



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2024 Mathematics 2025

Unit 17 Tasks – Part 1

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KING EDWARD VI
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KING EDWARD VI
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BIRMINGHAM

Year 10

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Unit 17 Tasks – Part 2

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KING EDWARD VI
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Year 10

2024 Mathematics 2025

Unit 17 Tasks – Part 3

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1 Bounds and Error Intervals

Fluency Practice

- 1) A number q , when rounded to the nearest 10 , is equal to 440
Find the upper and lower bound of q
- 2) A number p , when rounded to the nearest whole, is equal to 65
Find the upper and lower bound of p
- 3) A number x , when rounded to the nearest 100 , is equal to 4000
Find the upper and lower bound of x
- 4) A number x , when rounded to the nearest whole, is equal to 39
Find the upper and lower bound of x
- 5) A number y , when rounded to the nearest 10 , is equal to 420
Find the upper and lower bound of y
- 6) A number z , when rounded to the nearest 100 , is equal to 8300
Find the upper and lower bound of z
- 7) A number x , when rounded to the nearest 100 , is equal to 200
Find the upper and lower bound of x
- 8) A number x , when rounded to the nearest 10 , is equal to 380
Find the upper and lower bound of x
- 9) A number x , when rounded to the nearest 10 , is equal to 970
Find the upper and lower bound of x
- 10) A number p , when rounded to the nearest 10 , is equal to 30
Find the upper and lower bound of p

Fluency Practice

- 1) A number x , when rounded to 3 decimal places, is equal to 0.007
Find the upper and lower bound of x
- 2) A number x , when rounded to 1 decimal place, is equal to 18.4
Find the upper and lower bound of x
- 3) A number q , when rounded to 3 decimal places, is equal to 0.008
Find the upper and lower bound of q
- 4) A number p , when rounded to 3 decimal places, is equal to 0.055
Find the upper and lower bound of p
- 5) A number x , when rounded to 1 decimal place, is equal to 6.8
Find the upper and lower bound of x
- 6) A number z , when rounded to 1 decimal place, is equal to 44.9
Find the upper and lower bound of z
- 7) A number y , when rounded to 2 decimal places, is equal to 0.08
Find the upper and lower bound of y
- 8) A number x , when rounded to 3 decimal places, is equal to 0.007
Find the upper and lower bound of x
- 9) A number y , when rounded to 1 decimal place, is equal to 9.8
Find the upper and lower bound of y
- 10) A number z , when rounded to 1 decimal place, is equal to 78.9
Find the upper and lower bound of z

Fluency Practice

- 1) A number x , when rounded to 3 significant figures, is equal to 6.33
Find the upper and lower bound of x
- 2) A number q , when rounded to 1 significant figure, is equal to 8
Find the upper and lower bound of q
- 3) A number p , when rounded to 3 significant figures, is equal to 535
Find the upper and lower bound of p
- 4) A number y , when rounded to 3 significant figures, is equal to 5.61
Find the upper and lower bound of y
- 5) A number x , when rounded to 1 significant figure, is equal to 80
Find the upper and lower bound of x
- 6) A number x , when rounded to 3 significant figures, is equal to 9770
Find the upper and lower bound of x
- 7) A number p , when rounded to 2 significant figures, is equal to 0.86
Find the upper and lower bound of p
- 8) A number q , when rounded to 3 significant figures, is equal to 6.09
Find the upper and lower bound of q
- 9) A number q , when rounded to 3 significant figures, is equal to 0.988
Find the upper and lower bound of q
- 10) A number q , when rounded to 1 significant figure, is equal to 0.7
Find the upper and lower bound of q

Fluency Practice

- 1) A number y , when rounded to the nearest 10 , is equal to 20
Find the error interval for y
- 2) A number x , when rounded to 2 decimal places, is equal to 4.23
Find the error interval for x
- 3) A number x , when rounded to 1 decimal place, is equal to 12.5
Find the error interval for x
- 4) A number y , when rounded to the nearest 100 , is equal to 7500
Find the error interval for y
- 5) A number y , when rounded to 1 decimal place, is equal to 0.8
Find the error interval for y
- 6) A number y , when rounded to 1 decimal place, is equal to 88.4
Find the error interval for y
- 7) A number y , when rounded to the nearest 10 , is equal to 480
Find the error interval for y
- 8) A number y , when rounded to the nearest 100 , is equal to 8800
Find the error interval for y
- 9) A number x , when rounded to 1 decimal place, is equal to 9.9
Find the error interval for x
- 10) A number y , when rounded to 3 decimal places, is equal to 0.138
Find the error interval for y

Intelligent Practice

Value	Rounded to	Lower Bound	Upper Bound	Error Interval
6000	Nearest 1000			
6000	Nearest 100			
600	Nearest 100			
600	Nearest 10			
6000	Nearest 10			
60	Nearest 10			
60	To nearest whole			
6	To the nearest whole			
6000	To the nearest whole			
600	To the nearest whole			

Intelligent Practice

Value	Rounded to	Lower Bound	Upper Bound	Error Interval
400	1 significant figure			
400	3 significant figures			
400	2 significant figures			
40	2 significant figures			
4	2 significant figures			
4	1 significant figure			
0.4	1 significant figure			
0.3	1 significant figure			
30	2 significant figures			
0.03	1 significant figure			

Fluency Practice

Write these error intervals.

- (a) 50 cm to the nearest 10 cm
- (b) 150 cm to the nearest 10 cm
- (c) 800 g to the nearest 10 g
- (d) 800 g to the nearest 100 g
- (e) 500 m to the nearest 100 m
- (f) 500 m to the nearest 10 m
- (g) 500 m to the nearest m
- (h) 5 m to the nearest m
- (i) 55 g to the nearest g
- (j) 87 g to the nearest g

Write these error intervals.

- (a) 4.5 cm to 1 decimal place
- (b) 7.5 cm to 1 decimal place
- (c) 37.5 cm to 1 decimal place
- (d) 37.4 m to 1 decimal place
- (e) 30.4 g to 1 decimal place
- (f) 30.9 g to 1 decimal place
- (g) 30.0 g to 1 decimal place

Write the error intervals.

- (a) 500 g to 1 significant figure
- (b) 100 cm to 1 significant figure
- (c) 50 cm to 1 significant figure
- (d) 20 cm to 1 significant figure
- (e) 5 m to 1 significant figure
- (f) 2 m to 1 significant figure

Write the upper and lower bounds.

- (a) 347 cm to the nearest cm
- (b) 80 g to 1 significant figure
- (c) 54.6 m to 1 decimal place
- (d) 45.86 g to 2 decimal places
- (e) 52 cm to 2 significant figures

Fluency Practice

Maximum & Minimum : SF

Lower Bound	Rounded Number	Upper Bound
	200 (to the nearest 100)	
	500 (1 significant figure)	
	7000 (to the nearest 1000)	
	12 000 (2 significant figures)	
	80 (to the nearest 10)	
	80 (to the nearest 5)	
	40 (to the nearest 20)	
	450 (to the nearest 50)	
	830 (2 significant figures)	
	17 (to the nearest integer)	
	5280 (3 significant figures)	
	174 000 (3 sf)	
	0.1 (to the nearest tenth)	
	3.42 (3 sf)	

Maximum & Minimum : DP

Lower Bound	Rounded Number	Upper Bound
	5 (to the nearest integer)	
	0.2 (to the nearest tenth)	
	0.8 (to 1 decimal place)	
	7.1 (to 1 decimal place)	
	0.01 (nearest hundredth)	
	54.9 (to 1 decimal place)	
	0.17 (to 2 decimal places)	
	0.228 (to 3 dp)	
	5.0 (to the nearest tenth)	
	5.00 (nearest hundredth)	
	10.84 (to 2 dp)	
	2.000 (nearest thousandth)	
	9.40 (to 2 dp)	
	16.901 (to 3 dp)	

Fluency Practice

Try these Complete the following table:

	Measurement	Largest possible error	Upper bound	Lower bound
Height of a tree	50 m to the nearest m			
Mid-day temperature	28°C to the nearest degree			
Weight of a letter	32 g to the nearest g			
Time to complete task	40 minutes to the nearest minute			
Length of caterpillar	3.4 cm to 1 decimal place			
Patient's temperature	38.6°C to 1 decimal place			
Weight of parcel	2.9 kg to 1 decimal place			
Time to reach 60 mph	6.2 seconds to 1 decimal place			
Length of shelf	2.75 m to 2 decimal places			
Weight of fish	1.64 kg to 2 decimal places			
Sprint time	10.27 seconds to 2 decimal places			
Height of a hill	480 m to the nearest 10 m			
Width of drive	560 cm to the nearest 10 cm			
Weight of cake	1200 g to the nearest 10 g			
Weight of cake	1200 g to the nearest 100 g			
Length of a runway	1900 m to 2 significant figures			
Length of a runway	1900 m to 3 significant figures			
Weight of an aircraft	170 000 kg to 2 significant figures			
Weight of an aircraft	170 000 kg to 3 significant figures			

Fluency Practice

Here are some species of animal, and their numbers left in the wild (according to the WWF). Find their upper and lower bounds.

Animal	Number left	Rounding	Lower Bound	Upper Bound
African elephant	415,000	Nearest 1000		415,499
Amur leopard	60	Nearest 10		
Black rhino	5,500	Nearest 100		
Galápagos penguin	2,000	Nearest 1000		
Greater sage-grouse	100,000	Nearest 100,000		
Vaquita dolphin	10	Nearest 5		
Sundra tiger	"Under 400"	?!?		

Fluency Practice

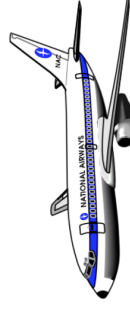
Question 1: Write down the lower bound and the upper bound for each of the following:

- (a) 4g measured to the nearest gram
- (b) 12cm correct to the nearest centimetre
- (c) 75 miles given to the nearest mile
- (d) 50kg measured to the nearest 10kg
- (e) 130 seconds given the nearest 10 seconds
- (f) 225km given to the nearest 5km
- (g) 400ml given to the nearest 100ml
- (h) 1900 hours correct to the nearest 10 hours
- (i) 2700mm measured to the nearest 100mm
- (j) 5000km correct to the nearest 100km
- (k) 28000kg measured to the nearest 10kg
- (l) 30000km/h given to the nearest 10000km/h



Question 2: Write down the lower bound and the upper bound for each of the following:

- (a) 80 people given to the nearest 10 people
- (b) £10 given to the nearest pound
- (c) 500 chairs correct to the nearest 100 chairs
- (d) 14000 flights given to the nearest 1000 flights
- (e) £29000 given to the nearest £100



Fluency Practice

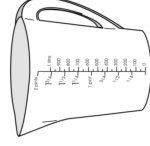
Question 3: Write down the lower bound and the upper bound for each of the following:

- (a) 3.8cm measured to the nearest 0.1 centimetre
- (b) 15.2 seconds to the nearest tenth of a second
- (c) 6.4g rounded to one decimal place
- (d) 515.9kg correct to one decimal place
- (e) 0.07 seconds rounded to two decimal places
- (f) 5.26mm measured to the nearest 0.01mm
- (g) 24.091kg correct to three decimal places
- (h) 8cm measured to the nearest 0.1cm



Question 4: Write down the lower bound and the upper bound for each of the following:

- (a) 4 miles correct to 1 significant figure
- (b) 30cm rounded to 1 significant figure
- (c) 900ml given to 1 significant figure
- (d) 0.2m given to 1 significant figure
- (e) 14 hours given to 2 significant figures
- (f) 280g correct to 2 significant figures
- (g) 42000km rounded to 2 significant figures
- (h) 748 gallons correct to 3 significant figures
- (i) 400m rounded to 2 significant figures
- (j) 8000kg given to 2 significant figures
- (k) 290000km/h given to 4 significant figures
- (l) 0.024 correct to 2 significant figures
- (m) 15.1 kilometres rounded to 3 significant figures
- (n) 100g correct to 1 significant figure



Apply



- Question 1: Declan is considering buying a sofa that is 207cm long. The space that Declan wants to the sofa is 210cm to the nearest 10cm. Should Declan buy the sofa?
- Question 2: Mr Jones wants to buy a notebook for every student in year 11. He knows there are 300 students in year 11 to the nearest 100. What is the greatest possible number of notebooks that he would have to buy?

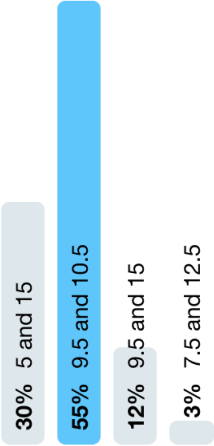
- Question 3: The length of a field is 400m to the nearest 10m. Rebecca says the lower bound is 350m and the upper bound is 450m. Is she correct?

1st Class		
Size	Weight up to and including	Price
Large Letter	100g	95p
	250g	£1.26
	500g	£1.68
	750g	£2.42

- Question 4: The table shows the prices of posting large letters. Gerard wants to post two large letters:
- a large letter weighing 230g to the nearest 10 grams
 - a large letter weighing 500g to the nearest 10 grams
- (a) What is the smallest possible price for posting both large letters?
- (b) What is the greatest possible price for posting both large letters?

- Question 5: Below is a question that was posted online and the results. Explain which answer you agree with.

A number has been rounded to 10, correct to 1 significant figure. What are the lower and upper bounds?



397 votes • Final results

learn by heart

Upper & Lower Bounds: The maximum & minimum values a measured quantity could have taken before rounding

examples

A length is measured to be 49.5cm to 1 d.p.
Write an inequality to show the error interval for this length.
 $49.45 \leq \text{length} < 49.55$

For continuous data, add and subtract half of the value to which the number has been rounded to find the upper and lower bounds.

The number of people at a football match was 300, correct to 1 significant figure. What are the upper and lower bounds for the actual number of people who attended?
 $250 \leq \text{people} \leq 349$

E.g. if the number is rounded to the tenths column, i.e. nearest 0.1, add half of this, 0.05, to find the upper bound

questions

1. Shade all the numbers in this grid which round to 600, to 1 significant figure:

64	521	653	580	700	550	599.99
679	620	500	60	593	650	549.5
6000	595	545	521	650.8	742	608.8

2. Each of these numbers has been rounded to the degree of accuracy shown. Write down the error interval for each measurement.

- a) 40.8 (1 d.p.)
- e) 90.9 (nearest tenth)
- b) 560 (nearest 10)
- f) 540 (nearest 20)
- c) 201 (nearest integer)
- g) 6000 (nearest 1000)
- d) 400 (nearest 100)
- h) 500 (nearest 10)

Fluency Practice

questions

3. Shade all the numbers in this grid which round to 0.5 to 1 significant figure.

0.8	0.56	0.407	0.55	0.491	0.4500	0.9
0.42	0.7	0.48	0.45	0.445	0.53	0.05
0.50	0.40	5	0.3689	0.500	0.545	0.491

4. Each of these numbers has been rounded to the degree of accuracy shown. Write down the error interval for each measurement.

- a) 55 (2 significant figures) e) 82.6 (3 significant figures)
- b) 0.8 (1 significant figure) f) 0.90 (2 significant figures)
- c) 4000 (1 significant figure) g) 2000 (3 significant figures)
- d) 8.23 (2 d.p.) h) 2000 (4 significant figures)
5. Which of these numbers has the largest upper bound?
- a) 500 (1 s.f.) b) 510 (2 s.f.) c) 516 (3 s.f.) d) 500 (2 s.f.)
6. Which of these numbers has the smallest lower bound?
- a) 30 (1 s.f.) b) 24 (2 s.f.) c) 22.5 (3 s.f.) d) 20 (2 s.f.)
7. The number of people attending a county fair was 1600, correct to two significant figures. What is the upper bound for the actual number?
- a) 1650 b) 1550 c) 1649 d) 1700
8. Which of these numbers is the same when rounded to either 1 significant figure or 2 significant figures?
- a) 452 b) 0.314 c) 495 d) 196

Problem Solving

To ride the SuperTwister rollercoaster you must be at least 120cm tall.

12 children measured their heights and recorded their findings.

Use the statements below to work out which children definitely can ride the SuperTwister, which ones definitely can't, and which ones might be able to.

Emma is 120cm tall, measured to the nearest cm.

Rakan is 125cm, to the nearest 5cm.

Paula is 122cm, to the nearest cm.

Ali is 121cm, to the nearest cm.

Caitlin is 1 meter, to the nearest meter.

Sheena is 130cm, to the nearest 10cm.

Jenny is 115cm, to the nearest 5cm.

Sarah is 120cm, to the nearest integer

Bob is 100cm, to the nearest 10cm.

Hayden is 1.2 meters tall, rounded to 1 d.p.

Carl is 110cm, to the nearest 10cm.

Harry is 150cm, to the nearest 50cm.

Definitely Can	can't be Sure	Definitely Can't

Problem Solving

Q. Choose two of these four numbers to make the **biggest** possible answer in each of the calculations below:

5.5

6.5

9.5

10.5

$$\square + \square$$

$$\square - \square$$

$$\square \times \square$$

$$\begin{array}{r} \square \\ - \\ \square \end{array}$$

Q. Now do it again, but this time try to make the **smallest** possible answer each time.

Fluency Practice

1) $p = qr$

$q = 0.095$ correct to 3 decimal places.

$r = 2000$ correct to 1 significant figure.

Work out the upper bound for the value of p

Give your answer correct to 3 decimal places when appropriate.

2) $a = 2b^2c$

$b = 0.2$ correct to 1 significant figure.

$c = 7.04$ correct to 3 significant figures.

Work out the upper bound for the value of a

Give your answer correct to 3 decimal places when appropriate.

3) $a = 2bc$

$b = 480$ correct to 2 significant figures.

$c = 9.73$ correct to 2 decimal places.

Work out the upper bound for the value of a

Give your answer correct to 3 decimal places when appropriate.

4) $a = 5bc$

$b = 63$ correct to 2 significant figures.

$c = 581$ correct to 3 significant figures.

Work out the lower bound for the value of a

Give your answer correct to 3 decimal places when appropriate.

5) $p = 5q^2r$

$q = 84.4$ correct to 1 decimal place.

$r = 0.48$ correct to 2 decimal places.

Work out the lower bound for the value of p

Give your answer correct to 3 decimal places when appropriate.

Fluency Practice

1) $a = 2b + 5c$

$b = 0.042$ correct to 3 decimal places.

$c = 0.7$ correct to 1 significant figure.

Work out the upper bound for the value of a

Give your answer correct to 3 decimal places when appropriate.

2) $x = y + 2z$

$y = 1.1$ correct to 2 significant figures.

$z = 3.6$ correct to 1 decimal place.

Work out the lower bound for the value of x

Give your answer correct to 3 decimal places when appropriate.

3) $p = 5q + 3r$

$q = 0.963$ correct to 3 decimal places.

$r = 77.9$ correct to 1 decimal place.

Work out the upper bound for the value of p

Give your answer correct to 3 decimal places when appropriate.

4) $p = 2q + 5r$

$q = 0.064$ correct to 3 decimal places.

$r = 2.35$ correct to 2 decimal places.

Work out the lower bound for the value of p

Give your answer correct to 3 decimal places when appropriate.

5) $p = 2q + r$

$q = 0.89$ correct to 2 decimal places.

$r = 590$ correct to 2 significant figures.

Work out the lower bound for the value of p

Give your answer correct to 3 decimal places when appropriate.

Fluency Practice

1) $a = 2b - 4c$

$b = 0.8$ correct to 1 significant figure.

$c = 9.69$ correct to 2 decimal places.

Work out the upper bound for the value of a

Give your answer correct to 3 decimal places when appropriate.

2) $x = 4y - z$

$y = 0.7$ correct to 1 significant figure.

$z = 0.4$ correct to 1 significant figure.

Work out the upper bound for the value of x

Give your answer correct to 3 decimal places when appropriate.

3) $a = 5b - 5c$

$b = 0.058$ correct to 2 significant figures.

$c = 8.4$ correct to 1 decimal place.

Work out the lower bound for the value of a

Give your answer correct to 3 decimal places when appropriate.

4) $a = 3b - 5c$

$b = 2.53$ correct to 2 decimal places.

$c = 3700$ correct to 2 significant figures.

Work out the upper bound for the value of a

Give your answer correct to 3 decimal places when appropriate.

5) $a = b - 4c$

$b = 0.64$ correct to 2 decimal places.

$c = 0.416$ correct to 3 decimal places.

Work out the lower bound for the value of a

Give your answer correct to 3 decimal places when appropriate.

Fluency Practice

1) $p = \frac{3q}{r}$

$q = 0.321$ correct to 3 decimal places.

$r = 0.007$ correct to 1 significant figure.

Work out the upper bound for the value of p

Give your answer correct to 3 decimal places when appropriate.

2) $x = \frac{4y}{z}$

$y = 0.188$ correct to 3 decimal places.

$z = 77.3$ correct to 1 decimal place.

Work out the upper bound for the value of x

Give your answer correct to 3 decimal places when appropriate.

3) $x = \frac{3y}{z^2}$

$y = 7.42$ correct to 2 decimal places.

$z = 500$ correct to 1 significant figure.

Work out the lower bound for the value of x

Give your answer correct to 3 decimal places when appropriate.

4) $p = \frac{5q}{r}$

$q = 0.32$ correct to 2 significant figures.

$r = 0.4$ correct to 1 significant figure.

Work out the upper bound for the value of p

Give your answer correct to 3 decimal places when appropriate.

5) $x = \frac{5y^3}{z}$

$y = 7.81$ correct to 2 decimal places.

$z = 0.37$ correct to 2 decimal places.

Work out the upper bound for the value of x

Give your answer correct to 3 decimal places when appropriate.

Fluency Practice

1) $a = \frac{b}{c+d}$

$b = 6.3$ correct to 1 decimal place.

$c = 7.49$ correct to 2 decimal places.

$d = 2.22$ correct to 2 decimal places.

Work out the lower bound for the value of a

Give your answer correct to 3 decimal places when appropriate.

2) $x = \frac{y}{z-w}$

$y = 45.5$ correct to 3 significant figures.

$z = 0.05$ correct to 1 significant figure.

$w = 1.73$ correct to 2 decimal places.

Work out the lower bound for the value of x

Give your answer correct to 3 decimal places when appropriate.

3) $x = \frac{y}{z-w}$

$y = 5.21$ correct to 2 decimal places.

$z = 2.7$ correct to 2 significant figures.

$w = 3.29$ correct to 2 decimal places.

Work out the lower bound for the value of x

Give your answer correct to 3 decimal places when appropriate.

4) $a = \frac{1}{bc}$

$b = 0.082$ correct to 3 decimal places.

$c = 0.081$ correct to 3 decimal places.

Work out the lower bound for the value of a

Give your answer correct to 3 decimal places when appropriate.

5) $p = \frac{5}{qr}$

$q = 383$ correct to 3 significant figures.

$r = 918$ correct to 3 significant figures.

Work out the lower bound for the value of p

Give your answer correct to 3 decimal places when appropriate.

Fluency Practice

The values for ***a*** & ***b*** are correct to the nearest ... ()...

	<i>a</i>	<i>b</i>
1	2 (integer)	4 (integer)
2	20 (10)	50 (10)
3	35 (5)	120 (10)
4	8 (1 sf)	40 (20)
5	300 (100)	350 (50)
6	7 (1 sf)	14 (2 sf)
7	1.4 (1 dp)	1 (integer)
8	6.0 (1 dp)	5.2 (2 sf)
9	1.48 (2 dp)	0.17 (2 dp)
10	2.00 (3 sf)	1.509 (3 dp)

Calculations with Error Intervals

Sum	
Lower Bound	Upper Bound

Difference	
Lower Bound	Upper Bound

Fluency Practice

$a = 30$ (nearest 10)	$b = 7$ (nearest integer)	$c = 2000$ (to 1 sf)	$d = 4.5$ (to 2 sf)	$e = 1.2$ (to 1 dp)	$f = 0.86$ (to 2 dp)

Find the LOWER BOUND of...

MIN.
MAX.

Find the UPPER BOUND of...



1) $a + b$

1) $b \times c$

2) $c - b$

2) $d + e$

3) $a - (b + d)$

3) $c - a$

4) $\frac{c}{a}$

4) $\frac{a + b}{e}$

5) $a - \frac{d}{e}$

5) $\frac{c}{ef}$

6) $\frac{e}{b - f}$

6) $\frac{de(c - f)}{b}$

Fluency Practice

Given that a is 40 to the nearest 10 and b is 8 correct to the nearest integer:	Given that e is 20 correct to the nearest five and f is 2.5 correct to 1 decimal place:	Given that p is 200 to 1 significant figure and q is 25 to 2 significant figures:	Given that x is 3 to the nearest integer, y is 1.5 to the nearest tenth and z is 12 to 2 significant figures:
(a)	(d)	(g)	(j)
Find the upper and lower bounds of $10a$	Find the upper and lower bounds of $e - f$	Find the upper and lower bounds of $\sqrt{p}$	Find the upper and lower bounds of $2(x + z - y)$
(b)	(e)	(h)	(k)
Find the upper and lower bounds of $a + b$	Find the upper and lower bounds of $\frac{e}{f}$	Find the upper and lower bounds of $\frac{1000}{pq}$	Find the upper and lower bounds of $\frac{z}{x-y}$
(c)	(f)	(i)	(l)
Find the upper and lower bounds of $a \times b$	Find the upper and lower bounds of e^2	Find the upper and lower bounds of $\sqrt{\frac{1}{p-q}}$	Find the upper and lower bounds of $z - x \times 2^y$

Problem Solving

Make each calculation
as small as possible

2

5

10

20

$$\boxed{} + \boxed{}$$

$$\boxed{} - \boxed{}$$

$$\boxed{} \times \boxed{}$$

$$\frac{\boxed{}}{\boxed{}}$$

$$(\boxed{})^2$$

$$\sqrt{\frac{\boxed{}}{\boxed{}}}$$

$$(\boxed{} - \boxed{})^2$$

$$\frac{\boxed{}}{\boxed{}}$$

More-Same-Less

Instructions: Calculate the lower bound and under bound of middle box. Complete the remaining boxes changing as little as possible.

Lower Bound of x

		Less	Same	More
<u>Lower bound of $x + y$</u>	More			
	Same		$2x + y$ $x = 2.3 \text{ to } 1 \text{ d.p.}$ $y = 4.9 \text{ to } 1 \text{ d.p.}$	
	Less			

Fluency Practice

Question 1: India has pieces of ribbon, each measuring 7cm to the nearest centimetre.

- (a) find the least total length of 10 pieces of ribbon.
- (b) find the greatest total length of 10 pieces of ribbon.

Question 2: Leon has four barrels, each with a mass of 30kg to the nearest 10 kg.

- (a) find the minimum possible mass of the four barrels.
- (b) find the maximum possible mass of the four barrels.

Question 3: A rectangle has a length of 14cm and width of 5cm, both to nearest centimetre.

Find (a) the maximum possible area and
 (b) the minimum possible area.



The diagram shows a yellow rectangle. To its left, the text '14cm' is written vertically. To its right, the text '5cm' is written vertically.

Question 4: The classes in a primary school have 20 students to the nearest 10.
There are 7 classes in the primary school.
Work out the greatest possible number of students that attend the school.

Question 5: Harry and Peter take part in a race.
It took Harry 30 seconds to the nearest 10 seconds to finish the race.
It took Peter 43 seconds to the nearest second to finish the race.
Work out the minimum possible difference between their finishing times.

Question 6: An average orange weighs 131g to the nearest gram.
A net contains 8 oranges.
The net weighs 10g to the nearest gram

What is the maximum possible weight of the net of oranges?

Question 7: Megan has 2 litres of fruit juice to the nearest litre.
She pours the fruit juice into glasses that hold 100ml to the nearest 10ml.
Work out the lowest possible number of glasses she can fill.

Fluency Practice

Question 8: A rectangular football pitch has a width of 72m, measured to the nearest metre. The length of the pitch is 105m, measured to the nearest 5 metres.

Work out the lower bound for the perimeter of the pitch.

Question 9: The lengths of time taken for 4 people to complete a puzzle are listed below. Each time is given to one decimal place.

20.8 seconds	35.1 seconds	19.7 seconds	41.3 seconds
--------------	--------------	--------------	--------------

- (a) Work out the greatest possible range
- (b) Work out the smallest possible mean.

Question 10: Mr Rodgers wants to keep 28 new maths textbooks on a shelf in his classroom. Each book has a mass of 700g correct to 1 significant figure. The shelf can hold up to 20kg to the nearest kilogram. Can the shelf safely hold the textbooks?

Question 11: The base of a triangle is 30cm, correct to 2 significant figures. The height of the triangle is 40cm, correct to 1 significant figure. Calculate the upper bound for the area of the triangle

Question 12: Kelly drove a distance of 120 miles, to the nearest 10 miles, in a time of 2 hours, to the nearest hour. Work out the difference between Kelly's greatest possible and lowest possible average speed.

Question 13: Rosie is buying strawberries, apples and grapes for a picnic. She buys 4kg of strawberries and 3kg of grapes, both to the nearest kilogram. Rosie buys 50 apples to the nearest 10 apples. A kilogram of strawberries costs £1.20 to the nearest 10p. A kilogram of grapes costs £1.30 to the nearest 10p. An apple costs 20p each to the nearest 10p. Work out the upper bound for the amount of money Rosie would have to pay

Question 14: A circle has an area of 600cm^2 to 2 significant figures. Work out the lower bound of the radius.

Question 15: $w = aT$

Given $a = 15$ correct to 2 significant figures
and $w = 700$ correct to 2 significant figures
Calculate the upper bound for T

Fluency Practice

Question 16: Shane estimated the distance between Cardiff and Swansea is 40 miles and that his average driving speed would be 60 mph.

He estimated the distance to the nearest 5 miles and the speed to the nearest 10mph.

Calculate the upper bound for the time the journey should take.
Give your answer to the nearest minute.

Question 17: A solid metal sphere has a radius of 4cm to 1 significant figure.
The sphere has a mass of 1200g to 2 significant figures.

Work out the lower bound for the density of the metal.

Question 18: The final velocity of a traveling object is given by the formula, $v = u + at$

where v is the final velocity
 u is the initial velocity
 a is the acceleration
and t is the time.

Given $u = 5.4\text{m/s}$ correct to 1 decimal place
 $a = 4.9\text{m/s}^2$ correct to 1 decimal place
 $v = 25.32$ correct to 2 decimal places

Calculate the upper bound for t .

Question 19: The population of a country is 6.4×10^6 to the nearest hundred thousand

The area of country is $8.4 \times 10^4 \text{ km}^2$ to the nearest 100km^2

Calculate the lower bound of the population density.

Fluency Practice

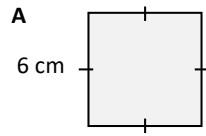
Calculations with Error Intervals

A = Area, P = Perimeter

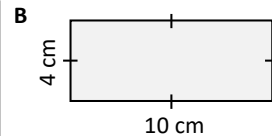
Unless stated otherwise, each number has been rounded to the nearest centimetre.

Round your answers to 2dp. Remember! The rounded answer must be **within** the error interval!

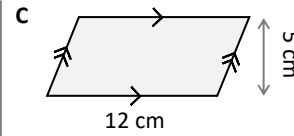
not drawn accurately



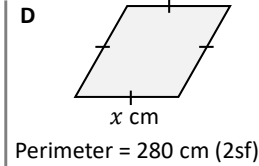
$$\leq P <$$



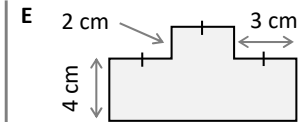
$$\leq A <$$



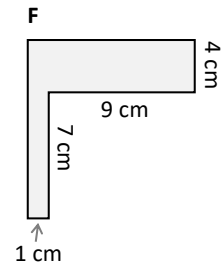
$$\leq P <$$



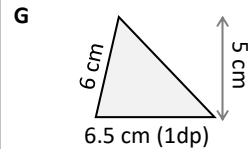
$$\leq x <$$



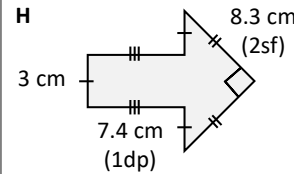
$$\leq A <$$



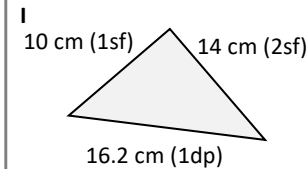
$$\leq A <$$



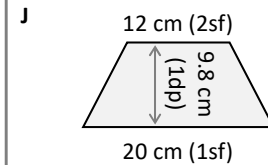
$$\leq A <$$



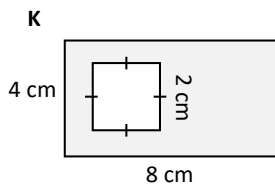
$$\leq A <$$



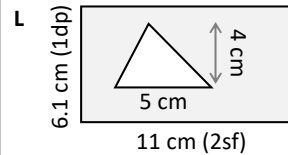
$$\leq P <$$



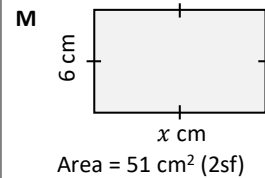
$$\leq A <$$



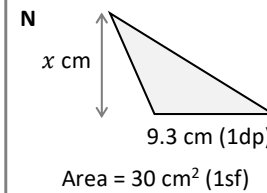
$$\leq A <$$



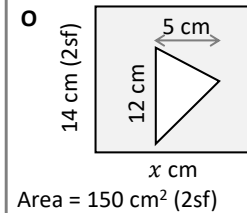
$$\leq A <$$



$$\leq x <$$



$$\leq x <$$



$$\leq x <$$

Fluency Practice

REHEARSE 1

Each value, x , has been rounded to the value in the brackets. Write down the error interval:

- a) 300 **(hundred)** b) 45 **(integer)**
 c) 140 **(ten)** d) 3000 **(thousand)**
 e) 600 **(ten)** f) 10 **(integer)**

REASON

Gemma's height is 190cm, correct to the nearest 10cm. Her height is 2m to the nearest metre. What is her minimum height? Which piece of information is most important?

REHEARSE 2

Each value, x , has been rounded to the value in the brackets. Write down the error interval:

- a) 3.6 **(1 dp)** b) 2.94 **(2 dp)**
 c) 8 **(1 dp)** d) 24.7 **(2 dp)**
 e) 430 **(2 dp)** f) 0.003 **(3 dp)**

EXPLAIN

Carly says that if a number is 500 correct to the nearest 100, then the error interval is

$$450 < x \leq 550$$

Do you agree? You must explain your answer

REHEARSE 3

Each value, x , has been rounded to the value in the brackets. Write down the error interval:

- a) 20 **(1 sf)** b) 2.9 **(2 sf)**
 c) 0.08 **(1 sf)** d) 0.47 **(2 sf)**
 e) 200 **(3 sf)** f) 100 **(1 sf)**

TOPIC +

The radius of a circle is measured as 3.2cm, correct to 1 decimal place. Calculate the error interval for the circumference of the circle, giving your answer in terms of π

example

Given $a = 3.4$ (correct to 1 d.p.) and $b = 4.6$ (correct to 1 d.p.),
work out the upper bound of $a \div b$

$\frac{a}{b}$ is greatest when a is largest and b is smallest.

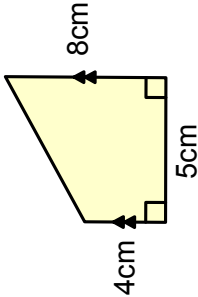
Upper bound = $\frac{3.45}{4.55} = 0.75824..$

questions

1. Complete the table to show the upper and lower bounds of each of these calculations:

a	b	calculation	upper bound	lower bound
2.8 (2 s.f.)	1.5 (2 s.f.)	$a + b$		
50 (1 s.f.)	30 (1 s.f.)	$a - b$		
0.5 (1 d.p.)	0.4 (1 d.p.)	$a - b$		
9.15 (2 d.p.)	3 (1 s.f.)	$a \times b$		
18 (2 s.f.)	5.1 (2 s.f.)	$a \div b$		
20 (1 s.f.)	0.8 (1 d.p.)	$a \div b$		

2. The measurements on this trapezium are all correct to the nearest cm.



Work out the upper bound for the area of the trapezium.

3. The radius of a circle is 3.84cm, correct to 3 significant figures.
Calculate the lower bound for its area. Give your answer to 3 d.p.
4. The area of a square is 42cm², correct to 2 significant figures.
What is the smallest possible side length it could have?
Give your answer to 2 d.p.

questions

In these questions, give your answers to 2 d.p. unless otherwise stated.

- 5. Given the formula $p = \frac{t}{s}$, work out the maximum value of p when t is measured to be 4.9 (1 d.p.) and s is measured as 1.65 (2 d.p.)
- 6. Given the formula $g = \frac{5}{tb}$, work out the maximum value of g when t is measured to be 0.8 (1 d.p.) and b is measured as 1.3 (1 d.p.)
- 7. Emma is painting a house. She is using tins of paint that cover 11m^2 to the nearest m^2 . If she has a total area of 300m^2 to paint, correct to 1 s.f., how many tins of paint should she buy to be certain she will cover the required area?
- 8. Dejaun is cutting small pieces from a length of rope which is 4m, correct to the nearest metre. Each small piece of rope is 2.8cm, to the nearest mm. What is the maximum number of pieces of rope that Dejaun might be able to cut from the rope?
- 9. 4800 people attended a football match, correct to 2 significant figures. Of these, 1800 were children correct to 2 significant figures. What is the maximum number of adults who could have attended?

errors in the error interval?

Which of these error intervals are wrong for the measurements given?

	Measurement	Error Interval
a)	A length, l , is 42.7cm (1 d.p.)	$42.65\text{cm} \leq l < 42.75\text{cm}$
b)	A weight, w , is 160kg (2 s.f.)	$159.5\text{kg} \leq w < 160.5\text{kg}$
c)	The number of people, p , is 400 (1 s.f.)	$350 \leq p \leq 449$
d)	A height, h , is 140cm (2 s.f.)	$135 \leq h < 144$
e)	An area, a , is 0.08m^2 (1 s.f.)	$0.075 \leq a < 0.085$
f)	A perimeter, p , is 100cm (2 s.f.)	$99.5 \leq p < 105\text{cm}$

Fluency Practice

Steve measures the length and width of a rectangle as 600 mm and 400 mm, both correct to 10 mm.

