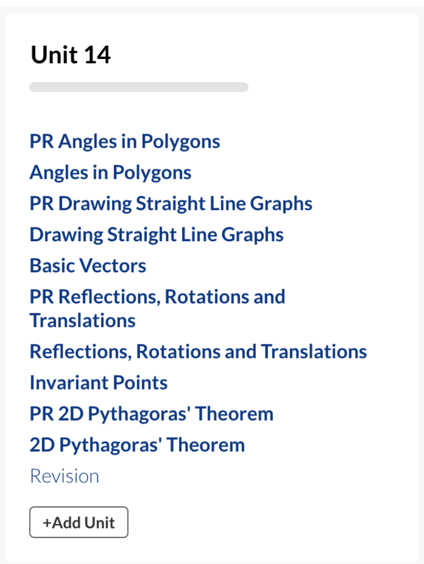
Year 9
Mathematics
Unit 14

Name: ____

Class: ____

Contents Page

See unit 14 course on drfrostmaths.com



1 Angles in Polygons

Frayer Model – Polygons

Definition	**Characteristics**
Examples	**Non-Examples**

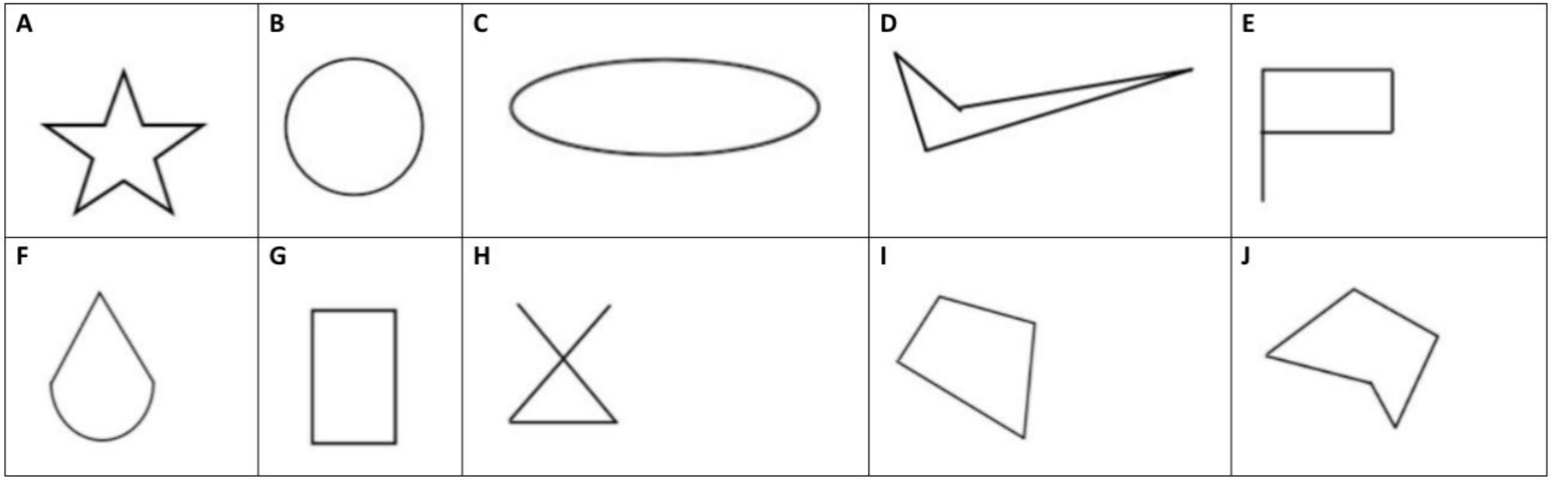
Frayer Model – Regular Polygons

Definition	**Characteristics**
Examples	**Non-Examples**

Fluency Practice

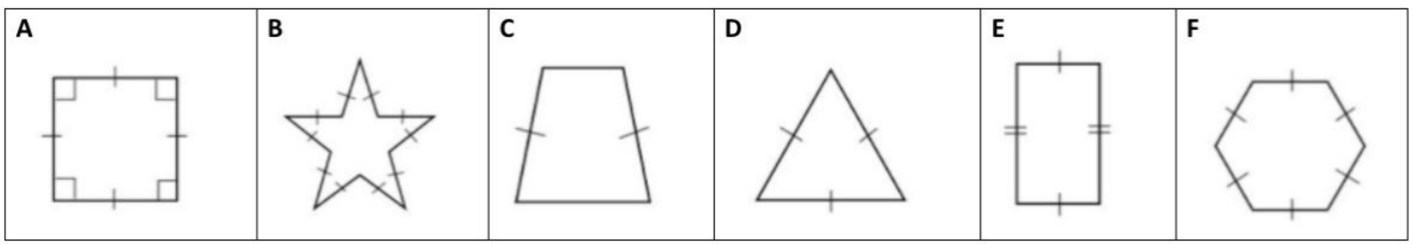
Polygons – Example or Non-Example

In each of the following diagrams decide whether the shape is a polygon or not. Label them 'Example' or 'Non-example'. For those that ARE polygons, give the name of the polygon.

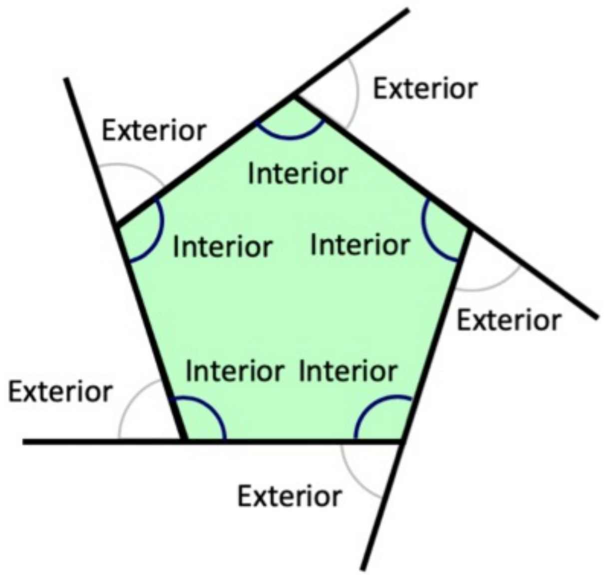


Polygons – Regular or Irregular

Which of the following are regular and which are irregular – how do you know?



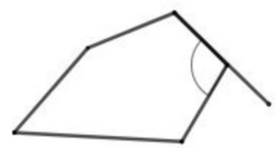
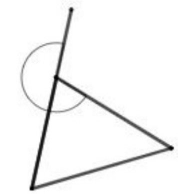
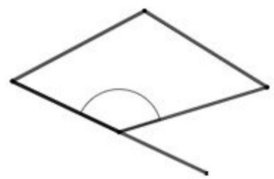
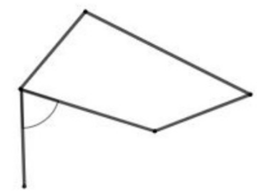
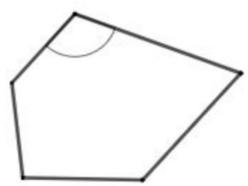
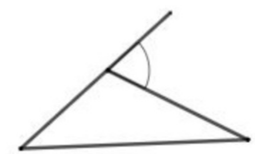
Interior and Exterior Angles



- The interior angles of a polygon are on the inside.
- The exterior angles of a polygon are on the outside.
- The interior and exterior angles form a straight line.

$$\text{Interior Angle} + \text{Exterior Angle} = 180°$$

Interior Angles

Examples	Nonexamples
	
	
	

Sum of Interior Angles

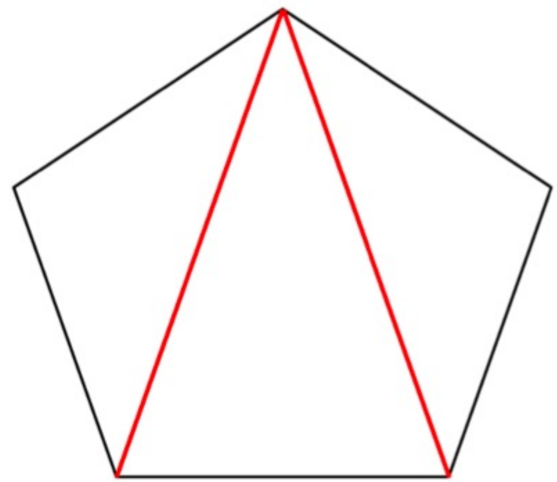
Number of Sides	Name of Shape	Interior Angle Sum
3	Triangle	180°
4	Quadrilateral	360°
5	Pentagon	540°
6	Hexagon	720°
7	Heptagon	900°
8	Octagon	1080°

Note: The polygon can be regular or irregular.

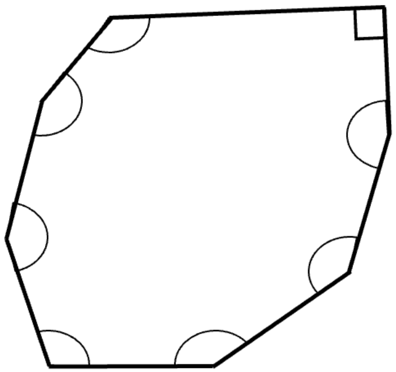
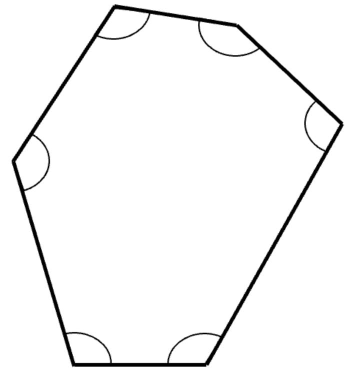
Sum of interior angles of a polygon $= (n - 2) \times 180°$
where n is the number of sides on the polygon

A polygon with n sides can be split into $n - 2$ triangles (with all triangle angles in the corners), and each triangle's angles add up to 180°.

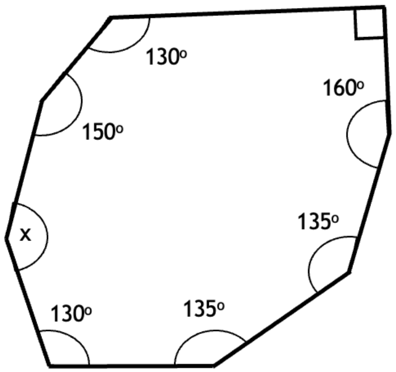
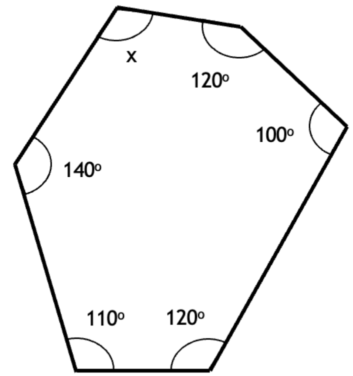
e.g.



5 sides
3 triangles
$3 \times 180° = 540°$
So the interior angles in a pentagon sum (add up) to 540°

Worked Example	Your Turn
Find the sum of interior angles of this polygon. 	Find the sum of interior angles of this polygon. 

Worked Example	Your Turn
Find the sum of the interior angles of a polygon with 30 sides.	Find the sum of the interior angles of a polygon with 60 sides.

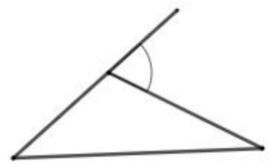
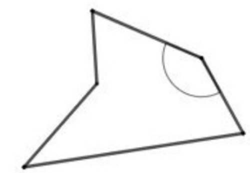
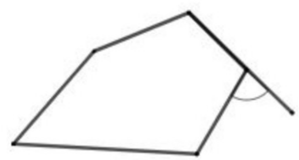
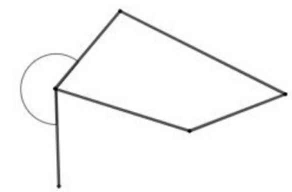
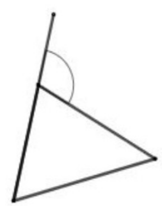
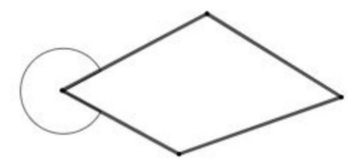
Worked Example	Your Turn
Find angle x.  	Find angle x. 

Worked Example	Your Turn
The sum of the interior angles of a polygon is $3240°$. How many sides does the polygon have?	The sum of the interior angles of a polygon is $6840°$. How many sides does the polygon have?

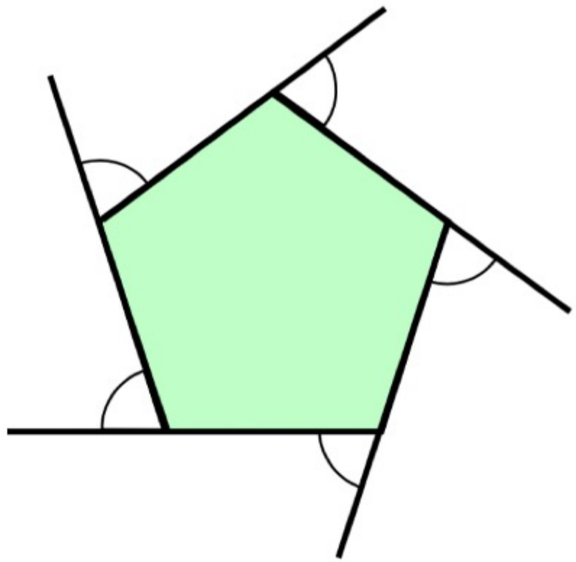
Fill in the Gaps

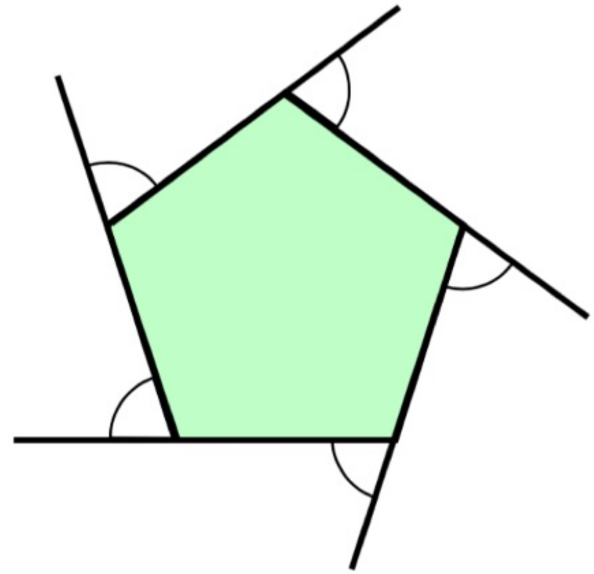
Number of sides	Sum of interior angles	Size of one interior angle in a regular polygon
3	180°	
	360°	
7		
9		
10		144°
	1800°	150°
13	1980°	
14		
	2700°	

Exterior Angles

Examples	Nonexamples
	
	
	

Sum of Exterior Angles

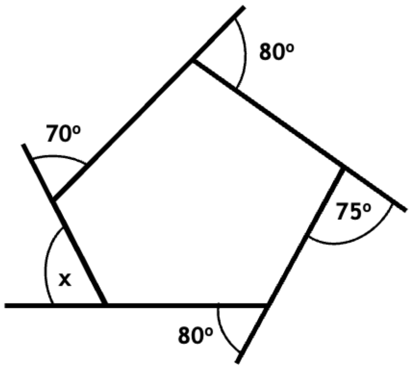
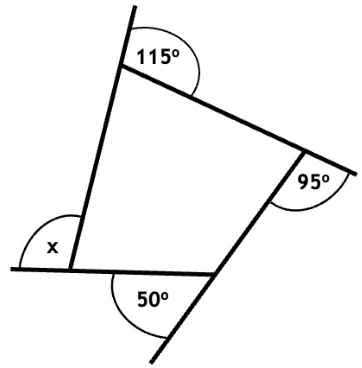


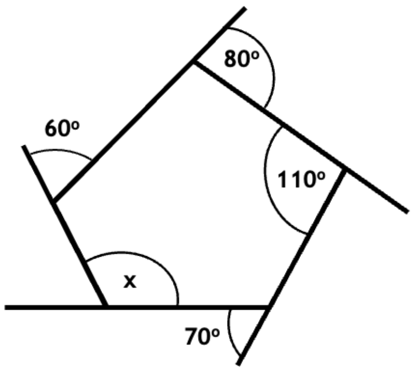
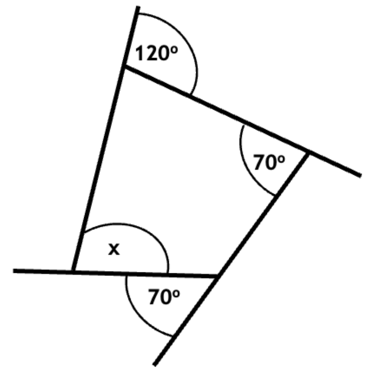


Note: The polygon can be regular or irregular.

$$\text{Sum of exterior angles of a polygon} = 360°$$

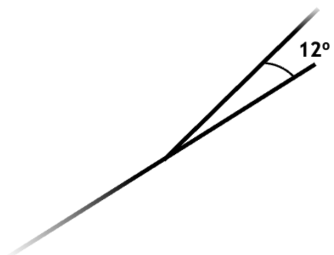
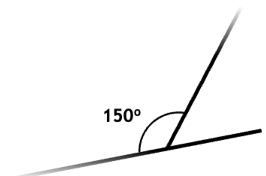
<u>Why?</u>
All the exterior angles can fit around a point, and angles around a point add up to 360°.

Worked Example	Your Turn
Find angle x. 	Find angle x. 

Worked Example	Your Turn
Find angle x. 	Find angle x. 

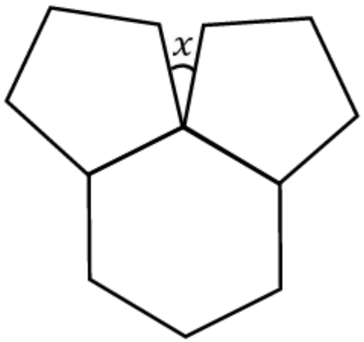
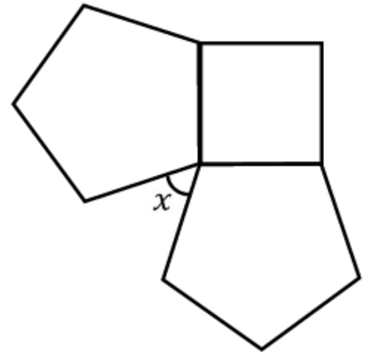
Worked Example	Your Turn
A regular polygon has 12 sides. Find the size of each exterior angle.	A regular polygon has 48 sides. Find the size of each exterior angle.

Worked Example	Your Turn
A regular polygon has 12 sides. Find the size of each interior angle.	A regular polygon has 48 sides. Find the size of each interior angle.

Worked Example	Your Turn
A section of a two different regular polygons are show below. How many sides do they each have?  	A section of a two different regular polygons are show below. How many sides do they each have?  

Worked Example	Your Turn
The interior angle of a regular polygon is $160°$. How many sides does the polygon have?	The interior angle of a regular polygon is $140°$. How many sides does the polygon have?

Worked Example	Your Turn
The size of each interior angle of a regular polygon is 9 times the size of each exterior angle. How many sides does the polygon have?	The size of each interior angle of a regular polygon is 11 times the size of each exterior angle. How many sides does the polygon have?

Worked Example	Your Turn
These are regular polygons. Find x.  	These are regular polygons. Find x. 

