



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2024 Mathematics 2025

Unit 19 Tasks – Part 1

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Unit 19 Tasks – Part 2

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Unit 19 Tasks – Part 3

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1 Advanced Indices

Fluency Practice

For the following terms,

a) Write with a positive exponent

b) Evaluate:

- | | | | | | | | |
|-----|--------------|-----|-------------|-----|--------------|-----|--------------|
| 1. | 26^{-1} | 2. | 2^{-1} | 3. | 10^{-2} | 4. | 2^{-2} |
| 5. | -26^{-1} | 6. | -2^{-1} | 7. | -10^{-2} | 8. | -2^{-2} |
| 9. | $(-26)^{-1}$ | 10. | $(-2)^{-1}$ | 11. | $(-10)^{-2}$ | 12. | $(-2)^{-2}$ |
| 13. | 2^{-5} | 12. | -7^{-3} | 13. | $(-8)^{-2}$ | 14. | $(-10)^{-5}$ |
| 15. | $(-4)^{-3}$ | 16. | 9^{-4} | 17. | -11^{-2} | 18. | $(-3)^{-3}$ |
| 19. | -3^{-4} | 20. | 25^{-2} | 21. | $(-2)^{-6}$ | 22. | 15^{-2} |

Decide if there are mistakes in the following and explain how to fix the answer:

- a) $4^{-2} = -16$ b) $10^{-3} = \frac{1}{30}$

Fluency Practice

1. Write in index form:

a) $\frac{1}{5^2}$

b) $\frac{1}{3^4}$

c) $\frac{1}{8^3}$

d) $\frac{1}{4^5}$

e) $\frac{1}{10^3}$

f) $\frac{1}{2^6}$

2. Write in the form 2^n :

a) $\frac{1}{2}$

b) $\frac{1}{4}$

c) $\frac{1}{8}$

d) $\frac{1}{32}$

e) $\frac{1}{64}$

f) $\frac{1}{256}$

3. Write in the form 5^n :

a) $\frac{1}{5}$

b) $\frac{1}{125}$

c) $\frac{1}{625}$

d) $\frac{1}{3125}$

e) $\frac{1}{78,125}$

f) $\frac{1}{390,625}$

4. Arrange in ascending order:

$$\frac{1}{50}$$

$$5^{-2}$$

$$\frac{3}{10}$$

$$2^{-3}$$

Fluency Practice

Simplify the following:

1. $\left(\frac{1}{5}\right)^{-1}$

2. $\left(\frac{2}{5}\right)^{-1}$

3. $\left(\frac{3}{5}\right)^{-1}$

4. $\left(\frac{4}{5}\right)^{-1}$

5. $\left(\frac{1}{4}\right)^{-1}$

6. $\left(\frac{3}{4}\right)^{-1}$

7. $\left(\frac{1}{3}\right)^{-2}$

8. $\left(\frac{2}{3}\right)^{-2}$

9. $\left(\frac{1}{5}\right)^{-2}$

10. $\left(\frac{2}{5}\right)^{-2}$

11. $\left(\frac{3}{5}\right)^{-2}$

12. $\left(\frac{4}{5}\right)^{-2}$

13. $\left(\frac{2}{3}\right)^{-3}$

14. $\left(\frac{4}{3}\right)^{-3}$

15. $\left(\frac{7}{8}\right)^{-2}$

16. $\left(-\frac{1}{10}\right)^{-4}$

17. $\left(-\frac{4}{9}\right)^{-3}$

18. $\left(-\frac{9}{10}\right)^{-2}$

19. $\left(-\frac{5}{3}\right)^{-3}$

20. $\left(-\frac{3}{2}\right)^{-4}$

21. $\left(-\frac{3}{10}\right)^{-3}$

22. $\left(-\frac{3}{2}\right)^{-3}$

23. $\left(-\frac{8}{5}\right)^{-3}$

24. $\left(-\frac{3}{8}\right)^{-2}$

25. $\left(-\frac{6}{5}\right)^{-2}$

Fluency Practice

Write down the reciprocal of:

- | | | | |
|-------------------|-------------------|-------------------|--|
| (a) 4 | (b) 10 | (c) 15 | |
| (d) -3 | (e) 0.4 | (f) 1 | |
| (g) $\frac{3}{4}$ | (h) $\frac{1}{8}$ | (i) $\frac{2}{7}$ | |

Evaluate:

- | | | | |
|-------------------------------------|--------------------------------------|--|--|
| (a) 4^{-1} | (b) 10^{-1} | | |
| (c) 15^{-1} | (d) $(-3)^{-1}$ | | |
| (e) a^{-1} | (f) $\left(\frac{2}{3}\right)^{-1}$ | | |
| (g) $\left(\frac{8}{5}\right)^{-1}$ | (h) $\left(1\frac{3}{7}\right)^{-1}$ | | |

Evaluate:

- | | | | |
|-------------------------------------|-------------------------------------|--|--|
| (a) 4^{-2} | (b) 10^{-3} | | |
| (c) 15^{-2} | (d) $(-3)^{-2}$ | | |
| (e) b^{-3} | (f) $\left(\frac{1}{3}\right)^{-2}$ | | |
| (g) $\left(\frac{2}{5}\right)^{-3}$ | (h) 0.1^{-2} | | |

Find x .

- | | | | |
|---------------------------|-----------------------------|--|--|
| (a) $\frac{1}{10} = 10^x$ | (b) $\frac{1}{1000} = 10^x$ | | |
| (c) $\frac{1}{2} = 2^x$ | (d) $\frac{1}{8} = 2^x$ | | |
| (e) $\frac{1}{16} = 2^x$ | (f) $\frac{1}{256} = 2^x$ | | |

Fluency Practice

1 Complete each collection of equivalent expressions.

Example

(a)

5^{-2}	$\frac{1}{25}$	$\frac{1}{64}$
$1 \div 5 \div 5$	$\frac{1}{5} \times \frac{1}{5}$	
	$\frac{1}{5^2}$	
	25^{-1}	

(b)

2^{-3}		

(c)

$1 \div 3 \div 3 \div 3 \div 3$

(d)

$\frac{1}{4} \times \frac{1}{4}$

(e)

$\frac{3}{16}$

(f)

$\frac{4}{16}$

(g)

$4^{-2} \times 3^{-1}$

Fluency Practice

positive, negative & zero indices

Match these calculations to their answers, which are jumbled up below. Record your answers in the table at the bottom.

^A $3^2 \times 2^0$	^B $3^{-1} \times 3$	^C $(-2)^2$	^D $6^2 \div 6^1$	^E 30×3^{-1}
^F 2×3^{-1}	^G $2^{-1} \times 6^2$	^H $5^{-1} \times 5^{-1}$	^I 10×5^{-1}	^J $-2^1 \times 2^1$
^K $2^2 \times 2^3$	^L $25 \div 5^0$	^M 11×2^0	^N $3^1 \times 3^0$	^O $5^{-1} \times 2^{-2}$
^P $2^{-1} \div 2^1$	^Q $(3^2)^{-1}$	^R $2^{-1} \times 2 \times 2^3$	^S $(-2 \times -3)^{-1}$	^T $2^{-2} \times 3$
^U $2 \times 5^2 \times 10^{-1}$	^V 63×3^{-2}	^W $12 \div 6^0$	^X $5^{-1} + 5^{-1}$	^Y $(4^{-2} \times 4^1)^2$

$\frac{2}{3}^1$ $\frac{1}{25}^6$ $\frac{1}{6}$ $\frac{1}{9}$ $\frac{1}{16}^{10}$ $\frac{3}{4}^5$ $\frac{2}{5}$
 32 18 7 9 3 25 11 $\frac{1}{20}$ 4 -4 12 $\frac{1}{4}$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Fluency Practice

TASK

Evaluate:

Calculations with Negative Indices

(a) $\left(\frac{3}{4}\right)^{-1} + 5^{-1}$

(d) $\left(\frac{2}{3} + \frac{4}{5}\right)^{-1}$

(g) $6^{-2} + 3^{-3}$

(b) $\left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{12}\right)^{-2}$

(e) $\left(2\frac{1}{3}\right)^{-2} - \left(1\frac{3}{4}\right)^{-2}$

(h) $\left(\frac{2}{x}\right)^{-3} + \left(\frac{5}{x}\right)^{-1}$

(c) $\left(\frac{4}{7}\right)^{-2} + \left(1\frac{5}{11}\right)^{-1}$

(f) $0.4^{-1} + 0.75^{-2}$

(i) $\left(\frac{1}{3x}\right)^{-2} + \left(\frac{3}{2x}\right)^{-3}$

Fluency Practice

TASK

Show that

Calculations with Negative Indices

(a) $\left(\frac{3}{4}\right)^{-1} + 5^{-1} = 1\frac{8}{15}$

(d) $\left(\frac{2}{3} + \frac{4}{5}\right)^{-1} = \frac{15}{22}$

(g) $6^{-2} + 3^{-3} = \frac{7}{108}$

(b) $\left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{12}\right)^{-2} = 152$

(e) $\left(2\frac{1}{3}\right)^{-2} - \left(1\frac{3}{4}\right)^{-2} = -\frac{1}{7}$

(h) $\left(\frac{2}{x}\right)^{-3} + \left(\frac{5}{x}\right)^{-1} = \frac{5x^3 + 8x}{40}$

(c) $\left(\frac{4}{7}\right)^{-2} + \left(1\frac{5}{11}\right)^{-1} = 3\frac{3}{4}$

(f) $0.4^{-1} + 0.75^{-2} = 4\frac{5}{18}$

(i) $\left(\frac{1}{3x}\right)^{-2} + \left(\frac{3}{2x}\right)^{-3} = \frac{8x^3 + 243x^2}{27}$

Fluency Practice

Rewrite the following with positive indices:

- | | | | | | | | | | |
|-----|--------------------|-----|----------------------|-----|----------------------|-----|---------------------|-----|---------------------|
| 1. | x^{-6} | 2. | x^{-7} | 3. | a^{-8} | 4. | a^{-10} | 5. | p^{-11} |
| 6. | k^{-14} | 7. | $2s^{-3}$ | 8. | $7z^{-3}$ | 9. | $3d^{-4}$ | 10. | $3d^{-7}$ |
| 11. | $14x^{-1}$ | 12. | $\frac{12}{3}x^{-2}$ | 13. | $\frac{16x^{-5}}{4}$ | 14. | $\frac{a^{-9}}{2}$ | 15. | $\frac{e^{-5}}{6}$ |
| 16. | $\frac{y^{-8}}{3}$ | 17. | $\frac{g^{-7}}{10}$ | 18. | $\frac{2x^{-4}}{3}$ | 19. | $\frac{4f^{-5}}{7}$ | 20. | $\frac{5x^{-8}}{9}$ |
| 21. | $(3x)^{-3}$ | 22. | $(-3x)^{-3}$ | 23. | $(5x)^{-2}$ | 24. | $(-5x)^{-2}$ | 25. | $(-7a)^{-3}$ |

Fluency Practice

Task 1

Rewrite the following with negative indices:

1. $\frac{1}{x^4}$

2. $\frac{1}{x}$

3. $\frac{1}{d^{10}}$

4. $\frac{1}{b^{13}}$

5. $\frac{1}{y^x}$

6. $\frac{1}{m^n}$

7. $\frac{3}{x^{13}}$

8. $\frac{5}{x^9}$

9. $\frac{7}{y^{10}}$

10. $\frac{3}{x^{11}}$

11. $\frac{4}{x^3}$

12. $\frac{12}{3x^2}$

13. $\frac{a}{x^n}$

14. $\frac{1}{2x^4}$

15. $\frac{1}{3x^7}$

16. $\frac{1}{8a^5}$

17. $\frac{1}{7b^6}$

18. $\frac{3}{4x^5}$

19. $\frac{2}{5x^{12}}$

20. $\frac{1}{8a^5}$

Task 2

Match the equivalent pairs of expressions

$(3x)^{-1}$

$(3x)^{-2}$

$3x^{-1}$

x^3

x^{-3}

$3x^{-2}$

$\frac{1}{x^3}$

$\frac{1}{x^{-3}}$

$\frac{1}{9x^2}$

$\frac{3}{x}$

$\frac{1}{3x}$

$\frac{3}{x^2}$

Fluency Practice

Simplify each of the following. Assume that all variables represent positive numbers. Present your answer without negative exponents.

1. 3^{-2}

2. $\frac{1}{2^{-3}}$

3. m^{-4}

4. $\frac{1}{x^{-5}}$

5. $a^8 \cdot a^{-1}$

6. $p^3 (p^{-7}) p^8$

7. $\frac{x^{-4}}{x^{-9}}$

8. $\frac{50a^{12}}{10a^{-3}}$

9. $\frac{t^{-3}}{t^4}$

10. x^0

11. $-x^0$

12. $(-x)^0$

13. $(b^{-5}) (b^2) (b^{-1})$

14. $\frac{1}{(b^{-5}) (b^2) (b^{-1})}$

15. $\frac{m^{-2}}{m^{-5}}$

16. $\frac{x^3 y^{-5}}{z^{-4}}$

17. $\frac{18q^3}{6q^{-3}}$

18. $\left(\frac{2}{3}\right)^{-3}$

19. $2y^{-3}$

20. $(2y)^{-3}$

21. $\left(-\frac{3}{5}\right)^{-2}$

22. $\frac{a^3 b^{-5}}{a^{-2} b^3}$

23. $(3m^3)^{-2}$

24. $(-2ab^{-3})^{-3}$

25. $\frac{(k^3)^{-3}}{(k^{-5})^2}$

26. $\left(\frac{2a^{-3}b^5}{-3a^3b^{-2}}\right)^{-2} (a^3b^{-5})^{-3}$

30. $\left(-\frac{x^3y^0x^{-5}}{y^{-3}}\right)^{-2}$

33. $\frac{(-2a^{-2})^{-2} b^3 a^0 (-aba^{-2}b^{-2})^{-3}}{2a^2 (-2a^{-2}b)^{-2} ab^0}$

27. $(-2a^{-3}) (-2a^{-2}b)^{-4}$

31. $\left(-\frac{x^3y^7x^{-5}}{y^{-3}}\right)^0$

34. $\left(\frac{-a^2 (b^{-1}a)^{-5}}{b^7 (-ab^2)^{-3}}\right)^{-2}$

28. $\frac{(-3p^3q^5)^2}{(2q^0p^3)^{-1}}$

32. $\frac{x^{-1} + y^{-1}}{x^{-2} - y^{-2}}$

35. $\frac{(x^{-2})^{-2} y^3 x^0 (-2yx^0y^{-2}x^{-2})^0}{yx^5 (y^{-2}x)^{-3} (2x^{-1}yx^3)^{-1}}$

29. $\left(\frac{2a^{-2}b^3}{-2^2 (a^{-1}b)^{-3}}\right)^{-2}$

36. Suppose that $x = 8.5 \cdot 10^{-12}$ and $y = 7.5 \cdot 10^7$. Perform each of the following operations. Present your answer using scientific notation.

a) xy

b) x^3

c) xy^2

d) $\frac{x}{y}$

e) $\frac{y}{x^5}$

Fluency Practice

Task 1

Expand and fully simplify:

1. $x(x^3 - 4)$
2. $x(x^2 - x^{-2})$
3. $2y^{-1}(3y^2 - 2y^3)$
4. $2y^2(3y^4 + 5y^{-2})$
5. $m(2m^{-1} - 4m^{-4})$
6. $3c(4c^3 - 6c^{-4})$
7. $4a^2(2a^{-1} + 3a^{-2})$
8. $t^{-2}(3t^{-2} - t^2)$
9. $4d^{\frac{1}{2}}(3d^{\frac{1}{2}} - d^{-\frac{1}{2}})$
10. $a^{-2}(a + a^{-1})$
11. $2w^5(\frac{1}{w} + 4w^{-2})$
12. $x^2(\frac{1}{x^2} + x^{\frac{1}{3}})$
13. $x^{\frac{1}{2}}(x^4 + x^3)$
14. $u^{\frac{1}{2}}(3u + u^3)$
15. $x^{-\frac{1}{4}}(x^8 + x^6)$
16. $3m^{\frac{3}{2}}(m^{\frac{3}{2}} + \frac{3}{m^{\frac{1}{2}}})$
17. $n^{\frac{1}{3}}(2n^{-\frac{4}{3}} - \frac{1}{n^{\frac{2}{3}}})$
18. $b^{\frac{1}{3}}(b^2 + 2b^{-1})$
19. $x^{-\frac{1}{3}}(x^{-2} + x^4)$
20. $x^{\frac{1}{2}}(\frac{3}{x^4} - \frac{2}{x^5})$
21. $p^3(p^{-2} + p^3)$
22. $x^{-3}(x^5 + x^2)$
23. $5x^{\frac{1}{2}}(2x^{\frac{1}{2}} + 3x^{\frac{3}{2}})$
24. $3a^{-1}(4a^3 + 2a)$
25. $2u^{-5}(u + 2u^5)$
26. $3m^2(2m^2 + 7m^{-4})$
27. $a^{\frac{1}{2}}(a^{\frac{1}{2}} + a^{-\frac{1}{2}})$
28. $p^{\frac{1}{3}}(p^{\frac{2}{3}} + p^{\frac{1}{3}})$
29. $e^{-\frac{2}{3}}(e^{\frac{7}{3}} - 2e^{\frac{2}{3}})$
30. $5n^4(n^{-2} + \frac{2}{n^3})$
31. $p^4(3p^{-4} - \frac{2}{p^3})$
32. $3a(a^{\frac{1}{2}} + 2a^{-2})$
33. $x^{\frac{1}{2}}(2x - 3)$
34. $2p^{\frac{4}{3}}(p^{\frac{1}{3}} - p)$
35. $x^{\frac{2}{3}}(x^{\frac{1}{2}} + x^{\frac{1}{4}})$
36. $x^{\frac{1}{5}}(2x^{\frac{9}{5}} + \frac{3}{x^{\frac{1}{5}}})$
37. $2a^{\frac{5}{3}}(\frac{1}{a^{\frac{2}{3}}} - 4a^{\frac{4}{3}})$
38. $x^{-4}(x^{-\frac{1}{2}} - x^2)$
39. $x^2(\frac{1}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{1}{3}}})$
40. $x^{-\frac{1}{2}}(\frac{1}{x^{\frac{1}{4}}} - \frac{3}{x^{\frac{1}{6}}})$

Task 2

Lauren writes down the following statement:

$$p^{\frac{1}{3}}(p^{\frac{2}{3}} - p^{-\frac{1}{3}}) = p - 1$$

Is this statement true? Justify your answer.

Fluency Practice

Expand and fully simplify:

1. $(m^{-2} + m^3)(m^{-2} + m)$

2. $(x^{-3} + x^{-6})(x^7 + x^{-2})$

3. $(x + x^{-1})^2$

4. $(b^{\frac{1}{2}} + 1)^2$

5. $(3k^{\frac{1}{2}} - 2)^2$

6. $\left(x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{2}}}\right)\left(x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{2}}}\right)$

7. $\left(c^{\frac{1}{2}} + \frac{1}{c^{\frac{1}{2}}}\right)^2$

8. $\left(x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{4}}}\right)\left(x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{4}}}\right)$

9. $\left(x^{\frac{2}{3}} + y^{\frac{1}{2}}\right)\left(x^{\frac{1}{3}} - y^{\frac{1}{2}}\right)$

10. $\left(5c^{\frac{3}{4}} + 9c^{\frac{1}{2}}\right)\left(6c^{\frac{1}{3}} - 2c^{\frac{1}{4}}\right)$

Fluency Practice

Expand and simplify

a $x(x^2 - x^{-1})$

b $2x^3(x^{-1} + 3)$

c $x^{-1}(3x - x^3)$

d $4x^{-2}(3x^5 + 2x^3)$

e $\frac{1}{2}x^2(6x + 4x^{-1})$

f $3x^{\frac{1}{2}}(x^{-\frac{1}{2}} - x^{\frac{3}{2}})$

g $x^{-\frac{3}{2}}(5x^2 + x^{\frac{7}{2}})$

h $x^{\frac{1}{3}}(3x^{\frac{5}{3}} - x^{-\frac{4}{3}})$

i $(x^2 + 1)(x^4 - 3)$

j $(2x^5 + x)(x^4 + 3)$

k $(x^2 - 2x^{-1})(x - x^{-2})$

l $(x^2 - x^{\frac{3}{2}})(x - x^{\frac{1}{2}})$

Fluency Practice

Task 1

Rewrite the following using the radical sign:

1. $x^{\frac{1}{5}}$ 2. $x^{\frac{1}{6}}$ 3. $x^{\frac{1}{7}}$ 4. $x^{\frac{1}{8}}$ 5. $x^{\frac{1}{9}}$ 6. $x^{\frac{1}{10}}$ 7. $x^{\frac{1}{m}}$
8. $x^{-\frac{1}{2}}$ 9. $x^{-\frac{1}{3}}$ 10. $x^{-\frac{1}{4}}$ 11. $x^{-\frac{1}{5}}$ 12. $x^{-\frac{1}{6}}$ 13. $x^{-\frac{1}{7}}$ 14. $x^{-\frac{1}{a}}$

Task 2

Rewrite the following using the radical sign:

1. $4x^{\frac{1}{3}}$ 2. $4x^{-\frac{1}{3}}$ 3. $\frac{1}{4}x^{\frac{1}{3}}$ 4. $\frac{1}{4}x^{-\frac{1}{3}}$
5. $15y^{\frac{1}{10}}$ 6. $15y^{-\frac{1}{10}}$ 7. $\frac{1}{15}y^{\frac{1}{10}}$ 8. $\frac{1}{15}y^{-\frac{1}{10}}$
9. $-14x^{\frac{1}{8}}$ 10. $-14x^{-\frac{1}{8}}$ 11. $-\frac{1}{14}x^{\frac{1}{8}}$ 12. $-\frac{1}{14}x^{-\frac{1}{8}}$
13. $18c^{\frac{1}{2}}$ 14. $18c^{-\frac{1}{2}}$ 15. $\frac{1}{18}c^{\frac{1}{2}}$ 16. $\frac{1}{18}c^{-\frac{1}{2}}$
17. $-20k^{\frac{1}{9}}$ 18. $-20k^{-\frac{1}{9}}$ 19. $-\frac{k^{\frac{1}{9}}}{20}$ 20. $-\frac{k^{-\frac{1}{9}}}{20}$
21. $3c^{\frac{1}{7}}$ 22. $3c^{-\frac{1}{7}}$ 23. $\frac{c^{\frac{1}{7}}}{3}$ 24. $\frac{c^{-\frac{1}{7}}}{3}$

Task 3

Rewrite the following using fractional exponents:

1. $\sqrt[9]{a}$ 2. $\sqrt[4]{k}$ 3. $\sqrt[5]{p}$ 4. $\sqrt[8]{x}$ 5. $\sqrt[3]{m}$
6. $\sqrt[7]{z}$ 7. $\sqrt[10]{w}$ 8. $\sqrt[14]{f}$ 9. $\frac{1}{\sqrt[9]{a}}$ 10. $\frac{1}{\sqrt[4]{k}}$
11. $\frac{1}{\sqrt[10]{l}}$ 12. $\frac{1}{\sqrt[7]{h}}$ 13. $\frac{1}{\sqrt[13]{p}}$ 14. $\frac{1}{\sqrt[7]{m}}$ 15. $\frac{1}{\sqrt[10]{d}}$

Fluency Practice

Task 1

Evaluate

1. $25^{\frac{1}{2}}$	2. $25^{-\frac{1}{2}}$	3. $16^{\frac{1}{2}}$	4. $16^{-\frac{1}{2}}$
5. $9^{\frac{1}{2}}$	6. $9^{-\frac{1}{2}}$	7. $100^{\frac{1}{2}}$	8. $100^{-\frac{1}{2}}$
9. $36^{\frac{1}{2}}$	10. $36^{-\frac{1}{2}}$	11. $8^{\frac{1}{3}}$	12. $8^{-\frac{1}{3}}$
13. $125^{\frac{1}{3}}$	14. $125^{-\frac{1}{3}}$	15. $1000^{\frac{1}{3}}$	16. $1000^{-\frac{1}{3}}$
17. $16^{\frac{1}{4}}$	18. $16^{-\frac{1}{4}}$	19. $32^{\frac{1}{5}}$	20. $32^{-\frac{1}{5}}$
21. $216^{\frac{1}{3}}$	22. $216^{-\frac{1}{3}}$	23. $512^{\frac{1}{3}}$	24. $512^{-\frac{1}{3}}$
25. $\left(\frac{25}{49}\right)^{\frac{1}{2}}$	26. $\left(\frac{25}{49}\right)^{-\frac{1}{2}}$	27. $\left(\frac{36}{121}\right)^{\frac{1}{2}}$	28. $\left(\frac{36}{121}\right)^{-\frac{1}{2}}$
29. $\left(\frac{625}{1296}\right)^{\frac{1}{4}}$	30. $\left(\frac{625}{1296}\right)^{-\frac{1}{4}}$	31. $\left(\frac{216}{343}\right)^{\frac{1}{3}}$	32. $\left(\frac{216}{343}\right)^{-\frac{1}{3}}$
33. $\left(\frac{343}{1000}\right)^{\frac{1}{3}}$	34. $\left(\frac{343}{1000}\right)^{-\frac{1}{3}}$	35. $\left(\frac{125}{729}\right)^{\frac{1}{3}}$	36. $\left(\frac{125}{729}\right)^{-\frac{1}{3}}$

Task 2

- Given that $y = x^{\frac{1}{3}}$, find y when $x = 64$.
- Given that $y = x^{\frac{1}{2}}$, find y when $x = 16$.
- Given that $y = x^{\frac{1}{4}}$, find y when $x = 81$.
- Given that $y = x^{\frac{1}{3}}$, find y when $x = 125$.

Fluency Practice

Task 3

For $a > 1$, put the following in order of size from smallest to largest:

$$a^0, a^2, a, a^{-2}, a^{\frac{1}{2}}$$

Task 4

Solve for a :

$$\frac{x \times x^4 \times x^a}{\sqrt{x}} = \sqrt{\frac{(x^a)^8}{\left(\frac{1}{x^4}\right)}}$$

Task 5

The statement $x > x^{\frac{1}{2}}$ is:

Always True

Sometimes True

Never True

Fluency Practice

Find the value of:

- | | | | |
|------------------|-----------------|--|--|
| (a) $4^{1/2}$ | (b) $100^{1/2}$ | | |
| (c) $196^{1/2}$ | (d) $49^{1/2}$ | | |
| (e) $64^{1/3}$ | (f) $512^{1/3}$ | | |
| (g) $1000^{1/3}$ | (h) $16^{1/4}$ | | |

Evaluate:

- | | | | |
|---|---|--|--|
| (a) $\left(\frac{4}{9}\right)^{1/2}$ | (b) $\left(\frac{1}{36}\right)^{1/2}$ | | |
| (c) $\left(\frac{225}{16}\right)^{1/2}$ | (d) $\left(\frac{25}{169}\right)^{1/2}$ | | |
| (e) $\left(\frac{64}{27}\right)^{1/3}$ | (f) $\left(\frac{8}{729}\right)^{1/3}$ | | |
| (g) $\left(\frac{1}{1000}\right)^{1/3}$ | (h) $\left(\frac{16}{10000}\right)^{1/4}$ | | |

Evaluate:

- | | | | |
|---|--|--|--|
| (a) $9^{-1/2}$ | (b) $225^{-1/2}$ | | |
| (c) $25^{-1/2}$ | (d) $512^{-1/3}$ | | |
| (e) $343^{-1/3}$ | (f) $8^{-1/3}$ | | |
| (g) $\left(\frac{8}{343}\right)^{-1/3}$ | (h) $\left(\frac{25}{9}\right)^{-1/2}$ | | |

Solve these equations.

- | | |
|-------------------|------------------------------|
| (a) $x^{1/2} = 9$ | (b) $x^{1/2} = 11$ |
| (c) $x^{1/3} = 5$ | (d) $x^{-1/2} = \frac{1}{4}$ |

Problem Solving

Which is bigger?

$$10^{\frac{1}{2}}$$

or

$$20^{\frac{1}{3}}$$

$$20^{\frac{1}{2}}$$

or

$$40^{\frac{1}{3}}$$

$$40^{\frac{1}{2}}$$

or

$$80^{\frac{1}{3}}$$

$$5^{\frac{1}{2}}$$

or

$$10^{\frac{1}{3}}$$

Fluency Practice

Task 1

Write in index form:

1. $\sqrt{a^3}$

2. $5\sqrt{a^3}$

3. $\frac{1}{\sqrt{a^3}}$

4. $\frac{5}{\sqrt{a^3}}$

5. $\sqrt{w^5}$

6. $4\sqrt{w^5}$

7. $\frac{1}{\sqrt{w^5}}$

8. $\frac{4}{\sqrt{w^5}}$

9. $\sqrt[3]{x^2}$

10. $9\sqrt[3]{x^2}$

11. $\frac{1}{\sqrt[3]{x^2}}$

12. $\frac{9}{\sqrt[3]{x^2}}$

13. $\sqrt[3]{w^4}$

14. $-6\sqrt[3]{w^4}$

15. $\frac{1}{\sqrt[3]{w^4}}$

16. $-\frac{6}{\sqrt[3]{w^4}}$

17. $\sqrt[5]{m^2}$

18. $-2\sqrt[5]{m^2}$

19. $\frac{1}{\sqrt[5]{m^2}}$

20. $-\frac{2}{\sqrt[5]{m^2}}$

21. $\sqrt[9]{k^4}$

22. $-12\sqrt[9]{k^4}$

23. $\frac{1}{\sqrt[9]{k^4}}$

24. $-\frac{12}{\sqrt[9]{k^4}}$

25. $\sqrt[6]{x^5y^7}$

26. $\sqrt[4]{a^7b^3c}$

27. $\sqrt[7]{p^2q^3r^4}$

28. $\sqrt[\frac{1}{7}]{p^2q^3r^4}$

Fluency Practice

Task 2

Write with a radical sign in simplest form:

1. $x^{\frac{3}{4}}$
2. $m^{\frac{7}{5}}$
3. $p^{\frac{3}{2}}$
4. $y^{\frac{5}{8}}$
5. $c^{\frac{4}{3}}$
6. $4\left(x^{\frac{3}{4}}\right)$
7. $9\left(m^{\frac{7}{5}}\right)$
8. $16\left(p^{\frac{3}{2}}\right)$
9. $5\left(y^{\frac{5}{8}}\right)$
10. $10\left(c^{\frac{4}{3}}\right)$
11. $x^{-\frac{3}{4}}$
12. $m^{-\frac{7}{5}}$
13. $p^{-\frac{3}{2}}$
14. $y^{-\frac{5}{8}}$
15. $c^{-\frac{4}{3}}$
16. $4\left(x^{-\frac{3}{4}}\right)$
17. $9\left(m^{-\frac{7}{5}}\right)$
18. $16\left(p^{-\frac{3}{2}}\right)$
19. $5\left(y^{-\frac{5}{8}}\right)$
20. $10\left(c^{-\frac{4}{3}}\right)$

Task 3

Find equivalent terms from the following:

x^{-3}	x^3	x^2	$\sqrt{x}$	$\sqrt[3]{x}$	x^{-1}	$2x^{-1}$	$2\sqrt{x}$
$x^{\frac{3}{2}}$	$x^{\frac{1}{2}}$	$\sqrt{x^3}$	$x\sqrt{x}$	$x^{-\frac{1}{3}}$	x^{-2}	$\frac{1}{2x^2}$	$x^{\frac{2}{3}}$
$x^{-\frac{1}{2}}$	$2x^{-2}$	$\frac{\sqrt{x}}{x}$	x^4	x^{-4}	$\frac{1}{2\sqrt{x}}$	x^0	$x^{-\frac{3}{2}}$
$\frac{1}{2}x^{-\frac{1}{2}}$	$\frac{1}{2}x^{-2}$	$x^{\frac{1}{3}}$	x	$\frac{2}{\sqrt{x}}$	$\frac{1}{x}$	$\frac{1}{2x}$	$\sqrt[3]{x^2}$

Fluency Practice

Task 1

Evaluate:

- | | | | | |
|--|---|---|---|--|
| 1. $8^{\frac{4}{3}}$ | 2. $27^{\frac{5}{3}}$ | 3. $64^{\frac{5}{6}}$ | 4. $81^{\frac{3}{2}}$ | 5. $625^{\frac{3}{4}}$ |
| 6. $9^{\frac{3}{2}}$ | 7. $100^{\frac{3}{2}}$ | 8. $3125^{\frac{2}{5}}$ | 9. $8^{\frac{5}{3}}$ | 10. $4^{\frac{5}{2}}$ |
| 11. $81^{\frac{3}{4}}$ | 12. $125^{-\frac{2}{3}}$ | 13. $243^{-\frac{2}{5}}$ | 14. $32^{-\frac{2}{5}}$ | 15. $16^{-\frac{3}{4}}$ |
| 16. $4^{-\frac{5}{2}}$ | 17. $64^{-\frac{2}{3}}$ | 18. $8^{-\frac{4}{3}}$ | 19. $32^{-\frac{4}{5}}$ | 20. $125^{-\frac{2}{3}}$ |
| 21. $1000^{-\frac{2}{3}}$ | 22. $81^{-\frac{3}{4}}$ | 23. $625^{-\frac{3}{4}}$ | 24. $(-8)^{\frac{4}{3}}$ | 25. $(-27)^{\frac{5}{3}}$ |
| 26. $(-3125)^{\frac{2}{5}}$ | 27. $(-8)^{\frac{5}{3}}$ | 28. $(-125)^{-\frac{2}{3}}$ | 29. $(-243)^{-\frac{2}{5}}$ | 30. $(-32)^{-\frac{2}{5}}$ |
| 31. $(-64)^{-\frac{2}{3}}$ | 32. $(-8)^{-\frac{4}{3}}$ | 33. $(-32)^{-\frac{4}{5}}$ | 34. $(-125)^{-\frac{2}{3}}$ | 35. $(-1000)^{-\frac{2}{3}}$ |
| 36. $\left(\frac{4}{25}\right)^{\frac{3}{2}}$ | 37. $\left(\frac{4}{25}\right)^{-\frac{3}{2}}$ | 38. $\left(\frac{8}{125}\right)^{\frac{2}{3}}$ | 39. $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$ | 40. $\left(\frac{49}{100}\right)^{\frac{3}{2}}$ |
| 41. $\left(\frac{49}{100}\right)^{-\frac{3}{2}}$ | 42. $\left(\frac{8}{125}\right)^{\frac{2}{3}}$ | 43. $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$ | 44. $\left(\frac{16}{100}\right)^{\frac{5}{2}}$ | 45. $\left(\frac{16}{100}\right)^{-\frac{5}{2}}$ |
| 46. $\left(\frac{64}{1000}\right)^{\frac{4}{3}}$ | 47. $\left(\frac{64}{1000}\right)^{-\frac{4}{3}}$ | 48. $\left(\frac{16}{25}\right)^{\frac{3}{2}}$ | 49. $\left(\frac{16}{25}\right)^{-\frac{3}{2}}$ | 50. $\left(\frac{32}{243}\right)^{\frac{6}{5}}$ |

Fluency Practice

Task 2

a) Arrange the following in ascending order:

$$25^{\frac{1}{2}}, 8^{\frac{2}{3}}, 27^{\frac{1}{3}}$$

b) Which is the odd one out? Explain your answer.

$$64^{\frac{1}{2}}, 16^{\frac{3}{4}}, 9^{\frac{2}{3}}, 4^{\frac{3}{2}}$$

Task 3

Gina has completed her homework.

Can you spot any mistakes?

Question 1

Work out $9^{\frac{1}{2}}$

4.5

Question 2

Work out $27^{\frac{2}{3}}$

18

Task 4

a) Given that $y = x^{\frac{3}{5}}$, find y when $x = 32$.

b) Given that $y = x^{\frac{3}{2}}$, find y when $x = 49$.

c) Given that $y = x^{\frac{2}{3}}$, find y when $x = 1000$.

d) Given that $y = x^{\frac{3}{4}}$, find y when $x = 81$.

e) Given that $y = x^{\frac{5}{6}}$, find y when $x = 64$.

f) Given that $y = x^{\frac{3}{4}}$, find y when $x = 10,000$.