(a) Find the upper and lower bounds of the **perimeter** of the rectangle.

(b) Find the upper and lower bounds of the **area** of the rectangle.

Milly measures the length and width of a field as 25 m and 20 m, both to the nearest m.

(a) Find the upper and lower bounds of the **perimeter** of the field.

(b) Find the upper and lower bounds of the **area** of the field.

Ola's weight is 47 kg, correct to the nearest kg. Tia's weight is 55 kg, also correct to the nearest kg.

(a) Find the upper and lower bounds of the **total weight** of the two girls.

(b) Find the upper and lower bounds of the **difference** in their weights.

A car travels 240 km in 4 hours, both measured to the nearest unit. Find the upper and lower bounds of the car's speed.

A rock has a mass of 5 kg to the nearest 0.5 kg, and a volume of 2.1 m^3 to 1 decimal place. Find the upper and lower bounds of the density of the rock.

A cuboid has dimensions of 8 cm by 10 cm by 12 cm, all measured to the nearest cm. Find the upper and lower bounds of the volume of the cuboid.

Fluency Practice

(a)	(b)	(c)	(d)
Find the upper and lower bounds of 286 metres to the nearest metre.	Find the upper and lower bounds of 21 cm to the nearest cm.	Find the upper and lower bounds of 7.8 cm to 1 decimal place.	Find the upper and lower bounds of 5.24 kg to 2 decimal places.
(e)	(f)	(g)	(h)
Find the upper and lower bound of 80 cm to 1 significant figure.	Find the upper and lower bound of 5.6 kg to 2 significant figures.	A square has a side length of 4.1 cm to 1 decimal place. Find the lower bound of the perimeter of the square.	A rectangle measures 10 cm by 15 cm, both to the nearest cm. Find the upper bound of the area of the rectangle.
(i)	(j)	(k)	(l)
$a = b - c$ $c = 18$ correct to 2 significant figures. $b = 4.7$ correct to 1 decimal place. Find the upper and lower bounds of a .	$p = \frac{q}{r}$ $q = 20$ correct to 1 significant figure. $r = 6.3$ correct to 1 decimal place. Find the lower bound of p to 3 significant figures.	$c = \frac{d - e}{f}$ $d = 46, e = 8.5, f = 15$, all correct to 2 significant figures. Find the upper bound of c to 2 decimal places.	$x = \frac{3a}{g - b}$ $a = 28, b = 12, g = 18$, all correct to 2 significant figures. Find the lower bound of x to 3 significant figures.

Fluency Practice

A1 Zoe weighs 62 kg, correct to the nearest kilogram. Write down the lower bound for Zoe's weight.	A2 The length of line $AB = 8.3$ cm, correct to 2 significant figures. Write down the upper bound for the length of AB .	A3 Anu weighs 83 kg, correct to the nearest <u>half</u> kilogram. Write down the upper bound for Anu's weight.	A4 The length of line $CD = 27$ cm, correct to the nearest 0.5 cm Write down the lower bound for the length of CD .
B1 Correct to the nearest millimetre, the length of a side of a regular hexagon is 3.6 cm Calculate the upper bound for the perimeter of the hexagon.	B2 The perimeter of a square is 24 cm, correct to the nearest half centimetre. Work out the lower bound for the length of a side.	B3 Correct to 1 significant figure, the area of a rectangle is 80 cm^2 . Correct to 2 significant figures, the length of the rectangle is 12 cm. Calculate the upper bound for the width.	B4 Correct to 2 significant figures the area of a square is 230 cm^2 . Calculate the lower bound for the perimeter of the square.
C1 $x = 1.8$ correct to 1 decimal place. Calculate the lower bound for the value of $4x + 1$	C2 Correct to 1 significant figure, $a = 20$ and $b = 5$ Work out the upper bound of $5(a - b)$	C3 $x = p(q - r)$ $p = 42$, $q = 24$ and $r = 14$ all correct to 2 significant figures. Work out the lower bound for the value of x .	C4 Correct to 2 significant figures, $w = 58$, $x = 28$ and $y = 18$ Calculate the upper bound of $\frac{w}{x - y}$
D1 Jada has 100 litres of oil, correct to the nearest litre. The oil is poured into tins of volume 1.5 litres, correct to one decimal place. Calculate the upper bound for the number of tins that can be filled.	D2 There are 300 sheets of paper in a pile, correct to the nearest 10 sheets. The height of the pile is 160 mm, correct to the nearest 10 mm. Calculate the upper bound for the thickness of one sheet.	D3 The distance to school is 2.8 km, correct to the nearest 0.1 km. Sam walks at a speed of 5 km/h, correct to the nearest km/h. Calculate the upper bound for the time Sam takes to walk to school.	D4 Correct to 2 decimal places, the volume of a solid cube is 42.88 cm^3 Calculate the lower bound for the surface area of the cube.

Fluency Practice

1.



The inside of a mug is cylindrical with height 8.5 cm and diameter 7.2 cm , both correct to the nearest mm .

- a) What is the maximum capacity of the mug?
- b) What is the minimum capacity?

2.



Wood flooring is made in the form of rectangles measuring 24 cm by 11 cm , measurements correct to the nearest cm . When arranged as shown on the left:

- a) What is the largest possible gap?
- b) What is the smallest possible gap?

3.

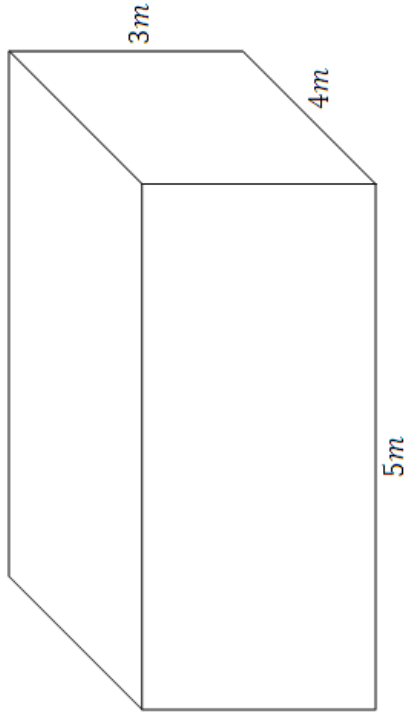


A restaurant provides a stick of butter to each table. The dimensions required by the restaurant chain's management are 3 cm by 3 cm by 8 cm , correct to the nearest cm .

- a) What is the largest possible volume?
- b) What is the smallest possible volume?
- c) What percentage saving would be made on butter if the smallest possible volume were provided compared to the largest?

Fluency Practice

The diagram below shows a room which is to be painted (walls and ceiling). Rough measurements have been taken and are correct to the nearest metre.



1. Calculate an estimate for the area to be painted, using the measurements given.
2. Paint comes in 2.5 litre tins which cost £18 and cover $25m^2$ of area. Find an upper bound for the cost of painting this room.
3. Calculate a minimum value for the amount of paint, in litres, needed for the room.
4. Enough tins of paint are bought to be certain of having enough. What is the maximum possible quantity of paint left over?

Fluency Practice

In each case you're trying to find a single suitable value of m to use (with justification), and you've calculated the upper and lower bounds.

1) $m_{lower} = 15.6241$
 $m_{upper} = 15.5935$

2) $m_{lower} = 148343$
 $m_{upper} = 151033$

3) $m_{lower} = 4.95013$
 $m_{upper} = 4.94502$

4) $m_{lower} = 1.29584$
 $m_{upper} = 1.29545$

Fluency Practice

- 1) $a = \frac{\sqrt{b}}{c}$
 $b = 0.018$ correct to 3 decimal places.
 $c = 0.032$ correct to 3 decimal places.
By considering bounds, work out the value of a , giving your answer to a suitable degree of accuracy.
- 2) $a = \frac{5}{bc}$
 $b = 5.54$ correct to 3 significant figures.
 $c = 0.437$ correct to 3 significant figures.
By considering bounds, work out the value of a , giving your answer to a suitable degree of accuracy.
- 3) $x = \frac{y}{z+w}$
 $y = 0.047$ correct to 3 decimal places.
 $z = 0.007$ correct to 1 significant figure.
- 4) $w = 0.016$ correct to 3 decimal places.
By considering bounds, work out the value of x , giving your answer to a suitable degree of accuracy.
- 5) $x = \frac{y^2}{z}$
 $y = 0.163$ correct to 3 decimal places.
 $z = 0.571$ correct to 3 significant figures.
By considering bounds, work out the value of x , giving your answer to a suitable degree of accuracy.
- 6) $p = \frac{q}{r-s}$
 $q = 8.7$ correct to 1 decimal place.
 $r = 0.02$ correct to 1 significant figure.
 $s = 0.894$ correct to 3 significant figures.
By considering bounds, work out the value of p , giving your answer to a suitable degree of accuracy.

Fluency Practice

example

Given that $p = \frac{t}{v}$ and $t = 42.8$ (1 d.p.) and $v = 16.6$ (1 d.p.), work out the value of p to a suitable degree of accuracy.

solution

1. Work out the upper bound of p , which is $\frac{42.85}{16.55} = 2.58912386...$
2. Work out the lower bound of p , which is $\frac{42.75}{16.65} = 2.567567567..$
3. Look at which digits agree, i.e. they are both **2.6** (1 d.p.)

questions

1. Given that $a = x + c$ and $x = 109.31$ (2 d.p.) and $c = 1.42$ (2 d.p.), work out a to a suitable degree of accuracy.
2. Given that $t = \frac{p}{v}$, and $p = 47.3$ (3 s.f.) and $v = 27.6$ (2 s.f.), work out t to a suitable degree of accuracy.
3. Sally is 1.8m tall, correct to 2 significant figures.
Asha is 1.65m tall, correct to 3 significant figures.
What is the maximum possible difference between their heights?
4. Given that $g = s^4$ and $s = 1.14$ (2 d.p.), work out the value of g to a suitable degree of accuracy.
5. Given that $p = t - q$, and $t = 4.6$ (1 d.p.) and $q = 3$ (nearest integer), use an inequality to write down an error interval for p .
6. Milo weighs 48.2kg and Ana 42.1kg, both correct to 3 significant figures.
Work out their maximum possible total combined weight.
7. Given that $b = \frac{\sqrt{a}}{4}$ and $a = 9.74$ (2 d.p.) work out the value of b to a suitable degree of accuracy.

Fluency Practice

- 1) Truncate 38.25369 to 1 decimal place.
- 2) Truncate 78.15659 to 1 decimal place.
- 3) Truncate 16.85692 to 3 decimal places.
- 4) Truncate 86.13870 to 1 decimal place.
- 5) Truncate 67.61322 to 1 decimal place.
- 6) Truncate 77.36573 to 1 decimal place.
- 7) Truncate 20.93772 to 3 decimal places.
- 8) Truncate 32.43570 to 3 decimal places.
- 9) Truncate 74.97334 to 1 decimal place.
- 10) Truncate 5.37433 to 2 decimal places.

learn by heart

Truncation: to shorten a decimal number without rounding

example

Truncate 42.89464 to 3 decimal places.
= 42.894

notice that this is
different to ROUNDING to
3 decimal places!

questions

1. Truncate 2.98456 to 3 d.p.
2. Truncate 43.00956 to 2 d.p.
3. Truncate 409.98 to the integer.
4. Truncate 5.03953 to 2 d.p.

5. Complete the table to compare different methods of rounding:

Number	Rounded to 1 significant figure	Rounded to 2 significant figures	Rounded to 2 decimal places	Truncated to 2 decimal places
408.6577				
0.48759				
989.00865				
500.506033				

6. Raul has chosen one of the numbers from the list below. Can you use the clues to work out which number he chose?

Clue 1: When rounded to 1 significant figure,
Raul's number is 3

2.68

2.50

Clue 2: When rounded to 1 decimal place,
Raul's number is 2.6

2.63

2.56

Clue 3: When truncated to 1 decimal place,
Raul's number is 2.5

2.52

3.54

Fluency Practice

- 1) A number p , when truncated to 2 decimal places, is equal to 0.77
Find the upper and lower bound of p
- 2) A number p , when truncated to 1 decimal place, is equal to 1.7
Find the upper and lower bound of p
- 3) A number z , when truncated to 1 decimal place, is equal to 1.4
Find the upper and lower bound of z
- 4) A number z , when truncated to 3 decimal places, is equal to 0.031
Find the upper and lower bound of z
- 5) A number x , when truncated to 3 decimal places, is equal to 0.555
Find the upper and lower bound of x
- 6) A number x , when truncated to 1 decimal place, is equal to 64.4
Find the upper and lower bound of x
- 7) A number x , when truncated to 3 decimal places, is equal to 0.026
Find the upper and lower bound of x
- 8) A number p , when truncated to 1 decimal place, is equal to 44.5
Find the upper and lower bound of p
- 9) A number p , when truncated to 1 decimal place, is equal to 81.3
Find the upper and lower bound of p
- 10) A number y , when truncated to 2 decimal places, is equal to 0.52
Find the upper and lower bound of y

Fluency Practice

Section A – Truncating Numbers

1. Truncate the following numbers to the unit:

- a) 25.3 b) 76.25 c) 10.9 d) 10.99 e) 10.999 f) 11.0001

2. Truncate the following numbers to the tens:

- a) 12 b) 58 c) 125 d) 972 e) 3684 f) 47209

3. Truncate the following numbers to the hundreds:

- a) 305 b) 395 c) 420 d) 4329 e) 670934 f) 1000001

4. Truncate the following numbers to 1 d.p.:

- a) 0.51 b) 0.58 c) 0.55 d) 1.452 e) 1.9538 f) 13.999999

5. Truncate the following numbers to 2 d.p.:

- a) 12.4534 b) 534.9872 c) 0.01684 d) 0.008452 e) 4.43920 f) 10.999123

Section B – The Error Interval for a Truncated Number

If each of the following numbers has been truncated as described, give the corresponding error interval:

1. 1 (to the unit)
2. 68 (to the unit)
3. 945 (to the unit)
4. 380 (to the tens)
5. 210 (to the tens)
6. 800 (to the hundreds)
7. 5400 (to the hundreds)
8. 10000 (to the hundreds)
9. 56.7 (to 1 d.p.)
10. 99.1 (to 1 d.p.)
11. 234.6 (to 1 d.p.)
12. 0.45 (to 2 d.p.)
13. 0.01 (to 2 d.p.)
14. 10.301 (to 3 d.p.)
15. 10.300 (to 3 d.p.)

Fluency Practice

example

When truncated to 2 decimal places,
 $p = 4.28$. Write down the error interval for p .

$$4.28 \leq p < 4.29$$

The smallest this number
could have been was
4.280000...

At its largest it could
have been 4.289999....

questions

1. Truncate 43.9859 to 2 decimal places.
 2. Truncate 109.49 to the integer.
 3. A distance, d , when truncated to 3 decimal places is 46.507.
Write down its upper and lower bounds.
 4. A weight, w , when truncated to 2 decimal places is 2.84.
Write down an inequality to show its error interval.
 5. A length, l , when truncated to 2 decimal places is 0.94.
Which of the following is the correct error interval for l ?
a) $0.94 < l < 0.949$
b) $0.935 \leq l < 0.945$
c) $0.94 \leq l < 0.95$
d) $0.94 < l < 0.95$
 6. Shade all of the numbers in this grid that truncate to 0.45.
- | | | | | | | | |
|--------|--------|-------|--------|--------|-------|--------|--------|
| 0.448 | 0.4512 | 0.4 | 0.5445 | 0.457 | 0.045 | 4.457 | 0.4501 |
| 0.4567 | 0.4499 | 0.458 | 0.4508 | 0.4051 | 0.544 | 0.4491 | 0.4 |
| 0.45 | 0.4 | 0.5 | 0.45 | 0.451 | 0.54 | 0.145 | 0.045 |
7. True or False: 1.85 rounded to 1 decimal place is greater than 1.85 truncated to 1 decimal place.
 8. True or False: 4.743 rounded to 1 decimal place is greater than 4.743 truncated to 2 decimal places.

Fluency Practice

Use inequalities to write down the error in these calculators with broken screens

a) $96______$

b) $63______$

c) $951______$

d) $0.05______$

e) $1.0______$

f) $81______$

(g) $6.2______$

(h) $0.______$

(i) $5_2______$

(j) $5______2______$

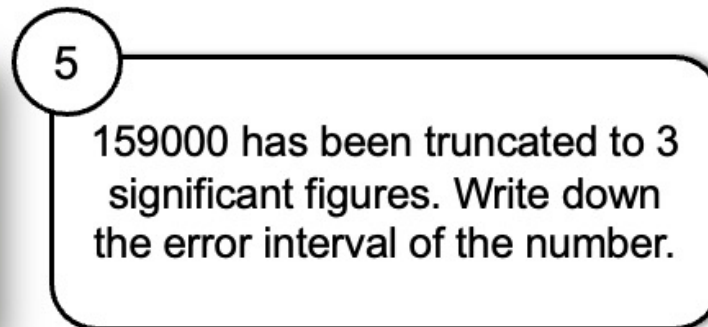
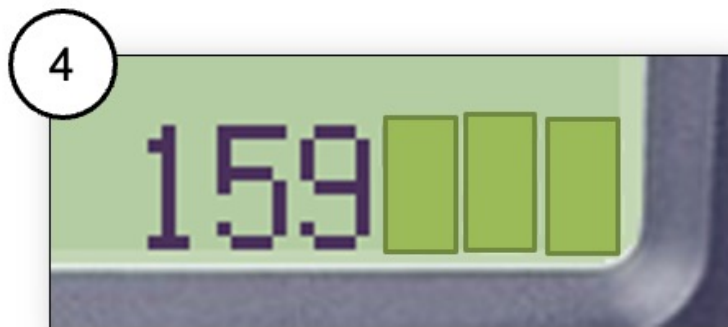
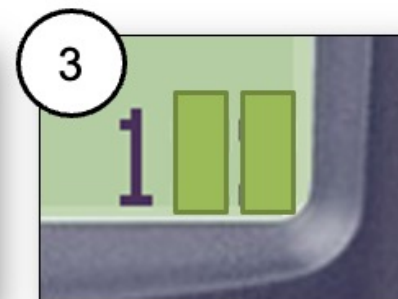
(k) $5______.2______$

(l) $0.05______2______$

Fluency Practice

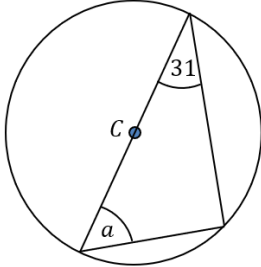
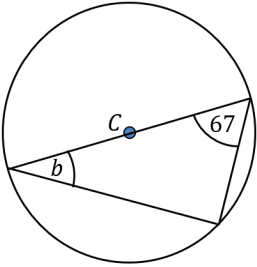
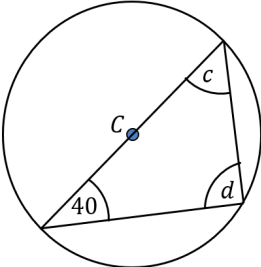
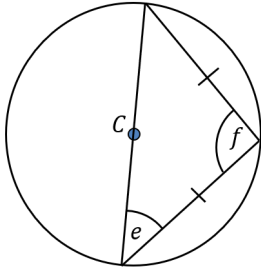
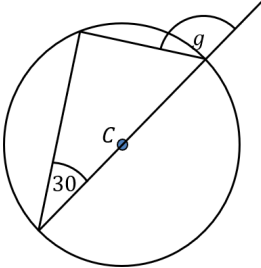
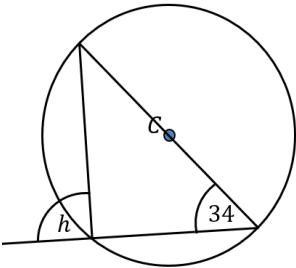
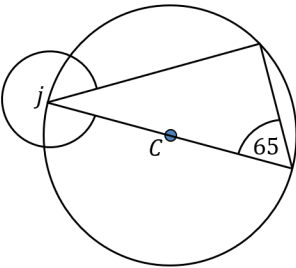
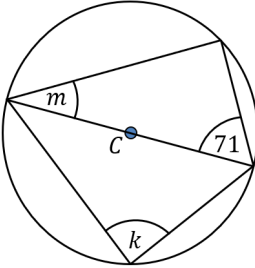
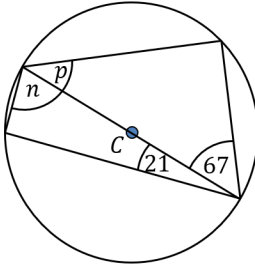
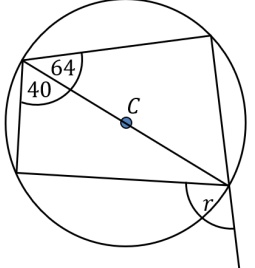
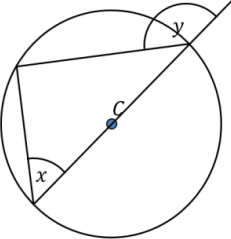
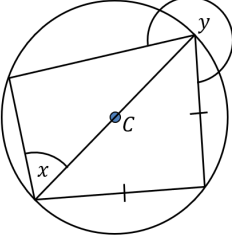
I've broken some calculators.

For each screen: what's the biggest the number could have been? What's the smallest? Write your answer as **error interval**.



2 Basic Circle Theorems

Fluency Practice

Right-Angle in a Semi-Circle Practice Grid			
(a)	(b)	(c)	(d)
			
(e)	(f)	(g)	(h)
			
(i)	(j)	(k)	(l)
		Find y in terms of x 	Find y in terms of x 

Fluency Practice

Angle at the Centre			
(a)	(b)	(c)	(d)
(e)	(f)	(g)	(h)
(i)	(j)	(k)	(l)
		Find y in terms of x	Find y in terms of x

Fluency Practice

Angles from the Same Segment			
(a)	(b)	(c)	(d)
(e)	(f)	(g)	(h)
(i)	(j)	(k)	(l)
		Find y in terms of x 	Find y in terms of x

Fluency Practice

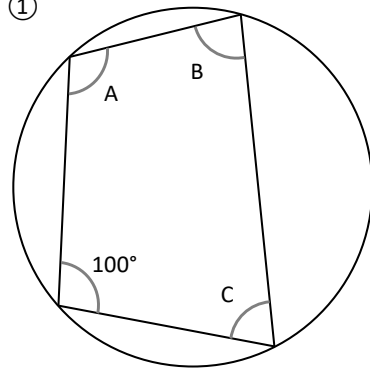
Cyclic Quadrilaterals			
(a)	(b)	(c)	(d)
(e)	(f)	(g)	(h)
(i)	(j)	(k)	(l)

Fluency Practice

Cyclic Quadrilaterals

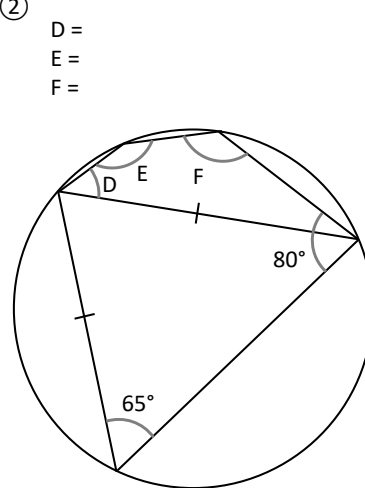
We have enough information to find **some** of the missing angles. Which ones can we find?

①



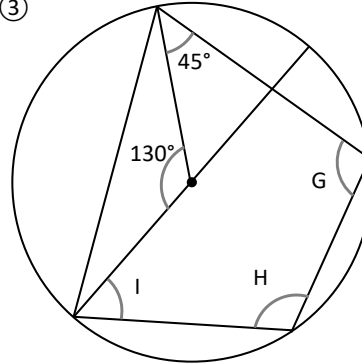
A =
B =
C =

②



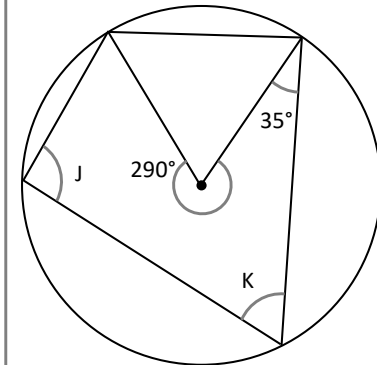
D =
E =
F =

③



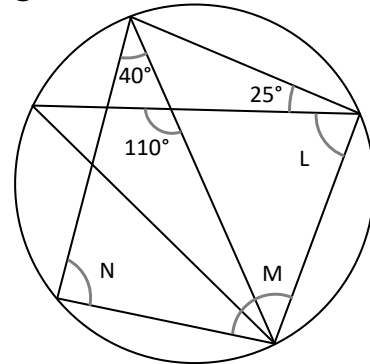
G =
H =
I =

④



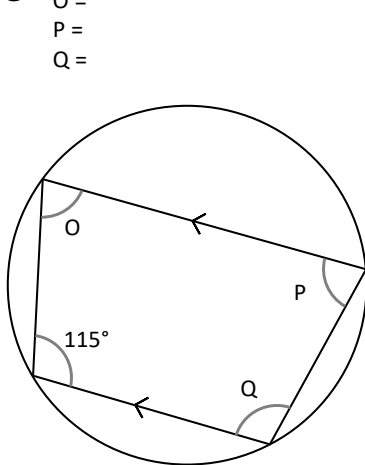
J =
K =

⑤



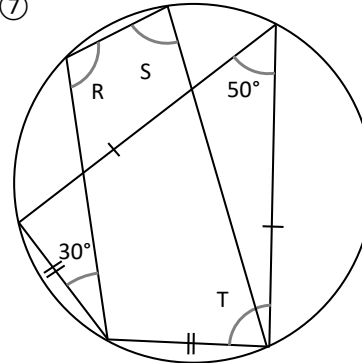
L =
M =
N =

⑥



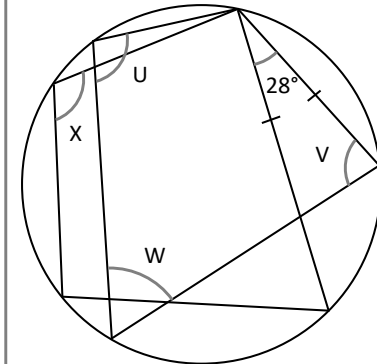
O =
P =
Q =

⑦



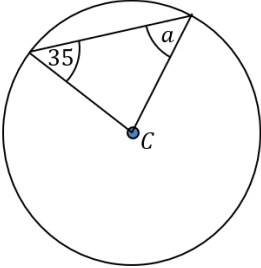
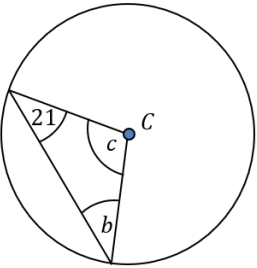
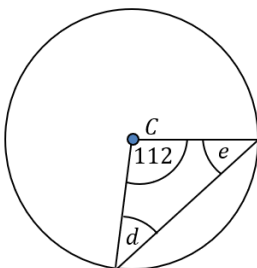
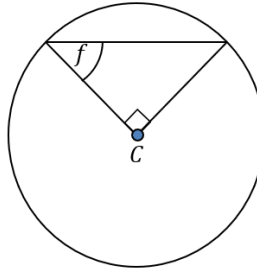
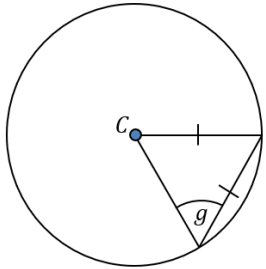
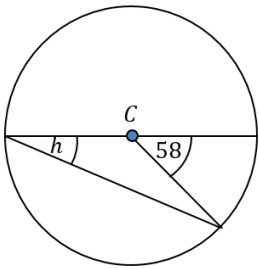
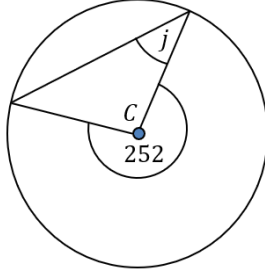
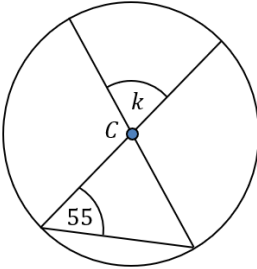
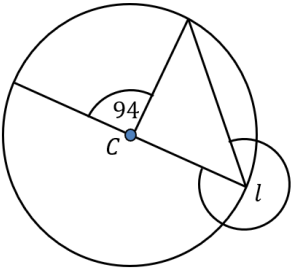
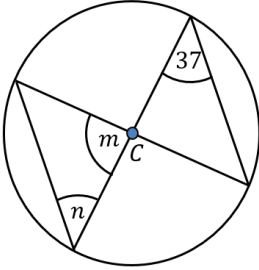
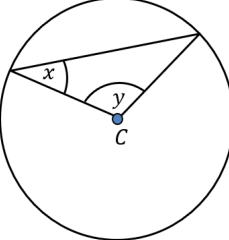
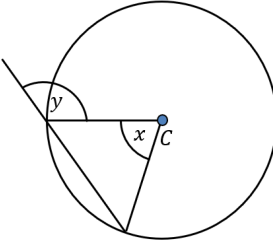
R =
S =
T =

⑧

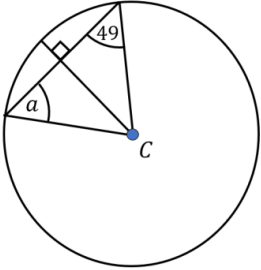
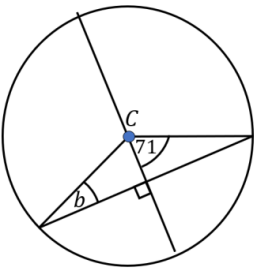
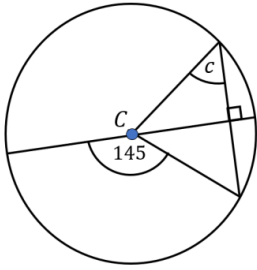
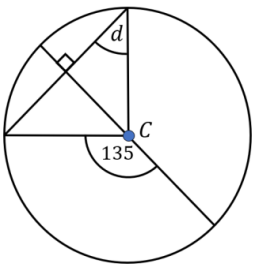
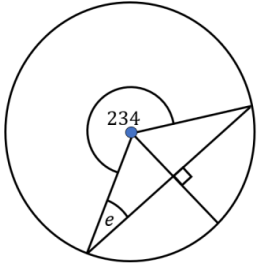
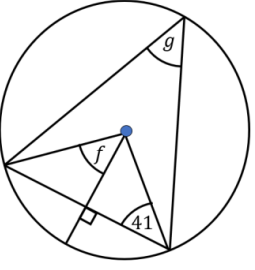
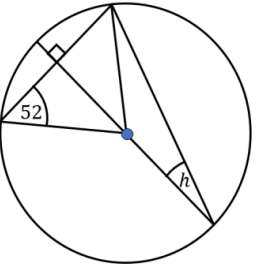
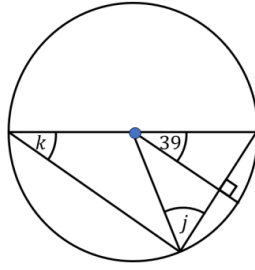
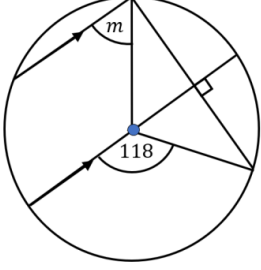
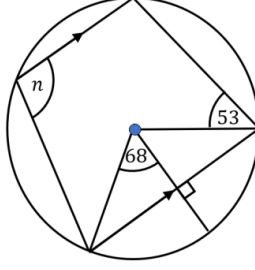
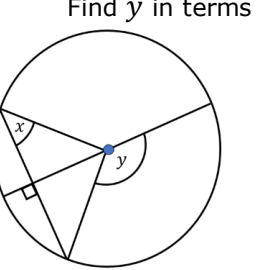
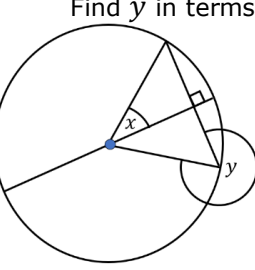


U =
V =
W =
X =

Fluency Practice

Isosceles Triangle in a Circle Practice Grid			
(a)	(b)	(c)	(d)
			
(e)	(f)	(g)	(h)
			
(i)	(j)	(k)	(l)
		Find y in terms of x 	Find y in terms of x 

Fluency Practice

Perpendicular from the Centre to a Chord			
(a)	(b)	(c)	(d)
			
(e)	(f)	(g)	(h)
			
(i)	(j)	(k)	(l)
		Find y in terms of x 	Find y in terms of x 

Fluency Practice

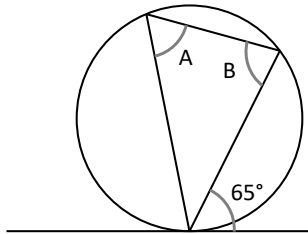
Alternate Segment Theorem			
(a)	(b)	(c)	(d)
(e)	(f)	(g)	(h)
(i)	(j)	(k)	(l)
		Find y in terms of x 	Find y in terms of x

Fluency Practice

Alternate Segment Theorem

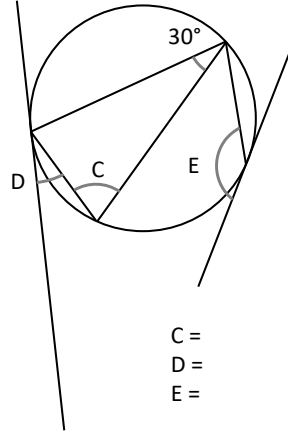
We have enough information to find **some** of the missing angles. Which ones can we find?

①



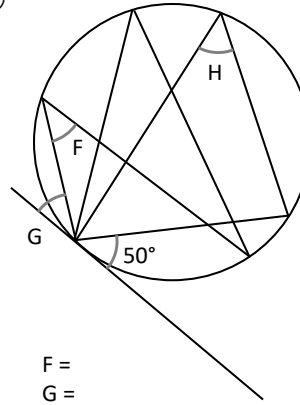
$$\begin{aligned} A &= \\ B &= \end{aligned}$$

②



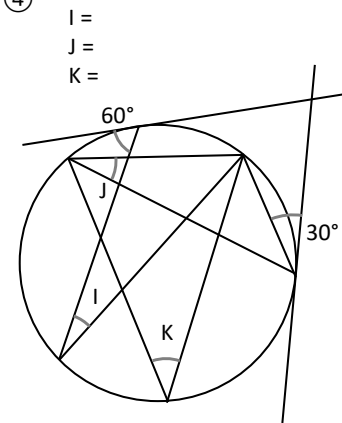
$$\begin{aligned} C &= \\ D &= \\ E &= \end{aligned}$$

③



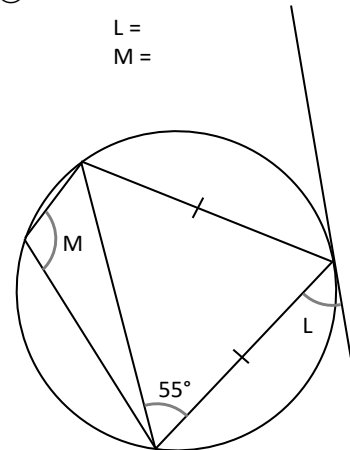
$$\begin{aligned} F &= \\ G &= \\ H &= \end{aligned}$$

④



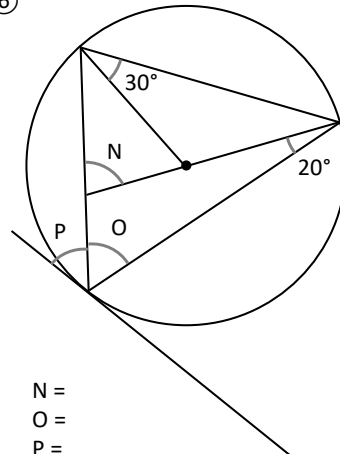
$$\begin{aligned} I &= \\ J &= \\ K &= \end{aligned}$$

⑤



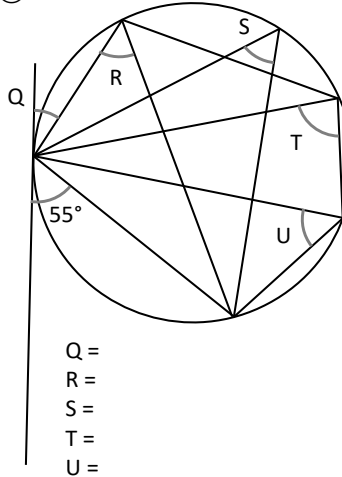
$$\begin{aligned} L &= \\ M &= \end{aligned}$$

⑥



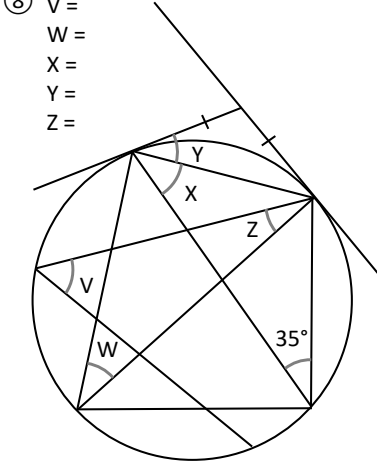
$$\begin{aligned} N &= \\ O &= \\ P &= \end{aligned}$$

⑦



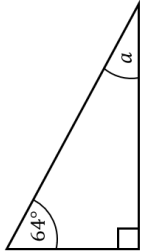
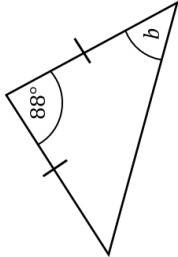
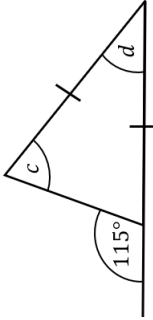
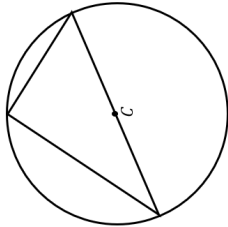
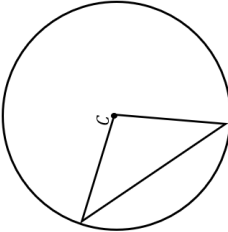
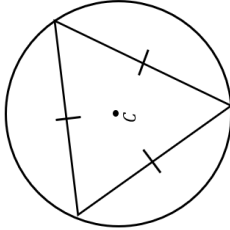
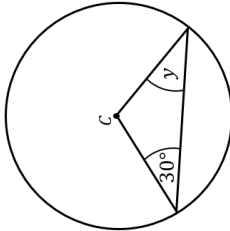
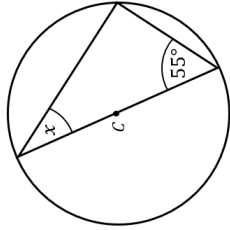
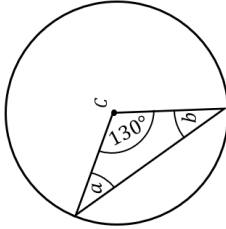
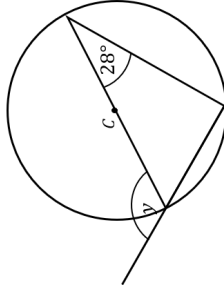
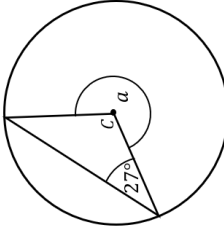
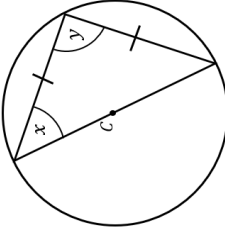
$$\begin{aligned} Q &= \\ R &= \\ S &= \\ T &= \\ U &= \end{aligned}$$

⑧

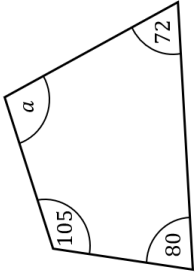
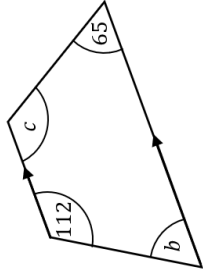
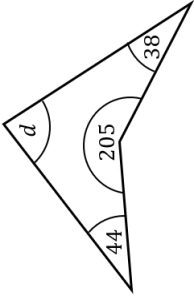
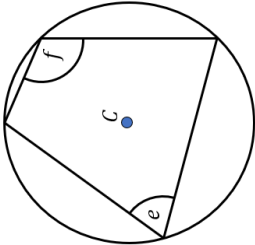
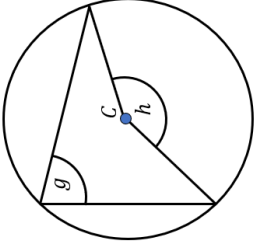
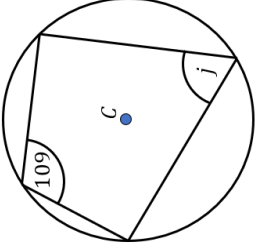
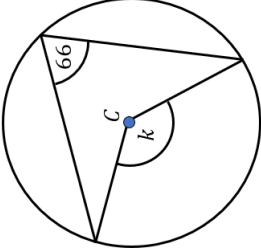
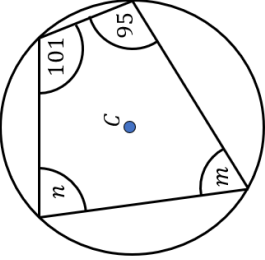
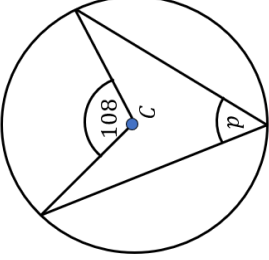
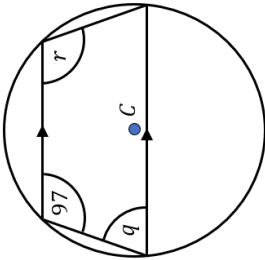
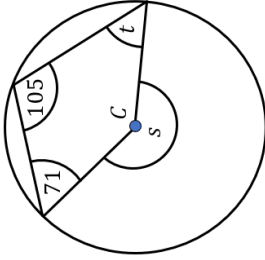
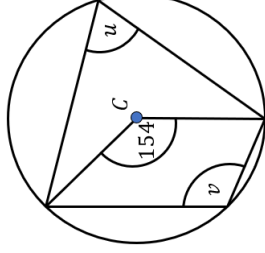


$$\begin{aligned} V &= \\ W &= \\ X &= \\ Y &= \\ Z &= \end{aligned}$$

Fluency Practice

Circle Theorems and Triangles		
(a)	(b)	(c)
Find the missing angle a .	Find the missing angle b .	Find the missing angles c and d .
		
