KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 9

2023 Mathematics 2024

Unit 12 Tasks – Part 1

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Unit 12 Tasks – Part 2

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1 Expanding Double Brackets

Fluency Practice

Question 1: Expand and simplify

(a) $(w + 4)(w + 2)$ (b) $(y + 1)(y + 2)$ (c) $(c + 2)(c + 5)$ (d) $(x + 6)(x + 7)$

(e) $(a + 5)(a - 3)$ (f) $(g + 7)(g - 4)$ (g) $(s - 4)(s + 5)$ (h) $(x + 1)(x - 3)$

(i) $(p - 3)(p - 2)$ (j) $(y - 4)(y - 4)$ (k) $(k - 5)(k - 6)$ (l) $(v + 4)(v + 3)$

(m) $(n + 8)(n - 10)$ (n) $(b - 3)(b + 7)$ (o) $(z - 9)(z - 3)$ (p) $(a - 5)(a + 7)$

(q) $(w + 2)(w - 8)$ (r) $(r + 7)(r + 7)$ (s) $(w - 11)(w + 1)$ (t) $(t - 8)(t - 7)$

Question 2: Expand and simplify

(a) $(8 + x)(2 + x)$ (b) $(9 + y)(4 - y)$ (c) $(1 + y)(3 + y)$ (d) $(10 - t)(4 - t)$

(e) $(4 - w)(w + 2)$ (f) $(6 - x)(x - 4)$ (g) $(2 - r)(8 - r)$ (h) $(x + 2)(8 - x)$

Question 3: Expand and simplify

(a) $(y + 2)(y - 2)$ (b) $(w + 7)(w - 7)$ (c) $(a + 1)(a - 1)$ (d) $(x - 10)(x + 10)$

(e) $(g - 8)(g + 8)$ (f) $(6 - x)(6 + x)$ (g) $(4 - r)(4 + r)$ (h) $(11 + y)(11 - y)$

Question 4: Expand and simplify

(a) $(2c + 1)(2c + 3)$ (b) $(5x + 1)(2x + 5)$ (c) $(3w + 2)(w + 1)$

(d) $(3p + 2)(2p - 1)$ (e) $(5g - 4)(g + 1)$ (f) $(2a - 3)(4a + 7)$

(g) $(4r - 5)(2r - 3)$ (h) $(2y - 3)(9y - 1)$ (i) $(5k - 4)(2k - 1)$

(j) $(2n + 3)(2n + 5)$ (k) $(3b + 4)(2b + 9)$ (l) $(2z - 9)(6z - 5)$

(m) $(4w - 3)(3w - 1)$ (n) $(4r + 3)(3r + 2)$ (o) $(5w - 1)(2w - 3)$

(p) $(3 + 2c)(5 + c)$ (q) $(9 + 2x)(3 - 2x)$ (r) $(9 - 4y)(2 + 3y)$

(s) $(3w + 2)(3w - 2)$ (t) $(2y + 3)(2y - 3)$ (u) $(5w - 1)(5w + 1)$

(v) $(9 - 5a)(9 + 5a)$ (w) $(1 - 2x)(1 + 2x)$ (x) $(3 + 2y)(3 - 2y)$

Question 5: Expand and simplify

(a) $(a + 2)^2$ (b) $(x + 7)^2$ (c) $(z - 9)^2$ (d) $(p + 1)^2$

(e) $(c - 5)^2$ (f) $(k + 4)^2$ (g) $(y - 3)^2$ (h) $(10 + r)^2$

(i) $(3g + 2)^2$ (j) $(2b - 1)^2$ (k) $(3m - 5)^2$ (l) $(2v + 9)^2$

(m) $(7 - a)^2$ (n) $(4 - 3s)^2$ (o) $(8 + 5h)^2$ (p) $(7 - 2p)^2$

Question 6: Expand and simplify

(a) $(a + 2)(a + 3) + (a + 4)(a + 1)$ (b) $(2w + 3)(w - 1) + (w - 3)(w - 2)$

(c) $(x + 9)(3x + 4) - (x + 3)(x - 1)$ (d) $2(x + 1)(x + 4) - (x + 1)(x + 2)$

(e) $(x + 4)^2 + (x + 1)^2$ (f) $(2x + 1)^2 - (x - 5)^2$

Question 1: Can you spot any mistakes in the questions below.

Expand and simplify $(5y - 1)(y - 2)$

$5y^2 - 10y - y - 2$

$5y^2 - 11y - 2$

Expand and simplify $(y - 7)^2$

$y^2 - 49$

Expand and simplify

(a) $(x+7)(x+3)$

(b) $(x+3)(x+6)$

(c) $(x+6)(x+5)$

(d) $(x+6)(x+1)$

(e) $(x+2)(x+1)$

Expand and simplify

(a) $(x+4)(x-2)$

(b) $(x-4)(x+2)$

(c) $(x-4)(x+6)$

(d) $(x+6)(x-3)$

(e) $(x-6)(x+3)$

Expand and simplify

(a) $(x-4)(x-2)$

(b) $(x-2)(x-4)$

(c) $(x-2)(x-10)$

(d) $(x-3)(x-10)$

(e) $(x-3)(x-5)$

Expand and simplify

(a) $(x+4)^2$ (b) $(x-4)^2$

(c) $(x+5)^2$ (d) $(x-5)^2$

(e) $(x+9)^2$ (f) $(x-11)^2$

Expand and simplify

(a) $(2x+1)(x+5)$

(b) $(2x+3)(3x+1)$

(c) $(2x-1)(x+6)$

(d) $(3x-2)(2x+7)$

(e) $(4x-1)(x-5)$

(f) $(2x-1)(5x-3)$

(g) $(4x+1)^2$

(h) $(4x-1)(4x+1)$

Extension

expanding brackets to obtain a quadratic expression

expand and simplify:

1) $(n + 4)(n + 6)$

2) $(k + 2)(k + 8)$

3) $(x + 5)^2$

4) $(m + 12)(m – 2)$

5) $(k + 13)(k – 3)$

6) $(a – 5)(a + 15)$

7) $(w + 14)(w – 4)$

8) $(2n + 1)(4n + 3)$

9) $(3h + 2)(8h – 2)$

10) $(2a + 7)(2a – 7)$

11) $(4b – 5)(4b + 5)$

12) $(9c + 1)(9c – 1)$

13) $(8d + 4)(2d – 1)$

14) $(9k – 3)(3k + 1)$

15) $(10p – 2)(5p + 1)$

16) $(12w + 6)(2w – 1)$

17) $(30n + 5)(6n – 1)$

18) $(25g – 10)(2 + 5g)$

Extension

quadratic expressions with patterns

(a) expand the brackets (showing your steps) and look for patterns

(1) $(m + 3)(m + 4)$

(2) $(m + 6)(m + 1)$

(3) $(a + 3)(a + 8)$

(4) $(a + 5)(a + 6)$

(5) $(t + 10)(t + 5)$

(6) $(t + 8)(t + 7)$

(7) $(h + 8)(h + 9)$

(8) $(h + 6)(h + 11)$

(9) $(z + 15)(z + 10)$

(10) $(z + 13)(z + 12)$

(11) $(y – 7)(y – 6)$

(12) $(y – 9)(y – 4)$

try to find some other examples that fit the pattern

(b) expand the brackets (showing your steps) and look for patterns

(1) $(a + 2)(a + 3)$

(2) $(a + 6)(a – 1)$

(3) $(b + 4)(b + 6)$

(4) $(b + 12)(b – 2)$

(5) $(c + 12)(c + 5)$

(6) $(c + 20)(c – 3)$

(7) $(d + 18)(d – 3)$

(8) $(d + 9)(d + 6)$

(9) $(e + 15)(e + 10)$

(10) $(e + 30)(e – 5)$

(11) $(f – 12)(f – 5)$

(12) $(f – 20)(f + 3)$

try to find some other examples that fit the pattern

Extension

1) Expand and simplify fully

a) $\left(2+\frac{3}{x}\right)\left(1+\frac{1}{x}\right)$

b) $\left(2a+\frac{1}{a}\right)\left(a+\frac{1}{a}\right)$

c) $\left(2+\frac{1}{x}\right)\left(1-\frac{1}{x}\right)$

d) $\left(n-\frac{1}{n}\right)\left(5n+\frac{1}{n}\right)$

e) $\left(1-\frac{1}{x}\right)\left(1+\frac{1}{x}\right)$

f) $\left(3c+\frac{1}{c}\right)\left(2c+\frac{1}{c}\right)$

g) $\left(x+\frac{1}{x}\right)\left(x+\frac{2}{x}\right)$

h) $\left(x-\frac{1}{x}\right)^2$

Expand and simplify

a $x(x^2-x^{-1})$ **b** $2x^3(x^{-1}+3)$ **c** $x^{-1}(3x-x^3)$ **d** $4x^{-2}(3x^5+2x^3)$

e $\frac{1}{2}x^2(6x+4x^{-1})$ **f** $3x^{\frac{1}{2}}(x^{-\frac{1}{2}}-x^{\frac{3}{2}})$ **g** $x^{-\frac{3}{2}}(5x^2+x^{\frac{7}{2}})$ **h** $x^{\frac{1}{3}}(3x^{\frac{5}{3}}-x^{-\frac{4}{3}})$

i $(x^2+1)(x^4-3)$ **j** $(2x^5+x)(x^4+3)$ **k** $(x^2-2x^{-1})(x-x^{-2})$ **l** $(x^2-x^{\frac{3}{2}})(x-x^{\frac{1}{2}})$

Problem Solving

$(x+3)$	$(x+6)$	$(x+5)$
$(x+2)$	$(x+9)$	$(x+3)$
$(x+7)$	$(x+6)$	$(x+1)$
$(x+7)$	$(x+3)$	$(x+4)$
$(x+9)$	$(x+1)$	$(x+6)$
$(x+5)$	$(x+2)$	$(x+1)$
$(x+8)$	$(x+7)$	$(x+4)$
$(x+3)$	$(x+5)$	$(x+6)$
$(x+4)$	$(x+2)$	$(x+8)$
$(x+2)$	$(x+9)$	$(x+7)$

(1) x^2+6x+8
(2) $x^2+8x+15$
(3) x^2+5x+4
(4) $x^2+11x+18$
(5) $x^2+13x+42$
(6) $x^2+11x+24$
(7) $x^2+16x+63$
(8) $x^2+7x+10$
(9) x^2+3x+2
(10) $x^2+14x+48$
(11) $x^2+13x+36$
(12) $x^2+12x+35$
(13) $x^2+12x+36$
(14) x^2+8x+7
(15) x^2+6x+9

Pick 2 expressions from the table on the left, multiply them together and try to get the answers on the right-hand side!

Fluency Practice

Expand and simplify:

1) $(5r^2 - 9r + 3)(3r^2 + 2)$

2) $(y^2 - 4y)(6y^2 + 9y + 1)$

3) $(9x - 1)(4x^2 + 7x + 3)$

4) $(9x^2 + 7x)(3x^2 + x + 6)$

5) $(7p^2 + 8)(8p^2 + p + 8)$

6) $(7p - 9)(6p^2 + p + 7)$

7) $(5q^2 + 7q + 7)(8q^2 + 7)$

8) $(3t^2 - 8t)(5t^2 + t + 4)$

9) $(5t^2 - 9)(8t^2 + 8t + 3)$

10) $(3t^2 - 2t + 7)(3t + 5)$

Fluency Practice

Expand and simplify:

1) $(3x-6)(x+2)+3(2x+3)$

2) $(x-5)(2x+2)+2(2x+3)$

3) $(2x+2)(3x-1)-5(3x+6)$

4) $(2x+5)(x+1)+5x(x-2)$

5) $(2x-2)^2-2x(2x+4)$

6) $(x-2)^2+5(3x-6)$

7) $(x+1)^2-4(3x-6)$

8) $(3x-3)^2+2x(3x+3)$

9) $(x-3)^2+5x(2x+6)$

10) $(x+3)(2x-2)-4x(2x-6)$

Fluency Practice

Expand and simplify:

1) $(2x + 3)(x + 4) - (3x + 2)(2x - 5)$

2) $(2x - 1)^2 - (x - 5)^2$

3) $(x + 6)^2 - (3x - 3)^2$

4) $(2x + 5)^2 - (2x - 5)^2$

5) $(2x + 4)^2 - (3x - 6)^2$

6) $(2x + 4)(x - 3) - (2x + 1)(x + 2)$

7) $(2x - 4)(3x + 6) - (2x - 4)(3x + 2)$

8) $(2x - 4)^2 - (x + 2)^2$

9) $(x - 3)^2 - (3x + 2)^2$

10) $(2x - 6)(3x + 3) - (3x + 4)(3x + 6)$

Extension

Remove the brackets and simplify:

1. $(x+4)^2$
2. $(x+2)^2$
3. $(x-2)^2$
4. $(2x+1)^2$
5. $(y-5)^2$
6. $(3y+1)^2$
7. $(x+y)^2$
8. $(2x+y)^2$
9. $(a-b)^2$
10. $(2a-3b)^2$
11. $3(x+2)^2$
12. $(3-x)^2$
13. $(3x+2)^2$
14. $(a-2b)^2$
15. $(x+1)^2+(x+2)^2$
16. $(x-2)^2+(x+3)^2$
17. $(x+2)^2+(2x+1)^2$
18. $(y-3)^2+(y-4)^2$
19. $(x+2)^2-(x-3)^2$
20. $(x-3)^2-(x+1)^2$
21. $(y-3)^2-(y+2)^2$
22. $(2x+1)^2-(x+3)^2$
23. $3(x+2)^2-(x+4)^2$
24. $2(x-3)^2-3(x+1)^2$

Extension

quadratic expression generalisations

find a simpler form for these expressions and possibly factorise the resulting expression

