



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2024 Mathematics 2025

Unit 20 Tasks – Part 1

DO NOT WRITE INSIDE



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SCHOOL FOR BOYS



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Unit 20 Tasks – Part 2

DO NOT WRITE INSIDE

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1 Graphical Simultaneous Equations

Fluency Practice

Solving Simultaneous Equations

Graphically

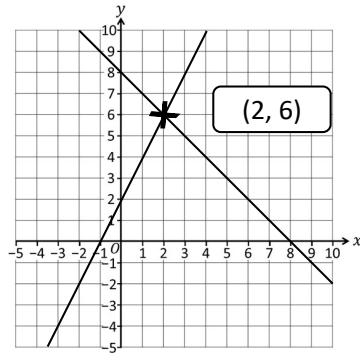
Plot each equation on the same grid.

You may want to rearrange the equations first.

The intersection shows the values (x, y) that satisfy both equations.

Check the solution by substituting values back into the equations.

$$y = 2x + 2 \quad y + x = 8$$



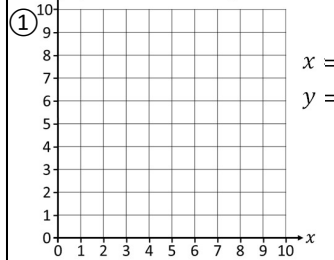
Solution: $x = 2 \quad y = 6$

Substitute to check:

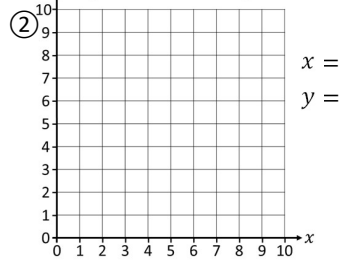
$$y = 2x + 2 \quad 6 = 2(2) + 2 \quad \checkmark$$

$$y + x = 8 \quad 6 + 2 = 8 \quad \checkmark$$

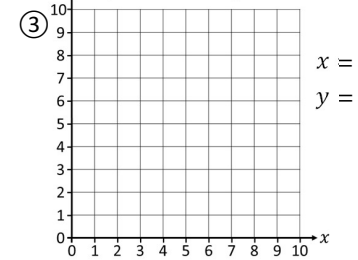
$$y = x + 1 \quad y + x = 9$$



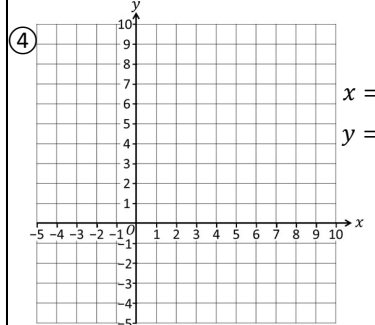
$$y = x + 3 \quad y + x = 7$$



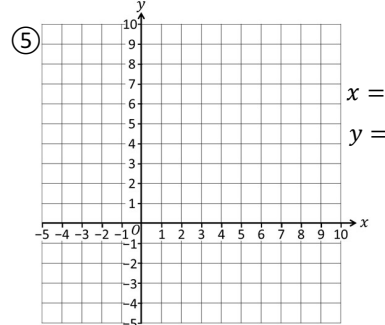
$$y = 2x \quad y + 2x = 8$$



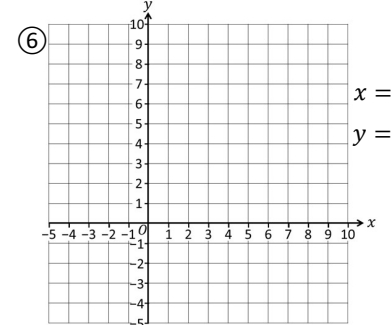
$$y = 3x - 3 \quad 2y + 3x = 12$$



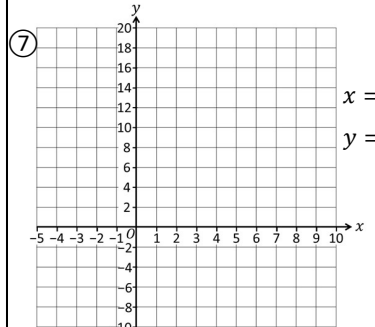
$$y = 5 - 2x \quad y = 0.5x - 5$$



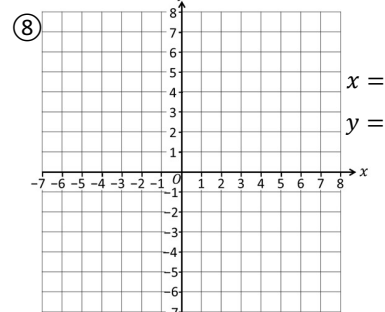
$$y - x = -3 \quad 2y - 3x = -9$$



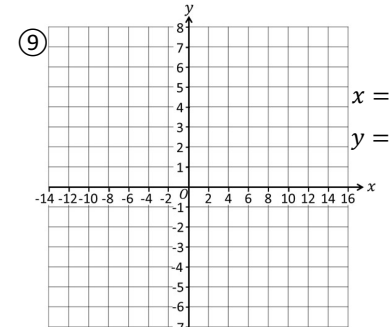
$$2y + 5x = 30 \quad y = 6x - 2$$



$$4y + 5x = -20 \quad y = \frac{1}{2}x + 5.5$$



$$2x = -24 - 6y \quad y - 10 = 2x$$



Fluency Practice

Plot both lines on the same graph. The point of intersection represents the solution.

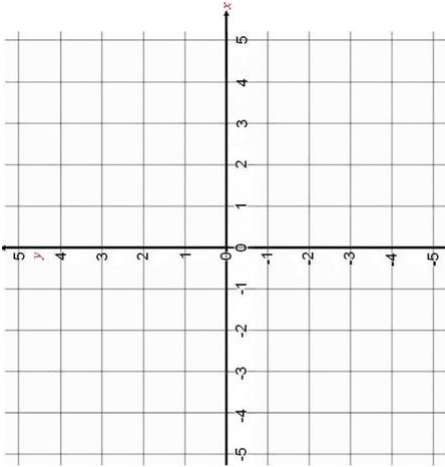
1. $y = 3x - 1$
 $y = 2x$

$y = 3x - 1$

x	-3	-2	-1	0	1	2	3
y							

$y = 2x$

x	-3	-2	-1	0	1	2	3
y							



Point of intersection: (____, ____)

Solution: $x =$ ____ $y =$ ____

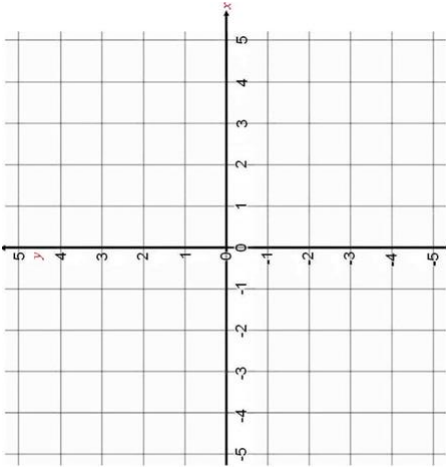
2. $y = 2x - 1$
 $y = x$

$y = 2x - 1$

x	-3	-2	-1	0	1	2	3
y							

$y = x$

x	-3	-2	-1	0	1	2	3
y							



Point of intersection: (____, ____)

Solution: $x =$ ____ $y =$ ____

Fluency Practice

3. $y = 3x - 2$

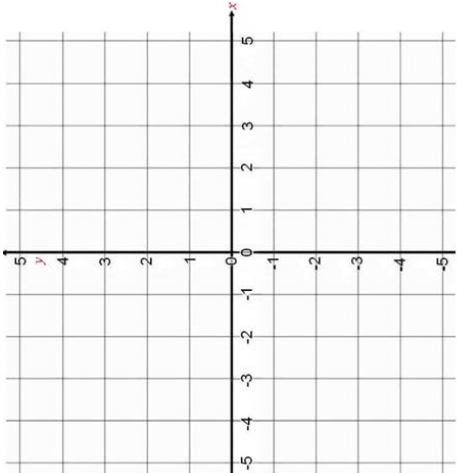
$y = x - 2$

$y = 3x - 2$

x	-3	-2	-1	0	1	2	3
y							

$y = x - 2$

x	-3	-2	-1	0	1	2	3
y							



Point of intersection: (____, ____)

Solution: $x =$ ____ $y =$ ____

4. $y = 5 - x$

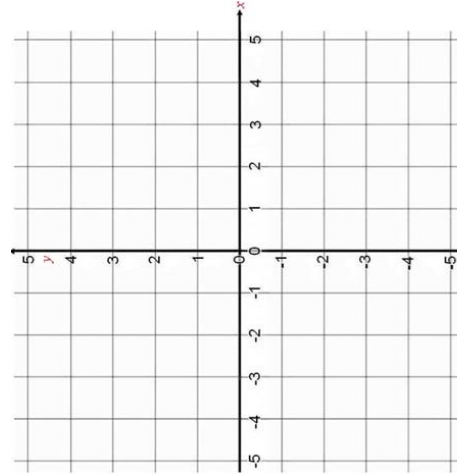
$y = 2x - 1$

$y = 5 - x$

x	-3	-2	-1	0	1	2	3
y							

$y = 2x - 1$

x	-3	-2	-1	0	1	2	3
y							



Point of intersection: (____, ____)

Solution: $x =$ ____ $y =$ ____

Fluency Practice

learn by heart

A function is a type of equation, because it contains an equals sign.

When we plot the graph of a function we can see all the pairs of values that make it true. These are also called the solutions of the equation.

Every function has an infinite number of solutions, but if we take any two functions it is possible that there is just one pair of values that solves both of them. To find this solution we can plot graphs of both functions and see where they cross. This is called solving the equations simultaneously - at the same time.

example

By plotting graphs, find estimates for the solution to the simultaneous equations:
 $y = 2x + 4$ and $y = 5 - 3x$

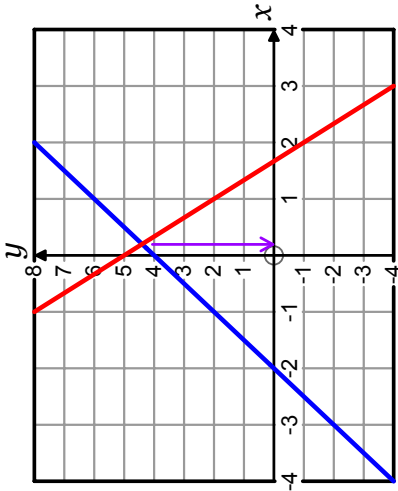
First create tables of values
and plot each function:

$y = 2x + 4$

x	y
-1	2
0	4
1	6
2	8

$y = 5 - 3x$

x	y
-1	8
0	5
1	2
2	-1



They cross when x is approximately 0.2, and y is approximately 4.2, so $x \approx 0.2$, $y \approx 4.2$ is an estimate of the solution.

exercise 70

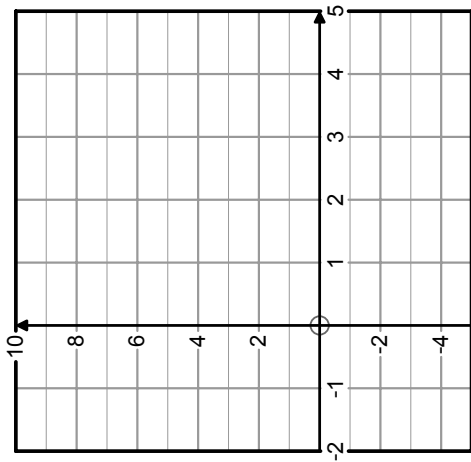
1. By plotting graphs, estimate the solution to the simultaneous equations:
 $y = 4x - 5$ and $y = 6 - 2x$

$y = 4x - 5$

x	y
0	
1	
2	
3	

$y = 6 - 2x$

x	y
-1	
0	
1	
2	



Fluency Practice

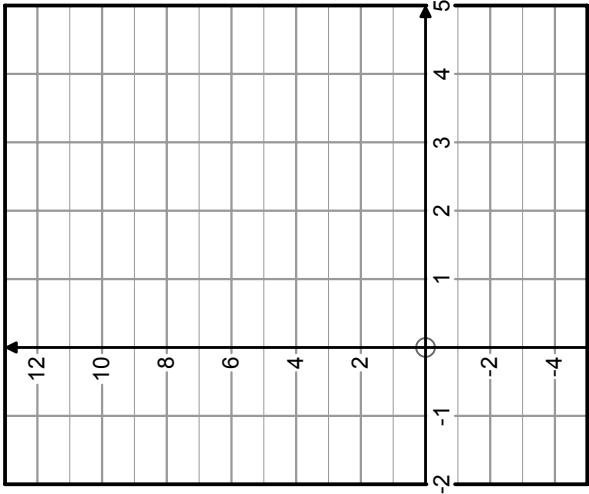
2. By plotting graphs, estimate the solution to the simultaneous equations:
 $y = 2x - 3$ and $y = 10 - x$

$y = 2x - 3$

x	y
-1	
0	
1	
2	

$y = 10 - x$

x	y
-1	
0	
1	
2	



4. By plotting graphs, estimate the solution to the simultaneous equations:
 $y = 8 - x$ and $y = 3x - 1$

$y = 8 - x$

x	y
0	
1	
2	
3	

$y = 3x - 1$

x	y
-1	
0	
1	
2	

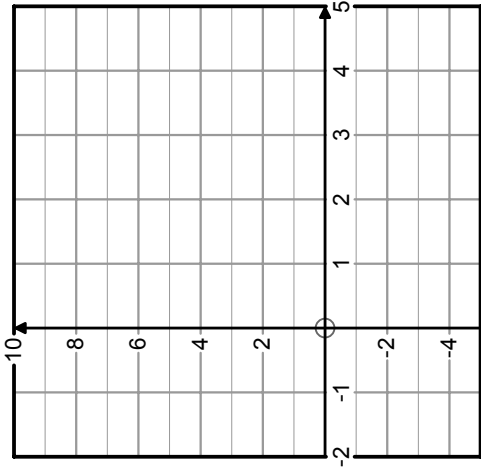
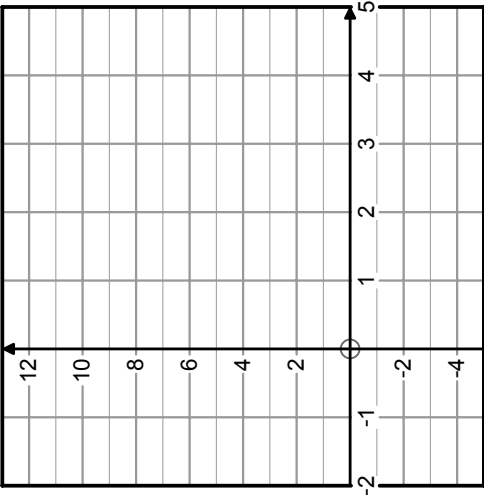
3. By plotting graphs, estimate the solution to the simultaneous equations:
 $y = 4x - 5$ and $y = 5 - 2x$

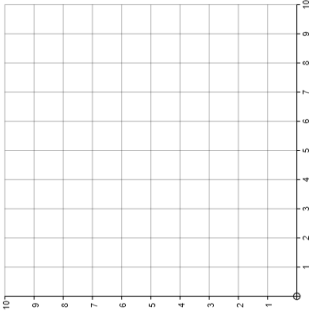
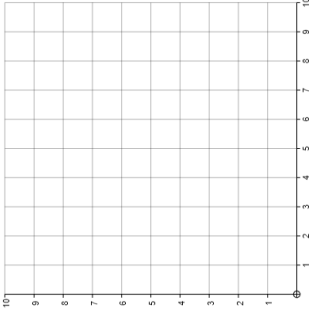
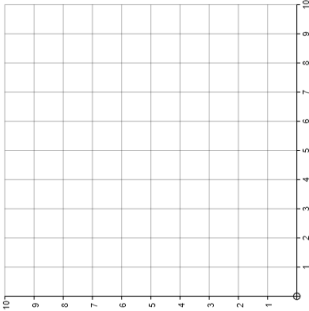
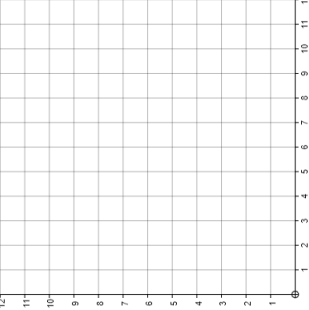
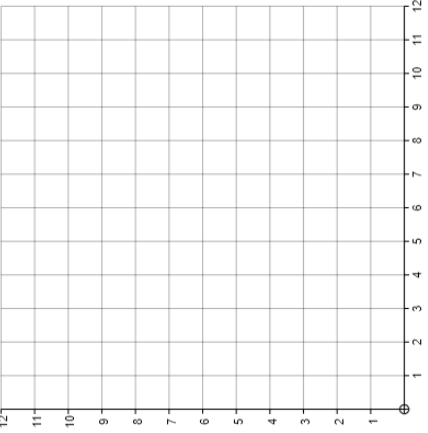
$y = 4x - 5$

x	y
0	
1	
2	
3	

$y = 5 - 2x$

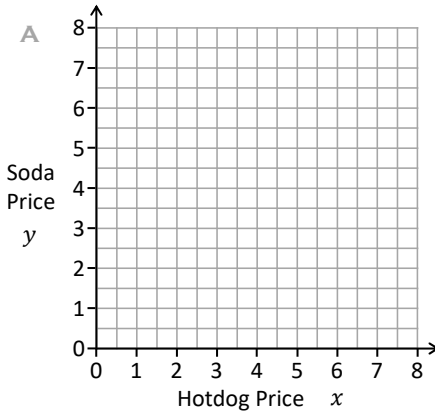
x	y
-1	
0	
1	
2	



Solving Simultaneous Equations Graphically	
(a)	(b)
Plot the graphs of $x + y = 5$ and $x + 3y = 9$ 	Plot the graphs of $x + y = 7$ and $5x + 2y = 20$ 
(c)	(d)
Find the solution to the simultaneous equations $x + y = 5$ and $x + 3y = 9$ Plot the graphs of $y = x + 1$ and $x + y = 9$ 	Find the solution to the simultaneous equations $y = 2x$ and $2x + y = 12$ 
(e)	
Plot the graphs of $x + y = 8$, $y = x$, $y = \frac{1}{2}x + 5$ and $x + y = 11$ Use the graphs to find the solutions of the simultaneous equations: (i) $x + y = 8$ and $y = x$ (ii) $x + y = 8$ and $y = \frac{1}{2}x + 5$ (iii) $y = x$ and $x + y = 11$ 	

Fluency Practice

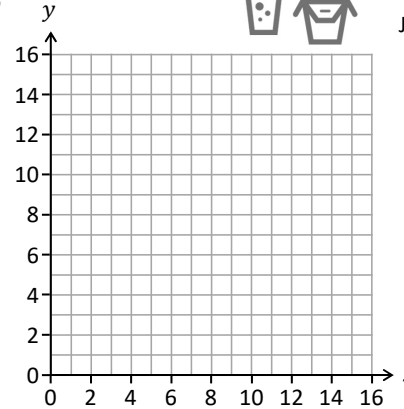
SIMULTANEOUS EQUATIONS



1 hotdog & 1 soda costs £6
 1 hotdogs & 2 sodas costs £8
 Write 2 equations in x & y
 to represent
 these purchases.

Plot both of the equations on the grid.
 What do the coordinates of the intersection tell us?

B

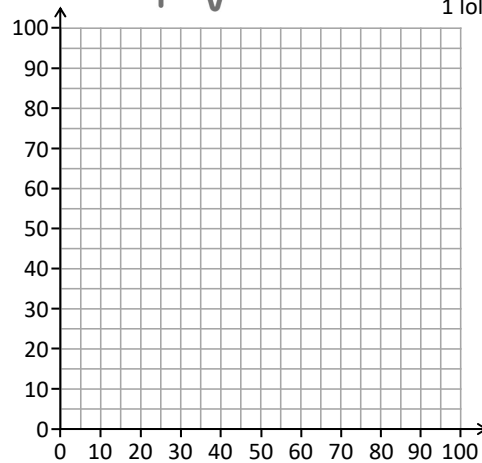


Jo buys 4 sodas and some noodles
 for £16 and Tye buys 2 sodas
 and some noodles for £12.

Plot and solve these purchases as
 simultaneous equations.

Hugh wants to buy
 2 sodas and 3 lots of noodles.
 How much will it cost?

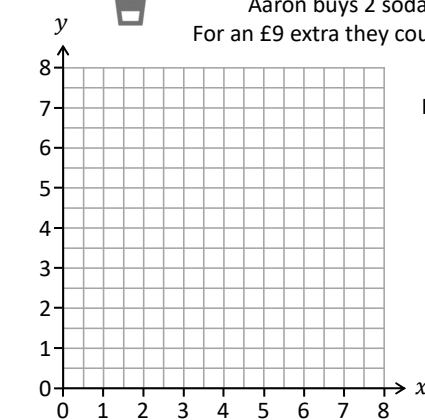
C



3 lollies and 1 cone costs 90p
 1 lolly and 2 cones costs 80p

How much will 5 lollies &
 4 cones cost?

D



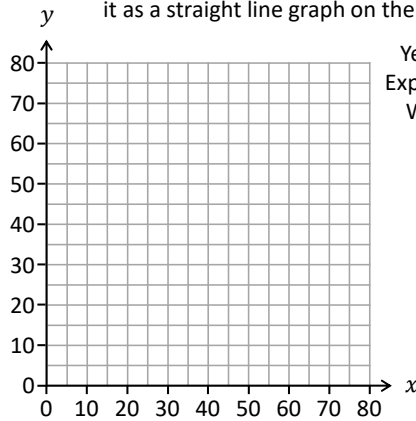
Aaron buys 2 sodas and 2 tacos for £15.
 For an £9 extra they could buy 4 sodas and 3 tacos.

How much does it cost to
 buy 3 sodas and 4 tacos?

Fluency Practice

Creating Simultaneous Equations

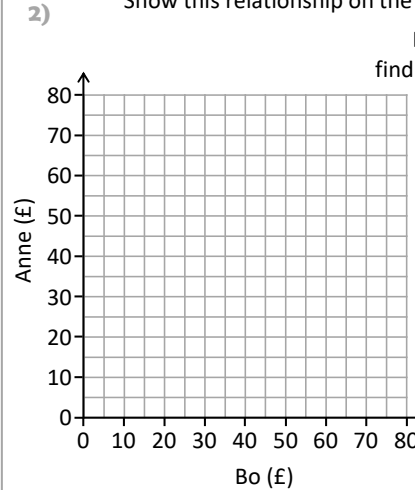
- 1) Xavier (x) and Yeliah (y) will share £60.
Create an equation to express this and plot it as a straight line graph on the grid.



Yeliah wants 5 times Xavier's share.
Express this as an equation and plot it.
What does the intersection show?

Instead, Yeliah's share is half the size of Xavier's share.
Plot the graph and find how much they should both get.

- Anne and Bo will share £80.
Show this relationship on the grid.



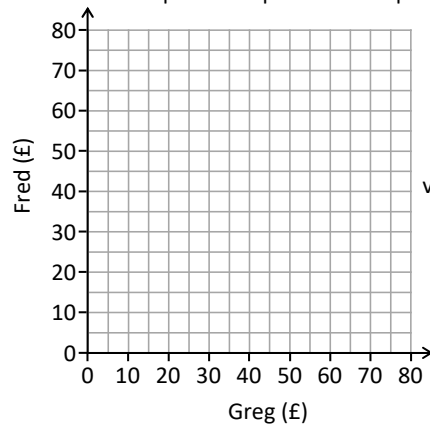
Express and plot these equations & find out how much each person would get.

Anne gets £20 more than Bo.

Bo gets £25.

Bo gets 3 times the money Anne gets.

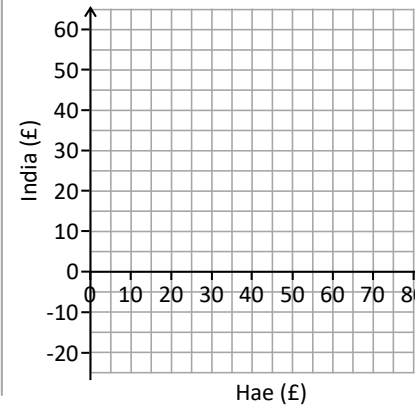
- 3) Fred and Greg will share £70.
Fred will get £10 more than double what Greg gets.
Express this problem as a pair of simultaneous equations.



Plot both equations to find the values that satisfy both equations.

If they shared £55 how much would each person get?

- 4) Hae and India will share £60.
How much do they get each if... ...India gets £15 more than Hae.



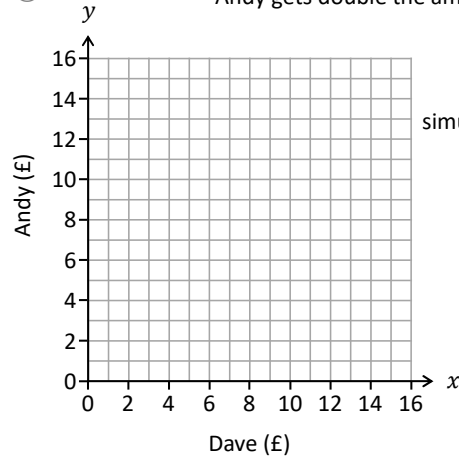
...Hae gets a third of what India gets.

...India gets £15 less than quadruple what Hae gets.

Fluency Practice

How much!?

①



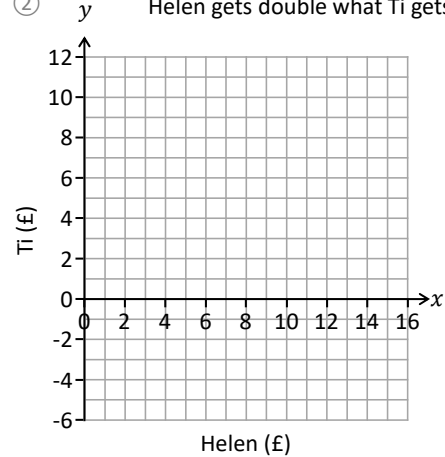
Andy gets £4 more than Dave.
Andy gets double the amount Dave gets



Represent these
two statements as a pair of
simultaneous equations on the grid.