(d)	(e)	(f)
What type of triangle is inside the circle?	What type of triangle is inside the circle?	What type of triangle is inside the circle?
		
(g)	(h)	(i)
Find the missing angle y .	Find the missing angle x .	Find the missing angles a and b .
		
(j)	(k)	(l)
Find the missing angle y .	Find the missing angle a .	Find the missing angles x and y .
		

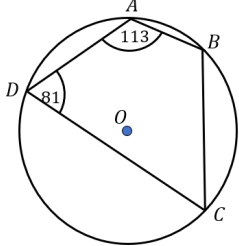
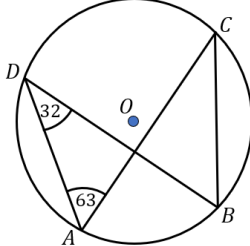
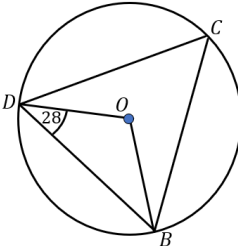
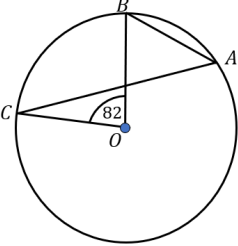
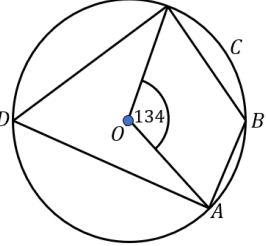
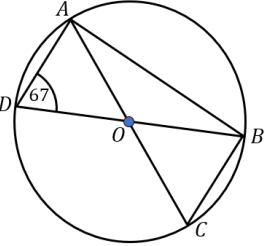
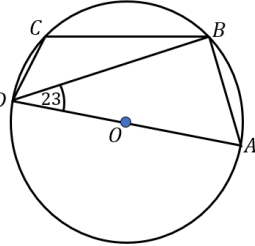
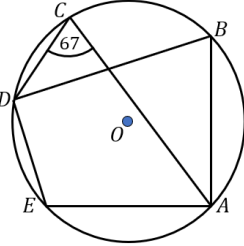
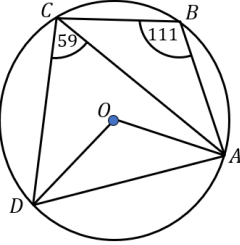
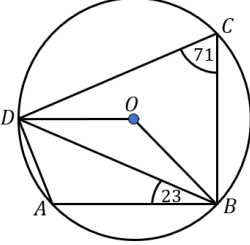
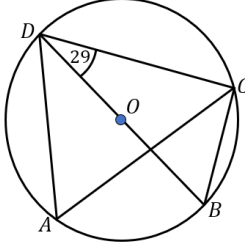
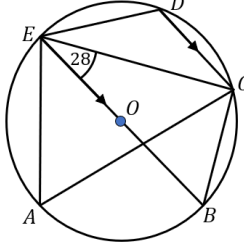
Fluency Practice

Circle Theorems and Quadrilaterals			
(a)	(b)	(c)	
Find the missing angle a .	Find the missing angles b and c .	Find the missing angle d .	
			
(d)	(e)	(f)	
What is the relationship between angles e and f ?	What is the relationship between angles g and h ?	Find the value of j .	
			
(g)	(h)	(i)	
Find the value of k .	Find the values of m and n .	Find the value of p .	
			
(j)	(k)	(l)	
Find the values of q and r .	Find the values of s and t .	Find the values of u and v .	
			

Fluency Practice

Circle Theorems and Tangents			
(a)	(b)	(c)	(d)
(e)	(f)	(g)	(h)
(i)	(j)	(k)	(l)

Fluency Practice

Mixed Circle Theorems			
(a) Find the size of angle BCD 	(b) Find the size of angle CBD 	(c) Find the size of angle DCB 	(d) Find the size of angle BAC 
(e) Find the size of angle ABC 	(f) Find the size of angle OBC 	(g) Find the size of angle BCD 	(h) Find the size of angle AED 
(i) Find the size of angle ODC 	(j) Find the size of angle ODA 	(k) Find the size of angle CAD 	(l) Find the size of angle CED 

Fluency Practice

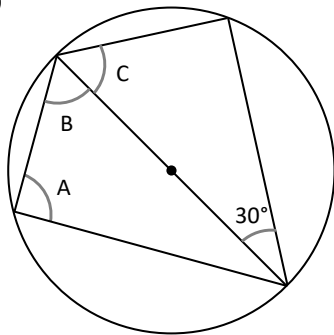
Mixed Circle Theorems with Tangents			
(a) Find the size of angle ABD 	(b) Find the size of angle EBA 	(c) Find the size of angle ABC 	(d) Find the size of angle BED
(e) Find the size of angle DEF 	(f) Find the size of angle BCD 	(g) Find the size of angle DBC 	(h) Find the size of angle ABE
(i) Find the size of angle ADC 	(j) Find the size of angle EDC 	(k) Find the size of angle DBC 	(l) Find the size of angle FEB

Fluency Practice

Angles at the Circumference

We have enough information to find **some** of the missing angles. Which ones can we find?

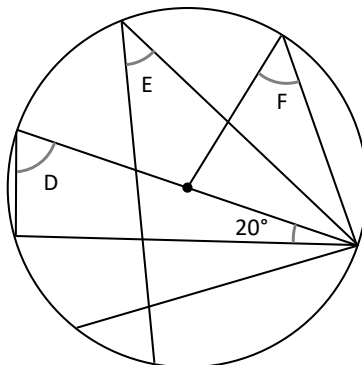
①



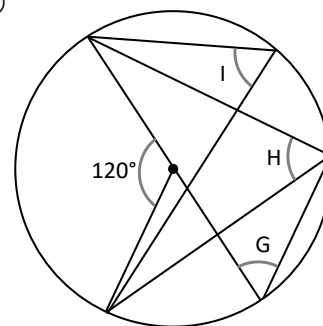
A =
B =
C =

②

D =
E =
F =



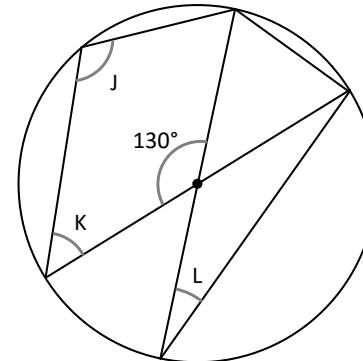
③



G =
H =
I =

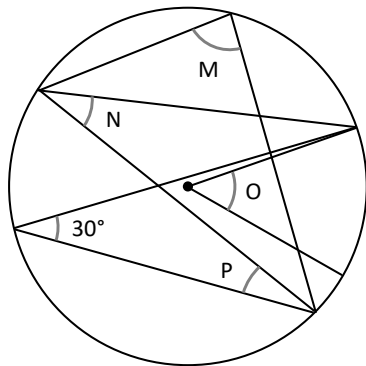
④

J =
K =
L =

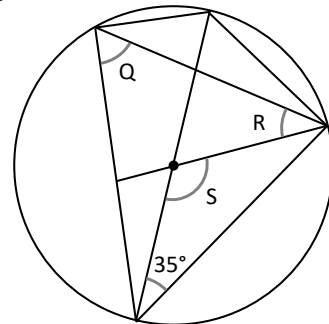


⑤

M =
N =
O =
P =



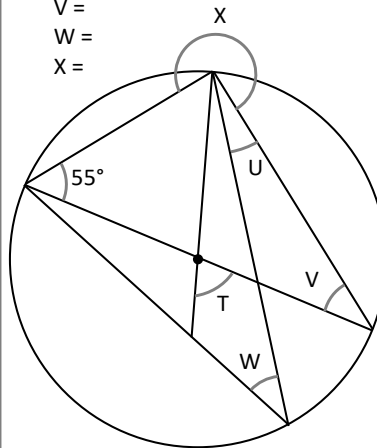
⑥



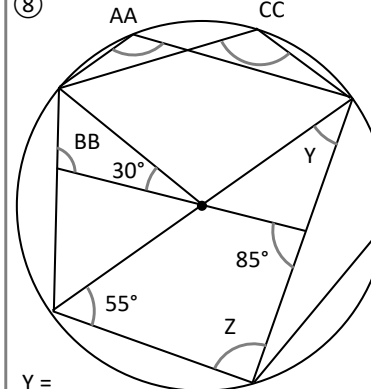
Q =
R =
S =

⑦

T =
U =
V =
W =
X =



⑧



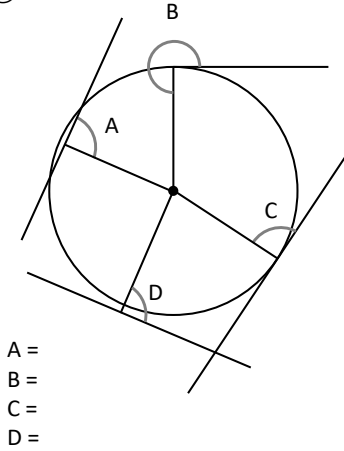
Y =
Z =
AA =
BB =
CC =

Fluency Practice

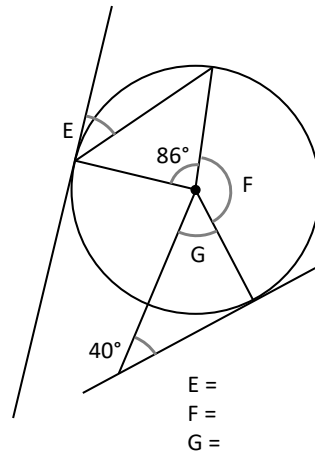
Tangents & Chords

We have enough information to find **some** of the missing angles & lengths. Which ones can we find?
(Assume lines that *look* like tangent are tangents.)

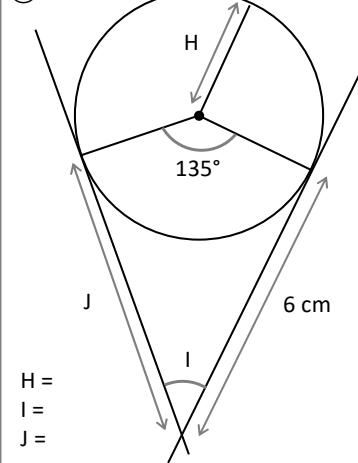
①



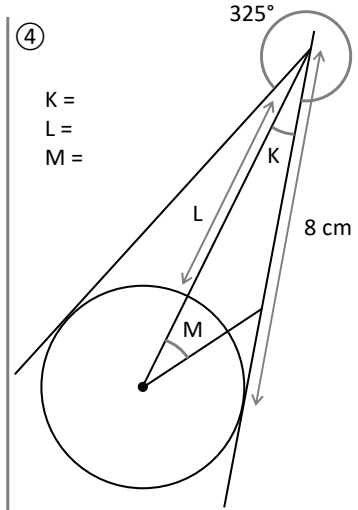
②



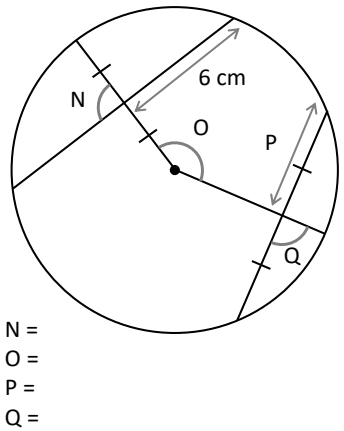
③



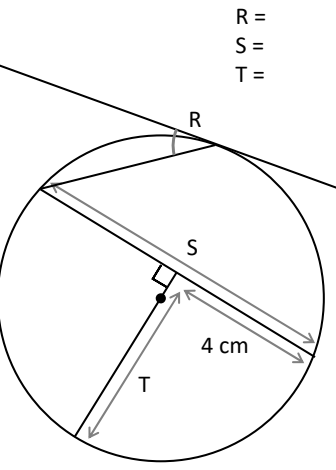
④



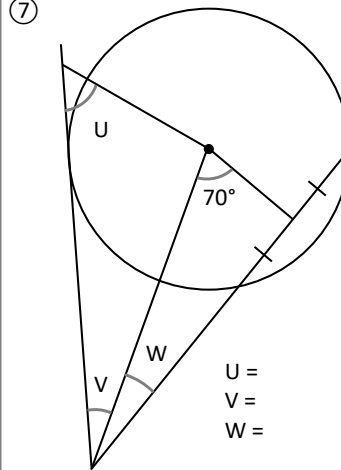
⑤



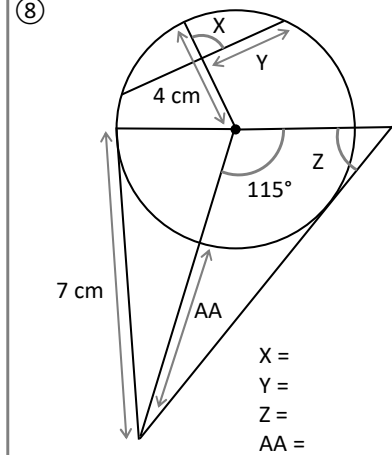
⑥



⑦



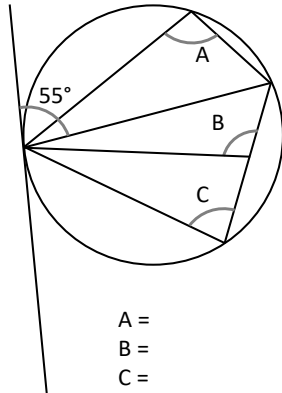
⑧



Fluency Practice

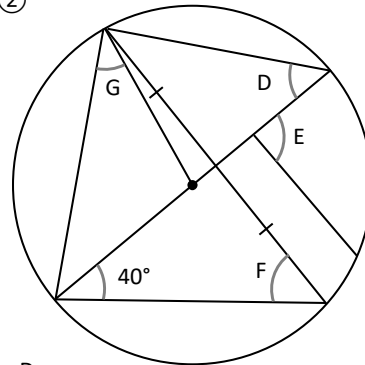
We have enough information to find **some** of the missing angles. Which ones can we find?

①



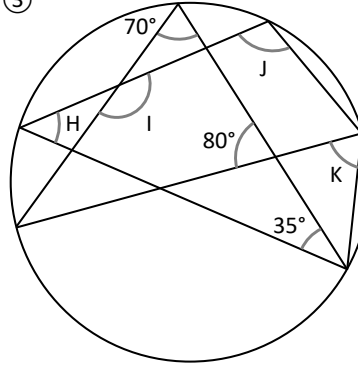
A =
B =
C =

②



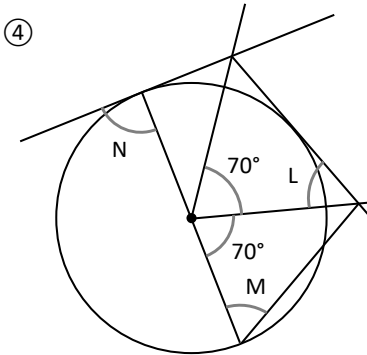
D =
E =
F =
G =

③



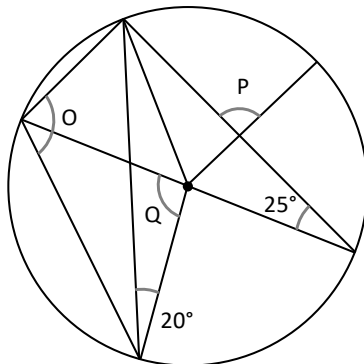
H =
I =
J =
K =

④



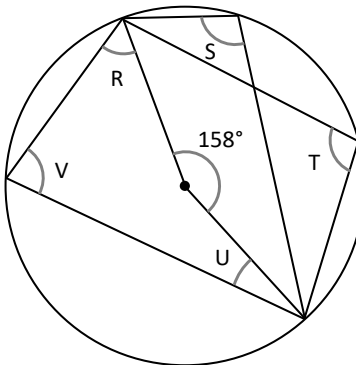
L =
M =
N =

⑤



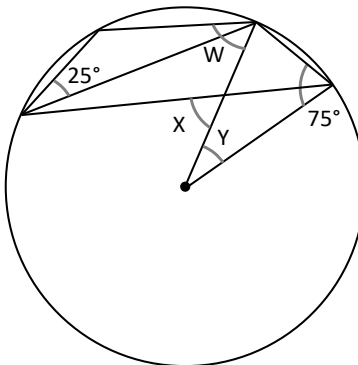
O =
P =
Q =

⑥



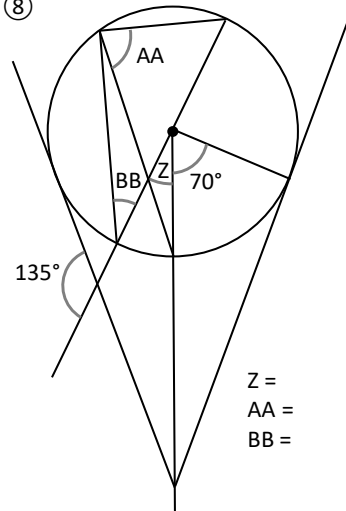
R =
S =
T =
U =
V =

⑦



W =
X =
Y =

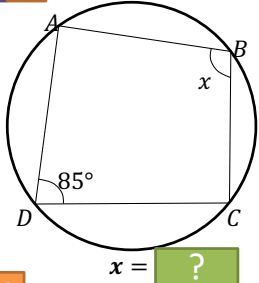
⑧



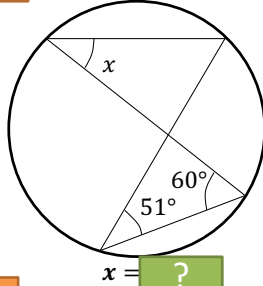
Z =
AA =
BB =

Fluency Practice

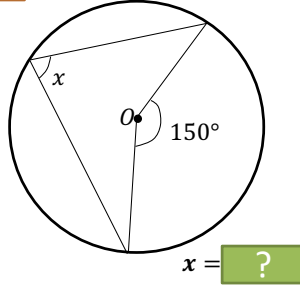
1 a



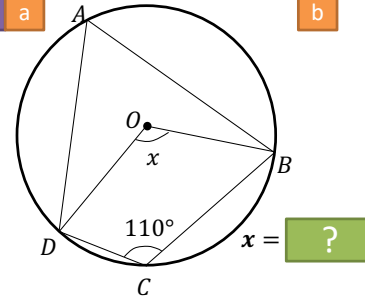
b



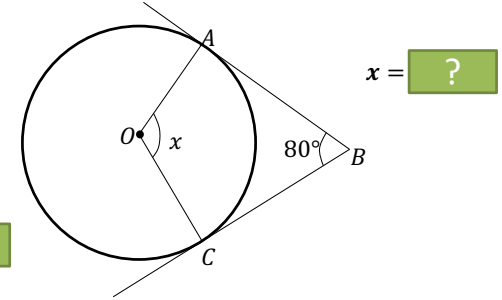
c



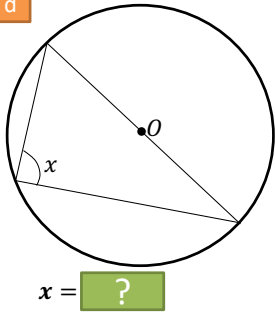
2 a



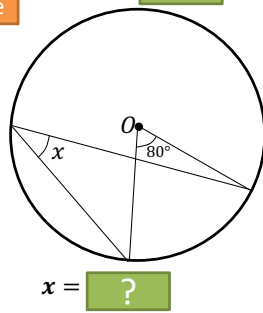
b



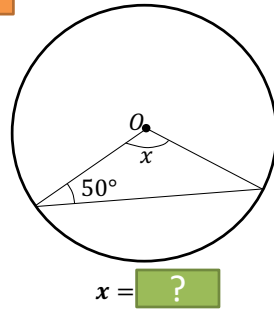
d



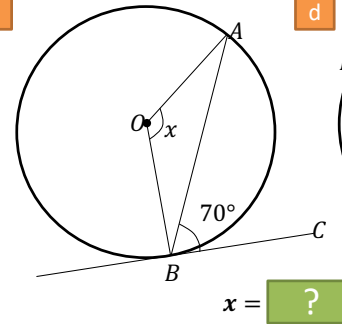
e



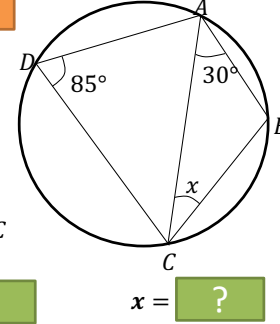
f



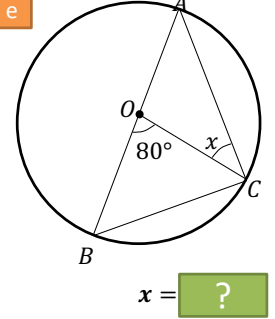
c



d

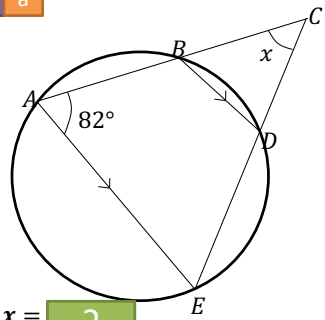


e



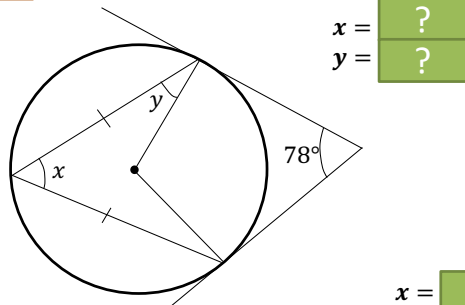
Fluency Practice

3 a



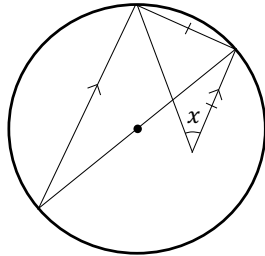
$$x = \boxed{?}$$

b



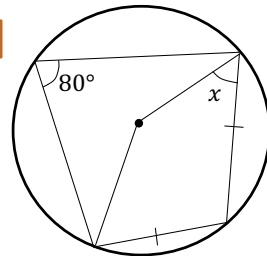
$$\begin{matrix} x = \boxed{?} \\ y = \boxed{?} \end{matrix}$$

c



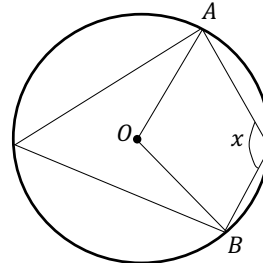
$$x = \boxed{?}$$

d

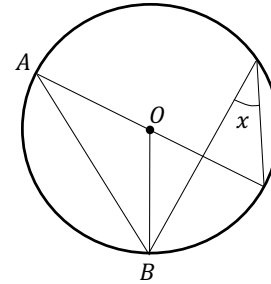


$$x = \boxed{?}$$

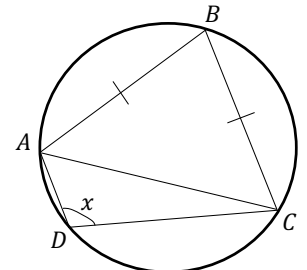
4 Determine each indicated angle in terms of x .



$$\angle AOB = \boxed{?}$$

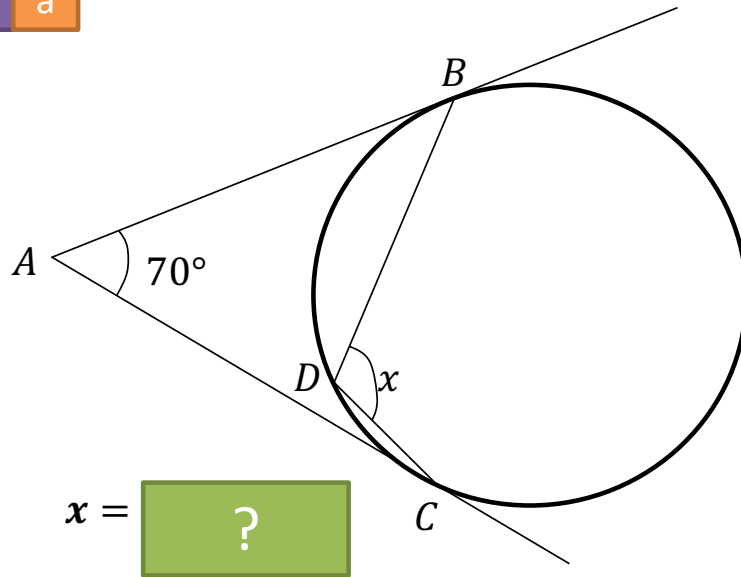


$$\angle AOB = \boxed{?}$$



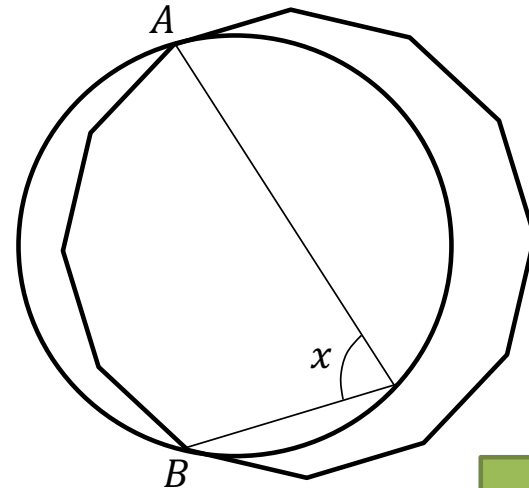
$$\angle BAC = \boxed{?}$$

Fluency Practice



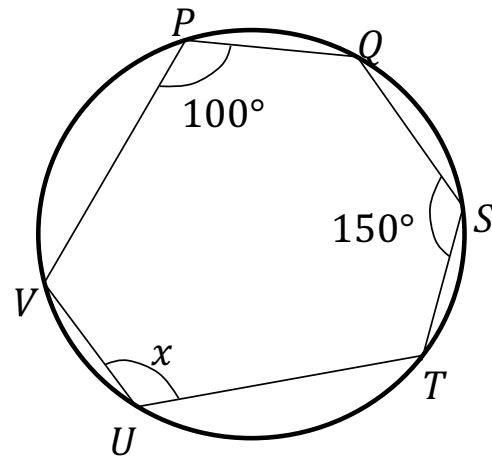
$x =$

?



$x =$

?

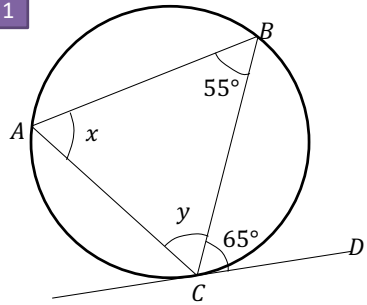


$x =$

?

Fluency Practice

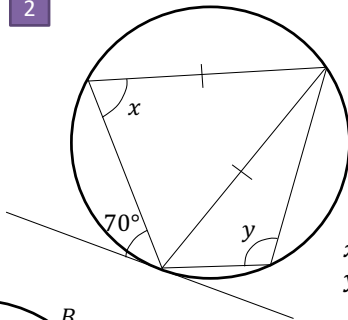
1



$$x = \boxed{?}$$

$$y = \boxed{?}$$

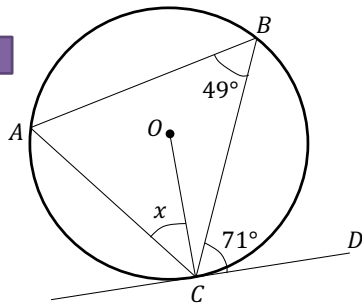
2



$$x = \boxed{?}$$

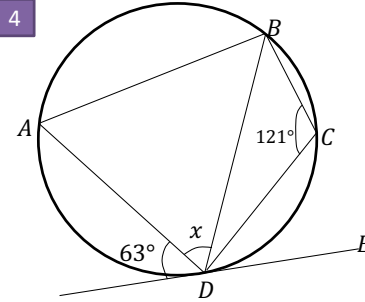
$$y = \boxed{?}$$

3



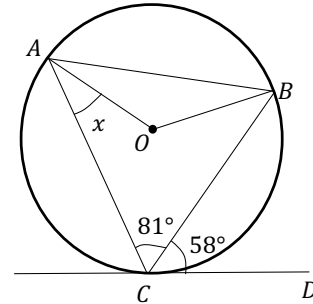
$$x = \boxed{?}$$

4



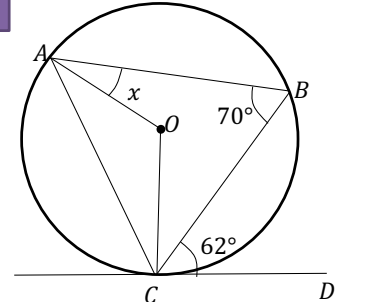
$$x = \boxed{?}$$

6



$$x = \boxed{?}$$

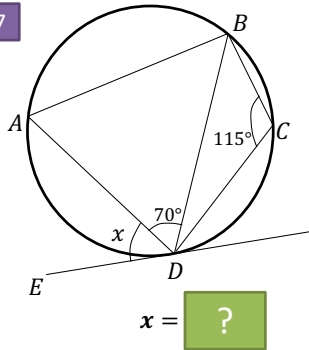
5



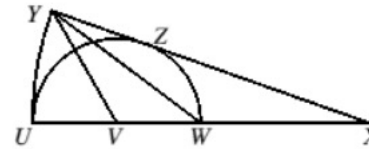
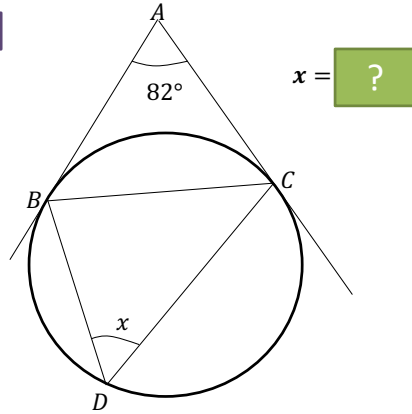
$$x = \boxed{?}$$

Fluency Practice

7

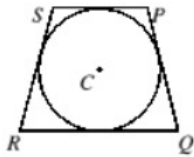


8



[SMC 2012 Q22] A semicircle of radius r is drawn with centre V and diameter UW . The line UW is then extended to the point X , such that UW and WX are of equal length. An arc of the circle with centre X and radius $4r$ is then drawn so that the line XY is a tangent to the semicircle at Z , as shown. What, in terms of r , is the area of triangle YVW ?

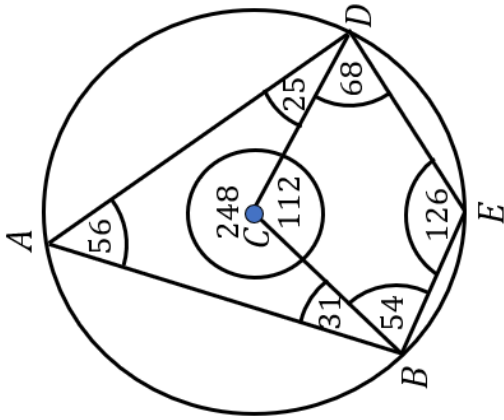
?



[SMC 2012 Q20] In trapezium $PQRS$, $SR = PQ = 25\text{cm}$ and SP is parallel to RQ . All four sides of $PQRS$ are tangent to a circle with centre C . The area of the trapezium is 600cm^2 . What is the radius of the circle?

