KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 9

2023 Mathematics 2024

Unit 11 Booklet

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

Contents

1 Fraction Arithmetic

Extra Notes

2 Highest Common Factor and Lowest Common Multiple

- The HCF is the largest integer which is a factor of two or more given positive integers.
- The HCF will be less than or equal to the smallest of the given numbers.

- The LCM is the smallest integer which is a multiple of two or more positive integers.
- The LCM will be greater than or equal to the largest of the numbers.

Worked Example	Your Turn
Find the HCF and LCM of $2^2 \times 3^2 \times 5^2 \times 11$ $2^3 \times 3 \times 5^2 \times 7$	Find the HCF and LCM of $2 \times 3^3 \times 5 \times 7^2$ $2^2 \times 3^2 \times 7^2 \times 11$

Worked Example	Your Turn
Find the HCF and LCM of 123 and 456	Find the HCF and LCM of 321 and 654

Worked Example	Your Turn
The HCF of two numbers is 6. The LCM of two numbers is 60. Write down two possible numbers.	The HCF of two numbers is 3. The LCM of two numbers is 36. Write down two possible numbers.

Worked Example	Your Turn
The HCF of two numbers is 5. The LCM of two numbers is a multiple of 12. Write down two possible numbers.	The HCF of two numbers is 8. The LCM of two numbers is a multiple of 5. Write down two possible numbers.

Worked Example	Your Turn
Two strings of different lengths, 240 cm and 318 cm are to be cut into equal integer lengths. What is the greatest possible length of each piece?	Two strings of different lengths, 212 cm and 360 cm are to be cut into equal integer lengths. What is the greatest possible length of each piece?

Worked Example	Your Turn
Two lighthouses flash their lights every 240 s and 318 s respectively. They both flash at the same time. After how many seconds will they next both flash at the same time.	Two lighthouses flash their lights every 212 s and 360 s respectively. They both flash at the same time. After how many seconds will they next both flash at the same time.

Extra Notes

3 Standard Form

Standard form is written in the form of $a \times 10^n$, where a is a number bigger than or equal to 1 and less than 10 (i.e. $1 \leq a < 10$). n can be any positive or negative whole number.

Note: a can be any positive or negative number.

In Standard Form	Not in Standard Form
7.3×10^3	$438{,}000$
1×10^{-3}	54×10^7
9.36×10^{18}	0.6×10^{-4}
4×10^1	389×10000
5.002×10^{-7}	$6 \times 10^{1.5}$
-1.729×10^{211}	0.000372

Why use standard form?

- It allows us to write really small or really big numbers concisely.
- It allows us to easily compare small and big numbers.

Intelligent Practice

Decide if the following numbers are in standard form

3×10^5

3×10^6

3×10^{67}

$3 \times 10^{6.7}$

$3 \times 10^{0.67}$

$3 \times 10^{0.7}$

3×10^7

3×10^{-7}

$3 \times 10^{-0.7}$

3×11^5

3×100^5

3×10.5^5

3×10.5^5

3×-10^5

$3 \times (-10)^5$

$3 \div 10^5$

$3 + 10^5$

$3 - 10^5$

4×10^5

40×10^5

46×10^5

4.6×10^5

0.46×10^5

3.46×10^5

3.46434561×10^5

-3.46434561×10^5

Fluency Practice

10^6	1 000 000	10 x 10 x 10 x 10 x 10 x 10
10^5		
10^4		
10^3		10 x 10 x 10
10^2		
10^1	10	
10^{-1}		
		$\frac{1}{10} \times \frac{1}{10}$
	$\frac{1}{1000}$	
10^{-4}		

Complete this part first.

Look for patterns in the columns to complete the table.

Worked Example	Your Turn
Write the following numbers in standard form a) $70{,}000$ b) $72{,}000$ c) $720{,}000$	Write the following numbers in standard form a) $63{,}000$ b) $630{,}000$ c) $60{,}000$

Worked Example	Your Turn
Write the following numbers in standard form a) 4367×10^{6} b) 0.125×10^{-6}	Write the following numbers in standard form a) 0.4367×10^{6} b) 125×10^{-6}

Worked Example	Your Turn
Write the following numbers in standard form a) 0.05 b) 0.005 c) 0.00572	Write the following numbers in standard form a) 0.006 b) 0.00683 c) 0.06

Worked Example	Your Turn
Write the following as an ordinary number 3.1×10^6	Write the following as an ordinary number 3.2×10^7

Worked Example	Your Turn
Write the following as an ordinary number 4.1×10^{-6}	Write the following as an ordinary number 4.2×10^{-7}