(1) $(n + 2)(n + 4) – (n + 1)(n + 5)$

(2) $(2n + 2)(2n + 3) – (2n + 1)(2n + 4)$

(3) $(n + 1)(n + 2) + (n + 2)^2 – (n + 1)(n + 6)$

(4) $(2n + 3)(n + 4) – (2n + 1)(n + 5)$

(5) $(n + 1)(n + 4) + (n + 3)^2 – (n + 5)^2$

(6) $n(n + 1) + (n + 1)(n + 2) + (n + 2)(n + 3) – (n + 3)(n + 4)$

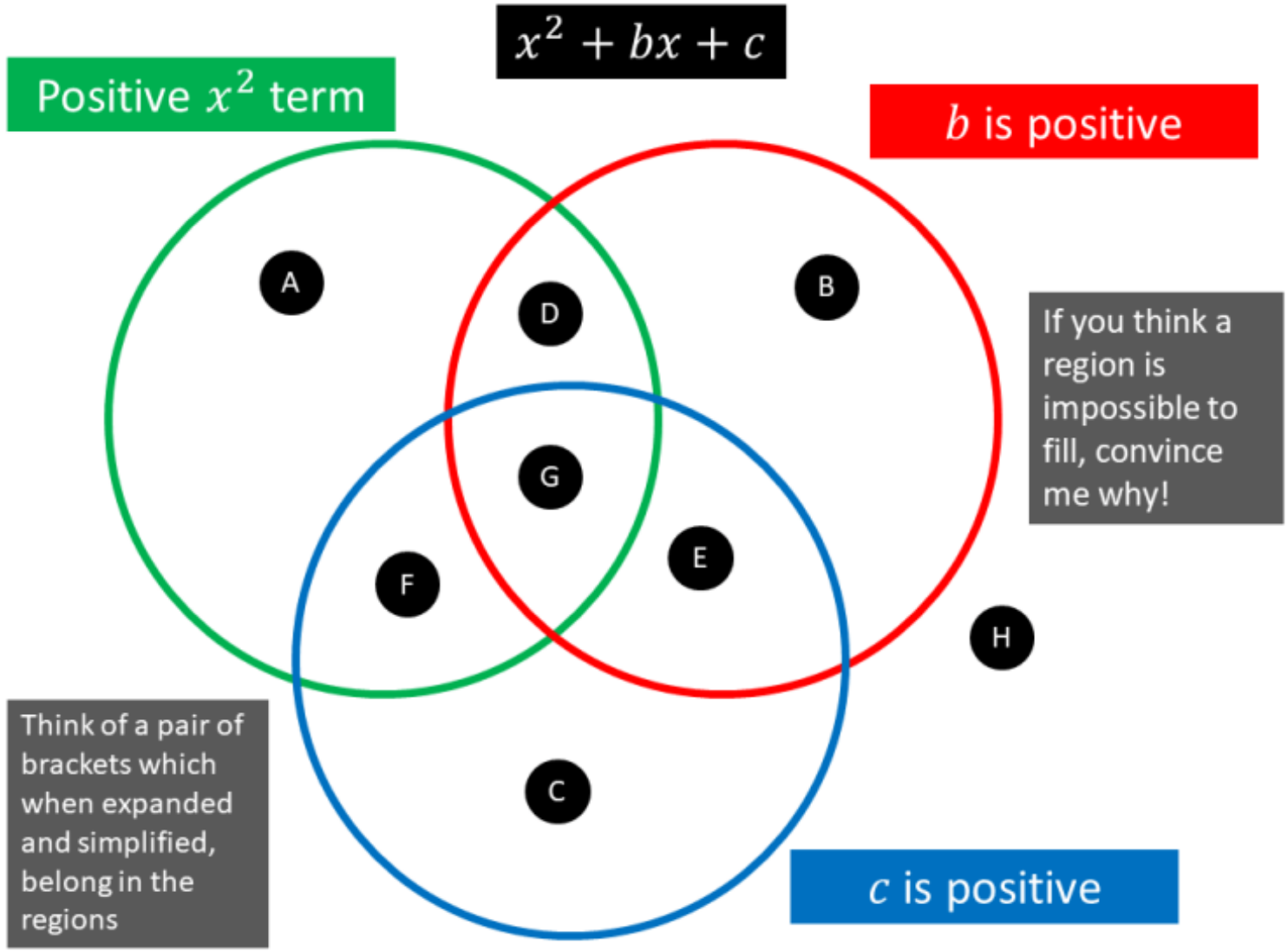
(7) $(3n – 7)(n + 2) – (n + 3)(2n – 5)$

(8) $(3n – 13)(n – 2) – (2n – 5)(n – 7)$

(9) $(2n – 1)(2n + 7) – (n + 4)(3n – 2)$

(10) $(2n – 3)(2n – 1) – (n – 2)(3n – 2)$

Maths Venns


$x^2 + bx + c$
Positive x^2 term
b is positive
A
D
B
If you think a region is impossible to fill, convince me why!
G
E
F
H
Think of a pair of brackets which when expanded and simplified, belong in the regions
C
c is positive

2 Factorising by Grouping

Intelligent Practice

Factorise:

1) $x(x+1)+3(x+1)$

2) $x(x+1)+2(x+1)$

3) $x(x+1)+1(x+1)$

4) $3x(x+1)+1(x+1)$

5) $3x(x+1)-1(x+1)$

6) $3x(2x+1)-1(2x+1)$

7) $3x(1+2x)-1(2x+1)$

8) $3x(1-2x)-1(1-2x)$

9) $2x(1-2x)-1(1-2x)$

10) $2x(1-2x)-1(2x-1)$

Intelligent Practice

Factorise:

1) $3x(x+1)+2(x+1)$

2) $3x(x+1)-2(x+1)$

3) $3x(x+1)-1(x+1)$

4) $3x(x+1)-(x+1)$

5) $3x(x+1)-4(x+1)$

6) $3x(x+1)^2-4(x+1)$

7) $3x(x+1)^2-2(x+1)$

8) $3x(x+1)^2-2x(x+1)$

9) $3x(x+1)^2-(2x+1)(x+1)$

10) $3x(x+1)^3-(2x+1)(x+1)^2$

Fluency Practice

1) Factorise fully
$(x+3)^2 + 4(x+3)$

2) Factorise fully
$4(x-5)^2 + 3(x-5)$

3) Factorise fully
$(p+q)^2 + 5(p+q)$

4) Factorise fully
$(2y+1)^2 + (x+y)(2y+1)$

5) Factorise fully
$(x+4)^3 + 2(x+4)^2$

6) Factorise fully
$2(x+y)^{10} + (x-y)(x+y)^9$

Now factorise the following expressions fully.

1. $x(x+4)-3(x+4)$.
2. $2x(x-3)+(x-3)$.
3. $4(2x-1)+3x(2x-1)$.
4. $x(x+2)+2(x+2)$.
5. $3x(y-1)+2(y-1)$.
6. $p(x+y)+q(x+y)$.
7. $4q(2x-y)-3p(2x-y)$.
8. $4x^2(x+2)^2-2x(x+2)$.
9. $10x^5(2x+1)+5x^4(2x+1)^2$.
10. $49y(y-4)^{10}-35y^2(y-4)^9$.
11. $(x+1)^4(x+2)-2(x+1)^3(x+2)^2$.
12. $20(2x+1)^4(x-2)^5-25(2x+1)^5(x-2)^4$.
13. $16x^2(x-1)-20x(x-1)^2$.

Factorise fully the following:

1. $x(x+1)+2(x+1)$.
2. $2x(x-1)-3(x-1)$.
3. $x(x+y)+y(x+y)$.
4. $p(p-2q)+q(p-2q)$.
5. $4x(x-3)+8(x-3)$.
6. $x^5(x-5)-x^4(x-5)$.
7. $7x^2(x+1)^5+14x(x+1)^6$.
8. $27x^8(x-1)^8-9x^9(x-1)^7$.
9. $2x^4(x-1)^4+8x^3(x-1)^3$.
10. $(2x-1)^7(2x+3)^4-(2x-1)^8(2x+3)^3$.
11. $10(t+2)^6(t+3)^4-5(t+2)^7(t+3)^3$.
12. $7x^3(x-1)^8(2x+1)^3-14x(x-1)^9(2x+1)^2$.
13. $27(2t-1)^2(t+1)^2-18(2t-1)(t+1)^3$.
14. $y^4(y+7)^4(2y-1)^2-2y^2(y+7)^3(2y-1)$.
15. $6(2z+5)^7(z-1)^{10}-12(2z+5)^6(z-1)^{11}$.

1. Consider the expression $ax + 2x + 3a + 6$. What do you notice about the first pair of terms and about the second pair of terms? Rewrite the expression in view of your discovery. You now have two terms. What have they in common? Now factorise the expression. If you are not sure about this result, refer to Exercise 77.
2. Consider the expression $ax + ay + bx + by$, and answer the same questions about this expression.
3. Consider the expression $ax + 6 + 2x + 3a$. Pick out two pairs of terms with common factors and hence factorise the expression. Compare the working out of this question with that of question 1.
4. Do the same for $ax + by + ay + bx$.

Factorise the following expressions where it is possible.

5. $ab + 2a + 3b + 6$.
6. $ac + 2a + 2c + 4$.
7. $bd + 3b + 2d + 6$.
8. $de + 5d + 3e + 15$.
9. $ef + 4e + 6f + 24$.
10. $ab + 8 + 2a + 4b$.
11. $ac + 12 + 2a + 6c$.
12. $de + 15 + 3d + 5e$.
13. $xy + 20 + 2x + 10y$.
14. $xy + 20 + 4x + 5y$.
15. $xy + xz + uy + uz$.
16. $pr + qr + ps + qs$.
17. $pr + qr + ps + rs$.
18. $rs + rt + us + ut$.
19. $sx + sy + vx + vy$.
20. $hk + lm + hl + km$.
21. $hk + lm + hm + kl$.
22. $x^2 + xy + xz + yz$.
23. $a^2 + ab + ac + bc$.
24. $e^2 + ef + eg + fg$.
25. $h^2 + lh + mh + lm$.
26. $h^2 + lh + mh + nh$.
27. $n^2 + pn + nq + pq$.
28. $ab - 2a + 3b - 6$.
29. $ac - 2a + 2c - 4$.
30. $bd + 3b - 2d - 6$.
31. $bc + 3b + 2d + 6$.
32. $de - 5d + 3e - 15$.
33. $ef + 4e - 6f - 24$.
34. $xy + x + y + 1$.
35. $pq + p + q + 1$.
36. $pq + 1 + p + q$.
37. $1 + cd + c + d$.
38. $x^3 + x^2 + x + 1$.
39. $r^3 + 1 + r^2 + r$.
40. $pq - p + q - 1$.
41. $st - 1 + s - t$.
42. $su + 1 + s - u$.
43. $uv - 1 - u + v$.
44. $c^2d - cd + c^2 - c$.
45. $k - p - ky + yp$.
46. $e - 5 - ex + 5x$.
47. $2ht - tz - 2hn + nz$.
48. $6cf - 7cp - 6kf + 7kp$.
49. $ap + aq + aps - 2aqr$.
50. $cy - dy + 5xy$.
51. $24cq - 8xq + 15cs - 5sx$.
52. $ah + hx - 3ab - 3bx$.
53. $bm - km - 6ab + 6ak$.
54. $br - kr - 6ab + 6ar$
55. $hx + hy + kx + ky + jx + jy$.
56. $ab + ac + ad + eb + ec + ed$.
57. $1 + x + y + z + xz + yz$.
58. $1 + a - b + x + ax - bx$.
59. $3 - r + s - 3u + ru - su$.
60. $t - v + st - sv + tx - vx$.

Fluency Practice

Factorise:

1) $ab - 2a + 4b - 8$

2) $ab + bc - 3a - 3c$

3) $ac + ad + bc + bd$

4) $y^2 + cy + dy + cd$

5) $x^2y - x^2 + 2y - 2$

6) $x^3 + x^2 + x + 1$

7) $a^3 - a^2 + a - 1$

8) $x^3 + x^2 - x - 1$

9) $x^3 + x^2 + xy^2 + y^2$

10) $x^3 - 2x^2 + 3x - 6$

Intelligent Practice

Factorise:

1) $x^2 + 2x + 3x + 6$

2) $x^2 + 3x + 2x + 6$

3) $3x^2 + 3x + 2x + 2$

4) $3x^2 + 3x + 4x + 4$

5) $3x^2 - 3x + 4x - 4$

6) $3x^2 - 3x - 4x + 4$

7) $3x^2 + 3x - 4x - 4$

8) $3x^2 - 4x + 3x - 4$

9) $5x^2 - 4x + 5x - 4$

10) $-5x^2 + 4x - 5x + 4$

3 Factorising Quadratics

Intelligent Practice

$_ \times _ = 18$ $_ + _ = 11$	$_ \times _ = 20$ $_ + _ = 9$	$_ \times _ = 180$ $_ + _ = -36$
$_ \times _ = -18$ $_ + _ = 7$	$_ \times _ = -20$ $_ + _ = -1$	$_ \times _ = -252$ $_ + _ = -36$
$_ \times _ = -18$ $_ + _ = -7$	$_ \times _ = -20$ $_ + _ = 1$	$_ \times _ = 252$ $_ + _ = 48$
$_ \times _ = 18$ $_ + _ = -11$	$_ \times _ = 20$ $_ + _ = -9$	$_ \times _ = 432$ $_ + _ = 48$
$_ \times _ = 18$ $_ + _ = -9$	$_ \times _ = -36$ $_ + _ = -9$	$_ \times _ = 576$ $_ + _ = 48$
$_ \times _ = -18$ $_ + _ = 3$	$_ \times _ = -36$ $_ + _ = 0$	$_ \times _ = -576$ $_ + _ = 0$

Number Pairs

What two numbers have a **sum** of 6 and a **product** of 8?

EXAMPLE

Sum = 6, Product = 8: 2 & 4

1) Sum = 7, Product = 12: 3 & ☐

2) Sum = 11, Product = 30: ☐ & ☐

3) Sum = 4, Product = 4: ☐ & ☐

4) Sum = 8, Product = 7: ☐ & ☐

5) Sum = 10, Product = 24: ☐ & ☐

How do we get a **negative product**?

6) Sum = 3, Product = −10: ☐ & ☐

7) Sum = 3, Product = −18: ☐ & ☐

8) Sum = 7, Product = −8: ☐ & ☐

9) Sum = 1, Product = −20: ☐ & ☐

10) Sum = −1, Product = −20: ☐ & ☐

11) Sum = −4, Product = −21: ☐ & ☐

12) Sum = 0, Product = −36: ☐ & ☐

A **negative** sum, but a **positive** product?

