Year 9
Mathematics
Unit 11

Name: _____

Class: _____

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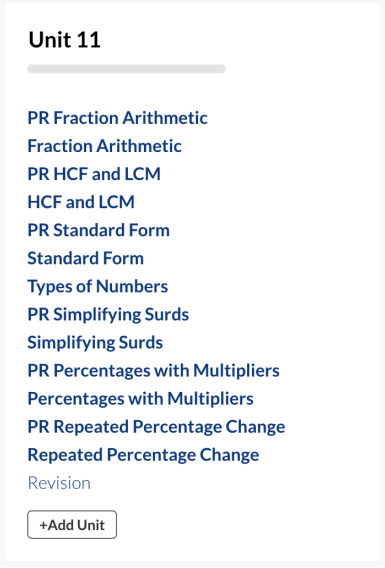
See unit 11 course on drfrostmaths.com



1 Fraction Arithmetic

Fluency Practice

Exercise 1M

Work out without a calculator and give the answer in its simplest form.

1 $\frac{1}{4}+\frac{3}{8}$ 2 $\frac{3}{5}+\frac{1}{10}$ 3 $\frac{2}{3}+\frac{1}{6}$ 4 $\frac{5}{12}+\frac{1}{4}$

5 $\frac{7}{8}-\frac{1}{2}$ 6 $\frac{1}{3}+\frac{1}{2}$ 7 $\frac{3}{5}-\frac{1}{4}$ 8 $\frac{4}{7}-\frac{1}{2}$

9 $\frac{2}{3}+\frac{1}{4}$ 10 $\frac{2}{5}+\frac{1}{3}$ 11 $\frac{1}{7}+\frac{1}{2}$ 12 $\frac{1}{5}-\frac{1}{6}$

13 $\frac{2}{3}-\frac{5}{12}$ 14 $\frac{7}{9}-\frac{1}{6}$ 15 $\frac{4}{5}-\frac{2}{7}$ 16 $\frac{7}{10}-\frac{1}{3}$

17 $1\frac{1}{4}-\frac{2}{5}$ 18 $1\frac{3}{4}-\frac{2}{3}$ 19 $3\frac{1}{4}+1\frac{3}{5}$ 20 $2\frac{5}{6}+1\frac{1}{4}$

Questions 21 to 40 involve either multiplying or dividing.

21 $\frac{2}{3}\times\frac{1}{5}$ 22 $\frac{3}{5}\times\frac{3}{4}$ 23 $\frac{5}{9}\times\frac{3}{4}$ 24 $1\frac{3}{4}\times\frac{1}{5}$

25 $\frac{3}{8}\times\frac{4}{5}$ 26 $\frac{2}{9}\times\frac{6}{7}$ 27 $\frac{5}{12}\times\frac{3}{10}$ 28 $\frac{5}{8}\times\frac{6}{15}$

29 $\frac{5}{6}\div\frac{1}{2}$ 30 $\frac{7}{8}\div\frac{2}{3}$ 31 $\frac{5}{9}\div\frac{3}{4}$ 32 $2\frac{1}{2}\div\frac{1}{5}$

33 $3\frac{1}{4}\times 2\frac{1}{2}$ 34 $\frac{5}{8}\div 1\frac{1}{2}$ 35 $\frac{5}{9}\div\frac{1}{3}$ 36 $\frac{3}{5}\div\frac{9}{100}$

37 $\frac{3}{5}\div 2$ 38 $\frac{4}{7}\div 3$ 39 $1\frac{1}{4}\div 4$ 40 $5\frac{1}{2}\div 3$

41 $\left(\frac{3}{5}\div\frac{1}{3}\right)+\left(1\frac{1}{4}\times\frac{1}{10}\right)$

42 $\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{9}\right)\div\left(\frac{1}{4}-\frac{1}{9}\right)$

Exercise 1E

Copy each square and fill in the missing numbers or symbols (+, –, ×, ÷). The arrows act as equals signs.

1

	÷	2	→	$1\frac{1}{2}$
÷		÷		
	×		→	2
↓		↓		
	×	8	→	

2

$\frac{1}{4}$	–	$\frac{1}{16}$	→	
		+		
$\frac{1}{8}$	÷		→	1
↓		↓		
$\frac{1}{8}$			→	$\frac{5}{16}$

3

	–	$\frac{1}{5}$	→	$\frac{7}{15}$
–		×		
	÷		→	$1\frac{1}{2}$
↓		↓		
	÷	$\frac{1}{20}$	→	

4

	–	$\frac{1}{5}$	→	$\frac{1}{20}$
×		÷		
2	÷		→	
↓		↓		
	×	$\frac{4}{5}$	→	

5

$\frac{2}{3}$	×	4	→	
		÷		
$\frac{1}{2}$	÷		→	$\frac{1}{16}$
↓		↓		
$\frac{1}{3}$			→	$\frac{5}{6}$

6

	×	$\frac{1}{3}$	→	$\frac{1}{8}$
		÷		
$\frac{1}{4}$			→	$\frac{11}{12}$
↓		↓		
$\frac{5}{8}$	–		→	$\frac{1}{8}$

Extra Notes

2 Highest Common Factor and Lowest Common Multiple

- The HCF is the largest integer which is a factor of two or more given positive integers.
- The HCF will be less than or equal to the smallest of the given numbers.

- The LCM is the smallest integer which is a multiple of two or more positive integers.
- The LCM will be greater than or equal to the largest of the numbers.