How much do they both get?
How much was shared?

②

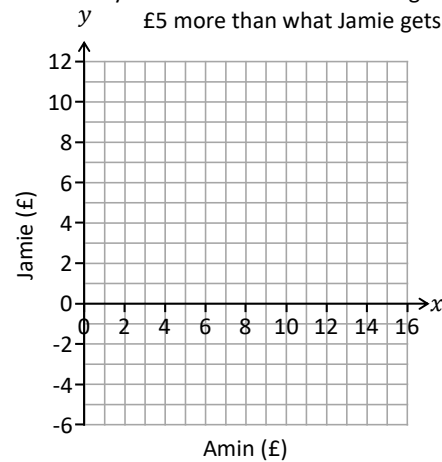


Ti gets £5 less than Helen.
Helen gets double what Ti gets

Represent these
two statements as a pair of
simultaneous equations
on the grid.

How much do they both get?
How much was shared?

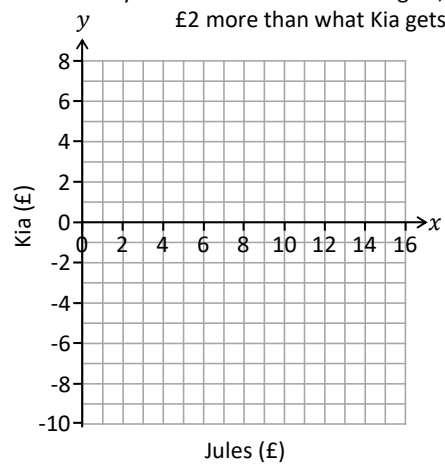
③



Jamie gets £2 more than Amin.
If you double the amount Amin gets, it is
£5 more than what Jamie gets.

How much do they both get?
How much was shared?

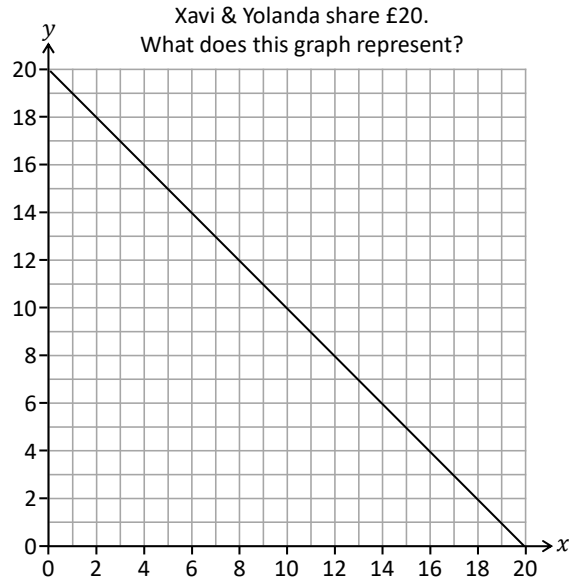
④



Kia gets £8 less than Jules.
If you halve the amount Jules gets, it is still
£2 more than what Kia gets.

How much do they both get?
How much was shared?

Fluency Practice



The money can be split in different ways.
Express and plot each split as an equation.
How much does each person get?

Xavi gets £6.

Xavi =

Yolanda =

Yolanda gets 4 times what Xavi gets.

Xavi =

Yolanda =

Yolanda gets £2 more than Xavi.

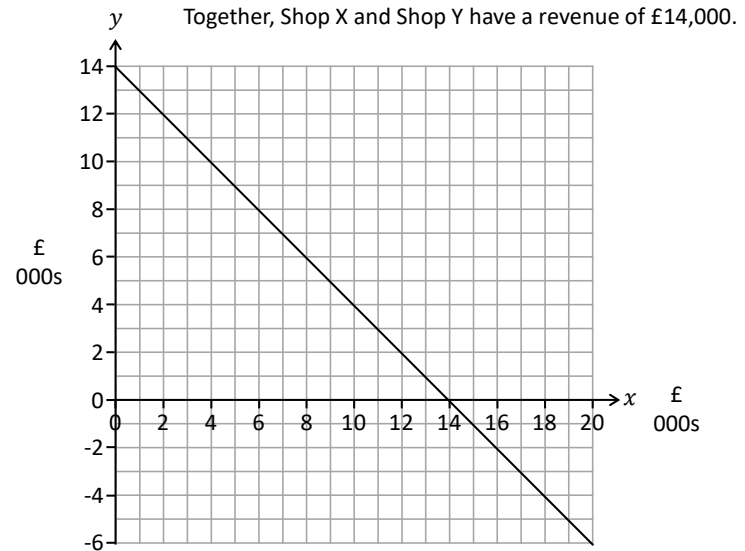
Xavi =

Yolanda =

Yolanda gets a quarter of what Xavi gets.

Xavi =

Yolanda =



Express each different split of the revenue as an equation on the grid.
Use intersections to solve the simultaneous equations.

Shop Y earnt £6000 less than Shop X.

X =

Y =

Shop X earnt a third of what Shop Y earnt.

X =

Y =

If you halve the amount Shop X earnt,
it is £5000 less than Shop Y.

X =

Y =

Shop X's revenue is £6000 greater than
triple Shop Y's revenue.

X =

Y =

Fluency Practice

Investigating Simultaneous Equations

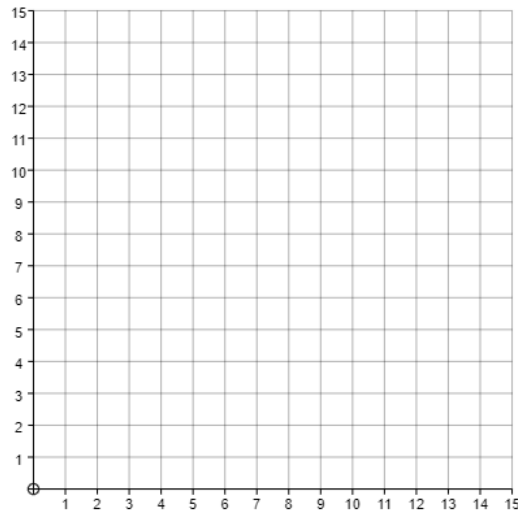
(a)

Solve the simultaneous equations

$$x + 2y = 10$$

$$2x + y = 14$$

Plot the equations of the lines $x + 2y = 10$ and $2x + y = 14$ on the same axes below. What do you notice?



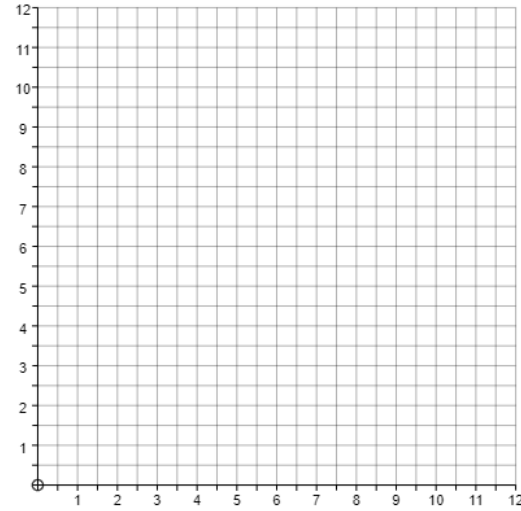
(b)

Solve the simultaneous equations

$$4x + 3y = 24$$

$$x + 2y = 11$$

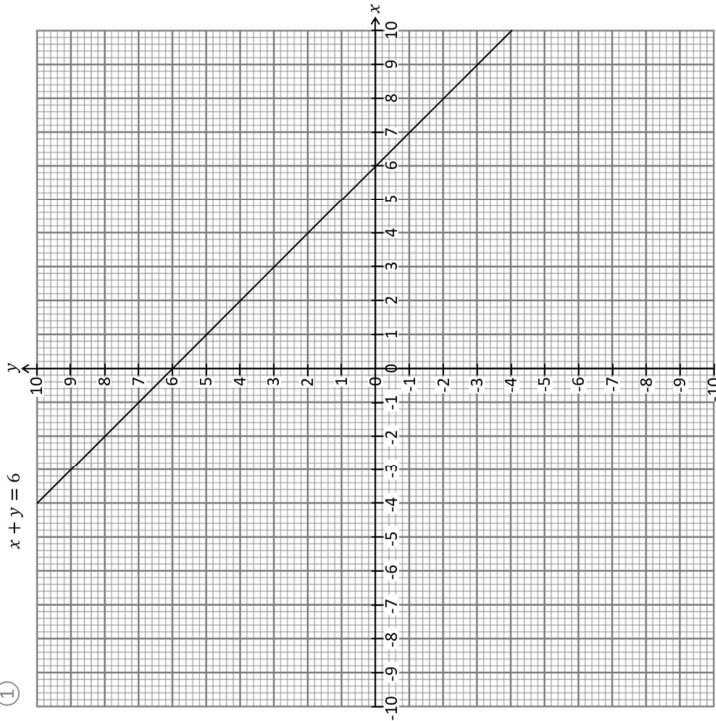
Plot the equations of the lines $4x + 3y = 24$ and $x + 2y = 11$ on the same axes below. What do you notice?



Fluency Practice

Plotting Simultaneous Equations

①



Plot & Solve...

$x + y = 6$
 $y = 1$

$x =$
 $y =$

$x + y = 6$
 $y = 2x$

$x =$
 $y =$

$x + y = 6$
 $y = 2x - 9$

$x =$
 $y =$

$y = 2x - 9$
 $x + y = -3$

$x =$
 $y =$

$x + y = -3$
 $y = 0.5x + 3$

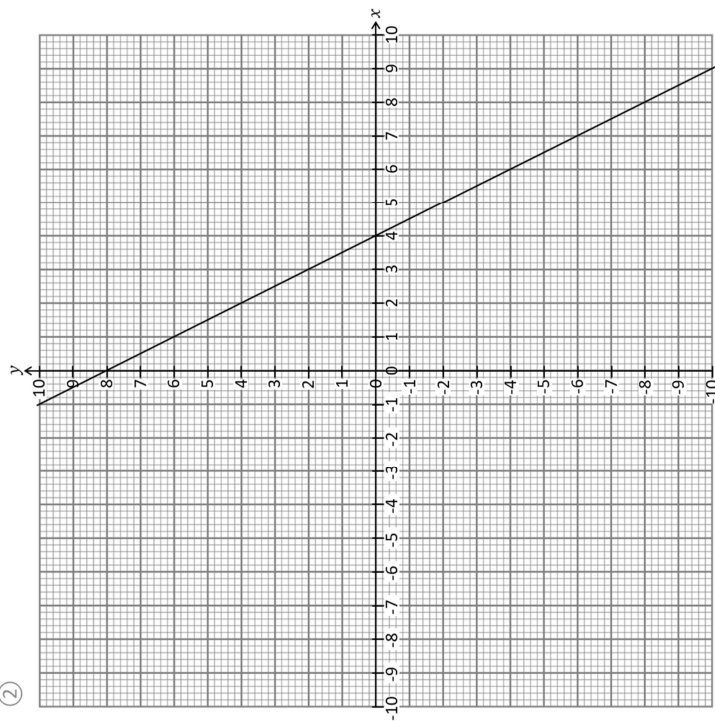
$x =$
 $y =$

$x + y = -3$
 $y + 2x = 2$

$x =$
 $y =$

Plotting Simultaneous Equations

②



Plot & Solve...

$2x + y = 8$
 $y = 2x - 4$

$x =$
 $y =$

$y = 2x - 4$
 $y + 2x = 4$

$x =$
 $y =$

$y + 2x = 4$
 $y = 0.5x + 9$

$x =$
 $y =$

$2y = x + 18$
 $3x + y = -5$

$x =$
 $y =$

$y - 2x + 4 = 0$
 $2x + 3y = -12$

$x =$
 $y =$

$2x + 3y = -12$
 $3x - 2y = -4$

$x =$
 $y =$

Fluency Practice

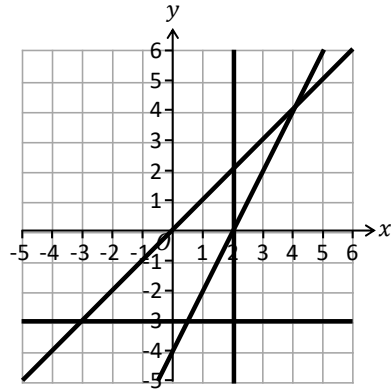
① The grid shows 4 equations:

$$y = x$$

$$x = 2$$

$$y = -3$$

$$y = 2x - 4$$



(2, 2) is the solution to which pair of simultaneous equations?

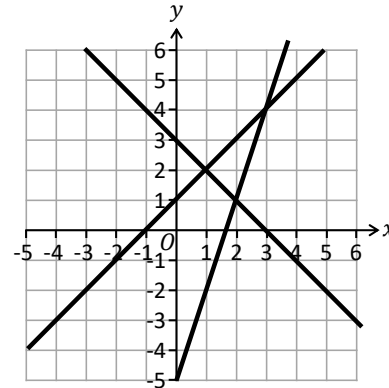
(0.5, -3) is the solution to which pair of simultaneous equations?

(4, 4) is the solution to which pair of simultaneous equations?

Identifying Simultaneous Equations

③

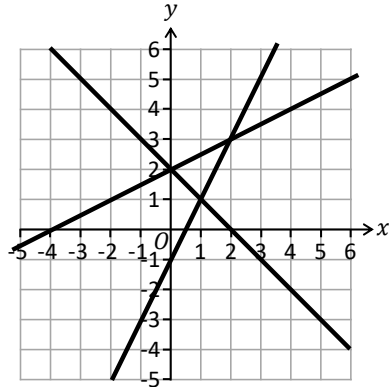
What pairs of simultaneous equations, and their solutions, are on the grid?



②

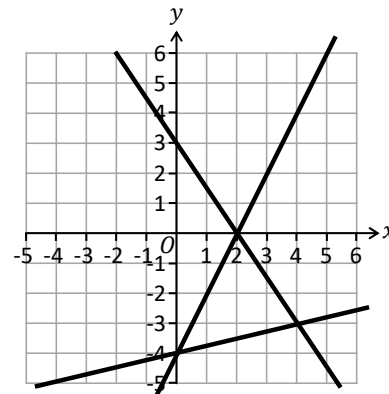
The grid shows 3 equations. $y = 2x - 1$ $y = \frac{x}{2} + 2$ $y + x = 2$

Write down the 3 pairs of simultaneous equations, and their solutions, shown on the grid.



④

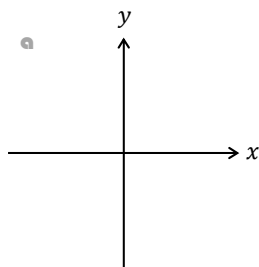
What pairs of simultaneous equations, and their solutions, are on the grid?



Fluency Practice

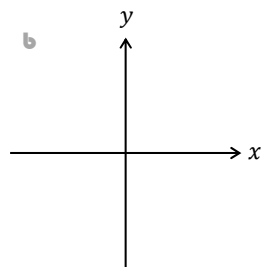
Solution Sketching

Sketch each pair of simultaneous equations on the grid. What method can you use to find the solutions?



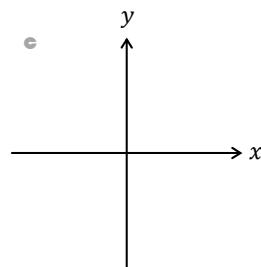
$$y = x$$

$$x = 4$$



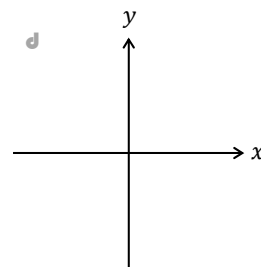
$$y = 2x$$

$$y = -4$$



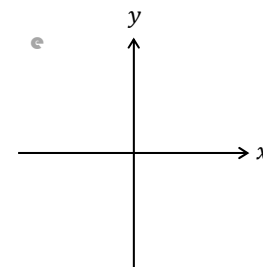
$$y = 6$$

$$y = x + 4$$



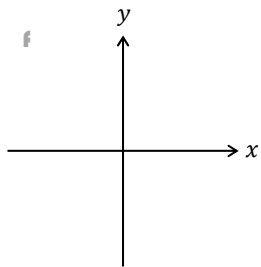
$$y = x$$

$$y = x - 2$$



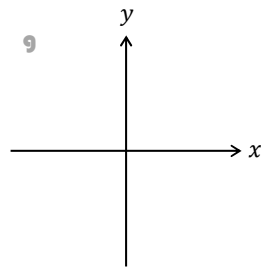
$$y = -x$$

$$y = x - 6$$



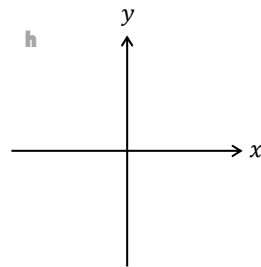
$$x + y = 8$$

$$y = 3x$$



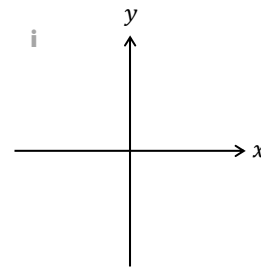
$$y = -2x$$

$$x + y = 2$$



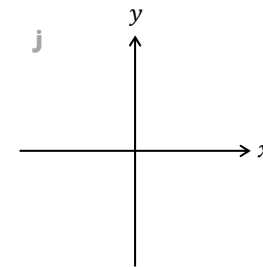
$$2y - 2x = 10$$

$$y = x + 5$$



$$x + y = -5$$

$$y = 2x - 2$$



$$2x + y = 10$$

$$y = 3x - 15$$

Fluency Practice

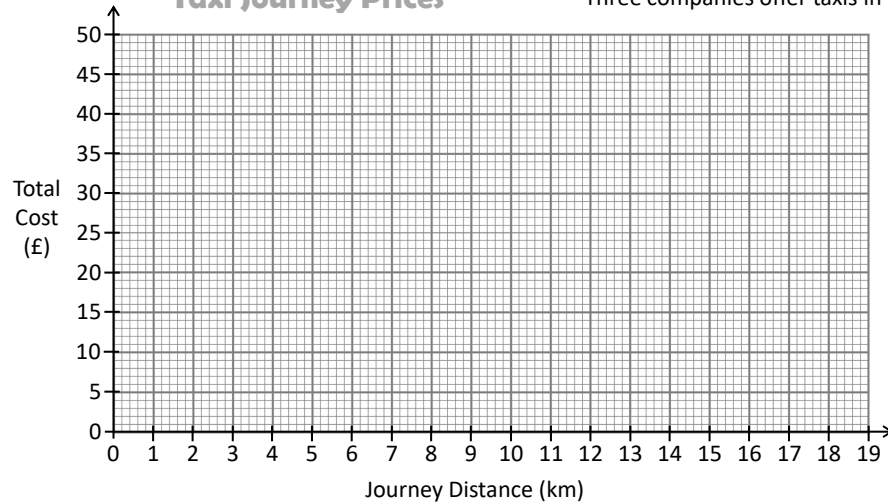
Taxi Journey Prices

Three companies offer taxis in London:

Quick Dash: £3 per mile

Zoom: £2 per mile with a £5 pick-up fee

Kar: 50p per mile with a £20 pick-up fee



Sketch these cost-relationships on the grid.

At what distance is...

... Zoom and Quick Dash the same price?

... Zoom and Kar the same price?

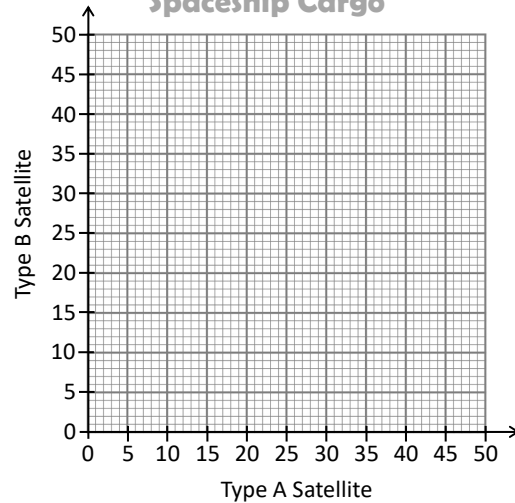
... Quick Dash and Kar the same price?

When is Zoom the cheapest option?

Uri has £22 to spend. Draw this graph and explain any intersections.

Lee wants to travel 13.5 km. *Estimate* the prices they could pay.

Spaceship Cargo



A spaceship will be loaded with 2 different types of mini-satellites (A & B).

In total the spaceship can carry 45 satellites.

Show this limit as a graph on the grid.

In discussion...

... Scientist G argues for an equal amount of A & B satellites.

... Scientist I argues that there should be 15 more B satellites than As onboard.

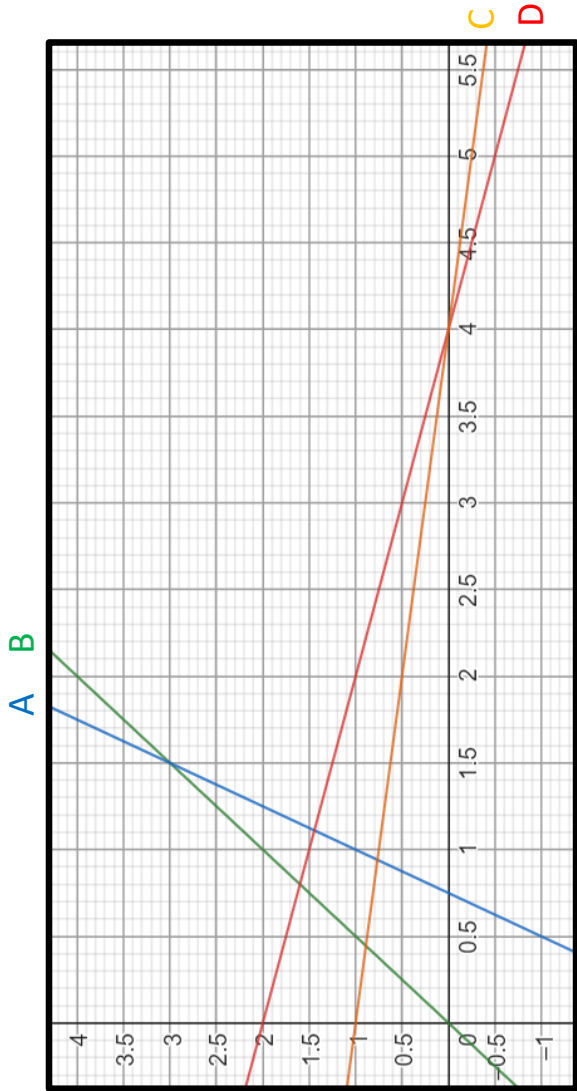
... Scientist H says there should be double the amount of A satellites compared to B satellites.

Draw each scientist's proposition as a graph and find solutions.

Before launch it is discovered the spaceship can only carry 39 satellites.

Show & find solutions for the new cargo possibilities.

Fluency Practice

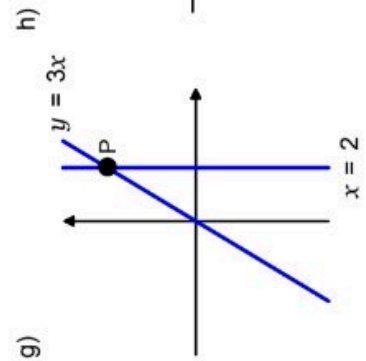
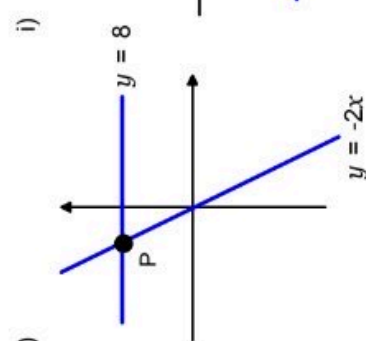
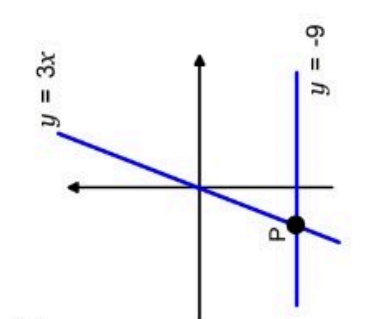
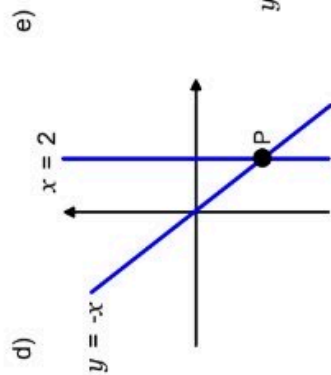
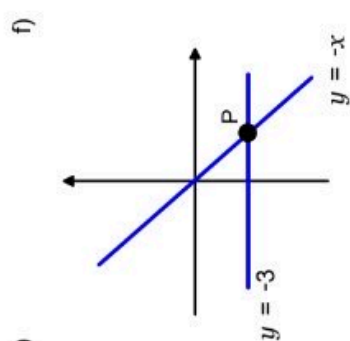
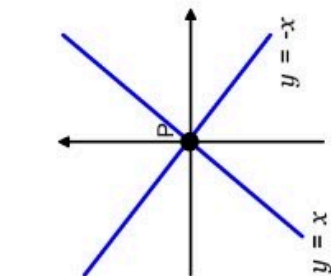
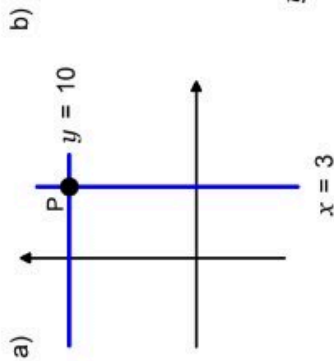
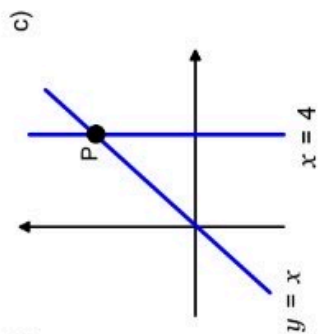
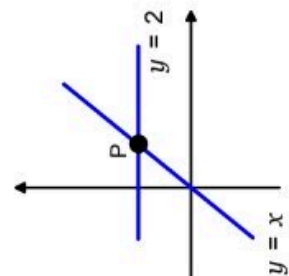


Using the lines above, complete the table below, give your solutions to one decimal place where necessary.