Worked Example	Your Turn
Put the following numbers in ascending order 5.77×10^6 8.85×10^6 6.35×10^6 2.6×10^5 3.9×10^5	Put the following numbers in ascending order 1.2×10^6 8.4×10^7 8.7×10^6 7×10^6 3.04×10^7

Worked Example	Your Turn
Work out a) $(3 \times 10^5) \times (2 \times 10^4)$ b) $(3 \times 10^5) \times (4 \times 10^{-4})$	Work out a) $(3 \times 10^5) \times (4 \times 10^4)$ b) $(3 \times 10^{-5}) \times (2 \times 10^4)$

Worked Example	Your Turn
Work out a) $(4 \times 10^9) \div (2 \times 10^3)$ b) $(2 \times 10^5) \div (8 \times 10^{-4})$	Work out a) $(2 \times 10^9) \div (4 \times 10^3)$ b) $(8 \times 10^5) \div (2 \times 10^{-4})$

Calculator

Use the $\times\mathbf{10}^{\boldsymbol{x}}$ button on your calculator to make calculations involving standard form. While you can explicitly write 3×10^7 using the $\boldsymbol{x}^{\boldsymbol{y}}$ button, it is faster to use the specialised standard form key.

Check the following using your calculator:
$(2.41 \times 10^{19}) \times (7.1 \times 10^{23}) = \mathbf{1.7111} \times \mathbf{10}^{\mathbf{43}}$

Worked Example	Your Turn
Work out a) $(3 \times 10^4) + (4 \times 10^4)$ b) $(3 \times 10^4) + (8 \times 10^4)$ c) $(3 \times 10^5) + (8 \times 10^4)$	Work out a) $(3 \times 10^7) + (2 \times 10^7)$ b) $(3 \times 10^7) + (9 \times 10^7)$ c) $(3 \times 10^8) + (9 \times 10^7)$

Worked Example	Your Turn
Work out a) $(7 \times 10^4) - (4 \times 10^4)$ b) $(7 \times 10^4) - (0.4 \times 10^4)$ c) $(7 \times 10^5) - (0.4 \times 10^4)$	Work out a) $(6 \times 10^7) - (2 \times 10^7)$ b) $(6 \times 10^7) - (0.2 \times 10^7)$ c) $(6 \times 10^7) - (0.2 \times 10^8)$

Worked Example	Your Turn
Work out a) $(4 \times 10^{-1}) + (3 \times 10^{-2})$ b) $(7 \times 10^{-3}) - (2 \times 10^{-4})$	Work out a) $(8 \times 10^{-2}) + (2 \times 10^{-3})$ b) $(2 \times 10^{-2}) - (5 \times 10^{-3})$

Worked Example	Your Turn
Calculate $\frac{(4.6 \times 10^4) + (1.5 \times 10^3)}{(2 \times 10^2)}$	Calculate $\frac{(4.5 \times 10^4) + (1.3 \times 10^2)}{(2 \times 10^2)}$

Standard Form: Percentages

Complete the table using standard form numbers.

100%	1%	5%	10%	20%	50%
4×10^5					
8×10^7					
			3×10^2		
	1×10^6				
					3×10^{-3}
		7.5×10^0			
				8.6×10^{-6}	