Fill in the Gaps

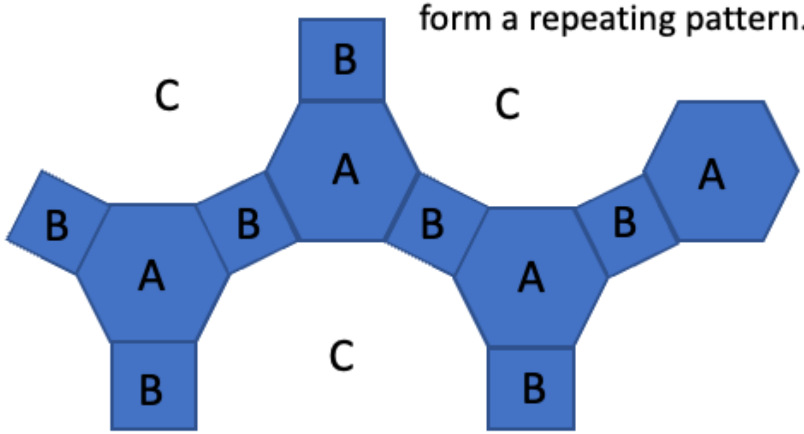
Name	Number of Angles	Sum of Interior Angles	Size of One Interior Angle in a Regular Polygon	Size of One Exterior Angle in a Regular Polygon
	3			
		360°	90°	
Octagon				45°
Hexadecagon		2520°		
Pentadecagon	15		156°	
				72°
		720°	120°	
	12			
		1620°		$\frac{360°}{11}$

Problem Solving with Interior and Exterior Angles

There are variety of skills that harder questions involving interior/exterior angles might involve:

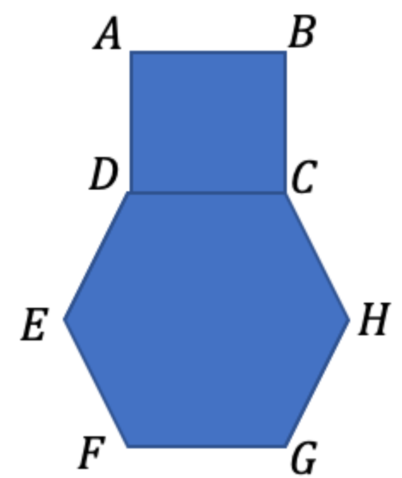
#1: Tessellation

Shapes 'tessellate' if they fit together, without overlap, to form a repeating pattern.

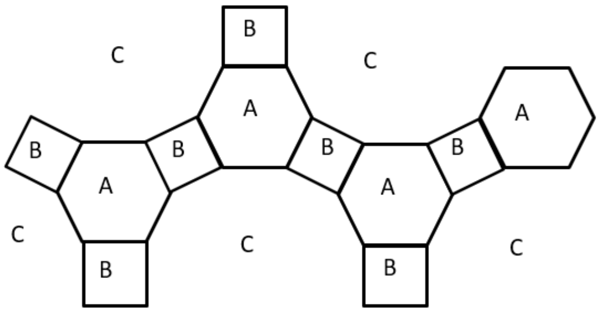
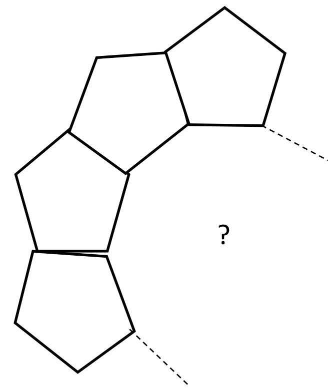


"The above repeating pattern consists of three regular polygons, A (hexagon), B (square) and C. Determine how many sides C has."

#2: Using isosceles triangles



"$ABCD$ is a square and $CDEFGH$ is a regular hexagon. Determine the angle CBH."

<table>
<tr><th>Worked Example</th><th>Your Turn</th></tr>
<tr><td>The repeating pattern consists of three regular polygons, A (hexagon), B (square) and C. Determine how many sides C has.

</td><td>The diagram shows 4 congruent regular pentagons that form the sides of an n-sided regular polygon. Determine the value of n.

</td></tr>
</table>

Worked Example

The diagram shows a regular pentagon. AB and CD are two of the lines of symmetry of the pentagon. Work out the size of the angle marked x.

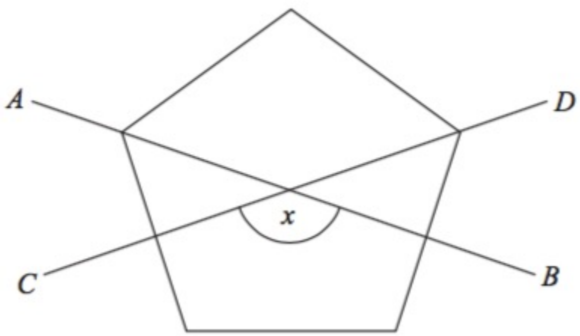
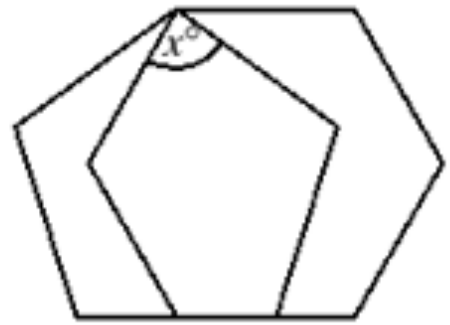
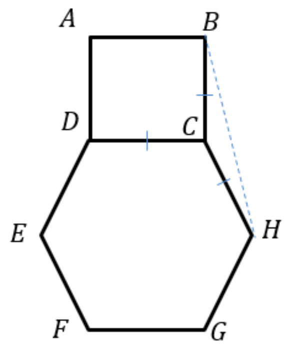
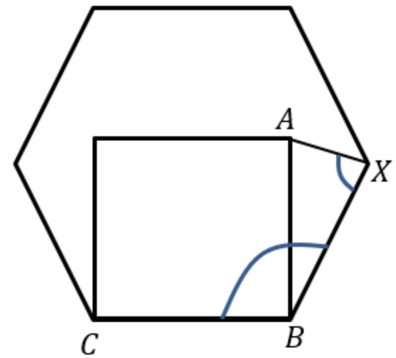


Diagram **NOT** accurately drawn

Your Turn

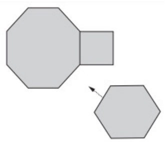
The diagram shows a regular pentagon and a regular hexagon which overlap. What is the value of x?



Worked Example	Your Turn
$ABCD$ is a square and $CDEFGH$ is a regular hexagon. Determine the angle CBH. 	The diagram shows a square inside a regular hexagon. What is the size of the marked angle at X? 

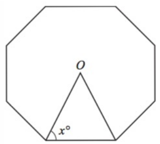
Fluency Practice

1 [KS3 SATs 2004 L6-L8 Paper 2 Q19 Edited]
A pupil has three tiles. One is a regular octagon, one is a regular hexagon, and one is a square. The side length of each tile is the same. The pupil says the hexagon will fit exactly like this. Is the pupil correct?



?

2



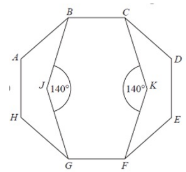
[Edexcel IGCSE Nov2009-3H Q3a]
The diagram shows a regular octagon, with centre O. Work out the value of x.

?

3 *[Edexcel IGCSE Nov-2010-4H Q13]*
The size of each interior angle of a regular polygon is 11 times the size of each exterior angle. Work out the number of sides the polygon has.

?

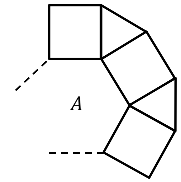
4



[Edexcel GCSE Nov2014-1H Q17] ABCDEFGH is a regular octagon. BCKFGJ is a hexagon. JK is a line of symmetry of the hexagon. Angle BJG = angle $CKF = 140°$. Work out the size of angle KFE.

?

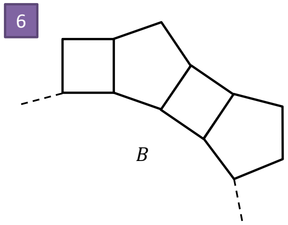
5



A regular polygon A is surrounded by squares and equilateral triangles in an alternating pattern, as shown. Show that A is a hexagon.

?

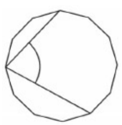
6



A regular polygon B with n sides is surrounded by squares and regular pentagons in an alternating pattern, as shown. Determine the value of n.

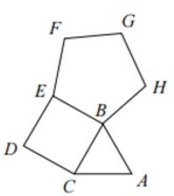
?

7 [IMC 2003 Q22] The diagram shows a regular dodecagon (a polygon with twelve equal sides and equal angles). What is the size of the marked angle?



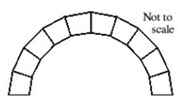
?

8 *[JMO 2014 B1]*
The figure shows an equilateral triangle ABC, a square BCDE, and a regular pentagon BEFGH.
What is the difference between the sizes of ∠ADE and ∠AHE?



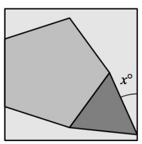
?

9 [IMC 2005 Q14] Ten stones, of identical shape and size, are used to make an arch, as shown in the diagram. Each stone has a cross-section in the shape of a trapezium with three equal sides. What is the size of the smallest angles of the trapezium?



?

10



[IMC 2018 Q18] The diagram shows a regular pentagon and an equilateral triangle placed inside a square. What is the value of x?

?

11 Find all regular polygons which tessellate (when restricted only to one type of polygon).

?

☠ By thinking about interior angles, prove that the regular polygons you identified above are the only regular polygons which tessellate.

?

Interior and Exterior Angle Rules

All Polygons	Regular Polygons
$Interior\ Angle + Exterior\ Angle = 180°$ $Sum\ of\ Interior\ Angles = (n-2) \times 180°$ $Sum\ of\ Exterior\ Angles = 360°$	$Each\ Exterior\ Angle = \frac{360°}{n}$ $Each\ Interior\ Angle = 180° - \frac{360°}{n}$

Fluency Practice

A1 Write down a formula that allows you to calculate the size of an exterior angle (E) of a regular polygon with n sides.	**A2** Write down a formula that relates the size of an exterior angle (E) and the size of an interior angle (I) of a polygon.	**A3** Write down a formula that allows you to calculate the sum (S) of the interior angles in a regular polygon with n sides.	**A4** Work out the size of an exterior angle of a regular polygon with 5 sides
B1 Work out the size of an interior angle of a regular polygon with 9 sides	**B2** Each exterior angle of a regular polygon is 15º. Work out the number of sides the polygon has.	**B3** Each interior angle of a regular polygon is 156º. Work out the number of sides the polygon has.	**B4** Find the **sum** of the interior angles of a polygon with 7 sides
C1 The size of each exterior angle of a regular polygon is 18°. Work out the **sum** of the interior angles of the polygon.	**C2** The sum of the interior angles of a polygon is 2700º. Work out the number of sides the polygon has.	**C3** The size of each interior angle of a regular polygon is 140º bigger than the size of each exterior angle. Work out the number of sides the polygon has.	**C4** The size of each interior angle of a regular polygon is 11 times the size of each exterior angle. Work out the number of sides the polygon has.
D1 The size of each interior angle of a regular polygon with n sides is 144°. Work out the size of each interior angle of a regular polygon with $2n$ sides.	**D2** An exterior angle of regular polygon **A** is 30º bigger than an exterior angle of regular polygon **B**. Polygon **A** has 9 sides. Find the number of sides of polygon **B**.	**D3** An interior angle of regular polygon **C** is 10º smaller than an interior angle of regular polygon **D**. Polygon **C** has 12 sides. Find the number of sides of polygon **D**.	**D4** The sum of the interior angles in polygon **E** is 900º more than the sum of the interior angles in polygon **F**. The total number of sides of the two polygons is 25. How many sides in each polygon?

Extra Notes

2 Drawing Straight Line Graphs

Worked Example

Plot the graph of $y = 2x + 1$ for the values $-2 \leq x \leq 2$

x					
y					

Your Turn

Plot the graph of $y = 4x + 2$ for the values $-2 \leq x \leq 2$

x					
y					

Worked Example	Your Turn

Worked Example

Plot the graph of $y = -2x + 1$ for the values $-2 \leq x \leq 2$

x					
y					

Your Turn

Plot the graph of $y = -4x - 2$ for the values $-2 \leq x \leq 2$

x					
y					

Fluency Practice

1) y = 2x + 3

x	-2	-1	0	1	2
y					

2) y = 2x + 4

x	-2	-1	0	1	2
y					

3) y = 2x + 5

x	-2	-1	0	1	2
y					

4) y = 3x + 5

x	-2	-1	0	1	2
y					

5) y = 3x + 1

x	-2	-1	0	1	2
y					

6) y = 3x - 1

x	-2	-1	0	1	2
y					

7) y = 3x - 2

x	-2	-1	0	1	2
y					

8) y = 3x - 3

x	-2	-1	0	1	2
y					

9) y = 3x - 5

x	-2	-1	0	1	2
y					

10) y = 4x - 5

x	-2	-1	0	1	2
y					

11) y = -4x - 5

x	-2	-1	0	1	2
y					

12) y = -2x - 5

x	-2	-1	0	1	2
y					

13) y = -½x - 5

x	-2	-1	0	1	2
y					

14) y = ½x + ¾

x	-2	-1	0	1	2
y					

Worked Example	Your Turn
Plot the graph of $2x + y = 8$ for the values $-2 \leq x \leq 2$	Plot the graph of $2x - y = 8$ for the values $-2 \leq x \leq 2$

x					
y					

x					
y					

Worked Example

Plot the graph of $x + 2y = 8$ for the values $-2 \leq x \leq 2$

x					
y					

Your Turn

Plot the graph of $x - 2y = 8$ for the values $-2 \leq x \leq 2$

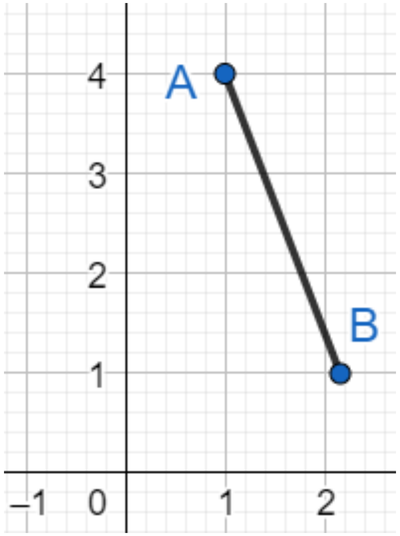
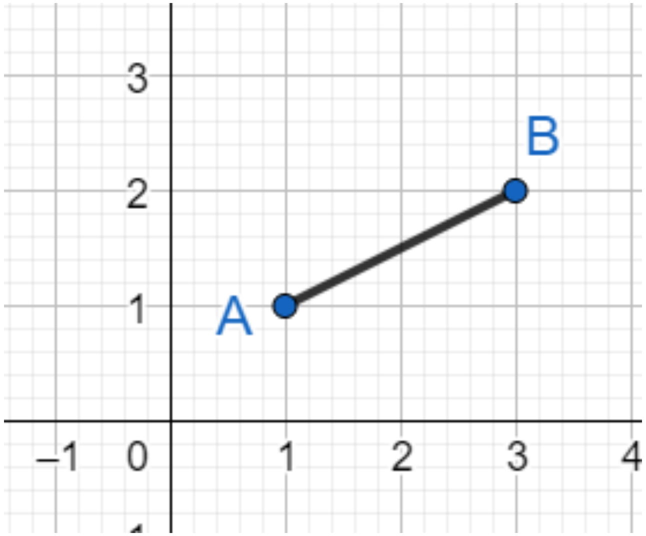
x					
y					

Extra Notes

3 Basic Vectors

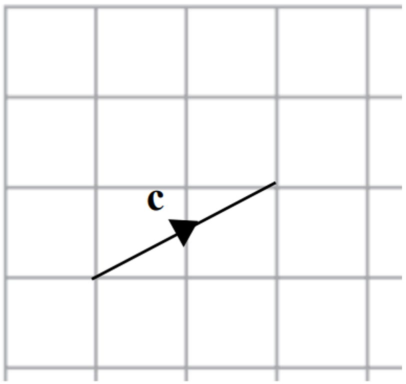
A vector has magnitude (how long it is) and direction.

Column Vector: $\begin{pmatrix} x \\ y \end{pmatrix}$ where x is movement right or left and y is movement up or down. Right and up are taken to be positive.

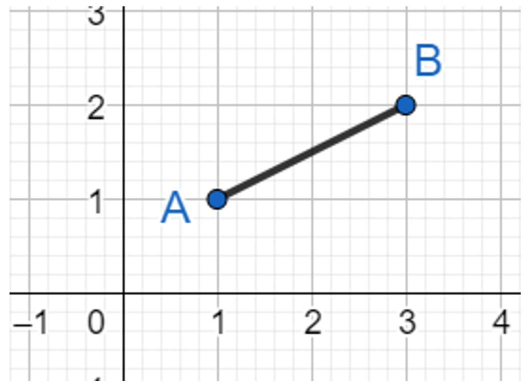
Worked Example	Your Turn
Write the vector $\overrightarrow{AB}$ in column form 	Write the vector $\overrightarrow{AB}$ in column form 

Fluency Practice

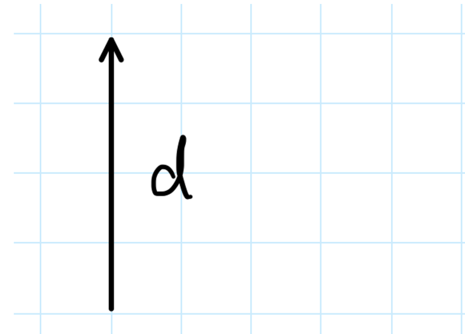
Write down the column vector **c**.



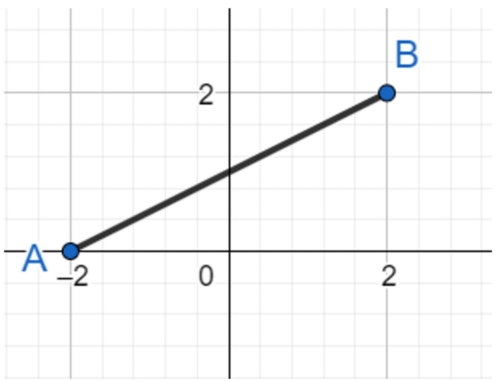
Write down the column vector $\overrightarrow{BA}$.



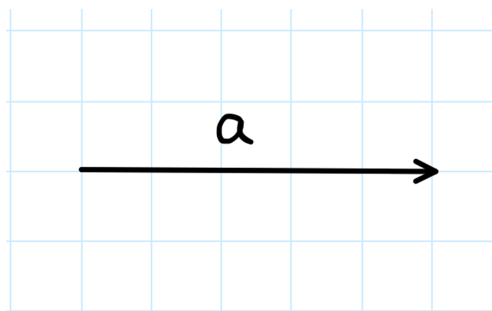
Write down the column vector **d**.



Write down the column vector $\overrightarrow{AB}$.



Write down the column vector **a**.