Line:	A	B	C	D
Equation:			$x + 2y = 4$	$x + 4y = 4$
Gradient				
Solution with A:				
Solution with B:				
Solution with C:				
Solution with D:				

Fluency Practice

10. For each graph, work out the co-ordinates of the point marked P.



2 Linear Simultaneous Equations

Fluency Practice

1. Solve:

- a) $4a + 3b = 44$
 $3a + 3b = 39$

b) $2a + b = 23$
 $a + b = 16$

c) $5a + 2b = 27$
 $5a + 4b = 49$
- d) $3a + 2b = 16$
 $4a + 6b = 28$

e) $2a + b = 14$
 $4a + 6b = 36$

f) $6a + 5b = 39$
 $2a + 3b = 17$

2. Explain why it is not possible to solve the pair of simultaneous equations:

$$\begin{matrix} x + y = 9 \\ 2x + 2y = 18 \end{matrix}$$

3. Select the correct pair of solutions to the simultaneous equations shown in the box:

$$\begin{matrix} p + q = 7 \\ 5p + 6q = 36 \end{matrix}$$

- a) $p = 3$
 $q = 4$
- b) $p = 6$
 $q = 1$
- c) $p = 1$
 $q = 6$

4. Solve:

- a) $2a + 5b = 25$
 $3a + 4b = 27$

b) $3a + 5b = 17$
 $5a + 2b = 22$

c) $4a + 3b = 33$
 $6a + 5b = 51$
- d) $2a + 3b = 5$
 $2a + 5b = 11$

e) $3a + 2b = -11$
 $5a + 4b = -21$

f) $3a + 7b = -6$
 $2a + 3b = 1$

5. Match pairs of equations with their solutions:

A $3x + y = 8$ $3x + 4y = 14$	B $2x + y = -2$ $3x + 2y = -1$	P $x = 3$ $y = 7$	Q $x = 1$ $y = 4$
C $3x + 2y = 11$ $2x + 3y = 14$	D $4x + 3y = -7$ $2x + 2y = -6$	R $x = 4$ $y = 2$	S $x = 2$ $y = 2$
E $6x + 5y = 34$ $4x + 3y = 22$	F $2x + 4y = -12$ $x + 3y = -9$	T $x = -3$ $y = 4$	U $x = -2$ $y = -1$
G $x + y = 10$ $2x + 5y = 41$	H $x + 2y = -4$ $2x + 3y = -7$	V $x = 2$ $y = -5$	W $x = 0$ $y = -3$

Fluency Practice

1. Solve:

a) $4p - 3q = 10$
 $3p + 6q = 24$

b) $8t - 3u = 61$
 $t - u = 7$

c) $2k + 3p = 30$
 $4k - 5p = 16$

d) $-x + 3y = -11$
 $4x + 2y = 16$

e) $-5c + 2d = 21$
 $2c - 3d = -15$

f) $-6x + 4y = -42$
 $-4x - 5y = -5$

2. Select the correct pair of solutions to the simultaneous equations shown in the box:

a) $g = 2$
 $h = -1$

b) $g = -6$
 $h = 4$

c) $g = 3$
 $h = 2$

$$\begin{array}{l} -2g + 3h = 0 \\ 3g - h = 7 \end{array}$$

3. Jake and Lucy are trying to solve the simultaneous equations shown.

Jake **adds** the two equations.
 Lucy **subtracts** the two equations.

$$\begin{array}{l} 4x + 3y = 20 \\ 3y - 3x = 6 \end{array}$$

Who has correctly eliminated one unknown?

4. Solve:

a) $3a + 2b = -13$
 $2a - 3b = -39$

b) $2c + 3d = 7$
 $-c + 6d = -26$

c) $g + 3h = -3$
 $h - 2g = -8$

d) $-3x = y + 8$
 $2x - y = -7$

e) $4u + 3w - 3 = 0$
 $3u - 2w = 32$

f) $-2x + 5y = -14$
 $-2y + 5x = -7$

5. Spot the mistake!

a) $4a + 3b = 18$
 $5a - 3b = 9$

b) $2a + 3b = 4$
 $4a + 5b = 10$

c) $-a + 3b = 3$
 $a - b = 1$

$$\begin{array}{r} 4a + 3b = 18 \\ 5a - 3b = 9 \quad - \\ \hline a = 9 \end{array}$$

$$\begin{array}{r} 4a + 6b = 4 \\ 4a + 5b = 10 \quad - \\ \hline b = -6 \end{array}$$

$$\begin{array}{r} -a + 3b = 3 \\ a - b = 1 \quad + \\ \hline 4b = 4 \end{array}$$

$$36 + 3b = 18$$

$$2a - 18 = 4$$

$$b = 1$$

$$a = 9, b = -6$$

$$a = 11, b = -6$$

$$a = 2, b = 1$$

Fluency Practice

Spot The Error*

*one error is the use of an inefficient method.

Solve **A**

$$3x - 2y = 8$$

$$x + y = 2$$

Solution

$$3x - 2y = 8$$

$$2x + 2y = 2 \quad +$$

$$5x = 10$$

$$x = 2$$

$$y = 0$$

Solve **B**

$$2x + y = 17$$

$$x - 2y = -1$$

Solution

$$2x + y = 17$$

$$2x - 4y = -2 \quad -$$

$$-3y = 15$$

$$y = -5$$

$$x = 11$$

Solve **C**

$$3x - 5y = 27$$

$$2x + 5y = 8$$

Solution

$$2x = 8 - 5y$$

$$3x = 1.5(8 - 5y)$$

$$1.5(8 - 5y) - 5y = 27$$

$$12 - 7.5y - 5y = 27$$

$$12 - 27 = 12.5y$$

$$-1.2 = y$$

$$2x - 6 = 8; x = 7$$

Solve **D**

$$3x + y = 10$$

$$5x - 4y = 12$$

Solution

$$y = 10 - 3x$$

$$5x - 4(10 - 3x) = 12$$

$$5x - 40 - 3x = 12$$

$$2x = 52, x = 26$$

$$130 - 4y = 12$$

$$130 - 12 = 4y$$

$$y = 29.5$$

When you have finished, work out the correct answer for each pair of equations.

Fluency Practice

Spot The Error*

Solve

E

$$\begin{aligned}x + 2y &= 18 \\ 3x - y &= 5\end{aligned}$$

Solution

$$x = 4$$

$$y = 7$$

Solve

F

$$\begin{aligned}8x + 12y &= 20 \\ 2x - 3y &= 3\end{aligned}$$

Solution

$$8x + 12y = 20$$

$$8x - 12y = 12 \quad -$$

$$24y = 8$$

$$y = 24/8 = 3$$

$$x = 6$$

Solve

G

$$\begin{aligned}2x + y &= 21 \\ x - 2y &= 8\end{aligned}$$

Solution

$$x = 8 + 2y$$

$$2(8 + 2y) + y = 21$$

$$16 + 4y + y = 21$$

$$16 + 5y = 21$$

$$y = 1$$

$$2x + 1 = 21$$

$$2x = 20; x = 10$$

Solve

H

$$\begin{aligned}3x + 7y &= 13 \\ 3x - 2y &= 7\end{aligned}$$

Solution

Different signs =

Add the equations:

$$5y = 20$$

$$y = 4;$$

$$3x - 8 = 7$$

$$3x = 15; x = 5$$

When you have finished, work out the correct answer for each pair of equations.

Fluency Practice

Simultaneous Equations

$$2x = y \quad y = x + 8$$

$$\underline{2x = y = x + 8}$$

$$2x = x + 8$$

$$x = 8$$

$$2x = y \quad 2(8) = 16$$

$$x \longrightarrow (8, 16) \longleftarrow y$$

a) $2x = y \quad y = x + 4$

$$2x = x + 4$$

b) $3x = y \quad y = x + 12$

c) $3x = y \quad y = x - 4$

d) $x + 3 = y$
 $y = 2x - 2$

e) $x + 7 = y$
 $y = 3x - 9$

f) $x + 10 = y$
 $y = 3x + 18$

g) $2x + 9 = y$
 $y = x + 12$

h) $2x + 6 = y$
 $y = 3x + 4$

i) $y = 2x + 8$
 $y = 4x - 6$

j) $y = 2x + 11$
 $y = 5 - x$

k) $y = \frac{x}{2}$
 $y = x - 6$

l) $y = \frac{x}{2} + 3$
 $y = x - 1$

m) $y = \frac{x}{3} + 2$
 $y = 2x - 3$

n) $2y = 3x + 3$
 $y = x + 4$

o) $2y = 2x + 10$
 $y = x + 5$

p) $y = x - 6$
 $3y = 2x - 16$

q) $2y = 2x - 22$
 $y = \frac{x}{2} - 7$

r) $x + 3 = y$
 $y = \frac{x + 11}{2}$

s) $2y + 10 = 2x$
 $y = 2x - 11$

t) $2y = 3x - 2$
 $3y = 2x + 7$

Fluency Practice

Manipulating Equations to Solve

Use $x + y$ to solve the other equations (the variable values will be different each time).

$x + y = 8$

$2x + 2y = 16$	$3x + 3y = 24$	$x + 2y = 13$	$x + 4y = 20$	$2x + y = 18$
$2x + y = 11$	$3x + 2y = 18$			
$x =$	$x =$	$x =$	$x =$	$x =$
$y =$	$y =$	$y =$	$y =$	$y =$

$x + y = 5$

$5x + 5y = 25$	$x + 6y = 15$	$2x + 4y = 10$	$3x + 6y = 6$	$x + 4y = 35$
$5x + y = 21$				
$x =$	$x =$	$x =$	$x =$	$x =$
$y =$	$y =$	$y =$	$y =$	$y =$

$2x + y = 12$

$4x + 2y = 24$	$6x + y = 28$	$x + 3y = 26$	$3x + 5y = 39$	$8x + 7y = 42$
$4x + y = 22$				
$x =$	$x =$	$x =$	$x =$	$x =$
$y =$	$y =$	$y =$	$y =$	$y =$

Fluency Practice

Substitution Solving

a

$$\begin{array}{ll}\text{If} & 3x + 2 = 14 \\ \text{and} & y + x = 9 \\ & y = ?\end{array}$$

d

$$\begin{array}{ll}\text{If} & 8 - x = 3 \\ \text{and} & 3y + 3x = 27 \\ & y = ?\end{array}$$

g

$$\begin{array}{ll}\text{If} & 3(2x + 5) = 33 \\ \text{and} & 3(y + x) = 36 \\ & y = ?\end{array}$$

b

$$\begin{array}{ll}\text{If} & 2x - 5 = 7 \\ \text{and} & y + 2x = 15 \\ & y = ?\end{array}$$

e

$$\begin{array}{ll}\text{If} & 19 - 2x = 7 \\ \text{and} & 2x + 2y = 16 \\ & y = ?\end{array}$$

h

$$\begin{array}{ll}\text{If} & 4x + 13 = 5 \\ \text{and} & 3y + 2x = 26 \\ & y = ?\end{array}$$

c

$$\begin{array}{ll}\text{If} & 5x - 8 = 7 \\ \text{and} & 2y + 2x = 18 \\ & y = ?\end{array}$$

f

$$\begin{array}{ll}\text{If} & 3(x + 5) = 21 \\ \text{and} & 6y + 5x = 10 \\ & y = ?\end{array}$$

i

$$\begin{array}{ll}\text{If} & \frac{3x + 1}{2} = 11 \\ \text{and} & 4x - 2y = 22 \\ & y = ?\end{array}$$

j

$$\begin{array}{ll}\text{If} & 2(5 - 2x) = 26 \\ \text{and} & 8 - 3x = 4y \\ & y = ?\end{array}$$

Fluency Practice

MIXED WRITTEN METHODS TO SOLVE SIMULTANEOUS EQUATIONS

Do you prefer a method?

ELIMINATE (SUBTRACT)	ELIMINATE (ADD)	BALANCE & ELIMINATE	SUBSTITUTE	REARRANGE & SUBSTITUTE
$3x + y = 19$ $x + y = 9$	$3x - y = 7$ $x + y = 13$	$3x + y = 9$ $x + 2y = 13$	$x = 2y + 3$ $x + 2y = 19$	$x - y = 6$ $x + 3y = 14$
$x =$ $y =$	$x =$ $y =$	$x =$ $y =$	$x =$ $y =$	$x =$ $y =$
$4x + 2y = 18$ $4x - y = 3$	$5x + 3y = 32$ $2x - 3y = -4$	$x - 2y = 6$ $3x + y = 32$	$3x + 2y = 14$ $y = 3x - 2$	$x = 9 - y$ $2x + 3y = 21$

$3x + 2y = 15$ $3x + y = 12$	$x + y = 8$ $3x + 2y = 18$	$3x + 5y = 28$ $2x + 5y = 22$	$x = 3y - 2$ $3x + 2y = 16$
$x + 3y = 2$ $2x + y = 9$	$2x + 3y = 23$ $6x + y = 29$	$x - y = 8$ $4x + 2y = 44$	$x + 4y = 22$ $3x - 4y = 18$

Fluency Practice

Ex)

$$\begin{aligned} x + y &= 12 \\ y &= x + 2 \\ x + (x + 2) &= 12 \\ 2x + 2 &= 12 \\ 2x &= 10 \\ x &= 5 \\ y &= (5) + 2 \\ y &= 7 \end{aligned}$$

Solving Simultaneous Equations using Substitution

a) $\begin{aligned} x + y &= 10 \\ y &= x + 4 \end{aligned}$

b) $\begin{aligned} x + y &= 10 \\ y &= x - 6 \end{aligned}$

c) $\begin{aligned} 2x + y &= 9 \\ y &= x - 3 \end{aligned}$

d) $\begin{aligned} x + y &= 13 \\ y &= 2x + 4 \end{aligned}$

e) $\begin{aligned} x + 2y &= 14 \\ y &= x + 1 \end{aligned}$

f) $\begin{aligned} x + 3y &= 14 \\ y &= x + 2 \end{aligned}$

g) $\begin{aligned} 2x - y &= 7 \\ y &= x - 2 \end{aligned}$

h) $\begin{aligned} x + 2y &= 17 \\ y &= 2x + 1 \end{aligned}$

i) $\begin{aligned} x + 3y &= 33 \\ y &= 2x - 3 \end{aligned}$

j) $\begin{aligned} 2x + 3y &= 30 \\ y &= 3x - 1 \end{aligned}$

k) $\begin{aligned} 2x + 3y &= 28 \\ x &= 4y + 3 \end{aligned}$

l) $\begin{aligned} 3x + 2y &= 15 \\ y &= 9 - 2x \end{aligned}$

m) $\begin{aligned} 4x + 2y &= 32 \\ y - 4x &= 4 \end{aligned}$

n) $\begin{aligned} 5x - y &= 12 \\ 2y - 6x &= -8 \end{aligned}$

Fluency Practice

Substitute & Solve

1) $a = b + 1$
 $a + b = 5$ $a =$ $b =$

2) $a = b + 3$
 $a + b = 9$ $a =$ $b =$

3) $a = 3b$
 $a + b = 8$ $a =$ $b =$

4) $a = b - 4$
 $a + b = 10$ $a =$ $b =$

5) $x = 2y$
 $2x + y = 20$ $x =$ $y =$

6) $a = 2b + 1$
 $a + b = 13$ $a =$ $b =$

7) $a = 3b + 2$
 $a + b = 6$ $a =$ $b =$

8) $g = h + 3$
 $2g + h = 21$ $g =$ $h =$

9) $b = 2a + 3$
 $a + 3b = 23$ $a =$ $b =$

10) $a - b = 4$
 $a + 3b = 12$ $a =$ $b =$

11) $a - b = 4$
 $2a + b = 17$ $a =$ $b =$

12) $m - n = 2$
 $2m - n = 10$ $m =$ $n =$

13) $a + b = 6$
 $a + 2b = 11$ $a =$ $b =$

14) $k + p = 6$
 $2k + p = 9$ $k =$ $p =$

15) $a + b = 10$
 $3a + 2b = 22$ $a =$ $b =$

16) $a + 2b = 13$
 $2a + b = 11$ $a =$ $b =$

17) $x - 2y = -2$
 $3x - y = 9$ $x =$ $y =$

18) $2a + 3b = 23$
 $3a + 2b = 17$ $a =$ $b =$

Fluency Practice

1) In each question say which variable we want to eliminate

a) $5x + 3y = 22$	b) $5x + 3y = 22$	c) $5x + 3y = 22$	d) $4x - y = 25$	e) $-3x - y = 25$
$2x + 3y = 10$	$5x + 2y = 10$	$-5x + 2y = 10$	$x + y = 10$	$-3x + 2y = 10$

2) For each part in question 1, would we add or subtract?

a)	b)	c)	d)	e)
----	----	----	----	----

3) Tom says that we need to add these to eliminate the x .
Mark says you need to subtract them to eliminate the y .
Who do you agree with? Why?

$$\begin{aligned} 3x - y &= 9 \\ -3x + y &= -9 \end{aligned}$$

4) Tarquin says, “*you add the equations because the equations have different signs in the middle.*”

$$\begin{aligned} 11x + 4y &= 19 \\ 11x - 3y &= 13 \end{aligned}$$

5) Now solve each pair of equations in question one.

Fluency Practice

1 Complete this table, to solve each pair of simultaneous equations.

Equations	Solved for 1st Variable	Solved for 2nd Variable
$3x - y = 1$ $x + y = 3$	$\begin{array}{r} 3x - y = 1 \\ + \quad x + y = 3 \\ \hline 4x \quad = 4 \\ x \quad = 1 \end{array}$	Sub $x = 1$ into either equation: $1 + y = 3$ $y = 2$
$2x - y = 2$ $x + y = 7$		
$4x + y = 9$ $2x - y = 3$		
$-x + 2y = 13$ $x + y = 8$		
$2x + y = 7$ $x + y = 4$		
$3x + y = 9$ $2x + y = 7$		
$5x - 2y = 13$ $3x + 2y = 3$		

Fluency Practice

2 Complete this table, to solve each pair of simultaneous equations. One of the equations will need to be 'scaled'.

Equations	Scaled	Solved for 1st Variable	Solved for 2nd Variable
$5x + 2y = 11$ $3x - 4y = 4$	$\times 2$ $\rightarrow$ $10x + 4y = 22$ $3x - 4y = 4$	$10x + 4y = 22$ $+ \quad 3x - 4y = 4$ $13x = 26$ $x = 2$	Sub $x = 2$ into any equation. $5(2) + 2y = 11$ $10 + 2y = 11$ $2y = 1$ $y = \frac{1}{2}$
$3x + 2y = 8$ $2x - y = 3$			
$3x + 2y = 11$ $2x - y = -3$			
$2x + 3y = 11$ $3x + y = 13$			
$2x - 3y = 8$ $x - 5y = 11$			
$x + 6y = 0$ $3x - 2y = -10$			
$5x - 4y = 24$ $2x = y + 9$			

Fluency Practice

3 Complete this table, to solve each pair of simultaneous equations. One of the equations will need to be 'scaled'.

Equations	Scaled	Solved for 1st Variable	Solved for 2nd Variable
$2x + 3y = 5$ $5x - 2y = -16$	$\begin{matrix} \times 2 & \text{red arrow} \\ \times 3 & \text{green arrow} \end{matrix}$ $4x + 6y = 10$ $15x - 6y = -48$	$4x + 6y = 10$ $+ \quad 15x - 6y = -48$ $19x = -38$ $x = -2$	Sub $x = -2$ into any equation. $2(-2) + 3y = 5$ $-4 + 3y = 5$ $3y = 9$ $y = 3$
$2x + 5y = 24$ $4x + 3y = 20$			
$2x + 3y = 11$ $3x + 4y = 15$			
$3x + 8y = 27$ $-4x - 3y = -13$			
$2x + 3y = 14$ $8x - 5y = 5$			
$9x - 4y = -20$ $5x - 6y = -13$			
$2y = 3x - 5$ $1 = 5x - 4y$			

Fluency Practice

A1 Solve $x + y = 1$ $x - y = 5$	A2 Solve $5x + y = 17$ $x - y = 3$	A3 Solve $6x - 5y = 9$ $6x + 3y = 33$	A4 Solve $x + 5y = -13$ $4x - 5y = 48$
B1 Solve $8x - y = 7$ $12x - 8y = 6$	B2 Solve $3x - 2y = 13$ $x - y = 5$	B3 Solve $2x - 3y = 3$ $3x + 6y = 1$	B4 Solve $3x + 5y = 7$ $9x + 11y = 13$
C1 Solve $5x + 3y = 14$ $2x + 2y = 4$	C2 Solve $4x + 5y = 13$ $3x - 2y = 27$	C3 Solve $2x + 7y = 12$ $3x + 8y = 13$	C4 Solve $3x + 2y = 7$ $4x - 3y = 15$
D1 Solve $y = 4x - 8$ $y = 3x - 5$	D2 Solve $y = 4x - 2$ $y = 9x - 12$	D3 Solve $3x + 2y = 61$ $y = 2x - 22$	D4 Solve $3x - 2y = -24$ $y = 5 - 2x$

Fluency Practice

(a) Solve $4x - y = 17$
 $x = y + 2$

(b) Solve $2x + y = 6$
 $y = 4x + 3$

(c) Solve $3x + 7y = 13$
 $y = x - 11$

(a) Solve $4x - 3y = 7$
 $3y = x + 5$

(b) Solve $y + 1 = 3x$
 $2x - 3y = 24$

(c) Solve $3x + 5y = 29$
 $y + 11 = 5x$

(a) Solve $4x + 6y = 74$
 $11 - y = 2x$

(b) Solve $y - 8 = 6x$
 $4x + 5y + 28 = 0$

(c) Solve $8 - x = 3y$
 $10 - 3x = 5y$

(a) Given that $7x = 2y + 34$ and
 $3x + 5y + 3 = 0$, find the value of
 $x^2 + y^2$

(b) Solve $\frac{3x+1}{2} = y$
 $5y - 4x = 13$

(c) Find the coordinates of intersection of
the straight lines with equations
 $y = 3x - 2$
 $x + 3y = 19$

Fluency Practice

Question 1: Solve the following simultaneous equations by using elimination.

- | | | |
|--|---------------------------------------|--|
| (a) $6x + y = 18$
$4x + y = 14$ | (b) $4x + 2y = 10$
$x + 2y = 7$ | (c) $9x - 4y = 19$
$4x + 4y = 20$ |
| (d) $2x + y = 36$
$x - y = 9$ | (e) $6x - 3y = 12$
$4x - 3y = 2$ | (f) $3x - 6y = 6$
$2x - 6y = 3$ |
| (g) $8x + 7y = 39$
$8x + 2y = 34$ | (h) $x + 3y = 38$
$x + 6y = 53$ | (i) $6x + 3y = 48$
$6x + y = 26$ |
| (j) $2x - 4y = 10$
$2x + 3y = 24$ | (k) $5x - 2y = 120$
$5x + y = 165$ | (l) $x - 2y = 8$
$x - 3y = 3$ |
| (m) $3x + 2y = 54$
$2x - 2y = 16$ | (n) $7x - 4y = 80$
$3x - 4y = -80$ | (o) $5x - 2y = -23$
$5x - 6y = -39$ |
| (p) $6x + 2y = -26$
$2x + 2y = -10$ | (q) $x - 5y = 65$
$2x - 5y = 85$ | (r) $10x - 10y = -40$
$10x + 4y = 16$ |

Question 2: Solve the following simultaneous equations by using elimination.

- | | | |
|---------------------------------------|--|---------------------------------------|
| (a) $3x + 2y = 23$
$2x - y = 6$ | (b) $3x - 3y = 9$
$2x + y = 12$ | (c) $4x + 2y = 34$
$3x + y = 21$ |
| (d) $9x - 4y = 59$
$2x - y = 12$ | (e) $2x + 8y = 43$
$x + 3y = 18$ | (f) $6x + 3y = 45$
$2x - 2y = 12$ |
| (g) $5x + 4y = 130$
$x + 6y = 130$ | (h) $10x - 15y = 25$
$x - 2y = 1$ | (i) $3x + 8y = 97$
$2x + 4y = 58$ |
| (j) $3x - y = 4$
$5x + 4y = 52$ | (k) $4x + 9y = 10$
$2x + 3y = 2$ | (l) $5x - 3y = 33$
$3x - 9y = 63$ |
| (m) $2x + 4y = -2$
$4x + 2y = -10$ | (n) $8x + 4y = -28$
$3x - 12y = 30$ | (o) $15x - 4y = 82$
$5x - 9y = 12$ |
| (p) $12x + 3y = 9$
$2x + 11y = -9$ | (q) $9x - 7y = 111$
$x - 2y = 16$ | (r) $8x - y = 4$
$3x + 8y = -166$ |

Fluency Practice

Question 3: Solve the following simultaneous equations by using elimination.

- | | | | | | |
|-----|----------------------------------|-----|-------------------------------------|-----|-----------------------------------|
| (a) | $2x + 2y = 14$
$5x - 3y = 19$ | (b) | $2x + 3y = 1$
$7x + 2y = -22$ | (c) | $5x + 3y = 22$
$2x + 4y = 20$ |
| (d) | $5x - 6y = 28$
$4x - 4y = 24$ | (e) | $3x + 2y = 7$
$2x + 9y = 43$ | (f) | $3x + 3y = -6$
$4x - 4y = -24$ |
| (g) | $3x + 8y = 31$
$5x + 3y = 31$ | (h) | $7x - 15y = 2.5$
$3x - 2y = 5.5$ | (i) | $3x + 2y = 53$
$2x + 5y = 72$ |
| (j) | $5x - 3y = 18$
$2x + 4y = 54$ | (k) | $2x + 9y = 11$
$9x + 3y = -63$ | (l) | $2x - 4y = 4$
$5x - 3y = 24$ |
| (m) | $3x + 3y = 42$
$2x + 4y = 38$ | (n) | $6x + 2y = -2$
$4x - 3y = 29$ | (o) | $4x - 4y = 8$
$5x - 3y = 18$ |
| (p) | $4x + 3y = 9$
$5x + 2y = 13$ | (q) | $4x - 2y = 18$
$2x - 3y = 15$ | (r) | $5x + 2y = 38$
$2x - 3y = 19$ |

Question 4: Solve the following simultaneous equations by rearranging and then using elimination.

