



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 9

2023 Mathematics 2024

Unit 15 Tasks – Part 1

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KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 9

2023 Mathematics 2024

Unit 15 Tasks – Part 2

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KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 9

2023 Mathematics 2024

Unit 15 Tasks – Part 3

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1 Percentages with Multipliers

Intelligent Practice

Percentage	To find	To increase by	To decrease by
40%			
50%			
60%			
6%			
7%			
8%			
18%			
28%			
48%			

Intelligent Practice

Percentage	To find	To increase by	To decrease by
88%			
98%			
108%			
118%			
218%			
21.8%			
2.18%			
2.08%			
0.08%			

Intelligent Practice

Percentage	To find	To increase by	To decrease by
	$\times 0.15$		
		$\times 1.25$	
			$\times 0.76$
			$\times 0.66$
	$\times 0.66$		
		$\times 2.66$	
		$\times 1.06$	
			$\times 0.994$
	$\times 0.606$		

Fluency Practice

Question 1: Calculate the following

- (a) 15% of 80ml (b) 9% of 205kg (c) 45% of £135 (d) 17% of 540km
(e) 53% of 700g (f) 14% of 12 hours (g) 31% of 280kg (h) 6% of 4GB
(i) 85% of 1250ml (j) 66% of 9.4 miles (k) 97% of \$54 (l) 13% of 0.5 tonnes

Question 2: Calculate the following

- (a) 2.5% of 60cm (b) 7.2% of 104ml (c) 24.5% of 30m (d) 47.9% of £3200
(e) 0.3% of 44km (f) 85.2% of 6000 marks (g) 0.25% of \$840 (h) 3.175% of 52g

Question 3: Calculate the following

- (a) 109% of 30m (b) 124% of 38 seconds (c) 186% of £40 (d) 196% of 20 miles
(e) 220% of 15g (f) 140.5% of 180kg (g) 371% of £60 (h) 1054% of 70 hours

Fluency Practice

Apply

- Question 1: In year 9, there are 150 students
16% of the students are left handed.
- (a) Work out how many students are left handed.
(b) What percentage of the students are right handed?
- Question 2: At a football match, 37% of the fans are children.
There are 12600 fans at the match.
Work out how many children went to the match?
- Question 3: During the last ten years, Donald has played 1200 games of chess.
Donald has drawn 6% of the games.
He has lost 33% of the games.
How many games of chess has Donald won?
- Question 4: Richard owns a coffee shop.
In one week, 68000 drinks are sold.
9% of the drinks sold are hot chocolates.
How many hot chocolates are sold?
- Question 5: Which is larger 20% of 7 or 7% of 20?
- Question 6: Maxwell is paid £460.
He spends 38% on his rent, 13% on his food and 20% on bills.
He saves the rest of the money.
How much money does Maxwell save?
- Question 7: Hannah and Kate each have a salary of £36400.
Hannah is given a pay rise of 4%.
Kate is given £125 extra each month.
Who is given the best pay rise?
- Question 8: There are 80 teachers in a school.
The headteacher says that exactly 89% of the teachers drive to work.
Explain why the headteacher is wrong.
- Question 9: Dorothy organises a charity raffle.
She sells 800 tickets for £2 each.
4% of the tickets win a prize that costs £20.
65% of the profit goes to Charity A and the rest goes to Charity B.
How much money does Dorothy raise for Charity B?
- Question 10: An adult ticket for the cinema costs £12.80
A child ticket is half the price of an adult ticket.
Mr and Mrs Henderson and their six children go to see a movie.
Mrs Henderson has a voucher for 22% off.
How much money does she save?



Fluency Practice

Convert these percentages to decimals:

- (a) 59%
- (b) 83%
- (c) 17%
- (d) 40%
- (e) 6%
- (f) 3%
- (g) 22.5%
- (h) 7.5%

Decide whether each of these statements is true or false:

- (a) Finding 36% of 75 is the same as calculating 0.36×75
- (b) Finding 8% of 45 is the same as calculating 0.8×45
- (c) Finding 91% of 60 is the same as calculating 60×0.91

Use your calculator to find:

- (a) 85% of 120
- (b) 22% of 250
- (c) 47% of 60
- (d) 59% of 320
- (e) 16% of 3450
- (f) 31% of 20
- (g) 3% of 6225
- (h) 8% of 43

Use your calculator to find:

- (a) 12.5% of 80
- (b) 72.5% of 55
- (c) 57.5% of 272
- (d) 4.5% of 62
- (e) 33.6% of 395
- (f) 7.1% of 30

- (a) A car costs £13650. To buy the car, Qasim needs to pay a deposit of 15%. How much is the deposit?
- (b) Florence earned \$42600 last year. She has to pay 27.5% tax on everything she earns. How much tax must she pay?
- (c) A theatre contains 400 people. 62.5% of these people are adults and the rest are children. 34% of the adults and 14% of the children are wearing glasses. How many people in the theatre are wearing glasses?

Fluency Practice

Method 1 Divide by 100 to find 1%, then multiply up.

Find 34% of 190

$$190 \div 100 \times 34 = 64.6$$

Method 2 Convert the percentage to a decimal, then multiply by the amount.

Find 34% of 190

$$0.34 \times 190 = 64.6$$

- Work out:
 - 12% of 350
 - 8% of 60
 - 80% of 60
 - 23% of 85
 - 60% of 172
 - 19% of 14
 - 6% of 60
 - 97% of 410
 - 9% of 4
- Which of the following calculations can be used to find 21% of 80?
Choose all that apply.
 - 2.1×80
 - $21 \div 80 \times 100$
 - 80×0.21
 - 0.21×80
 - $21 \div 100 \times 80$
 - $100 \div 21 \times 80$
- Work out 63% of a million.
- Which is larger? **80% of 45** or **8% of 455**
- Georgina books a holiday with a total cost of £740.
She must pay a deposit of 12% immediately and the rest of the cost later.
 - How much must Georgina pay for the deposit?
 - How much is left to pay after the deposit?
- Tariq has a bottle containing 250ml of medicine. He needs to measure out 40% of the medicine. How much is this in millilitres?
- Sara has an online charity fundraising page, where she is aiming to raise a total of £1200. The page shows that she has currently reached 73% of the target amount.
Work out how much **more** money Sara must raise in order to reach the target.
- Angela receives a bonus payment of 8% of £160.
Graham receives a bonus payment of 12.5% of £112.
Who receives the greater bonus, and by how much?

Problem Solving

% of

Use the digits 1, 2, 3, 4, 5, 6, 7, 8.
You can only use each digit once.

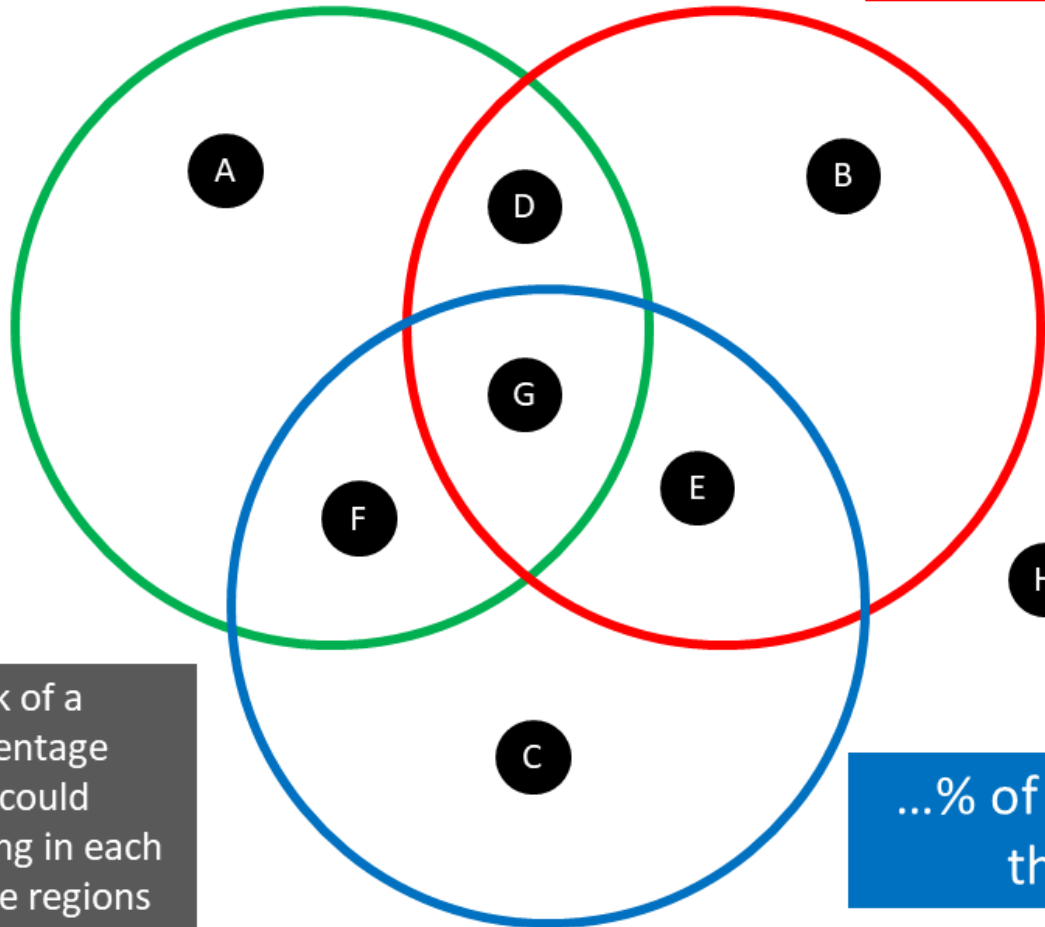
How close can you get to 100?

How close can you get to 500?

Maths Venns

...% of 150 is between 50 and 80

...% of 30 is less than 12



If you think a region is impossible to fill, convince me why!

Think of a percentage that could belong in each of the regions

...% of 60 is more than 16


Fill in the Gaps

	Starting amount	% increase	Write the percentage as a decimal (divide by 100)	Add one to the decimal. This is the multiplier .	Multiply the starting amount by the multiplier .	Answer (use a calculator and include units)
Example	£600	20%	0.2	1.2	600 x 1.2	£720
1	£600	50%				
2	£500	30%				
3	£500	25%				
4	50g	75%				
5	70m	5%				
6	150ml	2%				
7	£18		0.17			


Fill in the Gaps

	Starting amount	% decrease	Write the percentage as a decimal (divide by 100)	Subtract the decimal from one. This is the multiplier .	Multiply the starting amount by the multiplier .	Answer (use a calculator and include units)
Example	£600	20%	0.2	0.8	600 x 0.8	£480
1	£600	40%				
2	£500	50%				
3	£500	75%				
4	50g	25%				
5	70m	8%				
6	150ml	3%				
7	£20			0.88		


Fluency Practice


Question 3 


- (a) Increase 80ml by 9%
- (b) Increase 420g by 70%
- (c) Decrease 8 by 12%
- (d) Decrease £1250 by 38%
- (e) Increase 6000km by 23%
- (f) Decrease 48GB by 6%
- (g) Increase 204 by 98%
- (h) Decrease 149mm by 91%
- (i) Increase 88 by 185%

Question 4 

- (a) Decrease 90ml by 7.5%
- (b) Increase £670 by 1.2%
- (c) Increase 3 by 67.4%
- (d) Increase 750cm by 0.8%

Question 6:  Gabriel's salary is £24500.
Next year he is due to get a 9% increase.
What will his new salary be?

Question 7:  Iris spends £40 a month on water.
By changing company, Iris can save 16%.
How much would Iris pay each month?

Question 8:  An empty flowerpot has a mass of 800g.
The mass of the flowerpot increases to 4kg when filled with soil.

A different flowerpot is 25% lighter but holds 40% more soil.
Calculate the mass of this flowerpot when it is full of soil.

Question 9: Louis sees this special offer in a shop.




Louis buys both items.
How much does he pay?


Special Offer

iPad £489
Case £55

Buy both items and receive a 3% discount

Question 10:  An adult ticket for the cinema costs £13.40
A child ticket is half the price of an adult ticket.
Mr and Mrs Henderson and their six children go to see a movie.
Mrs Henderson has a voucher for 18% off.
Work out how much Mrs Henderson pays for the tickets.



Question 11:  Zara wants to buy 72 candles.
Each candle costs £4.80


There is a special offer

Special Offer

Candles £4.80 each

Buy 60 or more candles and
get 15% off the total cost.

Work out the cost of buying 72 candles
using the special offer.

Question 12:  When a tennis ball is dropped, it bounces and then rises.
The ball rises to 80% of the height from which it is dropped.
The ball is dropped from a height of 4 metres.

- (a) Calculate the height of the rise after the first bounce.
- (b) Calculate the height of the rise after the second bounce.

The ball carries on bouncing, each time rising to 80% of the last rise.

- (c) For how many bounces does the ball rise to a height greater than 10cm?

Fluency Practice

Increase £42 by 3%

$$\frac{100\% \text{ of } 42 + 3\% \text{ of } 42}{103\% \text{ of } 42} = 42 \times 1.03 = \text{£}43.26$$

Decrease £42 by 3%

$$\frac{100\% \text{ of } 42 - 3\% \text{ of } 42}{97\% \text{ of } 42} = 42 \times 0.97 = \text{£}40.74$$

- Write the decimal multiplier for each of these percentage changes:

a) 3% increase	b) 19% increase	c) 60% increase	
d) 34% increase	e) 90% increase	f) 45% increase	
- Work out the following percentage changes.

a) £35 increased by 22%	b) £160 increased by 60%		
c) 124 increased by 8%	d) 91,000 increased by 29%	e) £1.30 increased by 20%	f) £3.20 increased by 2.5%
- Write the decimal multiplier for each of these percentage changes:

a) 6% decrease	b) 14% reduction	c) 37% decrease	
d) 59% decrease	e) 30% decrease	f) 91% reduction	
- Work out the following percentage changes.


a) £80 decreased by 18%	b) £114 decreased by 20%		
c) 68 reduced by 5%	d) 8240 decreased by 35%	e) £2.60 decreased by 70%	f) £16 reduced by 4.5%
- Match each question to one of the answers.


A 78% of £120	P £92	S £93.60	
B £100 reduced by 8%	Q £74.40	T £104.50	
C £60 increased by 24%	R £66	U £19	
D £190 decreased by 90%			
E £30 increased by 120%			
F 95% of £110			


Fluency Practice


6. Bottles of lemonade usually contain 500ml.
A bottle with a special offer states **20% extra free**.
Work out the amount of lemonade in bottle with the special offer.
7. An exhibition was open for one weekend only.
On the Saturday, there were 360 visitors to the exhibition.
There were 15% fewer visitors on the Sunday than there were on Saturday.
Work out how many visitors there were on the Sunday.
8. The total number of people who voted in an election was 4,250.
The Green Party candidate received 14% of all the votes.
Work out how many people voted for the Green Party candidate.
9. In a sale, all items are to be sold at 35% off the marked price.
Work out the sale price of each of these items.
- | | | | | | | | | | | | |
|----------------------|---|------------------|----|---|----------------------|----|--|-------------------|----|--|---------------------|
| a) | <table border="1"><tr><td>Calculator
£8</td></tr></table> | Calculator
£8 | b) | <table border="1"><tr><td>Stationery
Set £6</td></tr></table> | Stationery
Set £6 | c) | <table border="1"><tr><td>Notebook
£2.80</td></tr></table> | Notebook
£2.80 | d) | <table border="1"><tr><td>Glue Stick
£1.40</td></tr></table> | Glue Stick
£1.40 |
| Calculator
£8 | | | | | | | | | | | |
| Stationery
Set £6 | | | | | | | | | | | |
| Notebook
£2.80 | | | | | | | | | | | |
| Glue Stick
£1.40 | | | | | | | | | | | |
10. John's rate of pay is £8.50 per hour. He is going to receive a pay rise of 12%.
Work out what John's rate of pay will be after the rise.
11. Hannah is booking a holiday with a total cost of £1260. She is required to pay a deposit of 15% of the total cost.
- a) Work out how much Hannah must pay for the deposit.
b) Work out how much of the total cost will need to be paid after the deposit is deducted.
12. Jack sees a special offer at a clothing store.
- | |
|---|
| T-shirts: £6.50 each
Buy 4 or more T-shirts,
Get 15% off the total price. |
|---|
- Jack wants to buy 6 T-shirts.
Work out how much this will cost.
13. Tom makes packs of greetings cards and sells them online.
In January, he sold 25 packs for £8.50 each. In February, he reduces the price by 22%.
How many packs must Tom sell in February to have the same income as for January?
14. The governing body of a school always has 15 members.
In 2010, 40% of the members were men and in 2015, this had risen to 60%.
Penny says "The number of men increased by 20% between 2010 and 2015."
Penny is wrong. By what percentage did the number of men increase?


Fluency Practice


Question 4:  An energy bar contains 2.1g of protein.
6% of the bar is protein.
What is the total mass of the bar?


Question 5:  Swansea is a city in Wales.
The population of Swansea is 240,000
This population is 7.5% of the total population of Wales.
What is the total population of Wales?


Question 7:  A chair is on sale at a price of £20.80
This is a 20% reduction of the normal price.
What was the price of the chair before the reduction?


Question 10:  Lucy has 68 books.
This number of books is 70% more than the number of books she had last year.
How many books did Lucy have last year?


Question 11:  Henry invested money into a bank account.
Each year the money in the account earns 3% interest.
After one year, the total amount of money in the account was £169.95
How much did Henry invest?

Question 12:  In a sale, the price of lawnmowers are decreased by 16%
Jude buys a lawnmower in the sale for £369.60
How much was the lawnmower before the sale?

Question 13:  Evie is given a 22% pay rise.
Her new salary is £21960
What was Evie's salary before the pay rise?


Question 14:  A limited edition bag of sugar contains 35% more than a standard bag.
The limited edition bag contains 702g of sugar.
How much sugar is in the standard bag?

Question 15:  An oil tank has sprung a leak and loses 77.5% of its contents.
There is now 333 litres of oil in the oil tank.
How much oil was in the oil tank before the leak?

Question 16:  Miss Jenkins buys a car costing £11895
This cost includes VAT at a rate of 20%
How much is the VAT?

Fluency Practice

Apply

Question 1:  In a sale, a shop reduces all its prices by 10%.
On the last day of the sale, the shop reduces the sale prices by 20%

On the last day of the sale, a mobile phone costs £432


How much was the mobile phone before the sale?

Question 2:  In 2008, Evan bought a car.


In 2010, Evan sold the car to Grace.
Evan made a loss of 25%

In 2018, Grace sold the car for £15225
Grace made a profit of 45%


Work out how much Evan bought the car for in 2008.


Question 3:  There are 1500 people at an ice hockey match.
The announcer says that this is exactly 30% more people than the previous match.


Explain why the announcer is wrong.

Question 4:  Gerard and Martin were both given a pay rise.
Gerard was given a 25% pay rise and Martin a 5% pay rise.
The ratio of Gerard's salary to Martin's salary is now 12:7
Martin is now paid £21000

Work out Gerard's pay before the pay rise.

Question 5:  Michael bought a hat and a coat.
The hat cost £10
He sold both items for a total of £90
Michael made 200% profit on the hat and 80% profit on the total cost.
Work out the percentage profit on the cost of the coat.

Question 6:  Leonie bought a hat and a coat.
The hat cost £6
She sold both items for a total of £45
Leonie made 300% profit on the hat and 125% profit on the total cost.
Work out her percentage profit on the cost of the coat.

Question 7:  Trevor is a car salesman.
He bought a car for £5000
Currently he is holding a sale with 35% off the price of all cars.
Trevor wants to sell the car so that he makes a 10% profit on the price he paid.
How much should Trevor advertise the car for?

Fluency Practice

Find 100% when:

- (a) 50% is 26 (b) 20% is 61
- (c) 17% is 40.8 (d) 6% is 1.5
- (e) 120% is 66 (f) 155% is 145.7

(a) Paul spends 28% of his monthly income on rent, which is £700. What is his total monthly income?

(b) 35% of Year 10 students take GCSE Spanish. If there are 63 Year 10 students who study Spanish, how many students are there in Year 10?

(c) A dress is reduced in a sale by 25%, which is £14. What is the original price of the dress? What is the sale price?

(d) A laptop costs £420 with 20% included. How much is the laptop excluding tax?

(e) In a sale, the price of a TV is reduced by 7% to £348.75. What was the original price of the TV?

(f) The population of a small village has fallen by 6% to 658. What was the population of the village before the decrease?

(g) An office worker receives a 6% pay increase, followed by an 8% pay increase. If her new salary is £1774.44, what was her original salary?

(h) The value of a house increases by 0.35% to £291015. What was the original value of the house?

Fluency Practice

Reverse Percentages																													
<p>(a)</p> <p>A van depreciates in value by 30% during its first year. Its value now is £8750. What was its original price?</p> <div style="text-align: center; margin-top: 10px;"> <table style="margin: auto;"> <tr> <td style="padding: 0 10px;">original</td> <td style="border: 1px solid black; padding: 2px 5px;">$\times 0.7$</td> <td style="padding: 0 10px;">now</td> </tr> <tr> <td style="border: 1px solid black; width: 60px; height: 25px;"></td> <td style="text-align: center; padding: 0 10px;">→</td> <td style="border: 1px solid black; padding: 2px 5px;">£8750</td> </tr> <tr> <td style="text-align: center; padding: 0 10px;">←</td> <td style="border: 1px solid black; padding: 2px 5px;">\div</td> <td></td> </tr> </table> </div>	original	$\times 0.7$	now		→	£8750	←	\div		<p>(b)</p> <p>A coat is reduced by 40% in a sale to £33.90. What was its original price?</p> <div style="text-align: center; margin-top: 10px;"> <table style="margin: auto;"> <tr> <td style="padding: 0 10px;">original</td> <td style="border: 1px solid black; padding: 2px 5px;">$\times 0.6$</td> <td style="padding: 0 10px;">now</td> </tr> <tr> <td style="border: 1px solid black; width: 60px; height: 25px;"></td> <td style="text-align: center; padding: 0 10px;">→</td> <td style="border: 1px solid black; padding: 2px 5px;">£33.90</td> </tr> <tr> <td style="text-align: center; padding: 0 10px;">←</td> <td style="border: 1px solid black; padding: 2px 5px;">\div</td> <td></td> </tr> </table> </div>	original	$\times 0.6$	now		→	£33.90	←	\div		<p>(c)</p> <p>After a pay rise of 4% Armaan is paid £8.58 per hour. How much was his hourly rate before the pay rise?</p> <div style="text-align: center; margin-top: 10px;"> <table style="margin: auto;"> <tr> <td style="padding: 0 10px;">original</td> <td style="border: 1px solid black; padding: 2px 5px;">$\times 1.04$</td> <td style="padding: 0 10px;">now</td> </tr> <tr> <td style="border: 1px solid black; width: 60px; height: 25px;"></td> <td style="text-align: center; padding: 0 10px;">→</td> <td style="border: 1px solid black; padding: 2px 5px;">£8.58</td> </tr> <tr> <td style="text-align: center; padding: 0 10px;">←</td> <td style="border: 1px solid black; padding: 2px 5px;">\div</td> <td></td> </tr> </table> </div>	original	$\times 1.04$	now		→	£8.58	←	\div	
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<p>(d)</p> <p>Property prices have risen by 11% in the last year. If Sheila's house is now worth £205350, how much was it worth last year?</p> <div style="text-align: center; margin-top: 10px;"> <table style="margin: auto;"> <tr> <td style="padding: 0 10px;">original</td> <td style="border: 1px solid black; padding: 2px 5px;">\times</td> <td style="padding: 0 10px;">now</td> </tr> <tr> <td style="border: 1px solid black; width: 60px; height: 25px;"></td> <td style="text-align: center; padding: 0 10px;">→</td> <td style="border: 1px solid black; width: 60px; height: 25px;"></td> </tr> <tr> <td style="text-align: center; padding: 0 10px;">←</td> <td style="border: 1px solid black; padding: 2px 5px;">\div</td> <td></td> </tr> </table> </div>	original	\times	now		→		←	\div		<p>(e)</p> <p>At Drink cafe all prices were increased by 5%. The new price of a cup of coffee is £3.15. What did it cost before the increase?</p> <div style="text-align: center; margin-top: 10px;"> <table style="margin: auto;"> <tr> <td style="padding: 0 10px;">original</td> <td style="border: 1px solid black; padding: 2px 5px;">\times</td> <td style="padding: 0 10px;">now</td> </tr> <tr> <td style="border: 1px solid black; width: 60px; height: 25px;"></td> <td style="text-align: center; padding: 0 10px;">→</td> <td style="border: 1px solid black; width: 60px; height: 25px;"></td> </tr> <tr> <td style="text-align: center; padding: 0 10px;">←</td> <td style="border: 1px solid black; padding: 2px 5px;">\div</td> <td></td> </tr> </table> </div>	original	\times	now		→		←	\div		<p>(f)</p> <p>A holiday cost £2012.50 including tax at 15%. What was the cost of the holiday without tax?</p> <div style="text-align: center; margin-top: 10px;"> <table style="margin: auto;"> <tr> <td style="padding: 0 10px;">original</td> <td style="border: 1px solid black; padding: 2px 5px;">\times</td> <td style="padding: 0 10px;">now</td> </tr> <tr> <td style="border: 1px solid black; width: 60px; height: 25px;"></td> <td style="text-align: center; padding: 0 10px;">→</td> <td style="border: 1px solid black; width: 60px; height: 25px;"></td> </tr> <tr> <td style="text-align: center; padding: 0 10px;">←</td> <td style="border: 1px solid black; padding: 2px 5px;">\div</td> <td></td> </tr> </table> </div>	original	\times	now		→		←	\div	
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<p>(g)</p> <p>The population of a small island has increased in the last ten years by 7% to 1498. What was the population ten years ago?</p>	<p>(h)</p> <p>Bruno sold his car for £7810. This was 45% less than he paid for it five years before. What did he pay for it?</p>	<p>(i)</p> <p>In a sale the price of everything is reduced by 7.5% rounded to the nearest penny. Susan is charged £41.61 for a blouse. What was its original price?</p>																											

Fluency Practice

The cost of a television is £540 including a 20% sales tax. Work out the cost of the television without tax.

$original\ price \times 1.2 = 540$
 $original\ price = 540 \div 1.2$
 $original\ price = \mathbf{\pounds 450}$

The cost of a holiday is reduced by 15% to £833. What was the original price of the holiday?

$original\ price \times 0.85 = 833$
 $original\ price = 833 \div 0.85$
 $original\ price = \mathbf{\pounds 980}$

1. The price of a coach ticket rises to £45.90. This is an 8% increase on the previous price. Work out the original price of the coach ticket.
2. In a sale, all items are reduced by 15%. The sale price of a pair of jeans is £30.60. Work out the price of the jeans before the sale.
3. The price of a washing machine is given as £450. This includes VAT of 20%. Work out the price of the washing machine before VAT was added.
4. A bottle of washing - up liquid reads '25% extra free' and contains 500ml of liquid. Work out the capacity of a normal - sized bottle.
5. The cost of a train ticket is reduced by 5% to £85.50. What was the original price of the ticket?
6. Fay sells her watch for £198, which is 10% less than she originally paid for it. How much did she buy the watch for?
7. The price of a laptop is reduced by $\frac{2}{5}$ to £564. What was the original price?
8. Mr. Cook sells his house for £319,200, which is 12% more than he paid for it. How much did Mr Cook buy his house for?
9. Jo invests some money in a savings account. At the end of the year she receives 3% interest on her savings, which brings her total to £3399. How much did she originally invest?

10. Complete the table:

	Original Amount	Percentage Change	New Amount
a)	£40		£32
b)		Increase by 12%	£24.64
c)		Decrease by 95%	£3
d)		Decrease by 62%	£121.60
e)		Increase by 80%	£1.62

Fluency Practice

11. All of the workers at a company receive a pay rise of the same percentage. Karen's salary increases from £32,400 to £34,344. Mark's salary increases to £27,348.
- Work out Mark's salary before the pay rise.
12. Sally sells a decorative vase for £138, making a 15% profit on the amount she originally paid for it.
- a) Work out how much Sally originally paid for the vase.
- b) What price would Sally have needed to sell the vase for in order to have made a 30% profit?
13. At a restaurant, Robert and Simone receive a bill for £37.95, which includes a 10% service charge. Work out their total bill without the service charge.
14. The table shows some information about the population of deer in a forest.
- | Year | Population |
|------|------------|
| 2005 | |
| 2010 | 1440 |
| 2015 | |
- a) Between 2005 and 2010, the population increased by 20%. Work out the deer population in 2005.
- b) Between 2010 and 2015, the population decreased by 20%. Work out the deer population in 2015.
- c) Zoe says "If the population increased by 20%, then decreased by 20%, it must be back where it started." Is Zoe correct?
15. A packet of cereal says '50% extra free'. The cereal packet weighs 750g and is priced at £2.40.
- a) Work out the price per 100g of the packet.
- b) Work out the price per 100g before the 50% extra free was added.
16. At 36 cm thick, the makers of Supercosy mattresses claim they are 20% thicker than a standard mattress.
- Work out the thickness of a standard mattress, according to the makers.
17. At a garden centre, Sarah buys a lawn mower and 8 identical fence panels. The price of the lawn mower is £185. Sarah receives a 30% discount on her entire bill. After the discount is applied, the total amount for Sarah to pay is £252.70.
- Work out the cost of each fence panel.
18. A report shows that the number of fish in a lake has increased by 300% to 240. Rich says there were originally 60 fish. Dave says there were originally 80 fish. Who is correct?

Fluency Practice

Reverse Percentage Change



① After a 20% price increase, a t-shirt costs £48. How much did it cost before the increase?

original quantity = _____

② After a 12% price increase, a TV costs £280. How much did it cost before the increase?

original quantity = _____

③ An 8% tax increases the cost of a computer to £594. What was the original price?

original quantity = _____

④ A tree is now 371 centimetres tall. Over the last year it has increased in height by 6%. How tall was the tree?

= _____

⑤ A bicycle is priced at \$640. This includes a tax of 14%. What is the price without this tax?

= _____

⑥ Over 1 year, an investment account gains 2% interest and is now worth £340. How much was the investment worth at the start of the year?

= _____

⑦ A shop decreases the price of a microwave by 30%. It is now on sale for £182. How much was it before the discount?

original quantity = _____

⑧ A laptop is on sale for £255. This includes a 25% student discount. What is the price without a student discount?

original quantity = _____

⑨ In March a farmer measures an average height for his corn. At 48 cm it is 15% shorter than last year's crop. What was the average height last year?

original quantity = _____

⑩ Over 10 years, a penguin population drops approximately 23% to 475 penguins. How many penguins were there 10 years ago?

= _____

⑪ The value of Ling's investment drops 5% to \$440. What was the value of the original investment?

= _____

⑫ A house increases in price by 5%. A month later the house increases in price by 8%. The house is now worth \$450,000. What was its original price?

= _____

10 Answers	250	56	561	340	350
	333	550	617	40	260

Fluency Practice

Question 10: The value of a painting rises from £24000 to £27120.
Work out the percentage increase in the value of the painting.



Question 11: Christy buys a book for £17.40
A year later she sells the book for £9.57
Calculate the percentage decrease in the value of the book.



Question 12: In a sale the price of a sofa is reduced from £2500 to £2290
What is the percentage decrease?



Question 13: The volume of juice in a bottle is increased from 500ml to 1.25 litres.
Work out the percentage increase.



Question 14: The population of Bristol in 1921 was 367,831.
In 2017, the population was 459,300.
Calculate the percentage increase.
Give your answer correct to one decimal place.



Question 15: A website had 80000 views in September.
It had 122400 views in October.
Work out the percentage increase in views.



Question 16: An empty bucket weighs 800g.
The weight of the bucket increases to 3.3kg when filled with water.
Calculate the percentage increase in the weight of the bucket.




Fluency Practice

Apply

Question 1:  A shop sells holidays.
The table shows the number of holidays sold each month from August to December.

Aug	Sept	Oct	Nov	Dec
28	40	54	80	111


Between which two consecutive months was the greatest percentage increase in the number of holidays sold.


Question 3:  Nancy goes to the Post Office to exchange money.





Exchange Rates
 £1 : \$1.31
 £1 : €1.14
*Commission Charged

Nancy changes \$759.80 and €342 into pounds sterling.
The Post Office deducts their commission and gives Nancy £827.20
What is the percentage commission?

Question 4:  The number of visitors to a museum in 2018 was 8.42×10^4
In 2019, the museum had 1.1×10^5 visitors.
Calculate the percentage increase

Question 5:  A shop normally sells their goods at 90% above cost price.
In a sale, the shop reduces the prices by 10%
What percentage profit does the shop make on clothes sold in the sale?

Question 6:  Maisie bought a house.
The value of the house decreased by 10% in 2016.
For three consecutive years, 2017, 2018 and 2019 the value of the house increased.
Each year the percentage increase in value was the same each time.
The value of the house at the end of 2019 was 55.52% more than he paid for the house.
Calculate the percentage increase in value of the house for each of the three consecutive years.

Question 7:  W is directly proportional to the cube root of X
X is increased by 50%
Work out the percentage increase in W.

Fluency Practice

- (a) In a school of 400 pupils, 260 are girls. What percentage of the pupils are girls?
- (b) The captain of a football team scored 17 out of the 85 goals scored that season. What percentage of the goals did he score?
- (c) A boy gets 29 out of 60 in his maths exam. What percentage did he score?
- (d) In September it rained for 14 days. What percentage of days did it rain for?
- (e) One day, 132 trains arrive at a railway station and 99 of them are late. What percentage of trains are on time?
- (f) A farmer has 120 sheep and 180 cows. What percentage of his animals are cows?
- (g) A painting is bought for £40 and sold for £50. What is the percentage profit?
- (h) A shop buys a shirt for £25, but it is damaged, and they have to sell it for £18. What is the percentage loss on the shirt?
- (i) In 2012 a house costs £210 000. In 2015 it is sold for £225 000. What is the percentage change in its price?
- (j) A man buys a car for £18000. After one year the car is worth £15500. What is the percentage change in value?
- (k) Currys buy TVs for £260 and sell them for £290. What is their percentage profit?
- (l) A painting was sold at auction for £15000. It originally cost 50p. What was the percentage profit?

Fluency Practice

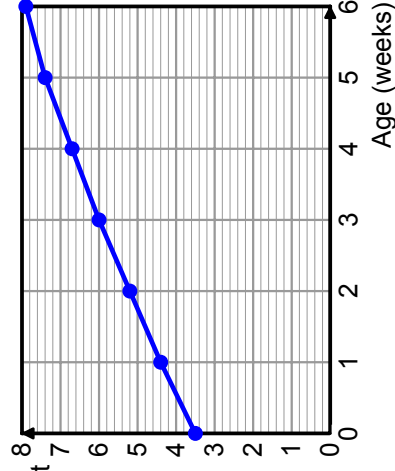
1. Work out the percentage change in each case:
 - a) From 140 to 189
 - b) From 60 to 97.2
 - c) From 320 to 288
 - d) From 45 to 18.9
 - e) From £48 to £49.92
 - f) From £24.50 to £17.15

2. In a sale, a dinner set is reduced from £40 to £24. Calculate the percentage reduction.

3. Anita invests £300 in a savings account. At the end of the year she has £342. Calculate the percentage increase in her savings.

4. A car depreciates in value from £9,200 to £8,400. Calculate the percentage reduction in price correct to 1 decimal place.

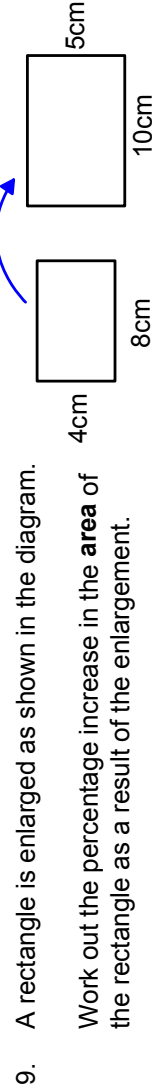
5. In February, the cost of a flight from London to Rome is £600. In June, the cost of the same flight is £960. Calculate the percentage increase from February to June.



6. The graph shows the weight of a baby from birth to 6 weeks old.
 - a) Work out the percentage increase in weight from 0 weeks to 3 weeks.
 - b) Work out the percentage increase in weight from 3 weeks to 6 weeks.

7. The price of a pair of running shoes is reduced from £45 to £32.40. The price of a tennis racket is reduced from £18 to £12.60. Which item has the larger percentage reduction?

8. The population of a town increased from 30,000 to 75,000. Work out the percentage increase.



Fluency Practice

1. Craig buys 10kg of sweets for £19.50. He puts the sweets into bags, each containing 400g. Craig sells all of the bags of sweets for £1.50 each. Work out the percentage profit.
2. Jenny buys individual items of stationary and makes them into sets to sell.

Each set contains:

- 1 pencil case
- 2 pens
- 1 pencil
- 1 ruler

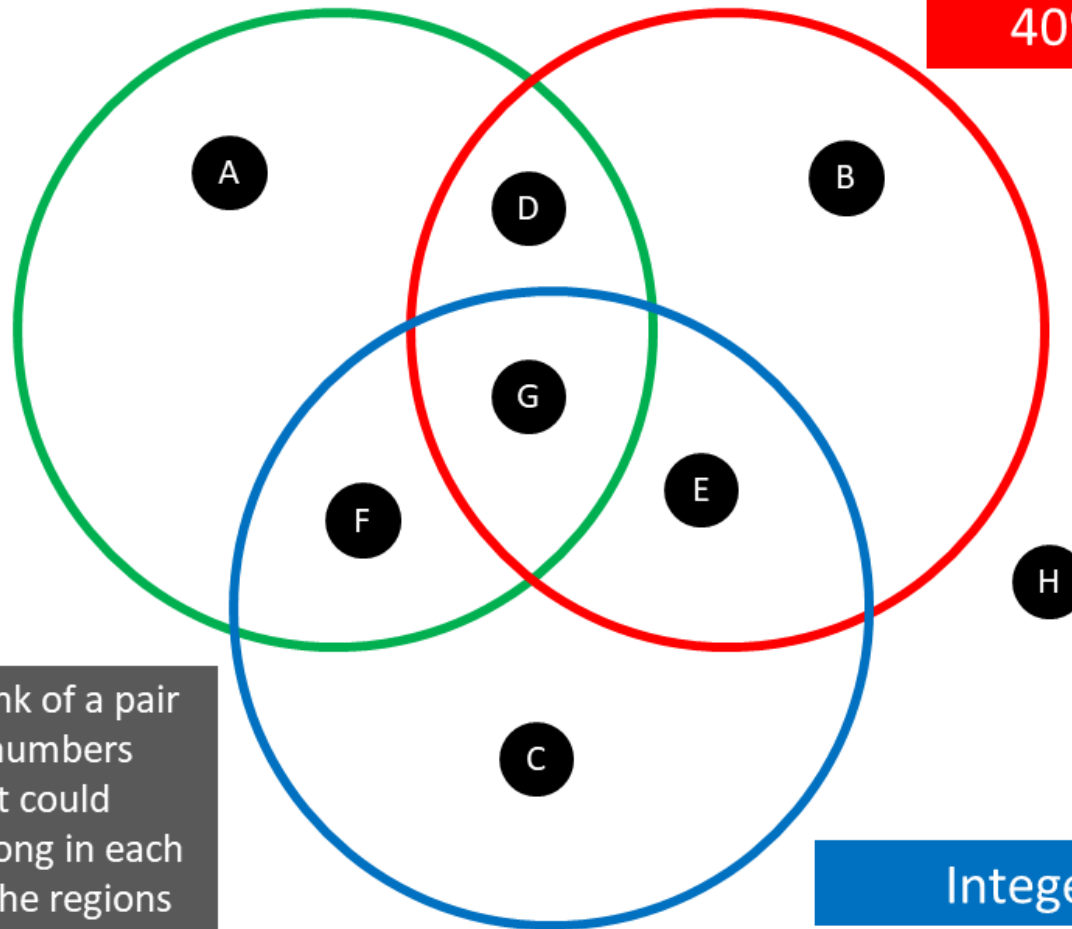
Item	Cost
Pencil Cases	£1.20 each
Pens	90p for a pack of 6
Pencils	£2.80 for a pack of 10
Rulers	72p each

- She buys enough items to make 30 stationary sets and sells them all for £3 each. Work out Jenny's percentage profit.
3. Callum is running a *hook-the-duck* game stall at a funfair. Players pay £3.50 for each turn and may win prizes of £10 or £50. On one day, 134 people pay to play. 17 of the players win £10 and 2 players win £50. Work out the percentage of the takings for the day that are left after the prizes are deducted.
 4. Simran makes fruit punch to sell. The ingredients cost a total of £24, on which she receives a 15% discount. Simran makes a total of 15 litres of the punch, and sells it in 300ml portions for 50p each. Work out the percentage profit.
 5. William buys a multipack of 24 drinks for £15 and a multipack of 18 ice lollies for £22.50. William sells the drinks individually for 75p each. He wants to make the **same** percentage profit on the ice lollies as he does on the drinks. How much should William charge for each individual ice lolly?
 6. Thomas is selling bicycles. His best-selling bike is currently priced at £89, which gives him a profit of £32 per bike.
 - a) Explain why £32 gives Thomas a 56% profit margin.
 - b) If Thomas wants to make a 60% profit on each bike, how much should he sell them for?

Maths Venns

Pair of numbers
with less than 30% change

Pair of numbers
with more than
40% change



If you think a
region is
impossible to
fill, convince
me why!

Think of a pair
of numbers
that could
belong in each
of the regions

Integers

Fluency Practice

A shop has a 20% discount on all normal prices. The normal price of a TV is £560.00. Work out the sale price.

Between 2017 and 2019, The number of students in a school rose by 5%. In 2019 there were 1260 pupils. How many were there in 2017?

A shop has a 10% discount on all normal prices. The sale price of a TV is £360.00. Work out the normal price.

My car depreciated in value this year by 20%. Last year it cost £14000. Calculate how much it costs this year

Next year I will receive a pay rise of 4%. This year I earn £36000. How much will I earn next year?

The price of a watch including 20% Value Added Tax (VAT) is £380.00. How much is it excluding VAT?

From 2018 to 2019, house prices went up by 2%. The price of a house in 2019 is £236000. What was it's price in 2018?

Last year I got a pay rise of 5%. This year I earn £27300. How much did I earn last year?

A limited edition bag of sweets contains 25% more than a standard bag. The limited edition bag contains 750g of sweets. How much do the sweets weigh in the standard bag?

Fluency Practice

Percentages

What is the question for this working?

Below are six calculations. Come up with a question involving 25% for which the answer to the question could be worked out by doing each of these calculations.

Q1. _____

Working: $0.25 \times \text{£}60$

Q2. _____

Working: $\text{£}80 \times 1.25$

Q3. _____

Working: $\text{£}120 \times 0.75$

Q4. _____

Working: $\text{£}175 \div 1.25$

Q5. _____

Working: $\text{£}39 \div 0.75$

Q6. _____

Working: $\text{£}26 \div 0.25$

2 Simple and Compound Interest

Fluency Practice

- 1) Amir invests £900 into an account that pays 3.5% simple interest per annum. Work out how much interest Amir will get after 9 years.
- 2) Alexandra puts £2885 into a bank account. The account pays 2.5% simple interest each year. Work out how much interest Alexandra will get after 8 years.
- 3) Alicia invests £920 into an account that pays 4% simple interest per year. Work out the total interest Alicia gets after 20 years.
- 4) Adam puts £365 into a bank account. The account pays 1.5% simple interest each year. Work out how much interest Adam will get after 16 years.
- 5) Alice invests £845 into an account that pays 0.5% simple interest per year. Work out the total interest Alice gets after 14 years.
- 6) Alice invests £500 into a savings account. Alice gets 2.75% per year simple interest. Work out the total interest Alice will get after 3 years.
- 7) Adam invests £3595 into an account that pays 1.25% simple interest per year. Work out the total interest Adam gets after 17 years.
- 8) Amelia invests £2660 into an account that pays 0.25% simple interest per annum. Work out how much interest Amelia will get after 23 years.
- 9) Adam invests £1530 into an account that pays 4% simple interest per annum. Work out how much interest Adam will get after 22 years.
- 10) Alfred invests £1530 into a savings account. Alfred gets 3% per year simple interest. Work out the total interest Alfred will get after 16 years.