- **Step 1:** Align numbers so that each prime factor has its own column.
- **Step 2a:** To calculate HCF, see 'what loses' for each of the prime factors (i.e., lowest power, where 'nothing' always loses against 'something').
- **Step 2b:** To calculate LCM, see 'what wins'.
  Note: A 'draw' counts as both a win and a loss.

Check for two numbers:
$\mathrm{HCF}(a, b) \times \mathrm{LCM}(a, b) = a \times b$

Worked Example	Your Turn
Find the HCF and LCM of: $2^2 \times 3^2 \times 5^2 \times 11$ $2^3 \times 3 \times 5^2 \times 7$	Find the HCF and LCM of: $2 \times 3^3 \times 5 \times 7^2$ $2^2 \times 3^2 \times 7^2 \times 11$

Worked Example	Your Turn
Find the HCF and LCM of: 123 and 456	Find the HCF and LCM of: 321 and 654

Worked Example	Your Turn
The HCF of two numbers is 6. The LCM of two numbers is 60. Write down two possible numbers.	The HCF of two numbers is 3. The LCM of two numbers is 36. Write down two possible numbers.

Worked Example	Your Turn
The HCF of two numbers is 5. The LCM of two numbers is a multiple of 12. Write down two possible numbers.	The HCF of two numbers is 8. The LCM of two numbers is a multiple of 5. Write down two possible numbers.

Worked Example	Your Turn
Two strings of different lengths, 240 cm and 318 cm are to be cut into equal lengths. What is the greatest possible length of each piece?	Two strings of different lengths, 212 cm and 360 cm are to be cut into equal lengths. What is the greatest possible length of each piece?

Worked Example	Your Turn
Two lighthouses flash their lights every 240 s and 318 s respectively. They both flash at the same time. After how many seconds will they next both flash at the same time.	Two lighthouses flash their lights every 212 s and 360 s respectively. They both flash at the same time. After how many seconds will they next both flash at the same time.

Fluency Practice

(1) as an aid effort, food and drink is distributed to each person in a refugee camp
if the total ration for a day is 136 apples, 204 oat biscuits and 340 small bottles of water, what is the most refugees there could be in the camp?

(2) what is the largest possible number of students if 294 blue smarties, 252 pink smarties and 210 yellow smarties so that each student has the same number of each colour, with none left over?

(3) in a number of boxes with the same number of chocolates in each there are, in total: 90 dark ginger, 198 white and 126 hazelnut; what is the largest possible number of boxes of chocolates?

(4) a person has a rectangular plot of land measuring 8.4 m by 5.6 m
to survey the numbers of dandelions they want to divide it equally into the minimum number of square plots; what is the size of each square plot and how many such squares will there be?

(5) someone wants to cut identical squares, as big as they can, from a piece of paper measuring 168 mm by 196 mm.; what is the length of each square?

(6) when on a school trip, 56 girls and 91 boys were divided into as many groups as possible so that there were the same number of girls and same number of boys in each group; how many groups?

(7) a small bus company has two bus services that both start at 9 am
bus number 801 leaves at 16-min intervals and bus 304 leaves at 20-min intervals
how many times do both bus services leave together between 9 am to 12 noon inclusive?

(8) Franz, Gertrude and Hugo are grasshoppers, jumping up a large flight of stairs:
- Franz jumps 2 steps at a time
- Gertrude 3 steps at time
- Hugo 4 steps at a time

if they all start at the bottom at the same time, on which step will they all land together for the first time?

(9) if Jamie helps wash up at a vegetarian restaurant every 9 days while his sister helps every 12 days, how often do they both help out?

(10) for Y7 sports, students can be put into smaller groups of either 6, 15 and 18 without anyone being left out; what is the smallest possible number of students?

(11) Jess has a stall at the market once every 64 days and
Carlo has a stall at the same market once every 72 days
when they are at the market on the same day they meet up for a coffee
if they meet one day, how many more days will it be until they meet again?

(12) three planks are to be cut into smaller pieces - all of the same size
the planks have lengths 84 cm, 156 cm and 180 cm
what is the greatest possible length of each of the smaller pieces?

(13) the front wheels of a toy truck are 9 cm in circumference
the back wheels are bigger, with a 12 cm in circumference
if the truck travels down a long slope, in a straight line and without slipping, how far will the truck have travelled when the front wheels have made 10 more revolutions than the back wheels?

(14) three amounts of money are to be split up into equal amounts
the amounts are: £441, £567 and £693
what is the greatest possible amount that these can be split into?

Fluency Practice

A1 Express 204 as a product of its prime factors. Show your working clearly.	**A2** Write 792 as a product of its prime factors. Show your working clearly.	**A3** $1400 = 2^p \times 5^2 \times 7$ Find the value of p.	**A4** Given that $120 = 2^3 \times 3 \times 5$ And that $n = 120 \times 108$ Write n as a product of powers of its prime factors.
B1 Find the highest common factor (HCF) of 90 and 252	**B2** Find the lowest common multiple (LCM) of 24 and 42	**B3** Find the highest common factor (HCF) and lowest common multiple (LCM) of 168 and 180	**B4** Find the highest common factor (HCF) and lowest common multiple (LCM) of 72, 180 and 540
C1 $A = 2^3 \times 3 \times 5^2$ $B = 2^2 \times 3$ Find the HCF and LCM of A and B.	**C2** $M = 2^4 \times 3^2 \times 7$ $N = 2^2 \times 3^2 \times 5$ Find the HCF and LCM of M and N.	**C3** $2520 = 2^3 \times 3^2 \times 5 \times 7$ $3024 = 2^4 \times 3^3 \times 7$ Find the HCF & LCM of 2520 and 3024. Write your answer as a product of prime factors.	**C4** $740\,880 = 2^4 \times 3^3 \times 5 \times 7^3$ $980\,100 = 2^2 \times 3^4 \times 5^2 \times 11^2$ Find the highest common factor (HCF) of 740 800 and 980 100
D1 The highest common factor (HCF) of 90 and x is 18 The lowest common multiple (LCM) of 90 and x is 540 Find the value of x.	**D2** The highest common factor (HCF) of x and 12 is 6 The lowest common multiple (LCM) of x and 12 is 180 Find the value of x.	**D3** Find two numbers between 100 and 150 that have a HCF of 22.	**D4** $360 = 2^4 \times 3^2 \times 5$ Write down three different factors of 360 with a sum between 90 and 100.