a) 20% of 6×10^9 =

b) 30% of 9×10^4 =

c) 90% of 5×10^7 =

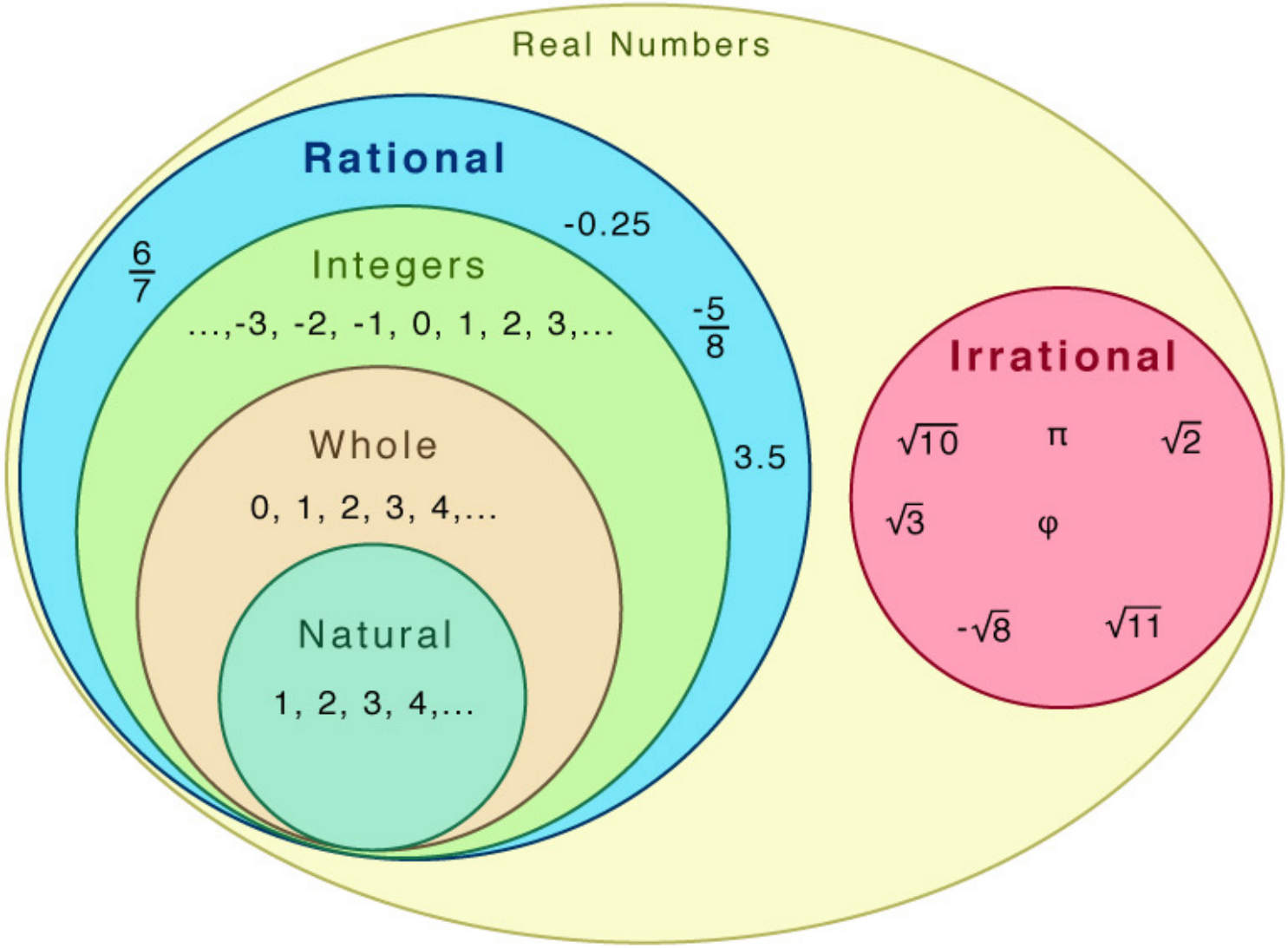
d) 2% of 1.7×10^7 =

e) 75% of 1×10^{-3} =

f) 120% of 9×10^6 =

Extra Notes

4 Types of Numbers



Classify each number below as either rational or irrational. If you believe your number is rational, prove your answer by writing it as a fraction. The first one is done for you.

	Rational or Irrational?	Fraction?
1) 0.8	Rational	$\frac{8}{10}$ or $\frac{4}{5}$
2) $-\frac{3}{10}$		
3) $\sqrt{40}$		
4) $\sqrt{81}$		
5) $2\frac{1}{3}$		
6) 0.35		
7) $0.33333\ldots$		
8) -9		
9) 3.4		
10) $\sqrt{2}$		

Directions: For each number shown, classify it as either rational or irrational, then tell whether or not it is terminating or repeating.

		(circle one)		*(circle one)*
11)	-0.6	rational or irrational		terminating, repeating, or neither
12)	$\sqrt{100}$	rational or irrational		terminating, repeating, or neither
13)	$\frac{2}{5}$	rational or irrational		terminating, repeating, or neither
14)	$-\frac{2}{3}$	rational or irrational		terminating, repeating, or neither
15)	$0.35217534\ldots$	rational or irrational		terminating, repeating, or neither

Extra Notes

5 Simplifying Surds

When the root (square root, cube root or higher root) of a number cannot be obtained exactly, the root is called a surd. A surd cannot be written as a fraction but can be written as a decimal, that goes on forever, without repeating (recurring) or ending (terminating). Hence, surds are irrational roots.

Surds	Not Surds
$\sqrt{8}$	8
$\sqrt{10}$	-12.05
$\sqrt{91}$	0.62
$\sqrt[3]{7}$	$\frac{3}{7}$
$\sqrt[3]{16}$	$7\frac{1}{2}$
$\sqrt[4]{73}$	$\sqrt{16}$
$2\sqrt{2}$	$\sqrt{25}$
$2+\sqrt{5}$	$\sqrt[3]{8}$
$(2+\sqrt{5})(3+\sqrt{5})$	$\sqrt{2.25}$
$\frac{1}{5-\sqrt{17}}$	$\frac{\sqrt{100}}{\sqrt{4}}$

