Year 9
Mathematics
Unit 15

Name: _____

Class: _____

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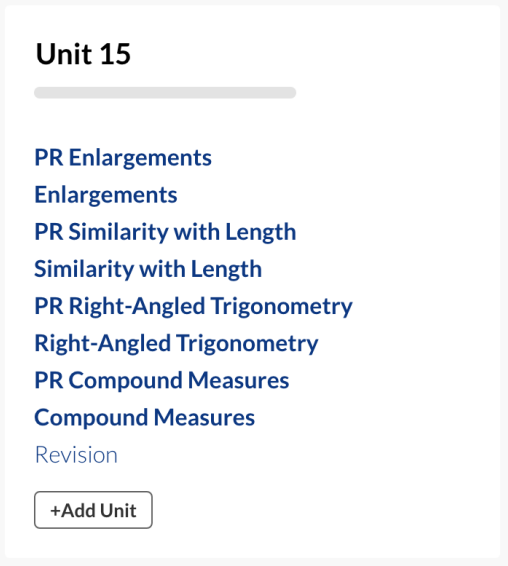
See unit 15 course on drfrostmaths.com



1 Enlargements

A transformation that moves all points a distance away from a centre point by applying a scale factor.

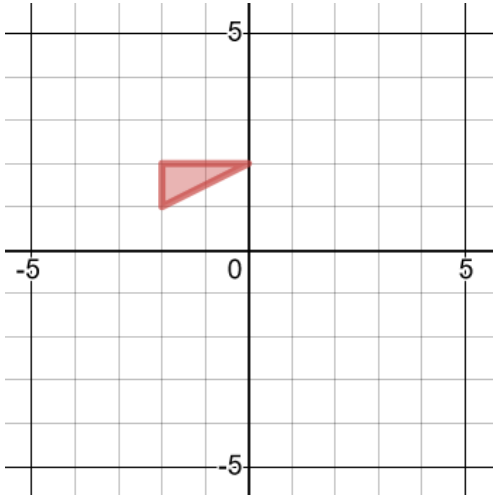
- Shapes change size.
- The scale factor multiplies distances, including the distance from the centre.

To fully describe an enlargement, we need to give three pieces of information:

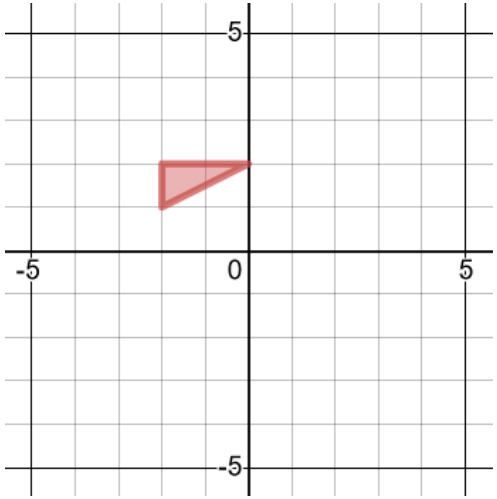
1. Type of Transformation: Enlargement
2. Scale Factor: Positive or Negative Number
3. Centre of Enlargement: Coordinate (x, y)

Worked Example

Enlarge about $(-4, 3)$, scale factor 2

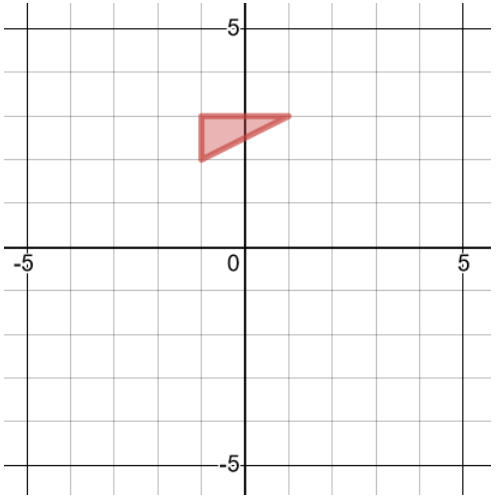


Enlarge about $(-2, 4)$, scale factor 3

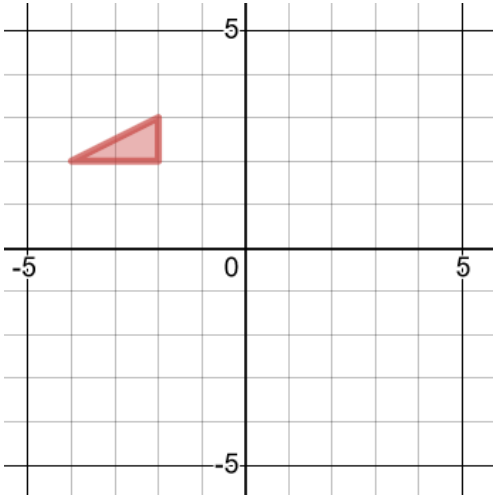


Your Turn

Enlarge about $(-3, 3)$, scale factor 2

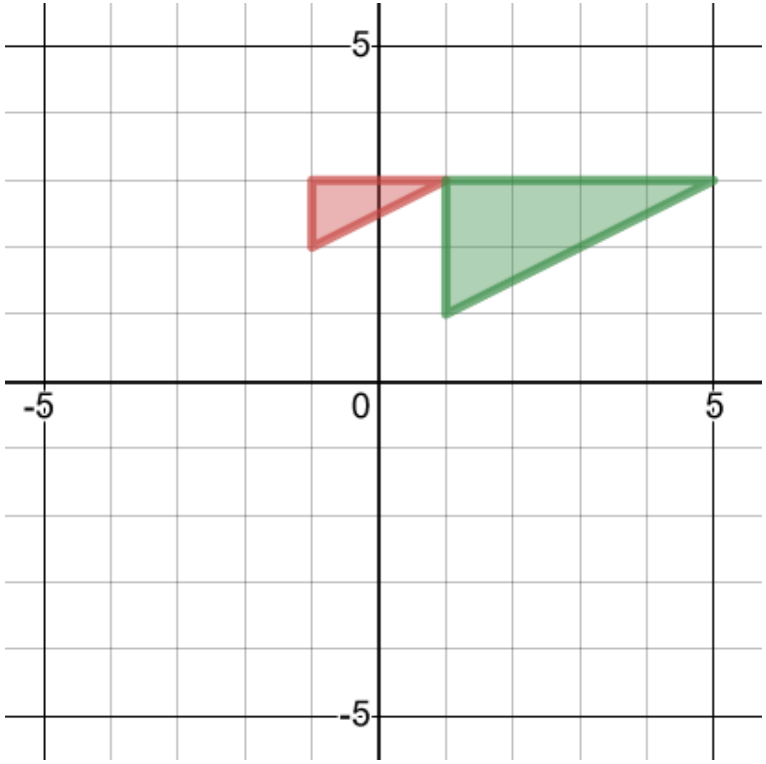


Enlarge about $(-4, 4)$, scale factor 3



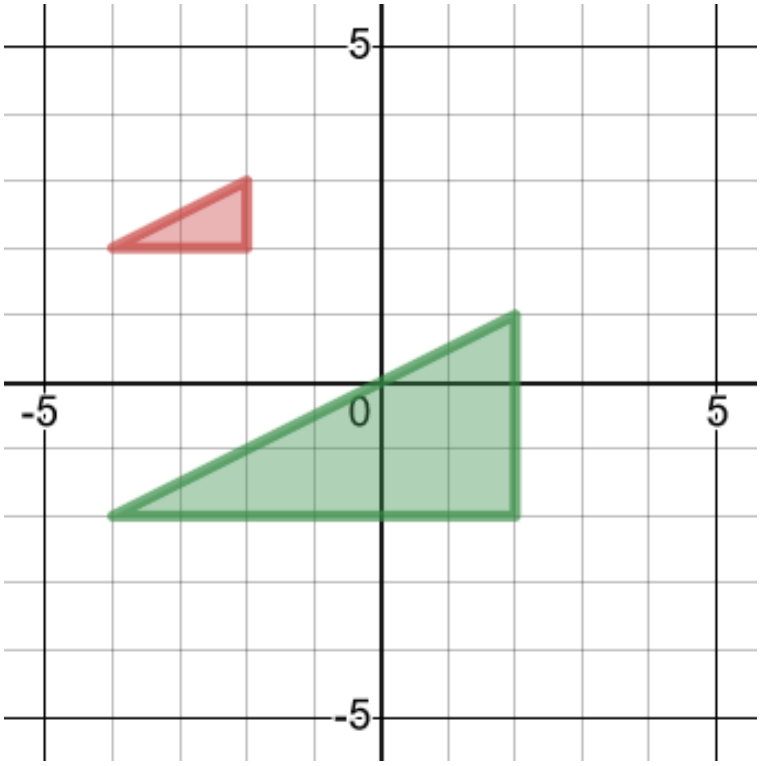
Worked Example

Describe the single transformation of the red object onto the green image



Your Turn

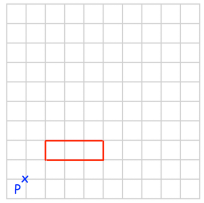
Describe the single transformation of the red object onto the green image



Fluency Practice

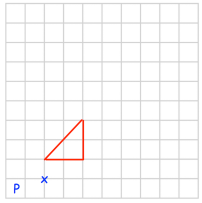
Question 1: Enlarge each shape by the scale factor given
Use P as the centre of enlargement.

(a)



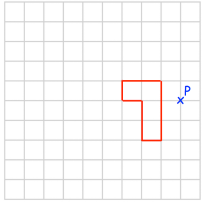
Enlarge by scale factor 2

(b)



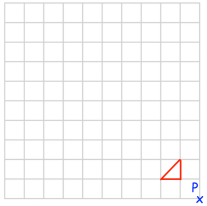
Enlarge by scale factor 3

(c)



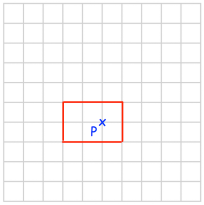
Enlarge by scale factor 2

(d)



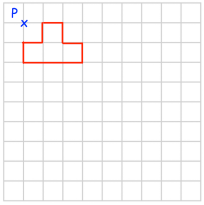
Enlarge by scale factor 4

(e)



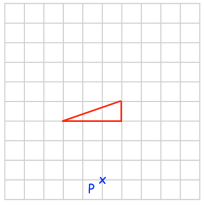
Enlarge by scale factor 2

(f)



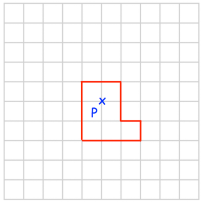
Enlarge by scale factor 3

(g)



Enlarge by scale factor 2

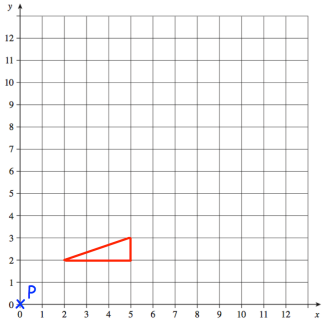
(h)



Enlarge by scale factor 2

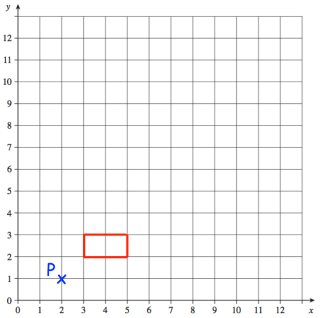
Question 2: Enlarge each shape by the scale factor given
Use P as the centre of enlargement.

(a)



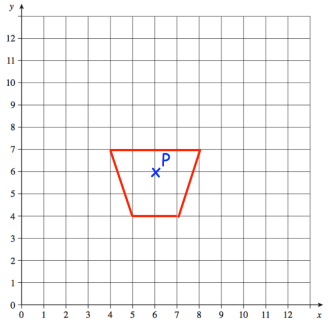
Enlarge by scale factor 2

(b)



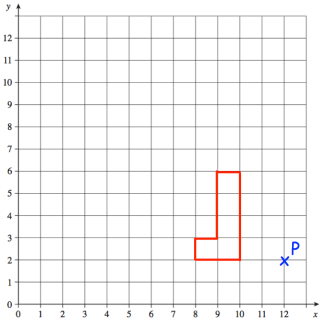
Enlarge by scale factor 3

(c)



Enlarge by scale factor 3

(d)

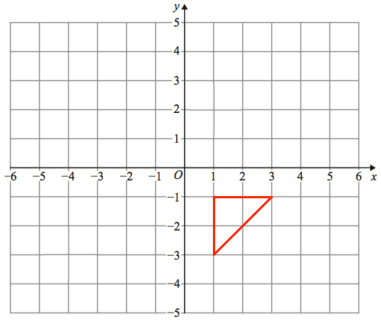


Enlarge by scale factor 2

Fluency Practice

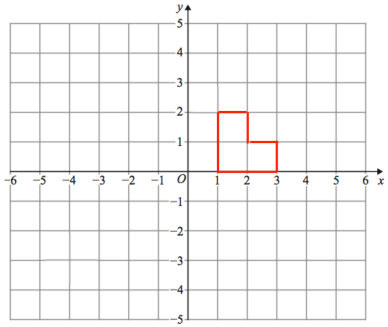
Question 3: Enlarge each shape by the scale factor given
The coordinates for each centre of enlargement are given.