Extra Notes

3 Standard Form

Standard form is written in the form of $a \times 10^n$, where a is a number bigger than or equal to 1 and less than 10 (i.e. $1 \leq a < 10$). n can be any positive or negative whole number.

Note: a can be any positive or negative number.

In Standard Form	Not in Standard Form
7.3×10^3	$438{,}000$
1×10^{-3}	54×10^7
9.36×10^{18}	0.6×10^{-4}
4×10^1	389×10000
5.002×10^{-7}	$6 \times 10^{1.5}$
-1.729×10^{211}	0.000372

Why use standard form?

- It allows us to write really small or really big numbers concisely.
- It allows us to easily compare small and big numbers.

Intelligent Practice

Decide if the following numbers are in standard form:

3×10^5

3×10^6

3×10^{67}

$3 \times 10^{6.7}$

$3 \times 10^{0.67}$

$3 \times 10^{0.7}$

3×10^7

3×10^{-7}

$3 \times 10^{-0.7}$

3×11^5

3×100^5

3×10.5^5

3×10.5^5

3×-10^5

$3 \times (-10)^5$

$3 \div 10^5$

$3 + 10^5$

$3 - 10^5$

4×10^5

40×10^5

46×10^5

4.6×10^5

0.46×10^5

3.46×10^5

3.46434561×10^5

-3.46434561×10^5

Converting Numbers to Standard Form

For the first number, keep dividing or multiplying by 10 until you get a number between 1 and 10.

For the power of 10, count how many times the decimal place moved leftwards or rightwards.

Worked Example	Your Turn
Write the following numbers in standard form: a) $70{,}000$ b) $72{,}000$ c) $720{,}000$	Write the following numbers in standard form: a) $60{,}000$ b) $63{,}000$ c) $630{,}000$

Worked Example	Your Turn
Write the following numbers in standard form: a) 0.05 b) 0.005 c) 0.00572	Write the following numbers in standard form: a) 0.06 b) 0.006 c) 0.00683

Converting Numbers from Standard Form

Recall that the index of the 10 tells us how many times we are multiplying by 10 (or if negative, dividing by 10). Therefore count the number of decimal place jumps, **adding 0's if necessary**.

Remember that we use negative powers for small numbers (numbers less than 1), positive powers for large numbers (numbers bigger than or equal to 1).

Worked Example	Your Turn
Write the following as an ordinary number: 3.1×10^6	Write the following as an ordinary number: 3.2×10^7

Worked Example	Your Turn
Write the following as an ordinary number: 4.1×10^{-6}	Write the following as an ordinary number: 4.2×10^{-7}

Ordering Numbers in Standard Form

Numbers written in standard form can be ordered by first looking at the power of 10, which tells you the size of the numbers. If two or more numbers have the same power of 10, use the first part of the number to decide the order.

Multiplying and Dividing in Standard Form

To multiply $(a \times 10^n) \times (b \times 10^m)$:

All the four things are being multiplied, so we can multiply in any order!

- Multiply $a \times b$
- Multiply $10^m \times 10^n$ (add the powers)
- Make sure the answer is in standard form

Division works in the same way.

Worked Example	Your Turn
Work out: a) $(3 \times 10^5) \times (2 \times 10^4)$ b) $(3 \times 10^5) \times (4 \times 10^{-4})$	Work out: a) $(3 \times 10^5) \times (4 \times 10^4)$ b) $(3 \times 10^{-5}) \times (2 \times 10^4)$

Worked Example	Your Turn
Work out: a) $(4 \times 10^9) \div (2 \times 10^3)$ b) $(2 \times 10^5) \div (8 \times 10^{-4})$	Work out: a) $(2 \times 10^9) \div (4 \times 10^3)$ b) $(8 \times 10^5) \div (2 \times 10^{-4})$

Calculator

Use the $\times\mathbf{10}^{\boldsymbol{x}}$ button on your calculator to make calculations involving standard form. While you can explicitly write 3×10^7 using the $\boldsymbol{x}^{\boldsymbol{y}}$ button, it is faster to use the specialised standard form key.

Check the following using your calculator:
$(2.41 \times 10^{19}) \times (7.1 \times 10^{23}) = \mathbf{1.7111} \times \mathbf{10}^{\mathbf{43}}$

Adding and Subtracting in Standard Form

If the powers are not the same, either:

- Convert both numbers to normal numbers first, then add or subtract, then convert back to standard form.
- Or better, change the number with the smaller power of 10 so it matches the power of the larger one.

