

## Year 9 <br> Mathematics Unit 13



Name:
:

Class:

## Contents Page

## 1 Interpreting Straight Line Graphs <br> 2 Linear Inequalities <br> 3 Properties of 3D Shapes <br> 4 Plans and Elevations <br> 5 Volume and Surface Area of Prisms <br> 6 Area and Volume Conversions <br> See unit 13 course on drfrostmaths.com

## Unit 13

PR Interpreting Straight Line Graphs
Interpreting Straight Line Graphs
PR Linear Inequalities
Linear Inequalities
Properties of 3D Shapes
Plans and Elevations
PR Volume and Surface Area of Prisms
Volume and Surface Area of Prisms
PR Area and Volume Unit Conversions
Area and Volume Unit Conversions
Revision
+Add Unit

## Horizontal and Vertical Lines



The relationship of the shared characteristic between points can be written as an equation.


All of these points have ay coordinate of 5 .

The straight line can be described as $y=5$ because this is true for every point on the line.


The relationship of the shared characteristic between points can be written as an equation.


All of these points have an $x$ coordinate of 1 .

The straight line can be describe as because this is true for every point on the line.

Find the equation of the line:


Find the equation of the line:



## Fluency Practice

Vertical and horizontal lines.

1) Fill in the boxes with the equations of the straight lines.

2) Write down the equation of the straight line
that would make the enclosed shape a square

3) Write the equation of the line that all of the following points will fall on
(a) $(4,5),(4,9),(4,0),(4,-3)$
(b) $(-10,2),(173,2),(10,2),(-0.3,2)$
(c) $(4.3,0.1),(0,0.1),(-9,0.1)$
(d) $\left(-\frac{1}{3}, 10\right),\left(-\frac{1}{3},-3\right),\left(-\frac{1}{3}, 0.5\right),\left(-\frac{1}{3},-0.1\right)$
4) Thinking carefully about the coordinates can rou find the equation of
(a) The $x$-axis
(b) The $y$-axis
5) A point has the coordinates of ( $3,-5$ ).
(a) What are the equations of the horizontal (a) What are the equations of the her
and vertical lines that this point is on?
(b) The line is vertical. Which of those two equations from (a) will it be?
6) A shape is made by the area enclosed by the lines $x=1, x=9, y=2$ and $y=5$.
(a) What is the shape?
(b) What is the area of the shape?

7) There is a rectangle below with a side missing. If the area of the rectangle is $24 \mathrm{~cm}^{2}$,

## Gradient

The gradient tells us how steep a line is, therefore the bigger the gradient the steeper the line is.

A positive gradient is a straight line which slopes up to the right.
A negative gradient is a straight line which slopes down to the right.




Fluency Practice

## Question 1: Find the gradient of each of these lines <br> (a) <br> (b)



(c)

(e)

(h)

(f)

(i)

Question 2: Draw lines with the following gradients
(a) 2
(b) 4
(c) 7
(d) -1
(e) -3
(f) -5
(g) $\frac{1}{2}$
(h) 10


Question 3: Find the gradient of each of these lines
(a)

(d)

(b)

(e)

(c)

(f)


Question 4: Draw lines with the following gradients
(a) $2 \frac{1}{2}$
(b) $\frac{1}{3}$
(c) $\frac{1}{5}$
(d) $-\frac{1}{6}$
(e) $\frac{3}{10}$
(f) $\frac{4}{5}$
(g) $1 \frac{1}{3}$
(h) $-\frac{3}{5}$


## Fluency Practice

Question 5: Find the gradient of each of these lines

(d)

(b)

(e)

(f)

(g)

(h)

(i)


Question 7: Work out the gradient of the line passing through these pairs of points
(a) $(1,4)$ and $(3,10)$
(b) $(0,0)$ and $(3,12)$
(c) $(5,-2)$ and $(9,14)$
(d) $(-8,6)$ and $(0,-2)$
(e) $(-5,-9)$ and $(1,3)$
(f) $(-7,-2)$ and $(1,-4)$
(g) $(-2,1)$ and $(8,-7)$
(h) $(-2,9)$ and $(4,7)$
(i) $(-4.5,3)$ and $(6,-7.5)$

Question 6: Find the gradient of each line shown below
(a)

(b)

(c)

(d)

(e)

(f)


The gradient connecting the two points $(2 a, 5)$ and $(7 a, 8)$ is 6 . Solve for $a$.

The gradient connecting the two points $(3 a, 7)$ and $(5 a, 12)$ is 6 . Solve for $a$.

The gradient connecting the two points $(2,10)$ and $(5, d)$ is 4 . Solve for d.

The gradient connecting the two points $(-3,-10)$ and $(2, d)$ is 12. Solve for d .

## Fluency Practice

Question 1: Alisha says that the gradient of the line is 2. Explain her mistake.