Intelligent Practice

£1000 is invested at 5% simple interest

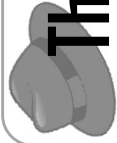
- 1) What is the value after 1 year
- 2) What is the value after 2 years
- 3) What is the value after 5 years
- 4) What is the value after 10 years

£1000 is invested at 5.5% simple interest

- 5) What is the value after 1 year
- 6) What is the value after 2 years
- 7) What is the value after 5 years
- 8) What is the value after 10 years

Intelligent Practice

- 1) £1000 is invested. The value after 1 year is £1200. What is the simple rate of interest?
- 2) £1000 is invested. The value after 2 years is £1200. What is the simple rate of interest?
- 3) £1000 is invested. The value after 5 years is £1200. What is the simple rate of interest?
- 4) £1000 is invested. The value after 10 years is £1500. What is the simple rate of interest?
- 5) £1000 is invested. The value after 10 years is £1200. What is the simple rate of interest?
- 6) £1000 is invested. The value after 10 years is £1100. What is the simple rate of interest?
- 7) £1000 is invested. The value after 10 years is £1050. What is the simple rate of interest?



The VIG

Organised crime earn a lot of their income through *usury* – lending money at **very high interest rates** – and by using **violence** to enforce repayment.

If someone borrows money they will have to pay *vig* on the principal – a weekly amount of interest. '3 points' *vig* is 3% *per week*.

- a) Ann borrows \$500 at 4 points. How much *vig* does she pay each week?
- b) Trey borrows \$700 at 8 points for 3 weeks. How much *vig* does he pay in total?
- c) Kay borrows \$400 and pays \$36 of *vig* per week. How many points is this?
- d) Dom borrows \$300 at 7 points. After 5 weeks he pays off the loan.
As a percentage of the principle, how much did the loan cost Dom?
- e) Dani lends Mika \$25 to buy a new top. Next week, Mika pays Dani back \$25 plus \$5 as a thank you. How much interest did Mika give Dani?
- f) Max borrows \$600. Over 8 weeks she pays \$408 of *vig*.
How many points is this?
- g) A gangster charges 8 points *vig*. Charlie borrows some money and pays the *vig*.
How many weeks is it until Charlie pays more in *vig* than the principal is worth?
- h) In real life, gangsters often charge 12 points *vig* to desperate people.
How much interest would a gangster earn on their money in a year?
- i) A credit card charges 30% interest over one year.
A gangster offers 4 points of *vig*.
If someone borrowed \$2000 for a year, how much more would they pay in interest to the gangster compared to the bank?
- j) A gangster charges 5 points on the principal and on any interest owed.
Teri borrows \$3000 but can't pay for the first week.
After the second week, how much does she now owe the gangster?
What is the actual interest rate per week?
- k) A different gangster charges 6 points on the principal and on any interest owed and increases the *vig* by 1 point each week.
Jak borrows \$800 but can't pay anything for 3 weeks.
How much does he now owe the gangster?
What yearly interest rate is this equal to?

Fill in the Gaps

Fill in the blanks. All amounts are in USD.

Initial amount	Interest Rate	Amount earned per year	Interest earned over 5 years	Total after 5 years
100	10%	10	50	150
100	1%	1		
100	2%			
100	4%			
200	4%			
600	4%			
600		600		
500	0.5%			
250				256.25

Fluency Practice

Question 1: Write down the multipliers that are equivalent to the following percentages

- (a) 50% (b) 80% (c) 10% (d) 25%
(e) 45% (f) 95% (g) 5% (h) 3%
(i) 7% (j) 36% (k) 71% (l) 44%
(m) 0% (n) 175% (o) 104% (p) 160%
(q) 7.5% (r) 1.2% (s) 0.8% (t) 0.01%

Question 2: Work out

- (a) 20% of 90cm (b) 70% of 3km (c) 15% of \$4500
(d) 57% of £58650 (e) 3.9% of 40cm (f) 106% of 8km

Question 3: Write down the multipliers that are used to calculate a:

- (a) 4% increase (b) 15% increase (c) 30% increase (d) 62% increase
(e) 29% increase (f) 70% increase (g) 1% increase (h) 100% increase
(i) 150% increase (j) 108% increase (k) 220% increase (l) 584% increase
(m) 1.5% increase (n) 2.8% increase (o) 0.3% increase (p) 0.09% increase

Question 4: Work out each of the following

- (a) 60ml increased by 70% (b) £940 increased by 8% (c) 143g increased by 19%
(d) 405 increased by 8.3% (e) 1150 increased by 131% (f) 9kg increased by 0.05%

Question 5: Write down the multipliers that are used to calculate a:

- (a) 2% decrease (b) 8% decrease (c) 12% decrease (d) 15% decrease
(e) 30% decrease (f) 60% decrease (g) 72% decrease (h) 23% decrease
(i) 100% decrease (j) 1.6% decrease (k) 2.9% decrease (l) 11.5% decrease
(m) 0.6% decrease (n) 0.9% decrease (o) 0.38% decrease (p) 0.08% decrease

Question 6: Work out each of the following

- (a) 80ml decreased by 4% (b) £480 decreased by 13% (c) 143g decreased by 40%
(d) 308 decreased by 1.2% (e) 2250 decreased by 0.5% (f) 9kg decreased by 6.03%

Apply

Question 1: 250 students attend a primary school.
94% of the students go on a visit to the zoo.
How many students went to the zoo?



Question 2: A car salesman receives 3% commission on his total weekly sales.
Last week his total sales were £28500
How much commission does he earn?



Question 3: 9400 people attend a rugby match between Bristol and Bath.
77% of the fans are male.
How many females attend the match?

Question 4: Over the past 10 years, the population of an island has decreased by 2%
The population of an island 10 years ago was 240000
What is the population of the island now?

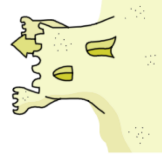


Question 5: Lily spends £60 a month on electricity.
By changing company, Lily can save 17%
How much would Lily pay each month?

Question 6: David's salary used to be £19,500 before he received a 8% pay rise.
Work out how much David is now paid?

Question 7: An empty sand pit weighs 6kg.
The weight of the sand pit increases to 50kg when filled with sand.

A different sand pit is 14% heavier but holds 22% less sand.
Calculate the weight of this sand pit when it is full of sand.



Question 8: A shop decreases all its prices by 10%.
A week later the shop increases all prices by 10%.
What is the overall change?

Question 9: Chloe is given a 10% pay rise.
The next year Chloe is given another 10% pay rise.
Her manager says that Chloe's pay has increased by 20% overall.
Explain why Chloe's manager is wrong.

Question 10: Gordon is building a wardrobe.
The wood costs 30% more than he had estimated.
He needs 40% more than he had estimated.
How much more than his original estimate does the material for the wardrobe cost?

Fluency Practice

Find the multiplier for each of these percentage increases:

- (a) 25% increase
- (b) 65% increase
- (c) 20% increase
- (d) 50% increase
- (e) 31% increase
- (f) 72% increase
- (g) 7% increase
- (h) 1% increase
- (i) 125% increase
- (j) 112% increase

Find the multiplier for each of these percentage decreases:

- (a) 15% decrease
- (b) 35% decrease
- (c) 40% decrease
- (d) 10% decrease
- (e) 18% decrease
- (f) 27% decrease
- (g) 6% decrease
- (h) 2% decrease
- (i) 90% decrease
- (j) 95% decrease

Find the multipliers for each of these percentage increases or decreases:

- (a) 165% increase
- (b) 12.5% increase
- (c) 17.5% decrease
- (d) 7.2% increase
- (e) 33.5% decrease
- (f) 4.5% decrease
- (g) 62.5% increase
- (h) 8.6% decrease

Find the multipliers for each of these percentage increase or decreases:

- (a) a 10% increase, followed by another 10% increase
- (b) a 25% increase, followed by a 10% increase
- (c) a 10% increase, followed by a 10% decrease

Fluency Practice

Section 2: reasoning and problem solving

- 1) Tarquin says “the order of percent change matters” do you agree? Explain your answer.
- 2) The sum of the percentage increases on the first five row is 45%. If you increase an amount by two percentages which add 45%, what is the biggest possible amount the original value it can be increase by?
- 3) Two percentages sum to 45%, one of the is a decrease and one is an increase. What is the biggest possible amount the original value can be increase by.
- 4) Rows 9, 10 and 11 all have a pattern cane you explain what is going on here?

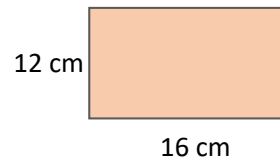
Section 3: Worded problem

- 1) A shop decreases all its prices by 15%. A week later the shop increases all prices by 17.5%. What is the overall change?
- 2) Amy is given a 7% pay rise. The next year Amy is given another 4.5% pay rise. Her manager says that Amy’s pay has increased by 11.5% overall. Explain why Amy’s manager is wrong.
- 3) Paul is building a wardrobe. The wood costs 30% more than he had estimated. He needs 40% more than he had estimated. How much more than his original estimate does the material for the wardrobe cost?
- 4) Create two word problems involving repeated percentage change of your own. Make one very difficult.

Section 4: Shapes

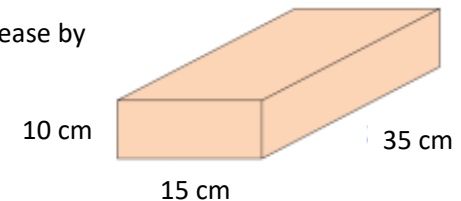
The sides lengths of this shape increase by 25%, what happens to the:

- a) Perimeter
- b) Area



The sides lengths of this shape increase by 20%, what happens to the:

- a) Surface area
- b) Volume



Intelligent Practice

- 1) A television costs £600
The price is increased by 10%
A month later the price is increased by another 20%
What is the final price of the television?
How much more is the television now?

- 2) A television costs £600
The price is increased by 20%
A month later the price is increased by another 30%
What is the final price of the television?
How much more is the television now?

- 3) A television costs £600
The price is decreased by 10%
A month later the price is decreased by another 20%
What is the final price of the television?
How much less is the television now?

- 4) A television costs £600
The price is decreased by 20%
A month later the price is decreased by another 30%
What is the final price of the television?
How much less is the television now?

Fluency Practice

- 1) Alice invests £1725 into an account that pays 3.75% compound interest per year. Work out how much Alice will have in the account after 12 years.
- 2) Alex buys a car for £15975 which depreciates in value at a rate of 4.25% per year. Work out how much Alex's car will be worth in 13 years.
- 3) Amir invests £2370 into an account that pays 3.25% compound interest per year. Work out how much Amir will have in the account after 21 years.
- 4) Alice buys a car for £12535 which depreciates in value at a rate of 1.75% per year. Work out how much Alice's car will be worth in 5 years.
- 5) Alice invests £1260 into a savings account. Alice gets 3.25% per year compound interest. Work out how much Alice will have in the account after 9 years.
- 6) Alfred buys a new car for £18735. The value of this car will depreciate by 4.5% each year. Work out how much Alfred's car will be worth in 16 years.
- 7) Alexandra puts £880 into a bank account. The account pays 0.5% compound interest each year. Work out how much Alexandra will have in the account after 21 years.
- 8) Amelia buys a car for £13425 which depreciates in value at a rate of 0.5% per year. Work out how much Amelia's car will be worth in 16 years.
- 9) Alicia puts £500 into a bank account. The account pays 4.75% compound interest each year. Work out how much Alicia will have in the account after 16 years.
- 10) Alice buys a new car for £18700. The value of this car will depreciate by 3.75% each year. Work out how much Alice's car will be worth in 17 years.

Fluency Practice

1. Either print the sheet or copy it into your book. Complete all empty boxes.

Question	Starting amount	Compound interest rate	Years	Calculation	End amount (from calculator, round to 2 dp)
Example	£800	37%	2	800×1.37^2	£1501.52
A	£900	18%	5		
B	£4000	25%	6		
C	£350	9%	8		
D	£148	10%	10		
E	£100,000	2%	10		
F	£500	2.5%	50		

2. Challenge:

- a) In Question A in the table above, how much interest was earned altogether?
- b) If the compound interest rate is 4%, the money is invested for 2 years, and the end amount is £2163.20, what was the starting amount?

3. Now open the Quiz attached to the assignment in Teams and type your answers in. This is so you can check they are correct and so I can see your score. Only type the numbers from the final column in the table. Use a pound sign and two decimal places.

Fill in the Gaps

Fill in the blanks. All amounts are in USD.

Initial amount	Interest Rate	Amount earned in year 1	Interest earned over 5 years	Total after 5 years
100	10%	10	161.05	261.05
100	1%	1	105.01	
100	2%	2		
100	4%			
200				
600				
600				
500				
500				

Fluency Practice

- 1) Alice invests £2085 for n years into a savings account. Alice was paid 3.75% per annum compound interest. At the end of the n years Alice had £3243.12 in the savings account. Work out the value of n .
- 2) Alicia invests £1735 for n years into a savings account. Alicia was paid 2.25% per annum compound interest. At the end of the n years Alicia had £2167.37 in the savings account. Work out the value of n .
- 3) Adam invests £3570 for n years into a savings account. Adam was paid 2.75% per annum compound interest. At the end of the n years Adam had £5661.86 in the savings account. Work out the value of n .
- 4) Amelia invests £695 for n years into a savings account. Amelia was paid 3.25% per annum compound interest. At the end of the n years Amelia had £1122.89 in the savings account. Work out the value of n .
- 5) Amelia invests £1960 for n years into a savings account. Amelia was paid 4.5% per annum compound interest. At the end of the n years Amelia had £3473.5 in the savings account. Work out the value of n .

Fluency Practice

- 1) Alicia invests some money into an account that pays 2.75% compound interest per annum. Work out after how many years Alicia will have trebled the investment.
- 2) Amir invests some money into an account that pays 0.5% compound interest per annum. Work out after how many years Amir will have trebled the investment.
- 3) Amelia invests some money into an account that pays 2.25% compound interest per annum. Work out after how many years Amelia will have trebled the investment.
- 4) Alfred invests some money into an account that pays 3.5% compound interest per annum. Work out after how many years Alfred will have trebled the investment.
- 5) Amir invests some money into an account that pays 0.5% compound interest per annum. Work out after how many years Amir will have trebled the investment.


Fill in the Gaps


	Start Amount	Annual Interest %	Multiplier	Period	£ Interest Total	Final Amount
1	£4,000	2%	0.02	3 years		
2	£4,000	4%		3 years		
3	£4,000	1.2%		3 years		
4	£2,500	3.25%		2 years		
5	£3,000			4 years	£600	
6	£2,500			3 years	£116.25	
7	£1,200			2 years		£1573.80
8		2.44%		3 years	£439.20	



Fluency Practice


- Question 1:  Paul leaves £4000 in the bank for two years.
It earns compound interest of 5% per year.
Calculate the total amount Paul has in the bank at the end of the two years.
- Question 2:  The population of birds on an island is estimated to increase by 10% every year.
The population of birds on the island is 20000.
Calculate an estimate for the population of birds in three years time.
- Question 3:  The value of a car decreases by 5% each year.
Sophie bought a car two years ago for £10000
Work out the value now.
- Question 4:  Sam invests £1800 in the bank for four years.
It earns compound interest of 4% each year.
Calculate the total amount Sam has in the bank at the end of four years.
- 
- Question 5:  A full water tank holds 500 litres.
The tank begins to leak water and is losing 14% of its contents every hour.
Find how much water is left in the tank after 8 hours.
- Question 6:  The height of a tree increases by 60% each year.
When planted the tree was 40cm tall.
How tall will the tree be in 5 years time.
- Question 7:  Carrie invests £800 for 4 years at 3% interest per year.
How much interest does she earn?
- Question 8:  A house was bought for £100,000
Its value appreciates by 7.5% each year for the first three years.
What was its value at the end of the three years?
- 


Fluency Practice



Question 9:  The number of people living on a remote island decreases by 9% every 10 years. In 1950 there were 18000 living on the island. Calculate how many less people will be living on the island in 2020.


Question 10:  A car was bought for £20,000. Its value depreciates by 31% each year for the first four years. What is its value at the end of the four years?


Question 11:  A tree is 80cm when planted. Each year the height of the tree increases by 22%. After how many complete years will the height of tree be at least 3m? 

Question 12:  The number of polar bears in a region is decreasing by 5% per year. There are 3000 polar bears in the region in 2017. What year will be the first year with less than 1000 polar bears in the region?

Question 13:  Michael has started working for a company on a salary of £15000. Each year he will be given a 6% pay rise. How many years will it take for Michael's salary to exceed £30000?

Question 14:  The value of a car decreases by 7.2% each year. When bought the car cost £6200. How many years will it take the car to have a value less than £1000? 

Question 15:  A full water tank has sprung a leak. 4% of the water is lost every minute. What percentage of water is left in the tank after twenty minutes?

Question 16:  A fish tank, that is full of water, has sprung a leak. 12% of the water is lost every hour. What percentage of the water is lost after three hours?

Fluency Practice

Apply

Question 1: Florence invests £200 for two years at 5% compound interest, paid yearly. Liam says that the interest that Florence will receive will be £20. Is Liam right?



Question 2: The value of a motorcycle was £14000 on 1st April 2014. Every three months the value of the motorcycle decreases by 2% of its value at the start of that three months. What was the value of the motorcycle on 1st April 2016?



Question 3: When a ball is dropped, it bounces and then rises. The ball rises to 90% of the height from which it is dropped. The ball is dropped from a height of 4m.



- (a) Calculate the height of the rise after the first bounce.
- (b) Calculate the height of the rise after the second bounce.

The ball carries on bouncing, each time rising to 90% of the last rise.
(c) For how many bounces does it rise to height greater than 1m?

Question 4: The population of a country is increasing by 5% a year. How many years will it take the population of the country to double?

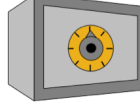


Question 5: Raheem and Ben invest money in 2010.
Raheem invests £1000 at Banks'R'us, who pay 3% interest per year.
Ben invests £1400 at Bank World, who pay 1% interest per year



In which year will Raheem's investment be worth more than Ben's?

Question 6: The population of a country increases by $x\%$ each year. In 2014 the population of the country was 24,000,000. Three years later, the population was 26,996,736. Find x .



Question 7: Charlotte invests £5000. The bank pays 10% interest for the first year and then $y\%$ every year after that. After three years, Charlotte has £5610.55. Calculate y .



Fluency Practice

- (a) £600 is invested at 6% per annum interest. How much money is in the bank after 3 years?
- (b) £1500 is invested at 5% per annum interest. How much money is in the bank after 4 years?
- (c) £4200 is invested at 4% per annum interest. How much money is in the bank after 7 years?

- (d) £500 is invested at 3.5% per annum interest. How much money is in the bank after 2 years?
- (e) £12000 is invested at 4.4% per annum interest. How much money is in the bank after 6 years?

- (f) Find the compound interest earned on £2300 invested for 2 years at 7% per annum.
- (g) Find the compound interest earned on £5200 invested for 3 years at 2.5% per annum.
- (h) Find the compound interest earned on £875 invested for 9 years at 0.5% per annum.

- (i) £12000 is invested for 2 years at 2% per annum, then at 3% for the next 3 years. How much money is in the bank at the end of the 5 years?
- (j) Find the compound interest is earned when £8000 is invested for 10 years at 1.1% per annum, then at 1.5% for the next 10 years.

- (k) Hannah invests £750 and after 4 years has £844.13. Find the compound interest rate.

Fluency Practice

- (a) A small plant is 5 cm tall. It increases its height by 10% each day. How tall will it be after one week?
- (b) The penguin population of an island in 2013 is 1500. It increases at 15% per year. What will the population be in 2019?
- (c) A sycamore tree is 40 cm tall. It grows at a rate of 8% per year. How tall is the tree after 10 years? Give your answer in metres.

- (d) A car is worth £18000 when new. It depreciates at a rate of 17% per year. What is its value after 5 years?
- (e) The population of Ainsworth is 500, and it is decreasing at a rate of 0.5% per year. What will the population be in 20 years' time?
- (f) A 12 kg lump of radioactive material loses mass at a rate of 1% per day as it decays. What is its mass after 90 days? Give your answer to the nearest gram.

- (g) A population of birds is growing at 9% per year. How many years does it take the population to double?
- (h) If the population of a country increases by 2% per year, by what percentage would the population increase over 20 years?
- (i) Mary invested £1200 in a savings account. She receives 5% per year compound interest. After n years she has £1608.11 in her account. Work out the value of n .

- (j) At 10am a scientist places a small number of bacteria in a petri dish. The number of bacteria doubles every hour. The petri dish is full by 7pm. At what time was the petri dish half full of bacteria?

Fluency Practice

(a) Aisha earns £13.50 per hour. Her hourly rate is increased by 4%. She is then given an additional pay increase of 2.5%. What is her hourly rate after these increases?

(b) In 2019, the population of Toytown was 2250. In 2020 it increased by 1.6% and in 2021 it increased by 2.1%. What was the population of Toytown at the beginning of 2022?

(c) A new motorbike costs £11650. Its value depreciates by 12% in the each of the first two years after it is bought, and 8.5% in the third year. How much is the motorbike worth after three years?

(d) An antique vase is valued at £885. Over three years, its value increases by 3.6% per year. It then gets damaged and its value decreases by 15%. How much is the vase now worth?

(e) An increase of 25% followed by an increase of 10% is equivalent to what overall percentage change?

(f) An increase of 25% followed by a decrease of 10% is equivalent to what overall percentage change?

(g) An increase of 25% followed by a decrease of 25% is equivalent to what overall percentage change?

(h) Giorgio invests £3900 at a compound interest rate of 2.5% for the first year, and $x\%$ for each of the second and third years. At the end of the three years Giorgio has earned £357.43 in interest. Find the value of x .

(i) A flask contains 5×10^{12} bacteria. The number of bacteria increases by a quarter every minute. After x minutes there are approximately 3.47×10^{14} bacteria in the flask. Find the value of x .

Fluency Practice

Repeated Percentage Change		
(a)	(b)	(c)
Aliza's salary will rise by 3% every year for the next 5 years. Her starting salary is £24500. What will she earn in 5 years' time?	Hassan invests £4000 at a compound interest rate of 2.4% per year. How much money will he have after 3 years?	A colony of rabbits is growing at a rate of 35% per year. Initially the colony has 20 rabbits. How many rabbits will it have after 10 years?
(d)	(e)	(f)
Lucas invested £6500 10 years ago at an compound interest rate of 2.5% per year. He needs £8500 to buy a new Ford Fiesta. Will he have enough money in his account?	Bank A has an interest rate of 1.2% and Bank B has an interest rate of 1.3%. If I invest £5000 in both banks, what is the difference between the two balances after 4 years?	When people have an overdraft at a bank they are charged interest. Sonny is £45 overdrawn. His bank charges 2% per month interest rate. How much will Sonny owe after 6 months, assuming he doesn't pay off any of his debt?
(g)	(h)	(i)
A credit card company charges interest at 3.2% per month on any outstanding balance. A balance of £1500 is left unpaid. What is the balance after 1 year?	The world population at the end of 2013 was 7.1 billion people. The current population growth rate is 1.1%. Assuming this rate stays the same, what will the population be in 2050?	There are 10 bacteria in a petri dish at the start of the day. The number doubles every hour. How many bacteria will there be after 24 hours?

Fluency Practice

compound interest

Where necessary, round your answers to the nearest penny.

1. Harry invests £2000 in a savings account that pays 3% compound interest each year. Work out how much Harry has in the account:
a) after 1 year b) after 3 years c) after 10 years
2. When she was born, Molly's parents invested £800 into a savings account with an interest rate of 2.5% compound annually. Work out how much money is in the account:
a) after 3 years b) after 10 years c) after 18 years
3. A savings account pays 4% compound interest per annum. Hassan invests £2500 into the account. Work out how much interest Hassan will earn in five years.
4. An account pays compound interest of $x\%$ per annum. The value of an investment £ V after t years is given by the formula: $V = 2000 \times 1.06^t$
a) How much was originally invested in the account?
b) Work out the value of x .
5. Ashley invests 1500 in a savings account that pays 1.5% compound interest per annum. Which of the following calculations gives the amount in Ashley's account after 12 years?
a) 1500×1.5^{12} b) 1500×1.15^{12} c) 1500×1.015^{12}
6. Simon invests £900 in a savings account that pays compound interest of 2.5%. A formula for the value £ V of Simon's investment after t years is $V = 900m^t$ where m is a constant. Work out the value of m .
7. Two banks have savings accounts:
Daniel invests £1200 with Premier Bank and Thomas invests £1150 with Superior Bank.
Work out who has more money in their account:
a) after 2 years b) after 6 years
8. Samira invests £2500 in an account that pays compound interest of 5% per annum. After how many complete years will the account have more than £3000?
9. Philip invests £8000 in an account that pays compound interest of 7.5% per annum. After how many complete years will the investment have increased by more than half of the original investment?

Superior Bank Compound Interest 5% for each year

Premier Bank Compound Interest 3.5% for each year
--

Fluency Practice

10. Matthew invests £5000 in an account that pays $x\%$ compound interest per annum. After 6 years, Matthew's investment is worth £6326.60. Work out the value of x .
11. Shanelle invests £1400 in an account that pays $x\%$ compound interest per annum. After 3 years, Shanelle has earned a total of £267.42 in interest. Work out the value of x .
12. Danielle invests some money in an account that pays 2.75% compound interest per annum. After 3 years, Danielle's investment is worth £4556.12. Work out how much money Danielle initially invested.
13. A bank offers a saving account with an introductory interest rate of 7% for the first year, and a follow-on rate of 5%, compounded annually. If £500 is invested in the account for 3 years, how much interest will be earned?
14. Angela wants to invest £4000 for 4 years in a savings account.

Solid Saver Account
Compound interest

3% for each year

Rocket Saver Account
Compound interest

6% for the first year
2% for each extra year

Which account will give Angela the most interest at the end of 4 years?

15. A savings account offers 5% interest during the first year, followed by 8% interest in the second year.
Work out the overall percentage increase on any money invested for 2 years.
16. £10,000 is invested into a savings account that pays compound interest. In the first year, the account earns 5% interest. In the second and third years, the account earns $x\%$ interest.
At the end of the third year, the investment is worth £11,247.86.
Work out the value of x .

17. Which of the following compound interest deals results in the greatest amount of interest earned over three years?

Steady Deal
3% interest each year

Riser Deal

First year: 1% interest
Second year: 3% interest
Third year: 5% interest

Fluency Practice

1. The number of cells in a test tube increases by 2% each hour. If there are originally 200 cells, how many are there after 6 hours?
2. The price of a car depreciates by 5% each year. If the car originally costs £2900, how much is it worth after 8 years?
3. Between 2013 and 2017 the share price of a company decreased by 3% each year. If a share was purchased in 2013 for £1200, how much was it worth in 2017?
4. A ball is dropped from a height of 2m. Each time it bounces it loses 50% of its height. What height will the ball reach on its 4th bounce?
5. Peter bought a new car for £15,500. The value £ V of the car after n years is given by the formula: $V = 15500 \times 0.88^n$. After how many years will Peter's car be worth less than 60% of the price he originally paid for it?
6. A shop is offering a 10% sale on all its products. This includes a washing machine that is already discounted by 25%. What is the overall percentage reduction in the price of the washing machine?
7. An online store increases all its prices by 10%. A month later, the store decreases all its prices by 10%. Noah says the prices are now all the same as they were originally. Show that Noah is wrong.
8. In January the number of people in the school chess club increased by $\frac{1}{5}$. In February the number decreased by 10%. Calculate the overall percentage change in the number of people attending the chess club during these 2 months.
9. John bought an antique vase. John sold the vase to Angela. He made a 15% loss. Angela then sold the vase for £714. She made a profit of 40%. Work out how much John paid for the vase.
10. Brian owns a boat, which depreciates in value by $x\%$ each year. In 2015, the boat was worth £3200 and in 2018, the boat was worth £1638.40. Work out the value of x .
11. After working at a company for 1 year, Afra's salary increased by 5%. After her second year, Afra received an 8% pay rise, taking her salary up to £28,000. How much was Afra's starting salary?
12. Joe invests £800 in a savings account that pays $x\%$ interest each year. At the end of the year, Joe pays tax on the interest at a rate of 20%. After paying tax, Joe receives £51.20. Work out the value of x .

Fluency Practice

Compound Growth



① £1000 is invested and earns 5% compound interest per annum. How much is the investment worth after 2 years?

$$\text{starting quantity} \times \text{multiplier}^{\text{periods}} = \text{final quantity}$$

$$1000 \times 1.05^2 =$$

② £1000 is invested and earns 4% compound interest per annum. How much is the investment worth after 2 years?

$$1000 \times 1.04$$

③ £1000 is invested and earns 4% compound interest per annum. How much is the investment worth after 5 years?

④ An investment account offers 7% interest per annum. How much is an investment of £2000 worth after 3 years?

⑤ Over 5 years a £3000 investment gains 3% of compound interest. How much is the investment now worth?

⑥ Gendale City has a population of 6000. It is expected this population will grow by 5% every year. What do we expect the population to be in 3 years?

⑦ A swarm of 6000 bees grows by 8% a week. How large is the swarm after 4 weeks?

⑧ Every week 120 tonnes of rubbish is collected from Hitown City. This is expected to increase by 6% a year. In 5 years, how many tonnes a week of rubbish will be collected every week?

⑨ A bank account offers 5.5% interest per annum. If an account now has \$506 in it, how much will it be worth in 3 years?

⑩ Every day a website increases its active users by 12%. If there are currently 9000 users. how many **new** users will be using the site in a week's time?

⑪ Some money is invested and earns 5% compound interest per annum for 4 years. As a percentage, how much does the original investment grow?

(What number can we use for the original investment?)

⑫ £1200 is invested and earns 4% interest. How many years does it take for the investment to be worth over £1500?

9 Answers (nearest integer)	6946	1103	594	1217	3478
	1082	10896	2450	8163	

Fluency Practice

Repeated Percentage Change



<p>A A £2000 investment earns 4% compound interest per year. How much is the investment worth after 2 years?</p>	<p>B A £2000 investment earns 4% interest per year. How much is the investment worth after 3 years?</p>	<p>C A colony of penguins has a population of 20,000. Every year it grows by 7%. How large is the colony after 5 years?</p>	<p>D A swarm of wasps grows by 12% every week. If the swarm had 2,000 members 4 weeks ago, how many wasps are there now?</p>	<p>E A city has a population of 25,000 citizens. How many citizens would be added if the population grows by 9% a year for 6 years?</p>
<p>F A £2000 investment loses 4% of its value every year. How much is the investment worth after 2 years?</p>	<p>G A £2000 investment loses 14% of its value every year. How much is the investment worth after 2 years?</p>	<p>H A £3000 investment earns 5% interest per year. How many years will it take for the investment to be worth over £4000?</p>	<p>I A £3000 investment loses 5% interest per year. How many years will it take for the investment to be worth less than £2500?</p>	<p>J A £3000 investment earns 7% interest per year for 4 years. Then it loses 7% per year for 4 years. What is its final value?</p>
<p>K After 3 years of 5% interest an investment is worth £2836. To the nearest pound, how much was invested?</p>	<p>L A population declines at 8% a year over 5 years. If the population decreases to 10,800, what did it start at?</p>	<p>M A company loses 14% of its value every year. How long does it take for the value of the company to halve?</p>	<p>N Over 2 years a £2000 investment becomes worth £2330. Calculate the yearly interest rate to the nearest integer.</p>	<p>O Over 3 months a bee population drops from 32,000 to 26,000. To 1 decimal place, what was the monthly percentage change in the population?</p>

13 Answers	5	3147	1479.20	2163.20	6	1843.20	2941.63
	2249.73	2450	4	16386	16928	28051	

Fluency Practice

(a)	(b)	(c)	(d)
Work out 78% of 240 cm	In a sale, all prices are reduced by 15%. The normal price of a necklace is 90 euros. Work out the sale price of the necklace.	Corey's pay increases from £12.36 per hour to £14.11 per hour. Find the percentage increase to 1 decimal place.	Dele invests \$6000 for 4 years at 3% per annum compound interest. Calculate the value of his investment at the end of 4 years.
(e)	(f)	(g)	(h)
The value of a car depreciates by 14% per annum. At the end of 2017 the car is worth \$17500. How much is it worth at the end of 2020?	A train company increases all its ticket prices by 12.5%. A ticket from Preston to Crewe currently costs £48. How much will it cost after the increase?	The value of Pierre's investment increases by 6% to \$1971.60. Calculate the value of his investment before the increase.	Express 2.1 million as a percentage of 3.7 million, giving your answer to 1 decimal place.
(i)	(j)	(k)	(l)
Nick bought a motorbike and then sold it for £7457.60. If he made an 18% profit, how much did he pay for the motorbike?	Zaneta invests \$650. Her investment earns 2.5% compound interest for the first 2 years, then 3.5% for the next 3 years. Work out the value of the investment after 5 years.	Aleeza invested some money at a compound interest rate of 3.5% per annum. After 3 years the investment was worth \$2660.92. How much did Aleeza invest?	A clothes shop has a 15% off sale. In the sale a jumper is reduced by £6. What was the original price of the jumper?

Fluency Practice

Section A: Percentage Change

1. The population of a village increased from 234 to 275 during one year. Find the percentage increase.
2. When a beaker of sand is dried in a hot oven its mass reduces from 1.2kg to 870g. Find the percentage reduction in its mass.
3. A battery was tested and found to power a camera for 12 hours before it needed recharging. An improved version of the battery powered the camera for an extra 30 minutes. Find the percentage increase in the life of the batteries.
4. The average cost of a local telephone call dropped by 8p to 27p. Find the percentage reduction in the average cost of a local call.

Section B: Increasing and Decreasing

1. In a sale, all the prices are reduced by 30%. Calculate the sale price of the following items:
 - a. a bike that cost £250
 - b. a pair of gloves that cost £3.20
2. In 2004, 180 parents applied to a school for a place for their child. The following year saw an increase of 35% in the number of applications. Find the number of applications in 2005.
3. Following the opening of a new supermarket nearby, the number of customers using a small store decreased by 21%. If 2,400 customers used to use the store each week, find the number of customers after the store opened.

4. A car costs £9,999.90 before VAT (value added tax). Work out the cost including VAT if it is charged at 20%.
5. Sally's investment of £450 has gone up by 30%, while Susie's investment of £650 has gone down by 10%. Who now has the larger amount of money, Sally or Susie?
6. A train company increases its rail fares by 4% one year and by 6.5% the following year. Find the percentage increase in cost over the two years.

Section C: Reverse Percentage Problems

1. A jacket is reduced by 12% to £66 in a sale. Find the original price.
2. A baby's weight increases by 8% over a month from birth to 4.05kg, what wasthe weight at birth?
3. Which product has the greatest original price? Show your working.

£? ~~£?~~
20% off! Now £2.00

A

£? ~~£?~~
30% off! Now £1.60

B

4. The air pressure increases by 1.2% to 1,214.4 mbar. What was the original air pressure?
5. A dress in a sale is reduced by 7% to £60.45. What is the original price?
6. A stereo system is sold for £1,998 and an 11% profit is made. Find the original cost of the stereo.
7. A shop sells a television to a man and makes a 15% profit. The man sells it to another man for £414 at a loss of 10%. Find the original price of the television.

Fluency Practice

Section D: Compound Interest

1. Jane invests £1,200 in a bank account which earns interest at the rate of 6% per annum. Find the value of a) the investment after 5 years b) the interest earned.
2. Craig puts £240 into a savings account. Each year the savings earn interest at 6% of the amount in the account at the start of the year. What will his savings be worth after 3 years? Give your answer to the nearest penny.
3. Each year a car loses value by 11% of its value at the start of the year. If it was worth £8000 when it was new, what will it be worth after 2 years?
4. A population of bacteria is estimated to increase by 12% every 24 hours. The population was 2000 at midnight on Friday. What was the population (to the nearest whole number) by midnight the following Wednesday?
5. Ambrose invested £3500 in a six-year bond that added 5% to the amount each year for the first three years and 7.5% each year for the next three years. What is the amount in the bond, to the nearest penny after six years?

Section E: Bonus Questions

1. Two numbers b and c are 20% and 28% less than the third number d. By what percentage is the number c less than the number b?
2. The ratio of two numbers is $\frac{5}{6} : \frac{2}{3}$. By what percentage is second number more or less than the first number?
3. 250% of x is increased by 250% to become 350. What is the value of x?

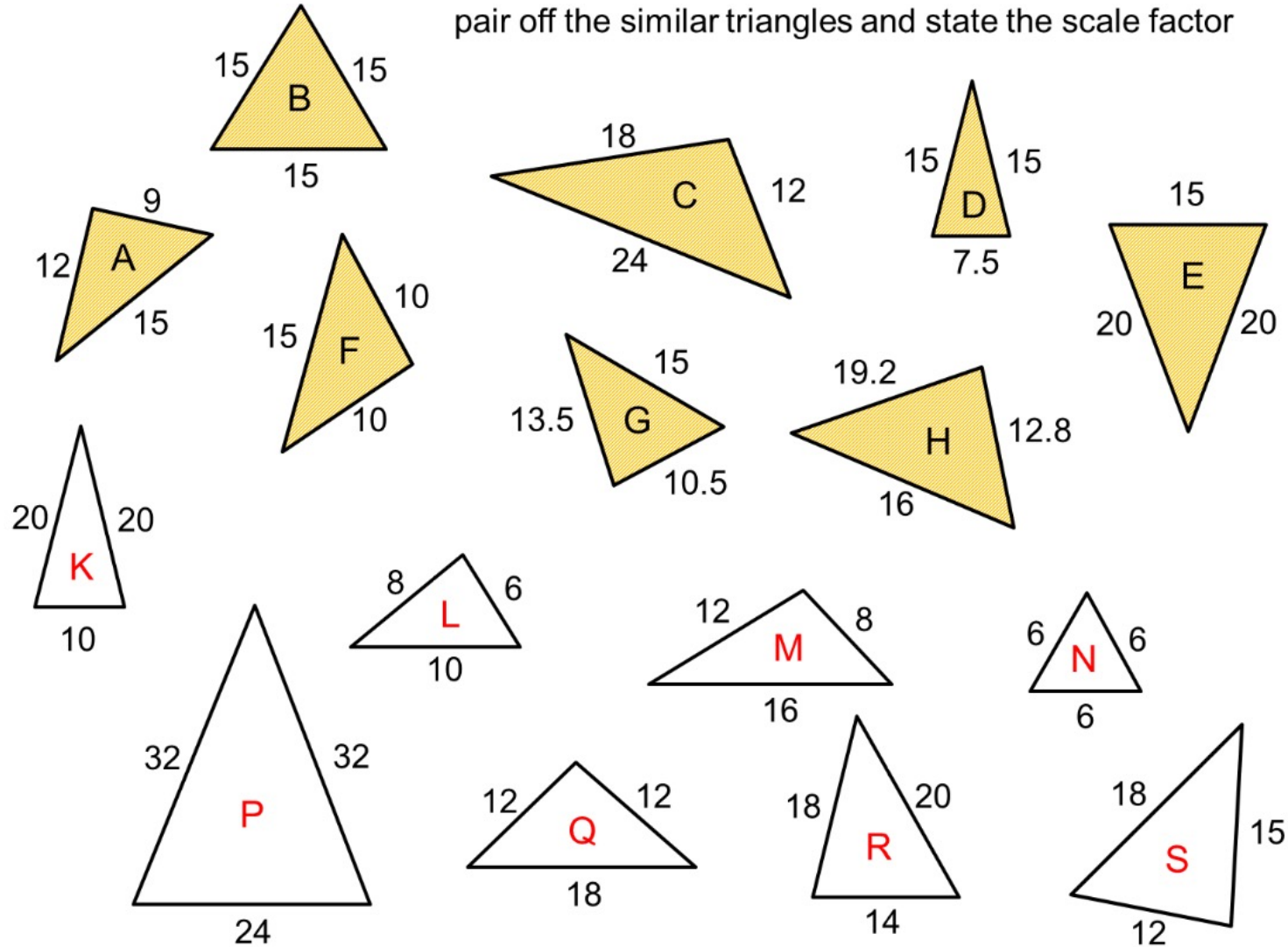
Fluency Practice

<p>A1 A bank pays 2.5% interest on its current account. Write 2.5% as a decimal.</p>	<p>A2 Rosie took a science test and scored 41 marks out of 45. Express 41 out of 45 as a percentage.</p>	<p>A3 A school has 80 staff. 15% of the staff wear glasses. Calculate the number of staff that wear glasses.</p>	<p>A4 56% of students in a school are girls. There are 420 girls in the school. Work out the total number of students in the school.</p>
<p>B1 Ayesha plays hockey. Last year Ayesha scored 8 goals. This year Ayesha scored 13 goals. Calculate the percentage increase in for the number of goals scored.</p>	<p>B2 Between 2001 and 2011, the population of a town increased by 8% In 2001 the population was 34 342. Calculate the population in 2011.</p>	<p>B3 In a sale, normal prices were reduced by 20%. The normal price of a camera was £180. Work out the sale price of the camera.</p>	<p>B4 Justin bought some clothes. The clothes should have cost £84.00 but he got a discount of 15%. Work out how much money Justin saved.</p>
<p>C1 Rohan invested £3000 for 4 years in a savings account. He was paid 2.5% per annum compound interest. How much did Rohan have in his savings account after 4 years?</p>	<p>C2 Susanna invested £2000 for 3 years at 4% interest per annum compound interest. Work out the amount of interest Susanna had earned after 3 years.</p>	<p>C3 Anya bought a car for £12 500. The car depreciates at a rate of 12% per year. Work out the value of the car after five years.</p>	<p>C4 The price of shoes was increased by 15%. However, customers were given a 20% discount if they bought two pairs at the same time. Work out the cost of two pairs of shoes that originally cost £68 each.</p>
<p>D1 In a sale, normal prices were reduced by 25%. The sale price of a computer was £442. Work out the normal price of the computer.</p>	<p>D2 In a sale, all prices are reduced by 15%. The sale price of a shirt is £22.40. Work out the original price of the shirt.</p>	<p>D3 The price of a new TV is £540, which includes 20% VAT. Find the cost of the TV excluding VAT.</p>	<p>D4 Natasha invested some money at 4% per annum compound interest. At the end of two years, the value of her investment was £3380. Find the amount of money that Natasha invested.</p>

3 Similarity with Length

Fluency Practice

pair off the similar triangles and state the scale factor



Fluency Practice

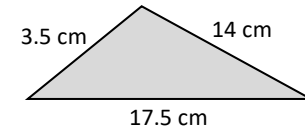
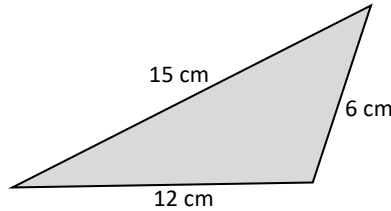
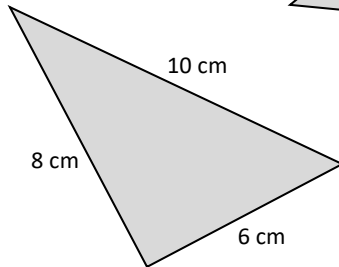
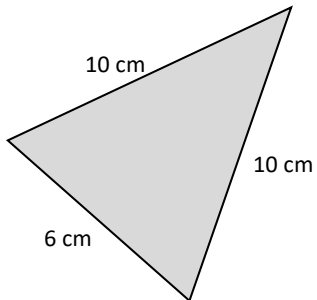
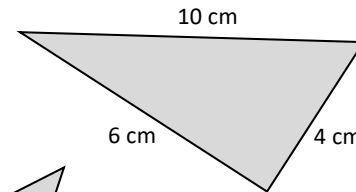
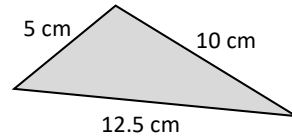
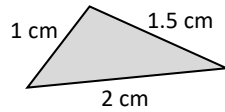
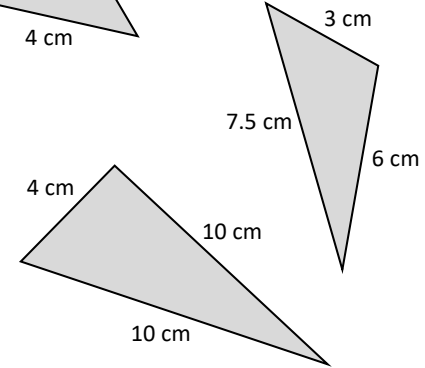
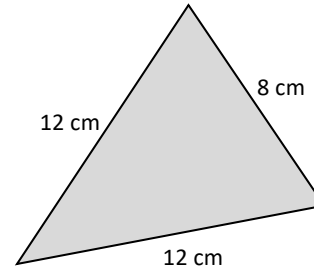
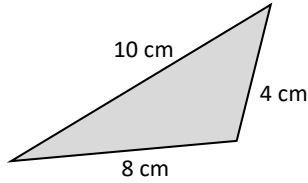
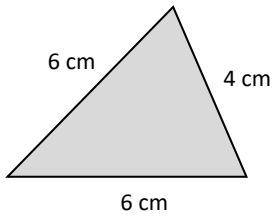
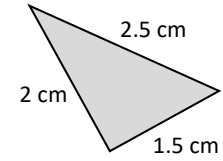
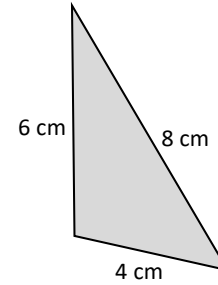
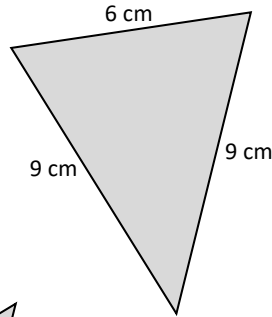
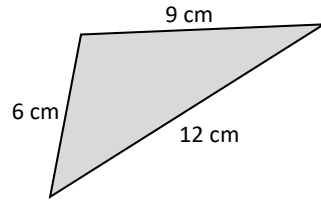
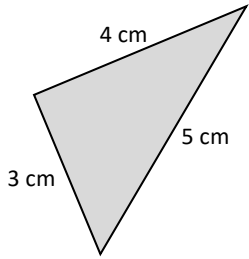
Which of these triangles are similar to each other?