(a)



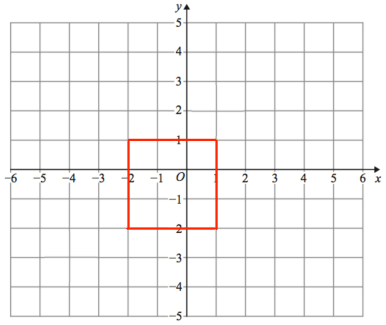
Enlarge by scale factor 2 using
(4, -3) as the centre of enlargement

(b)



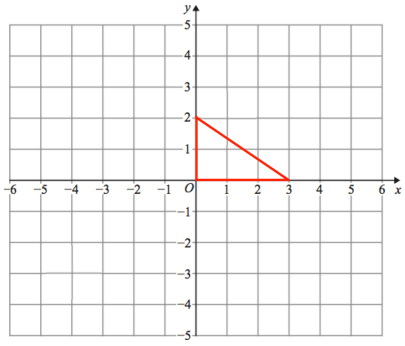
Enlarge by scale factor 3 using
(3, 2) as the centre of enlargement

(c)



Enlarge by scale factor 2 using
(0, -1) as the centre of enlargement

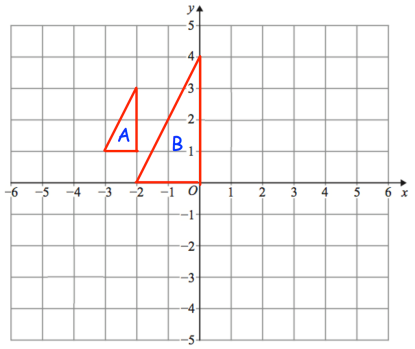
(d)



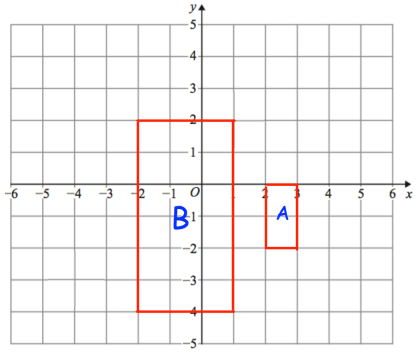
Enlarge by scale factor 2 using
the origin as the centre of enlargement

Question 4: Describe fully the single transformation that takes shape A to shape B.

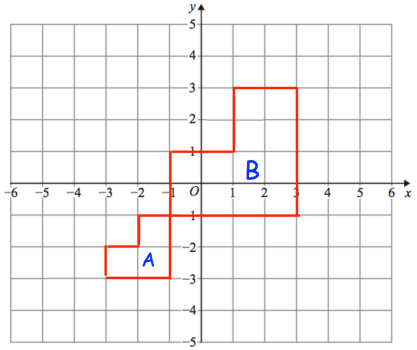
(a)



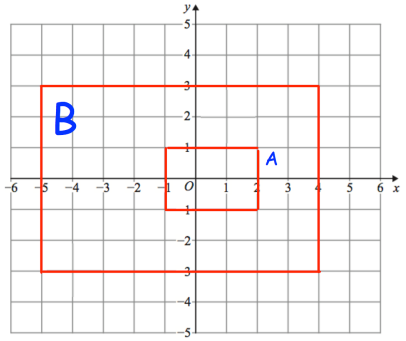
(b)



(c)

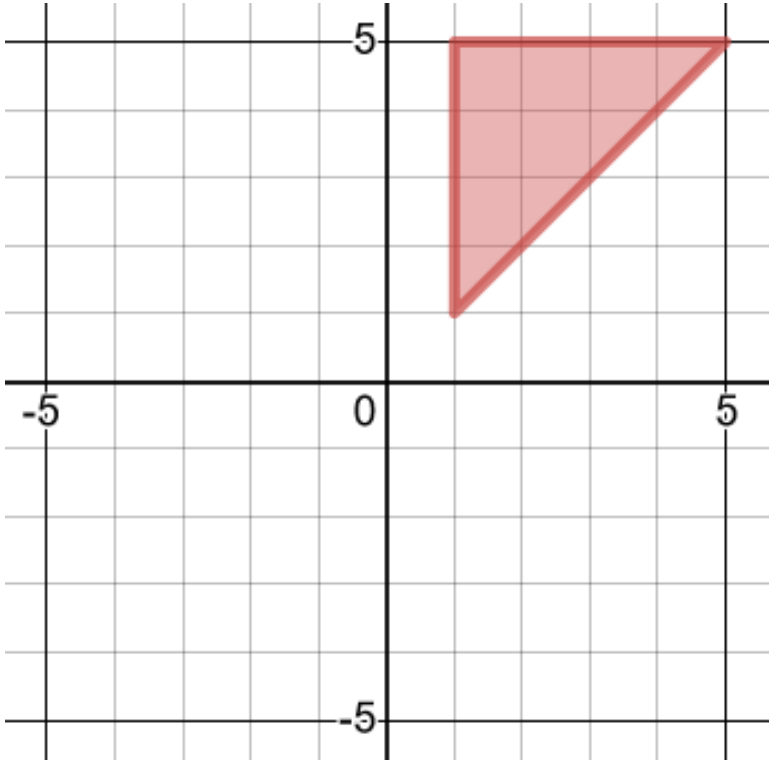


(d)



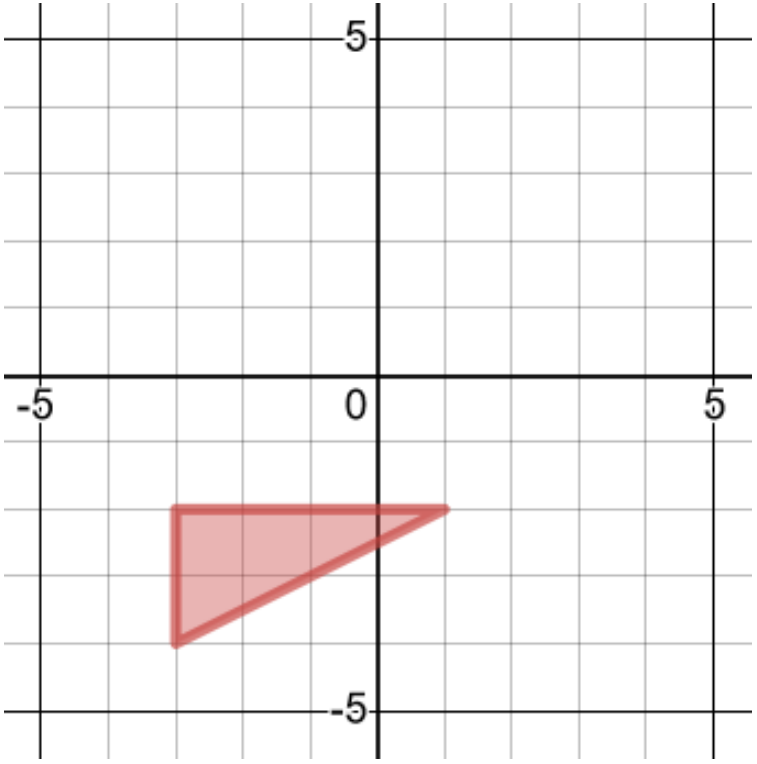
Worked Example

Enlarge about $(-3, -3)$, scale factor $\frac{1}{2}$



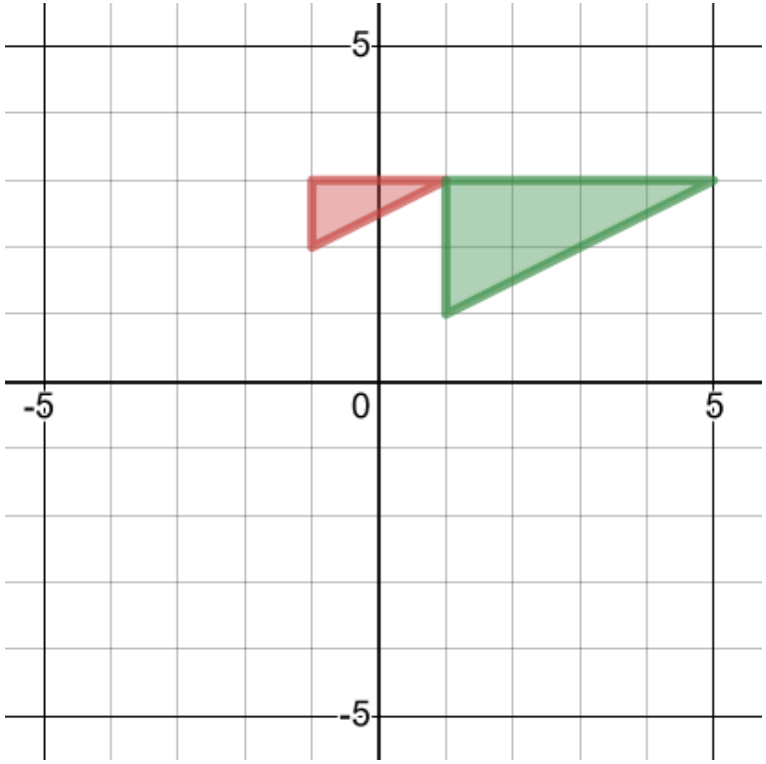
Your Turn

Enlarge about $(-1, 0)$, scale factor $\frac{1}{2}$



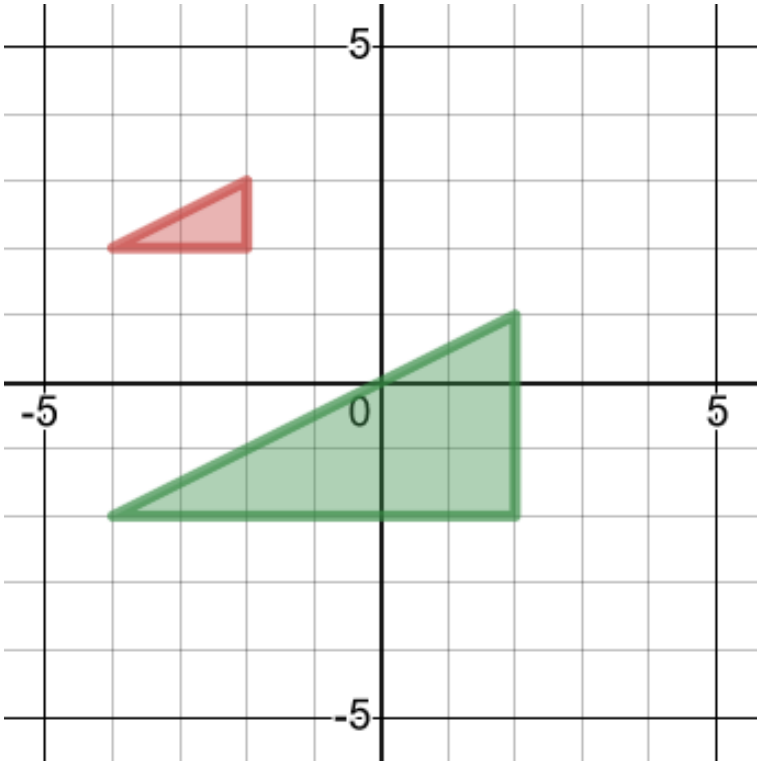
Worked Example

Describe the single transformation of the green object onto the red image



Your Turn

Describe the single transformation of the green object onto the red image



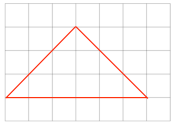
Fluency Practice

Question 1: Copy these shapes and then enlarge by the scale factor given.

(a)

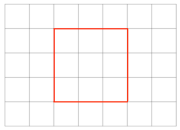
Enlarge by scale factor $\frac{1}{2}$

(b)



Enlarge by scale factor $\frac{1}{3}$

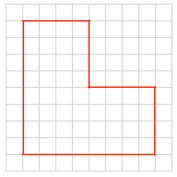
(c)



Enlarge by scale factor $\frac{2}{3}$

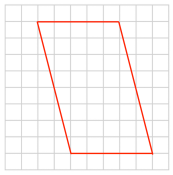
Question 2: Copy these shapes and then enlarge by the scale factor given.