13) Sum = −9, Product = 18: ☐ & ☐

14) Sum = −10, Product = 24: ☐ & ☐

15) Sum = −13, Product = 36: ☐ & ☐

16) Sum = −13, Product = 42: ☐ & ☐

17) Sum = −11, Product = 28: ☐ & ☐

18) Sum = −1, Product = −72: ☐ & ☐

19) Sum = −4, Product = −45: ☐ & ☐

Fluency Practice

① Factorise the following

	$2x^2 + 11x + 5$	$5x^2 + 8x + 3$	$3x^2 + 10x + 3$	$2x^2 + 5x + 3$
Identify a, b and c	a = 2, b = 13, c = 5			
Multiply a and c	2 x 5 = 10			
What multiples to give ac but adds to give b?	1 x 10 = 10 1+ 10 = 11			
Split the middle term	$2x^2 + 10x + x + 5$			
Factorise each pair of terms	$2x(x + 5) + 1(x + 5)$			
Complete the factorisation	$(2x + 1)(x + 5)$			

	$3x^2 + 13x - 10$	$3x^2 + 4x - 15$	$2x^2 - x - 1$	$2x^2 - 5x - 12$
Identify a, b and c				
Multiply a and c				
What multiples to give ac but adds to give b?				
Split the middle term				
Factorise each pair of terms				
Complete the factorisation				

Factorise:

(a) $2x^2 + 7x + 3$ (b) $3x^2 + 8x + 5$

(c) $2x^2 + 5x + 2$ (d) $7x^2 + 8x + 1$

(e) $6x^2 + 13x + 5$ (f) $4x^2 + 12x + 9$

(g) $3x^2 + 13x + 4$ (h) $8x^2 + 14x + 3$

Factorise:

(a) $16x^2 - 8x + 1$ (b) $6x^2 - 7x + 2$

(c) $3x^2 - 13x + 4$ (d) $7x^2 - 23x + 6$

(e) $4x^2 - 7x + 3$ (f) $6x^2 - 31x + 5$

(g) $12x^2 - 8x + 1$ (h) $9x^2 - 9x + 2$

Factorise:

(a) $2x^2 - x - 6$ (b) $3x^2 + 11x - 4$

(c) $5x^2 - 3x - 2$ (d) $4x^2 - 4x - 15$

(e) $7x^2 + 5x - 2$ (f) $6x^2 + 7x - 5$

(g) $8x^2 - 2x - 3$ (h) $9x^2 + 6x - 8$

Factorise

(a) $3x^2 + 19x + 20$

(b) $4x^2 - 23x + 15$

(c) $7x^2 - 19x - 6$

(d) $3x^2 - 8x - 60$

(e) $9x^2 + 48x + 28$

(f) $2x^2 - 21x + 52$

Fluency Practice

Question 1: Factorise each of the following

(a) $2x^2 + 7x + 5$ (b) $2x^2 + 11x + 15$ (c) $2x^2 + 9x + 10$

(d) $3x^2 + 13x + 4$ (e) $3x^2 + 4x + 1$ (f) $3x^2 + 8x + 4$

(g) $5x^2 + 13x + 6$ (h) $5x^2 + 26x + 5$ (i) $7x^2 + 10x + 3$

(j) $11x^2 + 47x + 12$ (k) $2x^2 + 17x + 36$ (l) $5x^2 + 62x + 24$

Question 2: Factorise each of the following

(a) $3x^2 + x - 4$ (b) $7x^2 + 20x - 3$ (c) $2x^2 - 13x + 15$

(d) $3x^2 - 17x + 10$ (e) $3x^2 - 16x - 12$ (f) $3x^2 - x - 4$

(g) $5x^2 - 13x - 6$ (h) $3x^2 + 8x - 3$ (i) $2x^2 - x - 10$

(j) $2x^2 - 3x - 44$ (k) $7x^2 - 22x + 16$ (l) $2x^2 + 15x - 38$

Question 3: Factorise each of the following

(a) $6x^2 + 13x + 6$ (b) $9x^2 + 9x + 2$ (c) $6x^2 + 13x + 2$

(d) $8x^2 + 41x + 5$ (e) $9x^2 + 6x + 1$ (f) $8x^2 + 26x + 15$

(g) $8x^2 + 29x + 15$ (h) $10x^2 + 9x + 2$ (i) $9x^2 + 27x + 20$

(j) $10x^2 + 17x + 7$ (k) $12x^2 + 13x + 3$ (l) $15x^2 + 32x + 16$

Question 4: Factorise each of the following

(a) $9x^2 - 12x - 5$ (b) $4x^2 - 4x - 3$ (c) $4x^2 - 11x + 6$

(d) $6x^2 - 7x + 2$ (e) $10x^2 - 91x + 9$ (f) $4x^2 + 25x - 56$

(g) $6x^2 - 35x + 49$ (h) $6x^2 - 7x - 10$ (i) $8x^2 + 10x - 3$

(j) $15x^2 + 31x + 10$ (k) $12x^2 + 5x - 3$ (l) $20x^2 - 23x + 6$

Question 1: A quadratic expression, $3x^2 + ax + 10$, can be factorised.
Find all possible values for a.
a can be positive or negative.

Question 2: A quadratic expression, $4x^2 + bx - 12$, can be factorised.
Find all possible values for b.
b can be positive or negative.

Question 3: A quadratic expression, $2x^2 - x + c$, can be factorised.
Find three possible values for c.

Question 4: Andrew has completed his homework on factorising quadratics.
Can you spot any mistakes?

Factorise $2y^2 + 7y - 15$

$(2y + 3)(Y - 5)$

Factorise $2w^2 - 9w + 4$

$(2w + 1)(w + 4)$

Factorise $3y^2 + 10y - 8$

$(3x - 2)(x + 4)$

Factorise $6w^2 - 11w - 10$

$(3w + 2)(2w - 5)$

Extension

factorise (1)

(1) $2x^2 + 7x + 6$

(2) $3x^2 + x - 14$

(3) $5x^2 + 13x - 6$

(4) $6x^2 + 5x + 1$

(5) $9x^2 + 12x + 4$

(6) $4x^2 + 21x + 5$

(7) $4x^2 - 5x - 6$

(8) $6x^2 + 17x + 5$

(9) $7x^2 - 19x - 6$

(10) $2x^2 - 35x + 150$

(11) $3x^2 + 19x + 20$

(12) $4x^2 - 23x + 15$

(13) $4x^2 + 5x - 9$

(14) $2x^2 - 31x + 42$

(15) $4x^2 + 21x - 18$

(16) $3x^2 - 34x + 80$

(17) $10x^2 - 43x - 9$

(18) $9x^2 - 65x - 56$

Extension

factorise (3)

(1) $4x^2 + 7x + 3$

(2) $2x^2 + 13x + 20$

(3) $20x^2 + 17x + 3$

(4) $6x^2 + 13x + 6$

(5) $4x^2 + 11x + 6$

(6) $12x^2 + 23x + 10$

(7) $2x^2 + 11x - 6$

(8) $6x^2 - 25x - 9$

(9) $8x^2 - 26x + 15$

(10) $20x^2 - 37x + 8$

(11) $4x^2 + 37x - 30$

(12) $9x^2 - 30x + 25$

(13) $9x^2 + 72x - 25$

(14) $12x^2 + 11x - 15$

(15) $10x^2 - 19x + 6$

(16) $12x^2 - 7x - 12$

(17) $30x^2 - 29x + 4$

(18) $12x^2 + 8x - 15$

Intelligent Practice

Factorise:

1) $3x^2 + 5x + 2$

2) $3x^2 + 8x + 4$

3) $3x^2 + 11x + 6$

4) $3x^2 + 7x - 6$

5) $3x^2 + 4x - 4$

6) $3x^2 + 1x - 2$

7) $3x^2 + x - 2$

8) $3x^2 - x - 2$

9) $3x^2 - 4x - 4$

10) $3x^2 - 7x - 6$

11) $3x^2 - 11x + 6$

12) $3x^2 - 8x + 4$

13) $3x^2 - 5x + 2$

14) $5x^2 - 3x + 2$

Intelligent Practice

Where possible, finish factorising:

1) $(2x + 4)(x + 5)$

2) $(x + 2)(2x + 10)$

3) $(x + 2)(2x + 11)$

4) $3x + 6)(2x + 11)$

5) $(9x + 6)(2x + 11)$

6) $(9x + 6)(2x + 10)$

7) $(9x + 6)(5x + 10)$

8) $(9x + 6)(15x + 10)$

9) $2(9x + 6)(15x + 10)$

10) $2(9x - 6)(15x + 10)$

11) $2(6 - 9x)(15x + 10)$

12) $2(6x - 9x^2)(15x + 10)$

13) $2x^3(6x - 9x^2)(15x + 10)$

14) $0.2(6 - 9x)(15x + 10)$

Intelligent Practice

Factorise:

1) $6x^2 + 10x + 4$

2) $6x^2 - 10x + 4$

3) $6x^2 + 2x - 4$

4) $6x^2 - 2x - 4$

5) $9x^2 - 3x - 6$

6) $9x^2 - 30x - 24$

Factorise:

(a) $x^2 + 7x + 10$ (b) $x^2 + 7x + 12$

(c) $x^2 + 5x + 4$ (d) $x^2 + 5x + 6$

(e) $x^2 + 10x + 16$ (f) $x^2 + 10x + 21$

(g) $x^2 + 22x + 21$ (h) $x^2 + 13x + 22$

(i) $x^2 + 13x + 42$ (j) $x^2 + 13x + 12$

Factorise:

(a) $x^2 - 7x + 10$ (b) $x^2 - 7x + 12$

(c) $x^2 - 8x + 12$ (d) $x^2 - 13x + 12$

(e) $x^2 - 13x + 30$ (f) $x^2 - 11x + 30$

(g) $x^2 - 11x + 18$ (h) $x^2 - 9x + 18$

(i) $x^2 - 9x + 20$ (j) $x^2 - 9x + 14$

Factorise:

(a) $x^2 + 3x - 10$ (b) $x^2 - 3x - 10$

(c) $x^2 + 3x - 28$ (d) $x^2 - 3x - 28$

(e) $x^2 - x - 30$ (f) $x^2 - x - 12$

(g) $x^2 + 4x - 12$ (h) $x^2 - 11x - 12$

(i) $x^2 + 2x - 15$ (j) $x^2 - 2x - 35$

Factorise:

(a) $x^2 + 18x + 81$ (b) $x^2 - 2x - 80$

(c) $x^2 - x - 42$ (d) $x^2 - 12x + 27$

(e) $x^2 - 2x + 1$ (f) $x^2 + 11x - 26$

(g) $x^2 - 12x + 32$ (h) $x^2 - 5x - 36$

(i) $x^2 + 13x - 30$ (j) $x^2 - 22x - 75$

Fluency Practice

Question 1: Factorise each of the following

(a) $x^2 + 7x + 12$ (b) $x^2 + 6x + 8$ (c) $x^2 + 5x + 6$ (d) $x^2 + 8x + 7$

(e) $x^2 + 4x + 4$ (f) $x^2 + 8x + 15$ (g) $x^2 + 6x + 9$ (h) $x^2 + 11x + 28$

(i) $x^2 + 10x + 25$ (j) $x^2 + 12x + 20$ (k) $x^2 + 25x + 24$ (l) $x^2 + 11x + 24$

(m) $x^2 + 9x + 14$ (n) $x^2 + 23x + 60$ (o) $x^2 + 29x + 100$ (p) $x^2 + 20x + 51$

Question 2: Factorise each of the following

(a) $x^2 + x - 12$ (b) $x^2 + 5x - 6$ (c) $x^2 + 3x - 10$ (d) $x^2 + 3x - 4$

(e) $x^2 + 2x - 48$ (f) $x^2 + 4x - 32$ (g) $x^2 + 2x - 35$ (h) $x^2 + 8x - 33$

Question 3: Factorise each of the following

(a) $x^2 - 3x - 10$ (b) $x^2 - x - 20$ (c) $x^2 - 6x - 27$ (d) $x^2 - 2x - 3$

(e) $x^2 - x - 12$ (f) $x^2 - 4x - 12$ (g) $x^2 - 4x - 21$ (h) $x^2 - 6x - 55$

Question 4: Factorise each of the following

(a) $x^2 - 6x + 9$ (b) $x^2 - 9x + 20$ (c) $x^2 - 9x + 14$ (d) $x^2 - 13x + 22$

(e) $x^2 - 9x + 8$ (f) $x^2 - 12x + 32$ (g) $x^2 - 15x + 36$ (h) $x^2 - 14x + 48$

Question 5: Factorise each of the following

(a) $x^2 - 9x + 8$ (b) $x^2 + 24x + 23$ (c) $x^2 - 5x - 14$ (d) $x^2 - 7x + 12$

(e) $x^2 + 12x + 36$ (f) $x^2 - 2x - 63$ (g) $x^2 + 14x + 24$ (h) $x^2 + 17x + 60$

(i) $x^2 - 11x + 30$ (j) $x^2 - 4x - 32$ (k) $x^2 - 2x - 63$ (l) $x^2 - 16x - 17$

(m) $x^2 - 11x + 18$ (n) $x^2 - 13x + 22$ (o) $x^2 + 18x + 56$ (p) $x^2 - 21x + 110$

(q) $x^2 - 16x + 64$ (r) $x^2 + 22x + 121$ (s) $x^2 - x - 72$ (t) $x^2 - 3x - 18$

(u) $x^2 - 4x - 45$ (v) $x^2 - 16x + 63$

Question 6: Factorise each of the following

(a) $x^2 + 8x - 105$ (b) $x^2 - 18x - 88$ (c) $x^2 - 75x + 350$ (d) $x^2 + 22x + 96$

(e) $x^2 + 25x + 154$ (f) $x^2 - 55x - 300$ (g) $x^2 - 29x + 180$ (h) $x^2 - x - 210$

Question 1: A quadratic expression, $x^2 + ax + 20$, can be factorised.
Find all possible values for a.
a can be positive or negative.

Question 2: A quadratic expression, $x^2 + bx + 16$, can be factorised.
Find all possible values for b.
b can be positive or negative.

Question 3: A quadratic expression, $x^2 - 6x + c$, can be factorised.
Find three possible values for c.

Question 4: Andrew has completed his homework on factorising quadratics.
Can you spot any mistakes?

Factorise $x^2 + x - 6$

$(x - 3)(x + 2)$

Factorise $x^2 + 10x + 9$

$(x + 3)(x + 3)$

Factorise $x^2 - 7x + 12$

$(x + 5)(x + 2)$

Factorise $x^2 + 8x + 16$

$(x + 4)(x + 4)$

Intelligent Practice

Factorise:

1) $x^2 + 10x + 24$

2) $x^2 + 10x - 24$

3) $x^2 - 10x - 24$

4) $x^2 - 10x + 24$

5) $x^2 - 11x + 30$

6) $x^2 - 110x + 3000$

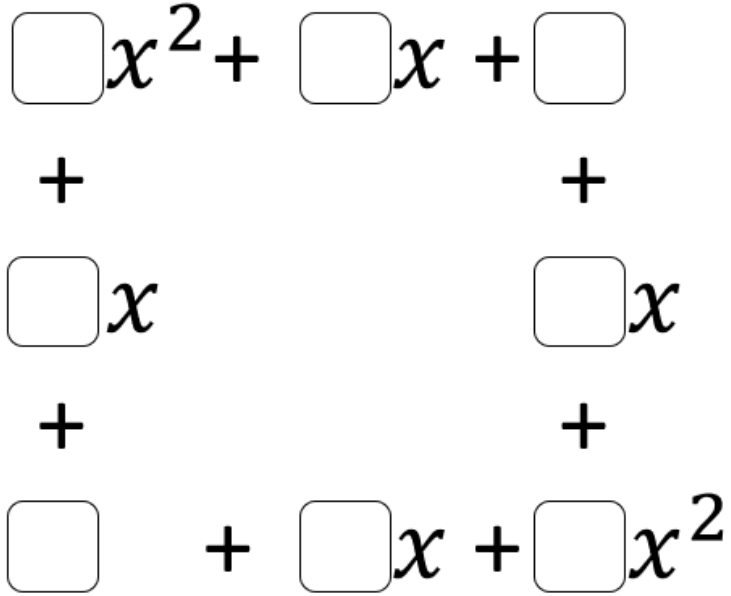
Problem Solving

Using the numbers 1-9, at most once each time, fill in the blanks so that each expression factorises.

$x^2 + \square x + 2$ $x^2 - \square x - 32$ $x^2 + 5x - \square$

$x^2 + 6x + \square$ $x^2 - 2x + \square$ $x^2 - \square x - 18$

$x^2 - \square x + 12$ $x^2 + \square x - 15$ $x^2 - \square x + 8$



Use the digits $1 - 8$ exactly once so that all four sides of the square factorise into pairs of linear expressions.

Can you prove that there must be multiple solutions?

