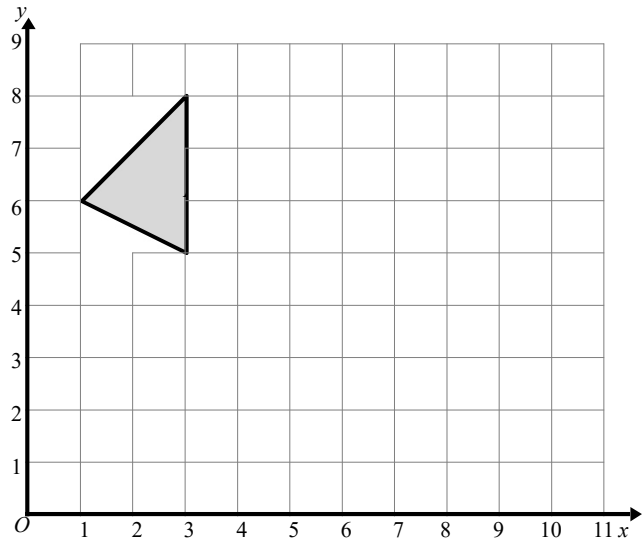


Rotate Shape **A**  $270^\circ$  anticlockwise about  $(3,-5)$ .



# EXAM QUESTIONS

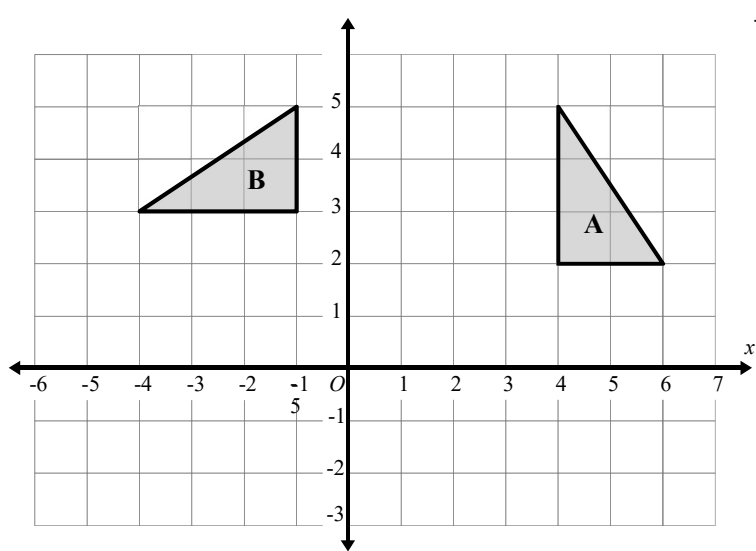
1



Rotate triangle A  $90^\circ$  clockwise about  $(4,3)$ .

(Total for question 1 is 2 marks)

2

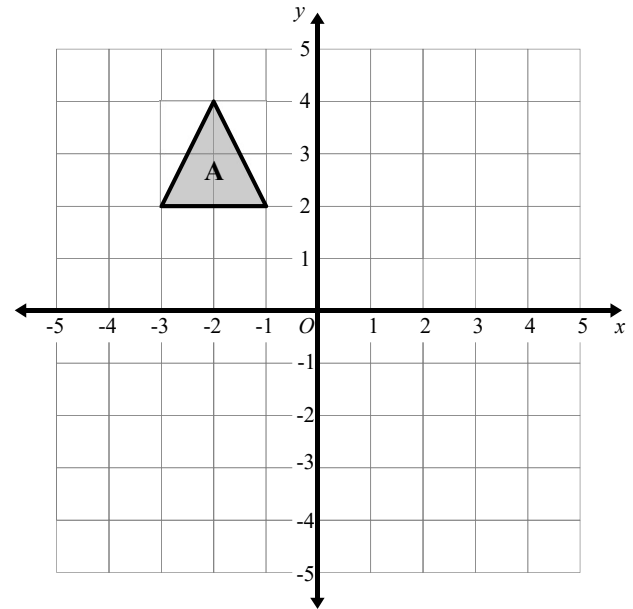


Describe fully the single transformation that maps triangle A on triangle B.

.....

(Total for question 2 is 2 marks)

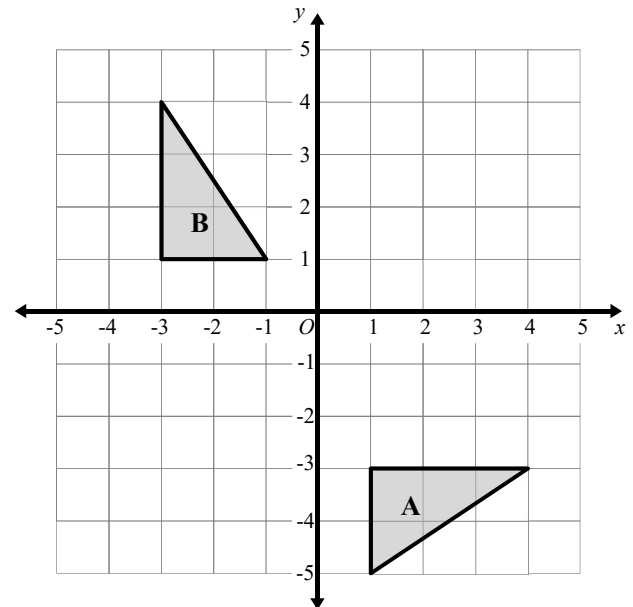
3



Rotate shape A  $180^\circ$  about  $(1, 0)$

(Total for question 3 is 2 marks)

4

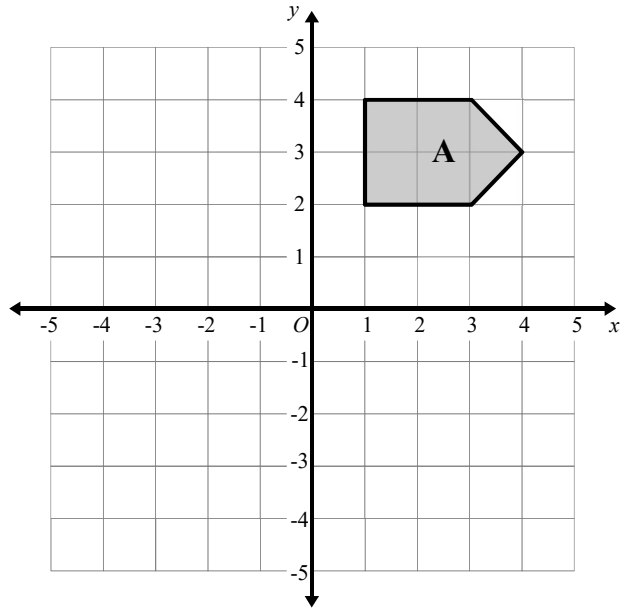


Describe fully the single transformation that maps triangle A on triangle B.

.....

(Total for question 4 is 2 marks)

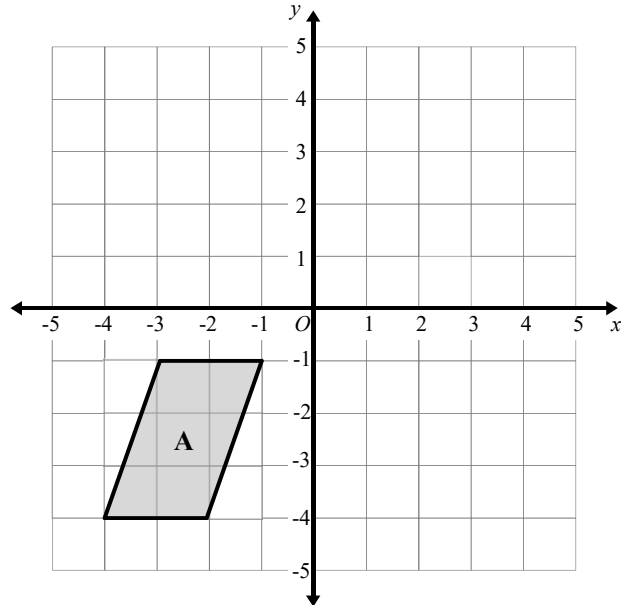
5



Rotate shape A  $180^\circ$  about  $O$ .

(Total for question 5 is 2 marks)

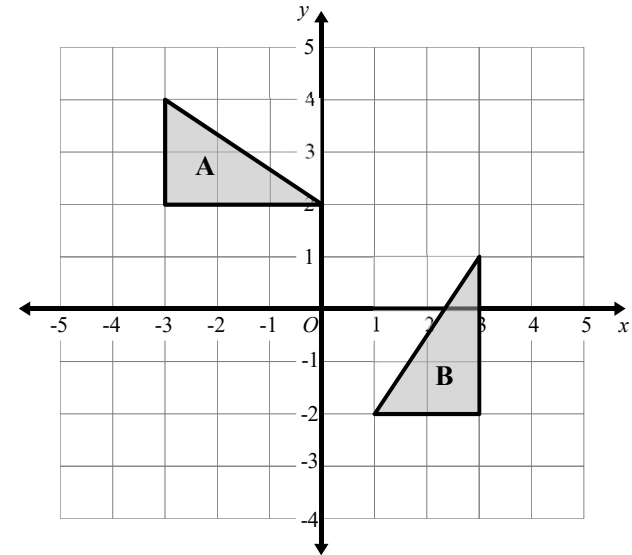
6



Rotate shape A  $90^\circ$  clockwise about centre  $O$ .

(Total for question 6 is 2 marks)

7

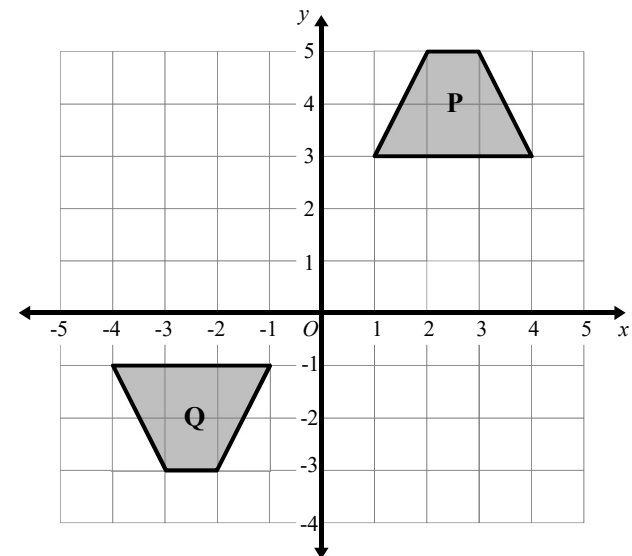


Describe fully the single transformation that maps triangle A on triangle B.

.....

(Total for question 7 is 2 marks)

8



Describe fully the single transformation that maps triangle P on triangle Q.

.....

(Total for question 8 is 2 marks)

## Translations

**A transformation that moves all points the same fixed distance.**

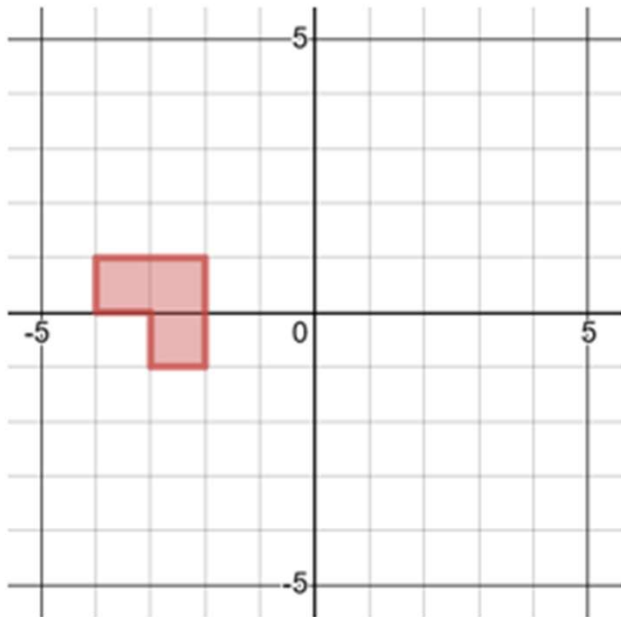
- Shapes move or “slide” a distance horizontally and/or vertically.
- On a rectangular grid, often described using a column vector.

To fully describe a translation, you need to give two pieces of information:

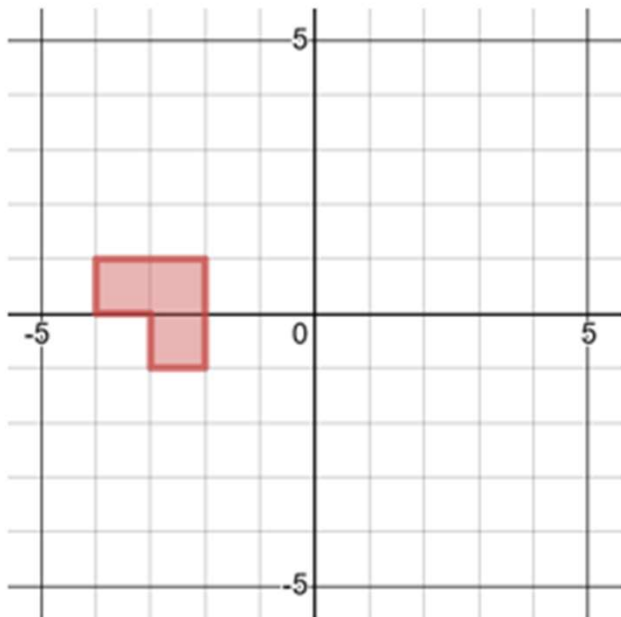
1. Type of Transformation: Translation
2. Column Vector:  $\begin{pmatrix} x \\ y \end{pmatrix}$  where  $x$  is movement right or left and  $y$  is movement up or down.  
Right and up are taken to be positive.