- | | | | | | |
|-----|----------------------------------|-----|-------------------------------------|-----|--|
| (a) | $x = 10 - y$
$2x + y = 17$ | (b) | $x - 4 = y$
$x + 3y = 12$ | (c) | $2x + 6y = 4$
$x = 12 + 2y$ |
| (d) | $3x = 10 + 5y$
$3y = 52 - 4x$ | (e) | $2x + y - 18 = 0$
$3y = 7x + 80$ | (f) | $6x + 2y + 6 = 0$
$7x - 5y - 93 = 10$ |

Apply

Question 1: The cost of buying a coffee and a tea in a cafe is £4.
The cost of buying a coffee and three teas in a cafe is £7.
Work out the cost of buying a coffee and the cost of buying a tea.

Question 2: The sum of Rosemary's age and Hannah's age is 102 years.
The difference between Rosemary's age and Hannah's age is 52 years.
Rosemary is older than Hannah.
Find the age of each woman by using simultaneous equations.

Question 3: Five adult tickets and three child tickets for a movie cost £58.
Two adult tickets and eight child tickets for a movie cost £47.
Find the cost of each type of ticket.

Fluency Practice

- Question 4: Four chairs and two tables cost £218.
Six chairs and seven tables cost £587.
Find the total cost of buying twenty chairs and five tables.
- Question 5: A plumber charges a price for each hour, £h, and a fixed charge, £c.
A 5 hour job costs £155 in total.
A 8 hour job costs £230 in total.
How much would a job that lasts 2 hours cost?
- Question 6: Barry buys 200 pieces of stationery for £76.
Of the 200 pieces of stationery, x of them are rulers that cost 50p each and y of them are pens that cost 20p each.
Find how many rulers Barry buys and how many pens he buys.
- Question 7: In a greengrocers, 4kg of bananas and 3kg of apples costs £7.50
In the same greengrocers, 3kg of bananas and 5kg of apples costs £8.10
How much would 2kg of bananas and 2kg of apples cost?
- Question 8: Can you spot any mistakes in the question below?

Solve the simultaneous equations

$$\begin{array}{l} 3x + 5y = 1 \quad \text{x2} \\ 2x - 3y = 7 \quad \text{x3} \end{array}$$

Do not use trial and improvement

$$\begin{array}{r} 6x + 10y = 2 \\ 6x - 9y = 21 \\ \hline 19y = 23 \\ y = 1.21 \end{array} \quad \begin{array}{l} 3x + (5 \times 1.21) = 1 \\ 3x + 6.05 = 1 \\ 3x = -5.05 \\ x = -1.68 \end{array}$$

$$x = \dots\dots\dots -1.68 \quad y = \dots\dots\dots 1.21 \quad (4)$$

Fluency Practice

Solve the following pairs of simultaneous equations:

1. $x + 4y = 1$,
 $x + 7y = 4$.
2. $x + 2y = 1$,
 $3x + 5y = 1$.
3. $5x + 3y = 1$,
 $7x + y = 11$.
4. $x - y = 5$,
 $3x - 4y = 16$.
5. $5x + 4y = 3$,
 $2x + 3y = 4$.
6. $3x + 4y = 1$,
 $5x + 3y = 9$.
7. $\frac{x}{3} + \frac{y}{2} = 4$, $x - \frac{y}{2} = 4$.
8. $\frac{x}{3} + \frac{y}{6} = 5$, $\frac{x}{2} - \frac{y}{9} = 1$.
9. $2x + y = 9$,
 $3x + 2y = 9$.
10. $7x - 5y + 5 = 0$,
 $3x + 2y - 2 = 0$.
11. $\frac{2}{x} + \frac{3}{y} = 2$, $\frac{4}{x} + \frac{3}{y} = 3$.
12. $\frac{6}{x} + \frac{1}{y} = 2$, $\frac{4}{x} + \frac{2}{y} = 2$.
13. $\frac{9}{x} - \frac{4}{y} = 5$, $\frac{3}{x} + \frac{2}{y} = 0$.
14. $\frac{5}{x} - \frac{2}{y} = 3$, $\frac{6}{x} + \frac{7}{y} = 13$.
15. $5/x - 1/y = 1$,
 $3/x + 2/y = 11$.
16. $\frac{2}{x} - \frac{1}{y} = 7$, $\frac{3}{x} + \frac{2}{y} = 28$.
17. $\frac{2}{x} + \frac{1}{y} = 1$, $\frac{5}{x} - \frac{3}{y} = 19$.
18. $x + \frac{3}{y} = 5$, $3x - \frac{2}{y} = 4$.
19. $5x + \frac{2}{y} = 4$, $13x + \frac{4}{y} = 11$.
20. $\frac{2}{x} - \frac{4}{y} = 2$, $\frac{8}{x} + \frac{12}{y} = 1$.
21. $\frac{8}{x} - 2y = 10$, $\frac{6}{x} - y = 6$.

Solve the following pairs of simultaneous equations by substituting the value of x given by the first equation into the second equation.

22. $x = y - 1$,
 $x + 2y = 8$.
23. $x = 3y - 2$,
 $x + 5y = 14$.
24. $x = y - 2$,
 $5x - y = 6$.
25. $x = 3y - 17$,
 $x + 2y = 8$.
26. $x = 3y + 5$,
 $x + y = 1$.
27. $x = 5y + 13$,
 $2x + y = 4$.

Solve the following pairs of simultaneous equations:

28. $x + y = 5 = 3x - 2y$.
29. $3x - y = 7 = 4x + y$.
30. $x = 5 - x - y = y - 2x$.
31. $3x + 2y = 5x - y = 13$.
32. $4x - 3y = 6 - 2x = y$.

Fluency Practice

33. If $3x - 2y = 11$, $x - 5y = 8$, find the values of $2x + y$ and $x - 2y$

34. If $2x + 5y = 19$, $3x + 4y = 11$, find the values of $x + 3y$ and $3x - y$

35. If $\frac{3}{x} + \frac{4}{y} = 3$, $\frac{9}{x} - \frac{4}{y} = 1$, find the values of $\frac{1}{x} + \frac{1}{y}$ and $\frac{1}{x} - \frac{1}{y}$.

Solve the following pairs of simultaneous equations:

36. $\frac{x}{y} = \frac{3}{4}$, $x + y = 14$.

37. $\frac{x+y}{2x+y} = \frac{3}{5}$, $x - y = 1$.

38. $\frac{x+2}{y} = 2$, $x + y = 10$.

39. $\frac{x+1}{x+y} = \frac{1}{2}$, $\frac{x}{y} = \frac{1}{2}$.

40. $\frac{x-3}{y} = 1$, $x + y = 13$.

Fluency Practice

Patrons are reminded to try to use the substitution method where possible. Look for an x or y to isolate from one equation. Then substitute this into the *other* equation. For example solve

$$\begin{aligned} 3x + y &= 7 \\ 2x + 5y &= 2 \end{aligned}$$

From the first we can see that $y = 7 - 3x$. Substituting this into the second we find $2x + 5(7 - 3x) = 2$ which solves to $x = \frac{33}{13}$. We then place this value into $y = 7 - 3x$ to discover $y = -\frac{8}{13}$. So $(x, y) = (\frac{33}{13}, -\frac{8}{13})$

Present answers in the form $(x, y) = (-1, \frac{1}{2})$. (Don't forget the brackets!)

$$\begin{aligned} 1. \quad x + y &= 3 \\ x - y &= 2 \end{aligned}$$

$$\begin{aligned} 13. \quad 5x + 4y &= 1 \\ x - 3y &= 0 \end{aligned}$$

$$\begin{aligned} 2. \quad x - y &= 4 \\ x + y &= 7 \end{aligned}$$

$$\begin{aligned} 14. \quad x + y &= -7 \\ 2x + 3y &= 4 \end{aligned}$$

$$\begin{aligned} 3. \quad 2x + y &= 3 \\ x - 2y &= 2 \end{aligned}$$

$$\begin{aligned} 15. \quad 2x + y &= 2 \\ x - 6y &= 1 \end{aligned}$$

$$\begin{aligned} 4. \quad x + 2y &= 4 \\ 3x - 2y &= -7 \end{aligned}$$

$$\begin{aligned} 16. \quad 3x + 2y &= 1 \\ y - 2x &= -3 \end{aligned}$$

$$\begin{aligned} 5. \quad 5x - y &= 4 \\ 4x - 5y &= 0 \end{aligned}$$

$$\begin{aligned} 17. \quad a - b &= 2 \\ 3a - 2b &= -4 \end{aligned}$$

$$\begin{aligned} 6. \quad 2x - 3y &= 5 \\ 3x + 2y &= 2 \end{aligned}$$

$$\begin{aligned} 18. \quad 4x + 7y &= 10 \\ 3x - y &= -2 \end{aligned}$$

$$\begin{aligned} 7. \quad 4x - 2y &= -9 \\ 3x + 5y &= 3 \end{aligned}$$

$$\begin{aligned} 19. \quad 2x + 3y &= 1 \\ 3x - 4y &= 2 \end{aligned}$$

$$\begin{aligned} 8. \quad x - 2y &= 2 \\ 3x + 2y &= 1 \end{aligned}$$

$$\begin{aligned} 20. \quad 5x + y &= 7 \\ 4x - \frac{1}{2}y &= 2 \end{aligned}$$

$$\begin{aligned} 9. \quad 3x - y &= 3 \\ 2x + 3y &= -1 \end{aligned}$$

$$\begin{aligned} 21. \quad x + 5y &= 0 \\ 3x + 4y &= -1 \end{aligned}$$

$$\begin{aligned} 10. \quad y - 2x &= 5 \\ 5x - 7y &= 2 \end{aligned}$$

$$\begin{aligned} 22. \quad \frac{x + y}{2} - \frac{x - y}{3} &= 1 \\ x - 2y &= 2 \end{aligned}$$

$$\begin{aligned} 11. \quad x + 3y &= 4 \\ 5x - 2y &= 6 \end{aligned}$$

$$\begin{aligned} 23. \quad \frac{x + 2y}{3} - \frac{x - 3y}{7} &= x \\ x + 3y &= 2 \end{aligned}$$

$$\begin{aligned} 12. \quad 4x - y &= 2 \\ 3x + 4y &= 1 \end{aligned}$$

$$\begin{aligned} 24. \quad x + ay &= 0 \\ 2x + 3y &= -1 \end{aligned}$$

Fluency Practice

$$25. \begin{cases} kx + y = 4 \\ 2x - 3y = 2 \end{cases}.$$

$$26. \begin{cases} ax + 4y = 6 \\ bx - y = 5 \end{cases}.$$

$$27. \begin{cases} kx + y = 1 \\ 5x - ky = m \end{cases}.$$

$$28. \begin{cases} x + y = 1 \\ ax + by = 1 \end{cases}.$$

$$29. \begin{cases} x + ay = 3 \\ ax + by = 4 \end{cases}.$$

$$30. \begin{cases} 4x + y = 3 \\ ax + by = c \end{cases}.$$

$$31. \begin{cases} x + by = 2 \\ \frac{x + ay}{2} - \frac{x - y}{3} = 1 \end{cases}.$$

$$32. \begin{cases} \frac{ax + 1}{2} + \frac{by + 2}{3} = 1 \\ \frac{5x + 1}{3} + \frac{ay + 1}{2} = 1 \end{cases}.$$

$$33. \begin{cases} 23x + 21y = 1 \\ 21x + 23y = -1 \end{cases}.$$

Solve and check each result:

1. $3x + 4y = 2a$

$5x + 6y = 4a$

2. $2ax + 3by = 4ab$

$5ax + 4by = 3ab$

3. $ax + by = 1$

$a'x + b'y = 1$

4. $x - y = 2n$

$mx - ny = m^2 + n^2$

5. $2bx + ay = 4b + a$

$abx - 2aby = 4b + a$

6. $ax - by = a^2 + b^2$

$bx + ay = 2(a^2 + b^2)$

7. $ax + by = c$

$mx + ny = d$

8. $bx + ay = a + b$

$ab(x - y) = a^2 - b^2$

9. $c^2x - d^2y = c - d$

$cd(2dx - cy) = 2d^2 - c^2$

10. $\frac{x + m}{y - n} = \frac{n}{m}$

$x + y = 2n$

11. $(a + 1)x - by = a + 2$

$(a - 1)x + 3by = 9a$

12. $(a - b)x - (a + b)y = a^2 + b^2$

$bx + ay = 0$

Fluency Practice

Simultaneous Equations

Find the value of each shape in each pair of equations.

$$\bigcirc + \square = 7$$

a)

$$\bigcirc + \square + \square = 12$$

$$\triangle + \triangle = 7$$

b)

$$\triangle + \triangle + \triangle = 15$$

$$\bigcirc + \square + \square = 16$$

c)

$$\bigcirc + \square + \square + \square = 18$$

$$\triangle + \triangle = 10$$

d)

$$\triangle + \triangle + \triangle + \triangle = 23$$

$$\triangle + \triangle = 13$$

e)

$$\triangle + \triangle + \triangle + \triangle + \triangle = 30$$

$$\triangle + \triangle = 12$$

f)

$$\triangle + \triangle = 18$$

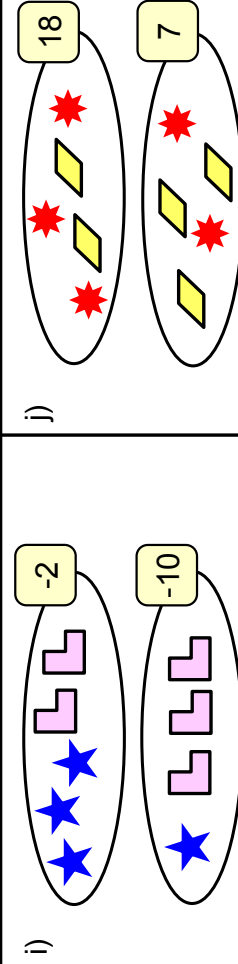
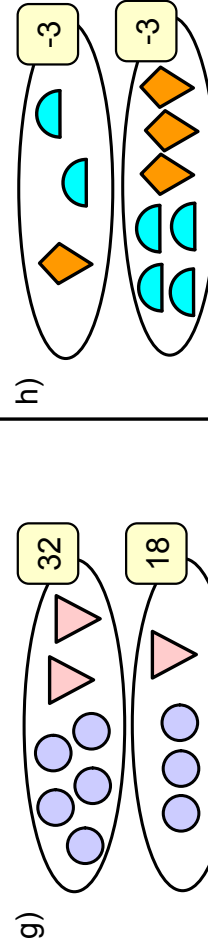
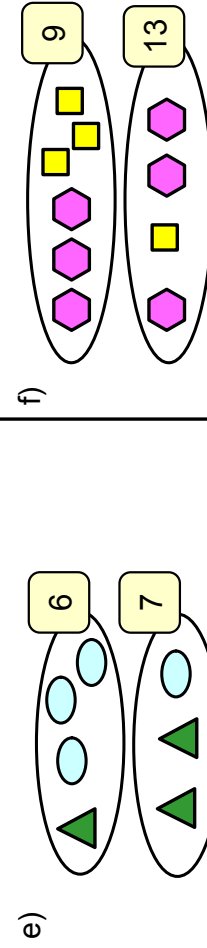
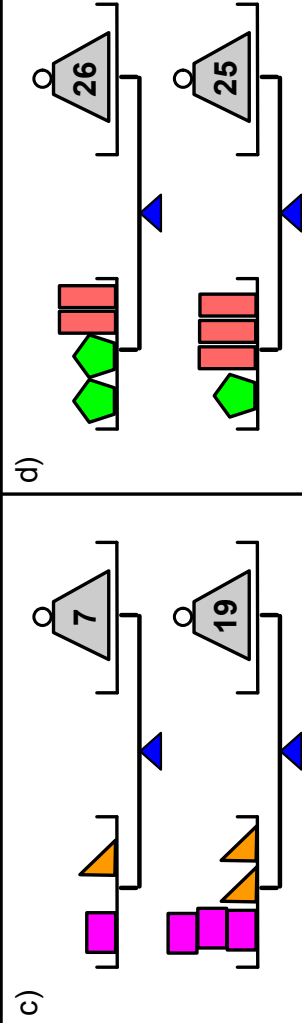
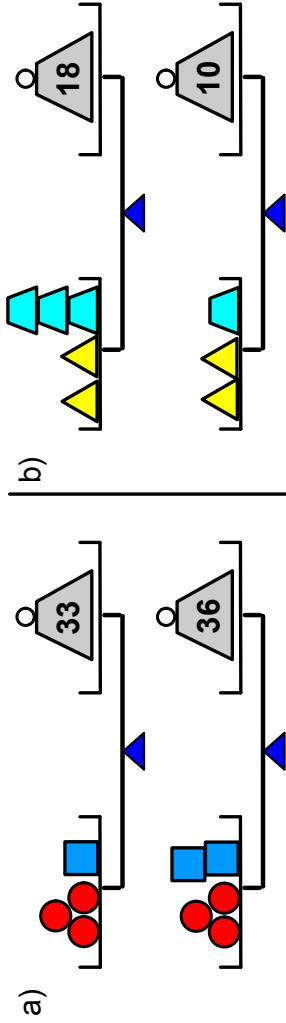
$$\triangle + \triangle + \triangle + \triangle = 18$$

g)

$$\triangle + \triangle + \triangle + \triangle = 17$$

Fluency Practice

Work out the value of each shape.



Fluency Practice

			16
			8
			8
			14

			12
			14
			13
			11

			32
			22
			30
			34

£	€	¥	26
€	£	£	40
\$	\$	£	46
¥	€	€	17

<i>a</i>	<i>b</i>	<i>c</i>	15
<i>a</i>	<i>b</i>	<i>c</i>	24
<i>d</i>	<i>a</i>	<i>a</i>	14
<i>d</i>	<i>d</i>	<i>d</i>	20

			18
			11
			18
			21

Fluency Practice

Forming & Solving Simultaneous Equations using Substitution

①

1 t-shirt and 1 jumper cost £25.
The jumper costs £5 more than the t-shirt.

②

1 scarf and 1 hat cost £16.
The hat costs £2 more than the scarf.

③

1 dress and 1 skirt cost £35.
The skirt is £5 cheaper than the dress.

④

1 coffee and 2 teas cost £11.
A tea is £1 more expensive than a coffee.

⑤

2 cupcakes and 1 muffin cost £8.
A muffin is double the price of a cupcake.

⑥

3 notebooks and 2 books cost £33.
A book costs £4 more than a notebook.

⑦

3 t-shirts and 2 hoodies cost £81.
For the cost of 1 hoodie you
could buy 3 t-shirts

⑧

2 pairs of sweatpants and 3 jumpers cost £89.
For the cost of 1 jumper you could
buy 2 pairs of sweatpants and
have £3 left over.

⑨

5 litres of washing up liquid and
2 litres of professional detergent cost £80.
The cost of 1 litre of detergent is £4 less than
the price of 3 litres of washing up liquid.

Fluency Practice

(a) Maryam buys 5 pears and 2 kiwi fruits for £3.90. At the same grocers, Dave buys 3 pears and 1 kiwi fruit for £2.20. Find the cost of one pear and the cost of one kiwi fruit.

(b) Peter buys 2 pencils and 3 pens for £1.70. Bola buys 4 pencils and 9 pens for £4.60. Find the cost of one pencil and the cost of one pen.

(c) Yusuf visits a café and orders 2 coffees and 3 cakes for £10.35. Maria orders 5 coffees and 2 cakes for £15.70 at the same café. How much would Tia pay for 3 coffees and 2 cakes at the café?

(d) Two numbers have a sum of 57 and a difference of 36. Find the two numbers.

(e) Cards are 65p each and gift wrap sheets are 45p each at a gift shop. Owen buys 21 items from the shop and spends £11.85. How many cards and how many gift wrap sheets did Owen buy?

(f) Terri has a loose change jar that contains only 5p and 10p coins. There are 134 coins in the jar and their total value is £9. Find the number of 5p coins and the number of 10p coins in the jar.

(g) Flour comes in bags of 1.5 kg or 2 kg. A baker has thirty bags of flour and their total weight is 51.5 kg. Find the number of 1.5 kg bags and the number of 2 kg bags the baker has.

(h) A farmer keeps ducks and pigs in the same field. The total number of animals in the field is 54. The difference between the number of pig's feet and the number of duck's feet is 72. Find the number of pigs and the number of ducks in the field.

Fluency Practice

1.

The sum of two numbers is 130, while their difference is 38. Find the two numbers.
2.

Becky and Peter's ages add to 53. If Becky is 3 years younger than Peter, what are their ages?
3.

A necklace is made from 164 purple and blue beads. There are 8 more purple beads than blue beads. How many of each colour bead are there?


4.

Ten Choco bars and six nutty bars cost £5. Three Choco bars and one Nutty bar cost £1.30. Find the cost of each chocolate bar.


5.

Some chickens and pigs are in a field. How many of each animal are there if there are 32 heads and 80 legs in total?


6.

Peter bought a mixture of large postcards (35p) and small postcards (29p). He bought 20 postcards in total, costing £6.28. How many of each size of postcard did he buy?

7. Find the value of each of the symbols in the grid:

				= 77
				= 78
				= 78
				= 79

8.

Five apples and three pears cost 86 pence. Seven apples and five pears cost £1.30. Find the cost of each piece of fruit.

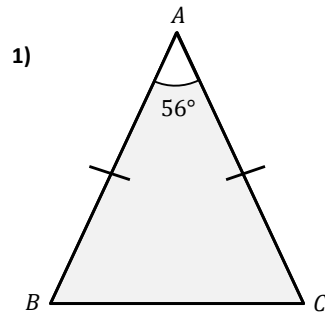


9.

Two adults and three children went to the cinema, and the total cost was £32.40. Three adults and five children cost £51.20. What were the individual prices of adult and child tickets?
10.

A slot machine takes only 20p and 50p coins. The machine contains a total of 140 coins worth £45.10. How many of each type of coin are in the machine?

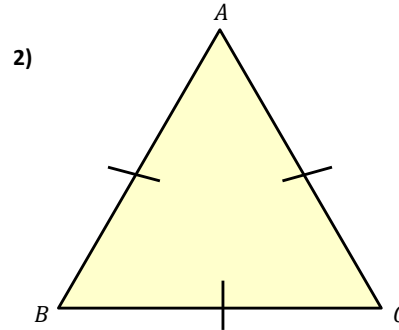
Fluency Practice



$$\angle ABC = 2x + 5y + 4$$

$$\angle ACB = 5x + 6y - 5$$

What are the values of x and y ?

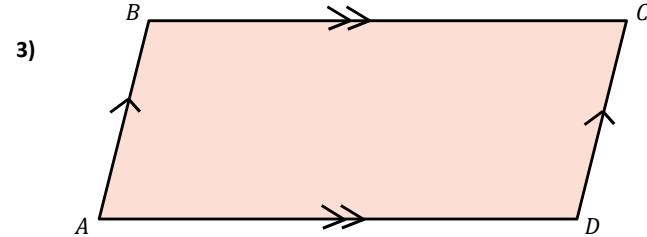


$$\angle ABC = 5x + 4y$$

$$\angle ACB = 8x - y + 1$$

$$\angle BAC = 4x + 3y + 13$$

What are the values of x and y ?



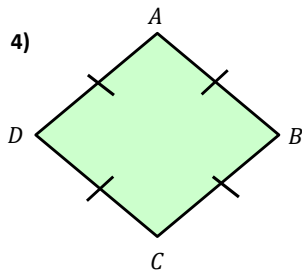
$$\angle ABC = 9x + 7y + 7$$

$$\angle BCD = 4x + y + 7$$

$$\angle CDA = 6x + 4y + 58$$

$$\angle DAB = 3x + 2y + 2$$

What is the size of each angle?



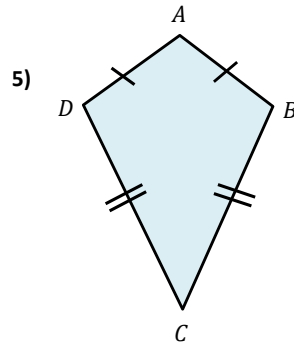
$$AB = 4x + y + 5$$

$$BC = 3x + 3y$$

$$CD = x + 4y + 2$$

$$DA = 2x + 4y - 1$$

How long is each side?



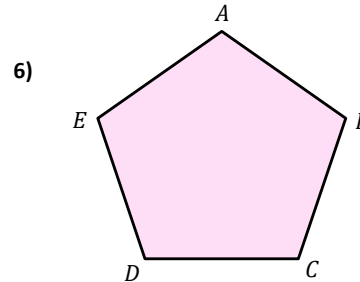
$$AB = 4x + y - 7$$

$$BC = 5x + 3y - 1$$

$$CD = 8x + 2y - 5$$

$$DA = 2x + y + 1$$

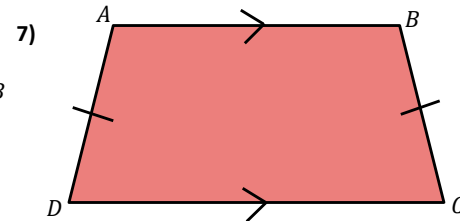
How long is each side?



$$\angle EAB = 5x + 7y + 14$$

$$AB = x + 2y$$

This is a regular pentagon. Its perimeter is 115cm. What are the values of x and y ?



$$\angle BCD = 15x + 6y + 16$$

$$\angle CDA = 10x + 7y + 22$$

$$AD = 3x + 3y$$

$$BC = 13x - 2y$$

What are the values of the given angles and sides?

Fluency Practice

subsections

a total of **60** people were observed

of these,

25% of the men had blue eyes

40% of the women had blue eyes

altogether there were **14** people with blue eyes

how many males and how many females
were observed?

98 people were interviewed

of these,

30% of the women were born in London

33 $\frac{1}{3}$ % of the men were born in London

altogether **31** people were born in London

how many females and how many males
gave information in the interviews?

