Year 9
Mathematics
UNIT 4

Name: _____

Class: _____

Contents Page

1 **PR Straight Line Graphs**
2 **Straight Line Graphs**
3 **Parallel and Perpendicular Lines**
4 **Linear Inequalities**
5 **Graphical Inequalities**

Please see unit 4 course on drfrostmaths.com

Gradient and y-intercept

Worked Example	Your Turn
$y = 2x - 1$ Gradient: y-intercept:	$y = 3x - 4$ Gradient: y-intercept:
$y = -2x + 6$ Gradient: y-intercept:	$y = -3x + 6$ Gradient: y-intercept:
$2x + 3y = 6$ Gradient: y-intercept:	$3x + 2y = 6$ Gradient: y-intercept:

Question 1: Write down the gradient of each of these lines.

(a) $y = 3x + 1$ (b) $y = 2x - 5$ (c) $y = 7x + 4$ (d) $y = 10x + 5$

(e) $y = x - 2$ (f) $y = 6x$ (g) $y = -4x + 3$ (h) $y = -3x - 7$

(i) $y = \frac{1}{2}x + 3$ (j) $y = -\frac{4}{5}x - 9$

Question 2: Write down where each of these lines cross the y-axis (y-intercept)

(a) $y = 2x + 3$ (b) $y = 7x + 1$ (c) $y = 3x - 2$ (d) $y = x - 5$

(e) $y = 2x$ (f) $y = -4x + 6$ (g) $y = -5x - 3$ (h) $y = -3x$

(i) $y = \frac{4}{3}x + \frac{2}{5}$ (j) $y = -\frac{2}{3}x - \frac{1}{2}$

Question 12: Find the gradients and the y-intercepts of each of these lines

(a) $x + y = 10$ (b) $x - y = 4$ (c) $2x + y = 6$

(d) $3x - y = -1$ (e) $8x + 2y + 9 = 0$ (f) $5x - 2y - 4 = 0$

(g) $7x = 1 - 2y$ (h) $15y - 6x = 8$ (i) $⅔x + 2y = 5$

(j) $⅕y - ½x = 1$ (k) $⅔x + ¾y = 1½$

Worked Example	Your Turn
Write in the form y= $mx + c$ the line with:	Write in the form $y = mx + c$ the line with:
Gradient 2 and y-intercept 3	Gradient 3 and y-intercept 4
Gradient $\frac{2}{3}$ and y-intercept -3	Gradient $-\frac{5}{6}$ and y-intercept -1
Gradient $-\frac{3}{2}$ and y-intercept 0	Gradient $\frac{3}{4}$ and y-intercept 0
Gradient 0 and y-intercept 4	Gradient 0 and y-intercept -5

$$y = \frac{4}{3}x + \frac{2}{5} \qquad\qquad y = -\frac{2}{3}x - \frac{1}{2}$$

Question 3:   Write down the equation of the lines below

(a) gradient of 3 and y–intercept of 6

(b) gradient of 2 and y–intercept of –1

(c) gradient of –4 and y–intercept of 3

(d) gradient of 8 and y–intercept of 4

(e) gradient of 1 and passing though (0, 4)

(f) passing through (0, –2) with gradient 4

(g) gradient of –5 and passing through the origin.
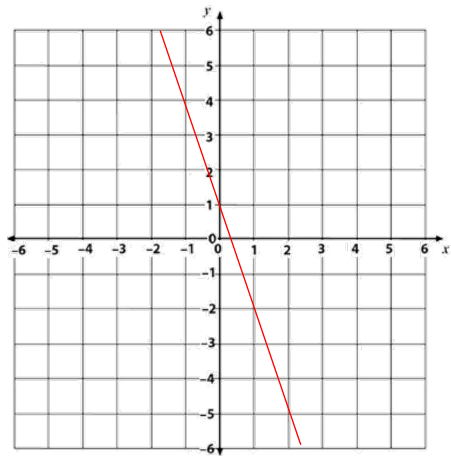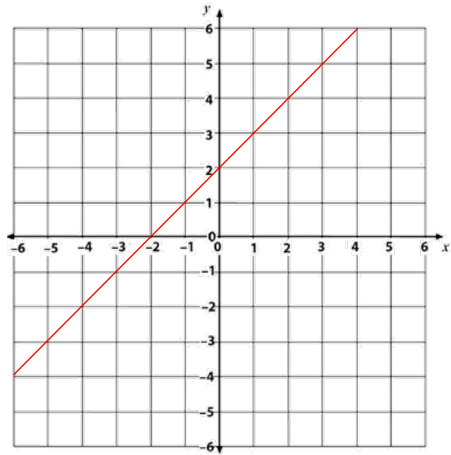
Equation of Straight Line Graphs

x	-3	-2	-1	0	1	2	3	4
y				c				

$+m$

$$y = mx + c$$

Worked Example	Your Turn
Find the equation of:	Find the equation of:
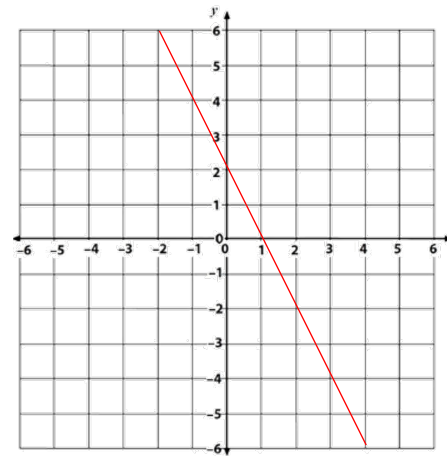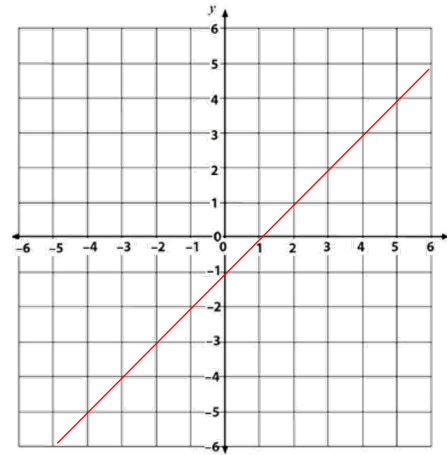
Worked Example	Your Turn
Find the equation of:	Find the equation of:

Worked Example	Your Turn
Find the equation of:	Find the equation of:

Equation of a Line

Videos 191, 194, 195 on www.corbettmaths.com
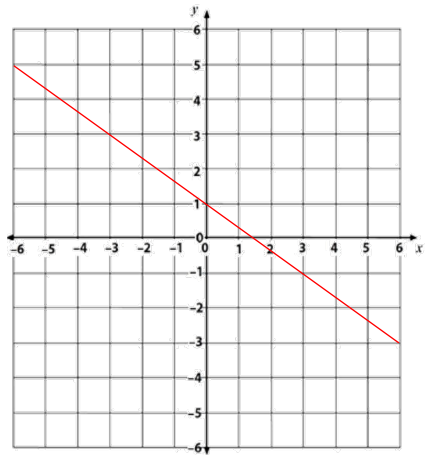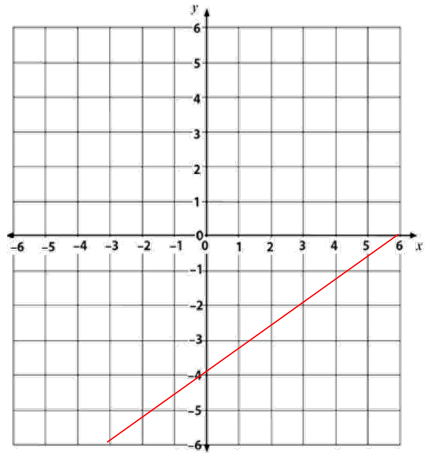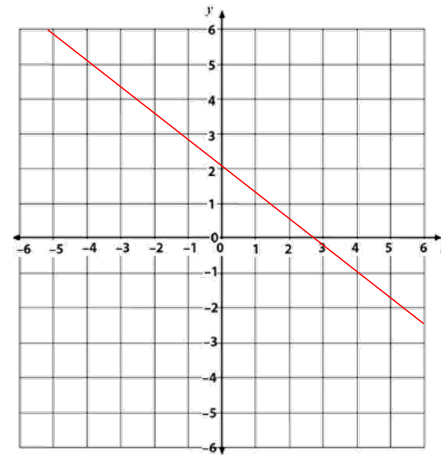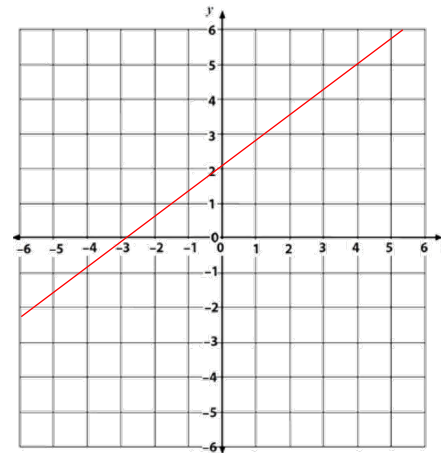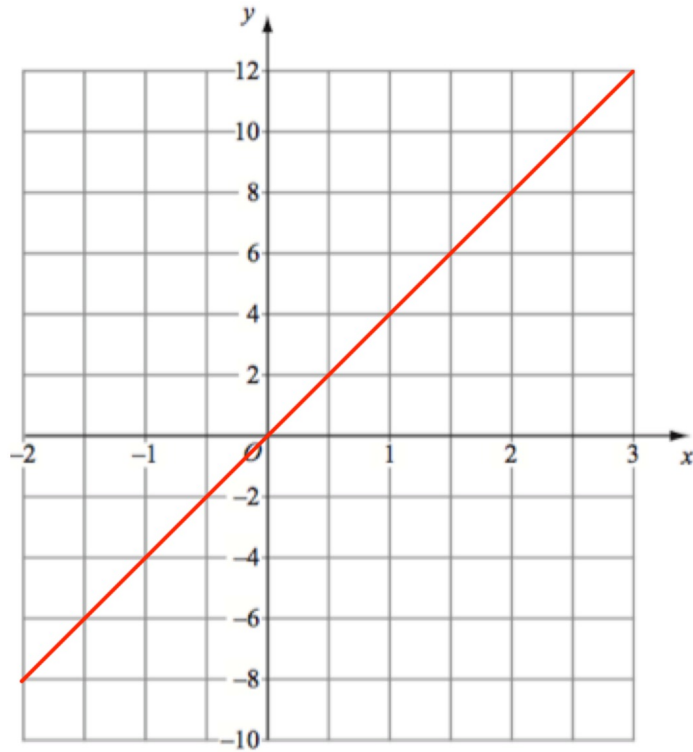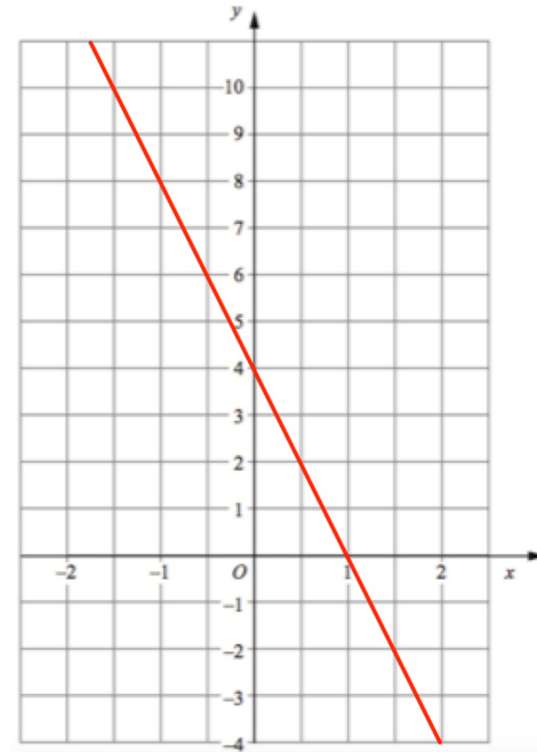
Question 5: Find the equation of each line

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

(l)

Corbett
mCths
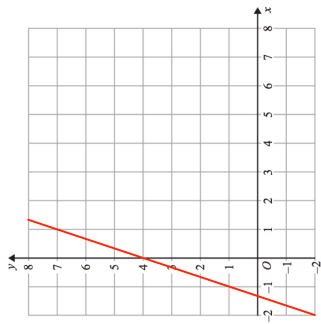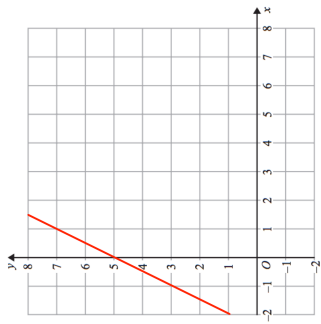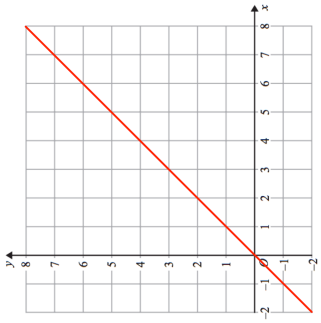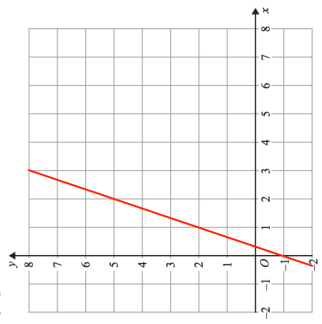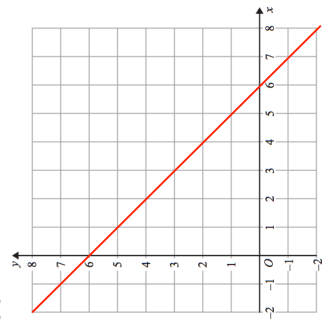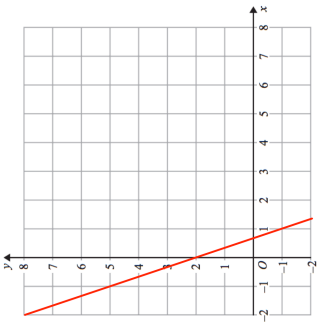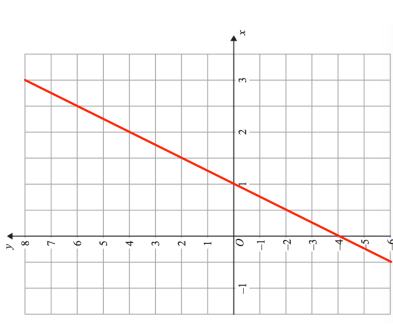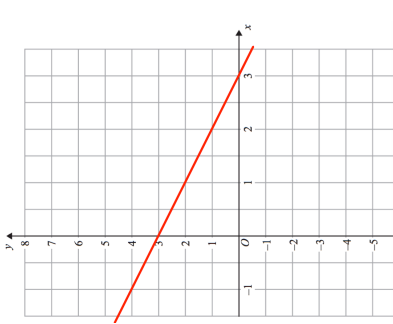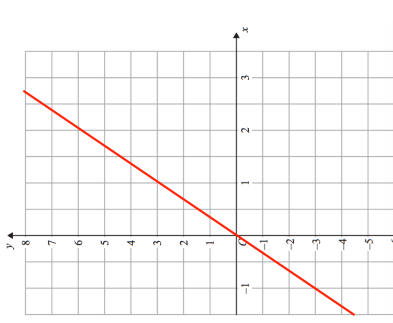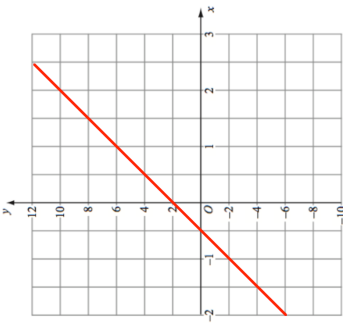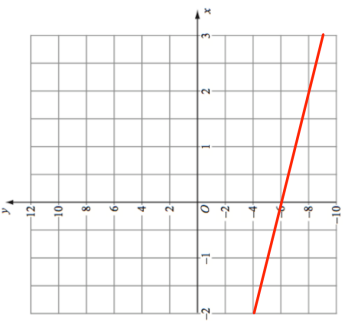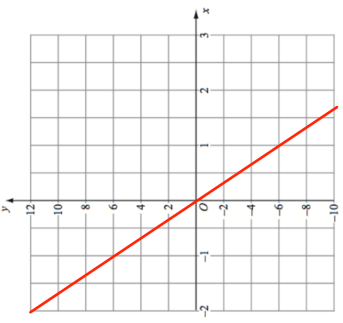
Question 6:   Find the equation of each line below.