## Worked Example

Translate by vector  $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

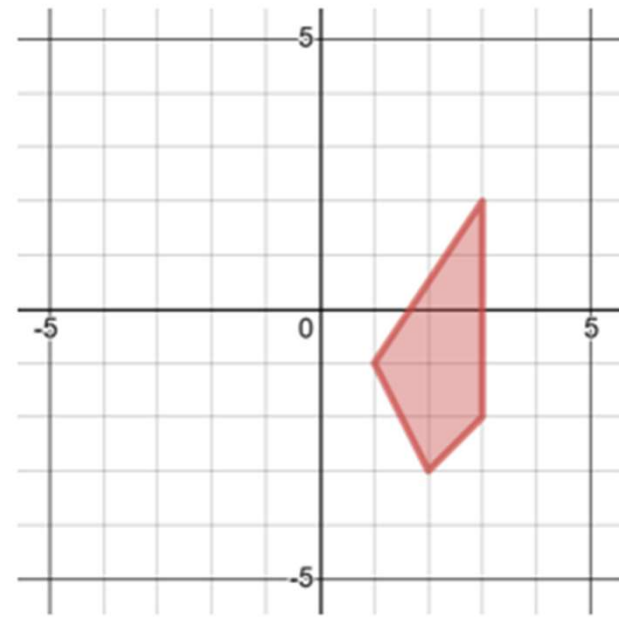


Translate by vector  $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

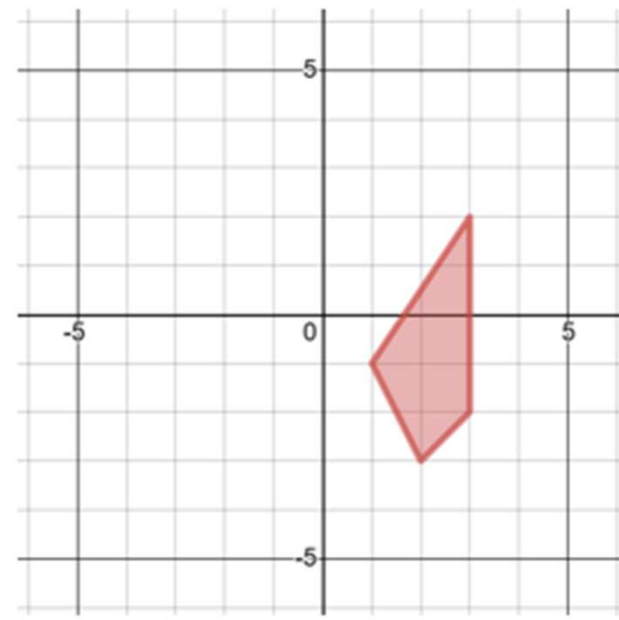


## Your Turn

Translate by vector  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

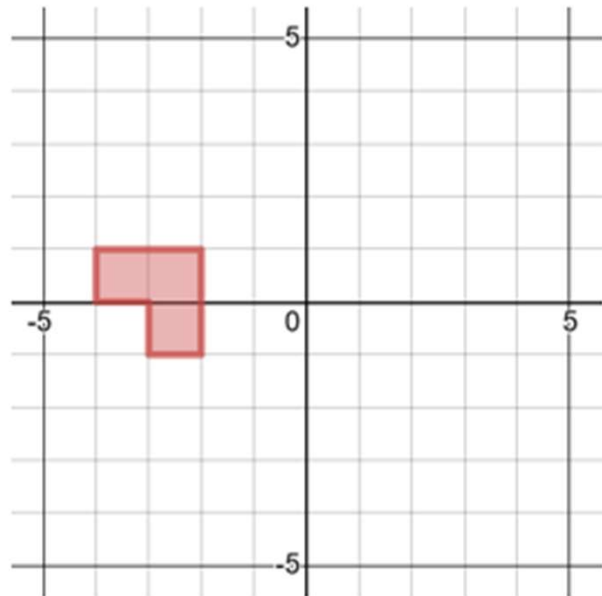


Translate by vector  $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

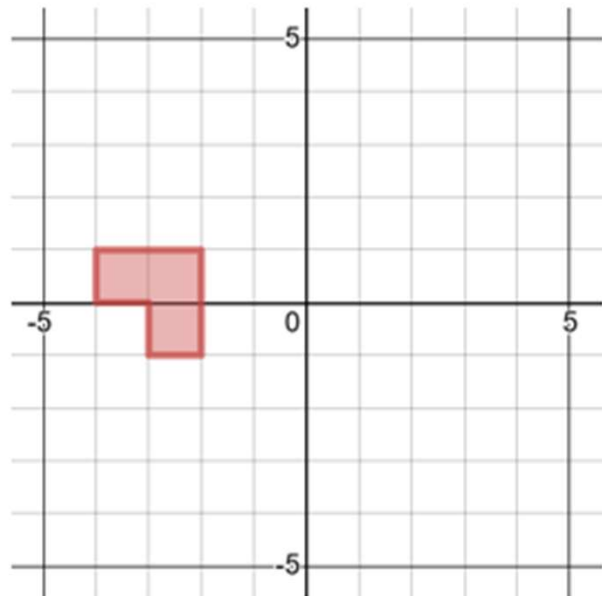


## Worked Example

Translate by vector  $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$

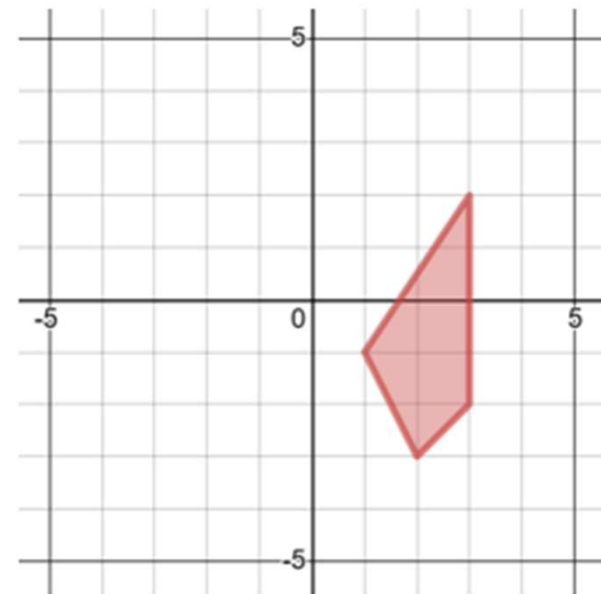


Translate by vector  $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$  and then by vector  $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$

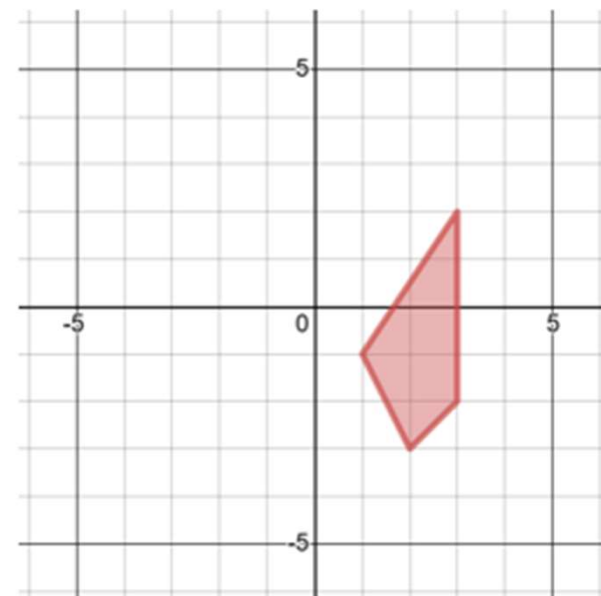


## Your Turn

Translate by vector  $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$

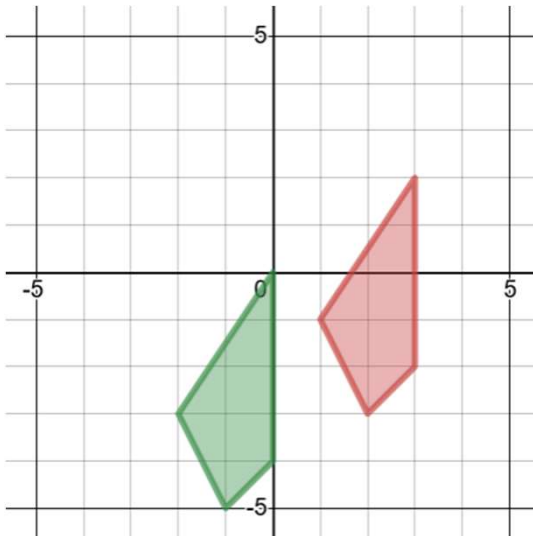
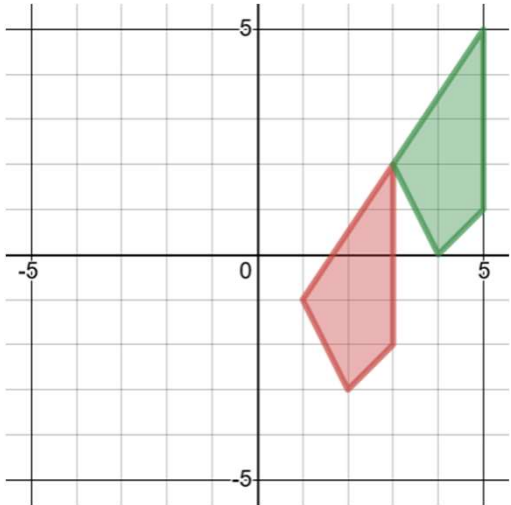


Translate by vector  $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$  and then by vector  $\begin{pmatrix} -5 \\ 4 \end{pmatrix}$



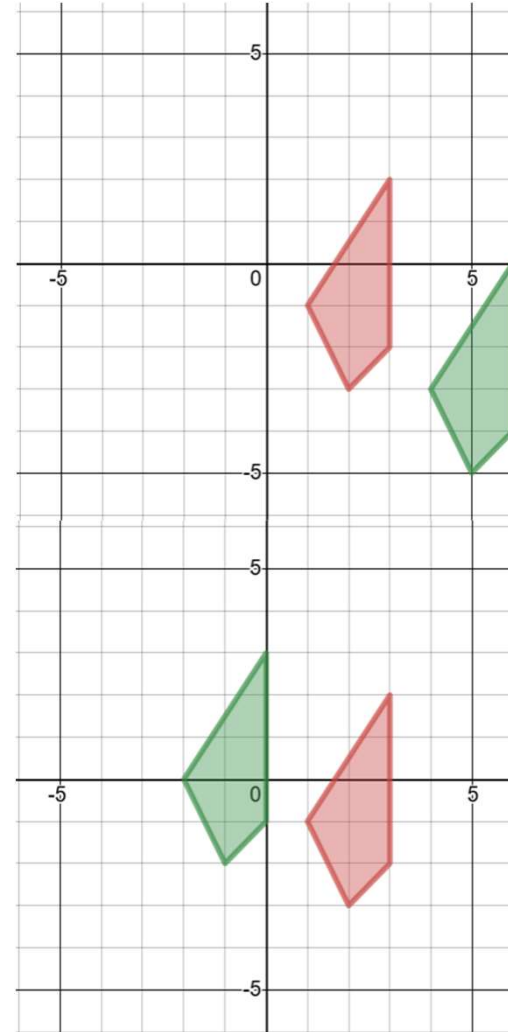
### Worked Example

Describe the single transformation of the red object onto the green image



### Your Turn

Describe the single transformation of the red object onto the green image





### Worked Example

A point  $(2, -5)$  is translated by the vector  $(-3, 7)$ . What is the image of the point after the transformation?

A point  $(11, -13)$  is translated by the vector  $(0, -5)$ . What is the image of the point after the transformation?

### Your Turn

A point  $(-2, 5)$  is translated by the vector  $(7, -3)$ . What is the image of the point after the transformation?

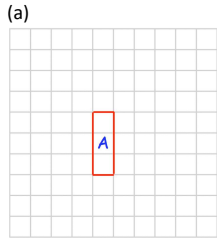
# Translations

Video 325, 326 on [www.corbettmaths.com](http://www.corbettmaths.com)

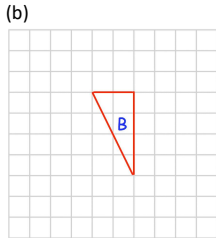


Scan here

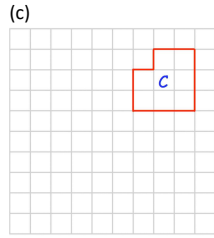
Question 1: Translate each of the shapes below as instructed.



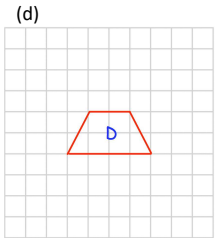
Translate A by  $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$



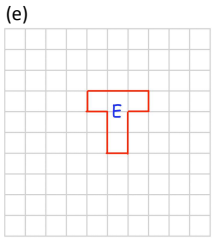
Translate B by  $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$



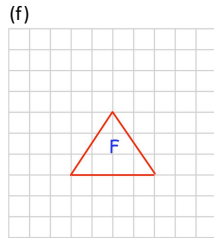
Translate C by  $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$



Translate D by  $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$

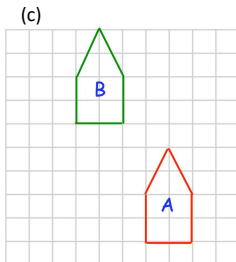
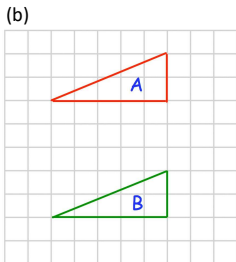
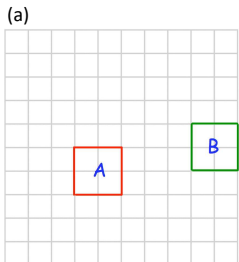


Translate E by  $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$



Translate F by  $\begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$

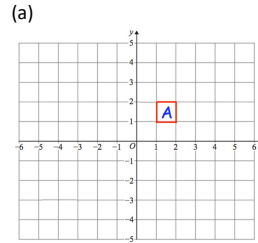
Question 2: Describe fully each translation that takes shape A to shape B



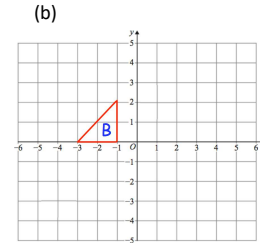
# Translations

Video 325, 326 on [www.corbettmaths.com](http://www.corbettmaths.com)

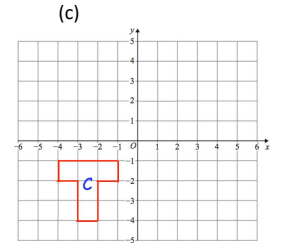
Question 3: Translate each of the shapes below as instructed.