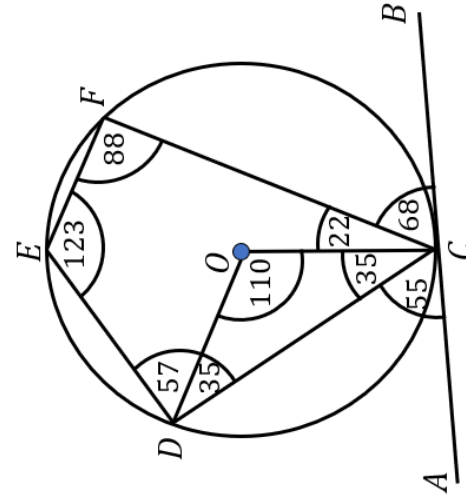
$$r = \boxed{?}$$

Fluency Practice



Write down the size of the following angles:

- (a) ABC
- (b) BED
- (c) BAD
- (d) EDC
- (e) CDA
- (f) EBA
- (g) the obtuse angle DCB
- (h) the reflex angle DCB

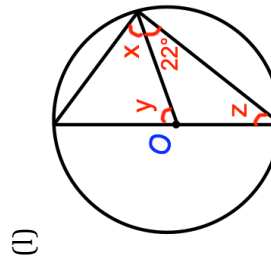
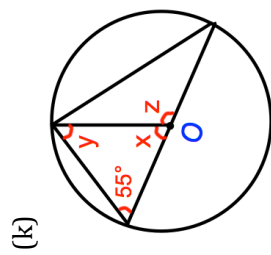
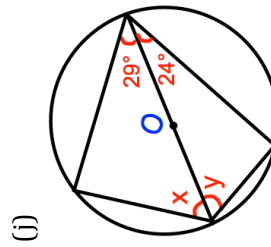
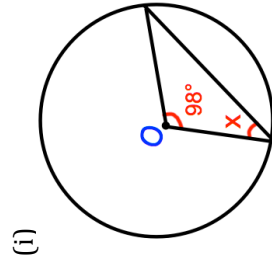
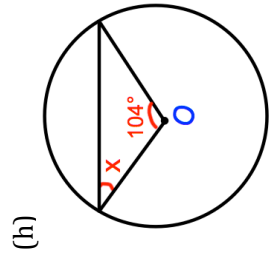
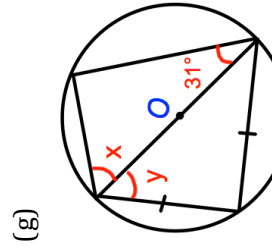
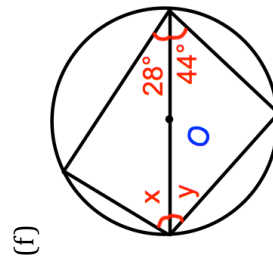
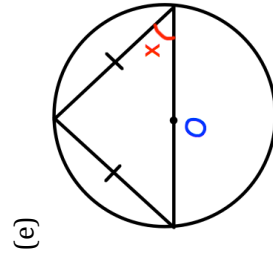
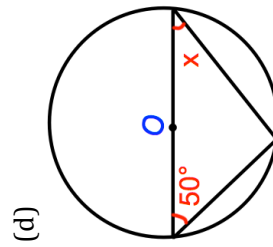
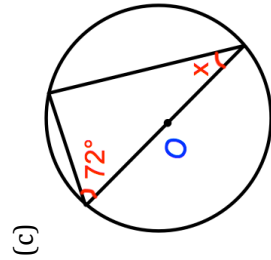
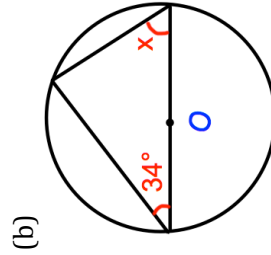
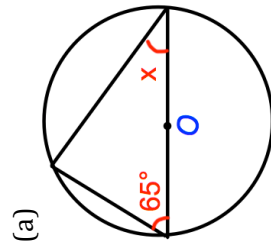


Write down the size of the following angles:

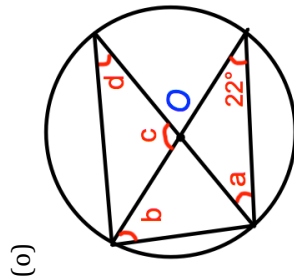
- (a) DEF
- (b) CFE
- (c) OCD
- (d) DCA
- (e) BCF
- (f) OCA
- (g) DCB
- (h) EDC

Fluency Practice

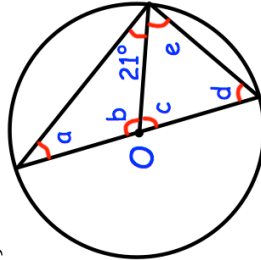
Question 1: Find the missing angles labelled in each of these circles



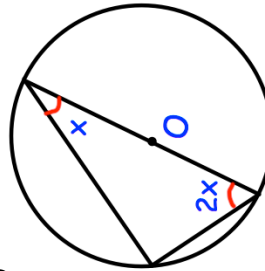
Fluency Practice



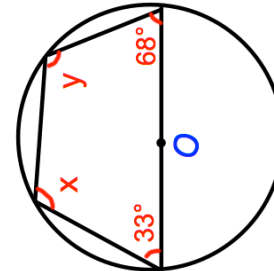
(o)



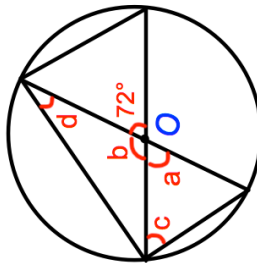
(r)



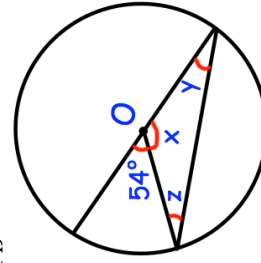
(u)



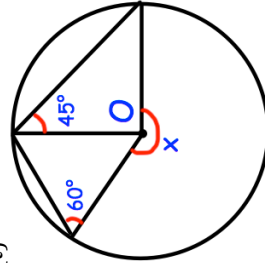
(x)



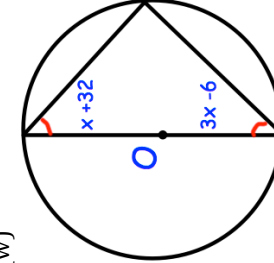
(n)



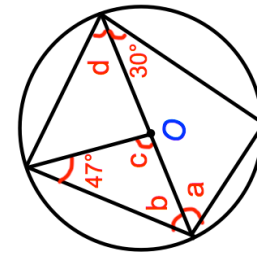
(q)



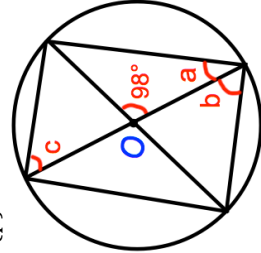
(t)



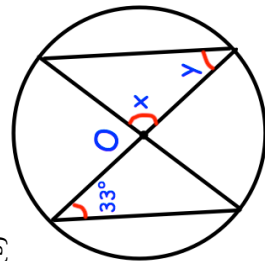
(w)



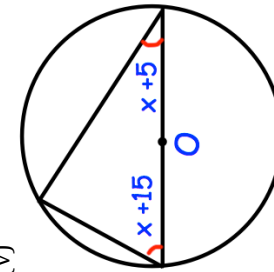
(m)



(p)



(s)

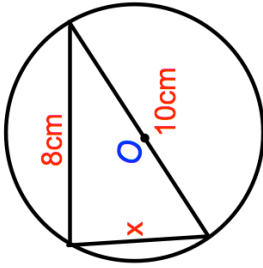


(v)

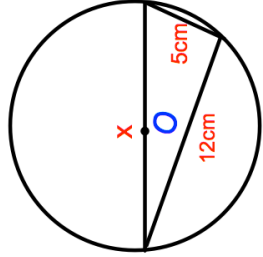
Fluency Practice

Question 2: Calculate the length of sides labelled in the circles below

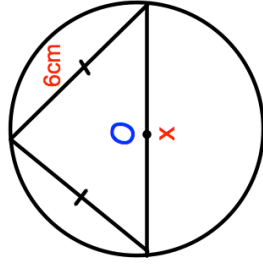
(a)



(b)

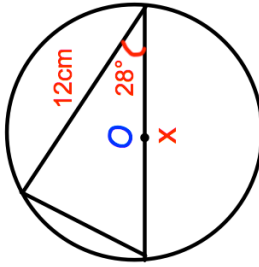


(c)

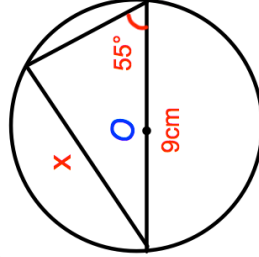


Question 3: Calculate the length of sides labelled in the circles below

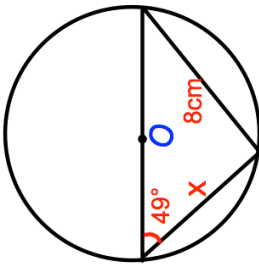
(a)



(b)

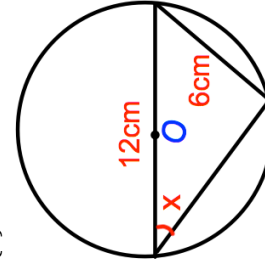


(c)

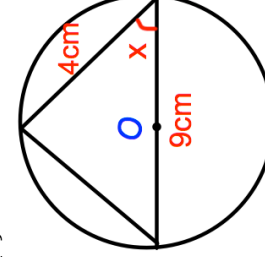


Question 4: Calculate the size of the missing angles

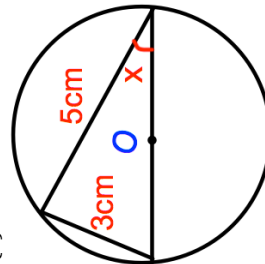
(a)



(b)

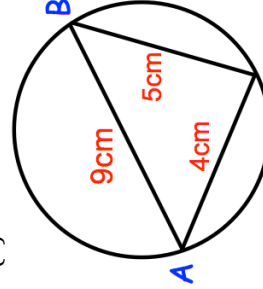


(c)

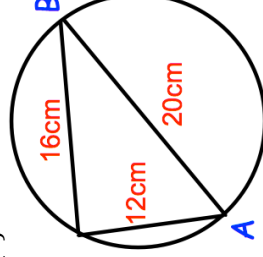


Question 5: State, with a reason, if AB is the diameter in each circle below.

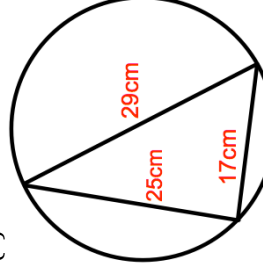
(a)



(b)



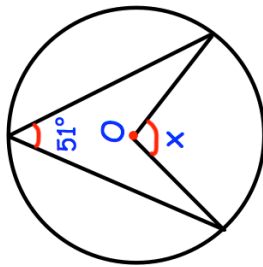
(c)



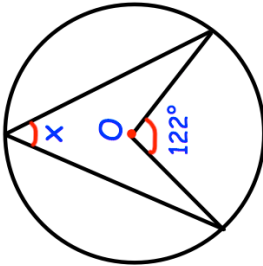
Fluency Practice

Question 6: Find the missing angles labelled in each of these circles

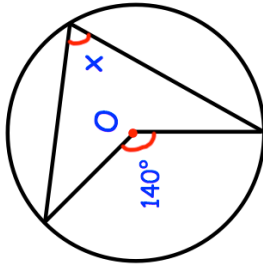
(a)



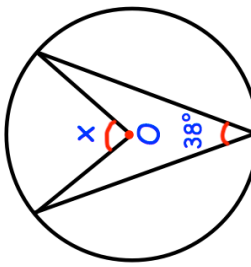
(b)



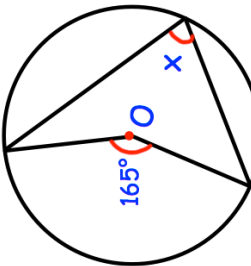
(c)



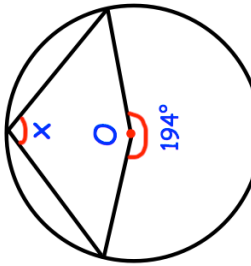
(d)



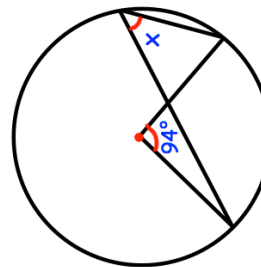
(e)



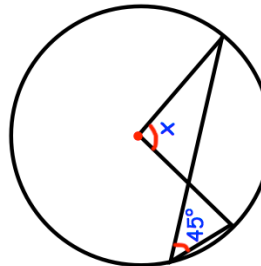
(f)



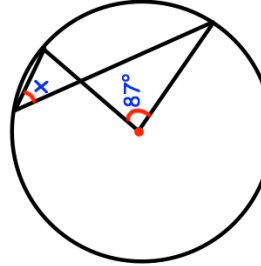
(g)



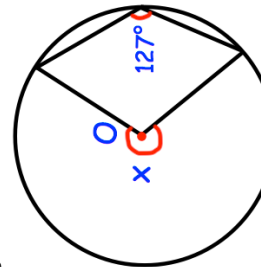
(h)



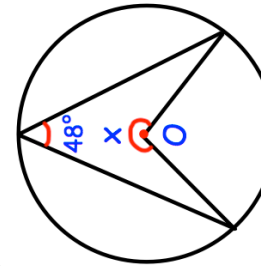
(i)



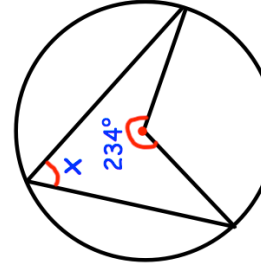
(j)



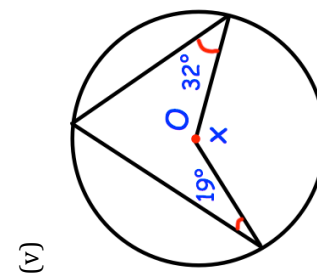
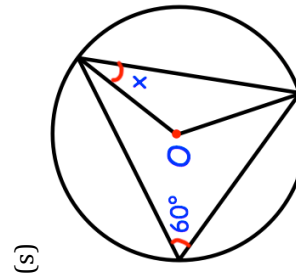
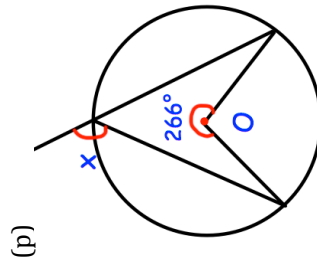
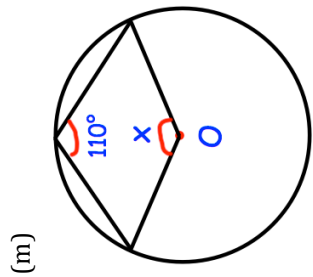
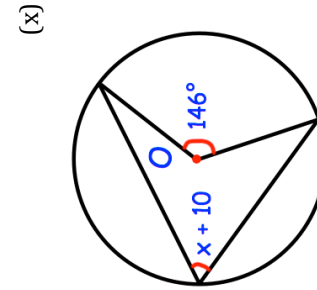
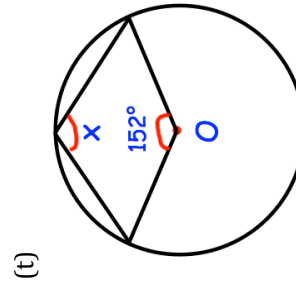
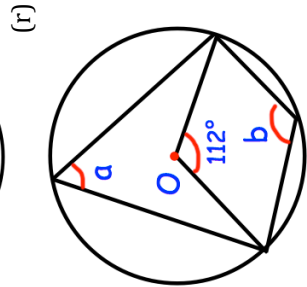
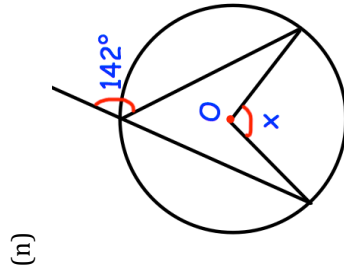
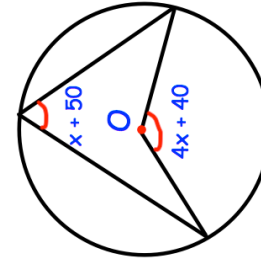
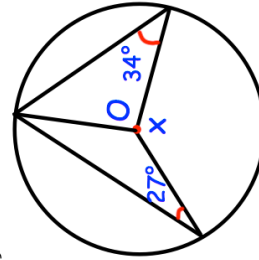
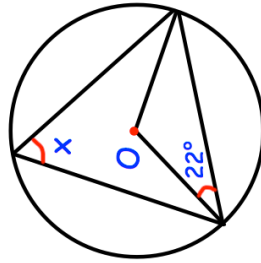
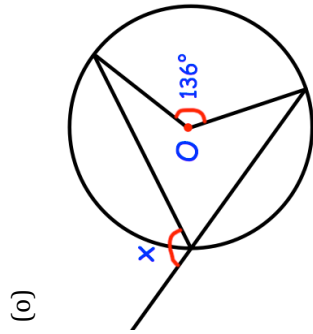
(k)



(l)



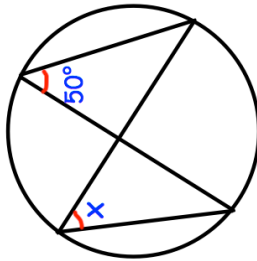
Fluency Practice



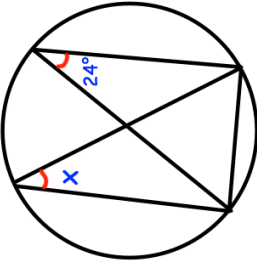
Fluency Practice

Question 7: Find the missing angles labelled in each of these circles

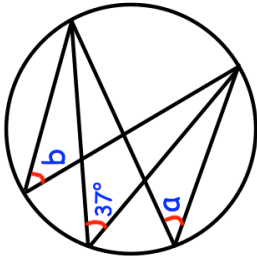
(a)



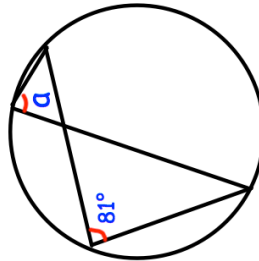
(b)



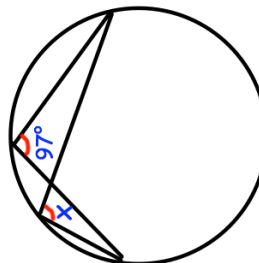
(c)



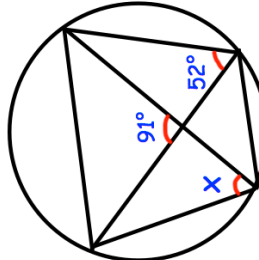
(d)



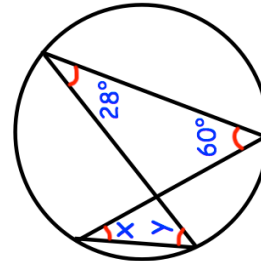
(e)



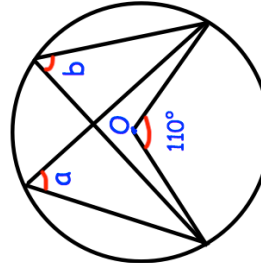
(f)



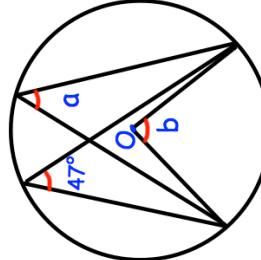
(g)



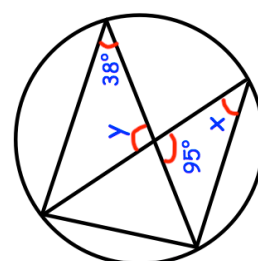
(h)



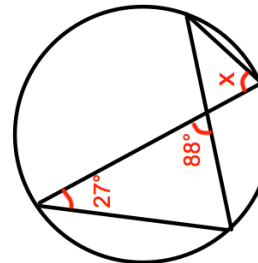
(i)



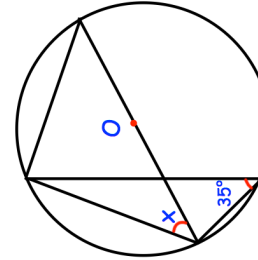
(j)



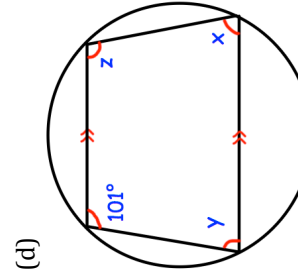
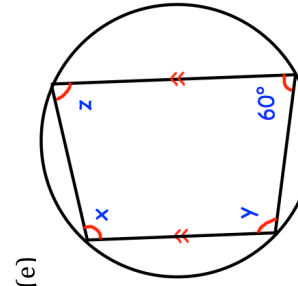
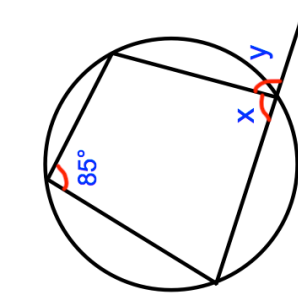
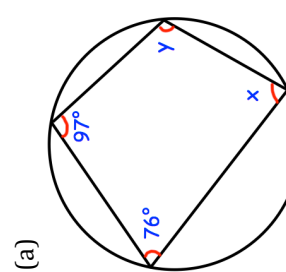
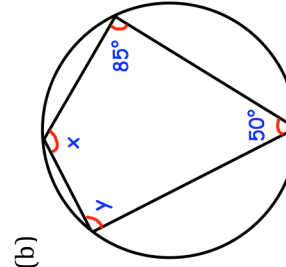
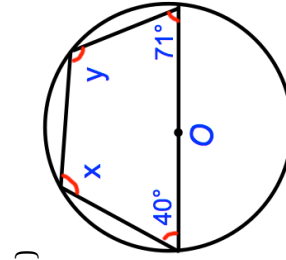
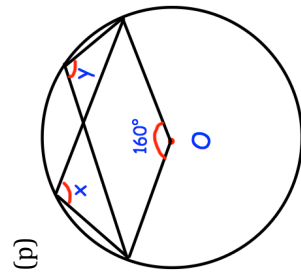
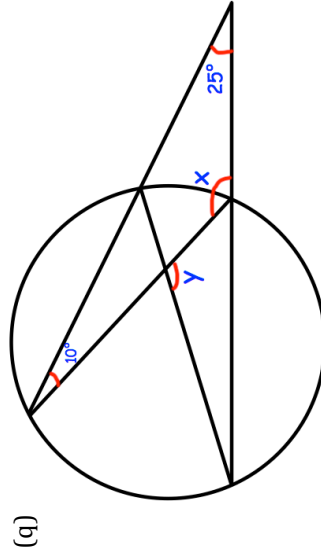
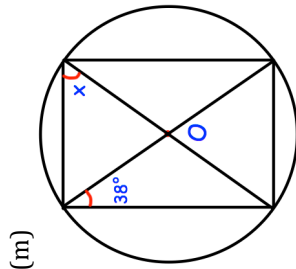
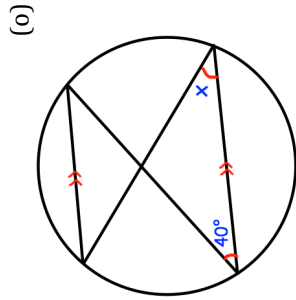
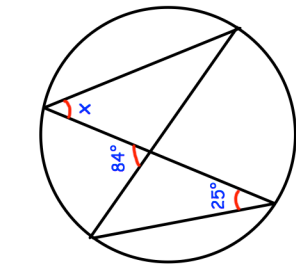
(k)



(l)

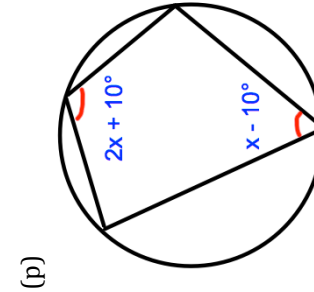
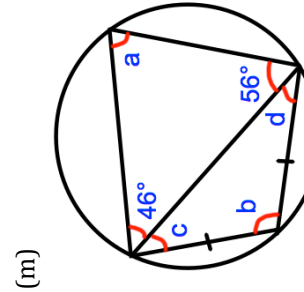
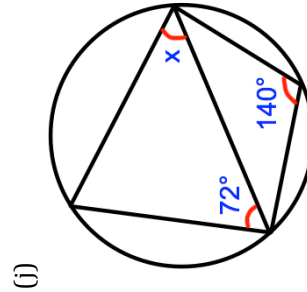
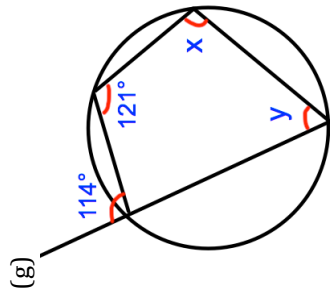
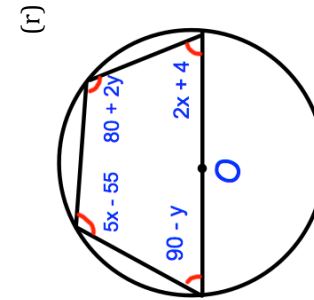
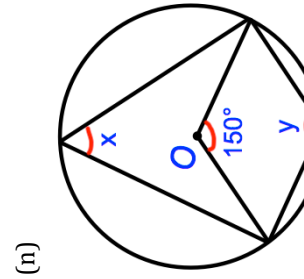
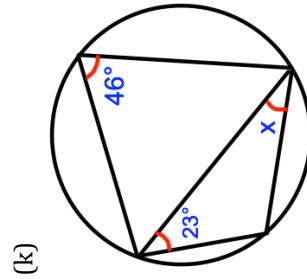
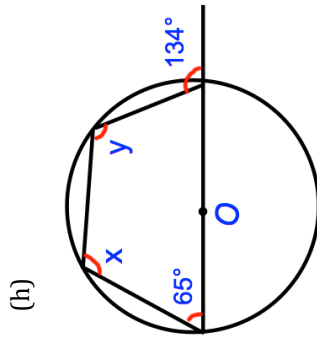
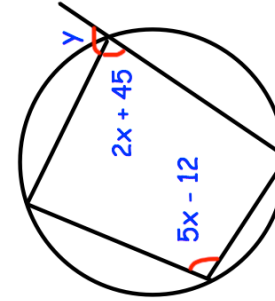
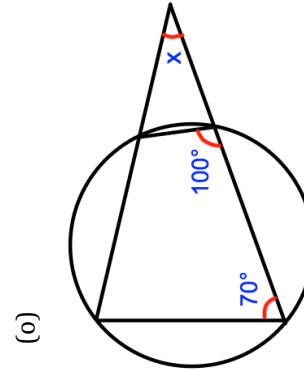
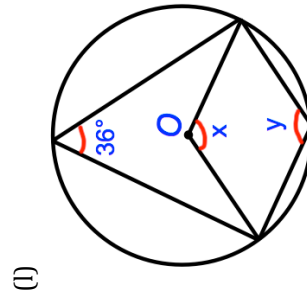
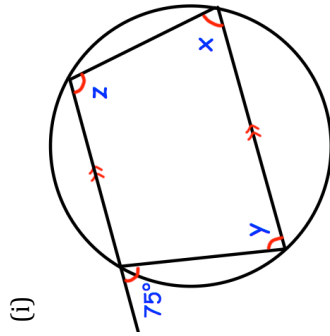


Fluency Practice



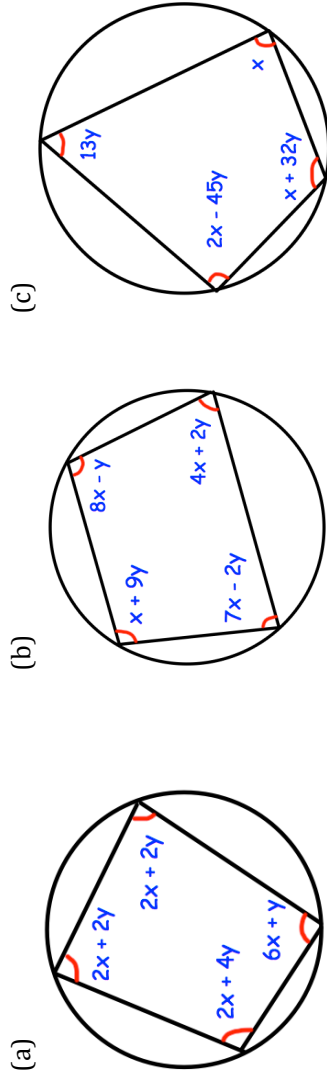
Question 8: Find the missing angles labelled in each of these circles

Fluency Practice

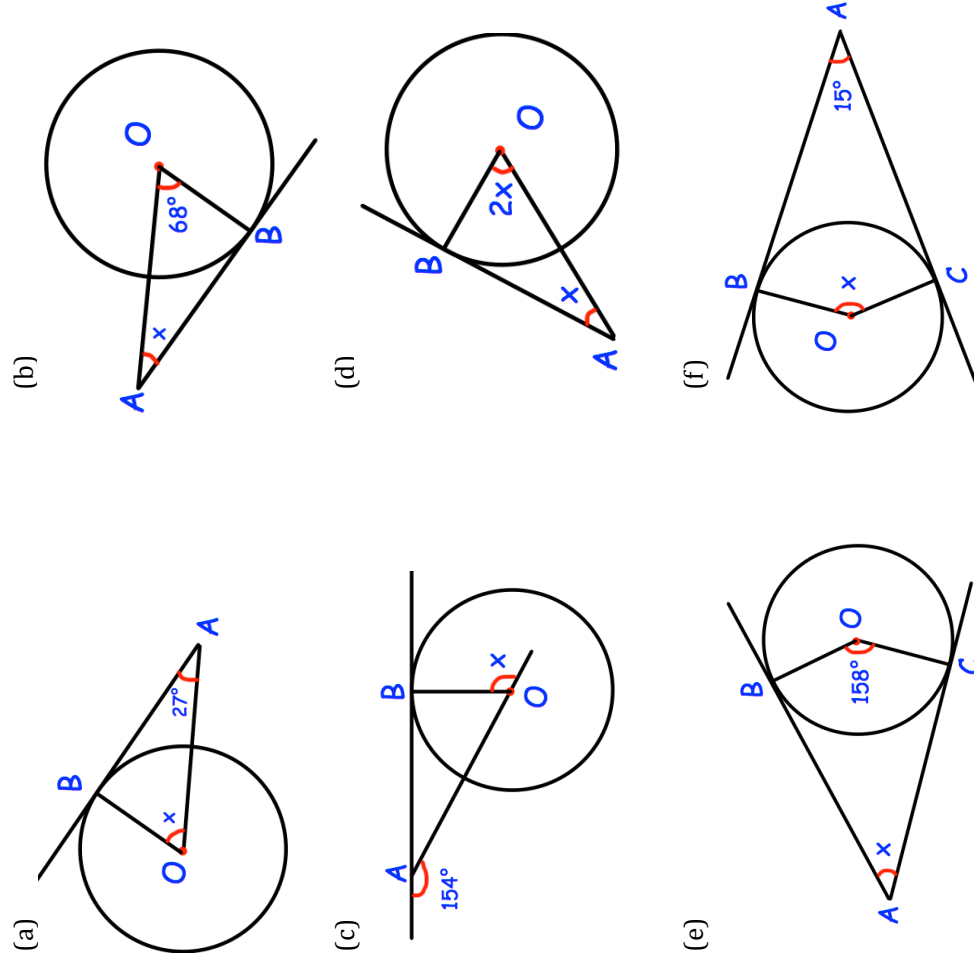


Fluency Practice

Question 9: Find the values of x and y

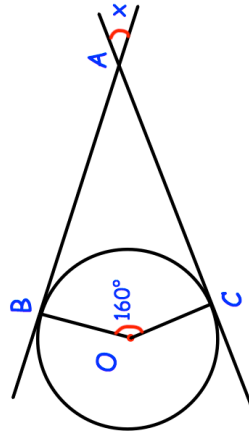


Question 10: Find the value of x in each diagram. The lines AB and AC are tangents.

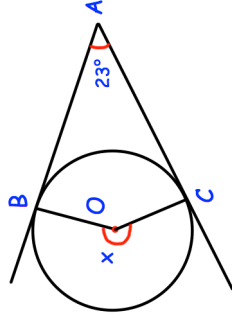


Fluency Practice

(g)

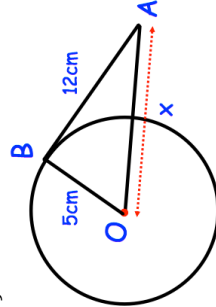


(h)

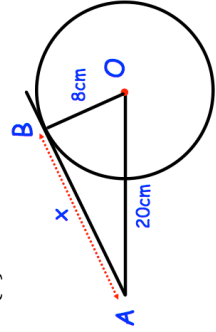


Question 11: Calculate the length of sides labelled in the circles below. The lines AB and AC are tangents.

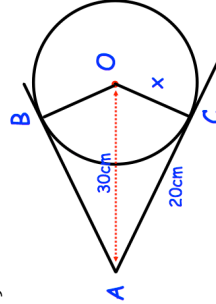
(a)



(b)

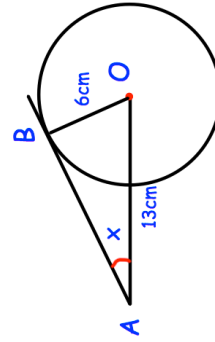


(c)

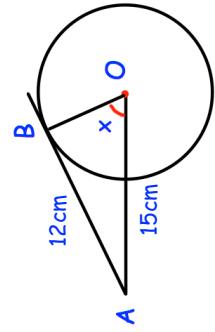


Question 12: Calculate the size of x in the circles below. The lines AB and AC are tangents.

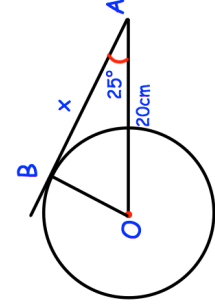
(a)



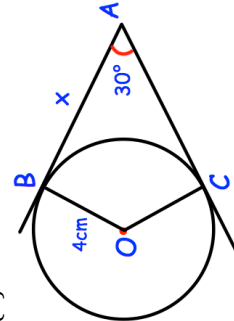
(b)



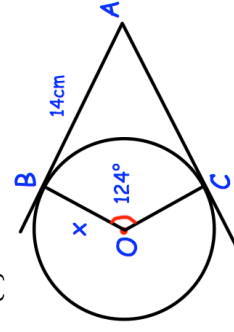
(c)



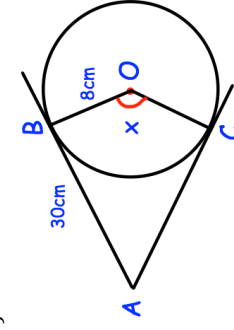
(d)



(e)

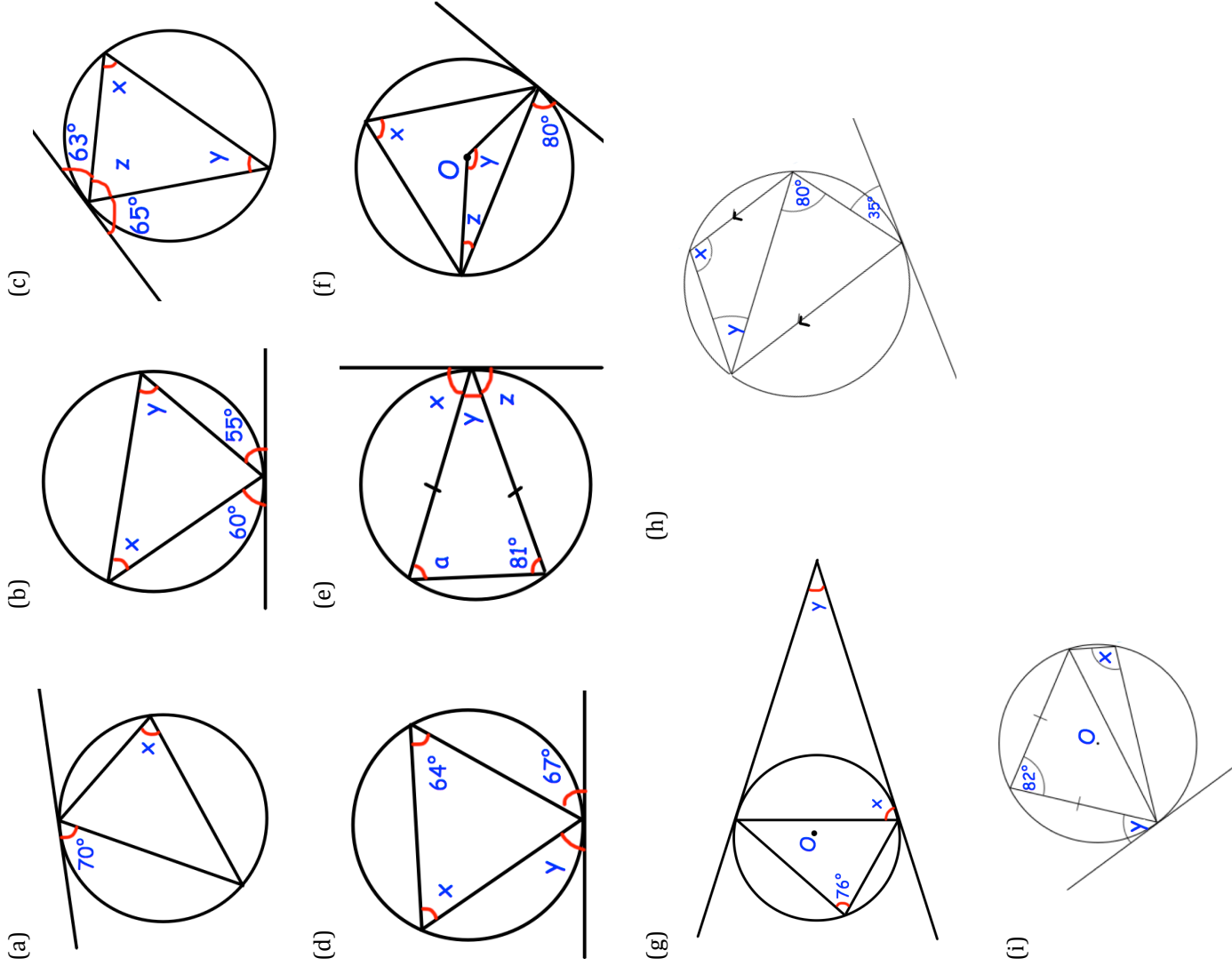


(f)



Fluency Practice

Question 13: Find the missing angles labelled in each of these circles



Fluency Practice

Trigonometry with Circle Theorems (assume tangents)

A

B

C

D

E

F

G

H

I

J

K

L

M

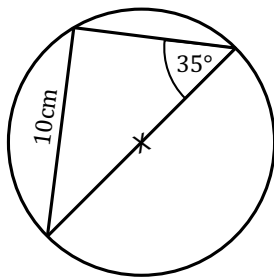
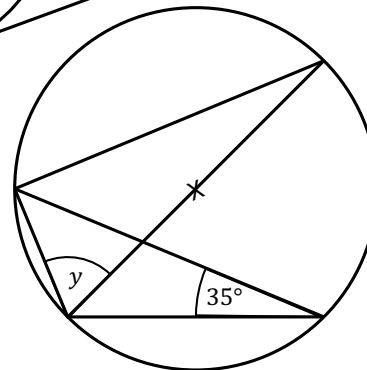
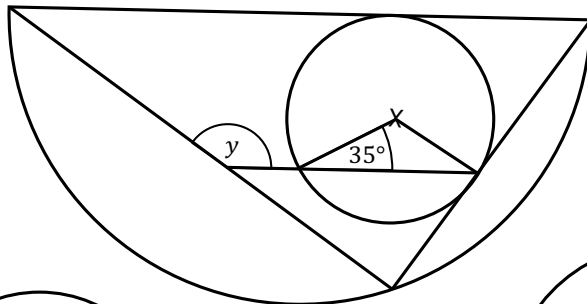
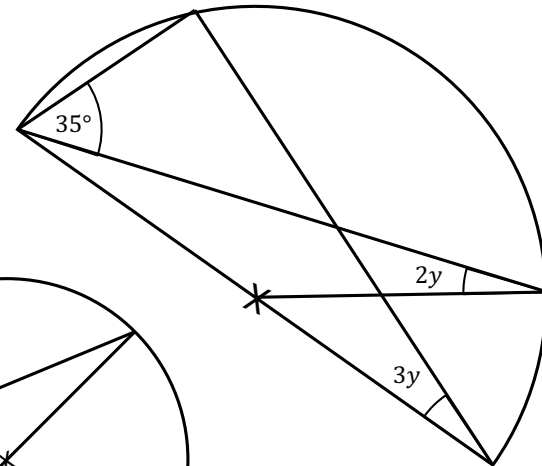
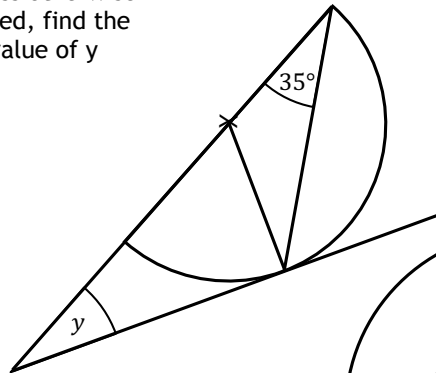
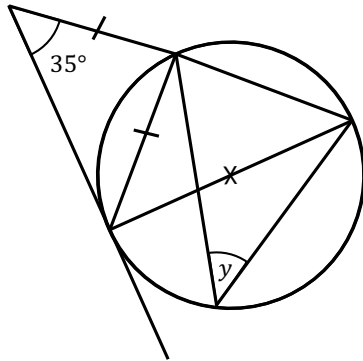
N