Fluency Practice

Task 5

Find equivalent pairs from the following:

$8^{\frac{2}{3}}$	$(-1)^{\frac{1}{3}}$	$9^{\frac{3}{2}}$	$16^{\frac{1}{4}}$	$\frac{2}{3}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{64}{125}$
$\left(\frac{1}{2}\right)^{-3}$	$\left(\frac{2}{5}\right)^{-2}$	$\left(\frac{1}{9}\right)^{-\frac{1}{2}}$	$36^{-\frac{1}{2}}$	$\frac{1}{3}$	4	1	$\frac{1}{2}$
3^{-2}	$\left(\frac{16}{25}\right)^{\frac{3}{2}}$	3^{-1}	$4^{-\frac{3}{2}}$	8	-1	$\frac{1}{9}$	3
$8^{-\frac{1}{3}}$	$(-1)^{-2}$	$\left(\frac{1}{5}\right)^{-1}$	$\left(\frac{4}{9}\right)^{\frac{1}{2}}$	5	$\frac{25}{4}$	27	2

Task 6

- $(4x)^{\frac{3}{4}}$
- $(9m)^{\frac{7}{5}}$
- $(16p)^{\frac{3}{2}}$
- $(5y)^{\frac{5}{8}}$
- $(10c)^{\frac{4}{3}}$
- $(4x)^{-\frac{3}{4}}$
- $(9m)^{-\frac{7}{5}}$
- $(16p)^{-\frac{3}{2}}$
- $(5y)^{-\frac{5}{8}}$
- $(10c)^{-\frac{4}{3}}$
- $\left(\frac{4x^6}{9y^4}\right)^{\frac{3}{2}}$
- $\left(\frac{243x^5}{32y^{20}}\right)^{\frac{3}{5}}$
- $\left(\frac{64x^3}{27y^6}\right)^{\frac{2}{3}}$

Fluency Practice

Task 7

Match up each expression from the first column with its partner from the second column, and write the answers in the table below

First column		Second column			
1	$\sqrt{x}$	11	$\frac{1}{x^2} \times \frac{1}{x^3}$	A	$x^{\frac{-3}{2}}$
2	$\frac{1}{x}$	12	$(\sqrt[3]{x})^2$	B	$\frac{1}{2}x^{-3}$
3	$\frac{1}{\sqrt{x}}$	13	$\sqrt{x^7}$	C	$\frac{1}{2}x^{-1}$
4	$\sqrt[3]{x}$	14	$\sqrt{\left(\frac{1}{x^8}\right)}$	D	x^{-4}
5	$\frac{1}{x^2}$	15	$\frac{1}{\sqrt{x^{-8}}}$	E	x^{-1}
6	$\frac{1}{2x^3}$	16	$x \times \sqrt{x \times x^6}$	F	$\frac{1}{2}x$
7	$\frac{2}{x^3}$	17	$\sqrt{\left(\frac{4}{x^2}\right)}$	G	x^4
8	$x\sqrt{x}$	18	$\sqrt{\left(\frac{1}{4x^2}\right)}$	H	x^2
9	$\frac{\sqrt{x}}{x^2}$	19	$\sqrt{\left(\frac{x^2}{4}\right)}$	I	$2x$
10	$\frac{1}{x^2}$	20	$\sqrt{4x^2}$	J	$x^{\frac{1}{5}}$
				K	$x^{\frac{-1}{2}}$
				L	$x^{\frac{9}{2}}$
				M	x^{-2}
				N	$x^{\frac{1}{2}}$
				O	$2x^{-1}$
				P	$x^{\frac{1}{2}}$
				Q	x^{-5}
				R	$x^{\frac{2}{3}}$
				S	$2x^{-3}$
				T	$x^{\frac{7}{2}}$

Table for Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Fluency Practice

$2^{\square}$ $\square^{\frac{3}{2}}$	$\square^2$ $\square^{\frac{2}{2}}$	16
$2^{\square}$ $\square^{\frac{3}{2}}$	$\square^3$ $\square^{\frac{3}{2}}$	8
$3^{\square}$ $\square^{\frac{3}{2}}$	$\square^3$ $\square^{\frac{3}{2}}$	27
$4^{\square}$ $\square^{\frac{3}{2}}$	$\square^2$ $\square^{\frac{2}{2}}$	4

$2^{\square}$ $\square^{\frac{3}{2}}$	$\square^2$ $\square^{\frac{2}{2}}$	16
$2^{\square}$ $\square^{\frac{3}{2}}$	$\square^3$ $\square^{\frac{3}{2}}$	8
$3^{\square}$ $\square^{\frac{3}{2}}$	$\square^3$ $\square^{\frac{3}{2}}$	27
$10^{\square}$ $\square^{\frac{3}{2}}$	$\square^2$ $\square^{\frac{4}{2}}$	$10,000$

Fluency Practice

Find the value of each of the following.

$$\left(25\right)^{\frac{1}{2}-\frac{1}{2}}_{(81)}$$

$$\left(\frac{125}{1000}\right)^{\frac{2}{3}-\frac{2}{3}}$$

$$\left(16\right)^{\frac{3}{4}-\frac{3}{4}}_{(81)}$$

$$\left(\frac{216}{512}\right)^{\frac{2}{3}-\frac{2}{3}}$$

$$\left(2\frac{1}{4}\right)^{\frac{1}{2}-\frac{1}{2}}$$

$$\left(3\frac{3}{8}\right)^{\frac{1}{3}-\frac{1}{3}}$$

1. Fill in the blanks



$$1. 100^{\frac{3}{2}} = (\sqrt{\quad})^{\quad} = 10^{\quad} = ?$$

$$2. 16^{\frac{5}{4}} = (\sqrt{\quad})^{\quad} = \quad^5 = ?$$

$$3. 64^{\frac{\quad}{\quad}} = (\sqrt[3]{64})^4 = \quad^{\quad} = ?$$

$$4. \quad^{\frac{\quad}{5}} = (\sqrt{\quad})^{\quad} = \quad^{\quad} = 8$$

$$5. \quad^{\frac{\quad}{\quad}} = (\sqrt[3]{\quad})^{\quad} = 3^5 = ?$$

$$6. \quad^{\frac{3}{2}} = (\sqrt{\quad})^{\quad} = \quad^{\quad} = 216$$

[Edexcel GCSE June 2003-6H Q17b]

Let $x = 2^p$, $y = 2^q$,

If $xy = 32$ and $2xy^2 = 32$, find the value of p and the value of q .

Fluency Practice

Evaluate:

(a) $27^{2/3}$ (b) $125^{2/3}$

(c) $8^{2/3}$ (d) $(-8)^{2/3}$

(e) $49^{3/2}$ (f) $100^{3/2}$

(g) $16^{3/4}$ (h) $4^{5/2}$

Evaluate:

(a) $64^{-2/3}$ (b) $343^{-2/3}$

(c) $64^{-3/2}$ (d) $1000^{-2/3}$

(e) $25^{-3/2}$ (f) $81^{-3/2}$

(g) $10000^{-3/4}$ (h) $32^{-2/5}$

Evaluate:

(a) $\left(\frac{8}{343}\right)^{-2/3}$ (b) $\left(\frac{1}{125}\right)^{-2/3}$

(c) $\left(\frac{343}{64}\right)^{-2/3}$ (d) $\left(\frac{4}{9}\right)^{-3/2}$

(e) $\left(\frac{100}{9}\right)^{-3/2}$ (f) $\left(\frac{27}{8}\right)^{-4/3}$

(g) $\left(\frac{25}{4}\right)^{-3/2}$ (h) $\left(\frac{16}{10000}\right)^{-3/4}$

Find x .

(a) $x^{2/3} = 4$ (b) $x^{3/2} = 64$

(c) $y^x \times y^{2/3} = y^3$

Fluency Practice

Fractional Indices

Patrons are reminded that $\sqrt{x} \equiv x^{\frac{1}{2}}$ and $\sqrt[n]{x} \equiv x^{\frac{1}{n}}$.
Also note that $\sqrt{4} = 2$. ($\sqrt{4} \neq \pm 2$.)

1. Without a calculator evaluate the following:

- | | |
|---------------------------|--|
| (a) $4^{\frac{1}{2}}$. | (o) $9^{-\frac{3}{2}}$. |
| (b) $16^{\frac{1}{2}}$. | (p) $8^{-\frac{2}{3}}$. |
| (c) $9^{\frac{1}{2}}$. | (q) $32^{\frac{4}{5}}$. |
| (d) $8^{\frac{1}{3}}$. | (r) $27^{-\frac{5}{3}}$. |
| (e) $64^{\frac{1}{6}}$. | (s) $(\frac{1}{4})^{\frac{1}{2}}$. |
| (f) $125^{\frac{1}{3}}$. | (t) $(\frac{1}{27})^{-\frac{1}{3}}$. |
| (g) $16^{\frac{3}{4}}$. | (u) $(\frac{1}{64})^{\frac{1}{3}}$. |
| (h) $81^{\frac{1}{4}}$. | (v) $(\frac{8}{27})^{-\frac{1}{3}}$. |
| (i) $32^{\frac{1}{5}}$. | (w) $(\frac{64}{9})^{\frac{3}{2}}$. |
| (j) $27^{\frac{1}{3}}$. | (x) $(\frac{27}{8})^{-\frac{5}{3}}$. |
| (k) $4^{\frac{3}{2}}$. | (y) $(\frac{1}{9})^{-\frac{1}{2}}$. |
| (l) $8^{\frac{2}{3}}$. | (z) $(64 \times 5^{-3})^{\frac{1}{3}}$. |
| (m) $4^{-\frac{1}{2}}$. | |
| (n) $4^{\frac{5}{2}}$. | |

2. Evaluate:

- | | |
|--|--|
| (a) $\frac{1}{(\frac{2}{3})^2 + (\frac{4}{9})^{\frac{1}{2}}}$. | (e) $\frac{\frac{2}{3} + (\frac{4}{9})^{\frac{1}{2}}}{1 + (\frac{8}{27})^{-\frac{1}{3}}}$. |
| (b) $\frac{2}{(\frac{4}{9})^{\frac{3}{2}} + (\frac{2}{3})^3}$. | (f) $\frac{4 + (\frac{9}{4})^{\frac{3}{2}}}{2 + (\frac{1}{9})^{\frac{1}{2}}}$. |
| (c) $\frac{1}{(\frac{8}{27})^{\frac{2}{3}} - (\frac{81}{4})^{-\frac{1}{2}}}$. | (g) $\frac{(\frac{27}{8})^{-\frac{1}{3}} + \frac{1}{6}}{(\frac{1}{4})^{\frac{1}{2}} - 9^{-\frac{1}{2}}}$. |
| (d) $\frac{4}{(\frac{81}{16})^{-\frac{3}{4}} + (\frac{4}{3})^3}$. | (h) $\frac{(\frac{9}{4})^{\frac{1}{2}} - (\frac{9}{16})^{-\frac{1}{2}}}{3 - (\frac{27}{8})^{\frac{1}{3}}}$. |

3. Simplify the following:

- | | |
|--|--|
| (a) $\frac{x}{\sqrt{x}}$. | (c) $x^{-2} \times x^{\frac{1}{2}}$. |
| (b) $\frac{x^2 \times \sqrt{x}}{x^{-1}}$. | (d) $\frac{x^2 \times x^{0.5}}{\sqrt[3]{x}}$. |

Fluency Practice

Simplify each of the following expressions.

1. $9^{1/2}$
2. $9^{-1/2}$
3. $-9^{1/2}$
4. $-9^{-1/2}$
5. $(-9)^{1/2}$
6. $9^{3/2}$
7. $8^{1/3}$
8. $8^{-1/3}$
9. $(-8)^{1/3}$
10. $16^{-3/4}$
11. $5^{1/2}$
12. $-4^{-5/2}$
13. $(-32)^{1/5}$
14. $32^{2/5}$
15. $36^{1/2}$
16. $36^{-1/2}$
17. $-36^{1/2}$
18. $(-36)^{1/2}$
19. $(-36)^{-1/2}$
20. $36^{-3/2}$
21. $-4^{-3/2}$
22. $(-4)^{-3/2}$
23. $(-8)^{1/3}$
24. $(-8)^{2/3}$
25. $-16^{1/2}$
26. $(-16)^{1/2}$
27. $8^{-2/3}$
28. $(32)^{-4/5}$
29. $(-32)^{-4/5}$
30. $81^{-3/4}$

Simplify each of the following. Present your final answers using positive exponents only. For example, $x^{-2/3} = \frac{1}{\sqrt[3]{x^2}}$. Assume that all variables represent positive numbers.

31. $(x^{1/3})^6$
32. $\frac{x^{2/5}x^{-3/4}}{x^{-1/2}}$
33. $(a^{-1/2})^{-4/5}$
34. $\frac{(m^{1/2})(m^{3/2})}{m^{-3}}$
35. $\sqrt[3]{\frac{x^2y^{-1}}{(-x^4y^{-2})^2}}$
36. $(-8a^6b^{-24})^{1/3}$
37. $\left(\frac{16x^4y^{\frac{2}{5}}}{36x^2y^2}\right)^{\frac{3}{2}}$
38. $(-3x^{-1/2}y^{3/8}x^{-7/2})^0$
39. $(A^{2/3})^{3/2}$
40. $\frac{(y^{-3/4})^{-8}}{(y^{-1/2})^{10}}$

Use exponential notation to simplify each of the following. Present your answer using positive integers only.

41. $\sqrt[8]{x^6}$
42. $(\sqrt[6]{y})^9$
43. $(\sqrt[6]{2})^{18}$
44. $(\sqrt{x})(\sqrt[3]{x})$
45. $\frac{\sqrt[5]{x}\sqrt[3]{x^2}}{x\sqrt[6]{x}}$
46. $\sqrt{\sqrt{\sqrt{\sqrt{x}}}}$

Problem Solving

always, sometimes, never true?

$$a^{\frac{1}{2}} \geq a^{\frac{1}{4}}$$

$$a^{\frac{4}{3}} = a^{\frac{2}{3}} \times a^{\frac{2}{3}}$$

$$4a^{\frac{1}{2}} = 2\sqrt{a}$$

$$2a^{\frac{1}{2}} \geq a^{\frac{1}{2}}$$

$$a^{\frac{1}{4}} = \sqrt{a^{\frac{1}{2}}}$$

$$(a^{\frac{1}{6}})^{\frac{1}{6}} = a^{\frac{1}{12}}$$

$$(a^{\frac{1}{2}})^2 = 1$$

$$a \neq 0$$

$$a > a^{\frac{1}{2}}$$

$$a^{\frac{1}{2}} > (a+1)^{\frac{1}{2}}$$

$$a < 0$$

$$a^{\frac{1}{2}} = \text{undefined}$$

$$a^{1.25} = \sqrt[4]{a^5}$$

$$a^3 \geq a^2$$

Fluency Practice

A1 State the value of 5^0	A2 State the value of 7^1	A3 Evaluate 4^{-3} Give your answer as a fraction in its simplest terms.	A4 Evaluate $\sqrt[3]{125}$
B1 Evaluate $49^{\frac{1}{2}}$	B2 Evaluate $27^{\frac{1}{3}}$	B3 Evaluate $64^{\frac{2}{3}}$	B4 Evaluate $16^{\frac{3}{4}}$
C1 Evaluate $8^{-\frac{1}{3}}$ Give your answer as a fraction in its simplest terms.	C2 Evaluate $36^{-\frac{1}{2}}$ Give your answer as a fraction in its simplest terms.	C3 Evaluate $81^{-\frac{3}{4}}$ Give your answer as a fraction in its simplest terms.	C4 Evaluate $216^{-\frac{2}{3}}$ Give your answer as a fraction in its simplest terms.
D1 Evaluate $\left(\frac{9}{16}\right)^{\frac{1}{2}}$ Give your answer as a fraction in its simplest terms.	D2 Evaluate $\left(\frac{27}{343}\right)^{\frac{2}{3}}$ Give your answer as a fraction in its simplest terms.	D3 Evaluate $\left(\frac{125}{8}\right)^{-\frac{1}{3}}$ Give your answer as a fraction in its simplest terms.	D4 Evaluate $\left(\frac{49}{36}\right)^{-\frac{3}{2}}$ Give your answer as a fraction in its simplest terms.

Fluency Practice

square and cube roots: give your answer as a fraction

(1) $\sqrt{0.36}$

(1) $\sqrt{2\frac{1}{4}}$

(2) $\sqrt{1\frac{24}{25}}$

(2) $\sqrt{1\frac{15}{49}}$

(3) $\sqrt{2\frac{14}{25}}$

(3) $\sqrt{1\frac{11}{25}}$

(4) $\sqrt{3.24}$

(4) $\sqrt[3]{1\frac{271}{729}}$

(5) $\sqrt{4.84}$

(5) $\frac{1}{2}\sqrt[3]{15.625}$

(6) $\sqrt{5\frac{19}{25}}$

(6) $2(2\frac{46}{49})^{-\frac{1}{2}}$

(7) $\sqrt[3]{1\frac{91}{125}}$

(7) $\sqrt{\frac{2}{7} \times 3\frac{3}{8} \times 1\frac{5}{16}}$

(8) $(1\frac{61}{64})^{-\frac{1}{3}}$

(8) $3(3.375)^{-\frac{1}{3}}$

no calculator

Fluency Practice

$$(2\frac{1}{4})^{-\frac{1}{2}} \times (3\frac{3}{8})^{\frac{1}{3}}$$

$$(1.25)^{-\frac{1}{3}} \times (10)^{\frac{1}{3}}$$

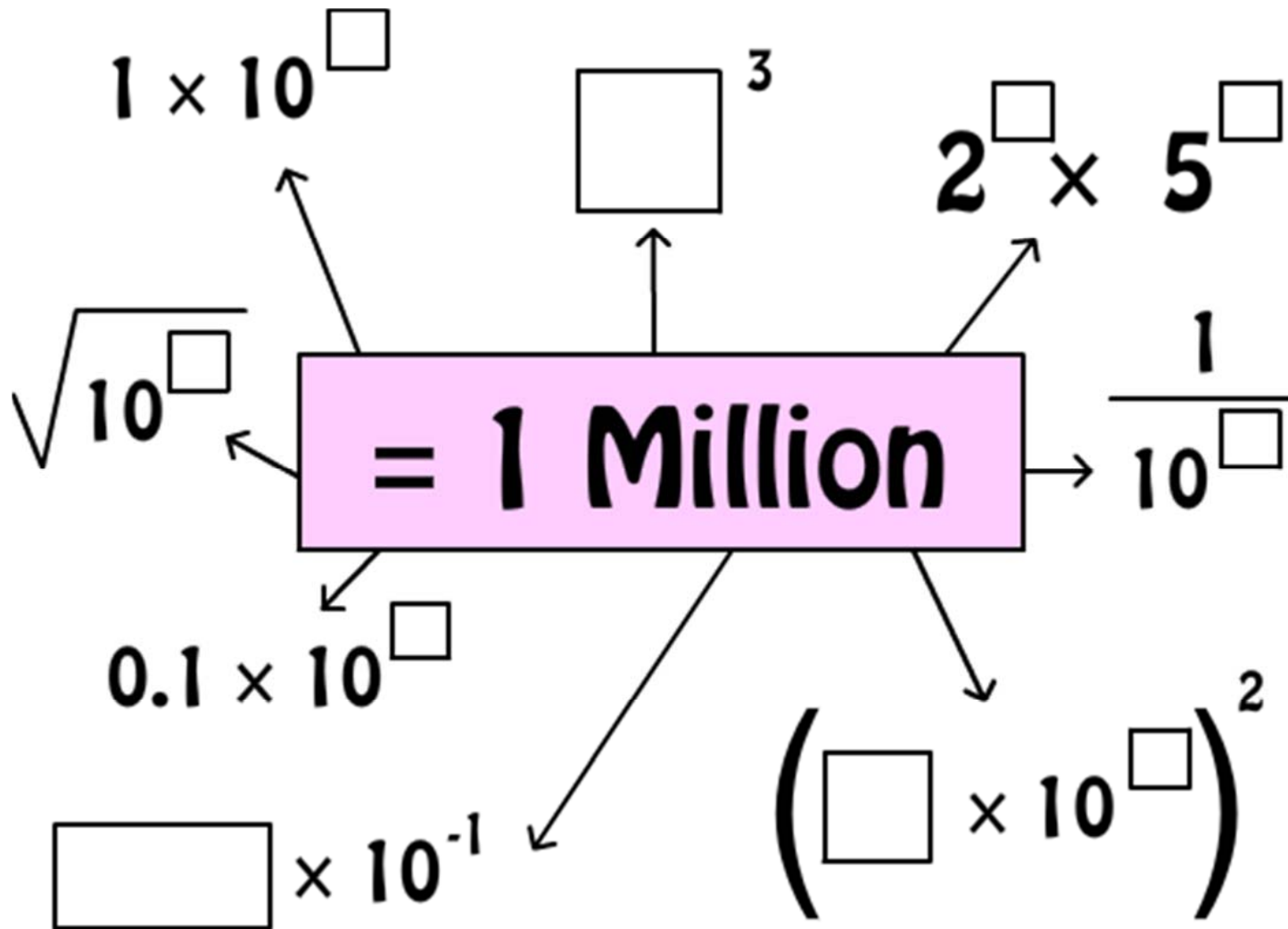
$$(1.5)^{-2} \times (16)^{-\frac{1}{2}} \times (81)^{\frac{3}{4}}$$

$$(48)^{\frac{1}{2}} \times (9)^{\frac{1}{6}} \times (3)^{-\frac{5}{6}}$$

$$(20)^{\frac{2}{3}} \div (100)^{\frac{1}{6}} \times (\frac{5}{2\sqrt{2}})^{\frac{2}{3}}$$

$$(12)^{\frac{1}{3}} \div (81)^{\frac{1}{6}} \times (162)^{\frac{1}{3}}$$

Fluency Practice



Fluency Practice

Write the following without negative or fraction powers.

① x^{-3}

② $3x^{-1}$

③ $5x^{-2}$

④ $\frac{2}{3}x^{-4}$

⑤ $\frac{4}{5}x^{-3}$

⑥ $\frac{1}{5}x^{-3}$

⑦ $\frac{x^{-3}}{5}$

⑧ $x^{\frac{1}{2}}$

⑨ $x^{\frac{3}{2}}$

⑩ $3x^{\frac{5}{2}}$

⑪ $x^{-\frac{1}{2}}$

⑫ $x^{-\frac{3}{4}}$

⑬ $3x^{-\frac{2}{3}}$

⑭ $\frac{1}{6}x^{-\frac{3}{4}}$

Write the following without roots and, if the x is on the denominator, with a negative power:

① $\frac{4}{x}$

② $\frac{4}{x^2}$

③ $\frac{3}{4x^2}$

④ $\frac{1}{4x^2}$

⑤ $\frac{4}{3x^2}$

⑥ $\frac{1}{5x}$

⑦ $\frac{5}{x^3}$

⑧ $\sqrt{x}$

⑨ $\sqrt[3]{x}$

⑩ $\sqrt[3]{x^2}$

⑪ $5\sqrt{x^3}$

⑫ $\frac{1}{\sqrt{x}}$

⑬ $\frac{1}{\sqrt[4]{x}}$

⑭ $\frac{3}{\sqrt[3]{x^2}}$

⑮ $\frac{2}{3\sqrt{x}}$

⑯ $\frac{1}{3\sqrt[3]{x^2}}$

⑰ $\frac{4}{5\sqrt{x^3}}$

⑱ $\frac{1}{4\sqrt{x}}$

Fluency Practice

Write as a single power of x :

- (a) $\sqrt{x^3}$ (b) $(\sqrt{x})^3$
 (c) $\sqrt[3]{x^2}$ (d) $\sqrt[4]{x}$
 (e) $(\sqrt[3]{x})^4$ (f) $\sqrt{x^5}$

Write as a single power of x :

- (a) $\frac{1}{\sqrt[3]{x}}$ (b) $\left(\frac{1}{\sqrt[3]{x}}\right)^2$
 (c) $\frac{1}{\sqrt{x^3}}$ (d) $\left(\frac{1}{\sqrt{x}}\right)^3$
 (e) $\left(\frac{1}{\sqrt[3]{x}}\right)^5$ (f) $\frac{1}{\sqrt[2]{x^7}}$

Write as a single power of x :

- (a) $x^2 \times \sqrt{x}$ (b) $\sqrt[3]{x} \times x$
 (c) $\frac{x^4}{\sqrt{x}}$ (d) $\frac{\sqrt[3]{x}}{x}$
 (e) $\sqrt{\frac{1}{x^5}}$ (f) $\frac{1}{x\sqrt{x}}$

(a) Given that

$$\frac{y^4 \times \sqrt{y}}{\sqrt{y^5}} = y^a$$

find the value of a .

(b) Given that

$$\frac{1}{\sqrt[3]{y^2}} \times (y\sqrt{y})^4 = y^b$$

find the value of b .

(c) Given that

$$\left(\sqrt[2]{y^3}\right)^3 \times \frac{1}{y^c} = \left(y^2 \times \sqrt[4]{y^3}\right)^{-2}$$

find the value of c .

Fluency Practice

Question 1: Evaluate each of the following

- (a) 5^{-2} (b) 2^{-1} (c) 2^{-3} (d) 4^{-2} (e) 3^{-3} (f) 6^{-1}
- (g) 10^{-2} (h) 2^{-4} (i) 9^{-2} (j) 3^{-4} (k) 10^{-1} (l) 7^{-2}
- (m) 2^{-5} (n) 5^{-3} (o) 2^{-6} (p) 10^{-4} (q) 6^{-3} (r) 10^{-6}

Question 2: Write each of the following in index form.

- (a) $\frac{1}{5^2}$ (b) $\frac{1}{3^4}$ (c) $\frac{1}{8^3}$ (d) $\frac{1}{4^5}$ (e) $\frac{1}{10^3}$ (f) $\frac{1}{2^6}$

Question 3: Write each of the following in the form 2^n

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{32}$ (d) $\frac{1}{8}$ (e) $\frac{1}{64}$ (f) $\frac{1}{256}$

Question 4: Write each of the following in the form 5^n

- (a) $\frac{1}{125}$ (b) $\frac{1}{25}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3125}$ (e) $\frac{1}{625}$ (f) $\frac{1}{15625}$

Question 5: Write each of the following as fractions

- (a) a^{-2} (b) y^{-1} (c) w^{-4} (d) 2^{-x} (e) 5^{-a} (f) x^{-n}

Question 6: Write each of the following in index form

- (a) $\frac{1}{w^2}$ (b) $\frac{1}{2^y}$ (c) $\frac{1}{x^5}$ (d) $\frac{1}{4^x}$ (e) $\frac{1}{y^x}$ (f) $\frac{1}{n^n}$

Question 7: Write each of the following as fractions

- (a) $5y^{-2}$ (b) $8c^{-1}$ (c) $(5x)^{-2}$ (d) $(2y)^{-3}$ (e) $2x^{-3}$ (f) $(10w)^{-3}$

Fluency Practice

Question 8: Write each of the following in index form

(a) $\frac{3}{x^2}$ (b) $\frac{5}{w^8}$ (c) $\frac{2}{3y^2}$ (d) $\frac{1}{4x^3}$ (e) $\frac{6}{y^m}$ (f) $\frac{a}{x^n}$

Question 9: Write each of the following as fractions

(a) $100^{-\frac{1}{2}}$ (b) $25^{-\frac{1}{2}}$ (c) $9^{-\frac{1}{2}}$ (d) $8^{-\frac{1}{3}}$ (e) $125^{-\frac{1}{3}}$ (f) $16^{-\frac{1}{4}}$

Question 10: Write each of the following as fractions

(a) $8^{-\frac{2}{3}}$ (b) $25^{-\frac{3}{2}}$ (c) $64^{-\frac{2}{3}}$ (d) $4^{-\frac{5}{2}}$ (e) $81^{-\frac{3}{4}}$ (f) $100000^{-\frac{2}{5}}$

Apply

Question 1: Arrange in order from smallest to largest.

$\frac{1}{50}$ 5^{-2} $\frac{3}{10}$ 2^{-3}

Question 2: Work out

(a) $4^{-2} \times 3^2$ (b) $10^{-1} \div 5^{-2}$ (c) $2^{-2} + 3^{-2} \times 2^3$

Question 3: Sally has completed her homework.
Can you spot any mistakes?

Question 2

Evaluate

Work out

4^{-2} 10^{-3} -16 $\frac{1}{30}$

Question 4: Given that $2^m + 2^n = \frac{9}{32}$

Work out mn

Fluency Practice

Question 5:

$$x^{-2} \quad x^0 \quad x \quad x^3$$

Put the expressions above in order, from smallest to largest, when:

(a) $x = 2$ (b) $x = 1$ (c) $x = 0.5$ (d) $x = -0.5$

(e) $x = -1$ (f) $x = -2$

Fluency Practice

Question 1: Evaluate each of the following

- (a) $25^{\frac{1}{2}}$ (b) $81^{\frac{1}{2}}$ (c) $4^{\frac{1}{2}}$ (d) $144^{\frac{1}{2}}$ (e) $8^{\frac{1}{3}}$ (f) $125^{\frac{1}{3}}$
- (g) $100^{\frac{1}{2}}$ (h) $1000^{\frac{1}{3}}$ (i) $49^{\frac{1}{2}}$ (j) $225^{\frac{1}{2}}$ (k) $64^{\frac{1}{2}}$ (l) $27^{\frac{1}{3}}$
- (m) $216^{\frac{1}{3}}$ (n) $64^{\frac{1}{3}}$ (o) $16^{\frac{1}{4}}$ (p) $1^{\frac{1}{4}}$ (q) $81^{\frac{1}{4}}$ (r) $625^{\frac{1}{4}}$

Question 2: Write each of the following in index form

- (a) $\sqrt{x}$ (b) $\sqrt{y}$ (c) $\sqrt[3]{a}$ (d) $\sqrt[4]{y}$ (e) $\sqrt[6]{x}$ (f) $\sqrt[8]{c}$

Question 3: Evaluate each of the following

- (a) $9^{\frac{3}{2}}$ (b) $4^{\frac{3}{2}}$ (c) $8^{\frac{2}{3}}$ (d) $27^{\frac{2}{3}}$ (e) $125^{\frac{2}{3}}$ (f) $49^{\frac{3}{2}}$
- (g) $4^{\frac{5}{2}}$ (h) $64^{\frac{2}{3}}$ (i) $9^{\frac{5}{2}}$ (j) $100^{\frac{3}{2}}$ (k) $16^{\frac{3}{2}}$ (l) $1000^{\frac{2}{3}}$
- (m) $100^{\frac{5}{2}}$ (n) $32^{\frac{2}{5}}$ (o) $4^{\frac{7}{2}}$ (p) $8^{\frac{5}{3}}$ (q) $16^{\frac{3}{4}}$ (r) $81^{\frac{3}{4}}$
- (s) $32^{\frac{3}{5}}$ (t) $27^{\frac{5}{3}}$ (u) $64^{\frac{5}{6}}$ (v) $10000^{\frac{3}{4}}$

Question 4: Write each of the following in index form

- (a) $\sqrt[3]{a}$ (b) $\sqrt{w^5}$ (c) $\sqrt[3]{x^2}$ (d) $\sqrt[3]{w^4}$ (e) $\sqrt[5]{m^2}$ (f) $\sqrt[9]{k^4}$

Question 5: Write each of the following in the form 9^n

- (a) 81 (b) 3 (c) 27

Question 6: Write each of the following in the form 64^n

- (a) 8 (b) 4 (c) 16

Fluency Practice

Question 7: Simplify each of the following

(a) $(9x^2)^{\frac{1}{2}}$ (b) $(4x^6)^{\frac{1}{2}}$ (c) $(25x^8)^{\frac{1}{2}}$ (d) $(4x^3)^{\frac{1}{2}}$ (e) $(8x^3)^{\frac{1}{3}}$ (f) $(125x^6)^{\frac{1}{3}}$

Question 8: Evaluate each of the following

(a) $(64x^3)^{\frac{2}{3}}$ (b) $(9x^4)^{\frac{3}{2}}$ (c) $(27x^6)^{\frac{2}{3}}$ (d) $(4x^6)^{\frac{5}{2}}$ (e) $(16x^8)^{\frac{3}{4}}$ (f) $(32x^{20})^{\frac{3}{5}}$

Question 9: Write each of the following as fractions

(a) $8^{-\frac{2}{3}}$ (b) $25^{-\frac{3}{2}}$ (c) $64^{-\frac{2}{3}}$ (d) $4^{-\frac{5}{2}}$ (e) $81^{-\frac{3}{4}}$ (f) $100000^{-\frac{2}{5}}$

Apply

Question 1: Arrange the following in order, smallest first.

$25^{\frac{1}{2}}$, $8^{\frac{2}{3}}$, $27^{\frac{1}{3}}$

Question 2: Which is the odd one out?
Explain your answer.

$64^{\frac{1}{2}}$, $16^{\frac{3}{4}}$, $9^{\frac{2}{3}}$, $4^{\frac{3}{2}}$

Question 3: Work out

(a) $64^{\frac{1}{3}} \times 2^3$ (b) $27^{\frac{2}{3}} \div 9^{\frac{3}{2}}$ (c) $(8^{\frac{2}{3}})^2$

Question 4: Gina has completed her homework.
Can you spot any mistakes?

Question 1

Work out $9^{\frac{1}{2}}$

4.5

Question 2

Work out $27^{\frac{2}{3}}$

18

Intelligent Practice

Write in terms of the given number:

- 1) 16 as a power of 2
- 2) 16 as a power of 4
- 3) 64 as a power of 4
- 4) 125 as a power of 5
- 5) 4^2 as a power of 2
- 6) 4^3 as a power of 2
- 7) 4^4 as a power of 2
- 8) 27^3 as a power of 3
- 9) 27^{3x} as a power of 3
- 10) 25^{3x} as a power of 5

Fluency Practice

Find the value of each of the following.

$$\sqrt[3]{4 \times 2 \times 10^6}$$

$$\sqrt[4]{9 \times 9 \times 10^8}$$

$$\sqrt[3]{4 \times 16 \times 10^6}$$

$$\sqrt[2]{3 \times 27 \times 10^4}$$

$$\sqrt[4]{125 \times 5 \times 10^{12}}$$

$$\sqrt[5]{16 \times 64 \times 10^{15}}$$

Intelligent Practice

Solve the following equations:

1) $3^x = 243$

2) $3^x = \frac{1}{243}$

3) $4^x = 64$

4) $4^x = \frac{1}{64}$

5) $5^x = 625$

6) $5^x = \frac{1}{625}$

Intelligent Practice

Solve the following equations:

1) $\left(\frac{1}{3}\right)^x = 81$

2) $\left(\frac{1}{3}\right)^x = \frac{1}{81}$

3) $\left(\frac{1}{3}\right)^x = \frac{1}{27}$

4) $\left(\frac{1}{3}\right)^x = 27$

5) $\left(\frac{1}{4}\right)^x = 64$

6) $\left(\frac{1}{4}\right)^x = 256$

7) $\left(\frac{1}{4}\right)^x = \frac{1}{256}$

8) $\left(\frac{5}{25}\right)^x = 625$

Intelligent Practice

Find the value of x that satisfies:

1) $2^{3x} \times 2^{x-3} = 32$

2) $2^{2x} \times 2^{x+3} = 64$

3) $3^{2x} \times 3^{x-6} = 81$

4) $3^{x+2} \times 3^{x-2} = 243$

5) $2^{4x} \div 2^{x-2} = 32$

6) $3^{6x} \div 3^{x-5} = 243$

Intelligent Practice

Find the value of x that satisfies:

1) $16^{\frac{1}{4}} \times 2^x = 8^{\frac{2}{3}}$

2) $81^{\frac{1}{4}} \times 3^x = 27^{\frac{2}{3}}$

3) $25^{\frac{1}{2}} \times 125^x = 5^{\frac{4}{2}}$

4) $27^{\frac{1}{4}} \times 3^x = 9^{\frac{2}{3}}$

Fluency Practice

Repeat addition Index problems

1) Simplify

a) $3^5 + 3^5 + 3^5$

b) $3^y + 3^y + 3^y$

c) $3^{2y+4} + 3^{2y+4} + 3^{2y+4}$

d) $2^3 + 2^3$

e) $2^3 + 2^3 + 2^3 + 2^3$

f) $2^3 + 2^3 + 3^3 + 3^3 + 3^3$

g) $2^3 + 2^3 + 3^3$

h) $2^x + 2^x + 2^y$

i) $2^x + 2^y + 2^x + 2^y$

j) $2^3 \times 2^3 + 2^6$

k) $2^5 + 2^{10} \div 2^5$

l) $(2^x)^2 + 2^{x^2} + 2^{2x}$

2) Solve

a) $4^5 + 4^5 + 4^5 + 4^5 = 4^x$

b) $4^5 + 4^5 + 4^5 + 4^5 = 4^{3x-5}$

c) $4^x + 4^x + 4^x + 4^x = 4^{10}$

d) $4^{3x+1} + 4^{3x+1} + 4^{3x+1} + 4^{3x+1} = 4^{10}$

e) $2^{3x+1} + 2^{3x+1} + 2^{3x+1} + 2^{3x+1} = 2^{10}$

f) $2^{3x+1} + 2^{3x+1} + 2^{3x+1} + 2^{3x+1} = 4^{10}$

g) $2^{5x-2} + 2^{5x-2} = 4^{10x-1}$

h) $2^{5x+3} + 2^5 \times 2^{5x-2} = 4^{10x-1}$

i) $4^3 + 4^3 = 2^x$

j) $8^3 + 8^3 + 8^3 + 8^3 = 2^x$

j) $8^3 + 8^3 + 8^3 + 8^3 = 4^x$

k) $8^{3x+2} + 8^{3x+2} + 8^{3x+2} + 8^{3x+2} = 2^x$

3) Solve

a) $2^3 + 2^3 = \frac{1}{2^x}$

b) $\frac{1}{2^3} + \frac{1}{2^3} = \frac{1}{2^x}$

c) $\frac{1}{8} + \frac{1}{4} \div 2 = \frac{1}{2^x}$

d) $\frac{1}{4^{3x}} + \frac{1}{4^{3x}} = \frac{1}{2^{7(x-2)}}$

e) $(\sqrt{2})^3 + (\sqrt{2})^3 = 2^x$

f) $\sqrt[3]{2^5} + \sqrt[3]{2^5} = 2^x$

g) $\sqrt[3]{4^5} + \sqrt[3]{4^5} = 8^x$

4) $2^{3x-2} + 2^{3x-2} = 4^{y+1}$

and $3^x \times (3^{x-4})^4 = \frac{1}{3^{3y}}$ find the value of x and y.

Fluency Practice

Find the value of m in each question

$$8^2 = 2^{2m-4}$$

$$16^5 = 2^{2m+4}$$

$$27^4 = 3^{5m-3}$$

$$81^5 = 3^{3m+2}$$

$$25^6 = 5^{7m-9}$$

$$125^7 = 5^{9m+3}$$

Fluency Practice

Find the value of x in each question

$$3^{-n} = 0.2$$

$$(3^4)^n = x$$

$$5^{-n} = 0.25$$

$$(5^3)^n = x$$

$$6^{-n} = 0.5$$

$$(6^5)^n = x$$

$$7^{-n} = 0.1$$

$$(7^3)^n = x$$

$$9^{-n} = 0.4$$

$$(9^2)^n = x$$

$$3^{-n} = 0.04$$

$$(3^{0.5})^n = x$$

Fluency Practice

- (a) Write 4^5 as a power of 2
- (b) Write 8^3 as a power of 2.
- (c) Write 16^3 as a power of 4.
- (d) Write 27^4 as a power of 3.
- (e) Write 125^3 as a power of 5.

- (a) $2^6 \times 2^4 = 4^n$. Find n .
- (b) $8^n = 2^3 \times 2^9$. Find n .
- (c) $3^n = 3^3 \times 9^2$. Find n .
- (d) $2^n = \frac{16}{2^3}$. Find n .
- (e) $\frac{3^8}{81} = 3^n$. Find n .

- (a) $4^3 \times 16 = 2^n$. Find n .
- (b) $243 \times 9^2 = 3^n$. Find n .
- (c) $2^n \times 4^3 = 1024$. Find n .
- (d) $\frac{9^2 \times 81^{1/2}}{3^2} = 3^n$. Find n .
- (e) $\frac{2^{2n} \times 64}{4^2} = 2^3$. Find n .

- (a) $128 = 4^{2x} \times 2^x$. Find x .
- (b) $\frac{1}{\sqrt[3]{9^4}} = 3^x$. Find x .
- (c) $16^{1/5} \times 2^x = 8^{3/4}$. Find x .
- (d) $\left(2^{1/2}\right)^x = \frac{32}{2^2}$. Find x .
- (e) $243 = \frac{3^x}{\sqrt[3]{81}}$. Find x .