(a)



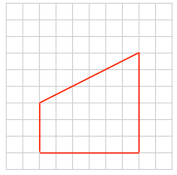
Enlarge by scale factor $\frac{1}{4}$

(b)



Enlarge by scale factor $\frac{1}{2}$

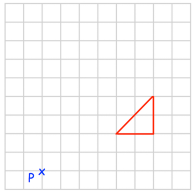
(c)



Enlarge by scale factor $1\frac{1}{3}$

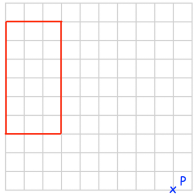
Question 3: Enlarge each shape by the scale factor given
Use P as the centre of enlargement.

(a)



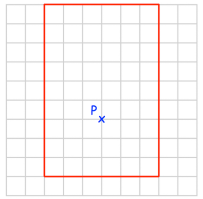
Enlarge by scale factor $\frac{1}{2}$

(b)



Enlarge by scale factor $\frac{1}{3}$

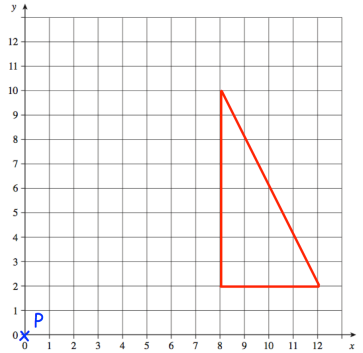
(c)



Enlarge by scale factor $\frac{2}{3}$

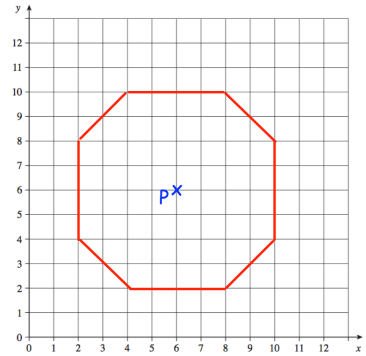
Question 4: Enlarge each shape by the scale factor given
Use P as the centre of enlargement.

(a)



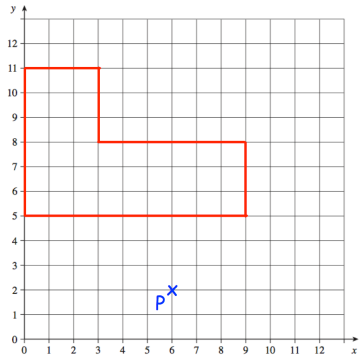
Enlarge by scale factor $\frac{1}{4}$

(b)



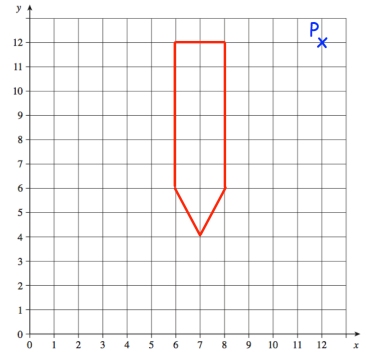
Enlarge by scale factor $\frac{1}{2}$

(c)



Enlarge by scale factor $\frac{2}{3}$

(d)

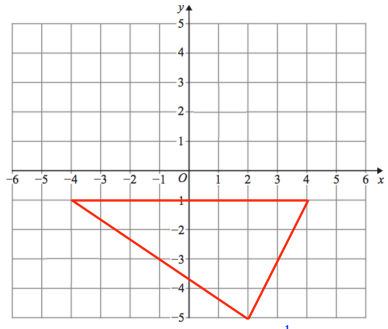


Enlarge by scale factor $1\frac{1}{2}$

Fluency Practice

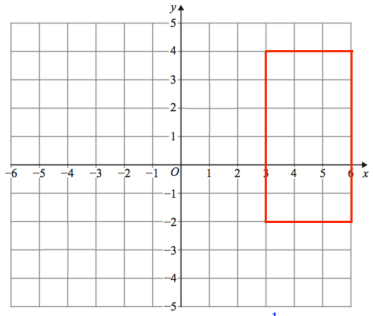
Question 5: Enlarge each shape by the scale factor given
The coordinates for each centre of enlargement are given.

(a)



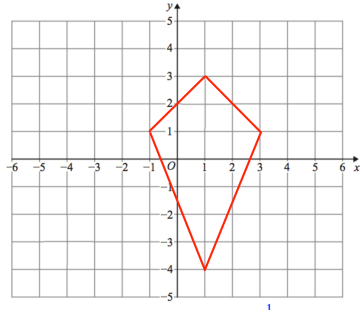
Enlarge by scale factor $\frac{1}{2}$ using
(0, 1) as the centre of enlargement

(b)



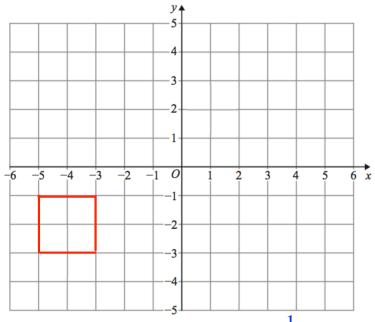
Enlarge by scale factor $\frac{1}{3}$ using
(-3, 1) as the centre of enlargement

(c)



Enlarge by scale factor $\frac{1}{2}$ using
(-5, -5) as the centre of enlargement

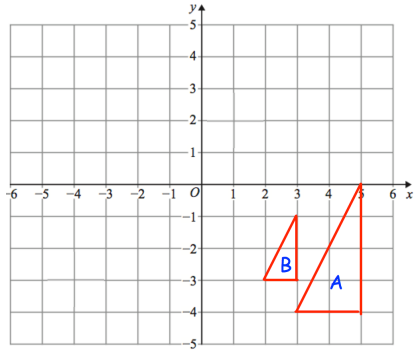
(d)



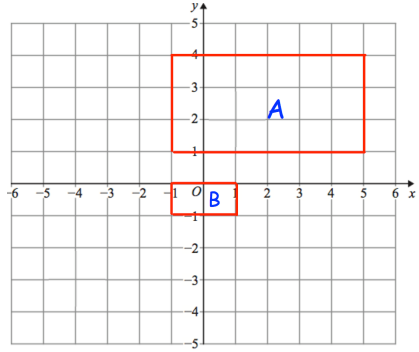
Enlarge by scale factor $2\frac{1}{2}$ using
(-5, -3) as the centre of enlargement

Question 6: Describe fully the single transformation that takes shape A to shape B.

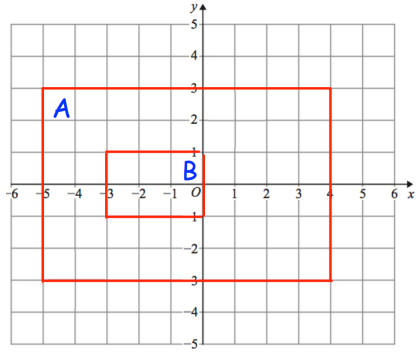
(a)



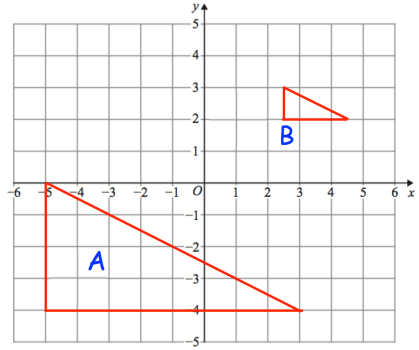
(b)



(c)

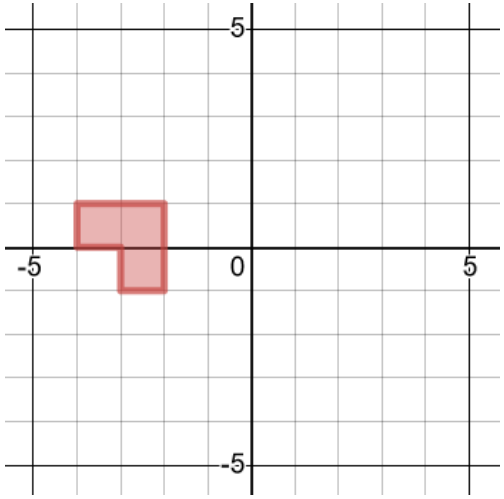


(d)