Question 2: Find the gradient of the line passing through the points $(4 a,-a)$ and $(6 a, 5 a)$

Question 3: The line passing through $(5,-2)$ and $(8, c)$ has a gradient of 3. Find c .

Question 4: The line passing through $(-8,-9)$ and $(-2, h)$ has a gradient of 4. Find $h$.

Question 5: The line passing through $(3,-4)$ and $(m, 10)$ has a gradient of 2. Find $m$.
Question 6: The line passing through $(-2,5)$ and $(2, n)$ has a gradient of $-1 / 2$ Find $n$.
Question 7: The line passing through $(1, p)$ and $(5,1)$ has a gradient of 0.75 Find p .
Question 8: Find the equation of the line shown


| Worked Example | Your Turn |
| :--- | :--- |
| $y=2 x-1$ | $y=3 x-4$ |
| Gradient: | Gradient: |
| $y$-intercept: | $y$-intercept: |
|  |  |
| $y=-2 x+6$ | $y=-3 x+6$ |
| Gradient: | Gradient: |
| $y$-intercept: | $y$-intercept: |
|  |  |
| $2 x+3 y=6$ | Gradient: |
| Gradient: | $y$-intercept: |
| $y$-intercept: |  |
|  |  |


| Worked Example | Your Turn |
| :--- | :--- |
| Write in the form $y=m x+c$ the line with: | Write in the form $y=m x+c$ the line with: |
| Gradient 2 and $y$-intercept 3 2 and $y$-intercept -3 | Gradient 3 and $y$-intercept 4 |
| Gradient $-\frac{3}{2}$ and $y$-intercept 0 | Gradient $-\frac{5}{6}$ and $y$-intercept -1 |
| Gradient 0 and $y$-intercept 4 | Gradient $\frac{3}{4}$ and $y$-intercept 0 |
|  |  |
|  | Gradient 0 and $y$-intercept -5 |

## Equation of Straight Line Graphs

Straight line graphs can be written in the form $y=m x+c$, where $m$ is the gradient, or steepness of the graph and $c$ is the $y$ intercept of the graph, where the graph cuts through the $y$-axis.

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ |  |  |  | $c$ |  |  |  |  |

$$
y=m x+c
$$

## $+m$





Fluency Practice

(d)

(b)

(e)

(c)

(f)

(h)

(k)

(i)

(1)


## Fluency Practice

Question 6: Find the equation of each line below.
(a)

(d)

(b)

(e)

(c)

(f)


Question 7: Find the equation of the straight line that passes through the points
(a) $(0,3)$ and $(4,19)$
(b) $(0,2)$ and $(6,20)$
(c) $(0,0)$ and $(1,4)$
(d) $(0,-9)$ and $(9,0)$
(e) $(0,-6)$ and $(7,8)$
(f) $(-8,-10)$ and $(0,14)$
(g) $(0,2)$ and $(10,7)$
(h) $(-4,1)$ and $(0,7)$
(i) $(-4,0)$ and $(0,18)$

| Worked Example | Your Turn |
| :--- | :--- |
| Find the equation of the line, given a point and the gradient: <br> $(-6,22)$ Gradient 3 | Find the equation of the line, given a point and the gradient: <br> $(-2,5)$ Gradient 4 |

## Your Turn

Write the equation of the line in the form $y=m x+c$ which
Write the equation of the line in the form $y=m x+c$ which passes through the points $(2,3)$ and $(5,-9)$

## Your Turn

Write the equation of the line in the form $y=m x+c$ which passes through the points $(2,-3)$ and $(7,-5)$

Write the equation of the line in the form $y=m x+c$ which passes through the points $(3,-2)$ and $(-7,5)$

Worked Example
Find where the line intercepts the axes:

| Line | $x$-intercept | $y$-intercept |
| :---: | :---: | :---: |
| $y=2 x+3$ |  |  |
| $y=2 x-3$ |  |  |

## Your Turn

Find where the line intercepts the axes:

| Line | $x$-intercept | $y$-intercept |
| :---: | :---: | :---: |
| $y=5 x-4$ |  |  |
| $y=5 x+4$ |  |  |

Worked Example
Find where the line intercepts the axes:

| Line | $x$-intercept | $y$-intercept |
| :---: | :---: | :---: |
| $y=3-2 x$ |  |  |
| $y=2-3 x$ |  |  |
| $2 x+3 y=6$ |  |  |

## Your Turn

Find where the line intercepts the axes:

| Line | $x$-intercept | $y$-intercept |
| :---: | :---: | :---: |
| $y=5-4 x$ |  |  |
| $y=4-5 x$ |  |  |
| $5 x+4 y=20$ |  |  |


| Worked Example | Your Turn |
| :--- | :--- |
| Does the point $(2,9)$ lie on the line $y=4 x+1$ ? | Does the point $(2,9)$ lie on the line $y=9-2 x ?$ |
|  |  |