(a)

B (3, 11)

A (0, 5)

(b)

B (2, 17)

A (0, 3)

(c)

B (5, 9)

A (0, -1)

(d)

A (0, 8)

B (8, 4)

(e)

A (-4, 20)

A (-

B (0, 0)

(f)

, -16)   A (-3, -16)

B (0, -10)
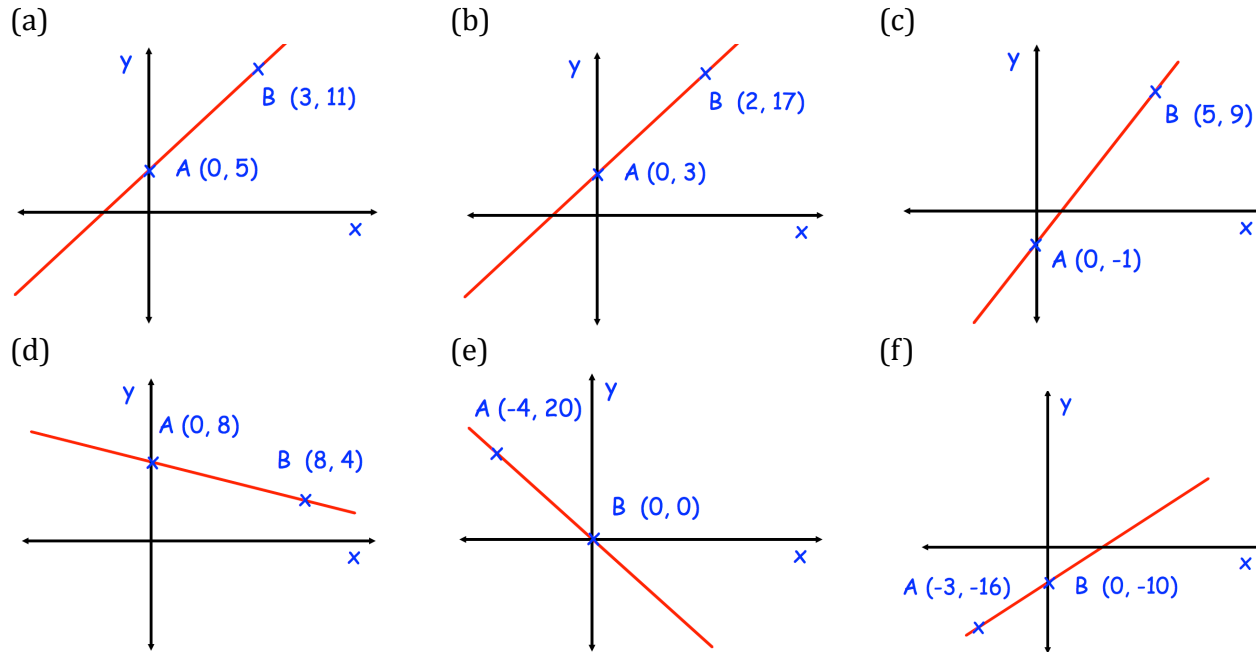
Question 7:   Find the equation of the straight line that passes through the points

(a) (0, 3) and (4, 19)

(b) (0, 2) and (6, 20)

(c) (0, 0) and (1, 4)

(d) (0, −9) and (9, 0)

(e) (0, −6) and (7, 8)

(f) (−8, −10) and (0, 14)

(g) (0, 2) and (10, 7)

(h) (−4, 1) and (0, 7)

(i) (−4, 0) and (0, 18)

Worked Example	Thinking	Your Turn
Find the equation of the line, given a point and the gradient: $(-6, 22)$ Gradient 3		Find the equation of the line, given a point and the gradient: $(-2, 5)$ Gradient 4

Question 8:   Find the equation of the straight line that:

(a) has a gradient of 4 and passes through the point (1, 10)

(b) has a gradient of 2 and passes through the point (−3, 3)

(c) has a gradient of 1 and passes through the point (5, 2)

(d) has a gradient of −3 and passes through the point (−2, 8)

(e) has a gradient of −5 and passes through the point (3, −1)

(f) has a gradient of ½ and passes through the point (4, 5)

(g) has a gradient of ⅖ and passes through the point (−5, −5)

(h) has a gradient of −⅔ and passes through the point (9, 15)

Worked Example	Thinking	Your Turn
Write the equation of the line in the form $y = mx + c$ which passes through the points $(2, 3)$ and $(5, -9)$		Write the equation of the line in the form $y = mx + c$ which passes through the points $(3, 10)$ and $(-5, 18)$

Worked Example	Thinking	Your Turn
Write the equation of the line in the form $y = mx + c$ which passes through the points $(2, -3)$ and $(7, -5)$		Write the equation of the line in the form $y = mx + c$ which passes through the points $(3, -2)$ and $(-7, 5)$

Question 9:   Find the equations of the lines below

(a)

B (3, 8)

A (-3, 2)

(b)

B (3, 19)

A (1, 13)

c)

B (11, 26)

A (5, 2)

(d)

A (-1, 5)

B (9, -25)

(e)

A (-6, -2)

B (-4, -8)

(f)

A (-4, 3)

B (8, 33)
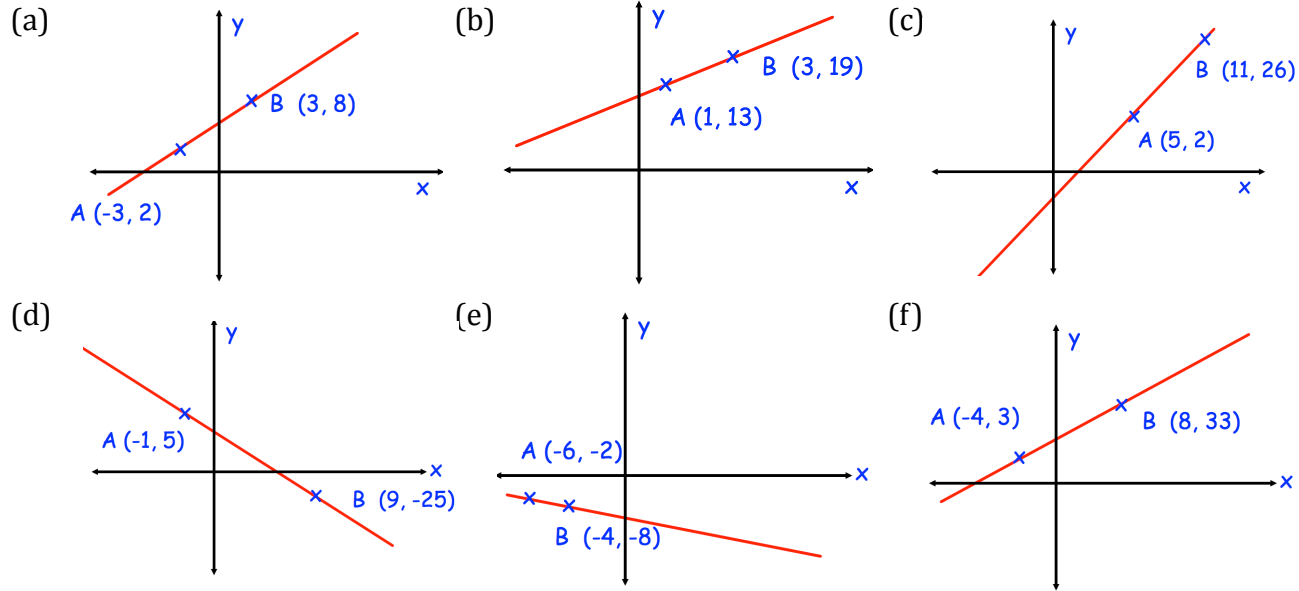
Question 10: Find the equation of the straight line that passes through these pairs of points

(a) (2, 5) and (4, 11)   (b) (−4, 2) and (1, 7)   (c) (−5, −8) and (−4, −4)

(d) (−1, −2) and (−6, 3)   (e) (−6, −4) and (−3, 2)   (f) (3, 5) and (4, 1)

(g) (−5, 4) and (5, 2)   (h) (1, 6) and (5, 4)   (i) (−10, −5) and (−7, 4)

Worked Example			Your Turn		

Worked Example

Find where the line intercepts the axes:

Line	x-intercept	y-intercept
$y = 2x + 3$		
$y = 3x + 2$		
$y = 3x - 2$		
$y = 2x - 3$		

Your Turn

Find where the line intercepts the axes:

Line	x-intercept	y-intercept
$y = 4x + 5$		
$y = 5x + 4$		
$y = 5x - 4$		
$y = 4x - 5$		

Worked Example			Your Turn		

Worked Example

Find where the line intercepts the axes:

Line	x-intercept	y-intercept
$y = 3 - 2x$		
$y = 2 - 3x$		
$2x + 3y = 6$		
$3x + 2y = 6$		

Your Turn

Find where the line intercepts the axes:

Line	x-intercept	y-intercept
$y = 5 - 4x$		
$y = 4 - 5x$		
$4x + 5y = 20$		
$5x + 4y = 20$		

Question 11: Find the coordinates where the following lines cross the x-axis

(a) $y = 2x + 6$

(b) $y = -x + 4$

(c) $y = 3x + 9$

(d) $y = x - 5$

(e) $y = 4x + 1$

(f) $y = -2x + 10$

(g) $y = -4x - 10$

(h) $y = 5x + 3$

(i) $y = \frac{1}{2}x + 3$

(j) $x + y = 8$

(k) $4x + 2y + 7 = 0$

(l) $3x + 2y - 8 = 0$

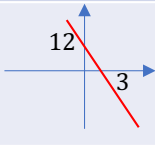| Worked Example	Thinking	Your Turn
Does the point $(2, 9)$ lie on the line $y = 4x + 1$?		Does the point $(2, 9)$ lie on the line $y = 9 - 2x$?

Question 4:

(a) Does the point $(2, 5)$ lie on the line $y = 3x - 1$?
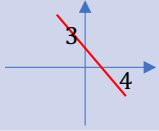
(b) Does the point $(4, 1)$ lie on the line $y = 3x + 1$?

(c) Does the point $(3, 1)$ lie on the line $y = x - 3$?

(d) Does the point $(5, 7)$ lie on the line $y = -3x + 22$?

(e) Does the point $(-4, -8)$ lie on the line $y = -2x$?

(f) Does the point $(-1, 8)$ lie on the line $y = 2x + 11$?

(g) Does the point $(12, 60)$ lie on the line $y = 7x - 18$?

© CORBETTMATHS 2019

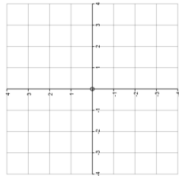| Worked Example	Thinking	Your Turn
$y = 5x + 10$ $ax + by = d$ Gradient: x intercept: y intercept: Sketch:		$y = 4x + 12$ $ax + by = d$ Gradient: x intercept: y intercept: Sketch:
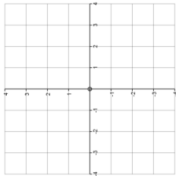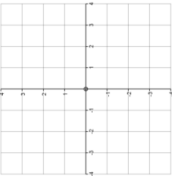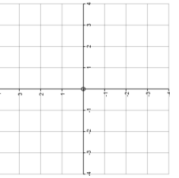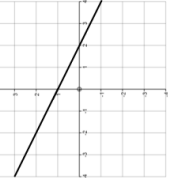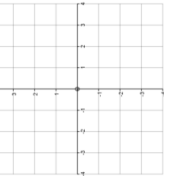
Fill in the Gaps

	$y = mx + c$	$ax + by = d$	Gradient	x intercept	y intercept	Sketch
1.	$y = 2x + 8$					
2.		$2x - y = -6$				
3.			3	$(-3, 0)$		
4.				$(3, 0)$	$(0, -9)$	
5.			4		$(0, -12)$	
6.						
7.				$(12, 0)$	$(0, 3)$	

Fill in the Gaps

	$y = mx + c$	$ax + by = d$	Gradient	x intercept	y intercept	Sketch
8.	$y = -\dfrac{1}{3}x + 4$					
9.		$4x + 3y = 12$				
10.						
11.			$\dfrac{3}{4}$	$(4, 0)$		
12.		$3x - 4y = 24$				
13.			$1\dfrac{3}{4}$	$(8, 0)$		
14.				No intercept	$(0, -14)$	

Fill in the Gaps

Equation of Straight Line	Graph	Gradient	Y-Intercept	A Point on the Line	Another Point on the Line
$y = x - 3$				$(-5, \square)$	$(\square, 10)$
$y = 1 + 2x$				$(2, \square)$	$(\square, -7)$
		-1	$(0, 2)$	$(2, \square)$	$(\square, -7)$
		-3		$(1, 0)$	$(\square, 9)$
				$(-8, \square)$	$(\square, 2)$
				$(1, 1)$	$(5, 13)$

Parallel Lines

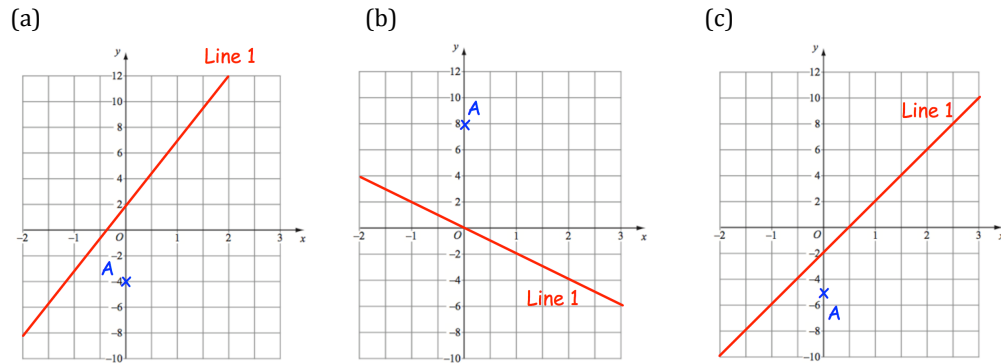| Worked Example	Thinking	Your Turn
a) Write down the equation of a line parallel to $y = 2x - 3$ b) Write down the equation of the line that is parallel to $y = 6x + 1$ and passes through $(0, 8)$		a) Write down the equation of a line parallel to $y = -2x + 3$ b) Write down the equation of the line that is parallel to $y = -6x - 1$ and passes through $(0, -8)$

Fluency Practice

Question 1: Write down the equation of a line parallel to each of the following

(a) y = 2x + 3   (b) y = 5x – 3   (c) y = –3x + 1   (d) y = x – 7

(e) y = –7x – 10   (f) y = –x   (g) y = 10x   (h) y = 4

(i) x + y = 5   (j) 2x + y – 1 = 0   (k) x – 2y + 5 = 0   (l) 3x – 4y – 9 = 0

Question 2: Write down the equation of each of the following lines

(a) Parallel to y = 3x + 5 and passing through (0, 2)

(b) Parallel to y = 4x – 1 and passing through (0, 6)

(c) Parallel to y = 5x and passing through (0, –3)

(d) Parallel to y = –2x + 10 and passing through the origin

(e) Parallel to x + y = 8 and passing through (0, –4)

(f) Parallel to x – 2y + 3 = 0 and passing through (0, 5)

Question 3: Write down the equation of the line parallel to Line 1 and passing through A.