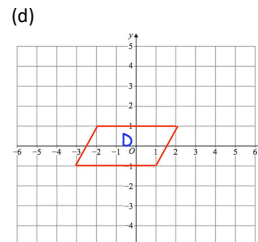
Translate A by  $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$



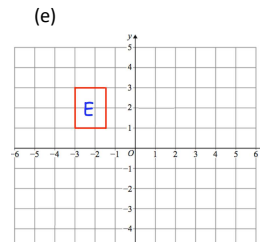
Translate B by  $\begin{pmatrix} 6 \\ 1 \end{pmatrix}$



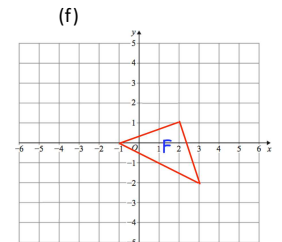
Translate C by  $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$



Translate D by  $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$

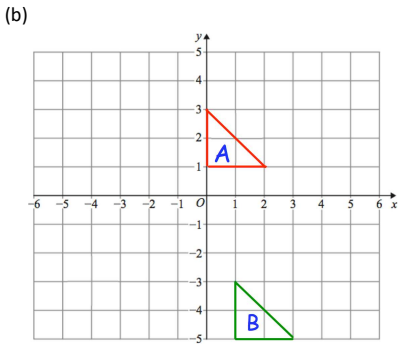
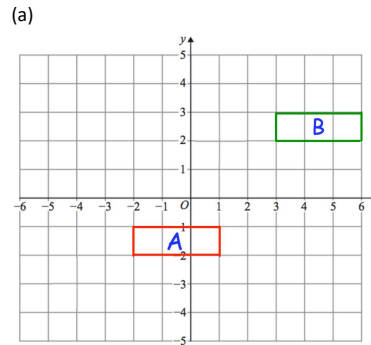


Translate E by  $\begin{pmatrix} 4.5 \\ -4 \end{pmatrix}$



Translate F by  $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

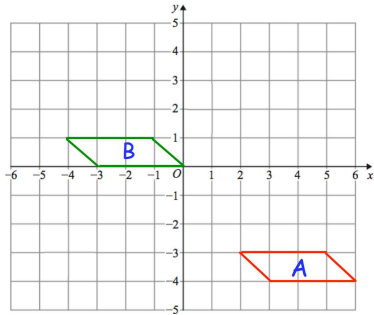
Question 4: Describe fully the single transformation that takes shape A to shape B



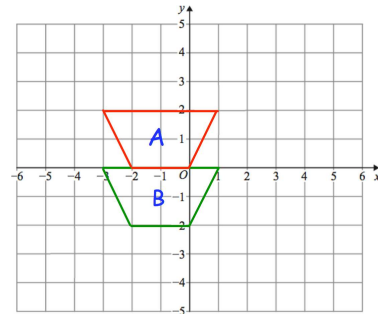
## Translations

Video 325, 326 on [www.corbettmaths.com](http://www.corbettmaths.com)

(c)

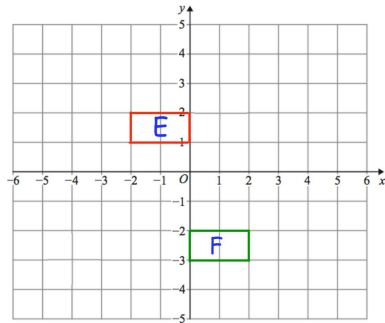


(d)



Question 5: The translation vector to take shape C to shape D is  $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$   
 What translation vector takes shape D to shape C?

Question 6: Edward has been asked to translate shape E by  $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$   
 He has labelled his answer shape F  
 Can you spot any mistakes?



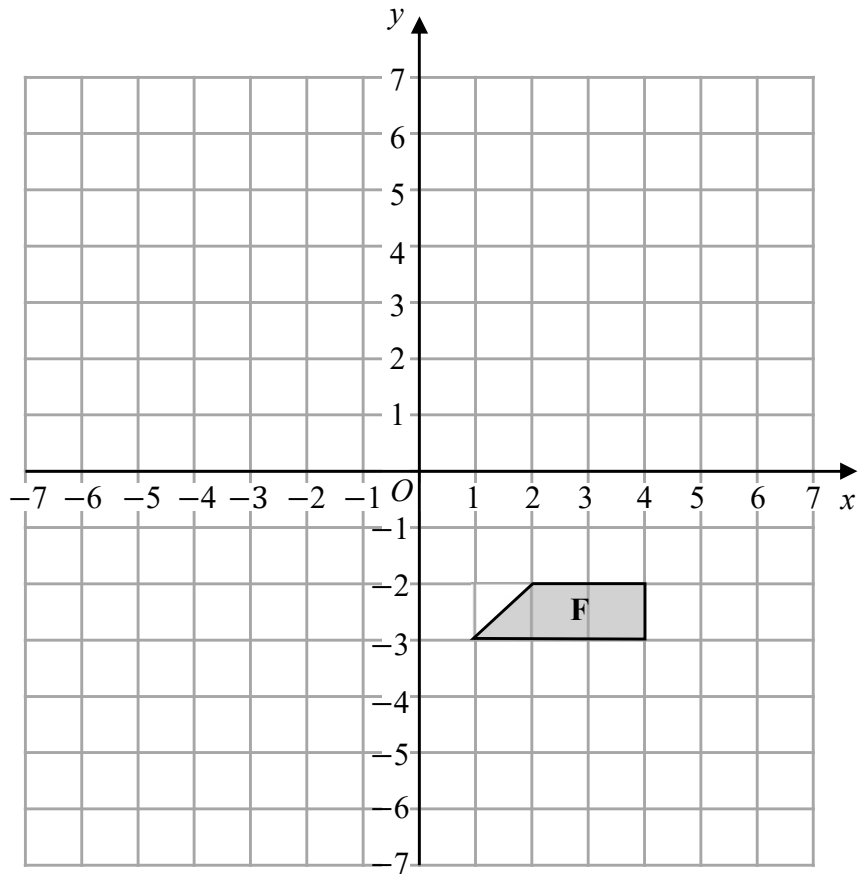
Answers



Scan here



1

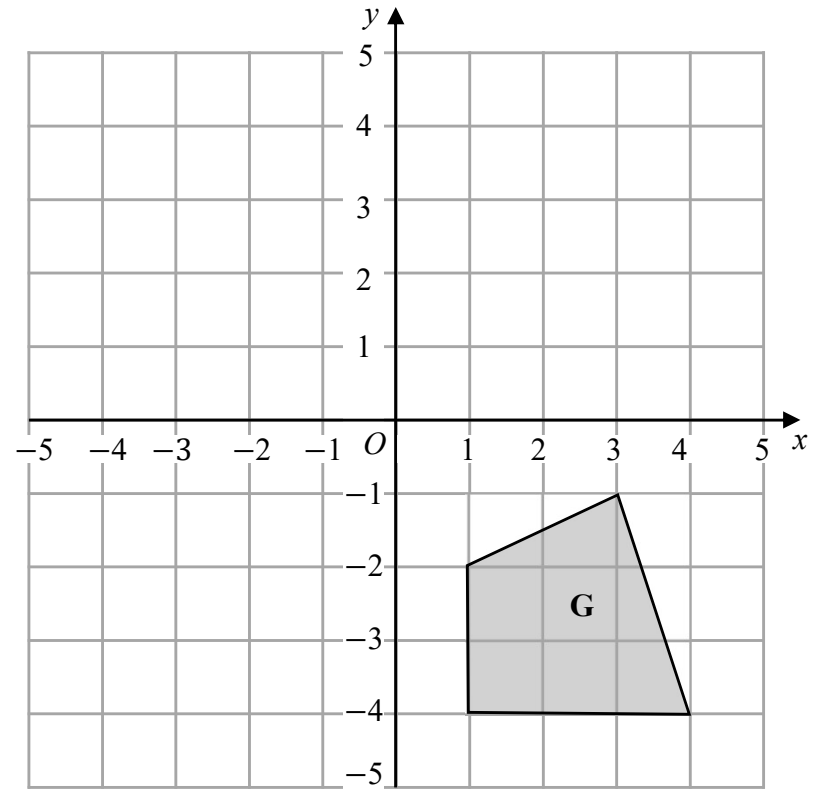


(a) Rotate trapezium **F**  $180^\circ$  about the origin.  
Label the new trapezium **A**. (1)

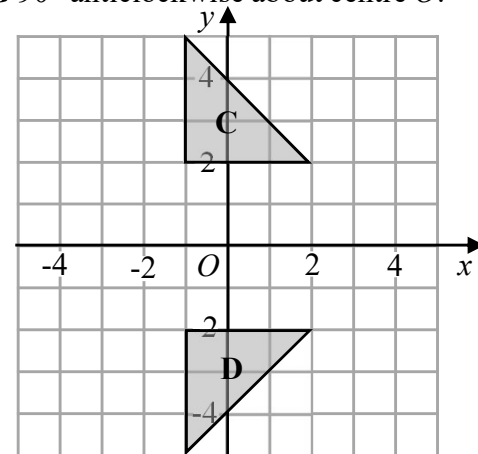
(b) Translate trapezium **F** by the vector  $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ .  
Label the new trapezium **B**. (1)

(Total for Question 1 is 2 marks)

1



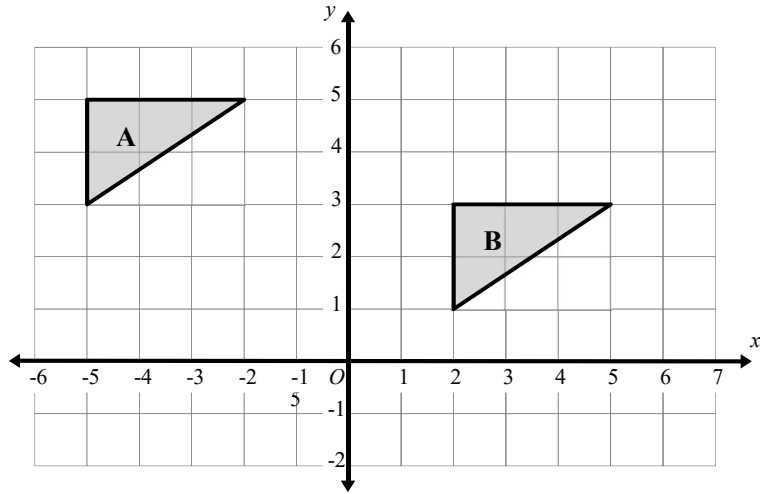
(a) Rotate shape **G**  $90^\circ$  anticlockwise about centre *O*. (2)



(b) Describe fully the single transformation that maps shape **C** onto shape **D**. (1)

(Total for Question 1 is 4 marks) <sup>(2)</sup>

1

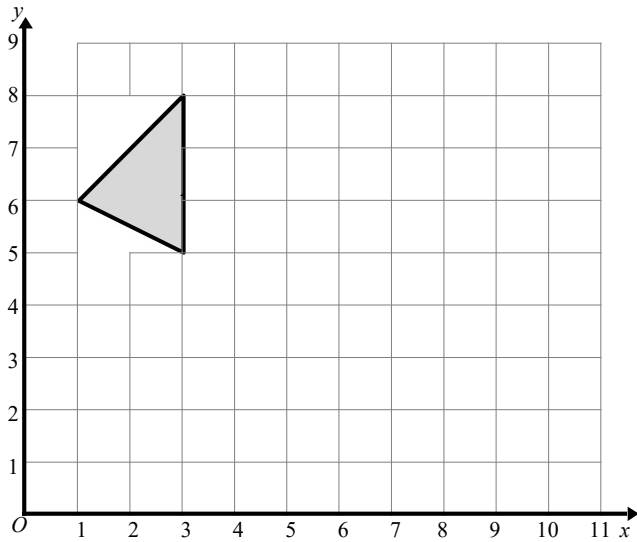


Describe fully the single transformation that maps triangle A on triangle B.

.....

**(Total for question 1 is 2 marks)**

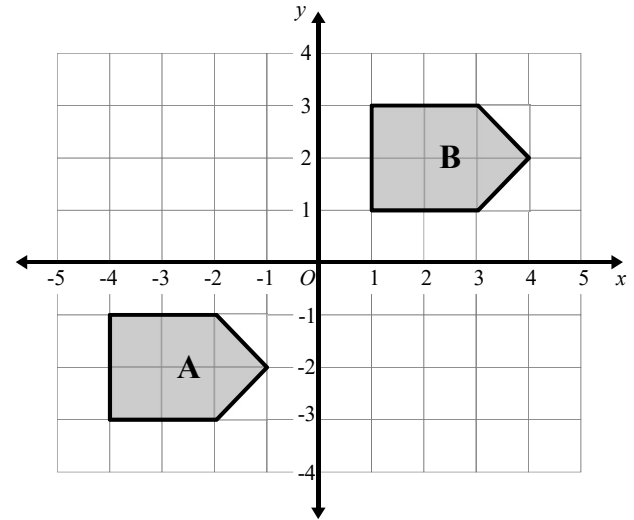
2



Translate triangle A by the vector  $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$

**(Total for question 2 is 2 marks)**

3

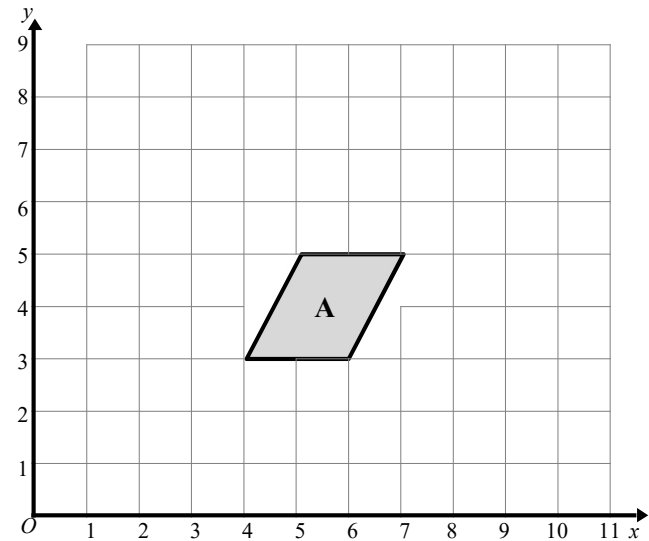


Describe fully the single transformation that maps shape A onto shape B.

.....