Problem Solving

		$x^2 + 7x + 6$
		$x^2 + 8x + 15$
$x^2 + 4x + 3$	$x^2 + 11x + 30$	**Question 1**

		$x^2 + 7x + 12$
		$x^2 + 6x + 8$
$x^2 + 8x + 16$	$x^2 + 5x + 6$	**Question 2**

		$x^2 + 9x + 14$
		$x^2 + 7x + 10$
$x^2 + 7x + 10$	$x^2 + 9x + 14$	**Question 3**

		$x^2 + 12x + 27$
		$x^2 + 13x + 42$
$x^2 + 16x + 63$	$x^2 + 9x + 18$	**Question 4**

		$x^2 + 14x + 48$
		$x^2 + 7x + 10$
$x^2 + 8x + 12$	$x^2 + 13x + 40$	**Question 5**

		$x^2 + 13x + 36$
		$x^2 + 13x + 40$
$x^2 + 17x + 72$	$x^2 + 9x + 20$	**Question 6**

		$x^2 + 18x + 81$
		$x^2 + 4x + 4$
$x^2 + 11x + 18$	$x^2 + 11x + 18$	**Question 7**

		$x^2 + 11x + 28$
		$x^2 + 10x + 21$
$x^2 + 7x + 12$	$x^2 + 14x + 49$	**Question 8**

		$x^2 + 12x + 35$
		$x^2 + 9x + 8$
$x^2 + 15x + 56$	$x^2 + 6x + 5$	**Question 9**

		$x^2 + 14x + 45$
		$x^2 + 12x + 36$
$x^2 + 15x + 54$	$x^2 + 11x + 30$	**Question 10**

Fill in each of the gaps in the grid with a bracket so that every row and column multiply to give the quadratic at the end.

There's an example to the right →

$(x + 6)$	$(x + 9)$	$x^2 + 15x + 54$
$(x + 1)$	$(x + 3)$	$x^2 + 4x + 3$
$x^2 + 7x + 6$	$x^2 + 12x + 27$	**Example**

Intelligent Practice

Factorise:

1) $-x^2 + 11x - 30$

2) $-x^2 + 13x - 30$

3) $-2x^2 + 13x - 15$

4) $-2x^2 + 13x + 15$

5) $-4x^2 + 26x + 30$

6) $-6x^2 - 39x + 45$

Factorise:

(a) $15 + 2x - x^2$

(b) $15 - 2x - x^2$

(c) $10 + 3x - x^2$

(d) $10 + 9x - x^2$

(e) $36 + 9x - x^2$

(f) $36 - 16x - x^2$

Factorise:

(a) $3 + 5x - 2x^2$

(b) $3 - 5x - 2x^2$

(c) $5 - 9x - 2x^2$

(d) $10 - x - 2x^2$

4 Difference of Two Squares

- The coefficients of the variables are square numbers.
- The powers of the variables must be even.
- The powers of the variables are **NEVER** odd numbers.
- One term will be negative **AND** the other term will be positive.
- If there is a number, then it must be a square number.

$a^2 - b^2 = (\mathrm{a} + \mathrm{b})(\mathrm{a} - \mathrm{b})$

Examples	Non-Examples
$4x^{10} - 36$	$4x^5 - 9$
$9x^{10} - 36$	$4x^6 + 9$
$-36 + 9x^{10}$	$3x^6 + 9$
$-36 + 9x^6$	$4x^6 - 8$
$9x^6 - \frac{16}{36}$	$4x^6 - 9y^5$
$9x^6 - \frac{1}{36}$	
$1x^6 - \frac{1}{36}$	
$1x^2 - \frac{1}{36}$	
$1x^2 - 36$	
$-36 + 1x^2$	
$36 - 1x^2$	
$36 - x^2$	
$x^2 - 36$	

Intelligent Practice

Factorise:

1) $x^2 - 4^2$
2) $4^2 - x^2$
3) $5^2 - x^2$
4) $5^2 - (2x)^2$
5) $7^2 - (3x)^2$
6) $8^2 - (3x)^2$
7) $64 - (3x)^2$

1) $x^2 - 144$
2) $144 - x^2$
3) $144 - (7x)^2$
4) $144 - 49x^2$
5) $144 - 25x^2$
6) $100 - 9x^2$
7) $100 - 49x^2$

1) $25x^2 - 121$
2) $121 - 25x^2$
3) $(11y)^2 - 25x^2$
4) $81y^2 - 25x^2$
5) $81y^4 - 25x^2$
6) $81y^6 - 25x^2$
7) $81y^6 - 25x^8$

1) $81w^2y^6 - 25x^8z^2$
2) $144w^2y^6 - 25x^8z^2$
3) $144w^6y^6 - 25x^{10}z^2$

Intelligent Practice

Factorise:

1) $2x^2 - 162$
2) $2x^2 - 98$
3) $98 - 2x^2$
4) $98 - 8x^2$
5) $98 - 18x^2$
6) $200 - 18x^2$
7) $200 - 2x^2$
8) $300 - 3x^2$

1) $12x^2 - 3y^2$
2) $3x^2 - 3y^2$
3) $18x^2 - 32y^2$
4) $32x^4 - 50y^6$
5) $48x^4 - 75y^6$
6) $80x^4 - 125y^6$
7) $80x^4 - 125y^6z^2$

Fluency Practice

Question 1: Factorise each of the following

(a) $x^2 - 25$ (b) $y^2 - 49$ (c) $w^2 - 100$ (d) $x^2 - 4$

(e) $c^2 - 64$ (f) $x^2 - 1$ (g) $x^2 - 900$ (h) $y^2 - 9$

(i) $16 - x^2$ (j) $1 - y^2$ (k) $81 - x^2$ (l) $144 - h^2$

(m) $x^2 - y^2$ (n) $a^2 - c^2$ (o) $9x^2 - 25$ (p) $4y^2 - 1$

(q) $49x^2 - 16$ (r) $100 - 81x^2$ (s) $9x^2 - 4y^2$ (t) $36a^2 - c^2$

(u) $121w^2 - 196y^2$ (v) $225 - 121y^2$

Question 2: Factorise **fully** each of the following

(a) $2x^2 - 32$ (b) $2y^2 - 18$ (c) $2x^2 - 200$ (d) $3x^2 - 75$

(e) $5c^2 - 20$ (f) $18x^2 - 2$ (g) $12x^2 - 147$ (h) $20y^2 - 320$

Question 3: Factorise each of the following

(a) $x^4 - 1$ (b) $y^4 - 16$ (c) $a^4 - 25$ (d) $x^4 - y^4$

(e) $h^2 - p^4$ (f) $16x^4 - 49$ (g) $y^6 - 36$ (h) $x^6 - 64$

(i) $81p^4 - x^6$ (j) $144x^8 - 1$

Question 1: Can you spot any mistakes?

Factorise $x^2 - 16$

$(x + 8)(x - 8)$

Factorise $x^2 - 25$

$(x - 5)(x - 5)$

Factorise fully $2y^2 - 50$

$2(y^2 - 25)$

Factorise $y^2 - 9w^2$

$(3w - y)(3w + y)$

Factorise:

(a) $x^2 - 4$ (b) $x^2 - 49$

(c) $x^2 - 9$ (d) $x^2 - 1$

(e) $x^2 - 64$ (f) $x^2 - 100$

(g) $x^2 - 144$ (h) $x^2 - 225$

(i) $100 - x^2$ (j) $16 - x^2$

Factorise:

(a) $4x^2 - 100$ (b) $4x^2 - 49$

(c) $16x^2 - 1$ (d) $9x^2 - y^2$

(e) $25 - 4x^2$ (f) $25y^2 - 81$

(g) $100x^2 - 4$ (h) $16x^2 - 25$

(i) $144x^2 - 16$ (j) $4x^2 - 64y^2$

Factorise:

(a) $2x^2 - 50$ (b) $2x^2 - 32$

(c) $10x^2 - 10$ (d) $10x^2 - 200$

(e) $5x^2 - 45$ (f) $5y^2 - 80$

(g) $8x^2 - 32$ (h) $8x^2 - 128$

(i) $20x^2 - 5$ (j) $75x^2 - 27$

Factorise:

(a) $x^3 - 25x$

(b) $x^4 - 49$

(c) $100x^2 - x^6$

(d) $4x^5 - 9x^9$

Extension

$d^2 - g^2$

$w^2 - 1$

$h^2 - 100$

$81 - k^4$

$t^2 - \frac{1}{4}$

$4t^2 - 9p^2$

$t^4 - e^2$

$d^6 - 25$

write these as a difference of two squares

$4t^2 - 1$

$a^8 - 9$

$v^{14} - p^{12}$

$25 - m^{20}$

$100g^{18} - 49$

$16g^6 - 49g^2$

$d^{100} - 2\frac{1}{4}$

$121b^{24} - 0.01$

$0.36r^2 - 0.04$

$(n + 1)^2 - (n - 1)^2$

Extension

difference of two squares: factorise fully

1) $1 - k^2 =$

2) $4 - 9r^2 =$

3) $49 - t^4 =$

4) $\frac{1}{4} - 4b^2 =$

5) $16g^6 - 4 =$

6) $a^{2n} - 1 =$

7) $1 - h^8 =$

8) $0.64 - 0.25d^2 =$

9) $64w^{10} - w^4 =$

10) $(d - 2)^2 - 9 =$

11) $e^2f^4 - 36 =$

12) $4a^2b^4c^8 - 4 =$

13) $18c^2 - 2 =$

14) $320 - 20m^2 =$

15) $12 - 75k^2 =$

16) $2a^3b - 8ab^3 =$

17) $(n + 2)^2 - (n + 1)^2 =$

18) $9(g + 2)^2 - 4(g - 3)^2 =$

19) $32m^2n - 50n^3 =$

20) $\frac{1}{4}d^2 - \frac{1}{9} =$

21) $\left(\frac{k+1}{2}\right)^2 - \left(\frac{k-1}{2}\right)^2$

22) $\left(\frac{k}{4}+1\right)^2 - \left(\frac{k}{4}-1\right)^2$

23) $\left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

24) $2\frac{1}{4}c^2 - 5\frac{4}{9} =$

Fluency Practice

difference of two squares: $a^2 - b^2 = (a - b)(a + b)$

use the difference of two squares to work out these calculations, without using a calculator:

1) $13^2 - 3^2$
2) $14^2 - 4^2$
3) $13^2 - 7^2$
4) $25^2 - 15^2$
5) $51^2 - 49^2$
6) $57^2 - 43^2$
7) $36^2 - 26^2$
8) $81^2 - 79^2$
9) $26^2 - 24^2$
10) $48^2 - 42^2$
11) $55^2 - 5^2$
12) $33^2 - 13^2$
13) $95^2 - 5^2$
14) $33^2 - 32^2$
15) $21^2 - 29^2$
16) $16^2 - 36^2$
17) $38^2 - 12^2$
18) $556^2 - 444^2$
19) $667^2 - 333^2$
20) $12345^2 - 12344^2$
21) $1.5^2 - 0.5^2$
22) $3.6^2 - 1.6^2$
23) $9.7^2 - 0.3^2$
24) $4.6^2 - 4.4^2$
25) $6.6^2 - 3.4^2$
26) $14.5^2 - 10.5^2$
27) $27.5^2 - 23.5^2$
28) $18.5^2 - 28.5^2$
29) $9.66^2 - 0.34^2$
30) $9.75^2 - 0.25^2$

try to write 55 as the difference between two square numbers – in two different ways

Fluency Practice

Factorise:

1) $3y^2 - 27$

2) $2x^2 - 32$

3) $x^3 + 4x^2 - 5x$

4) $ax^2 - ay^2$

5) $2x^3 - 8x$

6) $16x^2y - 36y$

7) $x^4 - 4x^3$

8) $x^{10} - 4x^9 + 4x^8$

9) $x^8 - 1$

Fluency Practice

THE FOUR BASIC METHODS OF FACTORING:

- HIGHEST COMMON FACTOR: *Always try this first.*
- DIFFERENCE OF SQUARES: This involves two terms.
- QUADRATICS: This involves three terms.
- GROUPING: This involves four or more terms.

Factoring should continue until each factor is *irreducible*, meaning that it cannot be factored further.

Use the techniques of the previous questions to factor each expression.

(a) $a^2 - 25$

(b) $b^2 - 25b$

(c) $c^2 - 25c + 100$

(d) $2d^2 + 25d + 50$

(e) $e^3 + 5e^2 + 5e + 25$

(f) $16 - f^2$

(g) $16g^2 - g^3$

(h) $h^2 + 16h + 64$

(i) $i^2 - 16i - 36$

(j) $5j^2 + 16j - 16$

(k) $4k^2 - 16k - 9$

(l) $2k^3 - 16k^2 - 3k + 24$

(m) $2a^2 + ab - 4a - 2b$

(n) $6m^3n^4 + 9m^2n^5$

(o) $49p^2 - 121q^2$

(p) $t^2 - 14t + 40$

(q) $3t^2 + 2t - 40$

(r) $5t^2 + 54t + 40$

(s) $5t^2 + 33t + 40$

(t) $5t^3 + 10t^2 + 15t$

(u) $u^2 + 15u - 54$

(v) $3x^3 - 2x^2y - 15x + 10y$

(w) $1 - 36a^2$

(x) $4a^2 - 12a + 9$

Fluency Practice

	Single Bracket	Double Bracket	Difference of two Squares	How do you know?	Factorised Form
$x^2 + 4x - 21$					
$4x^2 + 8x$					
$x^2 - 64$					
$x^2 - 14x + 48$					
$10xy + 25y^2$					
$144x^2 - 25$					
$3x^2 + 6x^3$					
$10x^2 + 5x$					
$x^2 + 6x - 7$					

Fluency Practice

(a)	(b)	(c)	(d)
Factorise $5x - 15$	Factorise $2x^2 + 7x$	Factorise fully $8x - 2x^2$	Factorise fully $15bc + 10abc$
(e)	**(f)**	**(g)**	**(h)**
Factorise fully $9a^3bc^2 + 15ab^2c$	Factorise $x^2 + 7x + 12$	Factorise $y^2 + 5y - 14$	Factorise $x^2 - 11x + 24$
(i)	**(j)**	**(k)**	**(l)**
Factorise $y^2 - 36$	Factorise $3x^2 + 5x + 2$	Factorise fully $2x^2 - 98$	Factorise $4x^2 + x - 14$
(m)	**(n)**	**(o)**	**(p)**
Factorise $25y^2 - 4x^2$	Factorise $6x^2 - 13x + 5$	Factorise $21 + 4x - x^2$	Factorise fully $2x^3 + 9x^2 - 5x$

Fluency Practice

1. $x^2 + 3x + 2$
2. $x^2 + 8x + 12$
3. $x^2 - 5x + 6$
4. $x^2 - 10x + 24$
5. $x^2 + 11x - 12$
6. $x^2 + x - 6$
7. $x^2 - 5x - 14$
8. $x^2 - 5x - 36$
9. $2x^2 + 6x + 4$
10. $2x^2 - 20x + 32$
11. $3x^2 - 12x - 36$
12. $x^3 - 4x^2 - 12x$
13. $7x^3 - 28x^2 - 84x$
14. $2x^2 + 7x + 5$
15. $3x^2 - 16x + 5$
16. $3x^2 + 8x - 3$
17. $2x^2 - 7x - 15$
18. $8x^2 + 30x + 7$
19. $3x^2 - 17x + 10$
20. $9x^2 - 24x + 16$
21. $8x^2 - 14x - 15$
22. $12x^2 - 2x - 30$
23. $12 - x - x^2$
24. $3 + 8x - 3x^2$
25. $cx^2 + 2bcx + b^2$
26. $3t^2 + 5qt + 2q^2$
27. $6r^2 - 5rt - 4t^2$
28. $12a^3 - 26a^2b + 4ab^2$

Factorising Quadratics

Factorise these expressions into two brackets:

A) $x^2 + 3x + 2$ B) $x^2 + 4x + 3$ C) $x^2 + 5x + 4$

D) $x^2 + 8x + 12$ E) $x^2 + 11x + 28$ F) $x^2 + 11x + 24$

G) $x^2 + 11x + 18$ H) $x^2 + 10x + 25$ I) $x^2 + 9x + 18$

J) $x^2 - 2x - 3$ K) $x^2 + 2x - 8$ L) $x^2 + 4x - 21$

M) $x^2 - 4x - 12$ N) $x^2 + 4x - 45$ O) $x^2 - 4x - 5$

P) $x^2 - 7x + 10$ Q) $x^2 - 11x + 30$ R) $x^2 - 4x + 4$

S) $x^2 - 14x + 49$ T) $x^2 - 13x + 42$ U) $x^2 - 13x + 40$

V) $x^2 + 9x - 52$ W) $x^2 + x - 30$ X) $x^2 - x - 72$

---

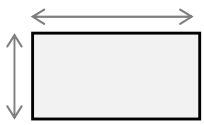
1) $x^2 + 2ax + a^2$ 2) $x^2 + mx - 2m^2$ 3) $x^2 - 9y^2$

4) The area of this square is $x^2 + 12x + 36$

What is the length of one side of the square?