O

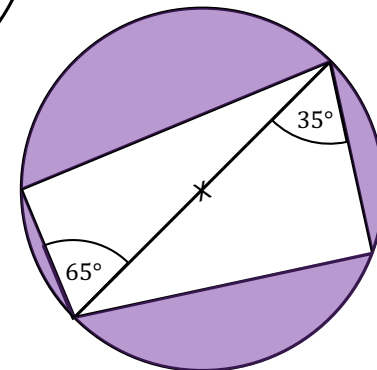
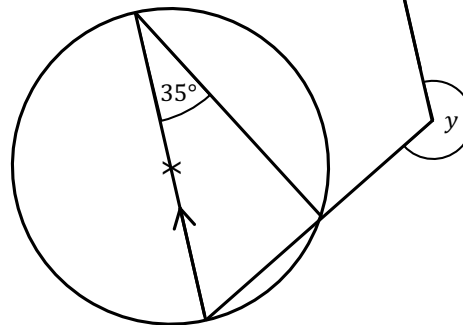
14 Answers	14.3	12.7	8.7	7.1	16.8	47.3	78.6
	53.1	16.5	40.0	9.4	299.5	8.9	34.8

Fluency Practice

Unless otherwise stated, find the value of y

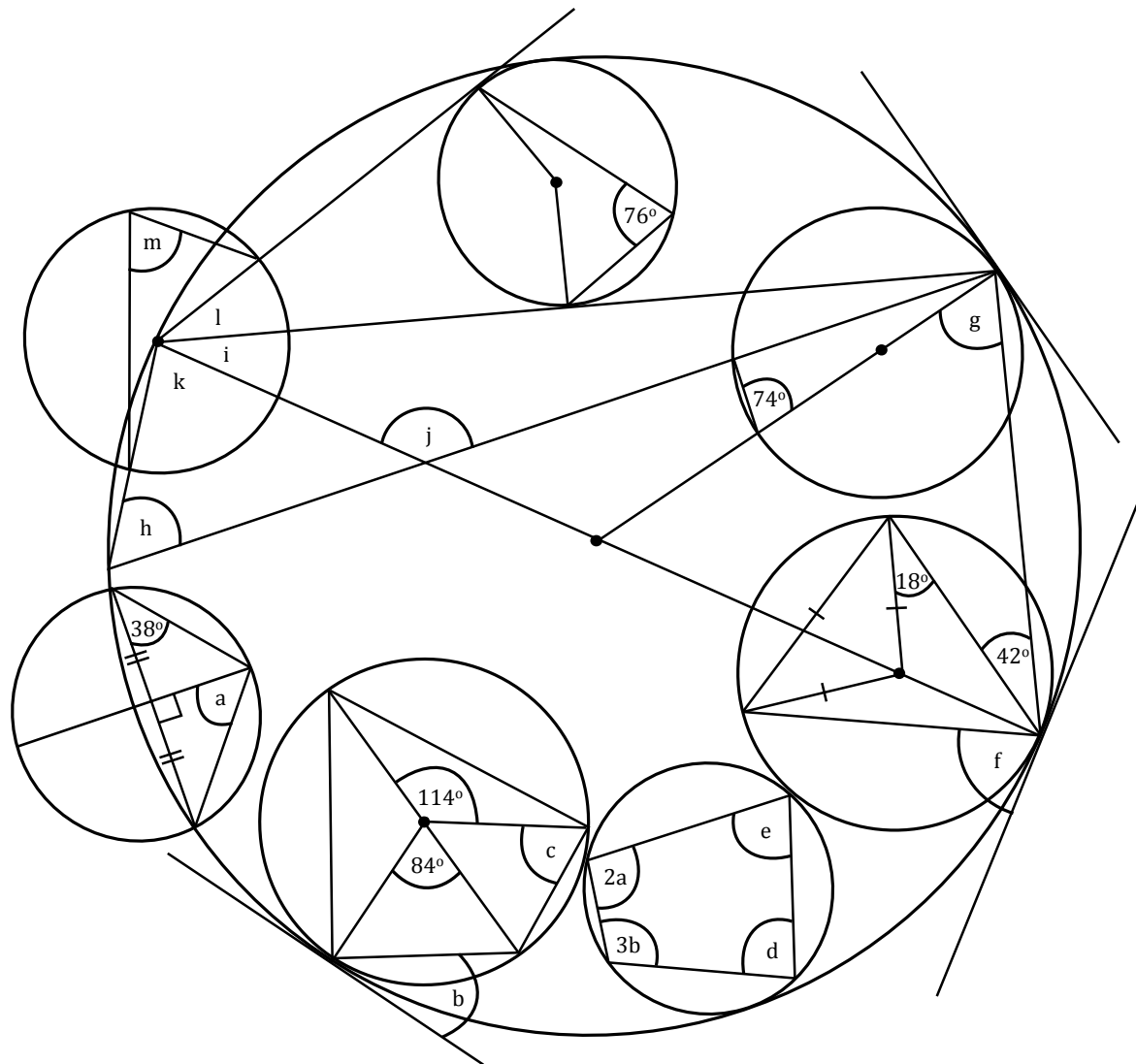


Find the area of the circle to 3sf



The area of the circle is $49\pi \text{ cm}^2$
Find the shaded area to 3sf

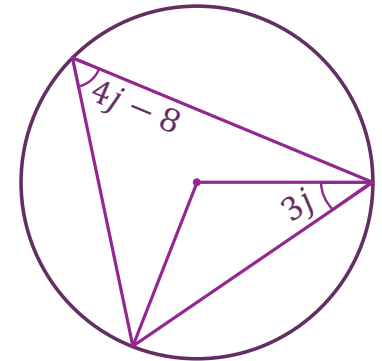
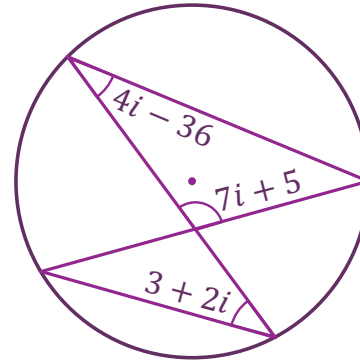
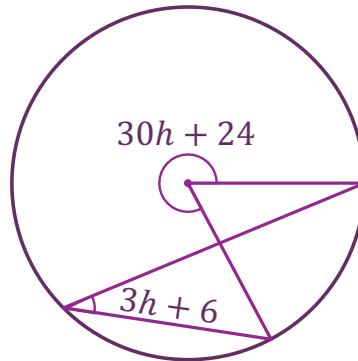
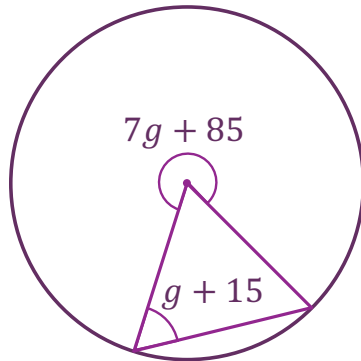
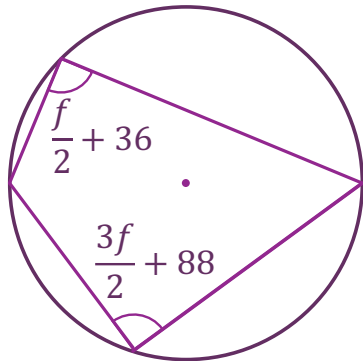
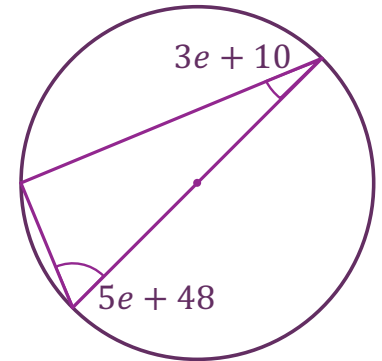
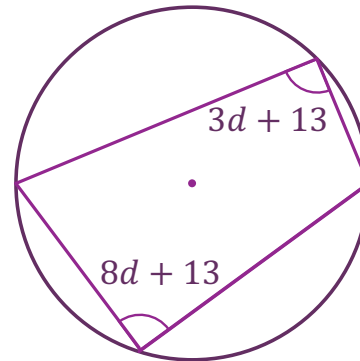
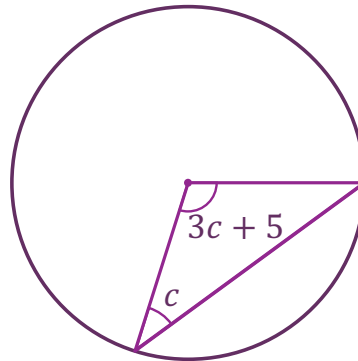
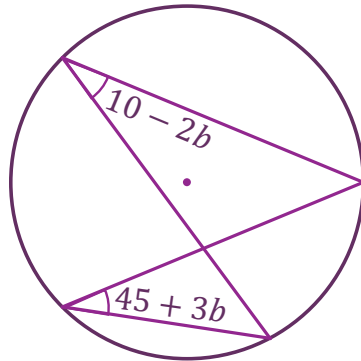
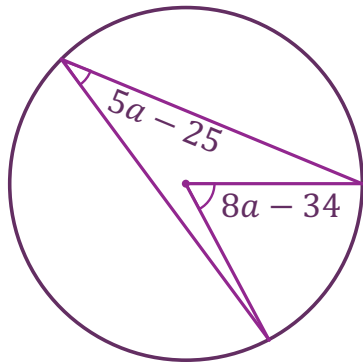
Fluency Practice



Solving Linear Equations from...

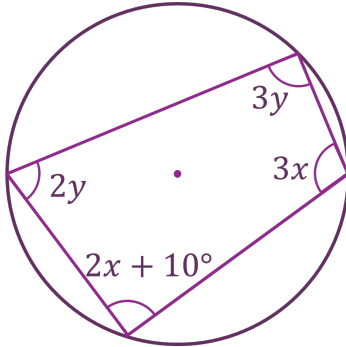
Find the value of each letter.

All angles are given in degrees.

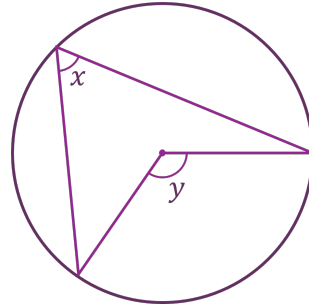


Solving further equations with... Circle Theorems

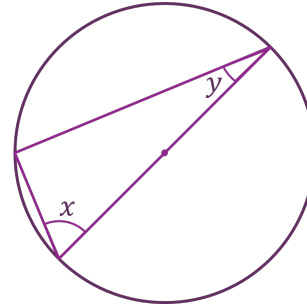
a) Find x and y



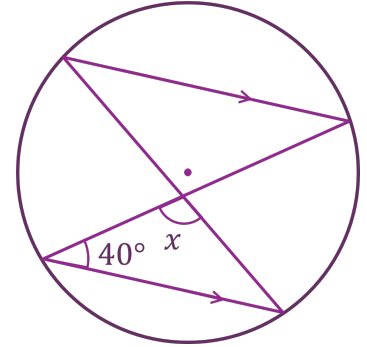
b) $x + y = 150^\circ$
Find x and y



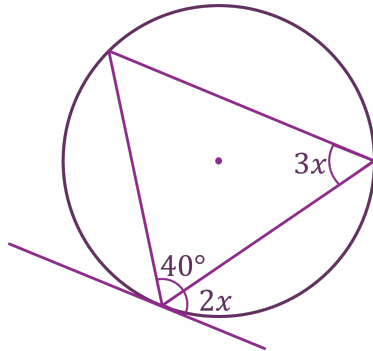
c) $x : y = 2 : 7$
Find x and y



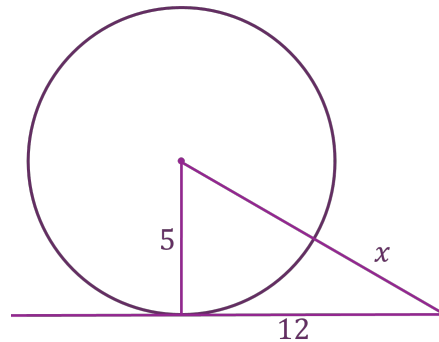
d) Find x



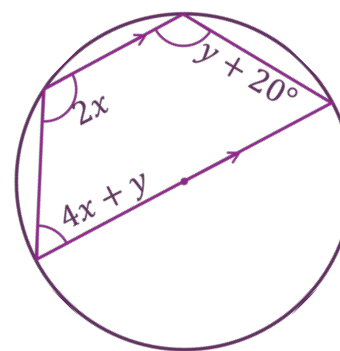
e) Find x



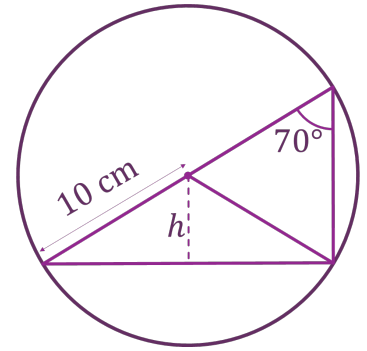
f) Find x



g) Find x and y

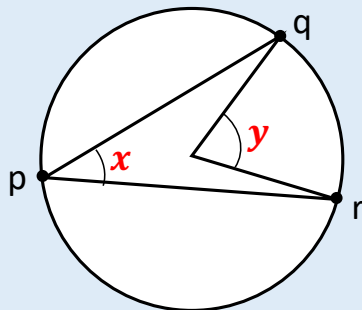


h) Find h

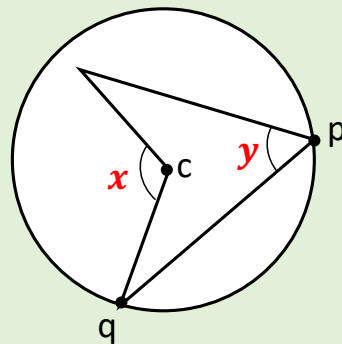


Problem Solving

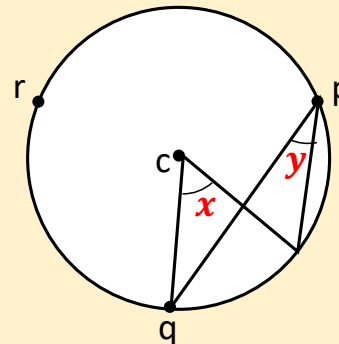
(1)



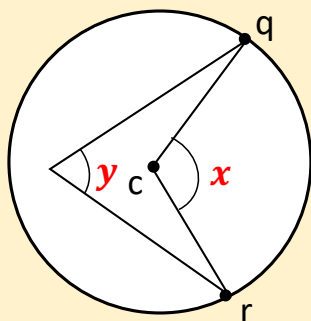
(2)



(3)



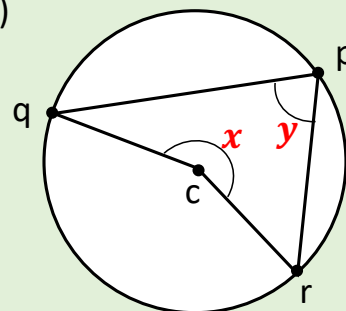
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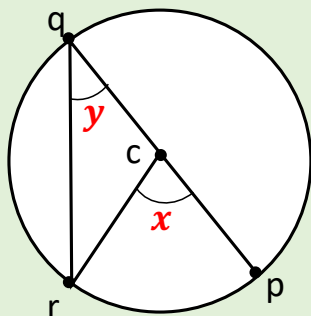
c is the centre of the circle
p, q and r are all on the
circumference of the circle

Determine in which of the
diagrams; x IS ALWAYS
TWICE y

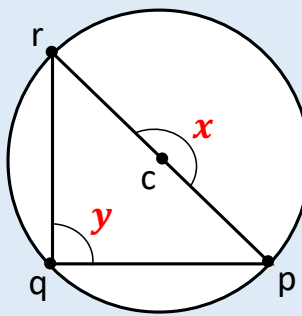
(5)



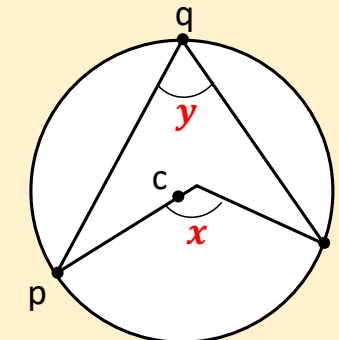
(6)



(7)



(8)



Problem Solving

χ° Create a Circle Theorem Question

Draw an question that needs knowledge of ... to solve.

angles subtended by
the same arc

angles within semicircles

cyclic quadrilaterals

radii & tangents

tangents to a circle &
external points

angles at the centre &
at the circumference



tangents & alternate segments

radii & chords

circle theorems &
vertically opposite angles

circle theorems &
isosceles triangles

circle theorems &
interior angles of a polygon

2 circle theorems

circle theorems &
parallel lines

circle theorems &
Pythagoras' theorem

Diagrams don't need to be drawn accurately!

3 Direct and Inverse Proportion

Fluency Practice

Question	General Equation	Find k	New Equation	Find Value using Equation
A is directly proportional to B , when $A = 10, B = 2$. Find A when $B = 12$.	$A = kB$	$10 = k \times 2$ so $k = 5$	$A = 5B$	$A = 5 \times 12 = 60$
(a) y is directly proportional to x , when $y = 55, x = 5$. Find y when $x = 9$				
(b) N is directly proportional to L , when $N = 1.8, L = 0.6$. Find N when $L = 2.5$				
(c) y is directly proportional to x . If $y = 5$ when $x = 10$, find y when $x = 60$				
(d) A is directly proportional to B and when $A = 12, B = 3$. Find A when $B = 20$	(e) h is directly proportional to V and $h = 36$ when $V = 8$. Find h when $V = 44$	(f) y is directly proportional to the x , and $y = 250$ when $x = 5$. Find x when $y = 7.5$	(g) y is directly proportional to x . When $x = 2, y = 64$. Find x when $y = 80$	

Fluency Practice

Question 1:



- A is directly proportional to B.
When $A = 12$, $B = 3$
- (a) Find a formula for A in terms of B.
 - (b) Find the value of A when $B = 5$
 - (c) Find the value of B when $A = 36$

Question 2:



- C is directly proportional to D.
When $C = 125$, $D = 5$
- (a) Find an equation for C in terms of D.
 - (b) Find the value of C when $D = 10$
 - (c) Find the value of D when $C = 75$

Question 3:



- E is directly proportional to F.
When $E = 2$, $F = 8$
- (a) Find an equation for E in terms of F.
 - (b) Find the value of E when $F = 30$
 - (c) Find the value of F when $E = 100$

Question 4:



- y is directly proportional to x.
When $x = 400$, $y = 10$
- (a) Find a formula for y in terms of x.
 - (b) Calculate the value of y when $x = 450$
 - (c) Find the value of x when $y = 200$

Question 5:



- y is directly proportional to x.
Complete the table.

x	4	9	12
y			72

Question 6:



- y is directly proportional to x
Complete the table.

x	2.5	8	
y	4		50

Question 7:



- The cost, C pounds, of hiring a car is directly proportional to the number of days, d, it is hired.
When $d = 5$, $C = 180$

- (a) Find the value of C when $d = 3$
- (b) Find the value of d when $C = 252$

Question 8:



- In a spring, the tension (T newtons) is directly proportional to the extension of the spring (y cm).
When the tension is 180 newtons, the extension is 4cm.
- (a) Find a formula for T in terms of y.
 - (b) Work out the tension when the extension is 3cm
 - (c) Work out the extension, when the tension is 585 newtons.

Fluency Practice

1. For each table of values, state whether or not $y \propto x$.

(a)

x	0	1	2	3
y	0	4	8	12

(b)

x	2	4	6	10
y	10	20	36	50

(c)

x	0	1	2	3
y	2	5	8	11

(d)

x	2	4	8	16
y	16	8	4	2

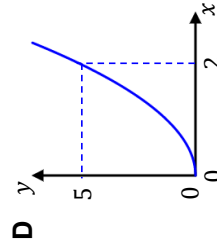
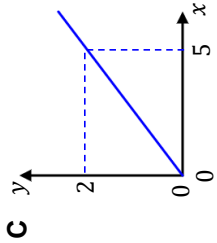
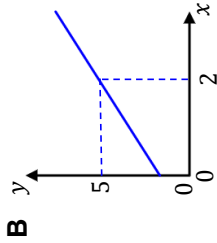
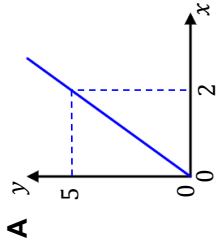
(e)

x	1	2	3	4
y	2.5	5	7.5	10

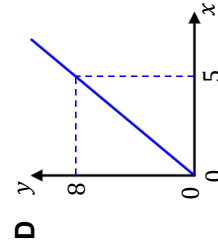
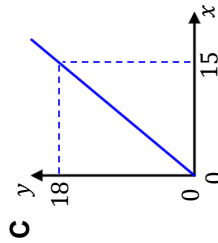
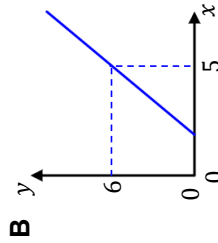
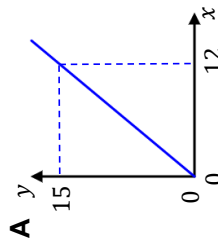
(f)

x	0	2	4	10
y	0	$\frac{n}{2}$	n	$\frac{5n}{2}$

2. x and y are variables such that $x \propto y$ and $y = 5$ when $x = 2$. Which is the correct graph for these variables?



3. x and y are variables such that $x \propto y$ and $y = 6$ when $x = 5$. Which is the correct graph for these variables?



4. In each table, $x \propto y$. Work out the missing values.

(a)

x	0	1	2.5	
y	0	6		30

(b)

x	2	4	6	8
y			6	

(c)

x		4	5
y	0	0.6	1.5

(d)

x	4		16	32
y		$\frac{1}{6}$	$\frac{1}{3}$	

Fluency Practice

5. John's weekly wage (£ w) is directly proportional to the number of hours he has worked in that week (h). One week, John works for 9 hours and earns £58.50.
- (a) Work out a formula for w in terms of h .
 - (b) How much would John earn if he worked for 15 hours?
 - (c) How many hours would John need to work to earn £130 ?
6. Sophie is filling a container from a tap. Water is pouring from a tap at a constant rate. The amount of water poured (V litres) is directly proportional to the time (t s) that the tap has been on. After 20 seconds, 1.6 litres of water have been poured.
- (a) Work out a formula for V in terms of t .
 - (b) How many litres of water will have been poured after 3 minutes?
 - (c) How long does the tap need to be on to fill a 50 litre container? Give your answer in minutes and seconds.
7. Fabric is sold from a roll. Customers can specify the length of fabric they would like to have cut from the roll. The cost of a patterned fabric (£ C) is directly proportional to the length of the fabric (x m). Rachel buys 1.5m of the fabric for £2.70.
- (a) Work out a formula for C in terms of x .
 - (b) How much would it cost to buy 2.5m of the fabric?
 - (c) What length of fabric could be bought for £6 ?
8. The distance travelled on a bicycle is directly proportional to the number of turns of its rear wheel. With 5 turns of the wheel, the bicycle travels 9.75m.
- (a) Work out a formula for D , the distance travelled by the bicycle in m, in terms of t , the number of turns of the wheel.
 - (b) How far will the bicycle travel with 12.5 turns of the wheel?
 - (c) How many times must the wheel turn to travel a distance of 100m?
9. Q is directly proportional to P . $Q = 40$ when $P = 15$.
- (a) Work out a formula for Q in terms of P .
 - (b) Given $P = 6$, work out the value of Q .
 - (c) Given $Q = 84$, work out the value of P .
10. y is directly proportional to x . $y = 54$ when $x = 36$.
- (a) Work out a formula for y in terms of x .
 - (b) Given $x = 30$, work out the value of y .
 - (c) Given $y = 27$, work out the value of x .

11. In the table, $x \propto y$.
Decide whether each statement is true or false.

x	2	3	q
y	5	p	r

- (a) $2y = 5x$
- (b) $\frac{q}{3} = \frac{r}{p}$
- (c) $\frac{q}{r} = 2.5$
- (d) The ratio 3 : p is equivalent to 2 : 5

Fluency Practice

Question	General Equation	Find k	New Equation	Find Value using Equation
A is directly proportional to B^2 , when $A = 45$, $B = 3$. Find A when $B = 7$	$A = k \times B^2$	$45 = k \times 3^2$ so $k = 5$	$A = 5B^2$	$A = 5 \times 7^2 = 245$
(a) y is directly proportional to x^2 , and $y = 270$ when $x = 3$. Find y when $x = 5$				
(b) N is directly proportional to L^3 , when $N = 1280$, $L = 4$. Find N when $L = 3$				
(c) A is directly proportional to $\sqrt{B}$ and when $A = 90$, $B = 9$. Find A when $B = 25$				
(d) A is directly proportional to B^2 and when $A = 8$, $B = 4$. Find A when $B = 0.5$	(e) h is directly proportional to $\sqrt{w}$ and $h = 15$ when $w = 4$. Find h when $w = 64$	(f) A is directly proportional to V^3 and when $A = 400$, $V = 2$. Find V when $A = 6250$	(g) y is directly proportional to $\sqrt[3]{x}$. When $x = 8$, $y = 64$. Find x when $y = 128$	

Fluency Practice

Question 1:



- A is directly proportional to B^2
When $A = 50$, $B = 5$
(a) Find a formula for A in terms of B.
(b) Find the value of A when $B = 3$
(c) Find the value of B when $A = 200$

Question 2:



- y is directly proportional to the square of x
When $y = 6.4$, $x = 4$
(a) Find a formula for y in terms of x
(b) Find the value of y when $x = 8$
(c) Find the value of x when $y = 78.4$

Question 3:



- W is directly proportional to P^3 .
When $P = 2$, $W = 32$
(a) Express W in terms of P
(b) What is the value of W when $P = 4$?
(c) What is the value of P when $W = 4000$?

Question 4:



- Z is directly proportion to $\sqrt{x}$
When $Z = 12$, $x = 36$
(a) Express Z in terms of x
(b) Work out the value of Z when $x = 121$
(c) Work out the value of x when $Z = 18$

Question 5:



- C is directly proportional to the cube of D
When $D = 5$, $C = 175$
(a) Work out the value of C when $D = 6$
(b) Work out the value of D when $C = 4725$

Question 6:



- y is directly proportional to the cube root of x
When $y = 7600$, $x = 4096$
(a) Find an equation connecting y and x.
(b) Calculate the value of y when $x = 125$
(c) Calculate the value of x when $y = 9975$

Question 7:



- The table shows a set of values for x and y.
y is directly proportional the the square root of x.

x	25	
y	9	36

Complete the table

Fluency Practice

Question 8: An object when dropped, falls d metres in t seconds.



d is directly proportional to the square of t .

The object falls 80 metres in 4 seconds.

Work out how far the object falls in 9 seconds.

Question 9: The cost of a circular table is directly proportional to the square of its radius.



A table with a radius of 40cm cost £90.

What is the cost of a table with a radius of 60cm?

Question 10: The mass of a paperweight is m grams.

The length of the paperweight is L centimetres.

m is directly proportional to the cube of L .

$m = 4968$ when $L = 12$



(a) Work out an equation connecting m and L

(b) Work out the mass of a paperweight with a length of 4 centimetres

Intelligent Practice

1) y is directly proportional to x .

When $y = 80$, $x = 2$.

Find y when $x = 4$

2) y is directly proportional to $2x$.

When $y = 80$, $x = 2$.

Find y when $x = 4$

3) y is directly proportional to $8x$.

When $y = 80$, $x = 2$.

Find y when $x = 4$

4) y is directly proportional to x^2 .

When $y = 80$, $x = 2$.

Find y when $x = 4$

5) y is directly proportional to $x + 2$.

When $y = 80$, $x = 2$.

Find y when $x = 4$

6) y is directly proportional to $(x + 2)^2$

When $y = 80$, $x = 2$.

Find y when $x = 4$

7) x is directly proportional to y

When $y = 80$, $x = 2$.

Find y when $x = 4$

8) x is directly proportional to $2y$

When $y = 80$, $x = 2$.

Find y when $x = 4$

9) y is directly proportional to $\sqrt{x + 2}$

When $y = 80$, $x = 2$.

Find y when $x = 4$

Fluency Practice

1. For each table of values, state whether $y \propto x$, $y \propto x^2$, $y \propto x^3$, or $y \propto \sqrt{x}$ and work out an equation for y in terms of x .

(a)

x	1	2	3	4
y	3	12	27	48

(b)

x	1	2	3	4
y	2	16	54	128

(c)

x	2	3	4	10
y	8	12	16	40

(d)

x	1	4	9	16
y	2	4	6	8

(e)

x	1	2	3	4
y	0.5	4	13.5	32

(f)

x	1	3	6	12
y	$\frac{1}{3}$	3	12	48

2. In each table, use the given proportional relationship to work out the missing values.

(a)

x	0	1	2	
$y \propto x^3$			24	648

(b)

x	0	1	4	9
$y \propto \sqrt{x}$				15

(c)

x	1	2		10
$y \propto x^2$		6	13.5	

(d)

x	2	3		6
$y \propto x^3$		18	$\frac{250}{3}$	

3. The mass of dough required to make a pizza is directly proportional to the square of the diameter of the pizza. A pizza with a diameter of 10 inches requires 500g of dough.

- (a) Work out a formula for m , the mass of dough required in grams, in terms of d , the diameter of the pizza in inches.
- (b) How much dough is required for a 12 inch pizza?
- (c) Amy has 320g of dough to make a pizza with. What diameter should her pizza have?

4. A stone is dropped from a hot air balloon. The distance travelled by the stone is proportional to the square of the time it has been falling. After 2 seconds, the stone has fallen a distance of 19.6m.

- (a) Work out a formula for the distance travelled by the stone (d m), in terms of time (t s).
- (b) How far will the stone have fallen after 4 seconds?
- (c) How long will it take for the stone to fall a total of 100m? Give your answer to the nearest tenth of a second.

5. David makes a range of similar pots, of varying sizes. The capacity of each pot is directly proportional to the cube of its height. One of the pots has a height of 20cm and a capacity of 400ml.

- (a) Work out a formula for the capacity of a pot (c ml), in terms of its height (h cm).
- (b) What is the capacity, in litres, of a pot with a height of 30cm?
- (c) What is the height of a pot with a capacity of 878.8ml?

Fluency Practice

6. The distance visible from a point to the horizon is directly proportional to the square root of the height above sea level. From a point on a lighthouse 9m above sea level, it is possible to see 10.8km to the horizon.

- (a) Work out a formula for d , the distance to the horizon in km, in terms of h , the height above sea level in m.
- (b) How far is it possible to see to the horizon at a height of 16m above sea level?
- (c) At what height above sea level is the distance to the horizon 18km?

7. A pendulum's period is the time taken for it to complete one full swing. The period is directly proportional to the square root of the pendulum's length. A pendulum with a length of 36cm has a period of 1.2s

- (a) Work out a formula for T , the period of a pendulum in s, in terms of l , the length of the pendulum in cm.
- (b) Work out the period of a pendulum with a length of 42cm, correct to the nearest tenth of a second.
- (c) What length of pendulum has a period of 1 second?

8. B is directly proportional to the square of A . $B = 128$ when $A = 4$.

- (a) Work out a formula for A in terms of B .
- (b) Given $A = 5$, work out the value of B .
- (c) Given $B = 32$, work out the value of A .

9. q is directly proportional to the cube of p . $q = 6$ when $p = 2$.

- (a) Work out a formula for q in terms of p .
- (b) Given $p = 6$, work out the value of q .
- (c) Given $q = 750$, work out the value of p .

10. y is directly proportional to the square root of x . $y = 12.5$ when $x = 25$.

- (a) Work out a formula for y in terms of x .
- (b) Given $x = 80$, work out the value of y as a surd in its simplest form.
- (c) Given $y = 5\sqrt{3}$, work out the value of x .

11. P is directly proportional to the cube of t . $P = 1$ when $t = \frac{2}{3}$.

- (a) Work out a formula for P in terms of t .
- (b) Given $t = 2$, work out the value of P .
- (c) Given $P = \frac{8}{3}$, work out the value of t .

12. A cuboid has dimensions x cm, x cm and $2x$ cm, where x is a variable.

- (a) Find a formula in terms of x for V , the volume of the cuboid.
- (b) Complete the statement: $V \propto \dots$
- (c) Find a formula in terms of x for A , the surface area of the cuboid.
- (d) Complete the statement: $A \propto \dots$
- (e) (Challenge) Find the proportional relationship between V and A .

Fluency Practice

The weight of a piece of wire is directly proportional to its length. A piece of wire is 25 cm long and has a weight of 6 grams. Another piece of the same wire is 30 cm long. Calculate the weight of the 30 cm piece of wire.

In a spring, the tension, T Newtons, is directly proportional to its extension, x cm. When the tension is 300 Newtons, the extension is 12 cm.

(a) Find a formula for T in terms of x .
(b) Calculate the tension, in Newtons, when the extension is 15 cm.

The time, T seconds, it takes a kettle to boil some water is directly proportional to the mass of water, m kg, in the kettle. When $m = 250$, $T = 300$. Find T when $m = 400$.

In a factory, chemical reactions are carried out in cylindrical containers. The time, T minutes, the chemical reaction takes is directly proportional to the square of the radius, R cm, of the cylindrical container. When $R = 12$, $T = 72$. Find the value of T when $R = 15$.

The amount of clay used to make a statue is directly proportional to the cube of the height of the statue. A statue which is 10 cm tall requires 500 cm^3 of clay. How much clay is required for a similar statue which is twice as tall?

Fluency Practice

direct proportion

- (1) the distance travelled by a Citroen 2CV6 in km (d , travelling at a steady speed) is directly proportional to the time taken (h hrs)
it travels 75 km in $2\frac{1}{2}$ hours
how far will it travel in 4 hours at the same steady speed?



- (2)  the amount of water a pig uses when taking a shower (w litres, at a steady flow) is directly proportional to the time it takes to shower (t minutes)
a pig uses 25.2 litres in 9 minutes
how much water would it use in half an hour?

- (3) the distance a cow falls (f metres) is directly proportional to the square of the time taken (t secs)
it falls 432m in 6 secs
how far will it fall in 10 secs?



- (4)  the pressure (p g/cm²) required to blow up a balloon at any time is directly proportional to the cube of its radius (r cm)
the pressure is 80 g/cm² when the radius is 5cm
what pressure is required at a time when this radius has doubled?



- (5) the slowest speed a bird can fly (f mph) is directly proportional to the square root of its length (L cm).
the slowest speed of a particular bird, 23cm long, is 24.5 mph
what are the slowest speeds of:
(i) a chaffinch, 15 cm long and (ii) a curlew, 56cm long?



Fluency Practice

Question	Equation	Find k	New Equation	Find Value using Equation
A is inversely proportional to B, when $A = 5, B = 6$. Find A when $B = 10$.	$A = \frac{k}{B}$	$5 = \frac{k}{6}$ so $k = 30$	$A = \frac{30}{B}$	$A = \frac{30}{10} = 3$
(a) y is inversely proportional to x, when $y = 12, x = 5$. Find y when $x = 4$.				
(b) N is inversely proportional to L, when $N = 2.5, L = 8$. Find N when $L = 4$.				
(c) y is inversely proportional to x. If $y = 5$ when $x = 8$, find y when $x = 20$				
(d) A is inversely proportional to B and when $A = 12, B = 3$. Find A when $B = 10$	(e) h is inversely proportional to V and $h = 36$ when $V = 8$. Find h when $V = 20$	(f) y is inversely proportional to x, and $y = 0.2$ when $x = 5$. Find x when $y = 25$	(g) y is inversely proportional to x. When $x = 2, y = 64$. Find x when $y = 80$.	

Fluency Practice

Question 1:  T is inversely proportional to N.
When $T = 30$, $N = 5$.

- (a) Find an equation connecting T and N.
- (b) Work out the value of T when $N = 10$
- (c) Work out the value of N when $T = 25$

Question 2:  w is inversely proportional to f
When $f = 12$, $w = 40$

- (a) Find a formula connecting w and f
- (b) Find the value of w when $f = 60$

Question 3:  B is inversely proportional to y
When $B = 0.8$, $y = 13$

- (a) Find an equation for B in terms of y.
- (b) Work out the value of B when $y = 5$

Question 4:  x is inversely proportional to y
Complete the table.

x	16	8	
y		10	20

Question 5:  The number of days, D, to complete research is inversely proportional to the number of researchers, R, who are working.
The research takes 125 days to complete when 24 people work on it.
Find out how many people are needed to complete the research in 60 days.

Question 6:  The volume, V litres, which a fixed mass of gas occupies is inversely proportional to its pressure, P pascals.

When the pressure is 200000 pascals, its volume is 6 litres.

- (a) Find an equation connecting V and P.
- (b) Find the volume of gas when the pressure is 150000
- (c) Find the pressure when the volume of gas is 20 litres.