**(Total for question 3 is 2 marks)**

4

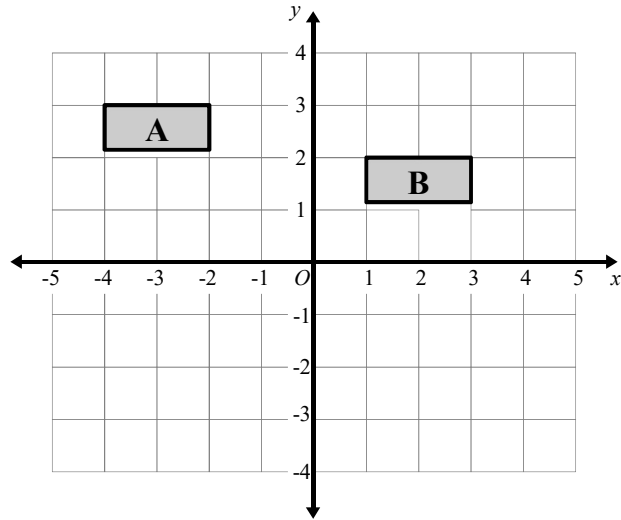


Translate shape A by the vector  $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

**(Total for question 4 is 2 marks)**

# EXAM QUESTIONS

5

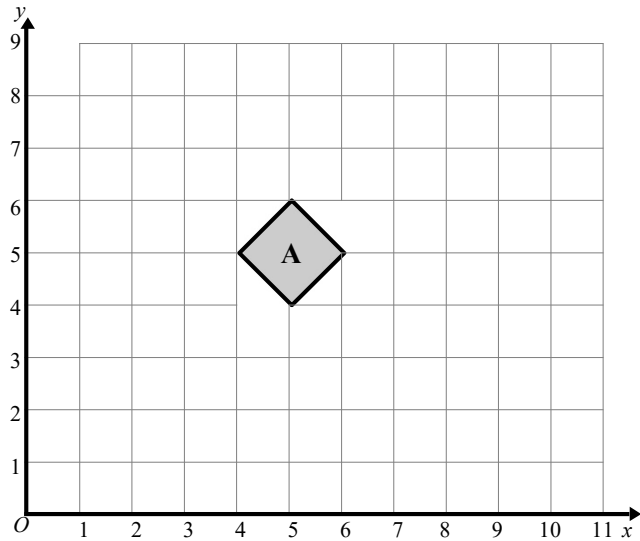


Describe fully the single transformation that maps shape A onto shape B.

.....

(Total for question 5 is 2 marks)

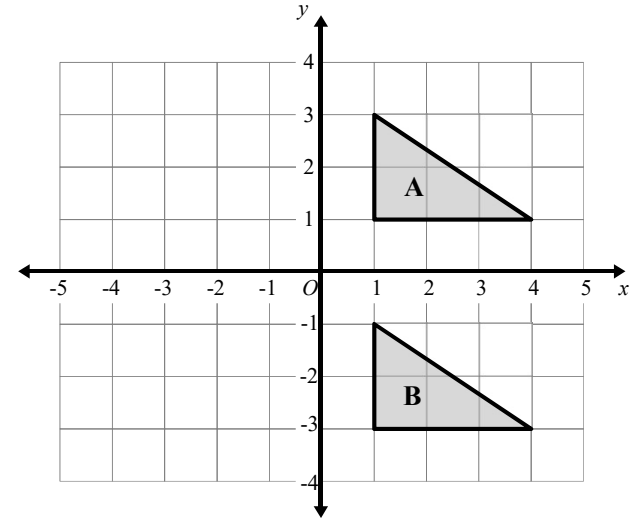
6



Translate shape A by the vector  $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$

(Total for question 6 is 2 marks)

7

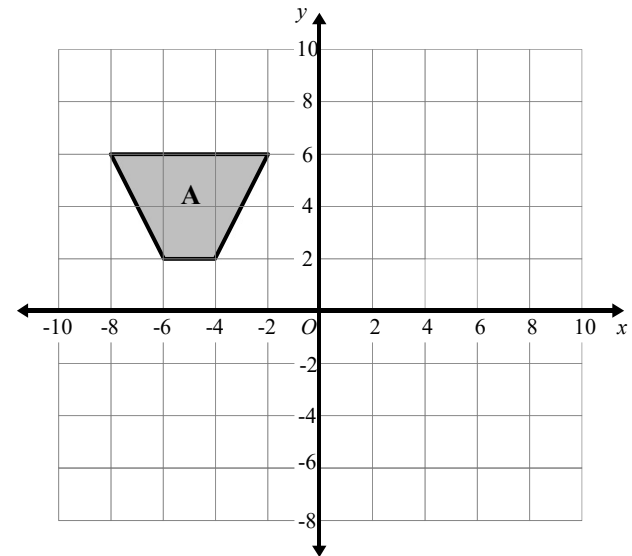


Describe fully the single transformation that maps triangle A on triangle B.

.....

(Total for question 7 is 2 marks)

8

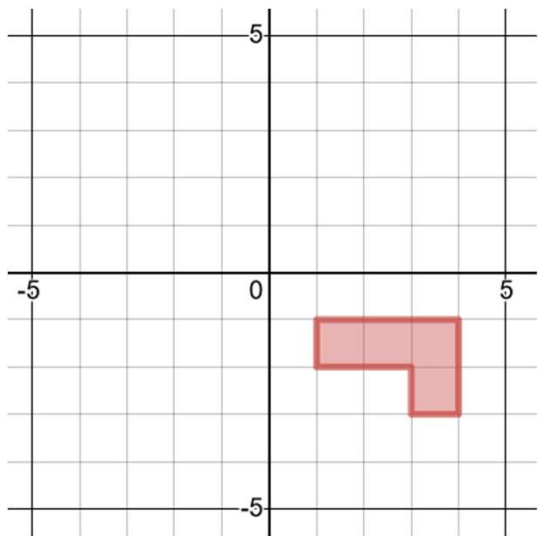
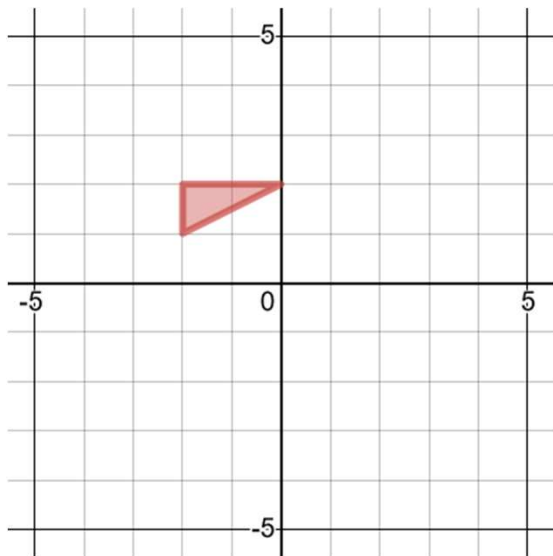


Translate shape A by the vector  $\begin{pmatrix} 8 \\ -1 \end{pmatrix}$

(Total for question 8 is 2 marks)

### Worked Example

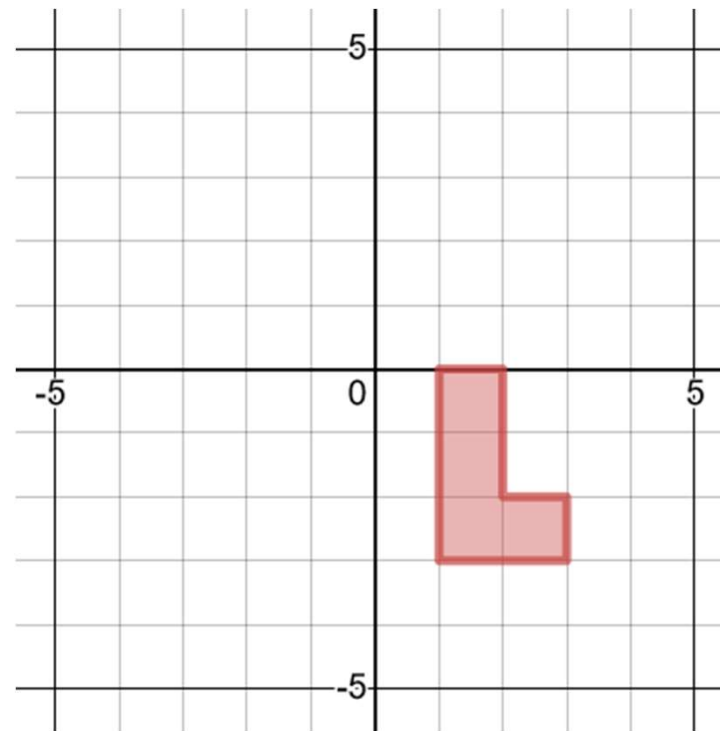
Rotate  $180^\circ$  about  $(1, -1)$   
Are there any invariant points?



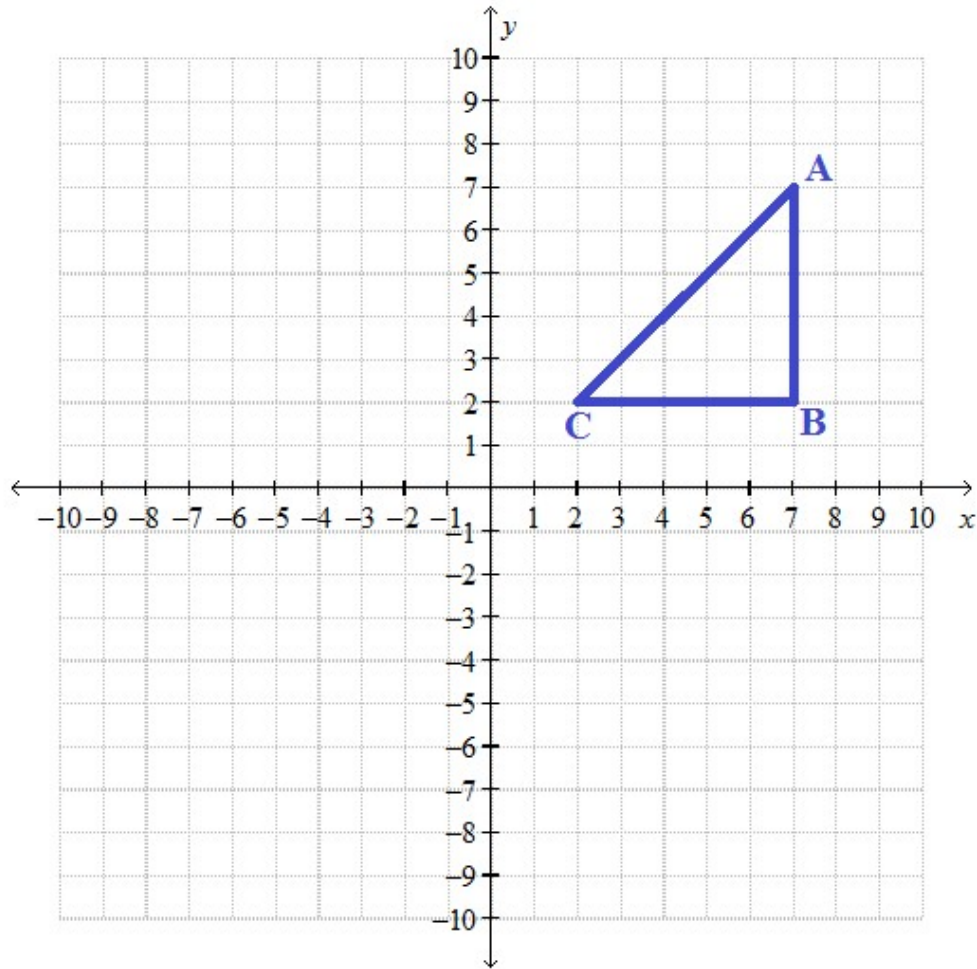
### Thinking

### Your Turn

Rotate  $180^\circ$  about  $(1, 0)$   
Are there any invariant points?



## Invariance activity



### Amber

Match the transformation to the invariant points for the triangle ABC

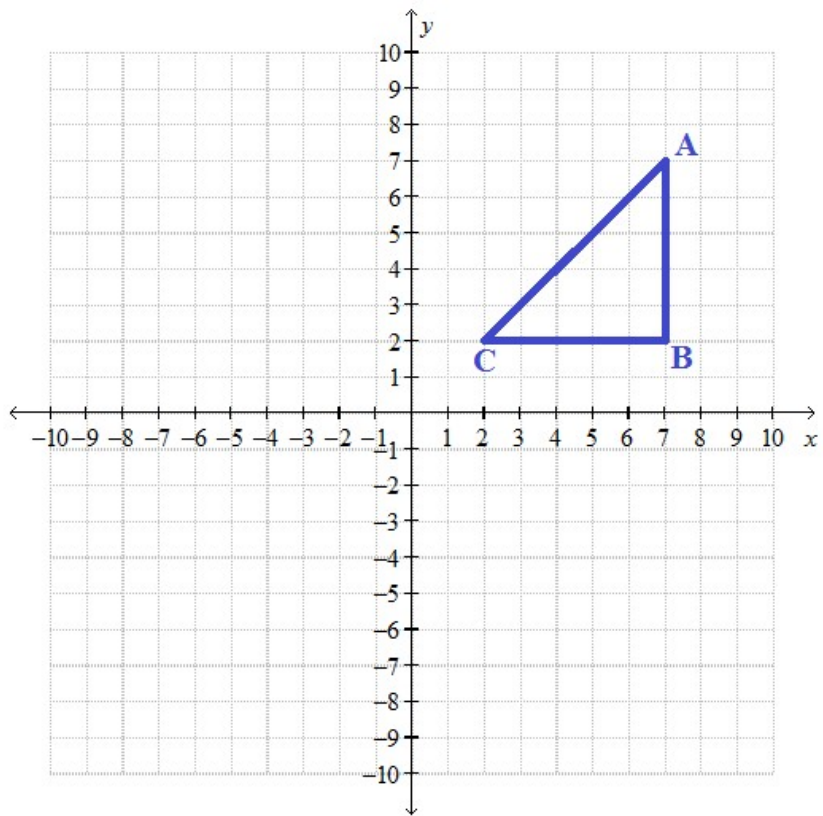
- |   |   |
|---|---|
| (a) Reflection in the line $y = 7$                |   |
| (b) Reflection in the line $y = x - 5$            | A |
| (c) Rotation around the centre $(7, 7)$           |   |
| (d) Reflection in the line $x + y = 4$            | B |
| (e) Reflection in the line $y = x$                |   |
| (f) Reflection in the line $x = 7$                | C |
| (g) Reflection in the line $y = 2x - 2$           |   |
| (h) Reflection in the line $y = \frac{1}{2}x + 1$ |   |

### Red

Complete these sentences:

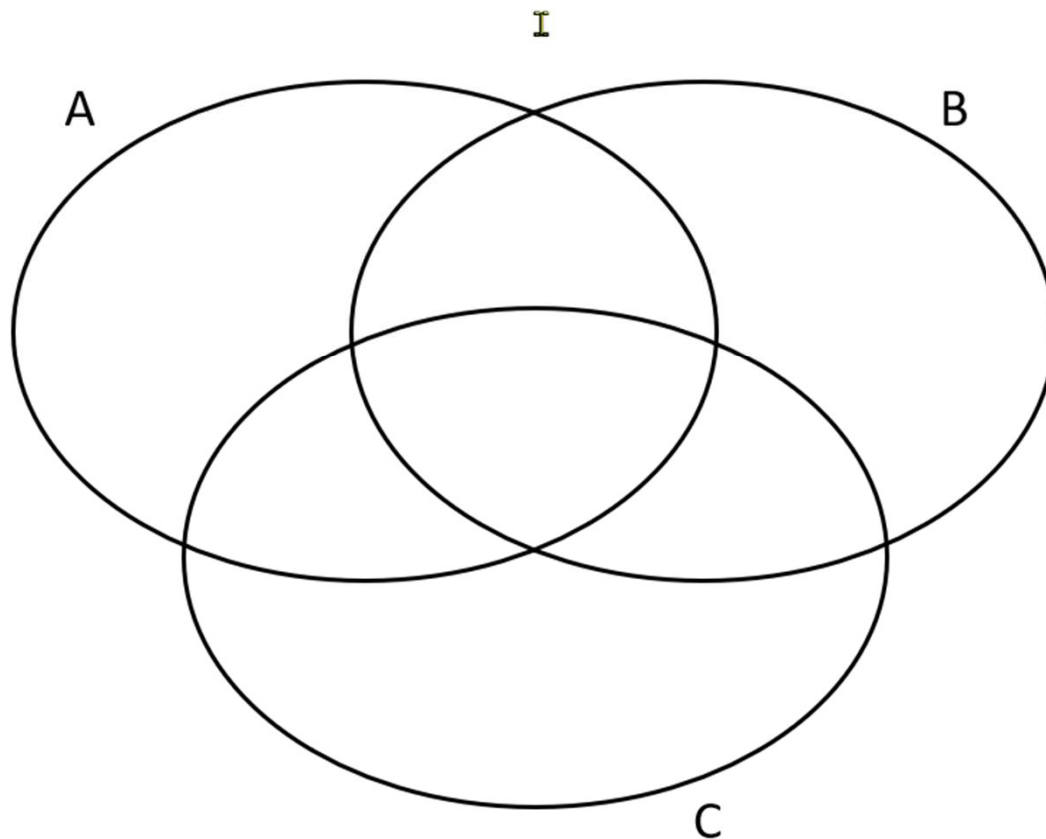
- When triangle ABC is reflected in the line  $y = 2$  the invariant points are ..... & .....
- When triangle ABC is rotated using centre  $(7, 2)$ , the invariant point is .....
- When triangle ABC is reflected in the line  $y = x$ , the invariant points are ..... & .....
- When triangle ABC is reflected in the line  $x + y = 9$ , the invariant point is .....
- When triangle ABC is.....the invariant points are A and B.
- When triangle ABC reflected in the line ..... the only invariant point is C.





Green

Write a transformation that would leave the correct points in the triangle ABC invariant for each region of the Venn diagram. Try and put at least one transformation in each region.



### Worked Example

Solve the following leaving your answer to 3s.f.

$$a) 10 + b^2 = 15$$

$$b) a^2 + 15 = 35$$

$$c) 5^2 + b^2 = 60$$

### Thinking

### Your Turn

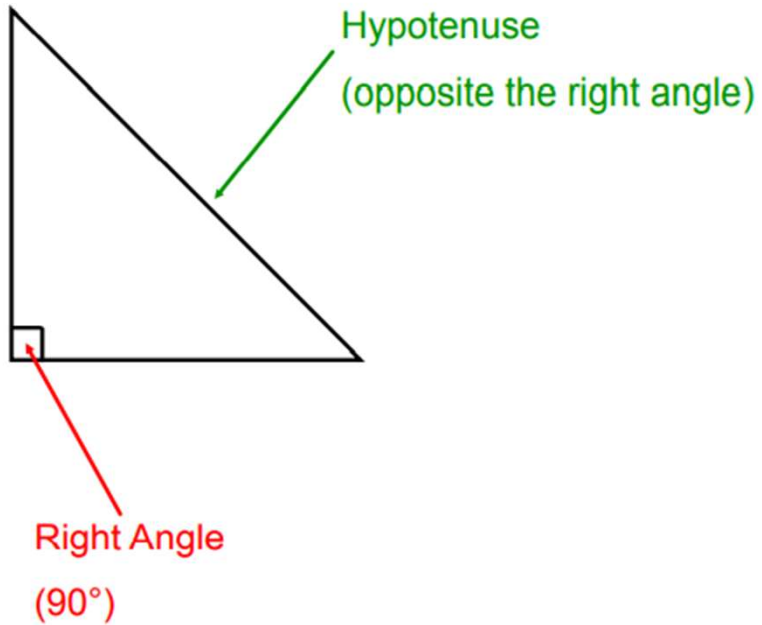
Solve the following leaving your answer to 3s.f.

$$a) 5 + b^2 = 15$$

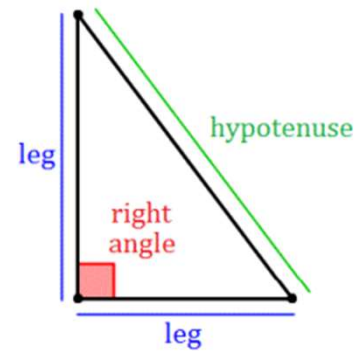
$$b) a^2 + 15 = 50$$

$$c) 8^2 + b^2 = 60$$

# KEY WORD: HYPOTENSUE - HY-POT-EN-USE

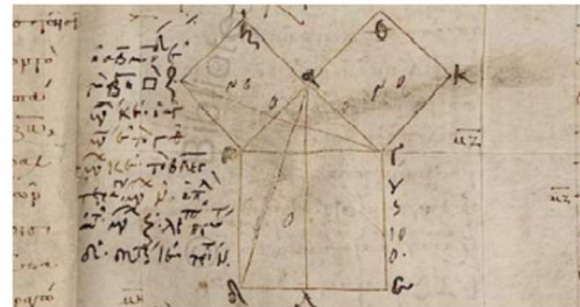


From the Greek derived *hypo* meaning 'under' and *teinein* meaning 'to stretch'.

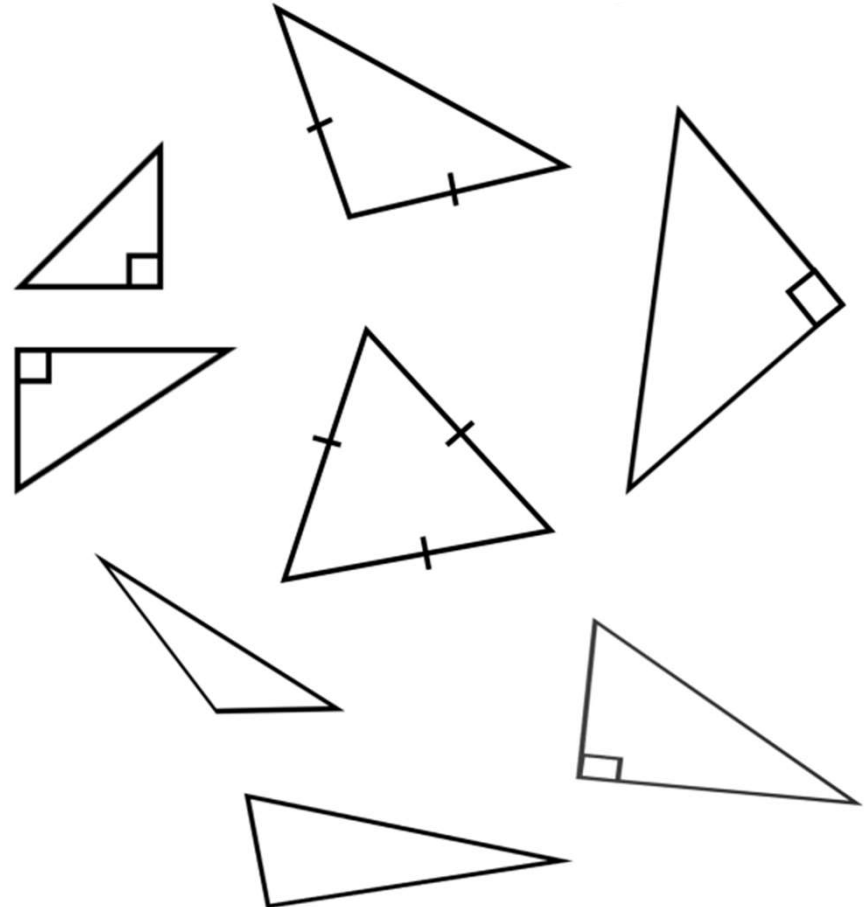
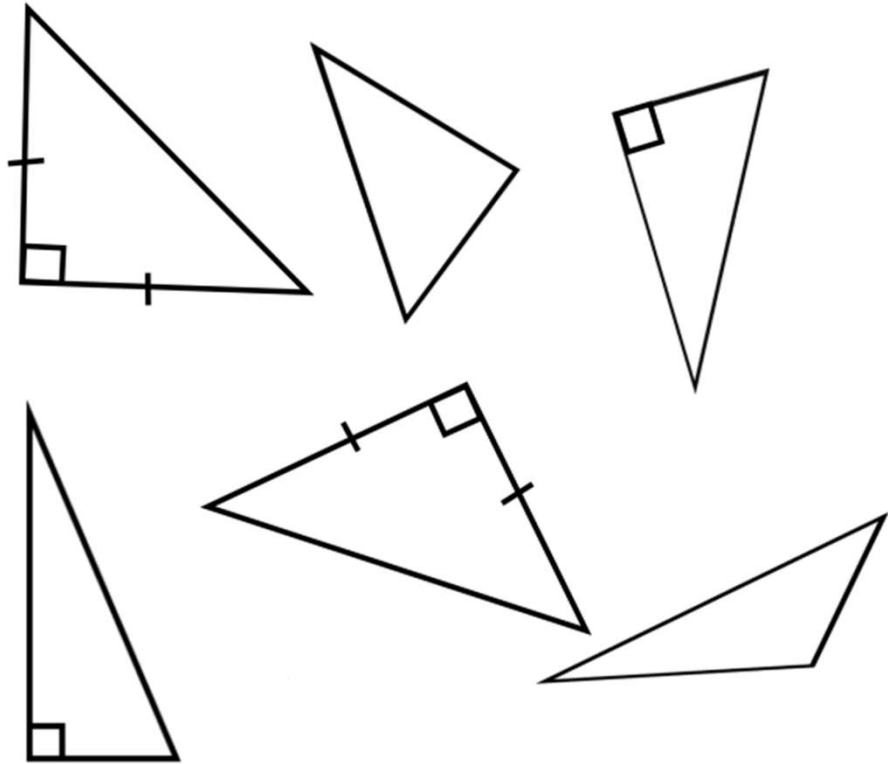


The two sides that aren't the hypotenuse are known as legs.

The hypotenuse is the side that stretches from one leg to another.

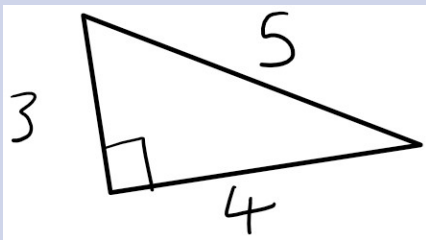
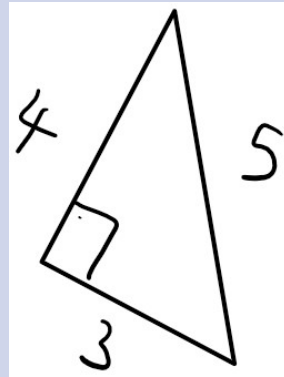
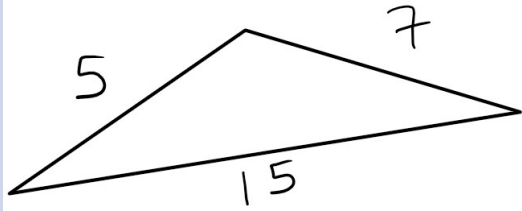


In each triangle that has a hypotenuse, label the hypotenuse with a letter h.

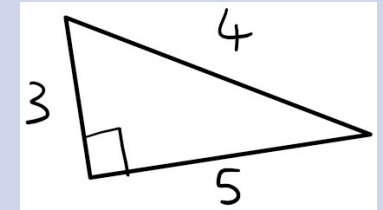
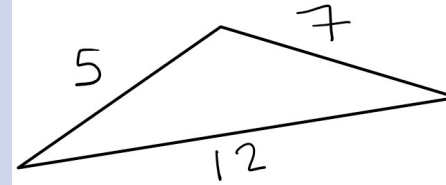


**Possible and impossible triangles** – can you explain why you know why the triangles drawn are possible or impossible? Can you add more examples?

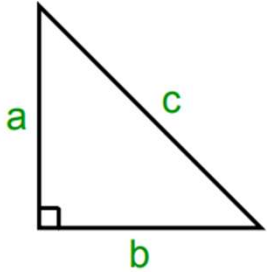
### POSSIBLE TRIANGLES



### IMPOSSIBLE TRIANGLES



## Pythagoras' Theorem

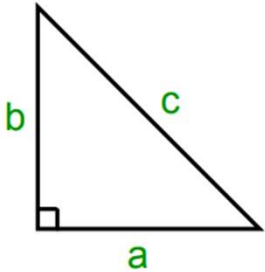


In any *right angled triangle*, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

In other words:

$$a^2 + b^2 = c^2$$

Note: a and b can be labelled in any order but c has to be the hypotenuse i.e the triangle could be labelled like this:

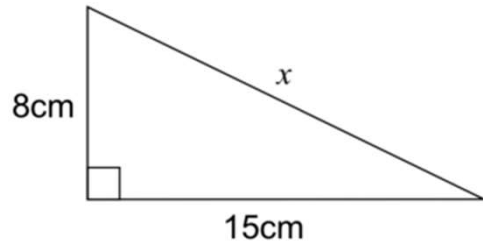


Can you think of triangles where the three lengths **DO NOT** obey Pythagoras's theorem?

## Worked Example

Calculate the unknown side in this triangle.