5) The area of this rectangle is $x^2 - 3x - 130$

What is the length & width of the rectangle in terms of x?



6) How many ways can you complete this identity?

$$x^2 + 8x \square \equiv (x + \square)(x + \square)$$

Fluency Practice

Multiply pairs of factors from the first table to make the quadratics in the second table.
Hint: It is much easier to do this by factorizing the quadratics first!
The first pair is done for you.

(x + 3)	(2x - 2)	(2x + 1)	(2x - 1)
(3x + 2)	(x + 3)	(2x - 1)	(x + 4)
(2x - 2)	(x - 1)	(x - 3)	(x + 4)
(2x + 1)	(x + 5)	(x - 3)	(2x - 3)
(x + 8)	(x - 7)	(x + 4)	(2x - 4)
(x + 2)	(x + 2)	2(x + 1)	(x - 2)
2(x - 2)	(3x - 3)	(x + 3)	(6x - 3)
(x - 5)	(x - 5)	(x + 3)	(x - 4)
(2x + 4)	(x - 1)	(x - 5)	(x - 2)
(2x + 3)	(2x - 5)	(3x + 1)	(x - 9)
(x - 3)	4	(x + 1)	(2x - 8)

(x + 3)	(x + 4)	$x^2 + 7x + 12$	$2x^2 + 3x + 1$
		$x^2 - 3x - 10$	$4x^2 - 1$
		$x^2 + 5x + 6$	$2x^2 - 15x - 27$
		$x^2 - 25$	$4x^2 + 2x - 12$
		$x^2 + 4x - 32$	$4x^2 - 12x + 8$
		$x^2 + 2x - 3$	$2x^2 - 4x - 6$
		$x^2 + 8x + 16$	$24x - 12$
		$x^2 - 7x + 10$	$4x^2 - 14x + 10$
		$x^2 - 4x + 3$	$2(3x^2 - 4x - 4)$
		$x^2 - 9x + 14$	$6x^2 - x - 1$
		$x^2 - 9$	$6x^2 - 30x + 24$

Fluency Practice

A1 Factorise: $12x + 4$	**A2** Factorise: $3x - 15$	**A3** Factorise: $x^2 + 4x$	**A4** Factorise: $2x^2 - 5x$
B1 Fully factorise: $2xy + 4x$	**B2** Fully factorise: $3xy - 9y$	**B3** Fully factorise: $4ab - 6abc$	**B4** Fully factorise: $2a^2b + 8ab^2$
C1 Factorise: $x^2 - 36$	**C2** Factorise: $x^2 - 81$	**C3** Factorise: $x^2 - 1$	**C4** Factorise: $x^2 - 9$
D1 Factorise: $x^2 + 3x - 54$	**D2** Factorise: $x^2 + 8x + 7$	**D3** Factorise: $x^2 - 4x - 32$	**D4** Factorise: $x^2 - 8x + 15$

Fluency Practice

A1 Factorise: $9x + 15$	**A2** Factorise: $4x^2 - 3x$	**A3** Factorise: $3x^2 + 15x$	**A4** Factorise: $9x^2y - 12xy^2$
B1 Factorise: $x^2 - 81$	**B2** Factorise: $x^2 - y^2$	**B3** Factorise: $4x^2 - 25$	**B4** Factorise: $9x^2 - 16y^2$
C1 Factorise: $x^2 + 13x + 22$	**C2** Factorise: $x^2 - 11x + 10$	**C3** Factorise: $x^2 + 3x - 18$	**C4** Factorise: $x^2 - x - 20$
D1 Factorise: $3x^2 + 8x + 4$	**D2** Factorise: $2x^2 - 19x + 42$	**D3** Factorise: $5x^2 + 21x - 20$	**D4** Factorise: $4x^2 - 4x - 15$

Intelligent Practice

1) $x^2 + 9x$
2) $x^2 - 9x$
3) $x^2 - 9$
4) $x^2 - 9y^2$
5) $x^2 + 9x + 8$
6) $x^2 + 9x + 20$
7) $x^2 - 9x + 20$
8) $x^2 - 9x - 22$
9) $x^2 + 9x - 22$
10) $2x^2 + 9x + 4$
11) $2x^2 - 9x + 4$
12) $2x^2 - 9x$
13) $2x^2y - 9xy^2$
14) $x^2 - y^2$
15) $4x^2 - 4y^2$
16) $6x^2 - 6y^4$
17) $6x^2y - 18y^4$
18) $6x^3y^2 - 18x^2y^3$
19) $6x^2 + 22x + 20$
20) $6x^2 + 2x - 20$
21) $6x^2 - 19x - 20$
22) $6x^2 - 150$
23) $3x^2 - 75$
24) $3x^2 - 40x - 75$
25) $3x^2 - 40xy - 75y^2$
26) $3x^6 - 75y^2$
27) $(x - 3)^2 - 49$
28) $(2x - 3)^2 - 49$
29) $4x^2 - 6x - 40$
30) $x^2 - 6x - 40$
31) $40 - 6x - x^2$
32) $40 - 3x - x^2$
33) $ax - 3a - 5x + 15$
34) $xy - 2x - 4y + 8$
35) $x^2 - 2x - 8$
36) $x^3 - 2x^2 - 8x$
37) $x^3y - 2x^2y - 8xy$
38) $10y^2 - 11y - 8$
39) $10y^4 - 11y^2 - 8$
40) $10y^5 - 11y^3 - 8y$

Fill in the Gaps

Expanded Expression	Factorised Expression
$2x + 8$	$2(x + 4)$
	$3(x - 2)$
	$x(x + 7)$
$5x + 35$	
$8x - 12$	
	$2x(x - 5)$
$x^2 - x$	
	$5x(3 - x)$
$10x^2 + 2x$	
$6x + 9y$	
	$4xy(x + 2)$
$6xy - 4y^2$	
	$(x + 2)(x + 3)$
	$(x + 5)(x - 3)$
$x^2 + 8x + 15$	
$x^2 + 3x + 2$	

Expanded Expression	Factorised Expression
$x^2 - 7x + 10$	
	$(x - 6)(x + 4)$
	$(x + 7)(x - 7)$
$x^2 + 2x - 15$	
$x^2 - 25$	
	$(2x + 1)(x + 5)$
$x^2 - x - 6$	
$x^2 + 3x$	
	$(3x - 1)(x - 2)$
$4x^2 - 25$	
	$(x + 5)^2$
$7x^2 + 10x + 3$	
	$(3x - 1)^2$
$4x^2 + 4x + 1$	
$5x^2 - 14x - 3$	
	$(x - 2)^3$

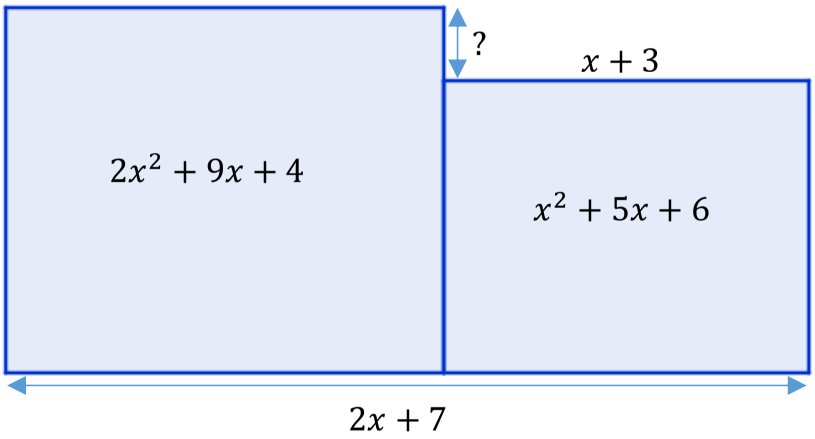
Extension

Solve the following equations:

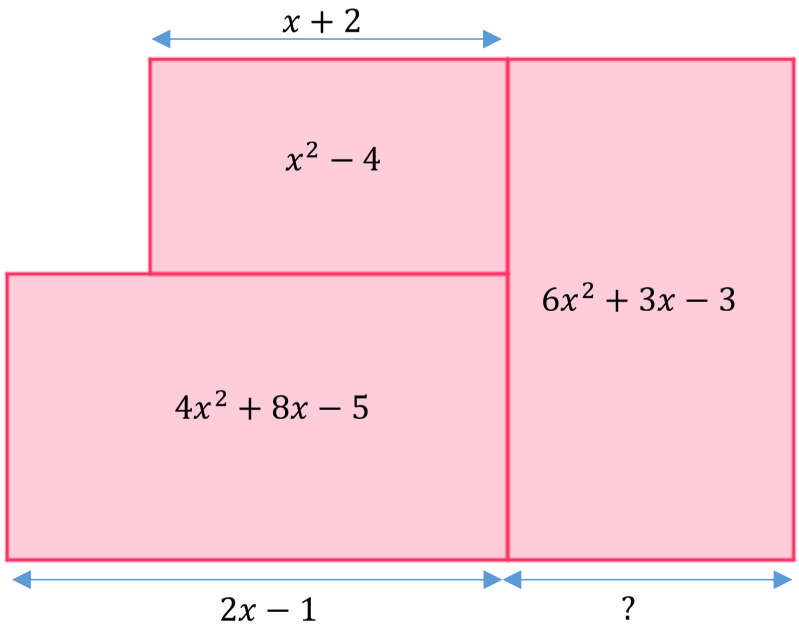
1. $(x+1)(x+2) = x^2 + 5$
2. $(x-1)(x+1) = x^2 + 3x + 17$
3. $(x+3)(x+4) = x^2 + 3x + 17$
4. $(x+3)(2x-3) = 2x^2 - 2x + 3$
5. $(x+2)(x+1) = x^2 + x - 2$
6. $(x+3)(x-4) = x^2 - 8x + 2$
7. $x^2 - 3x + \frac{1}{4} = (x+1)(x-1) - 4x + 2\frac{3}{4}$
8. $3x - x^2 + 5 = (5-x)(4+x) - 9$
9. $5 + 4x + x^2 = (5+x)(x+1)$
10. $6 + 3x - x^2 = (9-x)(x-6)$
11. $(2x+3)(x+2) + x = (x+2)(x+3) + x^2 + 9$
12. $(2x+3)(x-1) + x = (x+1)(2x-1) + 2x - 3$
13. $(x+3)(2x-1) + x^2 = (3x+2)(x-1) + 3x + 5$

In each of these questions, find the marked side or area in terms of x.

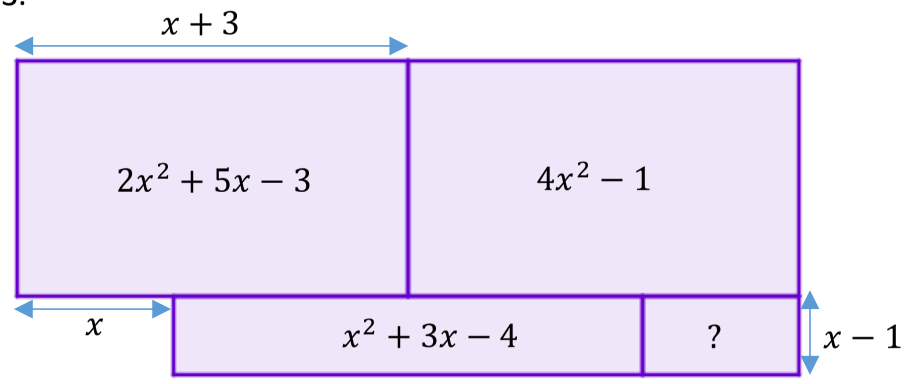
1.



2.

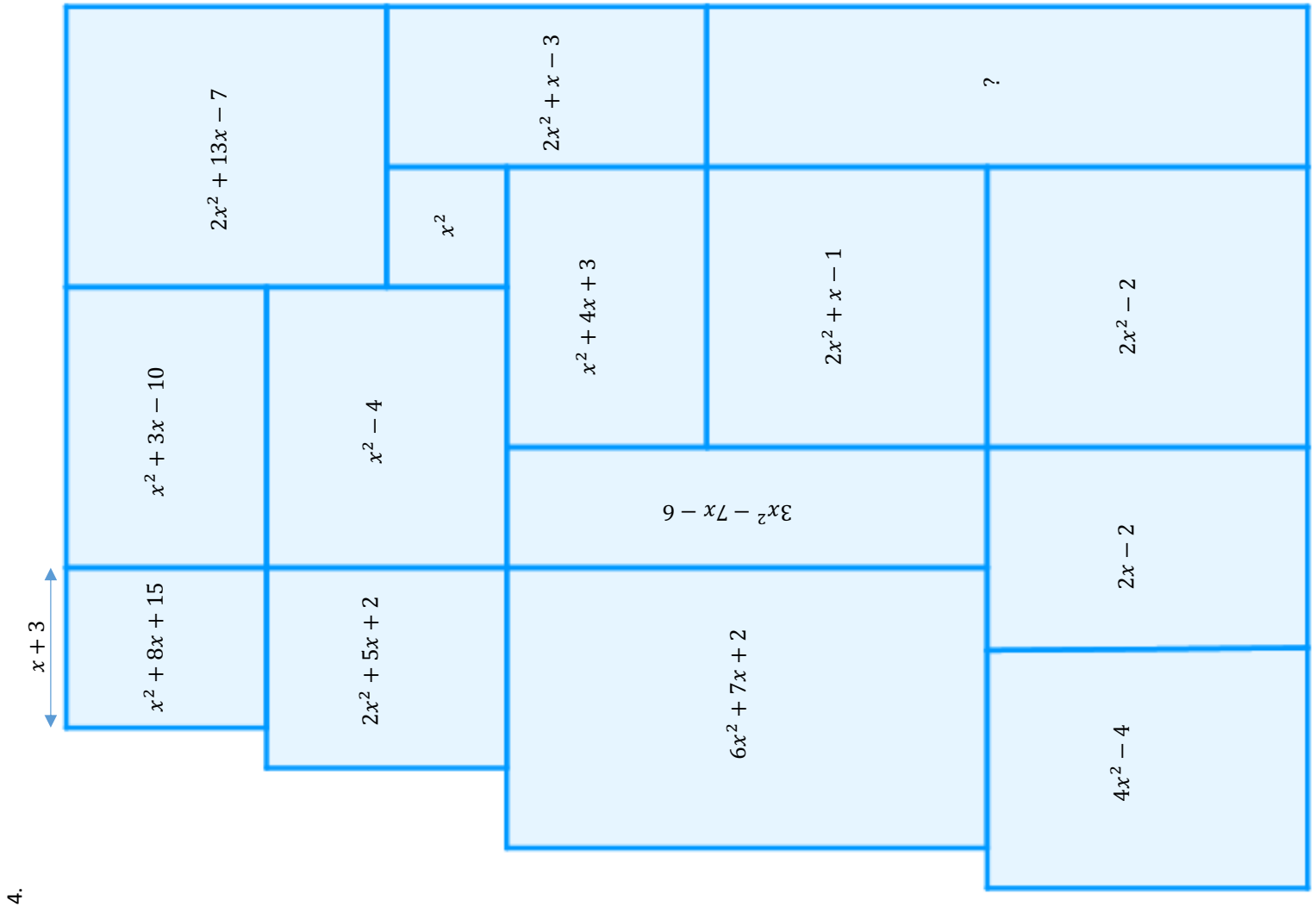


3.

