



### Year 9 Mathematics 2024 Unit 15 Booklet

**HGS Maths** 

2023



**Tasks** 



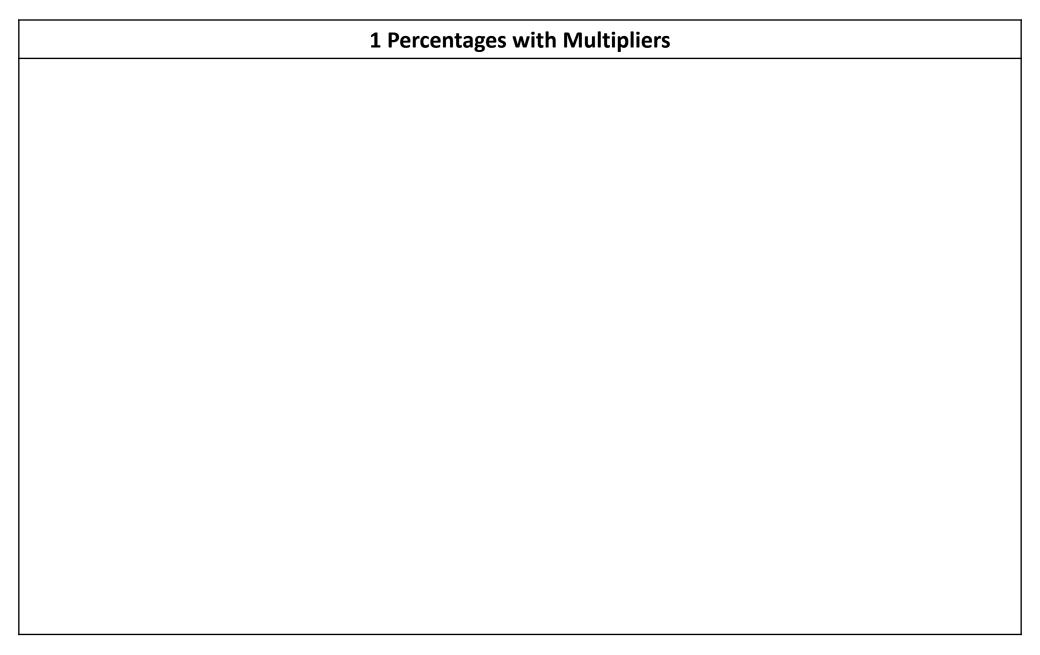
**Dr Frost Course** 



Name:			

Class: \_\_\_\_

### **Contents Page** Percentages with Multipliers 1 **Simple and Compound Interest** 2 Similarity with Length 3 **Right-Angled Trigonometry** 4



Worked Example	Your Turn
Write down the multiplier:	Write down the multiplier:
To find 20%	To find 30%
To increase by 20%	To increase by 30%
To decrease by 20%	To decrease by 30%
To decrease by 2070	10 decrease by 50 %

Worked Example	Your Turn
Write down the multiplier:	Write down the multiplier:
To find 12.5%	To find 0.5%
To increase by 12.5%	To increase by 0.5%
To decrease by 12.5%	To decrease by 0.5%

Worked Example	Your Turn	
Find 7% of 493.8	Find 2% of 34.32	

Worked Example		Your Turn
se 461.7 by 17% ase 461.7 by 17%	a) b)	Increase 295.6 by 18% Decrease 295.6 by 18%

### Fill in the Blanks

### Persenting Capycrease and Decrease

Original Amount	Percentage	Increase/ Decrease	Multiplier	Calculation	New Amount
£50	25%	Increase	1.25	£50 × 1.25	£62.50
£70	16%	Increase	1.16		
£89	15%	Decrease	0.85		
£125	76%	Increase			
£49	36%	Decrease			
£218	92%	Decrease			
£24	8%		1.08		
£92			1.83		
£48			0.73		
£75	12.5%	Increase			
£13	8.5%	Decrease			
£54			0.635		

Increasing & Decreasing by a Percentage

Q	Whole	Increase or	Cha	nge	Decimal Mu	tiplier	Result
Ų.	whole	Decrease	As a percentage	As a decimal	Calculation to Find		Result
Α	400	+	20%	0.2	1 + 0.2	1.2	
В	300	+	80%				
С	800	+		0.15			
D	700	+				1.12	
Е	900	+	3%				
F	600	-	30%				
G	200				1 - 0.15		
Н	1400					0.35	
1	500					0.93	
J	250						500
K	700					2.35	
L	140	+	0.5%				
M	550	+	14.5%				
0	820	-	0.5%				
Р	1600	-	32.8%				
Q	86	_	5.75%				

Worked Example	Your Turn
Worked Example  In a 24% sale, the price of a shirt is reduced by \$68.88. Find the original price of the shirt.	Your Turn  In a 3% sale, the price of a phone is reduced by \$2.82. Find the original price of the phone.

Worked Example	Your Turn
Worked Example  The price of a jumper is increased by 74% and now is \$581.16. Find the original price.	Your Turn  The price of a jumper is increased by 68% and now is \$717.36. Find the original price.

### ill in the Blanks

Reverse Percentages

### **Original Amount** £250 £25 1.15 0.88 Calculation 0.8 $\cdot | \cdot$ ·ŀ ·ŀ $\cdot | \cdot$ £275 £20 £92 528 Multiplier 1.08 0.84 0.8 Percentage Increase/ Decrease Decrease Decrease Increase Increase Increase10%20% 16%18%8% After an 8% pay rise, Omar earns £10.26 per hour. What was his hourly pay before the increase? A TV costs £258.42 including 18% tax. What was the price of the TV before the tax was added? puppy increases in weight by 7.5% to 3.87 kg. What was the previous weight of the The population of a village decreases by 16% to 1260. What was the population before the decrease? A painting is sold for £729, making a profit of 35%. What was the original cost of the In a sale, a coat is reduced by 33% to £43.55. What was its After an increase of 10%, the price of a computer is £275. What was its original price? 4 shirt is reduced by 20% to £20 in a sale. What was its original price? original price of the coat? Question painting? puppy? ⋖ ⋖

Worked Example	Your Turn
Worked Example  The price of a computer increases from £452 to £619.24.  Determine the percentage change.	The price of a train ticket decreases from £294 to £244.02. Determine the percentage change.

## Fill in the Blanks

# Percentage Change

Round your answers to  $1\ \mbox{decimal}$  place where necessary.

Question	Actual Change	Original Amount	Calculation	Percentage Change
A population of butterflies grows from 500 to 562. What is the percentage change?	62	200	$\frac{62}{500} \times 100$	
Ayesha buys a bike for £3000 and sells it for £3200. What is her percentage profit?	£200	£3000		
Hassan's savings increased from £150 to £167.50. Find the percentage increase.	£17.50			
Leia buys a painting for \$700 and sells it for \$642. Work out her percentage loss.	\$58			
Tony's wages increase from £14.25 per hour to £15.85 per hour. What is the percentage increase?				
Eric buys a laptop for £550 and after 1 year it is worth £325. What is the percentage loss in its value?				
The population of a town increases from 56500 to 58900. What is the percentage growth?				
The price of a book is reduced from £7.99 to £6.49. Find the percentage decrease.				
Noah buys an antique clock for £45 and sells it for £150. Find his percentage profit.				
			$\frac{1.65}{7.50} \times 100$	
			$\frac{4}{30} \times 100$	

Worked Example	Your Turn
Original Amount: 40 Percentage: 24%	Original Amount: 40 Percentage: 72%
As a fraction	As a fraction
Multiplier	Multiplier
Percentage of	Percentage of
Increased by	Increased by
Decreased by	Decreased by

	Original Amount	Percentage	As a fraction	Multiplier	Percentage of	Increased by	Decreased by
1.	60	20%					
2.	60		$\frac{3}{10}$				
3.	60			0.25			
4.		25%			7.5		
5.			$\frac{1}{40}$			30.75	29.25
6.	30				6.75		
7.				0.225	67.5		

	Original Amount	Percentage	As a fraction	Multiplier	Percentage of	Increased by	Decreased by
8.	300		$\frac{41}{200}$				
9.	60					72.3	47.7
10			$\frac{41}{40}$		61.5		
11	. 60			1.125			
12	6				0.675		
13	6					24.675	
14	6						-31.35

Fill in the gaps in the table. The first one is done already.

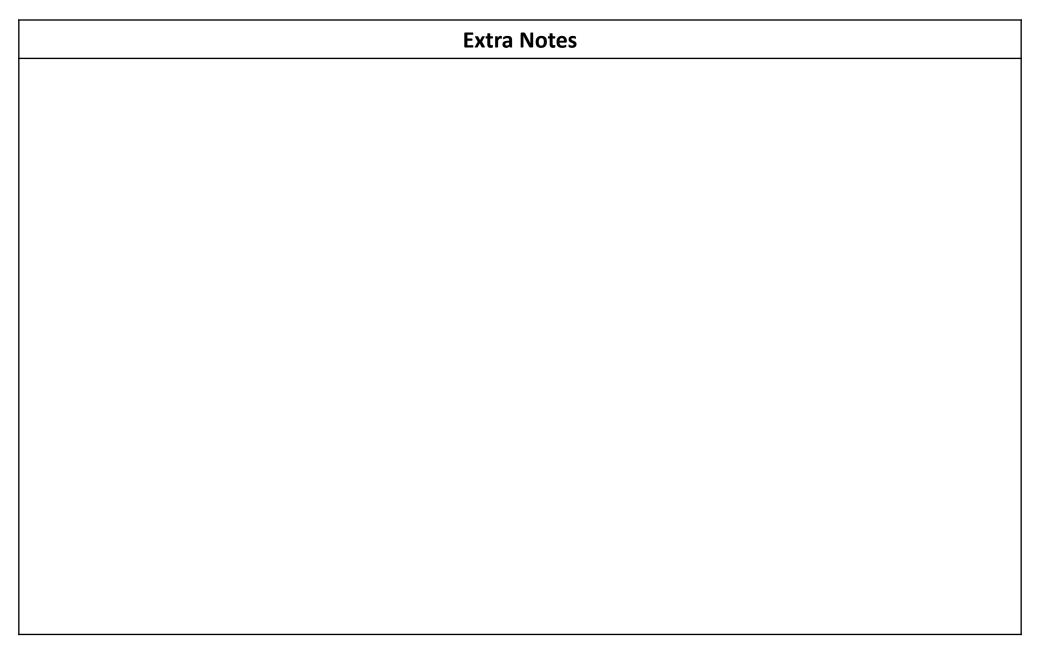
old price	new price	new price old price	what's happened?
£34.00	£50.00	1.47	47% increase
£6.50	£7.20		
£8.50	£8.10		
£241.00			41% decrease
£78.20			4% increase
£1.60		1.24	
£852.10		0.30	
£29.00			32% decrease
£43.80			90% increase
£329.35	£400.00		
£22.00	£10.00		
	£179.00	0.90	10% decrease
	£4.00	1.15	15% increase
	£11.00		16% increase
	£11.11		8% decrease
	£2.00		33% decrease
	£1,499.00		17% increase
	£8.50	1.09	
£8.00			10% decrease
	£543.00		17% decrease