*NON-CALC*

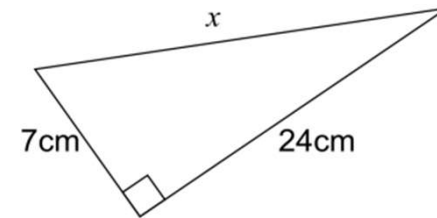


## Thinking

## Your Turn

Calculate the unknown side in this triangle.

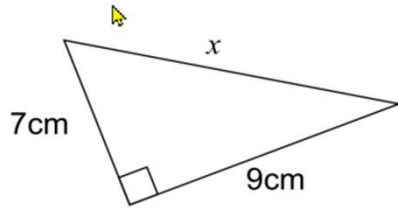
*NON-CALC*



## Worked Example

Calculate the unknown side in this triangle. Give your answer to 2 decimal places.

CALC

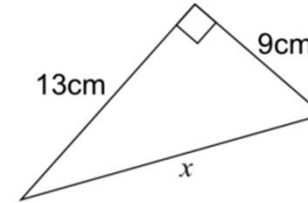


## Thinking

## Your Turn

Calculate the unknown side in this triangle. Give your answer to 2 decimal places.

CALC





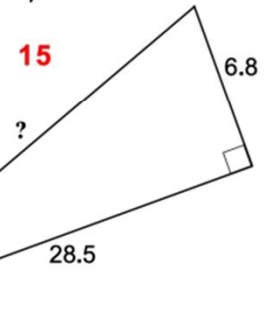
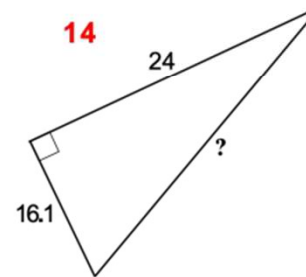
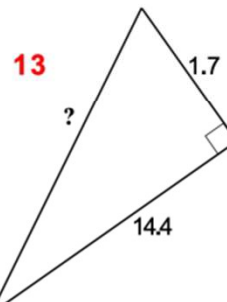
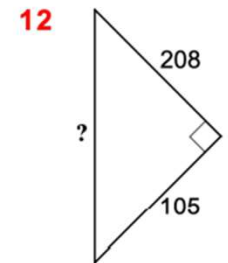
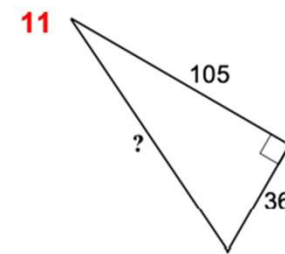
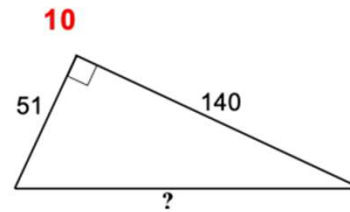
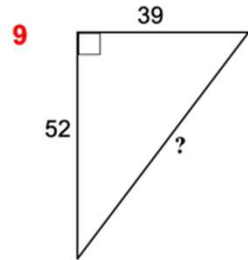
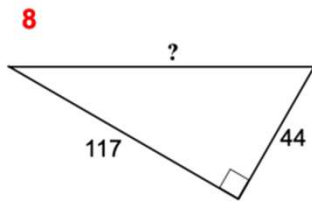
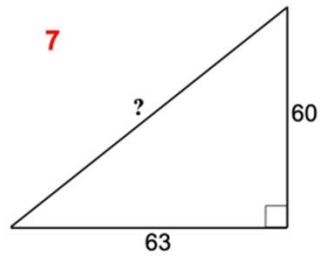
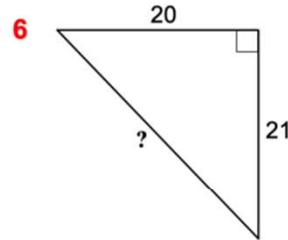
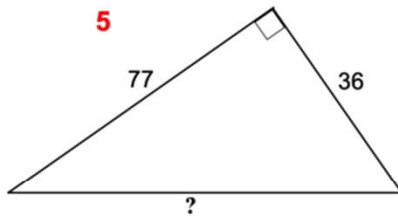
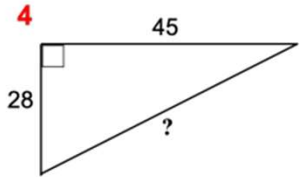
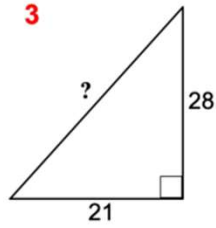
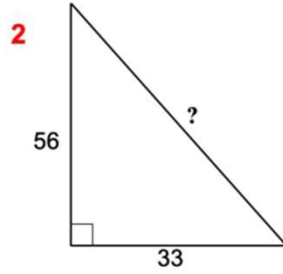
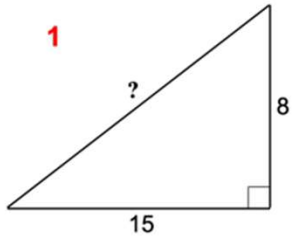
# Questions

## Trigonometry

T/2

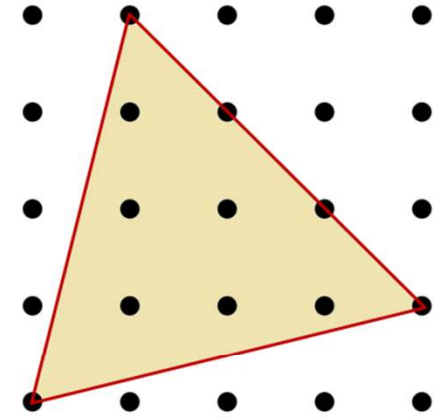
Use Pythagoras' theorem to find the length of the **hypotenuse** marked ? in each of these **right-angled** triangles.

*Drawings are NOT to scale.*



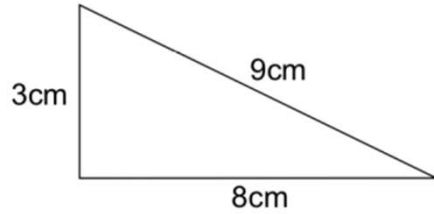
## Extension

equilateral triangle or not?



## Worked Example

Work out if this triangle is right-angled or not.

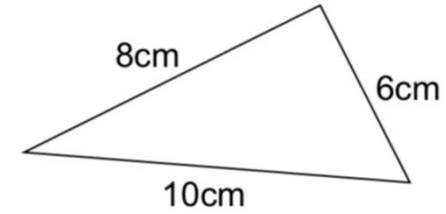


## Thinking

*Converse of Pythagoras' Theorem If Pythagoras' theorem holds true, then the triangle must be right-angled.*

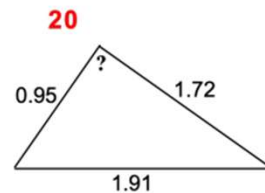
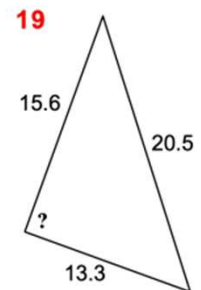
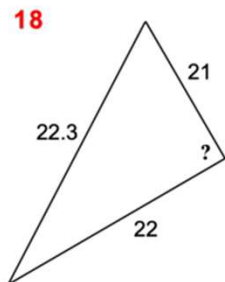
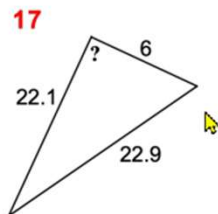
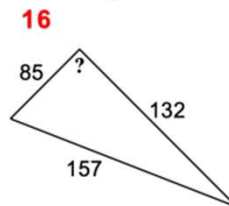
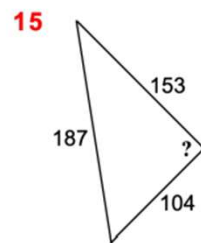
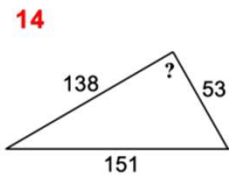
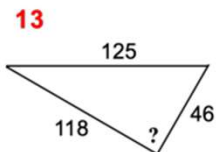
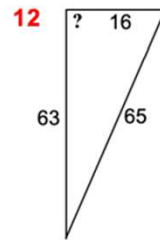
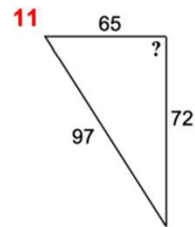
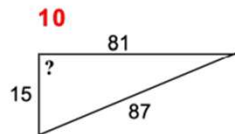
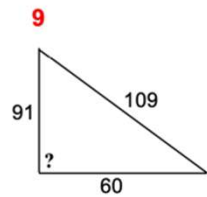
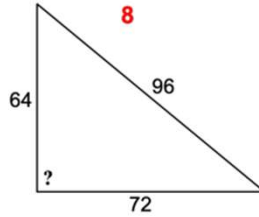
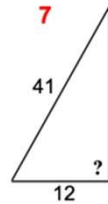
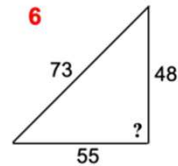
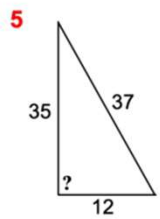
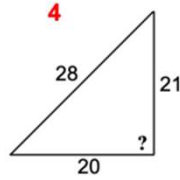
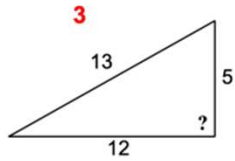
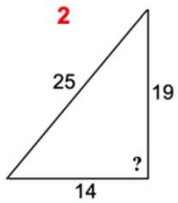
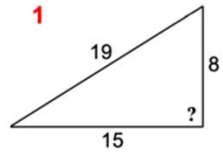
## Your Turn

Work out if this triangle is right-angled or not.



# EXERCISE:

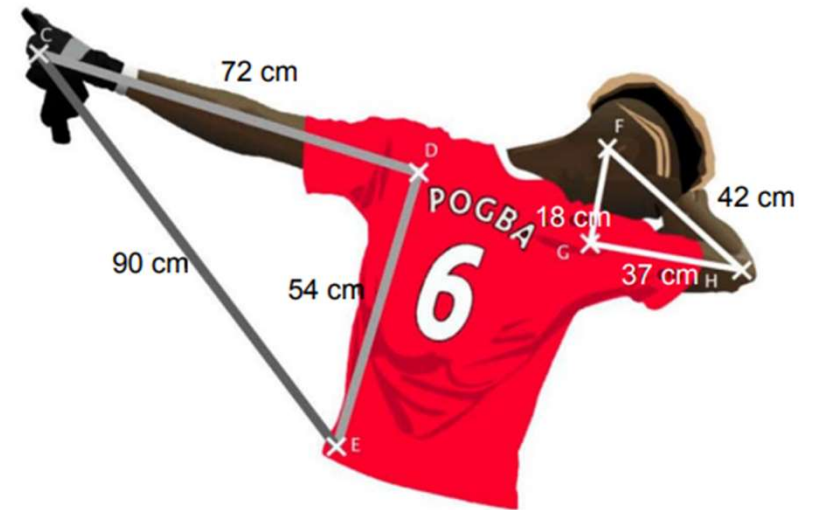
Use Pythagoras' theorem to decide whether each of these triangles is right-angled or not.  
*Drawings are NOT to scale.*



## Extension

Cristiano Ronaldo is jealous of Paul Pogba's dab, so Pogba tries to demonstrate that his dab is perfect. According to the book 'the Universal Declaration of the Rights of the Dab', a dab is only perfect if both triangles represented in the figure below are right angled.

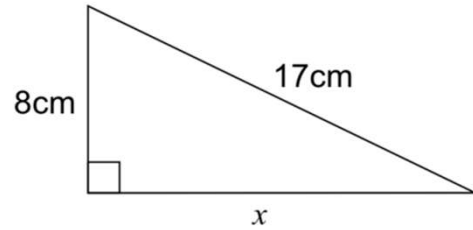
Is Paul Pogba's dab perfect?



## Worked Example

Calculate the unknown side in this triangle.

*NON-CALC*

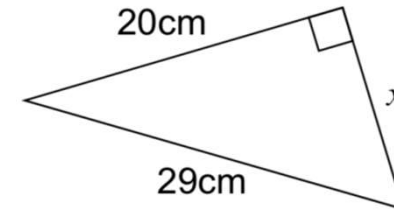


## Thinking

## Your Turn

Calculate the unknown side in this triangle.

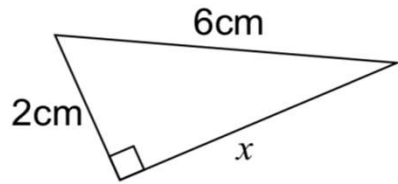
*NON-CALC*



## Worked Example

Calculate the unknown side in this triangle. Give your answer to 2 decimal places.