Each triangle is drawn at a different scale.

Express the side-length of each triangle as a ratio & simplify.

Similar triangles will have the same ratio of side lengths!

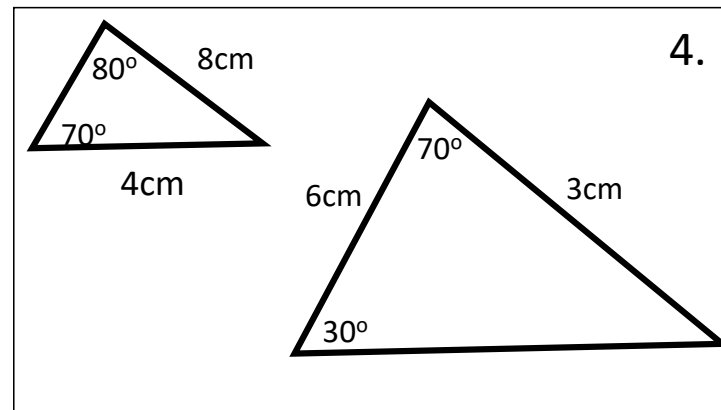
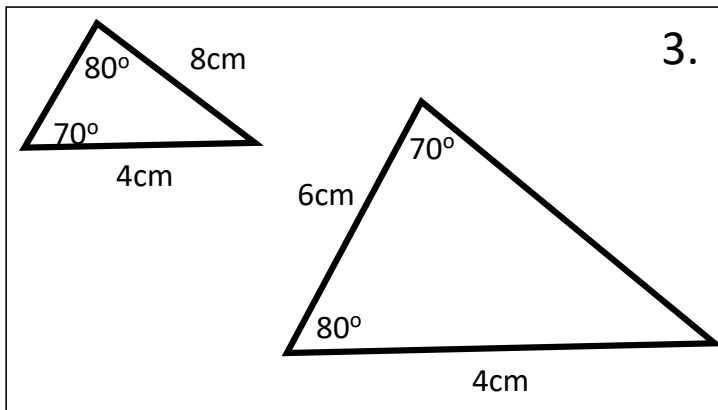
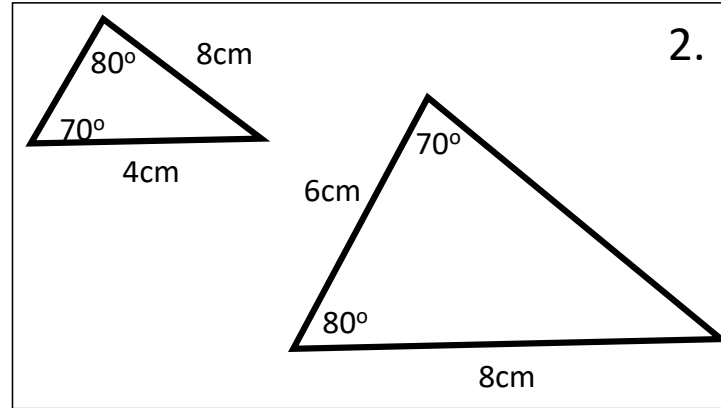
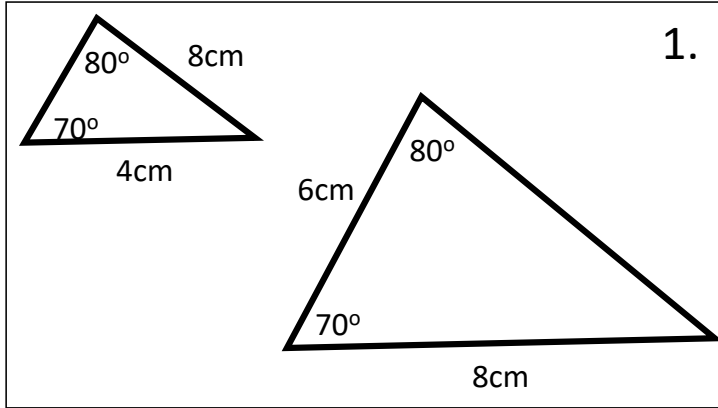
Not to scale.



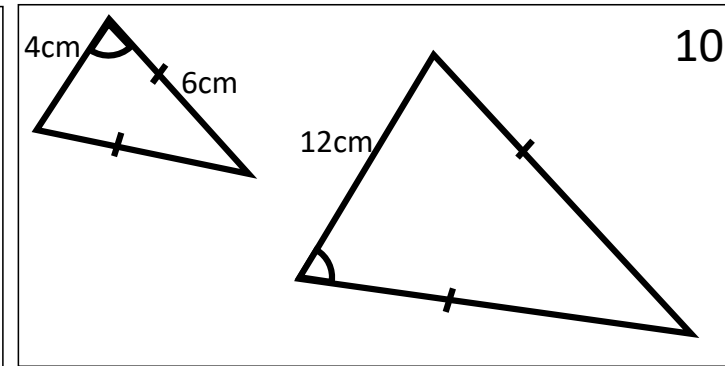
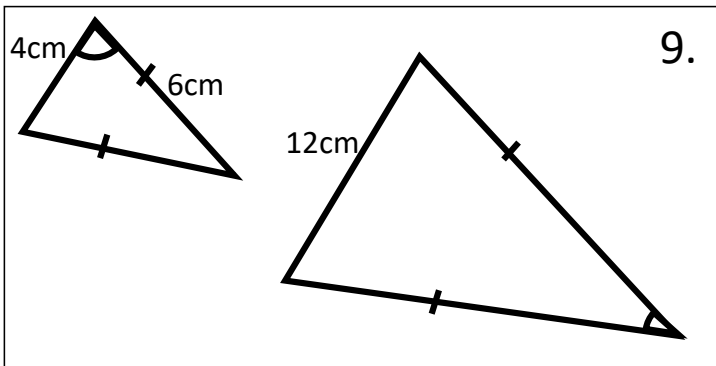
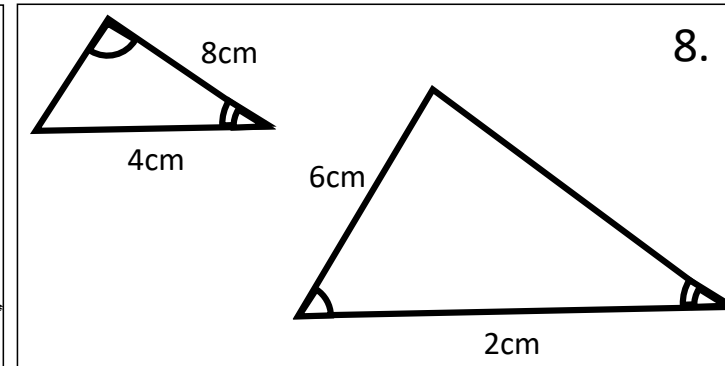
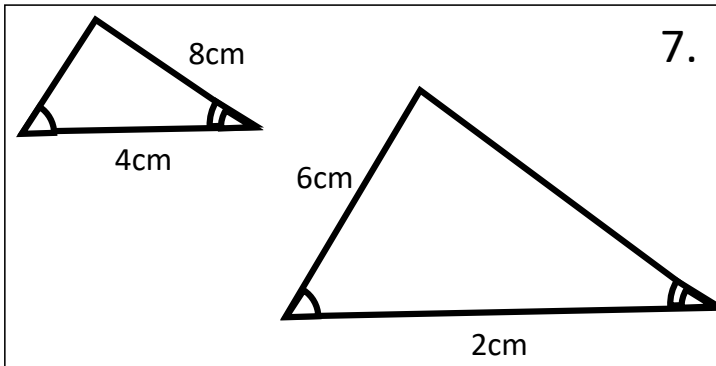
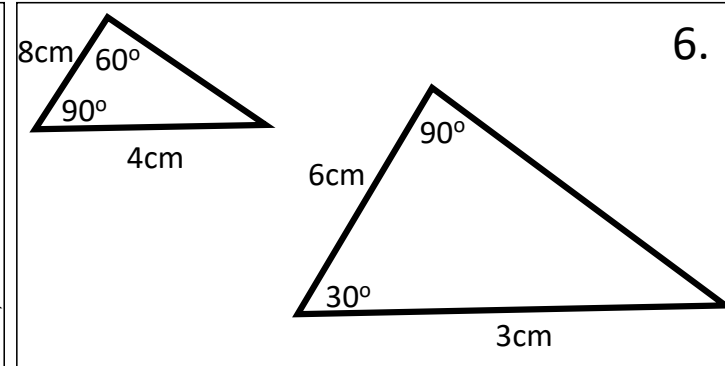
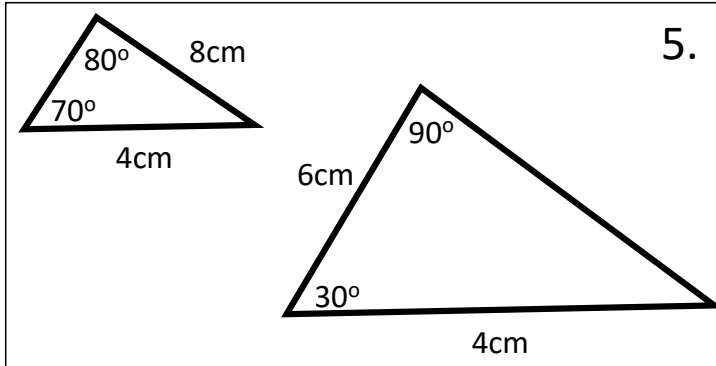
Intelligent Practice

Intelligent Practice – Find the length of every missing side

Triangles not drawn to scale

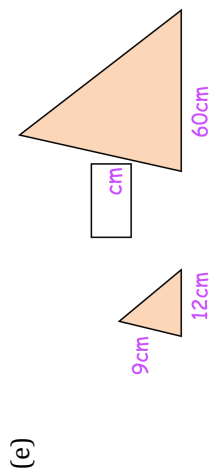
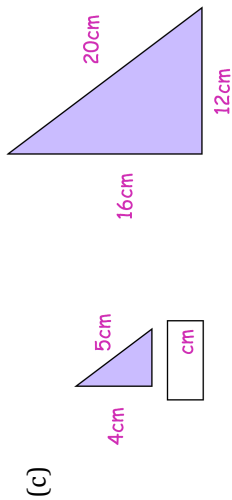
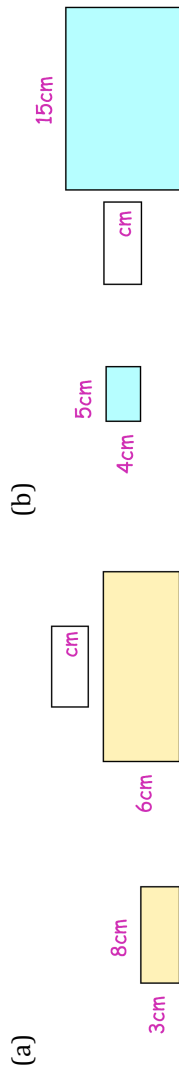


Intelligent Practice

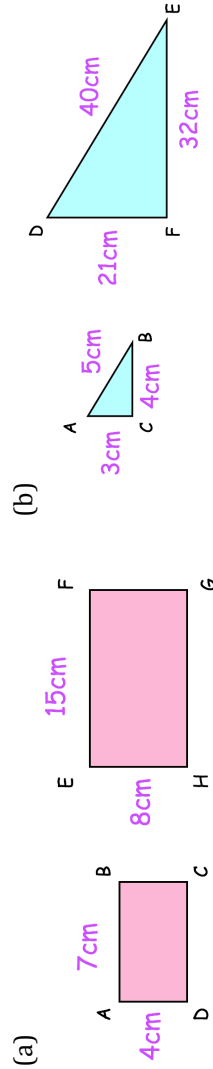


Fluency Practice

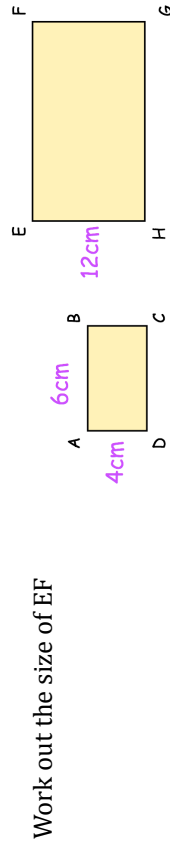
Question 1: Below are pairs of similar shapes.
Find the missing lengths.



Question 2: These pairs of shapes are **not** similar.
Explain why.

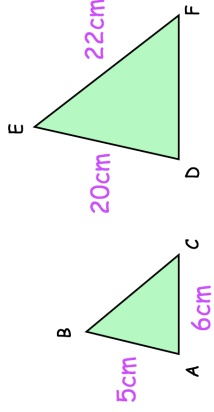


Question 3: Rectangles ABCD and EFGH are similar.



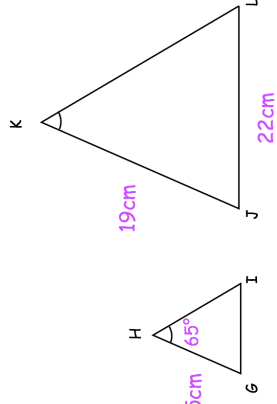
Fluency Practice

Question 4: Triangles ABC and DEF are similar.



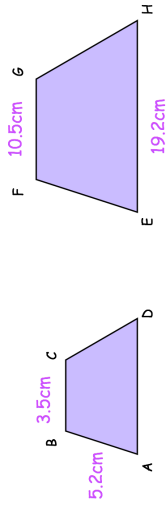
- (a) Work out the length of DF
- (b) Work out the length of BC

Question 5: Triangles GHI and JKL are similar.



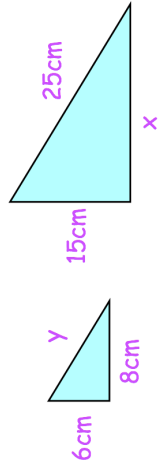
- (a) Write down the size of angle JKL
- (b) Work out the length of GI

Question 6: Trapezium ABCD and trapezium EFGH are similar.



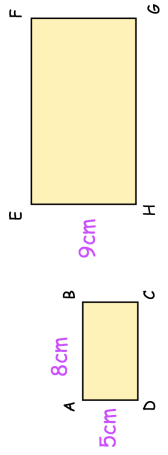
- (a) Work out the length of EF
- (b) Work out the length of AD

Question 7: The triangles below are similar



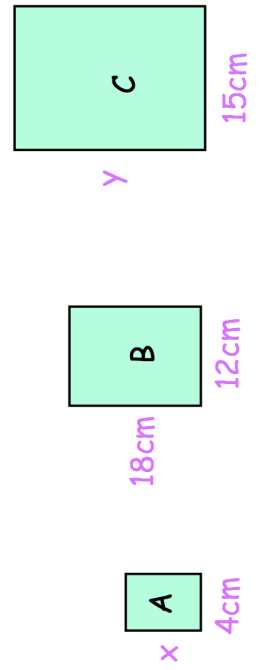
- (a) Find the size of x
- (b) Find the size of y

Question 8: Rectangles ABCD and EFGH are similar.



Work out the length of EF

Question 9: The diagram shows three similar rectangles.



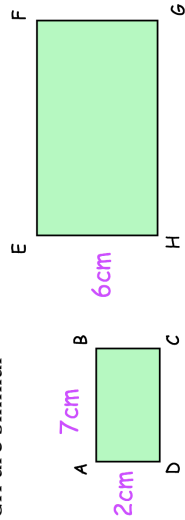
- (a) Work out the size of x
- (b) Work out the size of y

Fluency Practice

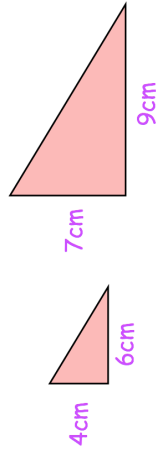
Apply

Question 1: Rectangles ABCD and EFGH are similar

Find the area of rectangle EFGH



Question 2: Here are two triangles



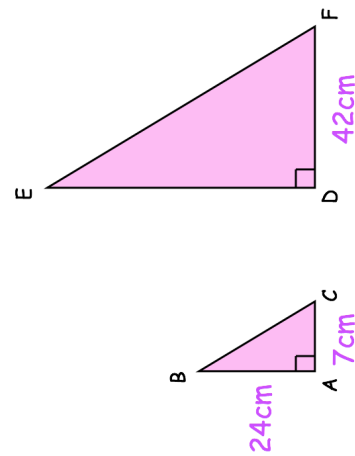
Finley says “the two triangles are similar because 3 cm has been added to both the height and base of the smaller triangle.”

Explain why Finley is incorrect.

Question 3: ABC and DEF are similar right angled triangles.

AB = 24cm AC = 7cm DF = 42cm

Work out the length of EF.



Fluency Practice

In each question, the 2 given shapes are similar. Work out the value of a .

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

(9)

(10)

(11)

(12)

(13)

(14)

(15)

(16)

(17)

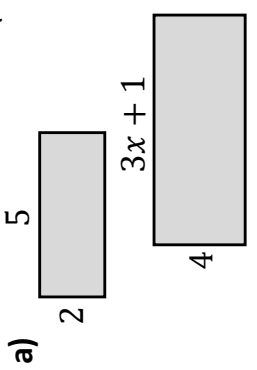
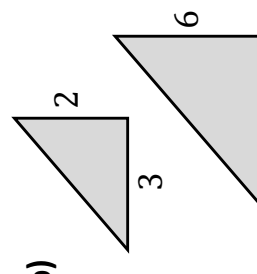
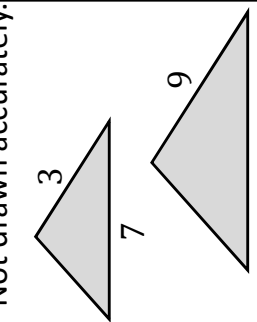
(18)

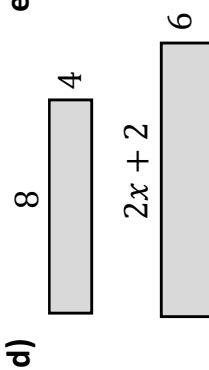
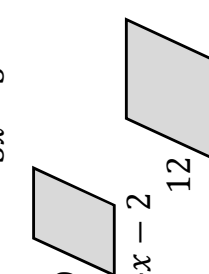
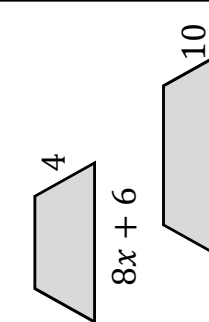
These ones require a little more thought...

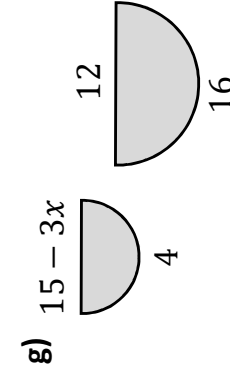
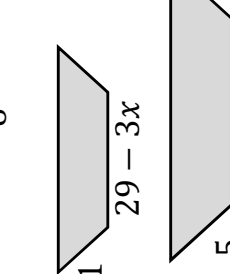
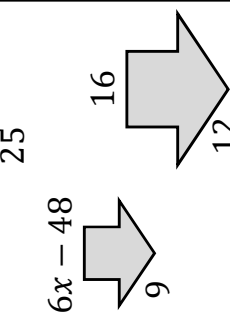
Fluency Practice

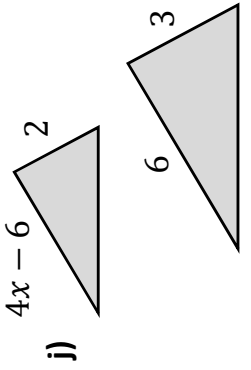
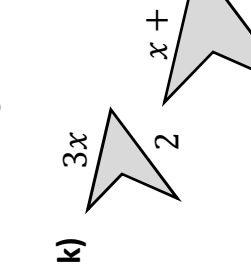
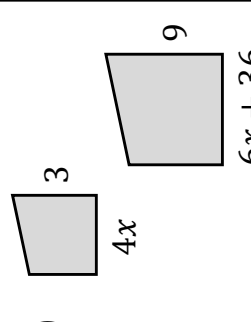
Similar Shapes

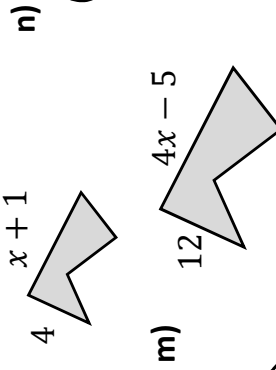
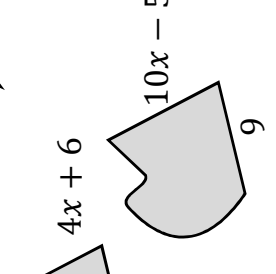
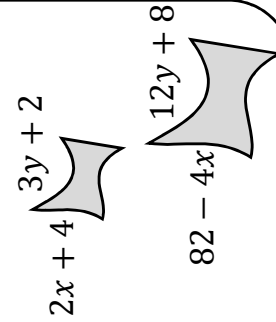
For each pair of shapes, form an equation and solve to find x .
(Find the scale factor first!) Not drawn accurately.

a)  **b)**  **c)** 

d)  **e)**  **f)** 

g)  **h)**  **i)** 

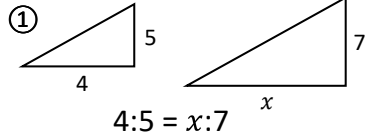
j)  **k)**  **l)** 

m)  **n)**  **o)** 

Fluency Practice

Finding Lengths of Similar Triangles by Comparing Ratios

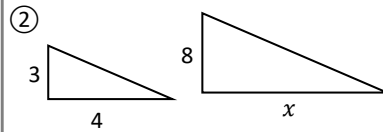
Pairs of triangles are similar and not to scale!



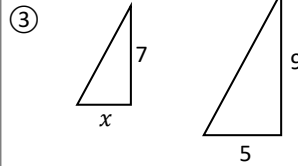
$$4:5 = x:7$$

$$\frac{4}{5} = \frac{x}{7}$$

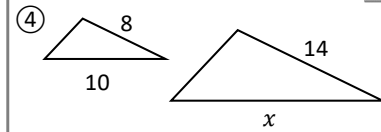
$$x =$$



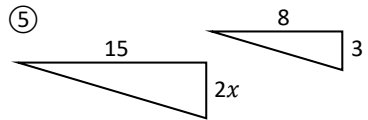
$$x =$$



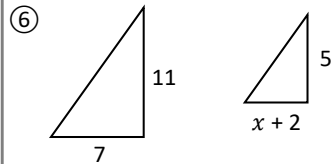
$$x =$$



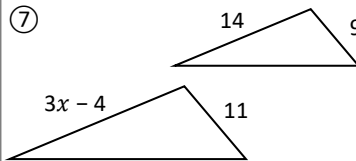
$$x =$$



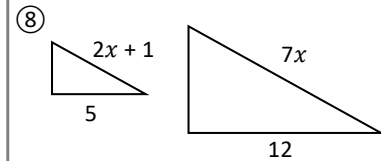
$$x =$$



$$x =$$



$$x =$$

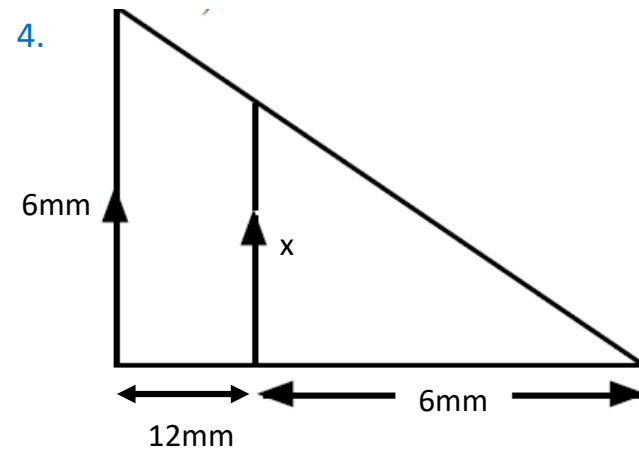
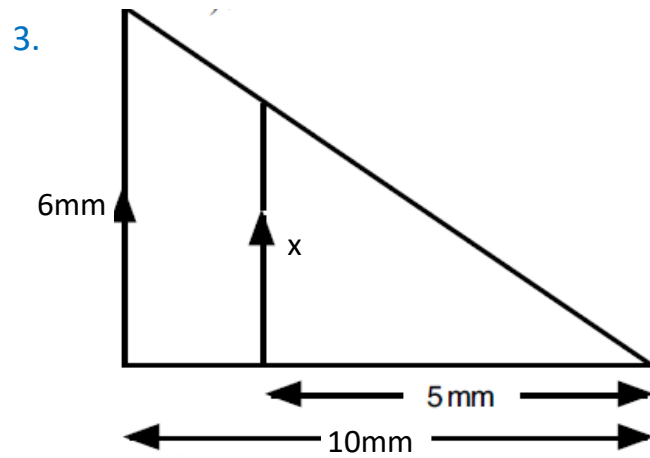
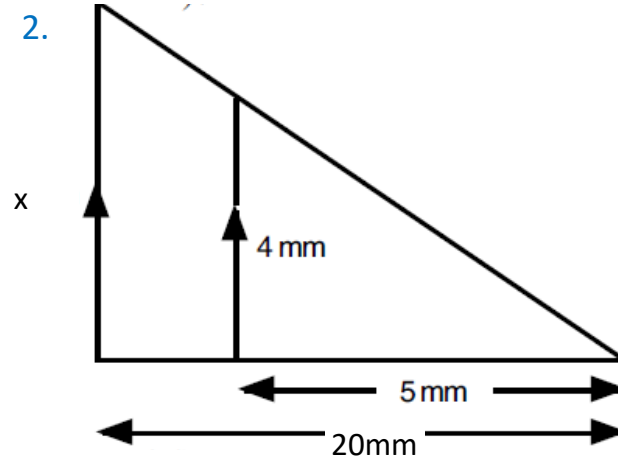
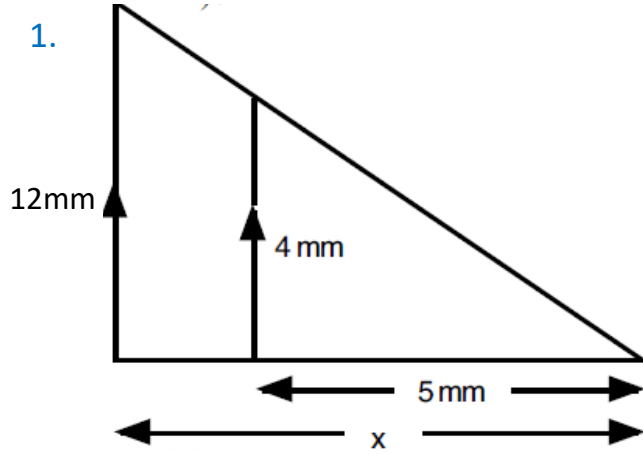


$$x =$$

Intelligent Practice

Intelligent Practice – Find the length of every missing side

Triangles not drawn to scale

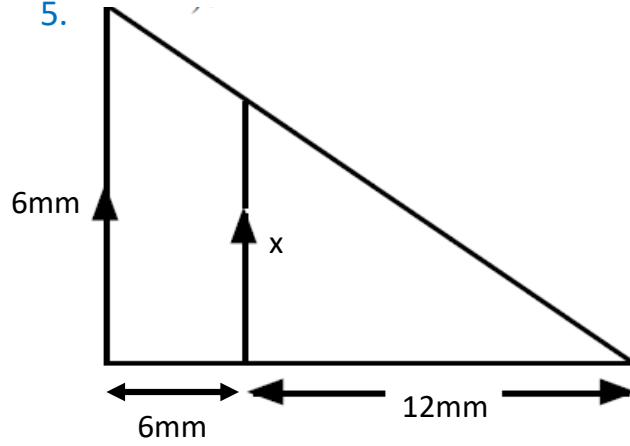


Intelligent Practice

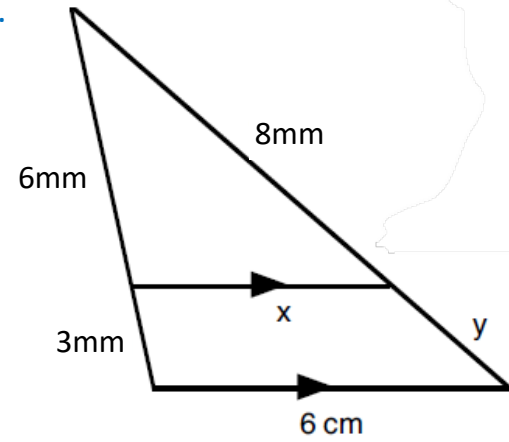
Intelligent Practice – Find the length of every missing side

Triangles not drawn to scale

5.



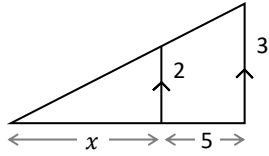
6.



Fluency Practice

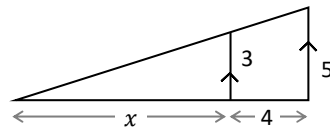
Finding Lengths of Similar Triangles using Equations

①



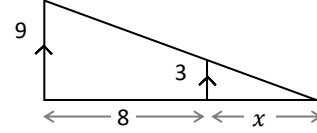
$x =$

②



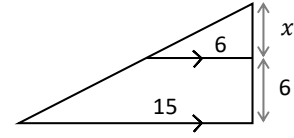
$x =$

③



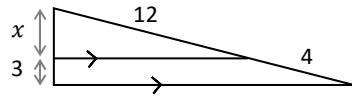
$x =$

④



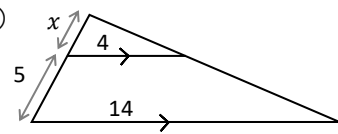
$x =$

⑤



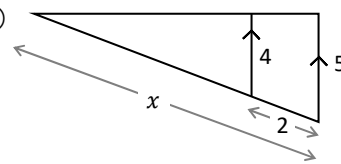
$x =$

⑥



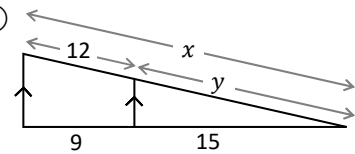
$x =$

⑦



$x =$

⑧

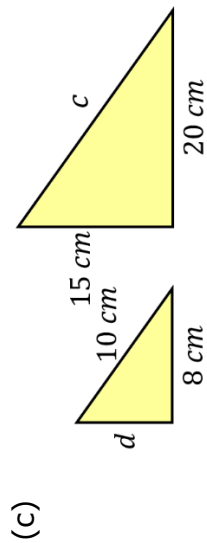
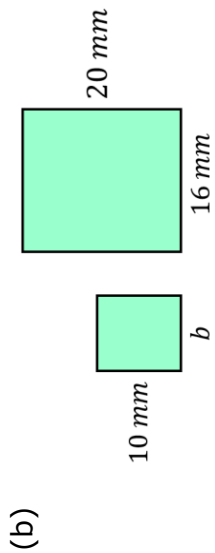
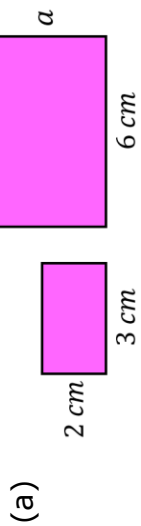


$x =$

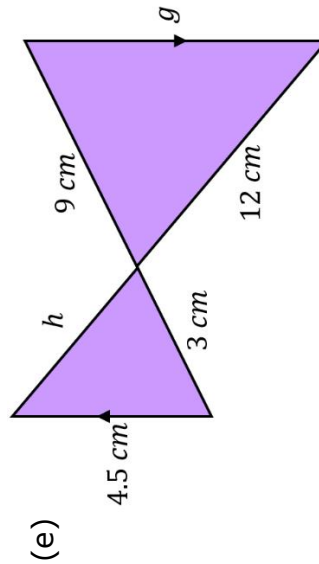
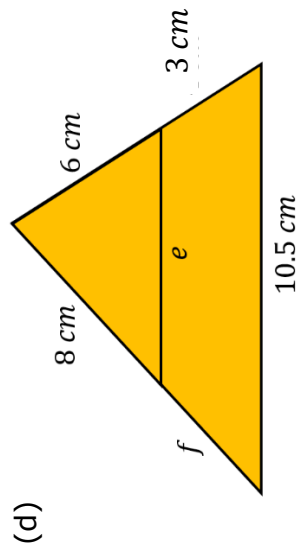
$y =$

Fluency Practice

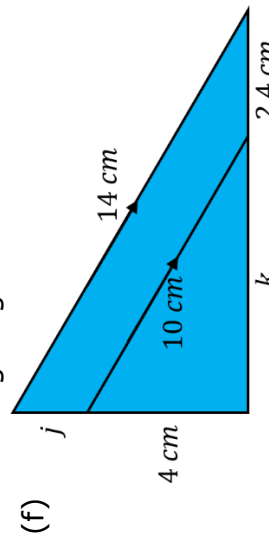
Find the missing lengths in these pairs of similar shapes.



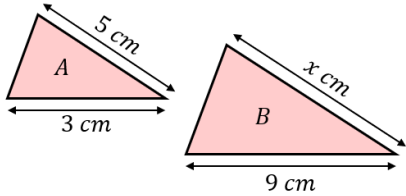
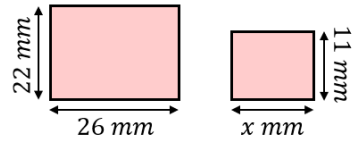
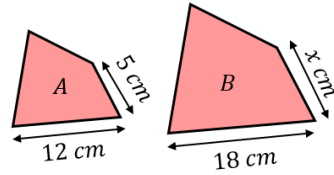
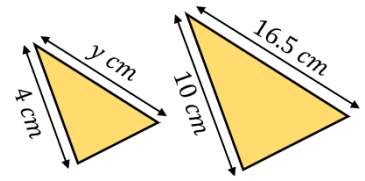
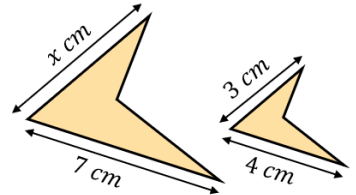
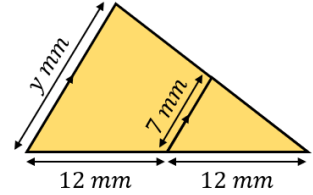
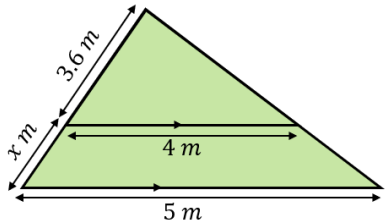
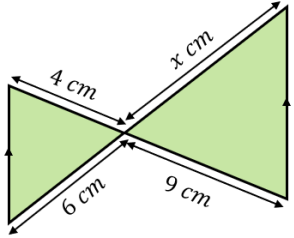
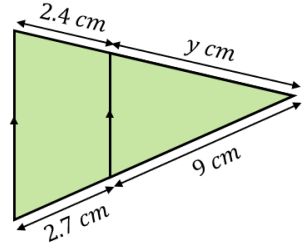
By drawing both shapes separately, find the missing lengths in these diagrams.



By drawing both shapes separately, find the missing lengths.



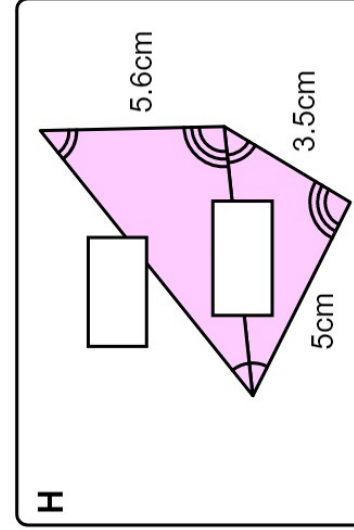
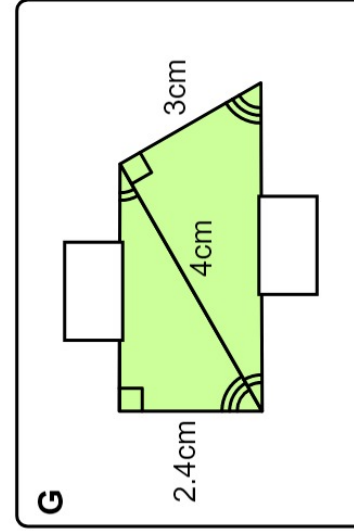
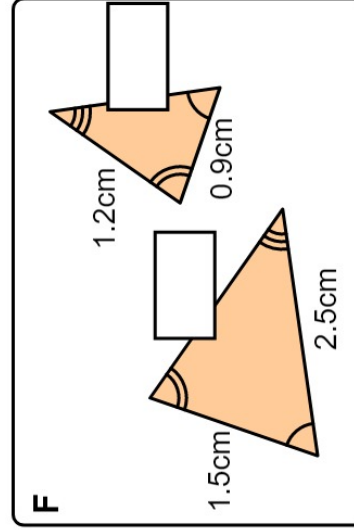
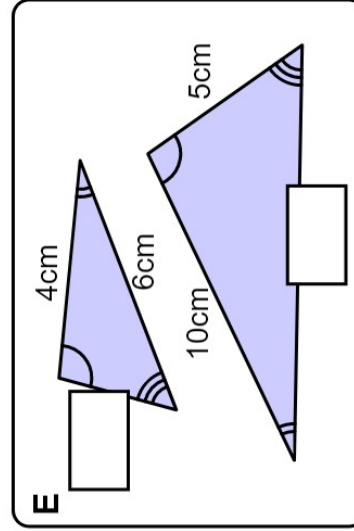
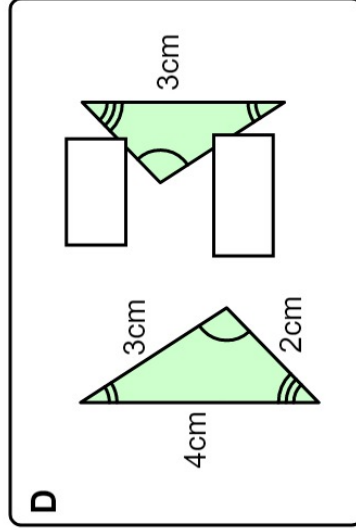
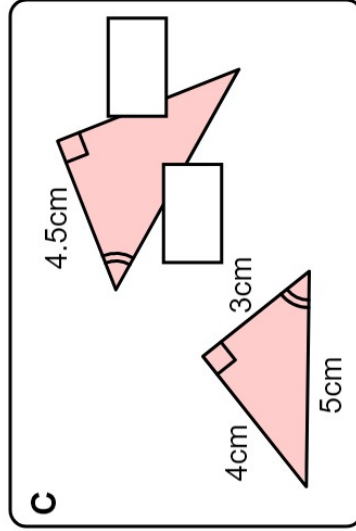
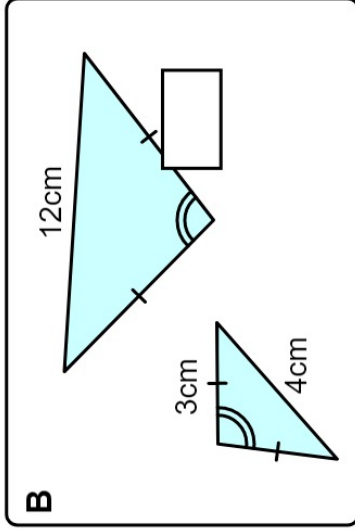
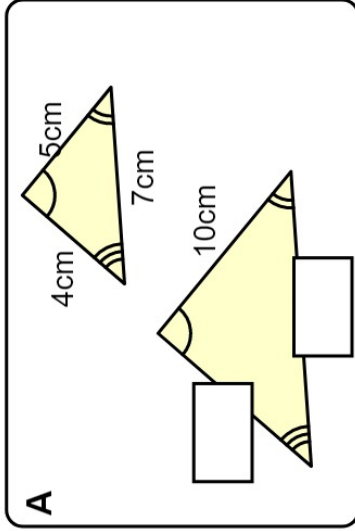
Fluency Practice

Similar Shapes		
<p>(a)</p> <p>Triangles <i>A</i> and <i>B</i> are similar. Find the missing length x.</p> 	<p>(b)</p> <p>The two rectangles shown are similar. Find the missing length x.</p> 	<p>(c)</p> <p>Shapes <i>A</i> and <i>B</i> are similar. Find the length x.</p> 
<p>(d)</p> <p>The two triangles shown are similar. Find the missing length y.</p> 	<p>(e)</p> <p>The two shapes shown are similar. Find the length x.</p> 	<p>(f)</p> <p>Find the missing length y cm.</p> 
<p>(g)</p> <p>Find the missing length x.</p> 	<p>(h)</p> <p>Find the missing length x in cm.</p> 	<p>(i)</p> <p>Find the missing length y.</p> 

Fluency Practice

similar triangles

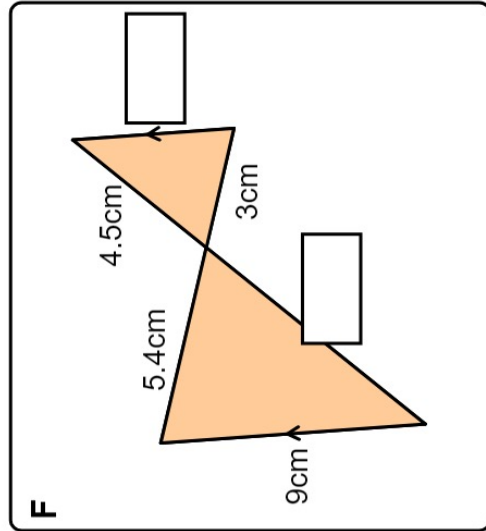
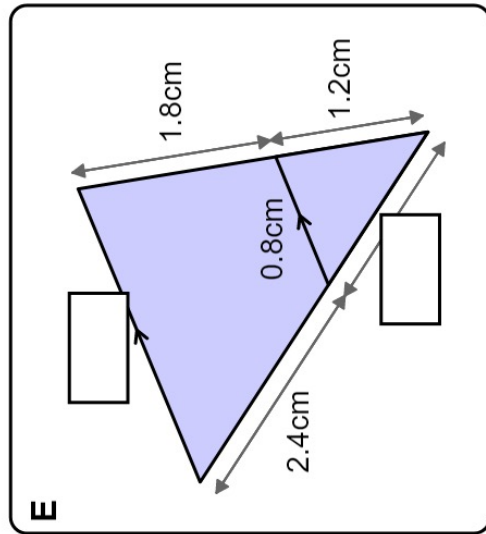
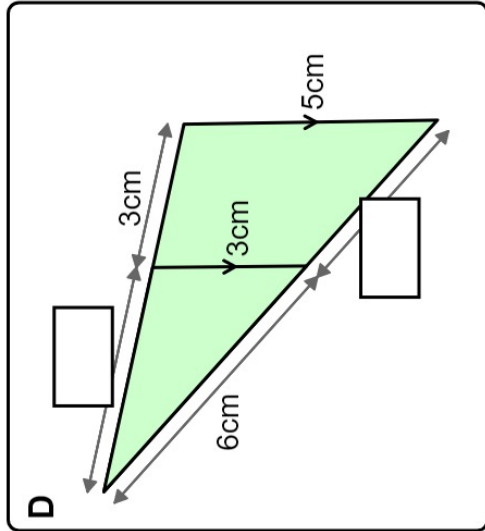
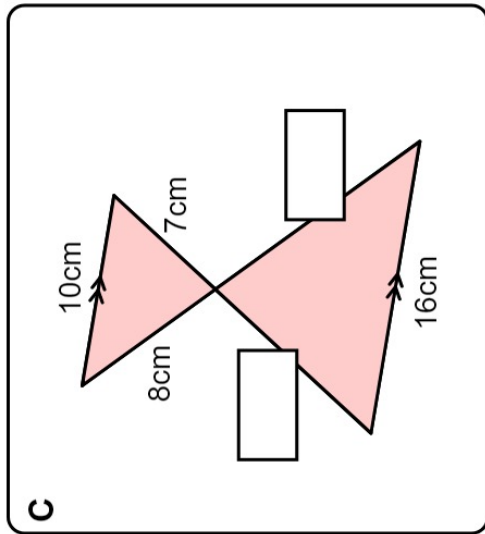
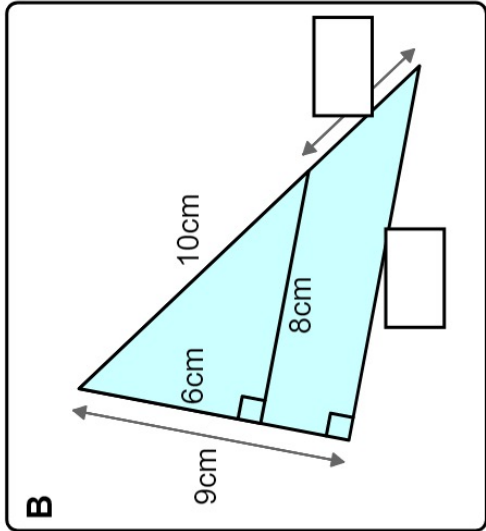
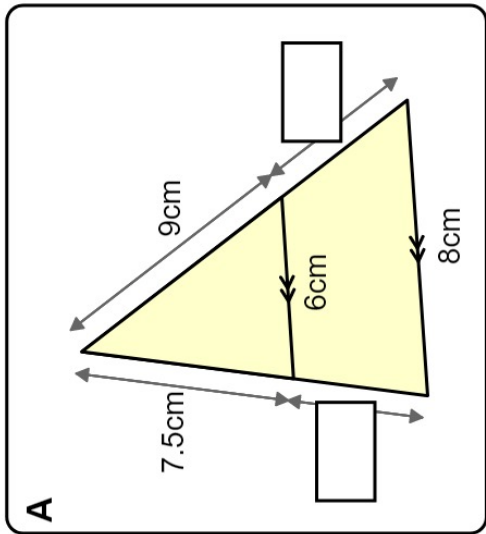
Work out the missing side lengths in each pair of similar triangles. Diagrams are not drawn accurately.