Fluency Practice

Change of Base Jumbled Answers

Write each question on the left as a single power of 5. Find your answer on the right.

A $125 = \underline{\hspace{2cm}}$	B $\frac{1}{25} = \underline{\hspace{2cm}}$	5^{14}	5^{13}
C $25^6 = \underline{\hspace{2cm}}$	D $125^{-3} = \underline{\hspace{2cm}}$	5^{12}	5^{10}
E $\frac{1}{25^2} = \underline{\hspace{2cm}}$	F $25^{\frac{1}{2}} = \underline{\hspace{2cm}}$	5^8	5^7
G $\left(\frac{1}{25}\right)^3 = \underline{\hspace{2cm}}$	H $(125^2)^{-4} = \underline{\hspace{2cm}}$	5^4	5^3
I $25 \times 25 = \underline{\hspace{2cm}}$	J $25^4 \times 25^3 = \underline{\hspace{2cm}}$	5^2	$5^{1.5}$
K $25^6 \times 5^{-5} = \underline{\hspace{2cm}}$	L $\frac{1}{125} \times \frac{1}{25} = \underline{\hspace{2cm}}$	5^1	$5^{\frac{3}{4}}$
M $25^7 \times \frac{1}{5} = \underline{\hspace{2cm}}$	N $\frac{1}{5^{-9}} = \underline{\hspace{2cm}}$	$5^{\frac{1}{2}}$	5^{-1}
O $125^{\frac{1}{4}} = \underline{\hspace{2cm}}$	P $\frac{1}{25^{-4}} = \underline{\hspace{2cm}}$	5^{-2}	5^{-3}
more challenging		5^{-4}	5^{-5}
Q $\sqrt{5} \times 5 = \underline{\hspace{2cm}}$	R $\frac{25^2}{5^4} \times 5^{-3} = \underline{\hspace{2cm}}$	5^{-6}	5^{-9}
S $\sqrt{25^{10}} = \underline{\hspace{2cm}}$	T $\frac{1}{\sqrt[3]{125}} = \underline{\hspace{2cm}}$	5^{-24}	$5^{\square}$
U $\frac{5}{\sqrt{5}} = \underline{\hspace{2cm}}$	V $\sqrt{625} = \underline{\hspace{2cm}}$		

Fluency Practice

$2^1 = 2$	$2^{11} = 2,048$	$2^{21} = 2,097,152$
$2^2 = 4$	$2^{12} = 4,096$	$2^{22} = 4,194,304$
$2^3 = 8$	$2^{13} = 8,192$	$2^{23} = 8,388,608$
$2^4 = 16$	$2^{14} = 16,384$	$2^{24} = 16,777,216$
$2^5 = 32$	$2^{15} = 32,768$	$2^{25} = 33,554,432$
$2^6 = 64$	$2^{16} = 65,536$	$2^{26} = 67,108,864$
$2^7 = 128$	$2^{17} = 131,072$	$2^{27} = 134,217,728$
$2^8 = 256$	$2^{18} = 262,144$	$2^{28} = 268,435,456$
$2^9 = 512$	$2^{19} = 524,288$	$2^{29} = 536,870,912$
$2^{10} = 1024$	$2^{20} = 1,048,576$	$2^{30} = 1,073,741,824$

Without using a calculator or paper, can you mentally compute these expressions?

- 32×16
- $4 \times 64 \times 1024$
- $4 \times 16 \times 32 \times 64$
- $1024 \div 64$
- $1,048,576 \div 32,768$
- $1,073,741,824 \div 64$
- 128^2
- $(1,024)^3$
- $\sqrt{65,536}$
- $\sqrt[3]{134,217,728}$
- $\sqrt[5]{1,073,741,824}$
- $(1,048,576)^{1/2}$
- $(262,144)^{2/3}$
- $\frac{524,288}{(16,384)^{\frac{1}{2}}}$
- $\frac{(67,108,864)^{1/2}}{16^3(256)^{1/4}}$
- $\frac{1}{4} \cdot (65,536)$

Equating Powers

Whenever you see 4 you should think 2^2 .

Whenever you see 8 you should think 2^3 .

Whenever you see 27 you should think 3^3 .

Whenever you see 64 you should think 4^3 or 8^2 or 2^6 .

Etcetera...

Also, pro tip; when you see 1 you should think 2^0 or 3^0 or 5^0 or ...

All of these problems reduce to

$$k^{\text{fish}} = k^{\text{dog}} \Rightarrow \text{fish} = \text{dog}.$$

Questions

1. Solve $2^{x+1} = 4^x$.

2. Solve $8^{3x} = 2^{x-2}$.

3. Solve $3^{2x-1} = 9^{4x}$.

4. Solve $27^{3-x} = 81^{2x}$.

5. Solve $2 \times 2^{2x+1} = 8^{x-1}$.

6. Solve $5^x \times 25^3 = 5^{2x-1}$.

7. Solve $4^x \times 8^{x-1} = 2 \times 4^{3x+1}$.

8. Solve $8 \times 2^{x-1} = 4^{2x-1}$.

9. Solve $16 \times 8^{2x+1} = 2 \times 16^x$.

10. Solve $32^x = \frac{8^x}{2^{x+1}}$.

11. Solve $\frac{2^{x+1}}{4} = \frac{4^{x+3}}{8^x}$.

12. Solve $\frac{9^x}{27^{x+1}} = \frac{3^x}{81^{1-x}}$.

13. Solve $\frac{5^{-2x+1}}{25^{3-x}} = \frac{125^x}{5^{x-4}}$.

14. Solve $8 \times \frac{2^{2-x}}{4^{3-2x}} = \frac{8^x}{4^{2+x}}$.

15. Solve $1 = 8^2 \times 4^{x-1} \times 2^{x+1}$.

16. Solve $\frac{7^x}{49^{6-x}} = \frac{343^{1-x}}{7^{2x-3}}$.

17. Solve $8^{ax} = 4^{bx+1}$.

18. Solve $27^{a+b} = 3^{cx-a}$.

19. Solve $9^{ax} \times 27^x \times 3^{ax} = 1$.

20. Solve $\frac{5^{ax-2}}{25^{b-x}} = 125^{cx+d}$.

21. Solve $\frac{2^{ax}}{2^{3-bx}} = \frac{4^{4+cx}}{8^{k-x}}$.

Only do the following if you've studied solving quadratics by factorisation.

22. Solve $4^{x+2} = 2^{x^2+5}$.

23. Solve $3^{x^2+2} = 27^x$.

24. Solve $5^{2x^2} = 25^{2x+3}$.

25. Solve $(x^2 + 5x + 5)^{x^2+11x+30} = 1$.

Indices

1. Simplify fully:

(a) $x^2 \times x^3 \times x^5$.

(b) $2 \times (x^2)^3$.

(c) $(x^{-3})^{-5}$.

(d) $(3x^2)^3$.

(e) $(12x^9) \div (4x^3)$.

2. Simplify fully:

(a) $\frac{(2x^{-2})^3 \times (2x^3)^2}{6^2 \times x^{-4}}$.

(b) $\frac{2x^8 \times (2x)^8}{2x \times (2x)^5}$.

(c) $\frac{(3x^{-1})^3 \times (2x^{-2})^4}{(6x^2)^3}$.

(d) $\frac{3x^3(2xy)^2}{6x^5y^2}$.

(e) $\frac{a^3b^7}{a^{-2}b^4}$.

(f) $\frac{64(x^{-3}y^7)^2x^6y}{(4x^4y^2)^2}$.

(g) $\frac{7pq^{-1}(2pq^3)^3}{14p(2p^2q)^2}$.

3. Evaluate without a calculator:

(a) 3^{-1} .

(b) 5^{-2} .

(c) 4^{-3} .

(d) 7^0 .

(e) $(\frac{2}{3})^{-1}$.

(f) $(\frac{4}{3})^{-2}$.

(g) $(2\frac{1}{3})^{-1}$.

(h) $(1\frac{3}{4})^{-3}$.

4. Evaluate without a calculator:

(a) $\frac{2^8 \times 2^{-3}}{2^4}$.

(b) $8^4 \div 4^6$.

5. Solve for x

(a) $2^x = 8$.

(b) $3^x = 81$.

(f) $4x \times (3x^{-2})^2$.

(g) $\frac{(3x^2)^3}{3x^{-2}}$.

(h) $\frac{12x^8}{6x^4} + 6x^4$.

(i) $2x^5(2x^5 + 2x^5)$.

(h) $\frac{(4m^7n)^3}{8n^3m^{-7}}$.

(i) $\frac{(uv)^2(2u^2v)^3}{(u^3v^8)^2(uv^{-1})^4}$.

(j) $\frac{3x(x^4y^8)^3}{x^3(3x^5y)^2}$.

(k) $\frac{6u^2v^3}{3u^{-2}v^5} \times \frac{4(u^3v^4)^2}{2uv^{-1}}$.

(l) $\frac{(2x^3)^3y}{3x^{-2}y^3} \div \frac{4(xy)^{-8}}{(3x^{-4})^2}$.

(m) $\frac{4x^3y}{7xy^2} \div \frac{8x(yz)^2}{14xz^2y^3}$.

(n) $\frac{\frac{6x}{y^2}}{(3xy^3)}$.

(i) $(\frac{1}{2})^{-2}$.

(j) $(0.6)^{-2}$.

(k) $(\frac{2}{3})^{-1} + (\frac{5}{2})^{-1}$.

(l) $(\frac{3}{2})^{-2} + (\frac{3}{2})^2$.

(m) $(\frac{1}{5})^{-2} - (\frac{2}{7})^{-1}$.

(n) $(2\frac{1}{2})^{-3} + (3\frac{2}{3})^{-1}$.

(o) $(0.4)^{-2} - (0.2)^{-1}$.

(c) $27^3 \div 9^4$.

(d) $\frac{64^2}{8 \times 16}$.

(c) $5^x = \frac{1}{25}$.

(d) $3^{-x} = \frac{1}{27}$.

Fluency Practice

Simplify:

(a) $4^5 \times 4^2$

(b) $4^5 \times 4^3$

(c) $4^3 \times 4^5$

(d) 4×4^5

(e) $4^5 \times 4^{-2}$

(f) $4^0 \times 4^3$

(g) $4^{2.5} \times 4^{0.5}$

(h) $4^5 \times 5^2$

(i) $5^{-3} \times 5^2$

(j) $5^{-5} \times 5^{-3}$

Simplify:

(a) $4^5 \div 4^2$

(b) $4^5 \div 4^3$

(c) $4^2 \div 4^5$

(d) $4^5 \div 4$

(e) $4^2 \div 4^0$

(f) $4^5 \div 4^{-2}$

(g) $4^5 \div 5^4$

(h) $5^{2.5} \div 5^{0.5}$

(i) $\frac{5^7}{5^2}$

(j) $\frac{(-5)^7}{(-5)^2}$

Simplify:

(a) $(3^4)^5$

(b) $(3^5)^4$

(c) $(3^2)^5$

(d) $(3^{-2})^5$

(e) $(3^4)^1$

(f) $3^3 \times (3^4)^{0.5}$

Simplify:

(a) $\frac{2^3 \times 2^8}{2^5}$

(b) $\frac{2^{-3} \times 2^8}{2^1}$

Find x:

(a) $5^x \times 5^4 = 5^7$

(b) $\frac{3^x \times 3^{-2}}{3^4} = 3^{10}$

(c) $10^2 \times 10^x = 1000000$

Fluency Practice

(a)	(b)	(c)	(d)
Write down the cube root of 27	Work out $3^5 - \sqrt{441}$	Write down the value of 5^0	Simplify $y^5 \times y^4$
(e)	(f)	(g)	(h)
Simplify $(x^{-3})^5$	Write as a power of 2 $\frac{2^{12}}{2^3}$	Simplify $(3a^2b^4)^3$	Simplify $\frac{a^5 \times a^2}{a^{-3}}$
(i)	(j)	(k)	(l)
Write $2\sqrt{2}$ as a single power of 2	Evaluate $\left(\frac{4}{9}\right)^{3/2}$	Evaluate $8^{-4/3}$	$\frac{4^{10} \times 4^x}{4^6} = 4^{-1}$ Find the value of x .
(m)	(n)	(o)	(p)
$\frac{2^{10}}{64} = 2^n$ Find the value of n .	Write $\frac{1}{\sqrt[3]{4}}$ as a single power of 2	$4^a = 16 \times 8^{2a}$ Find the value of a .	Given that $9^x = (27^a)^{1/2} \times 3^b$ find an expression for x in terms of a and b .

Fluency Practice

A1 Write 5^8 as a power of 25	A2 Write 8^6 as a power of 2	A3 Write 8^6 as a power of 4	A4 Write $8^2 \times 4^3$ as a power of 2
B1 Write 2 as a power of 8	B2 Write 4 as a power of 8	B3 Write $\sqrt{8}$ as a power of 2	B4 Express $3\sqrt{3}$ as a power of 9
C1 Express $\frac{1}{81}$ as a single power of 3	C2 Express $\frac{1}{\sqrt{3}}$ as a single power of 9	C3 Express $\frac{1}{4\sqrt{2}}$ as a single power of 2	C4 Express $3^7 + 9^4 + 15 \times 27^2$ as a power of 3
D1 Solve: $32^x = \frac{1}{16}$	D2 Solve: $2^x = 8^{\frac{1}{4}} \times 16^{\frac{1}{3}}$	D3 Find the value of x if: $9^{\frac{3}{4}} \times 27^x = 81^{\frac{2}{3}}$	D4 Find the value of m and n if: $6 \times 12^m = 9^4 \times 2^n$

Fluency Practice

Section a:

Simplify the following, giving your answer in index notation, or as a fraction or integer where possible:

- | | | |
|---------------------------|-------------------------------|------------------------------------|
| A) $3^4 \times 3^5$ | H) $(2c^2)^3$ | O) $6a^3 \div 2a$ |
| B) $4^4 \times 4^2$ | I) $(a^4b^2)^{-2}$ | P) $15m^3k^3 \div 3m^2k^5$ |
| C) $y^0 \times y^{-3}$ | J) $(2b^2)^3 \times (2b^2)^4$ | Q) $(ab^2)^3 \div ab^2$ |
| D) $(-2)^3 \times (-2)^2$ | K) $(5^2)^a \times (5^a)^4$ | R) $c^2 \times c^0 \div c$ |
| E) $3^{2a} \times 3^{4a}$ | L) $z^3 \div z^2$ | S) $27c^2 \times c^{-1} \div 9c^4$ |
| F) $(a^2)^5$ | M) $8^3 \div 8^{-3}$ | T) $3ab^4 \times 6a^{-5}b^2$ |
| G) $(2^2)^3$ | N) $z^3 \div z^{-4}$ | U) $-2x^3y^0 \times -2x^2y^{-5}$ |

Section b:

Simplify the following, giving your answer in index notation

- | | | |
|---------------------------|--|--|
| A) $12ab^3 \div 6a^2b^2$ | D) $\frac{2a^4b}{(3a^2)^2}$ | G) $\frac{(2xy)^3}{(6x^3y)^2} \times \frac{2xy}{3x^2}$ |
| B) $\frac{6x^3y}{3x^2}$ | E) $\frac{1}{a^2} \times \frac{2}{ab^2}$ | H) $\left(\frac{2}{x^2}\right)^3$ |
| C) $\frac{12a^2b}{4ab^2}$ | F) $\frac{2b^3c}{3ab} \times \frac{4ab}{2c}$ | I) $\left(\frac{ab^2}{8a}\right)^2$ |

Section c (hard):

Determine the value of the letters:

- | | | |
|-------------------------|-------------------------|--|
| A) $2^a = 4^2$ | F) $3^f \div 3^2 = 3^8$ | L) $\frac{2^3}{2^{-1}} = 2^l$ |
| B) $2^b = 8^2$ | G) $2^g \div 2^3 = 16$ | M) $\frac{2^m}{(2^m)^2} = \frac{1}{4}$ |
| C) $2^c \times 4 = 2^5$ | H) $(3^h)^2 = 3^6$ | N) $\frac{2 \times 3^{2n}}{2^2 \times 3^n} = \frac{18}{4}$ |
| D) $2^{3d} = 64$ | I) $26^i = 1$ | |
| E) $2^e \times 2 = 32$ | J) $(-4)^j = -64$ | |
| | K) $(2k)^2 = 100$ | |

Fluency Practice

1. Find the value of ...

a. $144^{\frac{1}{2}}$ b. $625^{\frac{1}{4}}$ c. $27^{\frac{1}{3}}$ d. $\left(\frac{1}{16}\right)^{0.25}$ e. $0.01^{\frac{1}{2}}$ f. $\left(\frac{125}{64}\right)^{\frac{1}{3}}$ g. $\left(\frac{1}{64}\right)^{-\frac{1}{6}}$ h. $81^{-\frac{1}{4}}$

2. Find the value of ...

a. $16^{\frac{3}{2}}$ b. $1000^{\frac{2}{3}}$ c. $32^{\frac{3}{5}}$ d. $\left(\frac{1}{4}\right)^{\frac{5}{2}}$ e. $\left(\frac{1}{81}\right)^{\frac{3}{4}}$ f. $\left(\frac{27}{125}\right)^{\frac{2}{3}}$ g. $\left(\frac{3125}{243}\right)^{\frac{4}{5}}$ h. $625^{-\frac{3}{4}}$

3. Find the missing values

a. $16^{\frac{3}{\square}} = 64$ b. $\square^{\frac{2}{5}} = 25$ c. $\left(\frac{\square}{81}\right)^{\frac{3}{4}} = \frac{8}{\square}$ d. $\left(\frac{100}{\square}\right)^{\frac{\square}{2}} = \frac{1000}{27}$ e. $\left(\frac{\square}{125}\right)^{\frac{2}{\square}} = 25$

4. Write in the form 9^n

a. 81 b. 3 c. 27 d. 243

5. Write in the form 64^n

a. 8 b. 4 c. 16 d. 512

6. Find the value of x

a. $9^{\frac{x}{4}} = 3$ b. $16^{\frac{x+2}{4}} = 2$ c. $125^{\frac{2x-1}{6}} = 5$ d. $27^{\frac{5x-7}{6}} = 81$
 e. $32^{\frac{2x-3}{10}} = 4$ f. $\left(\frac{4}{25}\right)^{5x-2} = \frac{2}{5}$ g. $2 \times 9^{\frac{3x-7}{4}} = 6$ h. $3 \times 4^{2x+8} = 24$

Fluency Practice

Part One - Consolidate

$$\begin{array}{ll} 1) 5^6 \times 5^3 = 5^? & 4) b^6 \times b^{-3} = b^? \\ 2) 5^{-3} \div 5^3 = 5^? & 5) h^{-4} \div h^{-2} = h^? \\ 3) 5^2 \times 5^{-4} = 5^? & 6) y^{\frac{1}{2}} \times y^2 = y^? \end{array}$$

Part Two – Develop

$$\begin{array}{ll} 1) y^5 \times y^? = y^3 & 4) y^{-3} \div y^? = 1 \\ 2) 6^2 \div ?^2 = 9 & 5) h^2 \times h = 1 \\ 3) 5^0 \times 5^? = 5 & 6) y^{\frac{1}{2}} \times y^? = y^2 \end{array}$$

Part Three – Enhance

$$\begin{array}{ll} 1) \frac{1}{36} \times \sqrt{6} = 6^? & 4) \left(\frac{1}{b}\right)^2 \times \sqrt[3]{b} = b^? \\ 2) 8^x \div 8^{3x} = 8^4 & 5) (\sqrt[3]{(h)^2})^{\frac{3}{2}} = h^? \\ 3) \sqrt[3]{125} \times 5^x = 25 & 6) y^x \times y^{\frac{2}{3}} = y^3 \end{array}$$

Problem Solving

Part Four - Investigate

x^2

x^{-3}

$\sqrt{x}^3$

$x^{-\frac{1}{2}}$

$x^{\frac{2}{3}}$

$\left(\frac{1}{x}\right)^2$

$x\sqrt{x}$

Here are seven cards: Assume that x is a positive integer greater than 1

Find:

Two different pairs of cards that when multiplied, equal x

A matching pair of cards

Two cards which multiply and give an answer that is on a different card also in the list!

The two cards who give the largest product

Finally – create one more card similar to above, and use three more from the list, so that you can complete this sum:

$$\boxed{} \times \boxed{} = \boxed{} \div \boxed{}$$

Fluency Practice

shifting bases (i)

use a calculator to work out the missing powers

$$(1) \quad 8 = 2^? = 4^? = 64^?$$

$$(2) \quad 64 = 2^? = 4^? = 8^?$$

$$(3) \quad 729 = 3^? = 9^? = 27^?$$

$$(4) \quad 4096 = 2^? = 4^? = 16^?$$

$$(5) \quad 15625 = 5^? = 25^? = 125^?$$

shifting bases (ii)

$$(1) \quad 9^2 = 3^?$$

$$(2) \quad 2^? = 8^2$$

$$(3) \quad 16^2 = 4^?$$

$$(4) \quad 81^3 = 3^?$$

$$(5) \quad 5^? = 625^2$$

$$(6) \quad 243^2 = 3^?$$

$$(7) \quad 78125^2 = 5^?$$

$$(8) \quad 3^? = 531441$$

$$(9) \quad 9^? = 43046721$$

$$(10) \quad 32^? = 33554432$$

Fluency Practice

multiple writings

(a) use a calculator to work out the missing powers:

1.) $9^2 = 3^?$

2.) $2^? = 8^2$

3.) $16^2 = 4^?$

4.) $81^3 = 3^?$

5.) $5^? = 625^2$

6.) $243^2 = 9^?$

7.) $78125^2 = 5^?$

8.) $3^? = 531441$

9.) $9^? = 43046721$

10.) $32^? = 33554432$

example:

$$25^2 = 5^?$$

$$625 = 5^4$$

(b) write these as powers of two different base numbers:

1.) 81

2.) 16

3.) 10,000

4.) 2401

5.) 512

6.) 4096

7.) 59049

8.) 0.001

9.) 390625

10.) 1771561

example:

$$625 = 5^4$$

$$625 = 25^2$$

(c) write these as a power of a power:

1.) 4096

2.) 729

3.) 10,000,000

4.) 15625

5.) 117649

examples:

$$46656 = (6^3)^2$$

$$531441 = (3^4)^3$$

Fluency Practice

Complete the tables below! The first row has been completed for you.

2^x	Evaluate
2^3	8
2^0	
	32
2^4	
	$\sqrt{2}$
2^{-1}	
	2
	$1\frac{1}{8}$
$2^{\frac{3}{2}}$	
2^{-6}	
	$\sqrt{128}$
	$1\frac{1}{\sqrt[3]{2}}$
$2^{-\frac{1}{2}}$	
	$8\sqrt{2}$
$2^{\frac{5}{2}}$	

3^x	Evaluate
3^2	9
	1
3^4	
	729
3^{-3}	
	$\sqrt[3]{3}$
	$1\frac{1}{3}$
$3^{0.5}$	
	$9\sqrt{3}$
$3^{0.2}$	
	$\sqrt{27}$
$3^{-1.5}$	
	$\frac{1}{243}$
	$\frac{81}{\sqrt{3}}$
	$27\sqrt[5]{3}$

Exam Questions

Complete the following exam style questions.

1) Find the value of the missing constant in the following equations.

a) $3^a = \frac{1}{27}$

b) $5^b = \frac{1}{\sqrt{5}}$

c) $7^c = \sqrt[3]{49}$

(3)

2) Solve the following equations to find x .

a) $\frac{8}{2^{3x}} = \sqrt{2}$

b) $4^x = \frac{\sqrt[3]{2}}{32}$

c) $125 = \frac{(\sqrt{5})^3}{25^{1-x}}$

(9)

3) Solve the equations below to find x and y .

$$\frac{16^x}{8^y} = \frac{1}{4}$$

$$4^x 2^y = 16$$

(6)

Problem Solving

$x\sqrt{x}$

$(x - 2)$

$(x + 3)$

$(x + 5)$

$(x + 2)$

$(x - 3)$

$(x + 1)$

$(x - 1)$

x^2

$\sqrt{x}$

$x^{-\frac{3}{2}}$

$\frac{1}{x}$

$x^2 + 2x + 4$

Select expressions from above which multiply to given the answers listed below.

You may not use an expression more than once in each product.

a) $x^2 + 8x + 15$

b) $x^3 + 2x^2$

c) $x^2 - 9$

d) x^2

e) $x^{\frac{3}{2}} - 3x^{\frac{1}{2}}$

f) 1

g) $\sqrt{x}$

h) $\frac{4}{x} + x + 2$

i) $x^2 + 6x + 5$

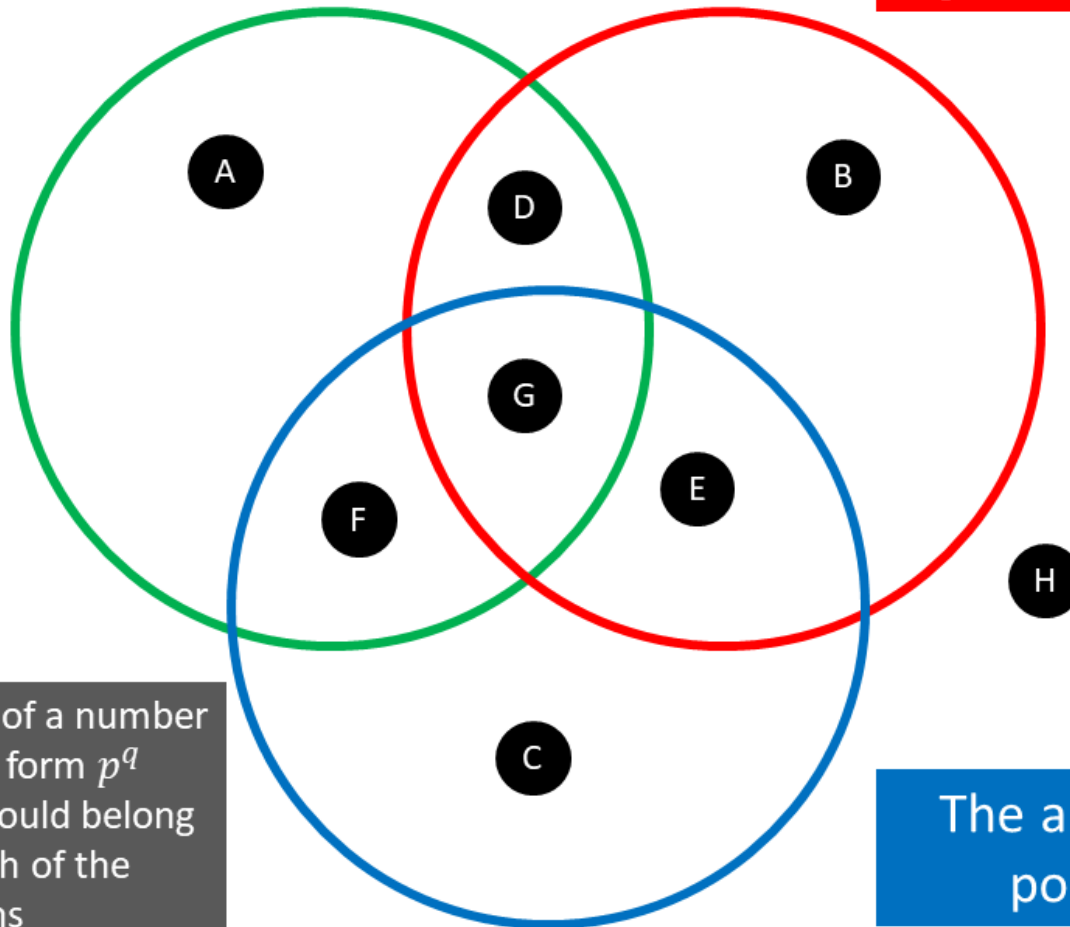
j) $x^3 + 2x^2 - 5x - 6$

k) $x^3 - 2x^2 - 5x + 6$

Maths Venns

The power (q) is negative

The base number (p) is a fraction



If you think a region is impossible to fill, convince me why!

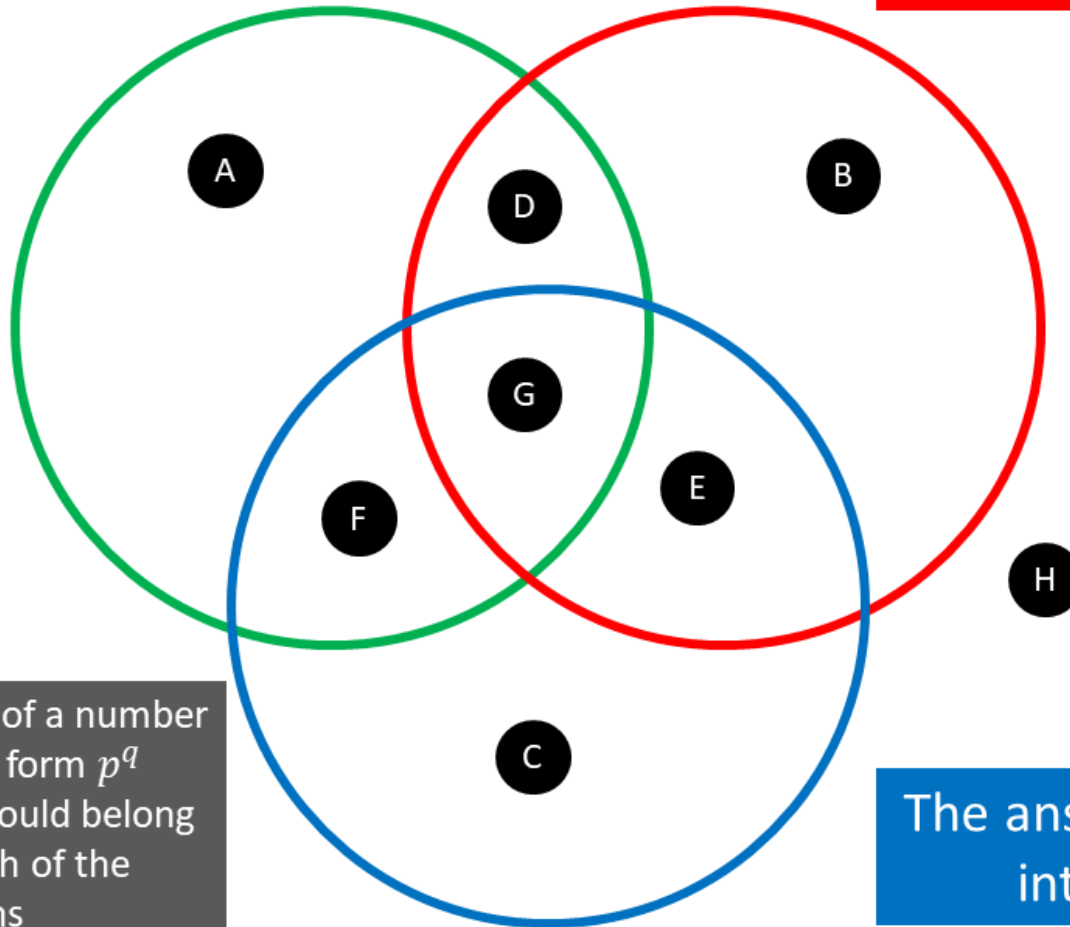
Think of a number in the form p^q that could belong in each of the regions

The answer is positive

Maths Venns

The power (q) is
a fraction

The base number
(p) is a fraction



If you think a
region is
impossible to
fill, convince
me why!

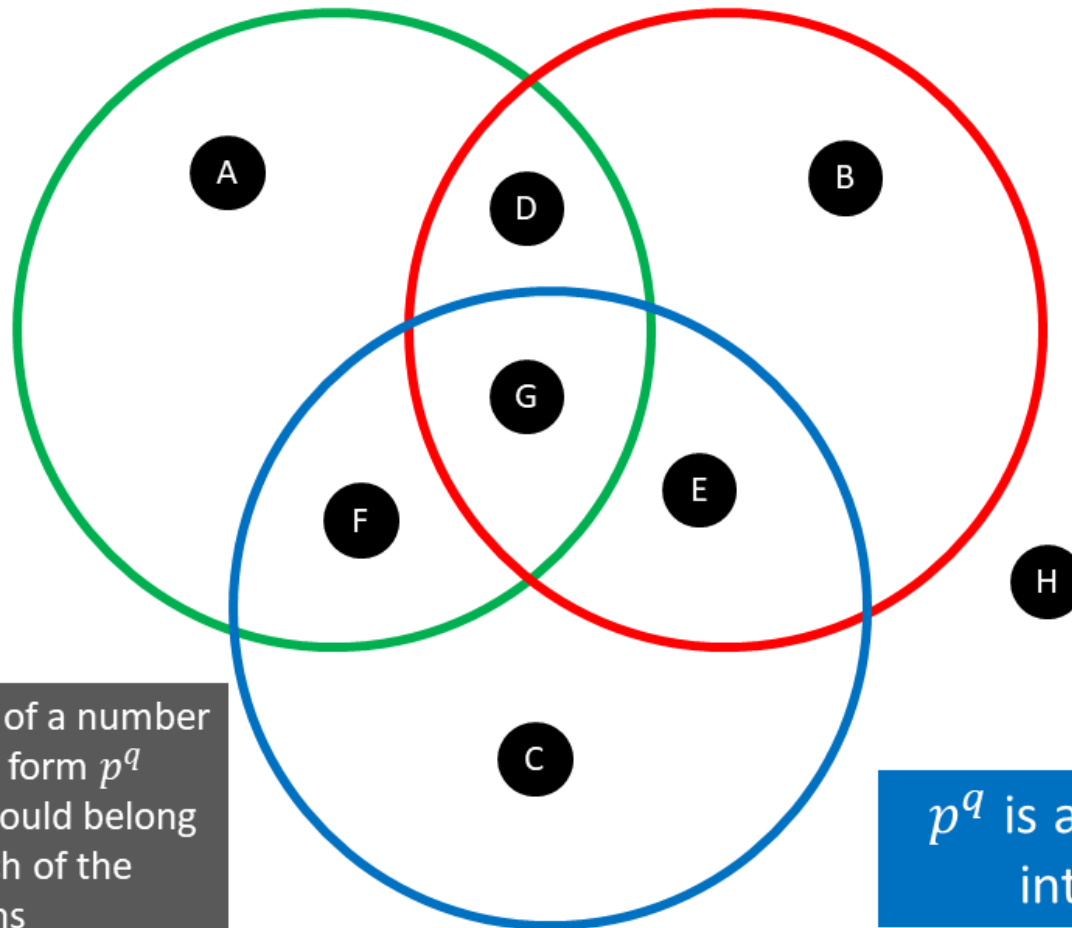
Think of a number
in the form p^q
that could belong
in each of the
regions

The answer is an
integer

Maths Venns

The power (q) is negative

The power (q) cannot be written as a unit fraction



If you think a region is impossible to fill, convince me why!

Think of a number in the form p^q that could belong in each of the regions

p^q is a positive integer

Problem Solving

$$2^{\textcolor{red}{3}} + 2^{\textcolor{red}{3}} + 3^{\textcolor{blue}{4}} + 3^{\textcolor{blue}{4}} + 3^{\textcolor{blue}{4}} = ?$$

☐ $2^3 + 3^4$

☐ 4^5

☐ $2^4 + 3^5$

☐ $2^6 + 3^{12}$

$$\sqrt{\textcolor{red}{9}\textcolor{green}{16}\textcolor{blue}{x}^2}$$

☐ 9^{4x}

☐ 9^{8x^2}

☐ 3^{4x}

☐ 3^{8x^2}

$$\frac{6^{\textcolor{red}{2007}} - \textcolor{red}{6}^{\textcolor{red}{2006}}}{\textcolor{green}{30}} = ?$$

☐ 30

☐ 0.2

☐ 6^{2005}

☐ 6^{2007}

☐ 36

$$27^{-\frac{x}{3}} + 81^{\frac{1-x}{4}}$$

If the expression above can be stated in the form of $\frac{a}{b^x}$ for positive integers a and b , what is the value of $a + b$?

$$\begin{cases} ab = a^b \\ \frac{a}{b} = a^{3b} \end{cases}$$

a and b are real numbers, such that $a > 1$ and $b \neq 0$, satisfying the above system.

Find b^{-a} .