Intelligent Practice

Decide if the following numbers are surds

$\sqrt{1}$

$\sqrt{4}$

$\sqrt{9}$

$\sqrt{36}$

$\sqrt{6}$

$\sqrt{24}$

$\sqrt{3}$

$2\sqrt{3}$

$3\sqrt{3}$

$3\sqrt{4}$

$\sqrt{5}$

$\sqrt{5^2}$

$\frac{1}{(\sqrt{5})^2}$

$\frac{\sqrt{1}}{\sqrt{4}}$

$\sqrt{\frac{1}{4}}$

$\sqrt{\frac{2}{8}}$

$\sqrt{\frac{2}{9}}$

$\sqrt{\frac{4}{9}}$

$\frac{2}{\sqrt{9}}$

$\frac{\sqrt{7}}{2}$

$\sqrt{0.25}$

$\sqrt{0.125}$

$\sqrt{0.01}$

$(\sqrt{2})^2$

$(\sqrt{2})^3$

$\sqrt{2}(\sqrt{2}+3)$

$(\sqrt{2}+3)(\sqrt{2}-3)$

$\frac{2}{\sqrt{2}}\sqrt{2}$

$\frac{2}{3+\sqrt{2}}$

$\frac{2}{\frac{3}{\sqrt{2}}+\sqrt{2}}$

Question	As a decimal or whole number	Is it a surd?	Question	As a decimal or whole number	Is it a surd?
$\sqrt{1}$	1	No	$\sqrt{16}$		
$\sqrt{2}$	1.4142135 …	Yes	$\sqrt{17}$		
$\sqrt{3}$	1.7320508 …	Yes	$\sqrt{18}$		
$\sqrt{4}$	2	No	$\sqrt{19}$		
$\sqrt{5}$			$\sqrt{20}$		
$\sqrt{6}$			$\sqrt{21}$		
$\sqrt{7}$			$\sqrt{22}$		
$\sqrt{8}$			$\sqrt{23}$		
$\sqrt{9}$			$\sqrt{24}$		
$\sqrt{10}$			$\sqrt{25}$		
$\sqrt{11}$			$\sqrt{26}$		
$\sqrt{12}$			$\sqrt{27}$		
$\sqrt{13}$			$\sqrt{28}$		
$\sqrt{14}$			$\sqrt{29}$		
$\sqrt{15}$			$\sqrt{30}$		

Worked Example	Your Turn
Simplify a) $\sqrt{60}$ b) $\sqrt{120}$	Simplify a) $\sqrt{50}$ b) $\sqrt{200}$

Fill in the Gaps

Square Numbers	1	4	9	16	25	36	49	64	81	100

Question	Largest Square Number Factor	Split into Two Surds	Rationalise the Square Number	Answer
$\sqrt{27}$	9	$\sqrt{9} \times \sqrt{3}$	$3 \times \sqrt{3}$	$3\sqrt{3}$
$\sqrt{24}$	4	$\sqrt{4} \times \sqrt{6}$		
$\sqrt{50}$	25			
$\sqrt{28}$				
$\sqrt{32}$				
$\sqrt{45}$				
$\sqrt{72}$				
$\sqrt{90}$				
$\sqrt{75}$				
$\sqrt{200}$				
$\sqrt{98}$				
$\sqrt{80}$				
		$\sqrt{9} \times \sqrt{7}$	$3 \times \sqrt{7}$	$3\sqrt{7}$
				$7\sqrt{3}$

Worked Example	Your Turn
Simplify a) $\sqrt{6} \times \sqrt{6}$ b) $\left(\sqrt{6}\right)^2$ c) $\left(2\sqrt{6}\right)^2$ d) $2\left(\sqrt{6}\right)^2$ e) $2\left(\sqrt{6}\right)^3$	Simplify a) $\sqrt{7} \times \sqrt{7}$ b) $\left(\sqrt{7}\right)^2$ c) $\left(2\sqrt{7}\right)^2$ d) $2\left(\sqrt{7}\right)^2$ e) $2\left(\sqrt{7}\right)^3$

Fill in the Gaps

Question	Surd as a Product of its Prime Factors	Simplify 'Repeated' Surds	Answer
$\sqrt{12}$	$\sqrt{2} \times \sqrt{2} \times \sqrt{3}$	$2 \times \sqrt{3}$	$2\sqrt{3}$
$\sqrt{45}$	$\sqrt{3} \times \sqrt{3} \times \sqrt{5}$		
$\sqrt{18}$	$\sqrt{2} \times \sqrt{3} \times \sqrt{3}$		
$\sqrt{75}$			
$\sqrt{20}$			
	$\sqrt{7} \times \sqrt{7} \times \sqrt{2}$		$7\sqrt{2}$
			$3\sqrt{7}$
$\sqrt{48}$	$\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{3}$	$2 \times 2 \times \sqrt{3}$	$4\sqrt{3}$
$\sqrt{72}$	$\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{3} \times \sqrt{3}$		
$\sqrt{200}$			
$\sqrt{162}$			
$\sqrt{675}$			
		$2 \times 3 \times \sqrt{5}$	$6\sqrt{5}$
			$10\sqrt{3}$
			$8\sqrt{7}$