Fluency Practice

similar triangles & parallel lines

Work out the missing lengths.
Diagrams are not drawn accurately.



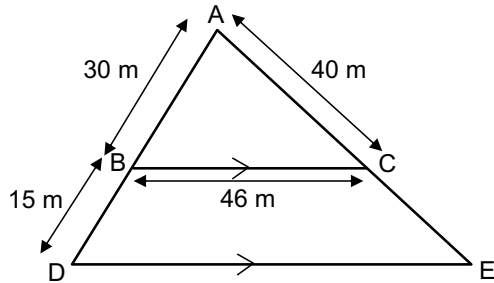
Fluency Practice

similarity questions

(1) BC is parallel to DE

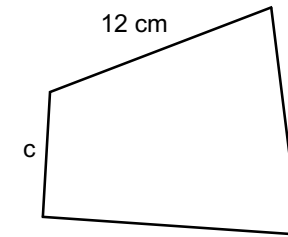
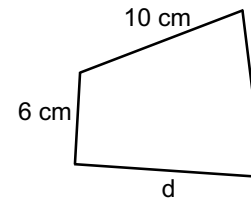
(a) the length DE

(b) the perimeter of the triangle ADE



(3)

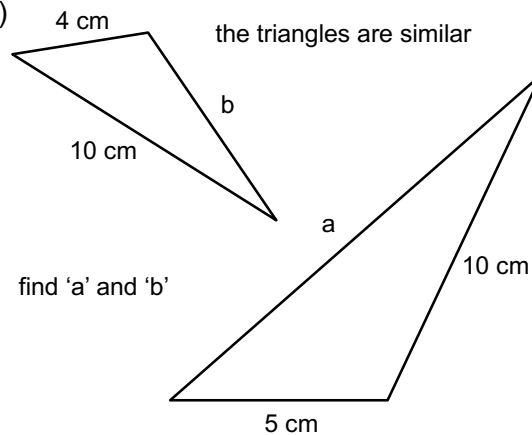
the quadrilaterals are similar



find 'c' and 'd'

(2)

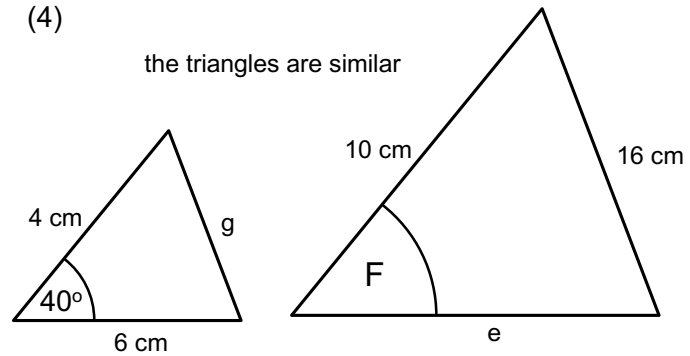
the triangles are similar



find 'a' and 'b'

(4)

the triangles are similar

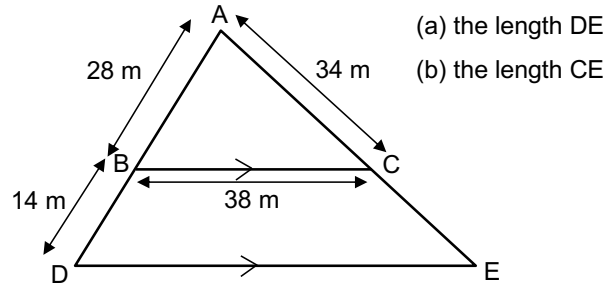


find 'e' and 'F' and 'g'

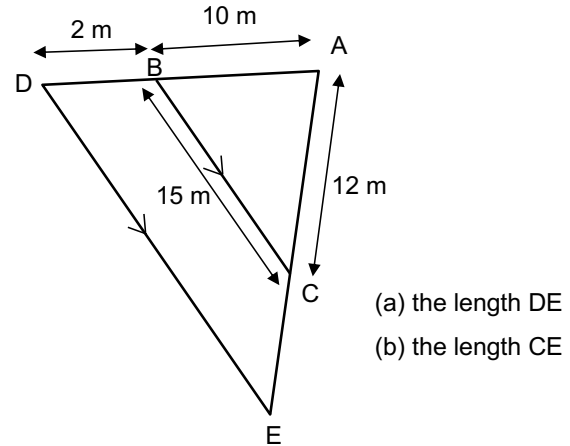
Fluency Practice

similar triangle questions

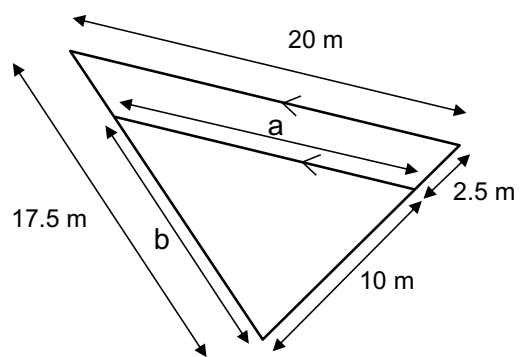
(1)



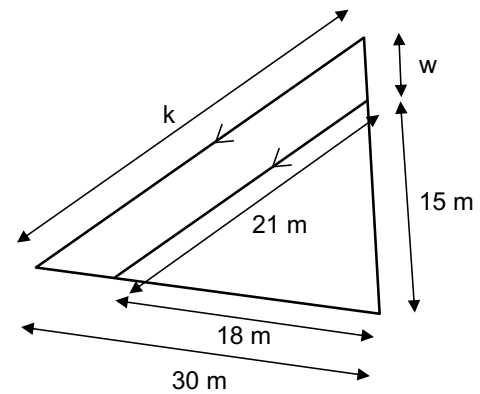
(2)



(3)



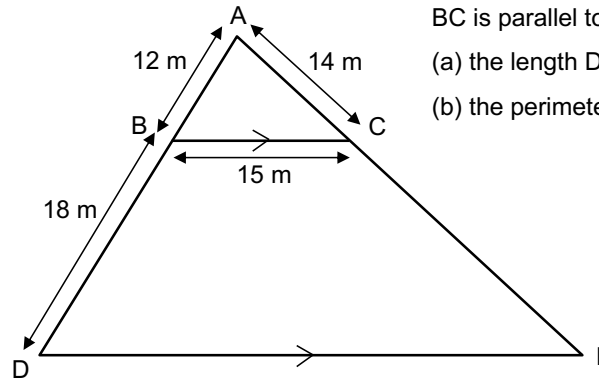
(4)



Fluency Practice

similar triangles

(6)



BC is parallel to DE, find:

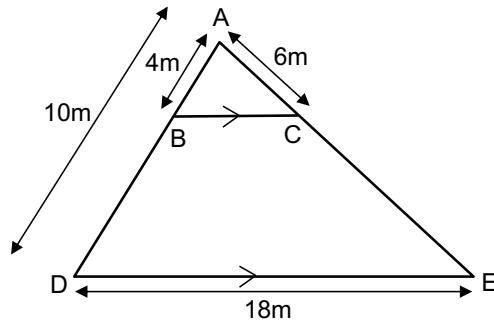
(a) the length DE

(b) the perimeter of the triangle ADE

(7) BC is parallel to DE

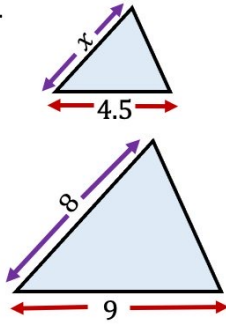
(a) the length CE

(b) the length BC

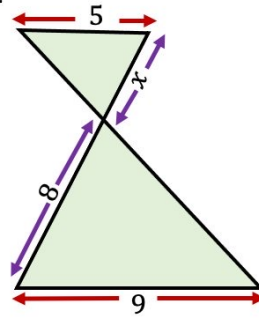


Fluency Practice

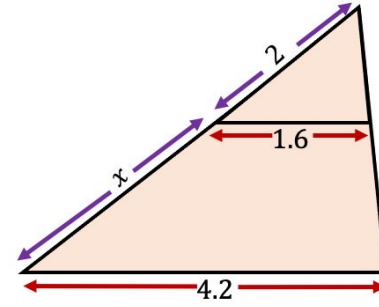
Q1



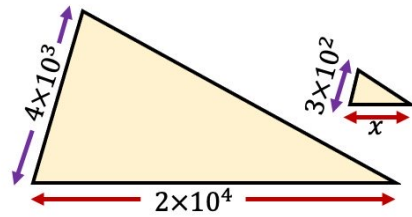
Q2



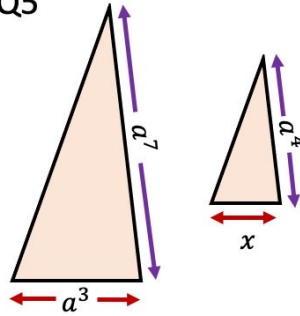
Q3



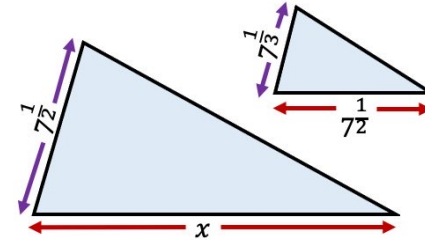
Q4



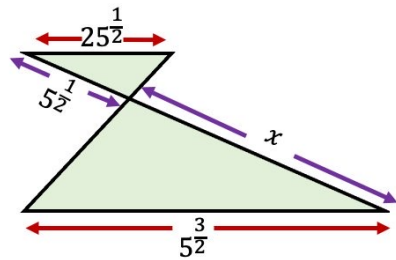
Q5



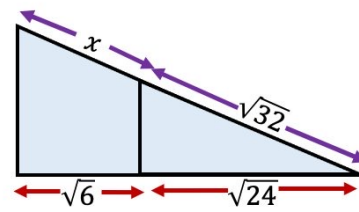
Q6



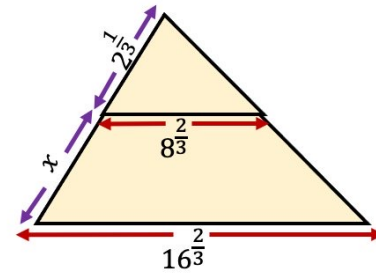
Q7



Q8

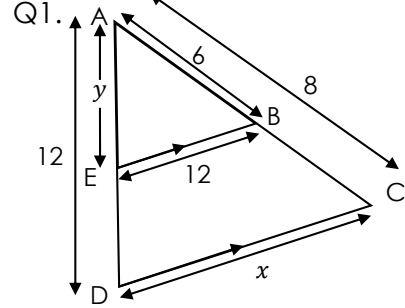


Q9



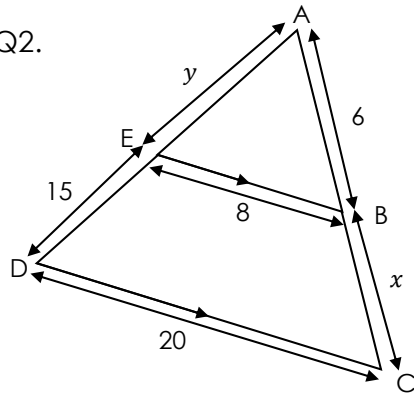
Fluency Practice

Similar Triangles

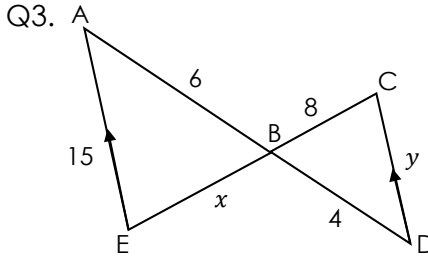


- i) Find the scale factor between triangle ABE and triangle ACD
- ii) Find the value of x
- iii) Find the value of y

Q2.

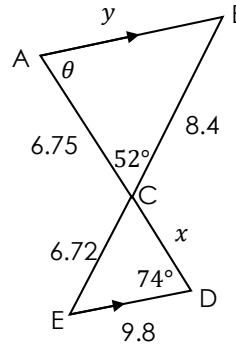


- i) Find the scale factor between ABE and ACD
- ii) Find the value of x
- iii) Find the value of y

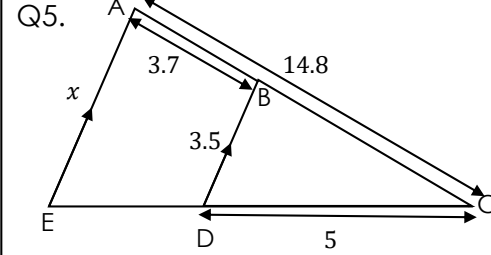


- i) How can you tell that ABE and CDE are mathematically similar?
- ii) Find the value of x
- iii) Find the value of y

Q4

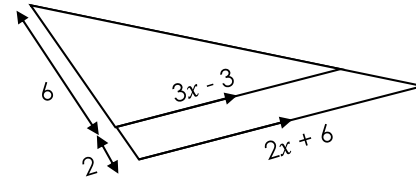


- i) Find the value of x
- ii) Find the value of y
- iii) Find the size of angle θ

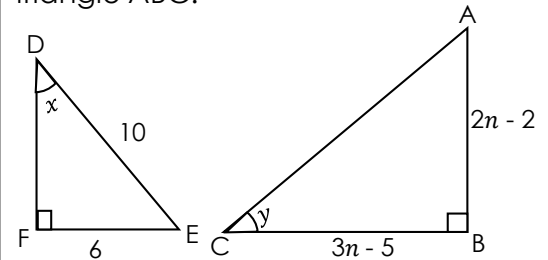


- i) Find the value of x
- ii) Hence, find the perimeter of trapezium ABDE

Q6. Find the value of x



Q7. [non-calculator] Given that $\tan(x) = \tan(y)$, find the area of triangle ABC.



Fluency Practice

Exam Questions around Similar shapes

Question 1

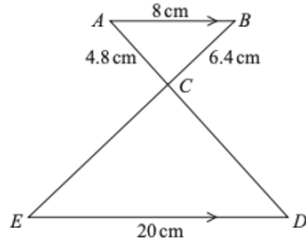


Diagram NOT accurately drawn

AB is parallel to ED .

ACD and BCE are straight lines.

$AB = 8\text{ cm}$

$AC = 4.8\text{ cm}$

$BC = 6.4\text{ cm}$

$ED = 20\text{ cm}$

Work out the length of BE .

..... cm (3 marks)

Question 3

The diagram represents a metal frame.

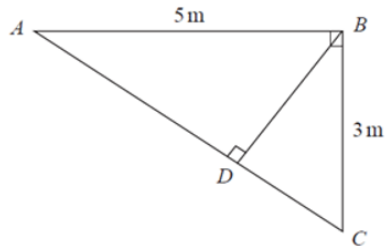


Diagram NOT accurately drawn

The frame is made from four metal bars, AB , AC , BC and BD .

Angle $ABC = \text{angle } ADB = 90^\circ$

$AB = 5\text{ m}$

$BC = 3\text{ m}$

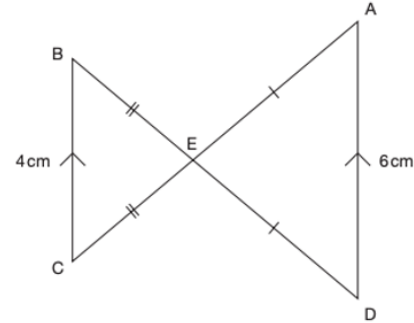
Work out the total length of the four metal bars of the frame.

Give your answer correct to 3 significant figures.

..... m (5 marks)

Question 2

The diagram shows five points joined with four straight lines. BC and AD are parallel. BCE and ADE are isosceles triangles.



Not drawn accurately

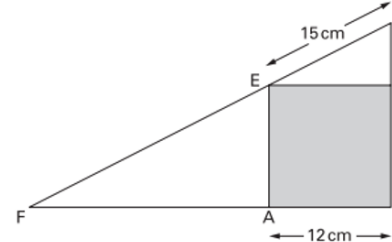
The total length of the four straight lines is 40 cm.

What is the length of EA ?

..... cm

Question 4

The diagram shows a square inside a triangle. DEF is a straight line.



Not drawn accurately

The side length of square $ABCE$ is 12 cm.

The length of DE is 15 cm.

Find the length of EF .

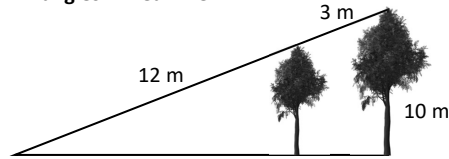
$EF = \dots\dots\dots\text{ cm}$

(3 marks)

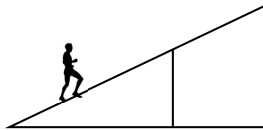
Fluency Practice

Similar Triangles in Real-Life

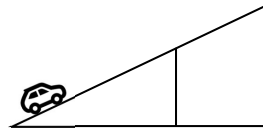
1) How tall is the smaller tree?



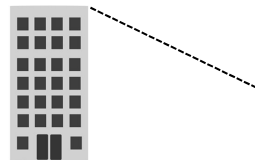
2) From a beach, Pablo is running up a hill. After running 300 metres he has gained 60 metres in height. At what height will Pablo be at after he runs another 500 metres? (Assume a constant gradient).



3) A car is travelling up an incline. After 5 minutes the car has travelled 3 km directly East and gained 500 metres in height. The top of the incline has a height of 900 m. How far East is the top from the base of the incline?

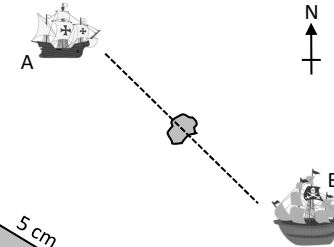


4) To estimate the height of a building, Ami walks 200 feet from its base and plants a 2 m tall stick. Ami lines up the top of the building and the top of the stick. At this point, Ami is 5 metres from the stick. Estimate the height of the building if Ami's eye level is the ground.

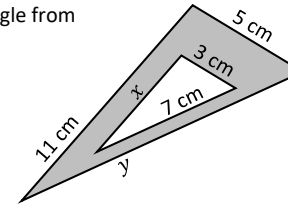


Extension) What if Ami's eye level is at 1.6 metres?

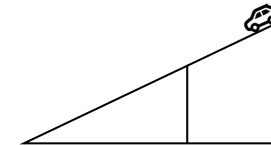
5) Ship A is 15 km from Treasure Island & 12 km North of the island. Ship B is 8 km South & 6 km East of the island. How far from the island is ship B? How far West of the island is ship A?



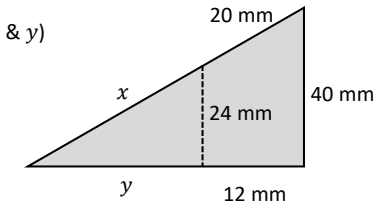
6) A metal logo is made from cutting a similar triangle from within a triangle. Find lengths x & y .



7) A car travels from the peak of an incline. After travelling 60 metres the car has lost 15% of its original height. How far will the car need to travel to reach the bottom of the hill?

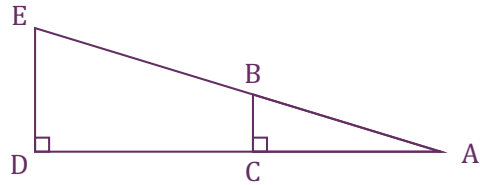


8) Jia wants to create a trapezium from a triangular piece of wood. What lengths (x & y) does she need to mark before cutting?



9) Rex is using distance-finding binoculars. The peaks of two mountains are in-line and 6 km & 8 km away. Rex knows the second mountain is 4.8 km high. He also knows the mountains are 1600 m apart. How tall is the smaller mountain? Horizontally, how far is the smaller mountain from Rex?

Pythagoras and Trigonometry with... Similar Shapes



AB	10 cm
BC	6 cm
AC	
Area of ABC	

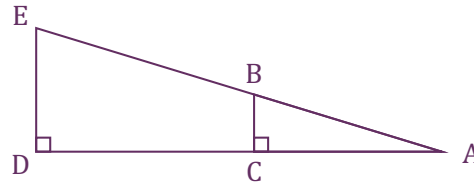
AE	25 cm
DE	
AD	
Area of ADE	

BE	
CD	
Area of BCDE	
Area of ABD	

BD	
CE	
Area of DCE	
Area of DBE	

Perimeter ABC	
Perimeter AED	
Perimeter BCDE	
Perimeter ABD	

Angle \widehat{BAC}	
Angle \widehat{BED}	
Angle \widehat{CBE}	
Angle \widehat{ABD}	



AB	
BC	7 cm
AC	
Area of ABC	

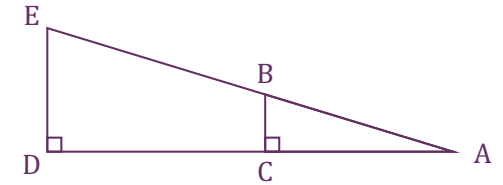
AE	
DE	
AD	
Area of ADE	

BE	
CD	24 cm
Area of BCDE	252 cm ²
Area of ABD	

BD	
CE	
Area of DCE	
Area of DBE	

Perimeter ABC	
Perimeter AED	
Perimeter BCDE	
Perimeter ABD	

Angle \widehat{BAC}	
Angle \widehat{BED}	
Angle \widehat{CBE}	
Angle \widehat{ABD}	



AB	
BC	
AC	
Area of ABC	

AE	68 cm
DE	
AD	60 cm
Area of ADE	

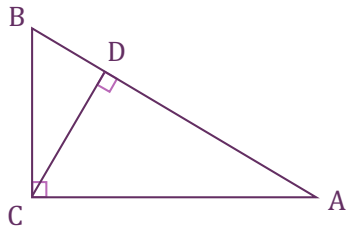
BE	
CD	
Area of BCDE	
Area of ABD	

BD	
CE	
Area of DCE	
Area of DBE	

Perimeter ABC	40 cm
Perimeter AED	
Perimeter BCDE	
Perimeter ABD	

Angle \widehat{BAC}	
Angle \widehat{BED}	
Angle \widehat{CBE}	
Angle \widehat{ABD}	

Pythagorean Areas with... Similar Shapes



Show that triangles ABC, ACD and CBD are similar.

BC	15 cm
CA	20 cm
AB	25 cm

AD	
DB	
CD	

Area of ABC	
Area of ACD	
Area of CBD	

Ratio of hypotenuses of each triangle	
---------------------------------------	--

Ratio of areas of each triangle	
---------------------------------	--

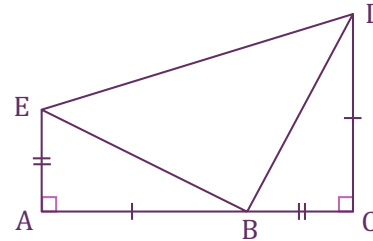
BC	a cm
CA	b cm
AB	c cm

AD	
DB	
CD	

Area of ABC	
Area of CAD	
Area of BCD	

Ratio of hypotenuses of each triangle	
---------------------------------------	--

Ratio of areas of each triangle	
---------------------------------	--



Show that angle \widehat{EBD} is a right angle.

EA	6 cm
AB	8 cm
BE	10 cm
Area of ABE	
Area of BCD	
Area of BDE	

Area of trapezium ACDE (two methods)	

EA	a cm
AB	b cm
BE	c cm
Area of ABE	
Area of BCD	
Area of BDE	

Area of trapezium ACDE (two methods)	

4 Right-Angled Trigonometry

Fluency Practice

Find the value of the following to 3 d.p..

- | | | | | |
|----------------------|----------------------|----------------------|----------------------|----------------------|
| 1). $\sin 10^\circ$ | 2). $\cos 45^\circ$ | 3). $\tan 45^\circ$ | 4). $\tan 62^\circ$ | 5). $\sin 14^\circ$ |
| 6). $\sin 69^\circ$ | 7). $\tan 14^\circ$ | 8). $\cos 32^\circ$ | 9). $\cos 5^\circ$ | 10). $\sin 85^\circ$ |
| 11). $\tan 68^\circ$ | 12). $\sin 55^\circ$ | 13). $\tan 4^\circ$ | 14). $\sin 15^\circ$ | 15). $\cos 75^\circ$ |
| 16). $\sin 90^\circ$ | 17). $\cos 90^\circ$ | 18). $\cos 12^\circ$ | 19). $\tan 78^\circ$ | 20). $\tan 9^\circ$ |

Calculate the following to 2 d.p..

- | | | | | |
|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| 1). $5 \tan 45^\circ$ | 2). $4 \sin 30^\circ$ | 3). $8 \cos 60^\circ$ | 4). $6 \sin 43^\circ$ | 5). $9 \cos 18^\circ$ |
| 6). $15 \tan 83^\circ$ | 7). $14 \cos 25^\circ$ | 8). $24 \cos 72^\circ$ | 9). $31 \sin 45^\circ$ | 10). $20 \cos 34^\circ$ |
| 11). $5 \cos 60^\circ$ | 12). $56 \sin 15^\circ$ | 13). $30 \tan 45^\circ$ | 14). $19 \sin 82^\circ$ | 15). $14 \tan 45^\circ$ |
| 16). $17 \tan 60^\circ$ | 17). $8 \cos 0^\circ$ | 18). $45 \tan 28^\circ$ | 19). $61 \sin 90^\circ$ | 20). $28 \tan 50^\circ$ |

Calculate the following to 2 d.p..

- | | | | | |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 1). $\frac{6}{\sin 34^\circ}$ | 2). $\frac{12}{\cos 83^\circ}$ | 3). $\frac{4}{\tan 16^\circ}$ | 4). $\frac{23}{\tan 45^\circ}$ | 5). $\frac{31}{\sin 30^\circ}$ |
| 6). $\frac{38}{\cos 18^\circ}$ | 7). $\frac{48}{\tan 80^\circ}$ | 8). $\frac{8}{\sin 54^\circ}$ | 9). $\frac{18}{\sin 15^\circ}$ | 10). $\frac{5}{\cos 51^\circ}$ |
| 11). $\frac{25}{\tan 52^\circ}$ | 12). $\frac{62}{\cos 71^\circ}$ | 13). $\frac{82}{\sin 68^\circ}$ | 14). $\frac{16}{\cos 8^\circ}$ | 15). $\frac{2}{\sin 12^\circ}$ |
| 16). $\frac{6}{\sin 75^\circ}$ | 17). $\frac{18}{\tan 45^\circ}$ | 18). $\frac{48}{\cos 50^\circ}$ | 19). $\frac{37}{\tan 12^\circ}$ | 20). $\frac{52}{\tan 84^\circ}$ |

Intelligent Practice

Find 'x'. Give your solution to 2 decimal places.

1) $\tan(30) = \frac{x}{2}$

2) $\tan(45) = \frac{x}{2}$

3) $\sin(45) = \frac{x}{2}$

4) $\sin(45) = \frac{x}{4}$

5) $\frac{x}{4} = \sin(45)$

6) $x \times \sin(45) = 4$

7) $x \times \sin(30) = 4$

8) $x \times \cos(30) = 4$

9) $x \times \cos(30) = 8$

10) $x \times \cos(31) = 8$

Find 'x'. Give your solution to 2 decimal places.

1) $\cos(30) = \frac{2}{x}$

2) $\cos(45) = \frac{2}{x}$

3) $\sin(45) = \frac{2}{x}$

4) $\sin(45) = \frac{4}{x}$

5) $\sin(45) = \frac{8}{x}$

6) $\tan(45) = \frac{8}{x}$

7) $\tan(45) = \frac{x}{8}$

8) $\cos(45) = \frac{x}{8}$

9) $\cos(45) = \frac{8}{x}$

10) $\frac{8}{x} = \cos(45)$

Fluency Practice

Q1. Rearrange to make c the subject.

a. $a = \frac{c}{b}$

b. $a = \frac{b}{c}$

c. $5 = \frac{c}{b}$

d. $20 = \frac{b}{c}$

e. $\sin A = \frac{c}{b}$

f. $\sin A = \frac{b}{c}$

g. $\sin 5 = \frac{c}{b}$

h. $\sin 20 = \frac{b}{c}$

i. $\cos A = \frac{c}{b}$

j. $\cos 28 = \frac{b}{c}$

k. $\tan A = \frac{b}{c}$

l. $\tan A = \frac{10}{c}$

Q2. Calculate a to 2dp.

a. $\sin 40 = \frac{a}{6}$

b. $\sin 31 = \frac{a}{8}$

c. $\cos 70 = \frac{20}{a}$

d. $\cos 46 = \frac{12a}{7}$

e. $\tan 20 = \frac{a}{27}$

f. $\tan 58 = \frac{67}{a}$

Q3. Calculate a to 3sf.

a. $\sin 36 = \frac{a}{9}$

b. $\sin 71 = \frac{a}{6}$

c. $\sin 29 = \frac{6}{a}$

d. $\sin 81 = \frac{75}{a}$

e. $\sin 205 = \frac{a}{11}$

f. $\cos 53 = \frac{29}{a}$

g. $\cos 101 = \frac{a}{61}$

h. $\tan 44 = \frac{a}{7}$

i. $\tan 18 = \frac{50}{c}$

Fluency Practice

Solve, giving your answers to 2 decimal places.

$$1) \sin(71) = \frac{13}{x}$$

$$2) \sin(71) = \frac{x}{13}$$

$$3) \cos(24) = \frac{9}{x}$$

$$4) \cos(24) = \frac{x}{9}$$

$$5) \tan(33) = \frac{9.4}{x}$$

$$6) \tan(33) = \frac{x}{9.4}$$

$$7) \sin(56) = \frac{x}{7.2}$$

$$8) \tan(7) = \frac{15}{x}$$

$$9) \cos(34) = \frac{8.9}{x}$$

$$10) \sin(19) = \frac{x}{56}$$

$$11) \tan(34.9) = \frac{56.5}{x}$$

$$12) \sin(71) = \frac{13}{x}$$

$$13) \tan(83) = \frac{x}{214}$$

$$14) \cos(53.2) = \frac{19.7}{x}$$

$$15) \sin(8.8) = \frac{140}{x}$$

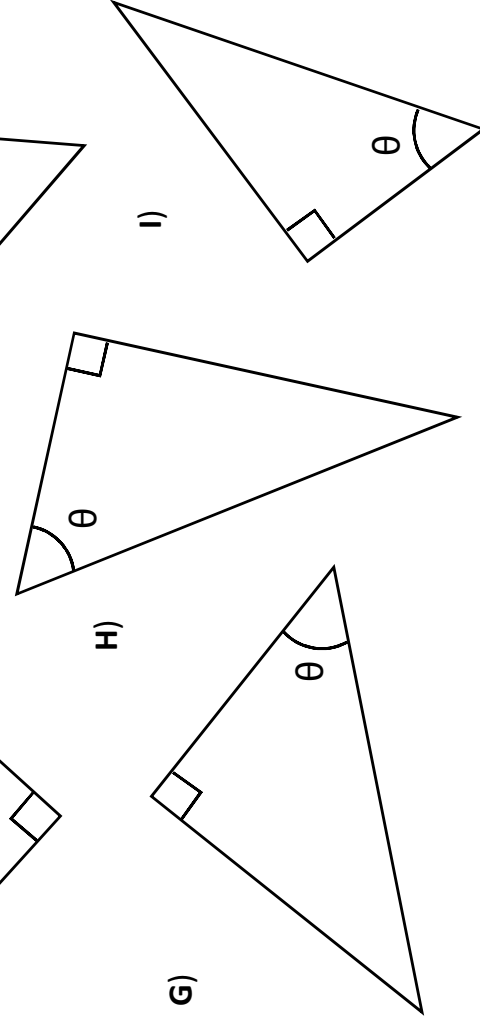
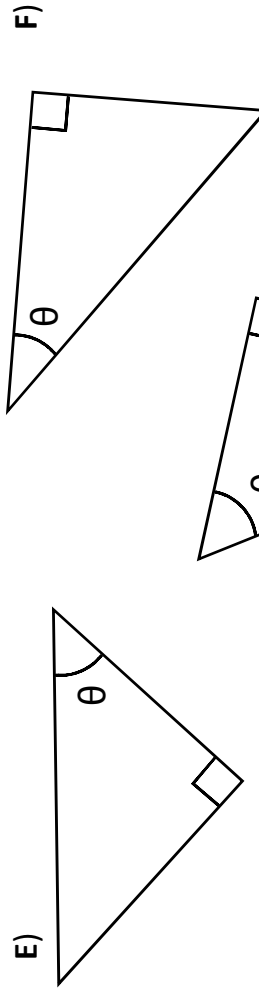
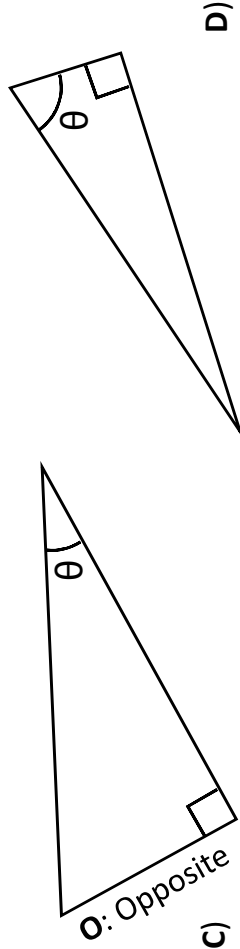
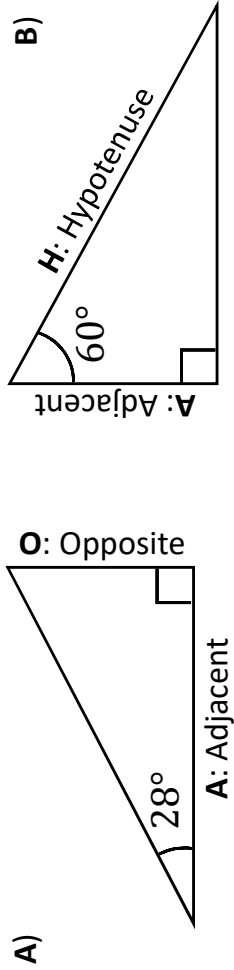
$$16) \cos(28.9) = \frac{x}{98.1}$$

Fluency Practice

Trigonometry: Labelling Right-Angled Triangles

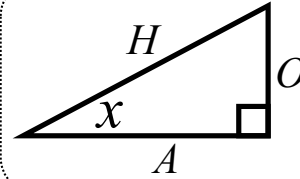
For each triangle, label each side with a letter:

- H:** Hypotenuse (the longest side)
- O:** Opposite (opposite the labelled angle)
- A:** Adjacent (next to the labelled angle)



Fluency Practice

Identifying Hypotenuse, Opposite and Adjacent



H = hypotenuse
O = opposite
A = adjacent

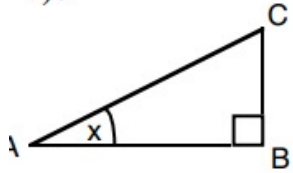
Relative to the angle marked x , label the sides as H, O and A

1. <p>A right-angled triangle with a right angle symbol at the bottom-left vertex. An angle of x is marked at the bottom-right vertex.</p>	2. <p>A right-angled triangle with a right angle symbol at the bottom-left vertex. An angle of x is marked at the top-left vertex.</p>	3. <p>A right-angled triangle with a right angle symbol at the top vertex. An angle of x is marked at the bottom-left vertex.</p>
4. <p>A right-angled triangle with a right angle symbol at the top vertex. An angle of x is marked at the top-right vertex.</p>	5. <p>A right-angled triangle with a right angle symbol at the bottom vertex. An angle of x is marked at the top-right vertex.</p>	6. <p>A right-angled triangle with a right angle symbol at the bottom vertex. An angle of x is marked at the top-left vertex.</p>

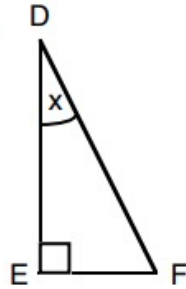
Fluency Practice

A. Name all the sides from the given angle, x° .

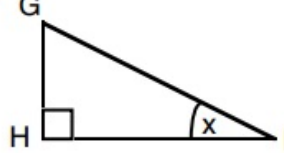
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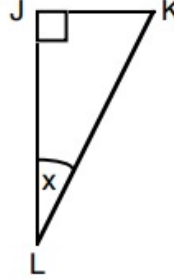
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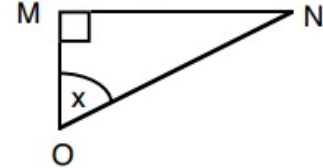
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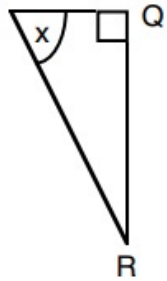
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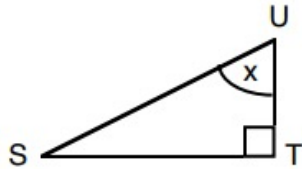
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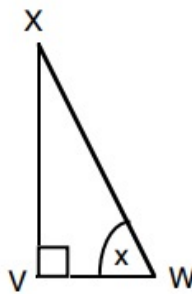
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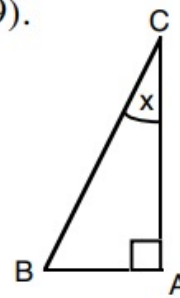
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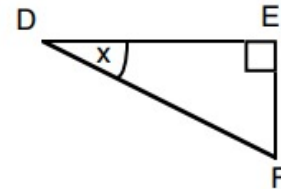
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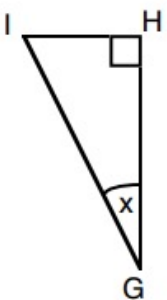
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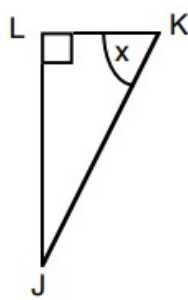
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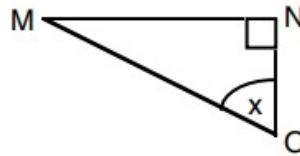
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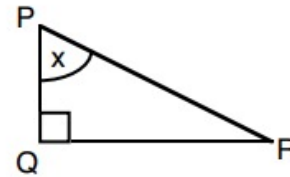
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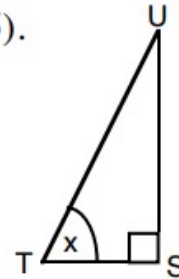
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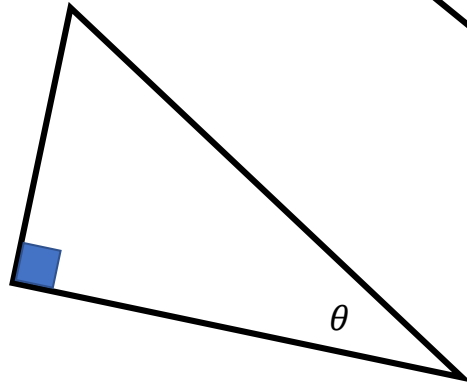
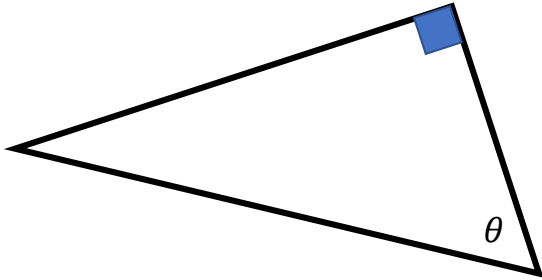
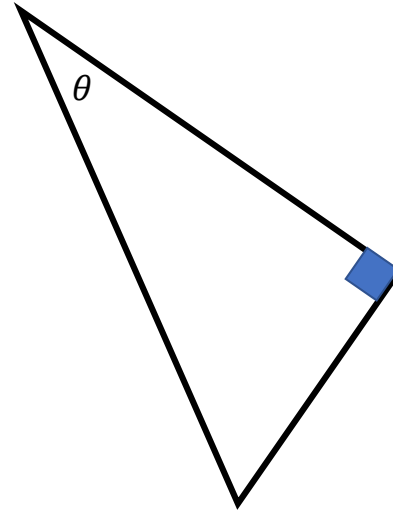
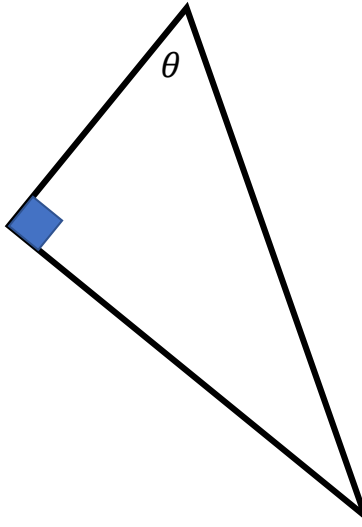
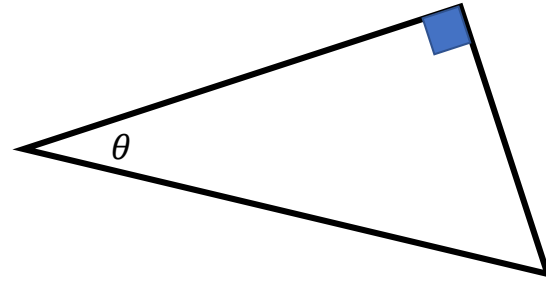
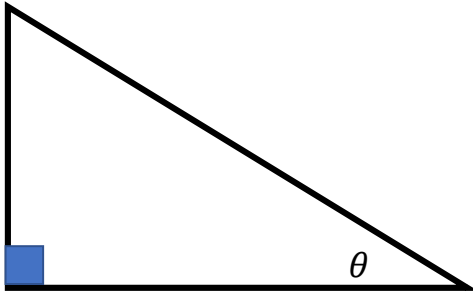


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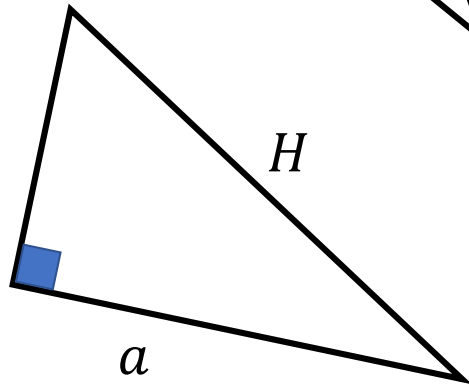
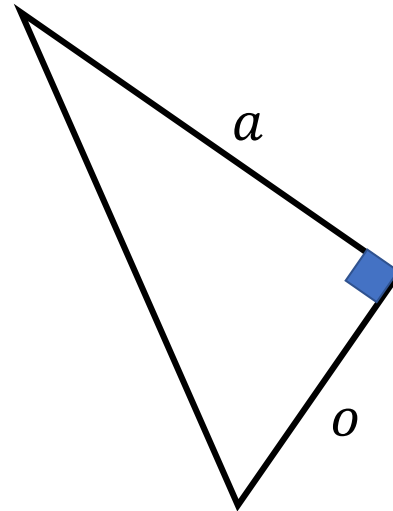
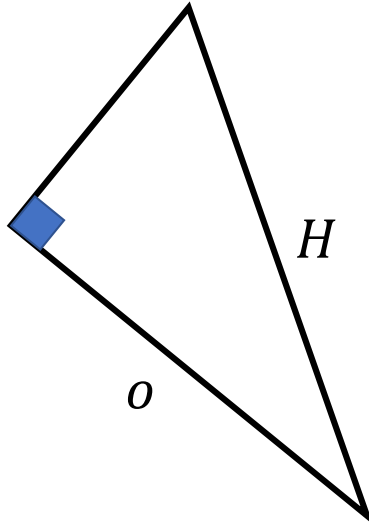
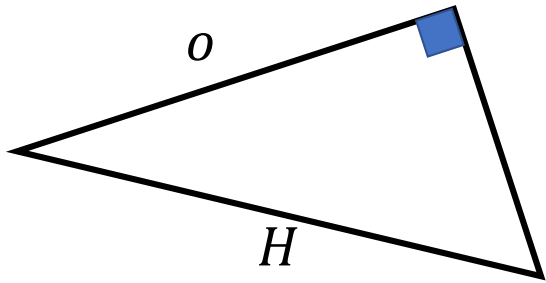
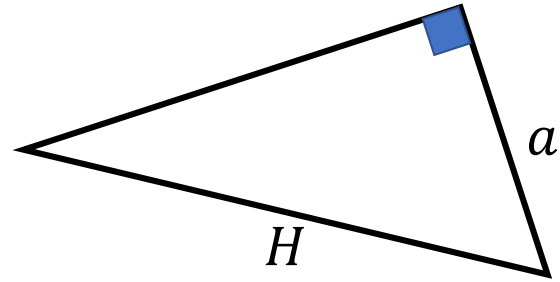
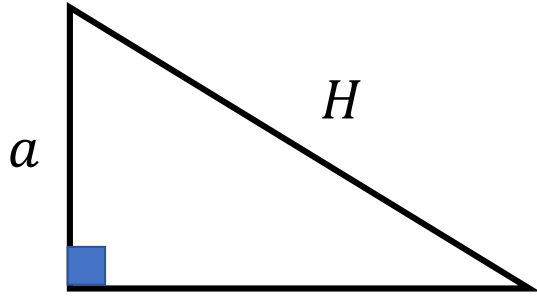
Fluency Practice

Label the triangles with hypotenuse, opposite and adjacent.