Fluency Practice

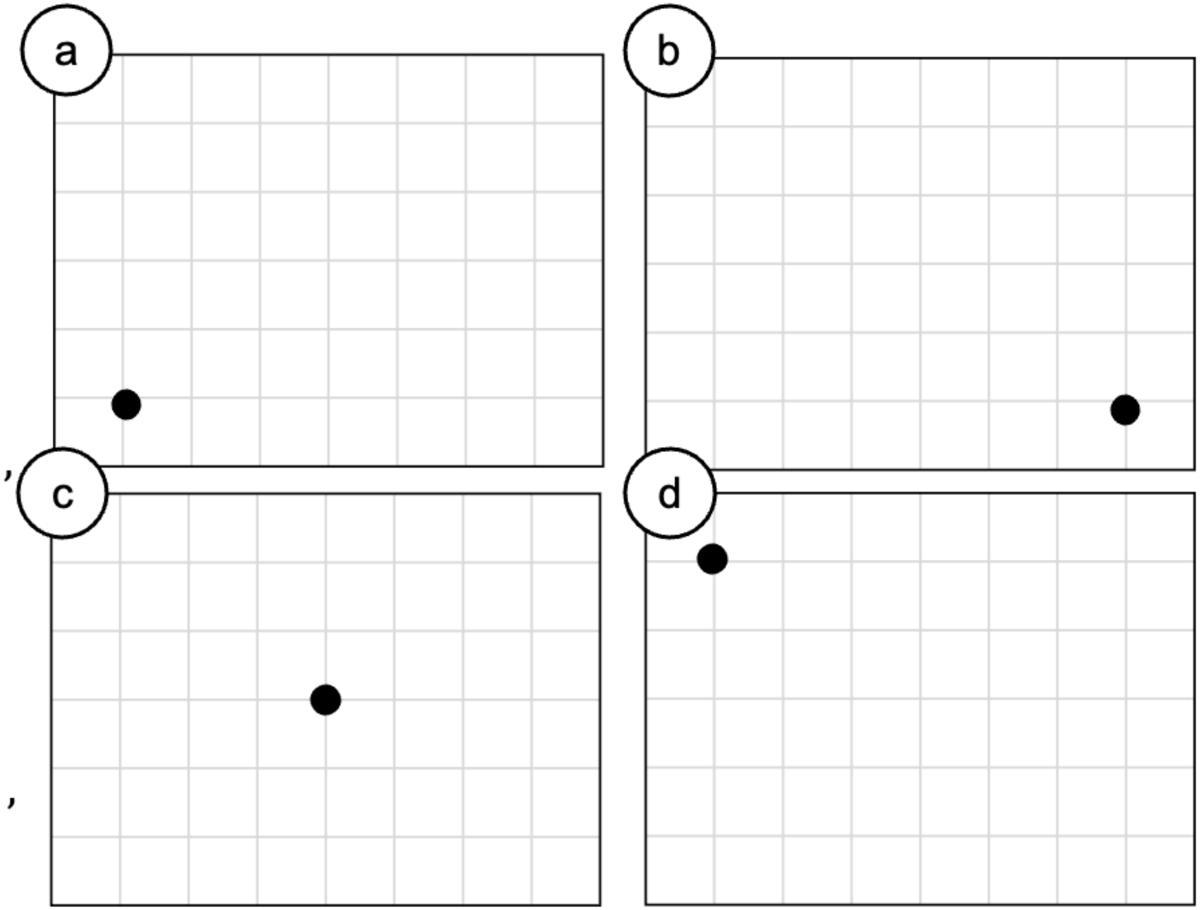
On each grid, start at the dot, then draw each vector in turn.

a) $\begin{pmatrix}0\\4\end{pmatrix}, \begin{pmatrix}3\\-2\end{pmatrix}, \begin{pmatrix}3\\2\end{pmatrix}, \begin{pmatrix}0\\-4\end{pmatrix}$

b) $\begin{pmatrix}-3\\4\end{pmatrix}, \begin{pmatrix}-3\\0\end{pmatrix}, \begin{pmatrix}0\\-4\end{pmatrix}, \begin{pmatrix}3\\0\end{pmatrix}, \begin{pmatrix}0\\2\end{pmatrix}, \begin{pmatrix}3\\-2\end{pmatrix}$

c) $\begin{pmatrix}-2\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\-1\end{pmatrix}, \begin{pmatrix}2\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\-1\end{pmatrix}, \begin{pmatrix}-2\\0\end{pmatrix}, \begin{pmatrix}0\\-1\end{pmatrix}, \begin{pmatrix}-2\\0\end{pmatrix}, \begin{pmatrix}4\\0\end{pmatrix}$

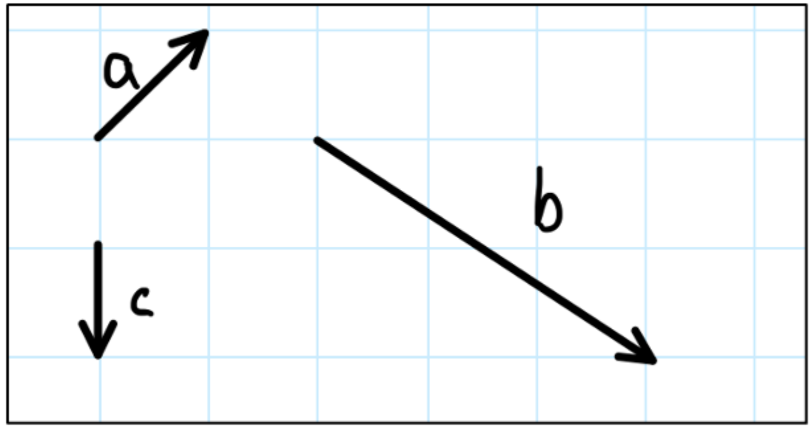
d) $\begin{pmatrix}0\\-2\end{pmatrix}, \begin{pmatrix}3\\2\end{pmatrix}, \begin{pmatrix}-3\\-4\end{pmatrix}, \begin{pmatrix}0\\6\end{pmatrix}, \begin{pmatrix}0\\4\end{pmatrix}, \begin{pmatrix}-1\\-2\end{pmatrix}, \begin{pmatrix}-2\\0\end{pmatrix}, \begin{pmatrix}0\\2\end{pmatrix}, \begin{pmatrix}-3\\0\end{pmatrix}$



Worked Example	Your Turn
$\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ Find 3**a**	$\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ Find -2**a**

Fluency Practice

Write each vector in column form



1) 2**a**	2) -4**a**	3) $\frac{1}{2}\boldsymbol{a}$	4) $\frac{3}{2}\boldsymbol{a}$	5) 2**c**
6) -2**c**	7) -**c**	8) -**b**	9) $-\frac{1}{2}\boldsymbol{b}$	10) $\frac{3}{2}\boldsymbol{b}$

Worked Example	Your Turn
$\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$ Find 2**a** - **b**	$\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$ Find 3**a + **2**b**

Extra Notes

4 Reflections, Rotations and Translations

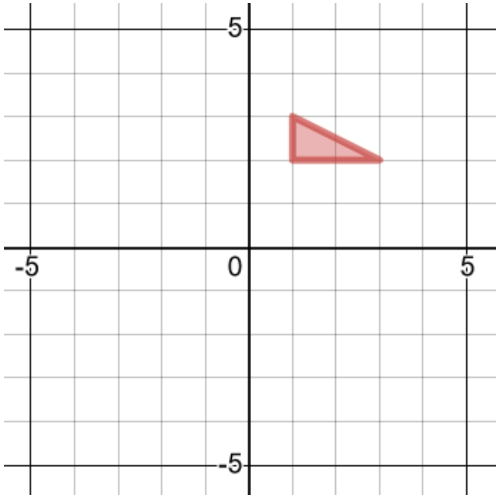
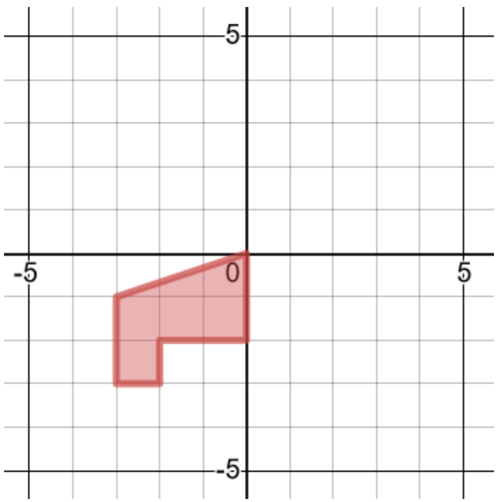
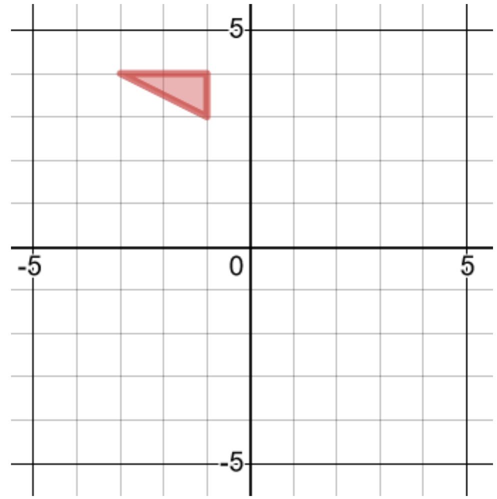
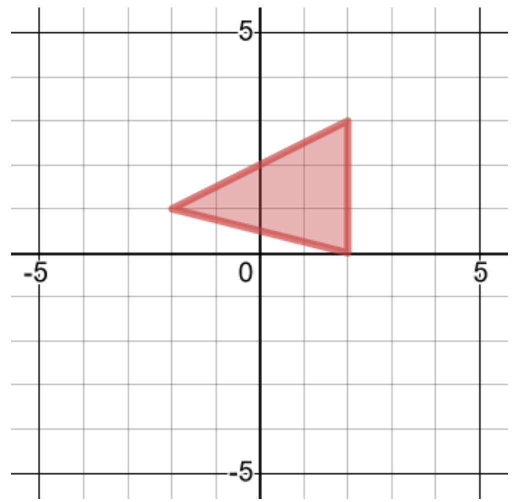
Reflections

A transformation that flips all points so that they are the same distance from a given mirror line as the original points, but in the opposite direction.

- Shapes flip over a mirror line.
- A shape and its reflection lie perfectly on top of each other if the page is folded in the mirror line.
- Produces a congruent shape.

To fully describe a reflection, you need to give two pieces of information:

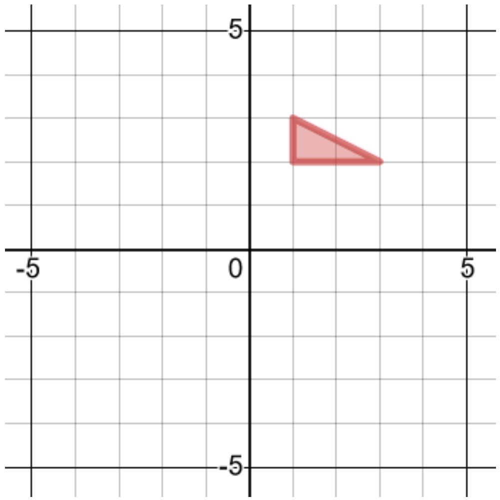
1. Type of Transformation: Reflection
2. The Line of Reflection:
    - $x-\text{axis}$ or $y-\text{axis}$
    - $y=\text{‘a number’}$ or $x=\text{‘a number’}$
    - $y=x$ or $y=-x$

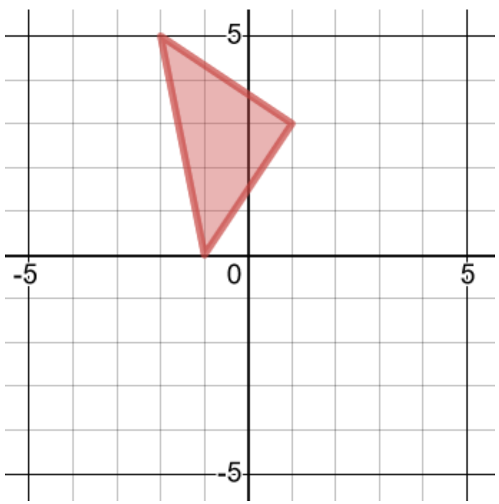
Worked Example	Your Turn
Reflect in the x-axis  	Reflect in the x-axis  

Worked Example	Your Turn
Reflect in the y-axis	Reflect in the y-axis

Worked Example

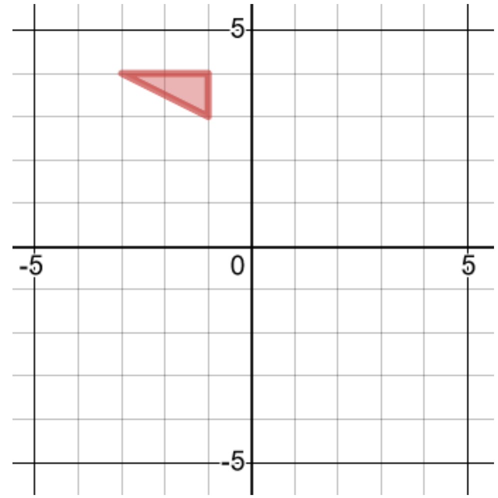
Reflect in the y-axis

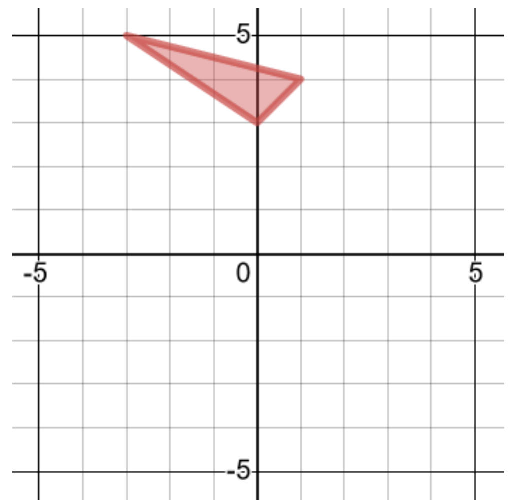




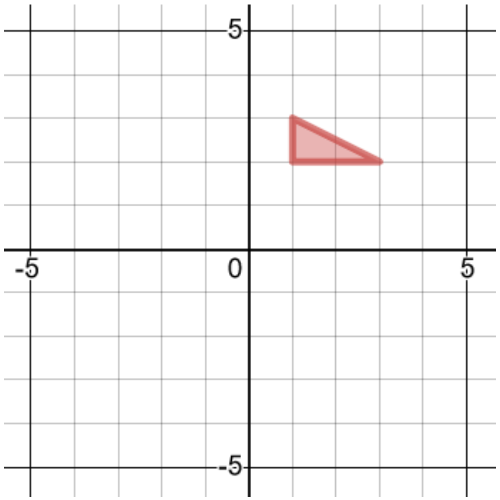
Your Turn

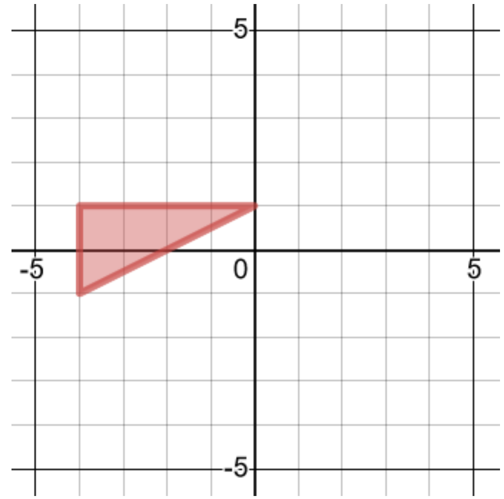
Reflect in the y-axis



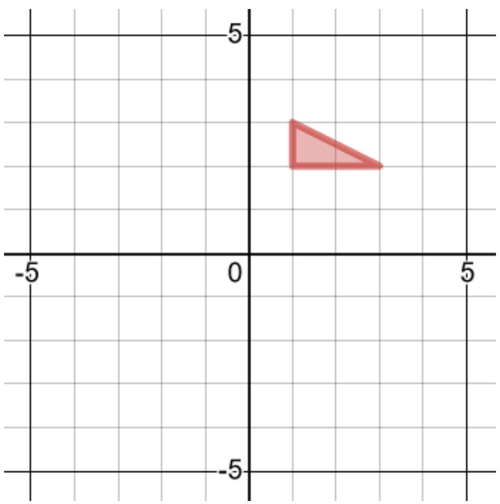


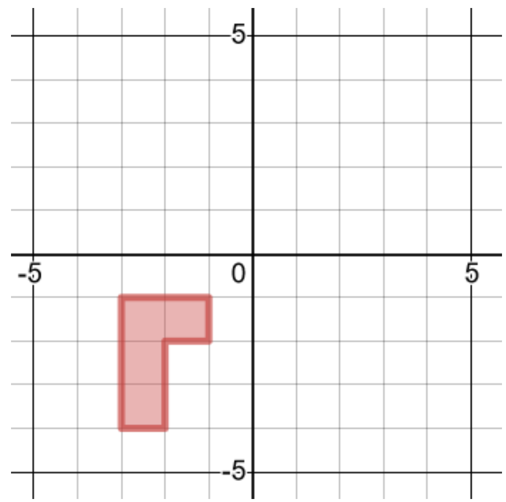
Worked Example	Your Turn
Reflect in the line $y = 1$	Reflect in the line $y = 2$

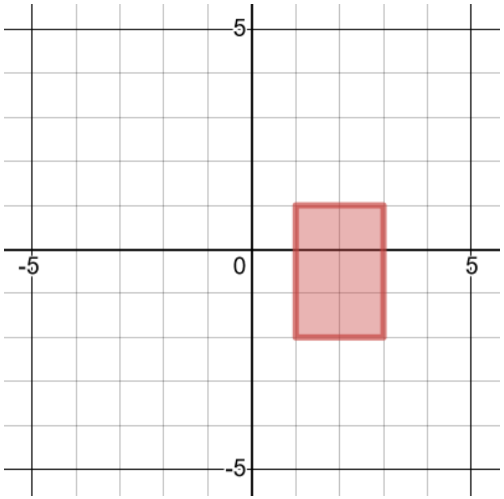
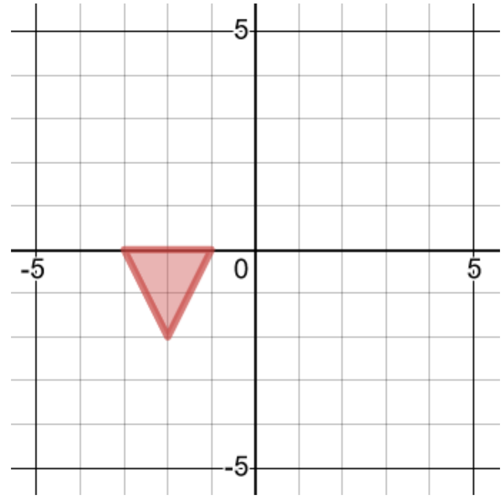
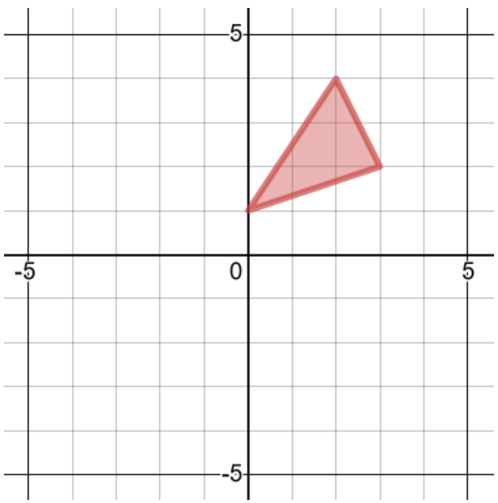
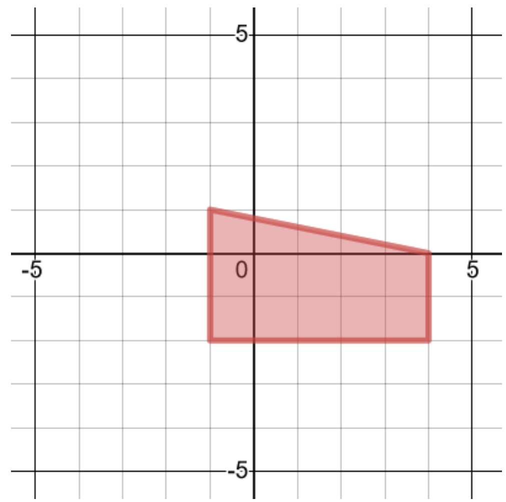


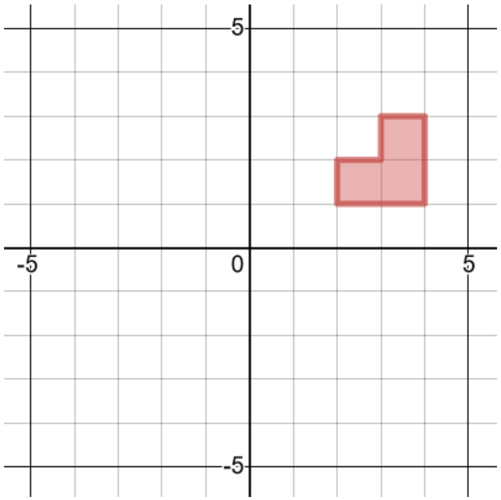
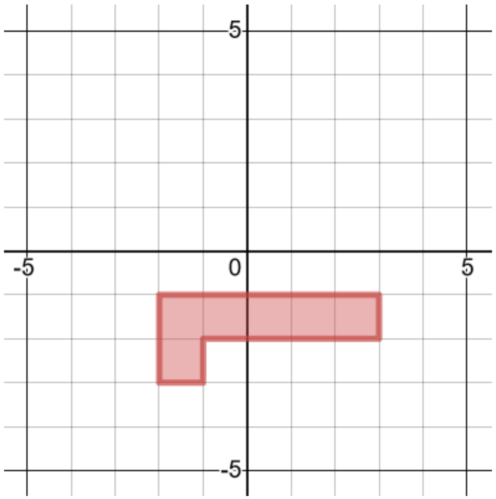
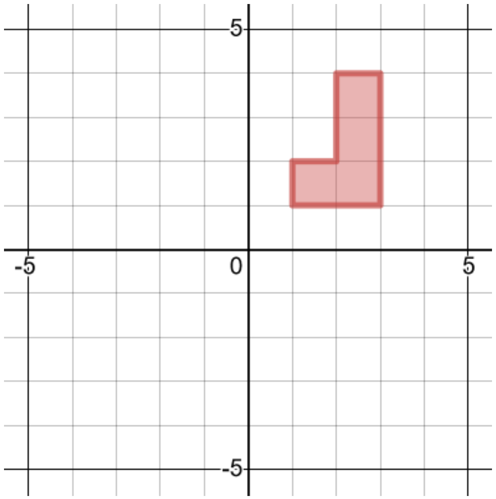
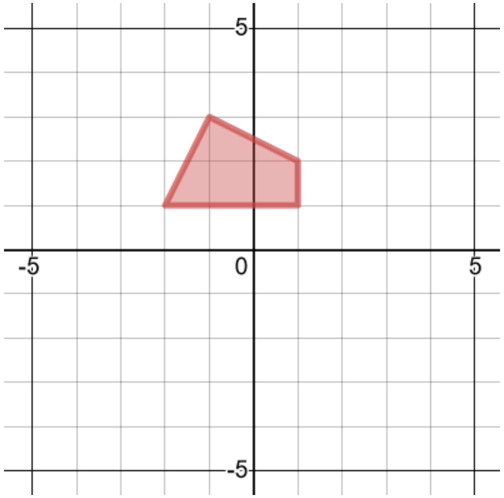