Fluency Practice

Solve

(a) $\sqrt{x} = 2$ (b) $5\sqrt[3]{x} = 3$

(c) $\sqrt{x} = \frac{16}{\sqrt{x}}$ (d) $x = \frac{27}{\sqrt{x}}$

(e) $2\sqrt{x} = \frac{64}{x^2}$ (f) $\frac{4}{\sqrt[3]{x}} = \frac{x}{4}$

Solve

(a) $x^{1/3} = -2$ (b) $5x^{1/2} - 2 = 13$

(c) $\frac{x^{3/2}}{20} = 6\frac{1}{4}$ (d) $x^{1/2} = \frac{9}{4x^{3/2}}$

(e) $x^{-2} = \frac{1}{36}$ (f) $48x^{-1/4} - 3 = 0$

Solve

(a) $(\sqrt{x} - 1)(\sqrt{x} - 5) = 0$

(b) $(\sqrt{x})^2 - 4\sqrt{x} + 3 = 0$

(c) $x - 7\sqrt{x} + 10 = 0$

(d) $8\sqrt{x} = x + 12$

(e) $8 - \frac{15}{\sqrt{x}} = \sqrt{x}$

(f) $4 - 2\sqrt{x} = \frac{2}{\sqrt{x}}$

Solve

(a) $(x^2 - 9)(x^2 - 1) = 0$

(b) $x^4 - 6x^2 + 8 = 0$

(c) $x^2 + \frac{4}{x^2} = 5$

(d) $x^6 - 27 = 26x^3$

(e) $x^3 = 7 + \frac{8}{x^3}$

2 Calculating with Surds

Intelligent Practice

Simplify:

1) $\sqrt{2} \times \sqrt{3}$

2) $\sqrt{4} \times \sqrt{3}$

3) $\sqrt{5} \times \sqrt{3}$

4) $\sqrt{6} \times \sqrt{3}$

5) $\sqrt{24} \times \sqrt{3}$

6) $\sqrt{12} \times \sqrt{3}$

7) $\sqrt{12} \times \sqrt{4}$

8) $\sqrt{11} \times \sqrt{4}$

9) $\sqrt{10} \times \sqrt{4}$

10) $\sqrt{10} \times \sqrt{8}$

11) $\sqrt{20} \times \sqrt{4}$

12) $\sqrt{20} \times \sqrt{5}$

13) $\sqrt{20} \times \sqrt{10}$

14) $\sqrt{20} \times \sqrt{20}$

15) $\sqrt{20} \times \sqrt{40}$

16) $\sqrt{20} \times \sqrt{40} \times \sqrt{2}$

17) $\sqrt{20} \times \sqrt{40} \times \sqrt{3}$

18) $(\sqrt{20} \times \sqrt{40} \times \sqrt{3})^2$

Intelligent Practice

Simplify:

1) $2\sqrt{2} \times \sqrt{3}$

10) $\sqrt{10} \times 2\sqrt{8}$

2) $2\sqrt{4} \times \sqrt{3}$

11) $\sqrt{20} \times 2\sqrt{4}$

3) $\sqrt{5} \times 2\sqrt{3}$

12) $5\sqrt{20} \times 3\sqrt{5}$

4) $\sqrt{6} \times 2\sqrt{3}$

13) $5\sqrt{20} \times 3\sqrt{10}$

5) $\sqrt{24} \times 3\sqrt{3}$

14) $5\sqrt{20} \times 3\sqrt{20}$

6) $2\sqrt{12} \times 3\sqrt{3}$

15) $2\sqrt{20} \times 3\sqrt{40}$

7) $2\sqrt{12} \times 3\sqrt{4}$

16) $2\sqrt{20} \times 2\sqrt{40} \times 2\sqrt{2}$

8) $2\sqrt{11} \times 3\sqrt{4}$

17) $2\sqrt{20} \times 2\sqrt{40} \times \sqrt{8}$

9) $\sqrt{10} \times 2\sqrt{4}$

18) $2\sqrt{20} \times 2\sqrt{40} \times 2\sqrt{8}$

Intelligent Practice

Simplify:

1) $\sqrt{6} \div \sqrt{3}$

2) $\sqrt{6} \div \sqrt{2}$

3) $\sqrt{12} \div \sqrt{2}$

4) $\sqrt{12} \div \sqrt{3}$

5) $\sqrt{15} \div \sqrt{3}$

6) $\sqrt{18} \div \sqrt{3}$

7) $\sqrt{72} \div \sqrt{3}$

8) $\sqrt{36} \div \sqrt{3}$

9) $\sqrt{36} \div \sqrt{4}$

10) $\sqrt{40} \div \sqrt{4}$

11) $\sqrt{40} \div \sqrt{40}$

12) $\sqrt{40} \div \sqrt{5}$

13) $\sqrt{100} \div \sqrt{5}$

14) $\sqrt{200} \div \sqrt{10}$

15) $\sqrt{400} \div \sqrt{20}$

16) $\sqrt{800} \div \sqrt{40}$

17) $\sqrt{800} \div \sqrt{40} \div \sqrt{2}$

18) $\sqrt{800} \div \sqrt{40} \div \sqrt{20}$

Intelligent Practice

Simplify:

1) $2\sqrt{6} \div \sqrt{3}$

2) $4\sqrt{6} \div \sqrt{2}$

3) $4\sqrt{12} \div 2\sqrt{2}$

4) $4\sqrt{12} \div 2\sqrt{3}$

5) $6\sqrt{15} \div 3\sqrt{3}$

6) $6\sqrt{18} \div 3\sqrt{3}$

7) $24\sqrt{72} \div 3\sqrt{3}$

8) $12\sqrt{36} \div 3\sqrt{3}$

9) $12\sqrt{36} \div 4\sqrt{4}$

10) $20\sqrt{40} \div 4\sqrt{4}$

11) $20\sqrt{40} \div 20\sqrt{40}$

12) $20\sqrt{40} \div 5\sqrt{5}$

13) $10\sqrt{100} \div 5\sqrt{5}$

14) $20\sqrt{200} \div 10\sqrt{10}$

15) $40\sqrt{400} \div 20\sqrt{20}$

16) $80\sqrt{800} \div 40\sqrt{40}$

17) $80\sqrt{800} \div 40\sqrt{40} \div 2\sqrt{2}$

18) $80\sqrt{800} \div 40\sqrt{40} \div 2\sqrt{20}$

Fluency Practice

Multiplication Madness

Think of this like a normal multiplication table, just with terms missing everywhere. Fill in all blanks.

	$-\sqrt{5}$	$\sqrt{2}$			$-\sqrt{6}$			$2\sqrt{3}$	$-\sqrt{2}$			-2
3												
		$\sqrt{10}$				$3\sqrt{5}$		$2\sqrt{15}$				
				$16\sqrt{2}$								$8\sqrt{2}$
												4
$\sqrt{7}$	$-\sqrt{35}$						$2\sqrt{14}$					
									$2\sqrt{6}$			
		6				$9\sqrt{2}$						
					$-9\sqrt{2}$							
			$4\sqrt{6}$					$8\sqrt{3}$				
$-2\sqrt{5}$											-20	
$\sqrt{2}$			$2\sqrt{3}$		$-2\sqrt{3}$					$-\sqrt{6}$		
		4										

Surds: Multiplication Squares

Can you fill in the missing numbers in these multiplication squares?

x	$\sqrt{10}$	$\sqrt{6}$
$\sqrt{8}$	$\sqrt{\quad}$	$2\sqrt{\quad}$
$\sqrt{6}$	$2\sqrt{\quad}$	

x	$\sqrt{6}$	$\sqrt{3}$
$\sqrt{12}$	$\sqrt{\quad} \sqrt{8}$	
$\sqrt{7}$	$\sqrt{3} \times \sqrt{\quad}$	$\sqrt{\quad}$

x	$3\sqrt{6}$	$2\sqrt{8}$
$\sqrt{6}$		$\sqrt{\quad} \sqrt{48}$
$\sqrt{3}$	$\sqrt{\quad} \sqrt{2}$	$4\sqrt{\quad}$

Fluency Practice

x	$\sqrt{9}$	$2\sqrt{3}$
$2\sqrt{3}$	$\sqrt{\quad}$	
$3\sqrt{3}$	$\sqrt{\quad}$	

x	$\sqrt{\quad}$	$\sqrt{\quad}$
$\sqrt{6}$	$\sqrt{48}$	$\sqrt{30}$
	$\sqrt{72}$	$\sqrt{45}$

x		$2\sqrt{5}$
	$\sqrt{60}$	10
	$4\sqrt{6}$	$4\sqrt{\quad}$

* Challenge: Make up one of your own!*

Fluency Practice

Multiplying, Dividing and Simplifying Surds				
(a)	(b)	(c)	(d)	(e)
Simplify $\sqrt{5} \times \sqrt{7}$	Simplify $\sqrt{3} \times \sqrt{27}$	Simplify $\sqrt{10} \div \sqrt{2}$	Simplify $\frac{\sqrt{27}}{\sqrt{3}}$	Simplify $\sqrt{2} \times \sqrt{10} \times \sqrt{5}$
(f)	(g)	(h)	(i)	(j)
Simplify $2\sqrt{3} \times 4\sqrt{7}$	Simplify $\sqrt{5} \times 8\sqrt{2}$	Simplify $3\sqrt{5} \times 4\sqrt{5}$	Simplify $\frac{9\sqrt{6}}{3\sqrt{2}}$	Simplify $\frac{\sqrt{48}}{\sqrt{3}}$
(k)	(l)	(m)	(n)	(o)
Simplify $\frac{10\sqrt{24}}{5\sqrt{6}}$	Simplify $\sqrt{12}$	Simplify $\sqrt{28}$	Simplify $\sqrt{35}$	Simplify $\sqrt{108}$
(p)	(q)	(r)	(s)	(t)
Express $\sqrt{90}$ in the form $a\sqrt{10}$ where a is an integer	Express $\sqrt{75}$ in the form $a\sqrt{3}$ where a is an integer	Express $\sqrt{125}$ in the form $a\sqrt{5}$ where a is an integer	Simplify $\frac{\sqrt{200}}{5}$	Simplify $\frac{\sqrt{112}}{2}$

Intelligent Practice

Simplify:

1) $2\sqrt{3} + 5\sqrt{3}$

2) $2\sqrt{3} + 5\sqrt{6}$

3) $2\sqrt{3} + 5\sqrt{12}$

4) $2\sqrt{3} + 5\sqrt{27}$

5) $2\sqrt{3} + 6\sqrt{27}$

6) $2\sqrt{30} + 6\sqrt{27}$

7) $2\sqrt{30} + 6\sqrt{120}$

8) $6\sqrt{30} + 2\sqrt{120}$

9) $6\sqrt{60} + 2\sqrt{240}$

10) $6\sqrt{6} + 2\sqrt{24}$

11) $6\sqrt{6} + 2\sqrt{24} + 5\sqrt{36}$

12) $6\sqrt{16} + 2\sqrt{24} + 5\sqrt{36}$

Intelligent Practice

Simplify:

$$1) \quad \frac{2\sqrt{3}+5\sqrt{3}}{\sqrt{3}}$$

$$2) \quad \frac{2\sqrt{3}+5\sqrt{12}}{\sqrt{3}}$$

$$3) \quad \frac{2\sqrt{3}+5\sqrt{27}}{\sqrt{3}}$$

$$4) \quad \frac{5\sqrt{27}-2\sqrt{3}}{\sqrt{3}}$$

$$5) \quad \frac{10\sqrt{27}-4\sqrt{3}}{2\sqrt{3}}$$

$$6) \quad \frac{10\sqrt{27}-30\sqrt{3}}{2\sqrt{3}}$$

$$7) \quad \frac{10\sqrt{270}+30\sqrt{30}}{2\sqrt{30}}$$

$$8) \quad \frac{10\sqrt{120}+30\sqrt{30}}{2\sqrt{30}}$$

$$9) \quad \frac{10\sqrt{120}+30\sqrt{30}}{50\sqrt{30}}$$

Fluency Practice

Work out:

(a) $4\sqrt{2} + 3\sqrt{2}$ (b) $10\sqrt{3} - 2\sqrt{3}$

(c) $-3\sqrt{5} + 7\sqrt{5}$ (d) $6\sqrt{2} - 8\sqrt{2}$

(e) $4\sqrt{3} + 7\sqrt{3} - \sqrt{3}$

(f) $2\sqrt{7} + 5\sqrt{7} - 8\sqrt{7}$

(g) $\frac{3}{2}\sqrt{5} + \frac{7}{2}\sqrt{5} - \frac{1}{2}\sqrt{5}$

Express as a single surd:

(a) $3\sqrt{2} + \sqrt{8}$ (b) $\sqrt{40} + \sqrt{10}$

(c) $6\sqrt{5} - \sqrt{20}$ (d) $-2\sqrt{3} + \sqrt{48}$

(e) $\sqrt{8} + \sqrt{32} - 10\sqrt{2}$

(f) $5\sqrt{3} - \sqrt{3} + 2\sqrt{12}$

(g) $-3\sqrt{10} - \sqrt{90} - 2\sqrt{160}$

Simplify:

(a) $5 + 2\sqrt{3} + 13 + 5\sqrt{3}$

(b) $5\sqrt{2} - 3 - 2\sqrt{2} + 11$

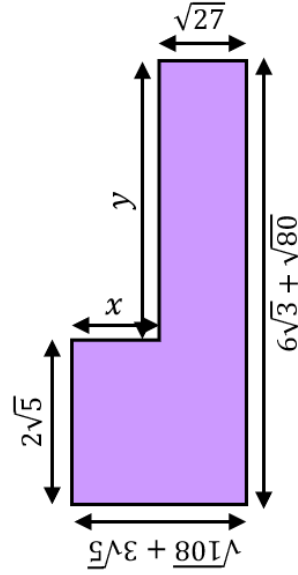
(c) $2\sqrt{3} + 3\sqrt{2} + 6\sqrt{3} - \sqrt{2}$

(d) $3\sqrt{5} - \sqrt{10} - 6\sqrt{10} - \sqrt{5}$

(e) $\sqrt{8} + \sqrt{20} + 6\sqrt{2} + 3\sqrt{5}$

(f) $\sqrt{200} - 3\sqrt{6} + 6\sqrt{2} - \sqrt{486}$

Find the values of x and y and the perimeter of the compound shape in the form $a\sqrt{3} + b\sqrt{5}$.



Fluency Practice

1) True or False?

Are these calculations correct?

Adding & Subtracting Surds

a) $\sqrt{8} + \sqrt{18} =$
 $2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$

b) $\sqrt{8} + \sqrt{50} =$
 $2\sqrt{2} + 5\sqrt{2} = 7\sqrt{2}$

c) $\sqrt{12} + \sqrt{27} =$
 $2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$

d) $\sqrt{90} - \sqrt{40} =$
 $3\sqrt{10} - 2\sqrt{10} = \sqrt{10}$

e) $\sqrt{80} - \sqrt{20} =$
 $8\sqrt{10} - 2\sqrt{10} = 6\sqrt{10}$

f) $\sqrt{125} + \sqrt{45} =$
 $5\sqrt{5} + 3\sqrt{5} = 8\sqrt{5}$

g) $\sqrt{63} + \sqrt{28} =$
 $3\sqrt{7} + 4\sqrt{7} = 7\sqrt{7}$

h) $\sqrt{150} - \sqrt{54} =$
 $5\sqrt{6} - 3\sqrt{6} = 2\sqrt{6}$

i) $\sqrt{75} - \sqrt{108} =$
 $5\sqrt{3} - 6\sqrt{3} = \sqrt{3}$

j) $\sqrt{180} + \sqrt{18} =$
 $10\sqrt{18} + \sqrt{18} = 11\sqrt{18}$

k) $\sqrt{72} + 2\sqrt{8} =$
 $6\sqrt{2} + 4\sqrt{2} = 10\sqrt{2}$

l) $2\sqrt{18} - \sqrt{32} =$
 $5\sqrt{2} - 4\sqrt{2} = \sqrt{2}$

2) Free Practice

a) $\sqrt{48} + \sqrt{27} =$

b) $\sqrt{360} + \sqrt{40} =$

c) $\sqrt{150} - \sqrt{96} =$

d) $\sqrt{200} - \sqrt{32} =$

e) $\sqrt{20} + \sqrt{40} =$

f) $\sqrt{48} - \sqrt{12} + \sqrt{27} =$

g) $\sqrt{8} + \sqrt{8} =$

h) $\sqrt{45} + \sqrt{45} =$

i) $\sqrt{18} + \sqrt{18} + \sqrt{32} =$

j) $\sqrt{90} + (2 \times \sqrt{40}) =$

k) $\sqrt{18} + 2\sqrt{18} =$

l) $3\sqrt{20} - \sqrt{180} =$

3) Extension

a) $2\sqrt{24} + \sqrt{54} =$

b) $2\sqrt{28} - \sqrt{63} =$

c) $2\sqrt{32} + 2\sqrt{18} =$

d) $2\sqrt{48} - 2\sqrt{117} =$

e) $3\sqrt{50} - \sqrt{200} =$

f) $2\sqrt{180} - 2\sqrt{125} =$

g) $3\sqrt{175} + 4\sqrt{28} =$

h) $3\sqrt{500} - 3\sqrt{245} =$

i) $2\sqrt{192} - 4\sqrt{75} =$

j) $2\sqrt{448} + 3\sqrt{640} =$

Fluency Practice

surd simplification

1.) $\sqrt{2} + \sqrt{32}$

2.) $\sqrt{3} + \sqrt{12}$

3.) $\sqrt{20} + \sqrt{125}$

4.) $\sqrt{18} + \sqrt{72}$

5.) $\sqrt{45} + \sqrt{20}$

6.) $\sqrt{24} + \sqrt{54}$

7.) $\sqrt{243} - \sqrt{48}$

8.) $\sqrt{176} - \sqrt{99}$

9.) $\sqrt{539} - \sqrt{275}$

10.) $\sqrt{800} - \sqrt{450}$

11.) $\sqrt{80} + \sqrt{125} + \sqrt{180}$

12.) $\sqrt{112} - \sqrt{63} + \sqrt{28}$

13.) $\sqrt{245} - \sqrt{125} + \sqrt{20}$

14.) $\sqrt{50} - 2\sqrt{72} + \sqrt{98}$

15.) $\sqrt{135} - \sqrt{540} + \sqrt{240}$

16.) $\sqrt{104} - \sqrt{234} + \sqrt{650}$

17.) $3\sqrt{50} - \sqrt{200} + 4\sqrt{50}$

18.) $5\sqrt{24} - 2\sqrt{54} + 3\sqrt{150}$

19.) $10\sqrt{20} - 3\sqrt{80} + 2\sqrt{605}$

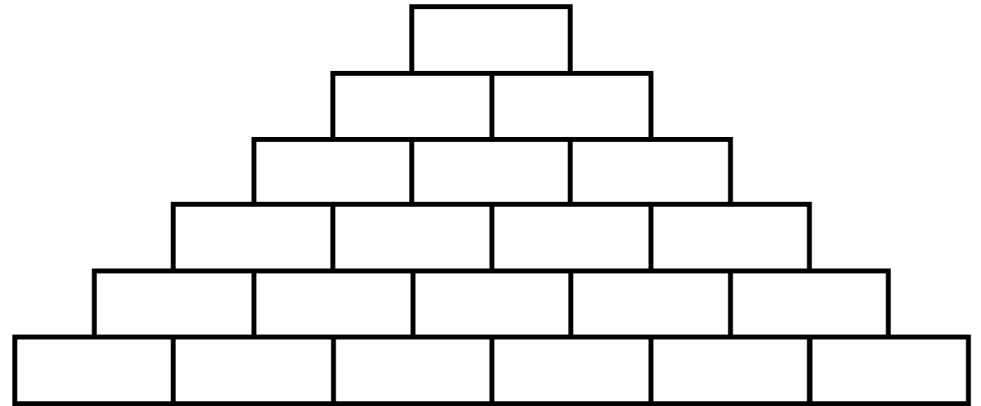
20.) $\sqrt{198} - \sqrt{1078} + \sqrt{352}$

Problem Solving

Pyramid II

Cut out the bricks below. Place the bricks in a pyramid shape so that each brick is the sum of the two bricks beneath it.

$7\sqrt{2} + \sqrt{147}$	$\sqrt{2} - \sqrt{75}$	$\sqrt{3}$
$2\sqrt{2} - \sqrt{432}$	$-5\sqrt{3}$	$\sqrt{3} + \sqrt{8}$
$-9\sqrt{3}$	$\sqrt{18} + \sqrt{48}$	$-3\sqrt{3} + \sqrt{722}$
$2\sqrt{2}$	$-\sqrt{2} - \sqrt{48}$	$12\sqrt{2} + 8\sqrt{3}$
$5\sqrt{2} + \sqrt{3}$	$2(\sqrt{3} + \sqrt{2})$	$-\sqrt{2} + \sqrt{3}$
$\sqrt{98} - \sqrt{363}$	$2\sqrt{3}$	$\sqrt{2} + \sqrt{12}$
$\sqrt{2}$	$\sqrt{8} - \sqrt{27}$	$\sqrt{27} + 4\sqrt{2}$

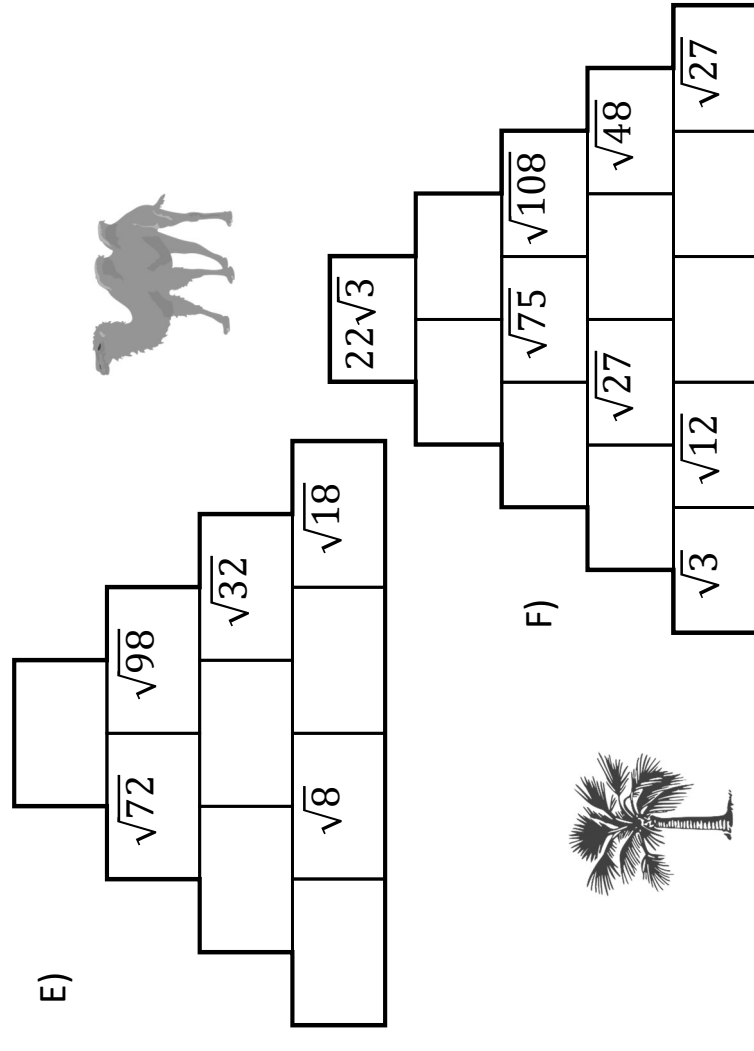
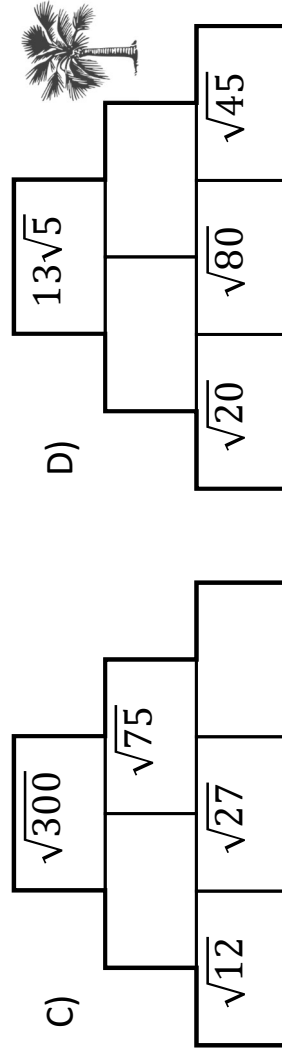
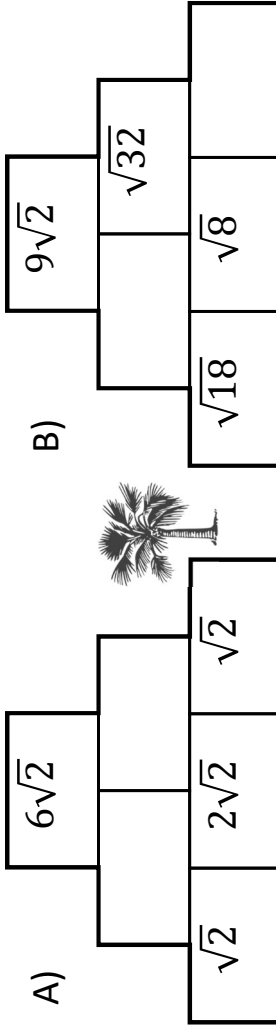


Fluency Practice

1

Surd Pyramids

Each brick is the two bricks below it added together.

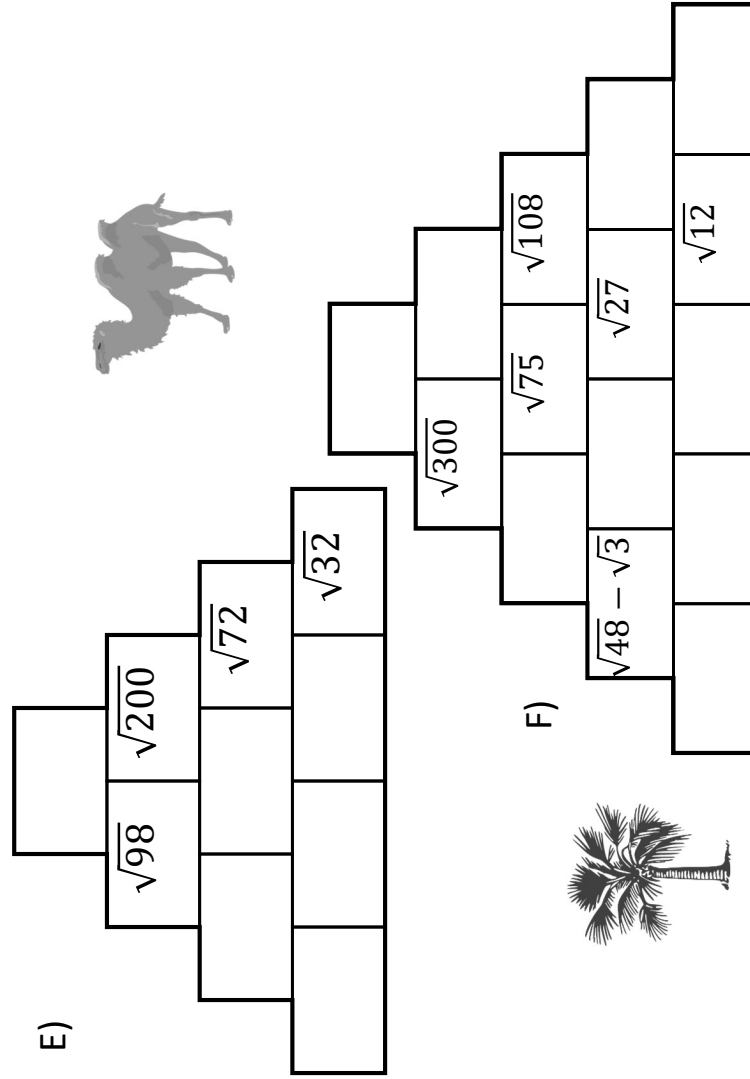
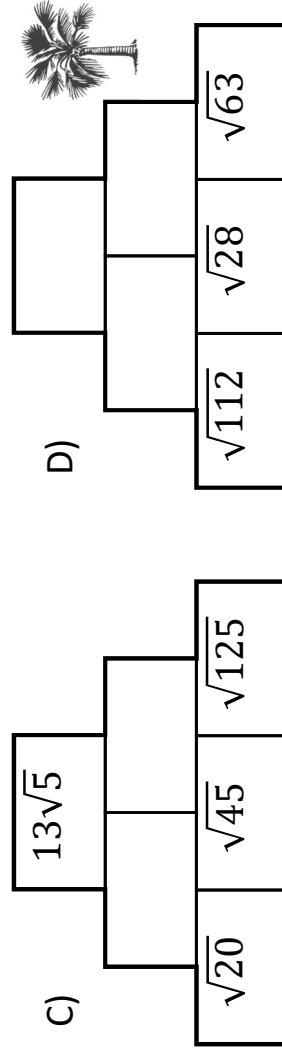
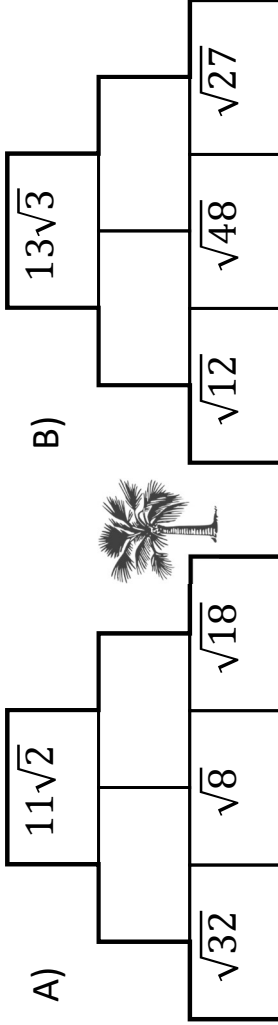


Fluency Practice

Surd Pyramids

2

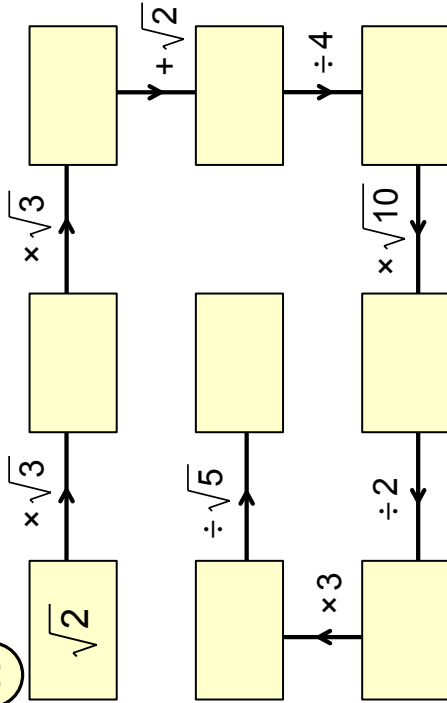
Each brick is the two bricks below it added together.



Problem Solving

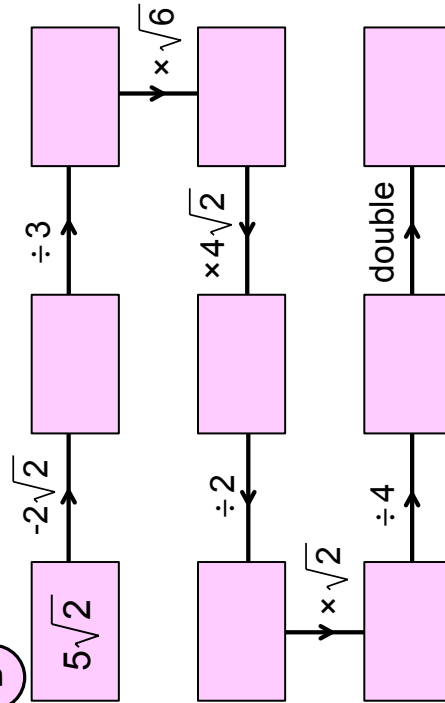
surds puzzles

A



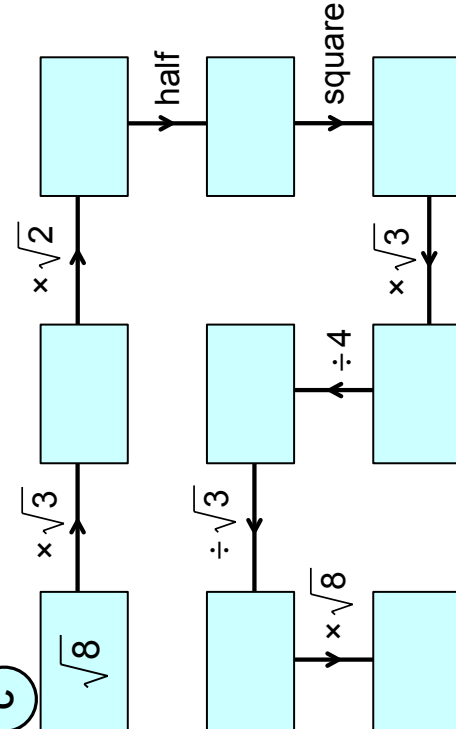
$\sqrt{5}$	$\sqrt{2}$
$4\sqrt{2}$	$\sqrt{6}$
$\sqrt{20}$	3
$3\sqrt{2}$	$3\sqrt{5}$

B



$8\sqrt{6}$	$3\sqrt{2}$
$2\sqrt{3}$	$4\sqrt{3}$
$\sqrt{2}$	$4\sqrt{6}$
$8\sqrt{3}$	$2\sqrt{3}$

C

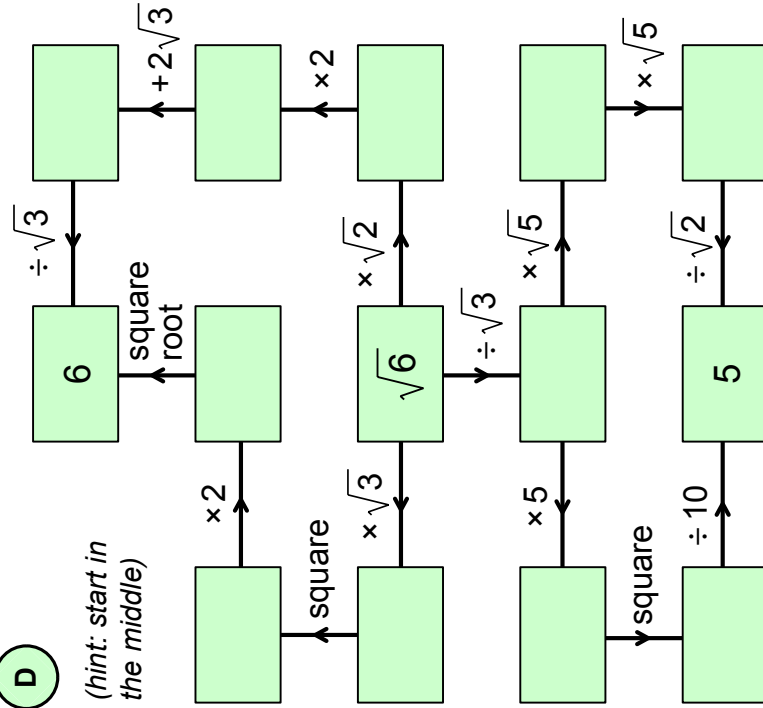


3	$2\sqrt{3}$
$2\sqrt{6}$	$12\sqrt{3}$
$3\sqrt{3}$	$4\sqrt{3}$
12	$6\sqrt{2}$

Problem Solving

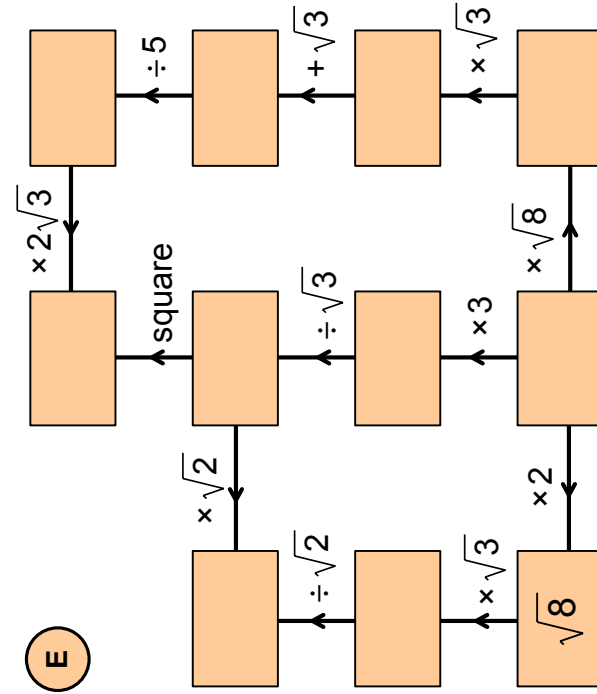
D

(hint: start in the middle)



$4\sqrt{3}$	5
50	$2\sqrt{3}$
$5\sqrt{2}$	36
$\sqrt{10}$	$\sqrt{2}$
$3\sqrt{2}$	$5\sqrt{2}$
$6\sqrt{3}$	18

E

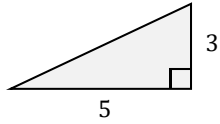


$\sqrt{6}$	$\sqrt{18}$
$2\sqrt{3}$	$\sqrt{3}$
6	$\sqrt{2}$
$4\sqrt{3}$	4
$5\sqrt{3}$	$2\sqrt{6}$

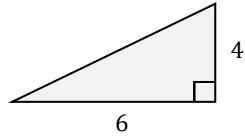
Fluency Practice

Find the missing lengths & simplify your answers.

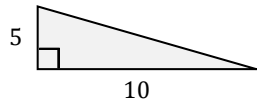
A



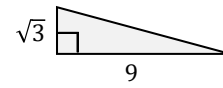
B



C

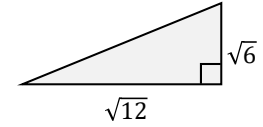


D

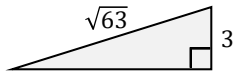


Diagrams not drawn accurately.

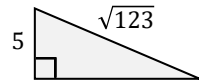
E



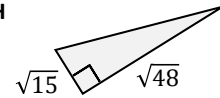
F



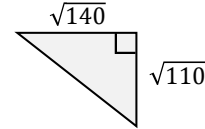
G



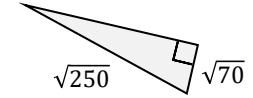
H



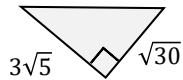
I



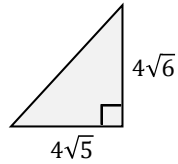
J



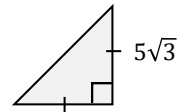
K



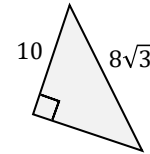
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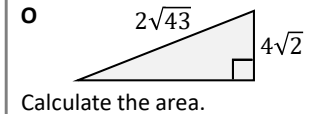
M



N



O



14 Answers	$3\sqrt{7}$	$3\sqrt{2}$	$5\sqrt{6}$	$\sqrt{34}$	$2\sqrt{23}$	$5\sqrt{5}$	$5\sqrt{3}$
	$2\sqrt{13}$	$4\sqrt{11}$	$2\sqrt{21}$	$5\sqrt{10}$	$7\sqrt{2}$	$6\sqrt{5}$	$3\sqrt{6}$

Averages with... Surds

For each set of numbers find: (i) the mean, (ii) the median, (iii) the range.