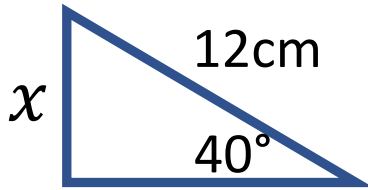
Fluency Practice

Highlight the relevant angle based on the labels

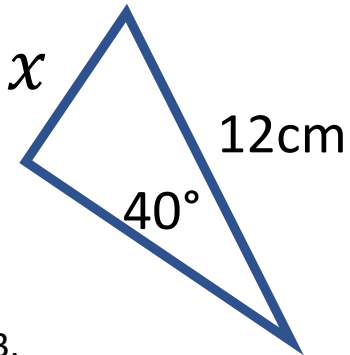


Fluency Practice

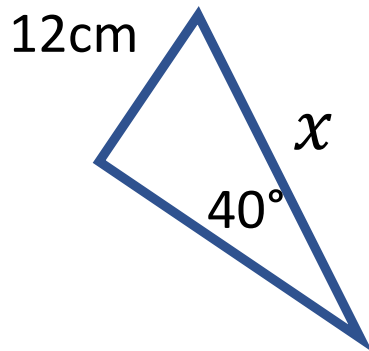
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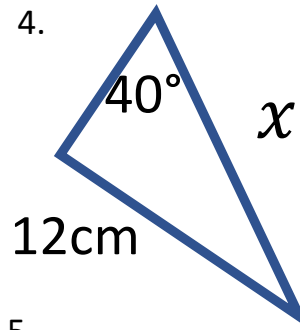
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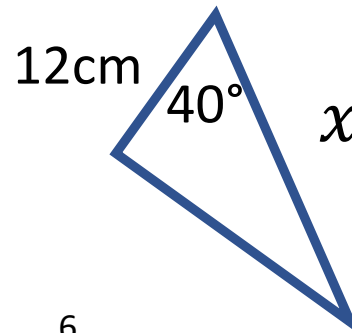
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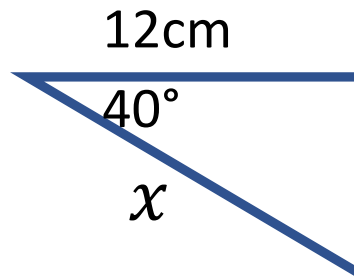
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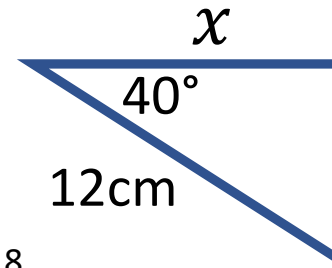
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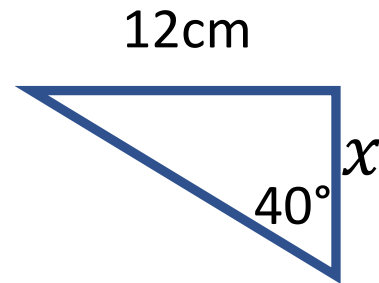
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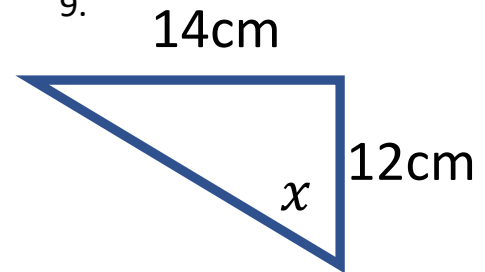
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8.



9.

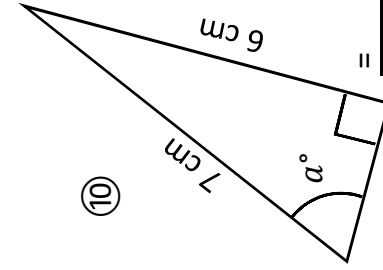
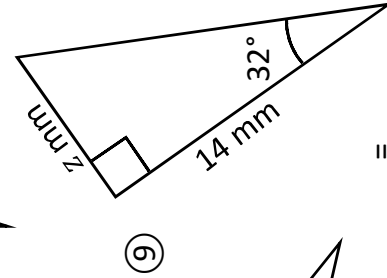
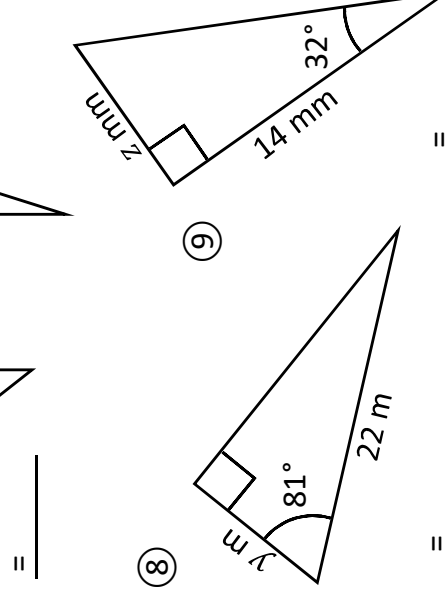
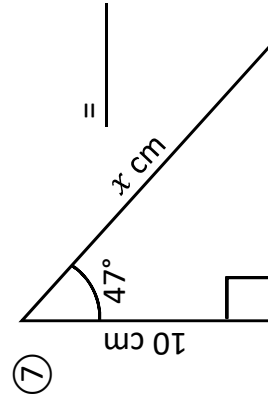
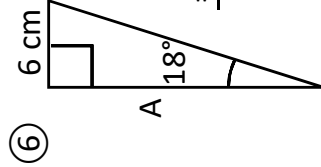
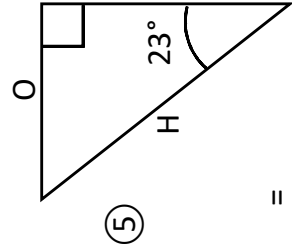
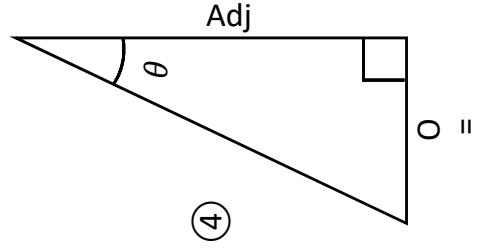
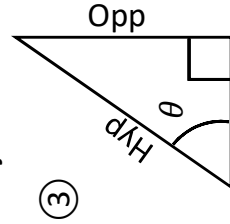
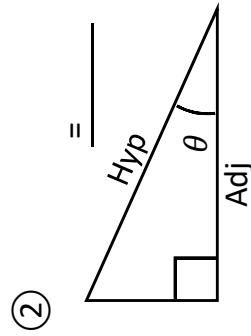
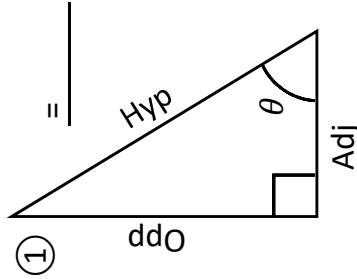
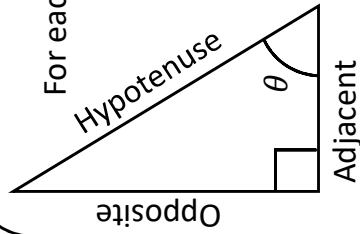


Fluency Practice

Choosing a Trigonometric Ratio to Use

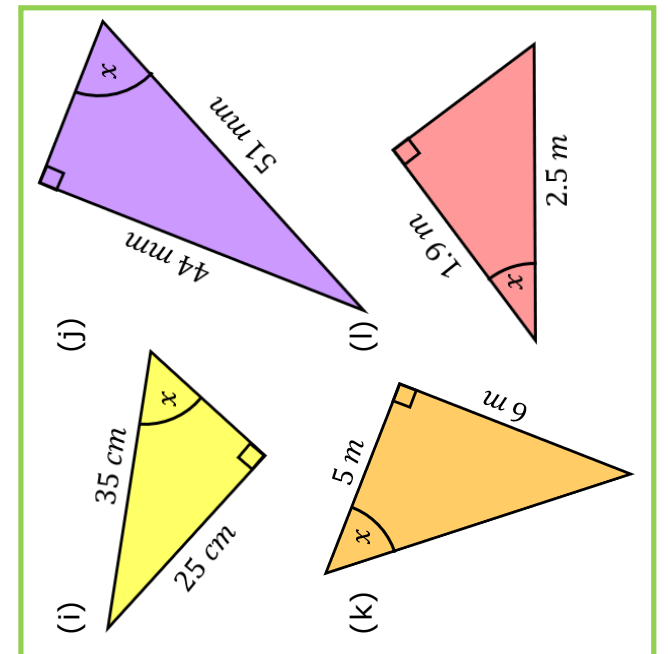
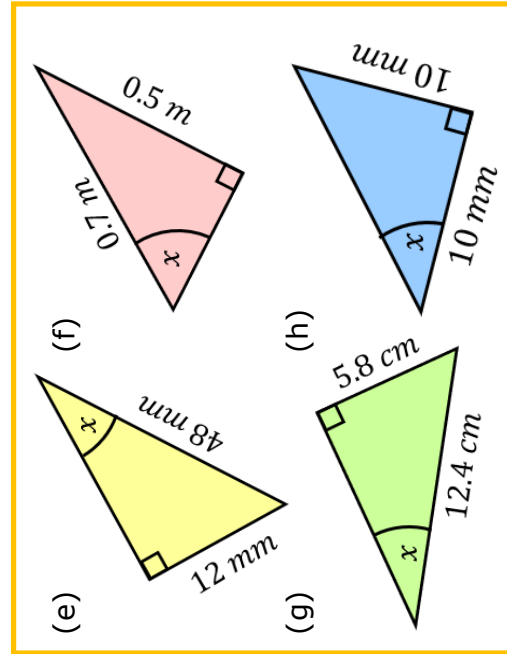
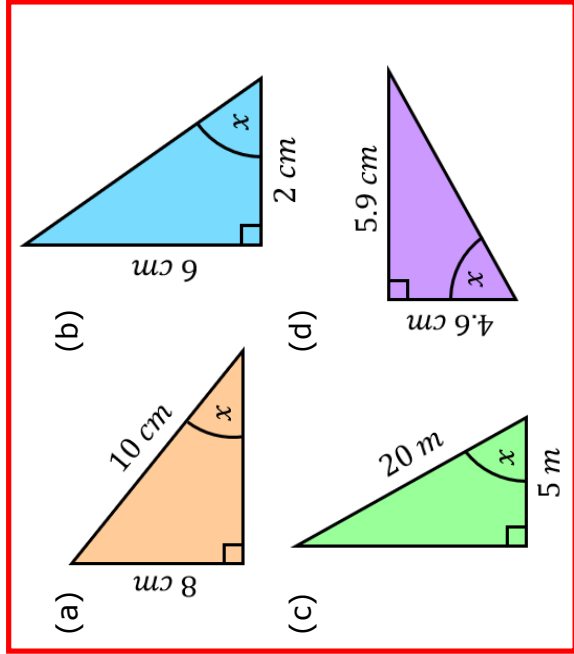
SOH
CAH
 or
TOA

For each triangle, decide whether you would use...



Fluency Practice

Label each of the triangles with opposite (O), adjacent (A) and hypotenuse (H). Use this to decide which ratio to use – sin (SOH), cos (CAH) or tan (TOA).

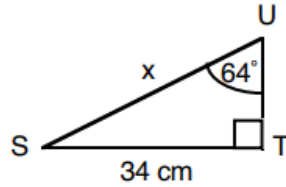


Fluency Practice

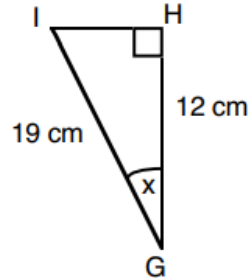
For each of the following questions look at the information given and the information you have to find. Which of the trigometrical ratios would you use to solve it for x ?

Do not try to solve the questions.

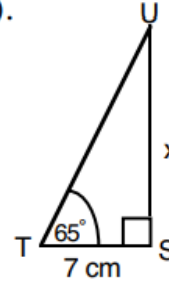
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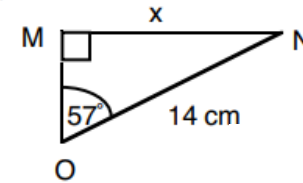
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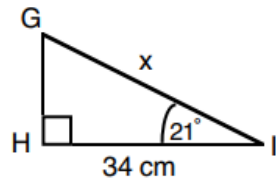
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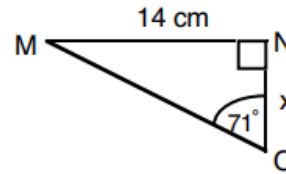
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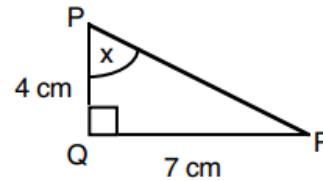
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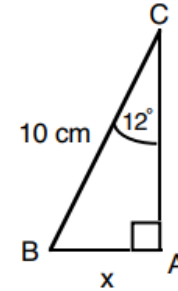
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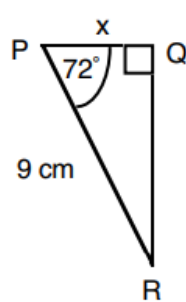
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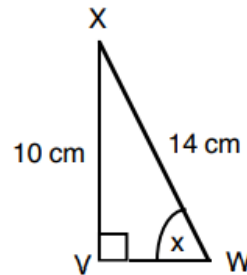
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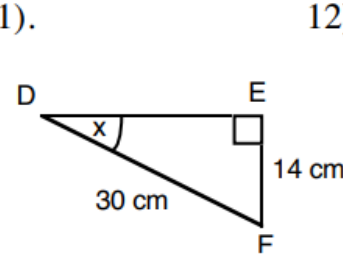
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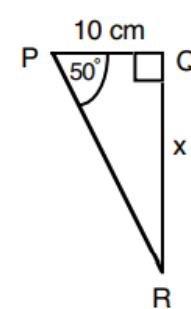
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11).

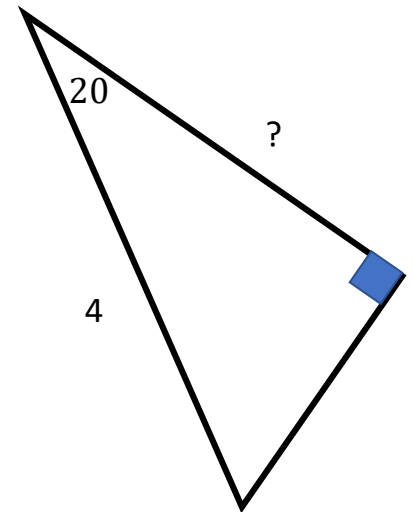
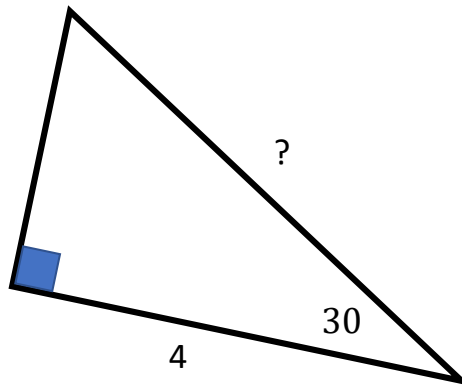
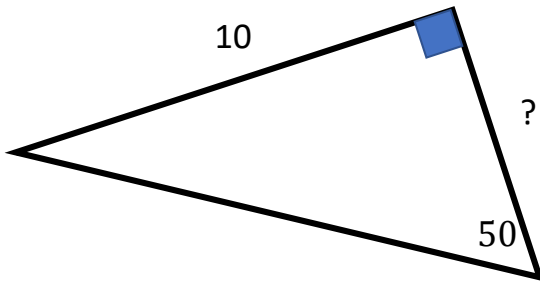
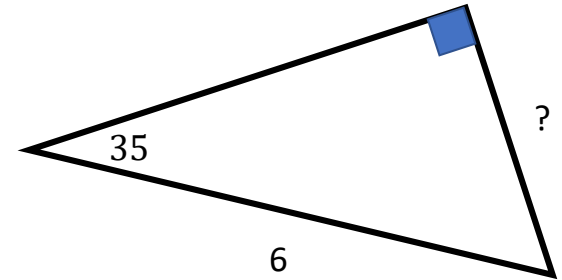
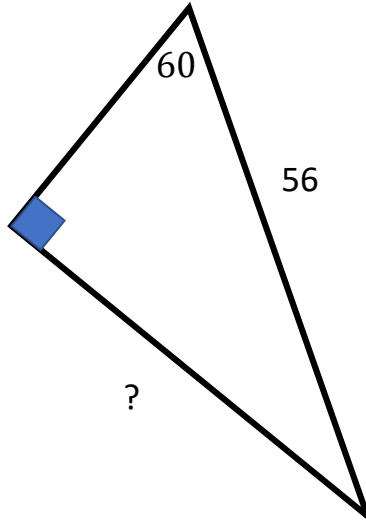
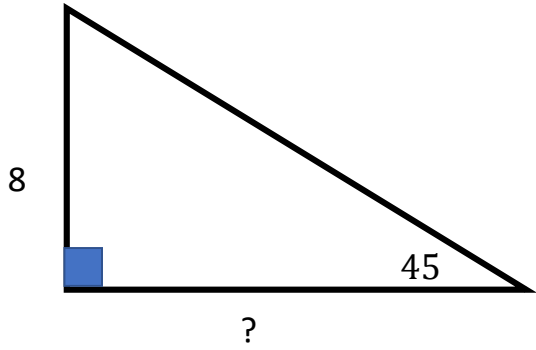


12).



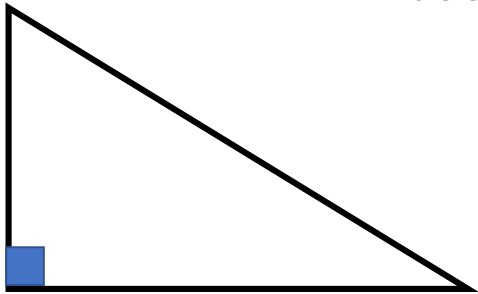
Fluency Practice

Set up your equation

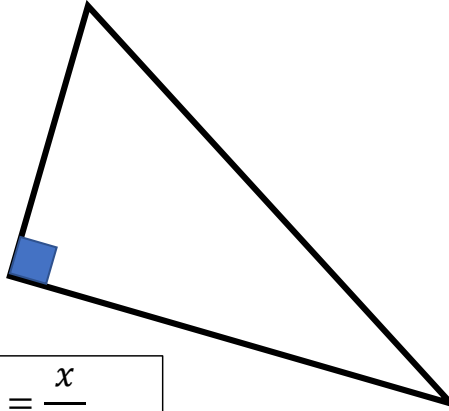


Fluency Practice

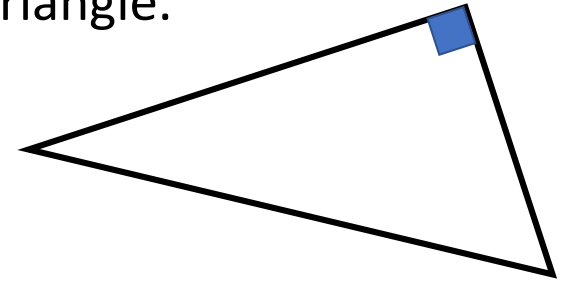
Based on the equation – label the triangle.



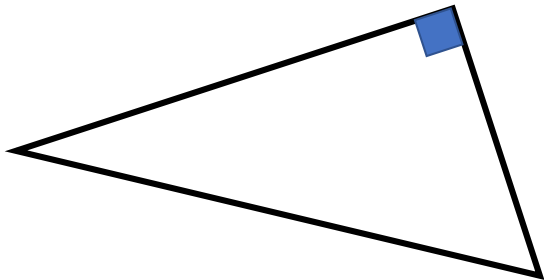
$$\sin 40 = \frac{x}{12}$$



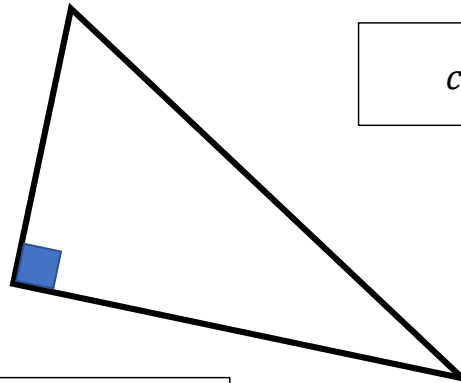
$$\cos 40 = \frac{x}{12}$$



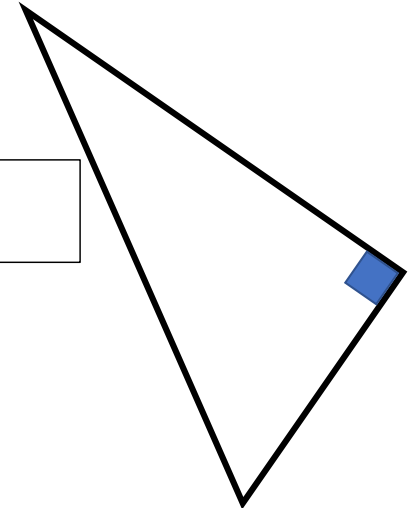
$$\tan 40 = \frac{x}{12}$$



$$\tan 20 = \frac{4}{x}$$



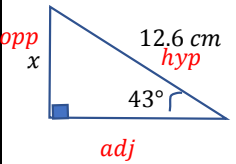
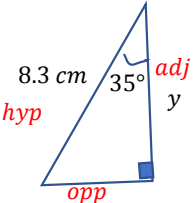
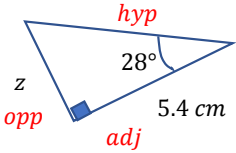
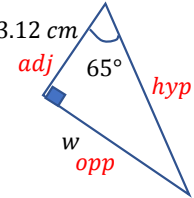
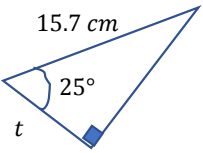
$$\sin 30 = \frac{4}{x}$$



$$\cos 35 = \frac{5}{x}$$

Fill in the Gaps

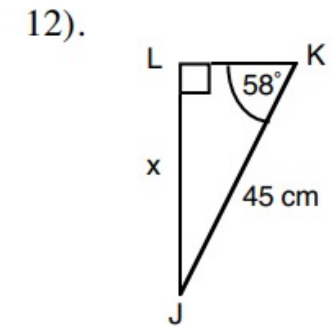
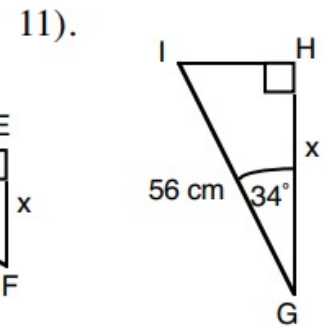
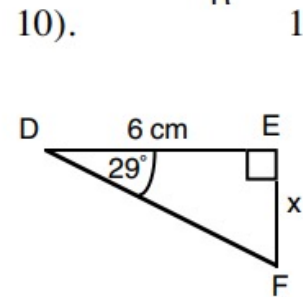
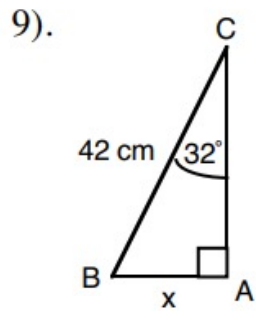
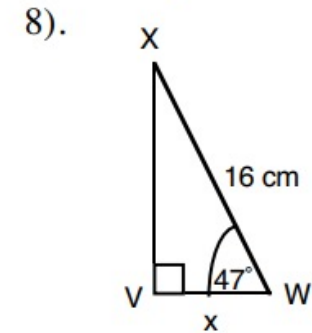
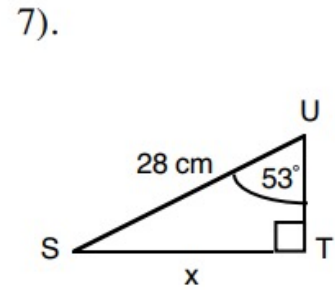
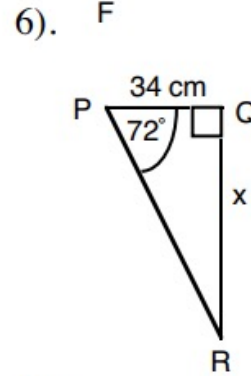
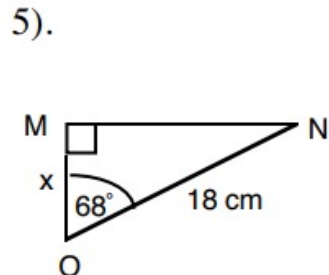
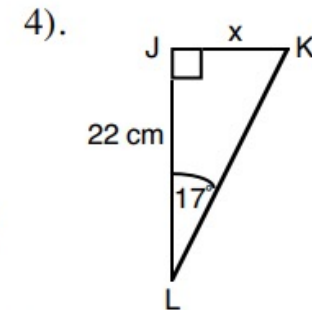
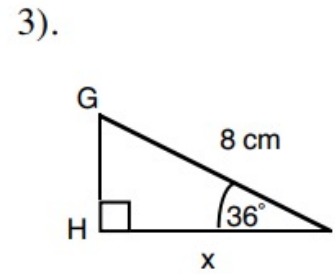
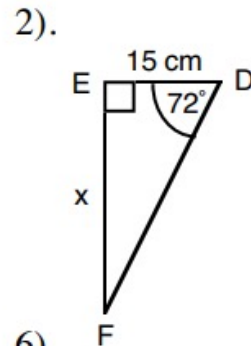
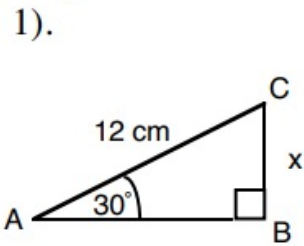
Finding Missing Lengths Part 1. Complete the examples in the table by finding the value of the labelled length. Round your answers to 3sf.

<p>Question Label diagram</p>					
<p>Choose the correct trigonometric ratio</p>	$\sin \theta = \frac{\text{opp}}{\text{hyp}}$	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$	$\tan \theta = \frac{\text{opp}}{\text{adj}}$	$\tan \theta = \frac{\text{opp}}{\text{adj}}$	
<p>Substitute in the values</p>	$\sin 43 = \frac{x}{12.6}$	$\cos 35 = \frac{y}{8.3}$	$\tan 28 = \frac{z}{5.4}$		
<p>Solve the equation</p>	$\times 12.6 \quad \times 12.6$ $x = 12.6 \sin 43$	$\times 8.3 \quad \times 8.3$ $y = 8.3 \cos 35$			
<p>Round your answer (where appropriate) and give units</p>	$x = 8.59 \text{ cm (3sf)}$				

Fluency Practice

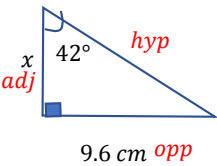
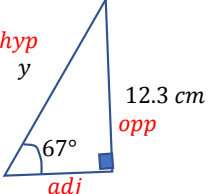
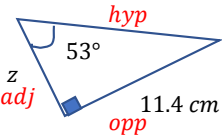
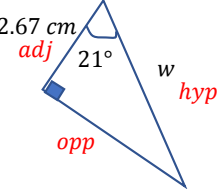
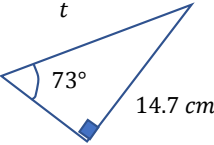
Find the length of the side marked x , leave all answers to 1 decimal place.

Diagrams not to scale.



Fill in the Gaps

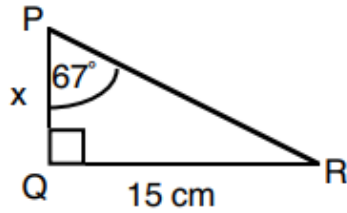
Finding Missing Lengths Part 2. Complete the examples in the table by finding the value of the labelled length. Round your answers to 3sf.

<p>Question</p> <p>Label diagram</p>					
<p>Choose the correct trigonometric ratio</p>	$\tan \theta = \frac{\text{opp}}{\text{adj}}$	$\sin \theta = \frac{\text{opp}}{\text{hyp}}$	$\tan \theta = \frac{\text{opp}}{\text{adj}}$	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$	
<p>Substitute in the values</p>	$\tan 42 = \frac{9.6}{x}$	$\sin 67 = \frac{12.3}{y}$	$\tan 53 = \frac{11.4}{z}$		
<p>Solve the equation (two steps to this)</p>	$\begin{aligned} x \tan 42 &= 9.6 \\ \div \tan 42 & \quad \div \tan 42 \\ x &= \frac{9.6}{\tan 42} \end{aligned}$	$\begin{aligned} y \sin 67 &= 12.3 \\ \div \sin 67 & \quad \div \sin 67 \\ y &= \frac{12.3}{\sin 67} \end{aligned}$			
<p>Round your answer (where appropriate) and give units</p>	$x = 10.7 \text{ cm (3sf)}$				

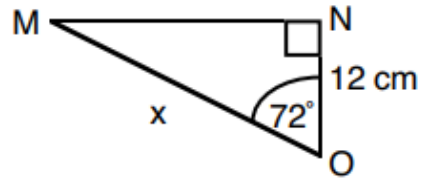
Fluency Practice

Find the length of the side marked x , leave all answers to 1 decimal place.
Diagrams not to scale.

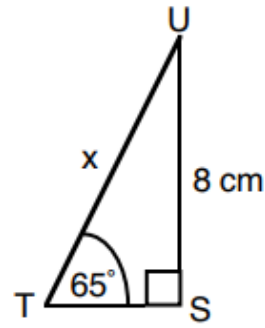
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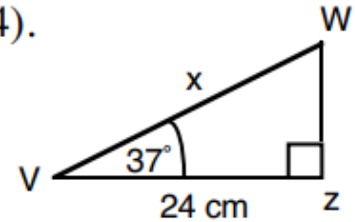
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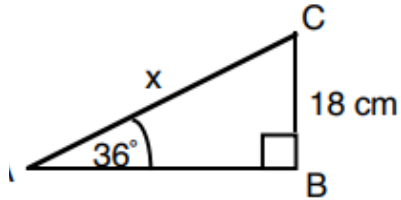
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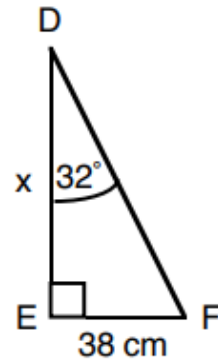
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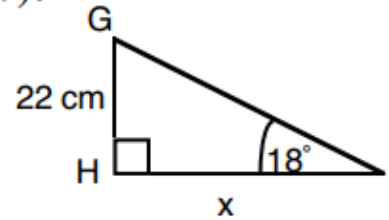
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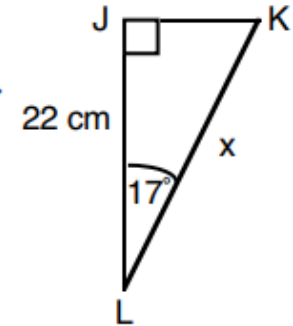
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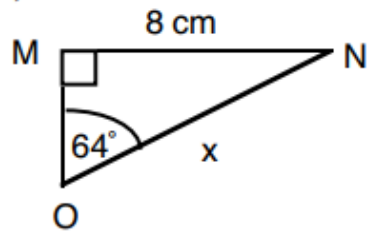
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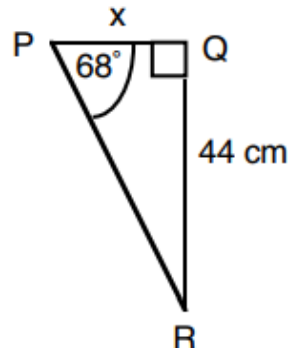
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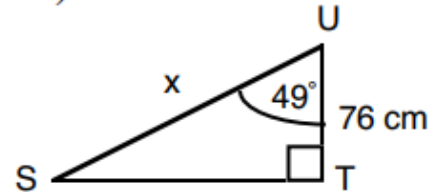
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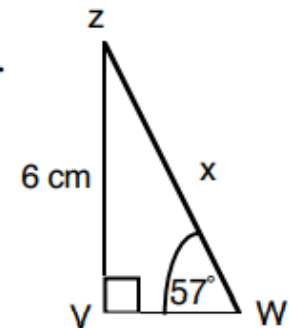
10).



11).

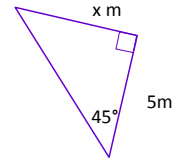
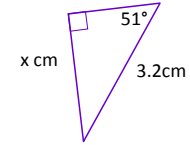
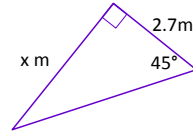
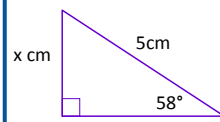
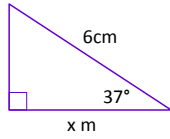
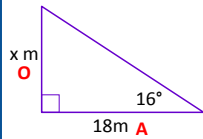


12).



Fill in the Gaps

Question:
(Label Sides)



SOH CAH TOA

~~SOH CAH TOA~~

Which Ratio?

$$\tan(x^\circ) = \frac{O}{A}$$

Substitute

$$\tan(16^\circ) = \frac{x}{18}$$

Rearrange

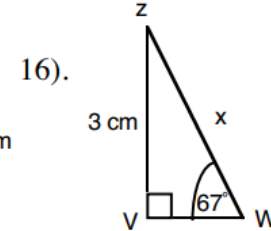
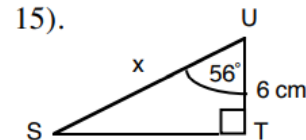
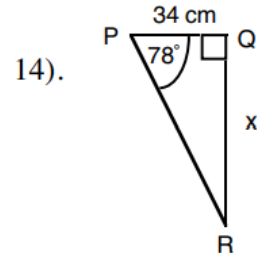
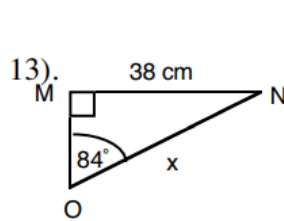
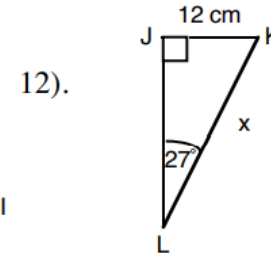
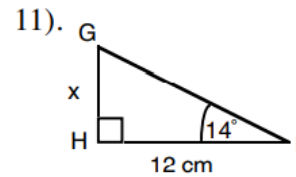
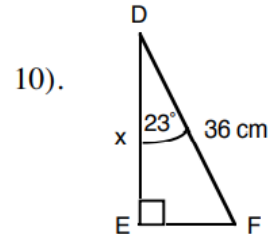
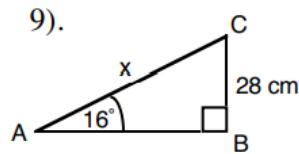
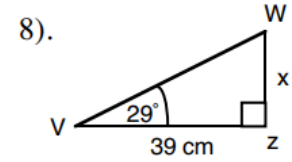
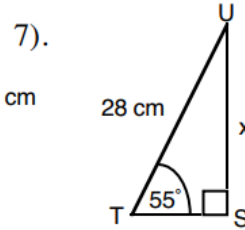
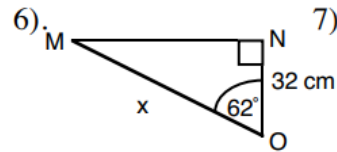
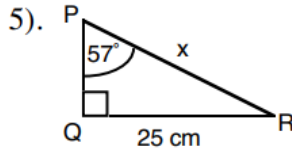
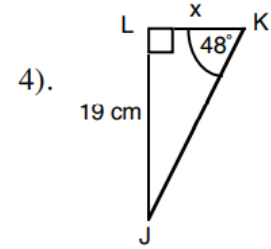
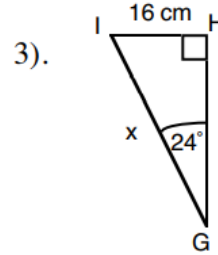
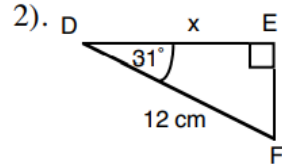
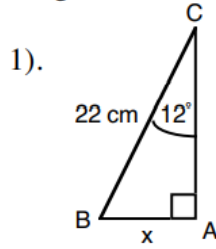
$$18 \tan(16^\circ) = x$$

Calculate
(Round to 2DP)

$$\underline{x = 5.16m}$$

Fluency Practice

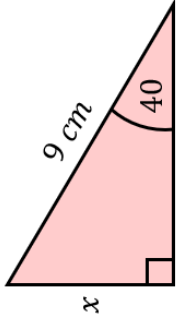
Find the length of the side marked x , leave all answers to 1 decimal place.
Diagrams not to scale.



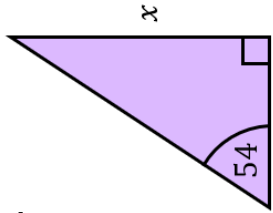
Fluency Practice

Find the missing lengths x .

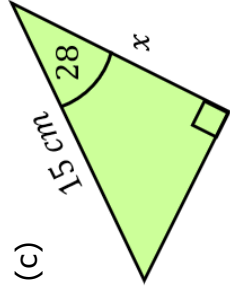
(a)



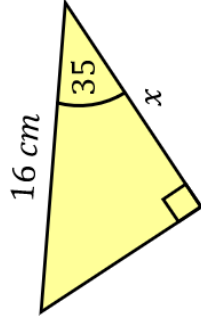
(b)



(c)

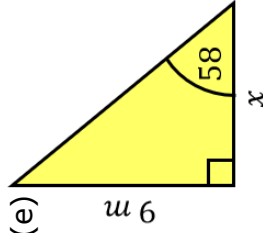


(d)

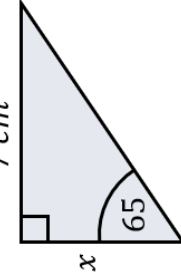


Find the missing lengths x .