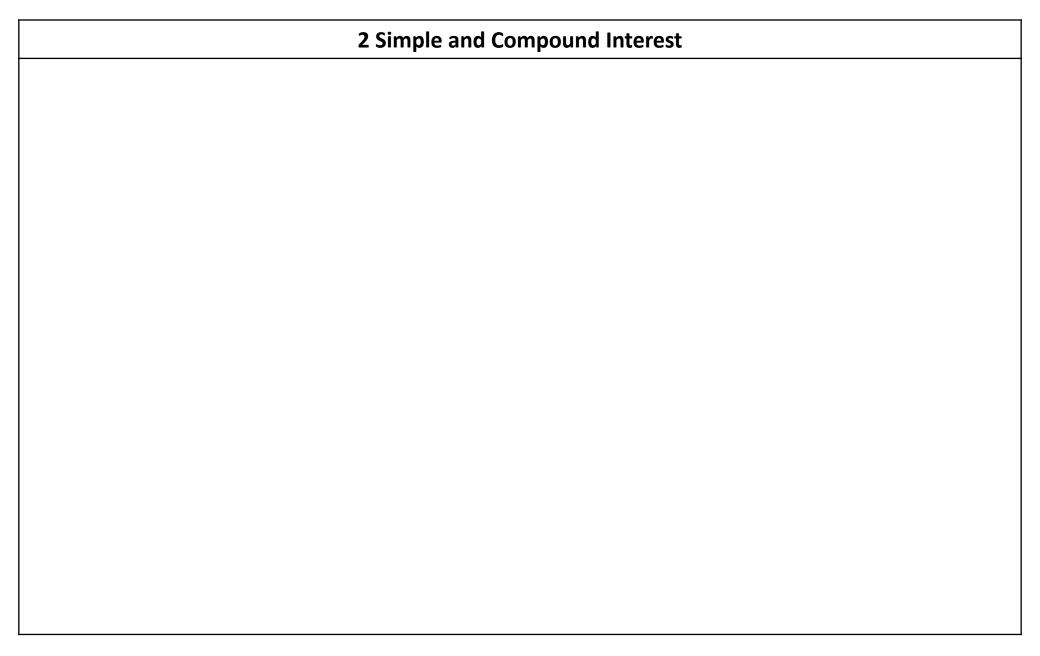
	Amount (A)	Percentage (P%)	P% of A	A increased by P%	A decreased by P%		Amount ( <b>A</b> )	Percentage ( <b>P%</b> )	<b>P%</b> of <b>A</b>	A increased by P%	A decreased by P%
1.	320	10 %	32	352	288	19.		10 %		88	
2.	320	25 %				20.	80		12		
3.	320	2.5 %				21.		80 %	12		
4.	320	1.25 %				22.			12	52	
5.	80	1.25 %				23.			12		48
6.	400	1.25 %				24.			12		-2
7.	125		5			25.		5 %	12		
8.		4 %	10			26.			12	13	
9.	250		20			27.	10			13	
10.	625	16 %				28.		25 %		13	
11.	1859	16 %				29.				13	12
12.	1234	16 %				30.	15				12
13.	609		97.44			31.		25 %			12
14.	84			97.44		32.			68		12
15.	116				97.44	33.				468	12
16.	116	160 %				34.		97.5 %			12
17.	116				-116	35.		2.5 %			468
18.	348	663/3 %				36.				328	312

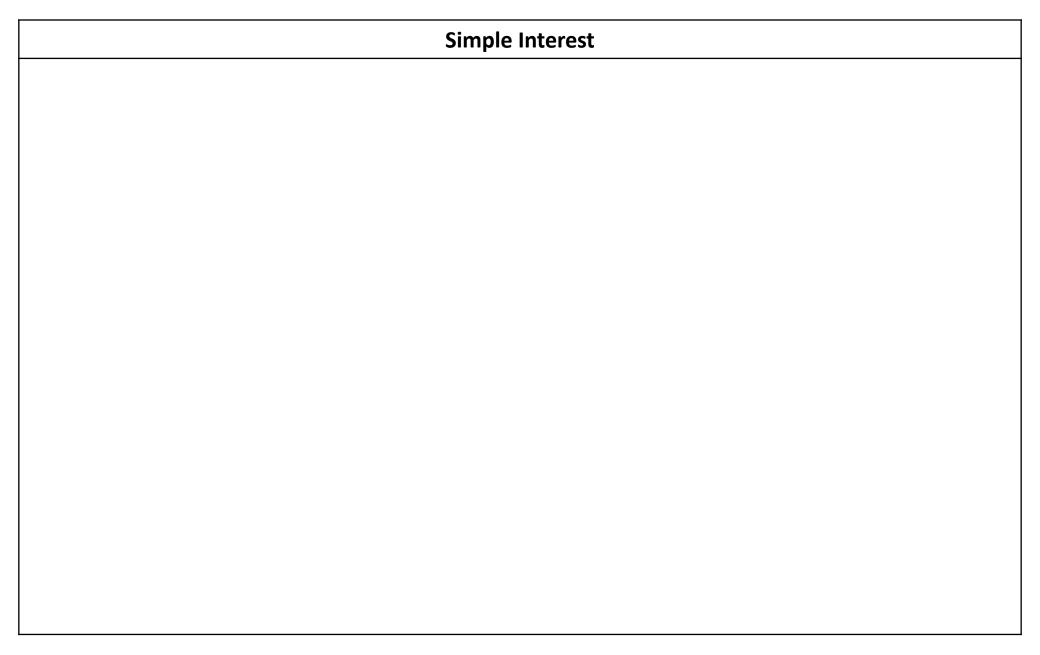
	Amount (A)	Percentage (P%)	<b>P%</b> of <b>A</b>	A increased by P%	A decreased by P%
1.		64 %	377 856		
2.		64 %		377 856	
3.		64 %			377 856
4.		42 %			
5.		42 %			
6.		42 %			
7.			313 344		
8.				313 344	
9.					313 344
10.					
11.					
12.					
13.					
14.					
15.					
16.					
17.					
18.					

- a) Complete rows 1-3. Why was 377 856 chosen for those rows?
- b) What number could be used in rows 4-6 to have the same effect?Use that number to complete those rows.
- c) What percentage could be used in rows 7-9 to have the same effect?Use that percentage to complete those rows.
- d) Find composite numbers for A and P such that P% of A is a prime number.Use such pairs to complete rows 10-12.
- e) Find composite numbers for A and P such that
  A increased by P% is a prime number.
  Use such pairs to complete rows 13-15.
- f) Find composite numbers for A and P such that
   A decreased by P% is a prime number.
   Use such pairs to complete rows 16-18.

	Q	uestic	on	New %	Multiplier	Calculation	Answer
а	Increase	15	by 54%	154%	1.54	1.54 × 15	
þ	Decrease	30	by 23%	77%	0.77	× 30	
С	Increase	14	by 65%	165%		×	
d	Decrease	35	by 34%			×	
е	Increase	22	by	105%		×	
f	Decrease		by		0.7	× 33	
g	Increase		by			1.1 × 21	
h			by			0.55 × 42	
į			by			1.155 × 20	
j	Decrease	25	by 7.6%			×	
k	Decrease	24	by 3.75%			×	
1	Increase	12	by 92.5%			×	
m		28	by			×	23.1
n			by 47.5%			×	23.1



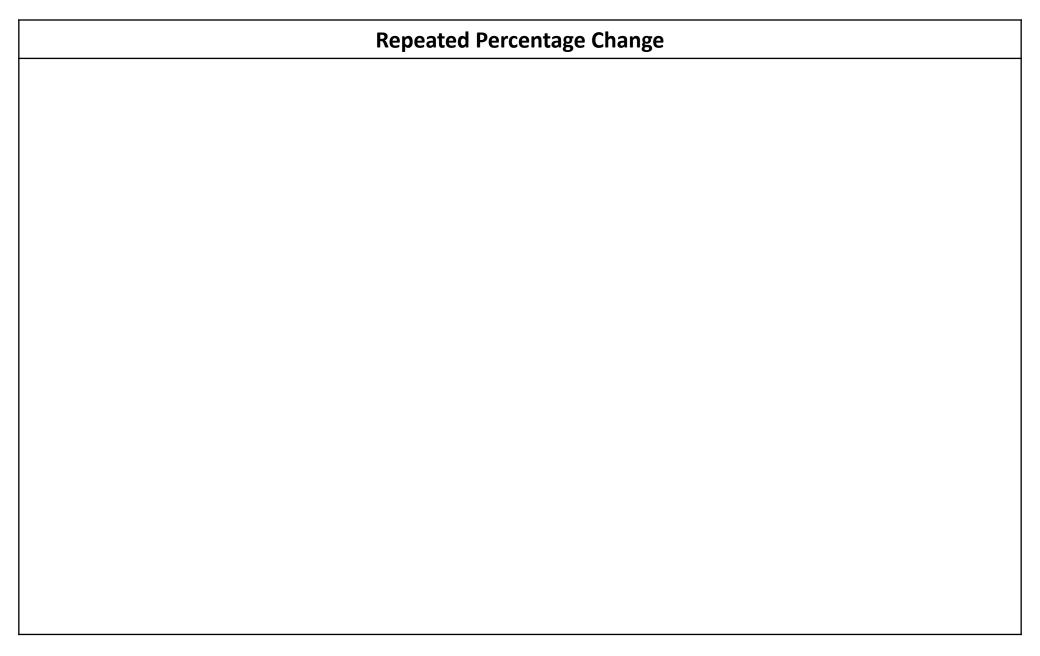




Worked Example	Your Turn
Worked Example  Mr Bansal invests £850 into a savings account.  Mr Bansal gets 3.75% per year simple interest.  Work out the total interest Mr Bansal will get after 12 years.	Wr Dhillow invests £2810 into a savings account.  Mr Dhillow gets 4.75% per year simple interest.  Work out the total interest Mr Dhillow will get after 12 years.