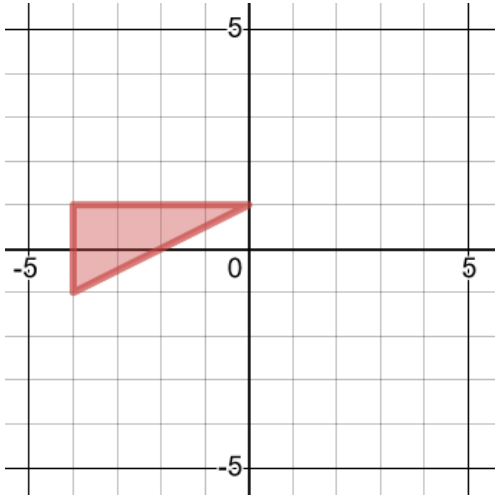


Worked Example

Enlarge about $(-2, 1)$, scale factor -2

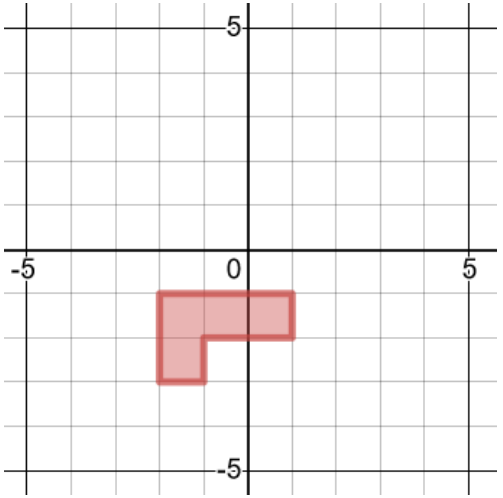


Enlarge about $(-2, -3)$, scale factor $-\frac{1}{2}$

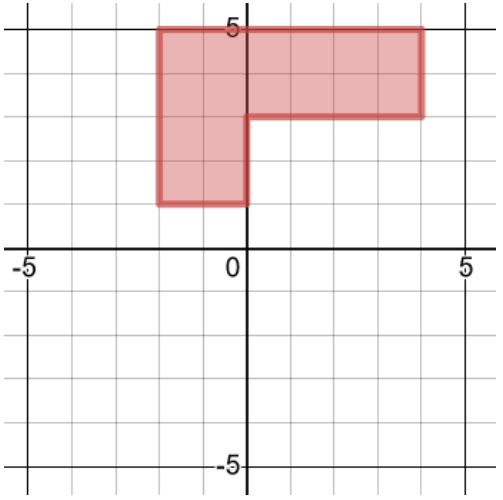


Your Turn

Enlarge about $(0, -1)$, scale factor -2

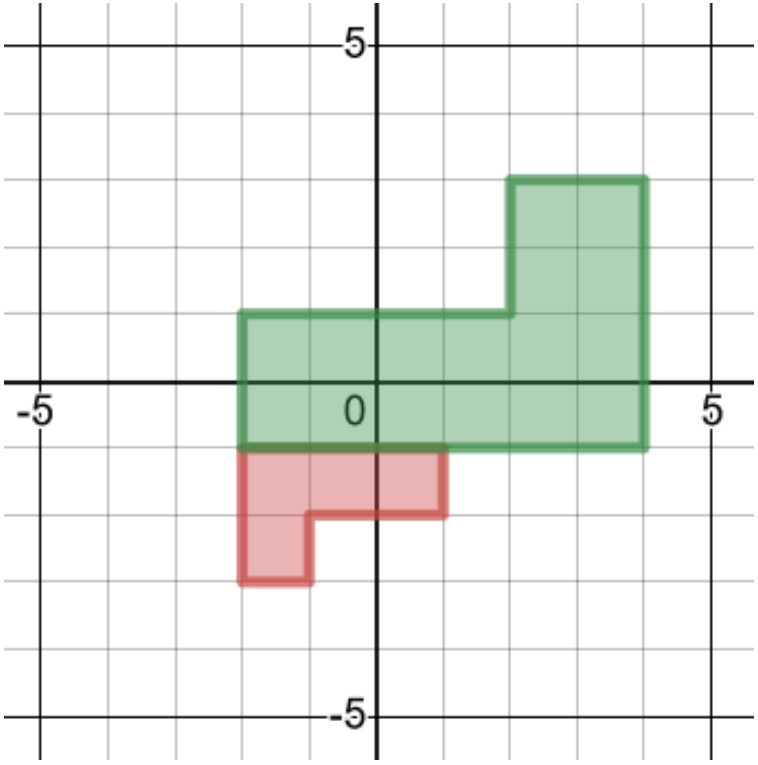


Enlarge about $(2, -1)$, scale factor $-\frac{1}{2}$



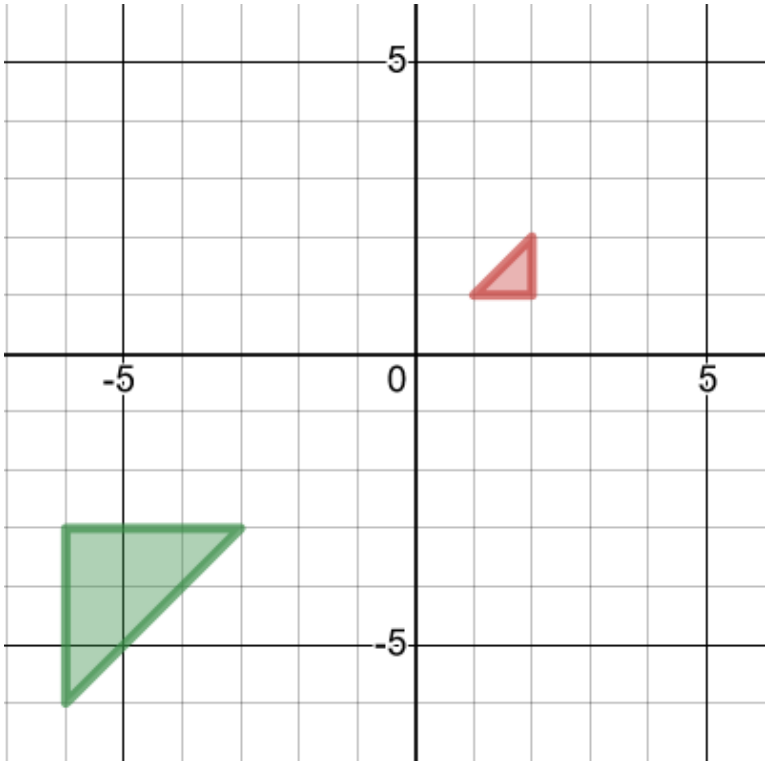
Worked Example

Describe the single transformation of the red object onto the green image



Your Turn

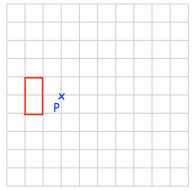
Describe the single transformation of the red object onto the green image



Fluency Practice

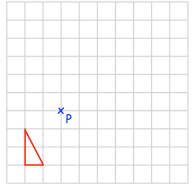
Question 1: Enlarge each shape by the scale factor given
Use P as the centre of enlargement.

(a)



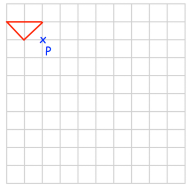
Enlarge by scale factor -3

(b)



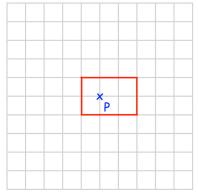
Enlarge by scale factor -2

(c)



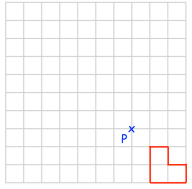
Enlarge by scale factor -4

(d)



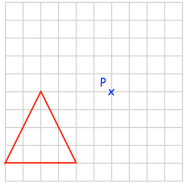
Enlarge by scale factor -2

(e)



Enlarge by scale factor -2

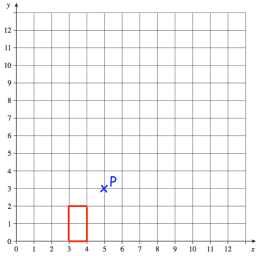
(f)



Enlarge by scale factor $-\frac{1}{2}$

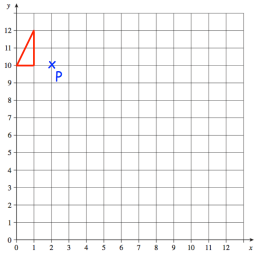
Question 2: Enlarge each shape by the scale factor given
Use P as the centre of enlargement.

(a)



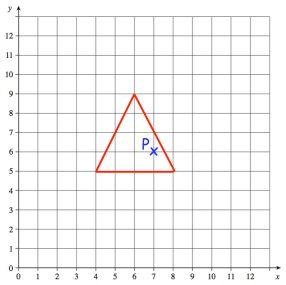
Enlarge by scale factor -3

(b)



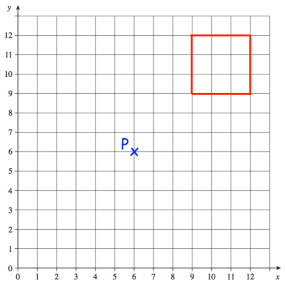
Enlarge by scale factor -4

(c)



Enlarge by scale factor -2

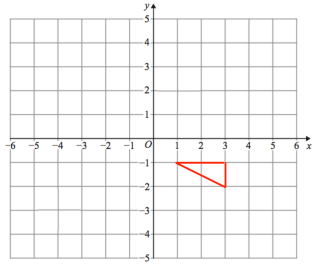
(d)



Enlarge by scale factor $-\frac{1}{3}$

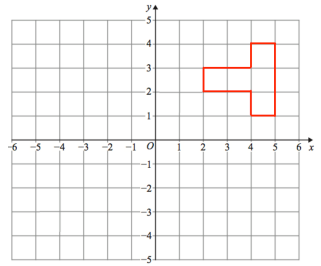
Question 3: Enlarge each shape by the scale factor given
The coordinates for each centre of enlargement are given.

(a)



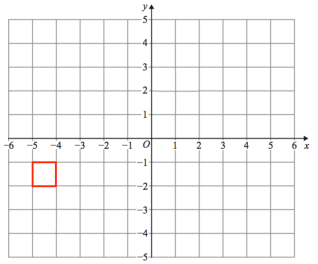
Enlarge by scale factor -2 using
(0, 0) as the centre of enlargement

(b)



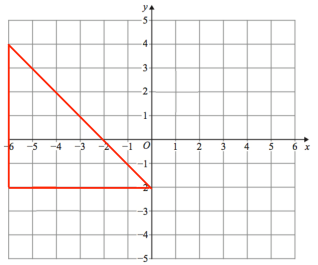
Enlarge by scale factor -2 using
(2, 2) as the centre of enlargement

(c)



Enlarge by scale factor -4 using
(-3, -1) as the centre of enlargement

(d)

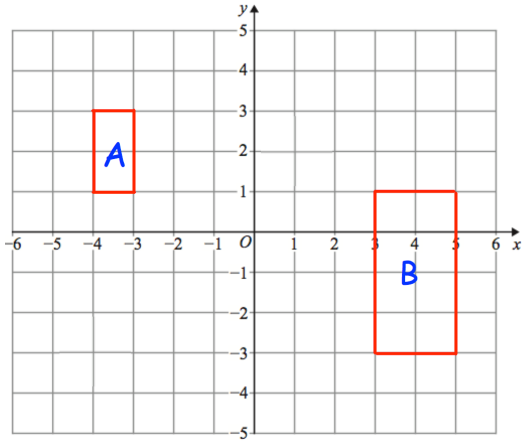


Enlarge by scale factor $-\frac{1}{2}$ using
(0, -2) as the centre of enlargement

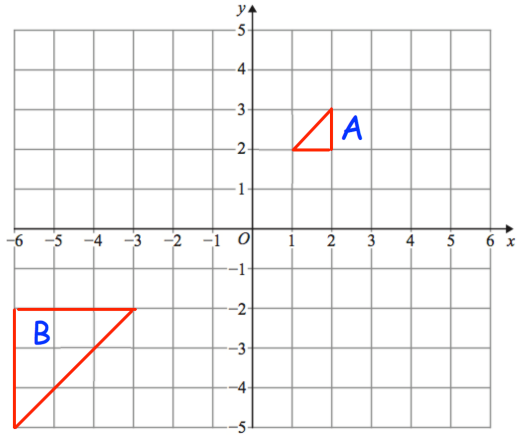
Fluency Practice

Question 4: Describe fully the single transformation that takes shape A to shape B.

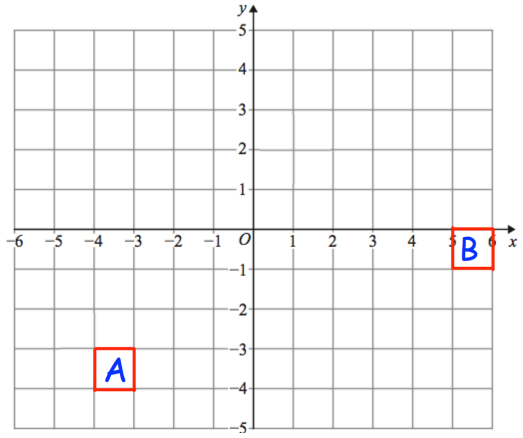
(a)



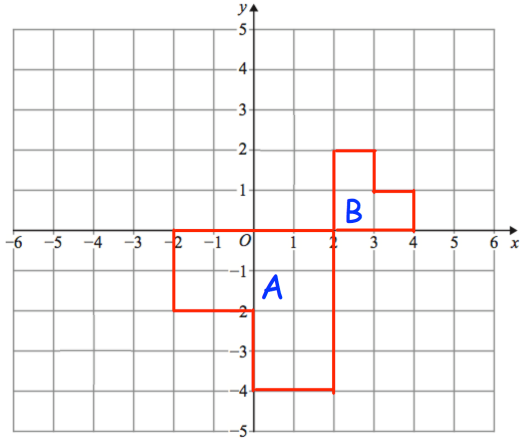
(b)



(c)

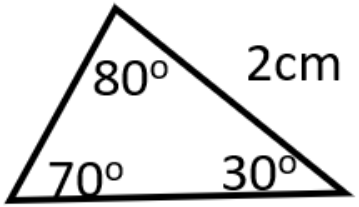
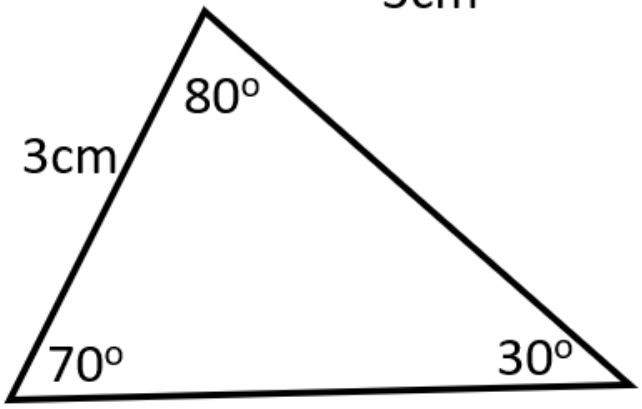
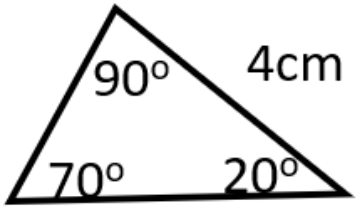
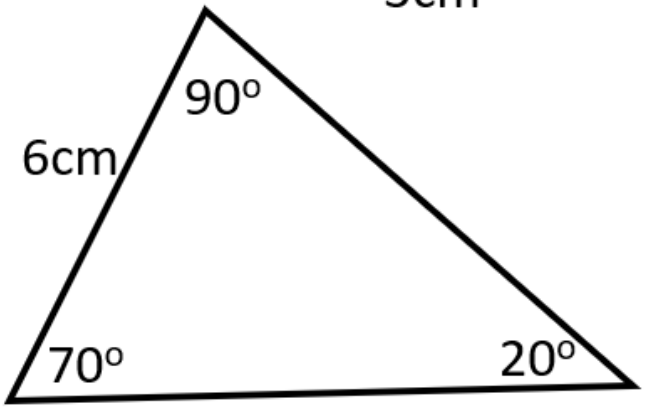