41 people bought a newspaper

of these,

25% of the men were over 60

40% of the women were over 60

altogether **14** people were over 60

how many females and how many males
bought newspapers?

a total of **76** people were asked about driving

of these,

20% of the women did not own a car

37 $\frac{1}{2}$ % of the men did not own a car

altogether, **25** people did not own a car

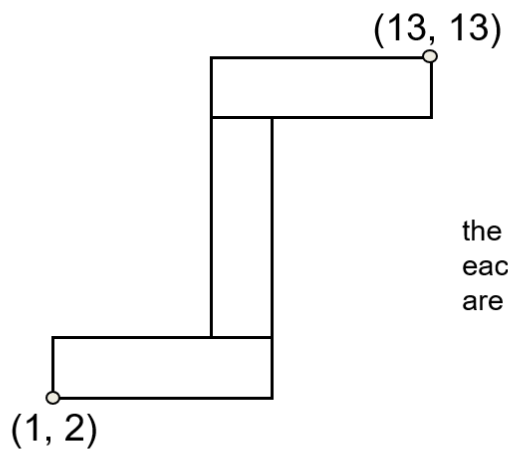
how many males and how many females
were asked?

Fluency Practice

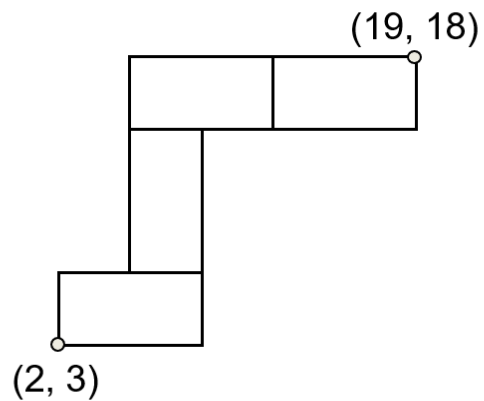
rectangles on a grid

how long and wide are each of the rectangles?

(1)

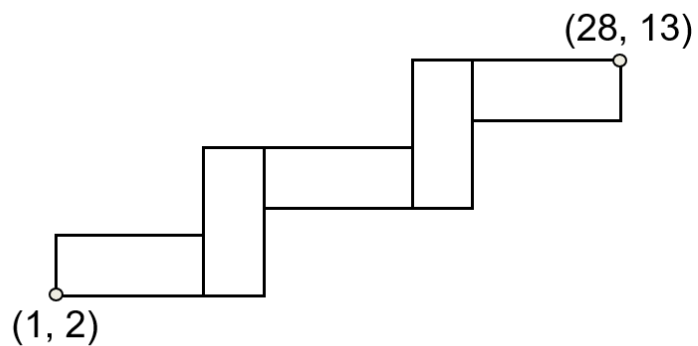


(2)

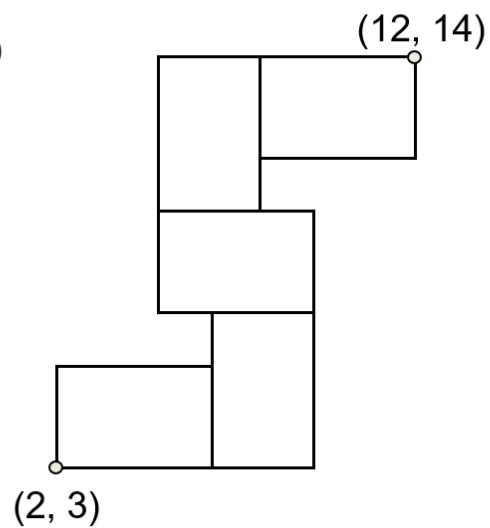


the rectangles in
each question
are all congruent

(3)

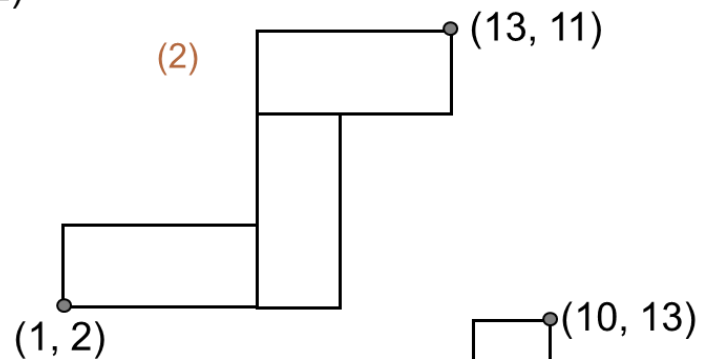
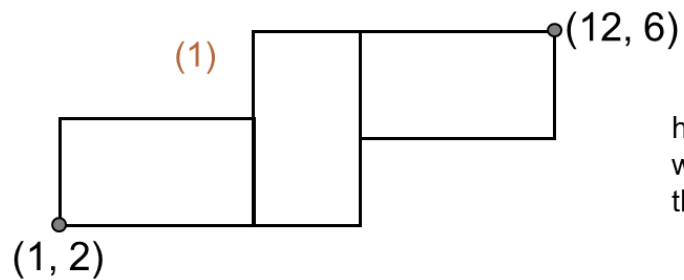


(4)

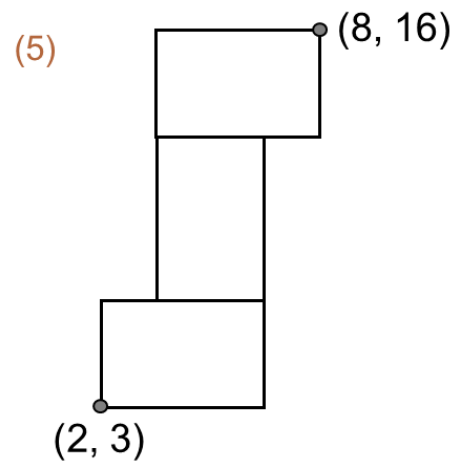
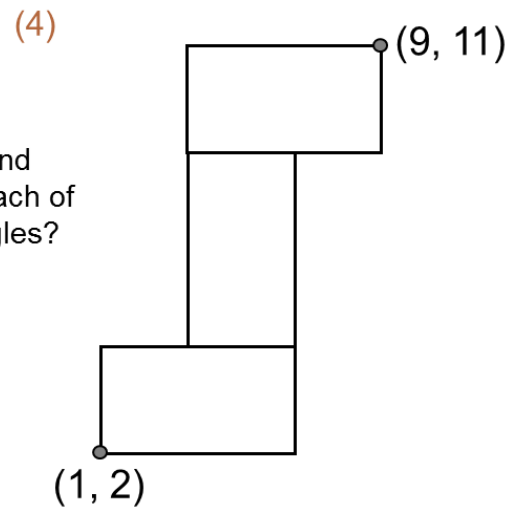
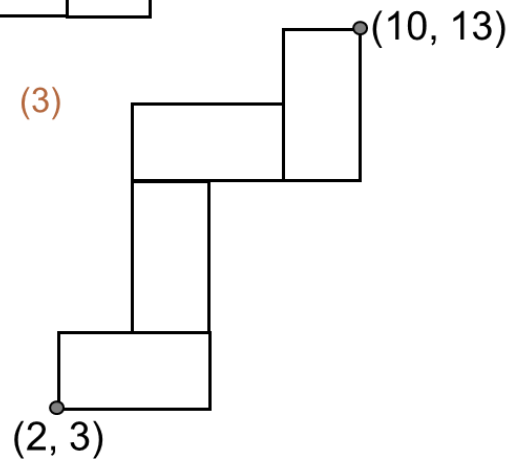


Fluency Practice

rectangles on a grid



the rectangles in
each question
are all congruent



how long and
wide are each of
the rectangles?

Fluency Practice

Solve:

(a) $2x + y = 10$

(b) $4x + y = 9$

$x + y = 7$

$x + y = 3$

(c) $3x + 2y = 22$

(d) $5x + 2y = 17$

$x + 2y = 10$

$3x + 2y = 11$

Solve:

(a) $2x + y = 14$

(b) $3x - y = 1$

$x - y = 1$

$x + y = 7$

(c) $3x + 2y = 25$

(d) $x - 3y = 2$

$x - 2y = 3$

$2x + 3y = 22$

Solve:

(a) $5x + y = 5$

(b) $6x - y = 9$

$3x + y = 7$

$5x - y = 7$

(c) $4x + 2y = 22$

(d) $x - 3y = 4$

$3x - 2y = 6$

$4x + 3y = 1$

(e) $x + y = 0$

(f) $5x + 2y = 13$

$x - y = 6$

$x + 2y = 9$

(g) $3x + 2y = 16$

(h) $3x - y = 9$

$x - 2y = 4$

$5x + y = 11$

(i) $4x + y = 8$

(j) $5x - 2y = 7$

$2x + y = 7$

$4x + 2y = 11$

David buys 5 biscuits and 3 cakes for £3.95. Samira buys 8 biscuits and 3 cakes for £5.15. Find the cost of one biscuit and the cost of one cake.

Jim is thinking of two numbers. When he adds them together he gets 20. When he subtracts one from the other he gets 30. What are Jim's two numbers?

Fluency Practice

Solve:

(a) $x + 2y = 7$ (b) $x + 3y = 11$

$3x + y = 6$ $4x + y = 22$

(c) $3x - y = 7$ (d) $x - y = 5$

$5x - 2y = 10$ $2x - 5y = 4$

Solve:

(a) $x + y = 5$ (b) $7x - y = 1$

$3x - 2y = 5$ $x + 3y = 19$

(c) $2x + 5y = 24$ (d) $4x - 2y = 14$

$3x - y = 2$ $3x + y = 23$

Solve:

(a) $x + 2y = 13$ (b) $7x - 4y = 5$

$2x + 3y = 20$ $x + 2y = 11$

(c) $2x + 5y = 5$ (d) $4x - 2y = 14$

$3x - 2y = 17$ $x - 3y = -4$

Solve:

(a) $5x + 2y = 31$ (b) $4x + y = 5$

$x - 4y = 4$ $2x + 3y = 10$

(c) $2x - 3y = 16$ (d) $x - 5y = 6$

$7x - 2y = 39$ $3x + 2y = 1$

(e) $3x + 4y = 14$ (f) $x + 7y = 15.5$

$x - y = -7$ $2x - 5y = -7$

Three apples and two oranges costs £2.53. Five apples and three oranges costs £4.12. Find the cost of one apple and the cost of one orange.

Fluency Practice

(a)	(b)	(c)	(d)
Solve $2x + 3y = 14$ $x + 3y = 10$	Solve $5x + 2y = 37$ $3x - 2y = 3$	Solve $x + 4y = 17$ $3x + 4y = 19$	Solve $2x + y = 4$ $3x + 2y = 5$
(e)	(f)	(g)	(h)
Solve $5x - y = 36$ $x + 3y = 4$	Solve $7x + 4y = 6$ $3x + 2y = 4$	Solve $6x - y = 4$ $2x - 4y = 5$	Solve $4x + 2y = 19$ $x + 3y = 16$
(i)	(j)	(k)	(l)
Solve $2x - 3y = 20$ $3x + 5y = 11$	4 burgers and 2 sausages costs £4.70. 3 burgers and 5 sausages costs £5.80. Find the cost of one burger and one sausage.	The sum of two numbers is 10.3. The difference between the same two numbers is 2.84. Find the two numbers.	Find the coordinates of the point where the lines $2x + 3y = 21$ and $3x - y = 4$ meet.

Problem Solving

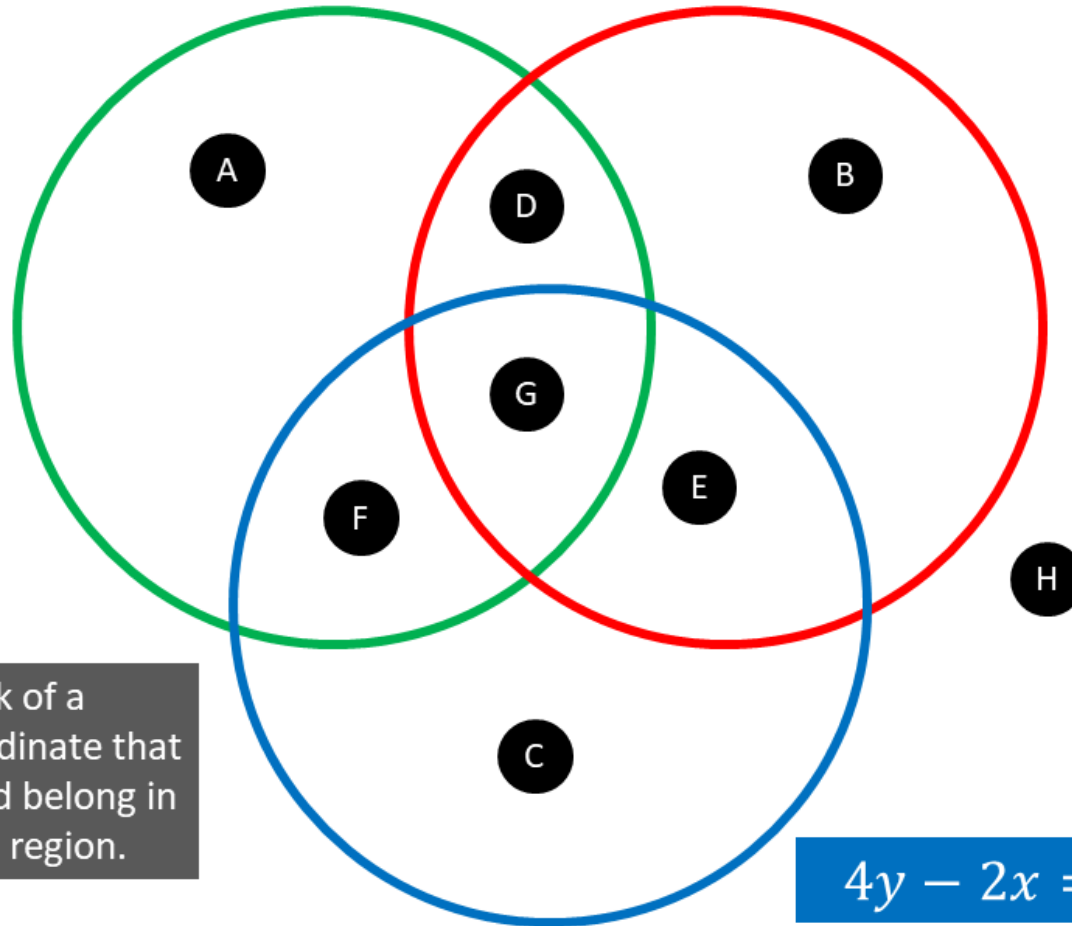
1. Write some **sets** of coordinates that fit the equations.
Place sets that fit both in the middle. The first is done for you.

1	$y = 2x$ $y = x + 4$ 	2	$y = 2x$ $y = x + 3$
3	$y = 2x$ $y = 3x - 2$ 	4	$y = 2x - 3$ $y = 3x - 2$
5	$y = 2x$ $x + y = 6$ 	6	$x + y = 5$ $x + 3y = 9$
7	$x + y = 5$ $3x + y = 9$ 	8	$4x + y = 26$ $3x - 2y = 25$

Maths Venns

$$3y - 6x = 15$$

$$2y - 4x = -8$$



If you think a region is impossible to fill, convince me why!

Think of a coordinate that could belong in each region.

$$4y - 2x = 10$$

3 Three Linear Simultaneous Equations (L2FM Only)

Fluency Practice

Solve the simultaneous equations:

$$x + y + z = 6$$

$$2x + 3y - z = 4$$

$$x + 5y + 2z = 13$$

Solve the simultaneous equations:

$$x + y + 2z = 12$$

$$5x - 2y + z = 22$$

$$x - y + z = 9$$

Solve the simultaneous equations:

$$x + y + z = 0$$

$$-2x + y + 2z = 7$$

$$4x - y - z = -5$$

1 banana, 1 apple and 1 orange costs 90p. 2 bananas, 3 apples and 1 orange costs £1.90. 4 bananas, 1 apple and 3 oranges costs £2.50. Work out the cost of each piece of fruit.

Explain why it is not possible to solve these three simultaneous equations.

$$x + y + z = 6$$

$$2x + 2y + 2z = 12$$

$$x + 3y - z = 10$$

Fluency Practice

three variables

find out the value of each letter

$$(1) \quad a + b = 12$$

$$b + c = 10$$

$$a + c = 16$$

$$(2) \quad d + 2e = 16$$

$$e + 2f = 16$$

$$f + 2d = 13$$

$$(3) \quad 2g + 3h + j = 10$$

$$g + 2h + 3j = 13$$

$$3g + h + 2j = 13$$

$$(4) \quad 2k + 3m + 4p = 38$$

$$4k + 2m + 3p = 32$$

$$3k + 4m + 2p = 38$$

Fluency Practice

Now solve the following simultaneous equations in three unknowns.

1.

$$\begin{aligned}x + y + z &= 1 \\2x + 3y + z &= 6 \\x - y + 2z &= -5\end{aligned}$$

2.

$$\begin{aligned}x + y + z &= 1 \\x - y + 2z &= 2 \\2x + 3y + 3z &= 3\end{aligned}$$

3.

$$\begin{aligned}2a + b + 3c &= -7 \\a - b + 2c &= -4 \\3a + 2b - c &= 11\end{aligned}$$

4.

$$\begin{aligned}2x + 3y - z &= 2 \\4x - y + 2z &= 5 \\2x + y - 3z &= -4\end{aligned}$$

5.

$$\begin{aligned}x - y + z &= 1 \\2x + 2y + 3z &= 1 \\x - y - 4z &= 2\end{aligned}$$

6.

$$\begin{aligned}a + 2b + c &= 3 \\2a + b + c &= 1 \\a - b + 2c &= 0\end{aligned}$$

7.

$$\begin{aligned}p + q + r &= -1 \\2p + q + 2r &= -1 \\p + 3q &= 1\end{aligned}$$

Fluency Practice

8.

$$x + y - z = 1$$

$$x - 2y + 3z = 0$$

$$x - y + 2z = -1$$

9.

$$4x - 5y + 2z = -2$$

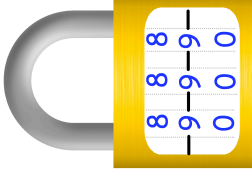
$$5x + 7y + 3z = 3$$

$$2x + 3y + z = 1$$

4 Combinations and Permutations

Fluency Practice

Question 1: Shown is a 3-digit combination padlock.
Each dial can be set to 0, 1, 2, 3, 4, 5, 6, 7, 8, 9



(a) Work out the total number of different combinations that can be used.

(b) Work out the total number of different combinations that have three different digits that can be used.

Question 2: A restaurant has 4 starters and 6 main course on its menu.
Hailey orders a starter and a main course.

How many different combinations of starters and main courses are there?

Question 3: A rugby coach is designing a new rugby strip.
She can choose from: 5 different pairs of socks
6 different pairs of shorts
and 14 different jerseys.

How many different strips are possible?

Question 4: Harry picks a 4 digit pin for his credit card.
Each digit is a number 0 to 9.
Harry can repeat digits.



(a) How many possible codes are there?

Harry chooses not to repeat any digits.

(b) How many possible codes are there now?

Question 5: Rosie picks a 4-digit **odd** number.

The first digit is 5.

The second digit is a 3 or a 4.

The third digit is prime.

How many different 4-digit numbers could Rosie pick?

Fluency Practice

Question 6: Oliver picks a 4-digit **even** number that is greater than 3000. The second digit is a multiple of 4.

How many different numbers could Oliver pick?

Question 7: Sophia is creating a 6-digit code to lock her iPad.

She only uses digits greater than 2.

She only uses each digit once.

How many possible codes can Sophia create?

Question 8: In a class, there are 10 boys and 9 girls.

The teacher has been asked to pick one boy and one girl to win a prize.

How many possible pairs of students can the teacher pick?

Question 9: Jason picks a 5-digit number that is less than 80000.

The first digit is odd.

The fourth and fifth digits are equal.

How many different numbers can Jason pick?

Question 10: A headteacher wants to survey two Year 7 students.

There are 100 students in Year 7.

How many possible pairs of students can the headteacher pick?

Question 11: How many even numbers greater than 40000 can be created using these digits?

2	9	1	6	8
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Apply

Question 1: On a school trip, students are given a packed lunch.

The students can choose one piece of fruit and one snack.

There are 8 different pieces of fruit and some different snacks.

Altogether there are 104 different ways to choose one piece of fruit and one snack

How many different snacks are there?

Fluency Practice

Question 2: At a summer camp, children pick a morning, an afternoon and an evening activity.

There are 4 morning and 7 evening activities to pick from.

Altogether there are 224 different ways to choose their activities.

How many afternoon activities are there?

Question 3: In a gym there are

12 exercise classes on a Monday

13 exercise classes on a Wednesday

7 exercise classes on a Friday

Katie is going to attend either

a class on Monday and a class on Friday

or a class on Wednesday and a class on Friday

or a class on Monday, Wednesday and Friday

Work out how many different ways there are to pick which exercises classes Katie is going to attend.

Question 4: A group of 10 people enter a room.

Each person shakes hands, once, with all the other people in the room.

How many handshakes are there in total?



Question 5: A pizza parlour sells 9 different toppings.

Michael orders a pizza with 2 different toppings.

(a) How many different pizzas can he choose from?

Beth orders a pizza with 3 different toppings.

(b) How many different pizzas can she choose from?

John orders a pizza with 4 different toppings.

(c) How many different pizzas can he choose from?

Fluency Practice

A. There are 8 runners in a race. In how many different ways could the three medals gold, silver and bronze, be allocated?	B. Car registration plates in Great Britain include a random sequence of three letters. Any letters other than I or Q can be used and the same letter can appear more than once. How many different three letter codes could be used?
C. 12 players take part in a singles table tennis competition. Each player must play each other player once. How many matches will there be in the competition?	D. On a school sports day, there are 6 different team events and 9 different individual events. Pupils must participate in two team events and one individual event. How many different ways are there of doing this?
E. In a class there are 8 boys and 11 girls. (a) Two students are to be selected at random from the class. How many different pairs can be chosen? (b) One boy and one girl are to be selected at random from the class. In how many ways can this be done?	F. Two cards are chosen from a standard deck of 52 cards. How many different possible pairs of cards are there?
G. Three cards are picked at random from this set. (a) In how many different ways could the digits picked be in the order <i>odd-even-odd</i>? (b) In how many different ways could the digits either be in the order <i>odd-even-odd</i> or <i>even-odd-even</i>?	1 2 3 4 5 6 7 8 9
H. A form group has 32 members. A form captain, a sports captain and a charity representative are to be chosen. No one can be in more than one of these positions. How many different combinations of results are possible?	I. A team of three is to be chosen from a group of 8 boys and 7 girls. There must be at least one boy and at least one girl in the team. How many possibilities are there?

Fluency Practice

A. Jake has 5 different pairs of trousers and 8 different shirts. How many different combinations of these could he wear?	B. A badminton club has 23 members, 8 of whom are female. In how many ways can the club select a mixed doubles team consisting of one male and one female?
C. UK banknotes are issued in denominations of £50, £20, £10 and £5. UK coins are issued in denominations of £2, £1, 50p, 20p, 10p, 5p, 2p and 1p. One coin and one banknote are to be selected at random. (a) In how many different ways could this be done? (b) In how many of the ways is the total at least £10.50?	
D. On a restaurant's menu, there are 5 different starters, 6 main courses and 4 desserts. For a set price, diners can choose either a starter and a main course or a main course and a dessert. How many different combinations are there?	E. In a class there are n boys. There are twice as many girls as boys. One boy and one girl are to be chosen at random. Write a simplified expression for the number of possible combinations.
F. How many different combinations are possible in this supermarket lunch deal?	 Sandwich (5 options) + Snack (8 options) + Drink (7 options) = £3
G. At a school, students are required to choose one language, one humanities subject and one technology subject to study. There are 4 languages to choose from, 3 humanities and 3 technologies. How many different combinations of subjects could be chosen?	H. An examination paper has three sections, each with six questions. The instructions read: 'Answer one question from each section.' How many different combinations of questions could be answered?

Fluency Practice

- (a) How many ways are there of arranging the word MATHS?
- (b) Ishaq wants to make a PIN code from the digits 1, 3, 5 and 7, using each digit once only. How many different PIN codes can he make?
- (c) How many different numbers can be made from the digits 2, 3, 4, 5 and 6?

- (a) 10 people are called for an interview. How many ways are there of arranging the order of the interviews?
- (b) 8 flags are to be flown outside a building hosting a world leaders' conference. How many ways are there for arranging the flags?
- (c) At a dog show, there is a gold, silver and bronze certificate for three top dogs. If 20 dogs enter, how many ways are there of awarding the certificates?

- (a) Using the digits 5, 6, 7 and 8 once each, how many possible numbers can be made that are multiples of 5?
- (b) Using the numbers 2, 3, 5, 7 and 9 once each, how many even numbers is it possible to make?

- Dr Austin randomly chooses 2 of her 24 students to compete in a quiz against Mrs Barber's class.
- (a) How many ways are there of Dr Austin selecting her 2 students?
- Mrs Barber has 26 students in her class and must also select 2 students at random.
- (b) How many different possible selections of the 4 students competing in the quiz are there?

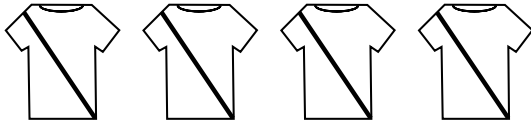
Fluency Practice

T-Shirt Designing (Permutations)

Jess designs T-shirts.

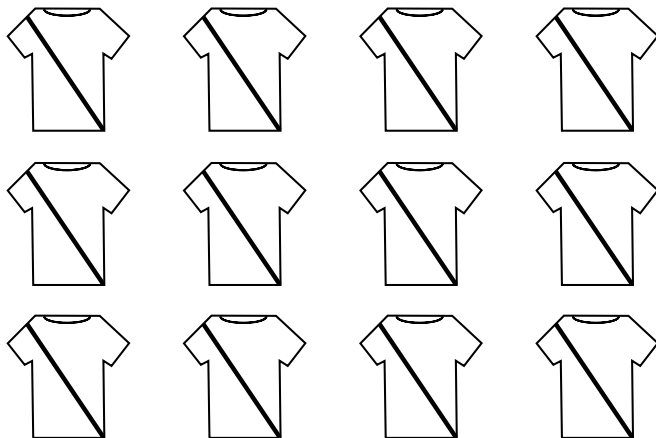
If Jess only uses red & yellow, she can make 4 different T-shirt designs.

Colour the T-shirts to show the different designs.