Worked Example	Your Turn			
£2000 is invested at 10% simple interest.	£4000 is invested at 10% simple interest.			
<ul><li>a) What is the value at the end of year 1?</li><li>b) What is the value at the end of year 2?</li><li>c) What is the value at the end of year 20?</li></ul>	<ul><li>a) What is the value at the end of year 1?</li><li>b) What is the value at the end of year 2?</li><li>c) What is the value at the end of year 20?</li></ul>			

	Worked Example		Your Turn
1)	£2000 is invested for 1 year. The value after 1 year is £2200. What is the simple rate of interest?	1)	£2000 is invested for 1 year. The value after 1 year is £2400. What is the simple rate of interest?
2)	£2000 is invested for 2 years. The value after 2 years is £2200. What is the simple rate of interest?	2)	£2000 is invested for 5 years. The value after 5 years is £2400. What is the simple rate of interest?



Worked Example	Your Turn
Write down the multiplier to increase by 20% then decrease by 20%	Your Turn  Write down the multiplier to decrease by 30% then increase by 30%

Q	Original amount	Percentage change 1	Percentage change 2	Overall percentage change	New amount
1	£200	Increase by 20%	Decrease by 20%		
2	£200	Decrease by 20%	Increase by 20%		
3	£200	Decrease by 20%			£200
4	£200	Decrease by 20%	Decrease by 20%		
5	£200	Increase by 20%	Increase by 20%		
6		Increase by 20%	Increase by 50%		£288
7		Increase by 20%		Increase by 50%	£288
8		Decrease by 20%	Decrease by 37.5%		£288
9	£576	Decrease by 20%		Increase by 50%	
10	£576	Increase by 20%		Decrease by 50%	
11	£576	Decrease by 50%			£576
12	£576	Increase by 50%	Decrease by 100%		

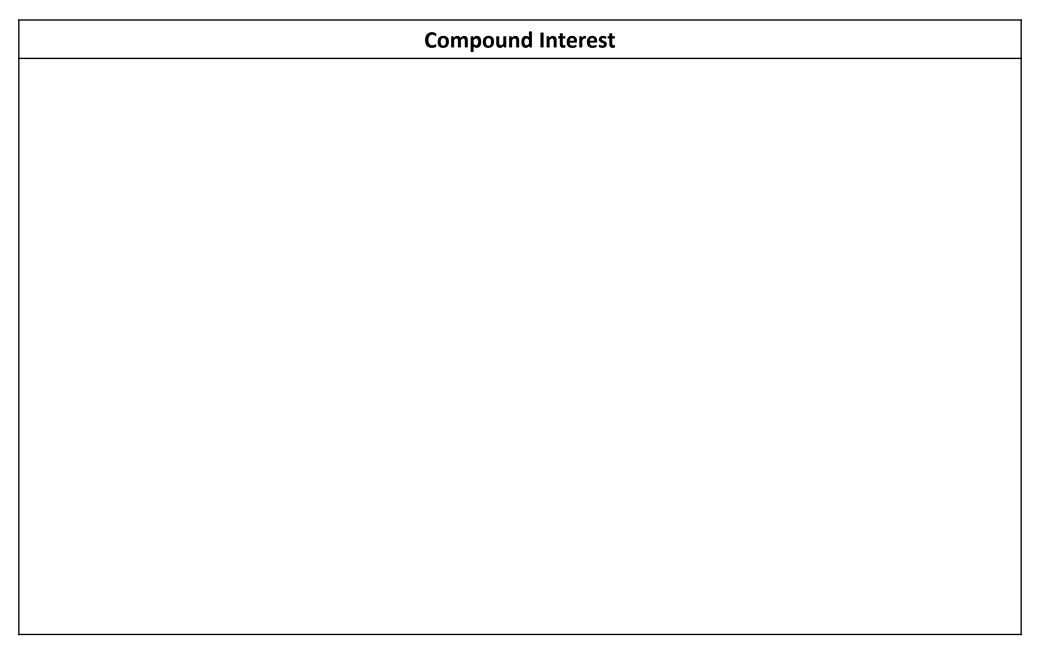
### Section 1: Complete the table

### Repeated percentage change

1 <sup>st</sup> percentage change	1 <sup>st</sup> percentage multiplier	2 <sup>nd</sup> percentage change	2 <sup>nd</sup> percentage multiplier	Overall percentage change	Overall percentage multiplier
30% increase	× 1.3	15% increase	× 1.15	49.5% increase	× 1.495
15% increase		30% increase			
20% increase		25% increase			
5% increase		40% increase			
7.5% increase			× 1.375		
	× 1.06		× 1.39		
	× 1.2				× 1.68
		50% increase		68% increase	
10% decrease		10% decrease			
20% decrease		20% decrease			
30% decrease		30% decrease			
30% decrease		30% increase			
30% increase		10% decrease			
	×0.85				× 1.19
			×0.92	35.6% decrease	

Worked Example	Your Turn
A television costs £500 The price is increased by 10% A month later the price is increased by another 10% What is the final price of the television? How much more is the television now?	A television costs £400 The price is increased by 10% A month later the price is increased by another 10% What is the final price of the television? How much more is the television now?

Worked Example	Your Turn
A television costs £500 The price is decreased by 10% A month later the price is decreased by another 10% What is the final price of the television? How much less is the television now?	A television costs £400 The price is decreased by 10% A month later the price is decreased by another 10% What is the final price of the television? How much more is the television now?



Worked Example	Your Turn
Mr Bansal buys a car for £17150 which depreciates in value at a rate of $4\%$ per year. Work out how much Mr Bansal's car will be worth in $14$ years.	Mr Dhillow buys a car for £14680 which appreciates in value at a rate of $1.25\%$ per year. Work out how much Mr Dhillow's car will be worth in 17 years.

Worked Example	Your Turn
A person invests £400 at 5% compound interest per annum. After $x$ years they have £463.05. Find the value of $x$ .	A person invests £400 at 6% compound interest per annum. After $x$ years they have £476.40. Find the value of $x$ .

Worked Example	Your Turn
Worked Example  Person A invests a sum of money. The account pays 5% compound interest per annum. After how many years will A have trebled their investment (as a whole number of years)?	Person A invests a sum of money. The account pays 6% compound interest per annum. After how many years will A have trebled their investment (as a whole number of years)?

Worked Example	Your Turn
A person invests £400 at 5% compound interest per annum.  How much interest has been earned after three years?	A person invests £400 at 3% compound interest per annum. How much interest has been earned after 5 years?

Worked Example	Your Turn
A person invests £400 at $x\%$ compound interest per annum. After 3 years they have £463.05. Find the value of $x$ .	A person invests £400 at $x\%$ compound interest per annum. After 3 years they have £476.40. Find the value of $x$ .

Worked Example	Your Turn
A person invests $\pounds x$ at 5% compound interest per annum. After 3 years they have $\pounds 463.05$ . Find the value of $x$ .	A person invests £ $x$ at 6% compound interest per annum. After 3 years they have £476.40. Find the value of $x$ .

#### **Compound Growth & Decay**

 $original \times multiplier^{years} = final$ 

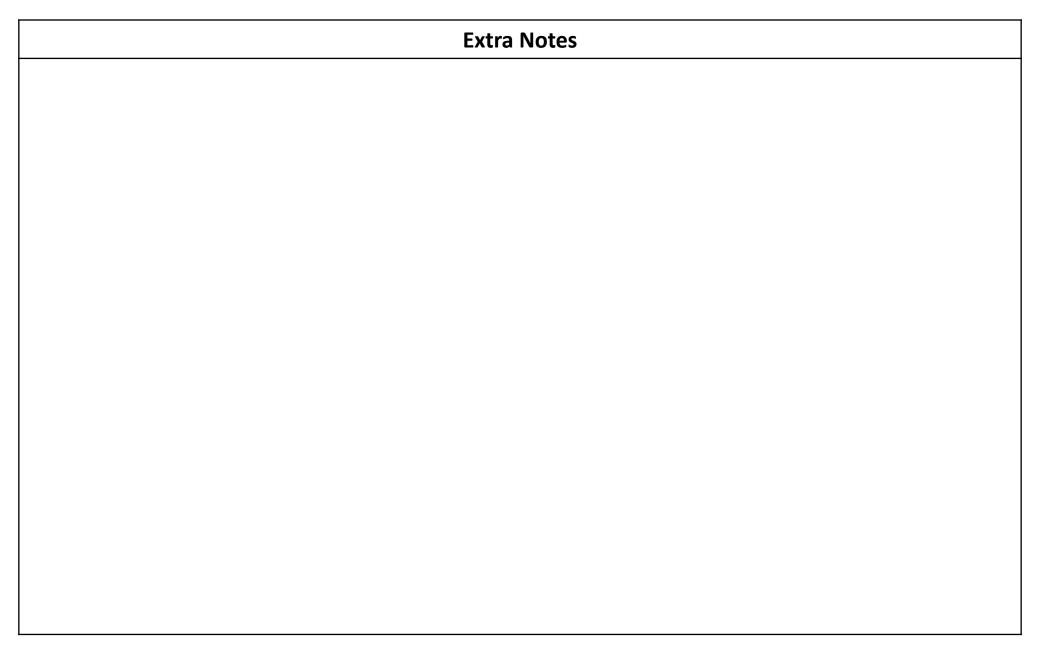
	Original Quantity	Yearly Growth Rate	Multiplier (M)	Years	Formula	Rearranged Formula (unknown as subject)	Final Quantity
а	400	+30%	1.3	2	$400 \times 1.3^2 = final$	Х	
b	400	+3%		2	$\times 1.03^2 = final$	х	
С	400	+12%		3		Х	
d	600		1.05	4		Х	
е	400	-20%			$400 \times 0.8^2 = final$	Х	
f	400	-2%		2		Х	
g					$400 \times 0.88^3 = final$	Х	
h	600	-33%		4		Х	
i		+20%			$original \times 1.2^2 = 720$	$original = 720 \div 1.2^2$	720
j		+8%			$original \times 1.08^3 =$		755
k			0.6	2			800
ı		-15%		3			430
m	800			2	$800 \times M^2 = 968$	$M = \sqrt[2]{968 \div 800}$	968
n	500			3			630