Worked Example	Your Turn
Simplify a) $2\sqrt{20}$ b) $4\sqrt{40}$	Simplify a) $3\sqrt{20}$ b) $4\sqrt{50}$

Worked Example	Your Turn
Write the following as a single root a) $2\sqrt{15}$ b) $2\sqrt{30}$	Write the following as a single root a) $5\sqrt{2}$ b) $10\sqrt{2}$

Splitting Surds All these roots are surds: complete the missing **radicands**.

a) $\sqrt{2} \times \sqrt{3} = \sqrt{\ \times\ } = \sqrt{6}$

b) $\sqrt{3} \times \sqrt{\ } = \sqrt{\ \times 5} = \sqrt{\ }$

c) $\sqrt{2} \times \sqrt{7} = \sqrt{\ } = \sqrt{\ }$

d) $\sqrt{5} \times \sqrt{6} = \sqrt{\ } = \sqrt{\ }$

e) $\sqrt{\ } \times \sqrt{\ } = \sqrt{3 \times 7} = \sqrt{\ }$

f) $\sqrt{6} \times \sqrt{\ } = \sqrt{\ } = \sqrt{42}$

g) $\sqrt{\ } \times \sqrt{\ } = \sqrt{\ } = \sqrt{55}$

h) $\sqrt{\ } \times \sqrt{\ } = \sqrt{\ } = \sqrt{34}$

i) $\sqrt{\ } \times \sqrt{\ } = \sqrt{\ } = 5$

j) $\sqrt{\ } \times \sqrt{\ } = \sqrt{\ } = \sqrt{77}$

k) $\sqrt{\ } \times \sqrt{\ } = \sqrt{\ } = \sqrt{65}$

l) $\sqrt{\ } \times \sqrt{\ } = \sqrt{\ } = \sqrt{102}$

m) $\sqrt{\ } \times \sqrt{\ } \times \sqrt{\ } = \sqrt{\ } = \sqrt{66}$

n) $\sqrt{\ } \times \sqrt{\ } \times \sqrt{\ } = \sqrt{\ } = \sqrt{210}$

Extra Notes

6 Angles in Polygons

Frayer Model – Polygons

Definition	**Characteristics**
Examples	**Non-Examples**

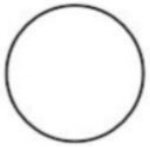
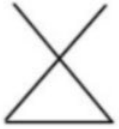
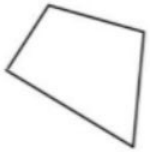
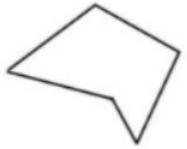
Frayer Model – Regular Polygons

Definition	**Characteristics**
Examples	**Non-Examples**

Fluency Practice

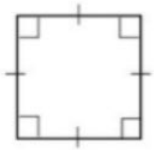
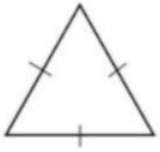
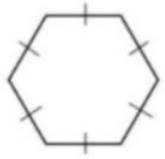
Polygons – Example or Non-Example

In each of the following diagrams decide whether the shape is a polygon or not. Label them 'Example' or 'Non-example'. For those that ARE polygons, give the name of the polygon.

A 	B 	C 	D 	E 
F 	G 	H 	I 	J 

Polygons – Regular or Irregular

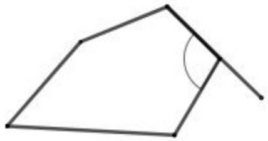
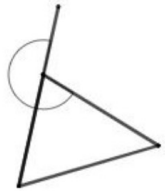
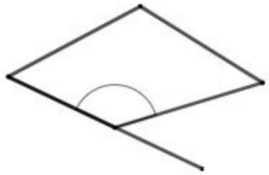
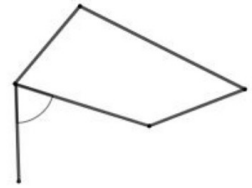
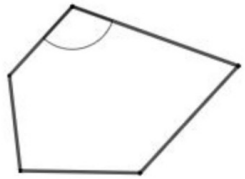
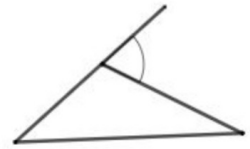
Which of the following are regular and which are irregular – how do you know?