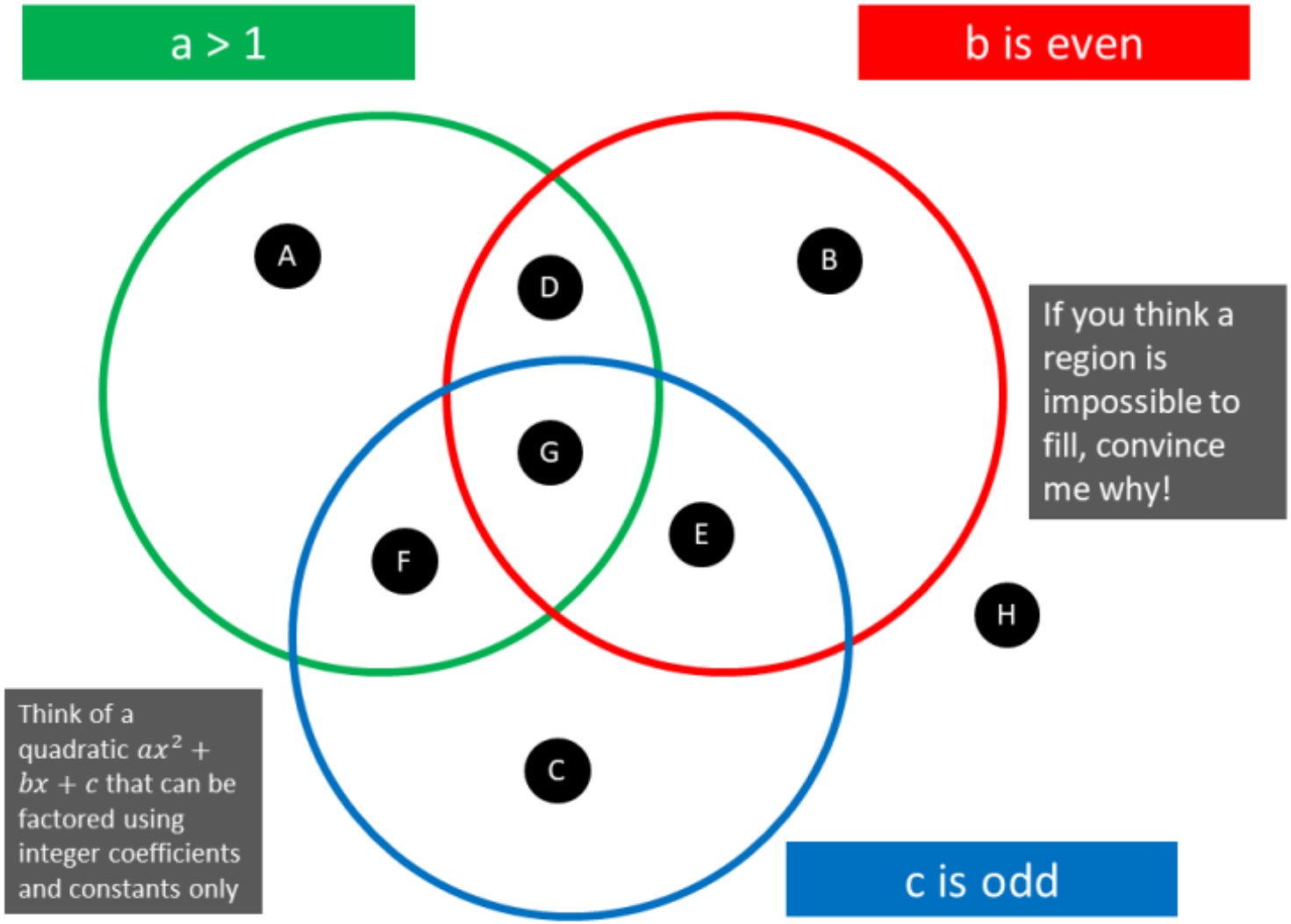


Problem Solving

4.



Maths Venns


a > 1
b is even
A
D
B
G
E
F
H
C
c is odd
If you think a region is impossible to fill, convince me why!
Think of a quadratic $ax^2 + bx + c$ that can be factored using integer coefficients and constants only

5 Basic Functions

Here is a number machine:

Input → x 3 → - 2 → Output

What is the output when the input is 7?

Here is an expression:

$3x - 2$

What is the value of this expression when $x = 7$

Here is a function:

$f(x) = 3x - 2$

Calculate the value of $f(7)$

Fluency Practice

Question 1: Given $f(x) = 3x + 5$

Work out the values of

(a) $f(2)$ (b) $f(8)$ (c) $f(0)$ (d) $f(-2)$

Question 2: Given $g(x) = \dfrac{2x + 9}{4}$

Work out the values of

(a) $g(6)$ (b) $g(-1)$ (c) $g(0)$ (d) $g(-10)$

Question 3: Given $h(x) = x^2 - 5$

Work out the values of

(a) $h(7)$ (b) $h(-1)$ (c) $h(-3)$ (d) $h(15)$

functions into functions

Work out the value of each of these functions.

Match them to the jumbled answers at the bottom.

$G(x)=\frac{x}{3}$	$y(x) = 2x - 3$	$p(x) = 10 - x$	$f(x) = \frac{1}{2}x^2$
$G(3)$	$y(3)$	$p(4)$	$f(4)$
$G(-9)$	$y(-3)$	$p(\frac{1}{4})$	$f(12)$
$G(\frac{1}{3})$	$y(?) = -3$	$p(-2)$	$f(-6)$
$G(x+4)$	$y(x+3)$	$p(-9)$	$f(2x)$
$G(6x + 6)$	$y(x-3)$	$p(x+2)$	$f(4x)$
$G(3x)$	$y(4x)$	$p(x-3)$	$f(\frac{x}{2})$
$G(9x^3)$	$y(\frac{x}{2})$	$p(x-7)$	$f(x^2)$

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jumbled answers

$8x^2$ | -3 | 6 | $13 - x$ | 3 | $2x^2$ | $8x - 3$ | $\frac{x^4}{2}$

0 | $2x - 9$ | $10 - (x+2)$ | $\frac{x+4}{3}$ | $2x + 3$ | -9 | 72 | 9.75 | $\frac{1}{9}$ | $x - 3$

12 | $17 - x$ | 1 | $2x+2$ | 18 | x | $3x^3$ | 8 | 19 | $\frac{x^2}{8}$

Fluency Practice

Question 4: The function f is such that $f(x) = 3x - 8$

Solve $f(x) = 7$

Question 5: The function g is such that $g(x) = 19 - 4x$

Solve $g(x) = 31$

Question 6: The function h is such that $h(x) = \dfrac{5x - 1}{2}$

Solve $h(x) = 32$

Question 1: Given $f(x) = 5x + 7$ and $g(x) = 3x - 18$

Find the value of a such that $f(a) = g(a)$

(a) $f(x) = 5x + 2$

Find (i) $f(3)$ (ii) $f(7)$

(iii) $f(-4)$ (iv) $f(-0.5)$

(b) $f: \rightarrow x^2 - 4$

Find (i) $f(4)$ (ii) $f(6)$

(iii) $f(-2)$ (iv) $f(0.9)$

(c) $g(x) = x^3 - 3x^2 - 2x + 1$

Find (i) $g(0)$ (ii) $g(1)$

(iii) $g(-1)$ (iv) $g(1.5)$

(d) $f(x) = \sqrt{2x + 5}$

Find (i) $f(2)$ (ii) $f(10)$

(iii) $f(-2)$ (iv) $f(-1.78)$

(e) The functions f and g are such that $f(x) = 3x - 5$ and $g(x) = 4x + 1$

(i) Find $f(-1)$ and $g(2)$

(ii) Find the value of x for which $f(x) = g(x)$.

(f) The functions f and g are such that $f(x) = 2x^2 - 1$ and $g(x) = 5x + 2$

(i) Find $f(-3)$ and $g(-5)$

(ii) Find the two values of x for which $f(x) = g(x)$.

Fluency Practice

Question 4: Given $f(x) = x^2 + 4x - 1$

Express the following in the form $ax^2 + bx + c$

(a) $f(x+2)$ (b) $f(x-1)$ (c) $f(2x)$

(d) $f(3x)$ (e) $f(2x-1)$ (f) $f(4x+3)$

Question 8: $f(x) = x^2 + 3x + 8$

Show that $f(x+1) - f(x) = 2x + 4$

1. The function f is such that $f(x) = 5x + 2$

   Find (a) f(3) (b) f(7) (c) f(−4)

   (d) f(−2) (e) f(−0.5) (f) f(0.3)

2. The function f is such that $f(x) = x^2 - 4$

   Find (a) f(4) (b) f(6) (c) f(−2)

   (d) f(−6) (e) f(−0.2) (f) f(0.9)

3. The function g is such that $g(x) = x^3 - 3x^2 - 2x + 1$

   Find (a) g(0) (b) g(1) (c) g(2)

   (d) g(−1) (e) g(−0.4) (f) g(1.5)

4. The function f is such that $f(x) = \sqrt{2x + 5}$

   Find (a) f(0) (b) f(1) (c) f(2)

   (d) f(−1) (e) f(−0.7) (f) f(1.5)

5. $f(x) = 3x^2 - 2x - 8$

   Express $f(x + 2)$ in the form $ax^2 + bx$

6. The functions f and g are such that

   $f(x) = 3x - 5$ and $g(x) = 4x + 1$

   (a) Find (i) f(−1) (ii) g(2)

   (b) Find the value of x for which $f(x) = g(x)$.

7. The functions f and g are such that

   $f(x) = 2x^2 - 1$ and $g(x) = 5x + 2$

   (a) Find f(−3) and g(−5)

Fluency Practice

Functions

1. If $f(x) = 3x + 5$, find

a) $f(5) =$ b) $f(2) =$ c) $f(-1) =$

2. If $g(x) = 4x^2 - 3x$, find

a) $g(3) =$ b) $g(-1) =$ c) $g\left(\frac{1}{2}\right) =$

3. If $f(x) = \frac{3x+5}{2x-1}$, find

a) $f(5) =$ b) $f(-1) =$ c) $f\left(\frac{5}{6}\right) =$

4. Given that $f(x) = 5x + 3$, find the value of x for

a) $f(x) = 9,$ b) $f(x) = -4,$

c) $f(x) = 2x + 7,$

5. Given that $f(x) = \frac{3x+4}{2x-5}$, find the value of x for

a) $f(x) = 4,$ b) $f(x) = -3,$

c) $f(x) = \frac{3}{4},$

7. If, $f(x) = 10 - 2x$, determine, by simplifying if possible

a) $f(3x) =$ b) $f(x + 3) =$

c) $f(x - 3) =$

8. If, $g(x) = x^2 - 2x$, determine, by simplifying if possible

a) $g(2x) =$ b) $g(-3x) =$

c) $g(x + 1) =$

9. If, $f(x) = 2x^2 - 5x$, determine, by simplifying if possible

a) $f(3x) =$ b) $f(-x) =$

c) $f(x - 4) =$

Extension

10. If $f(x) = 2x - 1$

a) Determine $f\big(1 + 2f(x - 1)\big) =$

b) Solve $f(x + 1) + f(x - 1) = 0,$

6 Changing the Subject

A formula is a mathematical equation containing two or more variables.

Suppose that you have the formula such as $2x = 3a$

We could write this formula as $x = \frac{3a}{2}$ in which case we would say that x is the subject of the formula, or that x is given/written in terms of a.

Note: x is the subject of the formula above as it appears on its own on one side of an equals sign.

Intelligent Practice

Make x the subject for each of the following formulae:

1) $y = ax + b$

2) $y = b^2 + ax$

3) $y = ax + bcd$

4) $y = d^2ef + ax$

5) $y = ax - b$

6) $y = -\sqrt{b} + ax$

7) $y = ax - bcd$

8) $y = -\sqrt{bcd} + ax$

Intelligent Practice

Make x the subject for each of the following formulae:

1) $y = \frac{x}{a} + c$

2) $y = -\frac{x}{a} + c$

3) $y = \frac{x}{a^2} - c$

4) $y = -\frac{x}{a^2} - c$

5) $y = \frac{x}{bc} + \sqrt{qrs}$

6) $y = -\frac{x}{bc} - \sqrt{qrs}$

Make x the subject of these equations.

There are two ways to rearrange each equation so there are two sets of answer boxes. You only need to fill in your answer once. Use the boxes where your answer fits best.

$a(x + b) = c$ $\quad x = \frac{\square}{\square} - \square$ or $x = \frac{\square}{\square}$

$2(x + a) = b$ $\quad x = \frac{\square}{\square} - \square$ or $x = \frac{\square}{\square}$

$a(x - 6) = b$ $\quad x = \frac{\square}{\square} + \square$ or $x = \frac{\square}{\square}$

$c(7 + x) = 2$ $\quad x = \frac{\square}{\square} - \square$ or $x = \frac{\square}{\square}$

Intelligent Practice

Make a the subject for each of the following formulae:

1) $2a^2 + b = c$

2) $2a^2 - b = c$

3) $6a^2 - b = c$

4) $2(a - 2b)^2 + b = c$

5) $2(a + 2b)^2 - b = c$

6) $6(a + 2b)^2 - b = c$

1) $2\sqrt{a} + b = c$

2) $2\sqrt{a} - b = c$

3) $6\sqrt{a} - b = c$

4) $2\sqrt{a - 2b} + b = c$

5) $2\sqrt{a + 2b} - b = c$

6) $6\sqrt{a + 2b} - b = c$

Fluency Practice

Question 1: Make y the subject of each of the following

(a) $y + w = c$ (b) $y - p = m$ (c) $m + y = s$

(d) $y - 2g = n$ (e) $3y = c$ (f) $ay = w$

(g) $\frac{y}{c} = w$ (h) $\frac{y}{a} = 2c$ (i) $a = y + p$

(j) $c = y - k$ (k) $y^2 = s$ (l) $y^3 = x$

(m) $\sqrt{y} = g$ (n) $\pi y = c$ (o) $n - y = t$

(p) $ry = c$ (q) $4\pi y = b$ (r) $y + 7t = c + r$

(s) $\frac{r}{y} = w$ (t) $y^2 = k + x$ (u) $A = xy$

Question 2: Make x the subject of the following formulae

(a) $4x + c = w$ (b) $dx - t = 8$ (c) $x^2 + 3 = h$

(d) $2x + 2y = P$ (e) $s = x^2 - 3$ (f) $y = xz + s$

(g) $\frac{x}{n} + 2 = w$ (h) $\frac{x}{6} - 5 = w$ (i) $\frac{x + 3}{c} = h$

(j) $3y = 4x + 1$ (k) $x^2 + a = v$ (l) $x^3 - 4 = 5y$

(m) $\frac{x + t}{m} = 2c$ (n) $\frac{w + x}{u} = 3z$ (o) $A = \pi x^2$

(p) $A = \frac{1}{2}bx$ (q) $V = abx$ (r) $v^2 = u^2 + 2ax$

(s) $\frac{a + b}{x} = r$ (t) $\frac{5cx}{b} = a$ (u) $\sqrt[3]{\frac{x}{k}} = w$

Question 3: Make c the subject of the following

(a) $(a + c)^2 = t$ (b) $v = u + ac$ (c) $v = \pi c^2 h$

Question 1: The circumference of a circle is given as $c = 2\pi r$
Make the radius, r, the subject of the formula.