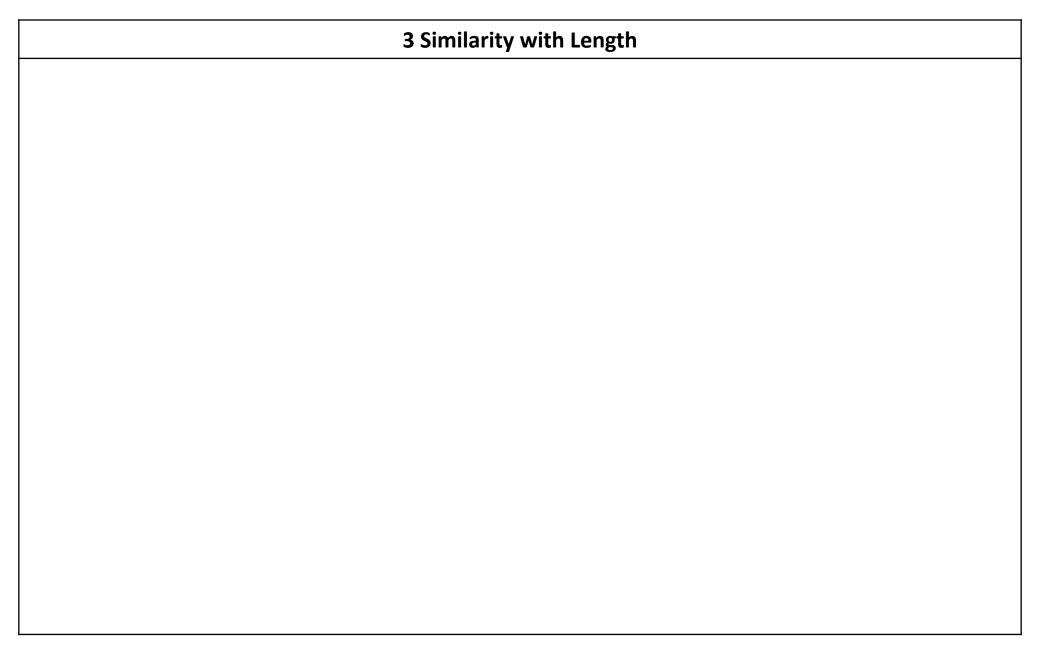
# Fill in the Blanks

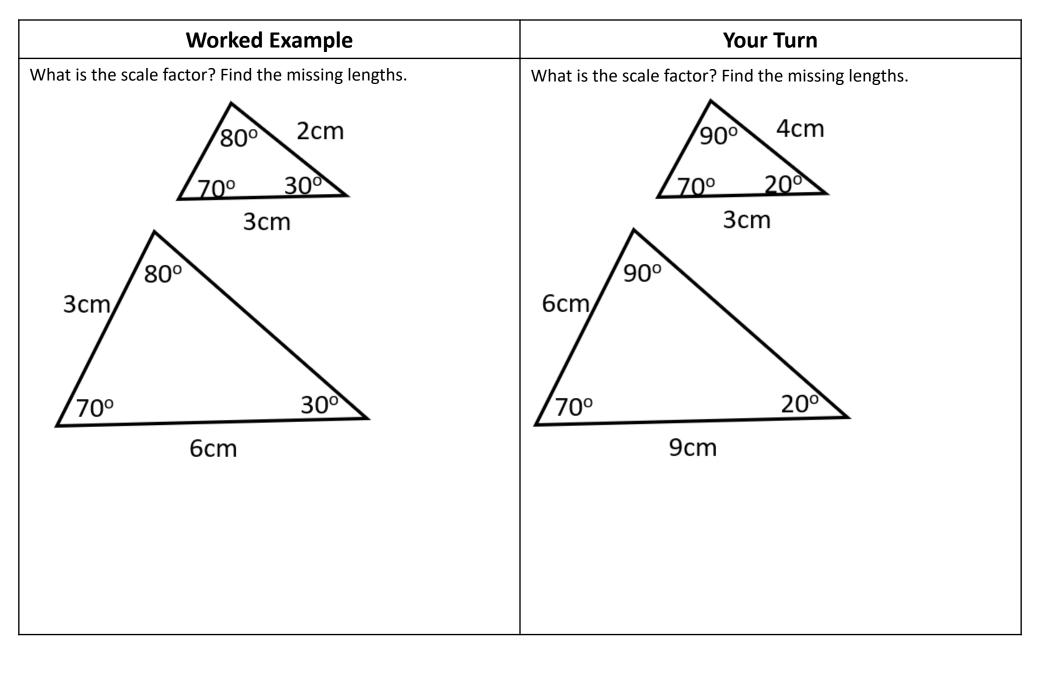
# Compound Interest

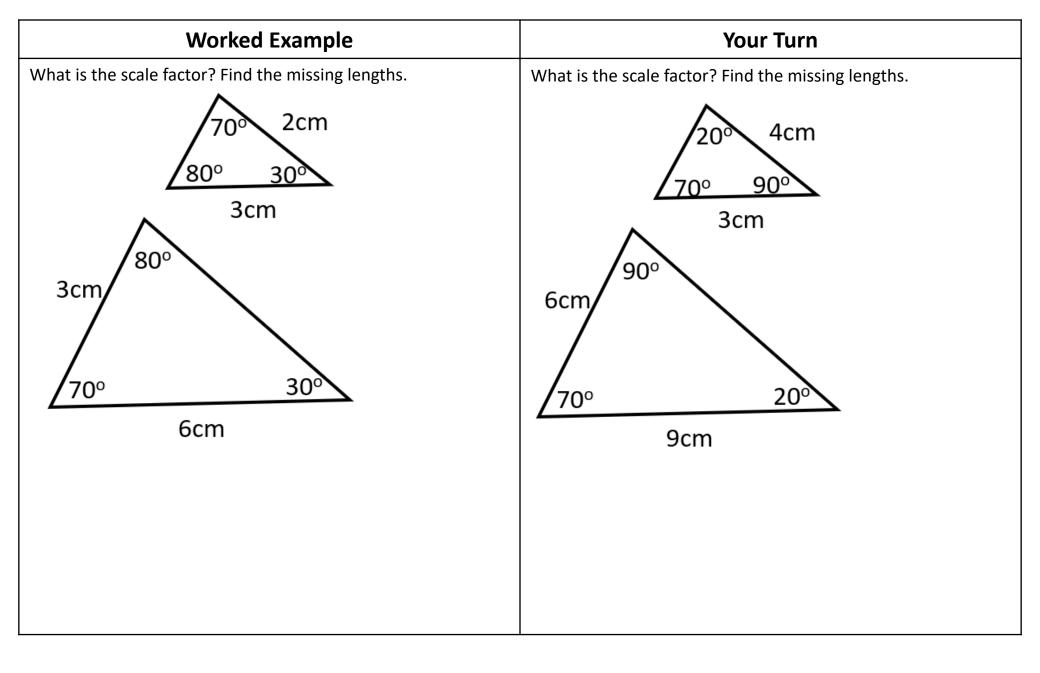
Original Amount	Interest Rate	Multiplier	Number of Years	Calculation	Final Amount
£100	2%	1.05	10	$100 \times 1.05^{10}$	£162.89
£100	4%	1.04	10		
£200	%9		8		
£250	3%		9		
£1200	%9		5		
0023	2.5%		8		
£500	1.5%		2		
£250		1.06	4		
£325		1.025	2		
				$£400 \times 1.03^{6}$	
				$£7000 \times 1.02^3$	
£400	%5				£510.51
£250	3%				£326.19
£600			8		£703
		1.025	9		£347.91
	6.5%		4		£932.69

Q	Yearly percentage change	Original Amount	Amount after 5 years	Amount after 10 years
1	100%	£1		
2	50%	£1		
3	5%	£1		
4	-50%	£2,048		
5	25%		£20	
6	-25%		£20	
7	7%		£1	
8	100%			£2,048
9	5%			£100
10	100%			£2,048
11		£20	£30	
12		£1	£5	
13		£1	£10	
14			£50	£100
15		£4		£64