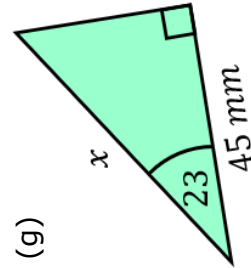
(e)



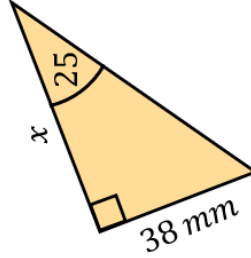
(f)



(g)

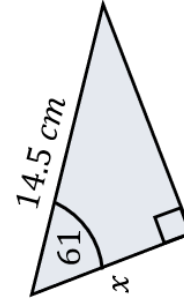


(h)

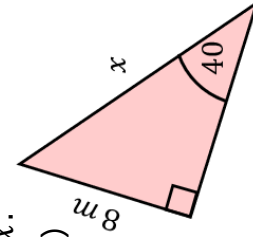


Find the missing lengths x .

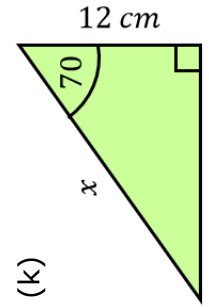
(i)



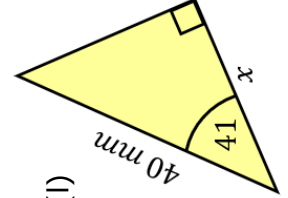
(j)



(k)



(l)

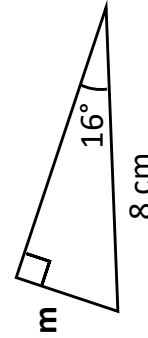
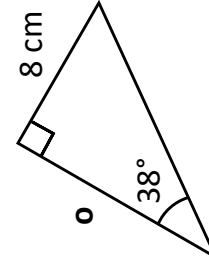
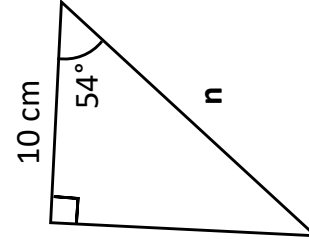
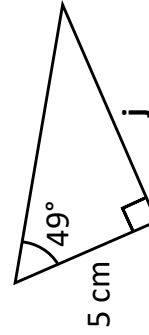
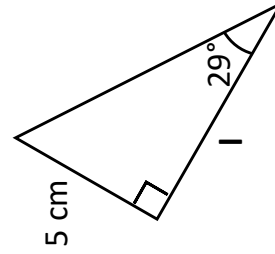
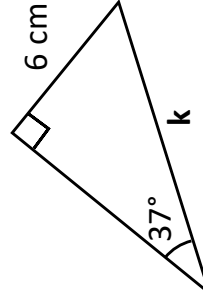
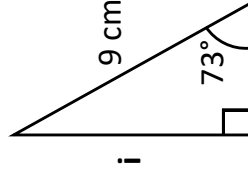
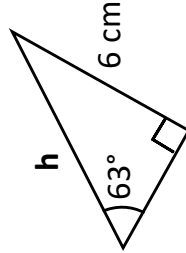
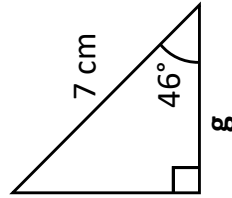
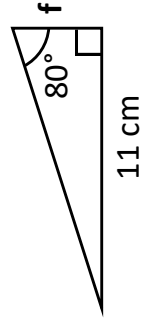
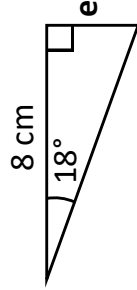
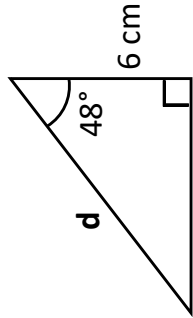
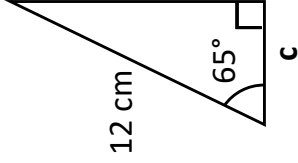
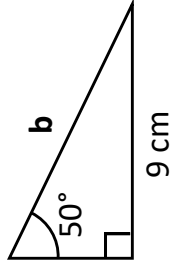
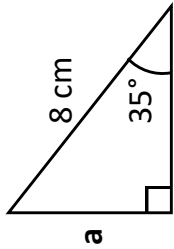


Fluency Practice

Trigonometry: Finding Lengths

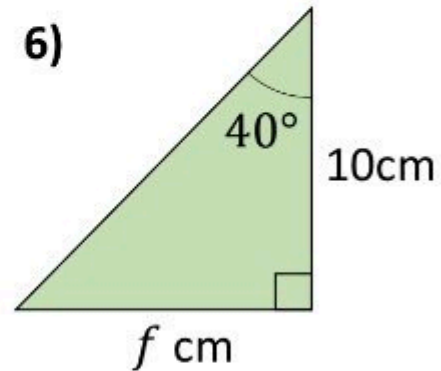
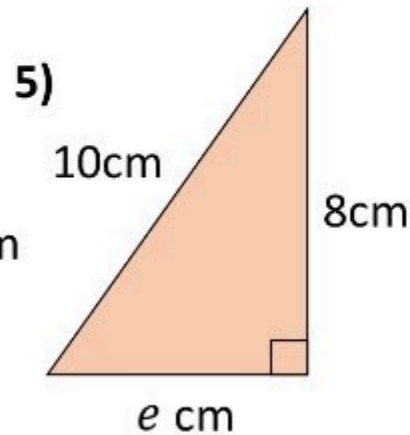
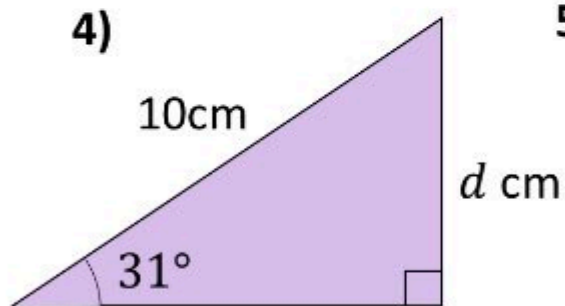
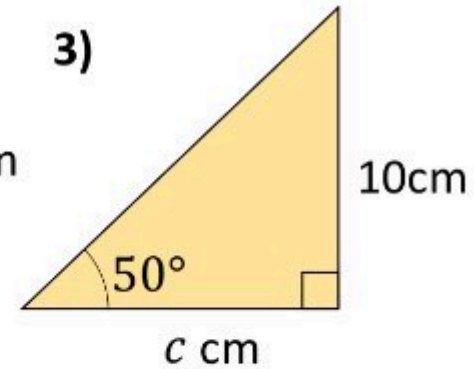
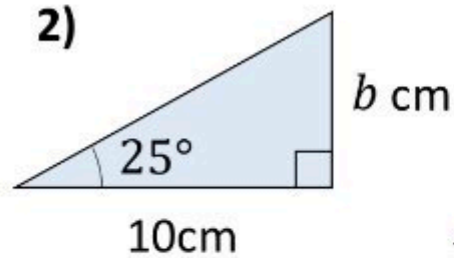
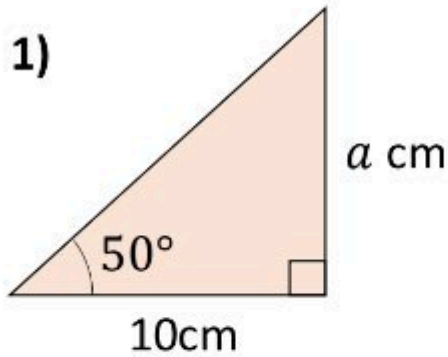
Find each missing length to 1 dp.

Not drawn accurately.



Fluency Practice

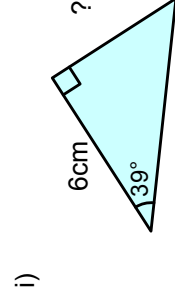
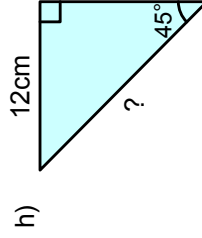
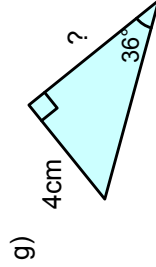
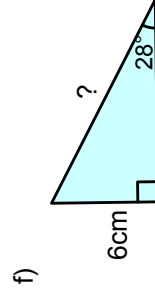
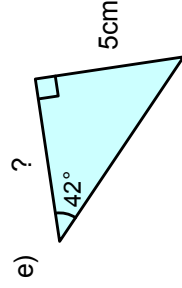
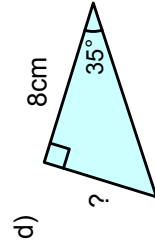
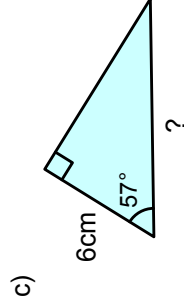
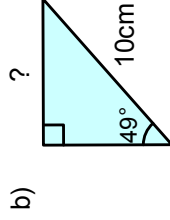
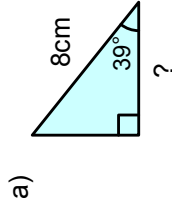
Answers to 2 decimal places



Ext) We've seen that $\tan(90)$ is undefined. Which other angles will have a tangent that is undefined?

Fluency Practice

1. Work out the missing side lengths correct to 1 decimal place.
Match your answers to those at the bottom.



5.6cm

5.5cm

4.9cm

7.5cm

5.6cm

12.8cm

11.0cm

6.2cm

17.0cm

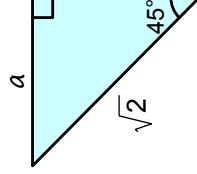
2. Which of these calculations will work out the length of the side marked a ?

a) $a = \tan(45^\circ) \times \sqrt{2}$

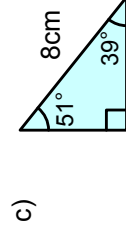
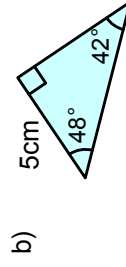
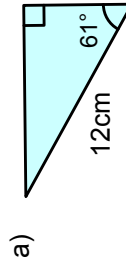
c) $a = \cos(45^\circ) \times \sqrt{2}$

b) $a = \sin(45^\circ) \times \sqrt{2}$

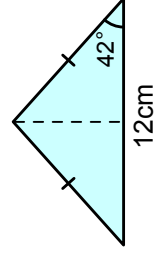
d) $a = \frac{\sin(45^\circ)}{\sqrt{2}}$



3. Work out the perimeter of each of these triangles, correct to the nearest centimetre.



4. a) Work out the height of the isosceles triangle.
Give your answer correct to the nearest 0.1 cm.



- b) Work out the area of the isosceles triangle.
Give your answer correct to the nearest 0.1 cm².

Problem Solving

Each calculation has been created from a diagram.
Draw each diagram.

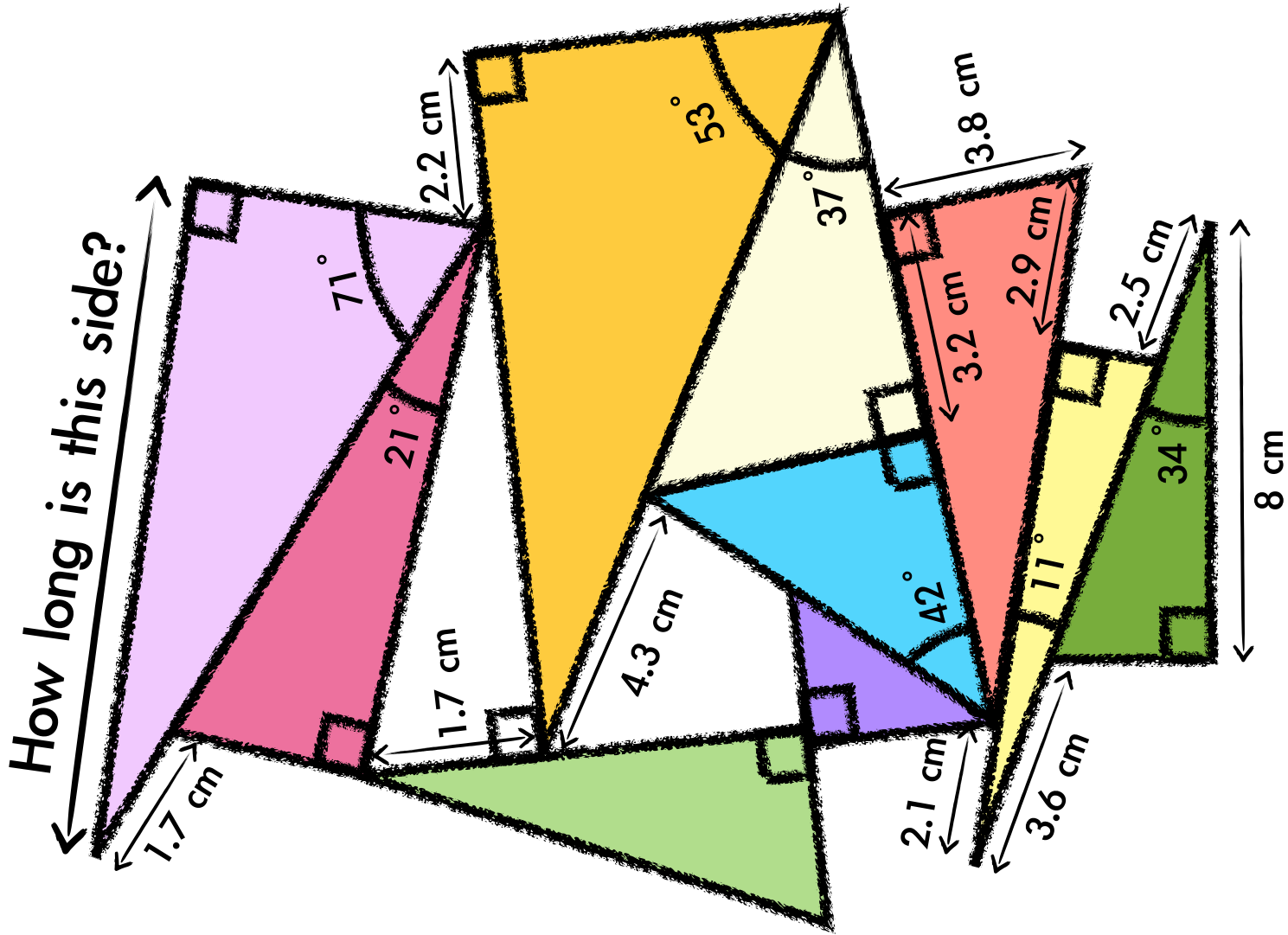
$$x = \frac{9}{\cos(38^\circ)}$$

$$x = 9 \times \cos(38^\circ)$$

$$y = \frac{9}{\sin(38^\circ)}$$

Thinker: $h = 12 \times \tan(45^\circ) + 6$

Fluency Practice



Intelligent Practice

Find ' x '. Give your solution to 2 decimal places.

1) $\sin(x) = 0$

2) $\sin(x) = \frac{1}{5}$

3) $\sin(x) = \frac{2}{5}$

4) $\sin(x) = \frac{3}{5}$

5) $\sin(x) = \frac{4}{5}$

6) $\sin(x) = 1$

7) $\cos(x) = 0$

8) $\cos(x) = \frac{1}{5}$

9) $\cos(x) = \frac{2}{5}$

10) $\cos(x) = \frac{3}{5}$

11) $\cos(x) = \frac{4}{5}$

12) $\cos(x) = 1$

Fluency Practice

Part ii: Solve, giving your answers to 2 decimal places.

$$1) \sin(x) = \frac{5}{13}$$

$$2) \cos(56) = \frac{x}{11}$$

$$3) \tan(72) = \frac{18}{x}$$

$$4) \cos(x) = \frac{4}{7}$$

$$5) \tan(x) = \frac{11}{5}$$

$$6) \sin(29) = \frac{45}{x}$$

$$7) \cos(63) = \frac{x}{28}$$

$$8) \sin(x) = \frac{14}{19}$$

$$9) \cos(57) = \frac{22}{x}$$

$$10) \cos(x) = \frac{5.3}{14.7}$$

$$11) \tan(83) = \frac{x}{34.6}$$

$$12) \tan(x) = \frac{72}{91}$$

$$13) \sin(8.6) = \frac{x}{140}$$

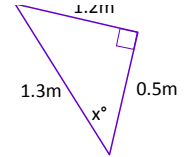
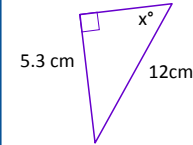
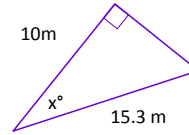
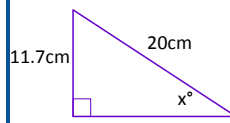
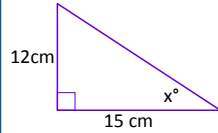
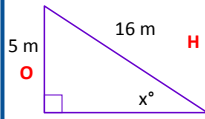
$$14) \cos(37) = \frac{325}{x}$$

$$15) \sin(x) = \frac{3.12}{9.57}$$

$$16) \cos(42.1) = \frac{842}{x}$$

Fill in the Gaps

Question:
(Label Sides)



SOH CAH TOA

~~SOH~~ CAH TOA

Which Ratio?

$$\sin(x^\circ) = \frac{O}{H}$$

Substitute

$$\sin(x^\circ) = \frac{5}{16}$$

Inverse

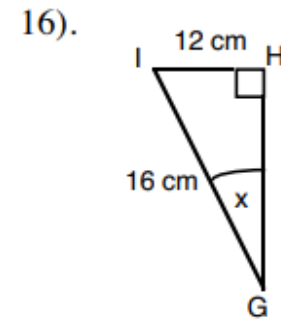
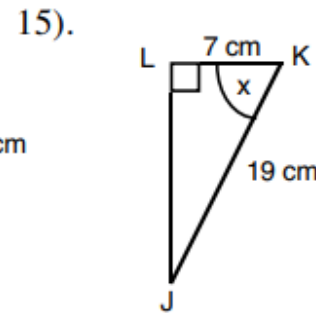
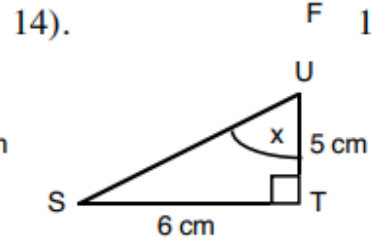
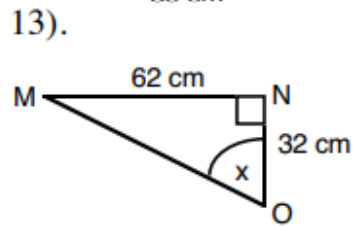
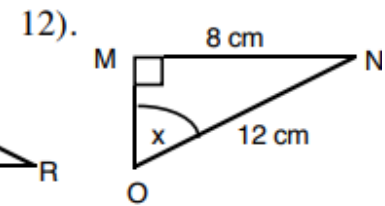
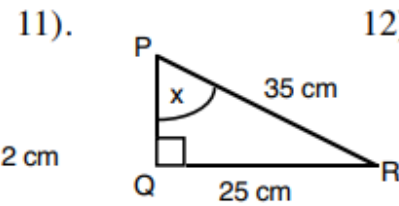
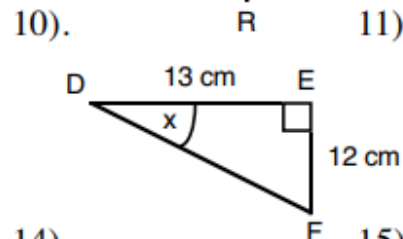
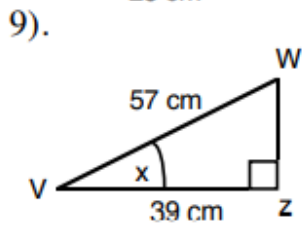
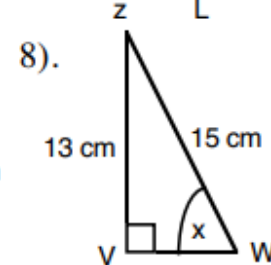
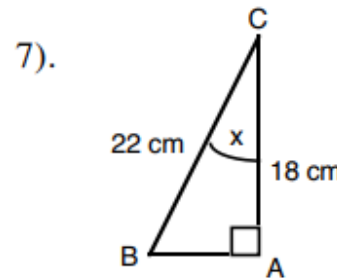
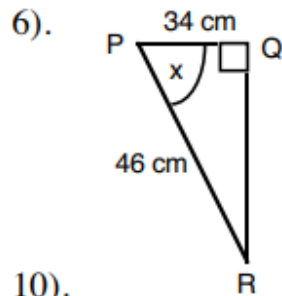
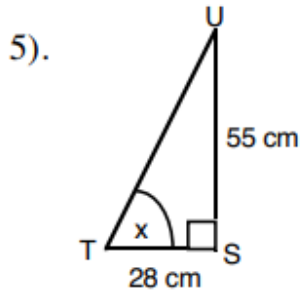
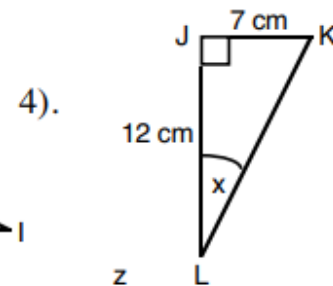
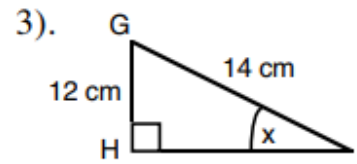
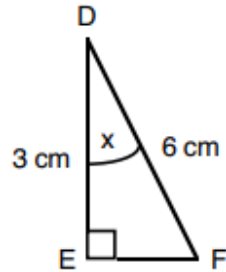
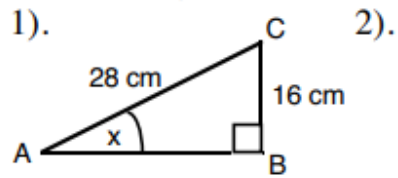
$$x^\circ = \sin^{-1}(5/16)$$

Calculate
(Round to 1DP)

$$x = \underline{18.2^\circ}$$

Fluency Practice

Find the angles marked x .

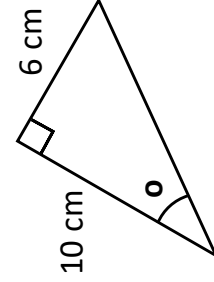
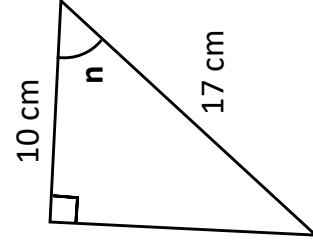
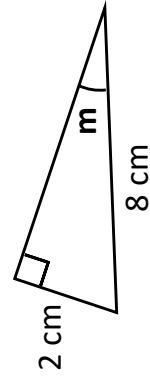
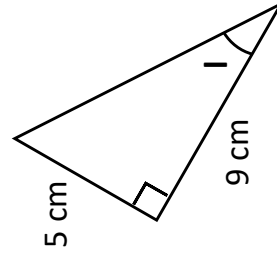
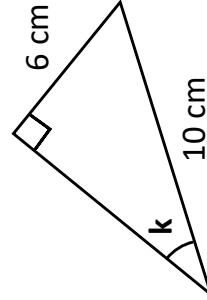
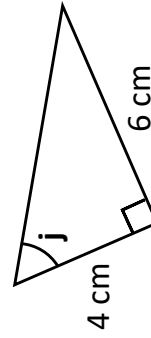
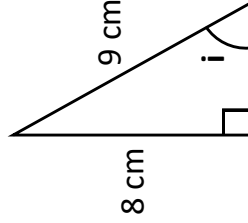
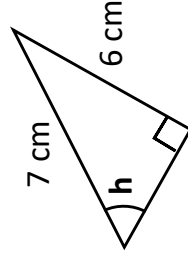
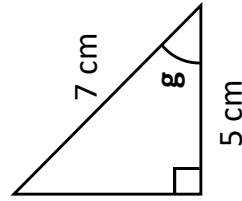
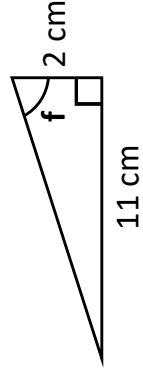
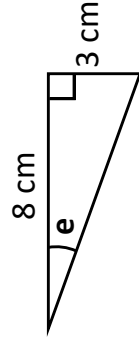
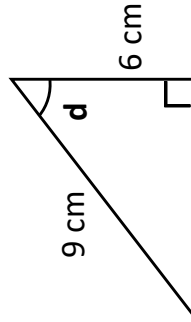
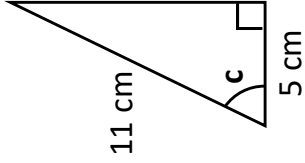
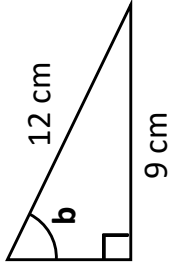
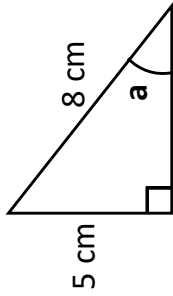


Fluency Practice

Trigonometry: Finding Angles

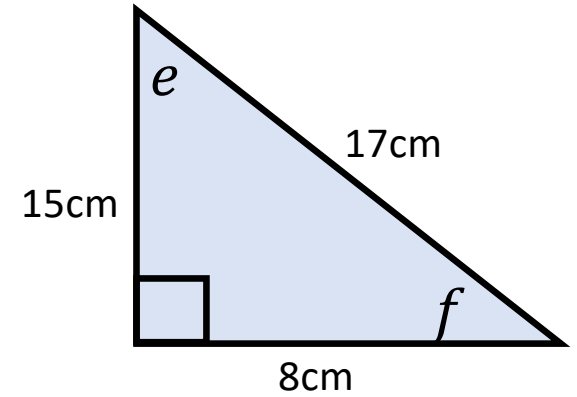
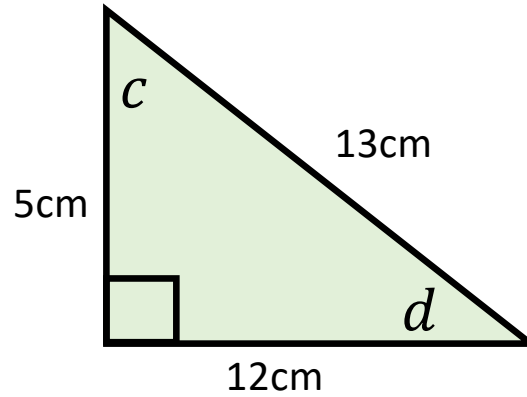
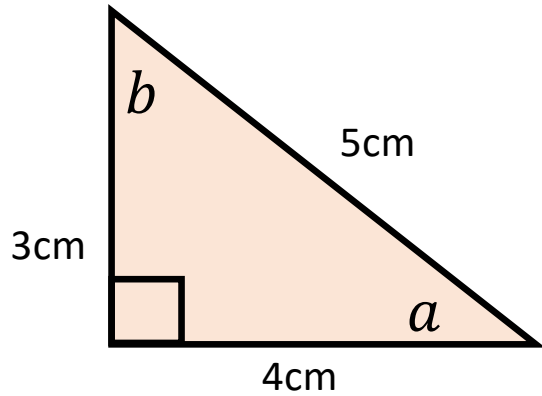
Find each angle to 1 dp.

Not drawn accurately.



Problem Solving

Which angle does each of the following equations calculate?



$$\sin x = \frac{3}{5}$$

$$\sin x = \frac{8}{17}$$

$$\cos x = \frac{8}{17}$$

$$\tan x = \frac{3}{4}$$

$$x = \tan^{-1} \frac{15}{8}$$

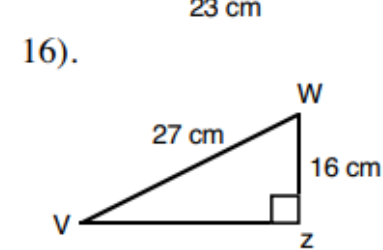
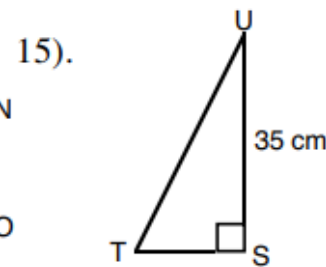
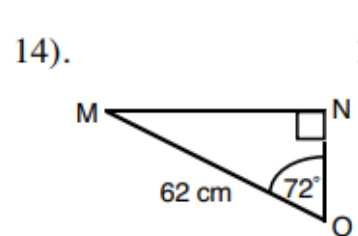
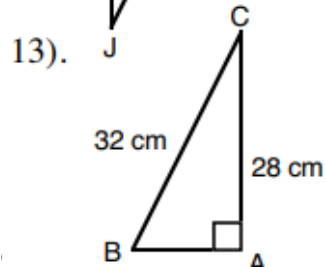
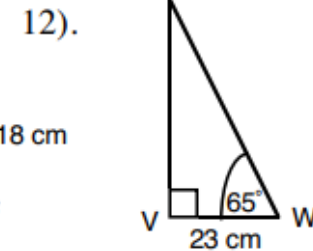
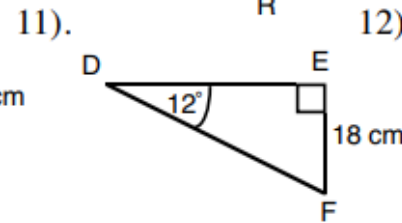
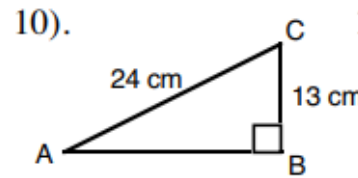
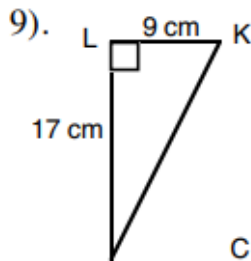
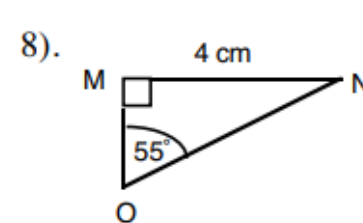
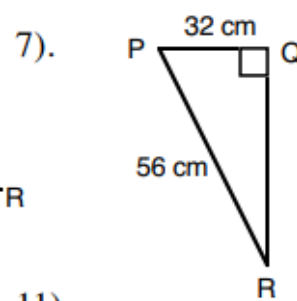
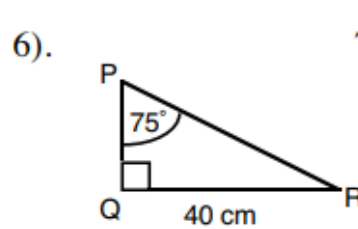
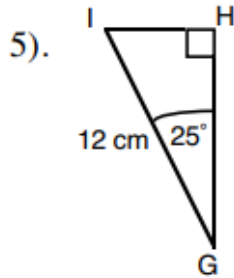
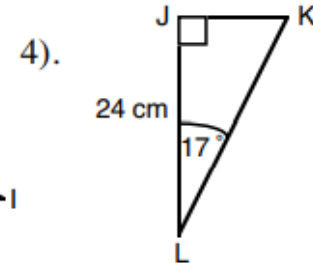
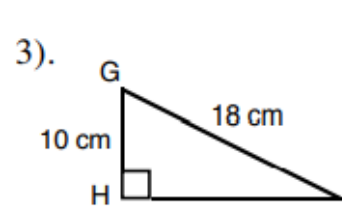
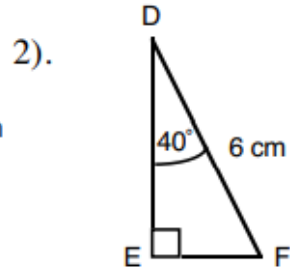
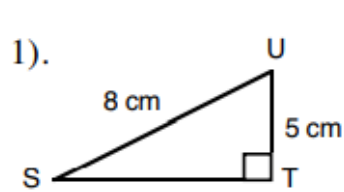
$$\tan x = \frac{15}{8}$$

$$x = \cos^{-1} \frac{3}{5}$$

$$\sin x = \frac{12}{13}$$

Fluency Practice

In the following triangles find **all the missing angles and sides.**

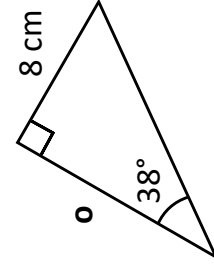
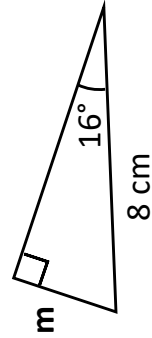
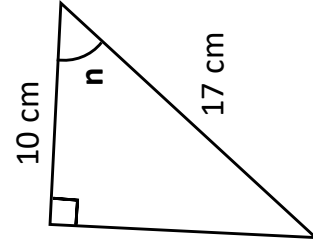
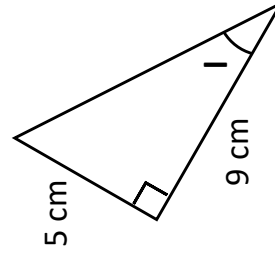
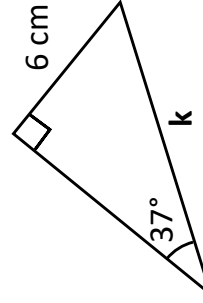
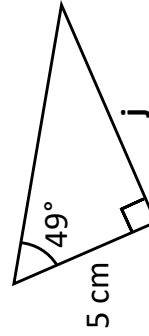
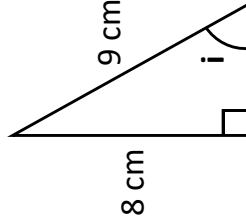
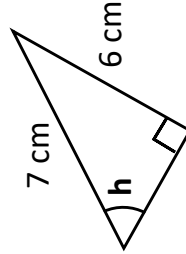
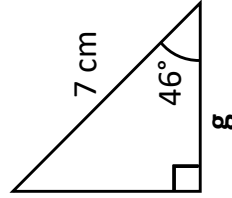
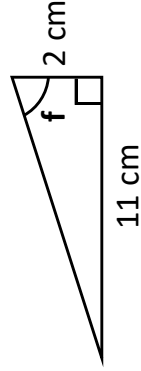
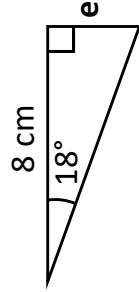
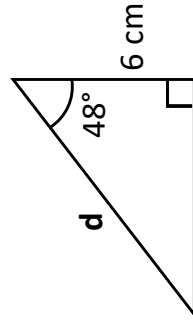
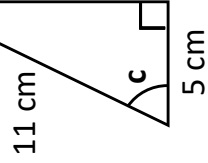
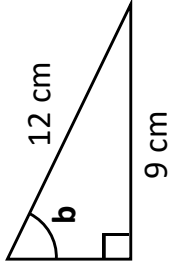
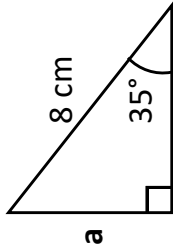


Fluency Practice

Trigonometry: Finding Angles & Lengths

Not drawn accurately.

Find each angle or length to 1 dp.

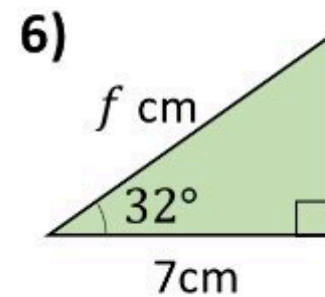
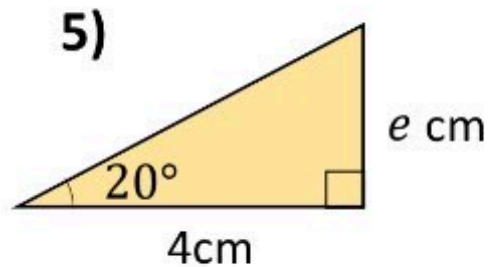
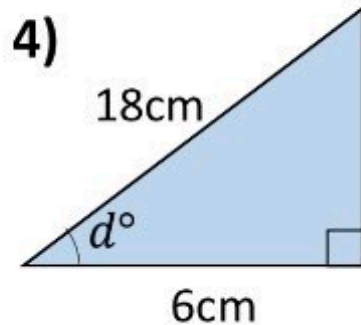
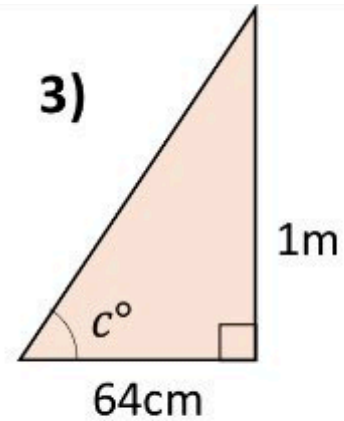
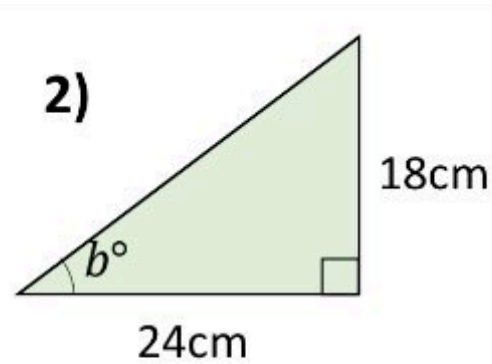
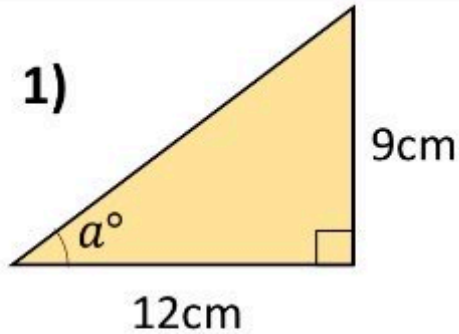


Fluency Practice

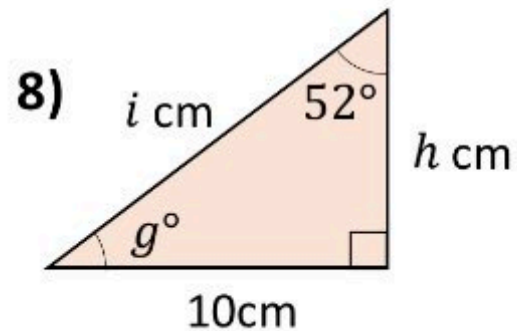
Mixed Trigonometry

(a)	(b)	(c)	(d)
(e)	(f)	(g)	(h)
(i)	(j)	(k)	(l)

Fluency Practice

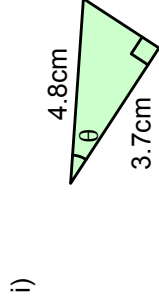
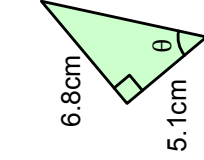
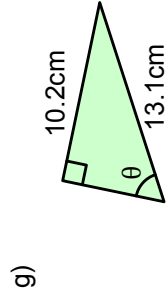
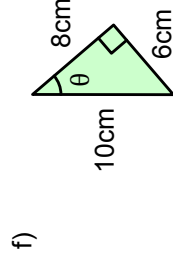
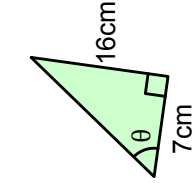
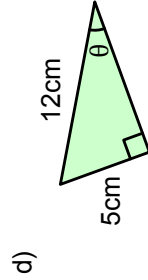
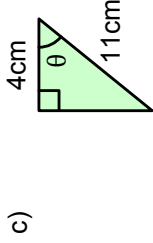
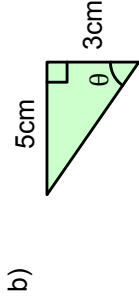
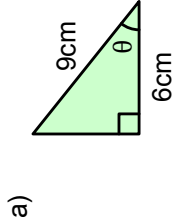


7) Draw a right angled triangle where the other angles are slightly above and slightly below 45°



Fluency Practice

1. Work out the angles marked θ in these triangles, correct to the nearest degree. Match your answers to those at the bottom.



- | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 59° | 53° | 25° | 48° | 51° | 66° | 69° | 40° | 37° |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|

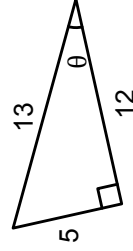
2. For the triangle to the right, which calculations are correct? Select all that apply.

a) $\theta = \sin^{-1}\left(\frac{12}{13}\right)$

b) $\theta = \cos^{-1}\left(\frac{5}{12}\right)$

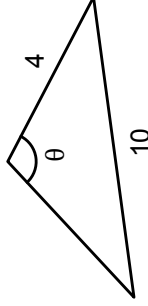
c) $\theta = \tan^{-1}\left(\frac{5}{12}\right)$

d) $\theta = \sin^{-1}\left(\frac{5}{13}\right)$



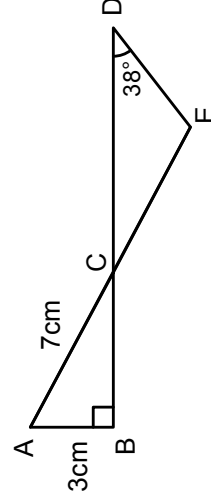
3. For the triangle to the right, why is this calculation not correct?

$\theta = \cos^{-1}\left(\frac{4}{10}\right)$



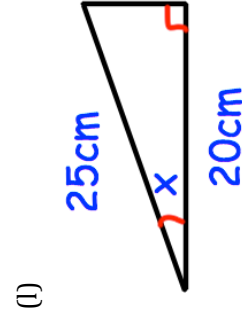
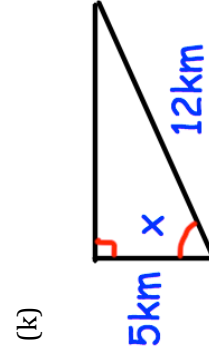
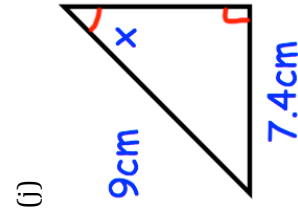
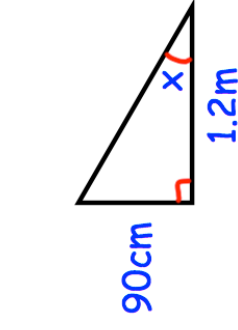
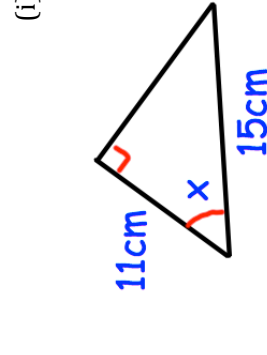
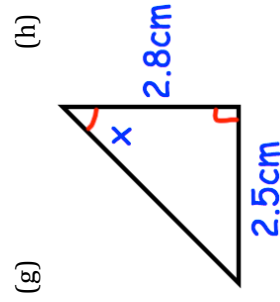
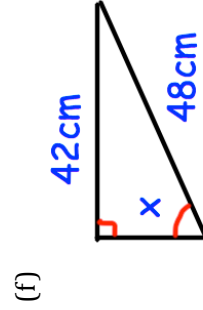
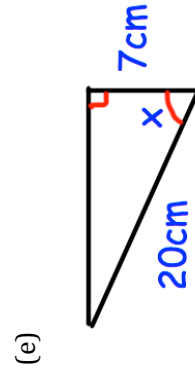
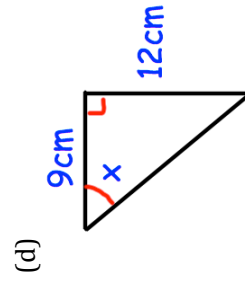
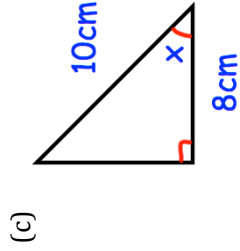
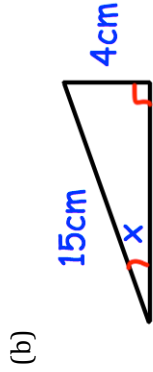
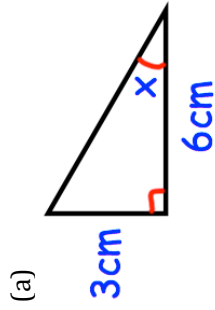
4. In the diagram, AE and BD are straight lines.