(a)   (b)   (c)

Worked Example	Thinking	Your Turn
Write down the equation parallel to $y = 4x + 1$ which passes through $(2, 17)$		Write down the equation parallel to $y = 8x + 5$ which passes through $(2, 26)$

Worked Example	Thinking	Your Turn
Find the equation of the line parallel to $y = -\frac{1}{3}x - 4$ that passes through $(-2, 5)$		Find the equation of the line parallel to $y = -\frac{1}{2}x - 3$ that passes through $(-2, 5)$

Intelligent Practice

Write down the equation:

Parallel to the line	Goes through the point	Answer
$y = 3x + 2$	$(0,7)$	
$y = 3x + 2$	$(3,0)$	
$y = 3x + 2$	$(3,7)$	
$y = 3x + 10$	$(3,7)$	
$y = 2x - 2$	$(3,7)$	
$y = 2x - 2$	$(-3,7)$	
$y = 2x - 2$	$(0,0)$	

Question 4:   Write down the equation of each of the following lines

(a)   Parallel to $y = 2x - 1$ and passing through (1, 8)

(b)   Parallel to $y = 3x + 2$ and passing through (1, 1)

(c)   Parallel to $y = 5x - 4$ and passing through (2, 9)

(d)   Parallel to $y = 3x - 7$ and passing through (4, 15)

(e)   Parallel to $y = 4x$ and passing through (−1, 3)

(f)   Parallel to $y = -2x + 5$ and passing through (−3, 0)

(g)   Parallel to $y = 6x + 3$ and passing through (10, 5)

(h)   Parallel to $y = -\frac{1}{2}x + 1$ and passing through (3, 0)

(i)   Parallel to $x + y = 10$ and passing through (4, 0)

(j)   Parallel to $x - 3y - 6 = 0$ and passing through (−9, −2)

Line A passes through the points (2, 1) and (5, 10)
Find the equation of the line parallel to A that passes through (2,5)   [3]

Line A passes through the points (2, 1) and (5, 10)
Line B passes through the points (4, 7) and (2, 1)
Show that Line A and Line B are parallel   [4]

Line A passes through the points (3, 6) and (5, -2)
Line B passes through the points (2, 5) and (8, k)
Line A and Line B are parallel. Find the value of k.   [4]

Negative Reciprocals

Worked Example	Thinking	Your Turn
Write the negative reciprocals of: a) 6 b) $\dfrac{1}{6}$ c) $\dfrac{5}{6}$		Write the negative reciprocals of: a) 7 b) $\dfrac{1}{7}$ c) $\dfrac{2}{7}$

Question 5:   Write down the negative reciprocal of each number below.

(a) 4 (b) $\dfrac{2}{3}$ (c) -6 (d) 8 (e) $\dfrac{1}{2}$ (f) 1

(g) $-\dfrac{1}{3}$ (h) $-\dfrac{2}{5}$ (i) $\dfrac{4}{7}$ (j) $1\dfrac{1}{2}$ (k) -1 (l) $-1\dfrac{3}{4}$

$$y = \frac{1}{2}x + 3 \qquad y = \frac{3}{4}x - 2 \qquad y = -\frac{1}{5}x + 1$$

$$y = -\frac{2}{3}x - 5$$

Perpendicular Lines

Worked Example	Thinking	Your Turn
a) Write down the equation of a line perpendicular to $y = 2x - 3$ b) Write down the equation of the line that is perpendicular to $y = \frac{1}{2}x + 3$ and passes through $(0, -1)$		a) Write down the equation of a line perpendicular to $y = -2x + 3$ b) Write down the equation of the line that is perpendicular to $y = -\frac{1}{2}x + 3$ and passes through $(0, 1)$

Fluency Practice

Question 6:   Write down the equation of a line perpendicular to each of the following

(a) y = 4x + 2   (b) y = 2x – 7   (c) y = –5x + 2   (d) y = x – 3

(e) y = –x + 1   (f) $y = \dfrac{1}{2}x + 3$ (g) $y = \dfrac{3}{4}x - 2$ (h) $y = -\dfrac{1}{5}x + 1$

(i) $y = -\dfrac{2}{3}x - 5$ (j) x + y = 12   (k) x – 2y + 8 = 0   (l) 5x – 3y – 3 = 0

Question 7:   Write down the equation of each of the following lines

(a)   Perpendicular to y = 2x + 4 and passing through (0, 3)

Parallel and Perpendicular Lines
Videos 196 and 197 on www.corbettmaths.com

(d) Perpendicular to $y = \dfrac{1}{3}x - 2$ and passing through the origin

(e)   Perpendicular to $y = -\dfrac{1}{5}x + 8$ and passing through (0, –2)

(f)   Perpendicular to $y = -\dfrac{2}{9}x - 10$ and passing through (0, 6)

Question 8:   Write down the equation of the line perpendicular to Line 1 & passing through A.

(a)   (b)   (c)

Page 80
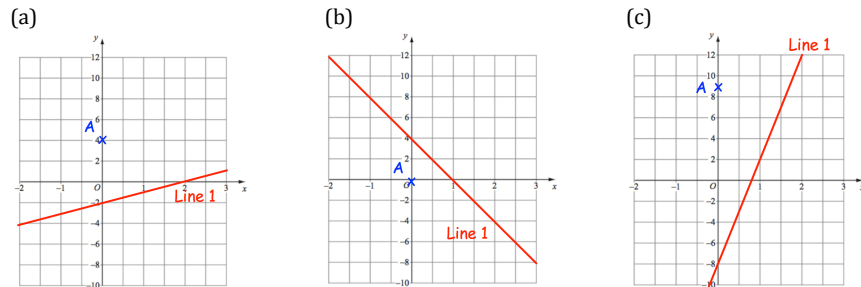
Worked Example	Thinking	Your Turn
Write down the equation perpendicular to $y = 4x + 1$ which passes through $(8, 17)$		Write down the equation perpendicular to $y = 8x + 5$ which passes through $(16, 26)$

Worked Example	Thinking	Your Turn
Find the equation of the line perpendicular to $y = \frac{1}{2}x - 4$ that passes through $(-2, 5)$		Find the equation of the line perpendicular to $y = -\frac{4}{3}x + 3$ that passes through $(-12, -5)$

Intelligent Practice

Write down the equation:

Perpendicular to the line	Goes through the point	Answer
$y = -x + 2$	$(0,7)$	
$y = 2x + 2$	$(0,7)$	
$y = 2x + 2$	$(12,7)$	
$y = 3x + 2$	$(12,7)$	
$y = 3x + 2$	$(10,7)$	
$y = 3x + 2$	$(-12,7)$	
$y = 2x + 2$	$(0,0)$	

Worked Example	Thinking	Your Turn
Find the equation of the line perpendicular to $3x + 2y = 5$ which passes through the point $(3, 7)$		Find the equation of the line perpendicular to $2x + 3y = 5$ which passes through the point $(4, 7)$

Question 9:   Write down the equation of each of the following lines

(a)   Perpendicular to $y = 2x - 1$ and passing through $(4, 0)$

(b)   Perpendicular to $y = -3x + 4$ and passing through $(6, 1)$

(c)   Perpendicular to $y = 4x$ and passing through $(-12, 5)$

(d)   Perpendicular to $y = -\frac{1}{2}x + 1$ and passing through $(3, -7)$

(e)   Perpendicular to $y = \frac{2}{3}x + 4$ and passing through $(-6, -4)$

(f)   Perpendicular to $y = -\frac{3}{5}x - 2$ and passing through $(9, 9)$

(g)   Perpendicular to $x + 4y - 6 = 0$ and passing through $(1, 8)$

Apply

Write down the equation of a line perpendicular to $y = 3x + 3$ [1]

Write down the equation of the line perpendicular to $y = \frac{1}{2}x + 4$ which passes through (0,7) [2]

Line A passes through the points (-3, -1) and (-1, 9).
Line B passes through the points (-2, 1) and (k, 4).
Line A and Line B are perpendicular.
Find the value of k. [4]

Worked Example	Thinking	Your Turn
A is the point $(3, 8)$ B is the point $(1, -2)$ C is the midpoint of AB Find the equation of the line perpendicular to AB which passes through C		A is the point $(3, 8)$ B is the point $(1, 4)$ C is the midpoint of AB Find the equation of the line perpendicular to AB which passes through C

Worked Example	Thinking	Your Turn
ABCD is a rhombus. A has coordinates $(5, 10)$ The equation of DB is $$y = \frac{1}{2}x + 5$$ Find an equation of diagonal AC 		ABCD is a rhombus. A has coordinates $(5, 11)$ The equation of DB is $$y = \frac{1}{2}x + 6$$ Find an equation of diagonal AC
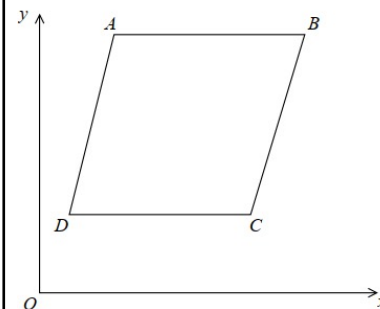
Apply

Question 1: Write down the equations of the lines, from the box, that are:

y = 2x	y = ½x + 1
	y = -5x
y = 3x + 2	
	y = -2x + 3
y = 5x - 4	y = 3x - 2

(a) parallel   (b) perpendicular   (c) cross the y–axis at (0, 3)

(d) pass through the origin   (e) pass through the point (1, 1)

Corbett maths

Parallel and Perpendicular Lines
Videos 196 and 197 on www.corbettmaths.com

Question 10: Two straight lines are shown.

Line 1 has equation $y = \dfrac{3}{2}x - 24$

(a) Find the equation of Line 2

(b) Are the lines perpendicular?

Line 1

O

Line 2
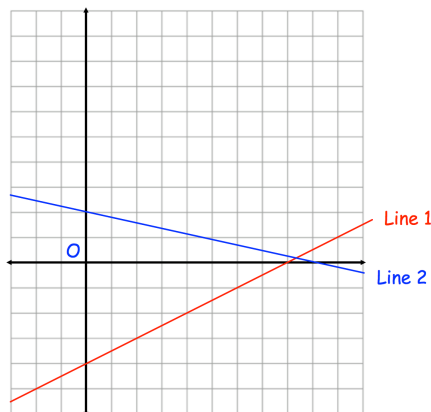
Question 2: Are the lines 2x + y = 8 and y = 2x + 5 parallel?

Question 3: Are the lines 4x – y – 5 = 0 and x + 4y + 1 = 0 perpendicular?

Question 4: A straight line passes through the points A(1, 4) and B(5, 16).

(a) Find the equation of the line parallel to AB that passes through (1, 7)

(b) Find the equation of the line perpendicular to AB that passes through the midpoint of AB.

Y

B (5, 16)

A (1, 4)

x
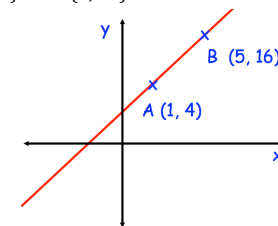
Question 5: The line L has equation y = 2x + 8
The line L crosses the x–axis at the point A.
The line M is perpendicular to Line L and passes through the point A

(a) Find the coordinates of the point A.
(b) Find equation of the Line M.

Question 6: The point A has coordinates (−12, −7) and the point B has coordinates (−8, 1)

Find the equation of the line parallel to AB and passing through (2, 5)

Question 7: The line L passes through the points (−2, 1) and (2, 3).
The line N passes through the points (4, 7) and (12, 11).

Bryan says that the lines L and N are parallel.
Is Bryan correct? Explain your answer.

Question 8: The point C has coordinates (2, −3) and the point D has coordinates (4, 6).

Find the equation of the line perpendicular to CD and passing through D.

Question 9: The line Q passes through the points (−10, −2) and (−8, −8)
The line R passes through the points (1, 2) and (10, a)

The lines Q and R are perpendicular.

Find a.

Answers

Inequalities

Where in real life might we use phrases like "at least", "more than", "less than" and "at most"?

Real-life scenario	How we could represent mathematically
"You can have at most 20 people at your party."	$x \leq 20$ (where x is number of people)
"I was chased by at least 10 zombies!"	$z \geq 10$ (where z is number of zombies)
"I'll visit next in less than a month."	$d < 31$ (where d is number of days)
"My cat's IQ is between 120 and 140."	$120 \leq x \leq 140$ (where x is my cat's IQ)

Definition	**Characteristics**
Relationship between two expressions that are not exactly equal.	Expressions can be connected with the following signs: • $>$ Greater than • $\geq$ Greater than or equal to • $<$ Less than • $\leq$ Less than or equal to • $\neq$ Not equal to

Examples	**Non Examples**
• $5 > -2$ • $x \leq 12$ • $-3 < y \leq 5$ • $x < -1, x \geq 8$ • $a \neq b$ • $2x - 7 < x + 6$	• $x = 5$ • $4x = 2x + 5$ • $-5 > -1$

Why we need Inequalities?

Inequalities are needed in mathematics when we need to represent a **range of values**.

Equation	Number of Solutions
$x + 5 = 7$	1 solution ($x = 2$)
$x^2 = 9$	2 solutions ($x = 3, -3$)
$x + 3 = x$	0 solutions
$x > 4$	∞ solutions ($x = 4.01, \ 5, \ 2000, ...$

A 'range' of values often involves infinitely possible many values. So we need inequalities to be able to represent them, as it's not possible to list all the values.

Reading Inequalities

Notice the symbol is taller on the side which is larger.

$x > 7$

Inequality	What It Means
$x > 7$	"x is greater than 7" This doesn't include 7 Examples: $7.2, 10$
$x \geq 7$	"x is greater than or equal to 7" or "x is at least 7" This does include 7 Examples: $7, 8, 100.5$
$x < 10$	"x is less than 10" Examples: $-3, 4, 9.2$
$x \leq 8$	"x is less than or equal to 8" or "x is at most 8" Examples: $8, -3, 4, 9.2$

$-1 \leq x < 3$

What does this mean in words?
"x is greater or equal to -1, and less than 3"

Or we could more simply say:
"x is between -1 and 3, inclusive of -1"

Inequalities on Number Lines

$x = 4$

We can use a filled circle on a number line to indicate we want to include the value.

But what about:

$x \leq 4$

We again use a filled circle to indicate that we want to include 4.
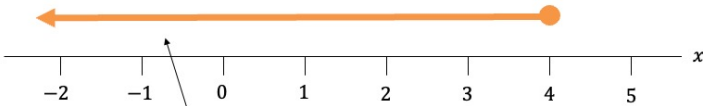
But we also have an arrow pointing left to say we also want any value less than 4.

$x \geq 1$

We again want to include 1, but our arrow is right this time to indicate values greater than 1.

$x < 2$

We again have an arrow left to indicate "less than 2", but this time we DON'T want to include 2 itself. We use an **unfilled circle** to indicate that 2 is excluded.
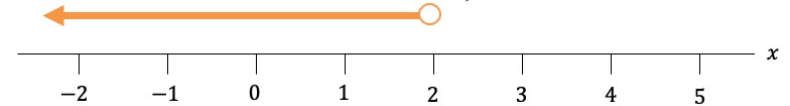
$-1 \leq x < 3$

What does this mean in words?
"*x is greater or equal to -1, and less than 3*"
Or we could more simply say:
"*x is **between** -1 and 3, inclusive of -1*"

We therefore tend to write inequalities in this form when we want to say a variable is **between** two values.

On a number line...
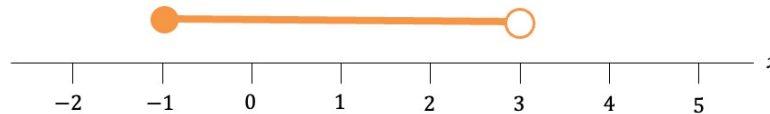
x is an integer (whole number) and $-1 \leq x < 4$. List all the numbers that x could be.
$-1, 0, 1, 2, 3$

If we said "3 to 5 inclusive", this would include the 3 and 5, i.e. $3 \leq x \leq 5$.
If we said "3 to 5 exclusive", this wouldn't include the 3 nor 5, i.e. $3 < x < 5$.

Fluency Practice

Question 4:   Write down the inequalities shown below

(a)
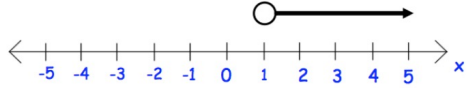
(b)

(c)
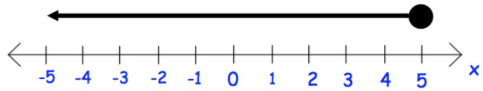
(d)
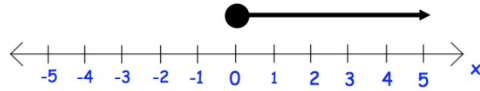
(e)
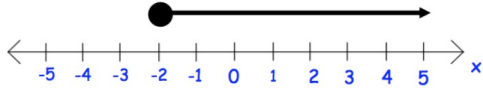
(f)

(g)
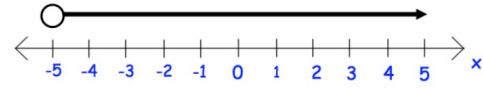
(h)
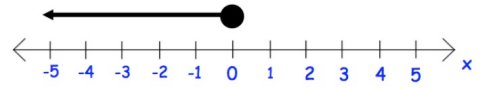
Question 5:   Show these inequalities on a number line.