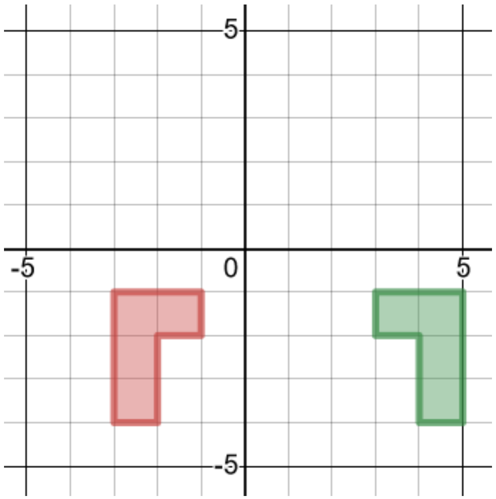
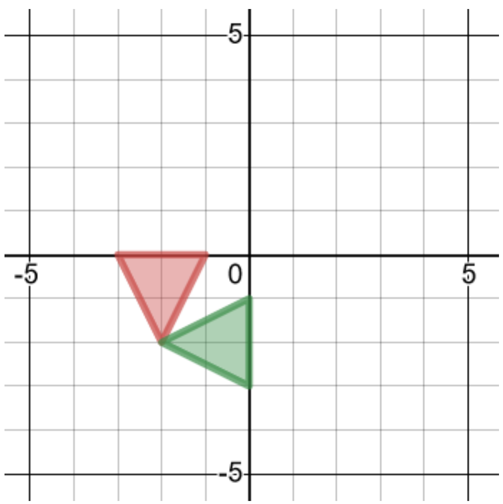
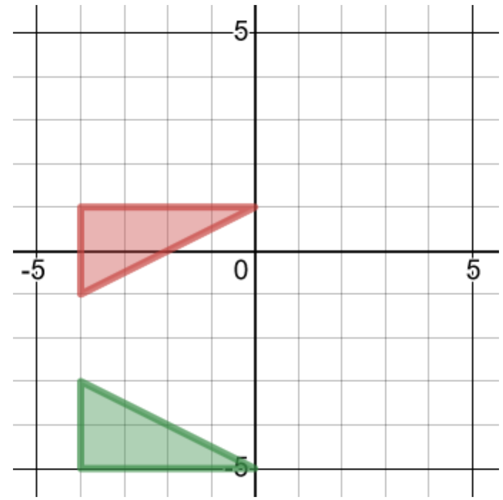
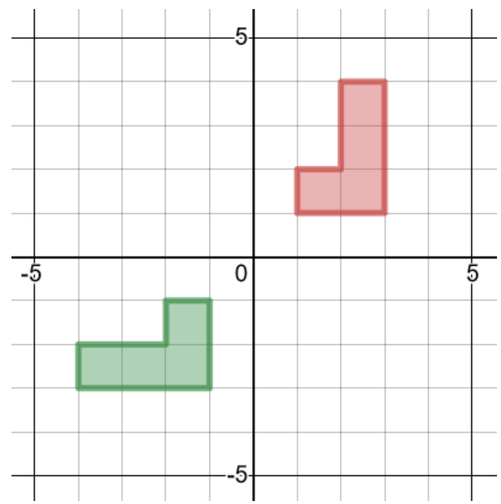
Reflect in the line $x = 3$	Reflect in the line $x = 1$





Worked Example	Your Turn
Reflect in the line $y = x$	Reflect in the line $y = x$
	
	

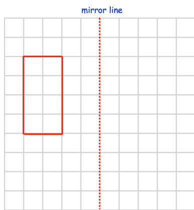
Worked Example	Your Turn
Reflect in the line $y = -x$  	Reflect in the line $y = -x$  

Worked Example	Your Turn
Describe the single transformation of the red object onto the green image   	Describe the single transformation of the red object onto the green image  

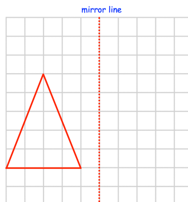
Fluency Practice

Question 1: Reflect each shape in the mirror line given

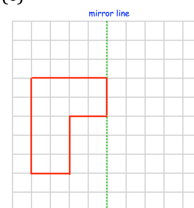
(a)



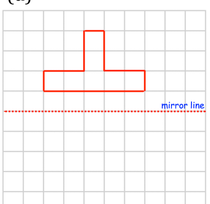
(b)



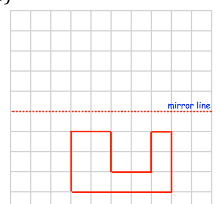
(c)



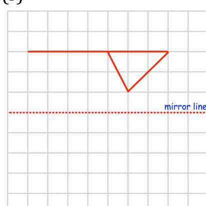
(d)



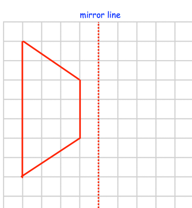
(e)



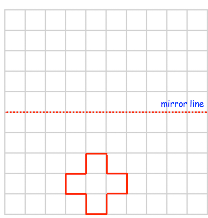
(f)



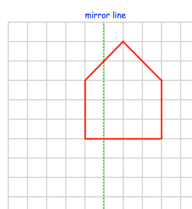
(g)



(h)

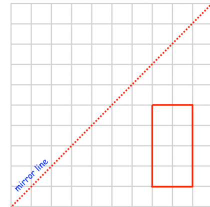


(i)



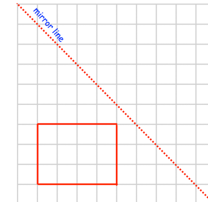
Question 2: Reflect each shape in the mirror line given

(a)

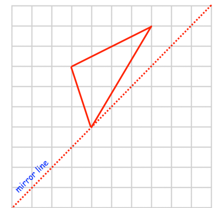


(b)

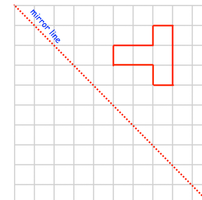
(c)



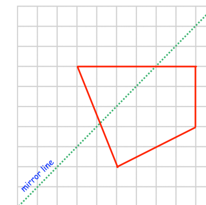
(d)



(e)

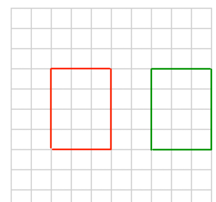


(f)

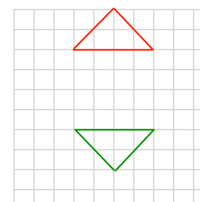


Question 3: Find the mirror line for each of the reflections below.

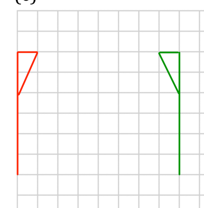
(a)



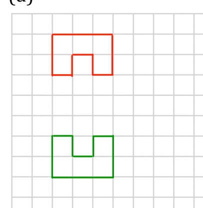
(b)



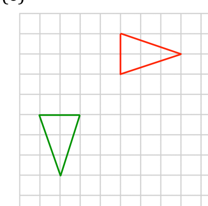
(c)



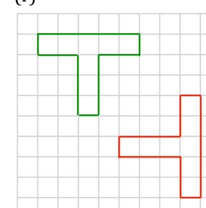
(d)



(e)



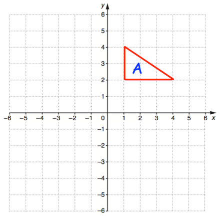
(f)



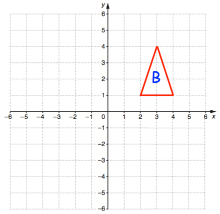
Fluency Practice

Question 4:

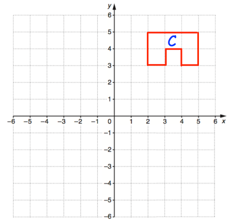
(a) Reflect triangle A in the x-axis



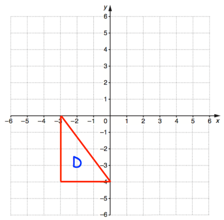
(b) Reflect triangle B in the y-axis



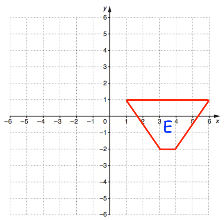
(c) Reflect shape C in the x-axis



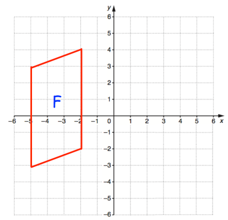
(d) Reflect shape D in the y-axis



(e) Reflect shape E in the y-axis

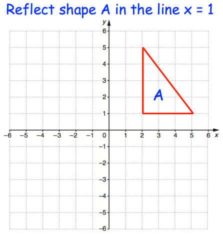


(f) Reflect shape F in the x-axis

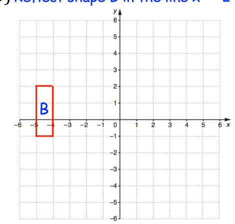


Question 5:

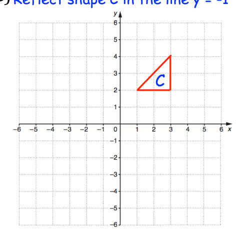
(a) Reflect shape A in the line x = 1



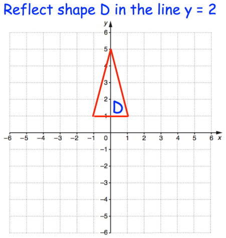
(b) Reflect shape B in the line x = -2



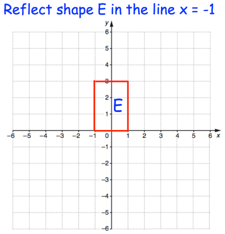
(c) Reflect shape C in the line y = -1



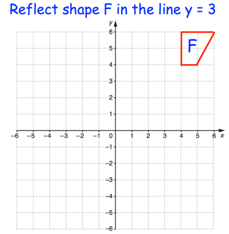
(d) Reflect shape D in the line y = 2



(e) Reflect shape E in the line x = -1

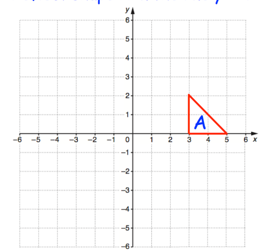


(f) Reflect shape F in the line y = 3

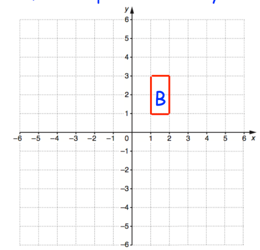


Question 6:

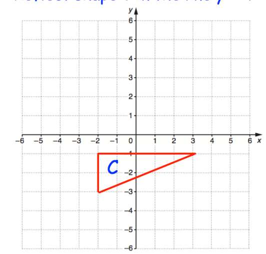
(a) Reflect shape A in the line y = x



(b) Reflect shape B in the line y = -x

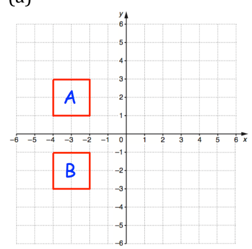


(c) Reflect shape C in the line y = x

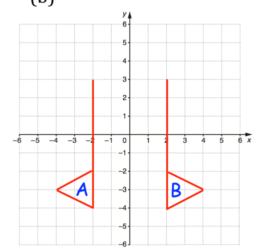


Question 7: Describe fully the single transformation that takes shape A to shape B.

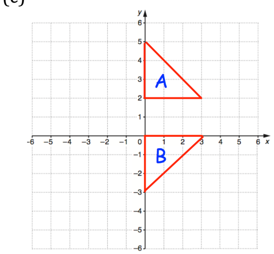
(a)



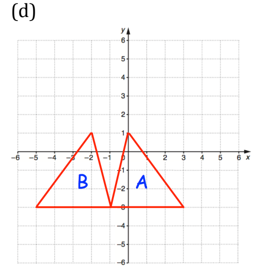
(b)



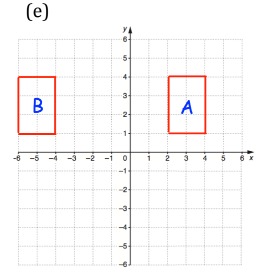
(c)



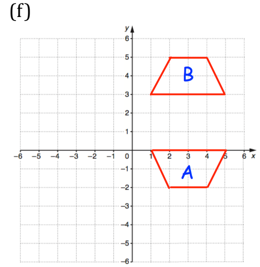
(d)



(e)

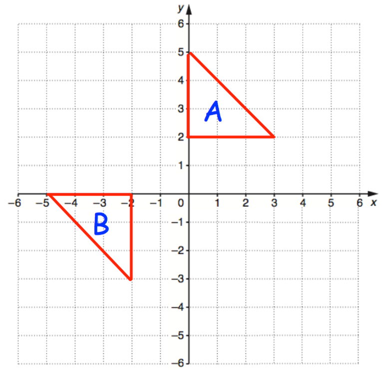


(f)

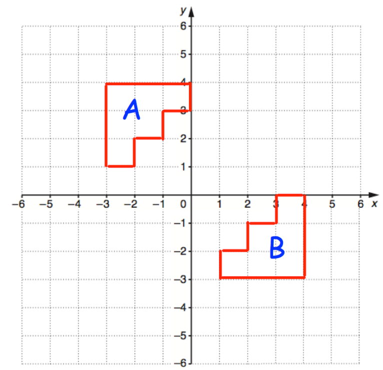


Fluency Practice

(g)



(h)



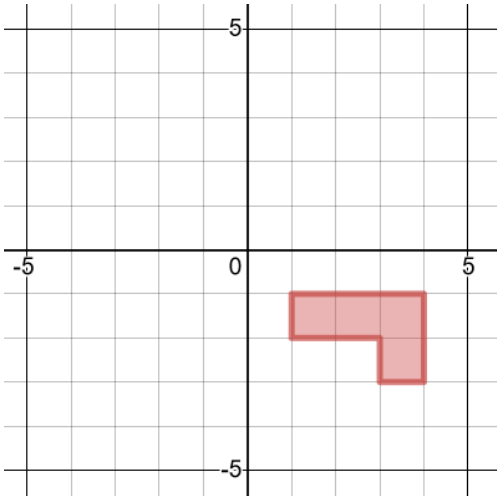
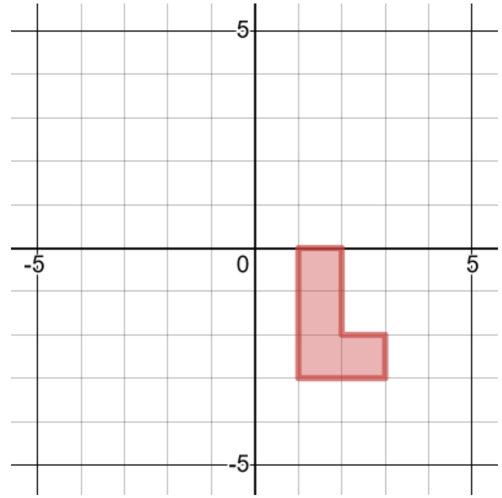
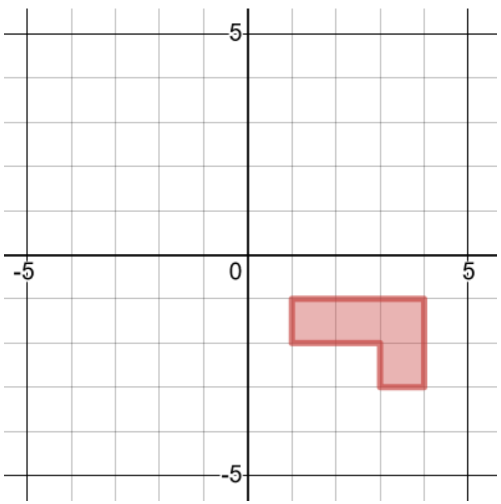
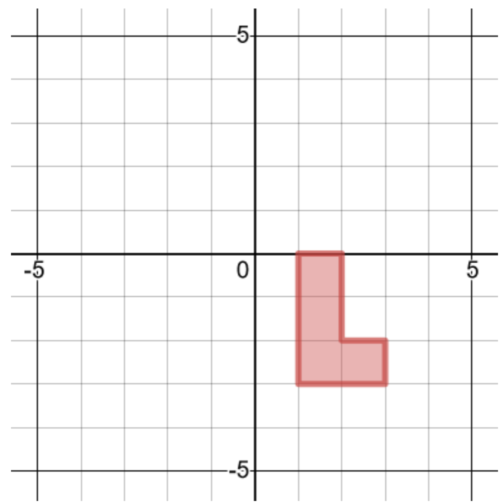
Rotations

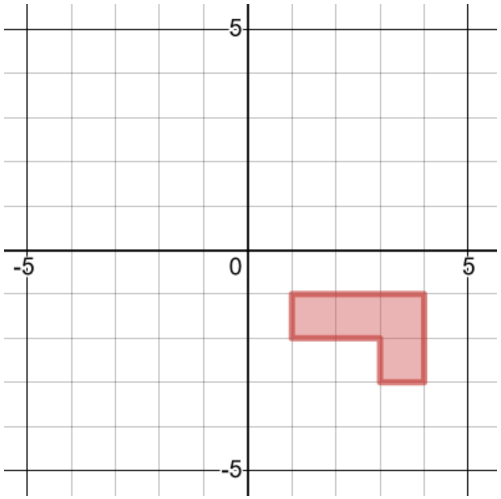
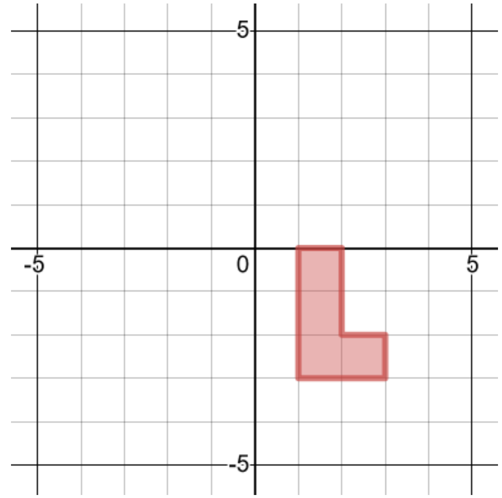
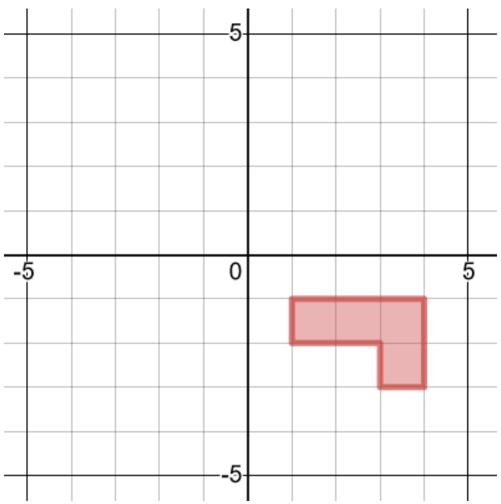
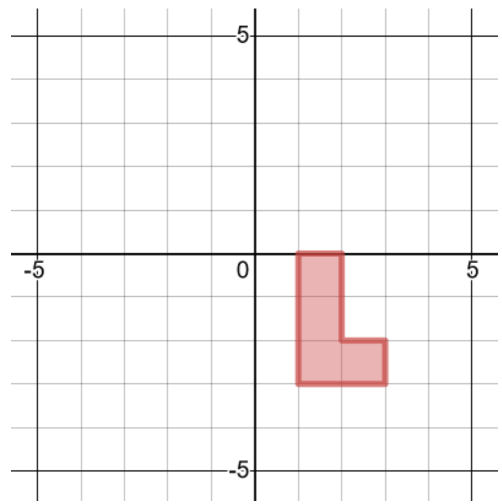
A transformation that turns all points through a given angle, in a given direction, around a given centre.

