Year 9
Mathematics
Unit 12

Name: _____

Class: _____

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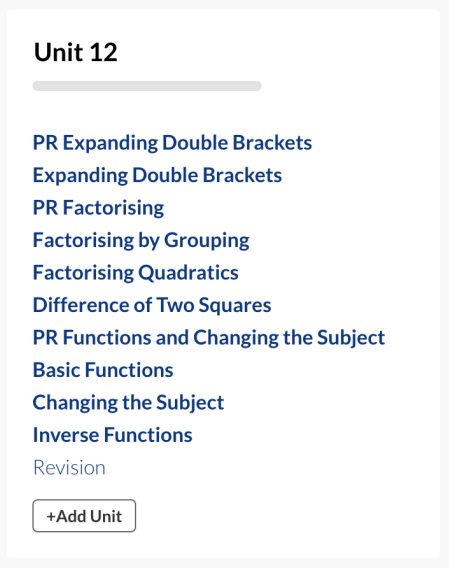
See unit 12 course on drfrostmaths.com



1 Expanding Double Brackets

Worked Example	Your Turn
Expand and simplify: a) $(x-3)(x-4)$ b) $(2x-3)(3x-4)$	Expand and simplify: a) $(x-3)(x-7)$ b) $(2x-3)(3x-7)$

Worked Example	Your Turn
Expand and simplify: a) $(x+3)^2$ b) $(2x-3)^2$	Expand and simplify: a) $(x-3)^2$ b) $(3x-7)^2$

Extra Notes

2 Factorising by Grouping

Worked Example	Your Turn
Factorise: a) $xy - 3y$ b) $x(x + 1) - 3(x + 1)$	Factorise: a) $2xy - 3y$ b) $2x(x + 1) - 3(x + 1)$

Worked Example	Your Turn
Factorise: a) $3x(x+1)-5(x+1)$ b) $3x(x+1)^2-5(x+1)$	Factorise: a) $5x(x+1)-3(x+1)$ b) $5x(x+1)^2-3(x+1)$

Worked Example	Your Turn
Factorise: $2x^2 + 2x - 3x - 3$	Factorise: $2x^2 - 2x - 3x + 3$

Extra Notes

3 Factorising Quadratics

Sum and Product

	Sum is Positive	**Sum is Negative**
Product is Positive	$\_\_ \times \_\_ = 14$ $\_\_ + \_\_ = 9$	$\_\_ \times \_\_ = 14$ $\_\_ + \_\_ = -9$
Product is Negative	$\_\_ \times \_\_ = -14$ $\_\_ + \_\_ = 5$	$\_\_ \times \_\_ = -14$ $\_\_ + \_\_ = -5$

	Sum is Positive	**Sum is Negative**
Product is Positive	Positive and Positive	Negative and Negative
Product is Negative	Positive and Negative where the size of the positive is greater than the size of the negative	Positive and Negative where the size of the negative is greater than the size of the positive

Worked Example	Your Turn
$\_\_ \times \_\_ = 140$ $\_\_ + \_\_ = -33$	$\_\_ \times \_\_ = 280$ $\_\_ + \_\_ = -43$

Quadratics

quadratum *v.* a square (Latin)

quadrat *n.*
1. A square frame, used to mark out an area of land to study its plants, animals, soil or other natural processes.

quadratic *n.*
1. A polynomial where the highest power of the variable is the second power (i.e. square).
Examples:
$x^2 + 3x - 5$ is a quadratic expression in x.
$t^2 - 9$ is a quadratic expression in t.
$5n^2 + n$ is a quadratic expression in n.

The general form of a quadratic expression is: $ax^2 + bx + c$
where a, b and c are numbers, $a \neq 0$ and x is the variable.

Quadratic expressions in x	Not quadratic expressions
x^2+4x+3	x^3+3x^2+6x+7
$2x^2+7x-\frac{13}{2}$	$3x^2+6x^{-1}+7$
$4x^2-3.5$	$x^2-6x^{\frac{1}{2}}+9$
$-x^2+3x$	
$-5x^2-x$	
$7-x^2$	
$7+\frac{x}{2}+5x^2$	

Monic means you have a single x^2, i.e. $a = 1$ in the general form $ax^2 + bx + c$

- The **coefficient** of an algebraic term is the number/constant in front of it. So the coefficient of $3x^2$ is 3 and the coefficient of $5x^3$ is 5.
- A **constant term** is one without any variables in it. So in $3x^2 + x + 5$, the constant term is 5.

