## Year 9 <br> Mathematics UNIT 5



Name:

Class:

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Please see unit 5 course on drfrostmaths.com
5. Reasoning with Proportion

PR Enlargement and Similarity
Enlargement and similarity
Right-angled Trigonometry
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Rates
Revision
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## PR Enlargement and Similarity



$$
\text { QU } 4 \text { - Enlargements }
$$

Enlarge by scale factor 1 :


QU 5 - Enlargements
QU 2 - Enlargements
Enlarge by scale factor 3 :


Enlarge by scale factor 1 :


QU 3 - Enlargements
Enlarge by scale factor 3 :


$$
\text { QU } 6 \text { - Enlargements }
$$

What is the scale factor to enlarge shape A ?:


## PR Enlargement and Similarity

QU 7 - Enlargements
What is the scale factor to enlarge shape $A$ ?:


QU 8 - Enlargements
What is the scale factor to enlarge shape A ?:


QU 9 - Enlargements
What is the scale factor to enlarge shape A ?:


QU 10 - Enlargements
What is the scale factor to enlarge shape A ?:


QU 11 - Enlargements
What is the scale factor to enlarge shape A ?:


## Enlargement

A transformation that moves all points a distance away from a centre point by applying a scale factor.

- Shapes change size.
- The scale factor multiplies distances, including the distance from the centre.

To fully describe an enlargement, we need to give three pieces of information:

1. Type of Transformation: Enlargement
2. Scale Factor: Positive or Negative Number
3. Centre of Enlargement: Coordinate $(x, y)$



*There are templates for questions 1,2 and 3 at the end of this exercise
Question 1: Enlarge each shape by the scale factor given Use P as the centre of enlargement.
(a)

(b)
Enlarge by scale factor 2
(d)

(g)

(e)

(h)

Enlarge by scale factor 2 © CORBETTMATHS 2019
(c)


Enlarge by scale factor 2 (f)


Enlarge by scale factor 2

## Enlargements: Centre of Enlargement Video 104a on www.corbettmaths.com

Question 2: Enlarge each shape by the scale factor given Use P as the centre of enlargement.


Enlarge by scale factor 2
(c)


Enlarge by scale factor 3
(b)


Enlarge by scale factor 3
(d)


Enlarge by scale factor 2

Enlargements: Centre of Enlargement Video 104a on www.corbettmaths.com

Question 3: Enlarge each shape by the scale factor given The coordinates for each centre of enlargement are given.
(a)


Enlarge by scale factor 2 using $(4,-3)$ as the centre of enlargement (c)


Enlarge by scale factor 2 using
$(0,-1)$ as the centre of enlargement
(b)


Enlarge by scale factor 3 using $(3,2)$ as the centre of enlargement


Enlarge by scale factor 2 using the origin as the centre of enlargement

Enlargements: Centre of Enlargement Video 104a on www.corbettmaths.com

Question 4: Describe fully the single transformation that takes shape A to shape B.
(a)

(c)

(b)

(d)


## Answers






*There are templates for questions 3,4 and 5 at the end of this exercise
Question 1: Copy these shapes and then enlarge by the scale factor given.
(b)
(a)


Enlarge by scale factor


Enlarge by scale factor $\frac{1}{3}$
(c)


Enlarge by scale factor $\frac{2}{3}$

Question 2: Copy these shapes and then enlarge by the scale factor given.
(a)

Enlarge by scale factor $\frac{1}{4}$
(b)
(c)

Enlarge by scale factor $1 \frac{1}{3}$

Question 3: Enlarge each shape by the scale factor given Use P as the centre of enlargement
(a)

Enlarge by scale factor $\frac{1}{2}$ © CORBETTMATHS 2016
(b)


Enlarge by scale factor
(c)


Enlarge by scale factor $\frac{2}{3}$

Enlargement: Fractional Scale Factor Video 107 on www.corbettmaths.com

Question 4: Enlarge each shape by the scale factor given Use P as the centre of enlargement.
(a)


$$
\text { Enlarge by scale factor } \frac{1}{4}
$$

(b)