## Extra Notes

2 Linear Inequalities

## Inequalities on Number Lines



| Worked Example | Your Turn |  |
| :--- | :--- | :--- |
| a) Plot $x<3$ on a number line | a) | Plot $x>14$ on a number line |
| b) $\quad$ Plot $x \geq 14$ on a number line | b) | Plot $x \leq-2$ on a number line |
|  |  |  |

## Fill in the Gaps

For each sentence, write an inequality then draw a number line representation.

| 1) $x$ is less than 7 | $x<7$ |  |
| :--- | :--- | :--- |
| 2) $x$ is less than or equal to 7 |  |  |
| 3) $x$ is more than 4 |  |  |
| 4) $x$ is more than 10 |  |  |
| 5) $x$ is more than 3.5 |  |  |
| 6) $x$ is more than or equal to 7.5 |  |  |
| 7) $x$ is less than or equal to 0 |  |  |
| 8) $x$ is more than or equal to 3.5 |  |  |



## Solving Linear Inequalities

Inequalities behave in a similar way to equations: whatever we do to one side of the equation, we have to do the same to the other.
'Solving an inequality' means to get $x$ on its own on one side of the equation. This is so that the range is then clear.

When you divide or multiply both sides of an inequality by a negative number, reverse the direction of the inequality.

## Why?

Consider the inequality $2<4$
This is clearly true as 2 is less than 4
But, if we multiple/divide by both sides by -1 , we get $-2<-4$, which is false.
However, if we reverse the inequality sign, we get $-2>-4$, which is true as -2 is more than -4 .

But it is probably easiest to avoid needing to divide by a negative number in the first place...

IF THERE IS A NEGATIVE COEFFICIENT OF THE VARIABLE THEN ADD TO BOTH SIDES TO GET A POSITIVE ONE.

| Worked Example |  | Your Turn |
| :--- | :--- | :--- |
| Solve: | Solve: |  |
| a) $2 x-8<16$ | a) $3 x-9>27$ |  |
| b) $2(4-x)<16$ | b) $3(3-x)>27$ |  |
|  |  |  |

## Your Turn

Solve:
a) $10(x+3)+3(2 x+6)<144$
b) $7(x+3)-3(2 x-6)=84$

Solve:
a) $5(x+3)+2(2 x-6) \leq 111$
b) $5(x-3)-2(2 x-6) \geq 111$

| Worked Example | Your Turn |
| :--- | :--- |
| Solve: | Solve: |
| a) $9 x+4<2 x+60$ | a) $5 x+7>2 x+22$ |
| b) $3 x-23 \leq 7-2 x$ | b) $2 x-23 \geq 9-2 x$ |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |


| Worked Example | Your Turn |
| :--- | :--- |
| Solve: | Solve: |
| a) $3(x+2)<2(x+3)$ | a) $7(x-3) \leq 2(x+7)$ |
| b) $3(x+8)>3(2-x)$ | b) $3(x-5) \geq 5(5-x)$ |
|  |  |
|  |  |
|  |  |


| Worked Example | Your Turn |
| :--- | :--- |
| Solve: | Solve: |
| a) $-1<2 x+3<9$ |  |
| b) $-1 \leq 2 x+6<9$ | a) $-9<2 x+3<1$ |
|  |  |
|  |  |
|  |  |

Fill in the Gaps

| Q | Inequality | Represent on a number line | Integer solutions |
| :---: | :---: | :---: | :---: |
| 1 | $x>3$ |  |  |
| 2 |  | 2 3 4 5 6 7 <br> 1 1 1 1 1 1 | $x=3,4,5 \ldots$ |
| 3 |  |  | $x=-3,-4,-5 \ldots$ |
| 4 | $-3 \leq x$ |  |  |
| 5 | $x-1>2$ |  |  |
| 6 |  | $\begin{array}{llllll} \hline-5 & -4 & -3 & -2 & -1 & 0 \\ \hline & 1 & 1 & 1 & & 1 \end{array}$ |  |
| 7 | $x+5 \leq 2$ |  |  |
| 8 |  |  |  |
| 9 |  |  | $\begin{gathered} x=4,5,6 \ldots o r \\ x=-1-2,-3 \ldots . \end{gathered}$ |
| 10 | $<x \leq$ |  | $x=-2,-1,0,1,2,3$ |
| 11 | $x \geq 1$ and $x<$ |  |  |
| 12 | $3 x>9$ |  |  |

## Combining Inequalities

We have already seen examples where we've combined inequalities together:


Values will only appear on our combined inequality if our finger is on BOTH lines. At

And for any value above 15 , our finger is still over both
the moment our finger is on neither.


| Worked Example | Your Turn |
| :--- | :--- |
| Solve: | Solve: |
| $3-x \leq 2<10-2 x$ | $1+x<5 \leq 7+5 x$ |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

## Extra Notes

## 3 Properties of 3D Shapes



Sphere


Cylinder


Square-based pyramid


Cone




## Fluency Practice

2. The net is folded to make a cube.

Two other vertices meet at $P$.