A 	B 	C 	D 	E 	F 

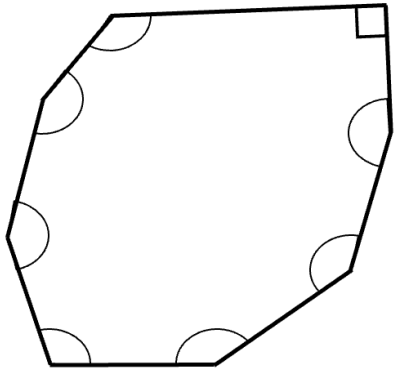
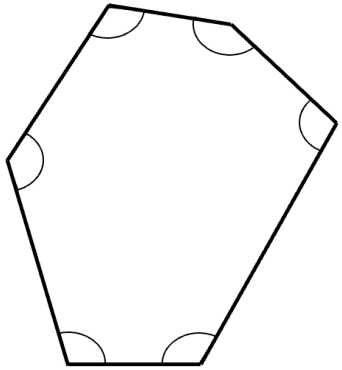
Interior and Exterior Angle Formulae

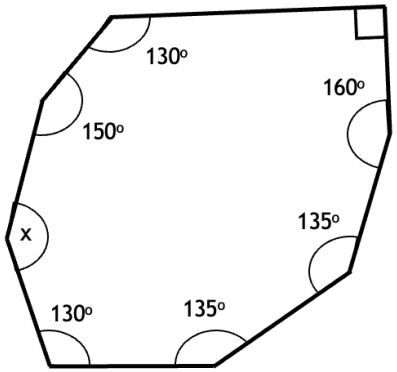
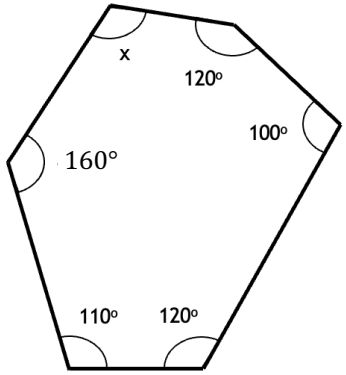
All Polygons	Regular Polygons
$\text{Interior Angle} + \text{Exterior Angle} = 180°$ $\text{Sum of Interior Angles} = (n-2) \times 180°$ $\text{Sum of Exterior Angles} = 360°$	$\text{Each Exterior Angle} = \frac{360°}{n}$ $\text{Each Interior Angle} = 180° - \frac{360°}{n}$

Interior Angles

Examples	Nonexamples
	
	
	

Worked Example	Your Turn
Find the sum of the interior angles of a polygon with 30 sides.	Find the sum of the interior angles of a polygon with 60 sides.

Worked Example	Your Turn
Find the sum of interior angles of this polygon. 	Find the sum of interior angles of this polygon. 

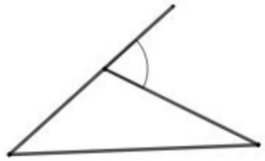
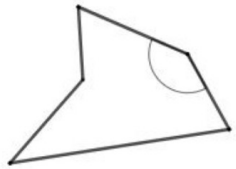
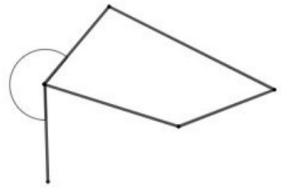
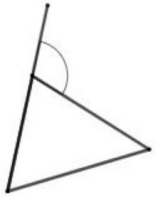
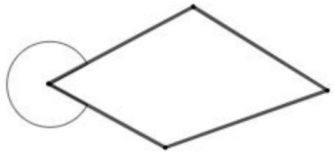
Worked Example	Your Turn
Find angle x.  	Find angle x. 

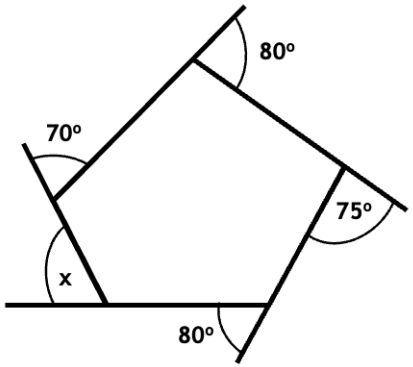
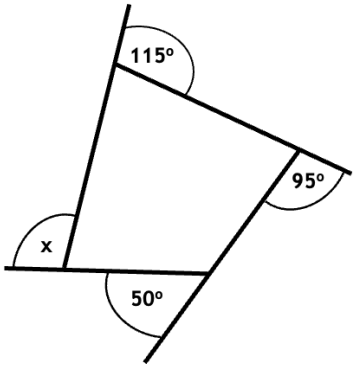
Worked Example	Your Turn
The sum of the interior angles of a polygon is $3240°$. How many sides does the polygon have?	The sum of the interior angles of a polygon is $6840°$. How many sides does the polygon have?