(c)


Enlarge by scale factor $\frac{2}{3}$
(d)


Enlarge by scale factor $1 \frac{1}{2}$ moths

Enlargement: Fractional Scale Factor Video 107 on www.corbettmaths.com

Question 5: Enlarge each shape by the scale factor given The coordinates for each centre of enlargement are given.
(a)


Enlarge by scale factor $\frac{1}{2}$ using
$(0,1)$ as the centre of enlargement
(b)


Enlarge by scale factor $\frac{1}{3}$ using $(-3,1)$ as the centre of enlargement
(d)


Enlarge by scale factor $2 \frac{1}{2}$ using $(-5,-3)$ as the centre of enlargement

Enlargement: Fractional Scale Factor Video 107 on www.corbettmaths.com

Question 6: Describe fully the single transformation that takes shape A to shape B.
(a)

(b)

(d)



Scan here


*There are templates for questions 1,2 and 3 at the end of this exercise
Question 1: Enlarge each shape by the scale factor given Use P as the centre of enlargement.



Enlarge by scale factor -2
(e)

(c)


Enlarge by scale factor - 4
(f)


Question 2: Enlarge each shape by the scale factor given
(a)
 Use P as the centre of enlargement
© col Enlarge by scale factor -3


Enlarge by scale factor - 4

## Enlargement: Negative Scale Factor Video 108 on www.corbettmaths.com



Enlarge by scale factor -2
(d)


Enlarge by scale factor $-\frac{1}{3}$

Question 3: Enlarge each shape by the scale factor given The coordinates for each centre of enlargement are given
(a)


Enlarge by scale factor - 2 using
$(0,0)$ as the centre of enlargement
(c)


Enlarge by scale factor - 4 using $(-3,-1)$ as the centre of enlargement © CORBETTMATHS 2019
(b)


Enlarge by scale factor -2 using $(2,2)$ as the centre of enlargement


Enlarge by scale factor $-\frac{1}{2}$ using $(0,-2)$ as the centre of enlargement

Question 4: Describe fully the single transformation that takes shape A to shape B.
(a)

(b)

(c)

(d)




## Similarity vs Congruence



They are the same shape and size
(flipping is allowed)

## Two shapes are similar if:



They are the same shape
(flipping is again allowed)





Intelligent Practice - Find the length of every missing side
Triangles not drawn to scale




Intelligent Practice - Find the length of every missing side
Triangles not drawn to scale


Intelligent Practice - Find the length of every missing side
Triangles not drawn to scale



| Worked Example | Your Turn |
| :--- | :--- |
| Calculate the length of PT | Calate the length of PT |

## Similar Triangles


i) Find the scale factor between triangle ABE and triangle ACD
ii) Find the value of $x$
iii) Find the value of $y$

i) Find the scale factor between $A B E$ and $A C D$
ii) Find the value of $x$
iii) Find the value of $y$

Q3.

i) How can you tell that $A B E$ and $B C D$ are mathematically similar?
ii) Find the value of $x$
iii) Find the value of $y$

Q4

i) Find the value of $x$
ii) Find the value of $y$
iii) Find the size of angle $\theta$

i) Find the value of $x$
ii) Hence, find the perimeter of trapezium ABDE

Q6. Find the value of $x$


Q7. [non-calculator] Given that $\tan (x)=\tan (y)$, find the area of triangle $A B C$.


## Trigonometry

https://youtu.be/1s7V7Ai3Eaw - story of trigonometry

We know that for any similar triangles:

- Corresponding angles are the same
- Corresponding lengths are enlargements of each other

We are going to look at the special case right-angled triangles and the relationship between the 3 sides and the 2 non-right angles.

Trigonometry
Using the idea of similar triangles complete the statements below:


How do we know what any of these ratios are?