Leave your answers in their simplest form.

a) $5\sqrt{3}$ $4\sqrt{3}$ $6\sqrt{3}$

b) $\sqrt{3}$ $\sqrt{108}$ $\sqrt{12}$

c) $2\sqrt{12}$ $4\sqrt{12}$ $3\sqrt{27}$

d) $2\sqrt{27}$ $3\sqrt{12}$ $\sqrt{243}$

e) $2\sqrt{75}$ $\sqrt{48}$ $5\sqrt{12}$

f) $\sqrt{243}$ $5\sqrt{27}$ $3\sqrt{75}$

g) $\sqrt{20}$ $\sqrt{20}$ $\sqrt{80}$ $\sqrt{320}$

h) $\sqrt{28}$ $\sqrt{63}$ $3\sqrt{63}$ $4\sqrt{7}$

i) $-\sqrt{99}$ $2\sqrt{44}$ $-\sqrt{11}$ $4\sqrt{44}$

j) 0 $-2\sqrt{44}$ $-4\sqrt{44}$ $\sqrt{44}$

k) $\sqrt{2}$ $-\sqrt{18}$ $\sqrt{48}$ $\sqrt{8}$

l) $2\sqrt{150}$ $-2\sqrt{54}$ $2\sqrt{24}$ $-2\sqrt{96}$

Intelligent Practice

Expand and simplify:

1) $3(4 + \sqrt{5})$

10) $\sqrt{5}(4 - \sqrt{5})$

2) $3(\sqrt{5} + 4)$

11) $2\sqrt{5}(4 - \sqrt{5})$

3) $6(\sqrt{5} + 4)$

12) $\sqrt{20}(4 - \sqrt{5})$

4) $6(\sqrt{5} - 4)$

13) $\sqrt{20}(\sqrt{4} - \sqrt{5})$

5) $6(4 - \sqrt{5})$

14) $\sqrt{20}(\sqrt{6} - \sqrt{5})$

6) $-6(4 - \sqrt{5})$

15) $\sqrt{20}(\sqrt{6} - \sqrt{10})$

7) $-6(-4 - \sqrt{5})$

16) $\sqrt{20}(\sqrt{12} - \sqrt{10})$

8) $6(4 + \sqrt{5})$

17) $\sqrt{10}(\sqrt{6} - \sqrt{5})$

9) $\sqrt{5}(4 + \sqrt{5})$

18) $\sqrt{10}(\sqrt{6} - \sqrt{5} + \sqrt{24})$

Intelligent Practice

Expand and simplify:

1) $(\sqrt{5} + 2)(\sqrt{5} + 1)$

2) $(2 + \sqrt{5})(1 + \sqrt{5})$

3) $(2 - \sqrt{5})(1 + \sqrt{5})$

4) $(2 - \sqrt{5})(1 - \sqrt{5})$

5) $(2 + 2\sqrt{5})(1 + \sqrt{5})$

6) $(\sqrt{5} + 2)(\sqrt{5} - 1)$

7) $(\sqrt{7} + 2)(\sqrt{7} - 1)$

8) $(2\sqrt{7} + 2)(3\sqrt{7} - 1)$

9) $(2\sqrt{7} + 2)(3\sqrt{7} - 6)$

10) $(\sqrt{7} + 6)(\sqrt{7} - 6)$

11) $2(\sqrt{7} + 6)(\sqrt{7} - 6)$

12) $(\sqrt{7} - 6)(\sqrt{7} - 6)$

13) $(\sqrt{7} - 6)^2$

14) $(\sqrt{7} - 6)^3$

Intelligent Practice

Expand and simplify:

1) $(2 + \sqrt{3})(4 + \sqrt{12})$

10) $(2\sqrt{54} - 3)(\sqrt{6} - 5)$

2) $(2 + \sqrt{3})(4 - \sqrt{12})$

11) $(2\sqrt{54} - 3)(4\sqrt{6} - 5)$

3) $(2 - \sqrt{3})(4 + \sqrt{12})$

12) $(2\sqrt{44} - 3)(4\sqrt{11} - 5)$

4) $(2 - \sqrt{3})(4 - \sqrt{12})$

13) $(3\sqrt{44} - 2)(5\sqrt{11} - 4)$

5) $(\sqrt{3} - 2)(\sqrt{12} - 4)$

14) $(3\sqrt{44} - 2)^2$

6) $(\sqrt{12} - 2)(\sqrt{3} - 4)$

7) $(\sqrt{27} - 2)(\sqrt{3} - 4)$

8) $(\sqrt{27} - 3)(\sqrt{3} - 5)$

9) $(\sqrt{54} - 3)(\sqrt{6} - 5)$

Fluency Practice

① 5×5

② $5 \times \sqrt{5}$

③ $\sqrt{5} \times \sqrt{5}$

④ $5 \times 5\sqrt{5}$

⑤ $\sqrt{5} \times 5\sqrt{5}$

⑥ $5\sqrt{5} \times 5\sqrt{5}$

⑦ $5(5 + \sqrt{5})$

⑧ $\sqrt{5}(5 + \sqrt{5})$

⑨ $5(5 + 5\sqrt{5})$

⑩ $5\sqrt{5}(5 + 5\sqrt{5})$

⑪ $5\sqrt{5}(\sqrt{5} + 5\sqrt{5})$

⑫ $(5 + \sqrt{5})(5 + \sqrt{5})$

⑬ $(5 + \sqrt{5})(5 - \sqrt{5})$

⑭ $(5 + \sqrt{5})(5 + 5\sqrt{5})$

⑮ $(5 + 5\sqrt{5})(5 + 5\sqrt{5})$

⑯ $(\sqrt{5} + 5\sqrt{5})(\sqrt{5} + 5\sqrt{5})$

Fluency Practice

(a)	(b)	(c)	(d)
Simplify $2\sqrt{5} + 3\sqrt{5}$	Simplify $6\sqrt{5} + 5\sqrt{5}$	Simplify $2\sqrt{5} - 3\sqrt{5}$	Simplify $7\sqrt{3} - 3\sqrt{3}$
(e)	(f)	(g)	(h)
Simplify $2\sqrt{3} + 3\sqrt{2}$	Simplify $7\sqrt{10} + 3\sqrt{10} - \sqrt{10}$	Expand $3(6 + \sqrt{5})$	Expand $\sqrt{2}(7 - \sqrt{3})$
(i)	(j)	(k)	(l)
Expand $5\sqrt{2}(3 - \sqrt{7})$	Expand and simplify $(3 + \sqrt{2})(6 + \sqrt{2})$	Expand and simplify $(3 + \sqrt{7})(5 - \sqrt{7})$	Expand and simplify $(4 + 3\sqrt{5})(2 - \sqrt{5})$

Fluency Practice

Simplify

(a) $4\sqrt{3} + 2\sqrt{3}$ (b) $4\sqrt{3} - 2\sqrt{3}$

(c) $2\sqrt{3} - 4\sqrt{3}$ (d) $2\sqrt{3} - 4\sqrt{5}$

(e) $-4\sqrt{2} + 2\sqrt{2}$ (f) $\sqrt{2} - 2\sqrt{2}$

(g) $6\sqrt{5} + 2\sqrt{5} - 3\sqrt{5}$

(h) $\sqrt{3} - 2\sqrt{3} + 7\sqrt{3}$

Expand and simplify where possible

(a) $5(2 + \sqrt{3})$ (b) $\sqrt{5}(2 + \sqrt{3})$

(c) $\sqrt{5}(\sqrt{2} - \sqrt{3})$ (d) $5(\sqrt{2} - \sqrt{3})$

(e) $\sqrt{3}(\sqrt{3} - 7)$ (f) $\sqrt{3}(2 + \sqrt{3})$

(g) $5\sqrt{2}(2 + \sqrt{3})$ (h) $\sqrt{5}(2\sqrt{3} + \sqrt{5})$

(i) $\sqrt{5}(\sqrt{2} + 2\sqrt{3})$ (j) $3\sqrt{5}(2\sqrt{2} + 3\sqrt{3})$

Expand and simplify where possible

(a) $(2 + \sqrt{3})(1 + \sqrt{3})$

(b) $(2 + \sqrt{3})(4 + \sqrt{3})$

(c) $(5 - \sqrt{5})(4 + \sqrt{5})$

(d) $(2 + \sqrt{5})(4 - \sqrt{5})$

(e) $(1 + 2\sqrt{3})(4 - \sqrt{3})$

(f) $(2 + 3\sqrt{5})(4 - 2\sqrt{5})$

Calculate the areas of these shapes, giving answers in their simplest form

(a) A square with side length $2 + \sqrt{5}$ cm

(b) A rectangle with length $\sqrt{7}$ cm and width $1 + \sqrt{3}$ cm

(c) A triangle with base $\sqrt{8}$ cm and height $2\sqrt{8}$ cm

Fluency Practice

(a) Show that $(5 - \sqrt{12})(2 + \sqrt{3})$ can be written in the form $a + \sqrt{b}$ where a and b are integers.

(b) Show that $(6 + \sqrt{2})(\sqrt{8} - 4)$ can be written in the form $a\sqrt{2} + b$ where a and b are integers.

(c) Show that $\sqrt{3}(\sqrt{12} - 2)^2$ can be written in the form $a\sqrt{3} + b$ where a and b are integers.

(d) Show that $(5 + \sqrt{5})(\sqrt{20} - 2)(3 - \sqrt{5})$ can be written in the form $a\sqrt{5} + b$ where a and b are integers.

(e) Show that $(1 + \sqrt{8})^3$ can be written in the form $p + \sqrt{q}$ where p and q are integers.

(f) Given that

$$(a + \sqrt{6})(10 - \sqrt{6}) = 24 + b\sqrt{6}$$

find the values of the integers a and b .

(g) Given that

$$(5 - \sqrt{8})(3 - a\sqrt{2}) = b - 21\sqrt{2}$$

find the values of the integers a and b .

(h) Given that

$$(p + 2\sqrt{q})^2 = 40 + 16\sqrt{q}$$

find the values of the integers p and q .

(i) Express $(6 + \sqrt{3})(a - 2\sqrt{3})(4 - \sqrt{12})$

in the form $b - 76\sqrt{3}$ where a and b are integers to be found.

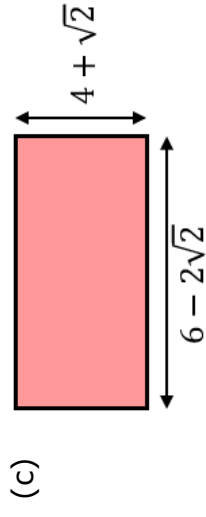
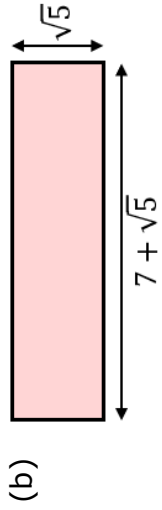
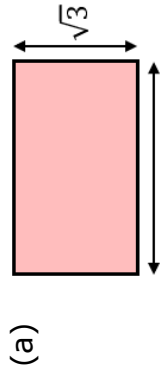
(j) Given that

$$(p + \sqrt{q})(p - 3\sqrt{q}) = 13 - 14\sqrt{q}$$

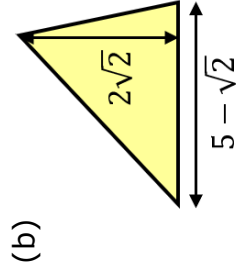
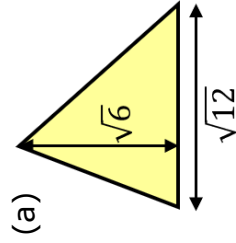
find the values of the integers p and q .

Fluency Practice

Find the area and perimeter of these rectangles.



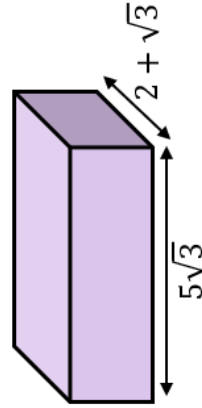
Find the area of these triangles.



The perimeter of this rectangle is $8\sqrt{2}$.
Find the area of the rectangle.



(a) The volume of the cuboid shown is $15 + 5\sqrt{3}$. Find the height of the cuboid.

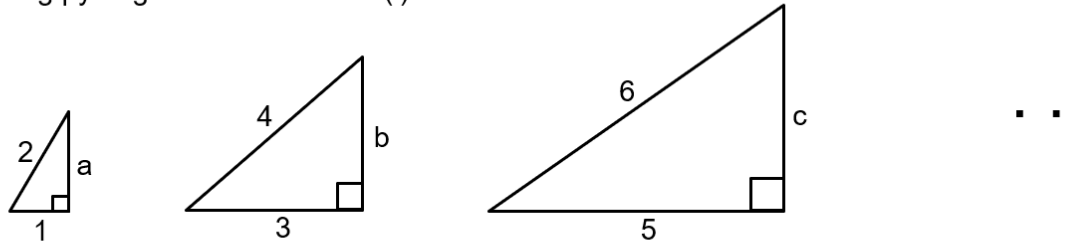


(b) The volume of a cylinder is $40\pi + 20\pi\sqrt{2}$. If the height of the cylinder is $2 + \sqrt{2}$, find the exact radius of the cylinder.

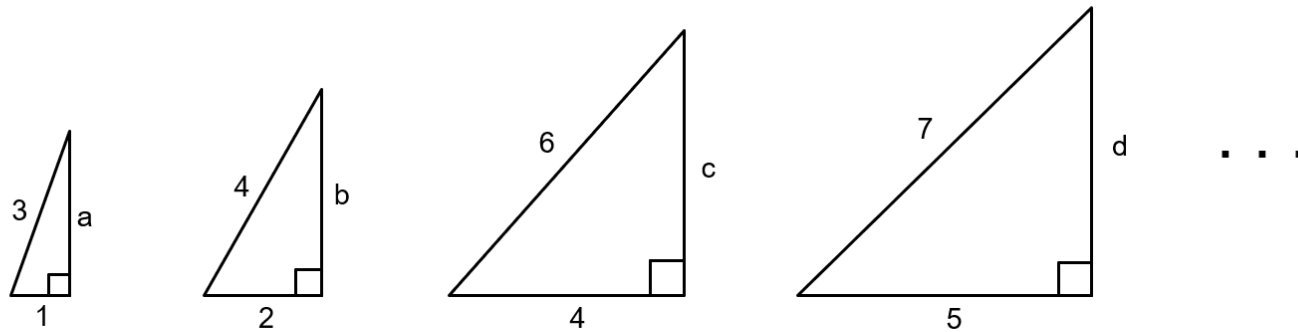
Fluency Practice

generalising pythagoras surd families (i)

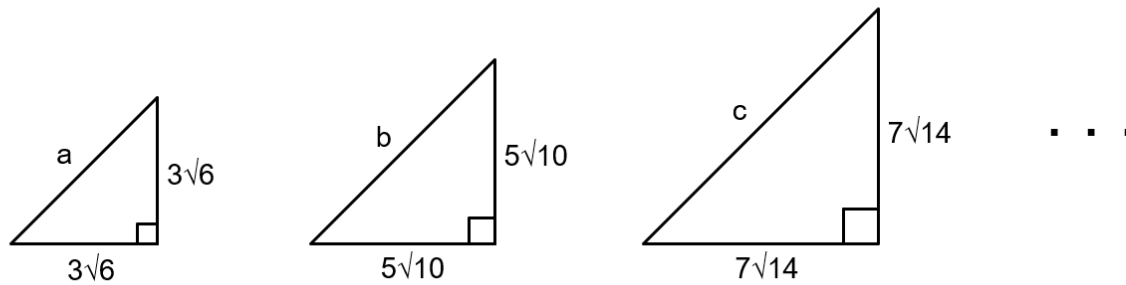
(1)



(2)



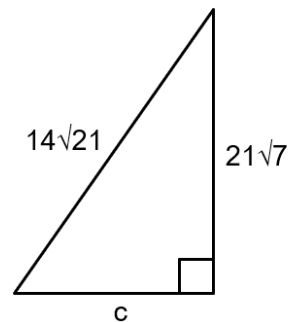
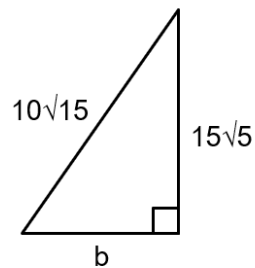
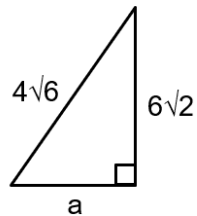
(3)



Fluency Practice

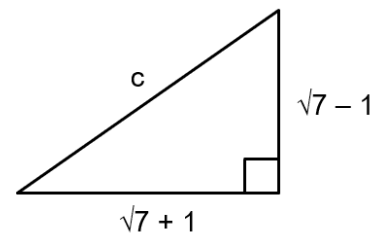
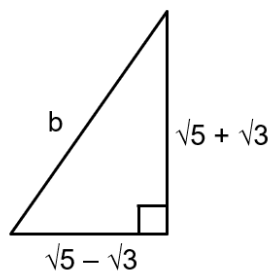
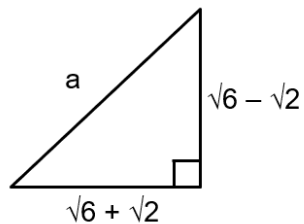
generalising pythagoras surd families (ii)

(1)



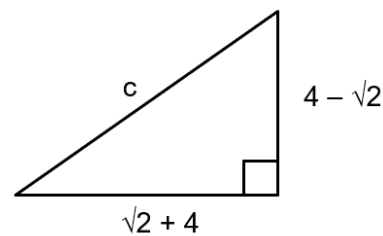
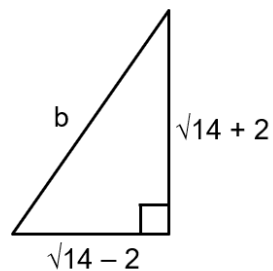
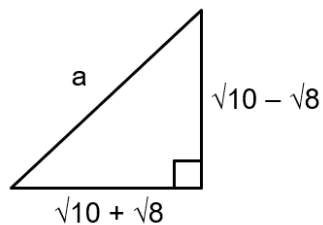
...

(2)



...

(3)

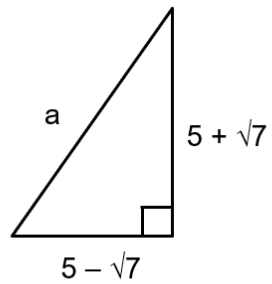


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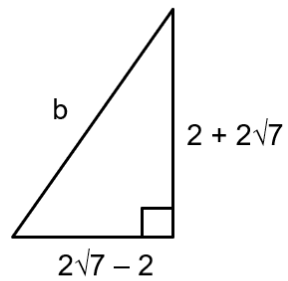
Fluency Practice

find the missing length

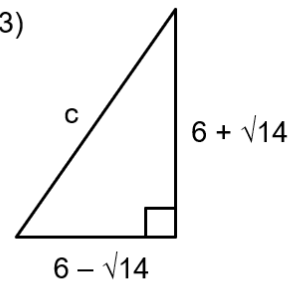
(1)



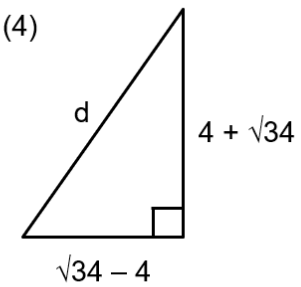
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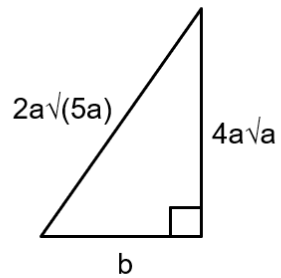
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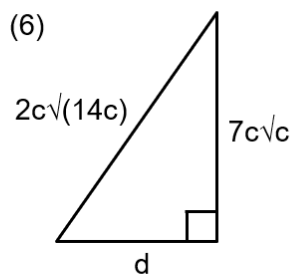
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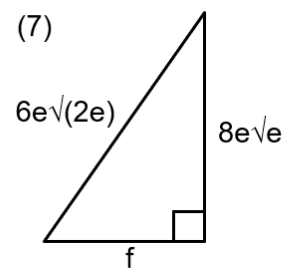
(5)



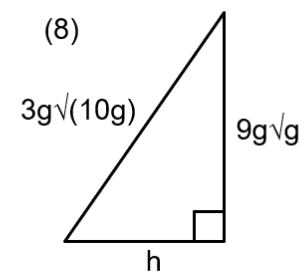
(6)



(7)



(8)



Problem Solving

making numbers by multiplying two surds

no calculator please

(A) (1) $(3 + \sqrt{7})(3 - \sqrt{7}) = ?$

(2) $(a + \sqrt{b})(a - \sqrt{b}) = 3$

(3) $(a + \sqrt{b})(a - \sqrt{b}) = 1$

(4) $(a + b\sqrt{c})(a - b\sqrt{c}) = 4$

(5) $(a + b\sqrt{c})(a - b\sqrt{c}) = 5$

b and c
cannot be
1 or 0

can you make the other
digits in this way?

in more than one way?

(B) (1) $(3 + b\sqrt{c})(3 - b\sqrt{c}) = 1$

(2) $(8 + b\sqrt{c})(8 - b\sqrt{c}) = 1$

(3) $(a + b\sqrt{3})(a - b\sqrt{3}) = 1$

(4) $(a + b\sqrt{5})(a - b\sqrt{5}) = 1$ b cannot be 0

(5) $(a + b\sqrt{11})(a - b\sqrt{11}) = 1$

can you make 1 in at
least three other ways?

(C) (1) $(b + \sqrt{b})(b - \sqrt{b}) = 6$

(2) $(a + \sqrt{a})(a - \sqrt{a}) = 42$

(3) $\phantom{(a + \sqrt{a})(a - \sqrt{a})} = 90$

(4) $\phantom{(a + \sqrt{a})(a - \sqrt{a})} = 110$

(5) $\phantom{(a + \sqrt{a})(a - \sqrt{a})} = 420$

(D) (1) $(a\sqrt{b} + \sqrt{c})(a\sqrt{b} - \sqrt{c}) = 5$

(2) $(a\sqrt{b} + \sqrt{c})(a\sqrt{b} - \sqrt{c}) = 13$

(3) $(a\sqrt{b} + \sqrt{c})(a\sqrt{b} - \sqrt{c}) = 29$

(4) $(a\sqrt{b} + \sqrt{c})(a\sqrt{b} - \sqrt{c}) = 38$

(5) $(a\sqrt{b} + \sqrt{c})(a\sqrt{b} - \sqrt{c}) = 43$

Intelligent Practice

Rationalise:

1) $\frac{5}{\sqrt{2}}$

10) $\frac{5}{10\sqrt{5}}$

2) $\frac{6}{\sqrt{2}}$

11) $\frac{5}{\sqrt{5}}$

3) $\frac{6}{\sqrt{3}}$

12) $\frac{8}{\sqrt{8}}$

4) $\frac{4}{\sqrt{3}}$

13) $\frac{10}{\sqrt{8}}$

5) $\frac{4}{\sqrt{6}}$

14) $\frac{10}{\sqrt{12}}$

6) $\frac{5}{\sqrt{6}}$

15) $\frac{10}{2\sqrt{3}}$

7) $\frac{5}{2\sqrt{6}}$

16) $\frac{10+\sqrt{3}}{2\sqrt{3}}$

8) $\frac{5}{3\sqrt{6}}$

17) $\frac{10-\sqrt{3}}{2\sqrt{3}}$

9) $\frac{5}{10\sqrt{6}}$

18) $\frac{12-\sqrt{3}}{2\sqrt{3}}$

Fluency Practice

Simplify each fraction, leaving the denominator as a rational number. Any surds in the numerator should also be fully simplified.

$\frac{2}{\sqrt{2}} =$	$\frac{3}{\sqrt{2}} =$	$\frac{4}{\sqrt{2}} =$	$\frac{5}{\sqrt{2}} =$	$\frac{6}{\sqrt{2}} =$	$\frac{7}{\sqrt{2}} =$	$\frac{8}{\sqrt{2}} =$
$\frac{\sqrt{2}}{\sqrt{2}} =$	$\frac{\sqrt{3}}{\sqrt{2}} =$	$\frac{\sqrt{4}}{\sqrt{2}} =$	$\frac{\sqrt{5}}{\sqrt{2}} =$	$\frac{\sqrt{6}}{\sqrt{2}} =$	$\frac{\sqrt{7}}{\sqrt{2}} =$	$\frac{\sqrt{8}}{\sqrt{2}} =$
$\frac{2}{2\sqrt{2}} =$	$\frac{3}{2\sqrt{2}} =$	$\frac{4}{2\sqrt{2}} =$	$\frac{5}{2\sqrt{2}} =$	$\frac{6}{2\sqrt{2}} =$	$\frac{7}{2\sqrt{2}} =$	$\frac{8}{2\sqrt{2}} =$
$\frac{2}{\sqrt{8}} =$	$\frac{3}{\sqrt{8}} =$	$\frac{4}{\sqrt{8}} =$	$\frac{5}{\sqrt{8}} =$	$\frac{6}{\sqrt{8}} =$	$\frac{7}{\sqrt{8}} =$	$\frac{8}{\sqrt{8}} =$
$\frac{2\sqrt{6}}{2\sqrt{2}} =$	$\frac{3\sqrt{6}}{2\sqrt{2}} =$	$\frac{4\sqrt{6}}{2\sqrt{2}} =$	$\frac{5\sqrt{6}}{2\sqrt{2}} =$	$\frac{6\sqrt{6}}{2\sqrt{2}} =$	$\frac{7\sqrt{6}}{2\sqrt{2}} =$	$\frac{8\sqrt{6}}{2\sqrt{2}} =$
$\frac{2\sqrt{6}}{2\sqrt{3}} =$	$\frac{3\sqrt{6}}{2\sqrt{3}} =$	$\frac{4\sqrt{6}}{2\sqrt{3}} =$	$\frac{5\sqrt{6}}{2\sqrt{3}} =$	$\frac{6\sqrt{6}}{2\sqrt{3}} =$	$\frac{7\sqrt{6}}{2\sqrt{3}} =$	$\frac{8\sqrt{6}}{2\sqrt{3}} =$

Fluency Practice

Express each fraction below as either:

- a rational number
- a fraction with a rational denominator, or
- a surd in simplest form

$$\frac{\sqrt{8}}{\sqrt{1}} \quad \frac{\sqrt{8}}{\sqrt{2}} \quad \frac{\sqrt{8}}{\sqrt{3}} \quad \frac{\sqrt{8}}{\sqrt{4}} \quad \frac{\sqrt{8}}{\sqrt{5}} \quad \frac{\sqrt{8}}{\sqrt{6}} \quad \frac{\sqrt{8}}{\sqrt{7}} \quad \frac{\sqrt{8}}{\sqrt{8}} \quad \frac{\sqrt{8}}{\sqrt{9}} \quad \frac{\sqrt{8}}{\sqrt{10}} \quad \frac{\sqrt{8}}{\sqrt{11}} \quad \frac{\sqrt{8}}{\sqrt{12}}$$

Intelligent Practice

Rationalise:

1) $\frac{2}{\sqrt{2}+3}$

2) $\frac{2}{3+\sqrt{2}}$

3) $\frac{2}{3-\sqrt{2}}$

4) $\frac{7}{3-\sqrt{2}}$

5) $\frac{7}{\sqrt{2}-3}$

6) $\frac{7}{\sqrt{7}-3}$

7) $\frac{\sqrt{7}}{\sqrt{7}-3}$

8) $\frac{\sqrt{5}}{\sqrt{5}-3}$

9) $\frac{\sqrt{5}}{5-\sqrt{3}}$

10) $\frac{2\sqrt{5}}{5-\sqrt{3}}$

11) $\frac{4\sqrt{5}}{5-\sqrt{3}}$

12) $\frac{4\sqrt{5}}{5-2\sqrt{3}}$

13) $\frac{4\sqrt{5}}{5-3\sqrt{3}}$

14) $\frac{4\sqrt{3}}{5-3\sqrt{3}}$

Fluency Practice

Rationalise the denominator.

(a) $\frac{1}{\sqrt{5}}$

(b) $\frac{2}{\sqrt{5}}$

(c) $\frac{2}{\sqrt{7}}$

(d) $\frac{\sqrt{2}}{\sqrt{7}}$

(e) $\frac{2\sqrt{2}}{\sqrt{5}}$

(f) $\frac{2}{3\sqrt{5}}$

Rationalise the denominator.

(a) $\frac{6\sqrt{2}}{\sqrt{6}}$

(b) $\frac{5\sqrt{2}}{\sqrt{10}}$

(c) $\frac{5\sqrt{3}}{\sqrt{6}}$

(d) $\frac{2\sqrt{6}}{\sqrt{15}}$

Rationalise the denominator.

(a) $\frac{1+\sqrt{2}}{\sqrt{5}}$

(b) $\frac{2-\sqrt{2}}{\sqrt{5}}$

(c) $\frac{2-3\sqrt{2}}{\sqrt{7}}$

(d) $\frac{5+\sqrt{5}}{\sqrt{3}}$

(e) $\frac{10-\sqrt{2}}{\sqrt{3}}$

(f) $\frac{2+\sqrt{3}}{3\sqrt{5}}$

Rationalise the denominator.

(a) $\frac{1}{2+\sqrt{5}}$

(b) $\frac{2}{2-\sqrt{5}}$

(c) $\frac{\sqrt{2}}{3+\sqrt{7}}$

(d) $\frac{2\sqrt{5}}{3-\sqrt{7}}$

(e) $\frac{10\sqrt{2}}{5+\sqrt{3}}$

(f) $\frac{2+\sqrt{3}}{3-\sqrt{5}}$

(g) $\frac{2+\sqrt{3}}{5+\sqrt{3}}$

(h) $\frac{2+3\sqrt{3}}{3-4\sqrt{5}}$

Fluency Practice

surds: rationalise the denominators

a) simple rationalisation

$$(1) \quad \frac{12}{\sqrt{6}}$$

$$(2) \quad \frac{\sqrt{24}}{\sqrt{6}}$$

$$(3) \quad \frac{10\sqrt{2}}{\sqrt{5}}$$

$$(4) \quad \frac{15}{5\sqrt{3}}$$

$$(5) \quad \frac{6\sqrt{3}}{\sqrt{12}}$$

$$(6) \quad \frac{\sqrt{8}}{\sqrt{72}}$$

b) harder rationalisation

$$(1) \quad \frac{4}{3 + \sqrt{5}}$$

$$(2) \quad \frac{8}{1 + \sqrt{3}}$$

$$(3) \quad \frac{12}{6 + \sqrt{12}}$$

$$(4) \quad \frac{5}{2 - \sqrt{3}}$$

$$(5) \quad \frac{3}{\sqrt{5} - 2}$$

$$(6) \quad \frac{7}{3 - \sqrt{2}}$$

Fluency Practice

rationalise the denominators

$$(1) \quad \frac{3 - \sqrt{3}}{2 - \sqrt{3}}$$

$$(2) \quad \frac{5 + \sqrt{7}}{3 + \sqrt{7}}$$

$$(3) \quad \frac{5 + \sqrt{7}}{4 - \sqrt{7}}$$

$$(4) \quad \frac{7 - \sqrt{5}}{3 - \sqrt{5}}$$

$$(5) \quad \frac{9 - \sqrt{11}}{4 - \sqrt{11}}$$

$$(6) \quad \frac{\sqrt{13} + 1}{\sqrt{13} - 3}$$

$$(7) \quad \frac{4 + \sqrt{10}}{\sqrt{10} - 2}$$

$$(8) \quad \frac{7 - 3\sqrt{5}}{3 + \sqrt{5}}$$

$$(9) \quad \frac{5 + 3\sqrt{5}}{5 - 2\sqrt{5}}$$

$$(10) \quad \frac{1 + 4\sqrt{11}}{2 + \sqrt{11}}$$

$$(11) \quad \frac{8 + 4\sqrt{10}}{\sqrt{10} + 2}$$

$$(12) \quad \frac{\sqrt{3} + 2}{\sqrt{12} + 4}$$

Fluency Practice

simplifying surds

rationalising the denominator

Write each expression in the form $a + b\sqrt{2}$, where $a, b \in \mathbb{Q}$.

$\frac{2 + \sqrt{2}}{4}$	
$\frac{2 + \sqrt{8}}{4}$	
$\frac{2 + 2\sqrt{8}}{4}$	
$\frac{2(2 + \sqrt{200})}{8}$	
$\frac{4 + \sqrt{32}}{4}$	
$\frac{(2 + \sqrt{2})^2}{4}$	
$\frac{\sqrt{50}}{20}$	
$\frac{1}{4\sqrt{2}}$	
$\frac{1}{1 + \sqrt{2}}$	
$\frac{1}{2 - \sqrt{2}}$	
$\frac{4}{4 - \sqrt{8}}$	
$\frac{4 - \sqrt{2}}{\sqrt{2} - 1}$	

$\frac{1}{2} + \frac{5}{2}\sqrt{2}$	$\sqrt{2} - 1$
$2 + 3\sqrt{2}$	$\frac{1}{2} + \frac{1}{2}\sqrt{2}$
$\frac{1}{2} + \sqrt{2}$	$1 + \sqrt{2}$
$\frac{1}{4}\sqrt{2}$	$\sqrt{2} - 2$
$1 + \frac{1}{2}\sqrt{2}$	$2 + \sqrt{2}$
$\frac{1}{8}\sqrt{2}$	$\frac{3}{2} + \sqrt{2}$
1	$\frac{1}{2} + \frac{1}{4}\sqrt{2}$

$\frac{\sqrt{50} - \sqrt{18}}{\sqrt{8}}$	
$\frac{\sqrt{8} - \sqrt{18}}{1 + \sqrt{2}}$	

Fluency Practice

Yay or Nay?

Each fraction has been standardised by
rationalising the denominator.

Which have been completed incorrectly?

a) $\frac{1}{4 + \sqrt{2}} \times \frac{4 - \sqrt{2}}{4 - \sqrt{2}} = \frac{1(4 - \sqrt{2})}{16 + 4\sqrt{2} - 4\sqrt{2} - 2} = \frac{4 - \sqrt{2}}{14}$

b) $\frac{2}{3 + \sqrt{3}} \times \frac{3 - \sqrt{3}}{3 - \sqrt{3}} = \frac{2(3 - \sqrt{3})}{9 + 3\sqrt{3} - 3\sqrt{3} - 3} = \frac{6 - \sqrt{6}}{6}$

c) $\frac{3}{6 - \sqrt{5}} \times \frac{6 + \sqrt{5}}{6 + \sqrt{5}} = \frac{3(6 + \sqrt{5})}{36 - 6\sqrt{5} + 6\sqrt{5} - 5} = \frac{18 - 3\sqrt{5}}{30}$

d) $\frac{\sqrt{5}}{2 - \sqrt{2}} \times \frac{2 + \sqrt{2}}{2 + \sqrt{2}} = \frac{\sqrt{5}(2 + \sqrt{2})}{4 - 2\sqrt{2} + 2\sqrt{2} - 2} = \frac{2\sqrt{5} + \sqrt{10}}{2}$

e) $\frac{\sqrt{3}}{4 + \sqrt{3}} \times \frac{4 - \sqrt{3}}{4 - \sqrt{3}} = \frac{\sqrt{3}(4 - \sqrt{3})}{16 + 4\sqrt{3} - 4\sqrt{3} - 3} = \frac{3\sqrt{4} - 3}{13}$

f) $\frac{\sqrt{3}}{5 + \sqrt{6}} \times \frac{5 - \sqrt{6}}{5 - \sqrt{6}} = \frac{\sqrt{3}(5 - \sqrt{6})}{25 - 5\sqrt{6} + 5\sqrt{6} - 6} = \frac{5\sqrt{3} - \sqrt{18}}{19}$

g) $\frac{\sqrt{7}}{6 - \sqrt{7}} \times \frac{6 + \sqrt{7}}{6 + \sqrt{7}} = \frac{\sqrt{7}(6 + \sqrt{7})}{36 - 6\sqrt{7} + 6\sqrt{7} - 7} = \frac{6\sqrt{7} + 7}{29}$

h) $\frac{2 + \sqrt{2}}{3 + \sqrt{2}} \times \frac{3 - \sqrt{2}}{3 - \sqrt{2}} = \frac{(2 + \sqrt{2})(3 - \sqrt{2})}{9 + 3\sqrt{2} - 3\sqrt{2} - 2} = \frac{4 + \sqrt{2}}{7}$

i) $\frac{4 - \sqrt{5}}{3 - \sqrt{5}} \times \frac{3 + \sqrt{5}}{3 + \sqrt{5}} = \frac{(4 - \sqrt{5})(3 + \sqrt{5})}{9 - 3\sqrt{5} + 3\sqrt{5} - 5} = \frac{7 - \sqrt{5}}{4}$

Fluency Practice

Rationalising the Denominator

To rationalise these denominators we will multiply by one term.

A) $\frac{2 + \sqrt{3}}{\sqrt{3}}$

B) $\frac{5 + \sqrt{2}}{\sqrt{2}}$

C) $\frac{\sqrt{5} + 3}{\sqrt{5}}$

D) $\frac{6 + \sqrt{2}}{\sqrt{3}}$

E) $\frac{\sqrt{5} + 8}{\sqrt{2}}$

F) $\frac{\sqrt{3} - 2}{\sqrt{3}}$

G) $\frac{3 + 2\sqrt{5}}{\sqrt{5}}$

H) $\frac{\sqrt{10} + 3}{\sqrt{2}}$

I) $\frac{3\sqrt{6} - 5}{\sqrt{3}}$

J) $\frac{4 - 3\sqrt{14}}{\sqrt{2}}$

To rationalise these denominators we will multiply by two terms within a bracket.

K) $\frac{5}{\sqrt{2} + 3}$

L) $\frac{8}{1 + \sqrt{3}}$

M) $\frac{1}{\sqrt{5} - 2}$

N) $\frac{\sqrt{2}}{-3 + \sqrt{2}}$

O) $\frac{2\sqrt{3}}{\sqrt{5} - 1}$

P) $\frac{5\sqrt{2}}{2\sqrt{5} - 3}$

Q) $\frac{3\sqrt{7}}{5\sqrt{3} - 4}$

R) $\frac{\sqrt{3} + 2}{\sqrt{3} - 1}$

S) $\frac{\sqrt{5} + 2\sqrt{3}}{4\sqrt{2} + 3}$

T) $\frac{5\sqrt{5} - 5}{-1 + 2\sqrt{3}}$

U) $\frac{5}{\sqrt{2}} + \frac{2}{3}$

V) $\frac{3}{4} + \frac{5}{\sqrt{3}}$

W) $\frac{\sqrt{2}}{\sqrt{5}} - \frac{3\sqrt{5}}{2}$

X) Work out the missing length. ?

$\sqrt{3} - 1$

Area
= $2\sqrt{5} + 5\sqrt{3}$

Fluency Practice

(a) Show that $\frac{5+\sqrt{3}}{2-\sqrt{3}}$ can be written in the form $a + b\sqrt{3}$, where a and b are integers to be found.

(b) Show that $\frac{8-\sqrt{2}}{2+\sqrt{2}}$ can be written in the form $a + b\sqrt{2}$, where a and b are integers to be found.