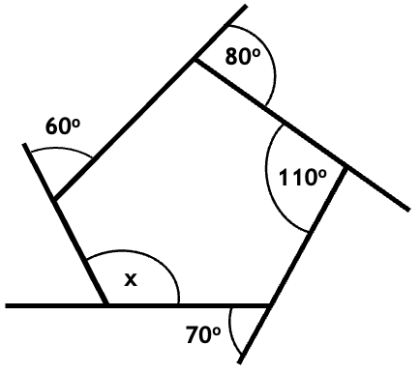
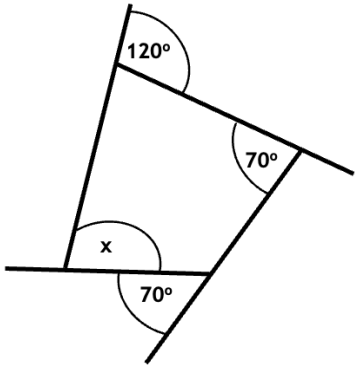
Fill in the Gaps

Number of sides	Sum of interior angles	Size of one interior angle in a regular polygon
3	180°	
	360°	
7		
9		
10		144°
	1800°	150°
13	1980°	
14		
	2700°	

Exterior Angles

Examples	Nonexamples
	
	
	

Worked Example	Your Turn
Find angle x.  	Find angle x. 

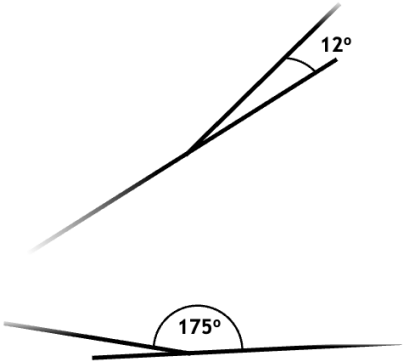
Worked Example	Your Turn
Find angle x.  	Find angle x. 

Worked Example	Your Turn
A regular polygon has 12 sides. Find the size of each exterior angle.	A regular polygon has 48 sides. Find the size of each exterior angle.

Worked Example	Your Turn
A regular polygon has 12 sides. Find the size of each interior angle.	A regular polygon has 48 sides. Find the size of each interior angle.

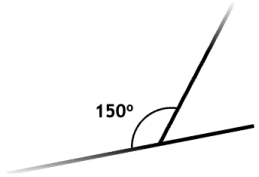
Worked Example

A section of a two different regular polygons are show below. How many sides do they each have?



Your Turn

A section of a two different regular polygons are show below. How many sides do they each have?

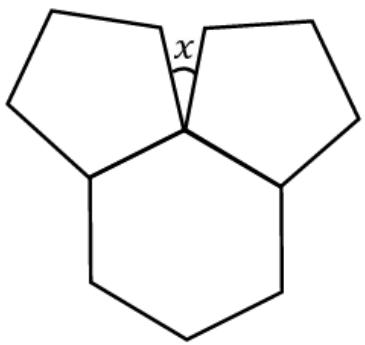
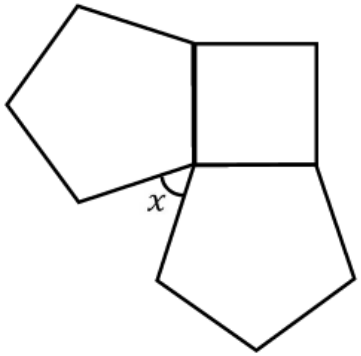


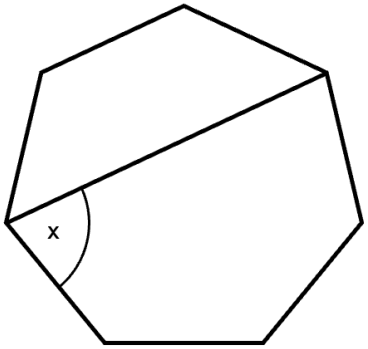
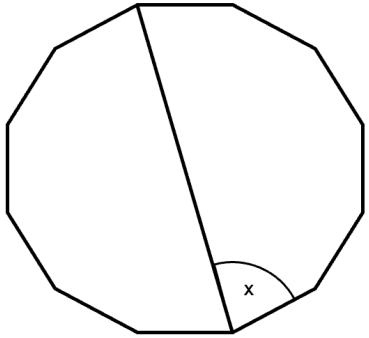
Worked Example	Your Turn
The interior angle of a regular polygon is $160°$. How many sides does the polygon have?	The interior angle of a regular polygon is $140°$. How many sides does the polygon have?