Fluency Practice

Question	Equation	Find k	New Equation	Find Value using Equation
A is inversely proportional to B^2 , and when $A = 6, B = 5$. Find A when $B = 2$	$A = \frac{k}{B^2}$	$6 = \frac{k}{5^2}$ so $k = 150$	$A = \frac{150}{B^2}$	$A = \frac{150}{2^2} = 37.5$
(a) y is inversely proportional to x^2 and when $y = 10, x = 2$. Find y when $x = 5$				
(b) y is inversely proportional to x^3 , and $y = 5$ when $x = 3$. Find y when $x = 10$				
(c) A is inversely proportional to $\sqrt{B}$ and when $A = 90, B = 9$. Find A when $B = 25$				
(d) h is inversely proportional to V^2 and $h = 3$ when $V = 8$. Find h when $V = 4$	(e) B is inversely proportional to $\sqrt{C}$, and when $B = 18, C = 16$. Find B when $C = 0.36$	(f) y is inversely proportional to x^3 , and $y = 20$ when $x = 6$. Find x when $y = 67.5$	(g) y is inversely proportional to $\sqrt[3]{x}$. When $x = 8, y = 4$, find x when $y = 0.8$	

Fluency Practice

Question 1:  y is inversely proportional to the square of x .

When $y = 200$, $x = 2$

- (a) Find an equation connecting y and x .
- (b) Work out the value of y when $x = 5$
- (c) Work out the value of x when $y = 50$

Question 2:  A is inversely proportional to the square of B .

When $A = 2$, $B = 3$

Find the value of A when $B = 2$

Question 3:  W is inversely proportional to $\sqrt{B}$

When $B = 9$, $W = 9$

- (a) Express W in terms of B
- (b) Work out the value of W when $B = 4$
- (c) Work out the value of B when $W = 1$

Question 4:  T is inversely proportional to the cube of L

When $L = 0.2$, $T = 5$

- (a) Write a formula connecting T and L .
- (b) Work out the value of T when $L = 0.5$
- (c) Work out the value of L when $T = 2$

Question 5:



- q is inversely proportional to the square of t .
When $q = 7.5$, $t = 1.6$
(a) Calculate the value of q when $t = 8$
(b) Calculate the value of t when $q = 1.875$

Question 6:



- y is inversely proportional to the cube root of x
When $y = 2500$, $x = 64$
(a) Find the value of y when $x = 8$
(b) Find the value of x when $y = 2000$

Question 7:



- The table shows a set of values for x and y .
 y is inversely proportional to the square of x .

x	1	2	3
y	7	$1\frac{3}{4}$	

- (a) Find the equation for y in terms of x
(b) Find the missing value

Question 8:



- The force, F newtons, exerted by a magnet on a metal object is inversely proportional to the square of the distance d cm
When the $d = 2$ cm, $F = 60$ N

- (a) Express F in terms of d
(b) Find the force when the distance between the magnet and the metal object is 10cm
(c) Find the distance between the magnet and the metal object when the force is 15N

Question 9:



- The time taken, t , for passengers to be checked-in for a flight is inversely proportional to the square of the number of staff, s , working
It takes 30 minutes for passengers to be checked-in when 5 staff are working.
(a) Find an equation connecting t and s .
(b) How long would it take to check-in the passengers with 3 staff working?

Intelligent Practice

Predict what you think the answer will be and then *work it out*.

‘y is inversely proportional to x.’ unless otherwise stated.

A

- 1) When $y = 5$, $x = 12$
Find y when $x = 3$
- 2) When $y = 12$, $x = 5$
Find y when $x = 3$
- 3) When $y = 5$, $x = 12$
Find y when $x = 6$
- 4) When $y = 10$, $x = 12$
Find y when $x = 3$
- 5) When $y = 10$, $x = 24$
Find y when $x = 3$
- 6) When $y = 10$, $x = 24$
Find y when $x = 1.5$
- 7) When $y = 10$, $x = 24$
Find y when $x = 30$
- 8) When $y = 5$, $x = 12$
Find x when $y = 15$
- 9) When $y = 5$, $x = 12$
Find x when $y = 30$
- 10) When $y = 12$, $x = 5$
Find y when $x = 120$

B

- 1) $y \propto \frac{k}{x}$
If $y = 6$ when $x = 2$, find the value of y when $x = 4$
- 2) When $y = 6$, $x = 12$
Find y when $x = 4$
- 3) $y \propto \frac{k}{x}$
If $y = 6$ when $x = 2$, find the value of y when $x = 1$
- 4) $y \propto \frac{k}{x}$
If $y = 6$ when $x = 2$, find the value of x when $y = 2$
- 5) When $y = 12$, $x = 12$
Find y when $x = 12$
- 6) $y \propto \frac{k}{x}$
If $y = 12$ when $x = 4$, find the value of y when $x = 2$

C

- 1) $y \propto \frac{k}{x^2}$
If $y = 15$ when $x = 2$, find the value of y when $x = 4$
- 2) $y \propto \frac{k}{x^2}$
If $y = 15$ when $x = 4$, find the value of y when $x = 8$
- 3) When $y = 10$, $x = 1.5$
Find y when $x = 3$
- 4) $y \propto \frac{k}{x}$
If $y = 6$ when $x = 2$, find the value of x when $y = 7$
- 5) $y \propto \frac{k}{x^3}$
If $y = 15$ when $x = 2$, find the value of y when $x = 4$
- 6) When $y = 1$, $x = 1$
Find y when $x = 576$

Intelligent Practice

1) y varies inversely with x

When $y = 5$, $x = 6$.

Find y when $x = 3$.

2) y varies inversely with $2x$

When $y = 5$, $x = 6$.

Find y when $x = 3$.

3) y varies inversely with x^2

When $y = 5$, $x = 6$.

Find y when $x = 3$.

4) y varies inversely with x^2

When $y = 5$, $x = 6$.

Find y when $x = 1$.

5) y varies inversely with $x + 3$

When $y = 5$, $x = 6$.

Find y when $x = 3$.

6) y varies inversely with $x + 3$

When $y = 5$, $x = 6$.

Find y when $x = 1$.

7) y varies inversely with $(x + 3)^2$

When $y = 5$, $x = 6$.

Find y when $x = 3$.

8) y varies inversely with $(x + 3)^2$

When $y = 5$, $x = 3$.

Find y when $x = 0$.

9) y varies inversely with $(2x)^2$

When $y = 5$, $x = 6$.

Find y when $x = 3$.

Fluency Practice

1. For each table of values, $y \propto \frac{1}{x}$. Work out the missing values.

(a)

x	2	3	6	8
y	24		8	

(b)

x	2	4	5	10
y		15		

(c)

x	0.5	1	1.5	2
y			12	

(d)

x		4		6
y	5		3	2.5

2. For each table of values, $y \propto \frac{1}{x}$. Work out an equation for y in terms of x .

(a)

x	2	4	5	8
y	12	6	4.8	3

(b)

x	1	2	3	4
y	2	1	$\frac{2}{3}$	$\frac{1}{2}$

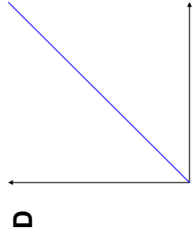
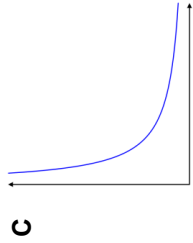
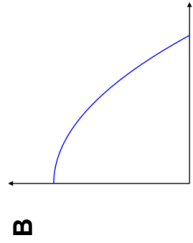
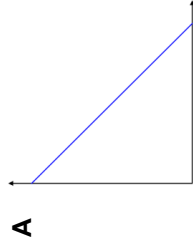
(c)

x	$\frac{1}{2}$	2	4	5
y	$\frac{2}{5}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{25}$

(d)

x	$\frac{1}{2}$	3	4	12
y	$\frac{3}{2}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{16}$

3. Which is the correct graph for variables that are inversely proportional?



4. Tim has a wheelbarrow full of sand. He can fill a 12 litre bucket with the sand 15 times.

- (a) How many times could he fill a bucket with a capacity of 18 litres, with the sand from the wheelbarrow?
- (b) Work out a formula for the number of times a bucket can be filled n , in terms of the capacity of the bucket c (litres).
- (c) Tim has a large bucket which he is able to fill $7\frac{1}{2}$ times with the sand.
Work out the capacity of this bucket.
5. The cost of renting a house is to be shared equally amongst its occupants. If there are 4 people living at the house, each of them will pay £360 per month.
- (a) Work out a formula for the amount each person will pay per month £ C , in terms of the number of people living in the house, n .
- (b) If there were 5 people living in the house, how much would each person have to pay per month?
- (c) If the amount each person paid for rent was £240, how many people would be living in the house?

Fluency Practice

6. P and Q are variables that are inversely proportional. $P = 3$ when $Q = \frac{1}{9}$.

(a) Work out a formula for Q in terms of P .

(b) Work out the value of Q when $P = 5$.

(c) Work out the value of Q when $P = \frac{1}{2}$.

(d) Work out the value of P when $Q = \frac{1}{21}$.

7. X and Y are variables that are inversely proportional. $Y = 6$ when $X = \frac{1}{8}$.

(a) Work out a formula for Y in terms of X .

(b) Work out the value of Y when $X = 3$.

(c) Work out the value of X when $Y = \frac{9}{10}$.

8. a is inversely proportional to b . Shade the correct statement(s):

If b is doubled, a is halved.	If 4 is added to a , 4 is subtracted from b .
If a is increased by 25%, b is decreased by 25%.	If a is increased by 25%, b is decreased by 20%.
a is directly proportional to the reciprocal of b .	The graph for a and b is a parabolic curve.

inverse proportion: $y \propto \frac{1}{x^2}$, $y \propto \frac{1}{x^3}$, $y \propto \frac{1}{\sqrt{x}}$

9. The force between two magnets is inversely proportional to the square of the distance between them. When the magnets are 0.5m apart, the force between them is 400 newtons.

(a) Work out a formula for the force between the magnets, F newtons, in terms of the distance between them, d metres.

(b) Work out F when $d = 2$.

(c) Work out d when $F = 2500$.

10. B is inversely proportional to the cube of A .

$B = 4$ when $A = 3$.

(a) Work out a formula for B in terms of A .

(b) Work out B when $A = 2$.

(c) Work out A when $B = 5$.

11. y is inversely proportional to the square root of x .

$y = 3$ when $x = 36$.

(a) Work out a formula for y in terms of x .

(b) Work out y when $x = 4$.

(c) Work out x when $y = 2$.

12. q is inversely proportional to the square of p . Shade the correct statement(s):

If p is doubled, q is halved.	If p is doubled, q is divided by 4.
The constant of proportionality is equal to pq^2 .	The constant of proportionality is equal to $(pq)^2$.

Fluency Practice

The shutter speed, S , of a camera varies inversely as the size of the aperture setting, f . When $f = 8$, $S = 125$.

- (a) Find a formula for S in terms of f .
- (b) Hence, or otherwise, calculate the value of S when $f = 4$.

The pressure of water from a hose is inversely proportional to the hose radius. For a hose of radius 2 cm, the water pressure is 40 Pa. What hose radius do you need for a pressure of 50 Pa?

The amount of diesel a van uses is inversely proportional to the number of miles it travels. When a van travels 320 miles, it uses 36 litres of diesel. How much diesel will it need to travel 200 miles?

In a science experiment, p is found to be inversely proportional to t . When $p = 42.8$, $t = 0.8$. Find t when $p = 23.6$. Give your answer to 2 decimal places.

The light intensity I on a surface is inversely proportional to the square of the distance x from the light source. When the surface is 6 cm from the light source, the intensity is 2400.

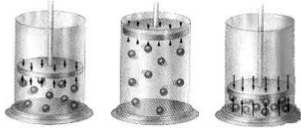
- (a) Find the light intensity when the surface is 15 cm from the light source.
- (b) If the light intensity is 600, how far is the surface from the light source?

Fluency Practice

inverse proportion

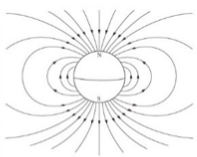
- (1) the amount of time (t , hours) it takes to travel to work is inversely proportional to the average speed (v , mph)
it takes 30 minutes in a car at an average speed of 27.4 mph
how long would it take on a bike at an average speed of 9.13 mph?



- (2)  at a constant temperature the volume of a gas (V) varies inversely as the pressure (P)
the pressure is 80 gm/cm^2 for a volume of 45 cc
what will the volume of gas be for a pressure of 25 gm/cm^2 ?
what will the pressure be for a volume of 75 cc?

- (3) the pull on a satellite (P) is inversely proportional to the square of the average distance from the centre of the Earth (d)
the pull on an 'Explorer' satellite is 900N when it is 9600km away
what will the pull be if it was 8000km away?



- (4)  the magnetic force (B) between two magnets is inversely proportional to the cube of the distance between them (d cm)
the force is 10 gauss for a distance of 10.7cm
what is the force for double this distance?

- (5) a person's pulse rate (p) is inversely proportional to the square root of their height (h cm).
for someone of height 50cm the pulse rate is 134 counts/minute
what is the pulse rate for someone of height 178cm?



Fluency Practice

A is directly proportional to B If $A = 5$ and $B = 100$ Find B when $A = 4$	C is inversely proportional to D If $C = 3$ and $D = 10$ Find C when $D = 6$	E is directly proportional to F If $E = 14$ and $F = 56$ Find F when $E = 10$	G is inversely proportional to H If $G = 8$ and $H = 5$ Find H when $G = 10$
J is directly proportional to K If $J = 7$ and $K = 56$ Find J when $K = 80$	L is directly proportional to M If $L = 10$ and $M = 12$ Find M when $L = 2$	N is directly proportional to P If $N = 3$ and $P = 300$ Find P when $N = 0.07$	Q is inversely proportional to R If $Q = 0.1$ and $R = 80$ Find R when $Q = 4$
S is directly proportional to T If $S = 6$ and $T = 2.4$ Find S when $T = 10$	U is inversely proportional to V If $U = 0.8$ and $V = 0.4$ Find U when $V = 0.2$	W is directly proportional to X If $W = 0.5$ and $X = 16$ Find W when $X = 64$	Y is inversely proportional to Z If $Y = 0.01$ and $Z = 4000$ Find Z when $Y = 0.2$

Fluency Practice

(a)	(b)	(c)	(d)												
y is directly proportional to x . When $x = 8, y = 40$. Find a formula for y in terms of x .	F is inversely proportional to t . When $F = 2.5, t = 4$. Find a formula for F in terms of t .	p is directly proportional to the square of q . When $q = 3, p = 90$. Find a formula linking p and q .	y is directly proportional to x^3 . When $x = 5, y = 2500$. Find a formula for y in terms of x .												
(e)	(f)	(g)	(h)												
Sketch the graph showing y is inversely proportional to x .	y is directly proportional to $\sqrt{x}$. When $x = 4, y = 0.5$. Find the value of y when $x = 64$.	d is inversely proportional to w^2 . When $w = 0.5, d = 12$. Find a formula for d in terms of w .	T is inversely proportional to $\sqrt{L}$. When $L = 16, T = 25$. Find the value of L when $T = 10$.												
(i)		(j)													
The distance d travelled by a ball is proportional to the square of the time taken, t . After 4 seconds the ball has travelled 40 m. (i) Find a formula linking d and t . (ii) Find the distance travelled after 7 seconds.		<table><tr><td>x</td><td>1</td><td>2</td><td>5</td><td>10</td><td>20</td></tr><tr><td>y</td><td>100</td><td>25</td><td>4</td><td></td><td></td></tr></table> (i) Find a formula for y in terms of x . (ii) Complete the table.		x	1	2	5	10	20	y	100	25	4		
x	1	2	5	10	20										
y	100	25	4												

Fluency Practice

direct and inverse proportion

- | | |
|--|--|
| (1) A is directly proportional to the square of L
when L is 8, A is 160

a) find A when L = 6
b) find L when A = 250 | (5) the force between objects (F) is inversely
proportional to the square of the distance
between them (D)
when the distance is 5, the force is 4
a) find F when D = 2
b) find D when F = 400 |
| (2) surface area (A) of a solid is directly
proportional to the square of its length (L)
L = 5, A is 150

a) find A when L = 3
b) find L when A = 2400 | (6) P is inversely proportional to R
when P is 8, R is 10

a) find P when R = 16
b) find R when P = $2\frac{1}{2}$ |
| (3) the radius (r) of a circle is directly proportional
to the square root of the area (A)
when the area is 100, the radius is 5.642

a) find r when A = 400
b) find A when r = 4 | (7) Q is inversely proportional to N
when Q is 8, N is 5

a) find Q when N = 16
b) find N when Q = 80 |
| (4) T is directly proportional to the cube of K
when K is 4, T is 320

a) find T when K = 2
b) find K when T = 40000 | (8) the length (L) of a rectangle of a fixed area is
inversely proportional to the width (W)
when the length is 16, the width is 1.5

a) what is the length when the width is 32?
b) what is the width when the length is 15? |

Fluency Practice

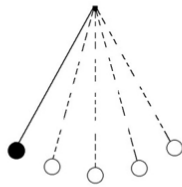
direct and inverse proportion in Physics

- (1) the kinetic energy (E) of a saloon car is directly proportional to the square of the speed of the car (S)
when the speed is 48 km/hr the energy is 141.51 Kjoules



what is the kinetic energy when the speed is doubled, to 96 km/hr?

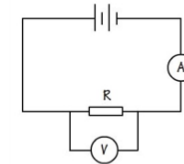
(2)



the period (T) of a pendulum swing is directly proportional to the square root of the length of the pendulum (L)
when the length is 25cm the period is 10.03 seconds

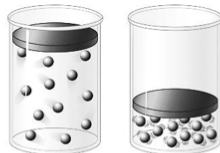
- a) what is the period when the length is 100cm?
b) what is the length for a period of 22.43 seconds?

- (3) in a circuit, the resistance (R ohms) is inversely proportional to the current (A amps)
when the resistance is 12 ohms the current is 9.6 amps



- a) what is the current when the resistance is 9 ohms?
b) what is the current when the resistance is 6.4 ohms?

(4)



at a constant temperature the volume of a gas (V) varies inversely as the pressure (P)

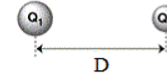
the pressure is 1×10^5 Pa for a volume of $12 \times 10^{-6} \text{ m}^3$

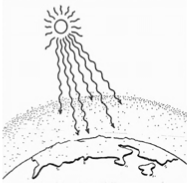
- a) what will the volume be for a pressure of 5×10^4 Pa?
b) what will the pressure be for a volume of $4 \times 10^{-6} \text{ m}^3$?

Fluency Practice

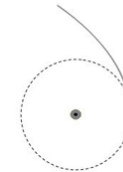
direct and inverse proportion in Physics

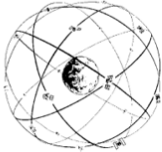
- (4) Coulomb's law states that the magnetic force (F) between two dipoles is inversely proportional to the square of the distance (D) between them
when the distance is 1.5m the force is 2.4×10^{10} N
what is the magnetic force when the distance between the dipoles is 3m?



- (5)  Stefan's law states that the total energy (E) radiated by a black body is directly proportional to the 4th power of the temperature (T) in degrees Kelvin
the temperature of the Earth is 288K; the energy is 390 Watts/m²
what is the energy for the Sun, with a temperature of 6000K?

- (6) the escape velocity of a rocket is inversely proportional to the square root of its distance from the centre of Earth
at the surface of the Earth (6378km from the centre) this velocity is 11.2 km/sec
what is the escape velocity 9000 km above the Earth's surface?



- (7)  the period of a satellite squared (T^2) is directly proportional to the cube of its distance from the centre of Earth
100 km above the surface of the Earth (i.e. 6478km from the centre) this period is 1.44 hours
what is the period of a satellite, 200 km above the Earth's surface?

- (8) in an experiment, measurements of 'h' are taken after time
2 secs, 5 secs and 6 secs with the results shown in the table

t	2	5	6
h	10.1	62.4	89.9

which rule fits the data best?

(a) $h \propto t$ (b) $h \propto t^2$ (c) $h \propto t^3$

Fluency Practice

- 1) x is directly proportional to y^2
 y is directly proportional to z^3
Given that $x = 2$ and $z = 5$ when $y = 6$ find a formula for x in terms of z
- 2) x is inversely proportional to y^3
 y is inversely proportional to z^3
Given that $x = 5$ and $z = 7$ when $y = 4$ find a formula for x in terms of z
- 3) x is inversely proportional to y
 y is inversely proportional to z
Given that $x = 5$ and $z = 4$ when $y = 6$ find a formula for x in terms of z
- 4) x is inversely proportional to y^3
 y is inversely proportional to z
Given that $x = 4$ and $z = 6$ when $y = 5$ find a formula for x in terms of z
- 5) x is inversely proportional to $\sqrt[3]{y}$
 y is inversely proportional to $\sqrt{z}$
Given that $x = 3$ and $z = 9$ when $y = 216$ find a formula for x in terms of z
- 6) x is inversely proportional to y^3
 y is directly proportional to z^3
Given that $x = 7$ and $z = 3$ when $y = 4$ find a formula for x in terms of z
- 7) x is directly proportional to y^2
 y is inversely proportional to z
Given that $x = 7$ and $z = 6$ when $y = 3$ find a formula for x in terms of z
- 8) x is directly proportional to $\sqrt{y}$
 y is directly proportional to z^3
Given that $x = 10$ and $z = 7$ when $y = 4$ find a formula for x in terms of z
- 9) x is directly proportional to $\sqrt{y}$
 y is inversely proportional to z
Given that $x = 3$ and $z = 3$ when $y = 16$ find a formula for x in terms of z
- 10) x is directly proportional to y^3
 y is directly proportional to z^3
Given that $x = 10$ and $z = 4$ when $y = 8$ find a formula for x in terms of z

Fluency Practice

A is proportional to B^2 and A is proportional to C^3 . When $A = 3, B = 1, C = 2$.

(a) Find the value of C when $B = 8$. (b) Find the exact value of B when $C = 1$.

A is proportional to the square root of B . A is proportional to C^3 .

When $A = 17, B = 16, C = 4$.

(a) Find B when $C = 1$.

(b) Find C when $B = 9$ (to 1 decimal place)

A is proportional to square root of B . A is inversely proportional to C .

When $A = 5, B = 4, C = 3$.

(a) Find B when $C = 2$.

(b) Find C when $B = 1$.

A is inversely proportional to B . A is inversely proportional to the square root of C .

(a) Find B when $C = 4$.

(b) Find C when $B = 10$

A is proportional to B^2 , and B is proportional to C^3 .

Show that A is proportional to C^d , where d is an integer value to be found. ($d = 6$)

A^2 is proportional to B^3 and B^2 is proportional to C . Show that A^n is proportional to C^m where n and m are integers to be found. ($n = 4, m = 3$)

The square root of A is proportional to cube of B . The cube root of B is proportional to C .

Show that A is proportional to C^n , where n is an integer to be found. ($n = 6$)

Fluency Practice

a is directly proportional to b^2. b varies directly with c. When $a = 320$, $b = 8$ when $c = 4$. Write a formula for a in terms of c	$d \propto e$, $e \propto f^3$ When $d = 48$, $e = 32$ when $f = 2$. Write a formula for d in terms of f	g varies directly with the square of h. g also varies directly with the cube of i. When $g = 9$, $h = \frac{3}{2}$ when $i = 3$. Write a formula for h in terms of i	$m \propto \sqrt{n}$, $n \propto p^2$ When $m = 10$, $n = 4$ when $p = 6$. Write a formula for m in terms of p
$a \propto b^2$ and $b \propto \frac{1}{\sqrt{c}}$ Given that $a = 3$, $b = 1$ when $c = 4$. Write a formula for a in terms of c	d varies inversely with the square of e. e is inversely proportional to f. When $d = 5$, $e = 2$ when $f = 15$. Write a formula for d in terms of f	$g \propto \frac{1}{\sqrt{h}}$ and $h \propto \frac{1}{i^2}$. Given that $g = 12$, $h = 9$ when $i = 2$. Write a formula for g in terms of i	m varies inversely with p^2. m is also inversely proportional to the cube of n. $m = 8$ when $n = 5$ and $p = 3$. Write a formula for p in terms of n
$a^2 \propto b^3$ and $b \propto \frac{1}{c}$ Given that $a = 9$, $b = 3$ when $c = 4$. Write a formula for a in terms of c	$d^2 \propto e$ and $d \propto \frac{1}{\sqrt{f}}$ Given that $d = 10$, $e = 20$ when $f = 49$. Write a formula for e in terms of f.	$a \propto b^2$, $b \propto \frac{1}{\sqrt{c}}$, $c \propto \frac{1}{d^3}$, $d \propto \sqrt[3]{e}$ Given that $a = 9$, $b = 30$, $c = \frac{1}{36}$, $d = 12$ when $e = 27$. Write a formula for a in terms of e	

Fluency Practice

- 1** $x \propto yz$. When $y = 2$ and $z = 3$, $x = 30$.
 - (a) Find the relation between x , y and z .
 - (b) Find x when $y = 4$ and $z = 6$.
- 2** $x \propto \frac{y}{z}$. $x = 27$ when $y = 9$ and $z = 2$.
 - (a) Find the relation between x , y and z .
 - (b) Find x when $y = 14$ and $z = 12$.
- 3** $p \propto \frac{q}{r^2}$ and $p = 3\frac{1}{3}$ when $q = 5$ and $r = 3$.
 - (a) Find the equation connecting p , q and r .
 - (b) Find p when $q = 9$ and $r = 1\frac{1}{2}$.
- 4** The height (h) of a cone varies directly as its volume (V) and inversely as the square of its radius (r). Use the constant k to show the relationship between h , V and r .
- 5** $A \propto BC$ and $A = 6$ when $B = 4$ and $C = 9$.
 - (a) Find A when $B = 3$ and $C = 10$.
 - (b) Find C if $A = 20$ and $B = 15$.
 - (c) By what percentage does A change if B is increased by 10% and C is decreased by 10%?
- 6** x varies directly with the square of y and with z . When $y = 2$ and $z = 3$, $x = 4\frac{1}{2}$.
 - (a) Find x when $y = 5$ and $z = 4$.
 - (b) Find y when $x = 21$ and $z = 3\frac{1}{2}$.
 - (c) What happens to x if y is doubled and z halved?
- 7** x , y and z are related quantities such that x varies directly as y and inversely as the square root of z . When $x = 300$ and $y = 65$, $z = 25$. Calculate the value of x when $y = 468$ and $z = 144$.
- 8** If $P \propto \frac{1}{V}$ and $V \propto R^2$, how does P vary with R ?
- 9** $x \propto y$ and $y \propto z^3$. How does x vary with z ?
- 10** $x \propto y^2$ and $y \propto z^2$. How does x vary with z ?
- 11** $A \propto BC$ and $B \propto \frac{1}{C^2}$. How does A vary with C ?
- 12** y varies directly as x and inversely as z . x varies inversely as y^2 . Prove that z^2 varies directly as x^3 .

Fluency Practice

1) y is inversely proportional to p^2
 p is decreased by 80%
Work out the percentage increase in y

2) z is proportional to $\sqrt[3]{x}$
 x is decreased by 78.4%
Work out the percentage decrease in z

3) y is inversely proportional to $\sqrt{z}$
 z is decreased by 96%
Find the percentage increase in y

4) z is proportional to q^3
 q is increased by 10%
Work out the percentage increase in z

5) p is proportional to z^2
 z is increased by 20%
Find the percentage increase in p

1) z is proportional to $\sqrt{p}$
 p is decreased by 96%
Work out the percentage decrease in z

2) t is proportional to x^2
 x is increased by 20%
Work out the percentage increase in t

3) x is proportional to p^3
 p is decreased by 60%
Work out the percentage decrease in x

4) p is proportional to z^3
 z is increased by 10%
Find the percentage increase in p

5) q is proportional to y^2
 y is increased by 30%
Find the percentage increase in q

Fluency Practice

c is proportional to d^3 . If d increases by 12%, what is the percentage increase of c (to 1dp)?

a is proportional to b^2 . If a increases by 21%, what is the percentage increase of b ?

p is proportional to the square of q . If q decreases by 3%, what is the percentage decrease of p (to 1dp)?

x is proportional to y^2 . If x decreases by 36%, what is the percentage decrease of y ?

c is proportional to $\sqrt{d}$. If c increases by 20%, what is the percentage increase of d ?

p^2 is proportional to q^3 . If p increases by 12%, what is the percentage increase of q (to the nearest percent)?

A is proportional to B^2 and B is proportional to C^3 . A increases by 50%. What is the percentage increase of C (to the nearest percent)?

Fluency Practice

Question 1:



C and D are positive numbers.
C is inversely proportional to D.
When $C = 7$, $D = 28$.
Find the value of C when $C = D$

Question 2:



The force, F newtons, exerted by a magnet on a metal object is inversely proportional to the square of the distance d cm
When $d = 4$ cm, $F = 300$ N.
Explain what happens to F , when d is halved.

Question 3:



W is directly proportional to the cube of y .
Harry says that when y is halved, the value of W is divided by 10.
Explain why Harry is wrong.

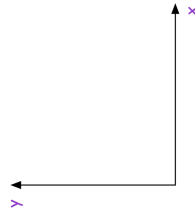
Question 4:



y is inversely proportional to x .
 y and x are positive quantities.
When $y = 200$, $x = 64$

(a) Express y in terms of x

(b) Sketch a graph of the relationship between y and x



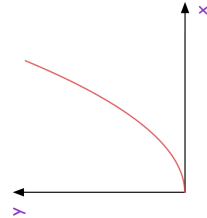
(c) Calculate the value of x , when y is twice as big as x .

Question 5:

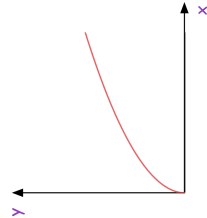


These graphs represent four different types of proportionality.
Match each type of proportionality to the correct graph.

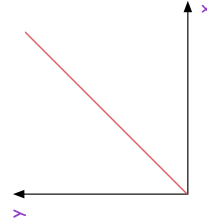
Graph 1



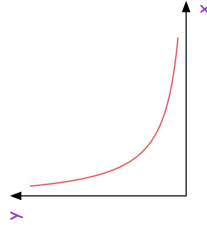
Graph 2



Graph 3



Graph 4



Graph	Type of Proportionality
	$y \propto x$
	$y \propto \sqrt{x}$
	$y \propto x^2$
	$y \propto \frac{1}{x}$

Fluency Practice

Question 6:  The time taken, t , for the waiters in a restaurant to serve food to the guest at a wedding is inversely proportional to the square of the number of waiters, w , working

It takes 90 minutes for the food to be served when 6 waiters are working.

What is the minimum number of waiters needed so that the time taken to serve the food is under 30 minutes?

Question 7:  A is directly proportional to the cube root of B.
B is increased by 60%.
Work out the percentage increase in A.

Question 8:  a is directly proportional to the cube of c.
w is inversely proportional to the square root of a

When $c = 2$, $a = 48$
When $a = 9$, $w = 2400$

Find the value of w when $c = 6$

Question 9:  x is directly proportional to w^2
When $w = 4$, $x = 48$
y is inversely proportional to x^3
When $x = 2$, $y = 14$

Find a formula for y in terms of w .
Give your answer in its simplest form.

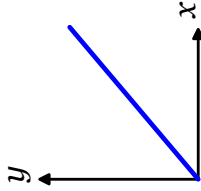
Fluency Practice

learn by heart+

If y is **directly** proportional to x ,

the graph is a **straight line through the origin**.

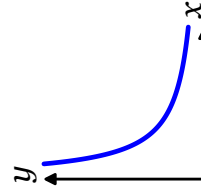
the equation is $y = kx$
where k is a number.



If y is **inversely** proportional to x ,

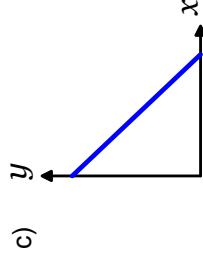
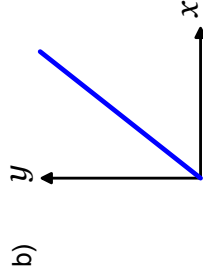
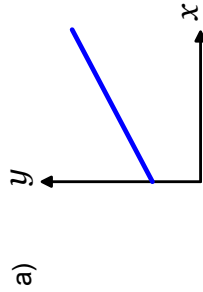
the graph is a **curve as shown**.

the equation is $y = \frac{k}{x}$
where k is a number.

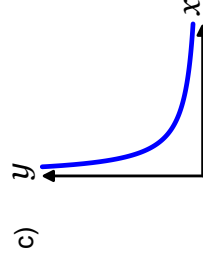
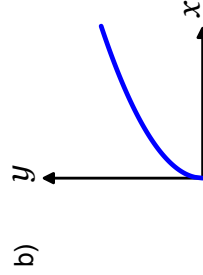
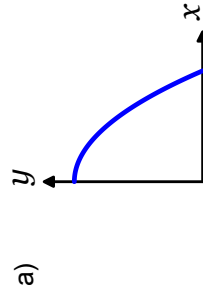


exercise

1. Which **one** of the following shows a graph where x and y are **directly** proportional?



2. Which **one** of the following shows a graph where x and y are **inversely** proportional?



3. For each equation, decide whether x and y are **directly proportional**, **inversely proportional**, or **neither**.

a) $y = 3x + 1$

b) $y = \frac{4}{x}$

c) $y = 0.5x$

d) $y = \frac{1}{2}x$

e) $y = 6x^2$

f) $y = \frac{2}{x}$

g) $y = \frac{3}{x} + 5$

h) $y = \frac{9}{x}$

i) $y = 5x$

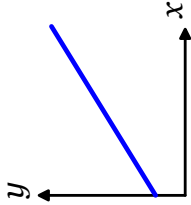
Fluency Practice

4. A graph is shown to the right.

Jessica says

" y is directly proportional to x "

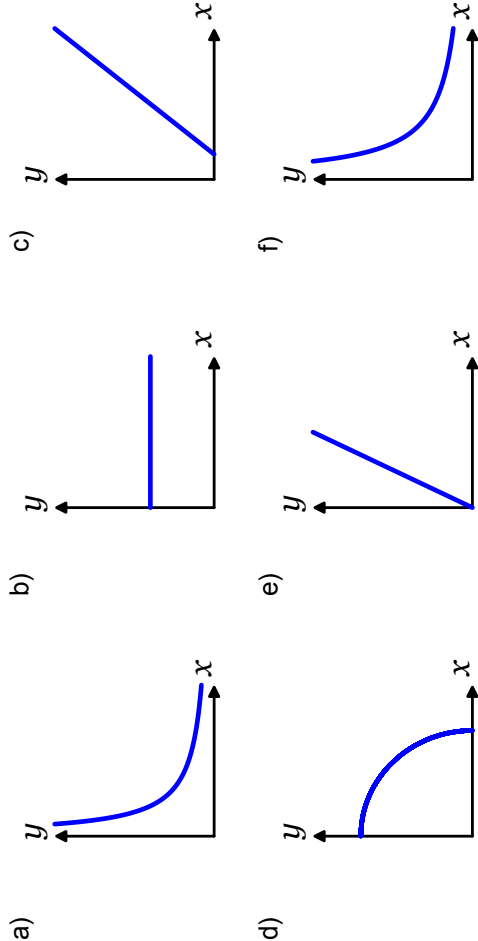
Explain how you can tell Jessica is wrong.



5. Four equations are shown.
 k is a constant in each equation.

$y = \frac{k}{x}$	$y = \frac{1}{x + k}$	$y = kx$	$y = k + x$
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- a) In which equation are x and y directly proportional?
b) In which equation are x and y inversely proportional?
6. For each graph, decide whether x and y are **directly proportional, inversely proportional, or neither**.



G Proportional limits

example

At a playgroup, at least 1 adult is needed for every 6 children.

- a) If there are 5 adults, work out the greatest possible number of children.
1 adult for up to 6 children
5 adults for up to 30 children
30 children
- b) If there are 20 children, work out the least number of adults required.
3 adults for up to 18 children
4 adults for up to 24 children
4 adults

4 Constructions and Loci

Fluency Practice

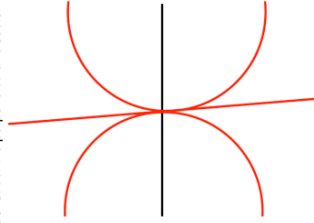
- Question 1: Draw a 8cm line and construct its perpendicular bisector
- Question 2: Draw a 6cm line and construct its perpendicular bisector
- Question 3: Draw the following angles and construct their angle bisectors.

- (a) 60° (b) 40° (c) 90° (d) 110°
- (e) 150° (f) 54° (g) 220°

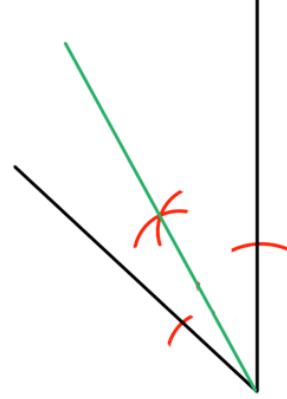
Apply

- Question 1: (i) Draw a triangle.
(ii) Construct the perpendicular bisector for each side of the triangle.
(iii) Using where the perpendicular bisectors meet as the centre, draw the circle that passes through each vertex of the triangle.
- Question 2: Create a storyboard to show how to construct the perpendicular bisector of a line. Clearly annotate each stage.
- Question 3: Create a storyboard to show how to construct the angle bisector of a given angle. Clearly annotate each stage.
- Question 4: Can you spot any mistakes in the questions below?