(c) Show that $\frac{4\sqrt{2}-1}{3+\sqrt{8}}$ can be written in the form $a\sqrt{2} + b$, where a and b are integers to be found.

(d) Show that $\frac{2+\sqrt{20}}{3-\sqrt{5}} \times 3\sqrt{5}$ can be written in the form $a + b\sqrt{5}$, where a and b are integers to be found.

(e) Show that $\frac{(2+2\sqrt{7})^2}{3-\sqrt{7}}$ can be written in the form $a + b\sqrt{7}$, where a and b are integers to be found.

(f) Show that $\frac{\sqrt{8}(4-3\sqrt{2})}{\sqrt{2}+1} + 5(4-\sqrt{2})$ can be written in the form $a + b\sqrt{2}$, where a and b are integers to be found.

(g) Given that $\frac{a+\sqrt{12}}{2-\sqrt{3}} = b + 10\sqrt{3}$ find the values of a and b .

(h) Given that $\frac{a}{(1+\sqrt{3})^2} + (a\sqrt{3})^3 = 2 + b\sqrt{3}$ find the values of a and b .

Fluency Practice

(a)	(b)	(c)	(d)
Write $\sqrt{108}$ in the form $k\sqrt{3}$	Write $\sqrt{45} + \sqrt{20}$ in the form $k\sqrt{5}$	Write $\sqrt{96} - \sqrt{24}$ in the form $k\sqrt{6}$	Expand $\sqrt{2}(5 + \sqrt{8})$
(e)	(f)	(g)	(h)
Expand and simplify $(7 + \sqrt{3})(4 - \sqrt{3})$	Expand and simplify $(5 + 2\sqrt{2})(6 - \sqrt{2})$	Expand and simplify $(5 + 3\sqrt{2})^2$	Rationalise the denominator and simplify fully $\frac{15}{\sqrt{18}}$
(i)	(j)	(k)	(l)
Rationalise the denominator and simplify fully $\frac{5 + 4\sqrt{3}}{\sqrt{3}}$	Express $\frac{\sqrt{3} + \sqrt{27}}{\sqrt{2}}$ as a single surd.	Rationalise the denominator and simplify fully $\frac{\sqrt{3} + 5}{2 - \sqrt{3}}$	$(4 + \sqrt{a})(7 - \sqrt{a}) = 23 + k\sqrt{a}$ Find the values of the positive integers a and k .

Fluency Practice

A1 Express $\sqrt{48}$ in the form $a\sqrt{b}$ where b is a prime number.	A2 Express $4\sqrt{5}$ in the form $\sqrt{a}$ where a is an integer.	A3 Express $\sqrt{175} - \sqrt{63}$ in the form $a\sqrt{7}$ where a is an integer.	A4 Express $\sqrt{75} + \sqrt{27}$ in the form $a\sqrt{b}$ where a and b are integers
B1 Expand $(3 + \sqrt{5})(2 + \sqrt{5})$ Give your answer in the form $a + b\sqrt{5}$ where a and b are integers	B2 Expand $(2 + 3\sqrt{7})^2$ Give your answer in the form $a + b\sqrt{7}$ where a and b are integers	B3 Show that $(4 + 2\sqrt{3})(5 - \sqrt{3}) = 14 + 6\sqrt{3}$ Show clear working out.	B4 Show that $(2 + \sqrt{8})(9 - 3\sqrt{2}) = 6 + 12\sqrt{2}$ Show clear working out.
C1 Show that $\frac{12}{\sqrt{45}} = \frac{4\sqrt{5}}{5}$ Show clear working out.	C2 Rationalise the denominator and simplify fully $\frac{9 + 2\sqrt{3}}{\sqrt{3}}$	C3 Simplify $\frac{2 + \sqrt{20}}{\sqrt{5} - 1}$ giving your answer in the form $a + b\sqrt{5}$	C4 A rectangle has an area of $\sqrt{80} \text{ cm}^2$ and a width of $1 + \sqrt{5} \text{ cm}$. Calculate the exact height of the rectangle, leaving your answer as a simplified surd.
D1 Given that $(a + \sqrt{b})^2 = 49 + 12\sqrt{b}$ Find the value of a and the value of b .	D2 Given that $(1 + \sqrt{e})(3 + \sqrt{e}) = f + 4\sqrt{5}$ Find the value of e and the value of f .	D3 Given that $(5 + 3\sqrt{2})^2 = p + \frac{q}{\sqrt{8}}$ Find the value of p and the value of q .	D4 Given that $(\sqrt{x} + \sqrt{8x})^2 = 54 + y\sqrt{2}$ Find the value of x and the value of y .

Fluency Practice

Question 1: Work out each of the following

- (a) $\sqrt{3} \times \sqrt{5}$ (b) $\sqrt{7} \times \sqrt{2}$ (c) $\sqrt{11} \times \sqrt{6}$ (d) $\sqrt{2} \times \sqrt{3}$
- (e) $\sqrt{8} \times \sqrt{2}$ (f) $\sqrt{3} \times \sqrt{3}$ (g) $\sqrt{5} \times \sqrt{6}$ (h) $\sqrt{5} \times \sqrt{2}$
- (i) $\sqrt{6} \times \sqrt{6}$ (j) $\sqrt{10} \times \sqrt{3}$ (k) $\sqrt{5} \times \sqrt{20}$ (l) $\sqrt{11} \times \sqrt{10}$
- (m) $\sqrt{2} \times \sqrt{5} \times \sqrt{3}$ (n) $\sqrt{2} \times \sqrt{6} \times \sqrt{3}$ (o) $\sqrt{10} \times \sqrt{7} \times \sqrt{3}$
- (p) $\sqrt{2} \times \sqrt{2} \times \sqrt{2}$ (q) $\sqrt{2} \times \sqrt{3} \times \sqrt{2} \times \sqrt{3}$

Question 2: Work out each of the following

- (a) $(\sqrt{3})^2$ (b) $(\sqrt{7})^2$ (c) $(\sqrt{10})^2$ (d) $(\sqrt{5})^2$

Question 3: Work out each of the following

- (a) $(\sqrt{2})^3$ (b) $(\sqrt{6})^3$ (c) $(\sqrt{10})^3$ (d) $(\sqrt{3})^3$
- (e) $(\sqrt{3})^4$ (f) $(\sqrt{5})^4$ (g) $(\sqrt{10})^4$ (h) $(\sqrt{2})^5$

Question 4: Work out each of the following

- (a) $2\sqrt{3} \times 3\sqrt{5}$ (b) $7\sqrt{2} \times 4\sqrt{11}$ (c) $2\sqrt{13} \times 2\sqrt{2}$ (d) $10\sqrt{2} \times 5\sqrt{3}$
- (e) $\sqrt{3} \times 6\sqrt{5}$ (f) $2\sqrt{2} \times \sqrt{7}$ (g) $4\sqrt{3} \times 2\sqrt{3}$ (h) $2\sqrt{2} \times 3\sqrt{8}$
- (i) $2\sqrt{2} \times \sqrt{5} \times 4\sqrt{3}$ (j) $3\sqrt{2} \times 2\sqrt{6} \times 2\sqrt{3}$

Question 5: Work out each of the following

- (a) $\sqrt{10} \div \sqrt{5}$ (b) $\sqrt{21} \div \sqrt{7}$ (c) $\sqrt{30} \div \sqrt{6}$ (d) $\sqrt{8} \div \sqrt{2}$
- (e) $\sqrt{80} \div \sqrt{5}$ (f) $\sqrt{56} \div \sqrt{7}$ (g) $\sqrt{15} \div \sqrt{3}$ (h) $\sqrt{72} \div \sqrt{8}$
- (i) $\sqrt{7} \div \sqrt{7}$ (j) $\sqrt{26} \div \sqrt{13}$ (k) $\sqrt{48} \div \sqrt{12}$ (l) $\sqrt{48} \div \sqrt{8}$

Fluency Practice

Question 6: Work out each of the following

- (a) $4\sqrt{6} \div 2\sqrt{2}$ (b) $12\sqrt{10} \div 3\sqrt{5}$ (c) $9\sqrt{15} \div 3\sqrt{3}$ (d) $7\sqrt{30} \div \sqrt{2}$
 (e) $20\sqrt{8} \div 5\sqrt{2}$ (f) $40\sqrt{80} \div 20\sqrt{5}$ (g) $51\sqrt{7} \div 3\sqrt{7}$ (h) $36\sqrt{54} \div 18\sqrt{6}$

Question 7: Work out each of the following

- (a) $\frac{\sqrt{6}}{\sqrt{3}}$ (b) $\frac{\sqrt{18}}{\sqrt{2}}$ (c) $\frac{15\sqrt{14}}{3\sqrt{2}}$ (d) $\frac{24\sqrt{72}}{3\sqrt{2}}$

Question 8: Simplify the following

- (a) $\sqrt{8}$ (b) $\sqrt{75}$ (c) $\sqrt{20}$ (d) $\sqrt{32}$ (e) $\sqrt{48}$ (f) $\sqrt{200}$
 (g) $\sqrt{300}$ (h) $\sqrt{80}$ (i) $\sqrt{50}$ (j) $\sqrt{98}$ (k) $\sqrt{63}$ (l) $\sqrt{800}$
 (m) $\sqrt{180}$ (n) $\sqrt{220}$ (o) $\sqrt{96}$ (p) $\sqrt{175}$ (q) $\sqrt{1000}$ (r) $\sqrt{60}$

Question 9: Simplify the following

- (a) $5\sqrt{8}$ (b) $2\sqrt{20}$ (c) $4\sqrt{50}$ (d) $3\sqrt{98}$ (e) $15\sqrt{32}$ (f) $10\sqrt{75}$

Question 10: Work out each of the following. Simplify each answer.

- (a) $\sqrt{6} \times \sqrt{10}$ (b) $\sqrt{15} \times \sqrt{10}$ (c) $\sqrt{30} \times \sqrt{6}$
 (d) $\sqrt{22} \times \sqrt{10}$ (e) $3\sqrt{2} \times \sqrt{6}$ (f) $\sqrt{14} \times 4\sqrt{2}$
 (g) $4\sqrt{6} \times 3\sqrt{15}$ (h) $2\sqrt{70} \times 3\sqrt{10}$ (i) $5\sqrt{10} \times 2\sqrt{30}$

Question 11: Work out the following additions/subtractions

- (a) $\sqrt{8} + \sqrt{18}$ (b) $\sqrt{50} + \sqrt{8}$ (c) $\sqrt{75} + \sqrt{27}$ (d) $\sqrt{200} - \sqrt{32}$
 (e) $\sqrt{8} + \sqrt{2} + \sqrt{72}$ (f) $\sqrt{300} - \sqrt{48}$ (g) $\sqrt{1000} + \sqrt{90}$ (h) $\sqrt{28} + \sqrt{63}$

Question 12: Work out the following additions/subtractions

- (a) $3\sqrt{8} + \sqrt{2}$ (b) $4\sqrt{27} - \sqrt{75}$ (c) $2\sqrt{50} + 5\sqrt{32}$ (d) $\sqrt{200} - 3\sqrt{18}$
 (e) $4\sqrt{80} + 3\sqrt{45}$ (f) $6\sqrt{75} - 2\sqrt{12}$ (g) $10\sqrt{7} + 2\sqrt{175}$

Fluency Practice

Question 13: Expand each of the following. Simplify if possible.

- (a) $\sqrt{2}(\sqrt{3} + 5)$ (b) $\sqrt{3}(\sqrt{5} + \sqrt{2})$ (c) $\sqrt{6}(2 - \sqrt{3})$ (d) $\sqrt{10}(5 + \sqrt{10})$
 (e) $\sqrt{2}(\sqrt{18} - \sqrt{2})$ (f) $\sqrt{5}(3\sqrt{2} - \sqrt{5})$ (g) $2\sqrt{3}(3\sqrt{2} + \sqrt{3})$ (h) $4\sqrt{11}(5\sqrt{2} + 2\sqrt{11})$
 (i) $\sqrt{27}(\sqrt{2} + \sqrt{3})$ (j) $\sqrt{12}(7 - \sqrt{3})$

Question 14: Expand each of the following. Simplify if possible.

- (a) $(2 + \sqrt{3})(1 + \sqrt{3})$ (b) $(\sqrt{2} + 5)(1 + \sqrt{2})$ (c) $(\sqrt{3} + 1)(\sqrt{3} + 4)$
 (d) $(3 + \sqrt{5})(4 - \sqrt{5})$ (e) $(\sqrt{7} - 1)(\sqrt{7} - 1)$ (f) $(5 - \sqrt{3})(5 + \sqrt{3})$
 (g) $(3 + \sqrt{2})(1 + \sqrt{3})$ (h) $(\sqrt{12} + \sqrt{3})(\sqrt{3} + 2)$ (i) $(4 - \sqrt{2})(3 + \sqrt{8})$
 (j) $(\sqrt{7} + \sqrt{2})(\sqrt{8} + \sqrt{7})$ (k) $(1 + 2\sqrt{2})(2 - \sqrt{2})$ (l) $(3\sqrt{5} + 7)(2\sqrt{5} + 1)$
 (m) $(3 + \sqrt{2})^2$ (n) $(1 + \sqrt{5})^2$ (o) $(10 - \sqrt{2})^2$
 (p) $(\sqrt{5} + \sqrt{2})^2$ (q) $(2\sqrt{3} - 1)^2$ (r) $(5\sqrt{2} + 3\sqrt{3})^2$

Question 15: Expand each of the following. Simplify if possible.

- (a) $(4 + \sqrt{3})(4 - \sqrt{3})$ (b) $(1 - \sqrt{6})(1 + \sqrt{6})$ (c) $(\sqrt{10} + 1)(\sqrt{10} - 1)$
 (d) $(\sqrt{7} + \sqrt{5})(\sqrt{7} - \sqrt{5})$ (e) $(2\sqrt{3} - 1)(2\sqrt{3} + 1)$ (f) $(5 - 4\sqrt{3})(5 + 4\sqrt{3})$

Question 16: Rationalise the denominators for each of the following

- (a) $\frac{2}{\sqrt{3}}$ (b) $\frac{5}{\sqrt{2}}$ (c) $\frac{7}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{10}}$
 (e) $\frac{4}{\sqrt{2}}$ (f) $\frac{9}{\sqrt{6}}$ (g) $\frac{\sqrt{2}}{\sqrt{3}}$ (h) $\frac{3}{2\sqrt{5}}$
 (i) $\frac{\sqrt{5}}{\sqrt{80}}$ (j) $\frac{5\sqrt{5}}{\sqrt{20}}$ (k) $\frac{\sqrt{2} + 1}{\sqrt{5}}$ (l) $\frac{2 - \sqrt{3}}{\sqrt{3}}$

Fluency Practice

Question 17: Rationalise the denominators for each of the following

(a) $\frac{6}{3 - \sqrt{2}}$ (b) $\frac{4}{1 + \sqrt{5}}$ (c) $\frac{8}{\sqrt{3} - \sqrt{2}}$ (d) $\frac{\sqrt{2}}{\sqrt{10} + \sqrt{8}}$

(e) $\frac{3\sqrt{6}}{\sqrt{6} - 3}$ (f) $\frac{1}{2\sqrt{3} + \sqrt{5}}$ (g) $\frac{1 + \sqrt{5}}{1 - \sqrt{5}}$ (h) $\frac{\sqrt{11} - \sqrt{3}}{\sqrt{11} + \sqrt{3}}$

Apply

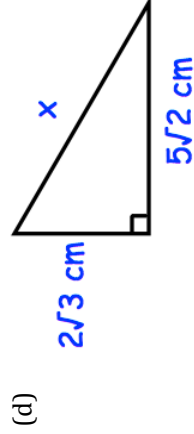
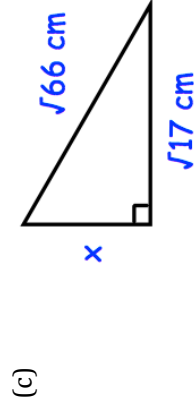
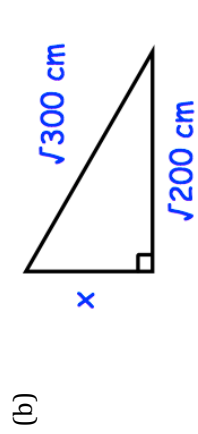
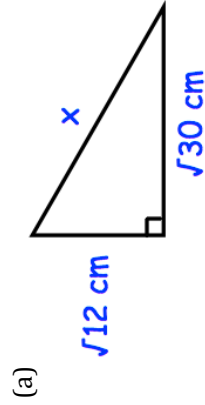
Question 1: Find the area of each of these rectangles

(a) $\sqrt{10}$ cm $\sqrt{5}$ cm $\sqrt{12}$ cm $9\sqrt{2}$ cm $2\sqrt{2}$ cm

Question 2: Find the perimeter of each of these rectangles

(a) $10\sqrt{3}$ cm $5\sqrt{3}$ cm $\sqrt{10} + \sqrt{2}$ cm $\sqrt{10} - \sqrt{2}$ cm $\sqrt{72}$ cm $\sqrt{18}$ cm

Question 3: Calculate x for each of the following right angled triangles. Give your answer as a simplified surd (or integer).



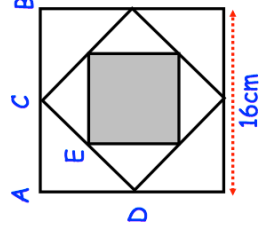
Fluency Practice

Question 4: A shed has dimensions, in metres, of height $\sqrt{5}$, width $\sqrt{6}$ and length $\sqrt{10}$. Find the volume of the shed.
Give your answer as a simplified surd.

Question 5: Mrs Jenkins is making decorations for a wedding. She needs $18\sqrt{5}$ metres of ribbon in total. Mrs Jenkins has 40 metres of ribbon. Does she have enough ribbon?



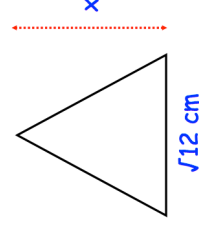
Question 6: The midpoints of the sides of a square of side 16cm are joined to form another square. This process is then repeated to create the shaded square.



Find the area of the shaded square.

Question 7: The area of a rectangle is $\sqrt{125} \text{ cm}^2$. The length of the rectangle is $(2 + \sqrt{5}) \text{ cm}$. Calculate the width of the rectangle.
Express your answer in the form $a + b\sqrt{5}$, where a and b are integers.

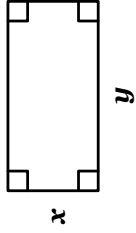
Question 8: The triangle below has an area of $2\sqrt{6} \text{ cm}^2$. Find the height of the triangle, x .
Give your answer as a simplified surd.



Fluency Practice

surds & rectangles 1

Complete each row with the side lengths x and y , the perimeter and area of a rectangle.



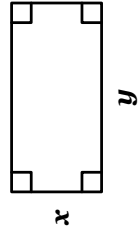
Write your answers in the simplest form, without fractions.

	Side length x	Side length y	Perimeter	Area
A	$3\sqrt{2}$	$\sqrt{2}$		
B	$\sqrt{3}$	$\sqrt{48}$		
C	$\sqrt{5}$		$6\sqrt{5}$	
D		$\sqrt{8}$	$\sqrt{200}$	
E		$2\sqrt{3}$		$8\sqrt{3}$
F	3			$\sqrt{27}$
G	$\sqrt{2}$	$\sqrt{2} + 3$		
H	$2\sqrt{5}$	$\sqrt{5} - 2$		
I		$\sqrt{3} + 2$	$4(\sqrt{3} + 1)$	
J	$2\sqrt{2}$		$6\sqrt{2} - 2$	
K	2			$4\sqrt{3} + 6$
L		$\sqrt{3}$		$\sqrt{75}$
M	$\sqrt{5}$			15
N		$2\sqrt{2}$		12
O	$\sqrt{3}$			$2\sqrt{6}$
P		$3\sqrt{3}$		18

Fluency Practice

surds & rectangles 2

Complete each row with the side lengths x and y , the perimeter and area of a rectangle.



Write your answers in the simplest form, without fractions.

	Side length x	Side length y	Perimeter	Area
A	$\sqrt{5} + \sqrt{2}$	$\sqrt{2}$		
B	$3\sqrt{2}$	$2\sqrt{3} - \sqrt{2}$		
C	$\sqrt{3} + 2\sqrt{2}$		$2\sqrt{3} + 8\sqrt{2}$	
D		$2\sqrt{7} - \sqrt{5}$	$4\sqrt{7}$	
E	$\sqrt{2} + 5$	$\sqrt{2} + 1$		
F	$2 + \sqrt{3}$	$2 - \sqrt{3}$		
G	$\sqrt{3} + 4$		$6\sqrt{3} + 6$	
H	$5 - 2\sqrt{2}$		$2(5 + \sqrt{2})$	
I		$\sqrt{3}$		$\sqrt{3} + 6$
J	$\sqrt{5}$			$4\sqrt{5} - 5$
K		$4\sqrt{3} - 2$	$16\sqrt{3}$	
L	$\sqrt{7} - 2$			3
M		$2 - \sqrt{2}$		4
N	$\sqrt{3}$			$3 + \sqrt{3}$
O		$3 + \sqrt{3}$		$2\sqrt{3}$
P	$\sqrt{5} + 3$			$2 + 2\sqrt{5}$

Fluency Practice

Given a choice between $\sqrt{8}$ and $2\sqrt{2}$ patrons should prefer $2\sqrt{2}$. Also square roots should never be left on the denominator of a fraction in a final answer.

1. Write the following in the form $a\sqrt{b}$ where b is as small as it can be:

- (a) $\sqrt{8}$.
- (b) $\sqrt{27}$.
- (c) $\sqrt{12}$.
- (d) $\sqrt{45}$.
- (e) $\sqrt{450}$.
- (f) $\sqrt{2}\sqrt{24}$.
- (g) $\sqrt{a^4b^2c}$.

- (h) $\sqrt{ab^4c^3d^{100}}$.
- (i) $\sqrt[3]{54}$.
- (j) $\sqrt{8} + \sqrt{18}$.
- (k) $\sqrt{12} + \sqrt{75} - \sqrt{48}$.
- (l) $\sqrt{20} + \sqrt{5} - \sqrt{45}$.
- (m) $\sqrt{200} - \sqrt{288} + \sqrt{128}$.

2. Write the following in the form $\sqrt{a}$:

- (a) $5\sqrt{2}$.
- (b) $4\sqrt{3}$.
- (c) $2\sqrt{5}$.
- (d) $7\sqrt{6}$.
- (e) $\pm^2\sqrt{\bullet}$.
- (f) $3\sqrt[3]{x+1}$.
- (g) $a\sqrt[4]{k}$.

- (h) $\sqrt{12} + 3\sqrt{3}$.
- (i) $\sqrt{8} + \sqrt{2}$.
- (j) $\sqrt{45} - \sqrt{20}$.
- (k) $\sqrt{108} + \sqrt{12} - \sqrt{3}$.
- (l) $\sqrt{44} - \sqrt{99} + \sqrt{176}$.
- (m) $\sqrt{3a^2} + \sqrt{12b^2} - \sqrt{3a^2 + 3b^2} - 6ab$.

3. Expand and simplify fully:

- (a) $(2\sqrt{3})^3$.
- (b) $(\sqrt{18})^3$.
- (c) $(2 - \sqrt{3})^2$.

- (d) $(1 - \sqrt{8})^2$.
- (e) $(4 + \sqrt{32})^2$.
- (f) $(3 + \sqrt{20})^3$.

4. Rationalise the denominator and simplify fully:

- (a) $\frac{4}{\sqrt{2}}$.
- (b) $\frac{1}{\sqrt{3}}$.
- (c) $\frac{25}{2\sqrt{5}}$.
- (d) $\frac{21}{6\sqrt{7}}$.
- (e) $\frac{ab}{c\sqrt{b}}$.
- (f) $\left(\frac{2}{\sqrt{3}}\right)^3$.

- (g) $\frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{3}$.
- (h) $\frac{3}{\sqrt{5}} - 7\sqrt{5}$.
- (i) $\frac{1}{\sqrt{8}} + \frac{1}{\sqrt{2}}$.
- (j) $\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{4}} + \sqrt{\frac{1}{8}}$.
- (k) $\frac{1}{\sqrt{2}-1}$.
- (l) $\frac{4}{\sqrt{3}+1}$.

Fluency Practice

$$(m) \frac{4 - \sqrt{3}}{2 + \sqrt{3}}.$$

$$(n) \frac{1 - \sqrt{5}}{\sqrt{5} + 2}.$$

$$(o) \frac{\sqrt{2} + 3}{1 - 2\sqrt{2}}.$$

$$(p) \frac{\sqrt{7} + 1}{\sqrt{7} - 1}.$$

$$(q) \frac{a}{b + c\sqrt{k}}.$$

$$(r) \frac{9}{\sqrt{3} - 1} + \frac{7}{\sqrt{3} + 1}.$$

$$(s) \frac{\sqrt{2} + 3}{\sqrt{2} - 1} + \frac{1}{\sqrt{2}}.$$

$$(t) \left(\frac{1}{1 - \sqrt{2}} \right)^2 + \left(\frac{2}{3 + \sqrt{2}} \right)^2.$$

$$(u) \frac{69}{1 + \sqrt{2} + \sqrt{3}}.$$

Sequences with... Surds

- 1) Assuming that each pair of numbers is the start of an arithmetic sequence, find:
(i) the next three terms, (ii) the n th term rule, (iii) the 200th term.
- 2) Assuming that each pair of numbers is the start of a geometric sequence, find:
(i) the next three terms, (ii) the ratio between the first and third terms,
(iii) the ratio between the second and fifth terms.

a) $\sqrt{2}$, $3\sqrt{2}$

b) $\sqrt{2}$, $\sqrt{32}$

c) $\sqrt{2}$, 2

d) $\sqrt{2}$, 1

e) $\sqrt{2}$, $2 + \sqrt{2}$

f) $\sqrt{27}$, $\sqrt{108}$

g) $\sqrt{48}$, $\sqrt{108}$

h) $\sqrt{3}$, $\sqrt{108}$

i) 6, $\sqrt{108}$

j) $-\sqrt{75}$, $\sqrt{108}$

Problem Solving

Show clearly that the mean and median of $\sqrt{3}$, $\sqrt{12}$, $\sqrt{48}$ and $\sqrt{75}$ are equal.

Simplify

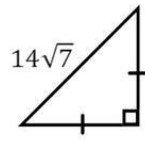
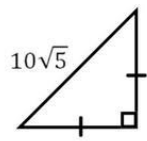
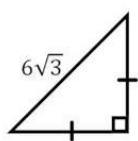
$$\sqrt{1 + \frac{1}{2}} \sqrt{1 + \frac{1}{3}} \sqrt{1 + \frac{1}{4}} \sqrt{1 + \frac{1}{5}} \sqrt{1 + \frac{1}{6}} \sqrt{1 + \frac{1}{7}}$$

Complete the multiplication table.

$\times$		
	$2\sqrt{6}$	$\sqrt{6} - 2$
	$2\sqrt{6} + 6$	1

On the number line shown below, a is a number between 0 and 1, and b is a number between 1 and 2. Mark possible positions on this line for

$$\sqrt{a}, \sqrt{b}, a^2, b^2 \text{ and } \sqrt{\frac{b}{a}}$$



These triangles all share a rather wonderful property. Find the shorter sides and you'll see!

Can you find other triangles with this property?

Find the sum of

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \text{and so on up to} + \frac{1}{\sqrt{99}+\sqrt{100}}$$

Fluency Practice

powers of surds

no calculator please

what are the surd square roots of these surds?

(1) $9 + 4\sqrt{5}$

(6) $36 + 16\sqrt{2}$

(11) $11 + 4\sqrt{6}$

(2) $25 + 4\sqrt{21}$

(7) $49 + 20\sqrt{6}$

(12) $8 - 2\sqrt{15}$

(3) $7 - 4\sqrt{3}$

(8) $29 - 12\sqrt{5}$

(13) $18 - 12\sqrt{2}$

(4) $8 + 2\sqrt{7}$

(9) $52 + 16\sqrt{3}$

(14) $55 + 30\sqrt{2}$

(5) $36 - 10\sqrt{11}$

(10) $64 - 24\sqrt{7}$

(15) $42 - 24\sqrt{3}$

what are the reciprocals of these surds?

(1) $2 + \sqrt{3}$

(1) $26 + 15\sqrt{3}$

(1) $(8 + 4\sqrt{3})^{-\frac{1}{2}}$

(2) $5 - 2\sqrt{6}$

(2) $7 - 5\sqrt{2}$

(2) $(9 + 6\sqrt{2})^{-\frac{1}{2}}$

(3) $3 + 2\sqrt{2}$

(3) $38 + 17\sqrt{5}$

(3) $(15 - 10\sqrt{2})^{-\frac{1}{2}}$

(4) $7 - 4\sqrt{3}$

(4) $54 - 30\sqrt{3}$

(5) $8 + 3\sqrt{7}$

(5) $9\sqrt{3} + 11\sqrt{2}$

try to find some other examples

Fluency Practice

- (a) Given that $(3 + \sqrt{a})(4 + \sqrt{a}) = 17 + b\sqrt{a}$ find the values of a and b .
- (b) Given that $(5 - \sqrt{x})^2 = y - 10\sqrt{2}$ find the values of x and y .
- (c) Given that $(c - \sqrt{d})^2 = 39 - 12\sqrt{d}$, find the values of c and d .

- (a) Find the n th term of the sequence
 $\sqrt{2} \quad \sqrt{8} \quad \sqrt{18} \quad \sqrt{32}$
- (b) Find the n th term of the sequence
 $20 \quad 17 + \sqrt{5} \quad 14 + \sqrt{20} \quad 11 + \sqrt{45}$

- (a) Show that $\sqrt{25a^2} + \frac{2ab\sqrt{75}}{\sqrt{3b^2}}$ is always a multiple of 5, given that a and b are integers.
- (b) Show that $\sqrt{2}(c\sqrt{8} - d\sqrt{50})$ is always even when c and d are integers.

- (a) Given that d is a prime number, rationalise the denominator of $\frac{5\sqrt{d}}{2+\sqrt{d}}$
- (b) Rationalise the denominator of $\frac{2e+\sqrt{f}}{e-\sqrt{f}}$ where e is an integer and f is a prime number.

Solve, giving your answer in its simplest form:

- (a) $3\sqrt{3} = \sqrt{3}x + 2$
- (b) $3x - \sqrt{5} = \sqrt{5}x + 1$
- (c) $x^2 - 2\sqrt{3}x + 3 = 0$
- (d) $2x^2 + \sqrt{7}x - 7 = 0$
- (e) $3x^2 + 5\sqrt{5}x = 10$

Fluency Practice

For each question express the answer as a surd in the form $a\sqrt{x}$ where a is an integer

$$\frac{6}{\frac{1}{\sqrt{5}} + \sqrt{5}}$$

$$\frac{8}{\sqrt{3} - \frac{1}{\sqrt{3}}}$$

$$\frac{18}{\frac{2}{\sqrt{7}} + \sqrt{7}}$$

$$\frac{5\sqrt{2}}{\frac{3}{\sqrt{2}} + \sqrt{2}}$$

$$\frac{3\sqrt{5}}{\sqrt{5} - \frac{2}{\sqrt{5}}}$$

$$\frac{\frac{4}{\sqrt{3}} + 2\sqrt{3}}{\sqrt{3} + \frac{2}{\sqrt{3}}}$$

Fluency Practice

Write each ratio in its simplest form.

$$\sqrt{20} : \sqrt{45}$$

$$\sqrt{63} : \sqrt{28} : \sqrt{175}$$

$$a = \sqrt{11} : \sqrt{c}$$

$$b = \sqrt{44} : \sqrt{d}$$

Given that $c:d$ is in the
ratio 1:4

Find the ratio $a:b$

$$a = \sqrt{3} : \sqrt{c}$$

$$b = \sqrt{48} : \sqrt{d}$$

Given that $c:d$ is in the
ratio 1:16

Find the ratio $a:b$

$$a = \sqrt{7} : \sqrt{c}$$

$$b = \sqrt{63} : \sqrt{d}$$

Given that $c:d$ is in the
ratio 1:9

Find the ratio $a:b$

$$a = \sqrt{12} : \sqrt{c}$$

$$b = \sqrt{192} : \sqrt{d}$$

Given that $c:d$ is in the
ratio 1:4

Find the ratio $a:b$

3 Algebraic Fractions

Intelligent Practice

Simplify:

1) $\frac{3}{12}$

2) $\frac{3x}{12}$

3) $\frac{3}{12x}$

4) $\frac{12x}{3}$

5) $\frac{12x}{6}$

6) $\frac{12x}{6x}$

7) $\frac{12xy}{6x}$

8) $\frac{6x}{12xy}$

9) $\frac{10x}{12xy}$

10) $\frac{10x}{12xy^2}$

11) $\frac{10xy}{12xy^2}$

12) $\frac{10x^2y}{12xy^2}$

13) $\frac{10x^2y^2}{12xy^2}$

14) $\frac{10x^2y^3}{12xy^2}$

15) $\frac{100x^2y^4}{120xy^2}$

Intelligent Practice

Simplify:

$$1) \quad \frac{2(x+3)}{3(x+3)}$$

$$10) \quad \frac{2x^2+14x+24}{x^2-5x-36}$$

$$2) \quad \frac{2(x+3)}{(x+3)}$$

$$11) \quad \frac{2x^2+14x+24}{3x^2-15x-108}$$

$$3) \quad \frac{(x+3)}{2(x+3)}$$

$$12) \quad \frac{2x^2+14x+24}{3x^2+4x-15}$$

$$4) \quad \frac{x+3}{2x+6}$$

$$13) \quad \frac{14x-24-2x^2}{3x^2+4x-15}$$

$$5) \quad \frac{(x+3)(x+4)}{2x+6}$$

$$14) \quad \frac{14x-24-2x^2}{3x^2-4x-15}$$

$$6) \quad \frac{(x+3)(x+4)}{2x+8}$$

$$15) \quad \frac{14x-24-2x^2}{4x+15-3x^2}$$

$$7) \quad \frac{x^2+7x+12}{2x+8}$$

$$8) \quad \frac{x^2+7x+12}{(x+4)(x-9)}$$

$$9) \quad \frac{x^2+7x+12}{x^2-5x-36}$$

Intelligent Practice

Simplify:

$$1) \frac{x+2}{(x+2)(x-1)}$$

$$2) \frac{x-1}{(x+2)(x-1)}$$

$$3) \frac{(x-1)^2}{(x+2)(x-1)}$$

$$4) \frac{(x+2)^2}{(x+2)(x-1)}$$

$$5) \frac{x^2-4}{(x+2)(x-1)}$$

$$6) \frac{x^2-4}{(x+2)(2x-5)}$$

$$7) \frac{4x^2-25}{2x^2-x-10}$$

$$8) \frac{4x^2-25}{4x^2+20x+25}$$

$$9) \frac{4x^2+8x-5}{4x^2+20x+25}$$

$$10) \frac{4ax^2+8ax-5a}{4a^2x^2+20a^2x+25a^2}$$

$$11) \frac{14x-24-2x^2}{3x^2+4x-15}$$

$$12) \frac{14x-24-2x^2}{3x^2-4x-15}$$

Fluency Practice

Q1 Simplify the following algebraic fractions.

[a] $\frac{6x + 10}{2}$

[e] $\frac{21x + 14}{7}$

[i] $\frac{12x + 18}{9}$

[b] $\frac{8x - 12}{4}$

[f] $\frac{8x - 12}{6}$

[j] $\frac{8x - 28}{12}$

[c] $\frac{5x + 15}{5}$

[g] $\frac{5x + 20}{10}$

[k] $\frac{22x - 44}{33}$

[d] $\frac{20x + 24}{4}$

[h] $\frac{6x - 12}{9}$

[l] $\frac{18x + 45}{27}$

Fluency Practice

Q2 Simplify the following algebraic fractions.

[a] $\frac{3(x + 4)}{x + 4}$

[e] $\frac{5(x - 3)}{x - 3}$

[i] $\frac{5x + 10}{x + 2}$

[m] $\frac{2x + 6}{3x + 9}$

[b] $\frac{3(x + 4)}{5(x + 4)}$

[f] $\frac{5(x - 3)}{6(x - 3)}$

[j] $\frac{x + 2}{5x + 10}$

[n] $\frac{3x - 21}{4x - 28}$

[c] $\frac{x + 4}{3(x + 4)}$

[g] $\frac{x - 3}{5(x - 3)}$

[k] $\frac{4x + 24}{x + 6}$

[o] $\frac{4x - 20}{3x - 15}$

[d] $\frac{x + 4}{3x + 12}$

[h] $\frac{x - 3}{5x - 15}$

[l] $\frac{x + 6}{4x + 24}$

[p] $\frac{2x + 12}{3x + 18}$

Fluency Practice

Q3 Simplify the following algebraic fractions.

[a] $\frac{4x + 8}{7x + 14}$

[e] $\frac{3x + 12}{2x + 8}$

[i] $\frac{2x + 2}{7x + 7}$

[m] $\frac{5x - 20}{8x - 32}$

[b] $\frac{5x + 25}{9x + 45}$

[f] $\frac{2x + 6}{10x + 30}$

[j] $\frac{2x - 20}{7x - 70}$

[n] $\frac{10x + 40}{9x + 36}$

[c] $\frac{5x - 40}{3x - 24}$

[g] $\frac{2x - 14}{9x - 63}$

[k] $\frac{8x + 24}{10x + 30}$

[o] $\frac{8x + 96}{9x + 108}$

[d] $\frac{3x - 15}{5x - 25}$

[h] $\frac{5x - 30}{7x - 42}$

[k] $\frac{10x - 80}{7x - 56}$

[p] $\frac{4x - 24}{6x - 36}$

Fluency Practice

Q4 Simplify the following algebraic fractions.

[a] $\frac{6x^2 + 5x}{x}$

[e] $\frac{12x + 6}{2x + 1}$

[i] $\frac{4x^4 + 8x^5}{12x^3}$

[b] $\frac{2x - 7x^3}{x}$

[f] $\frac{4x + 8}{x + 2}$

[j] $\frac{9x^5 - 6x^3}{18x^3}$

[c] $\frac{5t^2 - 4t^4}{t^2}$

[g] $\frac{6x + 10}{9x + 15}$

[k] $\frac{6x^2 + 15x}{2x + 5}$

[d] $\frac{9y^5 + 2y^3}{y^3}$

[h] $\frac{6x + 9}{8x + 12}$

[l] $\frac{(x - 3)(x + 5)}{(x + 5)}$

Fluency Practice

Q5 Simplify the following algebraic fractions.