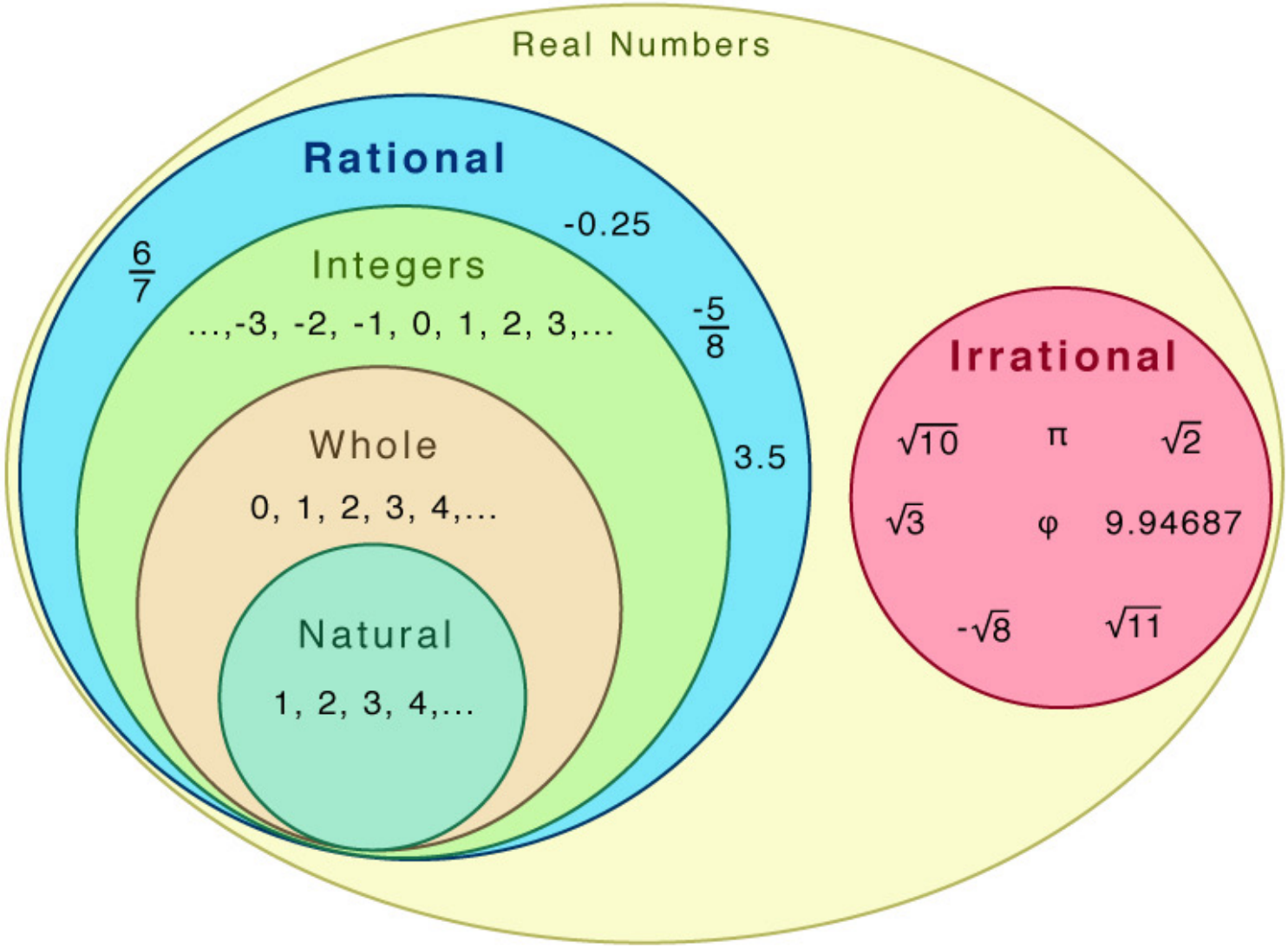
Worked Example	Your Turn
Work out: a) $(3 \times 10^4) + (4 \times 10^4)$ b) $(3 \times 10^4) + (8 \times 10^4)$ c) $(3 \times 10^5) + (8 \times 10^4)$	Work out: a) $(3 \times 10^7) + (2 \times 10^7)$ b) $(3 \times 10^7) + (9 \times 10^7)$ c) $(3 \times 10^8) + (9 \times 10^7)$

Worked Example	Your Turn
Work out: a) $(7 \times 10^4) - (4 \times 10^4)$ b) $(7 \times 10^4) - (0.4 \times 10^4)$ c) $(7 \times 10^5) - (0.4 \times 10^4)$	Work out: a) $(6 \times 10^7) - (2 \times 10^7)$ b) $(6 \times 10^7) - (0.2 \times 10^7)$ c) $(6 \times 10^7) - (0.2 \times 10^8)$

Worked Example	Your Turn
Work out: a) $(4 \times 10^{-1}) + (3 \times 10^{-2})$ b) $(7 \times 10^{-3}) - (2 \times 10^{-4})$	Work out: a) $(8 \times 10^{-2}) + (2 \times 10^{-3})$ b) $(2 \times 10^{-2}) - (5 \times 10^{-3})$

Extra Notes

4 Types of Numbers



Fluency Practice

Classify each number below as either rational or irrational. If you believe your number is rational, prove your answer by writing it as a fraction. The first one is done for you.

	Rational or Irrational?	Fraction?
1) 0.8	Rational	$\frac{8}{10}$ or $\frac{4}{5}$
2) $-\frac{3}{10}$		
3) $\sqrt{40}$		
4) $\sqrt{81}$		
5) $2\frac{1}{3}$		
6) 0.35		
7) $0.33333\ldots$		
8) -9		
9) 3.4		
10) $\sqrt{2}$		

Directions: For each number shown, classify it as either rational or irrational, then tell whether or not it is terminating or repeating.