Question 2: The formula to convert degrees Fahrenheit to degree Celsius is $\frac{5}{9}(F - 32) = C$

Find the formula to convert from degrees Celsius to degrees Fahrenheit by making F the subject.

Question 3: Can you spot any mistakes below?

Make *y* the subject of the formula:

$k = y^2 + a$

$\sqrt{k} = y + a$

$\sqrt{k} - a = y$

$y = \sqrt{k} - a$

Express *v* in terms of *t*

$t = \frac{v}{4} + 1$

$t - 1 = \frac{v}{4}$

$\frac{t - 1}{4} = v$

Question 1: Make x the subject of each of the following

(a) $A = \frac{1}{2}(x + y)$

(b) $A = \pi r^2 + 2\pi rx$

(c) $T = 3x^2 - y$

(d) $s = \frac{m}{ax}$

(e) $s = uy + \frac{1}{2}xy^2$

(f) $\frac{1}{3}w = \frac{1}{4}x + t$

(g) $j = \frac{x + 3}{d}$

(h) $g = \frac{t}{x - 2}$

(i) $p = 3(y + 2x)^2$

(j) $12w = \frac{3}{4}(2x + a)$

Fluency Practice

Rearranging One-Step Formulae. Make the subject the letter given in brackets.			
(a)	**(b)**	**(c)**	**(d)**
$V = I \times R \quad (R)$	$A = b \times h \quad (b)$	$P = \frac{F}{A} \quad (F)$	$D = \frac{M}{V} \quad (M)$
(e)	**(f)**	**(g)**	**(h)**
$E = C + D \quad (C)$	$a = b + c \quad (c)$	$S = U - V \quad (U)$	$y = x - 5 \quad (x)$
(i)	**(j)**	**(k)**	**(l)**
$y = x^2 \quad (x)$	$u = v^2 \quad (v)$	$T = \sqrt{S} \quad (S)$	$a = \sqrt{b} \quad (b)$
(m)	**(n)**	**(o)**	**(p)**
$y = mx + c \quad (c)$	$v = u + at \quad (u)$	$d = c^2 + b \quad (b)$	$V = R - I^2 \quad (R)$

Fluency Practice

Rearranging Two-Step Formulae. Make the subject the letter given in brackets.			
(a)	**(b)**	**(c)**	**(d)**
$v = u + at \quad (t)$	$y = mx + c \quad (m)$	$a = 2b - c \quad (b)$	$s = ut - d \quad (u)$
(e)	**(f)**	**(g)**	**(h)**
$a = \frac{b + c}{2} \quad (b)$	$t = \frac{u - v}{4} \quad (u)$	$d = \frac{a}{b} + c \quad (a)$	$h = \frac{e}{2} - f \quad (e)$
(i)	**(j)**	**(k)**	**(l)**
$T = m^2 + d \quad (m)$	$b = x^2 - a \quad (x)$	$d = 5c^2 \quad (c)$	$P = RI^2 \quad (I)$
(m)	**(n)**	**(o)**	**(p)**
$s = \frac{uv}{2} \quad (v)$	$a = b + c - d \quad (c)$	$v = \sqrt{e + f} \quad (e)$	$y = \sqrt{\frac{a}{b}} \quad (a)$

Fluency Practice

Rearranging Formulae. Make x the subject of these formulae.			
(a)	**(b)**	**(c)**	**(d)**
$y = x + a$	$y = bx$	$y = \frac{x}{c}$	$y = bx + a$
(e)	**(f)**	**(g)**	**(h)**
$y = \frac{x}{a} + b$	$y = \frac{x}{a} - b$	$y = \frac{x + b}{2}$	$y = 4x + a$
(i)	**(j)**	**(k)**	**(l)**
$y = x^2$	$y = x^2 + a$	$2y = x^2 - b$	$y = ax^2$
(m)	**(n)**	**(o)**	**(p)**
$y = ax^2 + b$	$2y = bx^2$	$y = \frac{3x + a}{5}$	$y = \sqrt{x + b}$

Fluency Practice

1	weight = mass × gravitational field strength	$W = m\,g$	Make m the subject	
2	work done = force × distance	$W = Fs$	Make s the subject	
3	force = spring constant × extension	$F = ke$	Make e the subject	
4	pressure = $\frac{\text{force normal to a surface}}{\text{area of that surface}}$	$p = \frac{F}{A}$	Make F the subject	
5	distance travelled = speed × time	$s = vt$	Make v the subject	
6	acceleration = $\frac{\text{change in velocity}}{\text{time taken}}$	$a = \frac{\Delta v}{t}$	Make Δv the subject	
7	resultant force = mass × acceleration	$F = ma$	Make a the subject	
8	momentum = mass × velocity	$p = mv$	Make m the subject	
9	power = $\frac{\text{work done}}{\text{time taken}}$	$P = \frac{W}{t}$	Make W the subject	
10	density = $\frac{\text{mass}}{\text{volume}}$	$\rho = \frac{m}{V}$	Make m the subject	
11	time period = $\frac{1}{\text{frequency}}$	$T = \frac{1}{f}$	Make f the subject	
12	power = (current)^2 × resistance	$P = I^2R$	Make I the subject	

Changing the Subject of a Formula

If you know all the other information, how do you find…

Volume of a Cuboid … the height of a cuboid?

$$V = whl$$

Area of a Triangle … the base length of a triangle?

$$A = \frac{bh}{2}$$

Volume of a Triangle-based Prism … the length of the prism?

$$V = \frac{1}{2}whl$$

Circumference of a Circle … the radius?

$$\mathrm{C} = 2\pi r$$

Area of a Trapezium … one of the parallel side lengths (b)?

$$A = \frac{1}{2}(a + b)h$$

Area of a Circle … the radius?

$$A = \pi r^2$$

Surface Area of a Sphere … the radius?

$$A = 4\pi r^2$$

Volume of a Square-based Pyramid … the base edge length (l)?

$$\mathrm{V} = \frac{1}{3}hl^2$$

Pythagoras' Theorem … the length of a shorter side (a)?

$$a^2 + b^2 = c^2$$

Volume of a Sphere … the radius?

$$V = \frac{4}{3}\pi r^3$$

Intelligent Practice

Make a the subject of the following formulae

1. $ab + ac = 3$
2. $3a + ay = x$
3. $a(3 + y) + 2a = x$
4. $2a = x + ak$
5. $ay + k = 3a$
6. $ay - x = ab$
7. $ay + b = 3b + ax$
8. $m(a + y) = x(a + b)$
9. $\frac{y-a}{x+a} = b$
10. $a - n = \frac{a+2}{n}$
11. $\sqrt{\frac{a+x}{a-x}} = 2$
12. $\sqrt{\frac{m(a+n)}{a}} = p$

Intelligent Practice

Make x the subject for each of the following formulae:

1) $5x + b = 2x + a$

2) $5x + bd = 2x + ac$

3) $bx + 5 = ax + 2$

4) $bx + 5d = ax + 2c$

5) $5(x + b) = 2(x + a)$

6) $5(x + bd) = 2(x + ac)$

7) $b(x + 5) = a(x + 2)$

8) $b(x + 5d) = a(x + 2c)$

Intelligent Practice

Make x the subject for each of the following formulae:

1) $w = \frac{5x+5b}{x+a}$

2) $w = \frac{5x+5b}{4x+4a}$

3) $w = \frac{5x+5b}{3x-3a}$

Fluency Practice

Question 2: Make m the subject of the following formulae

(a) $5(m + y) = 4(m - 3y)$

(b) $3(3m + 4) = 7(m + 2a)$

(c) $15(2m + 3c) = 5(m + 7c)$

(d) $9m + 4c = 2(a + 3m)$

(e) $a(c + m) = 2(c + 3m)$

(f) $w(m + n) = x(m - n)$

(g) $8 = \frac{m + 3c}{m - f}$

(h) $y = \frac{m + 4}{m + 5}$

(i) $y = 3mt - a^2m$

(j) $r(c + 7) = 3m + 5$

(k) $x = 4\pi m + am$

(l) $2 = \frac{m + k}{m - t}$

(m) $dm = y - em$

(n) $m(c + d) = m + f$

(o) $y - mp = np + 2y$

(p) $m(r + p) = r(h - m)$

(q) $\pi x = \frac{m + 8}{m - 1}$

(r) $\frac{3m + 2}{c} = \frac{m + 1}{a}$

Question 3: Make c the subject of the following

(a) $w = \frac{ac}{a - c}$

(b) $w = 6 + \frac{a}{c + 2}$

Fluency Practice

1. Make x the subject of each of the following formulae.

(a) $2x + a = x - b$

(b) $ax - b = cx - d$

(c) $xa - 4 = bx - 5$

(d) $3x - 6 = 4a + 2x$

(e) $b - 2x = c - 5x$

(f) $a - bx = c - dx$

(g) $2(x + 1) = a - x$

(h) $4(x - a) = 3(a - x)$

(i) $p(x + 1) = q(x - 1)$

(j) $\dfrac{x - a}{2} = \dfrac{x + b}{3}$

(k) $\dfrac{2x - a}{5} = x + 1$

(l) $\dfrac{x}{a} = \dfrac{x + b}{4}$

2. Make x the subject of each of the following formulae.

(a) $P = \dfrac{x}{x + 1}$

(b) $P = \dfrac{ax + b}{x}$

(c) $Q = \dfrac{x + b}{x - a}$

(d) $q^2 = \dfrac{x + y}{x - y}$

(e) $\dfrac{x - 2}{x + 3} = a$

(f) $\dfrac{x - b}{x - c} = 4$

(g) $p = \sqrt{\dfrac{x}{x + 1}}$

(h) $w = \sqrt{\dfrac{x - 2}{x}}$

(i) $w = \sqrt{\dfrac{x - 2}{x + 1}}$

(j) $p = \dfrac{x^2 + 2}{x^2}$

(k) $p = \dfrac{2 - x^2}{3 - x^2}$

(l) $g = \dfrac{x^2 - y}{x^2 + y}$

Make x the subject of each formula:

(a) $ax = bx + c$

(b) $ax = bx - c$

(c) $ax + c = bx + 2$

(d) $ax - d = bx + c$

(e) $ax - 2 = d - bx$

Make x the subject of each formula:

(a) $a = \frac{bx}{x-2}$

(b) $2 = \frac{bx}{a+x}$

(c) $a = \frac{x+b}{x}$

(d) $2a = \frac{x}{x-1}$

Make x the subject of each formula:

(a) $a(x + 2) = bx - c$

(b) $a(b - x) = c(x + d)$

(c) $2b = \frac{x-a}{c+x}$

(d) $a = \frac{x+b}{x-b}$

Make x the subject of each formula:

(a) $d = \sqrt{\frac{x-c}{x}}$

(b) $bx = c + \frac{x}{a}$

(c) $x = \sqrt{\frac{a+x^2}{b}}$

(d) $bx^3 = \frac{a^2-x^3}{c}$

Intelligent Practice

☆

Make x the subject of the formula

1) $y = x + a$

2) $y = ax$

3) $y = x - a$

4) $y = a - x$

5) $y = \frac{x}{a}$

6) $y = \frac{a}{x}$

7) $y = ax + b$

8) $y = abx$

☆☆

Make x the subject of the formula

1) $y = \frac{ax}{b}$

2) $y = x^2 + a$

3) $y = ax^2$

4) $y = \sqrt{x} + a$

5) $y = \sqrt{x - a}$

6) $y = a\sqrt{x}$

7) $y = \frac{x^2}{ab}$

8) $y = \frac{\sqrt{x}}{ab}$

☆☆☆

Make x the subject of the formula

1) $x + a = b - x$

2) $ax = b - x$

3) $x - b = ax + c$

4) $ax = by + x$

5) $a(x+y) = b - x$

6) $ax + y = bx + z$

7) $ax - y = bx + yz$

8) $a = \frac{bx}{x+c}$

Intelligent Practice

Make a the subject of the following formulae:

1) $2a = b$
2) $\frac{a}{2} = b$
3) $a + 2 = b$
4) $a - 2 = b$
5) $\frac{a}{c} = b$
6) $\frac{a}{c} + 2 = b$
7) $ac = b$
8) $ac - 2 = b$
9) $ac - d = b$
10) $\frac{a+2}{c} = b$
11) $\frac{2a}{c} = b$
12) $\frac{d(a+2)}{c} = b$
13) $2a = b + a$
14) $2a + c = b + a$
15) $2a + c = b - a$
16) $ae + c = b + a$
17) $ae + c = b + af$
18) $ae + c = b - af$

Make x the subject.

(a) $F = x^2 - 3$

(b) $p = 5(x - 3)$

(c) $2x + a = x + b$

(d) $y^2 = x^2 + a^2$

(e) $P = \frac{x^2}{4} + c$

(f) $y - 5 = 2 + \frac{x}{6}$

Make x the subject.

(a) $4x - 8 = ax$

(b) $ax = b - cx$

(c) $2(x + 3) = bx$

(d) $a - bx = dx - a$

(e) $4(x - 2y) = 3(2x - y)$

(f) $p(x - b) = q(x + b)$

Make x the subject.

(a) $A = 2xb^2 + cx$

(b) $x(c - d) = c(d - x)$

(c) $y = \frac{x+2}{x-2}$

(d) $2a = \frac{x-3}{4x}$

(e) $b = \frac{ax}{a+x}$

(f) $a = \frac{2+3x}{x-2}$

Fluency Practice

Make x the subject of each of the formulae.