(a) $x > 2$

(b) $x < 4$

(c) $x \geq 3$

(d) $x \leq 5$

(e) $x \geq 0$

(f) $x \leq -1$

(g) $x < -4$

(h) $x > -5$

(i) $x \geq -6$

(j) $x > 0$

(k) $x < -2$

(l) $x > -1$

Fluency Practice

Question 9: Write down the inequalities shown below

(a)

(b)

(c)

(d)

(e)

(f)

Question 1: The cost, c, of a TV is less than £300. Write this as an inequality.

Question 2: To go on a rollercoaster, a person's height, h, must be over 140cm. Write this as an inequality.

Question 3: The value of a house, v, is £100,000 or more. Write this as an inequality.
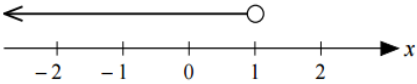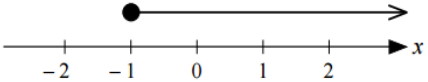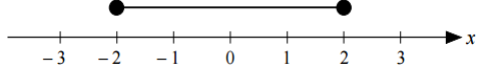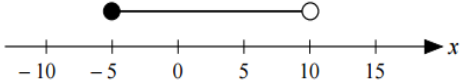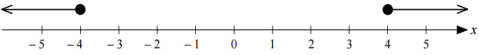
Question 4: There are 20 students in a class. The number of students present on a particular day is 20 or less. Write this as an inequality.

Question 5: Write down any integers (whole numbers) that satisfies **both** x > 4 and x ≤ 8

Question 6: Write down any integers (whole numbers) that satisfies **both** 2 < x ≤ 9 and x > 5

Plot the following on a number line:

1) $5 \leq n \leq 10$

2) $5 \leq n < 7$

3) $0 < n \leq 5$

4) $10 < n \leq 15$

5) $-5 \leq n \leq -2$

6) $-5 < n < -2$

7) $-3 \leq n < 3$

8) $-2 < n \leq \frac{1}{2}$

9) $\frac{1}{2} \leq n < 3$

10) $\frac{1}{2} \leq n < \frac{3}{2}$

11) $\frac{7}{5} \leq n \leq \frac{13}{5}$

12) $-\frac{7}{5} < n < \frac{11}{5}$

13) $3.5 \leq n \leq 4.5$

14) $-3.5 \leq n \leq 4.5$

15) $-9.1 < n \leq 1.1$

16) $-3.5 < n < -1.5$

Set Notation

Complete the table

	Number Line	Set Notation
1		
2		
3		
4		$\{x : -1 < x < 3\}$
5		$\{x : -2 < x \leq 2\}$
6		
7		$\{x : x < -3 \ or \ x > 3\}$
8		

Solving Linear Inequalities

Inequalities behave in a similar way to equations: whatever we do to one side of the equation, we have to do the same to the other.

'Solving an inequality' means to get x on its own on one side of the equation. This is so that the range is then clear.

When you divide or multiply both sides of an inequality by a negative number, **reverse the direction of the inequality**.

Why?
Consider the inequality $2 < 4$
This is clearly true as 2 is less than 4
But, if we multiple/divide by both sides by -1, we get $-2 < -4$, which is false.
However, if we reverse the inequality sign, we get $-2 > -4$, which is true as -2 is more than -4.

But it is probably easiest to avoid needing to divide by a negative number in the first place...

IF THERE IS A NEGATIVE COEFFICIENT OF THE VARIABLE THEN ADD TO BOTH SIDES TO GET A POSITIVE ONE.

Worked Example	Thinking	Your Turn
Solve: a) $2x - 8 < 16$ b) $2(4 - x) < 16$		Solve: a) $3x - 9 > 27$ b) $3(3 - x) > 27$

Worked Example	Thinking	Your Turn
Solve: $10(x + 3) + 3(2x + 6) < 144$		Solve: $5(x + 3) + 2(2x - 6) \leq 111$

Worked Example	Thinking	Your Turn
Solve: $7(x + 3) - 3(2x - 6) > 84$		Solve: $5(x - 3) - 2(2x - 6) \geq 111$

Fluency Practice

Solve the following inequalities:

1) $5x - 40 \leq 80$

2) $5x - 40 < 40$

3) $40 - 5x \geq 40$

4) $5(8 - x) < -40$

5) $5(8 - 2x) > -40$

6) $-5(8 - 2x) > -40$

7) $-2x + 5 < -35$

8) $5 - 2x < -35$

9) $-5 - 2x \leq -35$

10) $-7 - 2x \leq -35$

11) $-7 - 4x > -35$

12) $-7 - 7x > -35$

13) $4(x + 3) + 8(x + 1) < 44$

14) $7(x - 3) + 5(x + 2) \leq 37$

15) $3(x - 2) + 2(x - 5) > 24$

16) $2(2x - 1) - 4(3x - 1) > 26$

17) $5(2x + 3) - 6(x - 1) < 29$

18) $2(5x - 2) - 3(3x - 1) \geq 6$

Fluency Practice

Question 1: Solve each of the inequalities below

(a) $x + 4 > 9$ (b) $x - 3 < 2$ (c) $2x \leq 14$ (d) $8x < 16$

(e) $5x \geq 15$ (f) $\dfrac{x}{3} > 4$ (g) $\dfrac{x}{5} \leq 2$ (h) $x + 6 \geq 4$

Question 2: Solve each of the inequalities below

(a) $2x + 1 \leq 9$ (b) $3x - 5 > 16$ (c) $4x + 8 < 32$ (d) $5x - 2 \geq 68$

(e) $\dfrac{x}{2} + 1 \leq 5$ (f) $\dfrac{x}{9} - 6 > 4$ (g) $\dfrac{x + 3}{2} \geq 5$ (h) $\dfrac{x - 5}{4} > 2$

Question 3: Solve each inequality below and represent the solution on a number line.

(a) $4x + 7 < 11$ (b) $3x - 2 \geq 10$ (c) $\dfrac{x}{2} - 3 > 0$ (d) $\dfrac{x + 18}{4} \leq 5$

Question 4: Solve each of the inequalities below

(a) $5(x - 3) \geq 40$ (b) $6(x + 2) < 42$ (c) $2(5x + 1) \leq 36$

(d) $4(x - 2) < 18$ (e) $2(2x - 9) \geq 22$ (f) $3(2x + 7) \leq 9$

Worked Example	Thinking	Your Turn
Solve: a) $9x + 4 < 2x + 60$ b) $3x - 23 \leq 7 - 2x$		Solve: a) $5x + 7 > 2x + 22$ b) $2x - 23 \geq 9 - 2x$

Worked Example	Thinking	Your Turn
Solve: a) $3(x + 2) < 2(x + 3)$ b) $3(x + 8) > 3(2 - x)$		Solve: a) $7(x - 3) \leq 2(x + 7)$ b) $3(x - 5) \geq 5(5 - x)$

Intelligent Practice

Solve the following inequalities:

1) $5x + 3 < 3x + 13$

2) $5x + 2 \leq 3x + 44$

3) $11x + 2 \geq 5x + 44$

4) $11x + 44 \geq 5x + 2$

5) $11x + 39 > 5x + 21$

6) $8x + 39 > 5x + 21$

7) $8x + 39 < 2x + 21$

8) $8x - 39 < 21 - 2x$

9) $8x - 39 \leq 21 - 17x$

10) $8x - 39 \geq 6 - 7x$

11) $39 - 8x \geq 6 - 7x$

12) $39 - 10x \geq 6 - 7x$

13) $6 - 10x \leq 39 - 7x$

14) $6 - 18x \leq 39 - 7x$

Solve the following inequalities:

1) $3(x - 5) \leq 3(2x + 1)$

2) $3(x - 5) < -3(2x + 1)$

3) $-3(x + 5) \geq -3(2x + 1)$

4) $-3(x - 5) < -3(2x + 1)$

5) $-3(x - 5) > -3(2x - 1)$

6) $-3(2x - 1) > -3(x - 5)$

Extension 1

Why is it not possible to solve the following? Explain your answer.

$$3x + 3 < 15 + 3x$$

Extension 2

Explain your thinking process to solve the inequality

$$\frac{x}{4} - 2 < 3(2x - 7)$$

1. (a) (i) Solve the inequality

$$5x - 7 < 28$$

   (ii) On the number line, represent the solution set to part (i).

   (3)

2. (a) Solve $5x + 12 < 17$ **(2)**
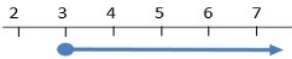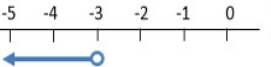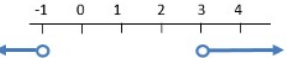
   (b) Solve the inequality $3(2y + 1) > 10$ **(2)**

3. (i) Solve the inequality $7x - 3 > 18$ **(2)**

   x is a whole number such that $7x - 3 > 18$

   (ii) Write down the smallest value of x. **(2)**

Fill in the Gaps

Q	Inequality	Represent on a number line	Integer solutions
1	$x > 3$		
2			$x = 3, 4, 5…$
3			$x = -3, -4, -5…$
4	$-3 \leq x$		
5	$x - 1 > 2$		
6			
7	$x + 5 \leq 2$		
8			
9			$x = 4, 5, 6 … or$ $x = -1 - 2, -3 ….$
10	$< x \leq$		$x = -2, -1, 0, 1, 2, 3$
11	$x \geq 1 \ and \ x < 3$		
12	$3x > 9$		

Worked Example	Thinking	Your Turn
Solve: a) $-1 < 2x + 3 < 9$ b) $-1 \leq 2x + 6 < 9$		Solve: a) $-9 < 2x + 3 < 1$ b) $-9 \leq 2x + 6 \leq 1$

Fluency Practice

Solve:

1) $4 < x + 1 < 10$

2) $4 < x - 1 < 10$

3) $4 < 2x - 1 \leq 10$

4) $-4 \leq 2x - 1 \leq 10$

5) $-10 \leq 2x - 1 \leq -4$

6) $-10 \leq 4x - 2 \leq -4$

7) $-10 \leq 4x - 2 \leq 4$

8) $-10 \leq 4x - 2 \leq 8$

9) $-20 \leq 4x - 2 \leq 8$

10) $-20 < 4x - 2 \leq 8$

11) $-20 < 4x \leq 8$

12) $-20 < \frac{1}{4}x \leq 8$

13) $-20 < \frac{1}{2}x \leq 8$

14) $-20 < \frac{1}{2}x - 8 \leq 8$

15) $-20 < \frac{x-8}{2} \leq 8$

16) $-20 < \frac{8-x}{2} < 8$

Question 8:   Solve each of the inequalities below

(a) $6 < x + 3 < 10$

(b) $4 \leq 2x \leq 7$

(c) $1 \leq 3x < 9$

(d) $4 < \frac{x}{5} < 6$

(e) $9 \leq 2x + 3 \leq 25$

(f) $-3 \leq \frac{x}{4} - 1 < 0$

Question 9:   Find the integers that satisfy each of the inequalities below

(a) $5 < x < 9$

(b) $-3 < x \leq 1$

(c) $4 \leq 2x \leq 8$

(d) $16 \leq 5x + 1 < 31$

(e) $0 \leq \frac{x-6}{2} < 2$

(f) $-9 < \frac{x}{4} - 1 < -8$

Combining Inequalities

We have already seen examples where we have combined inequalities together:

"x is greater than 2 **and** less than 5."

$x > 2$
$x < 5$

$2 < x < 5$

"Charles is at least 12 years old."

"Charles is at least 15 years old."
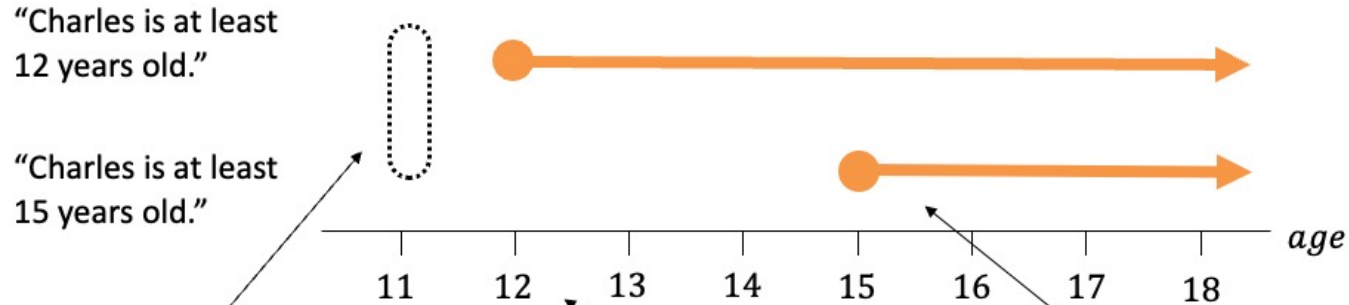
To combine these together, place your finger vertically up the page. We will gradually 'scan' our finger from left to right.

At 12, our finger is over the top line, but not the bottom. So 12 is not in our combined inequality.

However at 15, we're on both lines (recall the filled circle means 15 is included).
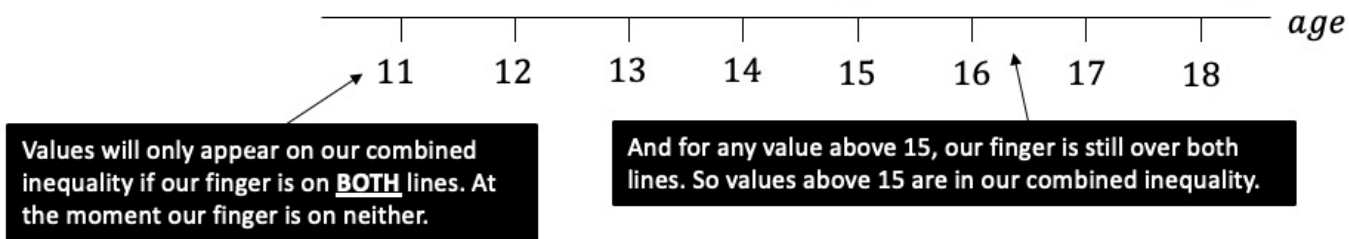
Values will only appear on our combined inequality if our finger is on **BOTH** lines. At the moment our finger is on neither.

And for any value above 15, our finger is still over both lines. So values above 15 are in our combined inequality.