		(circle one)		*(circle one)*
11)	-0.6	rational or irrational		terminating, repeating, or neither
12)	$\sqrt{100}$	rational or irrational		terminating, repeating, or neither
13)	$\frac{2}{5}$	rational or irrational		terminating, repeating, or neither
14)	$-\frac{2}{3}$	rational or irrational		terminating, repeating, or neither
15)	$0.35217534\ldots$	rational or irrational		terminating, repeating, or neither

Extra Notes

5 Simplifying Surds

When the root (square root, cube root or higher root) of a number cannot be obtained exactly, the root is called a surd. A surd cannot be written as a fraction but can be written as a decimal, that goes on forever, without repeating (recurring) or ending (terminating). Hence, surds are irrational numbers.

Surds	Not Surds
$\sqrt{8}$	8
$\sqrt{10}$	-12.05
$\sqrt{91}$	0.62
$\sqrt[3]{7}$	$\frac{3}{7}$
$\sqrt[3]{16}$	$7\frac{1}{2}$
$\sqrt[4]{73}$	$\sqrt{16}$
$2\sqrt{2}$	$\sqrt{25}$
$2+\sqrt{5}$	$\sqrt[3]{8}$
$(2+\sqrt{5})(3+\sqrt{5})$	$\sqrt{2.25}$
$\frac{1}{5-\sqrt{17}}$	$\frac{\sqrt{100}}{\sqrt{4}}$

Intelligent Practice

$\sqrt{1}$

$\sqrt{4}$

$\sqrt{9}$

$\sqrt{36}$

$\sqrt{6}$

$\sqrt{24}$

$\sqrt{3}$

$2\sqrt{3}$

$3\sqrt{3}$

$3\sqrt{4}$

$\sqrt{5}$

$\sqrt{5^2}$

$\frac{1}{(\sqrt{5})^2}$

$\frac{\sqrt{1}}{\sqrt{4}}$

$\sqrt{\frac{1}{4}}$

$\sqrt{\frac{2}{8}}$

$\sqrt{\frac{2}{9}}$

$\sqrt{\frac{4}{9}}$

$\frac{2}{\sqrt{9}}$

$\frac{\sqrt{7}}{2}$

$\sqrt{0.25}$

$\sqrt{0.125}$

$\sqrt{0.01}$

$(\sqrt{2})^2$

$(\sqrt{2})^3$

$\sqrt{2}(\sqrt{2}+3)$

$(\sqrt{2}+3)(\sqrt{2}-3)$

$\frac{2}{\sqrt{2}}\sqrt{2}$

$\frac{2}{3+\sqrt{2}}$

$\frac{2}{\frac{3}{\sqrt{2}}+\sqrt{2}}$

Purposeful Practice

Question	As a decimal or whole number	Is it a surd?	Question	As a decimal or whole number	Is it a surd?
$\sqrt{1}$	1	No	$\sqrt{16}$		
$\sqrt{2}$	1.4142135 …	Yes	$\sqrt{17}$		
$\sqrt{3}$	1.7320508 …	Yes	$\sqrt{18}$		
$\sqrt{4}$	2	No	$\sqrt{19}$		
$\sqrt{5}$			$\sqrt{20}$		
$\sqrt{6}$			$\sqrt{21}$		
$\sqrt{7}$			$\sqrt{22}$		
$\sqrt{8}$			$\sqrt{23}$		
$\sqrt{9}$			$\sqrt{24}$		
$\sqrt{10}$			$\sqrt{25}$		
$\sqrt{11}$			$\sqrt{26}$		
$\sqrt{12}$			$\sqrt{27}$		
$\sqrt{13}$			$\sqrt{28}$		
$\sqrt{14}$			$\sqrt{29}$		
$\sqrt{15}$			$\sqrt{30}$		

Multiplying Surds

To simplify $\sqrt{a} \times \sqrt{a}$:

- Use the fact $\sqrt{a} \times \sqrt{a} = (\sqrt{a})^2 = a$

Worked Example	Your Turn
Simplify: a) $\sqrt{6} \times \sqrt{6}$ b) $\left(\sqrt{6}\right)^2$ c) $\left(2\sqrt{6}\right)^2$ d) $2\left(\sqrt{6}\right)^2$ e) $2\left(\sqrt{6}\right)^3$	Simplify: a) $\sqrt{7} \times \sqrt{7}$ b) $\left(\sqrt{7}\right)^2$ c) $\left(2\sqrt{7}\right)^2$ d) $2\left(\sqrt{7}\right)^2$ e) $2\left(\sqrt{7}\right)^3$

Simplifying Surds

To simplify $\sqrt{x}$:

- Write down a list of all the square numbers up to x.
- Find the biggest square number which is a factor of x. Write x as a product using this square number.
- Now write $\sqrt{x}$ as this product.
- Use the fact $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$ to split the product.
- Simplify your answer.

OR

- Write x as the product of prime numbers.
- Use the fact $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$ to split the product.
- Use the fact $\sqrt{a} \times \sqrt{a} = a$ to simplify the product.