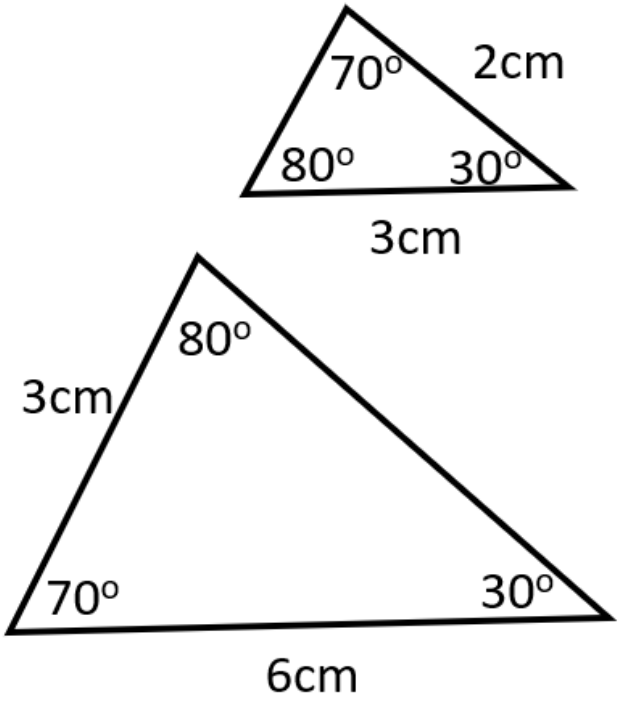
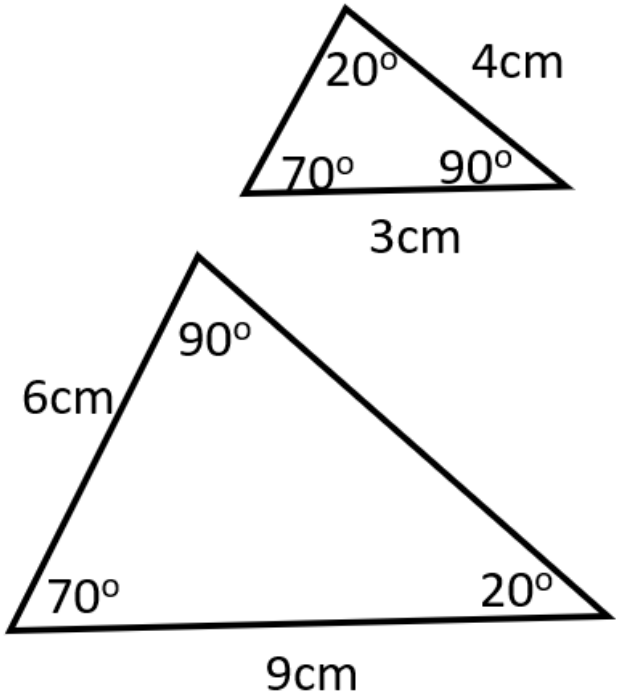
(d)



Extra Notes

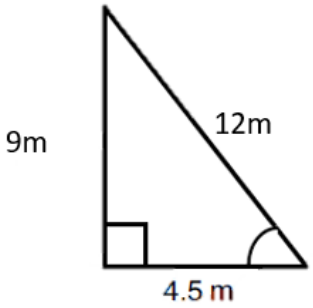
2 Similarity with Length

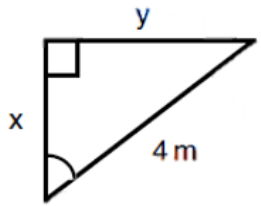
Worked Example	Your Turn
What is the scale factor? Find the missing lengths.  3cm  6cm	What is the scale factor? Find the missing lengths.  3cm  9cm

Worked Example	Your Turn
What is the scale factor? Find the missing lengths. 	What is the scale factor? Find the missing lengths. 

Worked Example

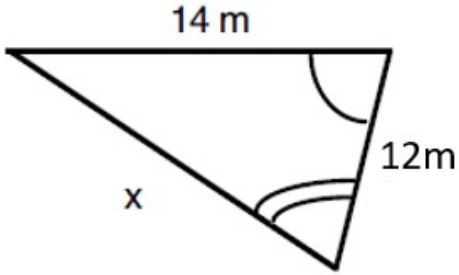
What is the scale factor? Find the missing lengths.

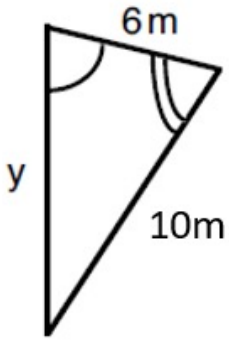


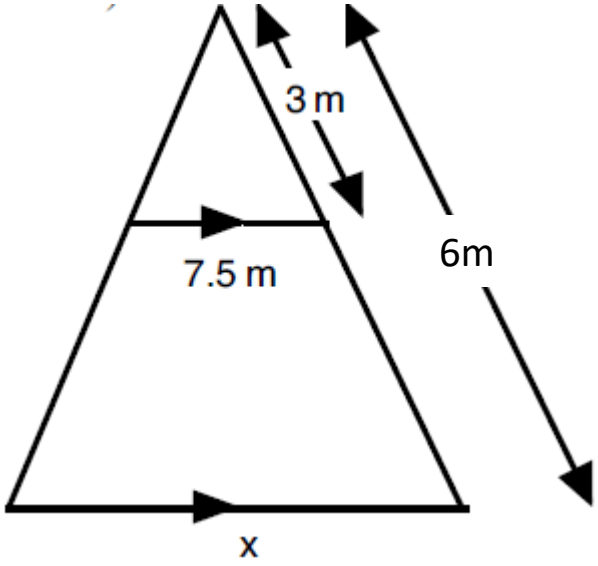
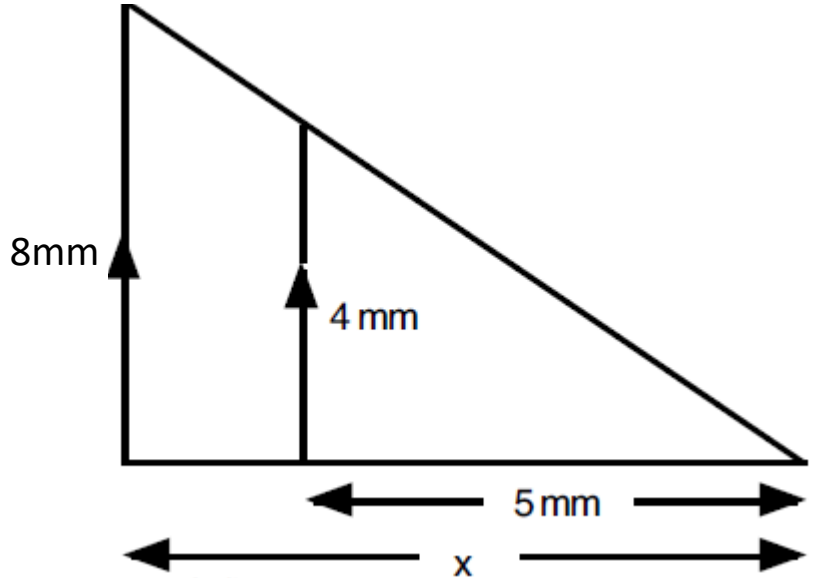


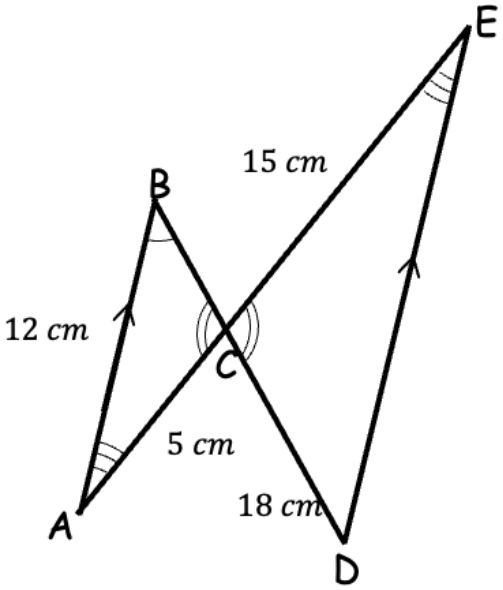
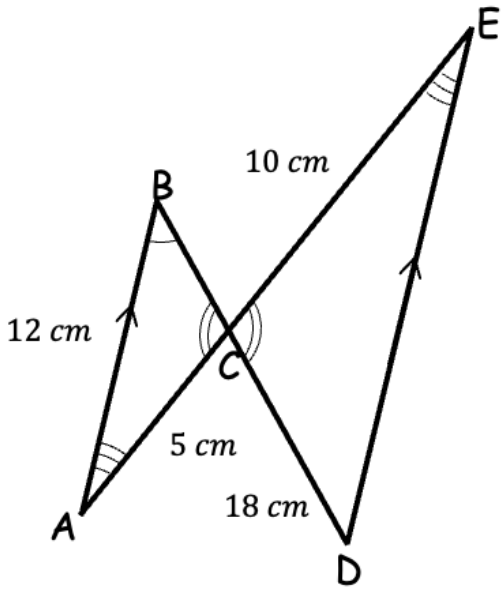
Your Turn

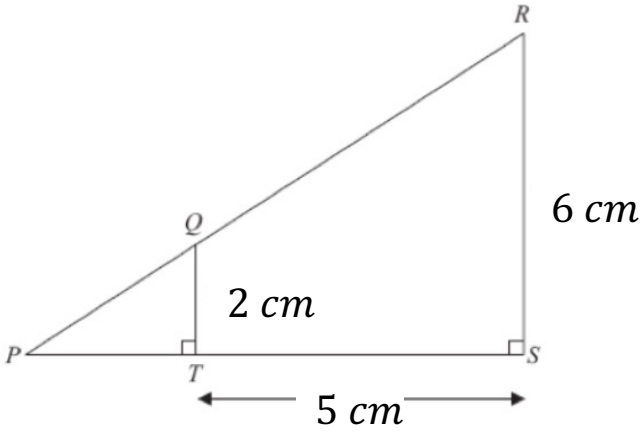
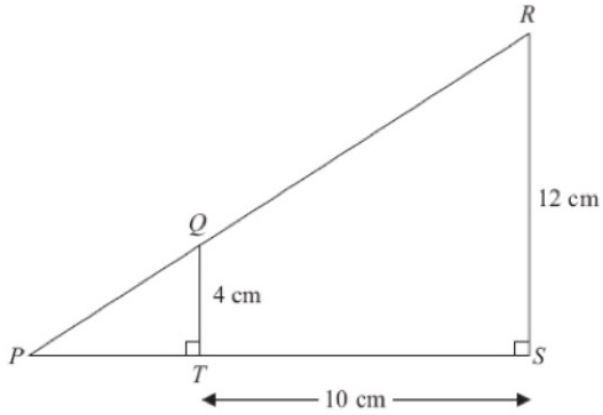
What is the scale factor? Find the missing lengths.





Worked Example	Your Turn
Find the length of every missing side 	Find the length of every missing side 

Worked Example	Your Turn
Calculate the missing lengths  	Calculate the missing lengths 

Worked Example	Your Turn
Calculate the length of PT  	Calculate the length of PT 

Extra Notes

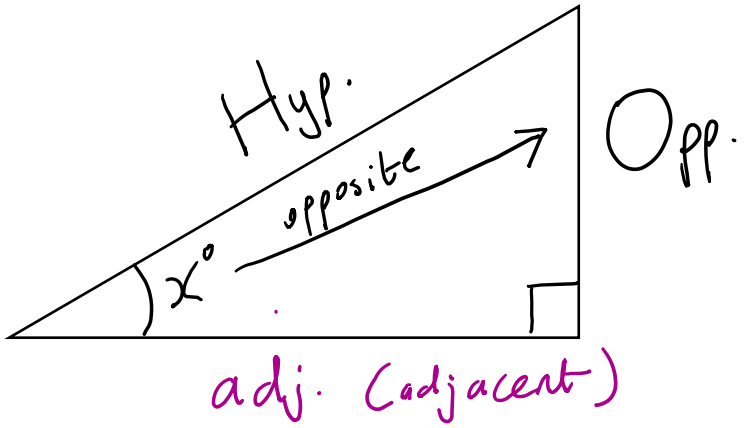
3 Right-Angled Trigonometry

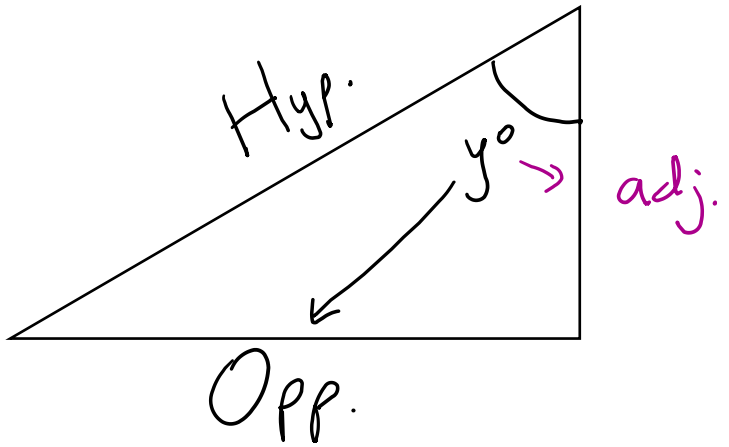
Trigonometric Functions

A function $f(x)$ takes an input x and outputs a value y. A **trigonometric function** takes an **angle x°** and outputs a **ratio of sides.**

For any **right-angled triangle** we **always** label the longest side as the hypotenuse *H.* For the purposes of trigonometry we label the other two sides **relative** to one of the non-right angles.