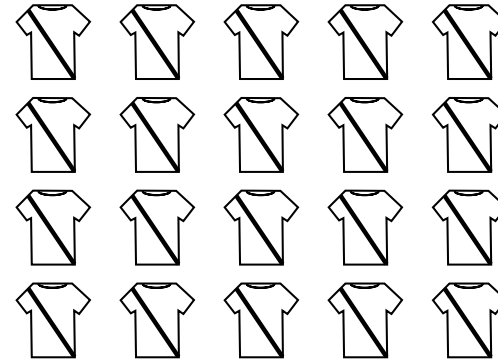
If Jess uses red, yellow & blue, how many different designs can she make?

(hint: it's less than 12)



Jess adds green.

How many different designs can she now make with 4 colours?



Jess uses the 4 colours – but she doesn't make any t-shirts with the same 2 colours (like red & red).

How many different designs can she make?

How many designs could Jess make if she added pink?

How many designs could Jess make with 7 colours, but she can only use a colour once?

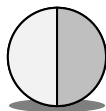


This design is made from 3 pieces of fabric.

If we use only red & green, how many different designs can we make? (including all the same colour).

If we use 3 colours, how many different designs can we make? (including all the same colour).

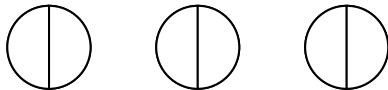
Fluency Practice



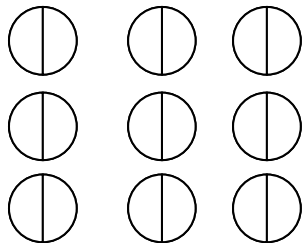
A company makes beachballs using 2 pieces of plastic.

If we use red & yellow plastic pieces, we can make 3 different designs. Colour those designs.

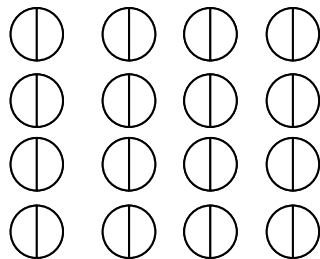
Why is it only 3 unique designs, not 4?



If we use red, yellow & blue pieces of plastic, how many **unique** designs can we make?



If we use red, yellow, blue & white pieces of plastic, how many **unique** designs can we make?



Choosing Combinations

The company adds green.
Complete the table to show all the possibilities for 5 colours.

		Left Side				
		Red	Yellow	Blue	White	Green
Right Side	Red	RR				
	Yellow	YR		YB		
	Blue					
	White					
	Green					

How many unique designs are there?

How many unique designs are there when a colour can't be used twice?

An ice cream shop has 6 flavours. For £1.99 you can get 2 scoops.

How many different 2-scoop combinations are there?



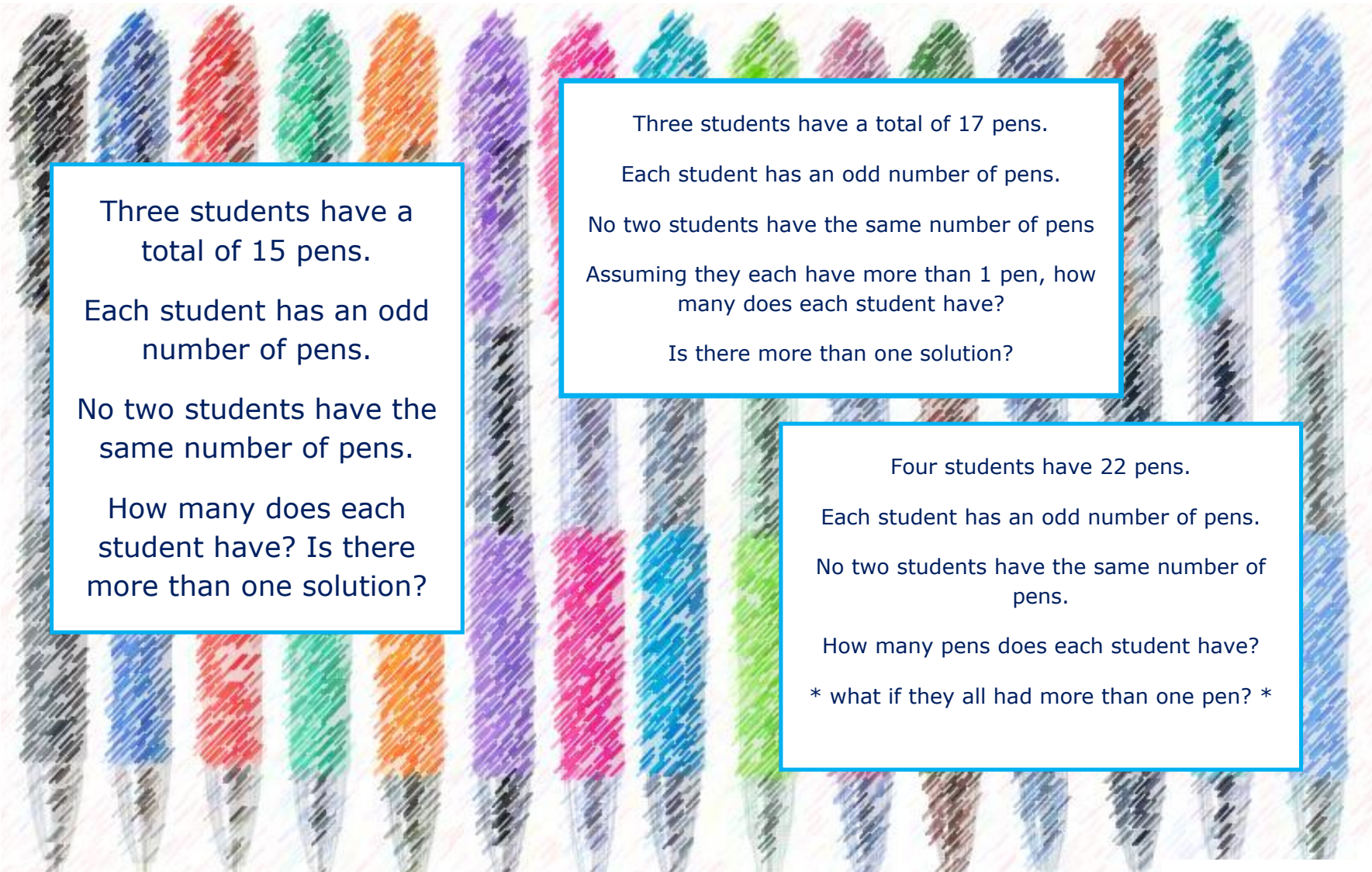
A pizza shop has 7 different types of topping.

You can choose two – but they must be different.

How many different types of 2-topping pizza are available?



Fluency Practice



Three students have a total of 15 pens.

Each student has an odd number of pens.

No two students have the same number of pens.

How many does each student have? Is there more than one solution?

Three students have a total of 17 pens.

Each student has an odd number of pens.

No two students have the same number of pens

Assuming they each have more than 1 pen, how many does each student have?

Is there more than one solution?

Four students have 22 pens.

Each student has an odd number of pens.

No two students have the same number of pens.

How many pens does each student have?

* what if they all had more than one pen? *

Fluency Practice

Product Rule for Counting

1 8 How many 2-digit numbers can you make with these 2 cards?

1 8 7 How does this change if we add a third card?

1 8 7 3

A) How many 4-digit numbers can you make using these cards?

* How many choices do you have for the 1st card?

* After you choose a 1st card,
how many choices do you have for the 2nd card, 3rd card & 4th card?

4 8 3 6 2

B) How many odd 5-digit numbers can you make with these cards?

* One number must be last, how does this affect our choices?

7 9 1 5 3

C) How many numbers greater than 40,000 can you make?

* How many choices are there for the 1st card?

5 6 8 3 4 2

D) How many odd 6-digit numbers can you make?

* How many choices are there for the last card?

7 1 6 5 2 9

E) How many numbers greater than 300,000 can you make?

1 6 5 2 7

F) How many 5-digit numbers that are a multiple of 2 can you make?

7 2 4 3 8

G) How many odd numbers greater than 50,000 can you make?

* Sum the choices for each starting card.

6 1 9 7 3 2

H) How many odd numbers greater than 500,000 can you make?

8 2 5 1 7 4

H) How many even numbers smaller than 600,000 can you make?

9 6 7 1

I) How many 2-digit numbers can we make using these cards?

J) How many 3-digit numbers can we make using these cards?

2 8 1 5 6

K) How many ways can we make 3 numbers from these cards?

L) The number 156 is made. How many different numbers use these same digits?

M) If we don't care about their order (the number they make):
how many ways can we pick 3 digits from the 5 cards?

5 Advanced Statistics

Fluency Practice

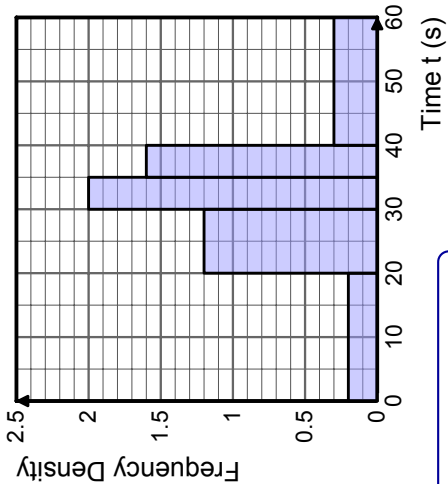
learn by heart

Histograms are used for **continuous data**. The frequency is represented by the **area** of each bar, and the vertical scale is **frequency density**, calculated for each bar using the formula:

$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$

example

Time, t (s)	Frequency	Class Width	Frequency Density
0 < t ≤ 20	4	20	0.2
20 < t ≤ 30	12	10	1.2
30 < t ≤ 35	10	5	2
35 < t ≤ 40	8	5	1.6
40 < t ≤ 60	6	20	0.3



The class width is the range of times included in the row.

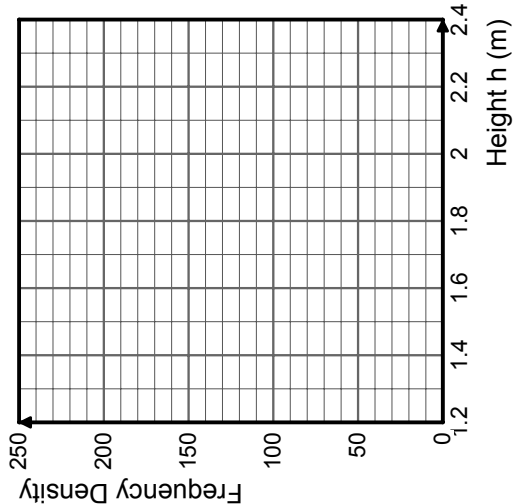
$$\text{Frequency Density} = \frac{\text{Frequency}}{\text{Class Width}}$$

exercise

1. The data in the table shows the heights of a group of people.

Work out the frequency densities and complete the histogram.

Height h (m)	Frequency	Frequency Density
1.4 ≤ h < 1.6	12	
1.6 ≤ h < 1.7	20	
1.7 ≤ h < 1.8	17	
1.8 ≤ h < 2.1	12	

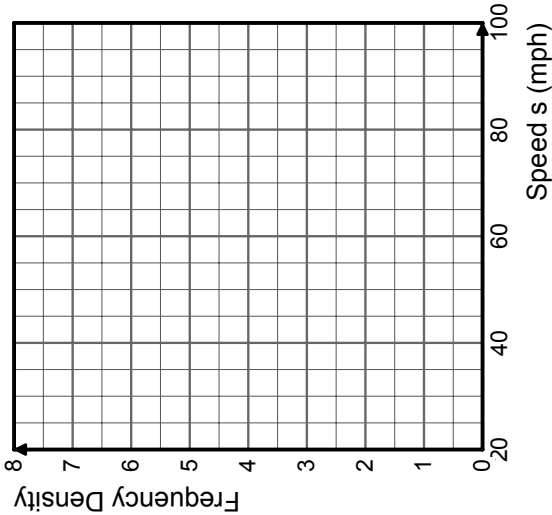


Fluency Practice

2. The data in the table shows the speeds of vehicles passing a point on a motorway.

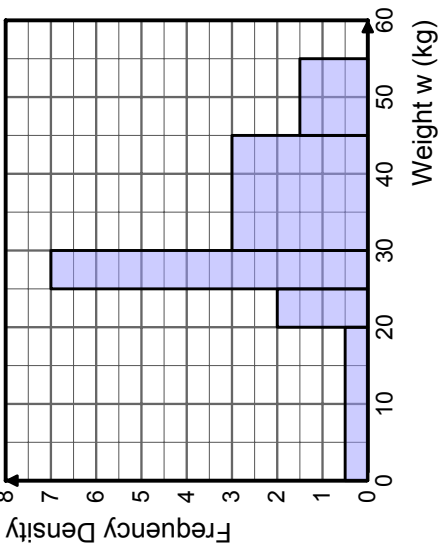
Complete the histogram.

Speed s (mph)	Frequency	Frequency Density
$30 < s \leq 55$	16	
$55 < s \leq 65$	23	
$65 < s \leq 70$	35	
$70 < s \leq 75$	32	
$75 < s \leq 95$	8	



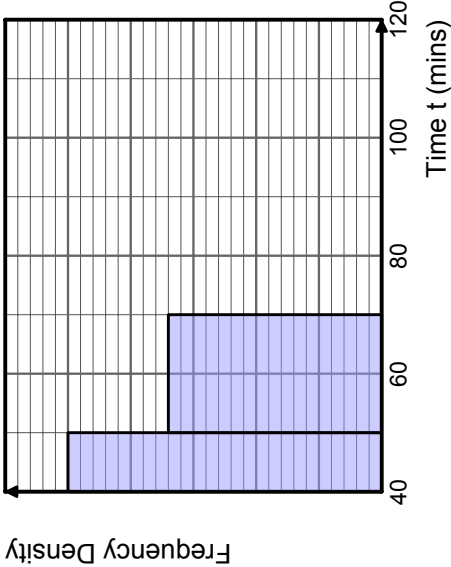
3. The histogram shows the weights of a number of children.
Complete the frequency table.

Weight w (kg)	Frequency
$0 < w \leq 20$	
$20 < w \leq 25$	
$25 < w \leq 30$	
$30 < w \leq 45$	
$45 < w \leq 55$	



4. The table and histogram show the time taken by runners to complete a race. Complete the table and histogram.

Time t (mins)	Frequency
$40 < t \leq 50$	25
$50 < t \leq 70$	
$70 < t \leq 100$	81
$100 < t \leq 120$	12



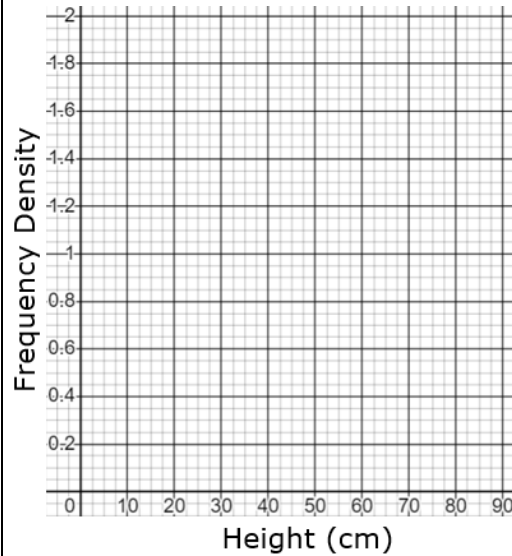
Fluency Practice

Plotting Histograms

(a)

Plot a histogram from the data shown in the table.

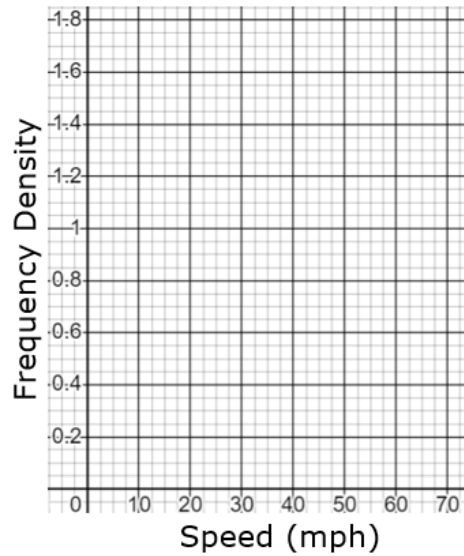
Height (cm)	Frequency	Frequency Density
$0 < h \leq 20$	24	1.2
$20 < h \leq 40$	32	1.6
$40 < h \leq 80$	44	
$80 < h \leq 90$	14	



(b)

Plot a histogram from the data shown in the table.

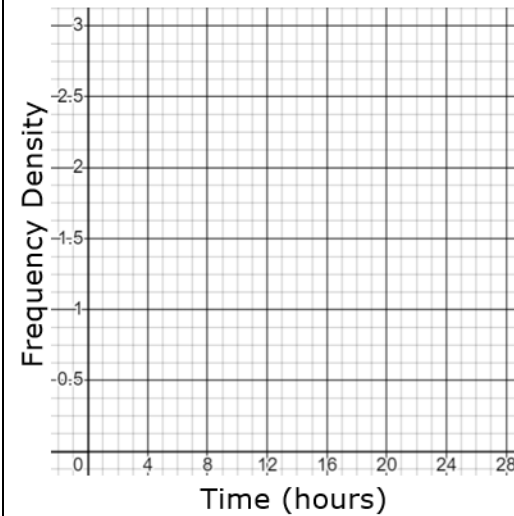
Speed (mph)	Frequency	Frequency Density
$0 < s \leq 10$	5	
$10 < s \leq 35$	30	
$35 < s \leq 50$	24	
$50 < s \leq 70$	16	



(c)

Plot a histogram from the data shown in the table.

Time (hours)	Frequency	Frequency Density
$0 < h \leq 4$	3	
$4 < h \leq 8$	5	
$8 < h \leq 16$	22	
$16 < h \leq 22$	15	
$22 < h \leq 25$	6	



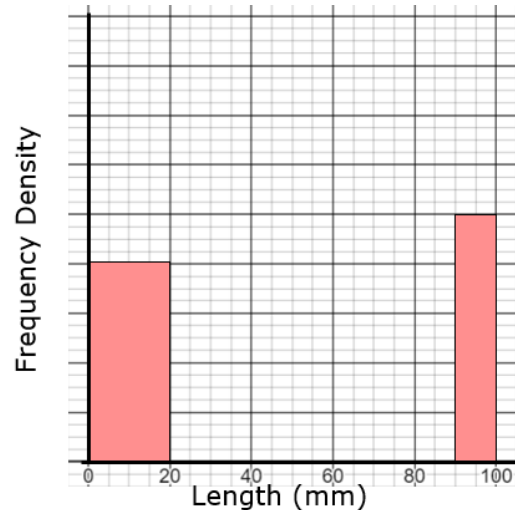
Fluency Practice

Plotting Incomplete Histograms

(a)

Complete the table and histogram.

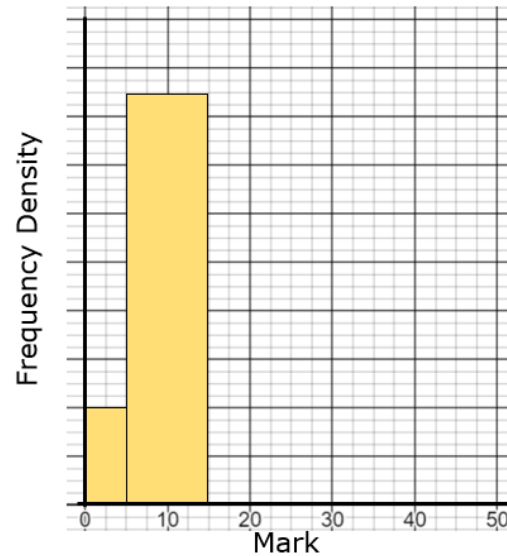
Length (mm)	Frequency	Frequency Density
$0 < h \leq 20$	16	0.8
$20 < h \leq 60$	48	
$60 < h \leq 90$	45	
$90 < h \leq 100$		



(b)

Complete the table and histogram.

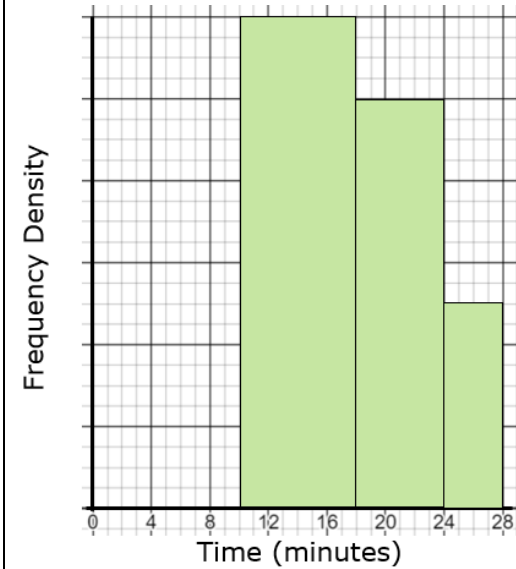
Mark	Frequency	Frequency Density
$0 < s \leq 5$	2	
$5 < s \leq 15$		
$15 < s \leq 30$	27	
$30 < s \leq 50$	22	



(c)

Complete the table and histogram.

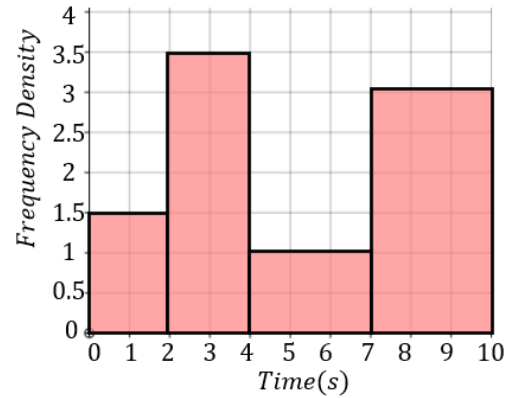
Time (minutes)	Frequency	Frequency Density
$0 < h \leq 4$	2	
$4 < h \leq 10$	9	
$10 < h \leq 18$		
$18 < h \leq 24$		
$24 < h \leq 28$	5	



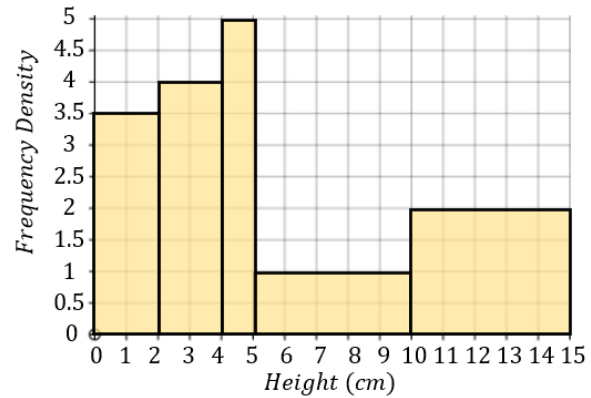
Fluency Practice

Finding the Median from a Histogram

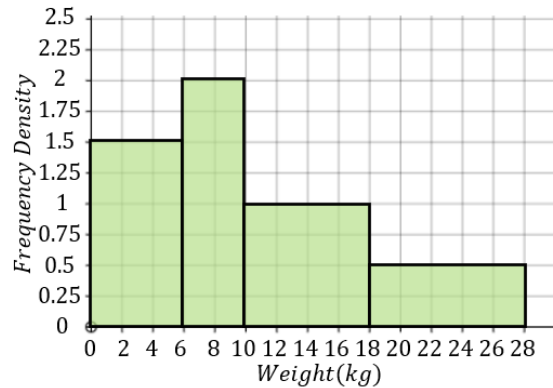
(a)



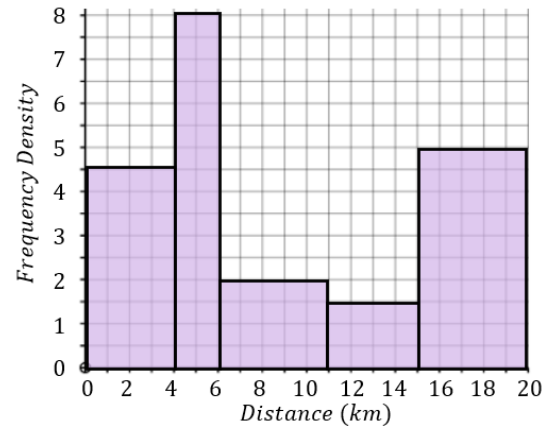
(b)



(c)

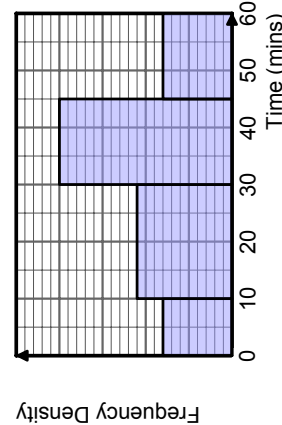


(d)



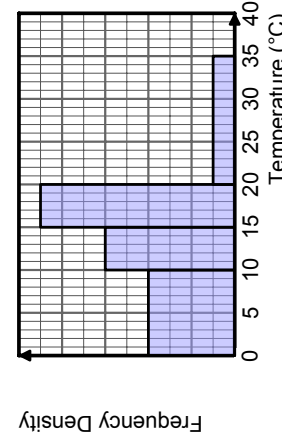
Fluency Practice

1.



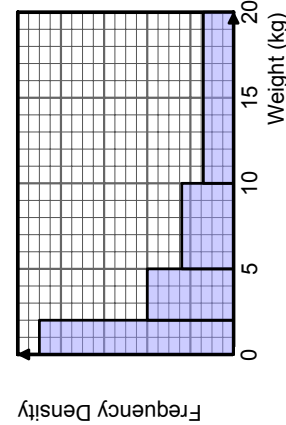
Twelve people waited more than 45 minutes.
How many people waited between 10 and 30 minutes?

2.



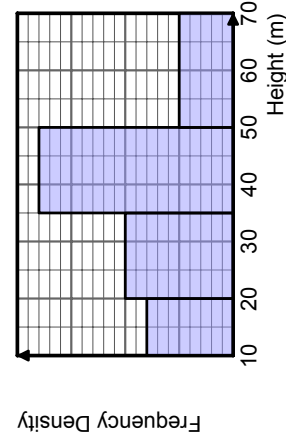
Twelve days had an average temperature between 10°C and 15°C.
How many days were surveyed in total?