C. Find the value of the following to 3 d.p..
1). $\sin 10^{\circ}$
2). $\cos 45^{\circ}$
3). $\tan 45^{\circ}$
4). $\tan 62^{\circ}$
6). $\sin 69^{\circ}$
7). $\tan 14^{\circ}$
8). $\quad \cos 32^{\circ}$
9). $\quad \cos 5^{\circ}$
13). $\tan 4^{\circ}$
14). $\sin 15^{\circ}$
11). $\tan 68^{\circ}$
12). $\sin 55^{\circ}$
18). $\cos 12^{\circ}$
19). $\tan 78^{\circ}$

D. Calculate the following to 2 d.p..
1). $5 \tan 45^{\circ}$
2). $4 \sin 30^{\circ}$
3). $8 \cos 60^{\circ}$
4). $6 \sin 43^{\circ}$
5). $9 \cos 18^{\circ}$
6). $15 \tan 83^{\circ}$
7). $14 \cos 25^{\circ}$
8). $24 \cos 72^{\circ}$
9). $31 \sin 45^{\circ}$
10). $20 \cos 34^{\circ}$
11). $5 \cos 60^{\circ}$
12). $56 \sin 15^{\circ}$
13). $30 \tan 45^{\circ}$
14). $19 \sin 82^{\circ}$
15). $14 \tan 45^{\circ}$
16). $17 \tan 60^{\circ}$ 17). $8 \cos 0^{\circ}$
18). $45 \tan 28^{\circ}$
19). $61 \sin 90^{\circ}$
20). $28 \tan 50^{\circ}$
E. Calculate the following to 2 d.p..
1). $\frac{6}{\sin 34^{\circ}}$
2). $\frac{12}{\cos 83^{\circ}}$
3). $\frac{4}{\tan 16^{\circ}}$
4). $\frac{23}{\tan 45^{\circ}}$
5). $\frac{31}{\sin 30^{\circ}}$
6). $\frac{38}{\cos 18^{\circ}}$
7). $\frac{48}{\tan 80^{\circ}}$
8). $\frac{8}{\sin 54^{\circ}}$
9). $\frac{18}{\sin 15^{\circ}}$
10). $\frac{5}{\cos 51^{\circ}}$
11). $\frac{25}{\tan 52^{\circ}}$
12). $\frac{62}{\cos 71^{\circ}}$
13). $\frac{82}{\sin 68^{\circ}}$
14). $\frac{16}{\cos 8^{\circ}}$
15). $\frac{2}{\sin 12^{\circ}}$
16). $\frac{6}{\sin 75^{\circ}}$
17). $\frac{18}{\tan 45^{\circ}}$
18). $\frac{48}{\cos 50^{\circ}}$
19). $\frac{37}{\tan 12^{\circ}}$
20). $\frac{52}{\tan 84^{\circ}}$

## KEY SKILL - rearrangements and calculator use:

Q1. Rearrange to make $\mathbf{c}$ the subject.
a. $\quad a=\frac{c}{b}$
b. $a=\frac{b}{c}$
c. $5=\frac{c}{b}$
d. $20=\frac{b}{c}$
e. $\sin A=\frac{c}{b}$
f. $\sin A=\frac{b}{c}$
g. $\sin 5=\frac{c}{b}$
h. $\sin 20=\frac{b}{c}$
i. $\cos A=\frac{c}{b}$
j. $\cos 28=\frac{b}{c}$
k. $\tan A=\frac{b}{c}$
I. $\tan A=\frac{10}{c}$

Q2. Calculate a to 2 dp .
a. $\sin 40=\frac{a}{6}$
b. $\sin 31=\frac{a}{8}$
c. $\cos 70=\frac{20}{a}$
d. $\cos 46=\frac{12 a}{7}$
e. $\tan 20=\frac{a}{27}$
f. $\tan 58=\frac{67}{a}$

Q3. Calculate a to 3sf.
a. $\sin 36=\frac{a}{9}$
b. $\sin 71=\frac{a}{6}$
c. $\sin 29=\frac{6}{a}$
d. $\sin 81=\frac{75}{a}$
e. $\sin 205=\frac{a}{11}$
f. $\cos 53=\frac{29}{a}$
g. $\cos 101=\frac{a}{61}$
h. $\tan 44=\frac{a}{7}$
i. $\tan 18=\frac{50}{c}$

| Worked Example |  | Your Turn |
| :--- | :--- | :--- |
| $\sin (30)=\frac{x}{5}$ | $\cos (45)=\frac{x}{4}$ |  |
|  |  |  |

Find ' $x$ '. Give your solution to 2 decimal places.