[a] $\frac{x^2 + 4x}{2x + 8}$

[e] $\frac{2x^2 + 6x^3}{6x + 2}$

[i] $\frac{(x + 2)(x + 3)}{4x + 12}$

[b] $\frac{6x^3 - x^2}{12x - 2}$

[f] $\frac{x^4 - 4x^3}{x^3 - 4x^2}$

[j] $\frac{5x - 20}{(x + 5)(x - 4)}$

[c] $\frac{x + 3x^2}{5 + 15x}$

[g] $\frac{3x^5 - 9x^4}{x^4 - 3x^3}$

[k] $\frac{9x + 27}{(x + 3)(x + 2)}$

[d] $\frac{x^4 - 5x^3}{6x - 30}$

[h] $\frac{4x^3 + 8x^2}{2x^3 + 4x^2}$

[l] $\frac{(x - 3)(x + 5)}{(x + 5)(x + 4)}$

Q1 Simplify the following algebraic fractions.

$$[a] \quad \frac{(x-5)(x+2)}{(x+2)(x+5)}$$

$$[e] \quad \frac{(x+6)(x-8)}{(x-8)(x-2)}$$

$$[b] \quad \frac{(x+4)(x+3)}{(x+5)(x+3)}$$

$$[f] \quad \frac{(x+3)(x-9)}{(x-5)(x+3)}$$

$$[c] \quad \frac{(x-1)(x+1)}{(x+6)(x-1)}$$

$$[g] \quad \frac{(x+4)(x-4)}{(x-4)(x+6)}$$

$$[d] \quad \frac{(x+7)(x+5)}{(x-9)(x+5)}$$

$$[h] \quad \frac{(x+2)(x-4)}{(x-4)(x+3)}$$

Q2 Simplify the following algebraic fractions.

$$[a] \quad \frac{(x-2)^2}{x(x-2)}$$

$$[b] \quad \frac{x^2 + 4x}{(x+4)(x+3)}$$

$$[c] \quad \frac{(x-1)(x+1)}{(x-1)^2}$$

$$[d] \quad \frac{x(x+6)^2}{x(x-9)(x+6)}$$

$$[e] \quad \frac{x-8}{(x-8)^2(x-2)}$$

$$[f] \quad \frac{(x+3)^3(x+2)}{(x+2)(x+3)^2}$$

$$[g] \quad \frac{(x+4)^2(x-4)}{(x-4)(x+4)^3}$$

$$[h] \quad \frac{(x+2)(x-3)(x+5)}{(x+5)(x-6)(x-3)}$$

Q3 Simplify the following algebraic fractions.

[a] $\frac{x^2 + 6x + 8}{x + 2}$

[e] $\frac{x + 2}{x^2 - x - 6}$

[b] $\frac{x^2 + 5x - 24}{x + 8}$

[f] $\frac{x^2 - 3x - 10}{x - 5}$

[c] $\frac{x^2 - 1}{x - 1}$

[g] $\frac{x + 4}{x^2 + 13x + 36}$

[d] $\frac{x + 4}{x^2 - 2x - 24}$

[h] $\frac{x^2 + 8x - 9}{x + 9}$

Q4 Simplify the following algebraic fractions.

[a] $\frac{x^2 + 3x + 2}{x^2 + 5x + 4}$

[b] $\frac{x^2 + 4x + 4}{x^2 + 6x + 8}$

[c] $\frac{x^2 + 6x + 5}{x^2 + 3x - 10}$

[d] $\frac{x^2 + 2x - 3}{x^2 - 6x + 5}$

[e] $\frac{x^2 + 4x - 45}{x^2 + 10x + 9}$

[f] $\frac{x^2 - 16}{x^2 + 7x + 12}$

[g] $\frac{x^2 - x - 20}{x^2 - 7x + 10}$

[h] $\frac{x^2 + 15x + 56}{x^2 + 3x - 40}$

Q5 Simplify the following algebraic fractions.

$$[a] \quad \frac{2x^2 - 18x}{x^2 - 2x - 63}$$

$$[b] \quad \frac{4x^2 + 16x}{x^2 + 12x + 32}$$

$$[c] \quad \frac{3x^2 + 24x}{x^2 + 2x - 48}$$

$$[d] \quad \frac{x^2 + 7x}{2x^2 + 17x + 21}$$

$$[e] \quad \frac{2x^2 + 4x}{2x^2 + 8x + 8}$$

$$[f] \quad \frac{2x^2 + 8x}{2x^2 + 20x + 48}$$

$$[g] \quad \frac{6x^2 + 40x - 14}{6x^2 - 2x}$$

$$[h] \quad \frac{6x^2 + 16x - 32}{6x^2 - 8x}$$

Q6 Simplify the following algebraic fractions.

[a] $\frac{3x^2 + 6x + 3}{x^2 - 4x - 5}$

[b] $\frac{2x^2 + 19x + 24}{x^2 + 4x - 32}$

[c] $\frac{2x^2 + 15x + 25}{x^2 - x - 30}$

[d] $\frac{3x^2 + 25x + 28}{x^2 - 49}$

[e] $\frac{5x^2 + 28x - 12}{x^2 + 4x - 12}$

[f] $\frac{3x^2 - 25x + 42}{x^2 - 12x + 36}$

[g] $\frac{5x^2 + 7x - 6}{x^2 - 5x - 14}$

[h] $\frac{4x^2 + 40x - 64}{x^2 - 64}$

Fluency Practice

A1 Simplify $\frac{x+3}{x^2+x-6}$	A2 Simplify $\frac{x+6}{x^2+10x+24}$	A3 Simplify $\frac{2x-18}{x^2-12x+27}$	A4 Simplify $\frac{x^2-7x}{x^2-2x-35}$
B1 Simplify $\frac{x^2-49}{x^2+2x-35}$	B2 Simplify $\frac{x^2-4}{x^2+6x-16}$	B3 Simplify $\frac{x^2-16}{x^2-2x-24}$	B4 Simplify $\frac{x^2-36}{x^2+3x-18}$
C1 Simplify $\frac{x^2+6x-16}{2x^2-x-6}$	C2 Simplify $\frac{x^2-7x+10}{3x^2-14x-5}$	C3 Simplify $\frac{x^2+3x}{5x^2+19x+12}$	C4 Simplify $\frac{x^2+4x-21}{4x^2-17x+15}$
D1 Simplify $\frac{2x^2-5x-12}{2x^2-11x+12}$	D2 Simplify $\frac{5x^2-24x-5}{5x^2-14x-3}$	D3 Simplify $\frac{4x^2-9}{2x^2+11x+12}$	D4 Simplify $\frac{6x^2-11x+4}{4x^2+8x-5}$

Fluency Practice

Simplifying Algebraic Fractions

a $\frac{3x+6}{4x+8} \rightarrow$

b $\frac{12x+4}{15x+5} \rightarrow$

c $\frac{2x^2+6}{x^2+5x+6} \rightarrow$

d $\frac{x^2+7x+12}{x^3+4x^2} \rightarrow$

e $\frac{x^2-7x+10}{2x^3-10x^2} \rightarrow$

f $\frac{x^2-2x-8}{x^2-6x+8} \rightarrow$

g $\frac{x^2-5x-6}{x^2+5x+4} \rightarrow$

h $\frac{8x^2-24x}{2x^2+4x-30} \rightarrow$

i $\frac{3x^2-3x-18}{3x^2-9x-30} \rightarrow$

j $\frac{x^2-1}{2x-2} \rightarrow$

k $\frac{x^2-4}{4x+8} \rightarrow$

l $\frac{5x+25}{x^2-25} \rightarrow$

m $\frac{x^2-9}{x^2-3x} \rightarrow$

n $\frac{3x^2+6x}{x^2-4} \rightarrow$

o $\frac{12-4x}{9-x^2} \rightarrow$

p $\frac{x^2-16}{3x^3+12x^2} \rightarrow$

q $\frac{2x^2-8}{2x^2-4x} \rightarrow$

r $\frac{x^2+2x-3}{x^2-1} \rightarrow$

s $\frac{x^2-16}{x^2-5x+4} \rightarrow$

t $\frac{x^2+5x+6}{x^2-9} \rightarrow$

u $\frac{x^2+12x+36}{x^2-36} \rightarrow$

v $\frac{4x^2-1}{6x^2+3x} \rightarrow$

w $\frac{3x^3+4x^2}{9x^2-16} \rightarrow$

x $\frac{2x^2-x-3}{4x^2-9} \rightarrow$

y $\frac{9x^2-4}{3x^2-8x+4} \rightarrow$

z $\frac{16x^2-25}{8x^2+2x-15} \rightarrow$

aa $\frac{6x^2-7x-20}{9x^2-16} \rightarrow$

Fluency Practice

Question 1: Simplify the following algebraic fractions

$$(a) \frac{42xyz}{56} \qquad (b) \frac{45ab}{60abc} \qquad (c) \frac{16mn}{18n} \qquad (d) \frac{40x^2y}{32xy}$$

$$(e) \frac{17cf}{34c^3} \qquad (f) \frac{8x^4}{2x^2} \qquad (g) \frac{33a^2b^2}{44a^3b} \qquad (h) \frac{12x^3}{20x^7}$$

Question 2: Simplify the following algebraic fractions

$$(a) \frac{6x+8}{2} \qquad (b) \frac{9x-12}{3} \qquad (c) \frac{35x^2+20}{5}$$

$$(d) \frac{7m-70n^3}{7} \qquad (e) \frac{10c+25}{15} \qquad (f) \frac{8w+2-4x}{12}$$

$$(g) \frac{9x^2+12x+33}{6} \qquad (h) \frac{3x^2+5x}{x} \qquad (i) \frac{3x^3-7x^2}{x}$$

$$(j) \frac{8x^6+x^4+3x}{x} \qquad (k) \frac{10x^7+15x^5-30x^4}{5x} \qquad (l) \frac{3c^6-15c^4}{6c}$$

$$(m) \frac{-8x^5-12x^4+2x^3}{-4x} \qquad (n) \frac{6c^9-12c^3}{3c^2} \qquad (o) \frac{6c^6+2c^2}{4c^4}$$

Fluency Practice

Question 3: Simplify the following algebraic fractions

$$(a) \frac{(x+6)(x+3)}{(x+3)}$$

$$(b) \frac{(x-1)(x+1)}{(x-1)}$$

$$(c) \frac{(x-3)}{(x-4)(x-3)}$$

$$(d) \frac{(x+7)^2}{(x+7)}$$

$$(e) \frac{(x-3)(x+2)}{(x+2)(x+9)}$$

$$(f) \frac{(x+2)(x+4)^2}{(x+4)}$$

$$(g) \frac{(x+1)(x+2)(x+3)}{(x+2)(x+3)(x+4)}$$

$$(h) \frac{x(x+3)^2}{x(x+1)(x+3)}$$

Question 4: Simplify the following algebraic fractions

$$(a) \frac{x^2 + 5x + 4}{x^2 + 4x + 3}$$

$$(b) \frac{x^2 + 6x + 9}{x^2 - 2x - 15}$$

$$(c) \frac{x^2 - 2x}{x^2 + 2x - 8}$$

$$(d) \frac{x^2 - 7x + 10}{x^2 + 3x - 10}$$

$$(e) \frac{x^2 + 8x + 15}{x^2 - x - 12}$$

$$(f) \frac{x^2 + 13x + 40}{x^2 + 14x + 48}$$

$$(g) \frac{x^2 - 2x - 8}{x^2 + 6x - 40}$$

$$(h) \frac{x^2 + 10x + 24}{x^2 - 36}$$

$$(i) \frac{x^2 + 4x - 45}{x^2 + 10x + 9}$$

$$(j) \frac{x^2 + 11x}{x^2 - 121}$$

$$(k) \frac{x^2 - 1}{x^2 + x}$$

$$(l) \frac{x^2 - 15x + 44}{x^2 - 16}$$

$$(m) \frac{x^2 - x - 6}{x^2 - 2x - 3}$$

Fluency Practice

Question 5: Simplify the following algebraic fractions

$$(a) \frac{3x^2 + 7x + 4}{x^2 + 3x + 2}$$

$$(b) \frac{x^2 - 2x - 8}{3x^2 + 7x + 2}$$

$$(c) \frac{5x^2 - 13x - 6}{x^2 - 9}$$

$$(d) \frac{2x^2 + 3x - 2}{2x^2 - 15x + 7}$$

$$(e) \frac{9x^2 - 1}{3x^2 - 13x + 4}$$

$$(f) \frac{x^2 + 17x + 70}{5x^2 + 38x + 21}$$

$$(g) \frac{3x^2 + 5x - 12}{12x^2 - 19x + 4}$$

$$(h) \frac{3x^2 + 11x + 6}{9x^2 + 21x + 10}$$

$$(i) \frac{4x^2 + x - 3}{4x^2 + 9x + 5}$$

$$(j) \frac{9x^2 - 30x + 25}{6x^2 + 5x - 25}$$

$$(k) \frac{10x^2 - 23x + 12}{4x^2 + 4x - 15}$$

$$(l) \frac{20x^2 + 21x + 4}{16x^2 - 1}$$

Fluency Practice

Simplify

$$(a) \frac{4x^2}{8x} \quad (b) \frac{4xy}{12y^2}$$

$$(c) \frac{10x^2y}{20xy} \quad (d) \frac{9x^2yz}{27y^2}$$

$$(e) \frac{15ab}{18a^2} \quad (f) \frac{36ab^2c}{6abd}$$

Simplify

$$(a) \frac{x(x+9)}{6x} \quad (b) \frac{2x(x-4)}{5(x-4)}$$

$$(c) \frac{x^2+6x}{5x} \quad (d) \frac{x^2-5x}{3x-15}$$

$$(e) \frac{3x-x^2}{6-2x} \quad (f) \frac{12x^2-6x}{14x-7}$$

Simplify

$$(a) \frac{x(x-5)}{(x+2)(x-5)} \quad (b) \frac{2x(x+1)}{4(x+2)(x+1)}$$

$$(c) \frac{x(x+5)}{x^2+7x+10} \quad (d) \frac{x^2+2x+1}{(x+1)(x-4)}$$

Simplify

$$(a) \frac{x^2+6x}{x^2+5x-6} \quad (b) \frac{x^2-x-6}{x^2+x-12}$$

$$(c) \frac{2x^2+14x}{x^2+10x+21} \quad (d) \frac{x^2-4}{x^2+2x-8}$$

Simplify

$$(a) \frac{x^2+7x+6}{2x^2+3x+1} \quad (b) \frac{3x^2+7x+2}{x^2+9x+14}$$

$$(c) \frac{5x^2+28x-12}{x^2+4x-12} \quad (d) \frac{2x^2+15x+25}{x^2-x-30}$$

$$(e) \frac{5x^2+7x-6}{x^2-5x-14} \quad (f) \frac{2x^2-32}{2x^2+9x+4}$$

Fluency Practice

tough algebra questions

- (1) simplify $(a + b)^2 - (a - b)^2$
- (2) expand $(a - b)(a^2 + ab + b^2)$
- (3) expand (i) $(n - 1)(n^2 + n + 1)$
(ii) $(n - 1)(n^3 + n^2 + n + 1)$
- (4) show that $(n + 1)(n + 2) + (n + 2)^2 - (n + 1)(n + 6)$ is a perfect square
- (5) if $\frac{1}{2}(\frac{2}{3}(2a + b) + a - 3b) = \frac{1}{4}(a + 2b)$ show that $b = \frac{7}{8}a$
- (6) simplify $\frac{a^2b + a}{b^2a + b}$
- (7) simplify $\frac{k^2 - 7k + 10}{k^2 - 5k}$
- (8) simplify $\frac{d^2 - 8d + 15}{2d^2 - 7d - 15}$
- (9) if $a = t - \frac{1}{t}$ and $b = t + \frac{1}{t}$ show that $b^2 - a^2 = 4$
- (10) if $a = \frac{b}{b + c}$ show that $\frac{a}{1 - a} = \frac{b}{c}$

Problem Solving

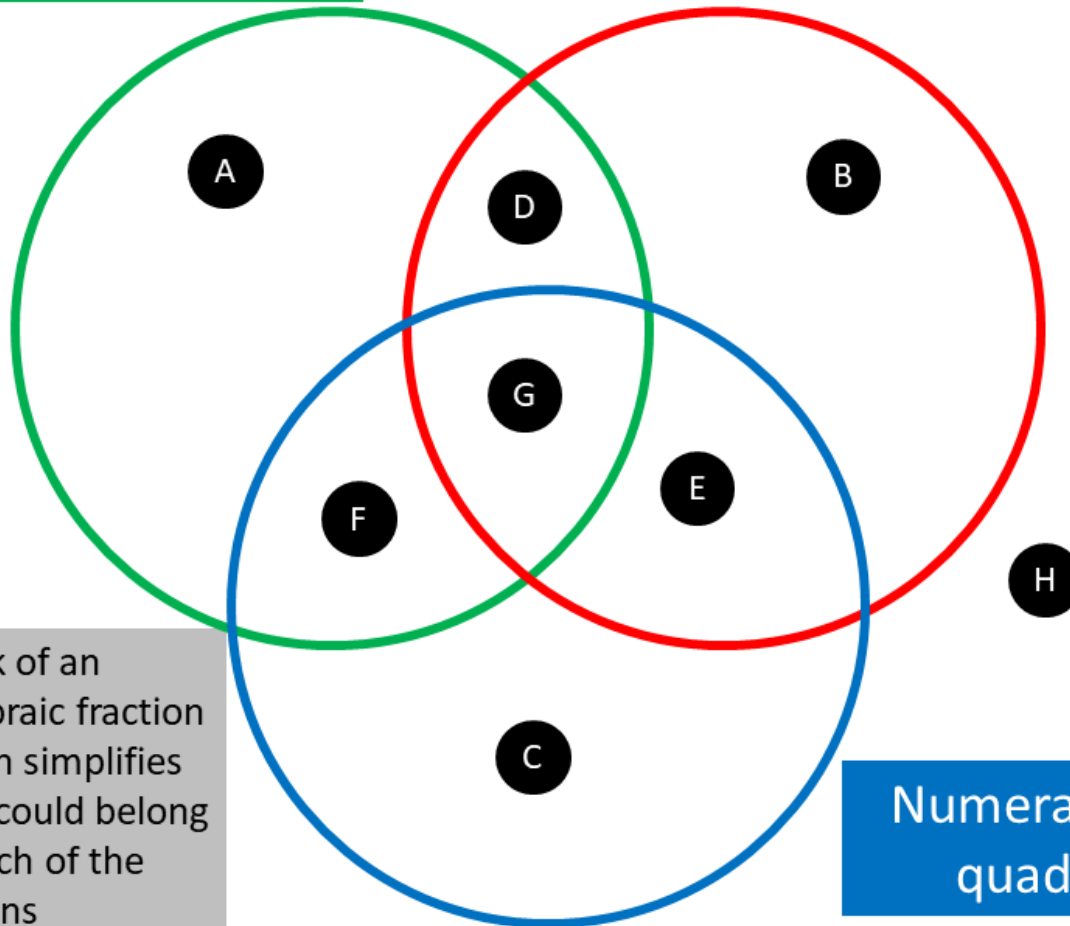
Use the digits 0-9 (at most one time each) to fill in the boxes below so that the rational expression on the left simplifies to the rational expression on the right.

$$\frac{x^2 + \boxed{}x + \boxed{}\boxed{}}{x^2 - \boxed{}\boxed{}} = \frac{x + \boxed{}}{x - \boxed{}}$$

Maths Venns

Denominator is
 $x^2 + 6x + 8$

Equivalent to $\frac{1}{x+4}$



If you think a region is impossible to fill, convince me why!

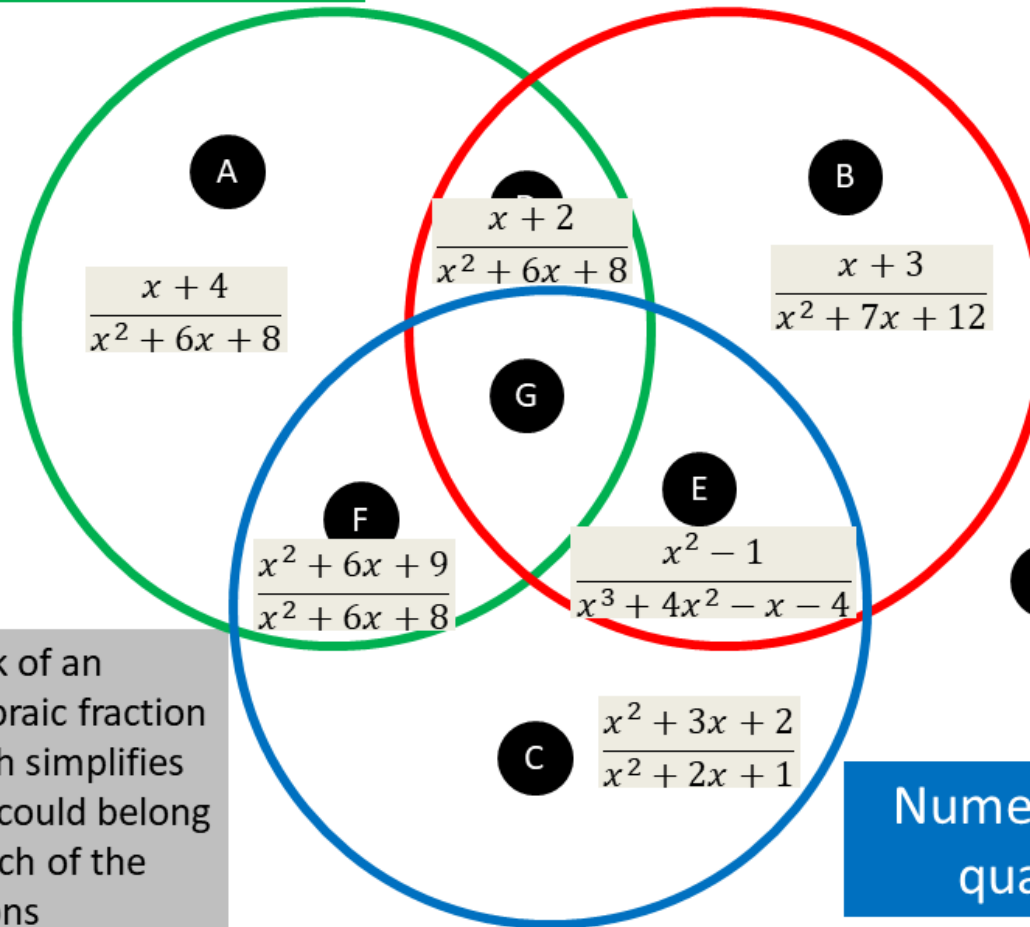
Think of an algebraic fraction which simplifies that could belong in each of the regions

Numerator is a quadratic

Maths Venns

Denominator is
 $x^2 + 6x + 8$

Equivalent to $\frac{1}{x+4}$



If you think a region is impossible to fill, convince me why!

Think of an algebraic fraction which simplifies that could belong in each of the regions

Numerator is a quadratic

Intelligent Practice

Simplify:

$$1) \quad \frac{3}{5} \times \frac{x}{y}$$

$$2) \quad \frac{x}{y} \times \frac{3}{5}$$

$$3) \quad \frac{x}{y} \times \frac{3}{6}$$

$$4) \quad \frac{x}{y} \times \frac{3}{6x}$$

$$5) \quad \frac{x}{xy} \times \frac{3}{6}$$

$$6) \quad \frac{7x}{xy} \times \frac{3}{6}$$

$$7) \quad \frac{7x}{xy} \times \frac{3}{6y}$$

$$1) \quad \frac{7x}{xy} \div \frac{3}{6y}$$

$$2) \quad \frac{7x}{xy} \div \frac{6y}{3}$$

$$3) \quad \frac{7x}{xy} \times \frac{6y}{3}$$

$$4) \quad \frac{7x}{xy} \times \frac{6y}{3} \times \frac{4y}{14}$$

$$5) \quad \frac{7x}{xy} \div \frac{6y}{3} \times \frac{4y}{14}$$

$$6) \quad \frac{7x}{xy} \div \frac{6y}{3} \div \frac{4y}{14}$$

$$7) \quad \frac{7x^2}{xy} \div \frac{6y}{3} \div \frac{4y}{14}$$

$$1) \quad \frac{7x^2}{xy} \div \left(\frac{6y}{3} \div \frac{4y}{14} \right)$$

$$2) \quad \frac{7x^2}{xy} \div \left(\frac{6y}{3} \times \frac{4y}{14} \right)$$

$$3) \quad \frac{7x^2}{xy} \div \left(\frac{6y}{9} \times \frac{4y}{14} \right)$$

Fluency Practice

Simplify:

$$(a) \frac{4x^2}{8x} \quad (b) \frac{4xy}{12y^2}$$

$$(c) \frac{9abc}{12bc} \quad (d) \frac{5a}{8a^2b}$$

$$(e) \frac{10x^2y}{20xy} \quad (f) \frac{9x^2yz}{27y^2}$$

Simplify:

$$(a) \frac{x}{2} \times \frac{x}{5} \quad (b) \frac{2x}{5} \times \frac{10}{3y}$$

$$(c) \frac{2x^2}{7} \times \frac{x}{5} \quad (d) \frac{5xy}{11} \times \frac{33}{2x}$$

$$(e) \frac{4ab}{9} \times \frac{3a}{8b^2} \quad (f) \frac{x^2}{y^2} \times \frac{y}{x}$$

$$(g) \frac{4}{9} \times \frac{abc}{8} \quad (h) \frac{2x^3}{5y} \times y$$

Simplify:

$$(a) \frac{x}{3} \div \frac{x}{7} \quad (b) \frac{2x}{5} \div \frac{10y}{3}$$

$$(c) \frac{2x^2}{7} \div \frac{x}{5} \quad (d) \frac{5x}{11y} \div \frac{3}{2x}$$

$$(e) \frac{3}{x} \div \frac{6y}{11} \quad (f) \frac{4}{5ab} \div \frac{10b}{3a}$$

$$(g) \frac{8a}{7b} \div 4 \quad (h) \frac{2x^2y}{5z} \div \frac{7x}{2yz}$$

Simplify:

$$(a) \frac{a^3b^4cd^2}{5a} \times \frac{10d}{ab^2c^3}$$

$$(b) \frac{20a^2b^5c^3}{15} \times \frac{3c^2}{a^5b^2}$$

Intelligent Practice

Simplify the following as far as possible:

$$1) \quad \frac{5}{x^2+x-2} \times \frac{x-1}{2}$$

$$2) \quad \frac{9}{x^2-4x+3} \times \frac{x-3}{2}$$

$$3) \quad \frac{7}{x-2} \div \frac{8}{x^2-x-2}$$

$$4) \quad \frac{3}{x-4} \div \frac{8}{x^2+x+20}$$

Simplify the following as far as possible:

$$1) \quad \frac{x^2-4x+3}{4x} \times \frac{3}{x^2-2x+1}$$

$$2) \quad \frac{x^2+3x-4}{7x} \times \frac{3x}{x^2+7x+12}$$

$$3) \quad \frac{x^2-2x-3}{2x} \div \frac{x^2-7x+12}{3}$$

$$4) \quad \frac{x^2+4x+3}{3x} \div \frac{x^2+3x+2}{5x}$$

Simplify the following as far as possible:

$$1) \quad \frac{x^2-2x+1}{x^2-1} \times \frac{x^2+4x+3}{x^2+2x-3}$$

$$2) \quad \frac{x^2+4x-5}{x^2-x-2} \times \frac{x^2-x-2}{x^2+6x+5}$$

$$3) \quad \frac{x^2+4x-5}{x^2+x-2} \div \frac{x^2+10x+25}{x^2+8x+15}$$

$$4) \quad \frac{x^2+x-2}{x^2-2x-8} \div \frac{x^2-4x+3}{x^2-6x+9}$$

Fluency Practice

Question 1: Express the following as a single fraction.

(a) $\frac{2}{g} \times \frac{3}{h}$

(b) $\frac{3}{c} \times \frac{a}{4}$

(c) $\frac{w}{x} \times \frac{3}{a}$

(d) $\frac{3a}{7} \times \frac{2c}{g}$

(e) $\frac{a}{e} \times \frac{f}{b}$

(f) $\frac{e}{8} \times \frac{d}{8}$

(g) $\frac{x}{2} \times \frac{x}{5}$

(h) $\frac{7}{y} \times \frac{2}{y}$

(i) $\frac{3}{w} \times \frac{x}{4} \times \frac{y}{w}$

(j) $\frac{2x}{5} \times \frac{3x}{7}$

(k) $\frac{x}{y} \times \frac{x}{y}$

(l) $\frac{6a}{7c} \times \frac{5a}{c}$

Question 2: Express the following as a single **simplified** fraction.

(a) $\frac{2x}{y} \times \frac{y}{4}$

(b) $\frac{3a}{c} \times \frac{5}{6}$

(c) $\frac{4}{5a} \times \frac{5w}{8}$

(d) $\frac{3a}{7} \times \frac{2c}{9}$

(e) $\frac{10g}{w} \times \frac{w}{5}$

(f) $\frac{4x}{5y} \times \frac{3y}{8x}$

(g) $\frac{2y}{3} \times \frac{2y}{wy}$

(h) $\frac{6x}{5y} \times \frac{4x}{3y}$

(i) $\frac{x^2}{a} \times \frac{a^2}{x^2}$

(j) $\frac{ab}{c} \times \frac{c}{ae}$

(k) $\frac{6c}{w^2} \times \frac{15w^3}{2c^2}$

(l) $\frac{2a^4}{3b^3} \times \frac{6b^2}{5a}$

(m) $\frac{2a^3b}{3} \times \frac{6}{ab^2}$

(n) $\frac{x^4y^4}{z^2} \times \frac{z}{x^6y}$

(o) $\frac{14a^2bc^3}{9} \times \frac{6b^3}{21a^3c}$

Fluency Practice

Question 3: Express the following as a single fraction. **Simplify** if possible.

$$(a) \quad \frac{x}{4} \times \frac{x-3}{2} \qquad (b) \quad \frac{x}{9} \times \frac{6}{x+7} \qquad (c) \quad \frac{x+1}{15} \times \frac{5}{x}$$

$$(d) \quad \frac{1}{x+3} \times \frac{2}{x+1} \qquad (e) \quad \frac{3x+2}{3} \times \frac{x+1}{3} \qquad (f) \quad \frac{x+4}{x-4} \times \frac{x-2}{x+5}$$

$$(g) \quad \frac{x+1}{x-7} \times \frac{x-5}{x+1} \qquad (h) \quad \frac{7}{2x+8} \times \frac{x+4}{14} \qquad (i) \quad \frac{4}{2x-1} \times \frac{6x-3}{x+7}$$

$$(j) \quad \frac{x+8}{15} \times \frac{10}{x^3+8x^2} \qquad (k) \quad \frac{4}{x-2} \times \frac{x^2-2x}{8}$$

$$(l) \quad \frac{x^2+5x+6}{4} \times \frac{2}{x+2} \qquad (m) \quad \frac{x^2+2x-8}{x^2+5x+6} \times \frac{x+2}{x+4}$$

$$(n) \quad \frac{x^2+x-6}{x^2-25} \times \frac{x^2+10x+25}{x^2-4} \qquad (o) \quad \frac{3x^2+8x-3}{25} \times \frac{30}{6x^2+13x-5}$$

Fluency Practice

Factorise $\Rightarrow$ **Cancel** $\Rightarrow$ **Calculate**

To make each calculation easier we can factorise out **binomial expressions** (e.g. $x + 2$) from each fraction.
At which stage should we factorise out any integers or single variables?

a) $\frac{3}{2x+2} \times \frac{3x+3}{4} \Rightarrow \frac{3}{2(x+1)} \times \frac{3(x+1)}{4} \Rightarrow \frac{3}{2} \times \frac{3}{4} \Rightarrow$

b) $\frac{5}{3x+6} \times \frac{2x+4}{3} \Rightarrow \frac{5}{3(\quad)} \times \frac{2(\quad)}{3} \Rightarrow \quad \Rightarrow$

c) $\frac{4x-12}{5} \times \frac{1}{2x-6} \Rightarrow \frac{4(\quad)}{5} \times \frac{1}{2(\quad)} \Rightarrow \quad \Rightarrow$

d) $\frac{2x+8}{7} \times \frac{3}{5x+20} \Rightarrow \frac{\quad}{7} \times \frac{3}{\quad} \Rightarrow \quad \Rightarrow$

e) $\frac{2x+5}{6} \times \frac{2}{6x+15} \Rightarrow \frac{\quad}{6} \times \frac{2}{\quad} \Rightarrow \quad \Rightarrow$

f) $\frac{3y}{12-8x} \times \frac{9-6x}{5y} \Rightarrow \quad \Rightarrow \quad \Rightarrow$

g) $\frac{x^2+3x+2}{3} \times \frac{8}{3x+6} \Rightarrow \frac{(x+1)(\quad)}{\quad} \Rightarrow \quad \Rightarrow$

h) $\frac{x^2+3x-10}{10} \times \frac{5}{2x+10} \Rightarrow \quad \Rightarrow \quad \Rightarrow$

i) $\frac{x^2-9}{7} \times \frac{3}{4x-12} \Rightarrow \quad \Rightarrow \quad \Rightarrow$

j) $\frac{x^2+4x-5}{3x^2-6x} \times \frac{2x^2-14x+20}{x^2-25} \Rightarrow \quad \Rightarrow \quad \Rightarrow$

Fluency Practice

Question 1: Express the following as a single **simplified** fraction.

(a) $\frac{x}{2} \div \frac{2}{3}$

(b) $\frac{a}{c} \div \frac{d}{5}$

(c) $\frac{3}{w} \div \frac{2}{a}$

(d) $\frac{c}{4} \div \frac{3}{c}$

(e) $\frac{3a}{4} \div \frac{6c}{7}$

(f) $\frac{4x}{9y} \div \frac{6x}{7}$

(g) $\frac{10x}{3y} \div \frac{15x}{y}$

(h) $\frac{ab}{3} \div \frac{2a}{b}$

(i) $\frac{4fg}{h} \div \frac{f}{2h}$

Question 2: Express the following as a single fraction. **Simplify** if possible.

(a) $\frac{x-4}{8} \div \frac{3x-12}{2}$

(b) $\frac{x+3}{x+2} \div \frac{x+1}{x+2}$

(c) $\frac{x+1}{2} \div \frac{2x+2}{3}$

(d) $\frac{3x+9}{2} \div \frac{x+3}{4}$

(e) $\frac{4}{x-2} \div \frac{3}{x^2-2x}$

(f) $\frac{11}{4x^2+2x} \div \frac{3}{2x+1}$

(g) $\frac{x+3}{x+1} \div \frac{x}{(x+1)^2}$

(h) $\frac{x^2}{7} \div \frac{6x^3+8x^2}{x^2-7x}$

(i) $\frac{x}{x+6} \div \frac{x+6}{x^2}$

(j) $\frac{x^2+7x+10}{2} \div \frac{x^2+4x-5}{4}$

(k) $\frac{14}{x^2-5x+6} \div \frac{7}{x^2+3x-10}$

(l) $\frac{x+4}{x+5} \div \frac{3x+12}{x^2-25}$

(m) $\frac{x^3-x}{x+2} \div \frac{x^2-x}{x^2-5x-14}$

Fluency Practice

Q7 Simplify the following algebraic fractions.

$$[\text{a}] \quad \frac{1}{2x^2 + x - 10} \div \frac{1}{6x^2 + 15x}$$

$$[\text{b}] \quad \frac{1}{4x^2 + 11x - 3} \div \frac{1}{2x^2 + 6x}$$

$$[\text{c}] \quad \frac{1}{2x^2 + 5x + 2} \div \frac{1}{x^2 + 2x}$$

$$[\text{d}] \quad \frac{1}{3x^2 - 5x + 2} \div \frac{1}{6x^2 - 4x}$$

$$[\text{e}] \quad \frac{4}{2x^2 + 5x - 3} \div \frac{2}{10x^2 - 5x}$$

$$[\text{f}] \quad \frac{6}{3x^2 + 10x - 8} \div \frac{3}{2x^2 + 8x}$$

$$[\text{g}] \quad \frac{4}{5x^2 - 19x - 4} \div \frac{1}{3x^2 - 12x}$$

$$[\text{h}] \quad \frac{1}{4x^2 + 4x - 15} \div \frac{9}{4x^2 + 10x}$$

Fluency Practice

Q9 Simplify the following algebraic fractions.

[a]
$$\frac{x^3 + 7x^2 + 12x}{x^2 + 4x}$$

[b]
$$\frac{x^3 + 9x^2 + 8x}{x^2 + x}$$

[c]
$$\frac{x^2 + 7x}{x^3 + 11x^2 + 28x}$$

Q10 Show that,

$$\frac{(x^2 - x - 6)(x^2 - 3x - 4)(x^2 + 5x)}{(x^2 - 2x - 8)(x^2 + 2x - 15)(3x + 3)} = \frac{x}{3}$$

Fill in the Gaps

Fill in the table. One row has multiple possible correct answers.