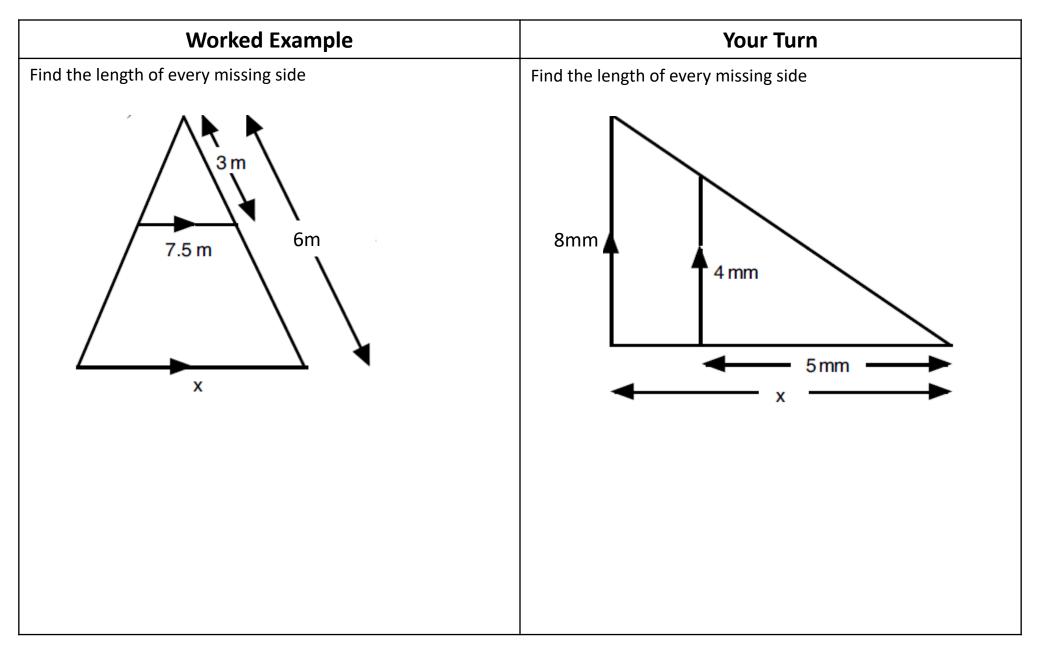


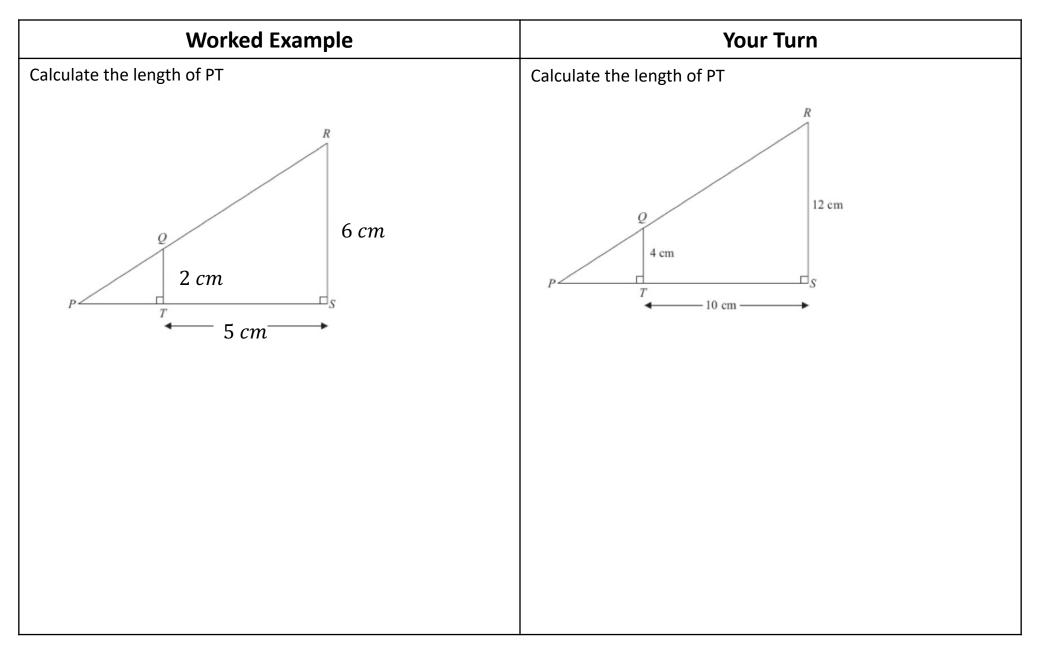


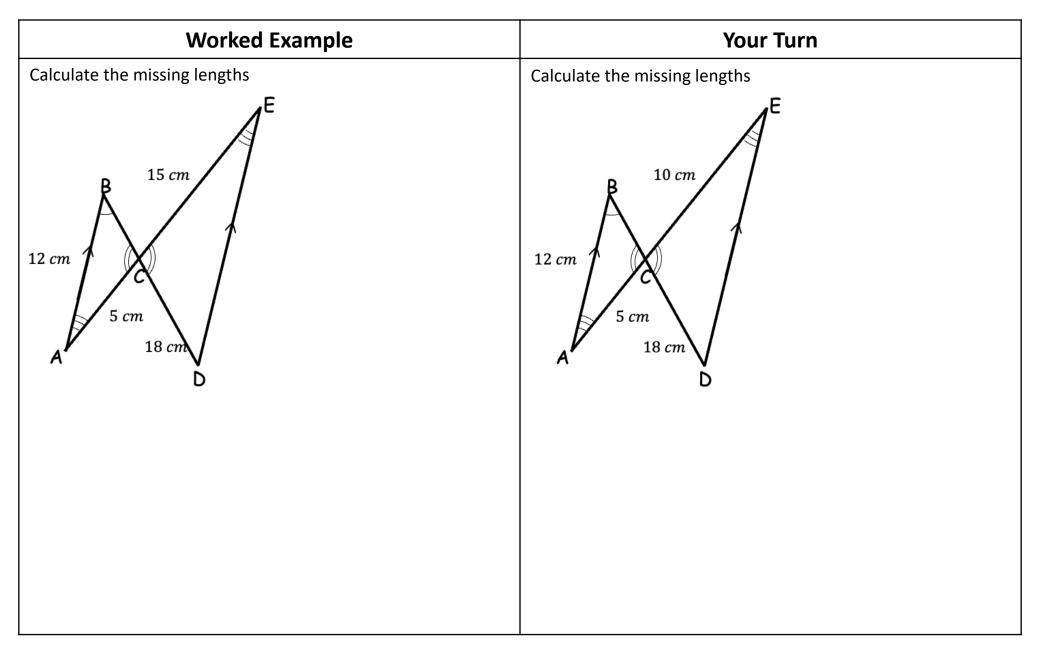


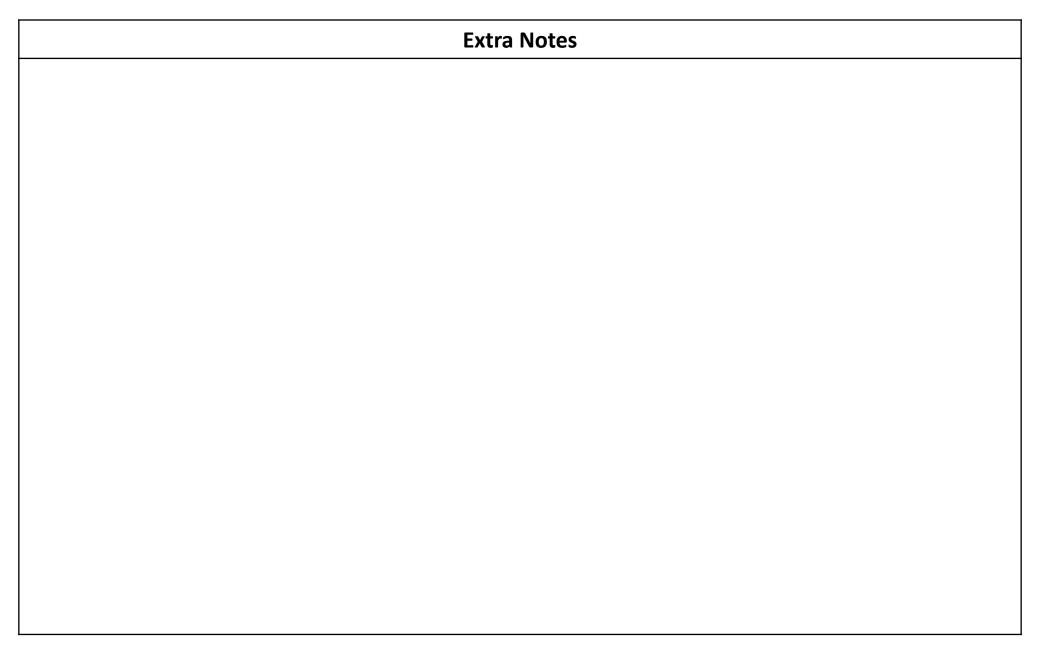


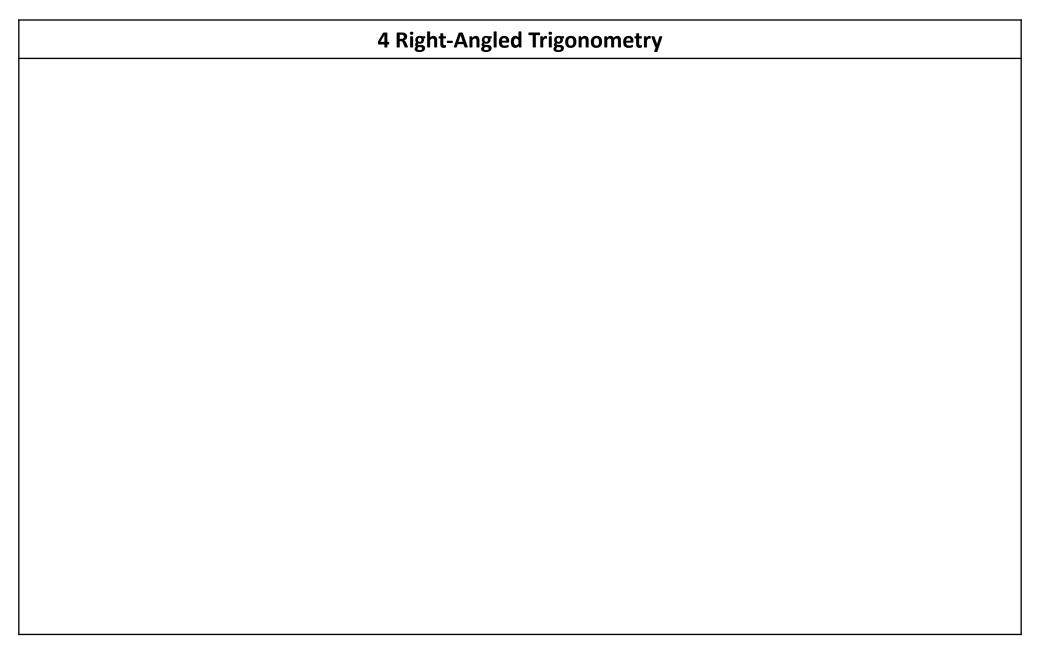
# **Worked Example Your Turn** What is the scale factor? Find the missing lengths. What is the scale factor? Find the missing lengths. 6m 14 m 12m Χ 9m 12m X 10m 4.5 m











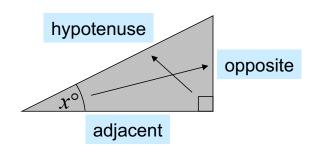
Worked Example	Your Turn
Find ' $x$ '. Give your solution to 2 decimal places if required.	Find ' $x$ '. Give your solution to 2 decimal places if required.
$sin(60) = \frac{x}{5}$	a) $sin(60) = \frac{x}{4}$
$\cos(45) = \frac{5}{x}$	a) $sin(60) = \frac{x}{4}$ b) $cos(45) = \frac{4}{x}$

#### **Trigonometric Functions**

A function f(x) takes an input x and outputs a value y. A trigonometric function takes an angle  $x^{\circ}$  and outputs a ratio of sides.

For any right-angled triangle we always label the longest side as the hypotenuse (H). For the purposes of trigonometry, we label the other two sides relative to one of the non-right angles.