Worked Example	Your Turn
Factorise: $3x^2 + 10x + 8$	Factorise: $3x^2 - 10x + 8$

Worked Example	Your Turn
Factorise: $3x^2 + 2x - 8$	Factorise: $3x^2 - 2x - 8$

Fill in the Gaps

① Factorise the following

	$2x^2 + 11x + 5$	$5x^2 + 8x + 3$	$3x^2 + 10x + 3$	$2x^2 + 5x + 3$
Identify a, b and c	a = 2, b = 13, c = 5			
Multiply a and c	2 x 5 = 10			
What multiples to give ac but adds to give b?	1 x 10 = 10 1+ 10 = 11			
Split the middle term	$2x^2 + 10x + x + 5$			
Factorise each pair of terms	$2x(x + 5) + 1(x + 5)$			
Complete the factorisation	$(2x + 1)(x + 5)$			

	$3x^2 + 13x - 10$	$3x^2 + 4x - 15$	$2x^2 - x - 1$	$2x^2 - 5x - 12$
Identify a, b and c				
Multiply a and c				
What multiples to give ac but adds to give b?				
Split the middle term				
Factorise each pair of terms				
Complete the factorisation				

Worked Example	Your Turn
Factorise: $6x^2 + 20x + 16$	Factorise: $6x^2 - 4x - 16$

Worked Example	Your Turn
Factorise: $x^2 + 20x + 96$	Factorise: $x^2 - 4x - 96$

Worked Example	Your Turn
Factorise: $-9x^2 + 30x + 24$	Factorise: $-9x^2 + 42x - 24$

Extra Notes

4 Difference of Two Squares

- The coefficients of the variables are square numbers.
- The powers of the variables must be even.
- The powers of the variables are **NEVER** odd numbers.
- One term will be negative **AND** the other term will be positive.
- If there is a number, then it must be a square number.

$a^2 - b^2 = (\mathrm{a} + \mathrm{b})(\mathrm{a} - \mathrm{b})$

Examples	Non-Examples
$4x^{10} - 36$	$4x^5 - 9$
$9x^{10} - 36$	$4x^6 + 9$
$-36 + 9x^{10}$	$3x^6 + 9$
$-36 + 9x^6$	$4x^6 - 8$
$9x^6 - \frac{16}{36}$	$4x^6 - 9y^5$
$9x^6 - \frac{1}{36}$	
$1x^6 - \frac{1}{36}$	
$1x^2 - \frac{1}{36}$	
$1x^2 - 36$	
$-36 + 1x^2$	
$36 - 1x^2$	
$36 - x^2$	
$x^2 - 36$	

Worked Example	Your Turn
Factorise: a) $x^2 - 9$ b) $9 - x^2$ c) $x^2 - 9y^6$ d) $16x^2 - 9y^6$	Factorise: a) $x^2 - 25$ b) $25 - x^2$ c) $x^2 - 25y^4$ d) $16x^2 - 25y^4$

Worked Example	Your Turn
Factorise: a) $2x^2 - 8$ b) $2x^2 - 8y^6$	Factorise: a) $2x^2 - 50$ b) $2x^2 - 50y^4$

Worked Example	Your Turn
$51^2 - 49^2 =$	$53^2 - 47^2 =$

Worked Example	Your Turn
Factorise: a) $x^3 - x$ b) $x^3 + 3x^2 + 2x$	Factorise: a) $8x^2 - 2$ b) $x^4 - x^3 - 6x^2$

Fill in the Gaps

Expanded Expression	Factorised Expression
$2x + 8$	$2(x + 4)$
	$3(x - 2)$
	$x(x + 7)$
$5x + 35$	
$8x - 12$	
	$2x(x - 5)$
$x^2 - x$	
	$5x(3 - x)$
$10x^2 + 2x$	
$6x + 9y$	
	$4xy(x + 2)$
$6xy - 4y^2$	
	$(x + 2)(x + 3)$
	$(x + 5)(x - 3)$
$x^2 + 8x + 15$	
$x^2 + 3x + 2$	

Expanded Expression	Factorised Expression
$x^2 - 7x + 10$	
	$(x - 6)(x + 4)$
	$(x + 7)(x - 7)$
$x^2 + 2x - 15$	
$x^2 - 25$	
	$(2x + 1)(x + 5)$
$x^2 - x - 6$	
$x^2 + 3x$	
	$(3x - 1)(x - 2)$
$4x^2 - 25$	
	$(x + 5)^2$
$7x^2 + 10x + 3$	
	$(3x - 1)^2$
$4x^2 + 4x + 1$	
$5x^2 - 14x - 3$	
	$(x - 2)^3$

Extra Notes

5 Basic Functions

Here is a number machine:

Input → [x 3] → [- 2] → Output

What is the output when the input is 7?