First Expression	Second Expression	Highest Common Factor	Lowest Common Multiple
$3x^2y^3$	$4x^4$	x^2	
$(x + 3)^2(y + 1)$	$(x + 3)(x + 4)(y + 1)$		$(x + 3)^2(x + 4)(y + 1)$
$9x^3yz$	$6x^2y^2z^2$		
$x^2 + 7x + 6$	$(x + 6)(x + 2)$		
$x^2 - 9$		$x - 3$	$(x + 3)(x - 3)$
	$2x^2y$	$2xy$	$14x^2y^3$
$2y^2 - 7y - 4$	$3y^2 - 11y - 4$		
		$5a^2b$	$5a^2b$
		1	$10(x - 3)(x + 2)(x - 1)$
$12x^3y^2$		$6xy$	$36x^3y^2z$
$96ab^3$		24	$288ab^3$

Intelligent Practice

Simplify:

1) $\frac{3x}{2} + \frac{4x}{5}$

2) $\frac{x}{2} + \frac{4x}{5}$

3) $\frac{x}{2} + \frac{x}{5}$

4) $\frac{x}{2} - \frac{x}{5}$

5) $\frac{x}{5} - \frac{x}{2}$

6) $\frac{5x}{5} - \frac{x}{2}$

7) $\frac{5}{x} + \frac{2}{x}$

8) $\frac{5}{x^2} + \frac{2}{x}$

9) $\frac{5}{2x^2} - \frac{2}{x}$

10) $\frac{6}{2x^2} - \frac{2}{x}$

11) $\frac{6}{2xy} + \frac{2}{x}$

12) $\frac{6}{2xy} + \frac{2}{3x}$

13) $\frac{6}{2xy} - \frac{2}{3x^2}$

14) $\frac{6y}{2xy} + \frac{2}{3x^2}$

15) $\frac{y}{2xy} - \frac{2}{3x^3}$

Fluency Practice

Simplify:

$$\begin{array}{ll} \text{(a)} \quad \frac{x}{5} + \frac{x}{6} & \text{(b)} \quad \frac{x}{3} + \frac{x}{8} \\ \text{(c)} \quad \frac{x}{2} - \frac{x}{3} & \text{(d)} \quad \frac{x}{4} - \frac{x}{9} \\ \text{(e)} \quad \frac{2x}{3} + \frac{x}{4} & \text{(f)} \quad \frac{3x}{5} + \frac{2x}{9} \\ \text{(g)} \quad \frac{3x}{5} - \frac{2x}{7} & \text{(h)} \quad \frac{4x}{3} - \frac{5x}{8} \end{array}$$

Simplify:

$$\begin{array}{ll} \text{(a)} \quad \frac{3}{x} + \frac{5}{x} & \text{(b)} \quad \frac{6}{y} - \frac{4}{y} \\ \text{(c)} \quad \frac{2}{x} + \frac{5}{2x} & \text{(d)} \quad \frac{6}{xy} - \frac{4}{x} \\ \text{(e)} \quad \frac{2}{x} + \frac{7}{x^2} & \text{(f)} \quad \frac{4}{xy} - \frac{2}{3x} \\ \text{(g)} \quad \frac{5}{4a} - \frac{1}{b} & \text{(h)} \quad \frac{7}{ab} - \frac{3}{2a} \end{array}$$

Simplify:

$$\begin{array}{ll} \text{(a)} \quad \frac{3y}{6x} + \frac{5}{9x} & \text{(b)} \quad \frac{5}{xy} - \frac{3x}{y} \\ \text{(c)} \quad \frac{3}{ab} + \frac{6b}{5a} & \text{(d)} \quad \frac{7}{x^2} - \frac{2}{x} \end{array}$$

Simplify:

$$\begin{array}{ll} \text{(a)} \quad \frac{3}{x} + \frac{5}{3x} + \frac{2}{3} & \\ \text{(b)} \quad \frac{8}{xy} + \frac{2y}{x} - \frac{x}{y} & \\ \text{(c)} \quad \frac{9}{a^2b} - \frac{3}{ab} - \frac{1}{ab^2} & \end{array}$$

Fluency Practice

Q1 Write the following as a fraction in its simplest form.

[a] $\frac{x}{3} + \frac{x}{9}$

[e] $\frac{x}{8} + \frac{2x}{3}$

[b] $\frac{x}{2} - \frac{x}{8}$

[f] $\frac{3x}{4} - \frac{x}{5}$

[c] $\frac{x}{4} - \frac{x}{6}$

[g] $\frac{5x}{6} - \frac{x}{9}$

[d] $\frac{x}{10} + \frac{x}{4}$

[h] $\frac{3x}{7} + \frac{2x}{5}$

Q2 Write the following as a fraction in its simplest form.

[a] $\frac{3}{x} + \frac{9}{x}$

[e] $\frac{8}{x} + \frac{3}{2x}$

[b] $\frac{8}{x} - \frac{2}{x}$

[f] $\frac{5}{x} - \frac{4}{3x}$

[c] $\frac{6}{x} - \frac{4}{x}$

[f] $\frac{9}{x} - \frac{6}{5x}$

[d] $\frac{10}{x} + \frac{4}{x}$

[h] $\frac{7}{3x} + \frac{5}{2x}$

Fluency Practice

Q3 Work out the following.

$$[\text{a}] \frac{3}{x} + \frac{5}{x^2}$$

$$[\text{e}] \frac{5}{x^2} - \frac{5}{2x^2y}$$

$$[\text{i}] \frac{5}{3s} + \frac{2}{s^2t}$$

$$[\text{b}] \frac{6}{x^2} + \frac{2}{x}$$

$$[\text{f}] \frac{1}{ab^2} - \frac{2}{b}$$

$$[\text{j}] \frac{3}{u^2v^2} - \frac{2}{uv}$$

$$[\text{c}] \frac{4}{xy} - \frac{3}{x}$$

$$[\text{g}] \frac{2}{3x} + \frac{1}{3t^2}$$

$$[\text{k}] \frac{3}{2r} - \frac{2}{3st}$$

$$[\text{d}] \frac{10}{xy} - \frac{8}{xyz}$$

$$[\text{h}] \frac{8}{p^2} + \frac{3}{2pq}$$

$$[\text{l}] \frac{2}{5x^3} + \frac{3x}{2}$$

Fluency Practice

Q4 Work out the following.

[a] $\frac{1}{x} + \frac{2}{y} + \frac{3}{z}$

[b] $\frac{5}{m} + \frac{4}{n} + \frac{1}{p}$

[c] $\frac{1}{2r} + \frac{2}{s} + \frac{3}{2t}$

[d] $\frac{1}{2a} + \frac{2}{3b} + \frac{3}{4c}$

Q5 Gemma says that,

$$\frac{2x}{5} + \frac{x}{7} = \frac{3x}{12}$$

Why is Gemma wrong?

Work out the correct answer.

Q6 Find the value of,

$$\frac{x}{2} + \frac{x}{3} + \frac{x}{6}$$

Q7 Write the following as a single fraction in its simplest form.

$$[a] \frac{x+2}{7} + \frac{x}{3}$$

$$[e] \frac{x+1}{2} - \frac{x+2}{4}$$

$$[b] \frac{x+5}{4} + \frac{2x}{3}$$

$$[f] \frac{x+8}{2} - \frac{x+3}{9}$$

$$[c] \frac{x+5}{2} + \frac{x+2}{5}$$

$$[g] \frac{3x+2}{10} + \frac{5x-4}{6}$$

$$[d] \frac{x+1}{6} - \frac{x-1}{7}$$

$$[h] \frac{3x+1}{6} + \frac{4x-1}{4}$$

Fluency Practice

Simplify the following as far as possible:

1) $\frac{6}{2xy} + \frac{2}{x}$

2) $\frac{6}{2xy} + \frac{2}{3x}$

3) $\frac{6}{2xy} - \frac{2}{3x^2}$

4) $\frac{6y}{2xy} + \frac{2}{3x^2}$

5) $\frac{y}{2xy} - \frac{2}{3x^3}$

Simplify the following as far as possible:

1) $\frac{1}{x^2-4} - \frac{1}{x-2}$

2) $\frac{x}{x^2-4} - \frac{1}{x-2}$

3) $\frac{x}{x^2-x-2} - \frac{1}{x-2}$

4) $\frac{x-3}{x^2-x-2} - \frac{1}{x-2}$

5) $\frac{x-3}{x^2-x-2} - \frac{2x}{x-2}$

Simplify the following as far as possible:

1) $\frac{2x+1}{x+4} - \frac{x-5}{x-2}$

2) $\frac{11x+27}{2x^2+11x-6} - \frac{3}{x+6}$

Challenge: Express the following as a fraction in its simplest form:

$$\frac{3}{x-1} + \frac{2}{x+1} + \frac{4}{x-3}$$

Fluency Practice

adding/subtracting algebraic fractions (simplification)

$$(1) \quad \frac{4m + 1}{4} + \frac{1 - 3m}{3}$$

$$(2) \quad \frac{4m - 1}{4} + \frac{5 - 6m}{6}$$

$$(3) \quad \frac{3m + 1}{3} - \frac{4m - 1}{4}$$

$$(4) \quad \frac{3m + 2}{6} + \frac{2 - 4m}{8}$$

$$(5) \quad \frac{3d + 4}{2} - \frac{8 + 6d}{4}$$

$$(6) \quad \frac{6d - 4}{8} - \frac{3d - 2}{4}$$

$$(7) \quad \frac{6d + 2}{4} - \frac{15d + 5}{10}$$

$$(8) \quad \frac{3d - 9}{9} - \frac{2d - 6}{6}$$

$$(9) \quad \frac{3k - 4}{5} + \frac{2 - k}{2}$$

$$(10) \quad \frac{6k - 3}{4} + \frac{3 - 4k}{3}$$

$$(11) \quad \frac{3k + 2}{3} - \frac{4k + 2}{5}$$

$$(12) \quad \frac{4k - 7}{6} - \frac{4k - 9}{7}$$

$$(13) \quad \frac{3f - 4}{4} + \frac{2f - 3}{5}$$

$$(14) \quad \frac{7f - 1}{5} + \frac{3f - 6}{4}$$

$$(15) \quad \frac{7f + 6}{5} - \frac{3f + 11}{4}$$

$$(16) \quad \frac{7f - 11}{4} - \frac{4f + 9}{5}$$

Q8 Write the following as a single fraction in its simplest form.

$$[\text{a}] \quad \frac{2}{x+2} + \frac{5}{x}$$

$$[\text{e}] \quad \frac{2}{x+1} - \frac{4}{x+2}$$

$$[\text{b}] \quad \frac{1}{x+5} + \frac{5}{2x}$$

$$[\text{f}] \quad \frac{5}{x+3} - \frac{9}{x+11}$$

$$[\text{c}] \quad \frac{3}{x+5} + \frac{4}{x+2}$$

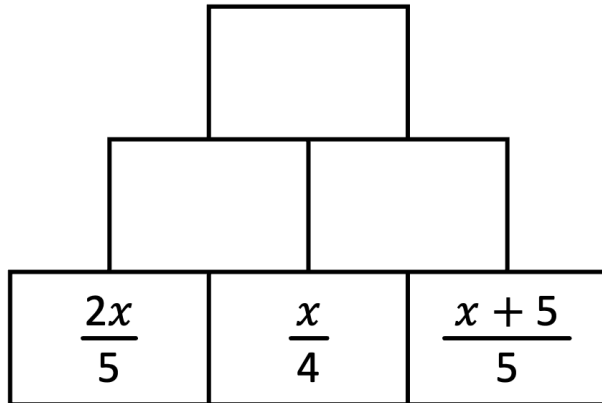
$$[\text{g}] \quad \frac{10}{3x+2} + \frac{2}{5x-4}$$

$$[\text{d}] \quad \frac{6}{x+1} - \frac{3}{x-1}$$

$$[\text{h}] \quad \frac{2}{2x+1} + \frac{5}{4x-1}$$

Fluency Practice

Q9 Fill in the missing boxes so that each block is the sum of the two below it.



Q10 Fill in the empty squares to make all the equations in this grid correct both horizontally and vertically.

	+		=	$\frac{2x}{3}$
+		+		+
$\frac{2x}{3}$	+	$\frac{2x}{5}$	=	
=		=		=
	+	$\frac{14x}{15}$	=	

Q8 Simplify the following algebraic fractions.

$$[a] \quad \frac{1}{3x^2 + 11x - 4} \div \frac{1}{3x + 12} + \frac{2}{3x + 1}$$

$$[b] \quad \frac{1}{5x^2 + 14x - 3} \div \frac{1}{2x + 6} + \frac{3}{2x - 5}$$

$$[c] \quad \frac{1}{2x^2 + 5x - 12} \div \frac{1}{5x + 20} - \frac{1}{4x + 3}$$

$$[d] \quad \frac{1}{6x^2 + x - 1} \div \frac{1}{9x - 3} - \frac{2}{3x - 2}$$

Fluency Practice

A1 Simplify

$$\frac{x}{5} + \frac{3x}{4}$$

A2 Simplify

$$\frac{3x}{2} - \frac{5x}{6}$$

A3 Simplify

$$\frac{x+3}{5} + \frac{x-5}{4}$$

A4 Simplify

$$\frac{2x-1}{3} - \frac{2}{5}$$

B1 Simplify

$$\frac{4}{x-1} + \frac{3}{x+1}$$

B2 Simplify

$$\frac{3}{x+1} - \frac{6}{2x+3}$$

B3 Simplify

$$\frac{3}{2x-3} - \frac{1}{x+2}$$

B4 Simplify

$$\frac{3}{x-1} - \frac{2}{x}$$

C1 Simplify

$$\frac{2}{x-1} + \frac{x-11}{x^2+3x-4}$$

C2 Simplify

$$\frac{2}{x+2} + \frac{x}{x^2+5x+6}$$

C3 Simplify

$$\frac{2}{x-1} + \frac{x}{x^2-4x+3}$$

C4 Simplify

$$\frac{5}{2x-6} - \frac{x+2}{x^2-4x+3}$$

D1 Simplify

$$1 + \frac{4}{(x+3)(x-2)}$$

D2 Simplify

$$1 + \frac{x^2+x-6}{(x+4)(x-2)}$$

D3 Simplify

$$2 + \frac{5-x}{x^2-2x-15}$$

D4 Simplify

$$5 - (x+2) \div \left(\frac{x^2-4}{x-3} \right)$$

Fluency Practice

1. Combine the following algebraic fractions, fully simplifying your answer.

$$(a) \frac{2}{x+4} + \frac{3}{x+1}.$$

$$(b) \frac{3}{x-2} + \frac{5}{x+4}.$$

$$(c) \frac{3}{x-1} - \frac{x+2}{x+2}.$$

$$(d) \frac{1}{x-5} - \frac{x-7}{x-7}.$$

$$(e) \frac{3}{2x+4} - \frac{1}{3x+6}.$$

$$(f) \frac{a}{x+b} + \frac{a}{x+c}.$$

$$(g) 2 + \frac{3}{x-5}.$$

$$(h) \frac{5}{2-x} + 3 + \frac{1}{x+1}.$$

$$(i) \frac{a}{x+k} + \frac{b}{x+2k}.$$

$$(j) \frac{5}{2x+3} - 2x.$$

$$(k) \frac{1}{(x-1)^2} - \frac{2}{x-1}.$$

2. Write the following as a single fraction.

$$(a) \frac{1}{\frac{2}{x-4} - 3}.$$

$$(b) \frac{3}{\frac{1}{x-2} + \frac{2}{x}}.$$

$$(c) \frac{\frac{1}{x}}{2 - \frac{3}{x}}.$$

$$(d) \frac{\frac{4}{x-1}}{1 - \frac{3}{x-1}}.$$

$$(e) \frac{2 + \frac{5}{x+3}}{\frac{1}{x+3} - 7}.$$

$$(f) \frac{\frac{a - c}{x+b}}{e + \frac{f}{x+b}}.$$

$$(l) \frac{5}{x^2+3x} + \frac{2}{x+3}.$$

$$(m) \frac{x}{x^2-x-6} - \frac{3}{2x+4}.$$

$$(n) \frac{x+1}{x^2-4x+3} - \frac{x-3}{x^2-1}.$$

$$(o) \frac{2}{x} - \frac{3}{x^2-x} - \frac{4}{x-1}.$$

$$(p) \frac{1}{x-1} - \frac{1}{x^2+x-2} + \frac{4}{x+2}$$

$$(q) \frac{2}{x^2-x} - \frac{1}{x^2-1}.$$

$$(r) \frac{3}{2x-3} + \frac{1}{x-4} - \frac{5x}{2x^2-11x+12}.$$

$$(s) \frac{a}{x+1} - \frac{b}{x^2+2x+1} - \frac{c}{(x+1)^3}.$$

$$(g) \frac{\frac{3}{x-2} + 2}{1 - \frac{4}{x}}.$$

$$(h) \frac{\frac{4}{3x}}{\frac{1}{1 - \frac{2}{x+1}}}.$$

$$(i) \frac{\frac{1}{x-1} + \frac{3}{x-2}}{\frac{1}{x-1} - \frac{2}{x+3}}.$$

$$(j) \frac{1 - \frac{2}{x-3}}{\frac{1}{x+2} - 1}.$$

$$(k) \frac{\frac{a-b}{b} + \frac{c-b}{c}}{\frac{a}{b} - \frac{b}{c}}.$$

$$(l) 1 + \frac{1}{1 + \frac{1}{1+\frac{1}{x}}}.$$

Fluency Practice

$$(m) \quad 3 - \frac{\frac{5}{x-1}}{x - \frac{4}{x-1}}.$$

$$(o) \quad \frac{2}{3 + \frac{4}{5 + \frac{6}{x}}} - \frac{1}{2 + \frac{3}{4 + \frac{5}{x}}}.$$

$$(n) \quad \frac{1}{\frac{1}{x} + 1} - \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}.$$

3. Cancel the following to their simplest form.

$$(a) \quad \frac{x^2 + 3x}{2x + 6}.$$

$$(h) \quad \frac{x^2 + 6x + 8}{x^2 - 4x + 3} \div \frac{2x + 8}{x - 3}.$$

$$(b) \quad \frac{x^2 - 1}{x^2 + 5x + 4}.$$

$$(i) \quad \frac{\pi x^2 - \pi x - 2\pi}{x^3 + x^2 - 6x} \div \frac{2x + 2}{3x + 9}.$$

$$(c) \quad \frac{2x^2 + 7x - 4}{2x^2 + 9x - 5}.$$

$$(j) \quad \frac{12x^2 + 10x + 2}{2x - 6} \div \frac{18x^2 + 15x + 3}{5x - 15}.$$

$$(d) \quad \frac{4x^2 - 8x - 12}{2x^2 - 2x - 12}.$$

$$(k) \quad \frac{2x^2 + 2x - 4}{3x^3 - 3x} \div \frac{4x + 8}{9 + 9x}.$$

$$(e) \quad \frac{4\pi x^2 - \pi}{12\pi x^2 - 10\pi x + 2\pi}.$$

$$(l) \quad \frac{4x^2 - 1}{2x^2 - 7x - 4} \div \frac{2x^2 + 5x - 3}{2x^2 - 2x - 24}.$$

$$(f) \quad \frac{2a^2x^2 + a^2x - 10a^2}{2ax^2 - 2ax - 4a}.$$

$$(m) \quad \frac{6\pi x^2 + 19\pi x + 10\pi}{3\pi x^3 + 4\pi x^2 - 7\pi x} \div \frac{6x + 15}{18x + 42}.$$

$$(g) \quad \frac{12x^4 + 17x^3 - 5x^2}{9x^3 - 25x}.$$

4. Find the value of the capital letters which make the following identities correct.

$$(a) \quad \frac{A}{x} + \frac{B}{x+1} \equiv \frac{5x+2}{x(x+1)}.$$

Fluency Practice

Q1

Solve the following equations.

$$[\text{a}] \frac{x+3}{4} + \frac{x+4}{5} = 2$$

$$[\text{e}] \frac{x+2}{2} + \frac{7x-7}{4} = 6$$

$$[\text{b}] \frac{x-1}{2} + \frac{x+3}{6} = 4$$

$$[\text{f}] \frac{x+5}{3} + \frac{2x+3}{4} = 2$$

$$[\text{c}] \frac{x-1}{4} - \frac{x+1}{6} = 1$$

$$[\text{g}] \frac{x+3}{6} + \frac{3x+8}{5} = 9$$

$$[\text{d}] \frac{x+2}{2} + \frac{x+3}{3} = 1$$

$$[\text{h}] \frac{x+2}{6} + \frac{7x+6}{2} = 6$$

Q2

Solve the following equations.

$$[\text{a}] \frac{3x+1}{2} + \frac{5x+7}{6} = 2$$

$$[\text{e}] \frac{8x-1}{5} + \frac{4x-5}{2} = 3$$

$$[\text{b}] \frac{6x-1}{3} - \frac{3x+1}{7} = 10$$

$$[\text{f}] \frac{3x-2}{6} + \frac{5x+6}{7} = 7$$

$$[\text{c}] \frac{2x+5}{2} - \frac{3x+1}{5} = 2$$

$$[\text{g}] \frac{9x+8}{6} + \frac{4x+3}{5} = 5$$

$$[\text{d}] \frac{2x+4}{7} + \frac{2x-3}{2} = 1$$

$$[\text{h}] \frac{2x+7}{6} + \frac{2x-8}{3} = 5$$

Fluency Practice

Q3

Solve the following equations.

$$[a] \frac{2}{x+6} + \frac{4}{x+8} = 2$$

$$[b] \frac{4}{x-5} + \frac{2}{x-8} = 3$$

$$[c] \frac{2}{x-9} + \frac{4}{x-7} = 2$$

$$[d] \frac{3}{x-1} + \frac{2}{x-3} = 1$$

$$[e] \frac{1}{2x-7} + \frac{6}{x+4} = 1$$

$$[f] \frac{1}{2x-5} + \frac{4}{x+2} = 1$$

$$[g] \frac{5}{2x-3} + \frac{2}{x-6} = 1$$

$$[h] \frac{2}{5x+2} + \frac{5}{x+4} = 1$$

Q4

Solve the following equations.

$$[a] \frac{4}{2x-1} + \frac{6}{4x-7} = 2$$

$$[b] \frac{5}{2x-6} + \frac{3}{2x+6} = 1$$

$$[c] \frac{6}{4x+1} + \frac{4}{2x+3} = 2$$

$$[d] \frac{2}{4x-9} + \frac{4}{2x-3} = 2$$

$$[e] \frac{6}{2x-3} + \frac{4}{3x-7} = 2$$

$$[f] \frac{3}{4x+7} + \frac{6}{2x+5} = 3$$

$$[g] \frac{3}{4x+9} + \frac{2}{2x+7} = 1$$

$$[h] \frac{3}{2x-1} + \frac{5}{2x-7} = 2$$

Fluency Practice

Question 1: Express the following as a single simplified fraction.

(a) $\frac{x}{3} + \frac{x}{5}$

(b) $\frac{c}{2} + \frac{c}{7}$

(c) $\frac{w}{3} + \frac{w}{9}$

(d) $\frac{x}{2} - \frac{x}{3}$

(e) $\frac{a}{5} - \frac{a}{9}$

(f) $\frac{m}{2} - \frac{m}{8}$

(g) $\frac{m}{3} + \frac{2m}{7}$

(h) $\frac{3x}{5} + \frac{x}{2}$

(i) $\frac{3c}{4} + \frac{5c}{9}$

(j) $\frac{m}{2} - \frac{2m}{5}$

(k) $\frac{3n}{4} - \frac{5n}{9}$

(l) $\frac{7h}{8} - \frac{5h}{12}$

Question 2: Express the following as a single simplified fraction.

(a) $\frac{2}{x^2} + \frac{5}{x}$

(b) $\frac{5}{6x} - \frac{1}{3x}$

(c) $\frac{2}{fg} - \frac{4}{g}$

(d) $\frac{6}{ac} + \frac{2}{3}$

(e) $\frac{9}{w} + \frac{wx}{4}$

(f) $\frac{d}{3} + \frac{2}{d^2}$

(g) $\frac{m^2}{6} - \frac{9}{4m}$

(h) $\frac{3}{4b^2} - \frac{1}{2b}$

(i) $\frac{ac}{5} + \frac{4}{c}$

(j) $\frac{x^3}{w} - \frac{2}{wx^2}$

(k) $\frac{2}{ab^2} - \frac{3}{b^3}$

(l) $\frac{xy}{5} - \frac{1}{x}$

Fluency Practice

Question 3: Express the following as a single simplified fraction.

(a) $\frac{x+4}{3} + \frac{x+1}{2}$ (b) $\frac{2x+1}{2} + \frac{x+3}{5}$ (c) $\frac{5x-2}{3} + \frac{2x+7}{4}$

(d) $\frac{3x+11}{2} + \frac{x-20}{3}$ (e) $\frac{5x-4}{2} + \frac{x+1}{3}$ (f) $\frac{x-4}{8} + \frac{2x-3}{2}$

(g) $\frac{7x+4}{2} - \frac{x+1}{3}$ (h) $\frac{9x+1}{5} - \frac{x+2}{2}$ (i) $\frac{3x+11}{2} - \frac{2x-3}{6}$

(j) $\frac{x-8}{3} - \frac{x-5}{7}$ (k) $\frac{4x-7}{10} - \frac{2x-9}{3}$ (l) $\frac{5-x}{6} - \frac{8-3x}{10}$

Question 4: Express the following as a single simplified fraction.

(a) $\frac{2}{x+5} + \frac{3}{x+1}$ (b) $\frac{2}{x+1} + \frac{1}{x+3}$ (c) $\frac{4}{x+5} - \frac{2}{x-1}$

(d) $\frac{x+1}{x-2} + \frac{x+3}{x+5}$ (e) $\frac{x+3}{2x+1} - \frac{x-2}{x-1}$ (f) $\frac{x}{x+7} + \frac{2x+5}{3x+1}$

(g) $\frac{3}{x+1} + \frac{x+7}{(x+1)(x+2)}$ (h) $\frac{1-x}{(x-7)(x+1)} - \frac{2}{x-7}$

Question 5: Solve the following equations

(a) $\frac{x+5}{3} + \frac{x+1}{2} = 8$ (b) $\frac{x+6}{5} + \frac{x-3}{2} = 13$

(c) $\frac{2x-1}{4} + \frac{x-3}{5} = 3$ (d) $\frac{x+5}{10} - \frac{x+4}{2} = 1$

(e) $\frac{x-3}{4} - \frac{x-8}{3} = 4$

Fluency Practice

Question 5: Solve the following equations

$$(a) \frac{x+5}{3} + \frac{x+1}{2} = 8$$

$$(b) \frac{x+6}{5} + \frac{x-3}{2} = 13$$

$$(c) \frac{2x-1}{4} + \frac{x-3}{5} = 3$$

$$(d) \frac{x+5}{10} - \frac{x+4}{2} = 1$$

$$(e) \frac{x-3}{4} - \frac{x-8}{3} = 4$$

Question 6: Solve the following equations

$$(a) \frac{4}{x+1} + \frac{2}{x-2} = 3$$

$$(b) \frac{2}{x-3} + \frac{1}{x-4} = 2$$

$$(c) \frac{2}{x-5} - \frac{2}{x-4} = 1$$

$$(d) \frac{2}{x+1} - \frac{10}{x+4} = -1$$

$$(e) \frac{3}{x-2} + \frac{3}{x+2} = 2$$

$$(f) \frac{11}{(x-1)(x+4)} + \frac{5}{x-1} = 1$$

Fluency Practice

Solve

(a) $\frac{x-1}{3} = 2$ (b) $\frac{2x-3}{5} = 7$

(c) $\frac{x+1}{10} = \frac{3x}{5}$ (d) $\frac{x-8}{6} = \frac{3-x}{4}$

Solve

(a) $\frac{x}{2} + \frac{x}{3} = 5$ (b) $\frac{8x}{15} - \frac{x}{5} = 6$

(c) $\frac{x+3}{8} + \frac{x+4}{3} = 4$

(d) $\frac{x+1}{3} + \frac{x}{4} = 5$

(e) $\frac{x+1}{2} + \frac{3x-1}{4} = 4$

(f) $\frac{x+13}{2} - \frac{12-3x}{3} = 1$

Solve

(a) $\frac{4}{x+1} + \frac{5}{x+2} = 2$

(b) $\frac{3}{x+2} + \frac{4}{x-3} = 2$

(c) $\frac{5}{3x+2} - \frac{3}{2x-3} = 4$

(d) $\frac{18}{4x-1} - \frac{1}{x+1} = 1$

(e) $\frac{3}{2x-1} - \frac{4}{3x-1} = 1$

Fluency Practice

simplifying and using algebraic fractions

express as a single fraction:

$$(1) \quad \frac{4}{2x-1} - \frac{4}{2x+1}$$

$$(2) \quad \frac{5}{x-1} + \frac{5}{x+1}$$

$$(3) \quad \frac{2x+1}{3} + \frac{x-1}{4}$$

$$(4) \quad \frac{x}{x+1} - \frac{x}{x+7}$$

$$(5) \quad \frac{x}{x-7} - \frac{x}{x-1}$$

$$(6) \quad \frac{x+1}{x+2} - \frac{x}{x+1}$$

solve the equations

$$(7) \quad \frac{x}{4} + \frac{x+5}{3} = 4$$

$$(8) \quad \frac{x+1}{2} + \frac{x+2}{5} = 3$$

$$(9) \quad \frac{2x+1}{3} - \frac{x-4}{5} = 3$$

$$(10) \quad \frac{5x-2}{7} - \frac{3x-4}{4} = \frac{1}{4}$$

$$(11) \quad \frac{3}{2x} + \frac{1-x}{x} = 4$$

$$(12) \quad \frac{x-1}{2x} + \frac{x+3}{4x} = \frac{5}{8}$$

Fluency Practice

solve the algebraic fraction equations (ii)

$$(1) \quad \frac{n}{12} + \frac{1}{4n} = \frac{1}{n}$$

$$(2) \quad \frac{n}{20} + \frac{1}{5n} = \frac{1}{n}$$

$$(3) \quad \frac{2}{n} - \frac{n}{5} = \frac{n}{20}$$

$$(4) \quad \frac{n}{6} + \frac{4}{n} = \frac{49}{6n}$$

$$(5) \quad \frac{1}{n+2} + \frac{2}{3n} = \frac{3}{2n}$$

$$(6) \quad \frac{2}{n-1} - \frac{n}{15} = \frac{n}{10}$$

$$(7) \quad \frac{1}{n+1} + \frac{1}{n} = \frac{5}{3n}$$

$$(8) \quad \frac{3}{n+1} - \frac{n}{5} = \frac{n}{20}$$

$$(9) \quad \frac{2}{n-8} + \frac{n}{2} = \frac{n}{3}$$

$$(10) \quad \frac{4}{n-3} + \frac{n}{10} = \frac{n}{2}$$

$$(11) \quad \frac{n+1}{10} + \frac{2n}{20} = \frac{3}{3n}$$

$$(12) \quad \frac{2}{n} - \frac{n}{5} = \frac{1}{15}$$

Fluency Practice

solve the algebraic fraction equations (iii)

$$(1) \quad \frac{a}{2} - \frac{2}{a+1} = 1$$

$$(2) \quad \frac{4}{b+3} + \frac{3(b-6)}{b-2} = 1$$

$$(3) \quad \frac{4}{c+4} + \frac{3}{2c+1} = 1$$

$$(4) \quad \frac{3}{d+2} + \frac{4}{2d-1} = \frac{7}{5}$$

$$(5) \quad \frac{5}{e+2} - \frac{3}{e-1} = 8$$

$$(6) \quad \frac{6}{12f+1} + \frac{1}{2f+1} = 1$$

$$(7) \quad \frac{6}{g+2} - \frac{g-1}{3g-6} = \frac{1}{2}$$

$$(8) \quad \frac{3}{h} + \frac{4}{h+2} = 1$$

$$(9) \quad \frac{4}{j-1} - \frac{2}{j+2} = 24$$

$$(10) \quad \frac{k}{k-1} - \frac{k-1}{k} = \frac{1}{12}$$

$$(11) \quad \frac{(m+1)^2}{6} + \frac{(m+3)^2}{4} = 1$$

$$(12) \quad \frac{24}{n^2+n-6} + \frac{6}{n+3} = \frac{5}{n-2}$$

Fluency Practice

algebraic fractions

past paper questions from WJEC

solve

express as a single fraction
in simplest form

$$(1) \quad \frac{3x-1}{4} - \frac{x+6}{3} = \frac{3}{2}$$

$$(6) \quad \frac{7}{x-3} - \frac{4}{3x+5}$$

$$(2) \quad \frac{4x-5}{2} + \frac{7x-10}{5} = 4$$

$$(7) \quad \frac{7x}{2x+3} + \frac{2}{3x-2}$$

$$(3) \quad \frac{8x-5}{3} + \frac{4x+5}{4} = \frac{149}{12}$$

$$(8) \quad \frac{x}{x-3} - \frac{x}{x+6}$$

$$(4) \quad \frac{8x+1}{3} - \frac{4x+7}{2} = \frac{1}{2}$$

$$(9) \quad \text{prove that} \\ \frac{2x+3}{4} - \frac{3x-2}{3} + \frac{1}{6} \equiv \frac{19-6x}{12}$$

$$(5) \quad \frac{12x-1}{5} - \frac{x}{2} = -4$$

$$(10) \quad \text{show that the following identity is true} \\ \frac{3x+2}{5} - \frac{5x-2}{4} + \frac{7}{10} \equiv \frac{32-13x}{20}$$

Fluency Practice

(a)	(b)	(c)	(d)
Simplify $\frac{x(x-5)}{x^2}$	Simplify $\frac{x^2+2x}{x^2+7x+10}$	Simplify $\frac{x^2-x-6}{x^2+6x+8}$	Simplify $\frac{3x^2-13x+4}{x^2-16}$
(e)	(f)	(g)	(h)
Simplify $\frac{x^2}{x-2} \times \frac{3x-6}{4x}$	Simplify $\frac{x(x-1)}{x^2+2x} \div \frac{x^2-1}{3x}$	Write as a single fraction $\frac{x}{3} + \frac{2x}{5}$	Write as a single fraction $\frac{5x}{6} - \frac{x}{4}$
(i)	(j)	(k)	(l)
Simplify fully $\frac{5}{x^2} + \frac{4}{3x}$	Simplify fully $\frac{3}{x+2} - \frac{2}{x-1}$	Write as a single fraction $\frac{2x-1}{5} + \frac{x+3}{4}$	Simplify fully $\frac{x}{2x+4} - \frac{3}{x^2+5x+6}$
(m)	(n)	(o)	
Solve $\frac{x}{3} + \frac{2x-1}{4} = 1$	Solve $\frac{2(x+1)}{5} = \frac{8+x}{6}$	Solve $\frac{5}{x+3} + \frac{3}{x-1} = 4$	

Fluency Practice

A1 Solve

$$\frac{3x}{5} + 3 = 7$$

A2 Solve

$$\frac{x+4}{3} = 11$$

A3 Solve

$$\frac{5x-11}{3} = x$$

A4 Solve

$$\frac{7x-16}{2} = 2x-3$$

B1 Solve

$$\frac{x+4}{2} = \frac{x+10}{3}$$

B2 Solve

$$\frac{x-2}{3} = \frac{x+4}{5}$$

B3 Solve

$$\frac{x-10}{10} = \frac{10-x}{3}$$

B4 Solve

$$\frac{2}{5x-2} = \frac{3}{6x+1}$$

C1 Solve

$$\frac{x-1}{2} + \frac{2x+3}{4} = 1$$

C2 Solve

$$\frac{2x-1}{4} + \frac{x-1}{5} = 2$$

C3 Solve

$$\frac{6x-1}{4} - \frac{5-2x}{2} = 1$$

C4 Solve

$$\frac{2x+1}{3} + \frac{x-5}{2} = 4$$

D1 Solve

$$\frac{6}{x-2} - \frac{6}{x+1} = 1$$

D2 Solve

$$\frac{3}{x+2} + \frac{4}{x-3} = 2$$

D3 Solve

$$\frac{3}{x+1} + \frac{2}{2x-3} = 1$$

D4 Solve

$$\frac{4}{2x+1} + \frac{1}{4x^2-1} = 3$$

Fluency Practice

Simplify

$$(a) \quad \frac{x}{2} \times \frac{x}{5} \qquad (b) \quad \frac{2x}{5} \times \frac{10}{3y}$$

$$(c) \quad \frac{2x^2}{7} \times \frac{x}{5} \qquad (d) \quad \frac{5xy}{11} \times \frac{33}{2x}$$

$$(e) \quad \frac{x}{3} \div \frac{x}{7} \qquad (f) \quad \frac{2x}{5} \div \frac{10y}{3}$$

$$(g) \quad \frac{2x^2}{7} \div \frac{x}{5} \qquad (h) \quad \frac{5x}{11y} \div \frac{3}{2x}$$

Simplify

$$(a) \quad \frac{x}{5} + \frac{x}{6} \qquad (b) \quad \frac{3x}{5} - \frac{2x}{9}$$

$$(c) \quad \frac{3}{x} + \frac{5}{2x} \qquad (d) \quad \frac{6}{xy} - \frac{4}{3x}$$

$$(e) \quad \frac{2}{x} + \frac{7}{x^2} \qquad (f) \quad \frac{x}{2} + \frac{x+1}{4}$$

$$(g) \quad \frac{2x}{3} + \frac{x+2}{9} \qquad (h) \quad \frac{x+4}{5} - \frac{x}{7}$$

Simplify

$$(a) \quad \frac{x-3}{5} \times \frac{15}{2x-6} \qquad (b) \quad \frac{x}{6} \div \frac{2x^2+x}{3}$$

$$(c) \quad \frac{x^2-1}{8} \div \frac{x+1}{4} \qquad (d) \quad \frac{5x+5}{4} \times \frac{x^2+2x+1}{2}$$

Simplify

$$(a) \quad \frac{x+2}{2} + \frac{x+3}{5} \qquad (b) \quad \frac{x-3}{3} + \frac{x+4}{7}$$

$$(c) \quad \frac{2x+1}{2} - \frac{3x+1}{5} \qquad (d) \quad \frac{7x+4}{2} - \frac{2x-1}{7}$$

Simplify

$$(a) \quad \frac{2}{x+1} + \frac{3}{x+2} \qquad (b) \quad \frac{5}{x-3} - \frac{3}{x+2}$$

$$(c) \quad \frac{5}{x-3} - \frac{3}{2x+1} \qquad (d) \quad \frac{8}{x-1} + \frac{2}{3x-2}$$

Intelligent Practice

1) Make r the subject of $\frac{6}{p} = \frac{7}{q} - \frac{6}{r}$

2) Make b the subject of $\frac{9}{a} = \frac{10}{b} - \frac{2}{c}$

3) Make z the subject of $\frac{1}{x} = \frac{3}{y} - \frac{3}{z}$

4) Make b the subject of $\frac{1}{a} = \frac{10}{b} + \frac{6}{c}$

5) Make q the subject of $\frac{6}{p} = \frac{1}{q} + \frac{9}{r}$

1) Make p the subject of $\frac{1}{10p} - \frac{4}{q} = \frac{7}{s}$
Give your answer in its simplest form.

2) Make x the subject of $\frac{1}{9x} - \frac{1}{y} = \frac{4z}{w}$
Give your answer in its simplest form.

3) Make a the subject of $\frac{4}{2a} - \frac{1}{b} = \frac{1}{d}$
Give your answer in its simplest form.

4) Make a the subject of $\frac{1}{2a} + \frac{5}{b} = \frac{9}{d}$
Give your answer in its simplest form.

5) Make a the subject of $\frac{3}{5a} + \frac{1}{b} = \frac{c}{d}$
Give your answer in its simplest form.