1. Match the 3D solids with their net


Mark each of these vertices with the letter $P$.

3. The net shown is folded to make a dodecahedron. Label the face which is opposite the shaded one

4. Using the grid provided with 1 square $=1 \mathrm{~cm}$, draw an accurate net of these solids


## Extra Notes

## 4 Plans and Elevations

The plan is the view from the top of a 3D solid.
Elevations are horizontal views of a 3D object:

- Front elevation: The view from the front of an object.
- Back elevation: The view from behind the object.
- Side elevation: The view from the side of an object.

front elevation

side
elevation

back elevation

side
elevation



## Fluency Practice

| Cutting | Plan $\downarrow$ |
| :---: | :---: |
| Cubes |  |
| $3 \times 3 \times 3$ | Front |

## Draw the plan and elevations for each cube.

## Solid line $=$ a visible edge Dashed line = a hidden edge

| a) | Plan | Side Elevation | Front Elevation |
| :---: | :---: | :---: | :---: |
|  |  | - | - |
| , |  |  |  |
| $\sqrt{V}$ |  |  |  |



## Fluency Practice

## Draw the plan and elevations for each cube.

Solid line = a visible edge
Dashed line = a hidden edge





Fluency Practice


## Extra Notes

## 5 Volume and Surface Area of Prisms

## Volume of Cuboids

Volume of Cuboid $=$ Length $\times$ Width $\times$ Height
Volume of Cuboid $=\mathrm{l} \times \mathrm{w} \times \mathrm{h}$






## Surface Area of Cuboids

Surface Area of Cuboid $=2 \times$ Length $\times$ Width $+2 \times$ Length $\times$ Height $+2 \times$ Width $\times$ Height Surface Area of Cuboid $=2 \mathrm{lw}+2 \mathrm{lh}+2 \mathrm{wh}$






## Prisms

A prism is a 3D shape which has the same cross-section along its length.


## Cross-Section

It is the shape made when a solid is cut through parallel to the base.


## Volume of Prisms

Volume of Prism $=$ Area of Cross Section $\times$ Length
Volume of Prism $=\mathrm{A} \times \mathrm{l}$






Surface Area of Prism $=2 \times$ Area of Cross Section + Length $\times$ Perimeter of Cross Section
Surface Area of Prism $=2 A+L P$






Volume of Cylinder $=$ Area of circle $\times$ height
Volume of Cylinder $=\pi \times$ radius $^{2} \times$ height
Volume of Cylinder $=\pi r^{2} h$








## Surface Area of Cylinders

Curved Surface Area of Cylinder $=2 \times \pi \times$ radius $\times$ height
Curved Surface Area of Cylinder $=2 \pi r h$

Total Surface Area of Cylinder $=2 \times \pi \times$ radius $\times$ height $+2 \times \pi \times$ radius $^{2}$

Total Surface Area of Cylinder $=2 \pi r h+2 \pi r^{2}$

Surface area of cylinder $=2 \pi r^{2}+2 \pi r h$






## Extra Notes

## 6 Area and Volume Unit Conversions

## Let's consider this square.



$$
\text { Area }=4 \times 4=16 \mathrm{~m}^{2}
$$

Imagine we want to convert the area of this shape into $\mathrm{cm}^{2}$. What scale factor would we use?

$$
\begin{gathered}
\text { Area }=400 \times 400 \\
\text { Area }=160,000 \mathrm{~cm}^{2}
\end{gathered}
$$

Is this what we expected?
Our scale factor is not 100 , but 10,000. 100²

| Worked Example | Your Turn |
| :---: | :---: |
| Convert: <br> a) $7 \mathrm{~cm}^{2}$ to $\mathrm{mm}^{2}$ <br> b) $\quad 2500 \mathrm{~cm}^{2}$ to $\mathrm{m}^{2}$ | Convert: <br> a) $7 \mathrm{~km}^{2}$ to $\mathrm{m}^{2}$ <br> b) $2500 \mathrm{~mm}^{2}$ to $\mathrm{cm}^{2}$ |

## Units of Volume

Let's now consider a cube of side 4 m .


| Worked Example |  |
| :--- | :--- |
| Convert: | Your Turn |
| a) $7 \mathrm{~cm}^{3}$ to $\mathrm{mm}^{3}$ | Convert: |
| b) $5 \mathrm{~mm}^{3}$ to $\mathrm{cm}^{3}$ | a) $7 \mathrm{~m}^{3}$ to $\mathrm{cm}^{3}$ |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

## Extra Notes