Page 135

Worked Example	Thinking	Your Turn

Worked Example

$12 \leq a \leq 17$

$a > 15$

Combined

age

11 12 13 14 15 16 17 18
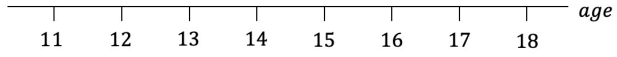
$x \leq 6$ or $x \geq 8$

$5 < x \leq 9$

Combined

age

4 5 6 7 8 9 10 11
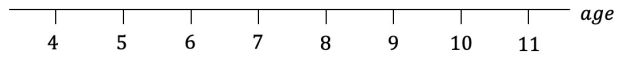
Your Turn

$x \leq 8$

$6 \leq x < 9$

Combined

x

4 5 6 7 8 9 10 11

$x \leq 8$

$6 \leq x < 9$

Combined

x

4 5 6 7 8 9 10 11
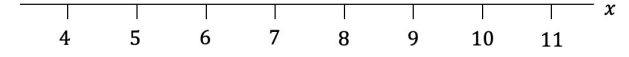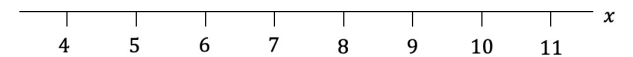
Worked Example	Thinking	Your Turn
Solve: $3 - x \leq 2 < 10 - 2x$		Solve: $1 + x < 5 \leq 7 + 5x$

Fluency Practice

1 By drawing suitable number lines or otherwise, combine the following inequalities.

a $\quad x < 3, \quad x < 4$ $\quad\Rightarrow$

b $\quad 0 < x < 3, \quad x < 2$ $\quad\Rightarrow$

c $\quad x \geq 4, \quad -1 < x < 5$ $\quad\Rightarrow$

d $\quad 0 < x < 2, \quad 1 < x < 3$ $\quad\Rightarrow$

e $\quad x < 1 \text{ or } x > 3, \quad x > 2$ $\quad\Rightarrow$

f $\quad x < 2 \text{ or } x > 4, \ x < 0$ $\quad\Rightarrow$

g $\quad x < 1 \text{ or } x > 3, \ 2 < x < 4$ $\quad\Rightarrow$

h $\quad x < 1 \text{ or } x > 3, \ 0 < x < 4$ $\quad\Rightarrow$

i $\quad x < 1 \text{ or } x > 3, \ x < 2 \text{ or } x > 4$ $\quad\Rightarrow$

j $\quad x < 2 \text{ or } x > 3, \ x < 1 \text{ or } x > 4$ $\quad\Rightarrow$
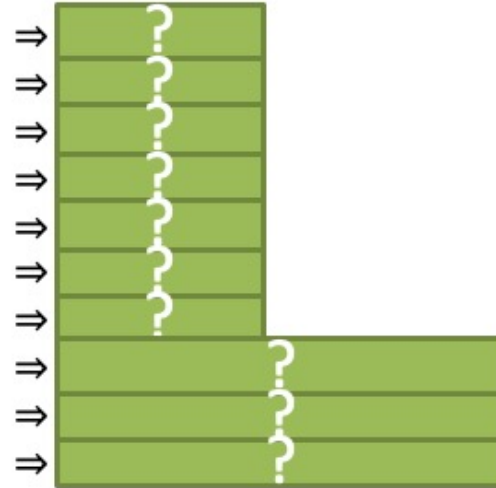
2 Solve the following inequalities.

a $\quad x + 1 < 5 < x + 7$ $\quad\Rightarrow$

b $\quad x - 2 \leq 6 < x + 1$ $\quad\Rightarrow$

c $\quad 3 - x < x \leq 2x + 1$ $\quad\Rightarrow$

d $\quad 2x + 1 > 4, \ x \leq 2$ $\quad\Rightarrow$

e $\quad 5 - 2x > 3, \ x + 1 > 0$ $\quad\Rightarrow$

f $\quad 2 - x < x < 8 - x, \quad x \leq 3$ $\quad\Rightarrow$

Graphical Inequalities

Worked Example	Thinking	Your Turn
Shade the region which satisfies the inequality: a) $x > 3$		Shade the region which satisfies the inequality: a) $x < 5$
b) $x \leq -2$		b) $x \geq -1$
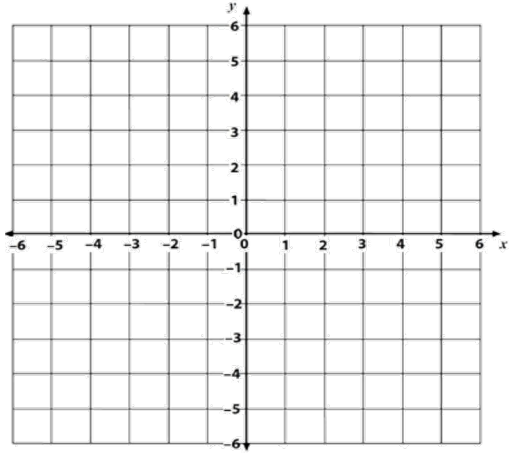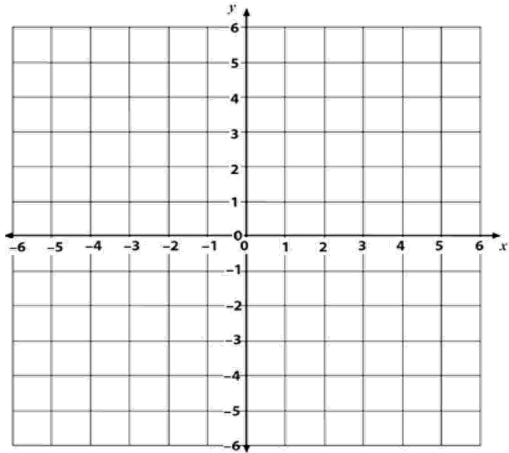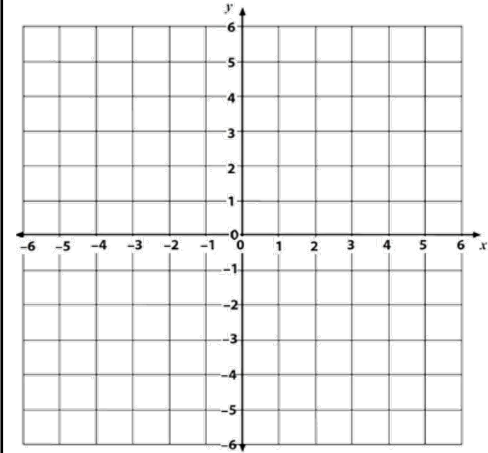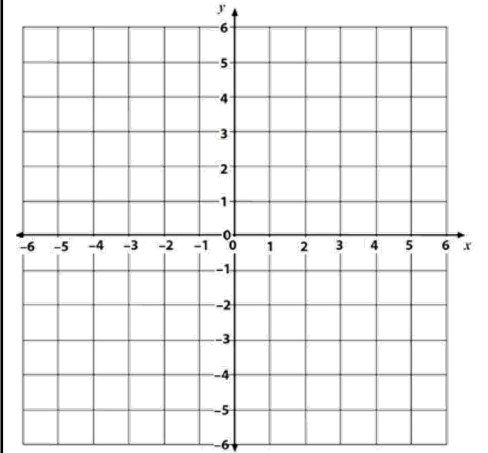
Worked Example	Thinking	Your Turn
Shade the region which satisfies the inequality: a) $y > 3$		Shade the region which satisfies the inequality: a) $y < 5$
b) $y \leq -2$		b) $y \geq -4$
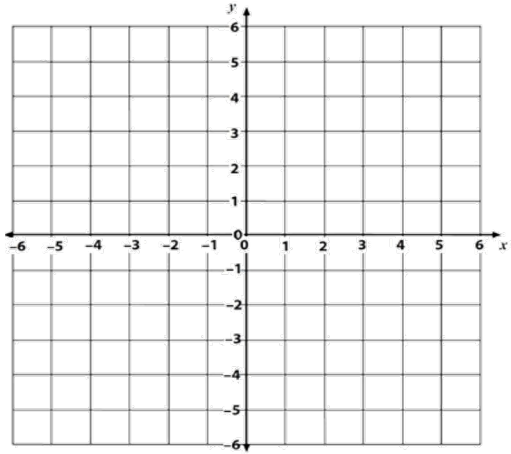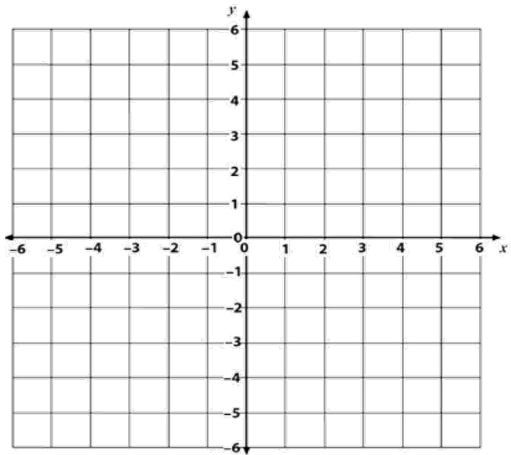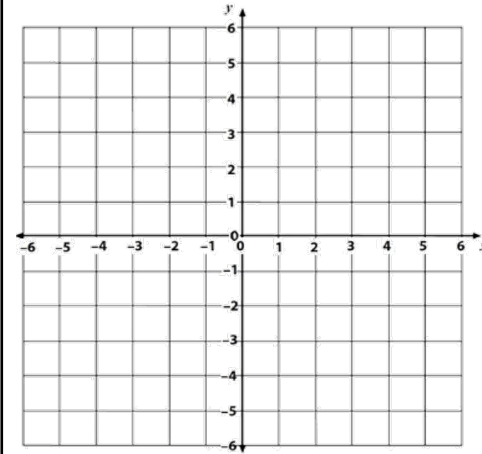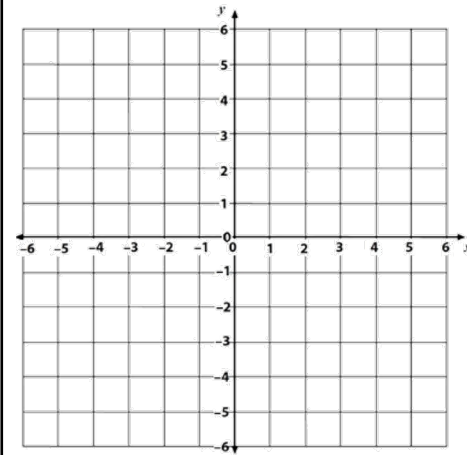
Worked Example	Thinking	Your Turn
Shade the region which satisfies the inequality: a) $-2 \leq x < 5$		Shade the region which satisfies the inequality: a) $-4 < x \leq 3$
b) $-2 \leq y < 5$		b) $-4 < y \leq 3$
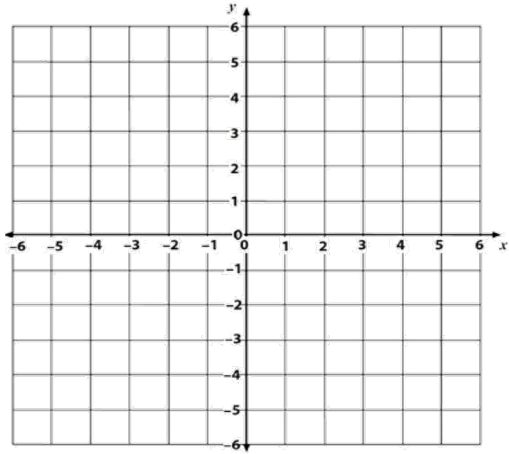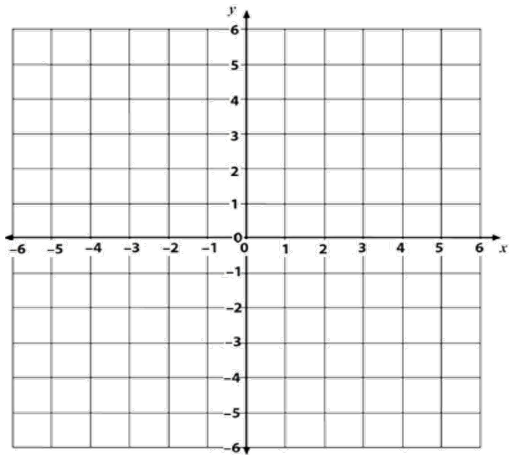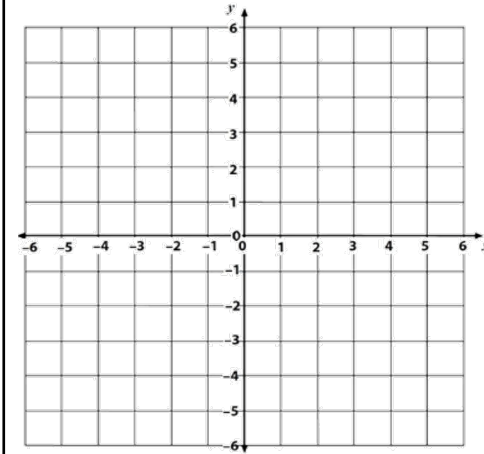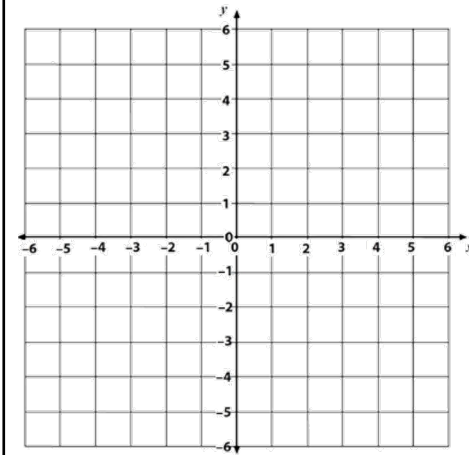
Worked Example	Thinking	Your Turn
Write the inequality that defines the red region:		Write the inequality that defines the red region:
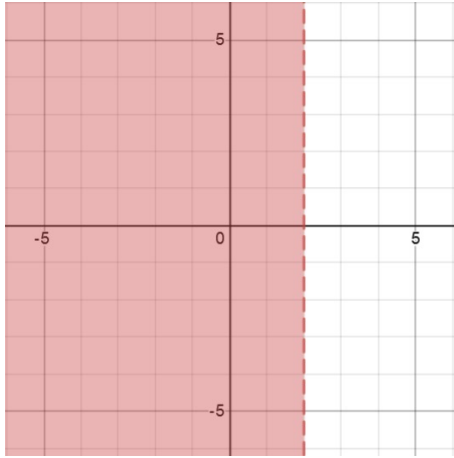
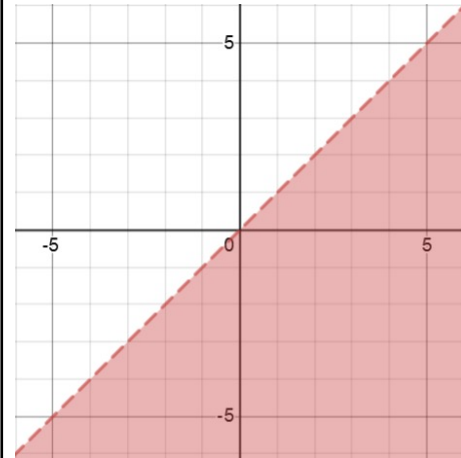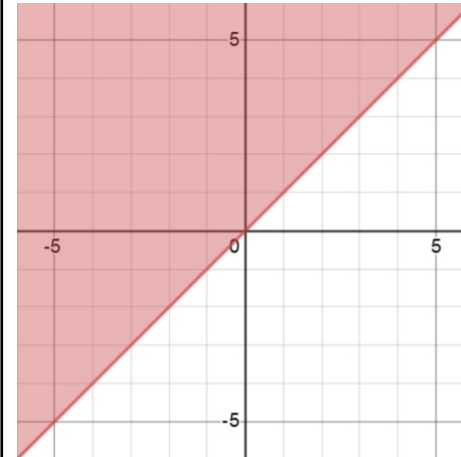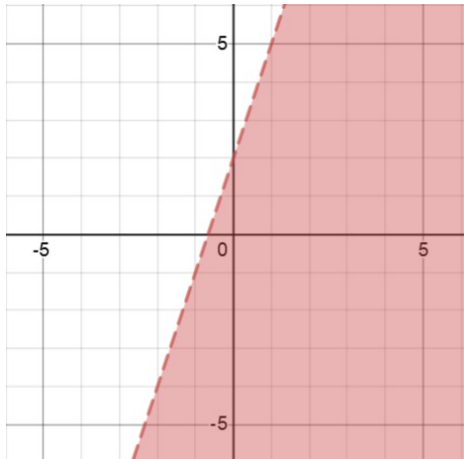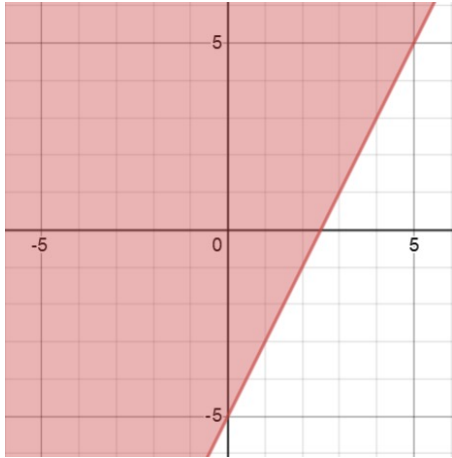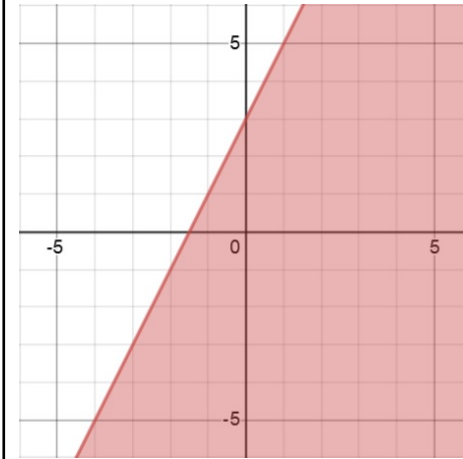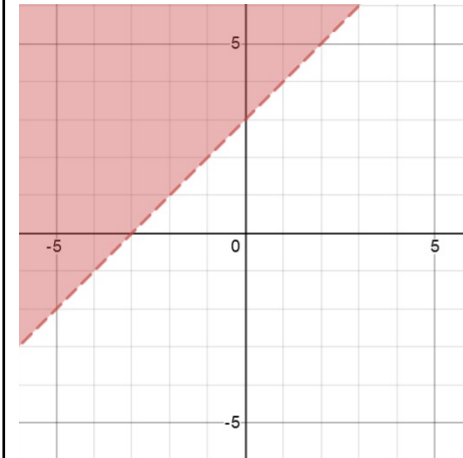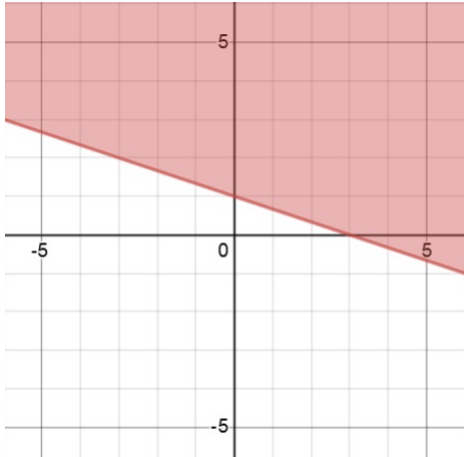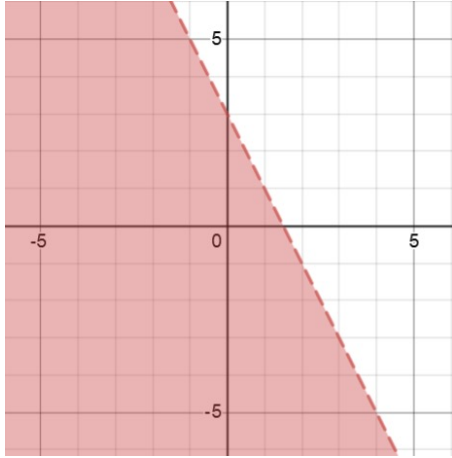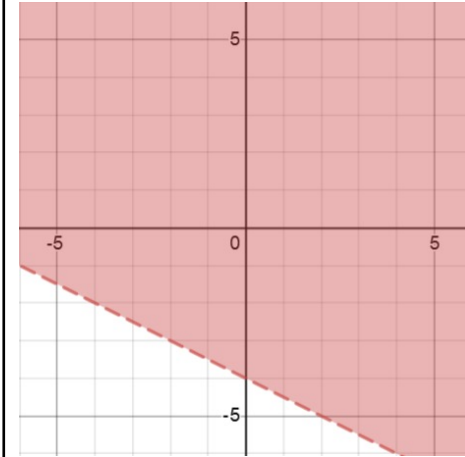| Worked Example	Thinking	Your Turn
Write the inequality that defines the red region:		Write the inequality that defines the red region:

Click here ———— Scan here ————

Fluency Practice Corbett maths
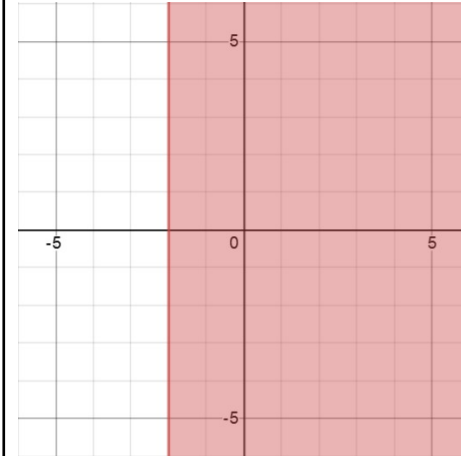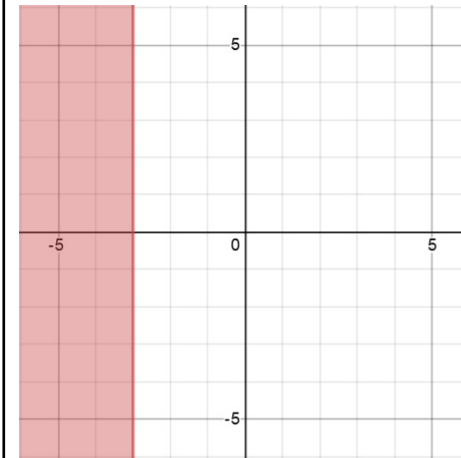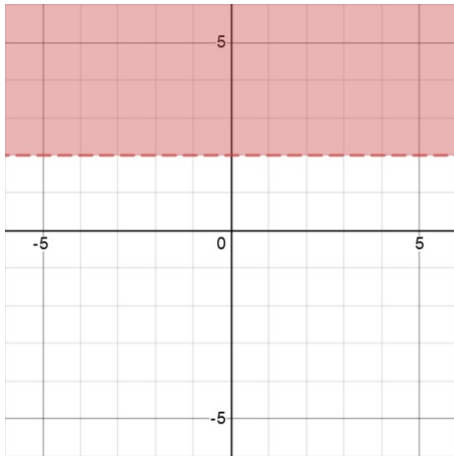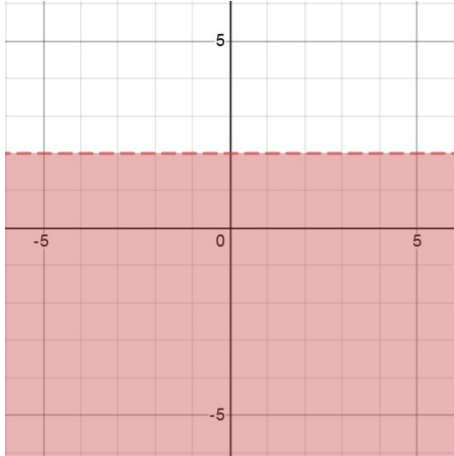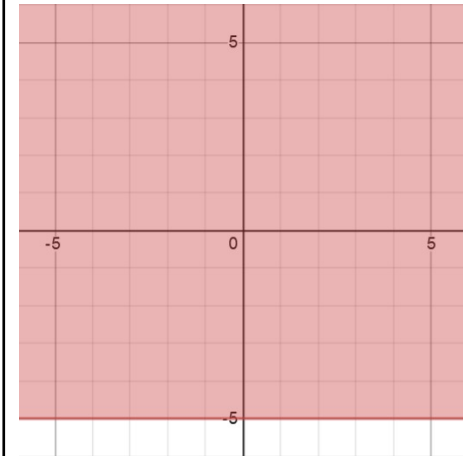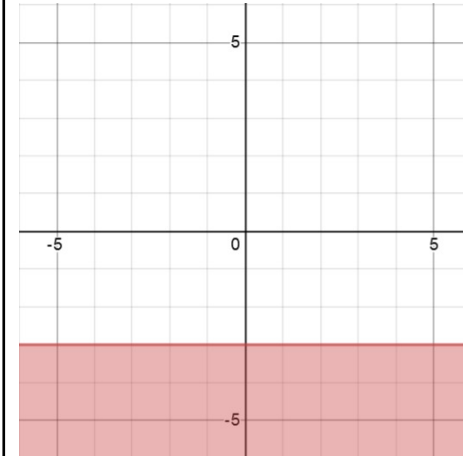
Question 1: On copies of the grid below, clearly indicate the region that satisfies each inequality.

(a) x > 2   (b) x < 4   (c) x ≤ −1   (d) x > 0

(e) x ≥ −3   (f) y < 1   (g) y ≥ −2   (h) y ≤ 4

(i) y > 2   (j) x ≥ 3   (k) y < 0   (k) x < − 5

Question 2: Write down the inequality represented in each diagram below.

(a)

(b)

(c)

(d)

(e)

(f)
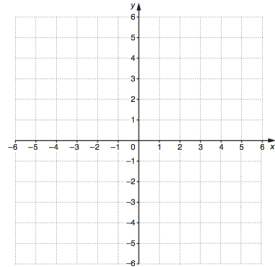
Question 3: On copies of the grid below, clearly indicate the region that satisfies each inequality.

(a) −4 < x < 1   (b) 0 ≤ x ≤ 5   (c) −3 ≤ x < 3

(d) −5 ≤ y ≤ −2   (e) −1< y < 4   (f) −1 < y ≤ 2.5

(g) −2 < x ≤ 3   (h) −4 ≤ y ≤ 2   (i) −2 ≤ y < 2
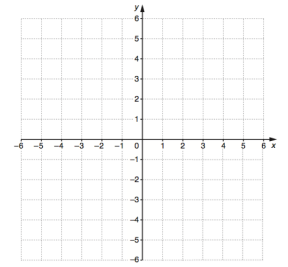
Question 4: Write down the inequality represented in each diagram below.

(a)

(b)

(c)

(d)

(e)

(f)
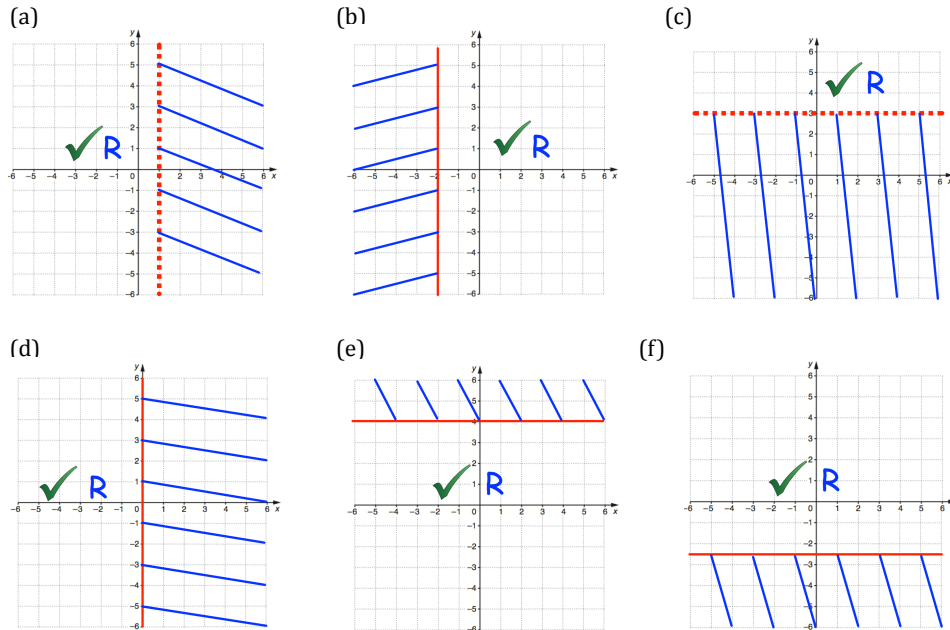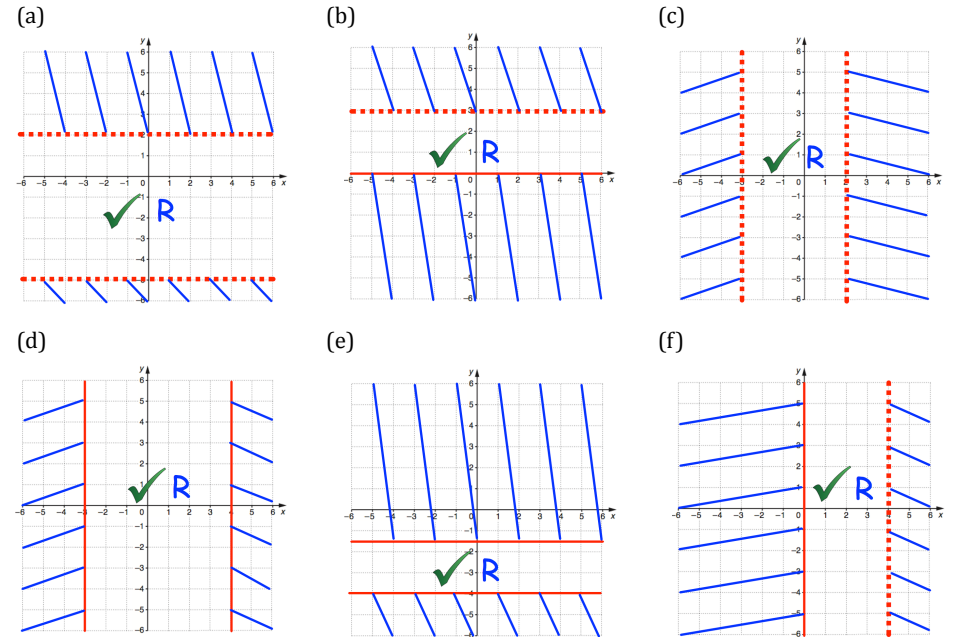
Question 5: On a grid, clearly indicate the region that satisfies the following inequalities.

(a) −2 < x < 3 and y ≥ −1   (b) −5 ≤ y ≤ 1 and x < 3   (c) 1 < x ≤ 3 and −2 ≤ y < 0