(a)	(b)	(c)	(d)
$a = b + x^2$	$a^2 = b^2 + x^2$	$b = \frac{x^3}{2a}$	$b = 2a + \sqrt{x}$
(e)	**(f)**	**(g)**	**(h)**
$b = \sqrt{\frac{x}{a}}$	$a + 5 = \sqrt{\frac{x}{b}}$	$ax = x + 5$	$ax + b = a + x$
(i)	**(j)**	**(k)**	**(l)**
$a = \frac{x}{x+1}$	$a(x - b) = bx$	$b = \frac{x+a}{x-a}$	$a = \sqrt{\frac{x}{x+b}}$

Fluency Practice

(a)	(b)	(c)	(d)
$y = x^2 + 2x$ Find the value of y when $x = 5$	$b = a^3 - 5a$ Find the value of b when $a = 3$	$w = 2d^2 + 5d$ Find the value of w when $d = -4$	$y = 3x^3 + 5x^2 - 6$ Find the value of y when $x = -2$
(e)	**(f)**	**(g)**	**(h)**
$d = 3a + 5b$ Find d when $a = 7$ and $b = -2$	$t = p^2 + pq$ Find t when $p = -6$ and $q = 2$	$f = \frac{2d + e^2}{de}$ Find f when $d = 5$ and $e = -2$	$y = \frac{3}{4}ab^2$ Find y when $a = 5$ and $b = -0.5$
(i)	**(j)**	**(k)**	**(l)**
Make b the subject of $a = 4b - 7$	Make x the subject of $y = x^2 + 5$	Make d the subject of $e = \frac{c + d}{5}$	Make a the subject of $x = 2a^2 - cd$
(m)		**(n)**	
Make x the subject of the formula $y = \frac{x}{x-3}$		Make a the subject of the formula $b = \frac{5-2a}{3a+2}$	

7 Inverse Functions

Here is a number machine:

Input → [x 3] → [- 5] → Output

What is the input when the output is 10?

Make x the subject:

$y = 3x - 5$

Given $f(x) = 3x - 5$

Find $f^{-1}(x)$

RULES FOR FINDING THE INVERSE $f^{-1}(x)$:

Step 1: Write out the function as $y = \cdots$

Step 2: Swap the x and y

Step 3: Make y the subject

Step 4: Instead of $y =$ write $f^{-1}(x) =$

Fluency Practice

Question 13: Find $f^{-1}(x)$ for each of the following:

(a) $f(x) = 2x$ (b) $f(x) = x - 6$ (c) $f(x) = \frac{x}{3}$

(d) $f(x) = 5x + 1$ (e) $f(x) = \frac{2x}{7}$ (f) $f(x) = \frac{x-2}{6}$

Question 14: Given $h(x) = \frac{x}{4}$

(a) Find $h^{-1}(x)$

(b) Calculate the value of $h^{-1}(1.5)$

Question 15: Given $f(x) = 2x - 3$

(a) Find $f^{-1}(x)$

(b) Calculate the value of $f^{-1}(7)$

Question 16: Given $g(x) = \frac{3x+1}{2}$

(a) Find $g^{-1}(x)$

(b) Calculate the value of $g^{-1}(11)$

Question 17: Given $f(x) = \frac{4x}{9} - 8$

(a) Find $f^{-1}(x)$

(b) Calculate the value of $f^{-1}(-10)$

1. Find the inverse function, $f^{-1}(x)$, of the following functions:

   (a) $f(x) = 3x - 1$ (b) $f(x) = 2x + 3$

   (c) $f(x) = 1 - 2x$ (d) $f(x) = x^2 + 5$

   (e) $f(x) = 6(4x - 1)$ (f) $f(x) = 4 - x$

   (g) $f(x) = 3x^2 - 2$ (h) $f(x) = 2(1 - x)$

   (i) $f(x) = \dfrac{2}{x+1}$ (j) $f(x) = \dfrac{x+1}{x-2}$

2. The function f is such that $f(x) = 7x - 3$

   (a) Find $f^{-1}(x)$.

   (b) Solve the equation $f^{-1}(x) = f(x)$.

3. The function f is such that $f(x) = \dfrac{8}{x+2}$

   (a) Find $f^{-1}(x)$.

   (b) Solve the equation $f^{-1}(x) = f(x)$.

4. The function f is such that $f(x) = \dfrac{1}{x+4}$, $x \neq -4$.

   Evaluate $f^{-1}(3)$. [Hint: First find $f^{-1}(x)$ and then substitute for $x = -3$]

5. $f(x) = \dfrac{x}{x+3}$, $x \in \mathbb{R}$, $x \neq -3$

   (a) If $f^{-1}(x) = -5$, find the value of x.

   (b) Show that $ff^{-1}(x) = x$

6. Functions f and g are such that

   $f(x) = 3x + 2$ $g(x) = x^2 + 1$

   Find an expression for $(fg)^{-1}(x)$ [Hint: First find $fg(x)$]

Fluency Practice

$a(x) = 2(x - 6)$

$S(x) = \frac{1}{4}x - 3$

$G(x) = \frac{1}{4}x^2$

$R(x) = \frac{1}{4}(4x)^2$

$n(x) = 4(x + 3)$

$C(x) = \sqrt{4x}$

$W(x) = (4x+2)^2$

$Q(x) = 16x^2$

$J(x) = \frac{\sqrt{x+2}}{4}$

$U(x) = \frac{1}{2}x - 3$

$f(x) = 2x - 6$

$e(x) = \frac{\sqrt{x}}{2}$

$m(x) = 4x - 12$

$D(x) = \frac{x + 6}{2}$

$y(x) = \frac{\sqrt{x} - 2}{4}$

$B(x) = \frac{\sqrt{x}}{4}$

$P(x) = (4x)^2 - 2$

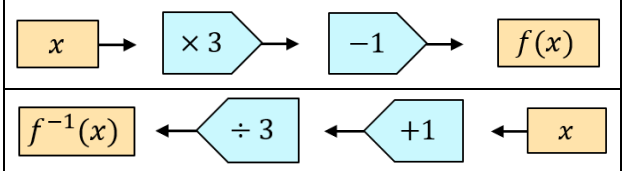
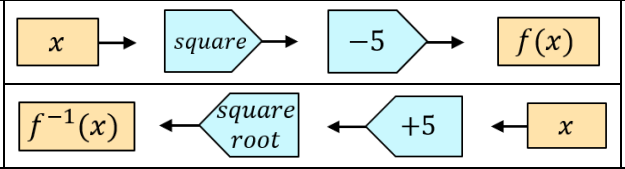
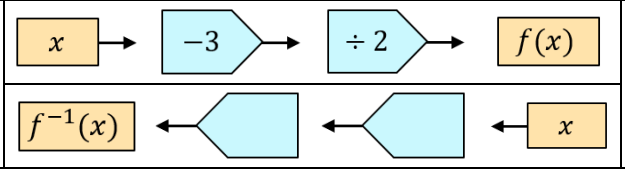
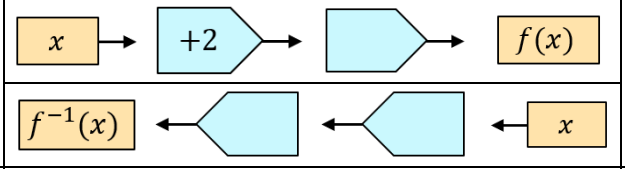
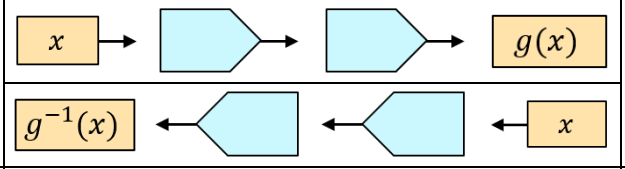
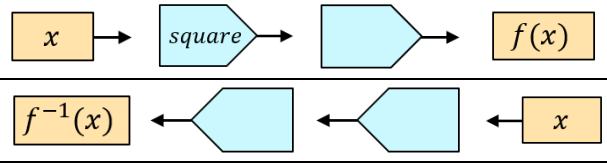
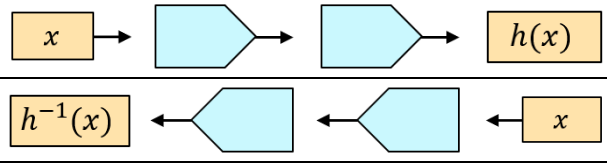
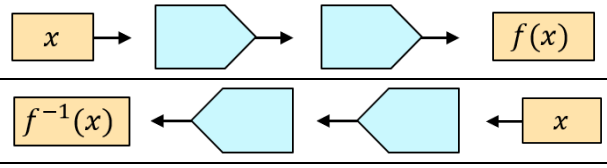
$K(x) = \frac{x}{4} + 3$

$H(x) = 2x + 6$

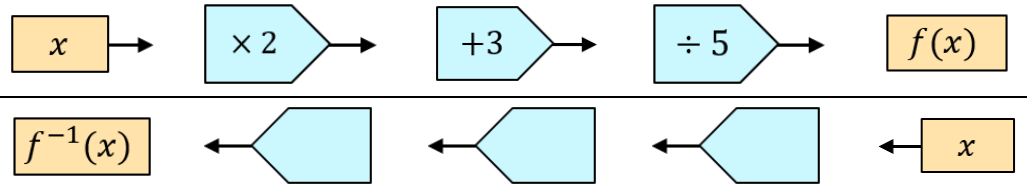
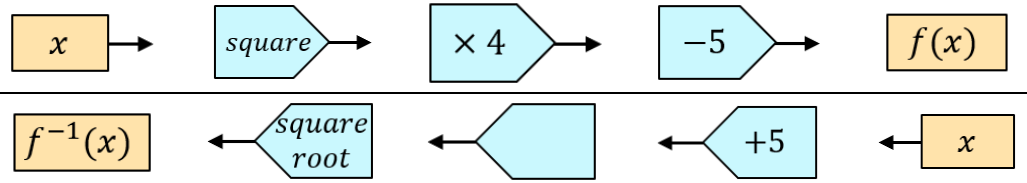
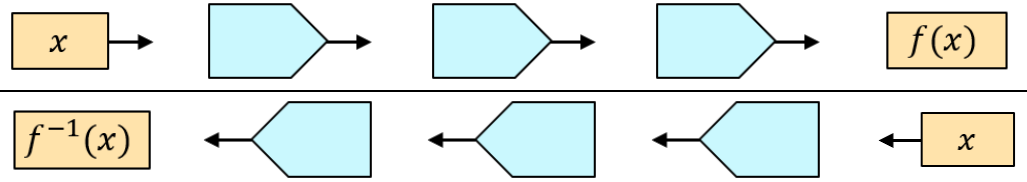
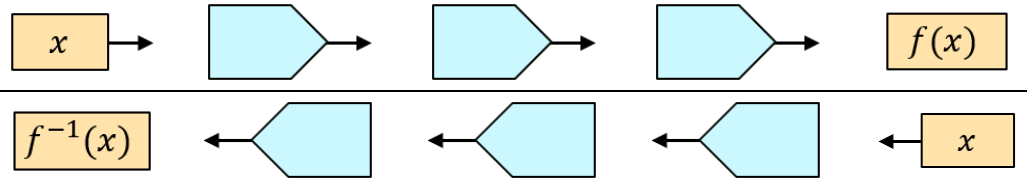
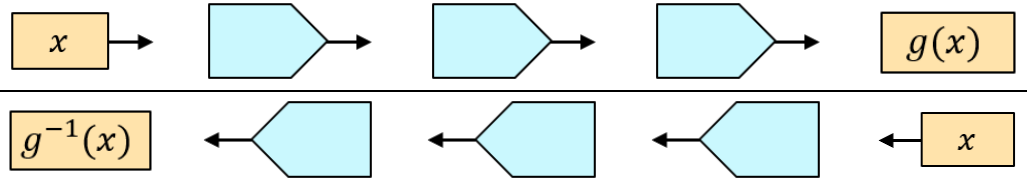
$t(x) = \frac{x}{2} + 6$

Match these functions into pairs that are inverses of each other

Function	Inverse

Question	Function Machines	Answer
$f(x) = 3x - 1$ Find $f^{-1}(x)$		$f^{-1}(x) = \frac{x+1}{3}$
$f(x) = x^2 - 5$ Find $f^{-1}(x)$		$f^{-1}(x) =$
$f(x) = \frac{x-3}{2}$ Find $f^{-1}(x)$		$f^{-1}(x) =$
$f(x) = 5(x + 2)$ Find $f^{-1}(x)$		$f^{-1}(x) =$
$g(x) = \frac{x}{4} + 7$ Find $g^{-1}(x)$		$g^{-1}(x) =$
$f(x) = 5x^2$ Find $f^{-1}(x)$		$f^{-1}(x) =$
$h(x) = \frac{1}{x} - 2$ Find $h^{-1}(x)$		$h^{-1}(x) =$
$f(x) = (x - 4)^3$ Find $f^{-1}(x)$		$f^{-1}(x) =$

Fluency Practice

Question	Function Machines	Answer
$f(x) = \frac{2x+3}{5}$ Find $f^{-1}(x)$		$f^{-1}(x) =$
$f(x) = 4x^2 - 5$ Find $f^{-1}(x)$		$f^{-1}(x) =$
$f(x) = 2\sqrt{x} + 1$ Find $f^{-1}(x)$		$f^{-1}(x) =$
$f(x) = \left(\frac{x-3}{2}\right)^2$ Find $f^{-1}(x)$		$f^{-1}(x) =$
$g(x) = \frac{4}{x} - 3$ Find $g^{-1}(x)$		$g^{-1}(x) =$

Find the inverse function, $f^{-1}(x)$, of the following functions:

(a) $f(x) = 3x - 1$

(b) $f(x) = 2x + 3$

(c) $f(x) = 1 - 2x$

(d) $f(x) = x^2 + 5$

(e) $f(x) = 6(4x - 1)$

(f) $f(x) = 4 - x$

(g) $f(x) = 3x^2 - 2$

(h) $f(x) = 2(1 - x)$

Find the inverse function, $f^{-1}(x)$, of the following functions:

(i) $f(x) = \frac{2x}{x+1}$

(j) $f(x) = \frac{x+1}{x-2}$

The function is such that $f(x) = 7x - 3$

(a) Find $f^{-1}(x)$

(b) Solve the equation $f^{-1}(x) = f(x)$

The function is such that $f(x) = \frac{8}{x+2}$

(a) Find $f^{-1}(x)$

(b) Solve the equation $f^{-1}(x) = f(x)$

The function is such that

$f(x) = \frac{1}{x+4}, \quad x \neq -4$

Evaluate $f^{-1}(-3)$

Inverse Functions

1. Find $f^{-1}(x)$ for the following functions,

a) $f(x) = 7x$

b) $f(x) = x - 12$

c) $f(x) = 9 - x$

d) $f(x) = 3x + 5$

e) $f(x) = \frac{x+10}{4}$

f) $f(x) = \frac{3x-4}{7}$

g) $f(x) = \sqrt{x}$

h) $f(x) = 4\sqrt{x} - 9$

i) $f(x) = \frac{3}{x+5}$

j) $f(x) = \frac{1}{2x-5}$

2. If $f(x) = 3x + 2$

a) Find $f^{-1}(x)$

b) Hence evaluate $f^{-1}(5)$

c) Solve $f(x) = f^{-1}(x)$

3. Find $f^{-1}(x)$ for the following functions,

a) $f(x) = \frac{x}{x-5}$

b) $f(x) = \frac{x+4}{x}$

c) $f(x) = \frac{x+5}{x-1}$

d) $f(x) = \frac{3x+2}{x-6}$

e) $f(x) = \frac{4-x}{2x+5}$

f) $f(x) = \frac{5x}{2x-7}$

4. If, $f(x) = \frac{2x}{x-5}$,

a) Find $f^{-1}(x)$

b) Evaluate $f^{-1}(1)$

c) Solve $f^{-1}(x) = 10$