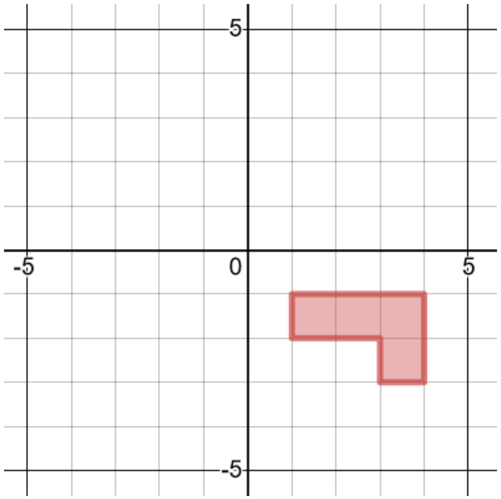
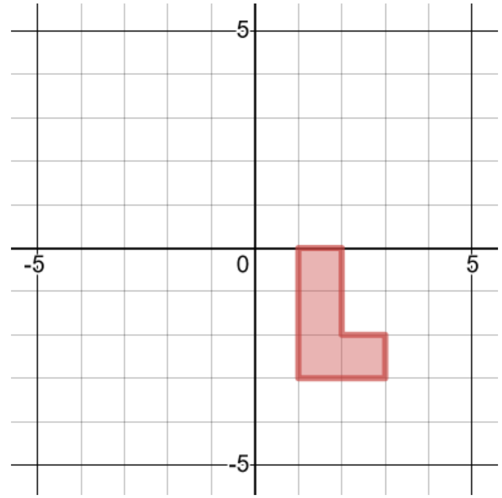
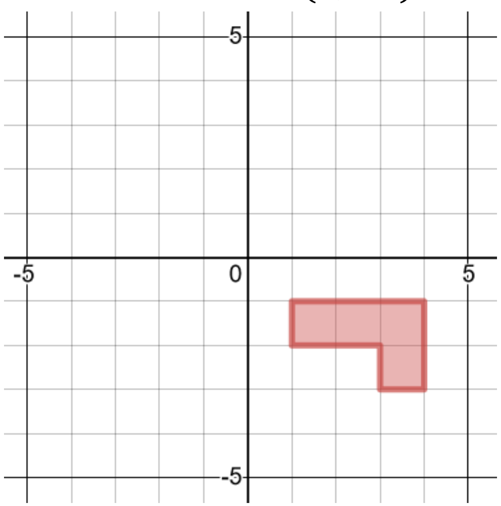
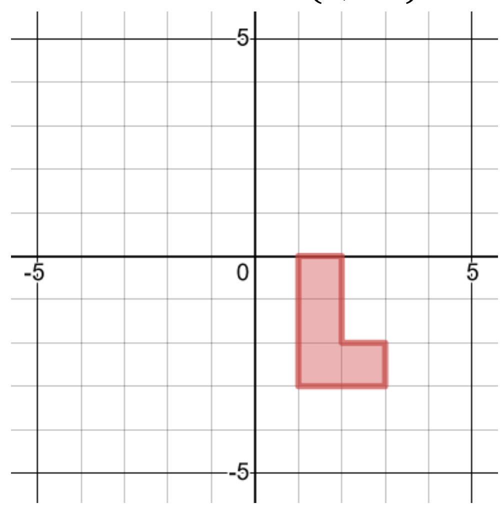
- Shapes turn around a centre point.
- Produces a congruent shape.

To fully describe a rotation, you need to give four pieces of information:

1. Type of Transformation: Rotation
2. Angle (in degrees): $90°, 180°, 270°$
3. Direction: Clockwise or Anticlockwise
4. Centre of Rotation: Coordinate (x, y)

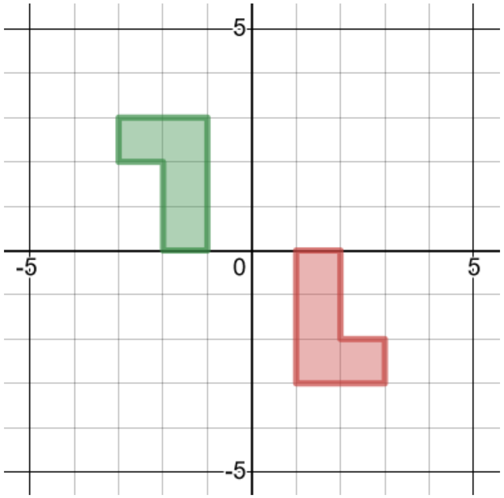
Worked Example	Your Turn
Rotate 90° clockwise about the origin 	Rotate 90° clockwise about the origin 
Rotate 90° anticlockwise about the origin 	Rotate 90° anticlockwise about the origin 

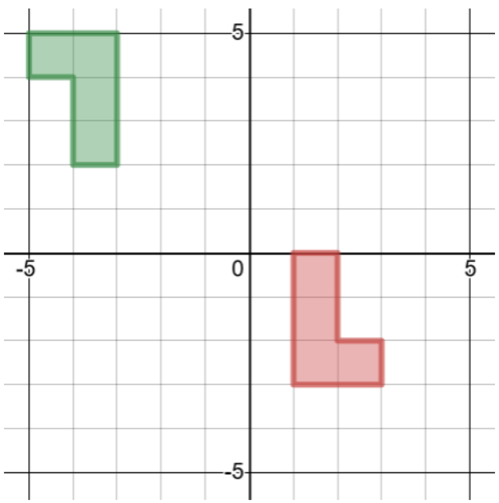
Worked Example	**Your Turn**
Rotate $90°$ clockwise about $(1, -1)$ 	Rotate $90°$ clockwise about $(1, -1)$ 
Rotate $90°$ anticlockwise about $(1, -1)$ 	Rotate $90°$ anticlockwise about $(1, -1)$ 

<table>
<tr><th>Worked Example</th><th>Your Turn</th></tr>
<tr><td>Rotate $180°$ about the origin
</td><td>Rotate $180°$ about the origin
</td></tr>
<tr><td>Rotate $180°$ about $(1, -1)$
</td><td>Rotate $180°$ about $(1, -1)$
</td></tr>
</table>

Worked Example

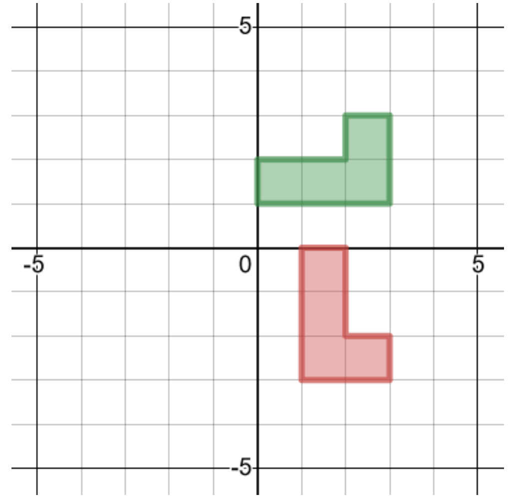
Describe the single transformation of the red object onto the green image.

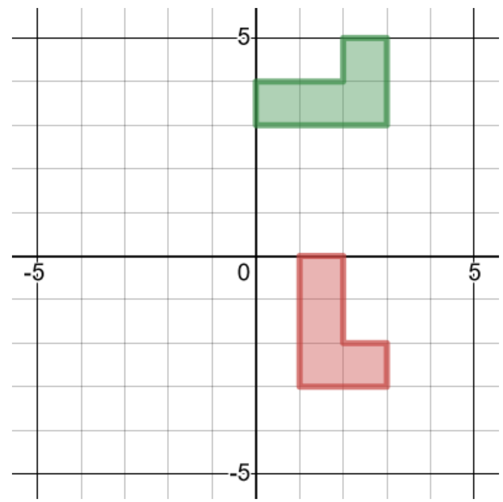




Your Turn

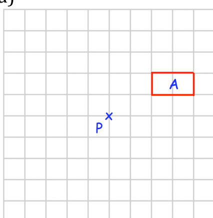
Describe the single transformation of the red object onto the green image.





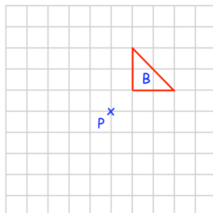
Fluency Practice

(a)



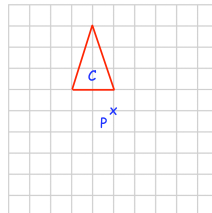
rotate 90° clockwise about P

(b)



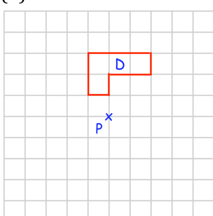
rotate 90° anticlockwise about P

(c)



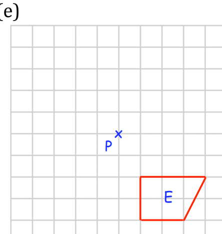
rotate 90° clockwise about P

(d)



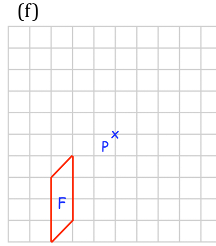
rotate 180° about P

(e)



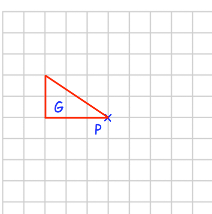
rotate 90° anticlockwise about P

(f)



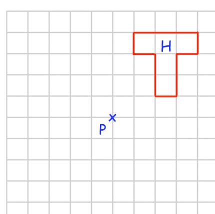
rotate 180° about P

(g)



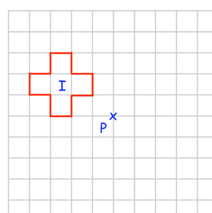
rotate 90° clockwise about P

(h)



rotate 270° clockwise about P

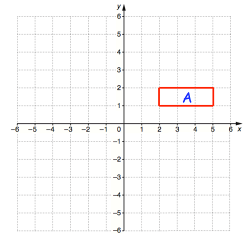
(i)



rotate 270° anticlockwise about P

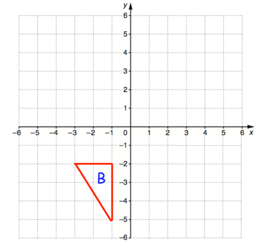
Question 2: Rotate each of the shapes below as instructed, using the origin, (0,0), as the centre of rotation.

(a)



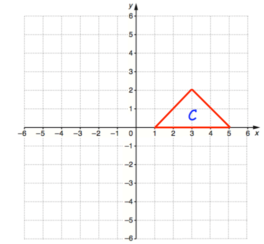
rotate 180° about (0, 0)

(b)



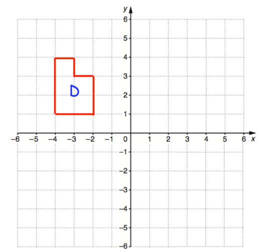
rotate 90° clockwise about (0, 0)

(c)



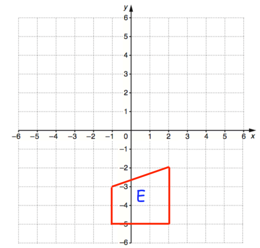
rotate 90° anticlockwise about (0, 0)

(d)



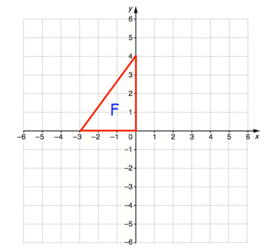
rotate 90° clockwise about (0, 0)

(e)



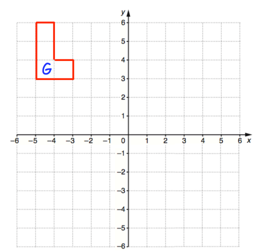
rotate 90° anticlockwise about (0, 0)

(f)



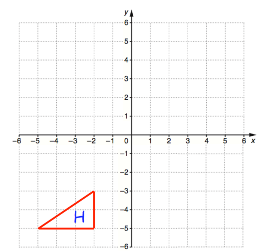
rotate 180° about (0, 0)

(g)



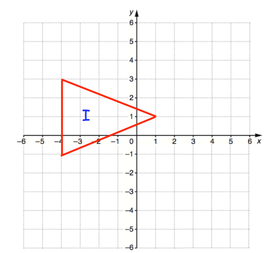
rotate 90° anticlockwise about (0, 0)

(h)



rotate 180° about (0, 0)

(i)

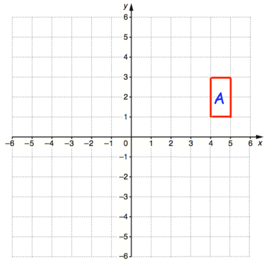


rotate 90° clockwise about (0, 0)

Fluency Practice

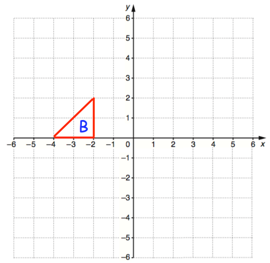
Question 3: Rotate each of the shapes below as instructed.

(a)



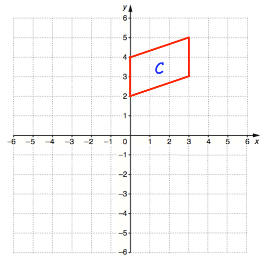
rotate 90° anticlockwise about (0, 1)

(b)



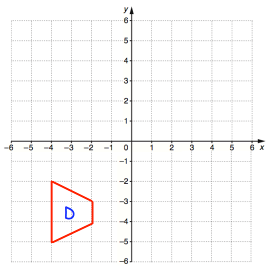
rotate 90° clockwise about (-1, -2)

(c)



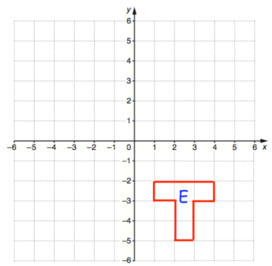
rotate 180° about (1, 1)

(d)



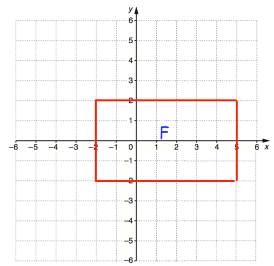
rotate 90° anticlockwise about (-4, 0)

(e)



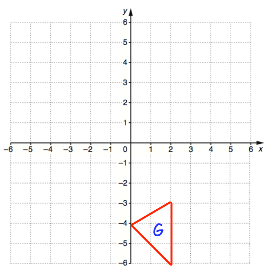
rotate 180° about (-1, 0)

(f)



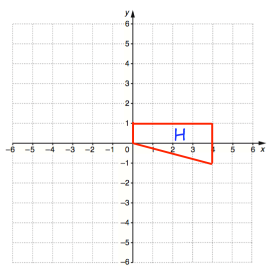
rotate 90° clockwise about (-1, 2)

(g)



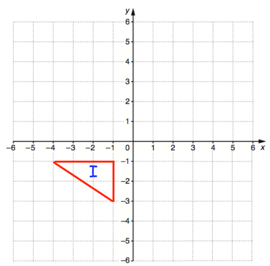
rotate 90° clockwise about (5, 0)

(h)



rotate 90° anticlockwise about (3, 0)

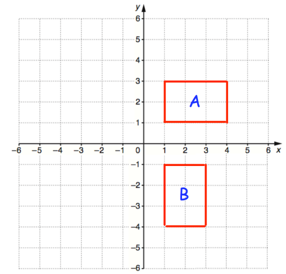
(i)



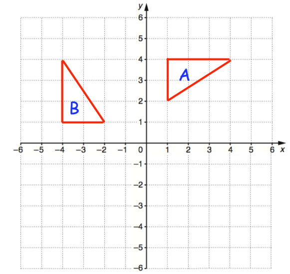
rotate 180° about (1, 1)

Question 4: Describe fully the single transformation that takes shape A to shape B.

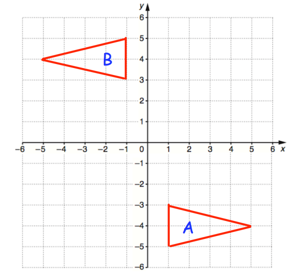
(a)



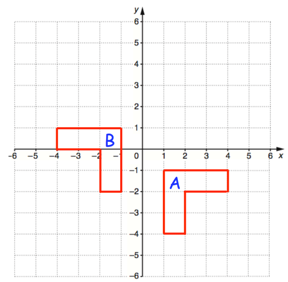
(b)



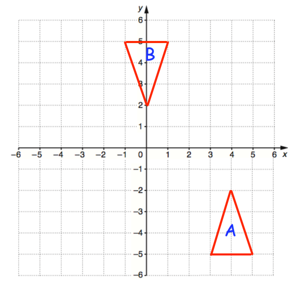
(c)



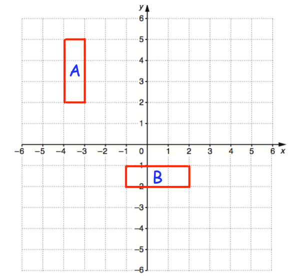
(d)



(e)



(f)



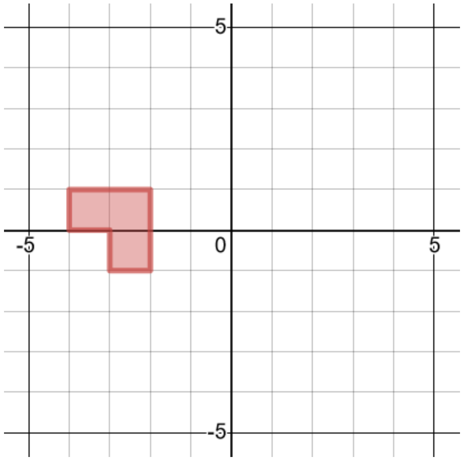
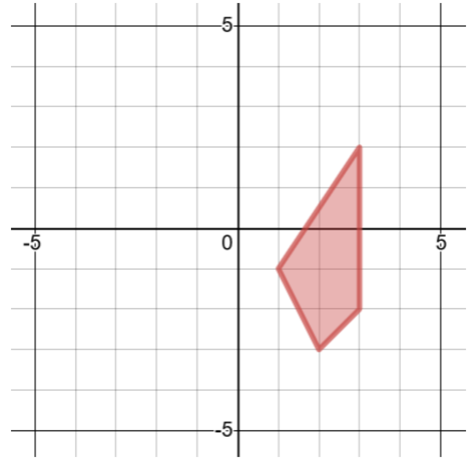
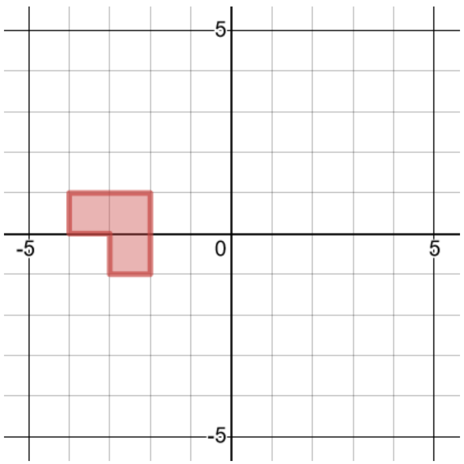
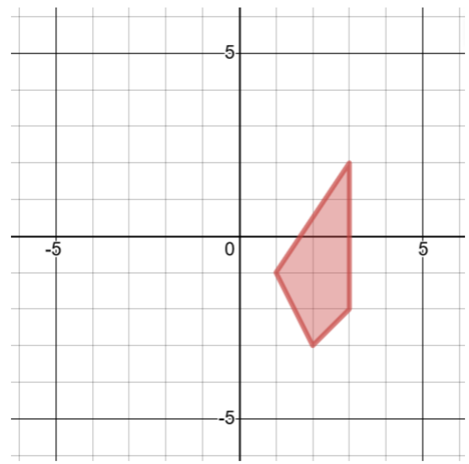
Translations

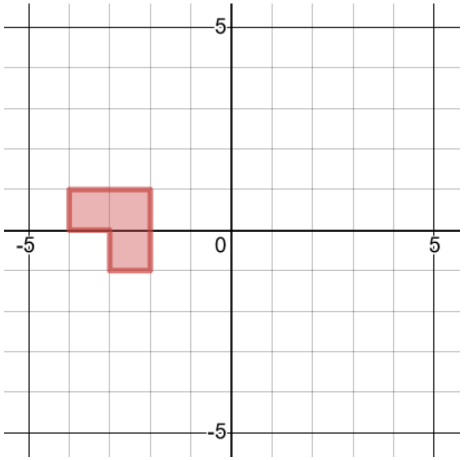
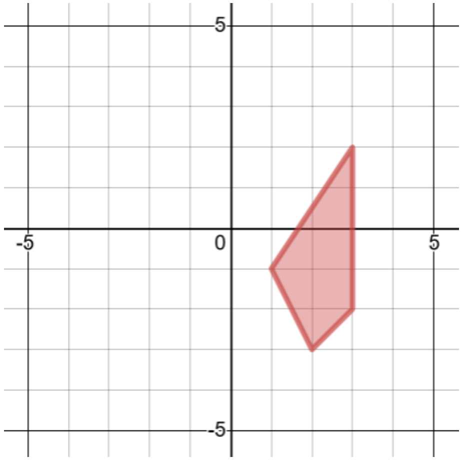
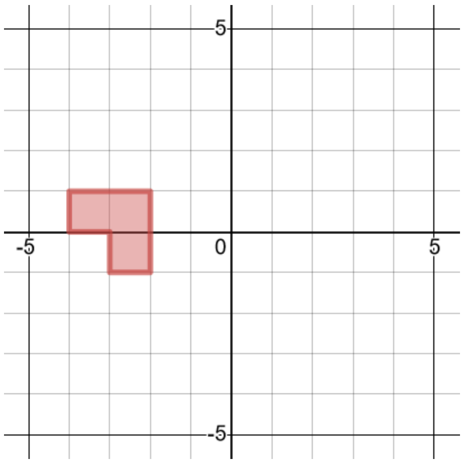
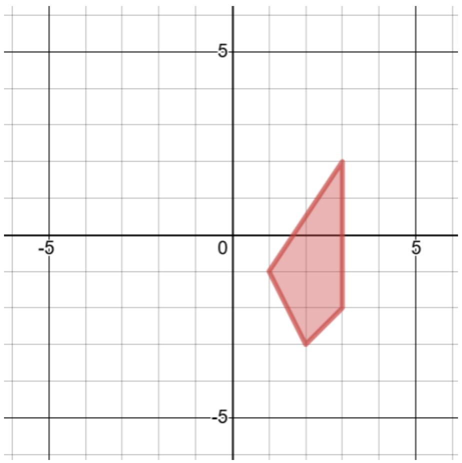
A transformation that moves all points the same fixed distance.