Answers

Click here
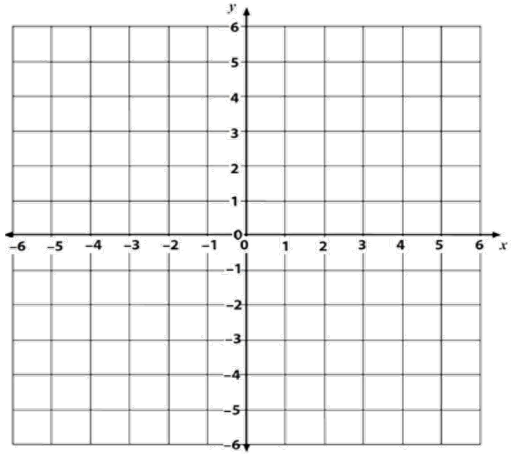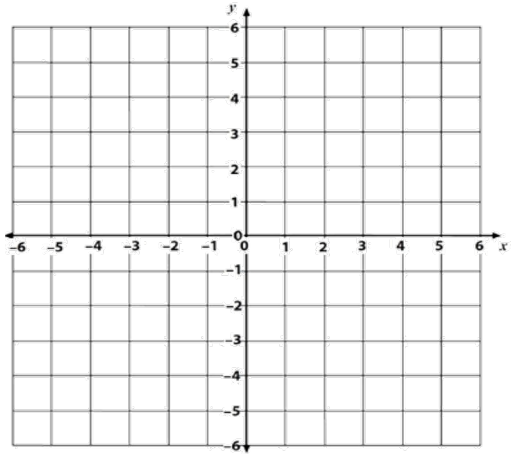
Worked Example	Thinking	Your Turn
Shade the region which satisfies the inequality: $y > 2x + 3$		Shade the region which satisfies the inequality: $y > 4x - 1$
$y < 2x + 3$		$y < 4x - 1$
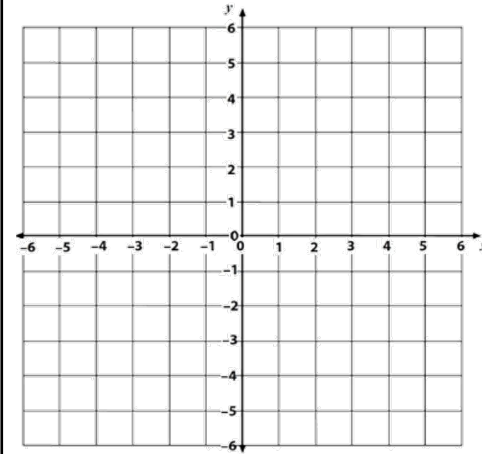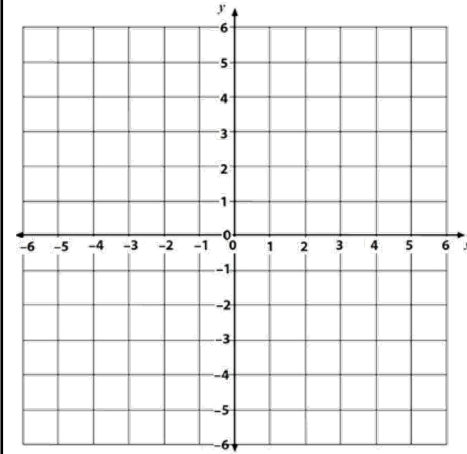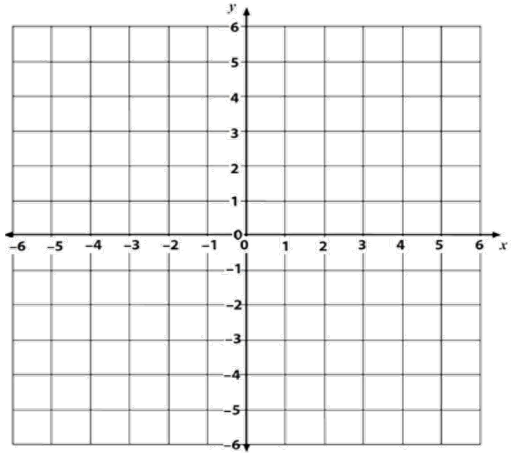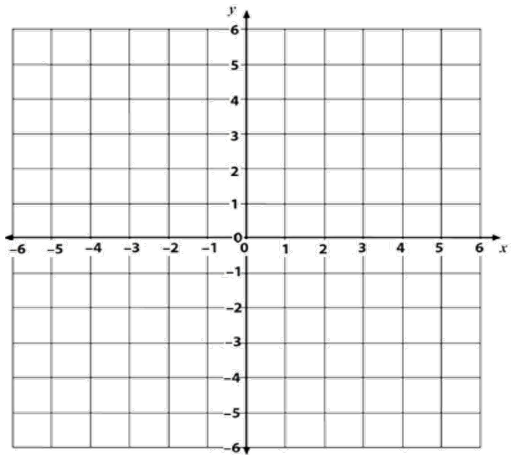
Worked Example	Thinking	Your Turn
Shade the region which satisfies the inequality: $y < -2x + 3$		Shade the region which satisfies the inequality: $y < -1 - 4x$
$y > -2x + 3$		$y > -1 - 4x$
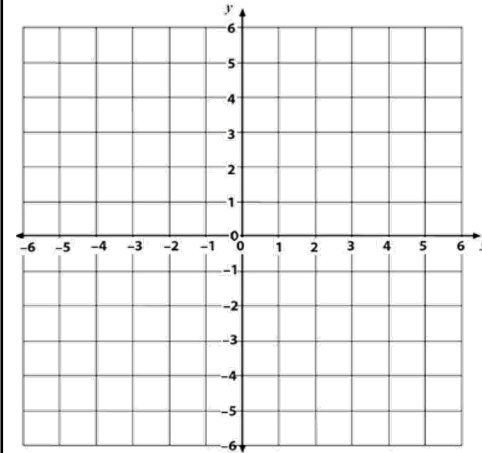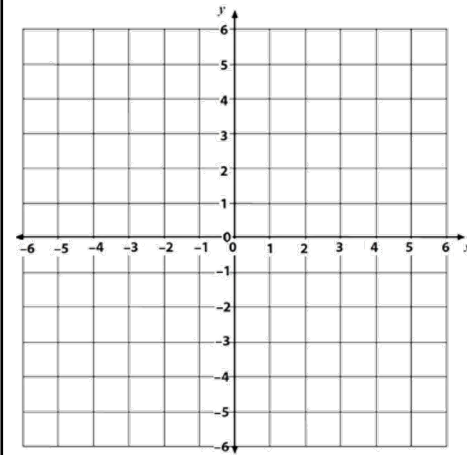
Worked Example	Thinking	Your Turn
Shade the region which satisfies the inequalities. Label it R. $x \leq 3, y > 1$ and $y \geq x + 3$		Shade the region which satisfies the inequalities. Label it R. $x < 4, y \geq 3, y \geq x + 2$
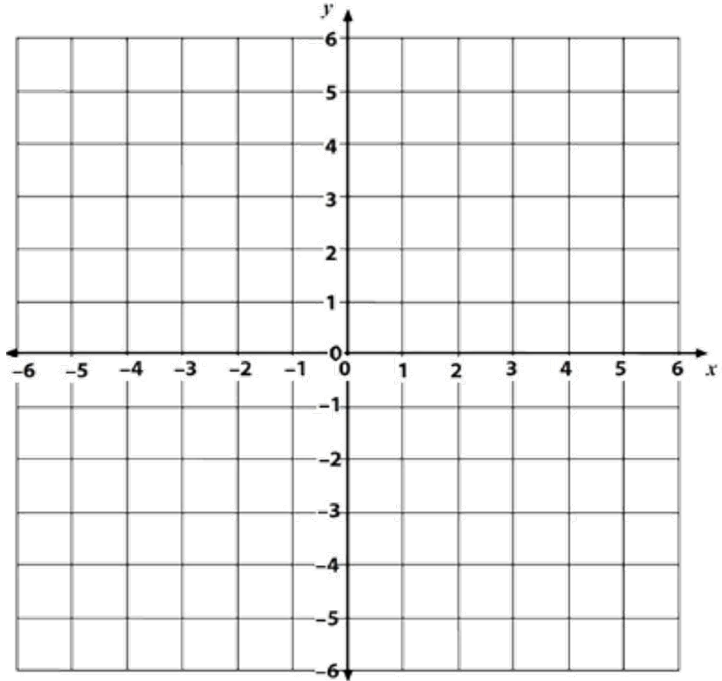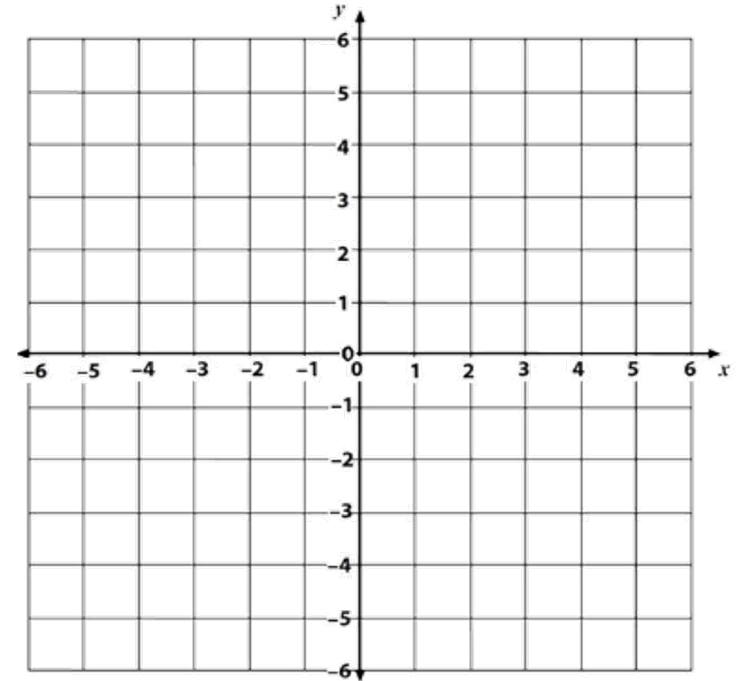
Worked Example	Thinking	Your Turn
Shade the region which satisfies the inequalities. Label it R. $x \geq -2, y < 1$ and $y < x - 1$		Shade the region which satisfies the inequalities. Label it R. $x > -3, y \leq 4$ and $y < x - 2$
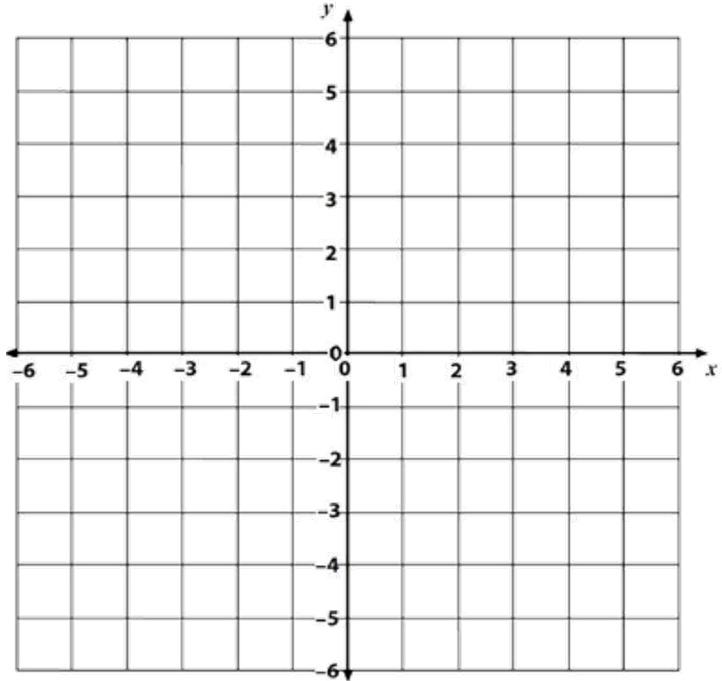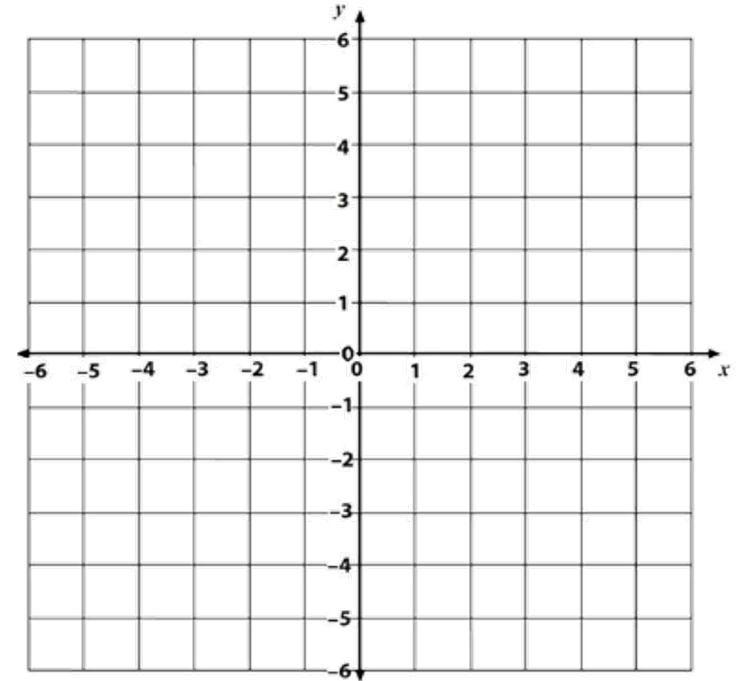
Worked Example	Thinking	Your Turn
Shade the region which satisfies the inequalities. Label it R. $x \geq 2, y > -1$ and $x + y \leq 5$		Shade the region which satisfies the inequalities. Label it R. $x \geq 2, y > 1$ and $x + y \leq 6$
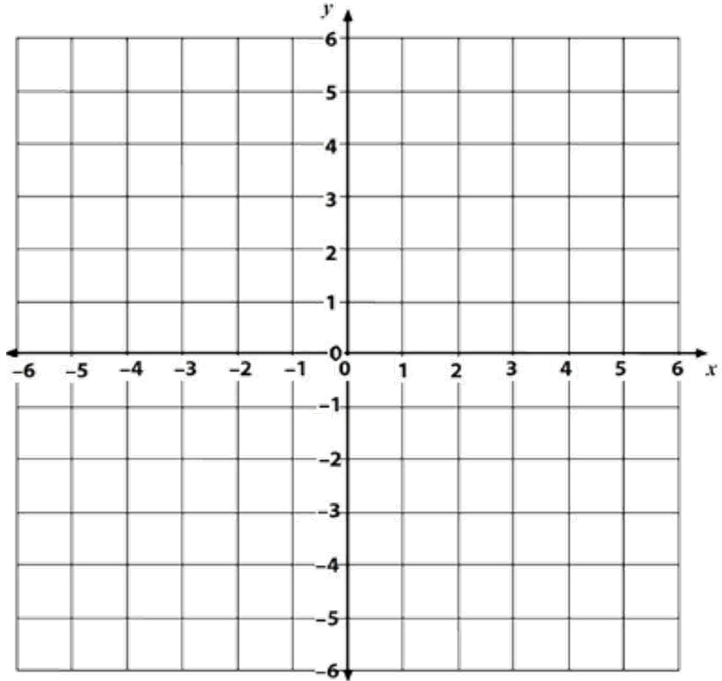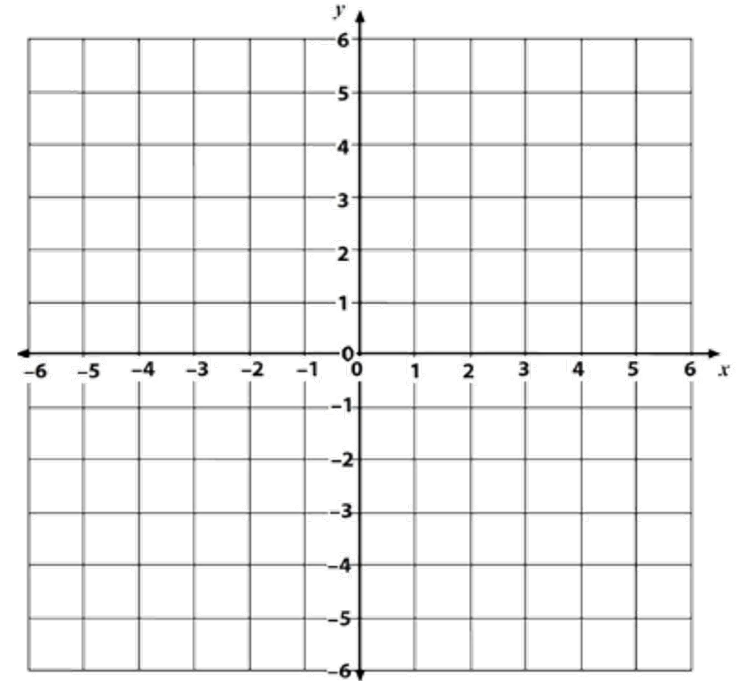
Worked Example	Thinking	Your Turn
Write the inequality that defines the red region:		Write the inequality that defines the red region:
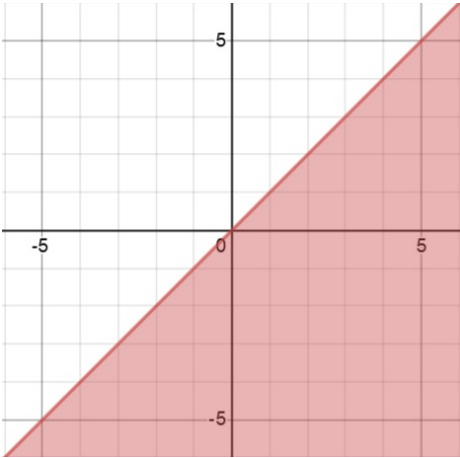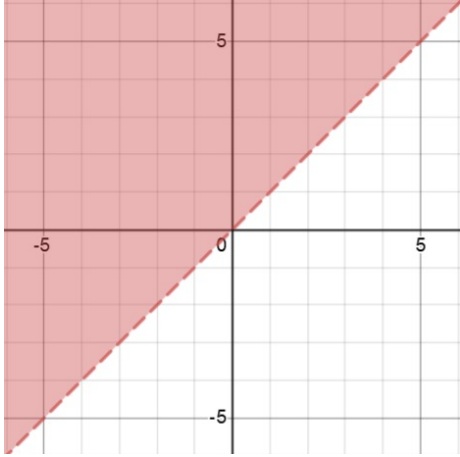
Worked Example	Thinking	Your Turn
Write the inequality that defines the red region:		Write the inequality that defines the red region:

Worked Example	Thinking	Your Turn
Write the inequality that defines the red region:		Write the inequality that defines the red region:

Page 162
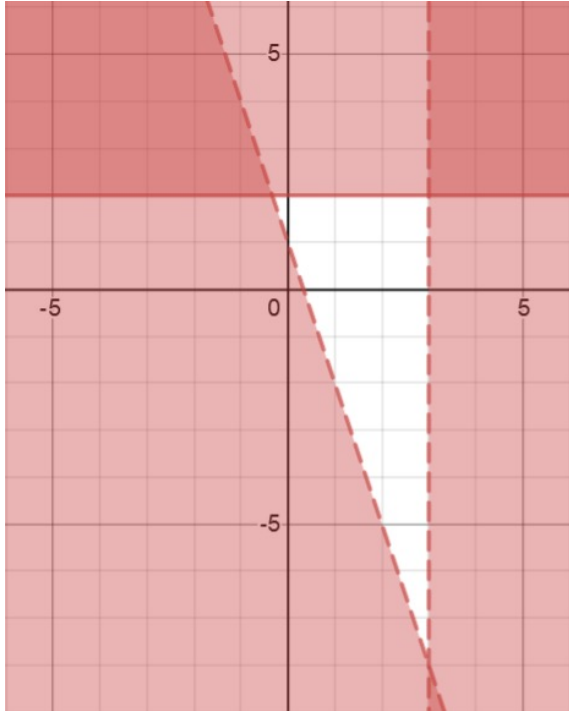
Worked Example	Thinking	Your Turn
Write the inequalities that define the unshaded region:		Write the inequalities that define the unshaded region:
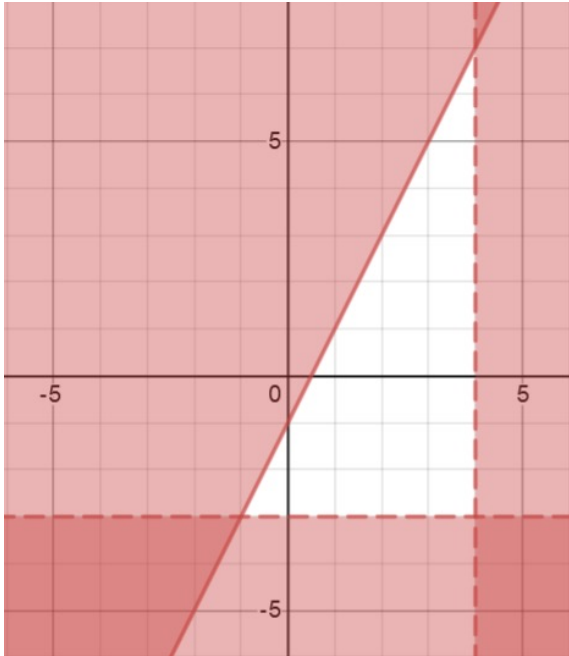
Worked Example	Thinking	Your Turn
Write the inequalities that define the unshaded region: 		Write the inequalities that define the unshaded region:

Fluency Practice

Corbett maths
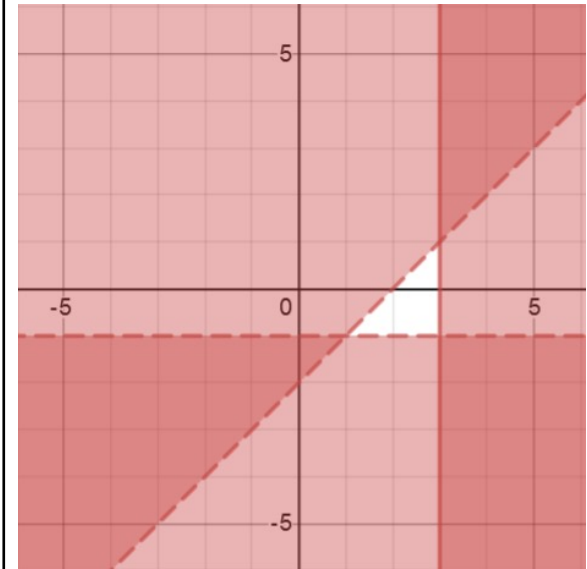
Question 1: On copies of the grid below, clearly indicate the region that satisfies each inequality.

(a) $y < x + 1$ (b) $y \le 2x + 2$ (c) $y > 3x - 1$

(d) $y \ge x + 3$ (e) $y > 2x$ (f) $y \le 4x$

(g) $y < -2x + 1$ (h) $y \ge \frac{1}{2}x + 2$ (i) $x + y < 4$
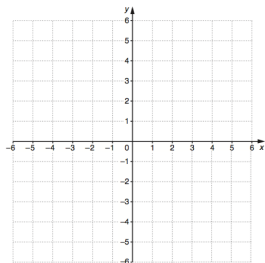
Question 2: On copies of the grid below, clearly indicate the region that satisfies each inequality.

(a) $y > 3x + 4$ (b) $y \ge 5x - 1$

(c) $y \le 4x + 1$ (d) $y < -2x + 5$

(e) $x + y < 2$ (f) $y > -x - 2$

(g) $y \ge 5 - 2x$ (h) $x + y \ge 7$

(i) $3x + y > 3$ (j) $5x + 2y > 4$
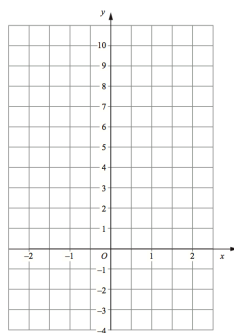
Question 3: Write down the inequality represented in each diagram below.

(a)

(b)

(c)

(d)

(e)

(f)
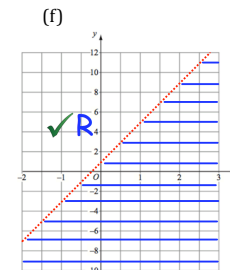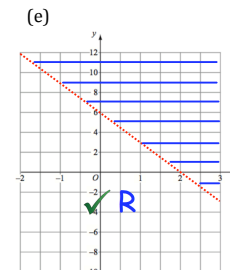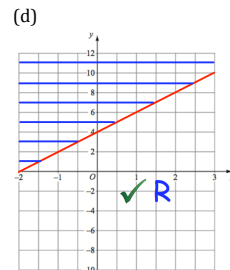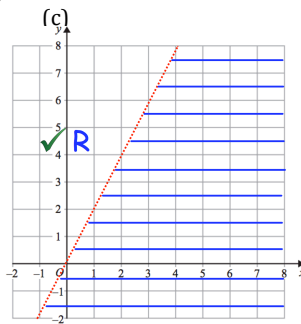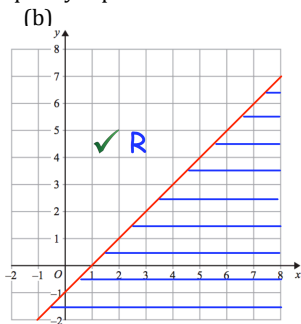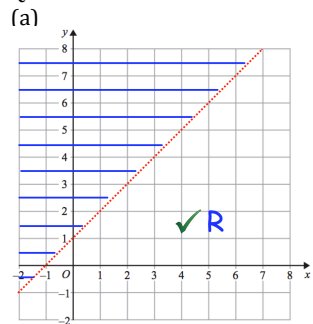
Question 4: On copies of the grid below, clearly indicate the region that satisfies the following inequalities.
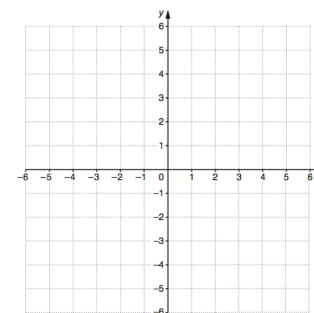
(a) $y > x - 1$, $x \ge -2$ and $y < 2$

(b) $y \le 2x$, $x \le 2$ and $y > -4$

(c) $y \le -2x + 2$, $x \ge 0$ and $y > x - 4$
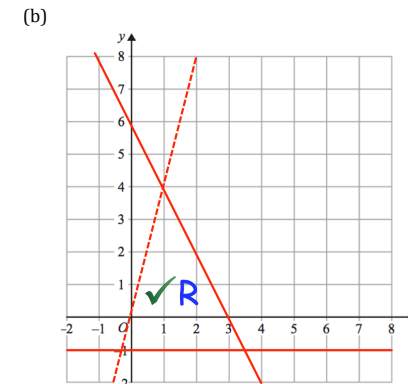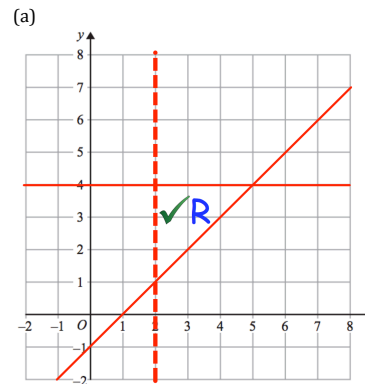
(d) $x + y < 3$, $-2 \le x < 3$ and $y \ge 0$

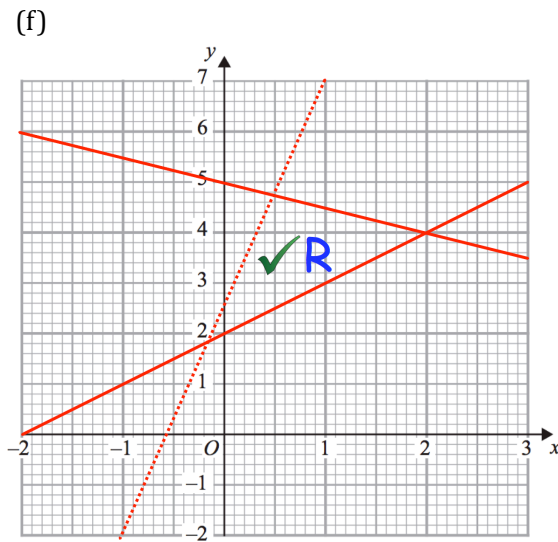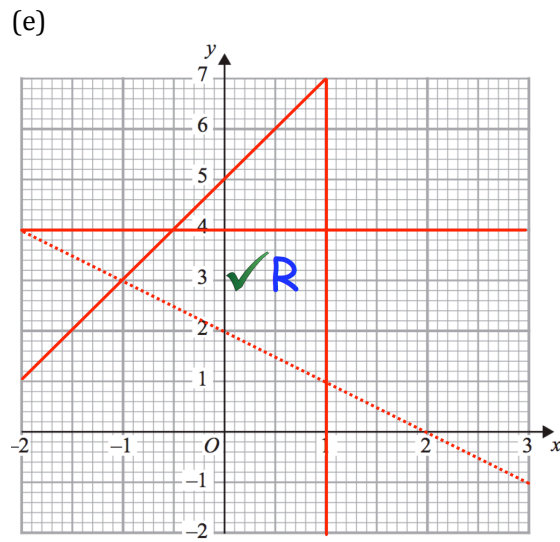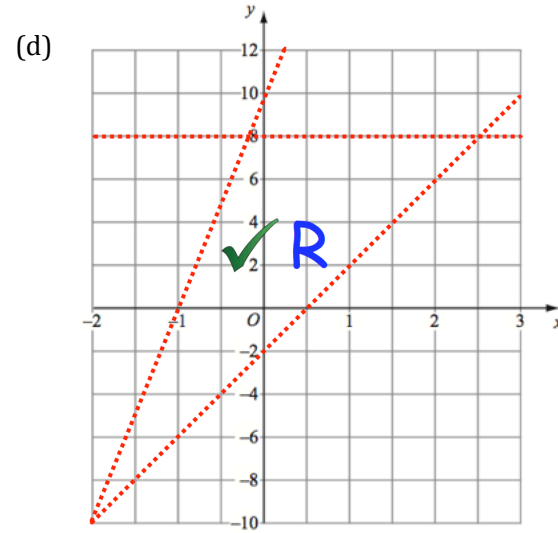(e) $y \le 5x - 4$, $y > x - 4$ and $y \le -\frac{1}{2}x + 2$
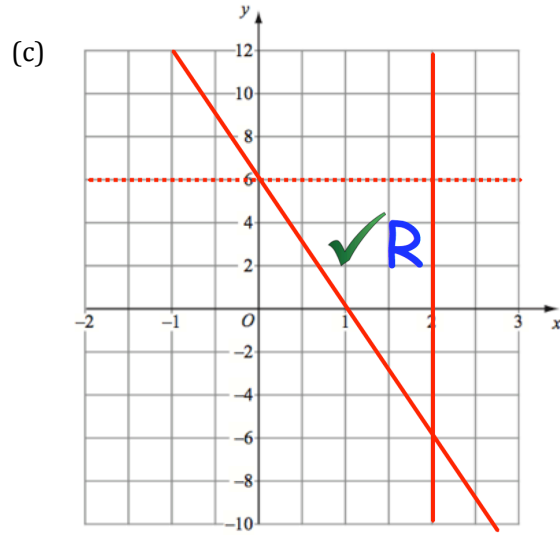
(f) $y \le -2x + 4$, $y < 2x - 6$ and $-4 < y < -3$

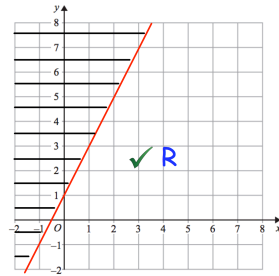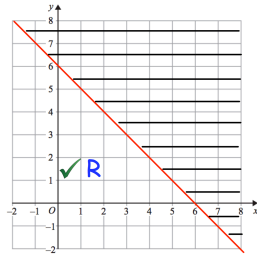Question 5: State the inequalities that the region labelled R satisfies.

(a)

(b)

Page 165

Corbett
maths

(c)

(d)

(e)

(f)

Apply

Question 1: Taylor has been asked to represent graphically y ≥ 2x + 1
Can you spot her mistake?

✓ R

Question 2: Conor has been asked to represent graphically x + y < 6
Can you spot his mistake?

✓ R

Question 3: At a fitness class, the maximum number of people who can attend is 20.
There are more men than women that attend the fitness class.
y = number of men that attend the fitness class.
x = number of women that attend the fitness class.
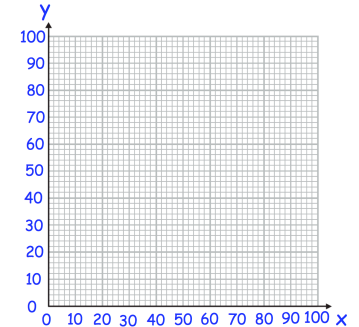Which region A, B, C or D represents the information above?

B
A   C
D
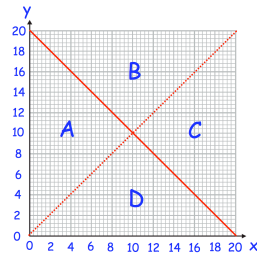
Question 4: A greengrocer sells apples and oranges.

One morning day he sells
up to 50 apples
up to 60 oranges
no more than a total of 90 pieces of fruit.

Let x be the number of apples sold.
Let y be the number of orange sold.

S   region th

Corbett
mɑths

80 90 100 x

Question 5: The region below shows information about the number first class passengers
and the number of economy passengers on a flight.

x = number of economy passengers and y = number of first class passengers

(a) Can 15 first class and 60 economy passengers
be on the flight?

(b) Can 30 economy and 40 first class passenger
be on the flight?

The profit made by the airline for each
economy passenger is £90 and for each
first class passenger is £200.

✓ R
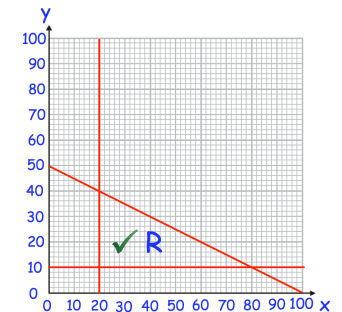
(c) What is the maximum profit the airline can make on one flight?

Question 6: A football stadium holds a maximum of 1000 fans.
Adult tickets cost £5 each and child tickets cost £2 each.
The football club needs to raise at least £3000 to cover costs.
The football club aims to sell at least one child ticket for two adult tickets sold
Let x = number of child tickets sold and y = number of adult tickets sold

Explain why: (a) x + y ≤ 1000   (b) 2x + 5y ≥ 3000   (c) y ≤ 2x

(d) Represent this information on a graph.

Answers