3.



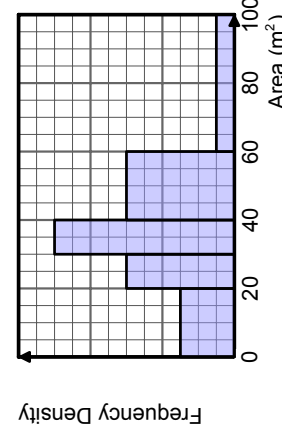
Twelve parcels weighed between 2kg and 5kg.
How many parcels weighed less than 5kg?

4.



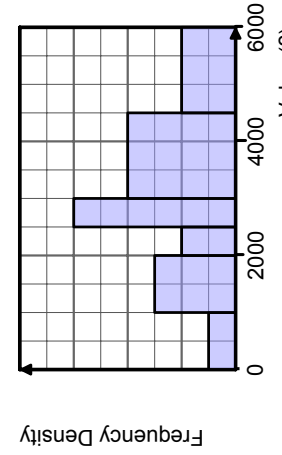
Ten rockets had a height greater than 50m.
How many rockets had a height between 20m and 50m?

5.



Forty rooms have a floor area between 20m² and 40m².
Estimate the number of rooms with a floor area greater than 50m².

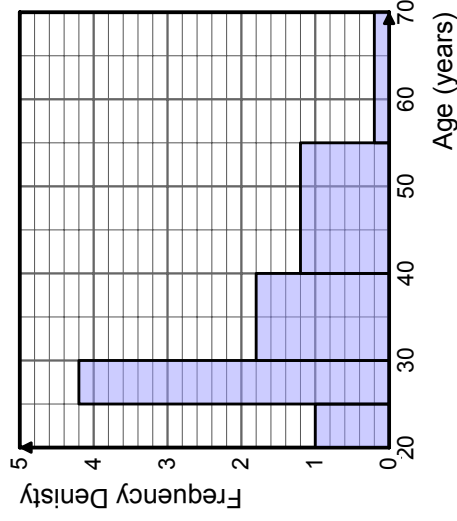
6.



Five used cars were valued at less than £1000.
Estimate the number of used cars valued at less than £4000.

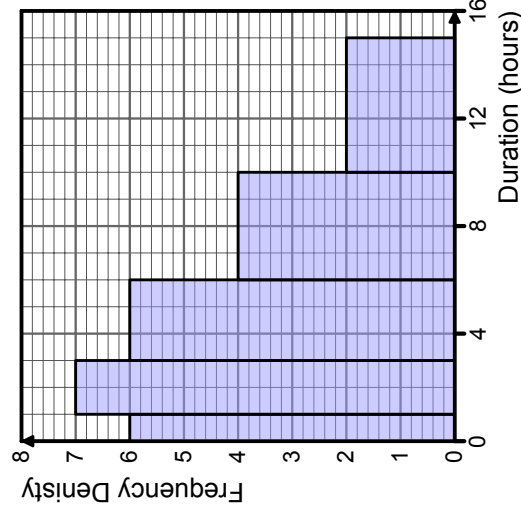
Fluency Practice

7. The histogram shows the ages of the workers at a company.



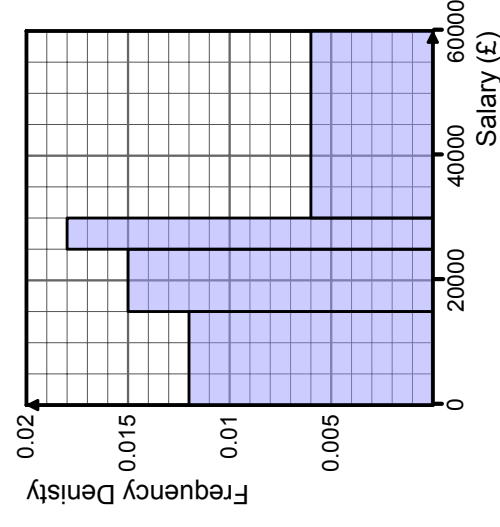
- How many workers are between 20 and 30 years old?
- True or false: there are more workers aged between 30 and 40 than between 40 and 55.
- 60% of the workers at the company are female. Work out the number of female workers.

8. The histogram shows the duration of 64 flights offered by an airline.



- How many of the flights are 3 hours or less in duration?
- Work out an estimate for the number of flights that take between 2 and 6 hours.
- Work out an estimate for the median flight time.

9. In a survey, 600 workers were asked about their salaries. The results are shown in the histogram.



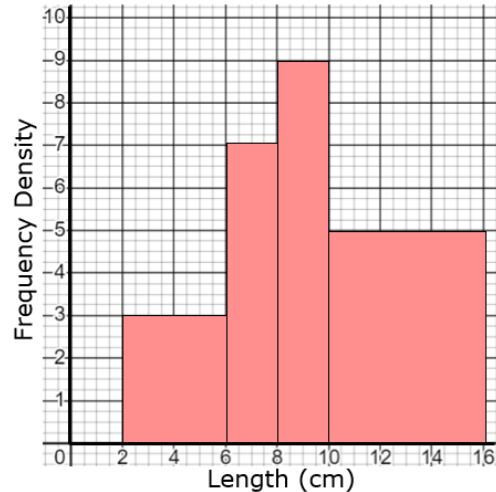
- Work out an estimate for the median salary of the workers.
- A worker is to be chosen at random.
- Work out the probability that their salary will be at most £15,000.
 - Work out an estimate for the probability that their salary is at least double the median salary.

Fluency Practice

Interpreting Histograms

(a)

The histogram shows some information about the lengths of some sycamore leaves.

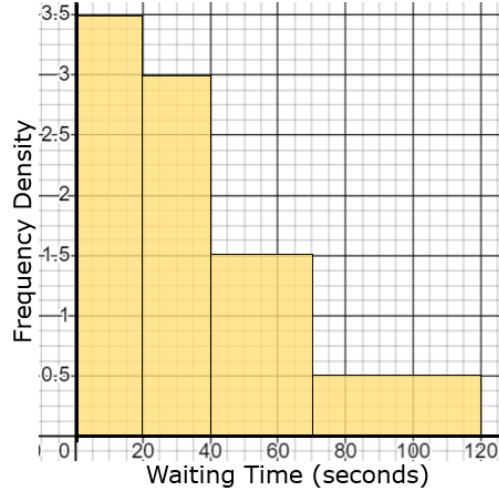


(a) Find the total number of leaves that were measured.

(b) Find an estimate of the number of leaves that were less than 7 cm in length.

(b)

The histogram shows some information about the waiting times in a queue at a supermarket.

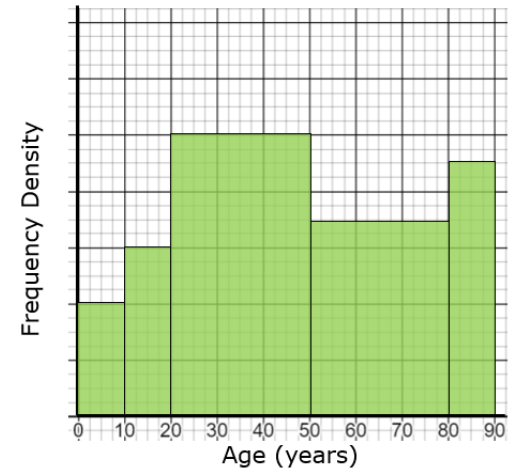


(a) Find an estimate of the proportion of customers who waited more than 50 seconds.

(b) Find an estimate of the median waiting time.

(c)

The histogram shows some information about the ages of people visiting a library. Ten of the people who visited the library were less than 20 years old.



(a) Find the number of people visiting the library who are over 50 years old.

(b) Find an estimate of the median age of the people visiting the library.

Fluency Practice

(a)

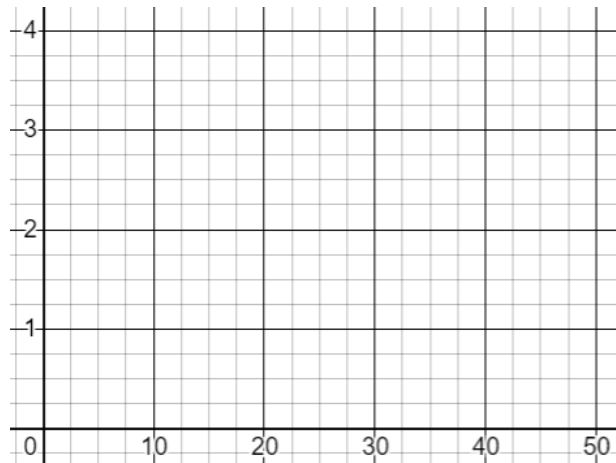
The distance travelled to work by 50 employees is recorded in a grouped frequency table.

(a) Calculate the frequency density for each class.

Distance (km)	Frequency		
$0 < d \leq 5$	12		
$5 < h \leq 10$	16		
$10 < h \leq 20$	10		
$20 < h \leq 30$	7		
$30 < h \leq 50$	5		

(b) Plot a histogram.

(c) Use your histogram to estimate the number of people who travel at least 12 km to work.



(b)

The house prices of 100 houses in a village are recorded in a grouped frequency table.

(a) Use the information in the table to calculate frequency densities and plot a histogram.

(b) Use your histogram to estimate the percentage of houses that cost less than £270 000.

House Price (£ thousands)	Frequency
$0 < p \leq 100$	6
$100 < p \leq 200$	22
$200 < p \leq 250$	18
$250 < p \leq 300$	35
$300 < p \leq 500$	15
$500 < p \leq 1000$	4

(c)

The time taken, in minutes, by 50 students to solve a maths puzzle is recorded in a grouped frequency table.

(a) Plot a histogram to represent this data.

(b) Use your histogram to find the median time taken.

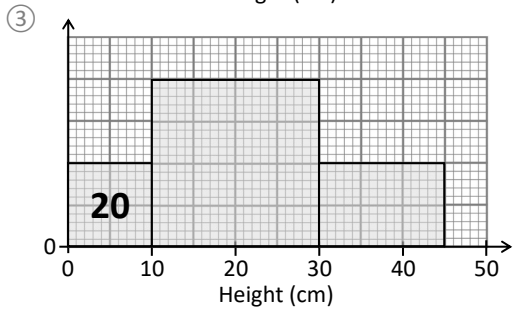
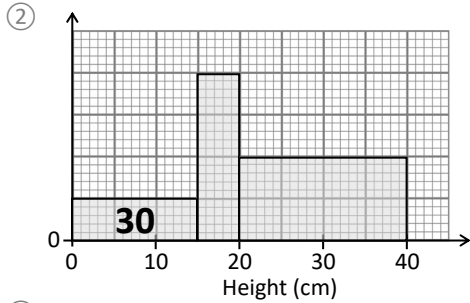
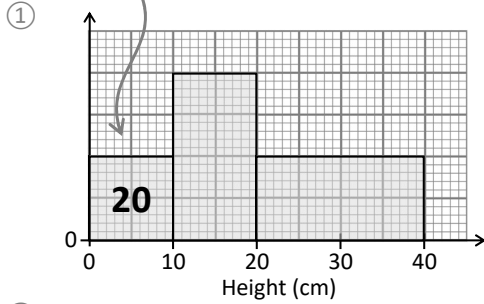
Time Taken (minutes)	Frequency
$0 < t \leq 2$	4
$2 < t \leq 4$	7
$4 < t \leq 5$	10
$5 < t \leq 6$	12
$6 < t \leq 7$	11
$7 < t \leq 12$	6

Fluency Practice

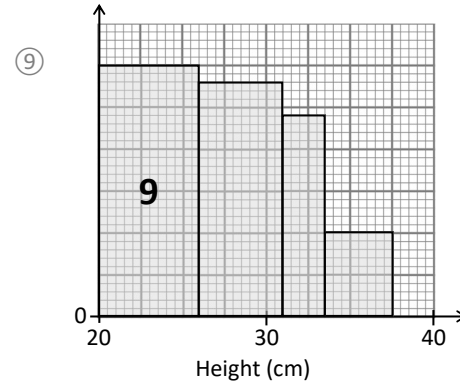
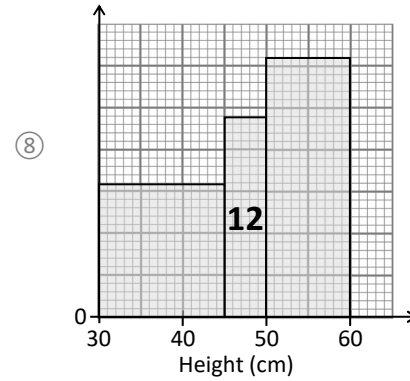
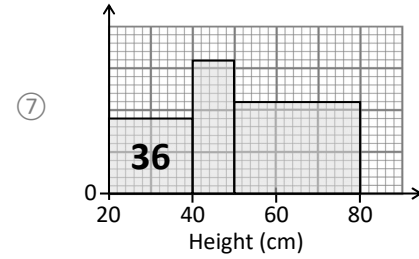
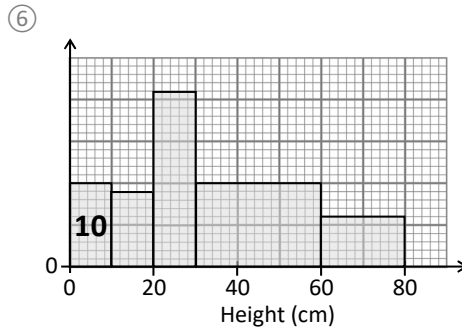
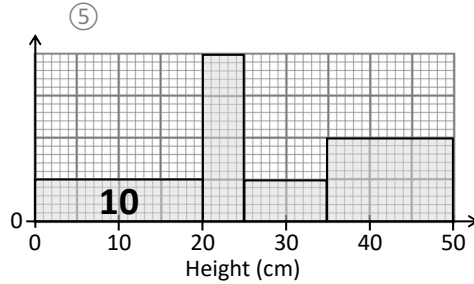
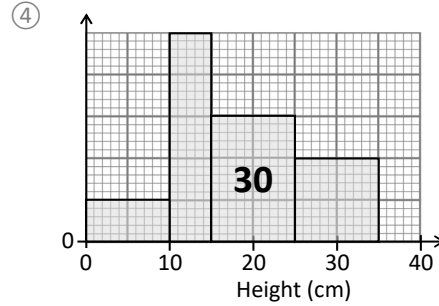


Total Pop.?

20 of the population had a height of 10 cm or less.



The **area** of a histogram represents **frequencies**.
On each histogram one group frequency has been given.
Find the **total population** for each data set.



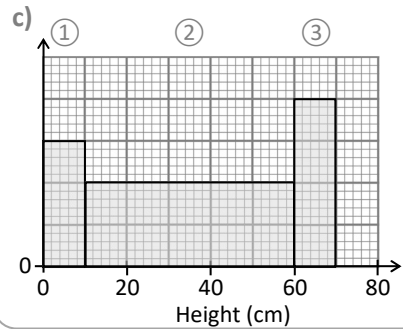
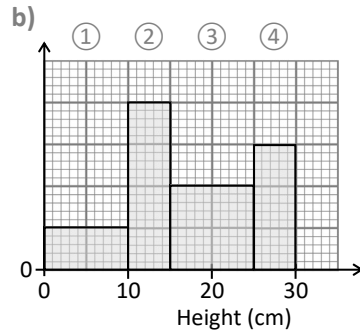
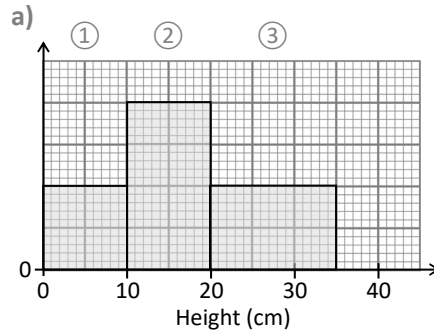
Fluency Practice



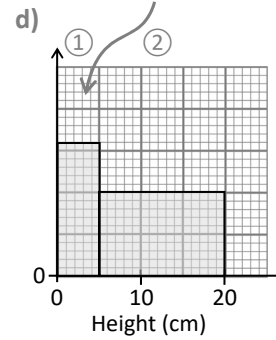
Workin' Backwards

Calculate the frequency for each group in each histogram.

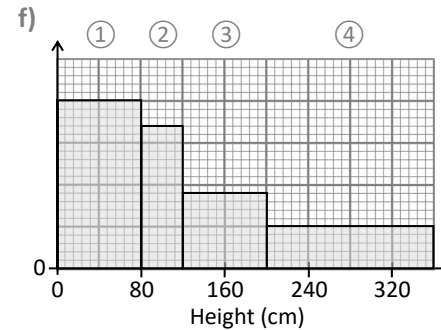
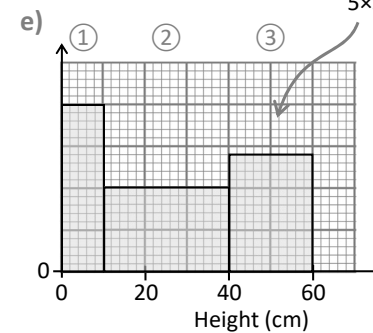
What *method* will you use?



d) How many large (5×5) squares are in this bar?
Express this as a decimal & sum the **total area**.



As a decimal, how many 5×5 squares are in this bar?



Fluency Practice

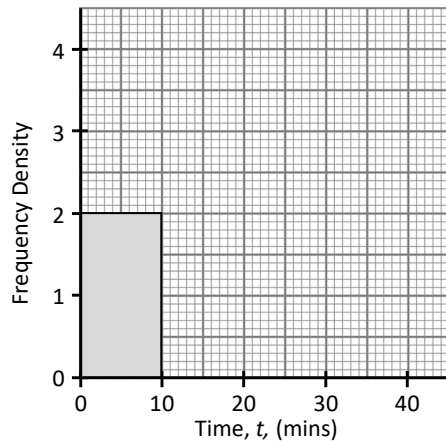
Histograms

1) People were surveyed about the time it took to get to their local shop.

Time, t , (mins)	Frequency	Class Width	Frequency Density
$0 < t \leq 10$	20		
$10 < t \leq 15$	15		
$15 < t \leq 20$	20		
$20 < t \leq 40$	30		

Notice that the classes (groups) are different sizes.
To represent them fairly, we must show how *relatively* dense the class is.
We do this using 'Frequency Density': Frequency $\div$ Class Width

- Complete Class Width & Frequency Density for each class.
- Use the data to complete the histogram.



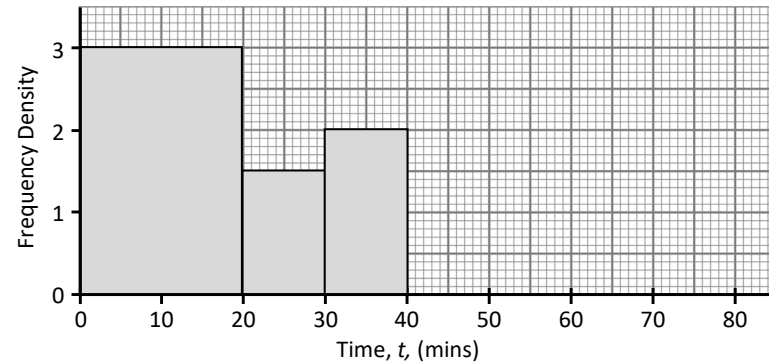
How many people is one square worth?

Estimate:

- How many people took 5 minutes or less to walk to work.
- How many people took between 30 & 40 minutes to walk to work.

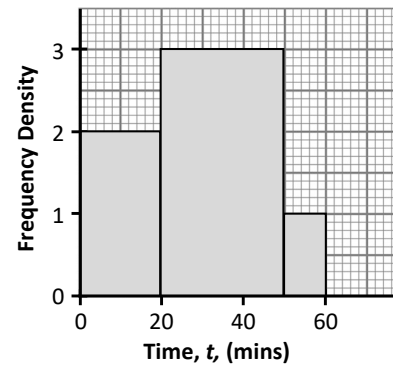
Why are these estimates?

2) Students were surveyed about the time they spent doing homework last night. Complete the graph & table.



Time, t , (mins)	Frequency	Class Width	Frequency Density
$0 < t \leq 20$			
$20 < t \leq 30$			
$30 < t \leq 40$			
$40 < t \leq 80$	40	40	1

3) As a logic test, adults were given a puzzle to complete.
The histogram shows the results. Complete the frequency table.



Time, t , (mins)	Freq.
$0 < t \leq 20$	
$20 < t \leq 50$	
$50 < t \leq 60$	

Which group is the median in?

How far into that group
is the median?

Mark the median on the histogram.

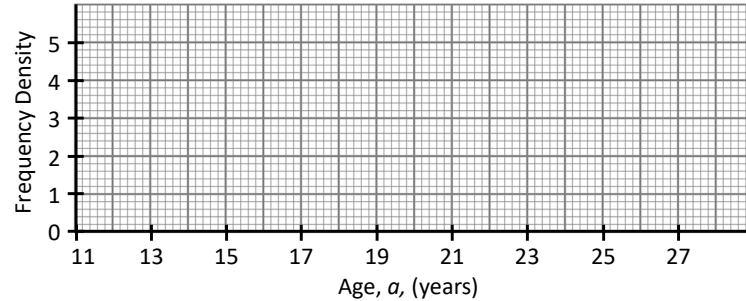
What is its value?

Fluency Practice

Histograms

1) Shoppers were surveyed about their age. Complete the histogram.

Age a , (years)	Frequency	Cumulative Frequency	Class Width	Frequency Density
$11 < a \leq 16$	5			
$16 < a \leq 24$	20			
$24 < a \leq 27$	15			



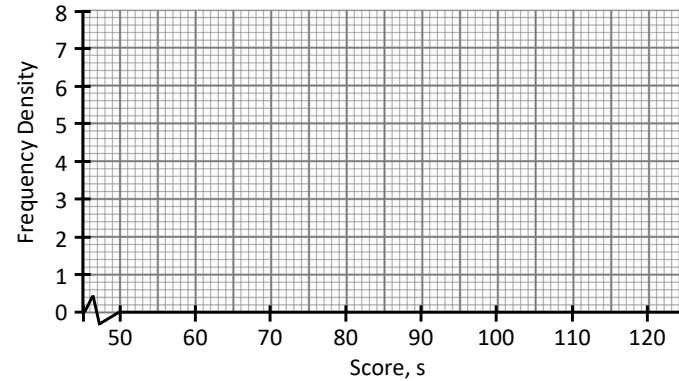
How many people is one square worth?

Complete the Cumulative Frequency column.

- In which class is the median value (20th value)?
How far into the class is the median value?
Mark the median value on the histogram. What is the median age?
Why is this an estimate?
- In which class is the lower quartile (10th)?
How far into the class is the lower quartile?
Use the histogram to estimate the lower quartile.
- In which class is the upper quartile?
How far into the class is the upper quartile?
Use the histogram to estimate the upper quartile.
- Estimate the inter-quartile range.

2) 200 students were given an English test.
Complete the histogram.

Score, s	Frequency	Cumulative Frequency	Class Width	Frequency Density
$50 < s \leq 60$	40			
$60 < s \leq 80$	20			
$80 < s \leq 90$	80			
$90 < s \leq 120$	60			



- What is the modal group?
 - Calculate an estimate for the median.
 - Calculate the lower and upper quartiles.
 - What is the interquartile range?
 - Estimate how many students got less than 55 marks
 - Estimate how many students got more than 115 marks.
- 3) 20% of the students got an 'outstanding' score.
- How many students got an outstanding score?
 - What score was the boundary for 'outstanding'?

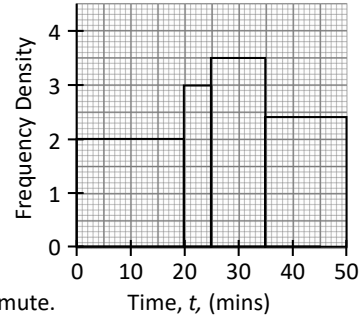
Fluency Practice

Histograms: Reading

1) The histogram shows the time taken by workers to commute.

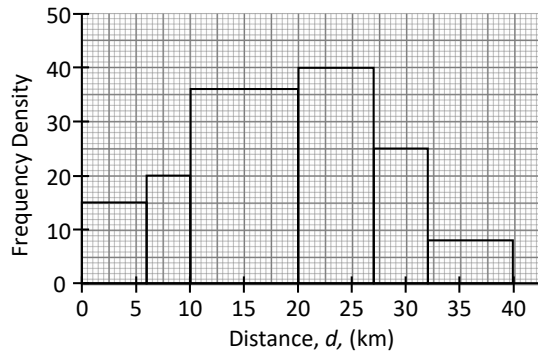
a) Complete the frequency table.

Time, t , (mins)	Freq.
$0 < t \leq 20$	
$20 < t \leq 25$	
$25 < t \leq 35$	
$35 < t \leq 50$	



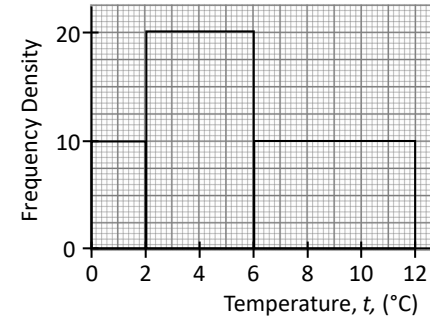
b) Calculate an estimate for the mean time taken to commute.

2) The histogram shows the distance workers commute.



- How many people commute ...less than 10 km?
- ...more than 20 km?
- ...between 10 km and 15 km?
- ...less than 2 km?
- ...more than 22 km?
- ...more than 7 km?

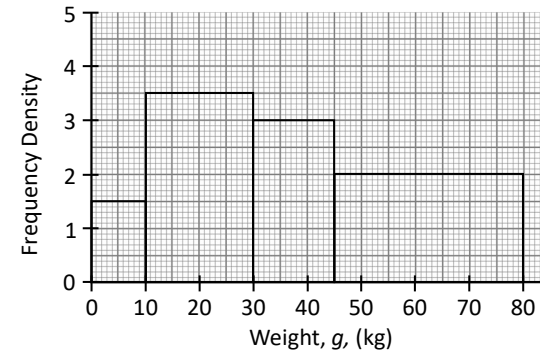
3) The histogram shows temperatures on different winter days.