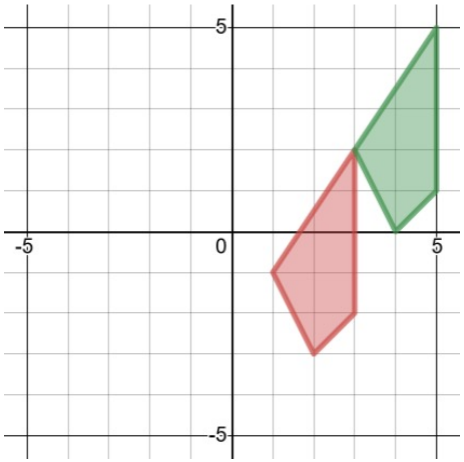
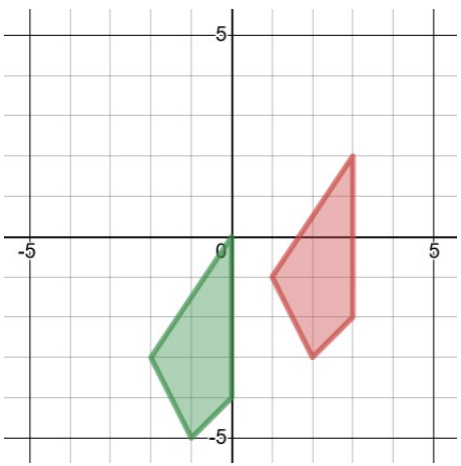
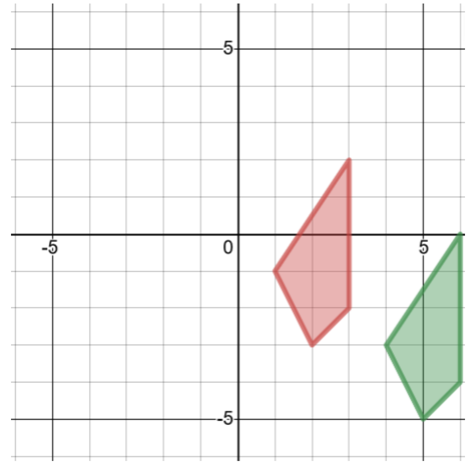
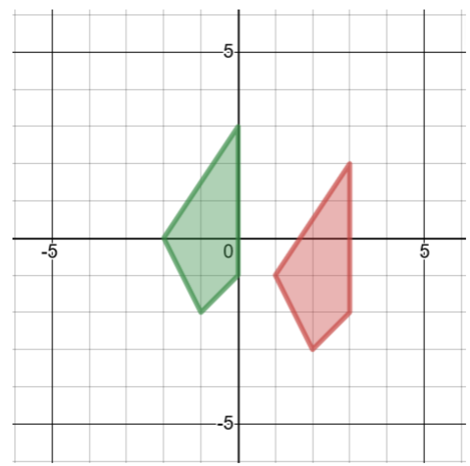
- Shapes move or “slide” a distance horizontally and/or vertically.
- On a rectangular grid, often described using a column vector.

To fully describe a translation, you need to give two pieces of information:

1. Type of Transformation: Translation
2. Column Vector: $\begin{pmatrix} x \\ y \end{pmatrix}$ where x is movement right or left and y is movement up or down. Right and up are taken to be positive.

Worked Example	Your Turn
Translate by vector $\binom{3}{2}$ 	Translate by vector $\binom{2}{3}$ 
Translate by vector $\binom{2}{-3}$ 	Translate by vector $\binom{3}{-2}$ 

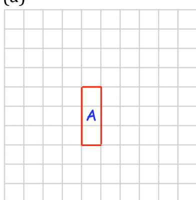
Worked Example	Your Turn
Translate by vector $\begin{pmatrix}-2\\-3\end{pmatrix}$ 	Translate by vector $\begin{pmatrix}-3\\-2\end{pmatrix}$ 
Translate by vector $\begin{pmatrix}3\\-2\end{pmatrix}$ and then by vector $\begin{pmatrix}-4\\5\end{pmatrix}$ 	Translate by vector $\begin{pmatrix}2\\-3\end{pmatrix}$ and then by vector $\begin{pmatrix}-5\\4\end{pmatrix}$ 

Worked Example	Your Turn
Describe the single transformation of the red object onto the green image.  	Describe the single transformation of the red object onto the green image.  

Fluency Practice

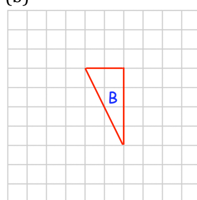
Question 1: Translate each of the shapes below as instructed.

(a)



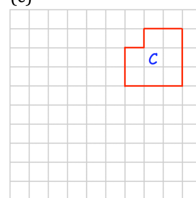
Translate A by $\begin{pmatrix}3\\1\end{pmatrix}$

(b)



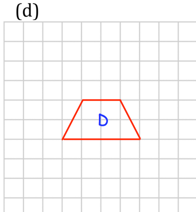
Translate B by $\begin{pmatrix}2\\-2\end{pmatrix}$

(c)



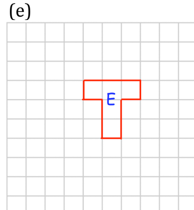
Translate C by $\begin{pmatrix}0\\-5\end{pmatrix}$

(d)



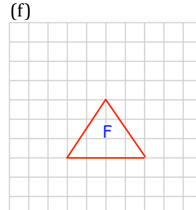
Translate D by $\begin{pmatrix}-3\\3\end{pmatrix}$

(e)



Translate E by $\begin{pmatrix}-2\\-4\end{pmatrix}$

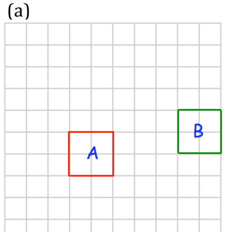
(f)



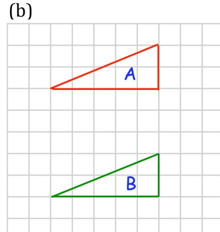
Translate F by $\begin{pmatrix}1.5\\0\end{pmatrix}$

Question 2: Describe fully each translation that takes shape A to shape B

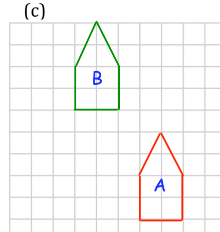
(a)



(b)

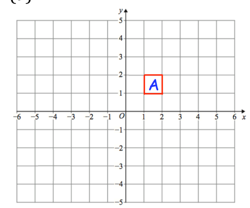


(c)



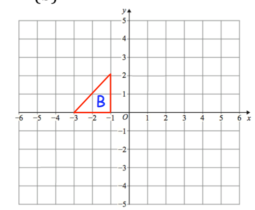
Question 3: Translate each of the shapes below as instructed.

(a)



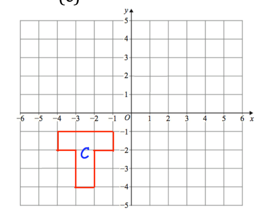
Translate A by $\begin{pmatrix}2\\-3\end{pmatrix}$

(b)



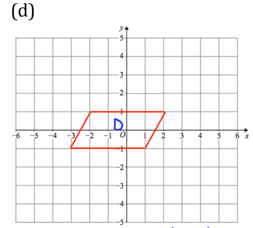
Translate B by $\begin{pmatrix}6\\1\end{pmatrix}$

(c)



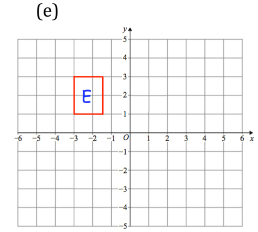
Translate C by $\begin{pmatrix}-1\\5\end{pmatrix}$

(d)



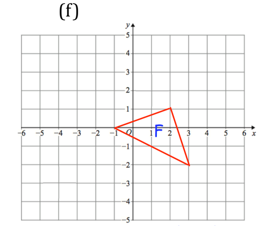
Translate D by $\begin{pmatrix}-3\\-2\end{pmatrix}$

(e)



Translate E by $\begin{pmatrix}4.5\\-4\end{pmatrix}$

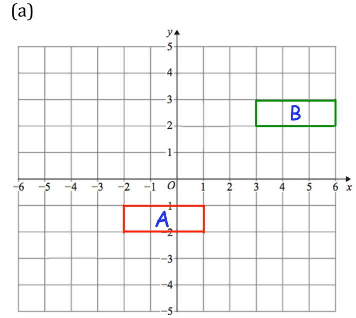
(f)



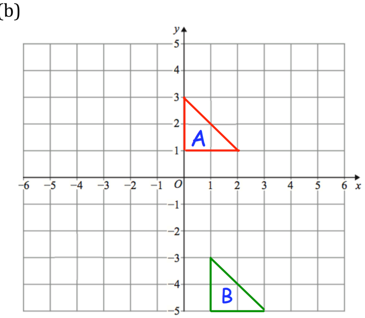
Translate F by $\begin{pmatrix}-1\\1\end{pmatrix}$

Question 4: Describe fully the single transformation that takes shape A to shape B

(a)

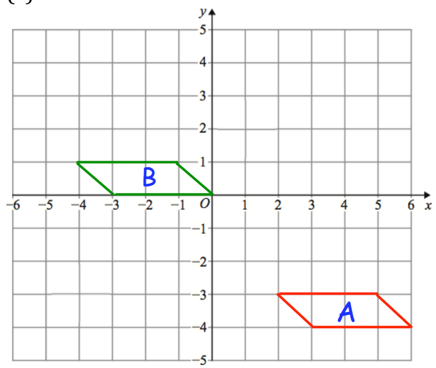


(b)

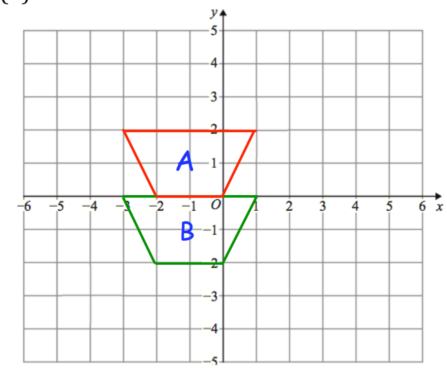


Fluency Practice

(c)



(d)

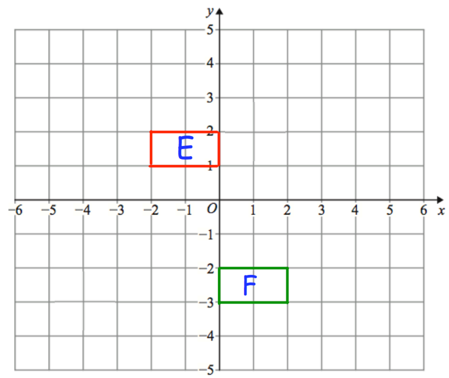


Question 5: The translation vector to take shape C to shape D is $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$

What translation vector takes shape D to shape C?

Question 6: Edward has been asked to translate shape E by $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

He has labelled his answer shape F
Can you spot any mistakes?

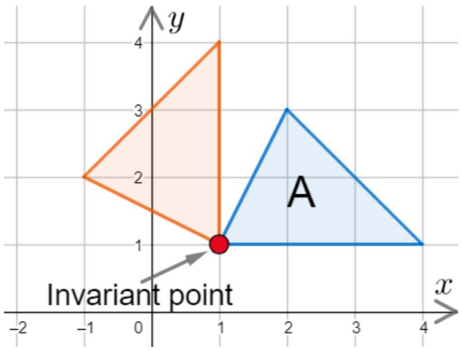


Extra Notes

5 Invariant Points

If something is invariant, then that means it does not change. In terms of transformations, **an invariant point is any point on the shape that hasn't moved after the transformation has been done**.

For example, when we rotated shape A, the bottom left corner of it did not move. As a result, the bottom left corner is an invariant point.

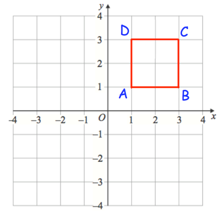


Fluency Practice

Question 1: ABCD is a square.

(a) Translate ABCD using vector $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$

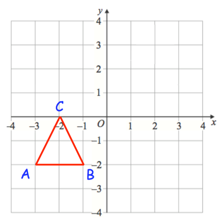
(b) Are there any invariant points?
If so, which point(s) are invariant?



Question 2: ABC is an isosceles triangle.

(a) Reflect ABC in the x-axis

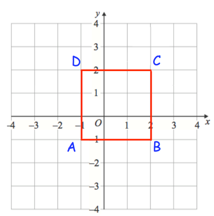
(b) Are there any invariant points?
If so, which point(s) are invariant?



Question 4: ABCD is a square

(a) Reflect ABCD in the line $y = x$

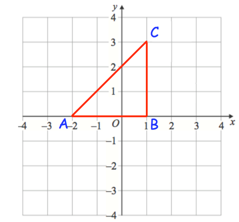
(b) Are there any invariant points?
If so, which point(s) are invariant?



Question 6: ABC is a triangle.

(a) Rotate ABC 90° clockwise about $(1, 0)$

(b) Are there any invariant points?
If so, which point(s) are invariant?



Question 7: A sketch of triangle ABC is shown

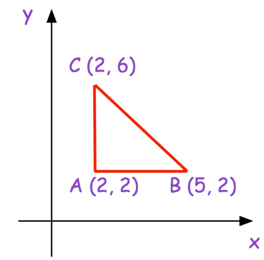
For each transformation below, write down the letter(s) of any vertices that are invariant.

(a) Rotation 180° about the point A

(c) Reflection in the line $x = 5$

(d) Reflection in the line $y = x$

(e) Reflection in the line $y = 2$



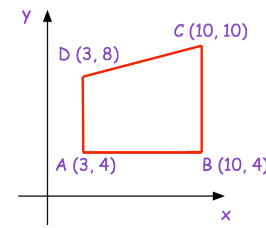
Question 8: A sketch of quadrilateral ABCD is shown.

For each transformation below, write down the letter(s) of any vertices that are invariant.

(a) Reflection in the line $y = 8$

(c) Reflection in the line $x = 3$

(d) Reflection in the line $y = x$



Fluency Practice

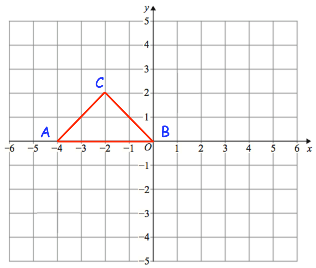
Question 1: ABC is a triangle.

Describe fully a **single** transformation of ABC so that:

(a) None of the vertices are invariant.

(b) Exactly one vertex is invariant.

(c) Exactly two vertices are invariant.



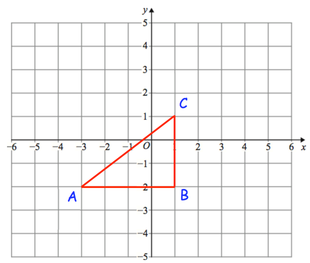
Question 2: Here is triangle ABC

Olivia says "if ABC is reflected in the line $x = -3$ there is one invariant point."

Amelia says "if ABC is reflected in the line $y = -2$ there are two invariant points."

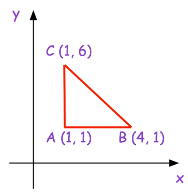
Isla says "if ABC is reflected in the line $x = 1$ there are two vertices that are invariant."

Which student is incorrect? Explain your answer.



Question 3: Here is a sketch of triangle ABC.

Describe fully a **single** transformation of ABC so that all the points on AC are invariant and the point B is not invariant.



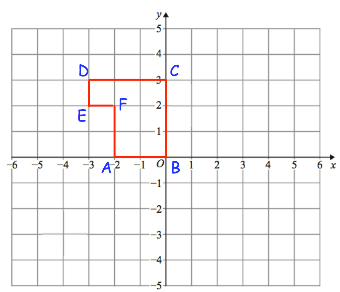
Question 4: Here is shape ABCDEF

Describe fully **single** transformations so that from the six vertices:

(a) only vertices B and C are invariant.

(b) only vertex F is invariant.

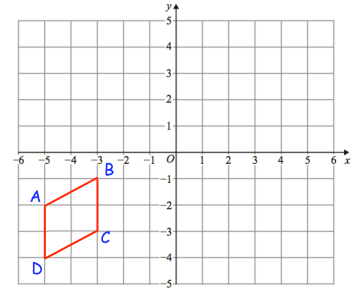
(c) only vertices B, D and F are invariant.



Question 5: Here is quadrilateral ABCD

ABCD is reflected in the line $x = -1$
followed by a reflection in the line $y = -x$
followed by a rotation of 180° about $(-1, -1)$

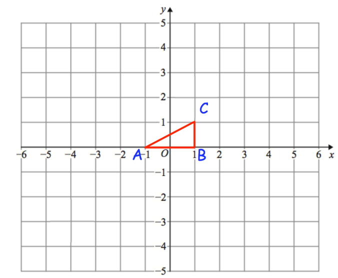
Which of the vertices are invariant?



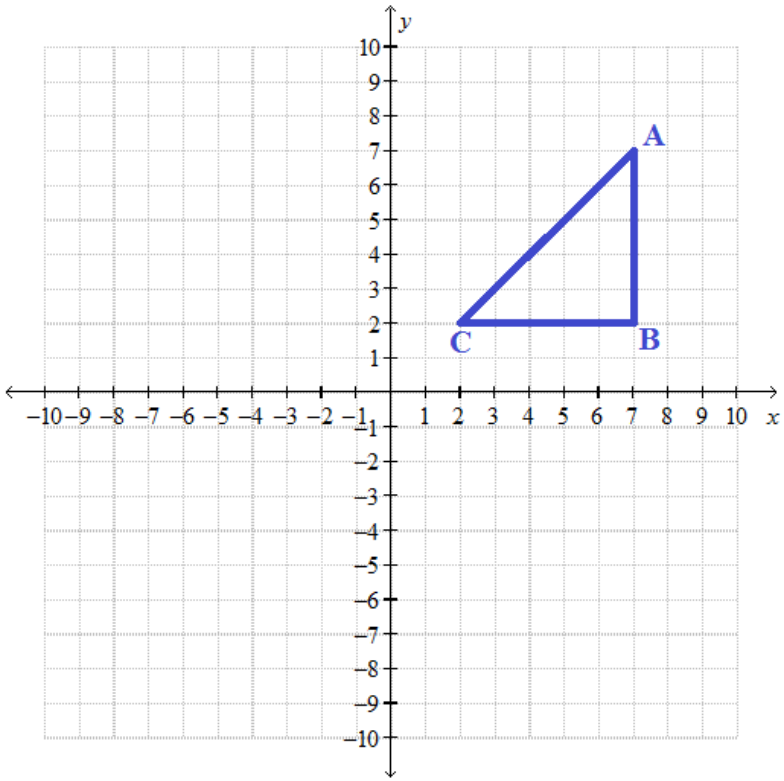
Question 6: Shown is triangle ABC

ABC is rotated 180° about $(-1, 2)$ and then translated by the vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

Write down the coordinate of the invariant point.



Fluency Practice



Red

Complete these sentences:

a) When triangle ABC is reflected in the line y = 2 the invariant points are ……… & …………

b) When triangle ABC is rotated using centre (7, 2), the invariant point is ……………………

c) When triangle ABC is reflected in the line y = x, the invariant points are ………. & …………..

d) When triangle ABC is reflected in the line x + y = 9, the invariant point is ……………………….

e) When triangle ABC is…………………………………………………… ………the invariant points are A and B.

f) When triangle ABC reflected in the line …………………………………… the only invariant point is C.