Work out the size of \hat{CED} , correct to the nearest degree.



Fluency Practice

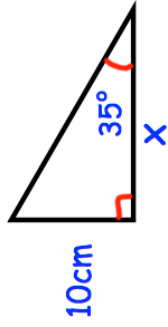
Question 1: Find the size of the missing angles in the triangles below.



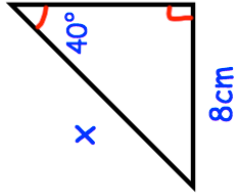
Fluency Practice

Question 2: Find the lengths of the sides labelled x below.

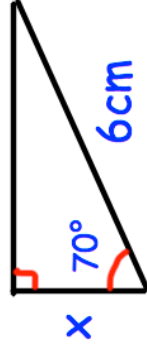
(a)



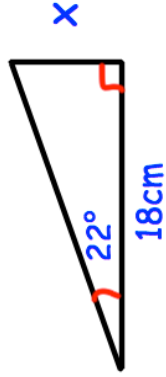
(b)



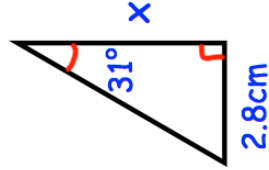
(c)



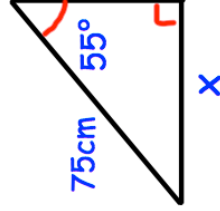
(d)



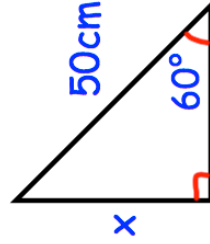
(e)



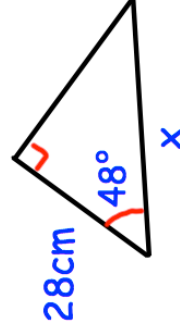
(f)



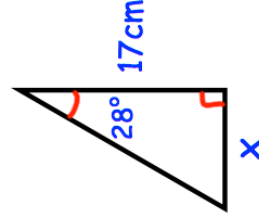
(g)



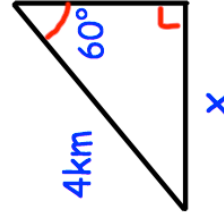
(h)



(i)



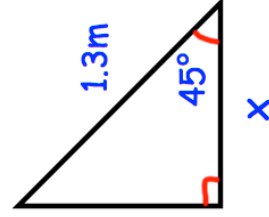
(j)



(k)

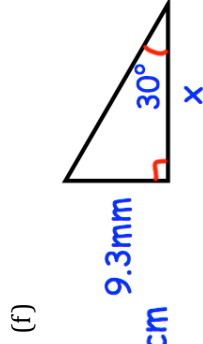
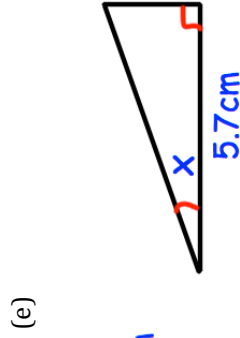
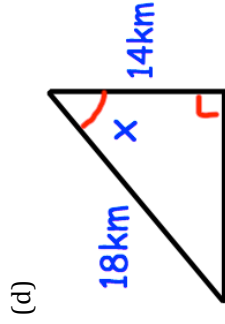
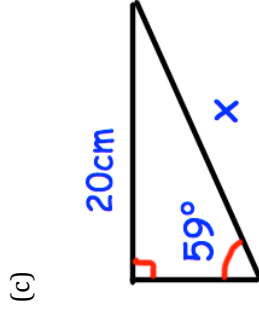
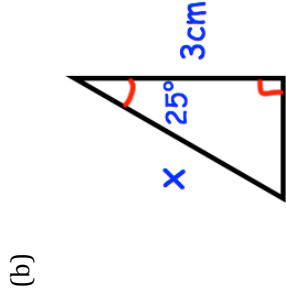
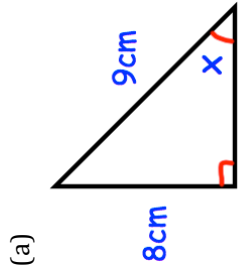


(l)



Fluency Practice

Question 3: Find the size of the missing angles/sides labelled x below.



Apply

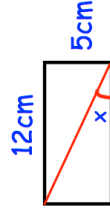
In each question, draw a diagram unless it has been given.

Question 1: A 4 metre long ladder is placed against a wall. The angle between the ladder and the ground is 75° . How far up the wall does the ladder reach?

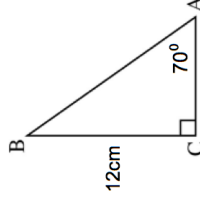
Question 2: A 5 metre long ladder is placed against a wall. It reaches 4.3 metres up the wall. What is the angle between the ladder and the ground?

Question 3: A ladder is placed against a wall.
The base of the ladder is 4 foot from the bottom of the wall.
The angle between the ladder and the ground is 80° .
What is the length of the ladder?

Question 4: A rectangle is 12cm long and 5cm wide. Find the size of the angle marked x.



Question 5:
(a) Find the length of AC.
(b) Find the length of AB.
(c) Find the perimeter of triangle ABC.
(d) Find the area of triangle ABC.

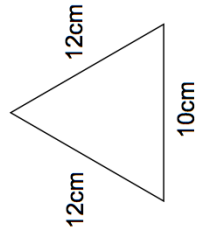


Fluency Practice

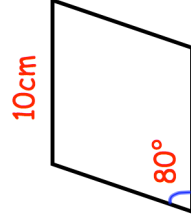
Question 6: A helicopter leaves A and flies 40 miles due east. Then the helicopter flies 10 miles due south and arrives at B. Work out the bearing of B from A.

Question 7: A boat leaves a port and sails 55km due west and then 30km due north and arrives at an oil rig. What is the bearing of the oil rig from the port?

Question 8: Shown is an isosceles triangle. Calculate its area.

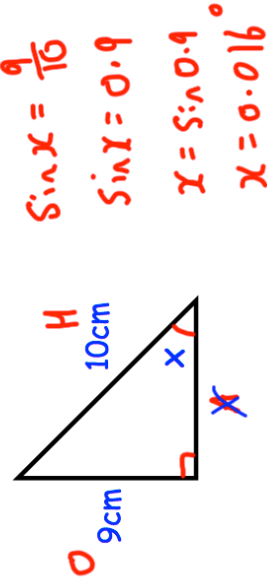


Question 9: Shown is a rhombus of side length 10cm. Calculate its area.



Question 10: Can you spot any mistakes in the question below?

Find the size of the angle x .



Problem Solving

In the table a line of working has been given.

Draw the corresponding triangle that would produce that line of working.

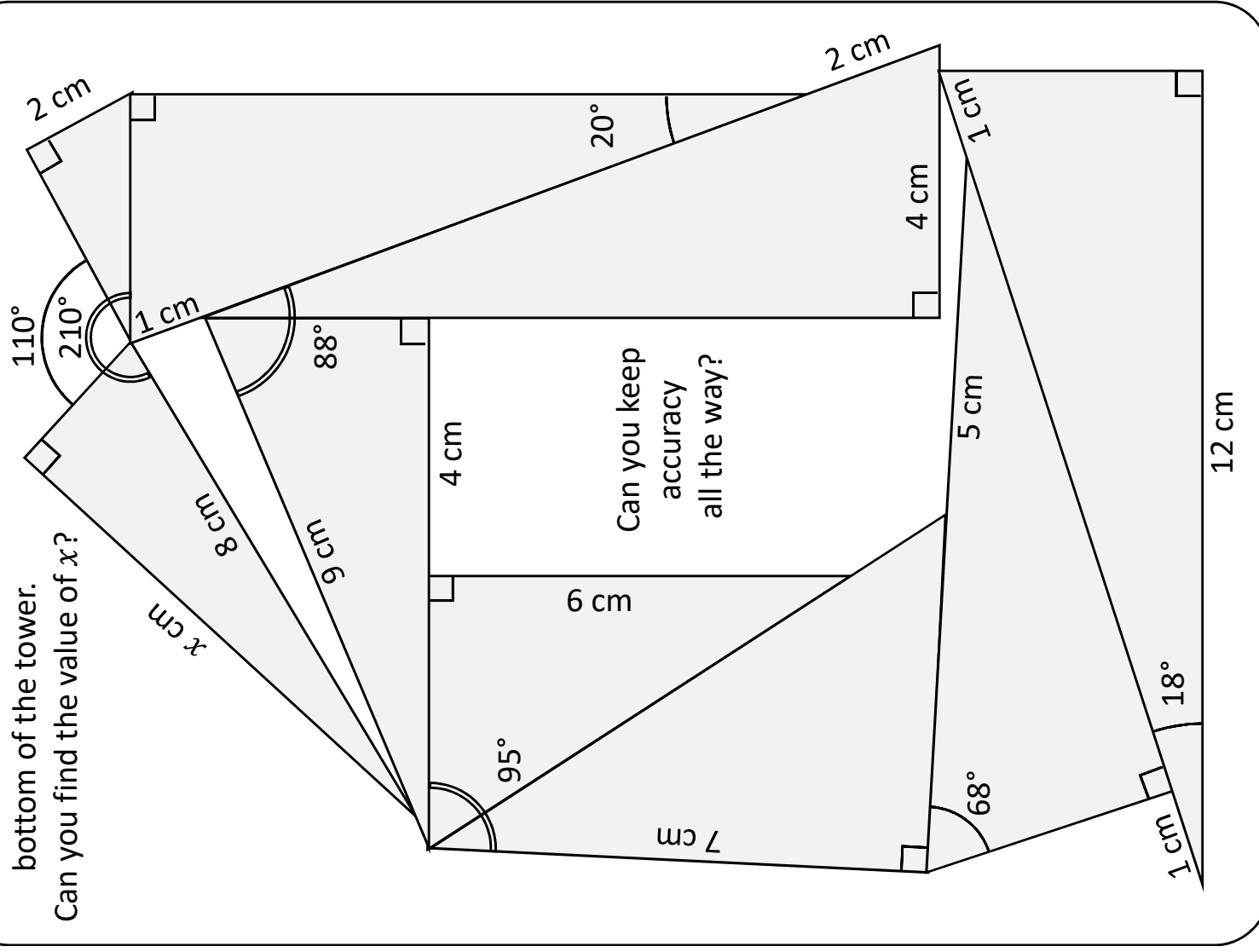
$x = 10\sin 12^\circ$	$y = \frac{16}{\cos 34^\circ}$	$x = \sin^{-1}\left(\frac{5}{6}\right)$
$x = \frac{4.3}{\tan 35^\circ}$	$\theta = \cos^{-1}\left(\frac{16}{34}\right)$	$x = \frac{10}{\sin 12^\circ}$
$y = \tan^{-1}\left(\frac{10}{9}\right)$	$x = 34\cos 16^\circ$	$x = \tan 67^\circ \times 6$

Fluency Practice

Start from the **Trig Tower**

bottom of the tower.

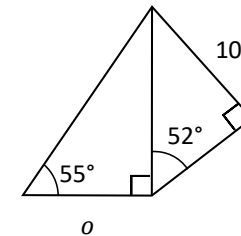
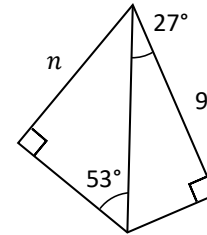
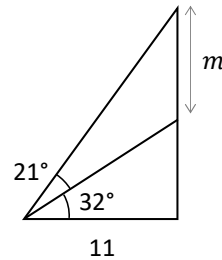
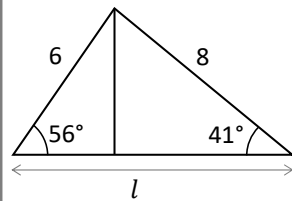
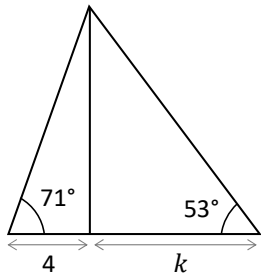
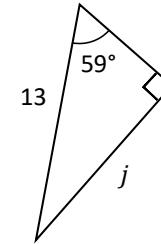
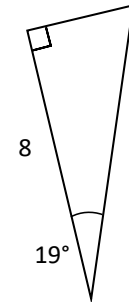
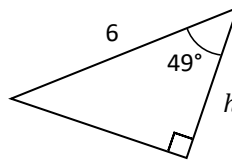
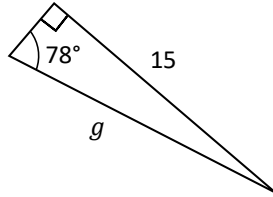
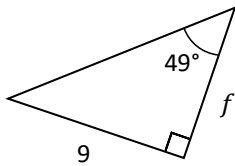
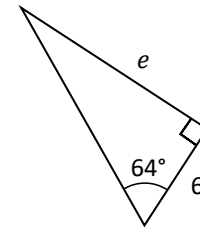
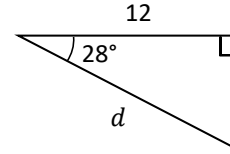
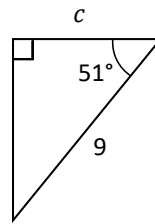
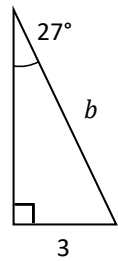
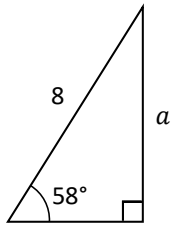
Can you find the value of x ?



Fluency Practice

Using Trigonometric Functions to Find Lengths

not drawn accurately

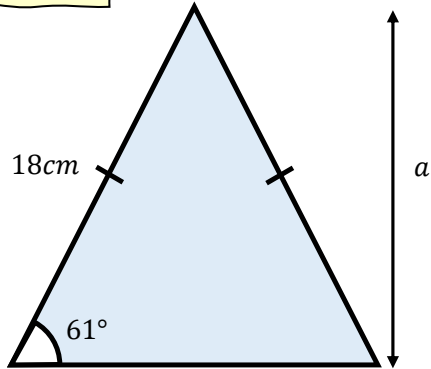


14 Answers	8.8	6.6	7.8	7.7	8.5	8.1	12.3
	13.6	11.1	3.9	5.7	15.3	6.8	9.4

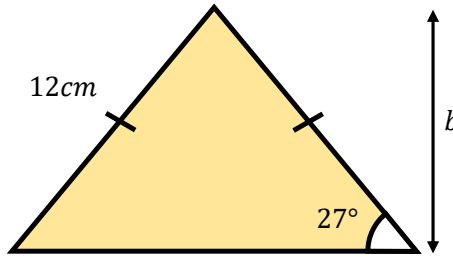
Fluency Practice

In each triangle, find the missing length indicated by a letter.

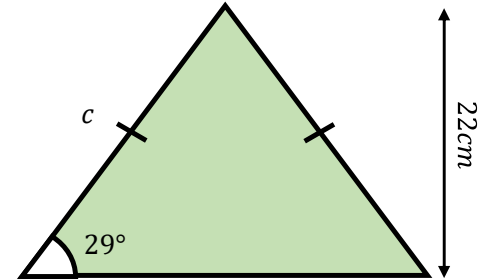
Question 1



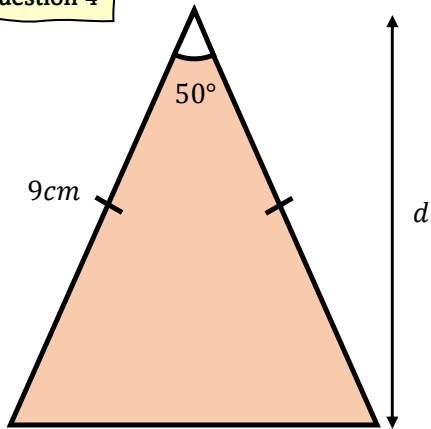
Question 2



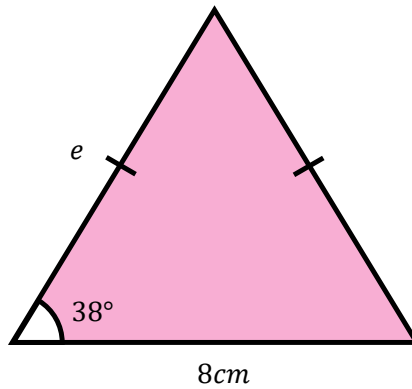
Question 3



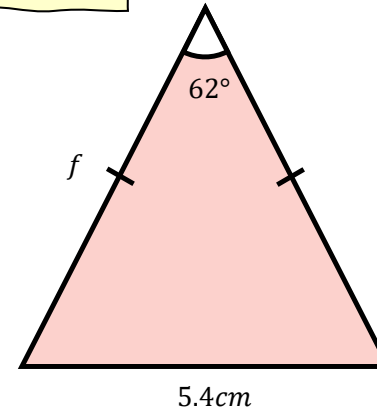
Question 4



Question 5



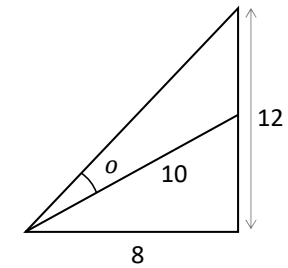
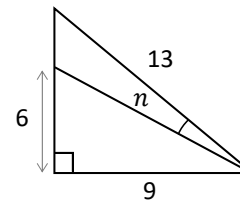
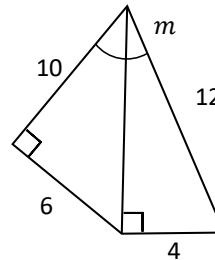
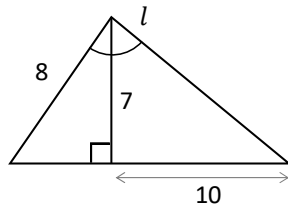
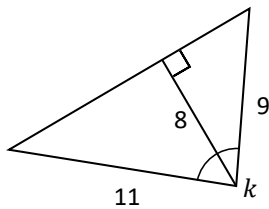
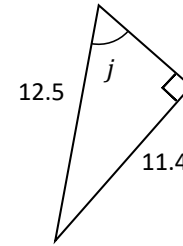
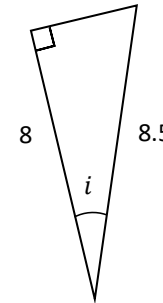
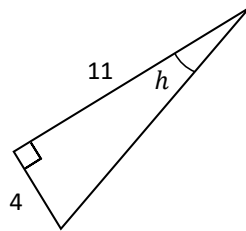
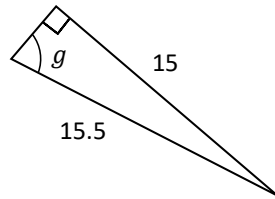
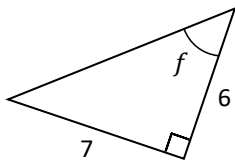
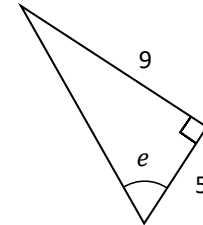
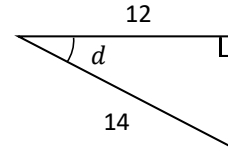
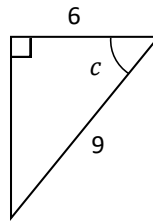
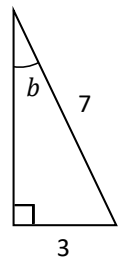
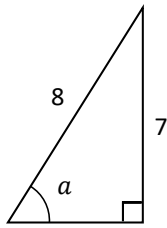
Question 6



Fluency Practice

Using Trigonometric Functions to Find Angles

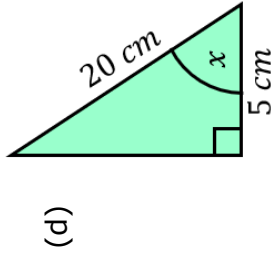
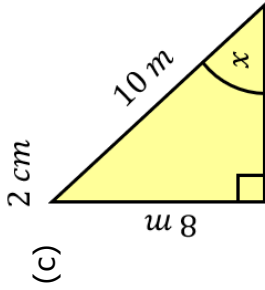
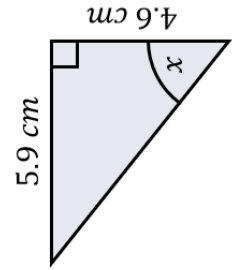
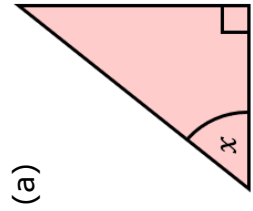
not drawn accurately



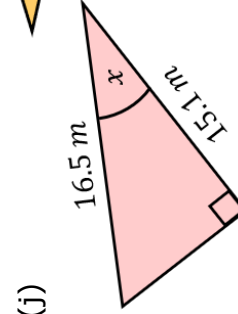
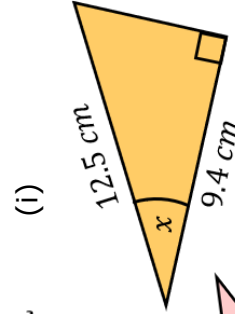
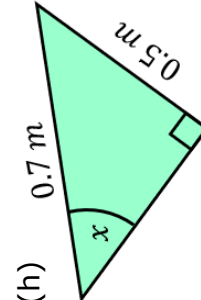
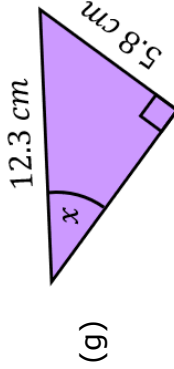
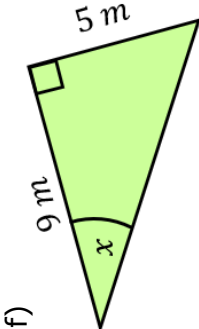
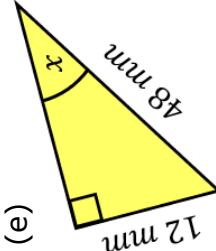
13 Answers	60.9	84.0	31.0	50.4	61.0	70.6	75.4
	65.8	25.4	20.0	49.4	19.7	48.2	

Fluency Practice

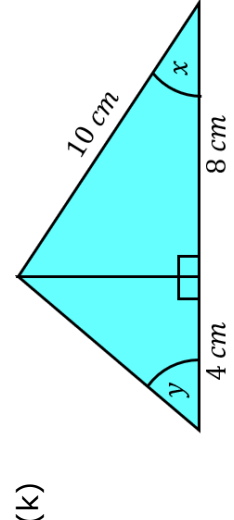
Find the value of the labelled angles.



Find the value of the labelled angles.



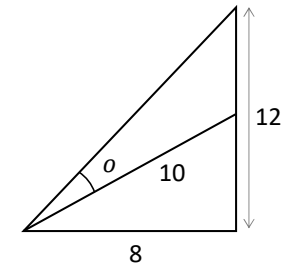
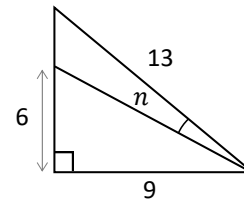
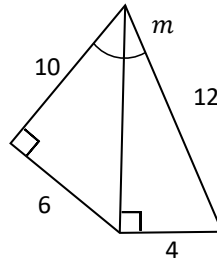
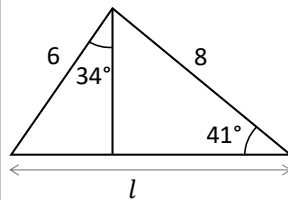
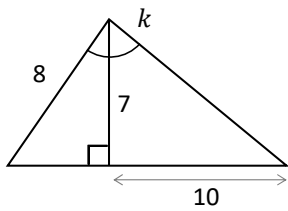
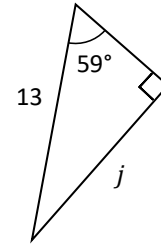
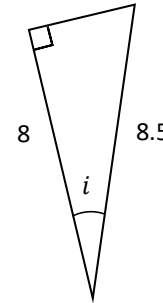
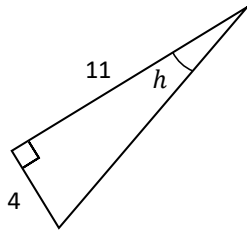
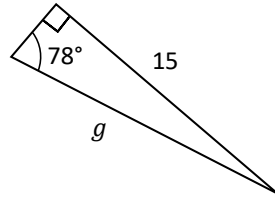
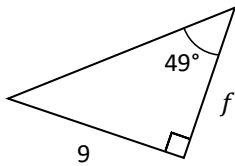
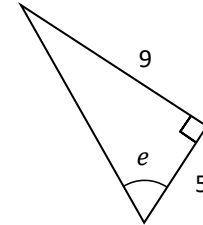
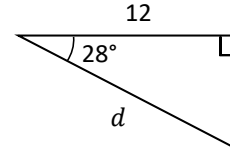
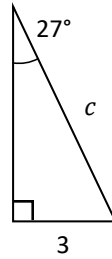
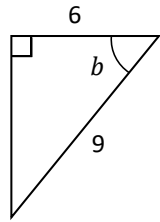
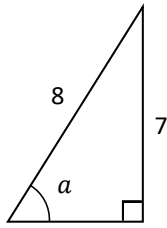
Find the missing angles.



Fluency Practice

Using Trigonometric Functions to Find Angles & Lengths

not drawn accurately



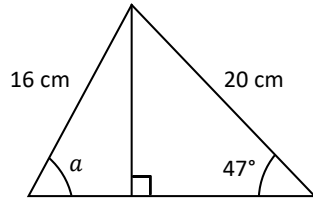
14 Answers	13.6	19.7	12.5	61.0	9.4	60.9	20.0
	11.1	6.6	15.3	84.0	7.8	50.4	48.2

Fluency Practice

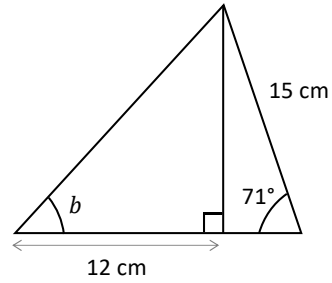
Solving Trigonometry Questions Involving Length & Angles

NOT TO SCALE

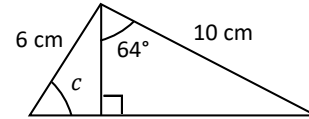
①



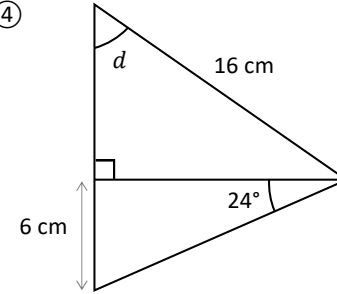
②



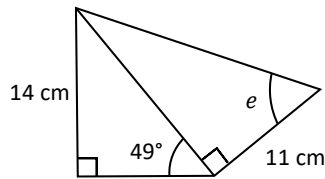
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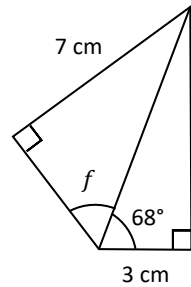
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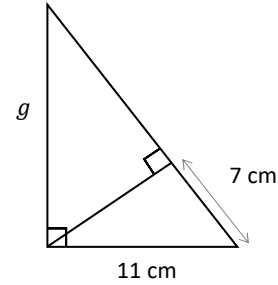
⑤



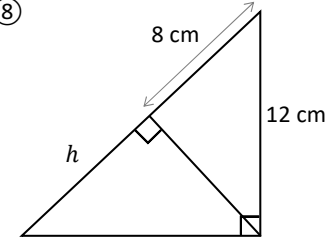
⑥



⑦

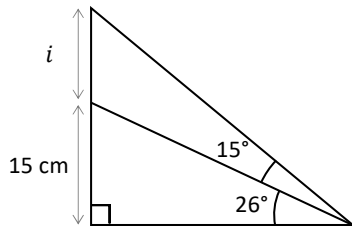


⑧

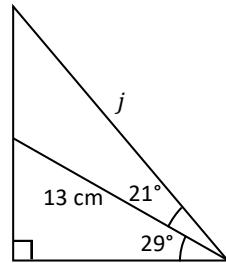


What statement can we make about these two triangles?

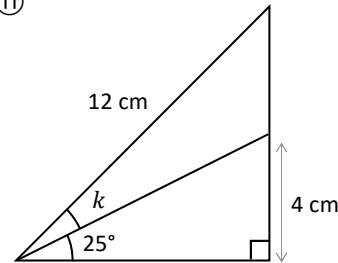
⑨



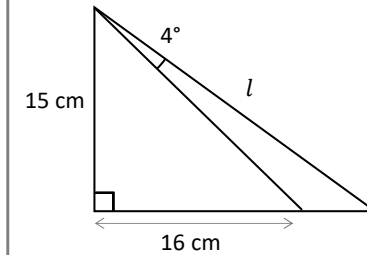
⑩



⑪

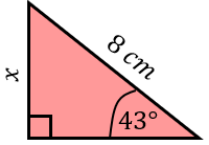
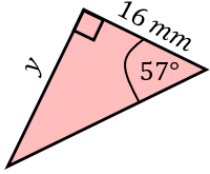
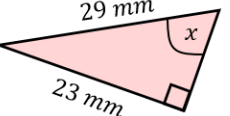
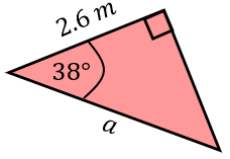
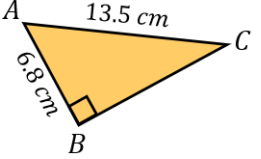
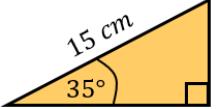
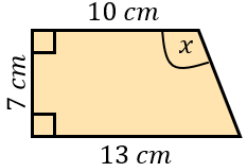
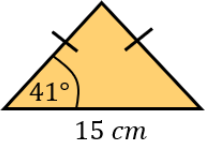
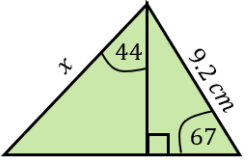
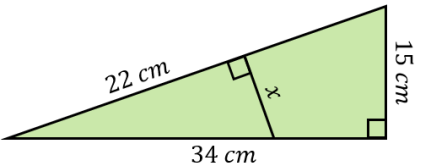
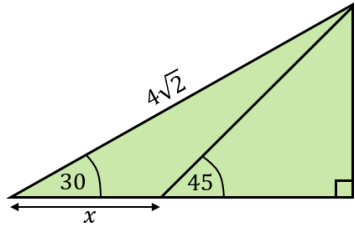


⑫



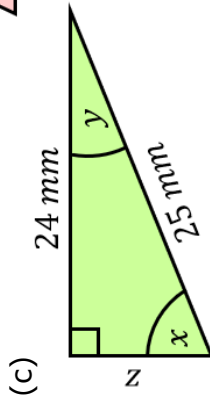
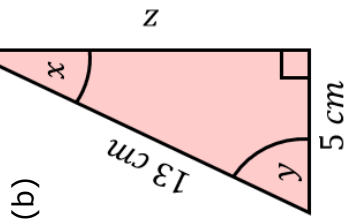
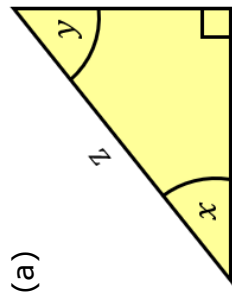
11 Answers	11.7	59.3	66.1	17.7	19.4	60.9
	49.8	46.9	10	13.3	57.4	

Fluency Practice

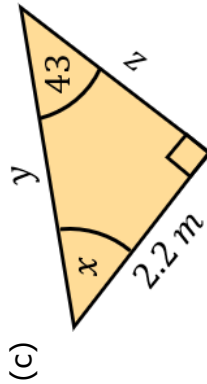
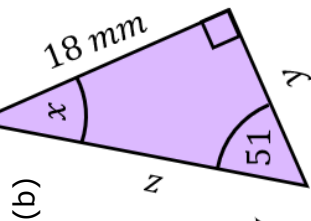
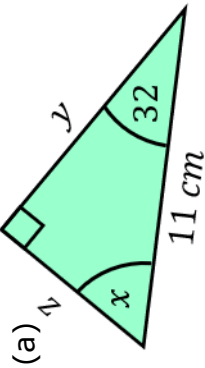
(a)	(b)	(c)	(d)
<p>Find the value of x, to 1 decimal place.</p> 	<p>Find the value of y to 3 significant figures.</p> 	<p>Find angle x, to 1 decimal place.</p> 	<p>Find the value of a, to 1 decimal place.</p> 
(e)	(f)	(g)	(h)
<p>Find angle ACB to 3 significant figures.</p> 	<p>Find the perimeter of the triangle to 1 decimal place.</p> 	<p>Find angle x to 3 significant figures.</p> 	<p>Find the area of the isosceles triangle to 1 decimal place.</p> 
(i)	(j)	(k)	
<p>Find the length x to 3 significant figures.</p> 	<p>Find the value of x, giving your answer to 3 significant figures.</p> 	<p>Find the exact value of x.</p> 	

Fluency Practice

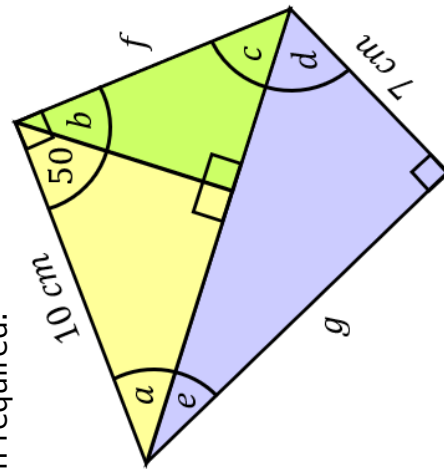
Find all the missing lengths and angles in each of these right-angled triangles, to 1 decimal place.



Find the values of x , y and z to 1 decimal place.

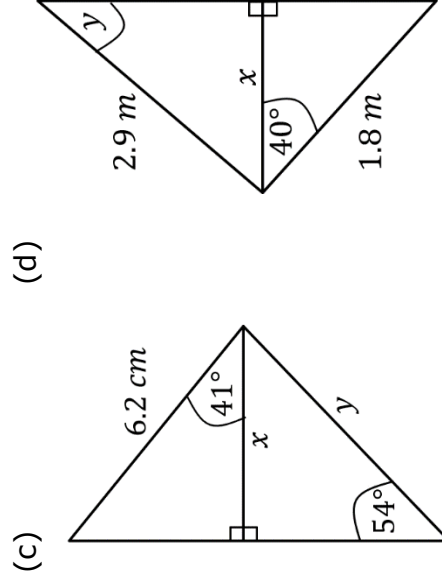
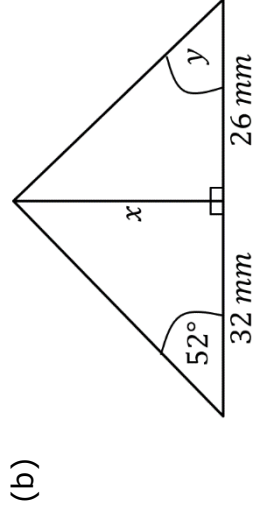
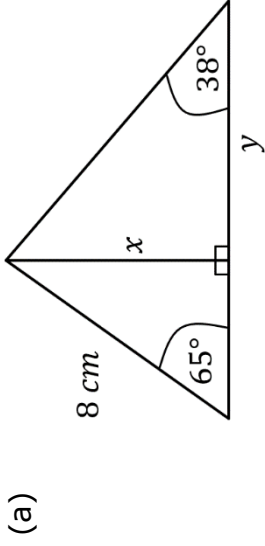


Find all the missing angles and lengths in this diagram, rounding to 1 decimal place when required.

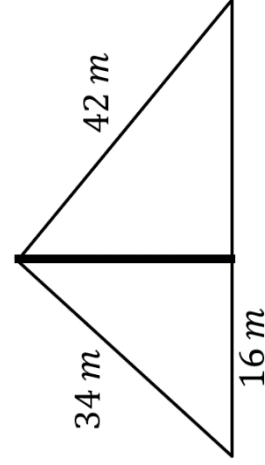


Fluency Practice

Find the missing lengths and angles in these diagrams.



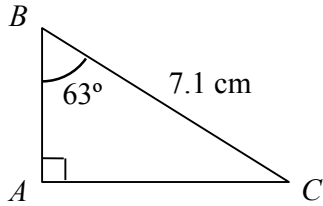
A vertical mast is held in position by two cables of lengths 34 m and 42m, as shown in the diagram.



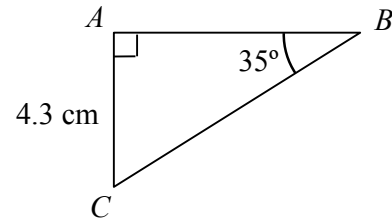
- (a) Calculate the height of the mast
- (b) Calculate the angle the horizontal makes with the 34 m cable.
- (c) Calculate the horizontal distance from the foot of the mast to the 42 m cable.

Fluency Practice

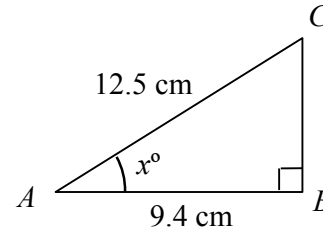
A1 Find length AB



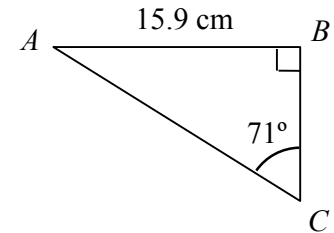
A2 Find length AB



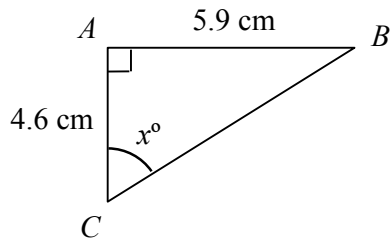
A3 Find angle BAC



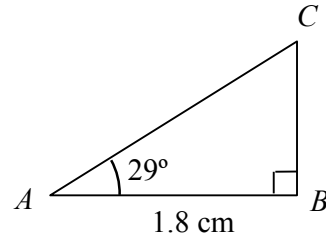
A4 Find length AC



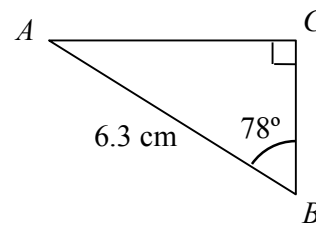
B1 Find angle ACB



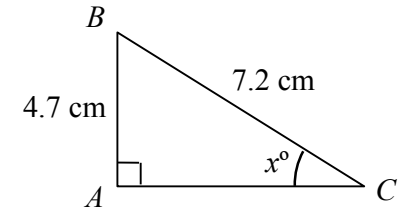
B2 Find length BC



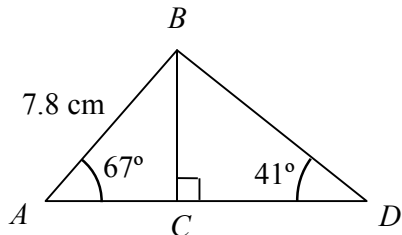
B3 Find length AC



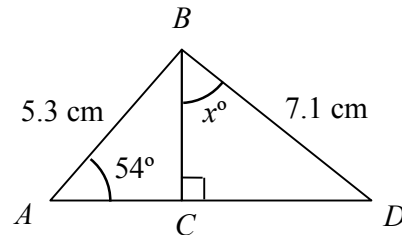
B4 Find angle ACB



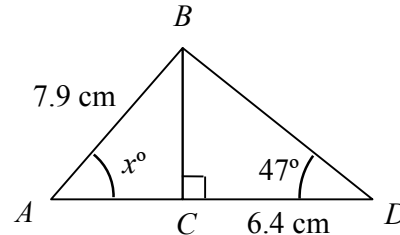
C1 Find length CD



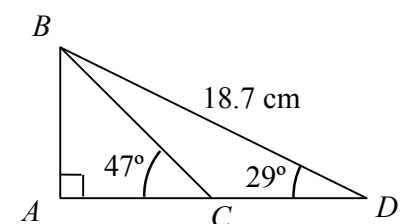
C2 Find angle CBD



C3 Find angle BAC



C4 Find length CD



Fluency Practice

(a) Mercy stands 50 m away from the foot of a tower. When she looks to the top of the tower, the angle of elevation is 41° . Find the height of the tower.

(b) Rob is a passenger on a boat. The boat is 450 m from the foot of a cliff, which is 110 m high. Find the angle of elevation of the top of the cliff from the boat.

(c) Talha is on plane, looking down at the airport with an angle of depression of 36° . The height of the plane from the ground is 5 km . Find the distance from the plane to the airport.

(d) A pigeon flies down to the ground from the top of a tree at an angle of depression of 28° . The distance the pigeon flies is 15.8 m . How tall is the tree?

(e) Salisbury cathedral spire is 123 m tall. Guy stands 46 m from the cathedral spire. What is the angle of elevation of the top of the spire from where Guy is standing?

(f) A plane passes overhead at a height of 8000 m . A short time later, it is at an angle of elevation of 71° . How far away is the plane from its original position?

(g) A prison officer watches prisoners from a guard tower which is 10.5 m tall. He looks due North and can see two prisoners. The angle of depression of each of the prisoners is 18° and 23° . How far apart along the ground are the two prisoners?

Fluency Practice

For each question, draw a diagram from the information given. Assume a right-angled triangle is used in each case.

Remember:

- make sure you include *all* information, including the unknown value and any labelling
- the diagrams do not need to be drawn to scale
- you are only interested in the triangle – you don't need to draw the whole picture!

1. A tree is 15m tall. At a certain time of day, it casts a shadow that is 25m long. A gardener wants to find angle x , which is the angle between the tip of the shadow and the top of the tree.

2. A pylon has been constructed 20m away from a farm gate. The farmer has worked out that the angle from the bottom of the farm gate (G) to the top of the pylon (P) is 42° . The farmer wants to know the height, h , of the pylon.

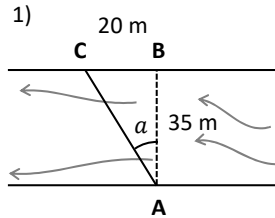
Fluency Practice

3. An abseiler has lowered 50m of rope from the top of a cliff (T). His friend (S) is standing away from the base of the cliff, holding the other end of the rope. The rope is making an angle of 70° with the ground. The abseiler wants to know the height of the cliff, c .

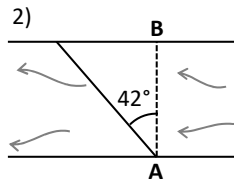
4. Bill the builder leans his 10m ladder against a house, at an angle of 47° . It touches the wall at point W , just below a window. Bill wants to know how high up the house (h) this is.

Fluency Practice

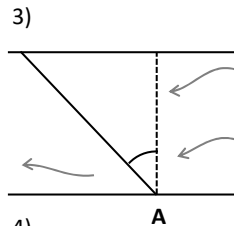
Practical Trigonometry Problems



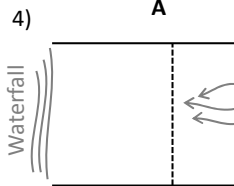
Jim tries to swim across this river from **A** to **B**.
The river is 35 m wide.
The current pushes him 20 m downstream to **C**.
What angle did Jim travel at? (a)



Rakesh tries to swim across this river from **A** to **B**.
The current pushes him 18 m downstream.
His angle of travel is 42° .
How wide is the river?

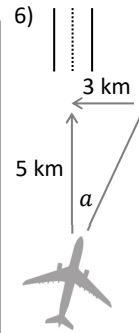


Hei tries to swim directly across a river from **A**.
The current creates an angle of travel of 56° .
In total she swims 43 m
How wide is the river?
How far is Hei pushed downstream from her target?



Jo tries to swim directly across a
58 m wide river, 38 m before a waterfall.
The current creates an angle of travel of 33° .
Does Jo make it across before the waterfall?

5)
A river's current has a speed of 0.5 metres per second.
Jo swims at 1.5 m/s.
At what angle (towards the current) does Jo
need to swim to travel perpendicular to the riverbank?