Worked Example	Your Turn
Simplify: a) $\sqrt{60}$ b) $\sqrt{120}$	Simplify: a) $\sqrt{50}$ b) $\sqrt{200}$

Fill in the Gaps

Square Numbers	1	4	9	16	25	36	49	64	81	100

Question	Largest Square Number Factor	Split into Two Surds	Rationalise the Square Number	Answer
$\sqrt{27}$	9	$\sqrt{9} \times \sqrt{3}$	$3 \times \sqrt{3}$	$3\sqrt{3}$
$\sqrt{24}$	4	$\sqrt{4} \times \sqrt{6}$		
$\sqrt{50}$	25			
$\sqrt{28}$				
$\sqrt{32}$				
$\sqrt{45}$				
$\sqrt{72}$				
$\sqrt{90}$				
$\sqrt{75}$				
$\sqrt{200}$				
$\sqrt{98}$				
$\sqrt{80}$				
		$\sqrt{9} \times \sqrt{7}$	$3 \times \sqrt{7}$	$3\sqrt{7}$
				$7\sqrt{3}$

Fill in the Gaps

Question	Surd as a Product of its Prime Factors	Simplify 'Repeated' Surds	Answer
$\sqrt{12}$	$\sqrt{2} \times \sqrt{2} \times \sqrt{3}$	$2 \times \sqrt{3}$	$2\sqrt{3}$
$\sqrt{45}$	$\sqrt{3} \times \sqrt{3} \times \sqrt{5}$		
$\sqrt{18}$	$\sqrt{2} \times \sqrt{3} \times \sqrt{3}$		
$\sqrt{75}$			
$\sqrt{20}$			
	$\sqrt{7} \times \sqrt{7} \times \sqrt{2}$		$7\sqrt{2}$
			$3\sqrt{7}$
$\sqrt{48}$	$\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{3}$	$2 \times 2 \times \sqrt{3}$	$4\sqrt{3}$
$\sqrt{72}$	$\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{3} \times \sqrt{3}$		
$\sqrt{200}$			
$\sqrt{162}$			
$\sqrt{675}$			
		$2 \times 3 \times \sqrt{5}$	$6\sqrt{5}$
			$10\sqrt{3}$
			$8\sqrt{7}$

Worked Example	Your Turn
Simplify: a) $2\sqrt{20}$ b) $4\sqrt{40}$	Simplify: a) $3\sqrt{20}$ b) $4\sqrt{50}$

Worked Example	Your Turn
Write the following as a single root: a) $2\sqrt{15}$ b) $2\sqrt{30}$	Write the following as a single root: a) $5\sqrt{2}$ b) $10\sqrt{2}$

Extra Notes

6 Percentages with Multipliers

Worked Example	Your Turn
Write down the multiplier: To find 20% ...	Write down the multiplier: To find 30% ...
To increase by 20% ...	To increase by 30% ...
To decrease by 20% ...	To decrease by 30% ...

Worked Example	Your Turn
Write down the multiplier: To find 12.5% …	Write down the multiplier: To find 0.5% …
To increase by 12.5% …	To increase by 0.5% …
To decrease by 12.5% …	To decrease by 0.5% …

Intelligent Practice

Percentage	To find	To increase by	To decrease by
40%			
50%			
60%			
6%			
7%			
8%			
18%			
28%			
48%			

Intelligent Practice

Percentage	To find	To increase by	To decrease by
88%			
98%			
108%			
118%			
218%			
21.8%			
2.18%			
2.08%			
0.08%			

Intelligent Practice

Percentage	To find	To increase by	To decrease by
	$\times 0.15$		
		$\times 1.25$	
			$\times 0.76$
			$\times 0.66$
	$\times 0.66$		
		$\times 2.66$	
		$\times 1.06$	
			$\times 0.994$
	$\times 0.606$		

Worked Example	Your Turn
Find 72% of 493.8	Find 2% of 34.32

Worked Example	Your Turn
a) Increase 461.7 by 17% b) Decrease 461.7 by 17%	a) Increase 295.6 by 18% b) Decrease 295.6 by 18%

Fill in the Gaps

Original Amount	Percentage	Increase/ Decrease	Multiplier	Calculation	New Amount
£50	25%	*Increase*	1.25	£50 × 1.25	£62.50
£70	16%	*Increase*	1.16		
£89	15%	*Decrease*	0.85		
£125	76%	*Increase*			
£49	36%	*Decrease*			
£218	92%	*Decrease*			
£24	8%		1.08		
£92			1.83		
£48			0.73		
£75	12.5%	*Increase*			
£13	8.5%	*Decrease*			
£54			0.635		

Worked Example	Your Turn
In a 24% sale, the price of a shirt is reduced by $\$68.88$. Find the original price of the shirt.	In a 3% sale, the price of a phone is reduced by $\$2.82$. Find the original price of the phone.

Worked Example	Your Turn
The price of a jumper is increased by 74% and now is $\$581.16$. Find the original price.	The price of a jumper is increased by 68% and now is $\$717.36$. Find the original price.

Fill in the Gaps

Question	Percentage Increase/ Decrease	Multiplier	Calculation	Original Amount
After an increase of 10%, the price of a computer is £275. What was its original price?	10% *Increase*	1.1	£275 ÷ 1.1	£250
After an 8% pay rise, Omar earns £10.26 per hour. What was his hourly pay before the increase?	8% *Increase*	1.08		
A shirt is reduced by 20% to £20 in a sale. What was its original price?	20% *Decrease*	0.8	£20 ÷ 0.8	£62.50
The population of a village decreases by 16% to 1260. What was the population before the decrease?	16% *Decrease*	0.84		
A TV costs £258.42 including 18% tax. What was the price of the TV before the tax was added?	18% *Increase*			
A painting is sold for £729, making a profit of 35%. What was the original cost of the painting?				
In a sale, a coat is reduced by 33% to £43.55. What was its original price of the coat?				
A puppy increases in weight by 7.5% to 3.87 kg. What was the previous weight of the puppy?				
			£92 ÷ 1.15	
			528 ÷ 0.88	

Worked Example	Your Turn
The price of a computer increases from £452 to £619.24. Determine the percentage change.	The price of a train ticket decreases from £294 to £244.02. Determine the percentage change.