One of these is **opposite** the angle and the other **adjacent** (meaning next to).

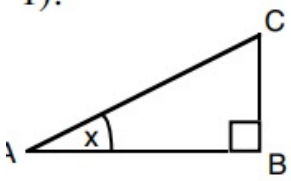




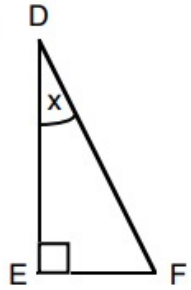
Fluency Practice

A. Name all the sides from the given angle, x°.

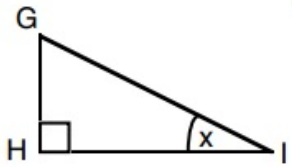
1).



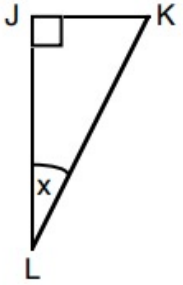
2).



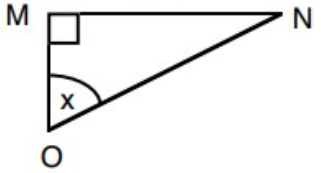
3).



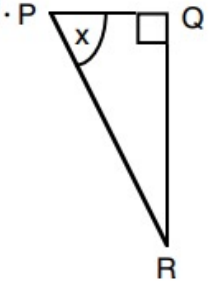
4).



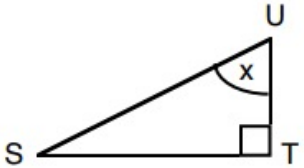
5).



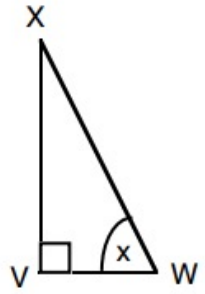
6).



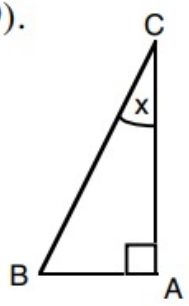
7).



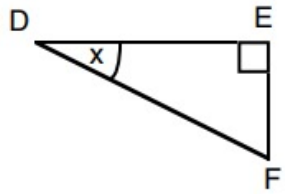
8).



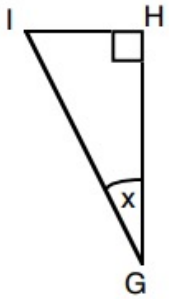
9).



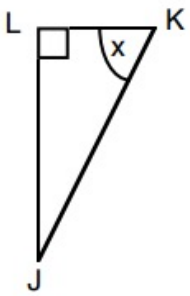
10).



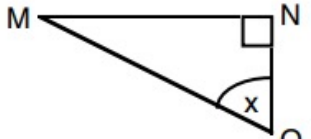
11).



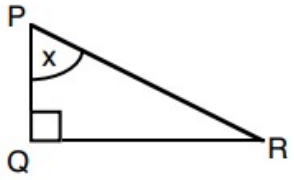
12).



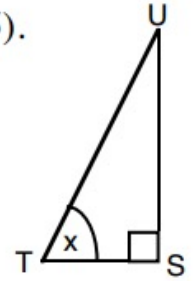
13).



14).



15).



Trigonometric Functions

A function $f(x)$ takes an input x and outputs a value y. A **trigonometric function** takes an **angle $x°$** and outputs a **ratio of sides.**

The three sides of right-angled triangles are:
O – Opposite
A – Adjacent
H – Hypotenuse

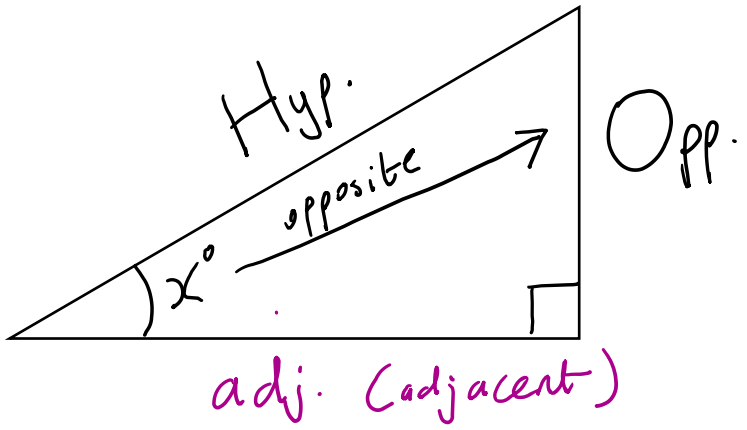
So the three ratios are: $\boldsymbol{O : H}$ **or** $\frac{O}{H}$ $\quad$ $\boldsymbol{A : H}$ **or** $\frac{A}{H}$ $\quad$ $\boldsymbol{O : A}$ **or** $\frac{O}{A}$

And so there are **three** trigonometric functions which **take any angles $x°$** and **output one of these ratios:**

$x° \longrightarrow \frac{O}{H}$ $\quad$ $x° \longrightarrow \frac{A}{H}$ $\quad$ $x° \longrightarrow \frac{O}{A}$

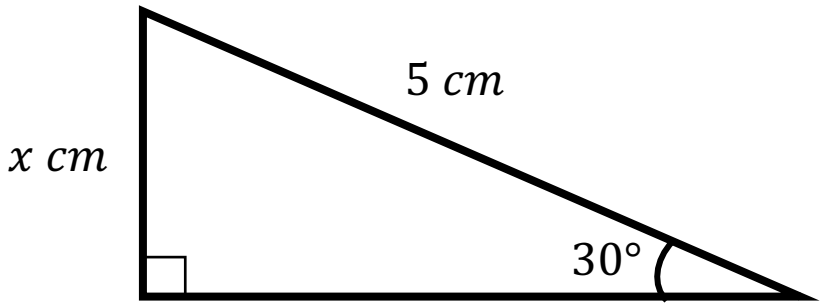
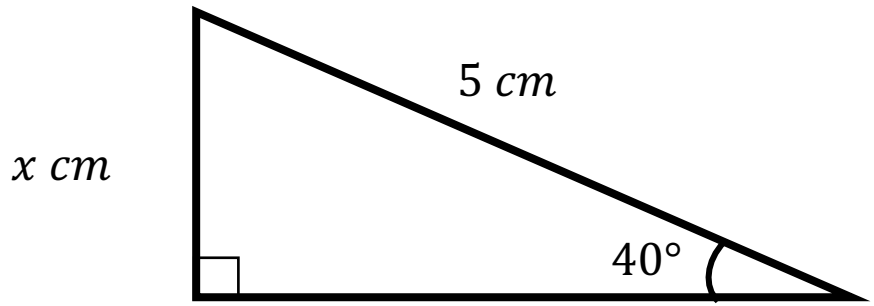
Sine (sin) $\quad$ Cosine (cos) $\quad$ Tangent (tan)

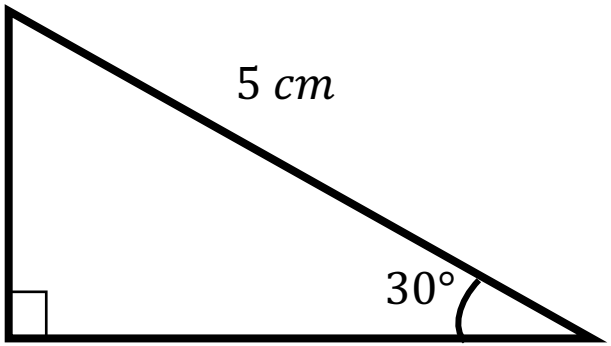
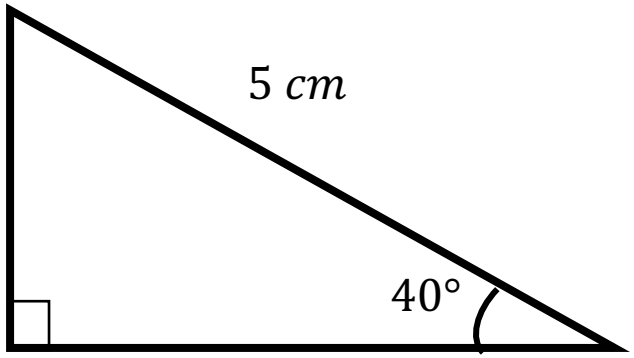
So altogether if we have:

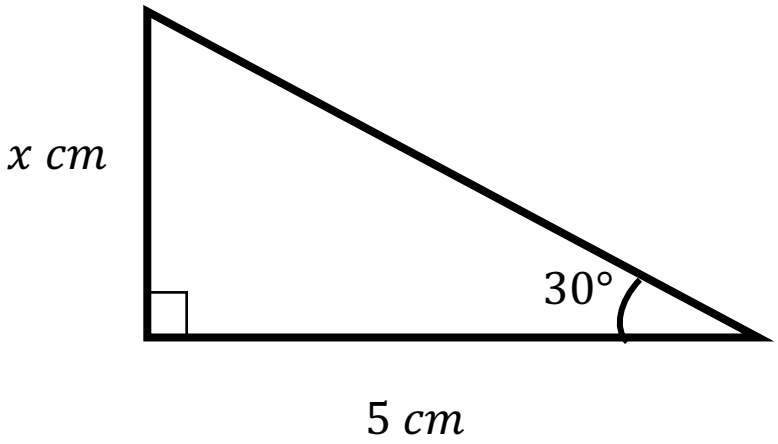
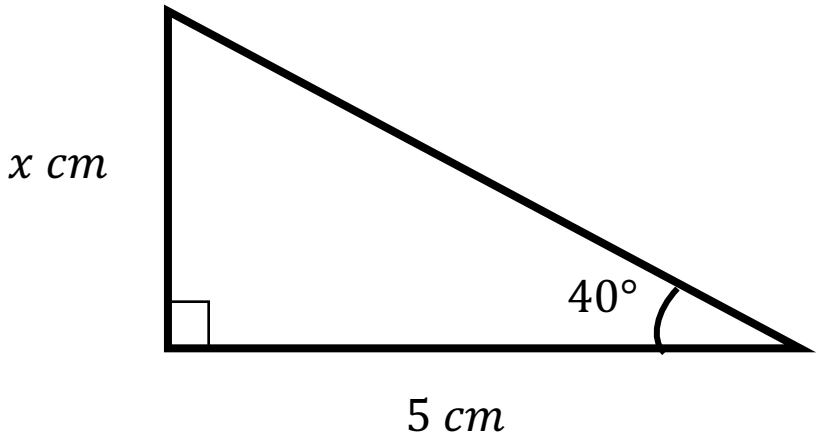


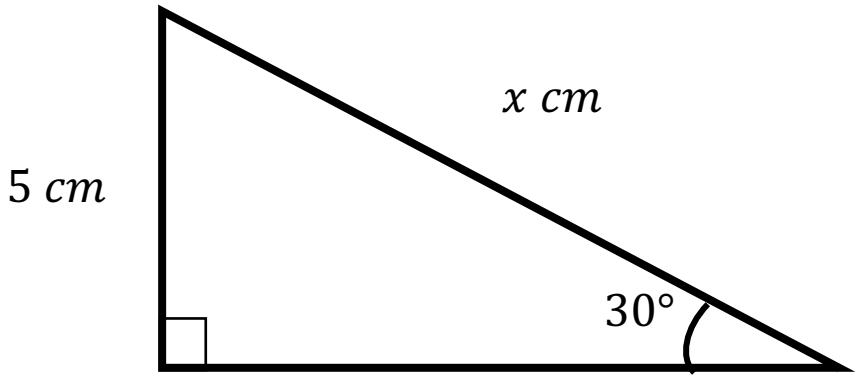
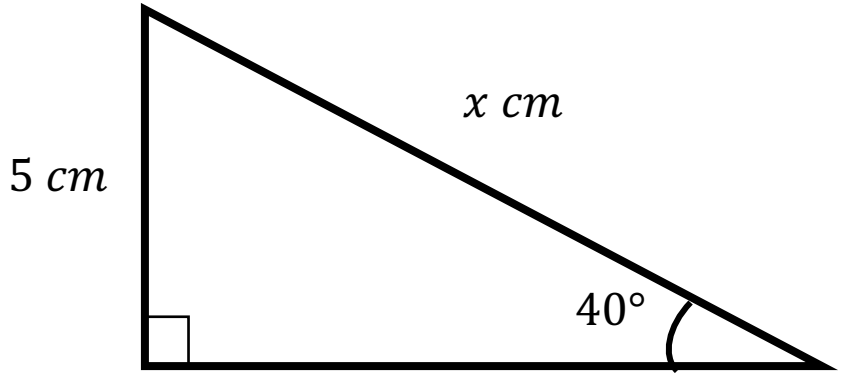
Then: $\sin(x°) = \frac{opp}{hyp}$ $\cos(x°) = \frac{adj}{hyp}$ $\tan(x°) = \frac{opp}{adj}$