Fill in the Gaps

Name	Number of Angles	Sum of Interior Angles	Size of One Interior Angle in a Regular Polygon	Size of One Exterior Angle in a Regular Polygon
	3			
		360°	90°	
Octagon				45°
Hexadecagon		2520°		
Pentadecagon	15		156°	
				72°
		720°	120°	
	12			
		1620°		$\frac{360°}{11}$

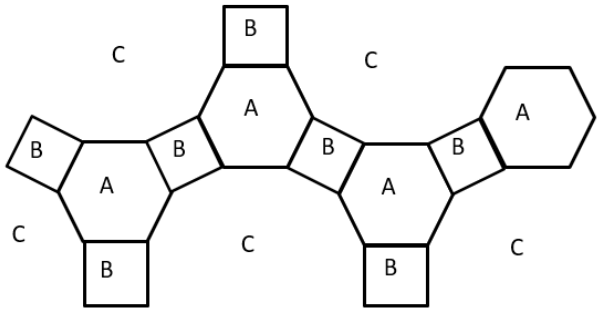
Worked Example	Your Turn
The size of each interior angle of a regular polygon is 9 times the size of each exterior angle. How many sides does the polygon have?	The size of each interior angle of a regular polygon is 11 times the size of each exterior angle. How many sides does the polygon have?

Worked Example	Your Turn
These are regular polygons. Find x. 	These are regular polygons. Find x. 

Worked Example	Your Turn
A line joins two vertices of a regular heptagon. Find angle x to 2dp. 	A line joins two vertices of a regular heptagon. Find angle x to 2dp. 

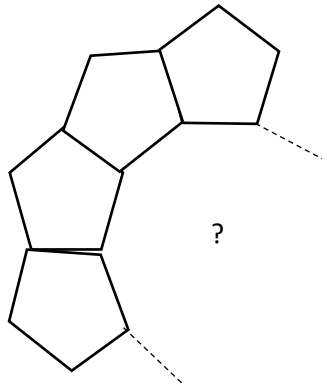
Worked Example

The repeating pattern consists of three regular polygons, A (hexagon), B (square) and C. Determine how many sides C has.



Your Turn

The diagram shows 4 congruent regular pentagons that form the sides of an n-sided regular polygon. Determine the value of n.



Worked Example

The diagram shows a regular pentagon. AB and CD are two of the lines of symmetry of the pentagon. Work out the size of the angle marked x.

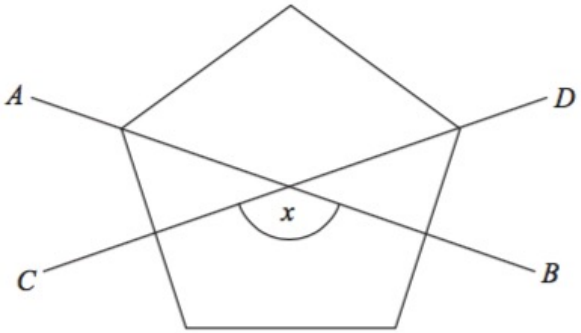
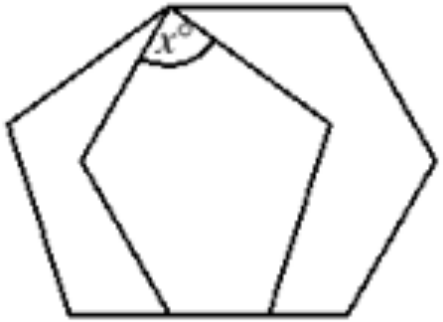


Diagram **NOT** accurately drawn

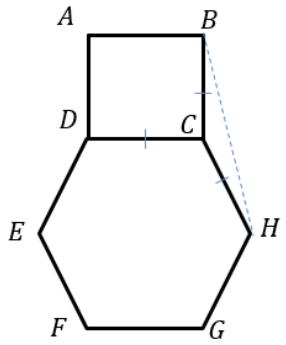
Your Turn

The diagram shows a regular pentagon and a regular hexagon which overlap. What is the value of x?



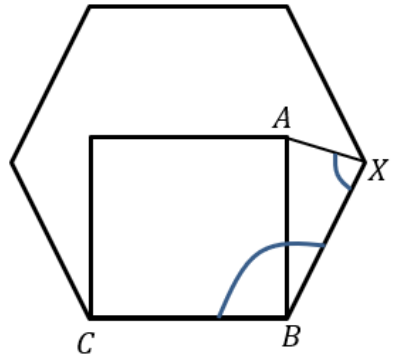
Worked Example

$ABCD$ is a square and $CDEFGH$ is a regular hexagon.
Determine the angle CBH.



Your Turn

The diagram shows a square inside a regular hexagon. What is the size of the marked angle at X?



Extra Notes