In order to understand and use some other rules connecting the sides & angle of right-angled triangles, we introduce a system for labelling the three sides:

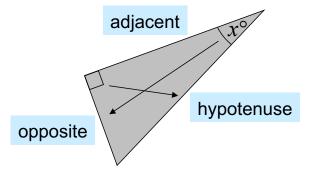


You must be able to correctly recognise the hypotenuse, opposite and adjacent side for any given right-angled triangle and angle

The **hypotenuse** is the longest side, always opposite the right-angle

The **opposite** is always from the angle  $\theta$ 

The **adjacent** is the remaini next to the angle  $\theta$ 



#### **Trigonometric Functions**

A function f(x) takes an input x and outputs a value y. A trigonometric function takes an angle  $x^{\circ}$  and outputs a ratio of sides.

The three sides of right-angled triangles are:

A – Adjacent 
$$X$$

$$H-Hypotenuse$$
 A

The next section considers the **ratios** between the hypotenuse, opposite and adjacent, relative to angle x, in a right-angled triangle.

The ratio of the opposite to the hypotenuse is called **sine** 

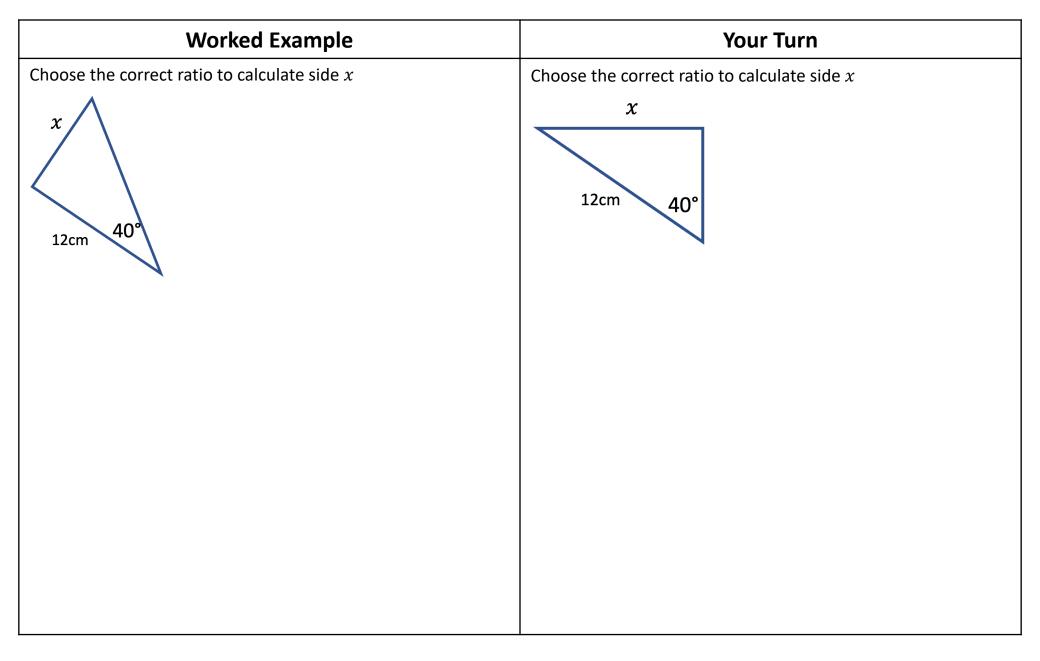
The ratio of the adjacent to the hypotenuse is called **cosine** 

The ratio of the opposite to the adjacent is called **tangent** 

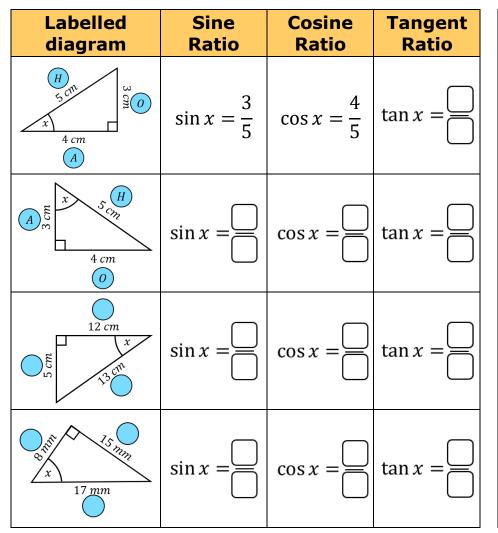
These are abbreviated as sin, cos and tan

$$\sin x = \frac{O}{H}$$
  $\cos x = \frac{A}{H}$   $\tan x = \frac{O}{A}$ 

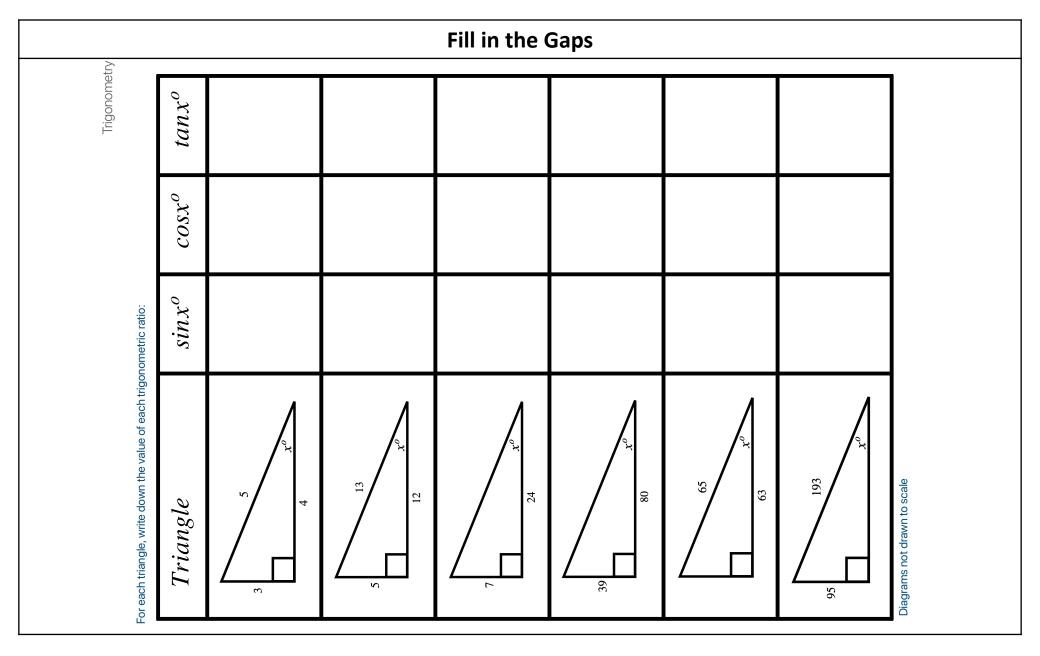
This is commonly given the acronym: SOHCAHTOA







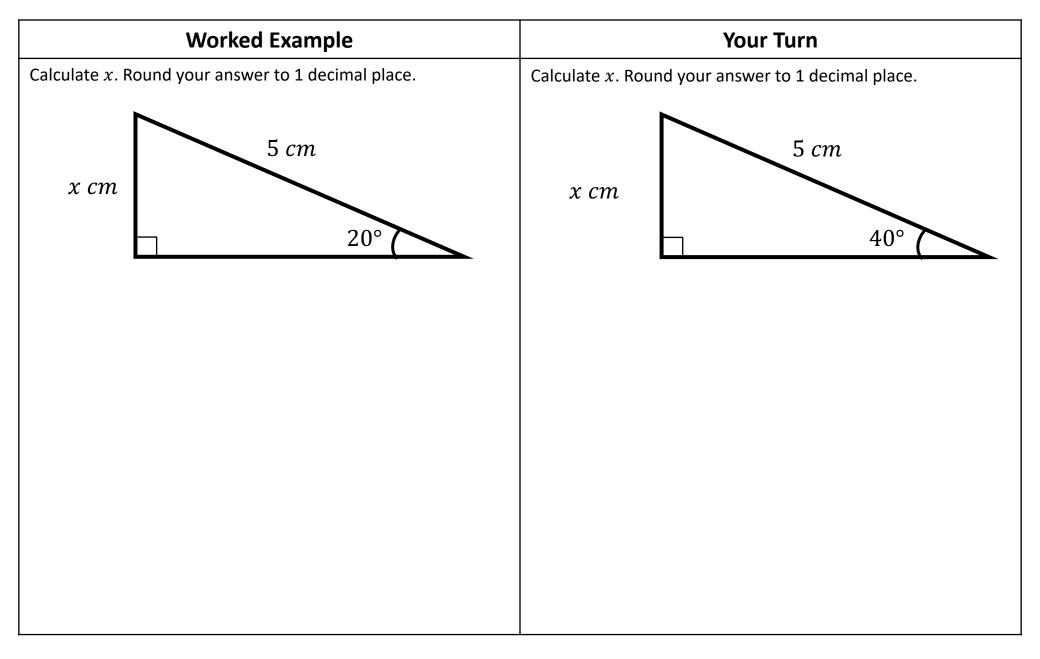
Labelled diagram	Sine Ratio	Cosine Ratio	Tangent Ratio
7.3 m x & & & & & & & & & & & & & & & & & &	$\sin x = $	$\cos x = $	$\tan x = \bigcirc$
√29 cm x √29 cm √3 ch	$\sin x = $	$\cos x = $	$\tan x = $
	$\sin x = $	$\cos x = $	$\tan x = \frac{9.9}{2}$
	$\sin x = \frac{4}{7}$	$\cos x = $	$\tan x = $