CALC

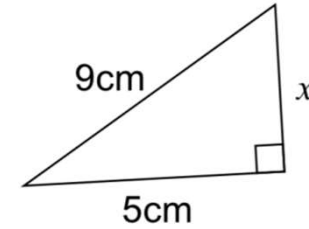


## Thinking

## Your Turn

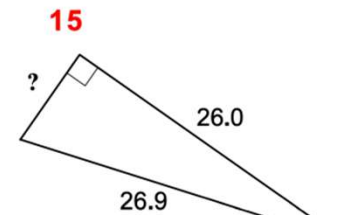
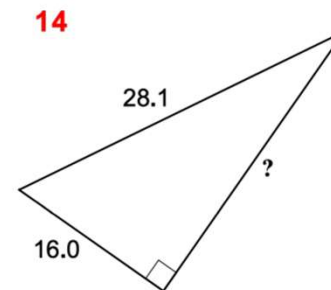
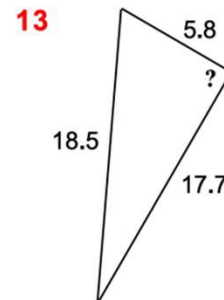
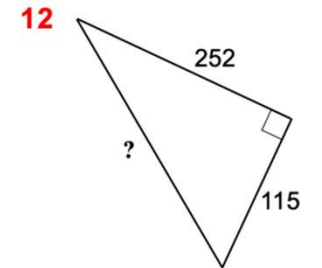
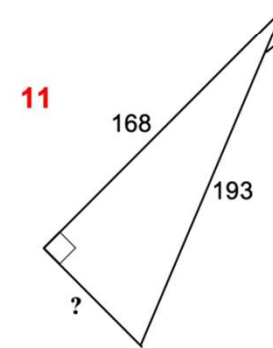
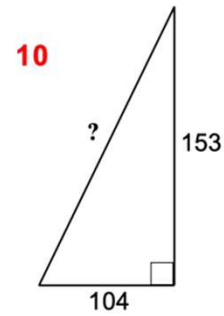
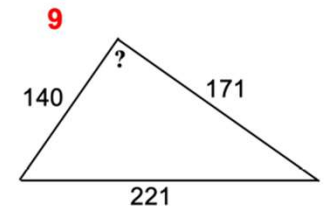
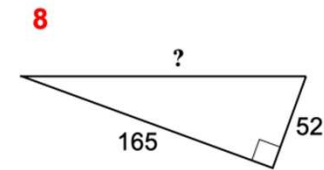
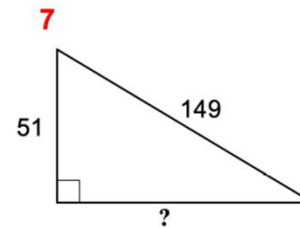
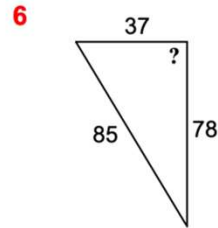
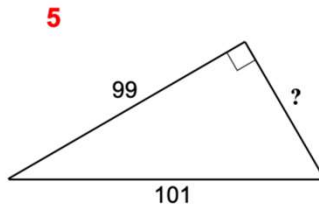
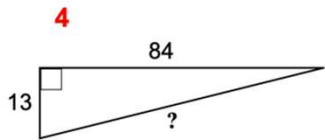
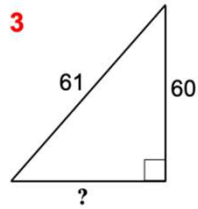
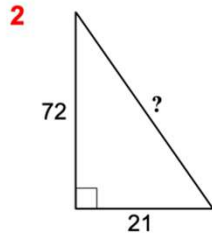
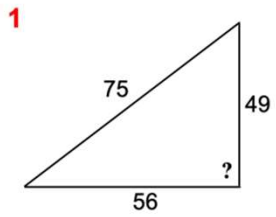
Calculate the unknown side in this triangle. Give your answer to 2 decimal places.

CALC



# EXERCISE:

Use Pythagoras' theorem to find the length of the edge marked ?, **OR** decide whether the triangle is right-angled or not  
*Drawings are NOT to scale.*



**Worked Example****Thinking****Your Turn**

Find the length of AB where  
A(-1, -4) and B(4, 3).

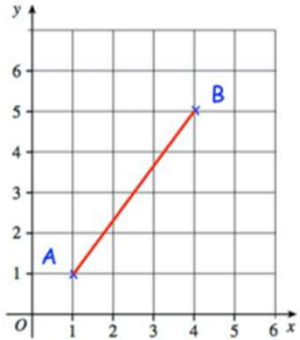
Find the length of AB where  
A(-2, -3) and B(8, 11).

# Questions

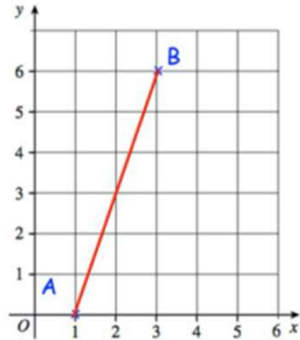
Give each answer to 2 decimal places.

Question 1: Calculate the length of the line joining the points A and B.

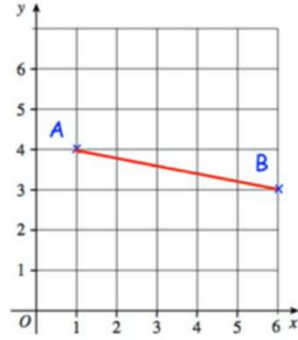
(a)



(b)

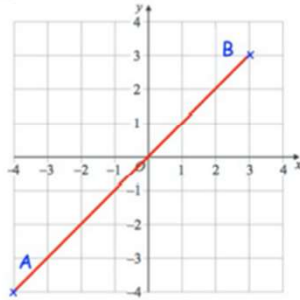


(c)

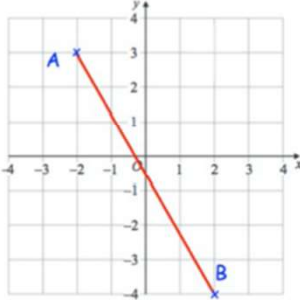


Question 2: Calculate the length of the line joining the points A and B.

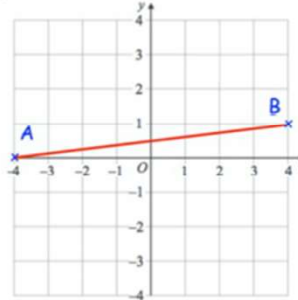
(a)



(b)



(c)



Question 5: Calculate the distance between the following pairs of coordinates

(a) (5, 1) and (9, 6)

(b) (1, 4) and (10, 10)

(c) (0, 0) and (6, 8)

(d) (2.5, 3) and (8, 0)

(e) (-6, 2) and (8, 3)

(f) (-5, -9) and (-3, 8)

(g) (-5, 7) and (-3, -2)

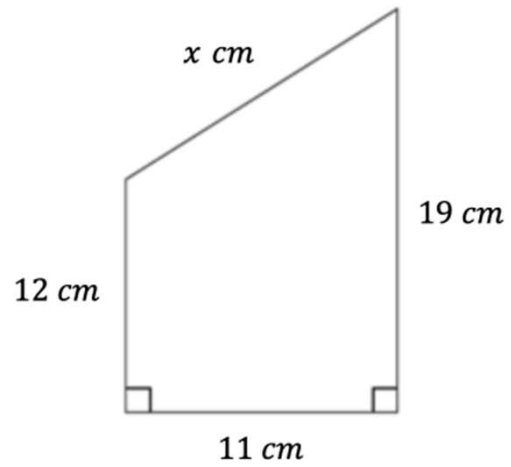
(h) (-9, -9) and (3, -20)

(i) (-4, 0) and (0, -4)



### Worked Example

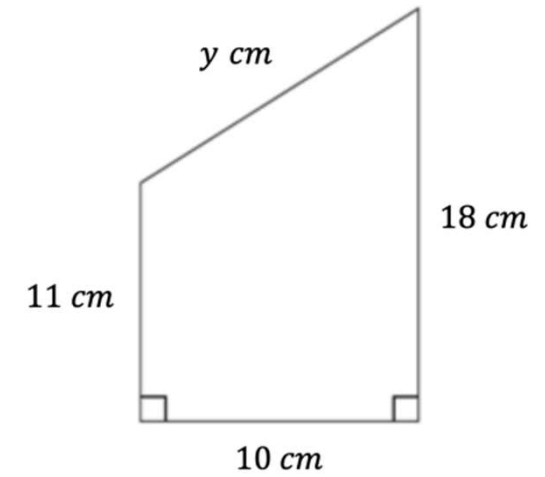
Calculate  $x$ .



### Thinking

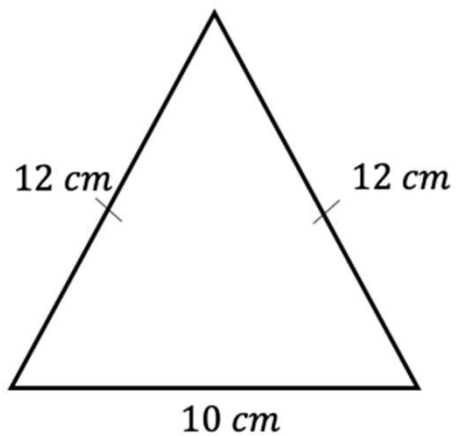
### Your Turn

Calculate  $y$ .



### Worked Example

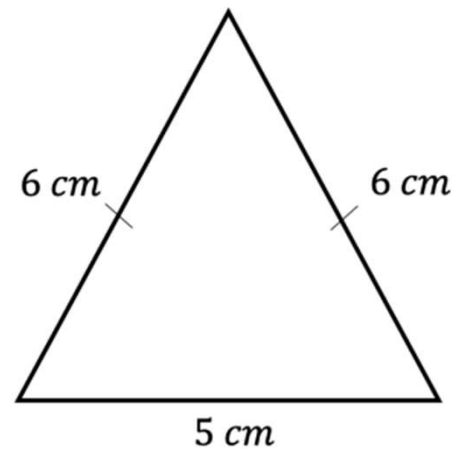
Find the area of this triangle.



### Thinking

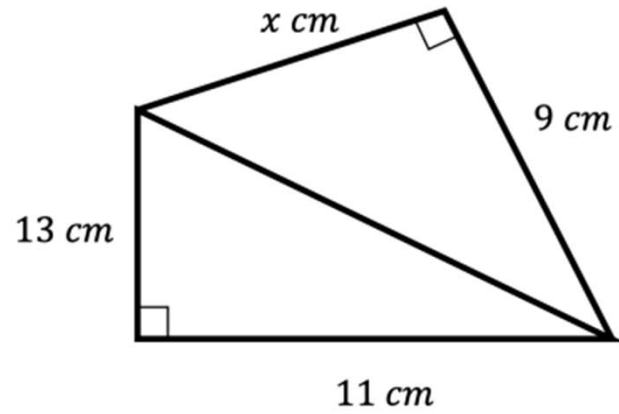
### Your Turn

Find the area of this triangle.



### Worked Example

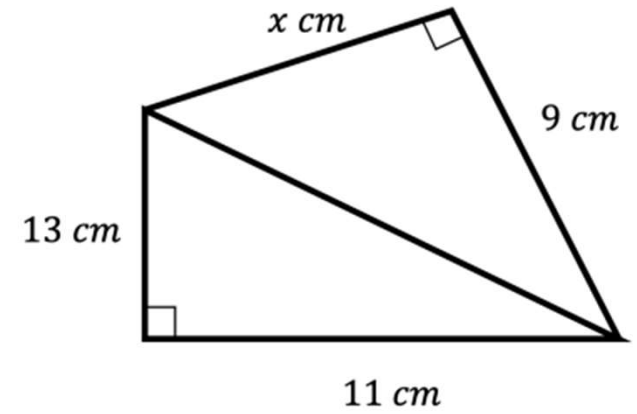
Calculate  $x$ .



### Thinking

### Your Turn

Calculate  $x$ .



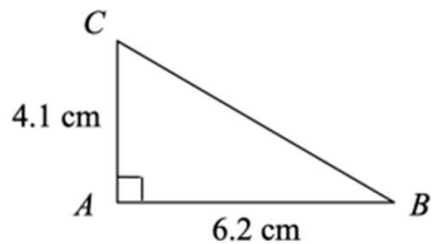


## PYTHAGORAS

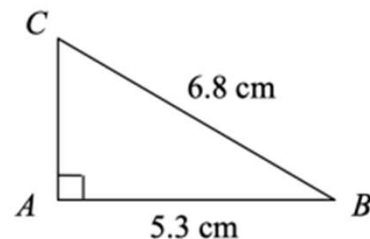
### MIXED QUESTIONS

Ref: G451. **1R1**

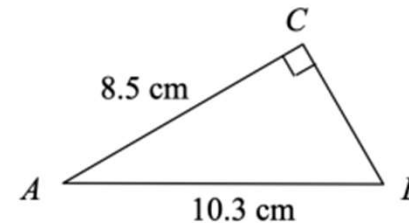
**A1** Find length  $BC$



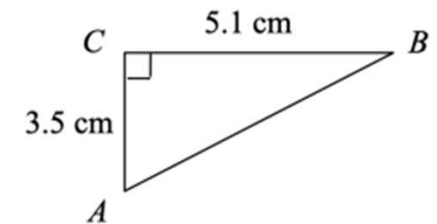
**A2** Find length  $AC$



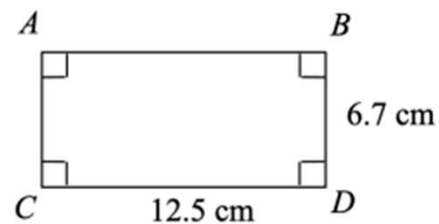
**A3** Find length  $BC$



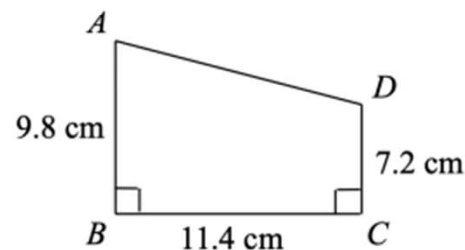
**A4** Find length  $AB$



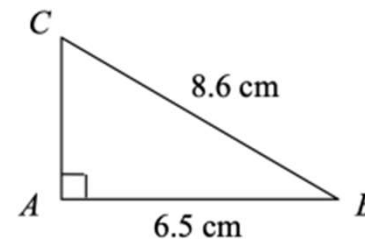
**B1** Find length  $BC$



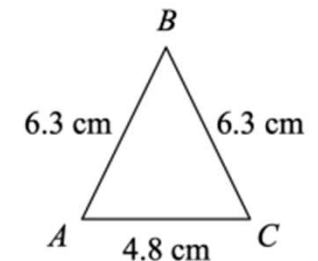
**B2** Find length  $AD$



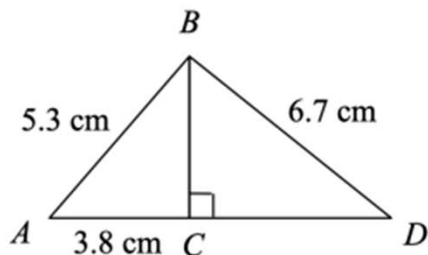
**B3** Find the area



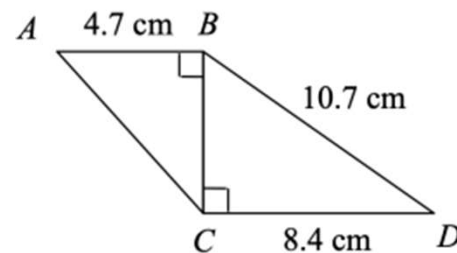
**B4** Find the area



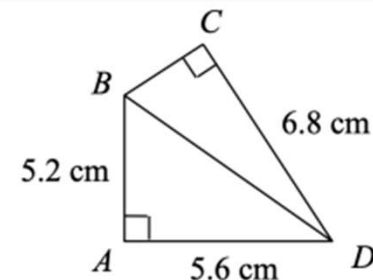
**C1** Find length  $CD$



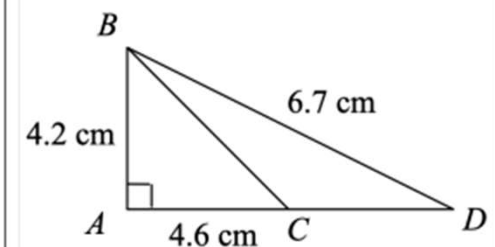
**C2** Find length  $AC$



**C3** Find length  $BC$



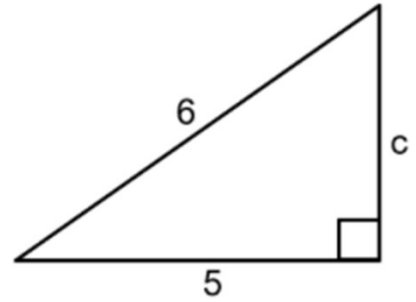
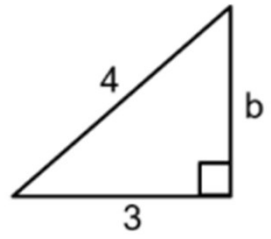
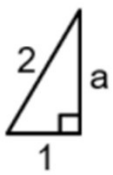
**C4** Find length  $CD$