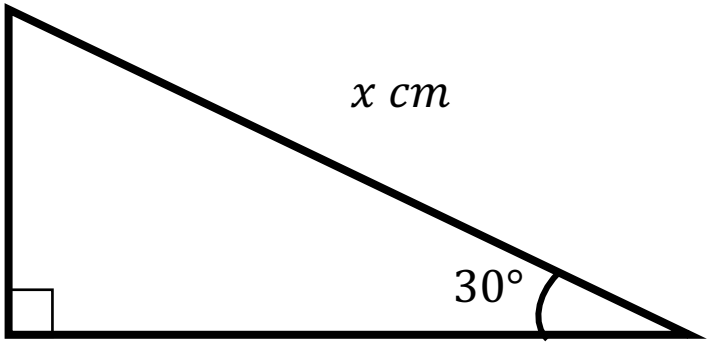
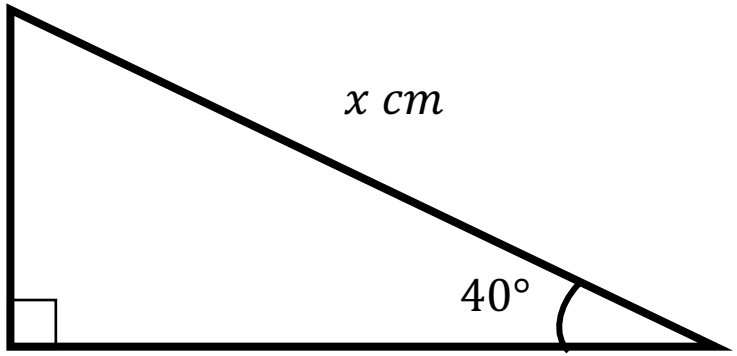
This is commonly given the acronym: **SOHCAHTOA**

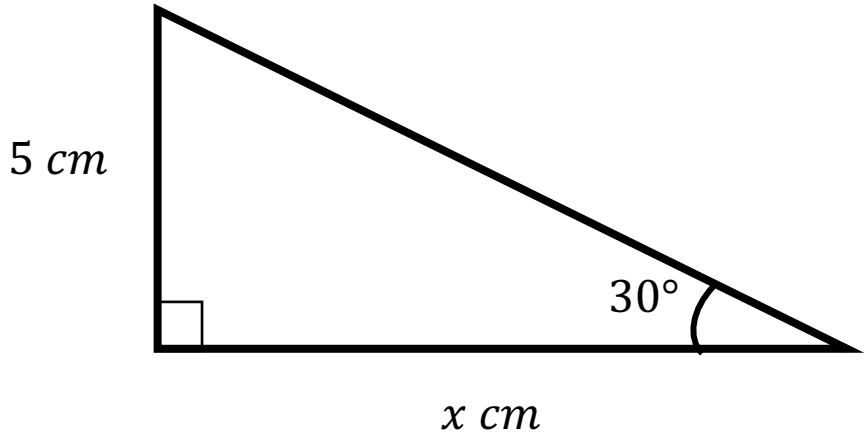
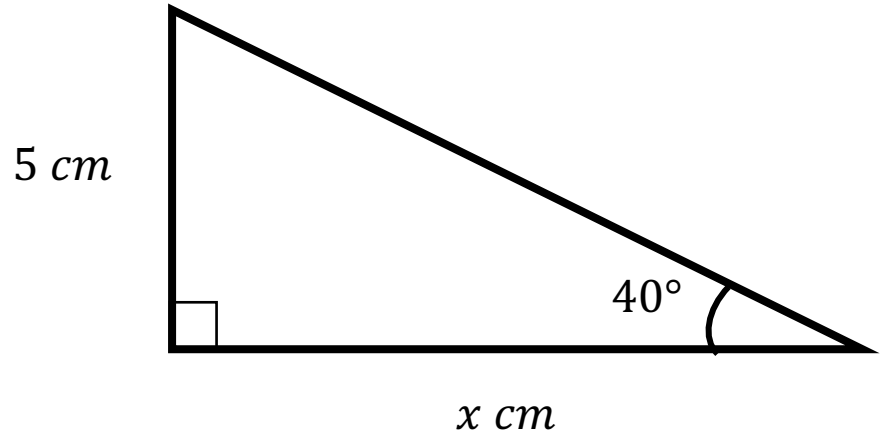
Worked Example	Your Turn
Calculate x:  	Calculate x: 

Worked Example	Your Turn
Calculate x:  	Calculate x: 

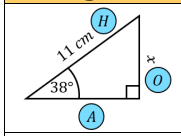
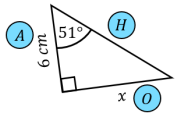
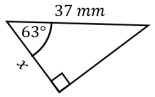
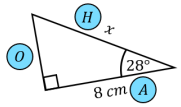
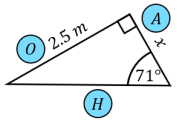
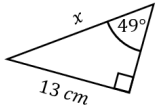
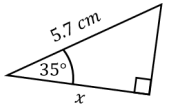
Worked Example	Your Turn
Calculate x:  	Calculate x: 

Worked Example	Your Turn
Calculate x:  	Calculate x: 

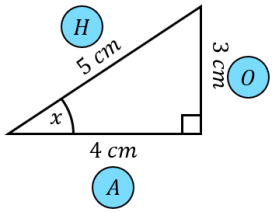
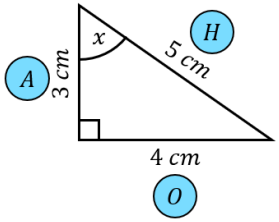
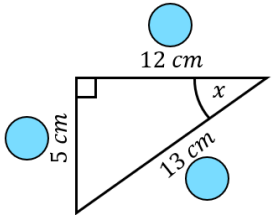
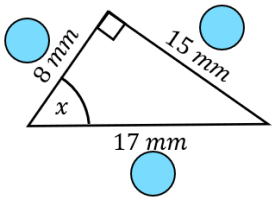
Worked Example	Your Turn
Calculate x:  $5\ cm$	Calculate x:  $5\ cm$

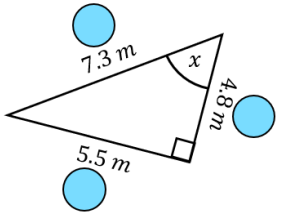
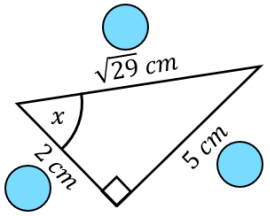
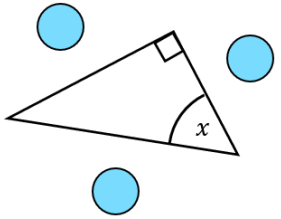
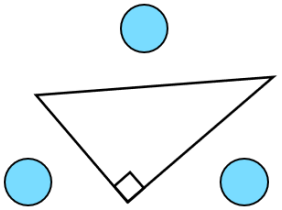
Worked Example	Your Turn
Calculate x:  	Calculate x: 

Fill in the Gaps

Labelled diagram	Choose ratio	Substitute into formula	Rearrange formula	Answer (1dp)
	sin	$\sin 38 = \frac{x}{11}$	$x = 11 \times \sin 38$	
	tan			
				
	cos	$\cos 28 = \frac{8}{x}$	$x = \frac{8}{\cos 28}$	
	tan			
				
				
		$\tan 68 = \frac{7}{x}$		

Fill in the Gaps

Labelled diagram	Sine Ratio	Cosine Ratio	Tangent Ratio
	$\sin x = \frac{3}{5}$	$\cos x = \frac{4}{5}$	$\tan x = \frac{\square}{\square}$
	$\sin x = \frac{\square}{\square}$	$\cos x = \frac{\square}{\square}$	$\tan x = \frac{\square}{\square}$
	$\sin x = \frac{\square}{\square}$	$\cos x = \frac{\square}{\square}$	$\tan x = \frac{\square}{\square}$
	$\sin x = \frac{\square}{\square}$	$\cos x = \frac{\square}{\square}$	$\tan x = \frac{\square}{\square}$

Labelled diagram	Sine Ratio	Cosine Ratio	Tangent Ratio
	$\sin x = \frac{\square}{\square}$	$\cos x = \frac{\square}{\square}$	$\tan x = \frac{\square}{\square}$
	$\sin x = \frac{\square}{\square}$	$\cos x = \frac{\square}{\square}$	$\tan x = \frac{\square}{\square}$
	$\sin x = \frac{\square}{\square}$	$\cos x = \frac{\square}{\square}$	$\tan x = \frac{9.9}{2}$
	$\sin x = \frac{4}{7}$	$\cos x = \frac{\square}{\square}$	$\tan x = \frac{\square}{\square}$

Inverse Trigonometric Functions

We have met the idea that: $f(x) = y$ so $f^{-1}(y) = x$

The trigonometric functions sin, cos and tan are all functions where the input is an angle giving an output which is a ratio of sides.

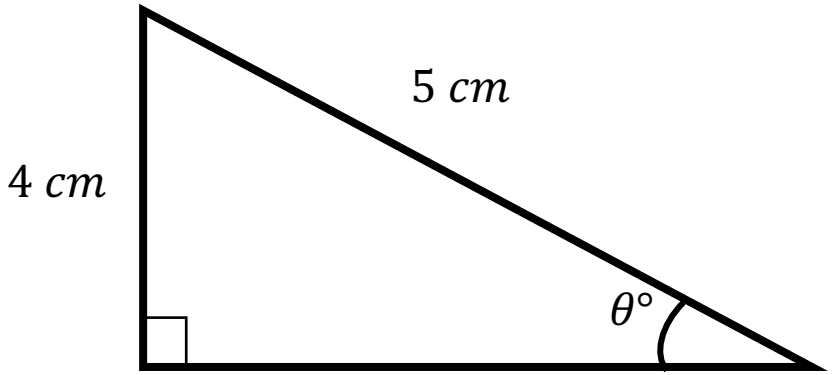
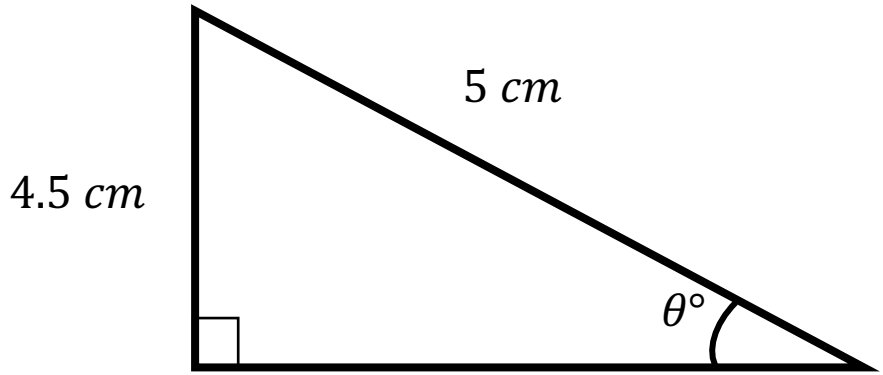
The inverse of these functions therefore does this in reverse.