A runway is 5 km directly North from a plane coming to land.
The wind is coming directly from the East.
If the pilot does not fly into the wind, the wind will push
the plane 3 km West & off course.
At what angle (from North) should the pilot fly?



A plane is 15 km from a runway. A *crosswind* will push the plane
5.5 km perpendicular to the runway if not corrected for.
At what angle (towards the crosswind) should the pilot fly?

8)

A plane's engine propels it forward at 200 m/s.
A crosswind pushes the plane directly sideways at 35 m/s.
How far does the plane actually travel in one second? (its groundspeed)
At what angle does the plane fly relative to where it is pointed?

9)

A plane maintains a groundspeed of 270 m/s directly North..
To maintain its heading, the plane is angled 15° **into** a crosswind.
What is the speed of the crosswind?

10)

A plane's engine propels it forward at 305 m/s.
The plane is angled **into** a 45 m/s crosswind.
What is the plane's angle relative to its direction of travel?
What is the plane's groundspeed?

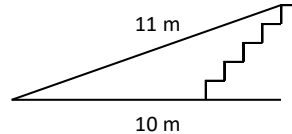
Fluency Practice

Practical Problems using Trigonometry

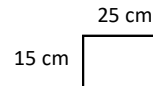
Diagrams are not to scale.

- 1) In the UK, a wheelchair ramp must have a maximum gradient of 1:12 (rise : run)
What angle should a ramp make with the horizontal?

- 2) An architect designs this ramp next to some stairs. Is the ramp legal?



- 3) The diagram shows one step from a set of 20 steps.
If a ramp starts at the bottom of the first step & has an angle of 4° , how long does it need to be?

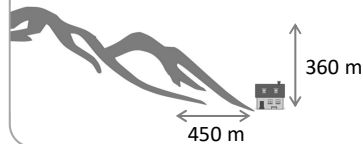


- 4) A volcano is 1867 m high. Jae starts at sea level and walks 4.6 km to the top.
At what average angle did he walk?

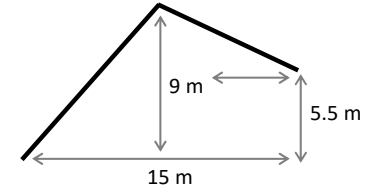
- 5) A palm tree will fall if it leans more than 28° from the vertical.
If a palm tree is 15 m tall, how far can its top overhang its base without it falling?



- 6) Heavy snowfall lands on the side of a mountain. An avalanche will happen if the slope angle (from the ground) is greater than 38° .
Will an avalanche hit the house?



- 7) In a village in the Alps, roofs must be built at least a 30° roof angle to let snow fall. An architect designs this roof with the 'ridgeline' in the middle.
Is the design legal?



- 8) In tropical climates, a roof should have a minimum slope of 7° to ensure rain runoff. If a building is 26 m wide,
what is the minimum length of the roof?

- 9) The basin of a walk-in shower needs to be angled at a minimum of 1.5° .
If the basin is square, with a length of 1.3 m & the drain is in one corner,
how high should the opposite corner be (in cm)?

- 10) Kim is directly West of the peak of a mountain. She walks 1 km directly south.
The mountain is now on a bearing of 084° (from North) from Kim.
What is the horizontal distance from Kim to the mountain?

The mountain peak is at an angle of elevation of 21° .
How high is the mountain?

Fluency Practice

Angles of Elevation & Depression

A zip wire starts on a hill 135 m high.
Assume it ends at a height of 0 m.
The wire is 440 m long.
At what angle of depression does a passenger travel?
Is this angle constant?

Jack is standing in a park – ignore his height.
He is 75 m from a tree. The top of the tree is 82 m away.
What is the angle of elevation from Jack to the top of the tree?

A cloud is 960 m East of Jack at a height of 380 m.
What is the angle of elevation from Jack to the cloud?

A control tower is monitoring two planes.
From the top of the tower (60 m) plane 1 has
an angle of elevation of 24° and is 450 m from the tower.
How high is plane 1?

Plane 2 is 132 m high at an angle of elevation of 8° from the tower.
How far away is plane 2 from the tower?

A UFO takes off, travels 7.2 km and
is at an angle of elevation of 76° from its launch pad.

A 2nd UFO is a horizontal distance of 8.4 km and
an angle of depression of 17° from the 1st UFO.
How high is the 2nd UFO?

Fluency Practice

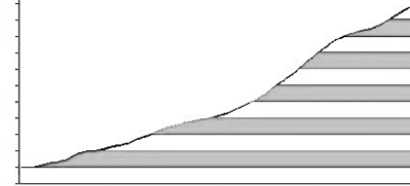
angle of the incline

the **Duquesne** incline in Pittsburgh, Pennsylvania



in 242m length of track
the height increase is 122m

the **Manitou** incline in Colorado



a steep hill, with some steps – a challenge for physically fit people to climb
in about $\frac{3}{4}$ mile horizontally (1200m)
the height increases by 610m

the **Langkawi** cable car in Malaysia



the safe angle for cable cars is between 20 and 45 degrees

rises 650m
for a horizontal distance of 1573m
is this a safe angle?

ladder

for safety, it is recommended that a ladder reaches 4 times as much up a wall as the distance of the base from the wall

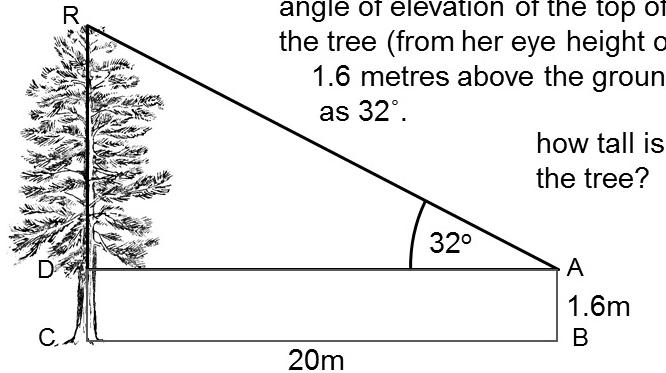


what (safe) angle does the ladder make with the ground?

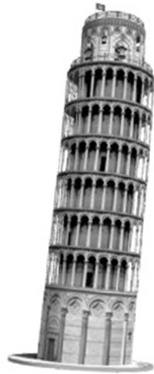
Fluency Practice

Phoebe stands on level ground, at B, 20 metres away from the foot C of a tree (CDR)

she whips out her handy pocket clinometer and measures the angle of elevation of the top of the tree (from her eye height of 1.6 metres above the ground) as 32° .



how tall is the tree?



Phoebe drops her pocket clinometer off the top of the leaning tower of Pisa

it falls to the ground, 3m from the base of the tower

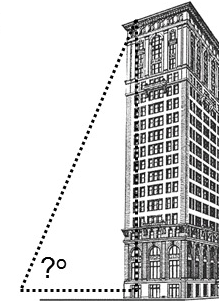
the tower was then leaning at an angle of 87.1° to the ground.

how far did it fall?

Phoebe still stands on level ground, a distance away from the foot of a building known to be 174.6m high

what will the angle of elevation be from a distance of:

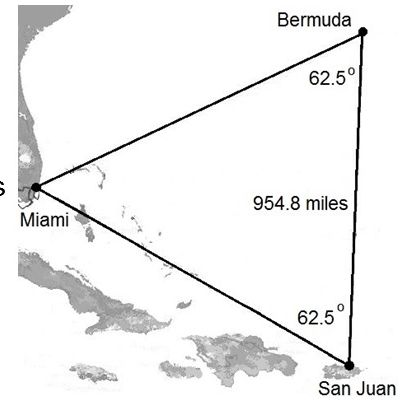
- (a) 50 metres away
- (b) 100metres away ?



the 'Bermuda triangle' (not recognised in any official way) is claimed to be 500,000 square miles in area

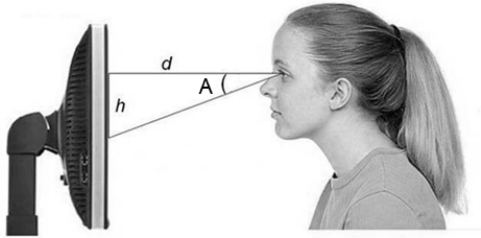
it is an isosceles triangle with base angles as shown

how far out is the area claim?



triangle joining Miami, San Juan in Puerto Rico and Hamilton, Bermuda

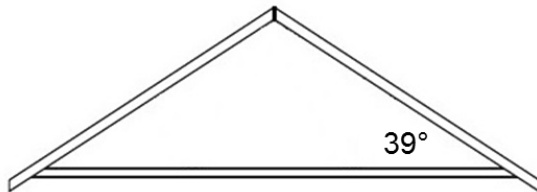
Fluency Practice



a person usually sits 50cm away from a screen

ease of reading guidance [ISO 9241 – 3] says that the font size (height) should be such that the angle subtended (angle A) should be between 0.33° and 0.37°

how high (in mm) should the font size be?



the pitch of a roof is to be 39 degrees

give some values for

(a) the width of a house

(b) the length of a roof

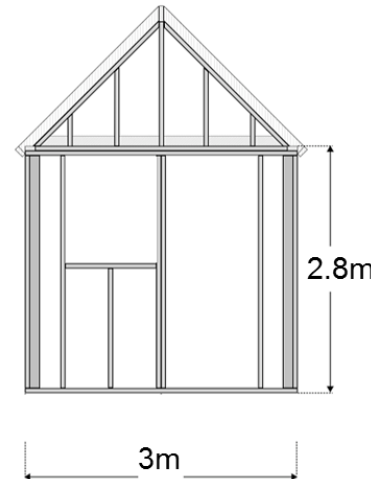
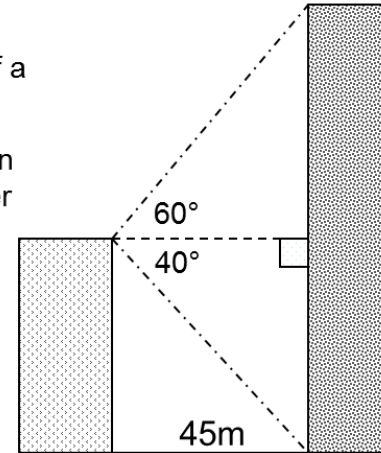
to give this pitch

from the top of one building the angle of inclination of the top of a taller building is 60°

the angle of depression of the base of this taller building is 40°

the gap between the buildings is 45m

how tall are both the buildings?



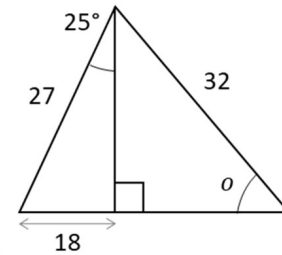
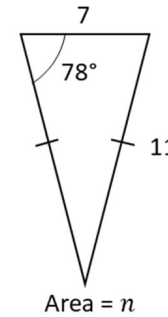
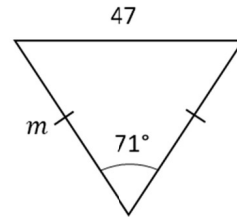
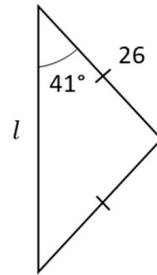
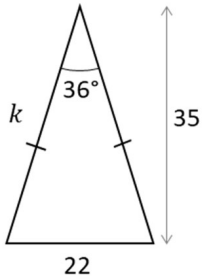
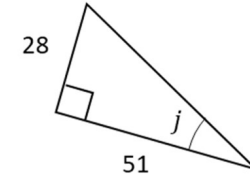
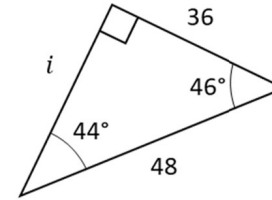
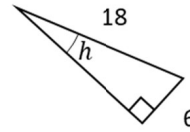
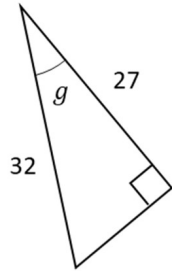
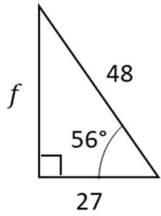
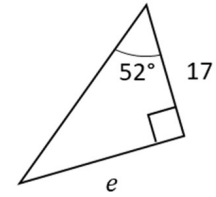
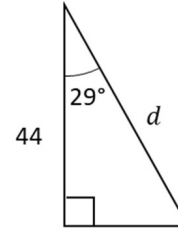
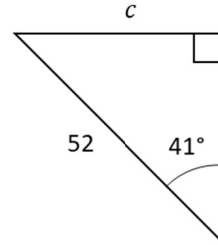
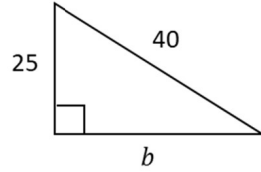
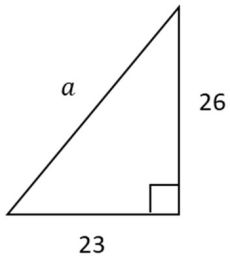
the pitch of the roof of a shed needs to be at least 23°

find the minimum height above ground level of the apex of the roof

Fluency Practice

Questions using Pythagoras' Theorem or Trigonometric Ratios

(lengths are centimetres) NOT TO SCALE



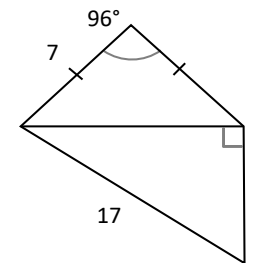
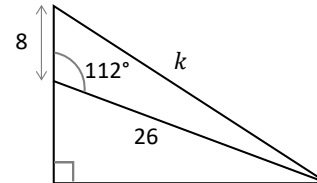
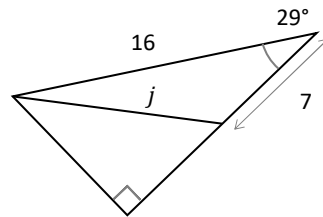
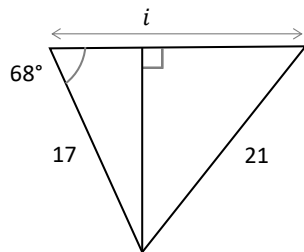
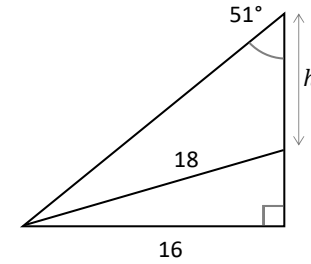
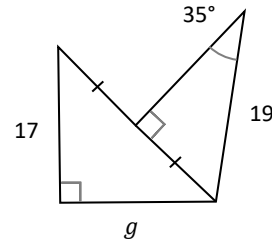
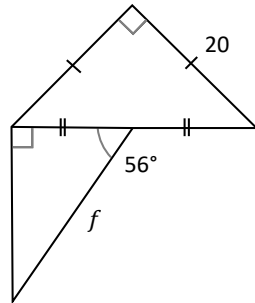
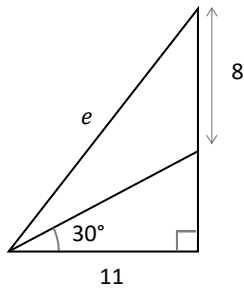
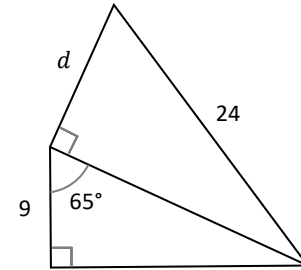
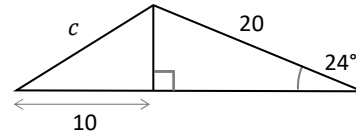
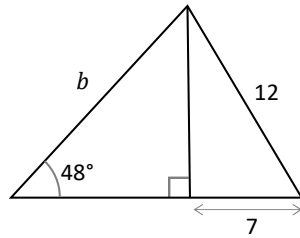
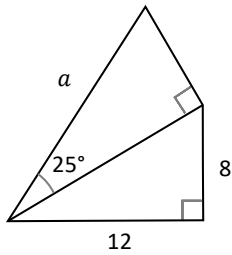
14 Answers	21.8	39.2	31.7	50.3	36.7	19.5	34.1
	39.7	31.2	36.5	32.5	34.7	28.8	40.5

Fluency Practice

Pythag. with Basic Trig. (Lengths)

Lengths are centimetres.

not to scale

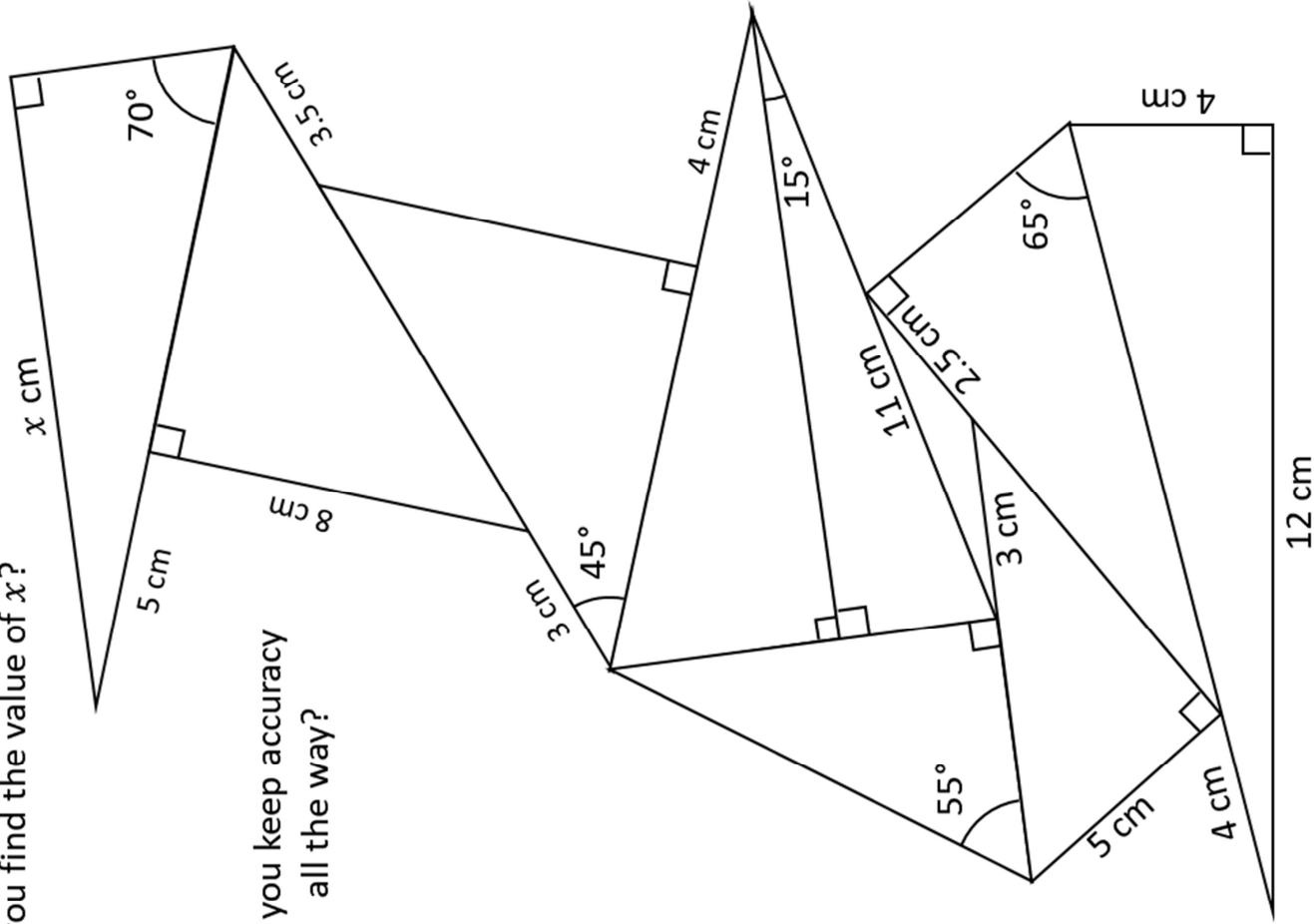


10 Answers	13.6	15.9	11.1	20.2	18.1
	12.9	10.4	25.3	13.1	4.7

Fluency Practice

Py-Trig Tower

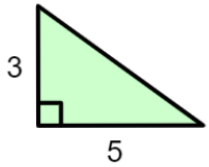
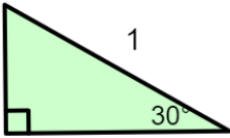
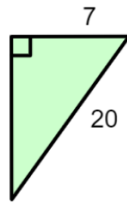
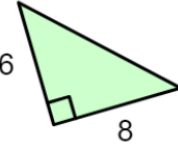
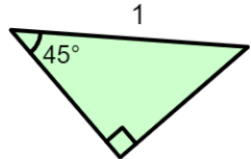
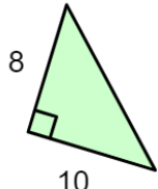
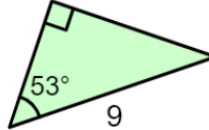
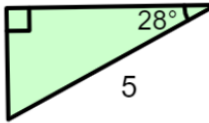
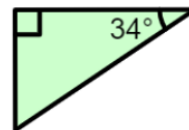
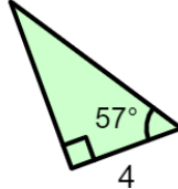
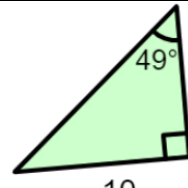
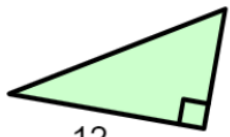
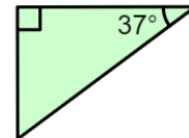
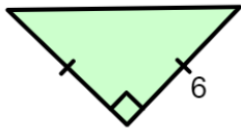

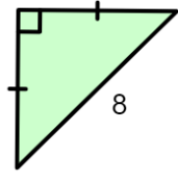
Start from the bottom of the tower. Can you find the value of x ?



Can you keep accuracy all the way?

perplexing perimeters?

Work out the perimeter of each triangle to 2 d.p. Cross of your answers from those on the right as you go.

<p>A</p> 	<p>B</p> 	<p>C</p> 	<p>D</p> 
<p>E</p> 	<p>F</p> 	<p>G</p> 	<p>H</p> 
<p>I</p> 	<p>J</p> 	<p>K</p> 	<p>L</p> 
<p>M</p> 	<p>N</p> 	<p>O</p> 	<p>P</p> 

2.37

2.41

11.76

13.83

17.08

17.50

19.31

20.49

21.60

24

24.05

30.81

31.94

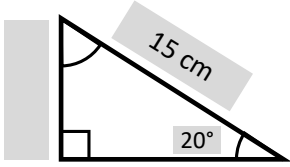
36

40.97

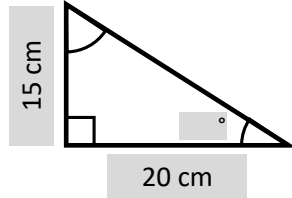
45.73

Fluency Practice

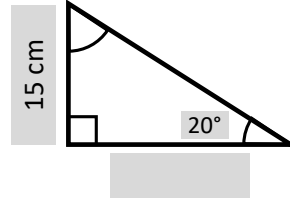
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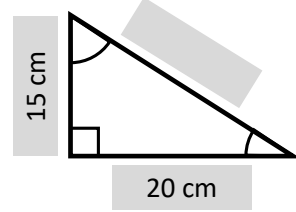
2.



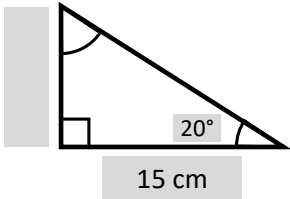
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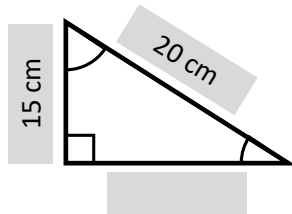
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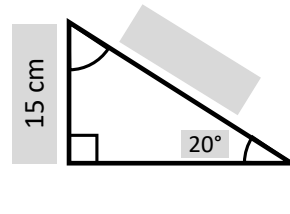
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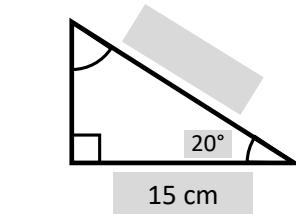
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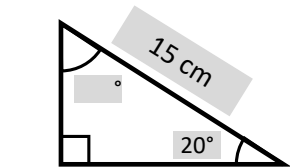
7.



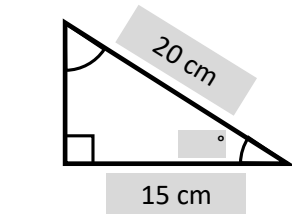
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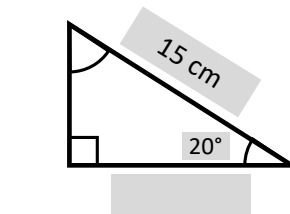
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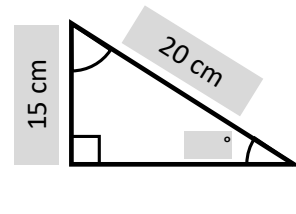
10.



11.

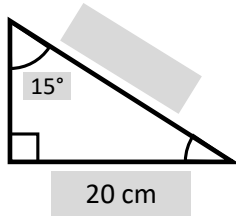


12.

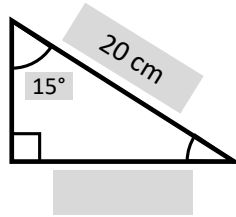


Fluency Practice

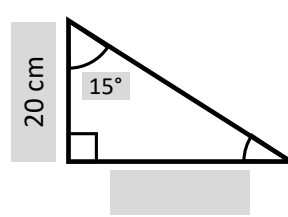
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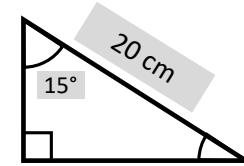
14.



15.

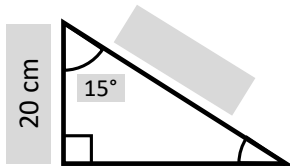


16.

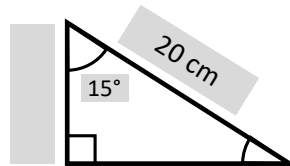


Perimeter =

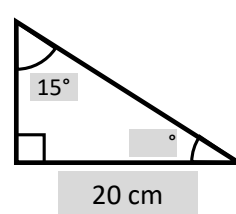
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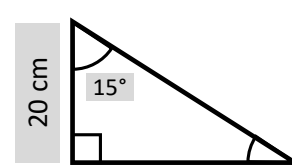
18.



19.

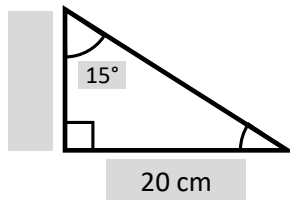


20.

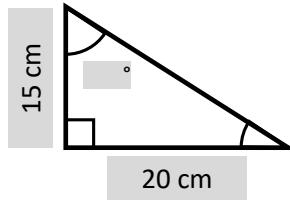


Area =

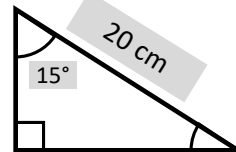
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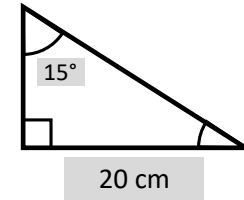
22.



23.



24.

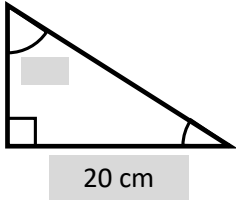


Area =

Perimeter =

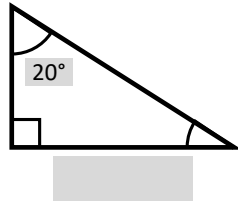
Fluency Practice

25.



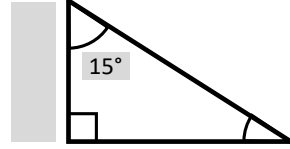
Area = 15 cm²

26.



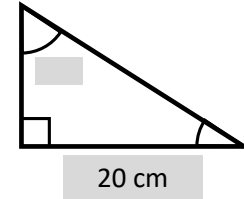
Perimeter = 15 cm

27.



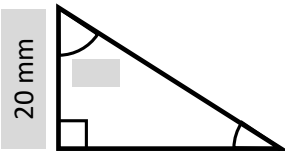
Area = 20 cm²

28.



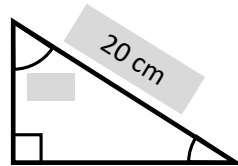
Perimeter = 15 m

29.



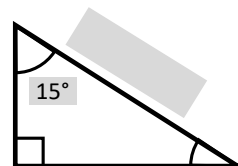
Perimeter = 15 cm

30.



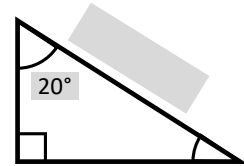
Area = 15 cm²

31.



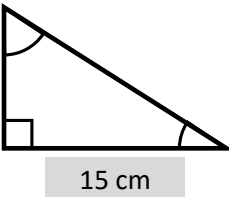
Perimeter = 20 cm

32.



Area = 15 cm²

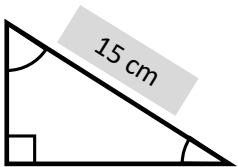
33.



Perimeter =

Area = 20 cm²

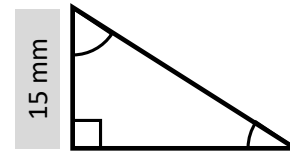
34.



Perimeter =

Area = 20 cm²

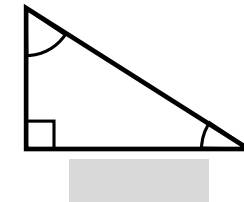
35.



Perimeter = 20 cm

Area =

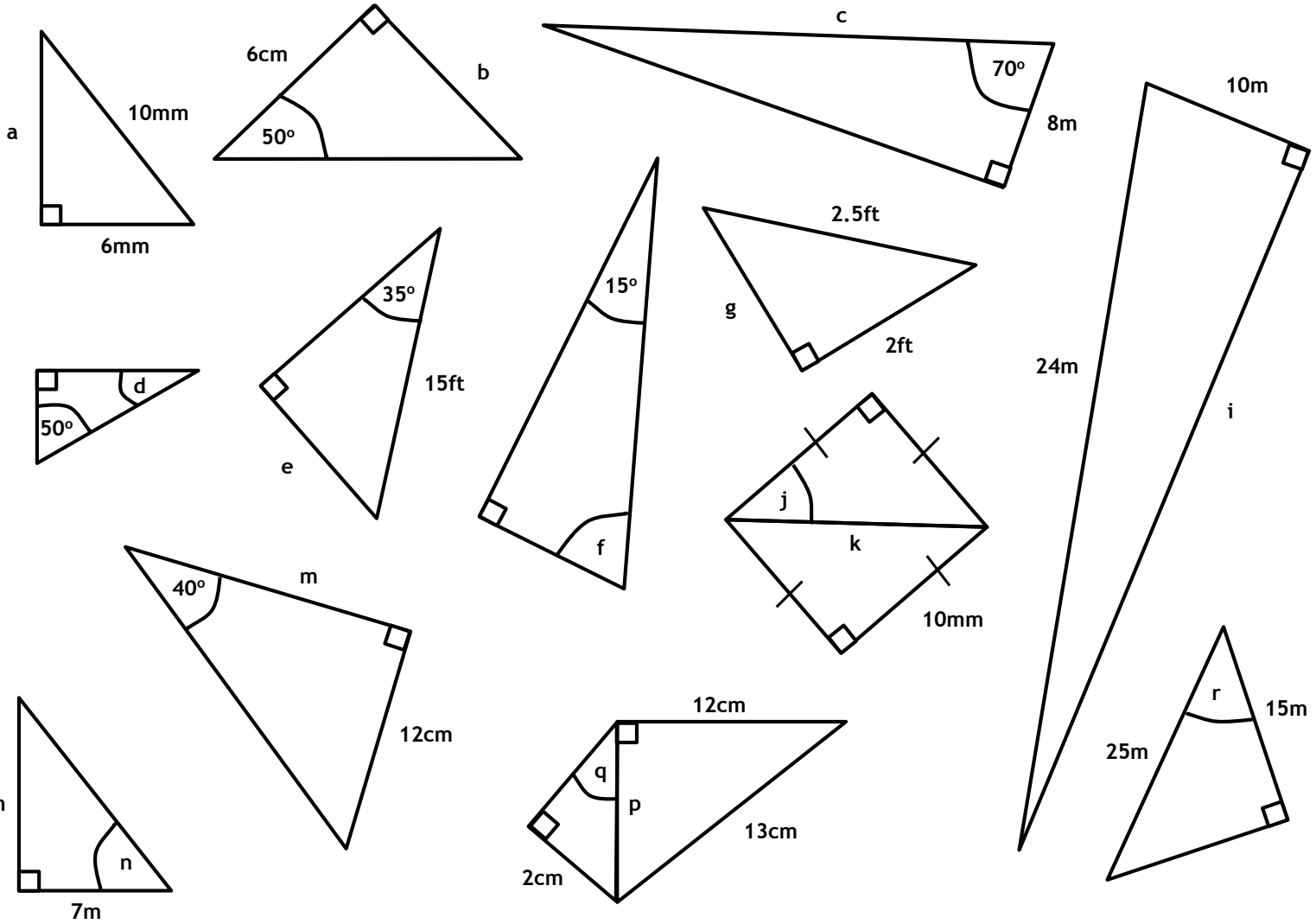
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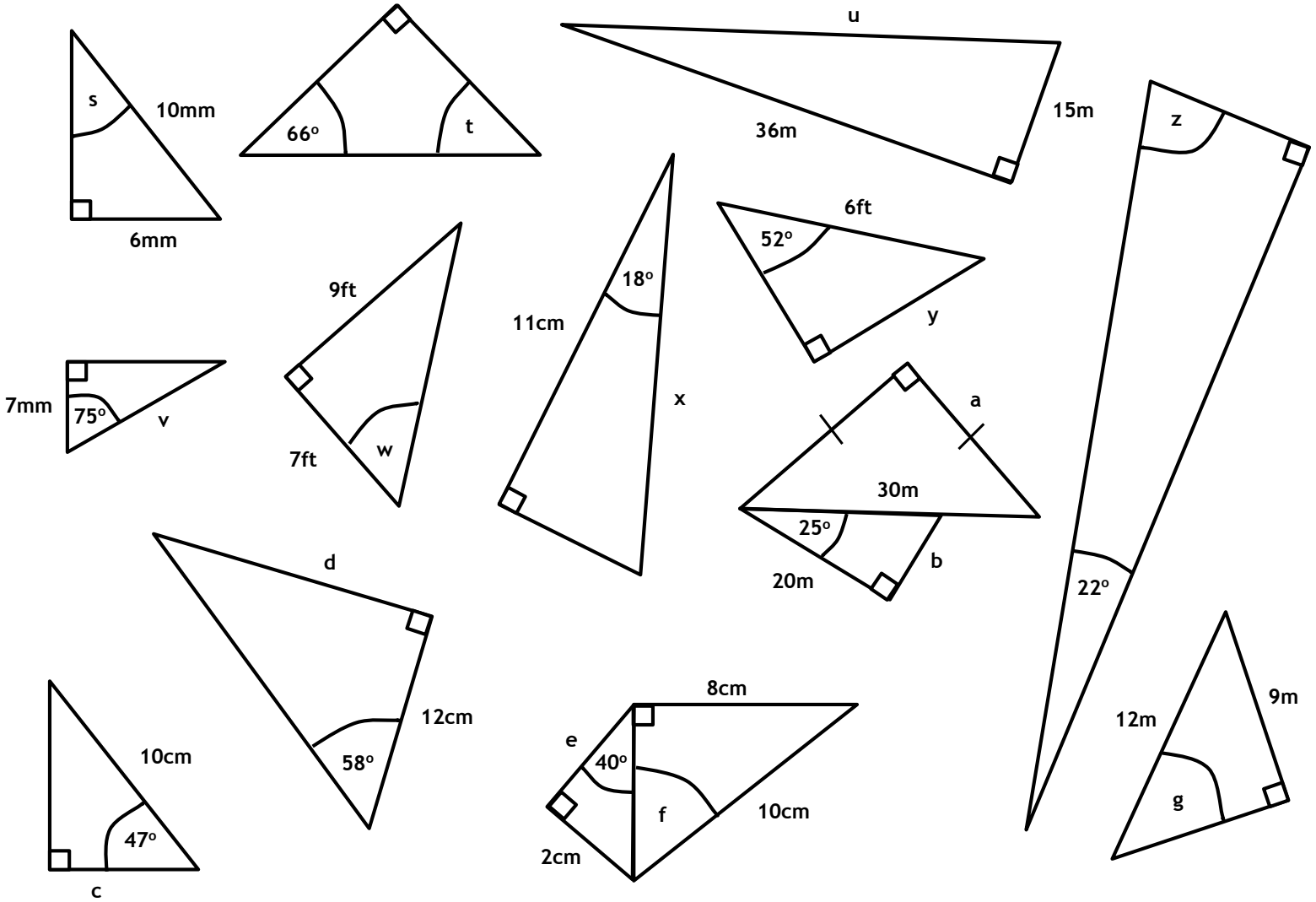
Perimeter = 20 cm

Area = 15 cm²

Fluency Practice

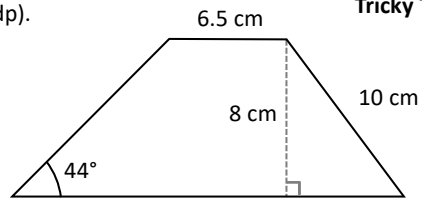


Fluency Practice



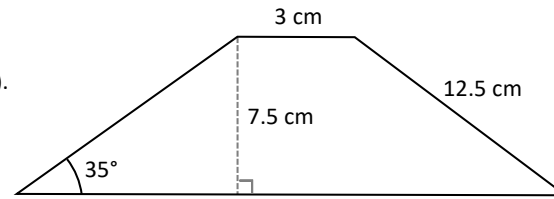
Fluency Practice

1) Find the area of this shape (2dp).

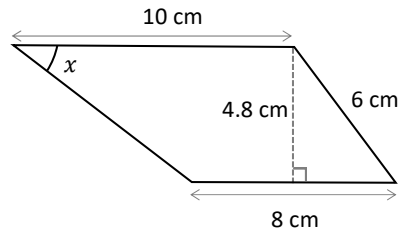


Tricky Trapeziums A

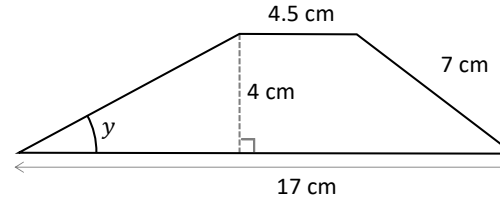
2) Find the area of this shape (2dp).



3) Find the size of angle x (2dp).

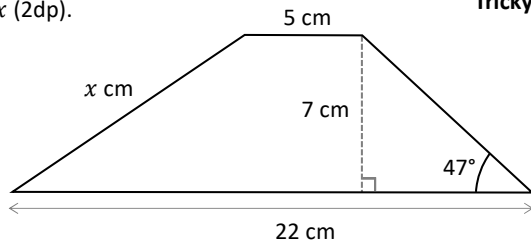


4) Find the size of angle y (2dp).



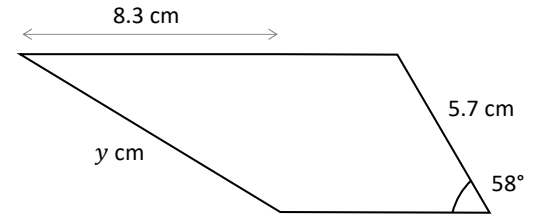
Fluency Practice

5) Find the value of x (2dp).

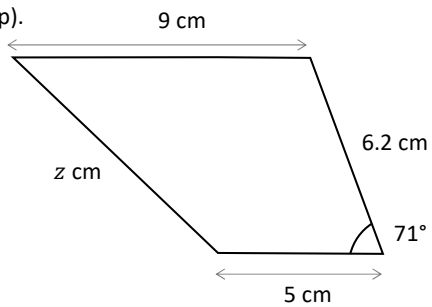


Tricky Trapeziums B

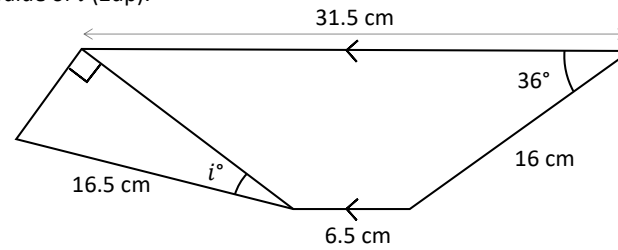
6) Find the value of y (2dp).



7) Find the value of z (2dp).

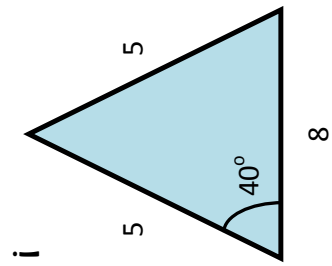
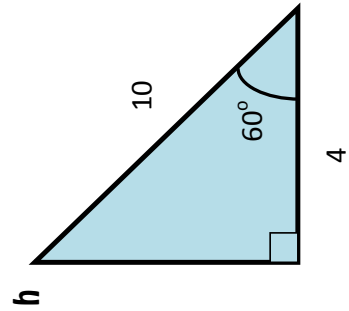
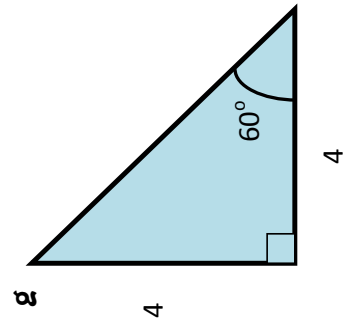
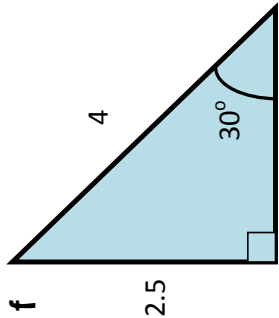
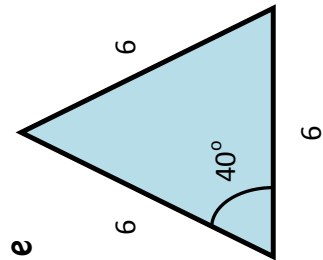
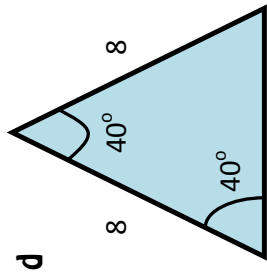
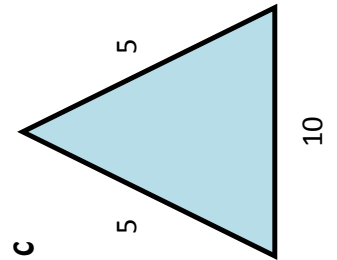
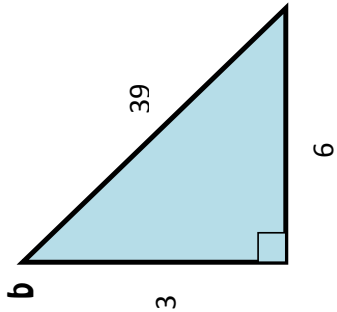
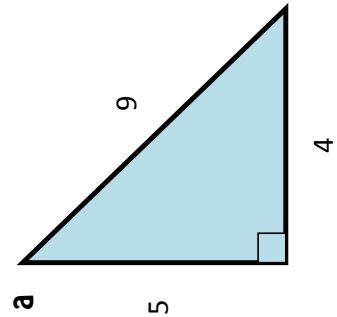


8) Find the value of i (2dp).



Problem Solving

Each of the following triangles cannot possibly exist. Can you explain why?



Challenge: Can you change just one number on each triangle so that they become possible?