1. $\tan (30)=\frac{x}{2}$
2. $\tan (45)=\frac{x}{2}$
3. $\sin (45)=\frac{x}{2}$
4. $\sin (45)=\frac{x}{4}$
5. $\frac{x}{4}=\sin (45)$
6. $x \times \sin (45)=4$
7. $x \times \sin (30)=4$
8. $x \times \cos (30)=4$
9. $x \times \cos (30)=8$
10. $x \times \cos (31)=8$

| Worked Example |  |
| :---: | :---: |
| $\sin (15)=\frac{5}{x}$ | $\cos (45)=\frac{5}{x}$ |
|  |  |

Find ' $x$ '. Give your solution to 2 decimal places.

1. $\cos (30)=\frac{2}{x}$
2. $\cos (45)=\frac{2}{x}$
3. $\sin (45)=\frac{2}{x}$
4. $\sin (45)=\frac{4}{x}$
5. $\sin (45)=\frac{8}{x}$
6. $\tan (45)=\frac{8}{x}$
7. $\tan (45)=\frac{x}{8}$
8. $\cos (45)=\frac{x}{8}$
9. $\cos (45)=\frac{8}{x}$
10. $\frac{8}{x}=\cos (45)$

Trigonometric Functions
A function $f(x)$ takes an input $x$ and outputs a value $y$. A trigonometric function takes an angle $x^{\circ}$ and outputs a ratio of sides.

For any right-angled triangle we always label the longest side as the hypotenuse $H$. For the purposes of trigonometry we label the other two sides relative to one of the non-right angles.

One of these is opposite the angle and the other adjacent (meaning next to).


## Labelling the sides exercise:

©
A. Name all the sides from the given angle, $\mathrm{x}^{\circ}$.
1).

2).

3).

4).


6).


8).


11).

12).

14).

15).


Trigonometric Functions
A function $f(x)$ takes an input $x$ and outputs a value $y$. A trigonometric function takes an angle $\boldsymbol{x}^{\circ}$ and outputs a ratio of sides.

The three sides of right-angled triangles are:
O -Opposite
A - Adjacent
H - Hypotenuse
So the three ratios are: $\boldsymbol{O}: \boldsymbol{H}$ or $\frac{\boldsymbol{O}}{\boldsymbol{H}} \quad \mathrm{A}: \boldsymbol{H}$ or $\frac{\boldsymbol{A}}{\boldsymbol{H}} \quad \mathrm{O}: \boldsymbol{A}$ or $\frac{\boldsymbol{o}}{\boldsymbol{A}}$
And so there are three trigonometric functions which take any angles $\boldsymbol{x}^{\circ}$ and output one of these ratios:

$$
\begin{array}{ccc}
x^{0} \rightarrow \frac{0}{H} & x^{0} \longrightarrow \frac{A}{H} & x^{0} \longrightarrow \frac{0}{A} \\
\begin{array}{c}
\text { sine } \\
(\sin )
\end{array} & \operatorname{cosine} & (\cos )
\end{array}
$$

## Trigonometric Functions

So altogether if we have:


Then: $\quad \sin \left(x^{\circ}\right)=\frac{o p p}{h y p} \quad \cos \left(x^{\circ}\right)=\frac{a d j}{h y p} \quad \tan \left(x^{\circ}\right)=\frac{o p p}{a d j}$


## Choosing the correct trignometric ratio exercise:

B. For each of the following questions look at the information given and the information you have to find. Which of the trigometrical ratios would you use to solve it for x ?
Do not try to solve the questions.
1).