Amber

Match the transformation to the invariant points for the triangle ABC

(a) Reflection in the line y = 7

(b) Reflection in the line y = x - 5   A

(c) Rotation around the centre (7, 7)

(d) Reflection in the line x + y = 4   B

(e) Reflection in the line y = x

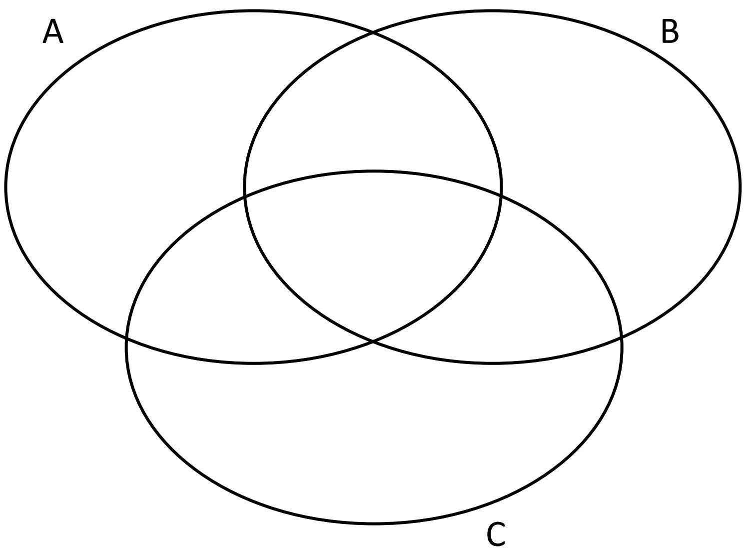
(f) Reflection in the line x = 7   C

(g) Reflection in the line y = 2x - 2

(h) Reflection in the line y = ½x + 1

Green

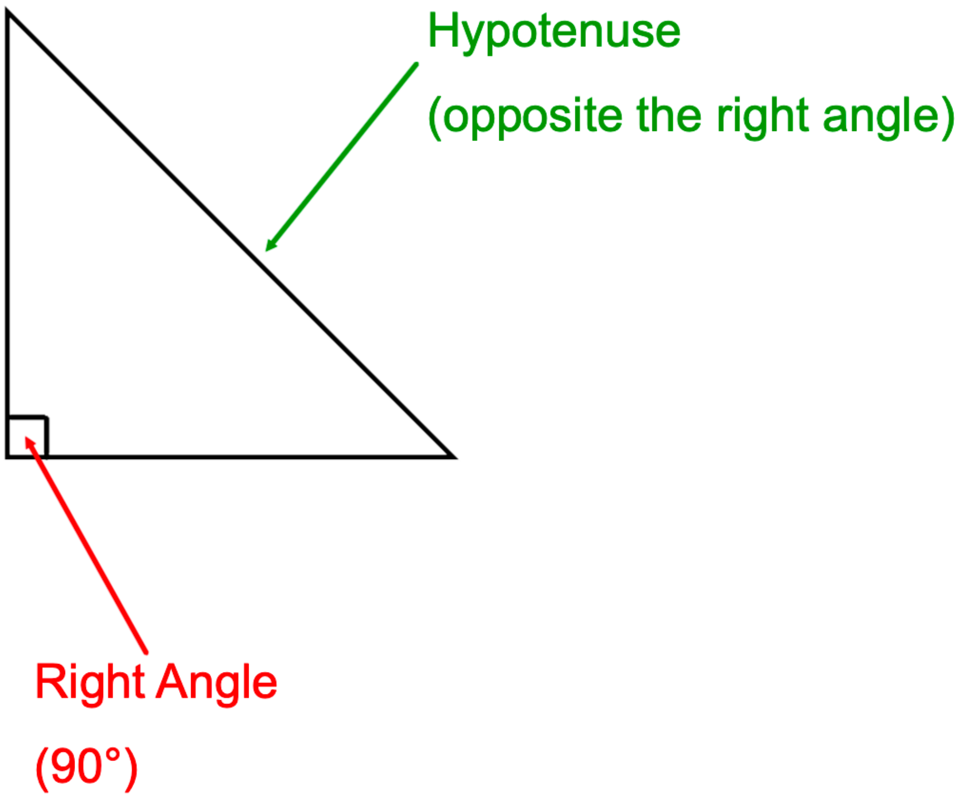
Write a transformation that would leave the correct points in the triangle ABC invariant for each region of the Venn diagram. Try and put at least one transformation in each region.



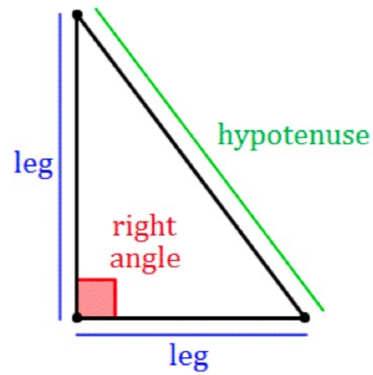
Extra Notes

6 2D Pythagoras' Theorem

Hypotenuse

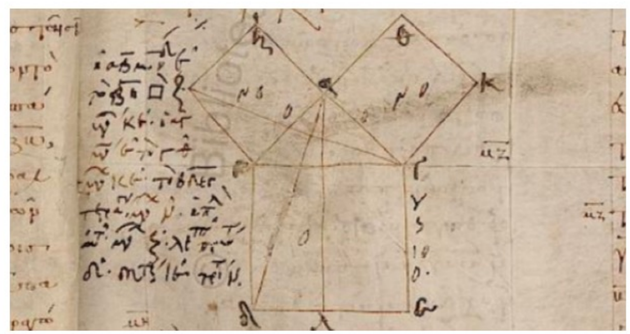


From the Greek derived *hypo* meaning 'under' and *teinein* meaning 'to stretch'.



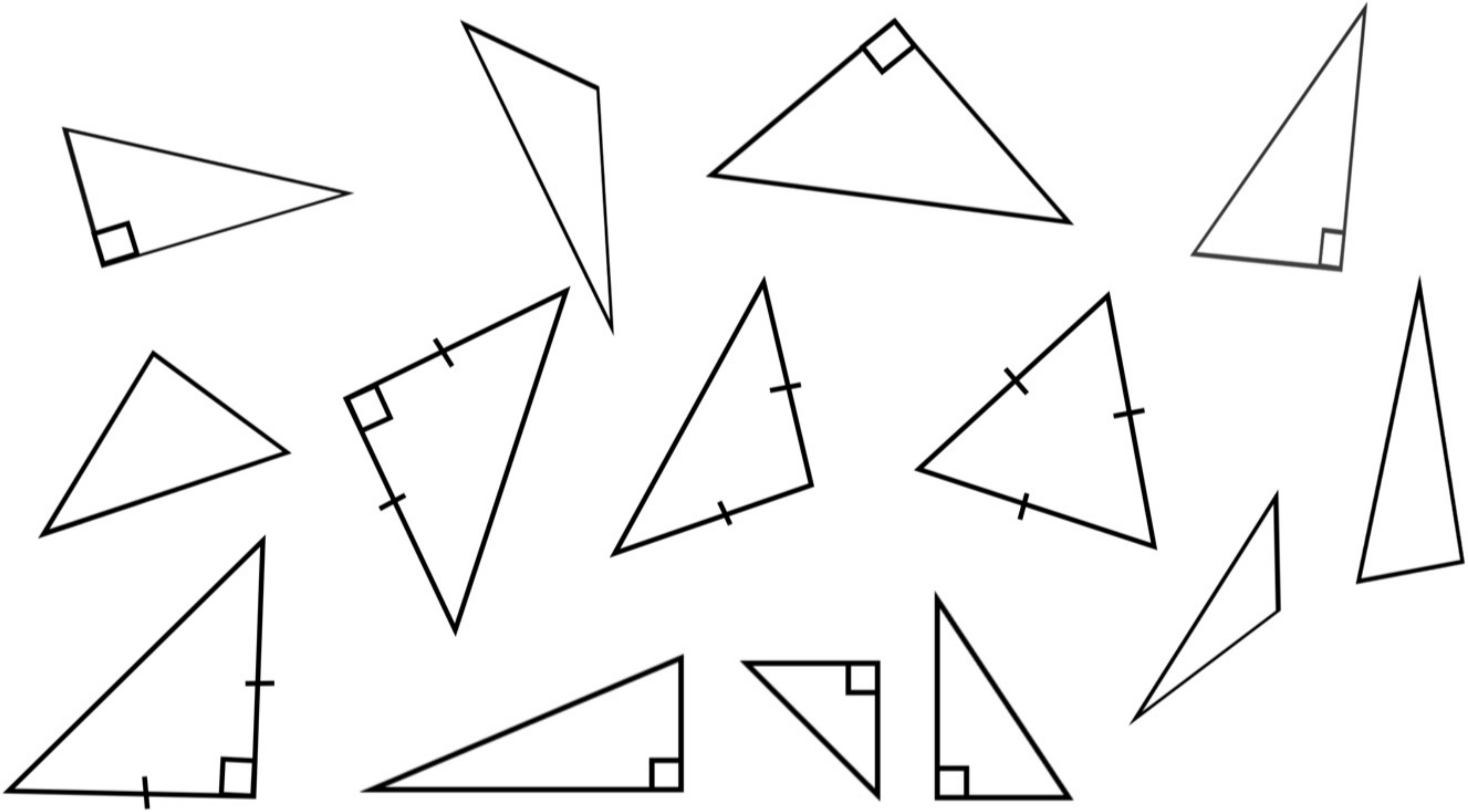
The two sides that aren't the hypotenuse are known as legs.

The hypotenuse is the side that stretches from one leg to another.



Fluency Practice

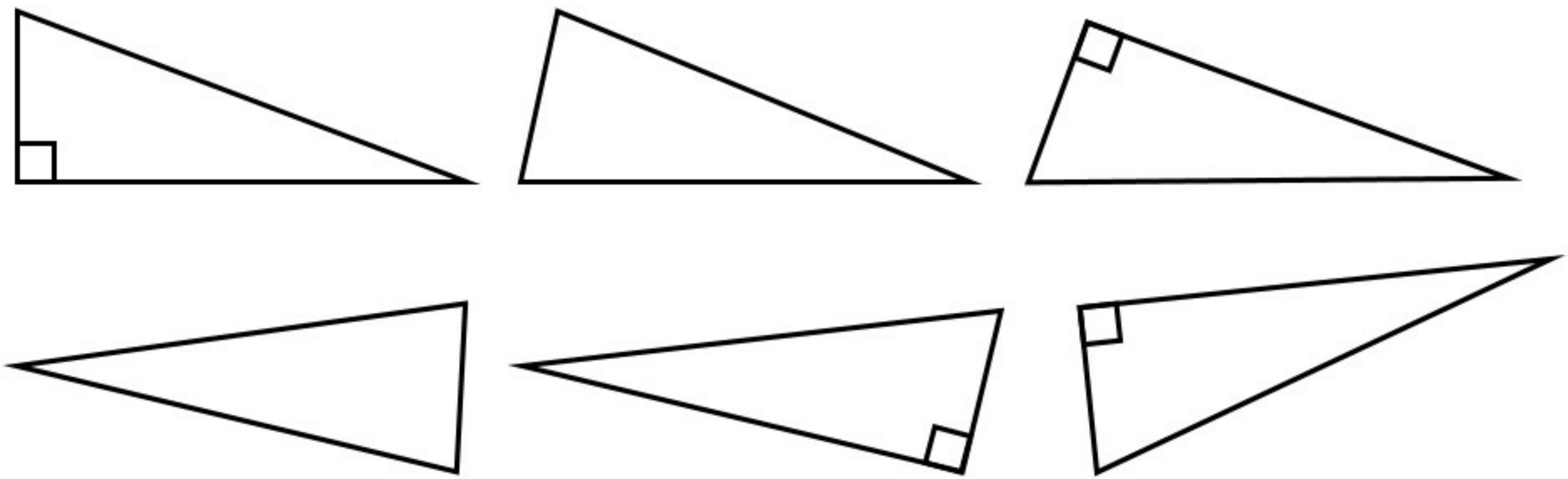
In each triangle that has a hypotenuse, label the hypotenuse with a letter h.



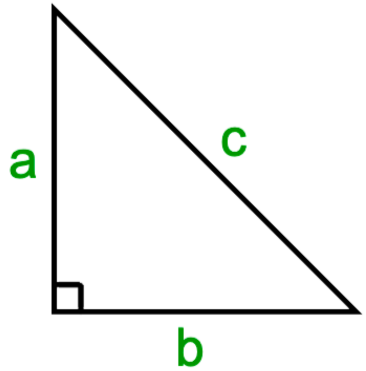
Fluency Practice

a) Cross out all shapes which Pythagoras' Theorem won't apply to.

b) In each remaining shape, label the hypotenuse h and the legs a and b.



Pythagoras' Theorem

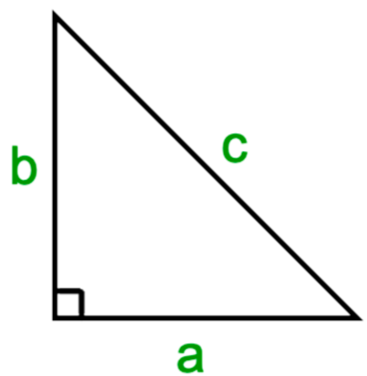


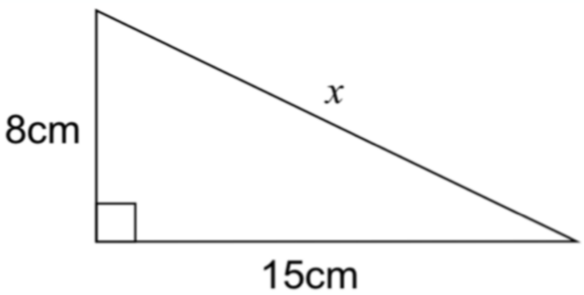
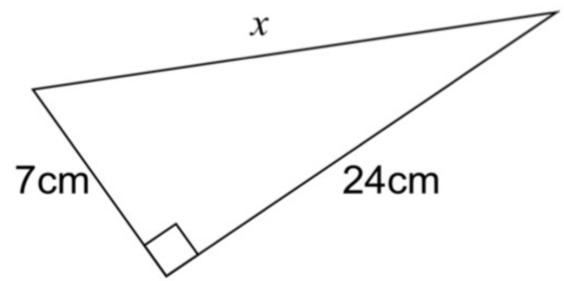
In any *right angled triangle*, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

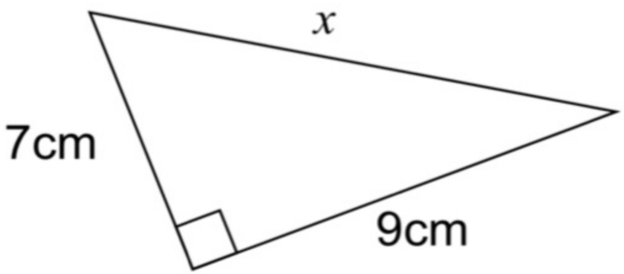
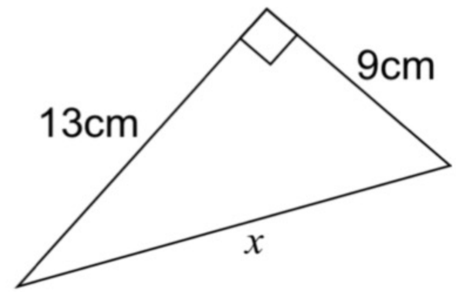
In other words:

$$a^2 + b^2 = c^2$$

Note: a and b can be labelled in any order but c has to be the hypotenuse i.e the triangle could be labelled like this:



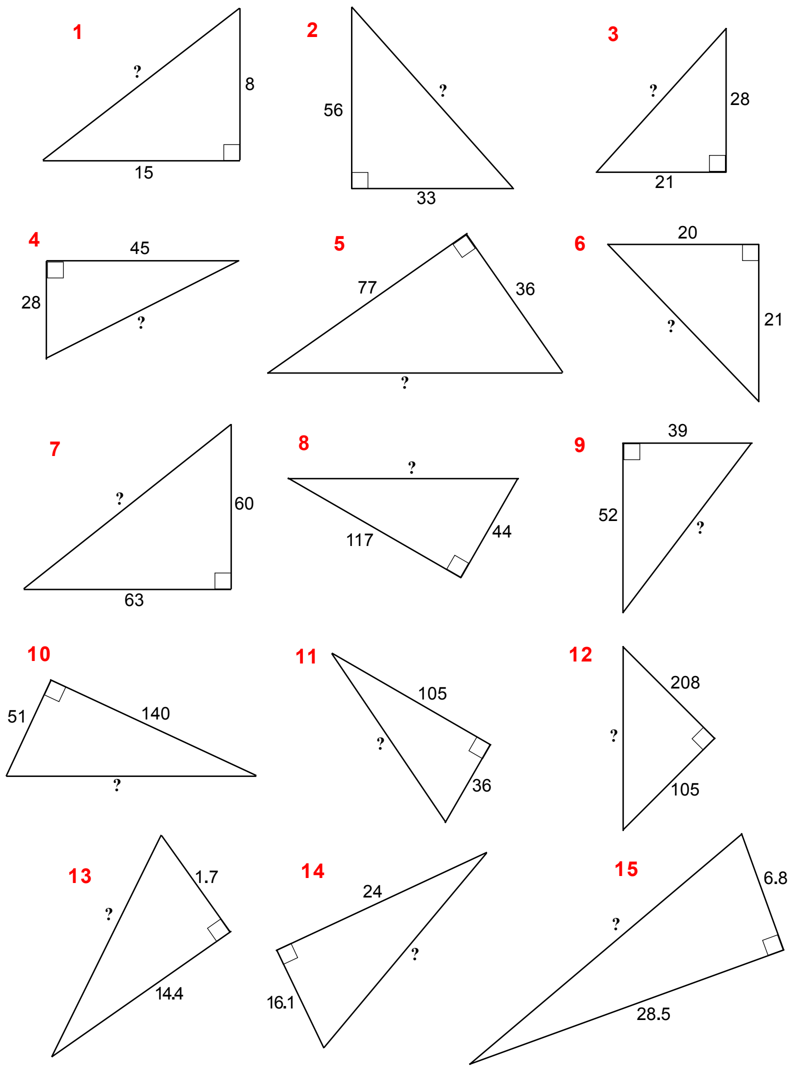
Worked Example	Your Turn
Calculate the unknown side in this triangle. 	Find the sum of interior angles of this polygon. 

Worked Example	Your Turn
Calculate the unknown side in this triangle. Give your answer to 2 decimal places.  	Calculate the unknown side in this triangle. Give your answer to 2 decimal places. 

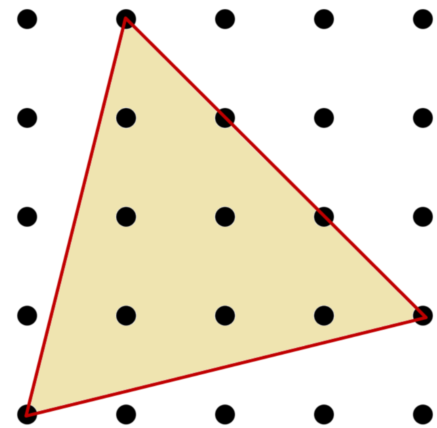
Fluency Practice

Use Pythagoras' theorem to find the length of the **hypotenuse** marked **?** in each of these **right-angled** triangles.

Drawings are NOT to scale.

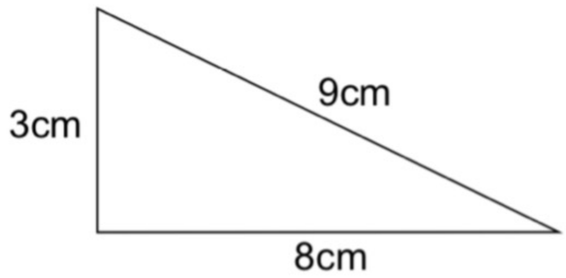
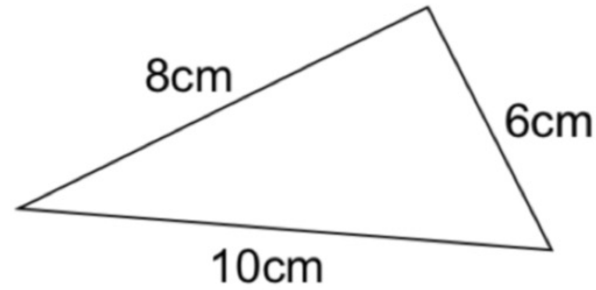


equilateral triangle or not?