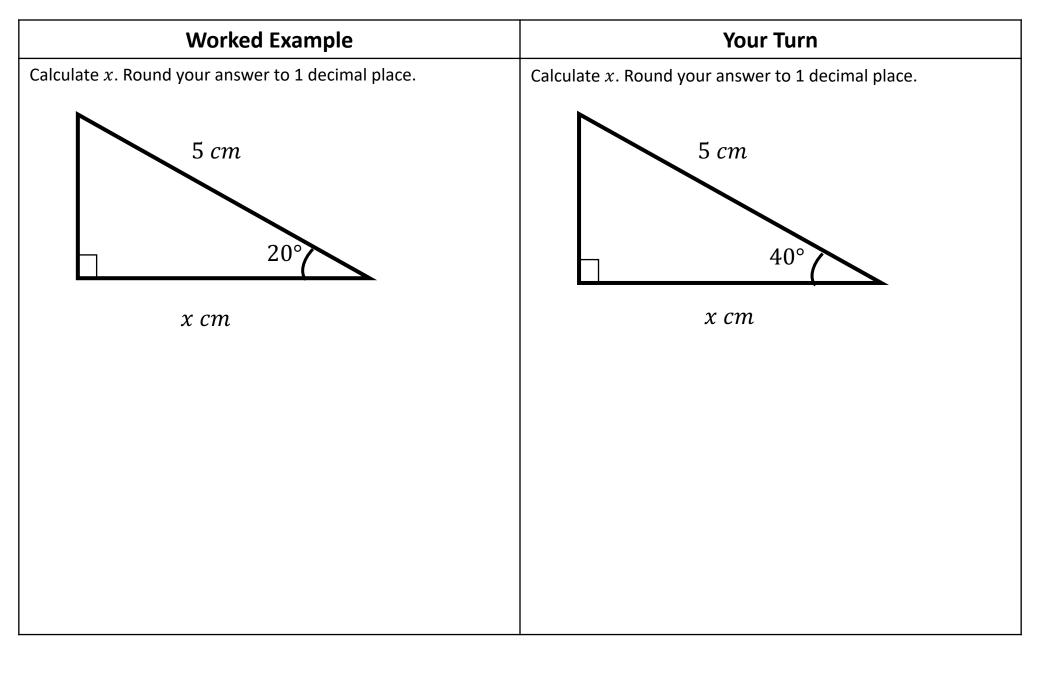


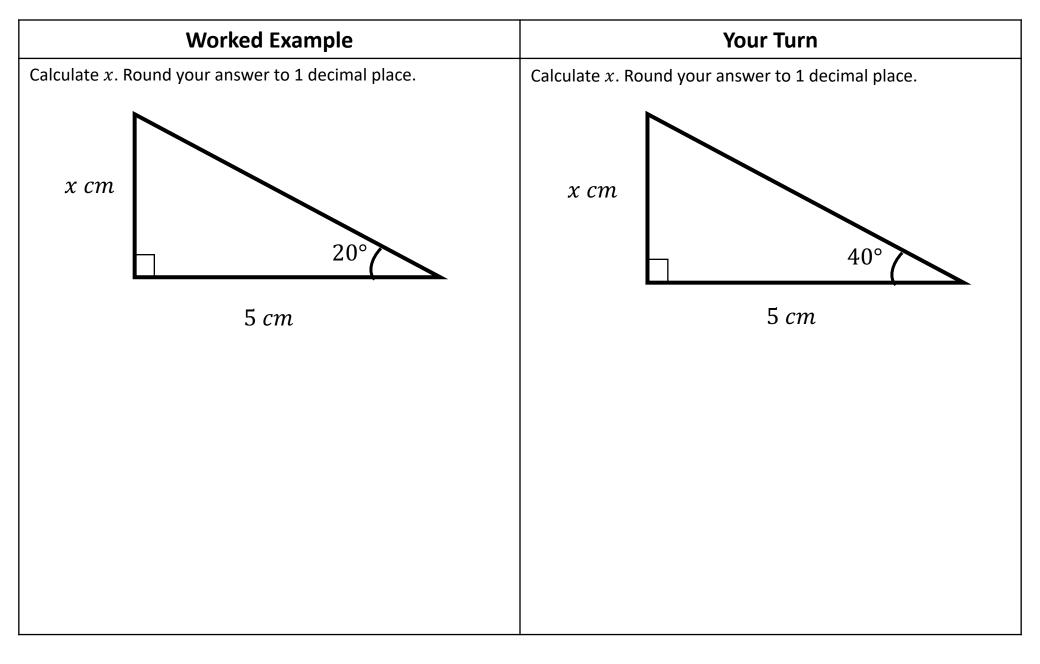
Fill	in	the	Gaps	
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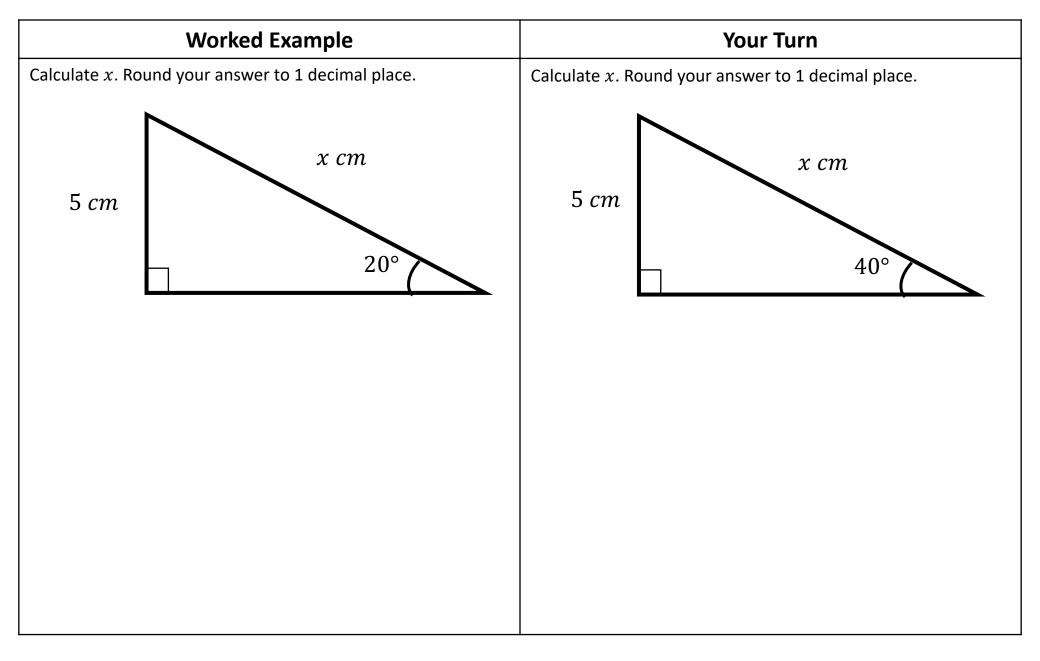
Trigonometry

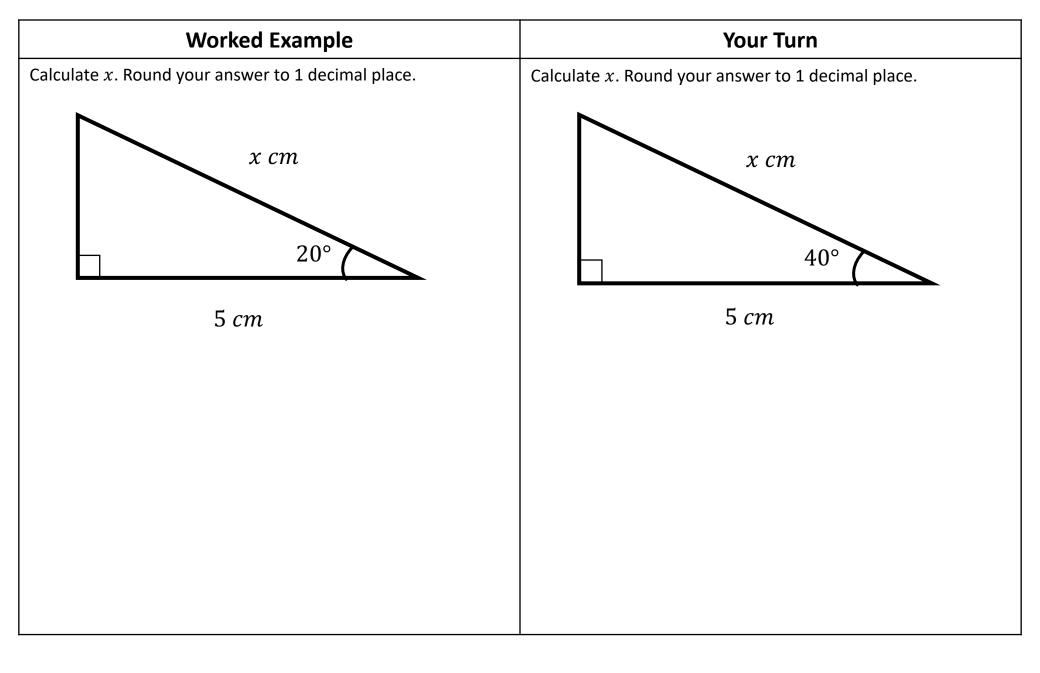
 $tanx^o$ 12  $4 \mid \omega$ 1 |  $cosx_o$  $\omega \mid \omega$  $\omega$  $sin x^o$ 15 5 13 Triangle Complete the table:

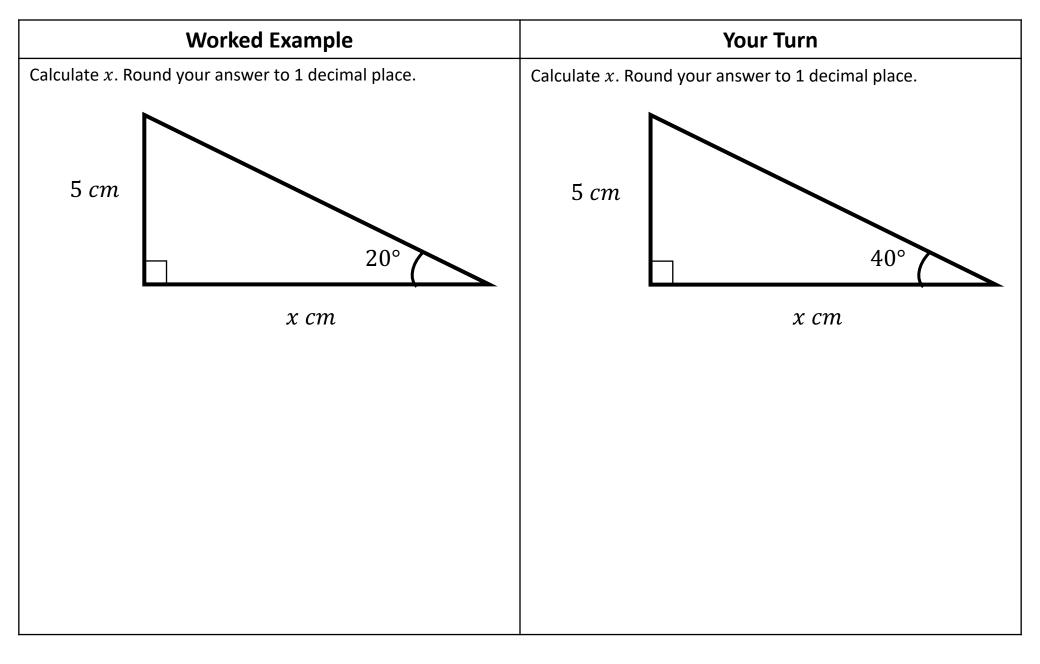














Labelled diagram	Choose ratio	Substitute into formula	Rearrange formula	Answer (1dp)
x 0 28° × 11 × 11 × 11 × 11 × 11 × 11 × 11 ×	sin	$\sin 38 = \frac{x}{11}$	$x = 11 \times \sin 38$	
# 9 9 x 0 0 x 0 0 0 0 0 0 0 0 0 0 0 0 0 0	tan			
37 mm				
8 cm (A)	SOO	$\cos 28 = \frac{8}{x}$	$x = \frac{8}{\cos 28}$	
0 25 m TT A	tan			
13 cm				
5.7 cm 35°				
		$\tan 68 = \frac{7}{x}$		