2).

3).

4).

5).


7).

10).

11).

12).
$P \sum_{R}^{50^{\circ}}{ }_{R}^{10 \mathrm{~cm}} \mathrm{Q}$

Label each of the triangles with opposite (O), adjacent (A) and hypotenuse (H). Use this to decide which ratio to use $-\sin (\mathrm{SOH})$, cos (CAH) or tan (TOA).


| Q | Diagram (label sides) | Correct trigonometric ratio? <br> (select sin / cos / tan) | Fill in formulae | rearrange | Answer (1 d.p) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | $\tan$ | $\tan (38)=\frac{y}{10}$ | $y=10 \tan (38)$ | 7.8 cm |
| 2 |  | cos |  |  |  |
| 3 | A |  |  |  |  |
| 4 |  |  |  | $x=8 \cos (33)$ |  |
| 5 |  |  | $\sin (32)=\frac{y}{6}$ |  |  |
| 6 |  |  | $\sin (48)=\frac{z}{10}$ |  |  |





| Worked Example |  | Your Turn |
| :--- | :--- | :--- |
| Calculate $x$ : |  |  |
|  |  |  |

F. Find the length of the side marked $x$, leave all answers to 1 decimal place.

Diagrams not to scale.
1).

5).

6).

$3)$.

4).

8).

9).

10).

11).

12).



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| Worked Example | Your Turn |
| :---: | :---: |
| Calculate $x$ : | Calculate $x$ : |

A. Find the length of the side marked $x$, leave all answers to 1 decimal place. Diagrams not to scale.

5).

$3)$.


$6)$.

10).

11).

12).

B. Find the length of the side marked x , leave all answers to 1 decimal place.
Diagrams not to scale.
1).

2).

4).

5).

6)


8).


10).

11).

12).

16).


| Inverse Trigonometric functions |
| :--- |
| We have met the idea that: $f(x)=y \quad$ so $\quad f^{-1}(y)=x$ |
| The e trigonometric functions sin, cos and tan are all functions where the input is an angle giving an <br> output which is a ratio of sides. <br> The inverse of these functions therefore does this in reverse. <br> if $\sin \left(30^{\circ}\right)=0.5$ then $\sin ^{-1}(0.5)=30^{\circ}$ <br> if $\cos \left(60^{\circ}\right)=0.5 \quad$ then $\cos ^{-1}(0.5)=60^{\circ}$ <br> if $\tan \left(45^{\circ}\right)=1 \quad$ then $\tan ^{-1}(1)=45^{\circ}$ |


| Worked Example |  | Your Turn |
| :--- | :--- | :--- |
| $\sin (x)=\frac{1}{2}$ | $\sin (x)=\frac{2}{5}$ |  |
|  |  |  |

Find ' $x$ '. Give your solution to 2 decimal places.

1. $\sin (x)=0$
2. $\cos (x)=0$
3. $\sin (x)=\frac{1}{5}$
4. $\cos (x)=\frac{1}{5}$
5. $\sin (x)=\frac{2}{5}$
6. $\cos (x)=\frac{2}{5}$
7. $\sin (x)=\frac{3}{5}$
8. $\cos (x)=\frac{3}{5}$
9. $\sin (x)=\frac{4}{5}$
10. $\cos (x)=\frac{4}{5}$
11. $\sin (x)=1$
12. $\cos (x)=1$


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| Worked Example | Your Turn |
| :---: | :---: |
| Calculate $\theta$ : | Calculate $\theta$ : |
| 4 cm | 4.5 cm |


| Worked Example |  | Calculate $\theta:$ |
| :--- | :--- | :--- |
| Calculate $\theta:$ |  |  |


| Q | Diagram (label sides) | Correct trigonometric ratio? (select $\sin / \cos / \tan$ ) | Fill in formulae | Inverse function | Answer in degrees (1 d.p) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | tan | $\tan y=\frac{5}{12}$ | $y=\tan ^{-1}\left(\frac{5}{12}\right)$ | 22.6 |
| 2 | $\underbrace{9 \mathrm{~cm}}_{(A)^{3} z^{\circ}}(\mathrm{M})$ | cos | $\cos z=\frac{3}{9}$ |  |  |
| 3 | $7 \mathrm{~cm} \quad$$2 / \mathrm{cm}$ <br> 10 cm |  |  |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  | $\cos y=\frac{3}{10}$ |  |  |
| 6 |  |  |  | $z=\sin ^{-1}\left(\frac{3}{12}\right)$ |  |