Here is an expression:

$3x - 2$

What is the value of this expression when $x = 7$

Here is a function:

$f(x) = 3x - 2$

Calculate the value of $f(7)$

Worked Example	Your Turn
If $f(x) = 3x + 4$, evaluate: a) $f(2)$ b) $f(-4)$	If $g(x) = -3x + 7$, evaluate: a) $g(5)$ b) $g(-2)$

Worked Example	Your Turn
If $f(x) = \frac{x}{3} + 2$, evaluate: a) $f(3)$ b) $f(-6)$	If $g(x) = \frac{x}{4} - 5$, evaluate: a) $g(24)$ b) $g(4)$

Worked Example	Your Turn
If $f(x) = x^2 + 3$, evaluate: a) $f(4)$ b) $f(-2)$	If $g(x) = x^2 - 4$, evaluate: a) $g(5)$ b) $g(-2)$

Worked Example	Your Turn
If $f(x) = 2x^2 + 3x$, evaluate: a) $f(4)$ b) $f(-2)$	If $g(x) = 3x^2 - 4x$, evaluate: a) $g(5)$ b) $g(-2)$

Solving Functions

Solve $3x + 2 = 26$

Here is a function:
$f(x) = 3x + 2$

Solve to find x:
$f(x) = 26$

Worked Example	Your Turn
If $f(x) = 3x + 4$, find x when $f(x) = 19$	If $g(x) = -3x + 7$, find x when $g(x) = 1$

Worked Example	Your Turn
If $f(x) = \frac{x}{3} + 2$, find x when $f(x) = 8$	If $g(x) = \frac{x}{4} - 5$, find x when $g(x) = -2$

Worked Example	Your Turn
If $f(x) = x^2 + 3$, find x when $f(x) = 19$	If $g(x) = x^2 - 4$, find x when $g(x) = 21$

Worked Example	Your Turn
If $f(x) = x^2 - 2$, evaluate: a) $f(x-2)$ b) $f(2x)$	If $g(x) = x^2 + 3$, evaluate: a) $g(x-3)$ b) $g(3x)$

Worked Example	Your Turn
If $f(x) = 3x^2 - 2$, evaluate: a) $f(x-2)$ b) $f(2x)$	If $g(x) = 5x^2 + 3$, evaluate: a) $g(x-3)$ b) $g(3x)$

Worked Example	Your Turn
If $f(x) = 3x^2 - 5x - 2$, evaluate $f(x - 2)$	If $g(x) = 5x^2 - 2x + 3$, evaluate $g(x - 3)$

Extra Notes

6 Changing the Subject

A formula is a mathematical equation containing two or more variables.

Suppose that you have the formula such as $2x = 3a$

We could write this formula as $x = \frac{3a}{2}$ in which case we would say that x is the subject of the formula, or that x is given/written in terms of a.

Note: x is the subject of the formula above as it appears on its own on one side of an equals sign.

Is a the subject?

$a = 3x + 1$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a + 1 = 3b + 2$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$4a = 3b + 2$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$4b + 2 = a$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a = 5a - 7b + 3$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a^2 = 3b + 2$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a = \frac{1}{2}b$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a = \frac{7b + 55c}{2}$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$\sqrt{b} = a$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$\sqrt{a} = b$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a + 0 = b$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject

Fluency Practice

Formula	Is a the subject?
$a = b + 3$	
$b + 3 = a$	
$a + 3 = b$	
$a + c = b$	
$ac = b$	
$a = bc$	
$a = bc - 6$	
$a = bc - x$	
$a = bc - a$	
$a = bc - a^2$	
$-a = b + 3$	
$\frac{1}{a} = b + 3$	

Formula	Is a the subject?
$a^2 = b + 3$	
$a = b^2 + 3$	
$2a = b^2 + 3$	
$\sqrt{a} = b^2 + 3$	
$a = \sqrt{\frac{b^2 + 3}{2}}$	
$\sqrt{\frac{b^2 + 3}{2}} = a$	
$\sqrt{\frac{b^2 + 3}{2a}} = a$	

Worked Example	Your Turn
Make x the subject of the following formulae: $y = mx + c$	Make x the subject of the following formulae: $y = abx + c$

Worked Example	Your Turn
Make x the subject of the following formulae: (a) $y = \frac{x}{m} + c$ (b) $y = -\frac{x}{ef} + c^2$	Make x the subject of the following formulae: (a) $y = \frac{x}{ab} + c$ (b) $y = -\frac{x}{cd} + e^2$