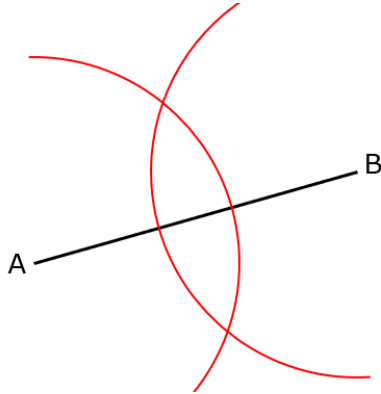
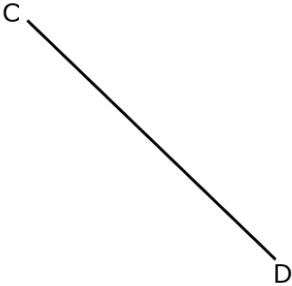
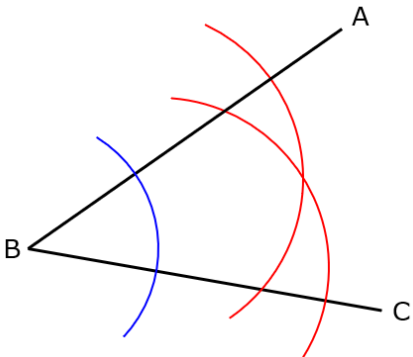
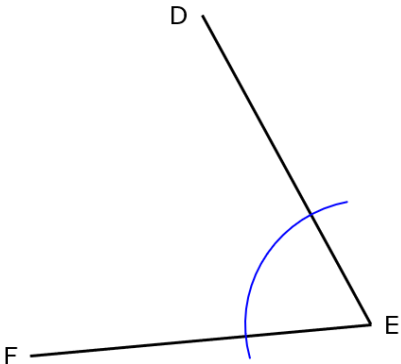
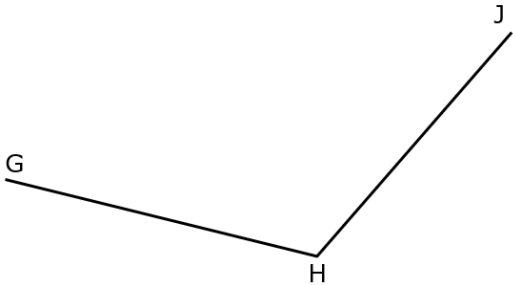
Construct the perpendicular bisector



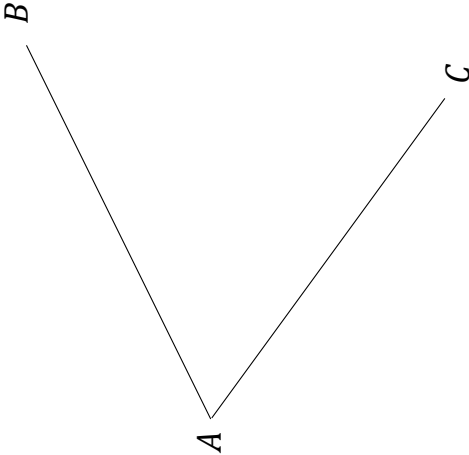
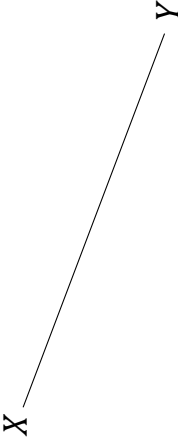
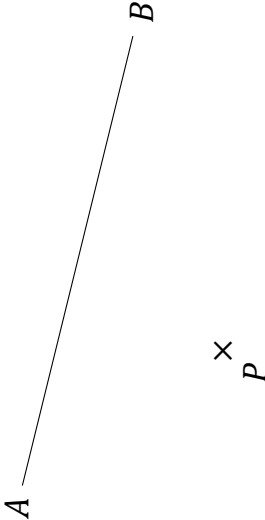
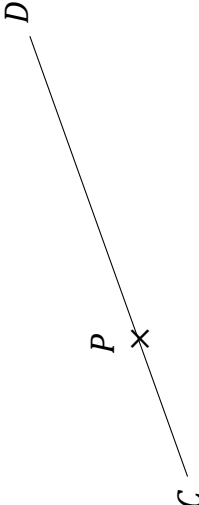
Construct the angle bisector



Fluency Practice

Perpendicular Bisectors and Angle Bisectors		
(a) Complete the perpendicular bisector of the line AB 	(b) Construct the perpendicular bisector of the line CD 	(c) Construct the perpendicular bisector of the line EF 
(d) Complete the bisector of the angle ABC 	(e) Construct the bisector of the angle DEF 	(f) Construct the bisector of the angle GHJ 

Fluency Practice

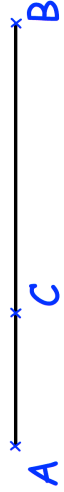
Constructing Perpendiculars and Bisectors	
(a)	(b)
Construct the bisector of the angle BAC.	Construct the perpendicular bisector of the line XY.
	
(c)	(d)
Construct the perpendicular to the line AB that passes through point P.	Construct the perpendicular to the line CD that passes through point P.
	

Fluency Practice

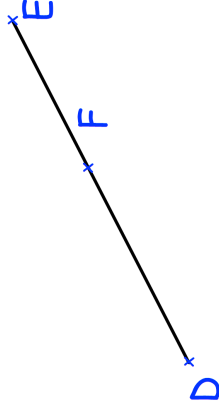
- Q10** Draw a near vertical line of length 6cm. Using a compass and ruler only, construct its perpendicular bisector.
- Q11** Draw a line 8cm in length. Using a compass and ruler only, construct its perpendicular bisector.
- Q12** Using a protractor, draw an angle of 64° . Using a compass and ruler only, construct its angle bisector. Check your answer by measuring the two angles formed.
- Q13** Using a protractor, draw an angle of 120° . Using a compass and ruler only, construct its angle bisector. Check your answer by measuring the two angles formed.
- Q14** Draw a triangle and construct the perpendicular bisector of each side. Draw the smallest possible circle that **does not enter** the triangle.
- Q15** Draw another triangle and construct the angle bisector of each vertex. Draw the largest possible circle that **does not exit** the triangle.

Fluency Practice

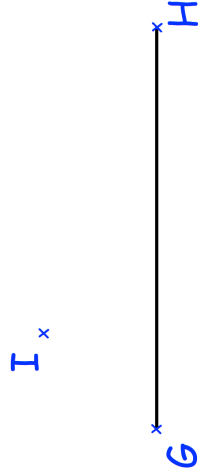
- Question 1:
- (a) Sketch the diagram below.
 - (b) Using a ruler and compasses, construct the perpendicular to AB that passes through the point C.



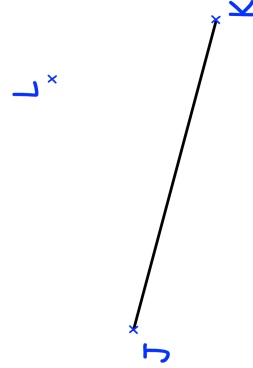
- Question 2:
- (a) Sketch the diagram below.
 - (b) Using a ruler and compasses, construct the perpendicular to DE that passes through the point F.



- Question 3:
- (a) Sketch the diagram below.
 - (b) Using a ruler and compasses, construct the perpendicular from the line GH to the point I.

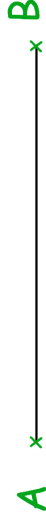


- Question 4:
- (a) Sketch the diagram below.
 - (b) Using a ruler and compasses, construct the perpendicular from the line JK to the point L.



Fluency Practice

- Question 1: (a) Sketch the diagram below.
(b) Using a ruler and compasses, construct the equilateral triangle ABC



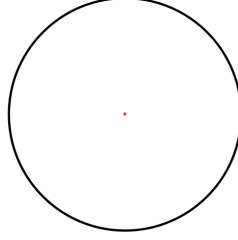
- Question 2: DEF is an equilateral triangle with side length 5cm.
Construct triangle DEF.

- Question 3: Construct an equilateral triangle with side length of 8cm.

- Question 4: ABC is an equilateral triangle with side length 9cm.

- (a) Construct ABC
- (b) Construct the perpendicular bisector of the line AB
- (c) Construct the perpendicular bisector of the line BC
- (d) Label the point where the two perpendicular bisectors met, D.

- Question 5: Use a ruler and compasses to construct a regular hexagon that is inscribed in the circle below.



- Question 6: (a) Using compasses, draw a circle with radius 5cm.
(b) Construct a regular hexagon that is inscribed in the circle.

- Question 7: Construct a regular hexagon with side length of 6cm.

Fluency Practice

Question 1: Construct an equilateral triangle with side length 8cm.

Question 2: Construct a 60° angle

Question 3: Construct a 90° angle

Question 4: Construct a 30° angle

Question 5: Construct a 120° angle

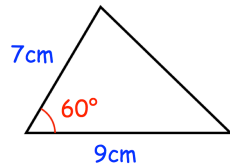
Question 6: Construct a 45° angle

Question 7: Construct a 15° angle

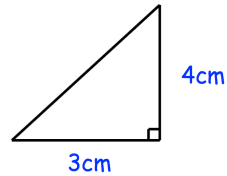
Question 8: Construct a 22.5° angle

Question 9: Construct the following triangles.
You may only use a pencil, ruler and compasses.

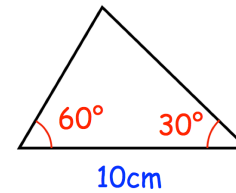
(a)



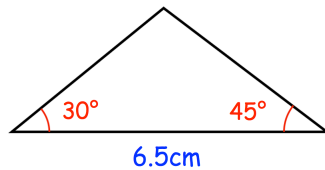
(b)



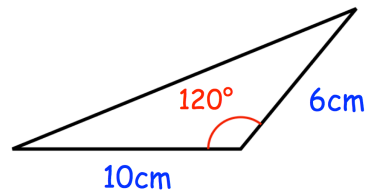
(c)



(d)



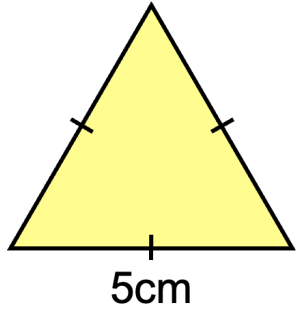
(e)



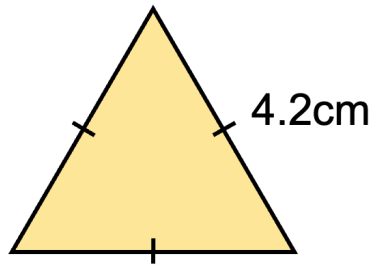
Fluency Practice

Q1 Use your compass and ruler only to construct the following **SSS** triangles.

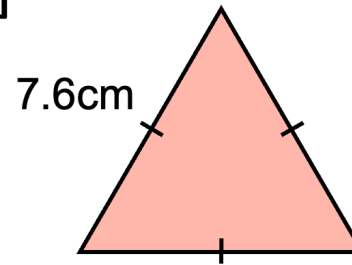
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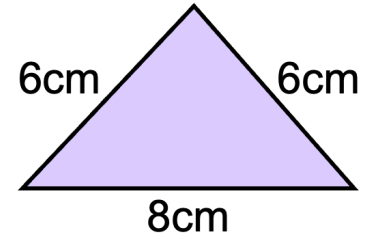
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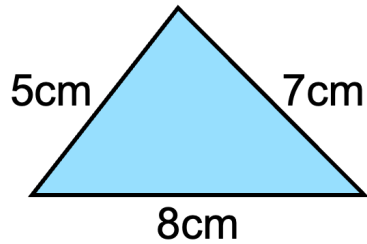
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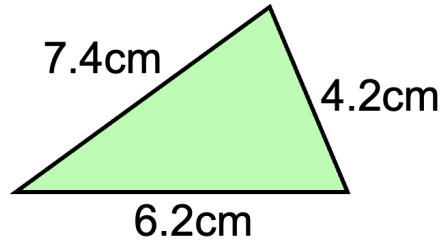
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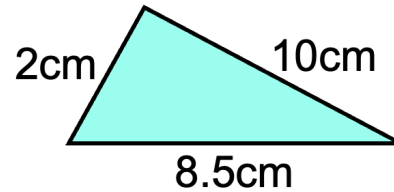
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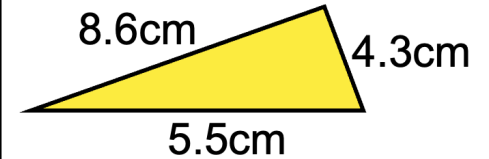
[f]



[g]



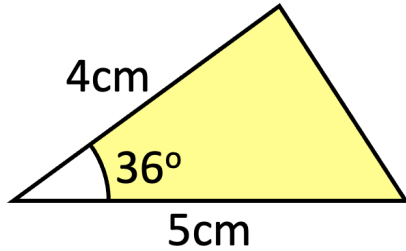
[h]



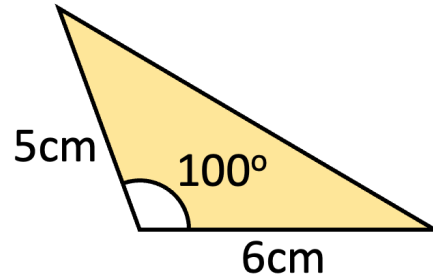
Fluency Practice

Q2 Use your compass and ruler only to construct the following triangles.

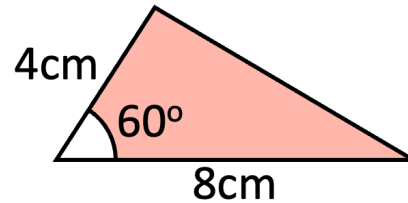
[a]



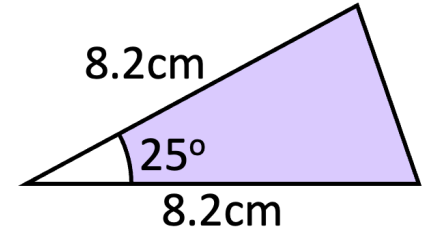
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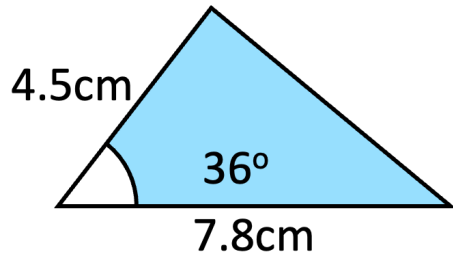
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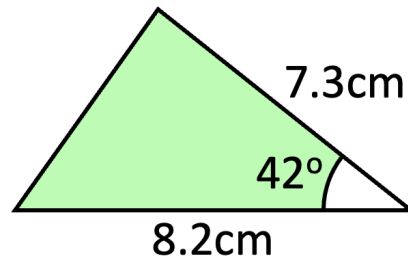
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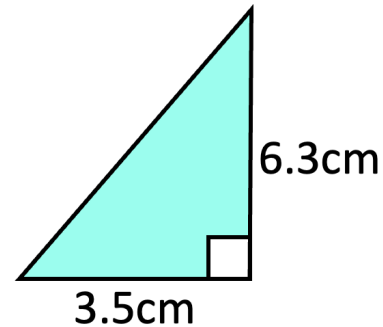
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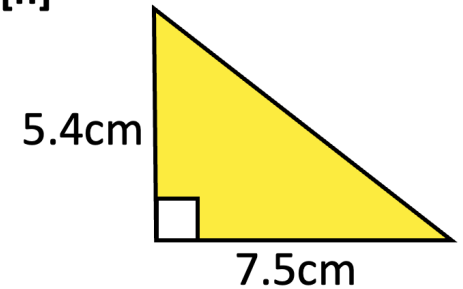
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[g]



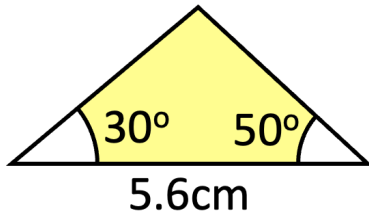
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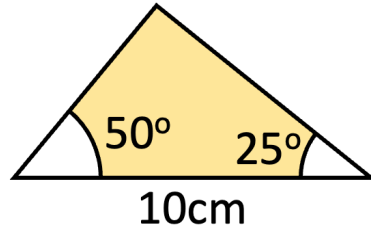
Fluency Practice

Q3 Use your compass and ruler only to construct the following triangles.

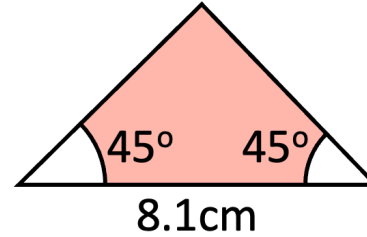
[a]



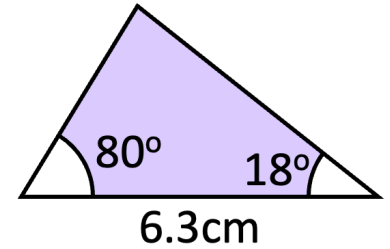
[b]



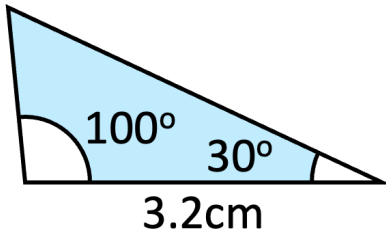
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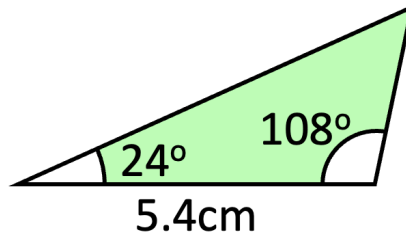
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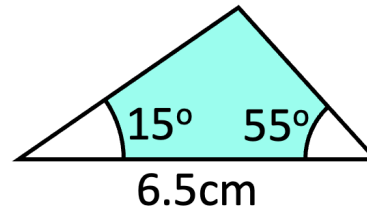
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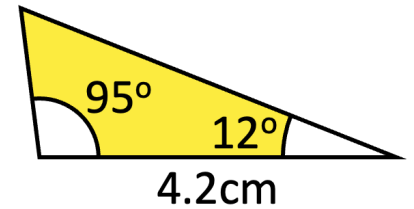
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[g]



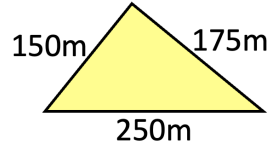
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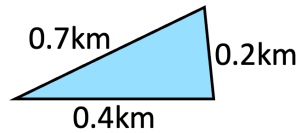
Fluency Practice

Q4 Use the scale of $1\text{cm} = 50\text{m}$, construct the following triangles.

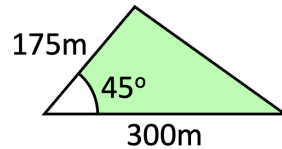
[a]



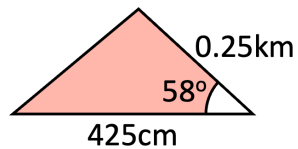
[b]



[c]



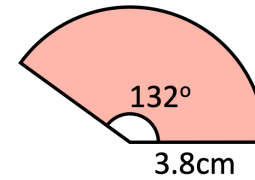
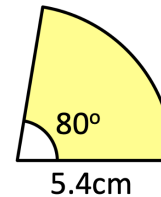
[d]



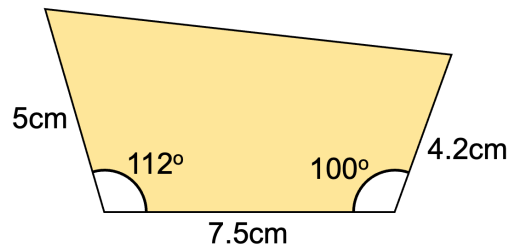
Q5 Accurately draw two different isosceles triangles with an angle of 40° .

Q6 Construct an equilateral triangle with side length of 7cm. By measuring its height, work out its area.

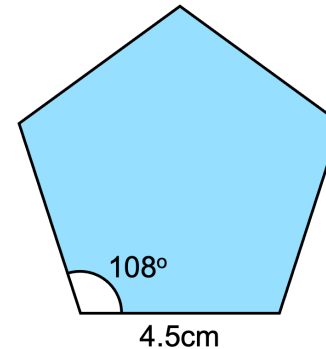
Q7 Accurately draw the sectors below.



Q8 Accurately construct the following quadrilateral and find the length of the missing side.

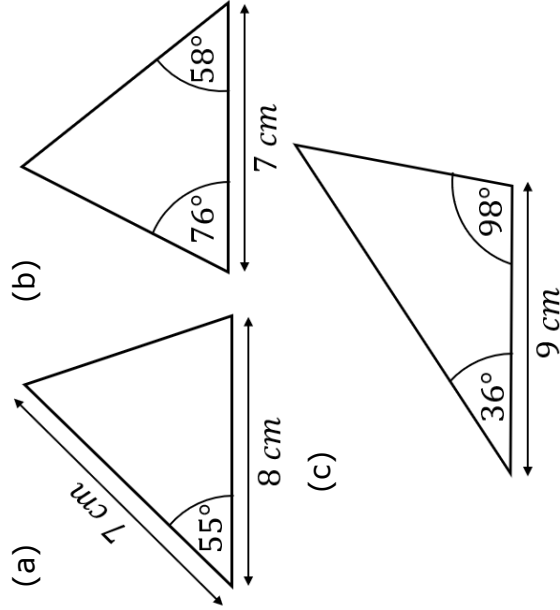


Q9 Use the information below to accurately construct a regular pentagon.

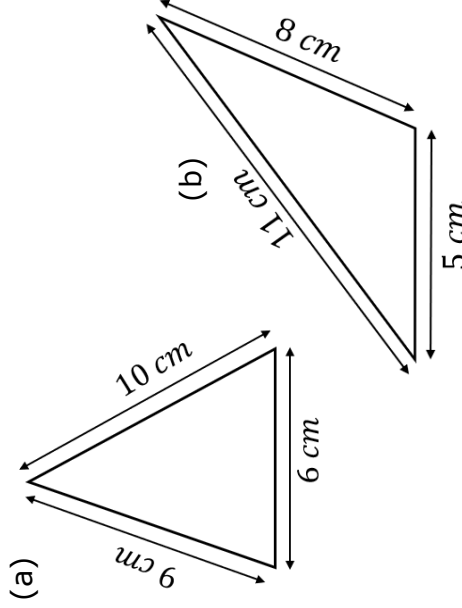


Fluency Practice

Construct the following triangles.



Construct the following triangles.



(a) In the triangle ABC , AB is 7 cm , BC is 6 cm and angle ABC is 65° . Make an accurate construction of the triangle.

(b) In the triangle DEF , DE is 6 cm , EF is 8 cm and DF is 10 cm . Make an accurate construction of the triangle. Measure the angle DEF .

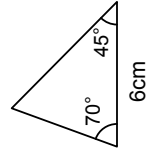
In the triangle KLM , KL is 7 cm , LM is 8 cm and angle LKM is 40° . Construct the two possible triangles accurately. Measure the angle KLM in both triangles. What do you notice?

Fluency Practice

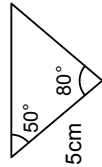
constructing triangles

Construct each triangle accurately.

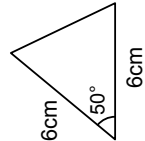
1. ASA



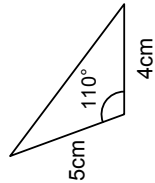
2. ASA



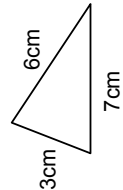
3. SAS



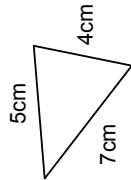
4. SAS



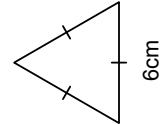
5. SSS



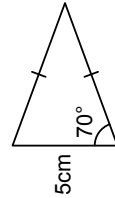
6. SSS



7.

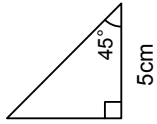


8.

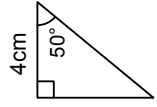


Fluency Practice

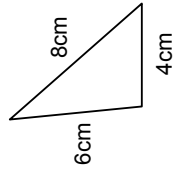
9.



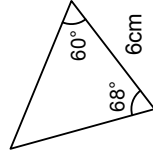
10.



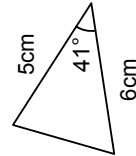
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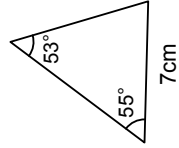
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13.

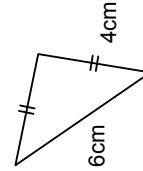


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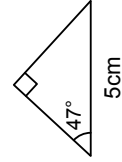


Hint: angles
in a triangle
add up to...

15.

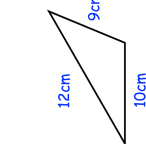
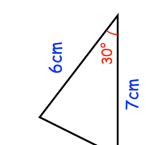
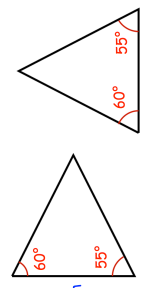


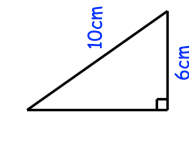
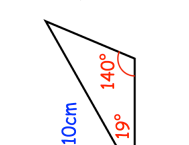
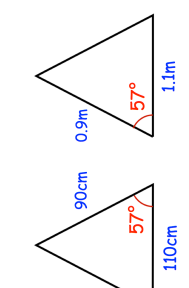
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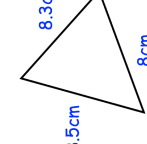
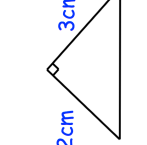
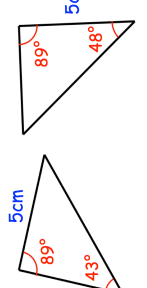


Fluency Practice

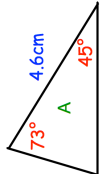
Question 1: The following pairs of triangles are congruent, state the condition that shows they are congruent.

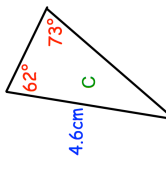
(a)  (b)  (c) 

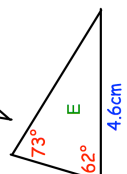
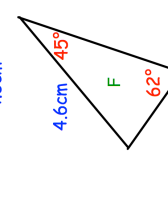
(d)  (e)  (f) 

(g)  (h)  (i) 

Question 2: Shown are six triangles.
Which triangles are congruent?

Triangle A:  Triangle B: 

Triangle C:  Triangle D: 

Triangle E:  Triangle F: 

Fluency Practice

Question 3: In triangle ABC, $AB = 7\text{cm}$, $\angle BAC = 50^\circ$ and $\angle ABC = 35^\circ$
 In triangle DEF, $EF = 7\text{cm}$, $\angle DEF = 35^\circ$ and $\angle DFE = 50^\circ$
 Are triangles ABC and DEF congruent? If they are, state the condition.

Question 4: In triangle GHI, $GH = 7\text{cm}$, $HI = 4\text{cm}$ and $GI = 5\text{cm}$.
 In triangle JKL, $JK = 7\text{cm}$, $KL = 4.5\text{cm}$ and $JL = 5\text{cm}$.
 Are triangles GHI and JKL congruent? If they are, state the condition.

Question 5: In triangle MNO, $\angle MNO = 50^\circ$, $\angle NOM = 60^\circ$ and $\angle OMN = 70^\circ$
 In triangle PQR, $\angle PQR = 50^\circ$, $\angle QRP = 60^\circ$ and $\angle RPQ = 70^\circ$
 Are triangles MNO and PQR congruent? If they are, state the condition.

Question 6: In triangle STU, $SU = 13\text{cm}$, $\angle TSU = 20^\circ$ and $\angle TUS = 30^\circ$
 In triangle VWX, $WX = 13\text{cm}$, $\angle WXV = 30^\circ$ and $\angle XVW = 20^\circ$
 Are triangles STU and VWX congruent? If they are, state the condition.

Apply

Question 1: Hannah and Chris each draw a triangle with one side of 3cm , one angle of 35° and one angle of 80° .

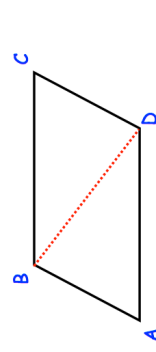
Hannah says their triangles **must** be congruent.
 Is Hannah correct?

Question 2: Paul and Greg each draw a triangle with one side of 3cm , one side of 9cm and one side of 10cm .

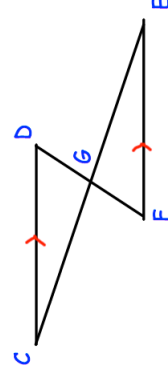
Greg says their triangles **must** be congruent.
 Is Greg correct?

Question 3: Carl and Michael each draw a triangle with one angle of 58° , one angle of 68° and one angle of 54° .

Carl says their triangles **must** be congruent.
 Is Carl correct?



Question 4: ABCD is a parallelogram.
 Prove that triangles ABD and BCD are congruent.



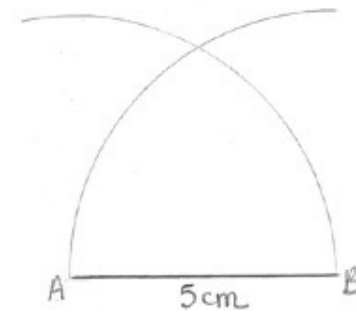
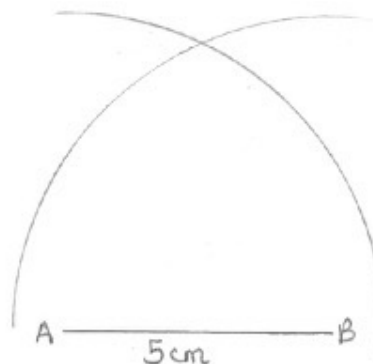
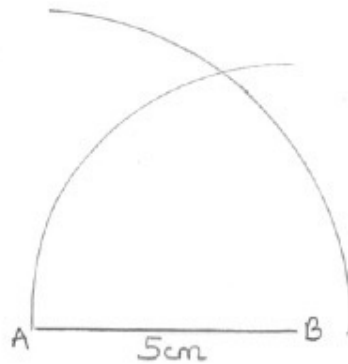
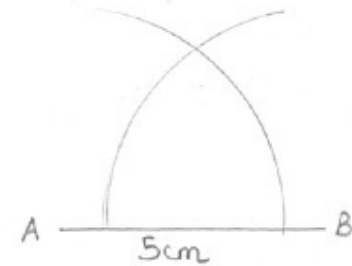
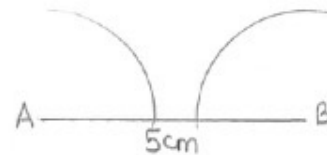
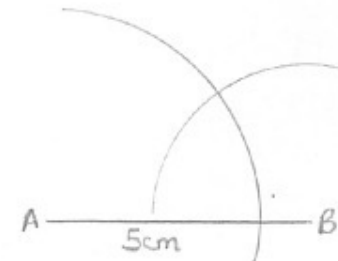
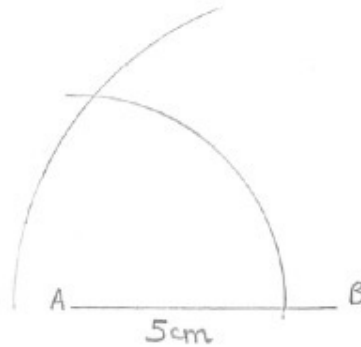
Question 5: In the diagram, the lines CE and DF intersect at G.
 CD and FE are parallel and $CD = FE$.
 Prove that triangles CDG and FEG are congruent.

Fluency Practice

Constructing Triangles

Determine which triangle matches with which information below and draw in the sides of the triangles and label them. Also decide which triangles are Equilateral, Isosceles, Scalene or right-angled and which triangle is impossible.

AB: 5cm, AC: 4cm, BC: 3cm	AB: 5cm, AC: 6cm, BC: 6cm
AB: 5cm, AC: 4cm, BC: 6cm	AB: 5cm, AC: 5cm, BC: 5cm
AB: 5cm, AC: 5cm, BC: 6cm	AB: 5cm, AC: 2cm, BC: 2cm
AB: 5cm, AC: 4cm, BC: 4cm	



Fluency Practice

constructions and congruence in triangles

three sides the same [SSS].

(1) 9 cm, 8 cm and 5 cm.

(2) 11 cm, 6 cm and 4.5 cm.

Will everyone's triangles look the same?

Reasons?

three angles the same [AAA].

(3) 45° , 70° , 65° .

(4) 80° , 40° , 70° .

Will all the triangles that people have drawn be identical (be congruent)?

Reasons?

two angles and an included side (between them) the same [ASA]

(5) 60° , 9cm, 40° .

(6) 100° , 8cm, 30° .

Will all the triangles drawn be congruent?

Reasons?

two sides and an angle [SAS] and [ASS]
a triangle PQR with:

(7) PQ = 9cm, QR = 8cm, angle P = 50° .

(8) PQ = 9cm, PR = 11cm, angle P = 50° .

(9) PQ = 8cm, PR = 6cm, angle R = 90° .

(10) PQ = 10cm, PR = 7cm, angle P = 50° .

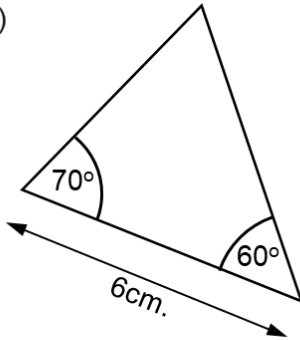
Will all the triangles drawn be congruent?

Reasons?

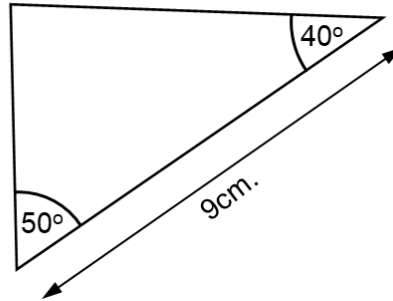
Fluency Practice

construct the following shapes accurately

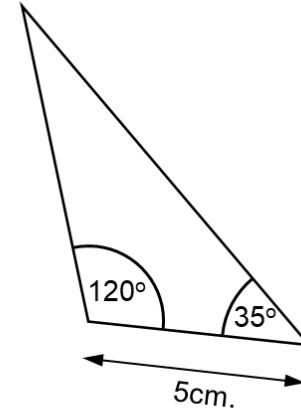
1)



2)



3)



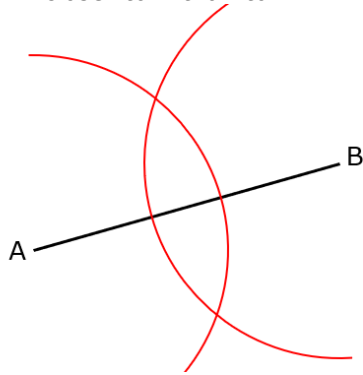
- 4) an equilateral triangle with a side length of 6cm
- 5) a square with a side length of 5.5cm
- 6) an isosceles triangle with equal sides of length 7cm and two equal angles of size 70°
- 7) a rhombus with side length 5cm and one angle of 80°
- 8) a right angled triangle with one angle of 60° and the longest side being 9cm
- 9) a square with a diagonal of length 8cm
- 10) a parallelogram with sides of length 8cm and 4.5cm with an angle of 55° between them

Fluency Practice

Constructing Loci with Bisectors

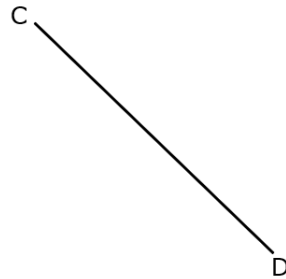
(a)

Complete the perpendicular bisector of the line AB and shade the region which is closer to B than to A



(b)

Construct the perpendicular bisector of the line CD and shade the region that is closer to C than to D



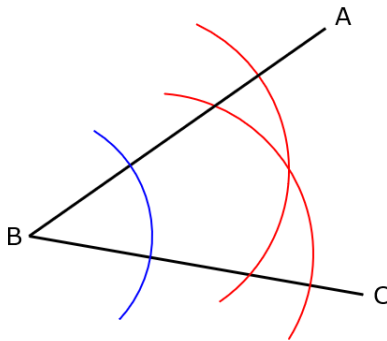
(c)

Shade the region that is closer to F than it is to E



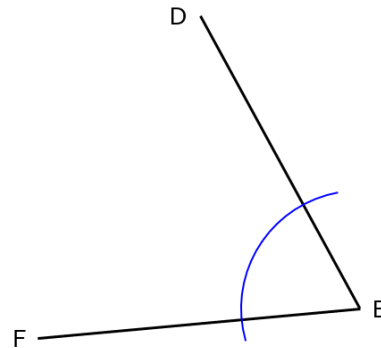
(d)

Complete the bisector of the angle ABC and shade the region which is closer to the line AB than to the line BC



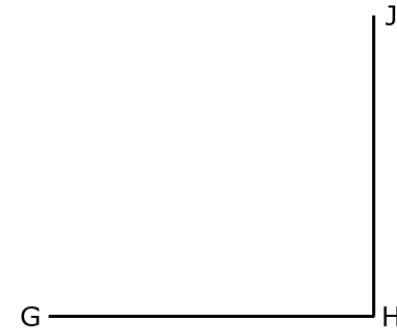
(e)

Construct the bisector of the angle DEF and shade the region which is closer to the line EF than to the line DE

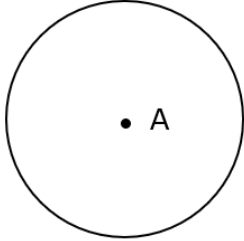
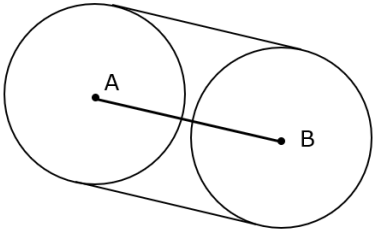


(f)

Shade the region that is closer to the line GH than it is to the line HJ



Fluency Practice

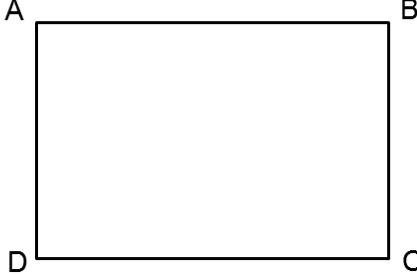
Constructing Loci with Circles		
(a) The circle drawn with centre A has radius 2 cm. Shade the region that is less than 2 cm from point A. 	(b) Draw a circle with centre B and radius 2 cm and shade the region which is more than 2 cm from B. 	(c) Shade the region that less than 2.5 cm from point C. 
(d) The two circles at points A and B have radii of 1.5 cm. Shade the region which is less than 1.5 cm from the line AB. 	(e) Draw two circles of radii 2 cm from C and D. Shade the region which is more than 2 cm from the line CD. 	(f) Shade the region that is less than 1.5 cm from the line EF. 