Fill in the Gaps

Round your answers to 1 decimal place where necessary.

Question	Actual Change	Original Amount	Calculation	Percentage Change
A population of butterflies grows from 500 to 562. What is the percentage change?	62	500	$\frac{62}{500} \times 100$	
Ayesha buys a bike for £3000 and sells it for £3200. What is her percentage profit?	£200	£3000		
Hassan's savings increased from £155 to £167.50. Find the percentage increase.	£17.50			
Leia buys a painting for $700 and sells it for $642. Work out her percentage loss.	$58			
Tony's wages increase from £14.25 per hour to £15.85 per hour. What is the percentage increase?				
Eric buys a laptop for £550 and after 1 year it is worth £325. What is the percentage loss in its value?				
The population of a town increases from 56500 to 58900. What is the percentage growth?				
The price of a book is reduced from £7.99 to £6.49. Find the percentage decrease.				
Noah buys an antique clock for £45 and sells it for £150. Find his percentage profit.				
			$\frac{1.65}{7.50} \times 100$	
			$\frac{4}{30} \times 100$	

Worked Example	Your Turn
Original Amount: 40 Percentage: 24% As a fraction Multiplier Percentage of… Increased by… Decreased by…	Original Amount: 40 Percentage: 72% As a fraction Multiplier Percentage of… Increased by… Decreased by…

Fill in the Gaps

	Original Amount	Percentage	As a fraction	Multiplier	Percentage of…	Increased by…	Decreased by….
1.	60	20%					
2.	60		$\frac{3}{10}$				
3.	60			0.25			
4.		25%			7.5		
5.			$\frac{1}{40}$			30.75	29.25
6.	30				6.75		
7.				0.225	67.5		

Fill in the Gaps

	Original Amount	Percentage	As a fraction	Multiplier	Percentage of…	Increased by…	Decreased by….
8.	300		$\frac{41}{200}$				
9.	60					72.3	47.7
10.			$\frac{41}{40}$		61.5		
11.	60			1.125			
12.	6				0.675		
13.	6					24.675	
14.	6						-31.35

Fill in the Gaps

	Amount (**A**)	Percentage (**P%**)	**P%** of **A**	**A** increased by **P%**	**A** decreased by **P%**
1.	320	10 %	32	352	288
2.	320	25 %			
3.	320	2.5 %			
4.	320	1.25 %			
5.	80	1.25 %			
6.	400	1.25 %			
7.	125		5		
8.		4 %	10		
9.	250		20		
10.	625	16 %			
11.	1859	16 %			
12.	1234	16 %			
13.	609		97.44		
14.	84			97.44	
15.	116				97.44
16.	116	160 %			
17.	116				-116
18.	348	66⅔ %			

	Amount (**A**)	Percentage (**P%**)	**P%** of **A**	**A** increased by **P%**	**A** decreased by **P%**
19.		10 %		88	
20.	80		12		
21.		80 %	12		
22.			12	52	
23.			12		48
24.			12		-2
25.		5 %	12		
26.			12	13	
27.	10			13	
28.		25 %		13	
29.				13	12
30.	15				12
31.		25 %			12
32.			68		12
33.				468	12
34.		97.5 %			12
35.		2.5 %			468
36.				328	312

Extra Notes

7 Repeated Percentage Change

Simple Interest

Interest is money that is paid regularly at a particular percentage, usually when money has been lent or borrowed. For example, a bank will give its customers interest to reward them for saving money with them, but it will also charge interest to anyone who has borrowed money from them.

As the name suggests, **simple interest** is a quick way of calculating interest. Simple interest is worked out by calculating the percentage amount and multiplying it by the number of periods that the money will be invested for.

Worked Example	Your Turn
Mr Bansal invests £850 into a savings account. Mr Bansal gets 3.75% per year simple interest. Work out the total interest Mr Bansal will get after 12 years.	Mr Dhillow invests £2810 into a savings account. Mr Dhillow gets 4.75% per year simple interest. Work out the total interest Mr Dhillow will get after 12 years.

Compound Interest

Interest is money that is paid regularly at a particular percentage, usually when money has been lent or borrowed. For example, a bank will give its customers interest to reward them for saving money with them, but it will also charge interest to anyone who has borrowed money from them.

Compound interest means that each time interest is paid onto an amount saved or owed, the added interest also receives interest from then on. Put simply, compound interest changes the amount of money in the bank each time and a new calculation has to be worked out.

Worked Example	Your Turn
Mr Bansal buys a car for £17150 which depreciates in value at a rate of 4% per year. Work out how much Mr Bansal's car will be worth in 14 years.	Mr Dhillow buys a car for £14680 which appreciates in value at a rate of 1.25% per year. Work out how much Mr Dhillow's car will be worth in 17 years.

Worked Example	Your Turn
A person invests £400 at 5% compound interest per annum. After x years they have £463.05. Find the value of x.	A person invests £400 at 6% compound interest per annum. After x years they have £476.40. Find the value of x.