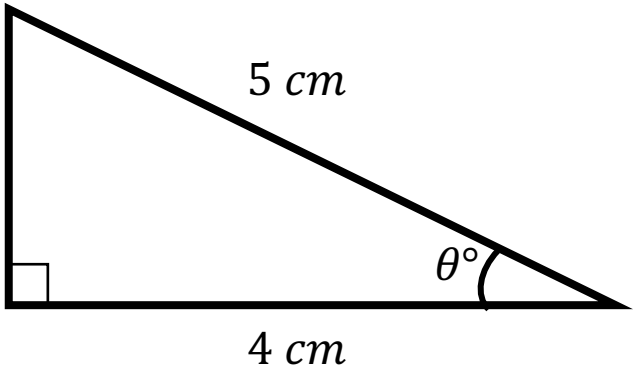
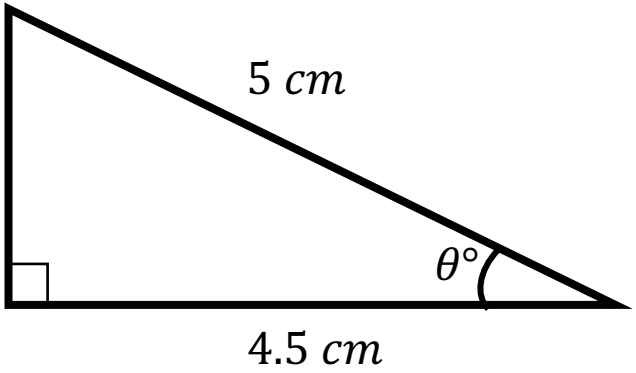
if $\sin(30°) = 0.5$ then $\sin^{-1}(0.5) = 30°$

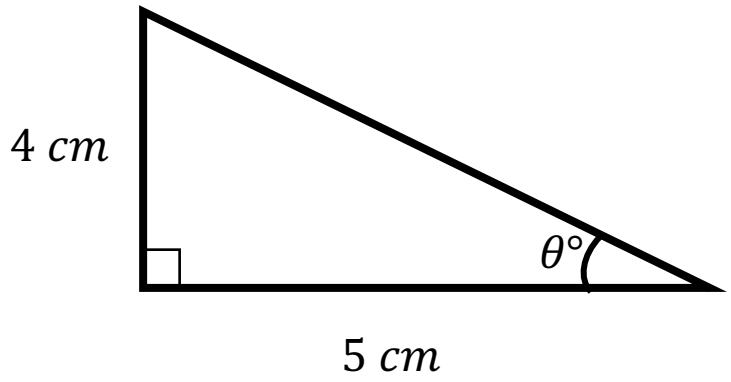
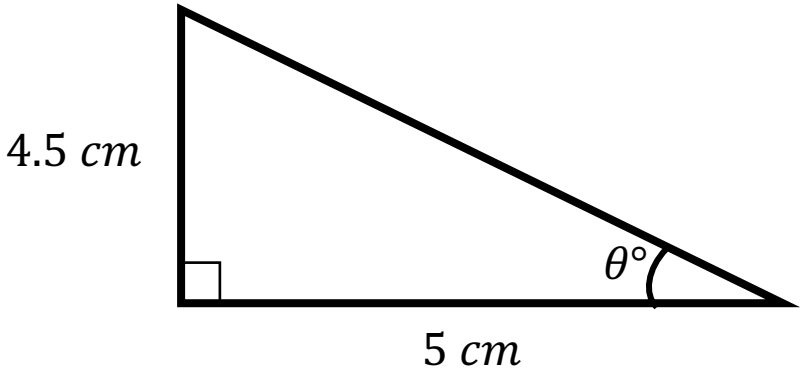
if $\cos(60°) = 0.5$ then $\cos^{-1}(0.5) = 60°$

if $\tan(45°) = 1$ then $\tan^{-1}(1) = 45°$

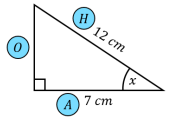
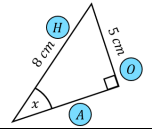
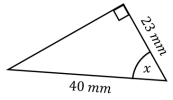
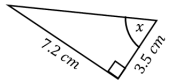
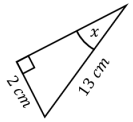
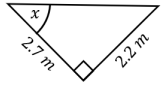
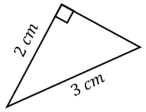
Worked Example	Your Turn
Find 'x'. Give your solution to 2 decimal places. $\sin(x) = \frac{1}{2}$	Find 'x'. Give your solution to 2 decimal places. $\sin(x) = \frac{2}{5}$

Worked Example	Your Turn
Calculate θ:  	Calculate θ: 

Worked Example	Your Turn
Calculate θ:  	Calculate θ: 

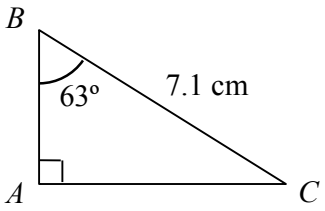
Worked Example	Your Turn
Calculate θ:  	Calculate θ: 

Fill in the Gaps

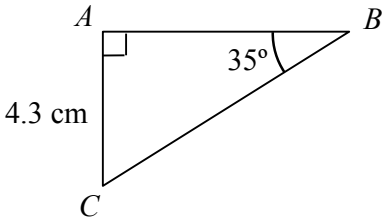
Labelled diagram	Choose ratio	Substitute into formula	Rearrange formula	Answer (1dp)
	cos	$\cos x = \frac{7}{12}$	$x = \cos^{-1}\left(\frac{7}{12}\right)$	
	sin			
				
				
				
				
		$\cos x = \frac{2}{3}$		
			$x = \tan^{-1}\left(\frac{15}{11}\right)$	

Fluency Practice

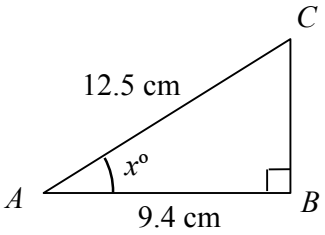
A1 Find length *AB*



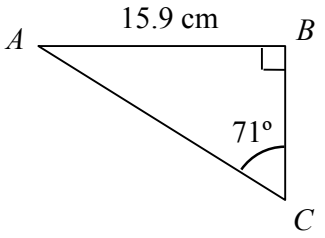
A2 Find length *AB*



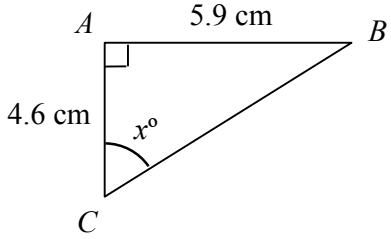
A3 Find angle *BAC*



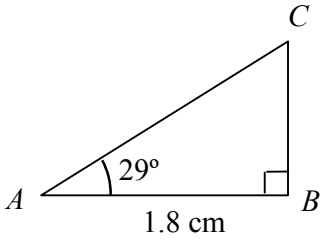
A4 Find length *AC*



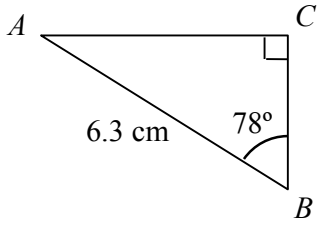
B1 Find angle *ACB*



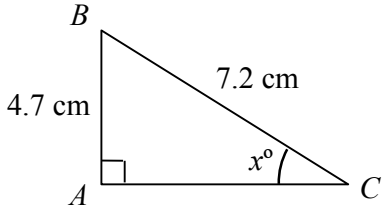
B2 Find length *BC*



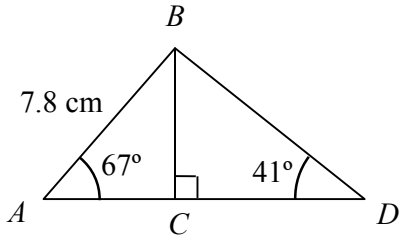
B3 Find length *AC*



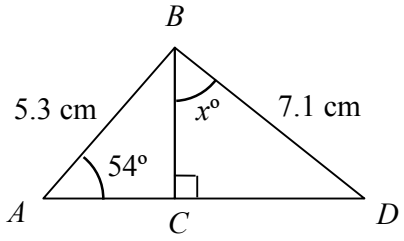
B4 Find angle *ACB*



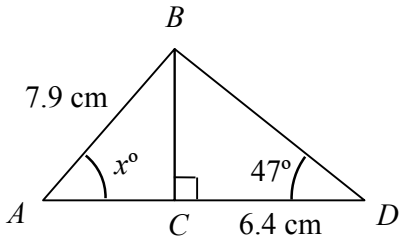
C1 Find length *CD*



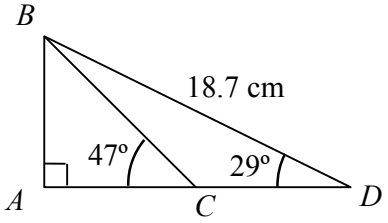
C2 Find angle *CBD*



C3 Find angle *BAC*



C4 Find length *CD*



Extra Notes

4 Compound Measures

Compound measures are measures that rely on other measures:

- Speed
- Density
- Pressure

Speed

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Worked Example	Your Turn
A car travels 50 miles in 2 hours. What speed does it travel at?	A car travels 60 miles in 2 hours. What speed does it travel at?
A car travels at $50mph$ (miles per hour) for 2 hours. How far does it travel?	A car travels at $60mph$ (miles per hour) for 2 hours. How far does it travel?
A car travels 50 miles at $25mph$ (miles per hour). How long does it take?	A car travels 30 miles at $60mph$ (miles per hour). How long does it take?

Worked Example	Your Turn
The distance from A to B is 5 miles. The distance from B to C is 9 miles. Person X drives from A to B then B to C. X leaves A at $10:00$. X drives from A to B at an average speed of 40 miles per hour. X wants to get to C at $10{:}35$. Work out the average speed X must drive from B to C.	The distance from A to B is 10 miles. The distance from B to C is 18 miles. Person X drives from A to B then B to C. X leaves A at $10:00$. X drives from A to B at an average speed of 40 miles per hour. X wants to get to C at $10{:}35$. Work out the average speed X must drive from B to C.

Fill in the Gaps

Distance	Time	Speed	Units of Speed
120 km	4 $hours$		km/h
55 m	5 $seconds$		m/s
8000 m	2 $hours$		km/h
450 km	180 $minutes$		km/h
	20 $seconds$	10	m/s
	3 $hours$	25	km/h
900 cm	3 $seconds$		m/s
132 m		12	m/s
640 km		80	km/h
	120 $minutes$	65	km/h
30 m	1 $minute$		m/s
1750 cm		2.5	m/s
	150 $minutes$	88	km/h
	1.5 $minutes$	8.5	m/s
20000 m	30 $minutes$	40	

Density

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Worked Example	Your Turn
The mass of an object is $50\ g$. The volume is $10\ cm^3$. What is the density of the object?	The mass of an object is $100\ g$. The volume is $25\ cm^3$. What is the density of the object?
The density of an object is $10\ g/cm^3$. The volume is $5\ cm^3$. What is the mass?	The density of an object is $10\ g/cm^3$. The volume is $25\ cm^3$. What is the mass?
The density of an object is $10\ g/cm^3$. The mass is $50\ g$. What is the volume?	The density of an object is $10\ g/cm^3$. The mass is $25\ g$. What is the volume?

Worked Example	Your Turn
A drink is made from: $100\ g$ of liquid A. $150\ g$ of liquid B. Liquid A has density $1.05\ g/cm^3$. Liquid B has density $0.8\ g/cm^3$. Work out the density of the drink.	A drink is made from: $200\ g$ of liquid A. $150\ g$ of liquid B. Liquid A has density $2.1\ g/cm^3$. Liquid B has density $0.4\ g/cm^3$. Work out the density of the drink.

Worked Example	Your Turn
A drink is made from: $100\ cm^3$ of liquid A. $150\ cm^3$ of liquid B. Liquid A has density $1.05\ g/cm^3$. Liquid B has density $0.8\ g/cm^3$. Work out the density of the drink.	A drink is made from: $200\ cm^3$ of liquid A. $150\ cm^3$ of liquid B. Liquid A has density $2.1\ g/cm^3$. Liquid B has density $0.4\ g/cm^3$. Work out the density of the drink.

Pressure

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

Worked Example	Your Turn
The force exerted by an object on a surface is $50N$. The surface area in contact with the object is $10cm^2$. What is the pressure exerted by the object?	The force exerted by an object on a surface is $100N$. The surface area in contact with the object is $25cm^2$. What is the pressure exerted by the object?
The pressure exerted on a surface by an object is $50N/cm^2$. The surface area in contact with the object is $10cm^2$. What is the force exerted?	The pressure exerted on a surface by an object is $100N/cm^2$. The surface area in contact with the object is $25cm^2$. What is the force exerted?
The pressure exerted on a surface by an object is $50N/cm^2$. The force exerted on the surface is $10N$. What is the surface area in contact with the object?	The pressure exerted on a surface by an object is $100N/cm^2$. The force exerted on the surface is $25N$. What is the surface area in contact with the object?

Extra Notes