D). In the following triangles find all the missing angles and sides.


3).

4).

5).

6).

7).



11).

14).



# perPLeXing perimeters? 

Work out the perimeter of each triangle to 2 d.p. Cross of your answers from those on the right as you go.

| A |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\begin{aligned} & 19.31 \\ & 20.49 \\ & 21.60 \end{aligned}$ |
|  |  |  |  | $\begin{gathered} 24 \\ 24.05 \\ 30.81 \end{gathered}$ |
| M ${ }^{8}$ |  |  |  | $\begin{gathered} 31.94 \\ 36 \\ 40.97 \\ 45.73 \end{gathered}$ |




## Compound Measures

Compound measures are measures that rely on other measures:

- Speed
- Density
- Pressure


## Speed

$$
\begin{aligned}
\text { Speed } & =\frac{\text { Distance }}{\text { Time }} \\
\text { Distance } & =\text { Speed } \times \text { Time } \\
\text { Time } & =\frac{\text { Distance }}{\text { Speed }}
\end{aligned}
$$

| Worked Example | Your Turn |
| :--- | :--- |
| An object travels 40 miles in 2 hours. <br> Calculate its speed in mph? | An object travels 40 miles in 30 minutes. <br> Calculate its speed in mph? |
|  |  |


| Worked Example | Your Turn |
| :--- | :--- |
| An object travels at 40 mph for 2 hours. <br> How far has it travelled in miles? | An object travels at 40 mph for 30 minutes. <br> How far has it travelled in miles? |
|  |  |


| Worked Example | Your Turn |
| :--- | :--- |
| An object travels 80 miles at 40 mph. <br> How long does the journey take in hours? | An object travels 20 miles at 40 mph. <br> How long does the journey take in hours? |
|  |  |

## 

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| :---: | :---: | :---: | :---: |
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## Density

$$
\begin{gathered}
\text { Density }=\frac{\text { Mass }}{\text { Volume }} \\
\text { Mass }=\text { Density } \times \text { Volume } \\
\text { Volume }=\frac{\text { Mass }}{\text { Density }}
\end{gathered}
$$

| Worked Example | Your Turn |
| :--- | :--- |
| Work out the density of copper. <br> 150 g of a copper block has a volume of $17 \mathrm{~cm}^{3}$. <br> Round your answer to 2 decimal places. | Work out the density of gold. <br> 97 g of gold has a volume of $5 \mathrm{~cm}^{3}$. <br> Round your answer to 2 decimal places. |

## Pressure

$$
\text { Pressure }=\frac{\text { Force }}{\text { Area }}
$$

## Force $=$ Pressure $\times$ Area

$$
\text { Area }=\frac{\text { Force }}{\text { Pressure }}
$$

| Worked Example | Your Turn |
| :--- | :--- |
| An object with an area of $5 \mathrm{~m}^{2}$ exerts a force of 10 N. <br> Find the pressure. | An object with an area of $2 \mathrm{~m}^{2}$ exerts a force of 10 N. <br> Find the pressure. |

## Worked Example

## Your Turn

An object with a cross-sectional area of $2 \mathrm{~m}^{2}$ exerts a pressure of $40 \mathrm{~N} / \mathrm{m}^{2}$.
Find the force.

An object with a cross-sectional area of $2 \mathrm{~m}^{2}$ exerts a pressure of $10 \mathrm{~N} / \mathrm{m}^{2}$.
Find the force.