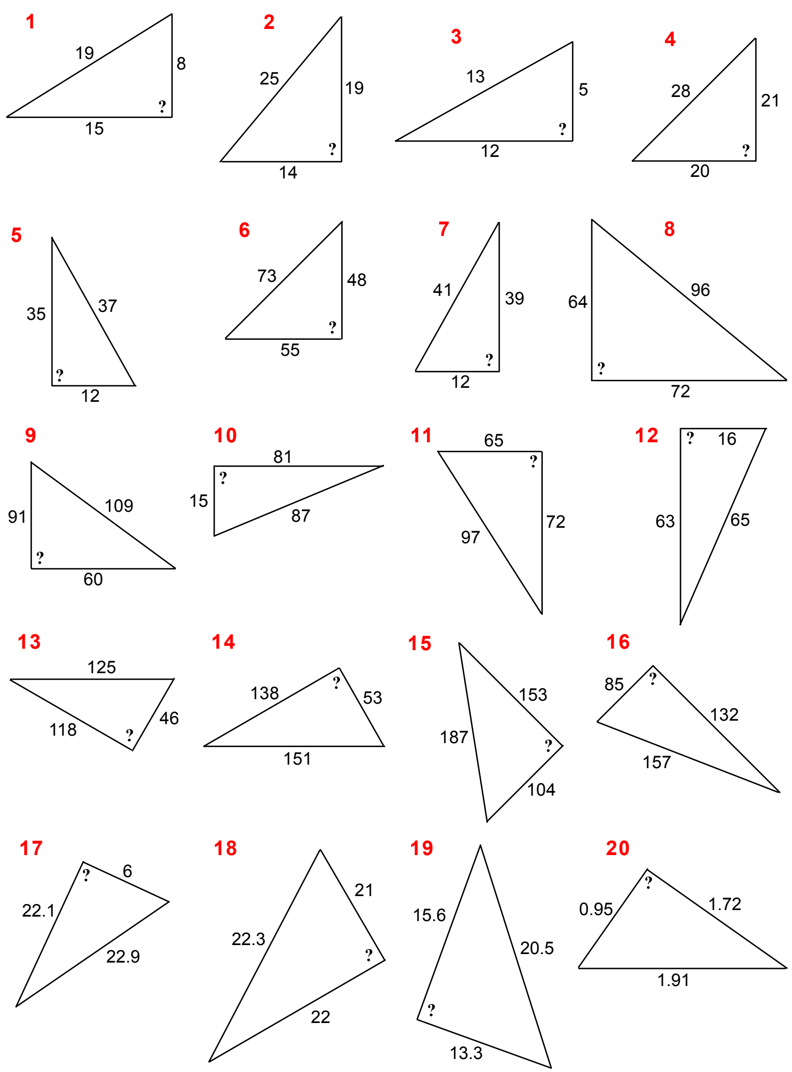
Converse of Pythagoras' Theorem

If Pythagoras' theorem holds true (i.e. if $a^2 + b^2 = c^2$) then the triangle must be right-angled.

Worked Example	Your Turn
Work out if this triangle is right-angled or not.  	Work out if this triangle is right-angled or not. 

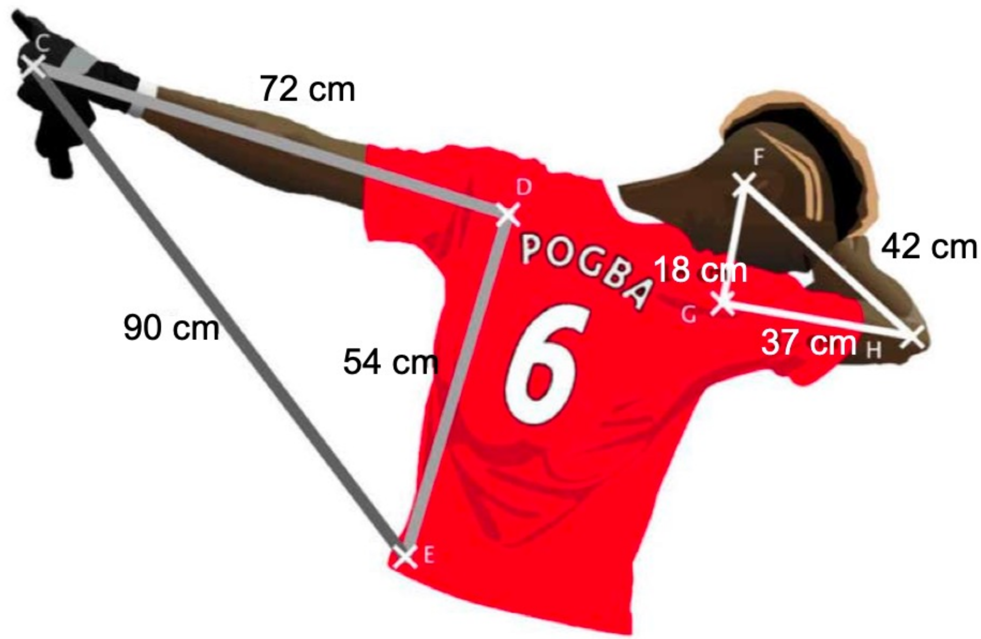
Fluency Practice

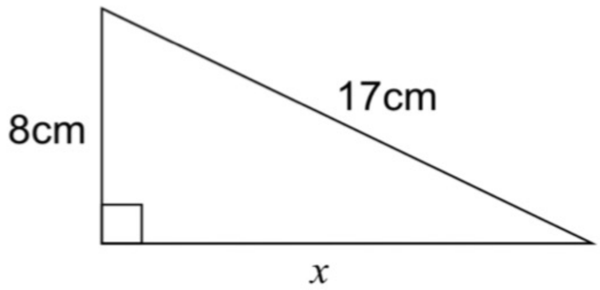
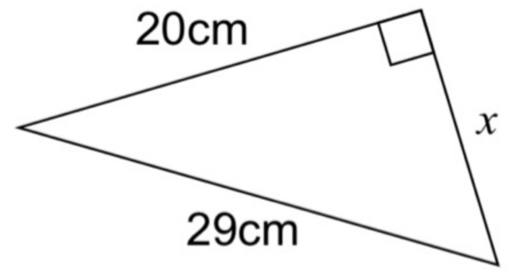
Use Pythagoras' theorem to decide whether each of these triangles is right-angled or not.
Drawings are NOT to scale..

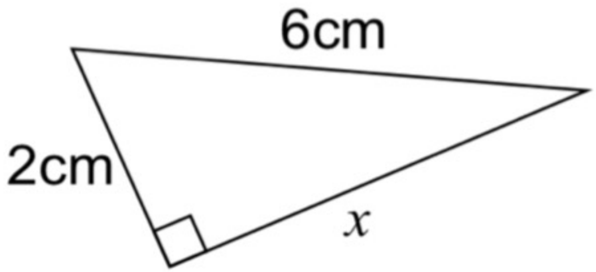
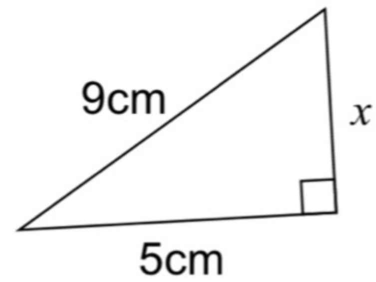


Cristiano Ronaldo is jealous of Paul Pogba's dab, so Pogba tries to demonstrate that his dab is perfect. According to the book 'the Universal Declaration of the Rights of the Dab', a dab is only perfect if both triangles represented in the figure below are right angled.

Is Paul Pogba's dab perfect?



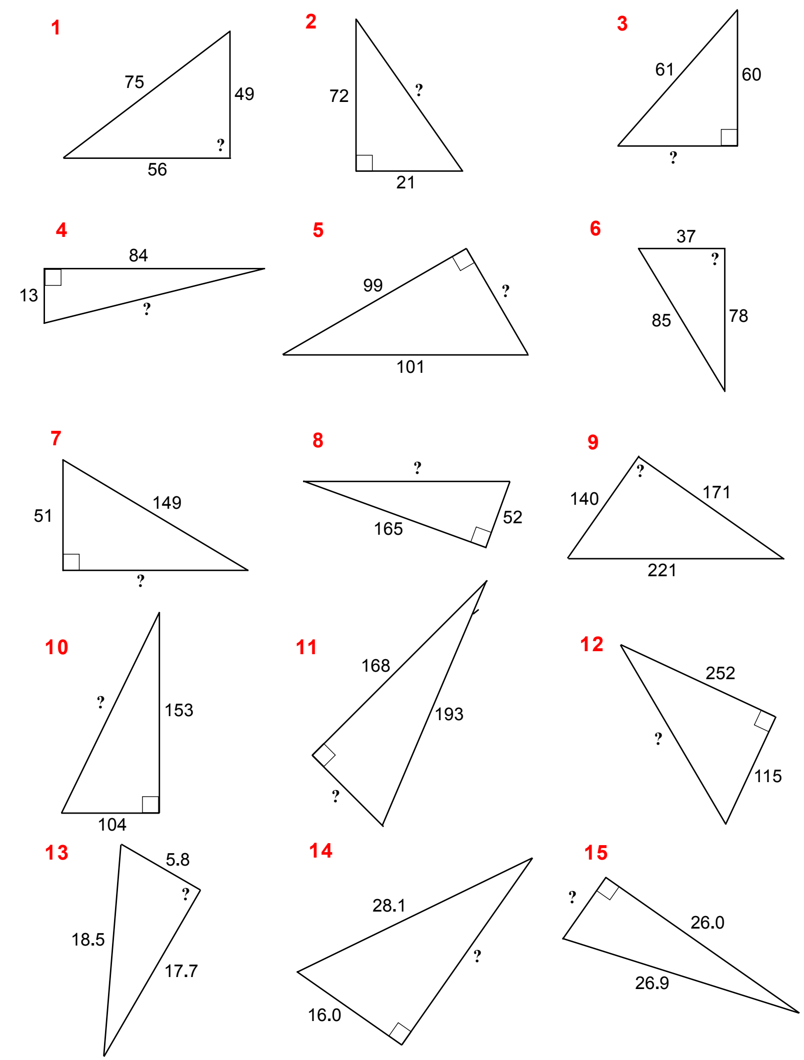
Worked Example	Your Turn
Calculate the unknown side in this triangle.  	Calculate the unknown side in this triangle. 

Worked Example	Your Turn
Calculate the unknown side in this triangle. Give your answer to 2 decimal places.  	Calculate the unknown side in this triangle. Give your answer to 2 decimal places. 

Fluency Practice

Use Pythagoras' theorem to find the length of the edge marked **?**, **OR** decide whether the triangle is right-angled or not

Drawings are NOT to scale.



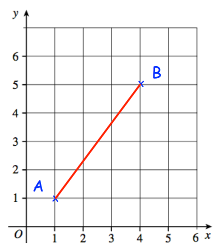
Worked Example	Your Turn
Find the length of AB where A$(-1, -4)$ and B$(4, 3)$.	Find the length of AB where A$(-2, -3)$ and B$(8, 11)$.

Fluency Practice

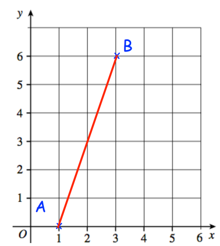
Round answers to 2dp

Question 1: Calculate the length of the line joining the points A and B.

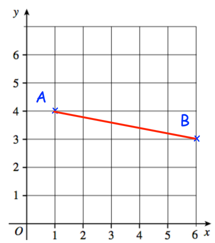
(a)



(b)

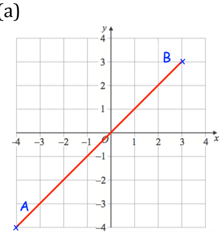


(c)

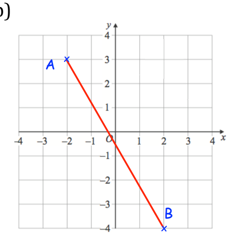


Question 2: Calculate the length of the line joining the points A and B.

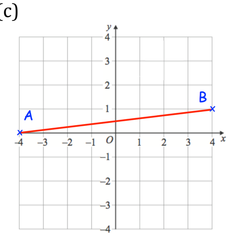
(a)



(b)

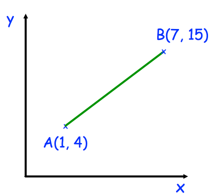


(c)

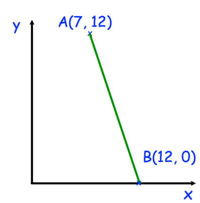


Question 3: Calculate the length of the line joining the point A and B.

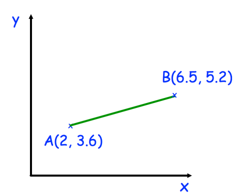
(a)



(b)

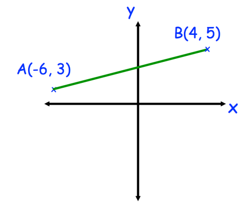


(c)

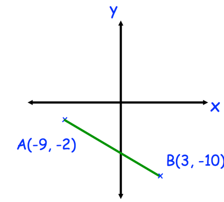


Question 4: Calculate the length of the line joining the points A and B

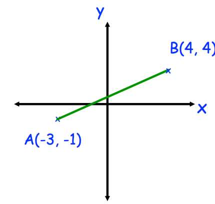
(a)



(b)



(c)

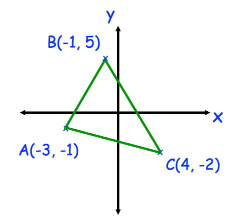


Question 5: Calculate the distance between the following pairs of coordinates

(a) (5, 1) and (9, 6)
(b) (1, 4) and (10, 10)
(c) (0, 0) and (6, 8)
(d) (2.5, 3) and (8, 0)
(e) (−6, 2) and (8, 3)
(f) (−5, −9) and (−3, 8)
(g) (−5, 7) and (−3, −2)
(h) (−9, −9) and (3, −20)
(i) (−4, 0) and (0, −4)

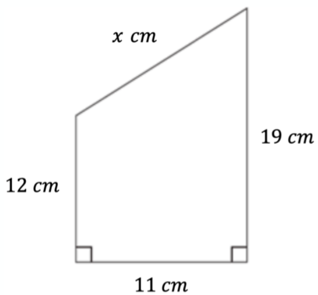
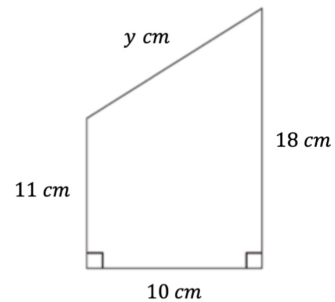
Apply

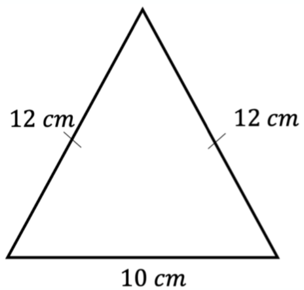
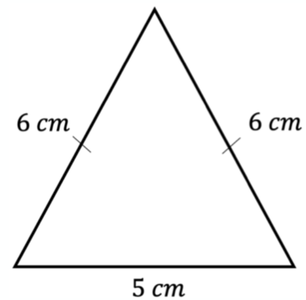
Question 1: Calculate the perimeter of triangle ABC.

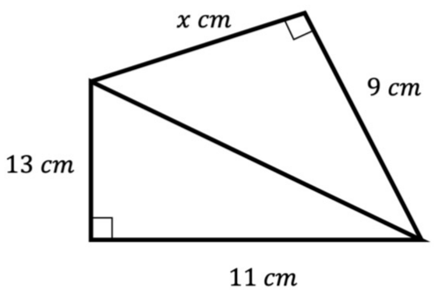
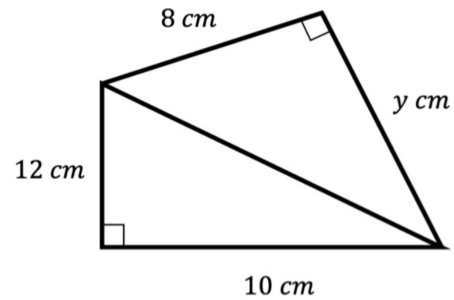


Question 2: The distance between the points (1, 2) and (16, p) is 17.
Find the possible values of p.

Question 3: The distance between the points (−3, −4) and (q, 5) is 15.
Find the possible values of q.

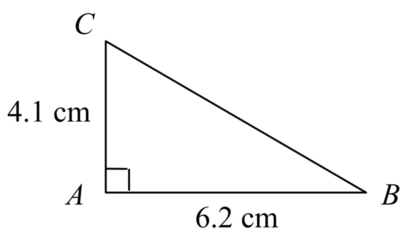
Worked Example	Your Turn
Calculate x. 	Calculate y. 

Worked Example	Your Turn
Find the area of this triangle. 	Find the area of this triangle. 

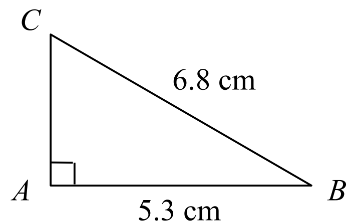
Worked Example	Your Turn
Calculate x.  	Calculate y. 

Fluency Practice

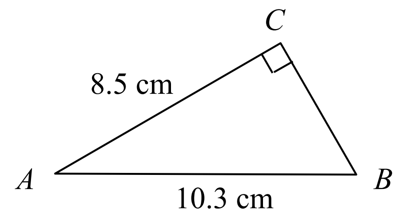
A1 Find length *BC*



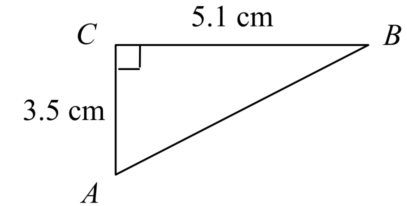
A2 Find length *AC*



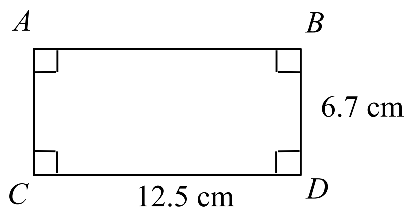
A3 Find length *BC*



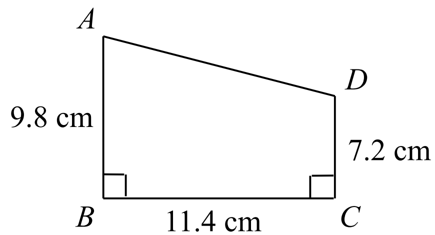
A4 Find length *AB*



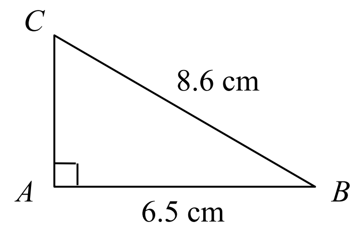
B1 Find length *BC*



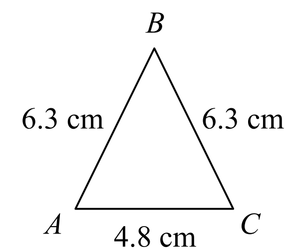
B2 Find length *AD*



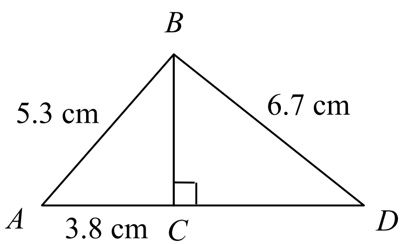
B3 Find the **area**



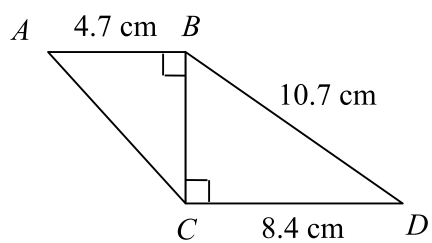
B4 Find the **area**



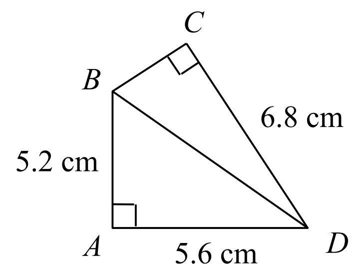
C1 Find length *CD*



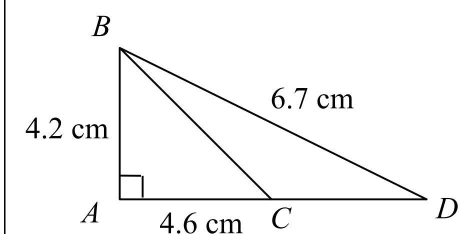
C2 Find length *AC*



C3 Find length *BC*



C4 Find length *CD*



Extra Notes