Fluency Practice

loci and regions (inside the rectangle)

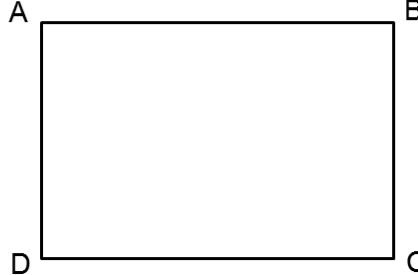
shade the region (hatching: )

(1)



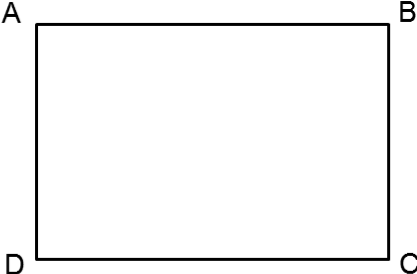
more than 3 cm from AB
more than 4 cm from AD

(2)



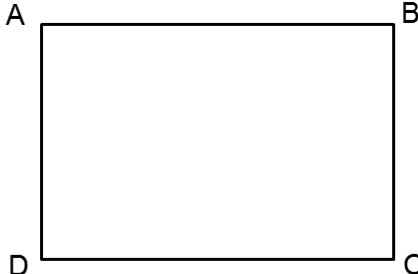
less than 2 cm from AB
less than 3 cm from C

(3)



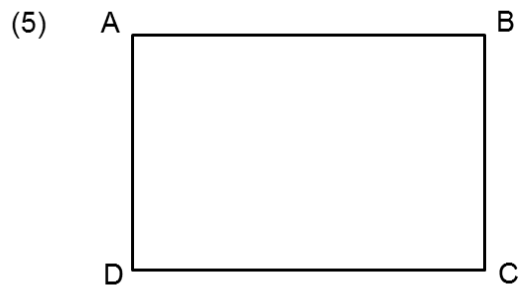
more than 4 cm from A
closer to B than C

(4)

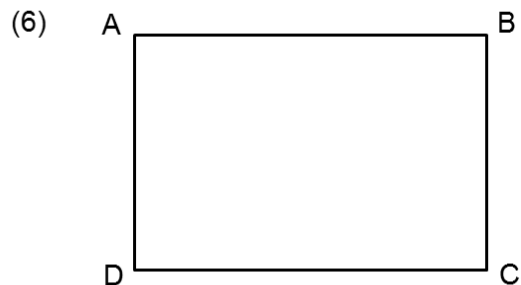


closer to AD than AB
more than 3 cm from AD

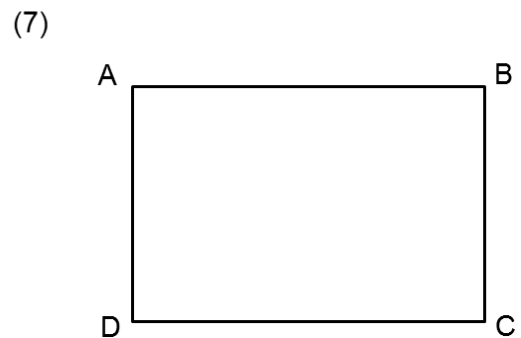
Fluency Practice



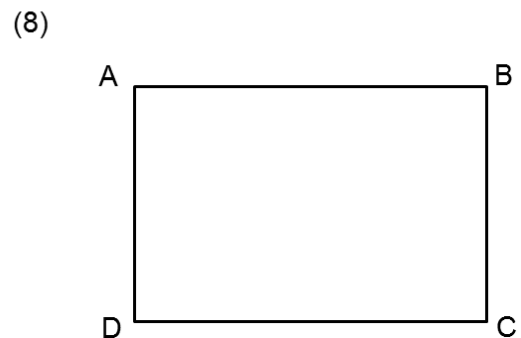
closer to AB than AD
more than 4 cm from C



closer to B than A
closer to BC than BA



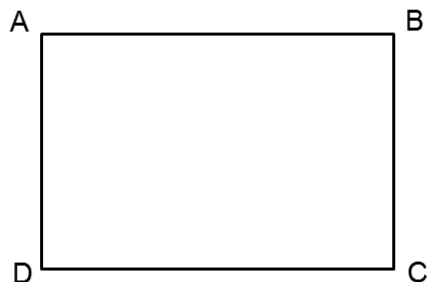
closer to CB than CD
closer to C than A



closer to CB than CD
less than 5 cm from D

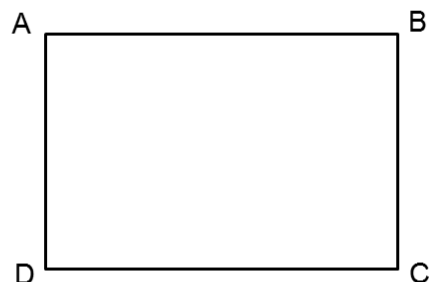
Fluency Practice

(9)



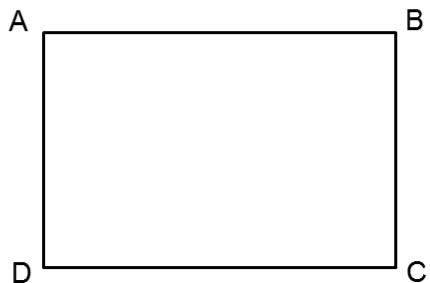
less than $3\frac{1}{2}$ cm from D
less than 5 cm from B

(10)



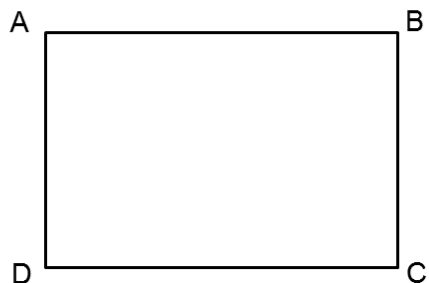
closer to D than B
more than 4 cm from C

(11)



closer to AB than AD
less than 4 cm from A
closer to B than A

(12)



closer to AB than BC
less than 6 cm from D
closer to AB than DC

Intelligent Practice

1. Show all the points that are 4cm from point A

A



2. Show all the points that are 4cm from point A and 6cm from point B

A



B



3. Show all the points that are the same distance from points A and B

A



B



4. Show all the points that are closer to point A than point B

A



B

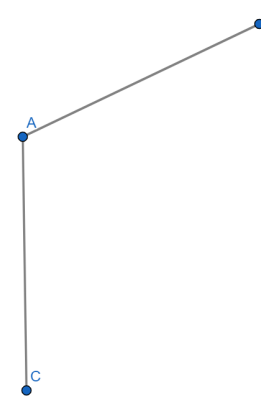


Intelligent Practice

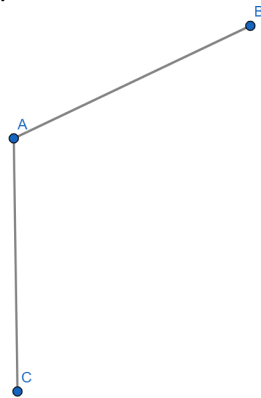
5. Show all the points that are 4cm from the line segment AB



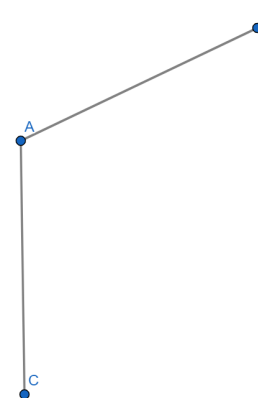
6. Show all the points that are the same distance from the line segments AB and AC



7. Show all the points that are the same distance from the line segments AB and AC, and the same distance from points A and B



8. Show all the points that are closer to line segment AB than AC, and less than 4cm from point A



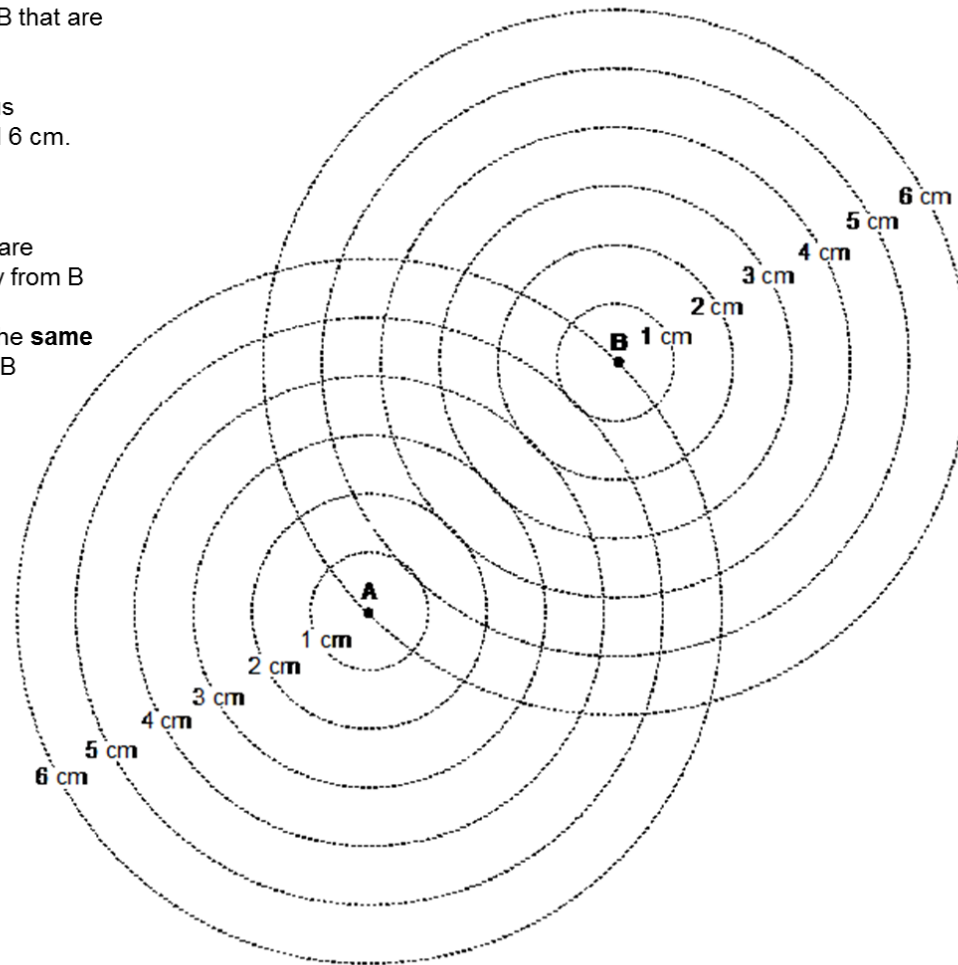
Fluency Practice

1. whirls

the diagram shows two points A and B that are 6 cm. apart

around each point are circles of radius 1 cm., 2 cm., 3 cm., 4 cm., 5 cm. and 6 cm.

- (a) mark with a cross two points that are 4 cm away from A **and** 4 cm away from B
- (b) draw the locus of points that are the **same distance** from A as they are from B



Fluency Practice

2. tree (i)

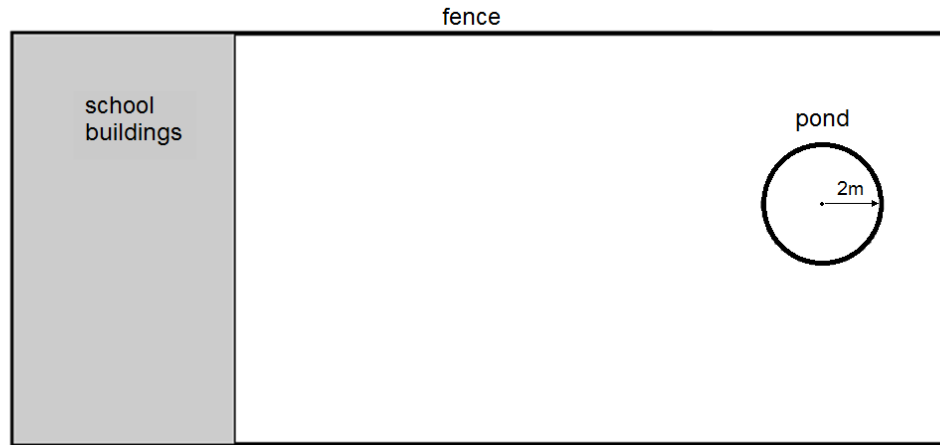
students are planning to plant a tree in the school garden

it must be at least **10m.** from the school buildings and

it must also be at least **8m.** from the centre of the circular pond

shade in the **region** to show accurately where the tree could be planted

scale 1 cm. = 2 m.



3. tree (ii)

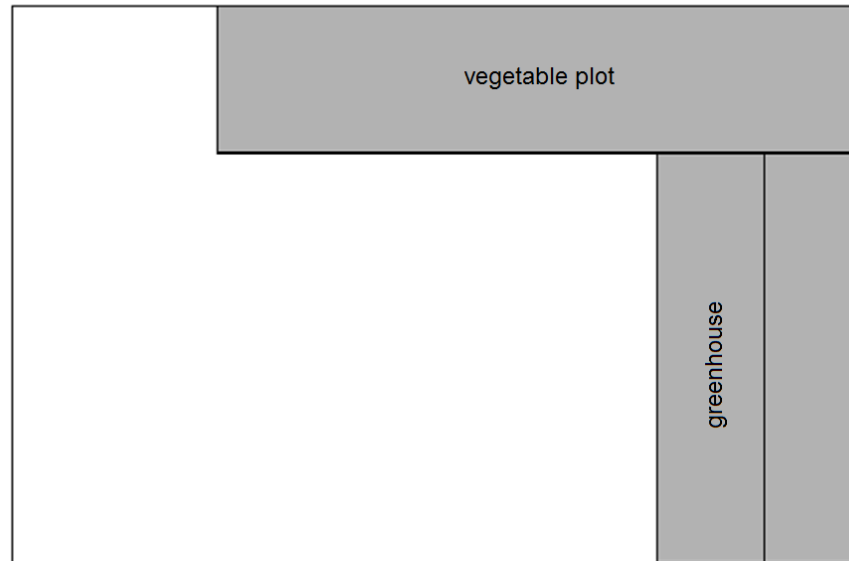
a gardener wants to plant a tree

they want it to be **more than 8m.** away from the **vegetable plot**

they want it to be **more than 18m.** away from the **greenhouse**

the plan shows part of the garden
the scale is **1cm.** to **4m.**

show **accurately** on the plan the region of the garden where she can plant the tree and label this region **R**



Fluency Practice

4. fence

in the scale drawing, the shaded area is a semi-circular lawn

there is a fence **all around** the lawn

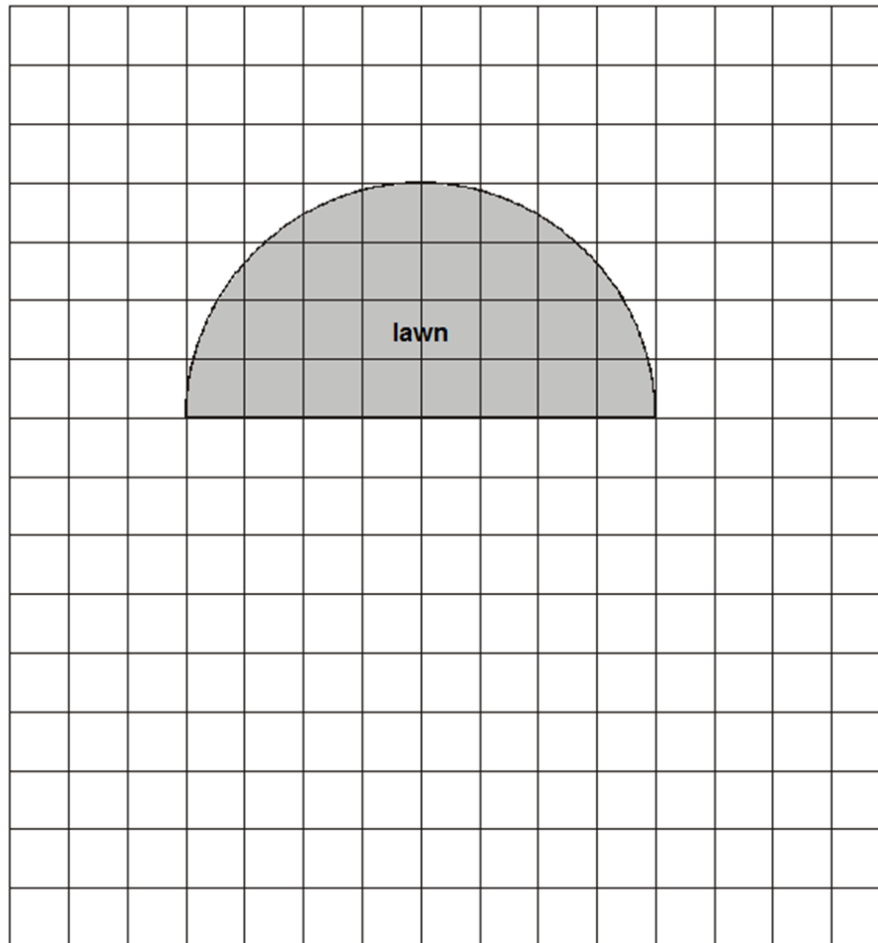
the shortest distance from the fence to the edge of the lawn is **always 6m**.

on the diagram, draw the fence **accurately**

scale: 1 cm = 3 m

3 m

3 m



Fluency Practice

5. mast

the plan shows the position of three towns,
each marked with an ×

the scale of the plan is **1 cm. to 10 km.**

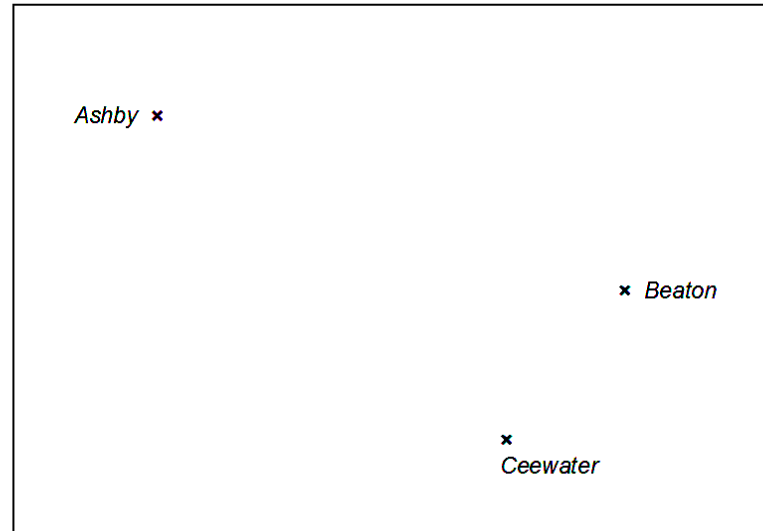
the towns need a new phone mast

the new mast must be:

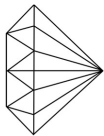
- nearer to *Ashby* than *Ceewater*
- less than 45 km from *Beaton*

show on the plan the region where the new mast
can be placed

leave in your construction lines to show how you
found the region



Fluency Practice



Diamond
HEIST!

You're planning to steal the diamond from the Grand Casino Vault!
Find a route from the vault door to the diamond avoiding
the lasers and motion detectors!

Lasers

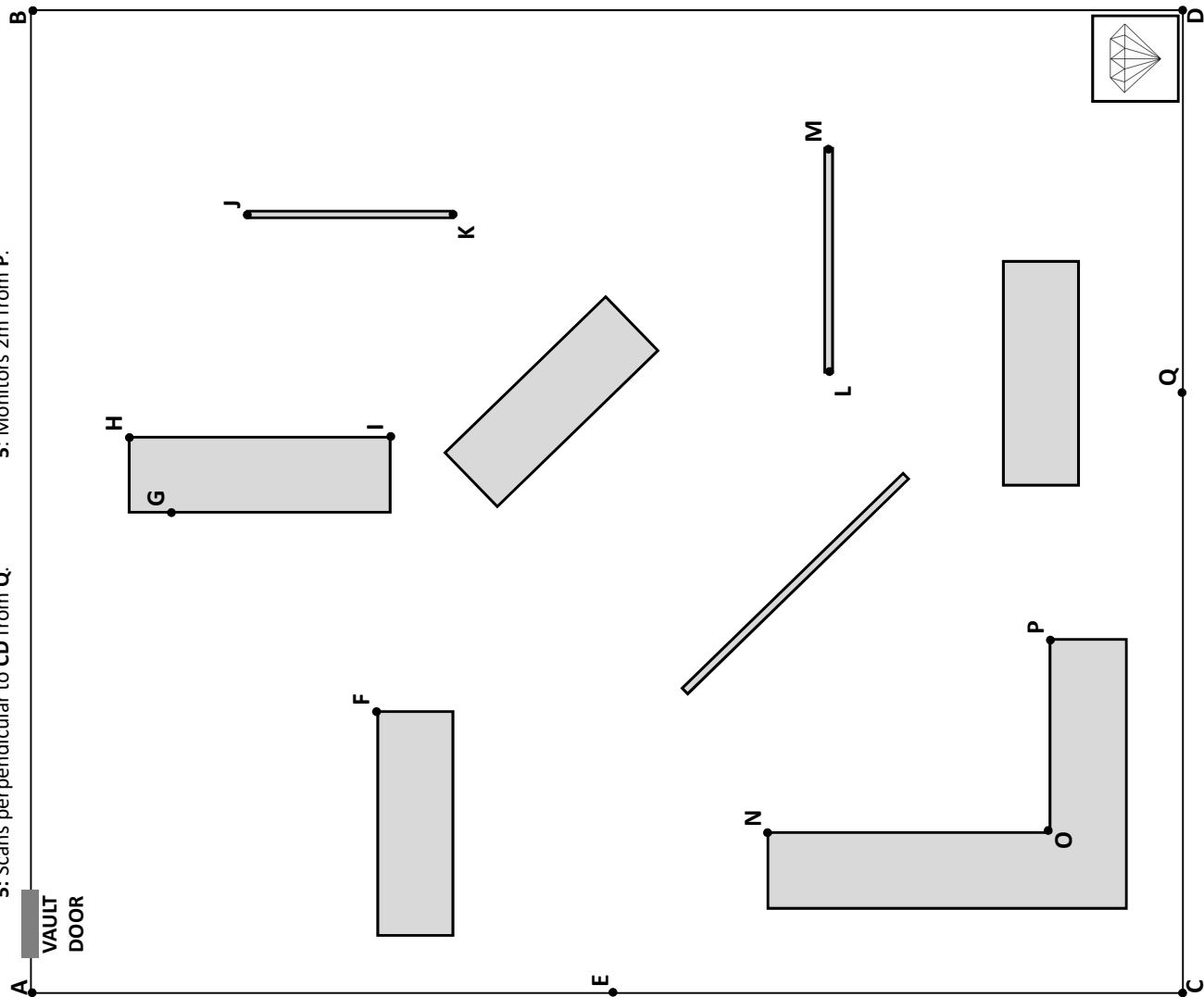
- 1: Scans 3m from AC from the north wall.
- 2: Scans from E perpendicular to AC.
- 3: Scans east equidistant from HI.
- 4: Scans from O, equidistant between ON & OP.
- 5: Scans perpendicular to CD from Q.

Motion Detectors

- 1: Monitors 3m from F.
- 2: Monitors 2m from G.
- 3: Monitors 3.5m from the wall JK.
- 4: Monitors 3m from the wall LM.
- 5: Monitors 2m from P.



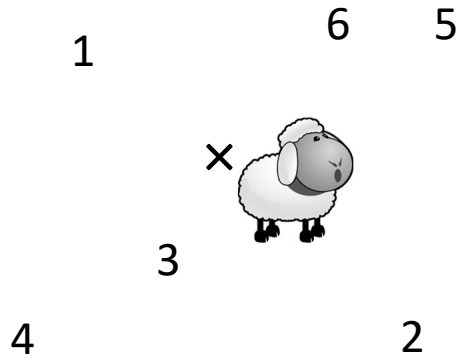
1 cm = 1m



Fluency Practice

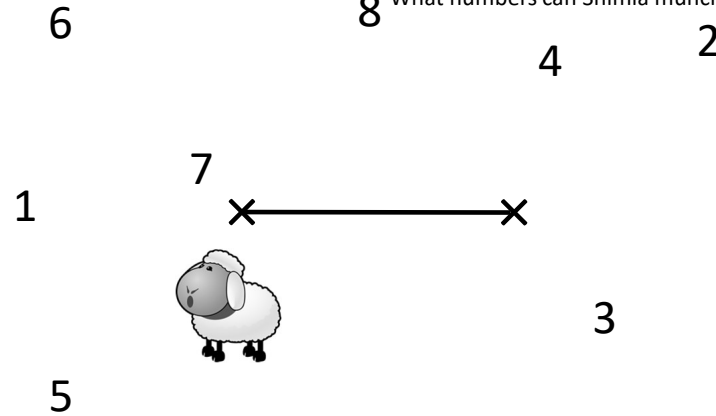
1. 1 cm = 1 m

Shimla is attached to a 4 metre rope that is attached to a stake in the ground. What numbers can Shimla munch?



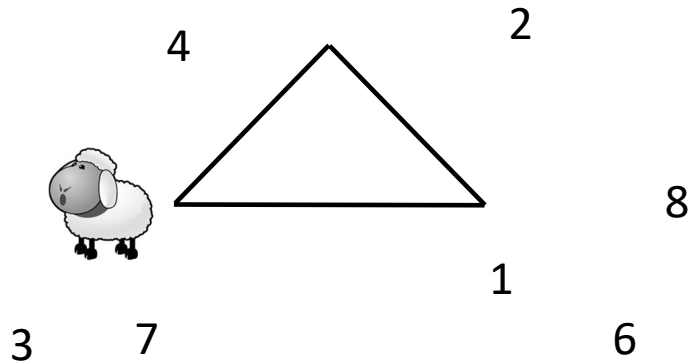
2.

Shimla is attached to a 4 metre rope that is attached to a rail on top of a fence. What numbers can Shimla munch?



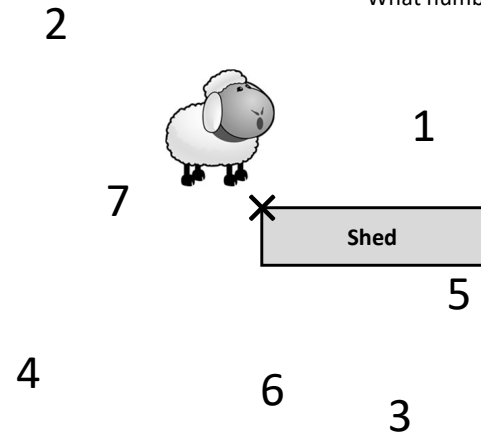
3.

Shimla is attached to a 3 metre rope that is attached to a rail on top of a triangular fence. What numbers can Shimla munch?



4.

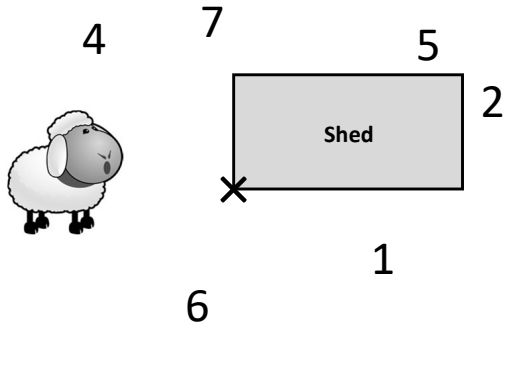
Shimla is attached to a 4 metre rope that is attached to the corner of a shed. What numbers can Shimla munch?



Fluency Practice

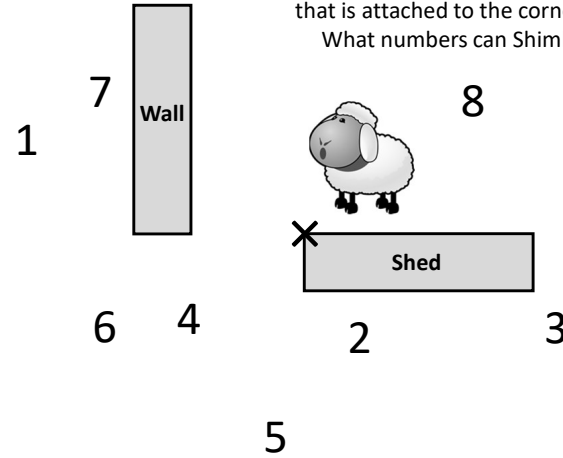
5. 1 cm = 1 m

Shimla is attached to a 5 metre rope that is attached to the corner of a shed. What numbers can Shimla munch?



6.

Shimla is attached to a 5 metre rope that is attached to the corner of a shed. What numbers can Shimla munch?



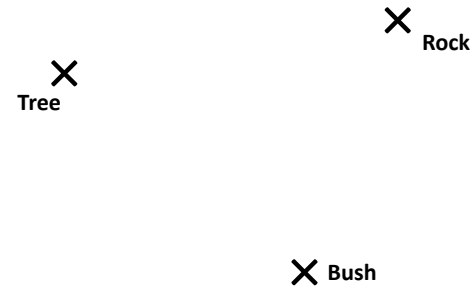
7.

Shimla is attached to a 4 metre rope and Shawn is attached to a 4 metre rope. Where can they **both** graze?



8.

A pirate has buried some treasure! It is 4 metres from the tree. It is **equal distance** from the rock and the bush. Where could the treasure be?

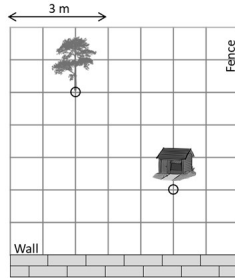


Fluency Practice

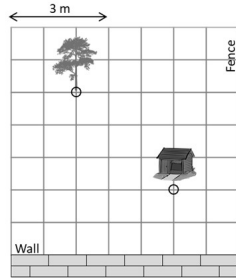
Loci

Max is deciding where to plant a new tree.
On each diagram, mark or shade **all** the possible locations according to the rules.

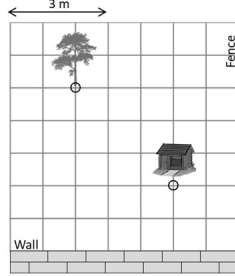
A) 4m away from the wall.



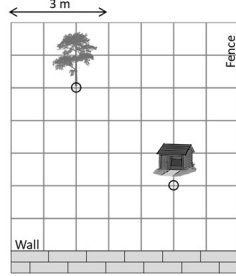
B) 2m or less from the fence.



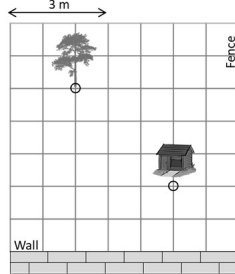
C) 1m from the wall & 5m from the fence.



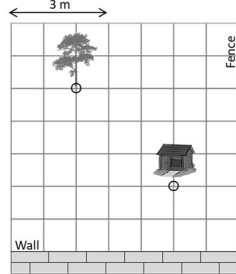
D) 4m away from the wall & 2m from the shed.



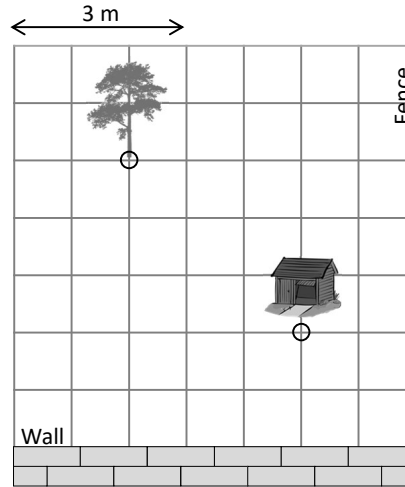
**E) 4m or more from the wall
3m or less from the fence.**



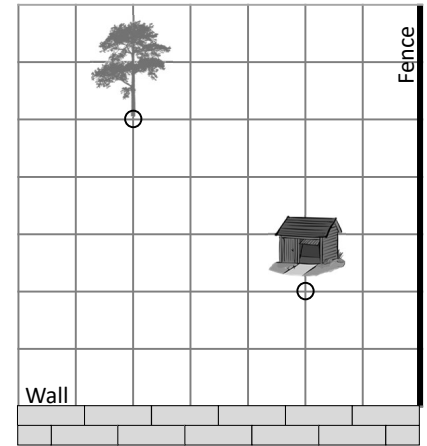
F) Equidistance from the fence & the wall.



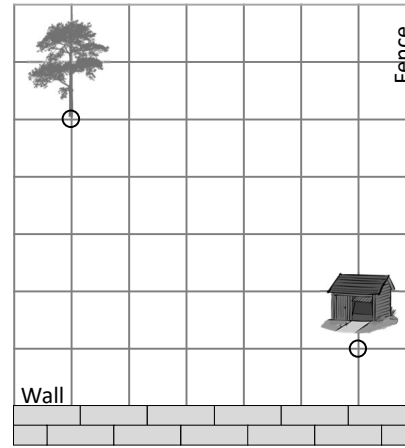
G) 5m or less from the shed.



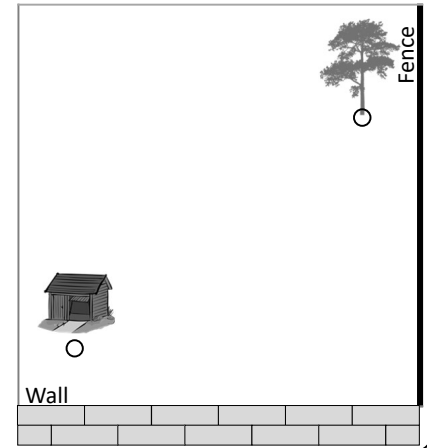
H) 5m away from the tree & 1m from the fence.



I) 4m away from both the tree & the shed.



J) Equidistant from the tree and the shed.



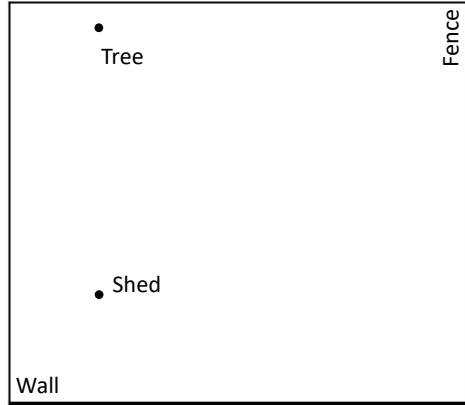
Fluency Practice

Loci

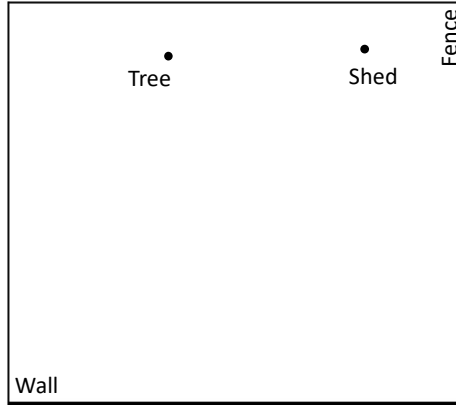
Jenny is deciding where to dig a well. On each diagram (1cm = 2m), mark or shade **all** the possible locations according to the rules. Use a **dotted line** if those points are **not** included.

2

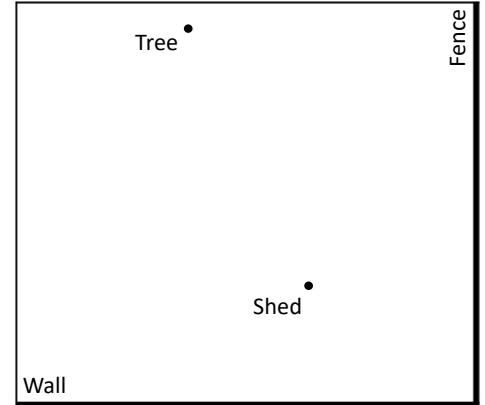
- A) 4m or less from the fence.
12m or less from the tree.



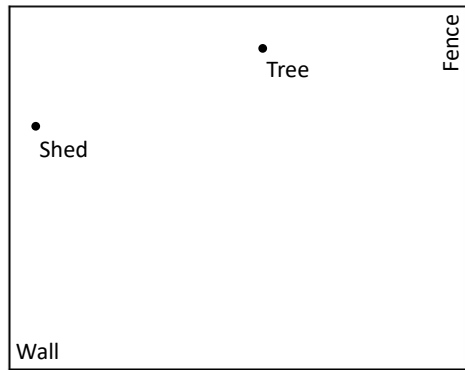
- B) 10m or more from the shed.
Less than 6m from the wall.



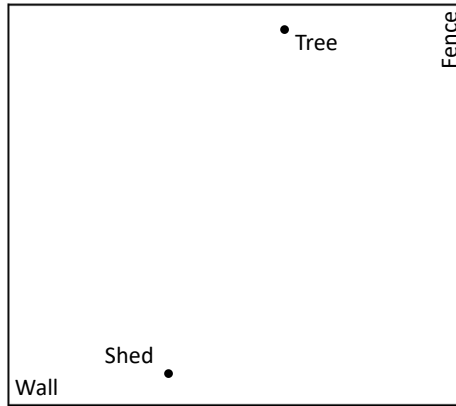
- C) Equidistant from the tree and the shed
and more than 5m from the fence.



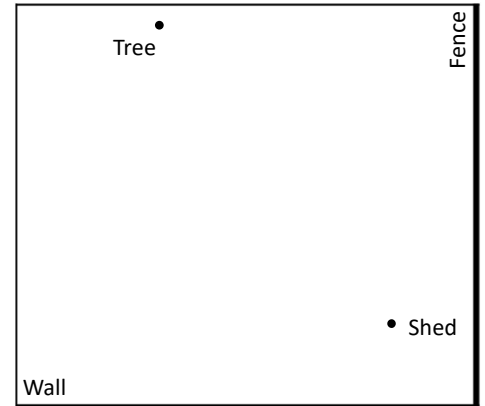
- D) Equidistant from the fence and the wall.
11m or less from the shed.



- E) Less than 8m from the tree,
less than 9m from the shed and
more than 8m from the fence.

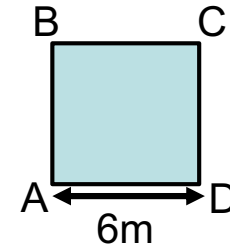


- E) 2 metres closer to the wall
than to the fence and equidistant from
the tree and the shed.



Problem Solving

A treasure chest is buried somewhere on a plot of land surrounded by fencing. The land is a square 6m in length. Four people each have a different treasure map detailing where the treasure is buried. Which map has smallest area that needs to be checked?



The treasure is closer to the fence AB than the fence AD

The treasure is exactly 5m from corner D

The treasure is no further than 3.2m from the fence CD

The treasure is exactly 2m from the centre