Worked Example	Your Turn
Person A invests a sum of money. The account pays 5% compound interest per annum. After how many years will A have trebled their investment (as a whole number of years)?	Person A invests a sum of money. The account pays 6% compound interest per annum. After how many years will A have trebled their investment (as a whole number of years)?

Worked Example	Your Turn
A person invests £400 at 5% compound interest per annum. How much interest has been earned after three years?	A person invests £400 at 3% compound interest per annum. How much interest has been earned after 5 years?

Worked Example	Your Turn
A person invests £400 at $x\%$ compound interest per annum. After 3 years they have £463.05. Find the value of x.	A person invests £400 at $x\%$ compound interest per annum. After 3 years they have £476.40. Find the value of x.

Worked Example	Your Turn
A person invests £x at 5% compound interest per annum. After 3 years they have £463.05. Find the value of x.	A person invests £x at 6% compound interest per annum. After 3 years they have £476.40. Find the value of x.

Fill in the Gaps

Original Amount	Interest Rate	Multiplier	Number of Years	Calculation	Final Amount
£100	5%	1.05	10	100×1.05^{10}	£162.89
£100	4%	1.04	10		
£200	6%		8		
£250	3%		6		
£1200	6%		5		
£700	2.5%		3		
£500	1.5%		7		
£250		1.06	4		
£325		1.025	5		
				$£400 \times 1.03^{6}$	
				$£7000 \times 1.02^{3}$	
£400	5%				£510.51
£250	3%				£326.19
£600			8		£703
		1.025	6		£347.91
	6.5%		4		£932.69

Fill in the Gaps

Q	Yearly percentage change	Original Amount	Amount after 5 years	Amount after 10 years
1	100%	£1		
2	50%	£1		
3	5%	£1		
4	-50%	£2,048		
5	25%		£20	
6	-25%		£20	
7	7%		£1	
8	100%			£2,048
9	5%			£100
10	100%			£2,048
11		£20	£30	
12		£1	£5	
13		£1	£10	
14			£50	£100
15		£4		£64

Fill in the Gaps

1st percentage change	1st percentage multiplier	2nd percentage change	2nd percentage multiplier	Overall percentage change	Overall percentage multiplier
30% increase	× 1.3	15% increase	× 1.15	49.5% increase	× 1.495
15% increase		30% increase			
20% increase		25% increase			
5% increase		40% increase			
7.5% increase			× 1.375		
	× 1.06		× 1.39		
	× 1.2				× 1.68
		50% increase		68% increase	
10% decrease		10% decrease			
20% decrease		20% decrease			
30% decrease		30% decrease			
30% decrease		30% increase			
30% increase		10% decrease			
	× 0.85				× 1.19
			× 0.92	35.6% decrease	

Fill in the Gaps

Q	Original amount	Percentage change 1	Percentage change 2	Overall percentage change	New amount
1	£200	Increase by 20%	Decrease by 20%		
2	£200	Decrease by 20%	Increase by 20%		
3	£200	Decrease by 20%			£200
4	£200	Decrease by 20%	Decrease by 20%		
5	£200	Increase by 20%	Increase by 20%		
6		Increase by 20%	Increase by 50%		£288
7		Increase by 20%		Increase by 50%	£288
8		Decrease by 20%	Decrease by 37.5%		£288
9	£576	Decrease by 20%		Increase by 50%	
10	£576	Increase by 20%		Decrease by 50%	
11	£576	Decrease by 50%			£576
12	£576	Increase by 50%	Decrease by 100%		

Fluency Practice

A1 A bank pays 2.5% interest on its current account. Write 2.5% as a decimal.	**A2** Rosie took a science test and scored 41 marks out of 45. Express 41 out of 45 as a percentage.	**A3** A school has 80 staff. 15% of the staff wear glasses. Calculate the number of staff that wear glasses.	**A4** 56% of students in a school are girls. There are 420 girls in the school. Work out the total number of students in the school.
B1 Ayesha plays hockey. Last year Ayesha scored 8 goals. This year Ayesha scored 13 goals. Calculate the percentage increase in for the number of goals scored.	**B2** Between 2001 and 2011, the population of a town increased by 8% In 2001 the population was 34 342. Calculate the population in 2011.	**B3** In a sale, normal prices were reduced by 20%. The normal price of a camera was £180. Work out the sale price of the camera.	**B4** Justin bought some clothes. The clothes should have cost £84.00 but he got a discount of 15%. Work out how much money Justin saved.
C1 Rohan invested £3000 for 4 years in a savings account. He was paid 2.5% per annum compound interest. How much did Rohan have in his savings account after 4 years?	**C2** Susanna invested £2000 for 3 years at 4% interest per annum compound interest. Work out the amount of interest Susanna had earned after 3 years.	**C3** Anya bought a car for £12 500. The car depreciates at a rate of 12% per year. Work out the value of the car after five years.	**C4** The price of shoes was increased by 15%. However, customers were given a 20% discount if they bought two pairs at the same time. Work out the cost of two pairs of shoes that originally cost £68 each.
D1 In a sale, normal prices were reduced by 25%. The sale price of a computer was £442. Work out the normal price of the computer.	**D2** In a sale, all prices are reduced by 15%. The sale price of a shirt is £22.40. Work out the original price of the shirt.	**D3** The price of a new TV is £540, which includes 20% VAT. Find the cost of the TV excluding VAT.	**D4** Natasha invested some money at 4% per annum compound interest. At the end of two years, the value of her investment was £3380. Find the amount of money that Natasha invested.

Extra Notes