Worked Example	Your Turn
Make x the subject of the following formulae: $y = p(x + q)$	Make x the subject of the following formulae: $y = p(x - q)$

Worked Example	Your Turn
Make a the subject of the following formulae: a) $2(a+b)^2 = c$ b) $2\sqrt{a-b} = c$	Make a the subject of the following formulae: a) $3(a-b)^2 = c$ b) $3\sqrt{a+b} = c$

Fill in the Gaps

Q	$\mathbf{a} =$	$\mathbf{b} =$	$\mathbf{c} =$
1	$a = b + c$		
2	$a = b - c$		
3		$b = ac$	
4			$c = \frac{2b}{a}$
5	$a = 2b + c$		
6		$b = \frac{a + c}{2}$	
7		$b = \frac{a}{2} + c$	
8			$c = b^2 - \frac{a}{2}$
9		$b = \frac{a}{2} + \sqrt{c}$	
10	$a = \frac{2b - 2\sqrt{c}}{3}$		

Worked Example	Your Turn
Make a the subject of the following formula: $ax + ay = 3$	Make a the subject of the following formula: $ak + am = 5$

Worked Example	Your Turn
Make a the subject of the following formula: $ax + 2y = 5y + am$	Make a the subject of the following formula: $ab + 3y = 7y + ak$

Worked Example	Your Turn
Make x the subject of the following formula: $ax + ay = cx + by$	Make x the subject of the following formula: $yx + wz = 3xz + 3yz$

Worked Example	Your Turn
Make x the subject of the following formula: $w = \frac{x + a}{x - a}$	Make x the subject of the following formula: $w = \frac{x + 2y}{x - y}$

Extra Notes

7 Inverse Functions

Here is a number machine:

Input → [x 3] → [- 5] → Output

What is the input when the output is 10?

Make x the subject:

$y = 3x - 5$

Given $f(x) = 3x - 5$

Find $f^{-1}(x)$

RULES FOR FINDING THE INVERSE $f^{-1}(x)$:

Step 1: Write out the function as $y = \cdots$

Step 2: Swap the x and y

Step 3: Make y the subject

Step 4: Instead of $y =$ write $f^{-1}(x) =$

Worked Example	Your Turn
Find the inverse function: $f(x) = 3x - 5$	Find the inverse function: $g(x) = 4x + 2$

Worked Example	Your Turn
Find the inverse function: $f(x) = 3(x - 2)$	Find the inverse function: $g(x) = 5(x + 4)$

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{2x + 3}{4}$	Find the inverse function: $g(x) = \frac{4x - 3}{2}$

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{x}{2} - 3$	Find the inverse function: $g(x) = \frac{x}{5} + 4$

Worked Example	Your Turn
Find the inverse function: $f(x) = 2 + \frac{x}{3}$	Find the inverse function: $g(x) = 5 + \frac{x}{4}$

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{2}{x} - 3$	Find the inverse function: $g(x) = \frac{5}{x} + 4$

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{2}{x-3}$	Find the inverse function: $g(x) = \frac{5}{x-4}$

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{3}{2-5x}$	Find the inverse function: $g(x) = \frac{4}{5-3x}$

Worked Example	Your Turn
Find the inverse function: $f(x) = 3\sqrt{x}$	Find the inverse function: $g(x) = 4\sqrt{x}$

Worked Example	Your Turn
Find the inverse function: $f(x) = 3\sqrt{x} - 2$	Find the inverse function: $g(x) = 4\sqrt{x} - 5$

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{2x - 3}{x + 2}$	Find the inverse function: $g(x) = \frac{4x - 5}{x - 3}$

Worked Example	Your Turn
Find the inverse function: $f(x) = \sqrt{\frac{3x-2}{x-4}}$	Find the inverse function: $g(x) = \sqrt{\frac{5x-4}{x+3}}$

Fill in the Gaps

$f(x)$	Write as $y = \cdots$	Swap x and y	Make y the subject	Write as $f^{-1}(x) = \cdots$
$f(x) = 3x - 1$	$y = 3x - 1$	$x = 3y - 1$	$x + 1 = 3y$ $\quad \frac{x+1}{3} = y$	$f^{-1}(x) = \frac{x+1}{3}$
$f(x) = 2x + 5$				
$f(x) = x^2 + 8$				
$f(x) = \sqrt{x-3}$	$y = \sqrt{x-3}$	$x = \sqrt{y-3}$	$x^2 = y - 3$	
$f(x) = \frac{x+2}{7}$				
$f(x) = \frac{x}{3} - 5$				
$f(x) = \frac{9}{x}$				
$f(x) = \frac{4}{x+3}$				

Extra Notes