Surds and Pythagoras

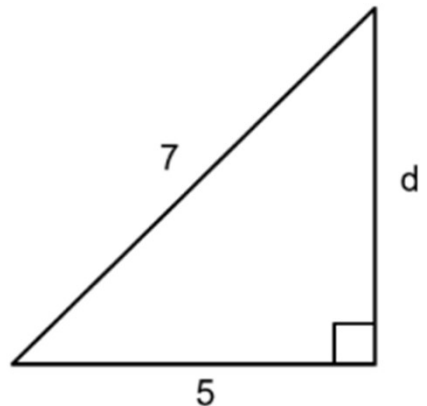
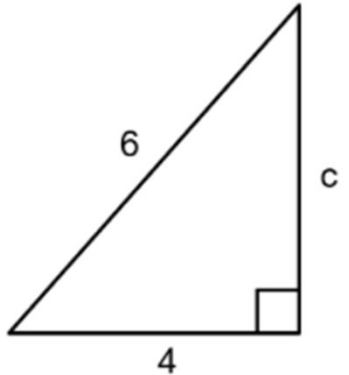
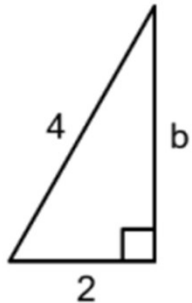
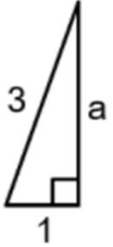
generalising pythagoras surd families (i)

(1)



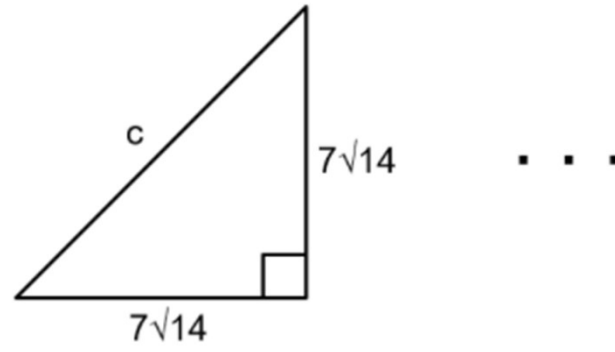
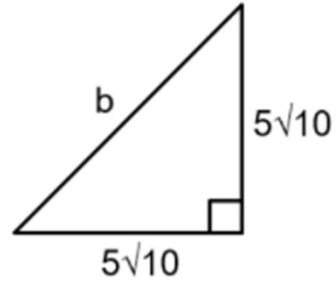
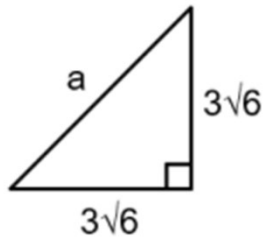
...

(2)



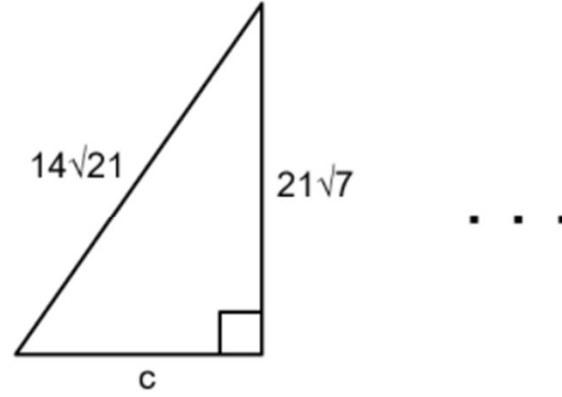
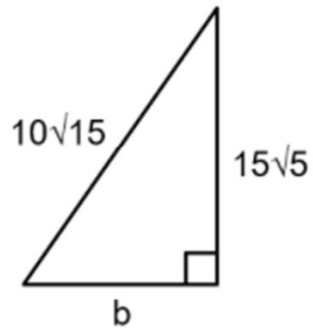
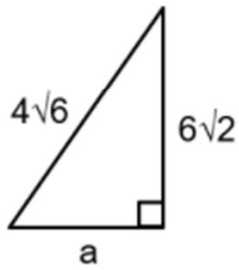
...

(3)

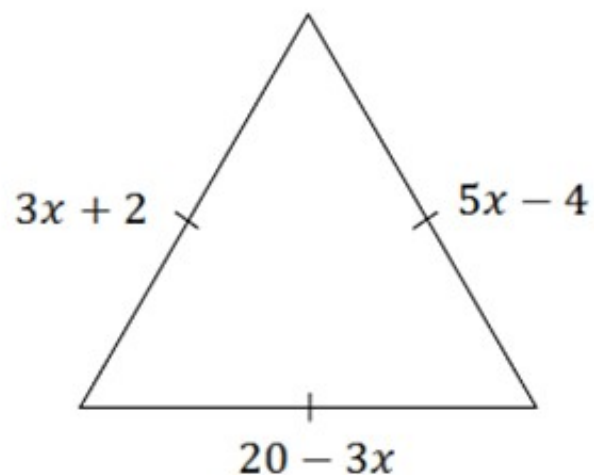


generalising pythagoras surd families (ii)

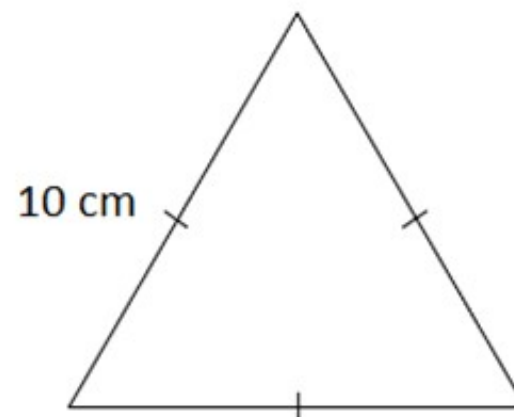
(1)



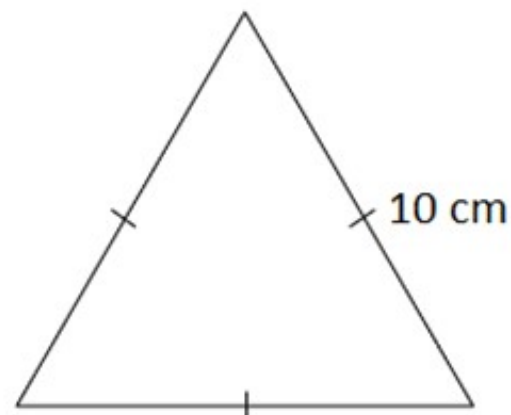
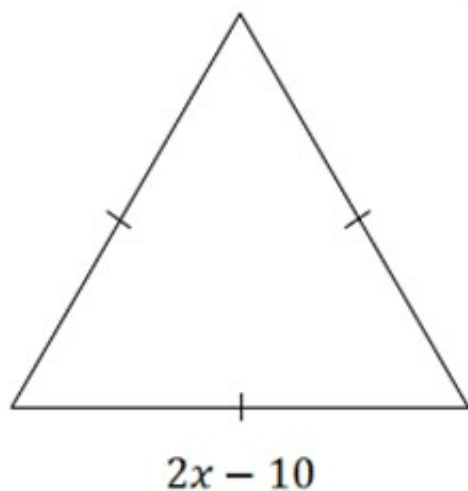
Calculate the perimeter



Calculate the exact area, giving your answer in simplified form



### The Equilateral Triangle



The perimeter is 21m, what is the value of  $x$ ?

A square has the same perimeter as the triangle. Find the area of the square.

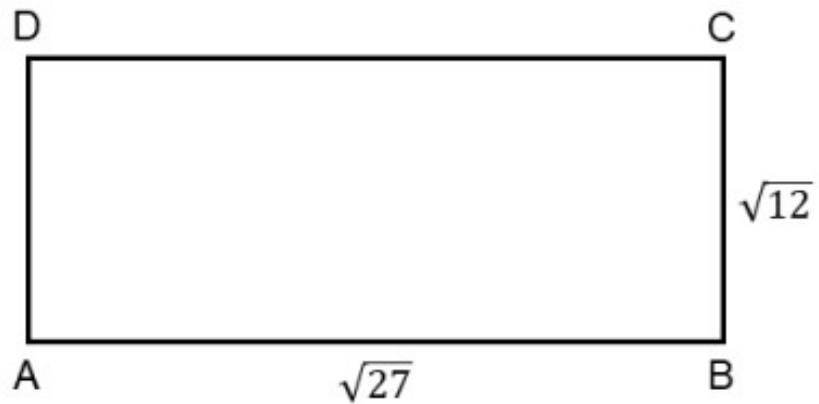
Calculate the perimeter of the rectangle.



Calculate the area of the rectangle.



Calculate the length AC.



Write the ratio of AB:BC in its simplest form.







- 1 A square with sides of length  $x$  cm, is inside a circle.  
Each vertex of the square is on the circumference of the circle.

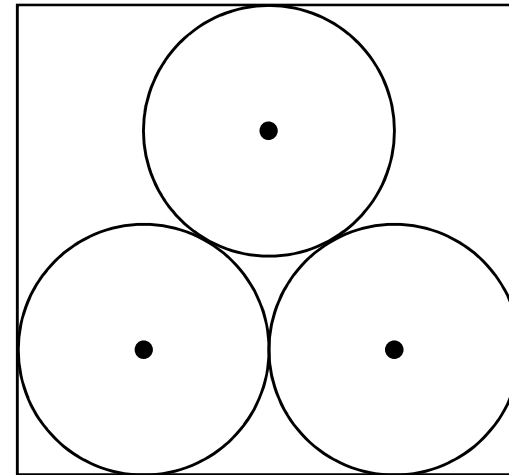
The area of the circle is  $36 \text{ cm}^2$ .

Work out the value of  $x$ .

Give your answer correct to 3 significant figures.



- 1 The diagram shows 3 identical circles inside a rectangle.  
Each circle touches the other two circles and the sides of the rectangle,  
as shown in the diagram.



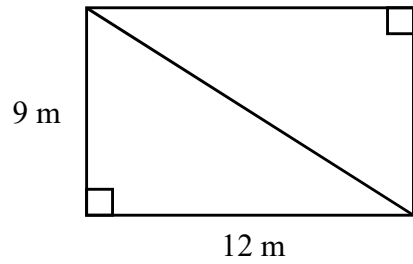
The radius of each circle is 16 mm.

Work out the area of the rectangle.

Give your answer correct to 3 significant figures.



1 This rectangular frame is made from 5 straight pieces of wood.

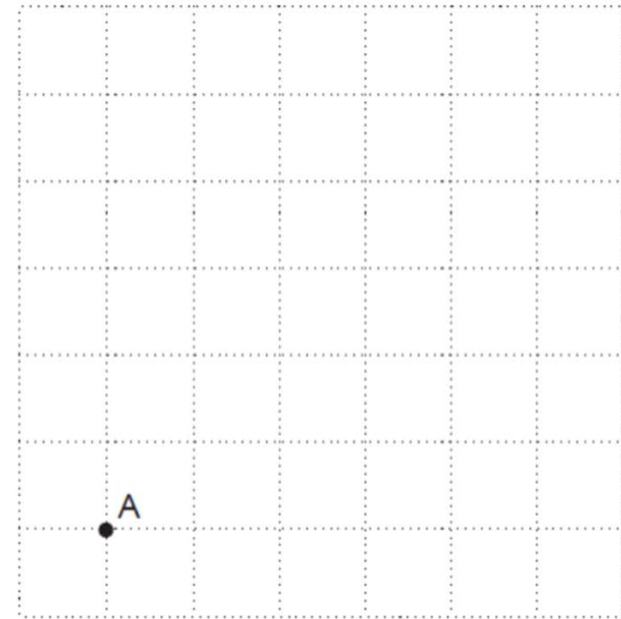


The weight of the wood is 2.5 kg per metre.  
Work out the total weight of the wood in the frame.

The point A is shown on the unit grid below.

The point B is  $2\sqrt{5}$  units from A and lies on the intersection of two grid lines.

Mark one possible position for B.

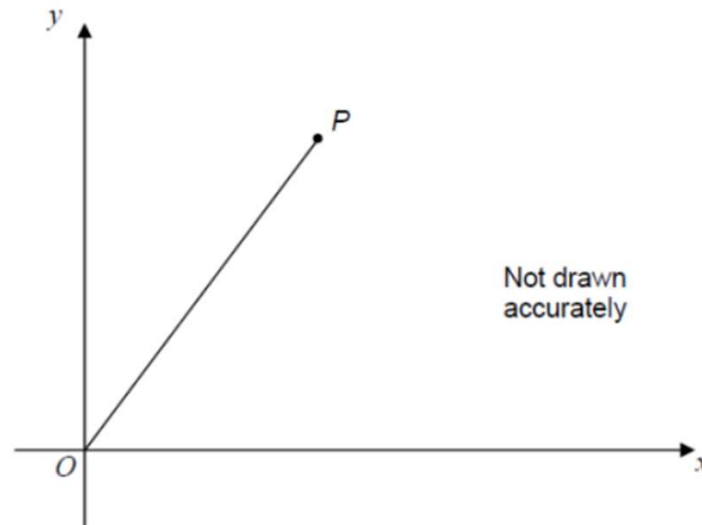


The diagram shows a line joining O to P.

The gradient of the line is 2

The length of the line is  $\sqrt{2645}$

Work out the coordinates of P.



The area of a right-angled, isosceles triangle is  $4 \text{ cm}^2$

Work out the perimeter of the triangle in centimetres. Give your answer in the form  $a + b\sqrt{c}$ , where  $a, b$  and  $c$  are integers.