#### **Inverse Trigonometric Functions**

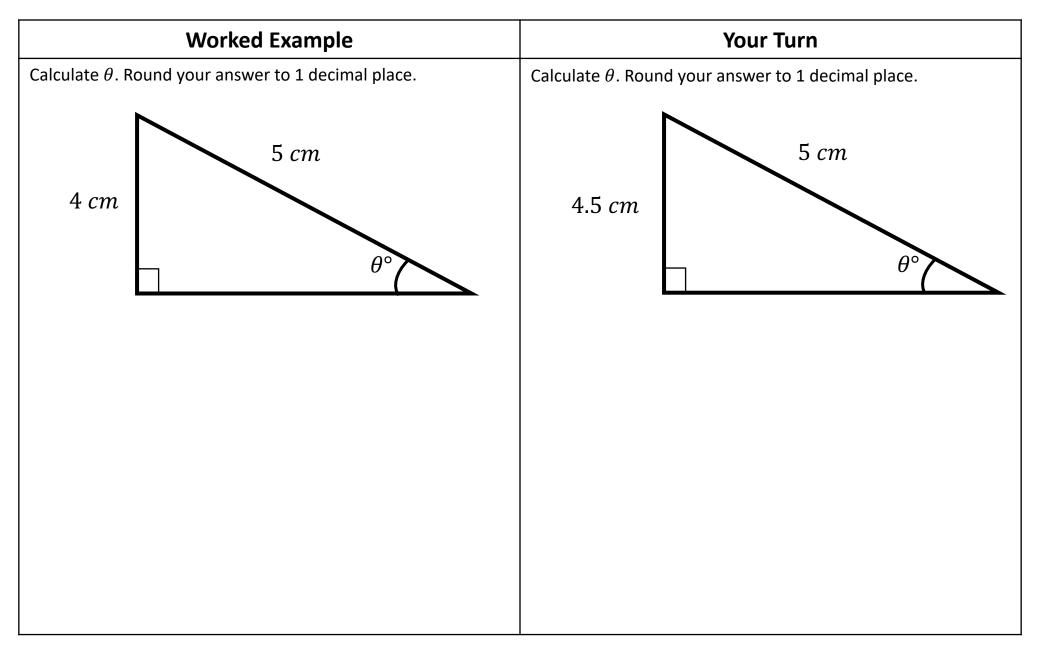
We have met the idea that if f(x) = y then  $f^{-1}(y) = x$ 

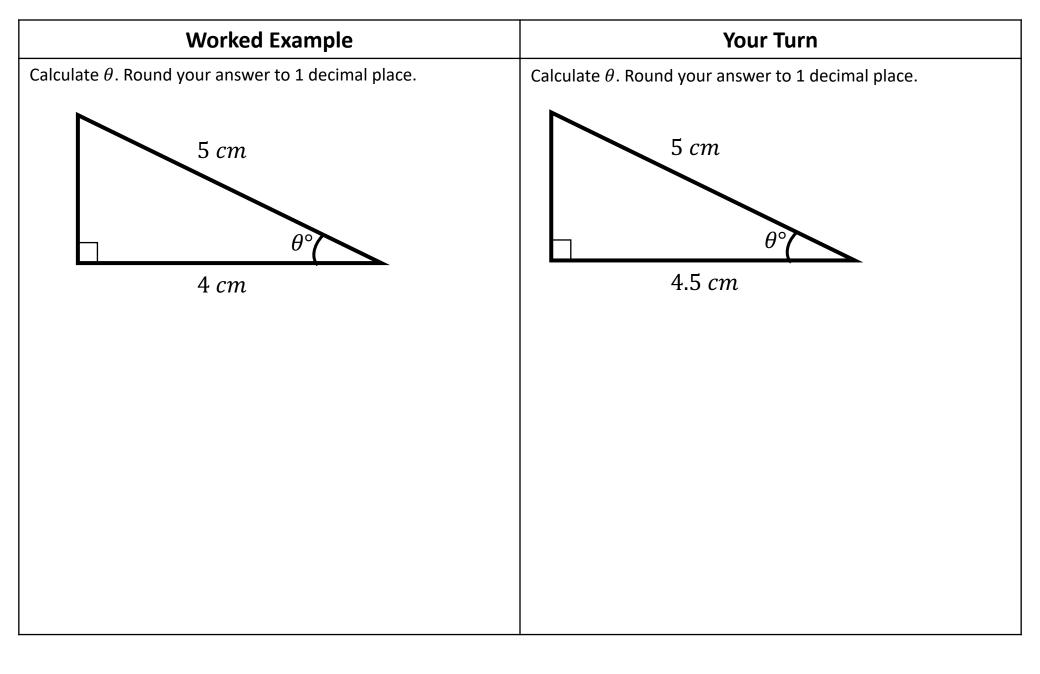
The trigonometric functions sin, cos and tan are all functions where the input is an angle giving an output which is a ratio of sides.

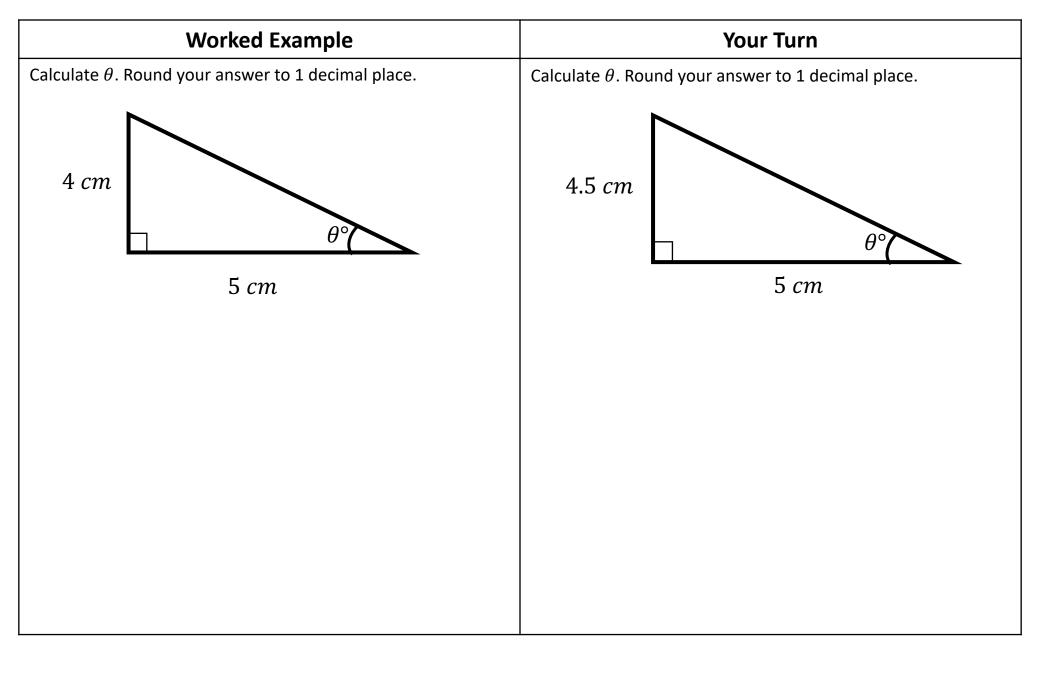
The inverse of these functions therefore does this in reverse.

If 
$$\sin(30^\circ) = 0.5$$
 then  $\sin^{-1}(0.5) = 30^\circ$   
If  $\cos(60^\circ) = 0.5$  then  $\cos^{-1}(0.5) = 60^\circ$   
If  $\tan(45^\circ) = 1$  then  $\tan^{-1}(1) = 45^\circ$ 

Worked Example	Your Turn  Find ' $x$ '. Give your solution to 2 decimal places.				
Find ' $x$ '. Give your solution to 2 decimal places.					
$sin(x) = \frac{2}{3}$	$cos(x) = \frac{2}{3}$				









Answer (1dp)								
Rearrange formula	$x = \cos^{-1}\left(\frac{7}{12}\right)$							$x = \tan^{-1}\left(\frac{15}{11}\right)$
Substitute into formula	$\cos x = \frac{7}{12}$						$\cos x = \frac{2}{3}$	
<b>Choose</b> ratio	SOO	sin						
<b>Labelled</b> diagram	(H) 2/2/2/2/2/2/2/2/2/2/2/2/2/2/2/2/2/2/2/	S CM B	40 mm	14.0 S & S & S & S & S & S & S & S & S & S	The state of the s	# 2. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.	10 E	



### **Worked Example Your Turn** Calculate the length of BD: Calculate the length of BD: 10~cm20~cm60° 30° 50° 30° BD BD

## **Worked Example Your Turn** BC is 4.2 *cm*. BC is 8.4 *cm*. Calculate the length of *AD*: Calculate the length of AD: 38° 19° D В 8.4~cmВ 4.2~cm

# **Worked Example Your Turn** BC is 12 cm. BC is 24 cm. Calculate $\theta$ Calculate $\theta$ 15~cm7.5 *cm* DB24~cmB12~cm

# **Worked Example Your Turn** Calculate the angle of elevation of T from A: Calculate the angle of elevation of ${\it T}$ from ${\it A}$ : 5 cm 10 m 7 cm 14 m 25 m 12.5 m

Worked Example	Your Turn
A tree is a horizontal distance of 5 $m$ away from where you are standing. Using a clinometer, you measure the angle of elevation from the ground to the top of the tree. This angle is $30^\circ$ . How tall is the tree?	A tree is a horizontal distance of 5 $m$ away from where you are standing. Using a clinometer, you measure the angle of elevation from the ground to the top of the tree. This angle is $60^{\circ}$ . How tall is the tree?