Use the histogram to estimate...

- ...the median.
- ...the lower quartile.
- ...the upper quartile.
- What is the inter-quartile range?

4) The histogram shows the weight of different packages.



- Estimate the median.
- Estimate the inter-quartile range.
- Estimate the mean.

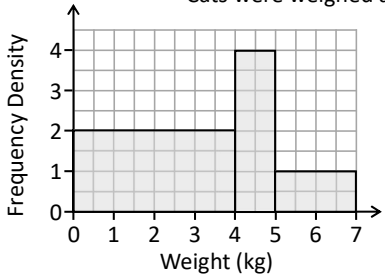
Fluency Practice



Histogram Populations Find the total size of each population.

①

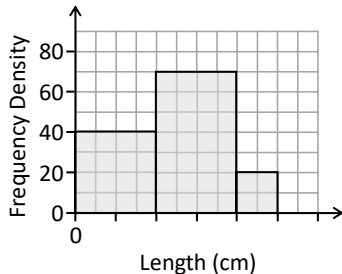
Cats were weighed at a rescue centre.



Weight w (kg)	Frequency	Class Width	Frequency Density
$0 < w \leq 4$		4	2
$4 < w \leq 5$			
$5 < w \leq 7$			

④

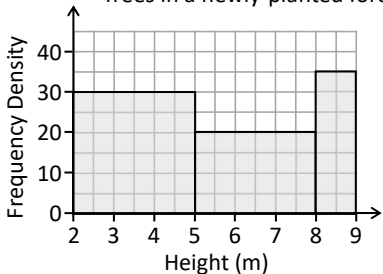
A sample of the snail population on St. Hilda Island was measured.



Length l (cm)	Freq.	CW	FD
$0 < l \leq 5$	80		
$5 < l \leq 7$			
$7 < l \leq 8$			

②

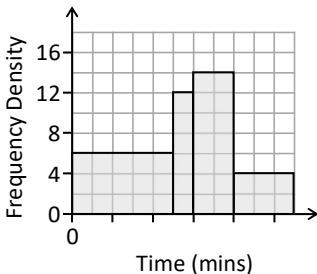
Trees in a newly-planted forest were measured.



Height h (m)	Freq.	Class Width	Frequency Density
$2 < h \leq 5$			
$5 < h \leq 8$			
$8 < h \leq 9$			

⑤

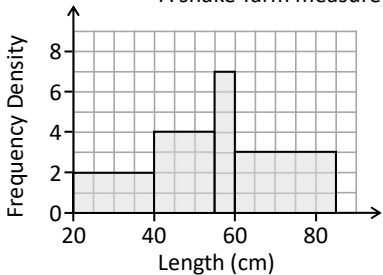
Participants in a global experiment were timed completing a complex puzzle.



Time t (mins)	Freq.	CW	FD
$0 < t \leq 3$	300		
$3 < t \leq 4$			
$4 < t \leq 5$			
$5 < t \leq 6$			

③

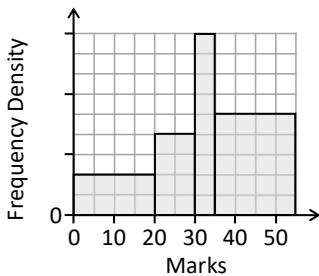
A snake-farm measured its serpents.



Length l (m)	Freq.	CW	FD
$20 < l \leq 40$			
$40 < l \leq 55$			
$55 < l \leq 60$			
$60 < l \leq 80$			

⑥

Local schools compiled English test scores.



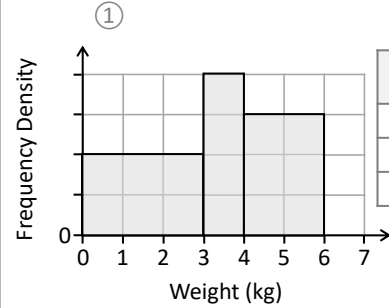
Marks, m	Freq.	CW	FD
$0 < m \leq 10$	120		
$10 < m \leq 20$			
$20 < m \leq 30$			
$30 < m \leq 50$			

Fluency Practice

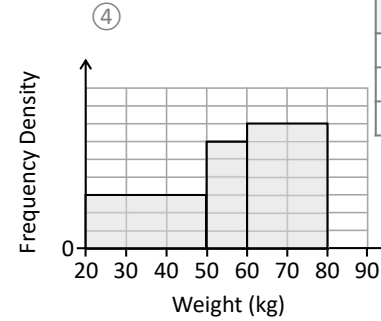


Histogram Populations: Missing Frequency Density

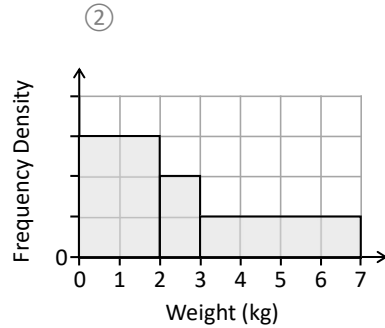
Use frequency tables to find the **total population** from each graph.



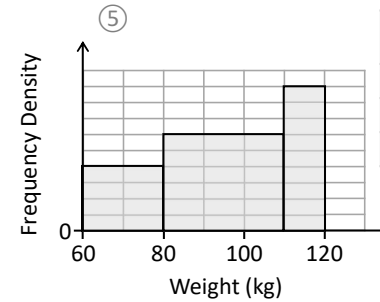
Weight w (kg)	Frequency	Class Width	Frequency Density
$0 < w \leq 3$	6		
$3 < w \leq 4$			
$4 < w \leq 6$			



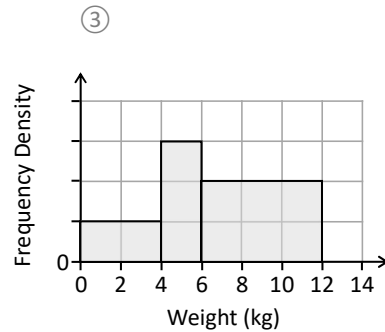
Weight w (kg)	F	CW	FD
$20 < w \leq 50$			
$50 < w \leq 60$	30		
$60 < w \leq 80$			



Weight w (kg)	Freq.	CW	FD
$0 < w \leq 2$	12		
$2 < w \leq 3$			
$3 < w \leq 7$			



Weight w (kg)	F		
$60 < w \leq 80$			
$80 < w \leq 110$	36		
$110 < w \leq 120$			



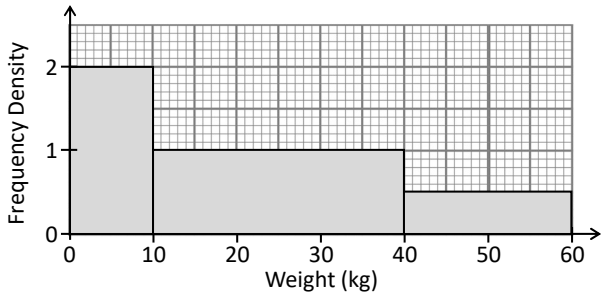
Weight w (kg)	Freq.	CW	FD
$0 < w \leq 4$			
$4 < w \leq 6$	30		
$6 < w \leq 12$			



Fluency Practice

A scientist records catch-weights on a fishing boat.
Estimate the median.

Weight w (kg)	Frequency	Class Width	Frequency Density
$0 < w \leq 10$			
$10 < w \leq 40$			
$40 < w \leq 60$			

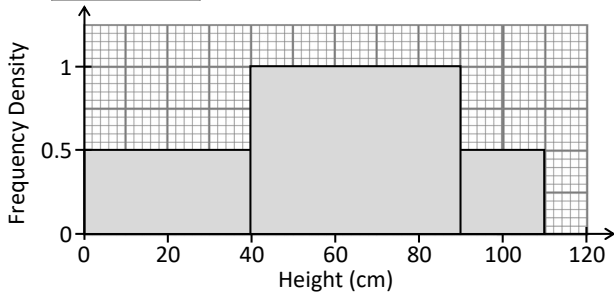


Histograms: Quartiles



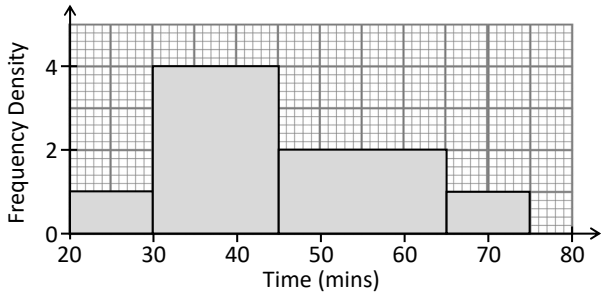
A scientist samples corn heights from a field.
Estimate the median.

Height, h (cm)	Frequency	CW	FD
$0 < h \leq 40$			
$40 < h \leq 90$			
$90 < h \leq 110$			



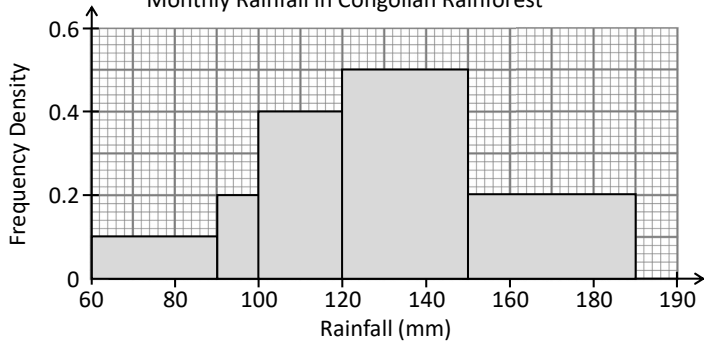
A fast-food delivery company checks delivery times.
Estimate the interquartile range.

Time, t (mins)	Frequency		



① ②
③ ④

Monthly Rainfall in Congolian Rainforest



Estimate the interquartile range.

Fluency Practice

Overspeed



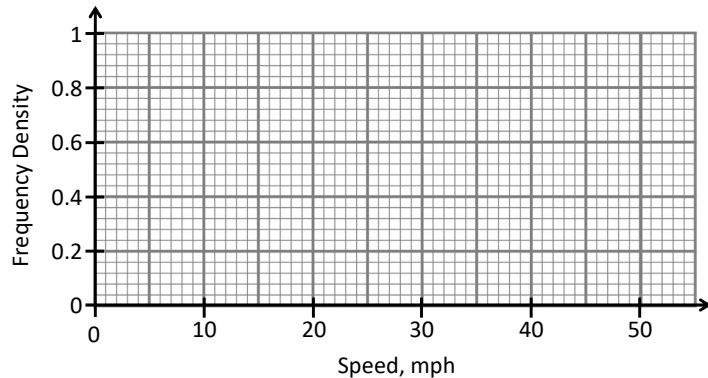
A local council samples car speeds within a 30 mph zone.



27	20	10	26	16	15	3
36	11	28	18	12	8	14
21	7	26	31	42	24	48
22	6	29	30	9	35	

Tally & summarise the data in the frequency table.
Use the information to create a histogram.

	Speed, s (mph)	Tally	Frequency	Class Width	Frequency Density
Underspeed	$0 < s \leq 10$				
Expected	$10 < s \leq 30$				
Acceptable	$30 < s \leq 35$				
Speeding	$35 < s \leq 50$				



The council may impose a 20 mph speed limit. Use the graph to **estimate** how many cars were travelling above this speed.

How can we calculate an estimate for the mean?

Overspeed



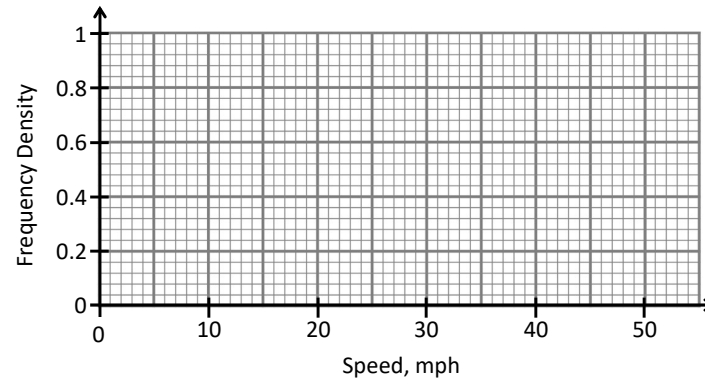
A local council samples car speeds within a 30 mph zone.



27	20	10	26	16	15	3
36	11	28	18	12	8	14
21	7	26	31	42	24	48
22	6	29	30	9	35	

Tally & summarise the data in the frequency table.
Use the information to create a histogram.

	Speed, s (mph)	Tally	Frequency	Class Width	Frequency Density
Underspeed	$0 < s \leq 10$				
Expected	$10 < s \leq 30$				
Acceptable	$30 < s \leq 35$				
Speeding	$35 < s \leq 50$				



The council may impose a 20 mph speed limit. Use the graph to **estimate** how many cars were travelling above this speed.

How can we calculate an estimate for the mean?

Fluency Practice

Sugar Rush



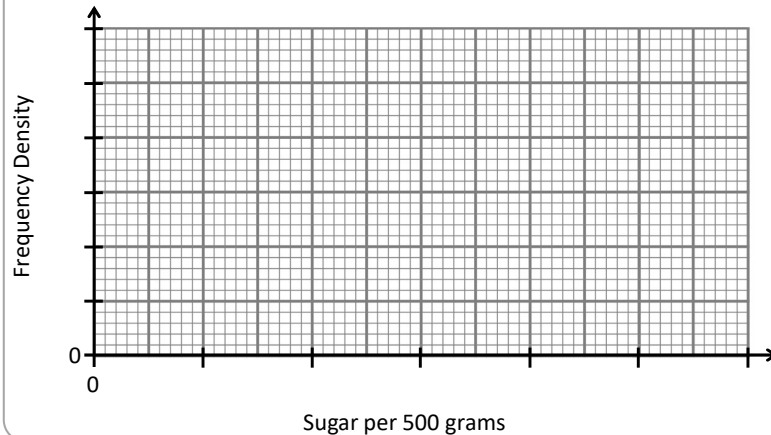
Food scientists measure the grams of sugar in 500 grams of different snacks.



31	81	26	14	42	37	93	11
87	28	96	23	80	12	6	95
33	10	18	100	35	38		

Tally & summarise the data in the frequency table.
Use the information to create a histogram.

	Sugar, s (g)	Tally			
Negligible	$0 < s \leq 10$				
Acceptable	$10 < s \leq 40$				
Unhealthy	$40 < s \leq 80$				
Unacceptable	$80 < s \leq 100$				



According to health experts, snacks should have less than 5% sugar.
Use the graph to estimate the percentage of snacks above this limit.

Mark an estimate for the mean sugar level on the histogram.

Sugar Rush



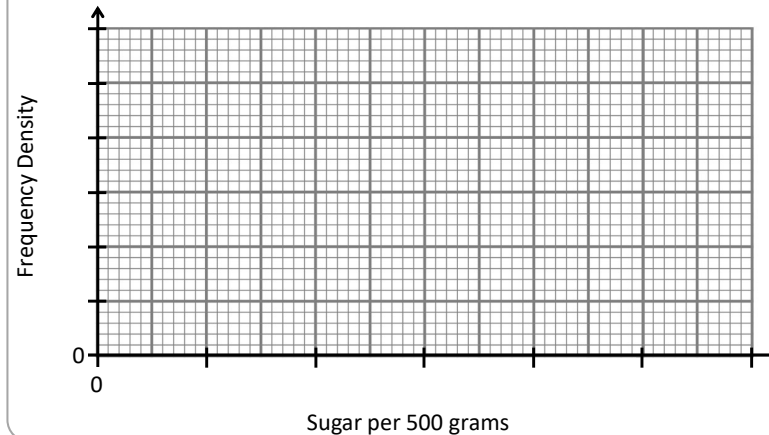
Food scientists measure the grams of sugar in 500 grams of different snacks.



31	81	26	14	42	37	93	11
87	28	96	23	80	12	6	95
33	10	18	100	35	38		

Tally & summarise the data in the frequency table.
Use the information to create a histogram.

	Sugar, s (g)	Tally			
Negligible	$0 < s \leq 10$				
Acceptable	$10 < s \leq 40$				
Unhealthy	$40 < s \leq 80$				
Unacceptable	$80 < s \leq 100$				



According to health experts, snacks should have less than 5% sugar.
Use the graph to estimate the percentage of snacks above this limit.

Mark an estimate for the mean sugar level on the histogram.

Fluency Practice

Question 1

Lloyd collects the lengths of 98 animals and records the data in the table below.

Length (z cm)	Frequency
$10 < z \leq 20$	16
$20 < z \leq 35$	45
$35 < z \leq 45$	11
$45 < z \leq 50$	10
$50 < z \leq 70$	16

A histogram was drawn and the class $35 < z \leq 45$ was represented by a rectangle of width 2 cm and height 5.5 cm.

Calculate the width and the height of the rectangle representing the class $20 < z \leq 35$.

Question 2

Lloyd collects the running times of 167 athletes and records the data in the table below.

Time (y seconds)	Frequency
$0 < y \leq 10$	34
$10 < y \leq 15$	32
$15 < y \leq 35$	20
$35 < y \leq 50$	30
$50 < y \leq 55$	6
$55 < y \leq 70$	45

A histogram was drawn and the class $50 < y \leq 55$ was represented by a rectangle of width 2 cm and height 9 cm.

Calculate the width and the height of the rectangle representing the class $0 < y \leq 10$.

Question 3

Lenny collects the heights of 115 plants and records the data in the table below.

Height (y cm)	Frequency
$20 < y \leq 35$	30
$35 < y \leq 40$	19
$40 < y \leq 45$	16
$45 < y \leq 55$	14
$55 < y \leq 60$	29
$60 < y \leq 80$	7

A histogram was drawn and the class $55 < y \leq 60$ was represented by a rectangle of width 1 cm and height 7.25 cm.

Calculate the width and the height of the rectangle representing the class $45 < y \leq 55$.

Question 4

Lesley collects the heights of 119 plants and records the data in the table below.

Height (x cm)	Frequency
$60 < x \leq 80$	22
$80 < x \leq 90$	12
$90 < x \leq 105$	15
$105 < x \leq 110$	25
$110 < x \leq 125$	45

A histogram was drawn and the class $105 < x \leq 110$ was represented by a rectangle of width 2.5 cm and height 6.25 cm.

Calculate the width and the height of the rectangle representing the class $60 < x \leq 80$.

Fluency Practice

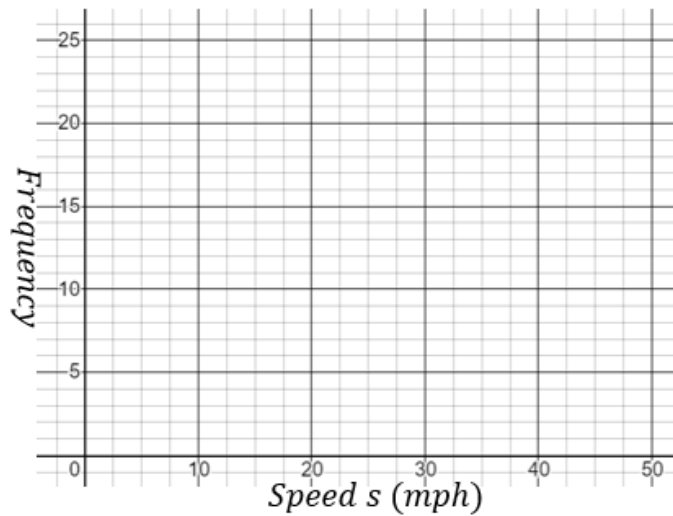
Plotting Frequency Polygons

(a)

The table shows the speeds of 60 motorbikes on a road.

Speed s (mph)	Frequency
$0 < s \leq 10$	3
$10 < s \leq 20$	11
$20 < s \leq 30$	24
$30 < s \leq 40$	15
$40 < s \leq 50$	7

Draw a frequency polygon to represent this information.

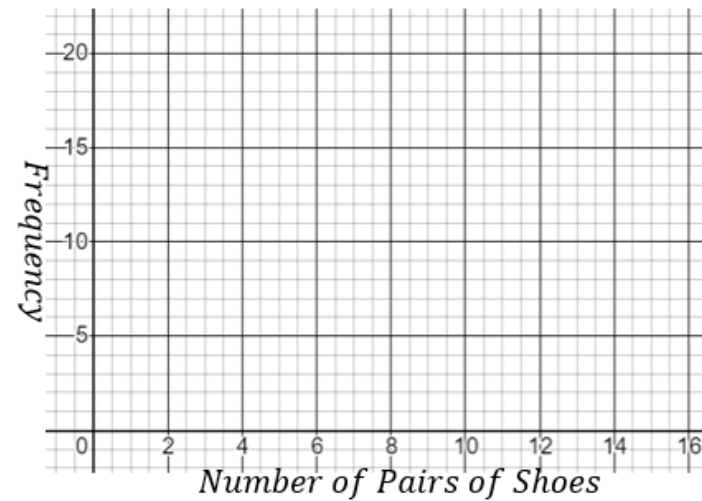


(b)

The table shows the number of pairs of shoes owned by 50 students.

Number of Pairs of Shoes	Frequency
1 – 3	6
4 – 6	16
7 – 9	18
10 – 12	7
13 – 15	3

Draw a frequency polygon to represent this information.



Fluency Practice

①

Calculating Averages From Graphs



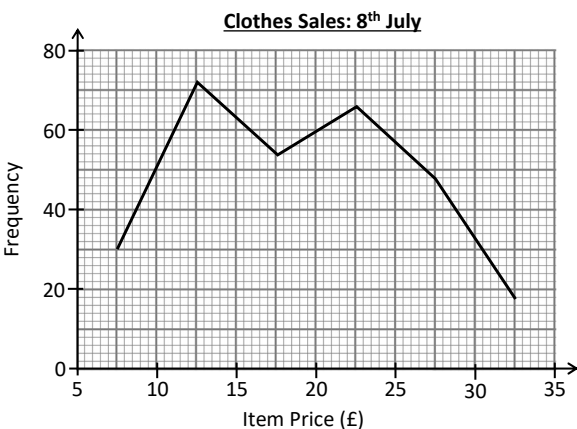
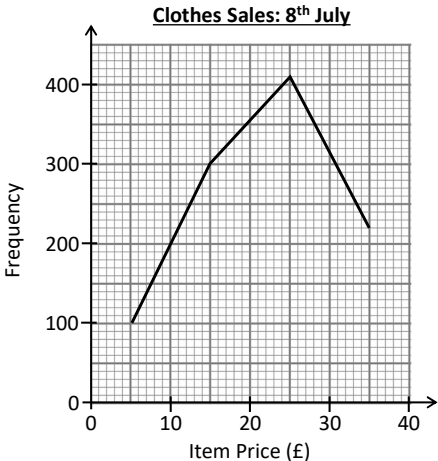
②

Calculating Averages From Graphs



New Trend Clothing Inc. record sales in their stores over one day.

The New Trend store in Forest Valley Mall records its sales on one day.



Use the frequency polygon to complete the table & calculate the **mean** item price.

Item Price, p (£)	Frequency	Midpoint	$f \times m$
$0 < p \leq 10$			
$10 < p \leq 20$			
$20 < p \leq 30$			
$30 < p \leq 40$			
Totals			

In which group is the median?

Which is the modal group?

State the **modal** group, the **mean** item price, and the group that contains the **median** item price.

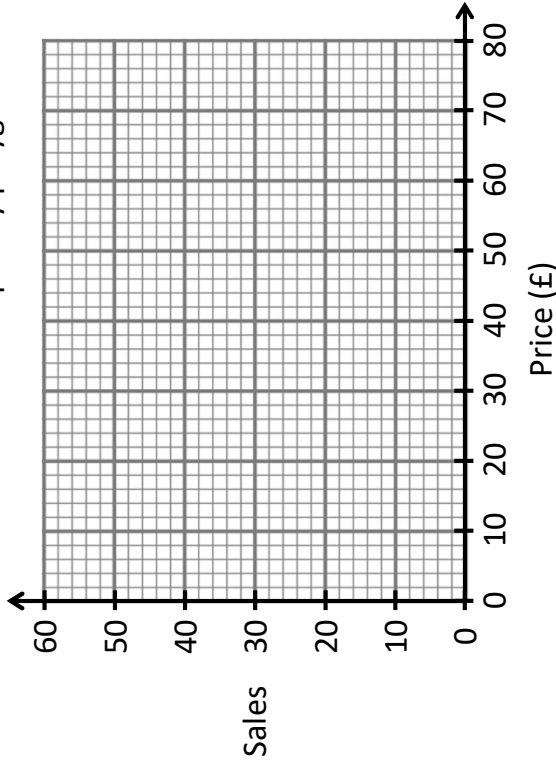
Shop Sales

A clothes shop has 4 price groups: Bargain, Value, Everyday & Luxury.
Which of the groups earnt the shop the most amount of money on Monday?
(multiply sales by the mid-group value to find the total group value)

Monday

	Price p , (£)	Sales	Mid-Group Value (£)	Total Group Value
Bargain	$0 < p \leq 20$	46		
Value	$20 < p \leq 40$	28		
Everyday	$40 < p \leq 60$	32		
Luxury	$60 < p \leq 80$	14		
Totals				

Calculate the total income & the mean price of the items sold.
Plot the data as a frequency polygon.



Plot the data for Friday on the same grid. Calculate a mean price.

Friday

	Price p , (£)	Sales	
Bargain	$0 < p \leq 20$	26	
Value	$20 < p \leq 40$	36	
Everyday	$40 < p \leq 60$	40	
Luxury	$60 < p \leq 80$	56	

What comments can you make comparing the sales on Monday & Friday?

Snack Stores

3 food shops in a shopping centre record their sales over a Saturday.
Complete the tables to find estimates for their total profits.

Pete's Pizza 'n' Slices

Profit p , (£)	Sales	
$0 < p \leq 2$	30	
$2 < p \leq 4$	34	
$4 < p \leq 6$	28	
$6 < p \leq 8$	16	

Plot the information as three frequency polygons on the grid.

What comments can you make about the data distributions?

Which shop was most profitable?

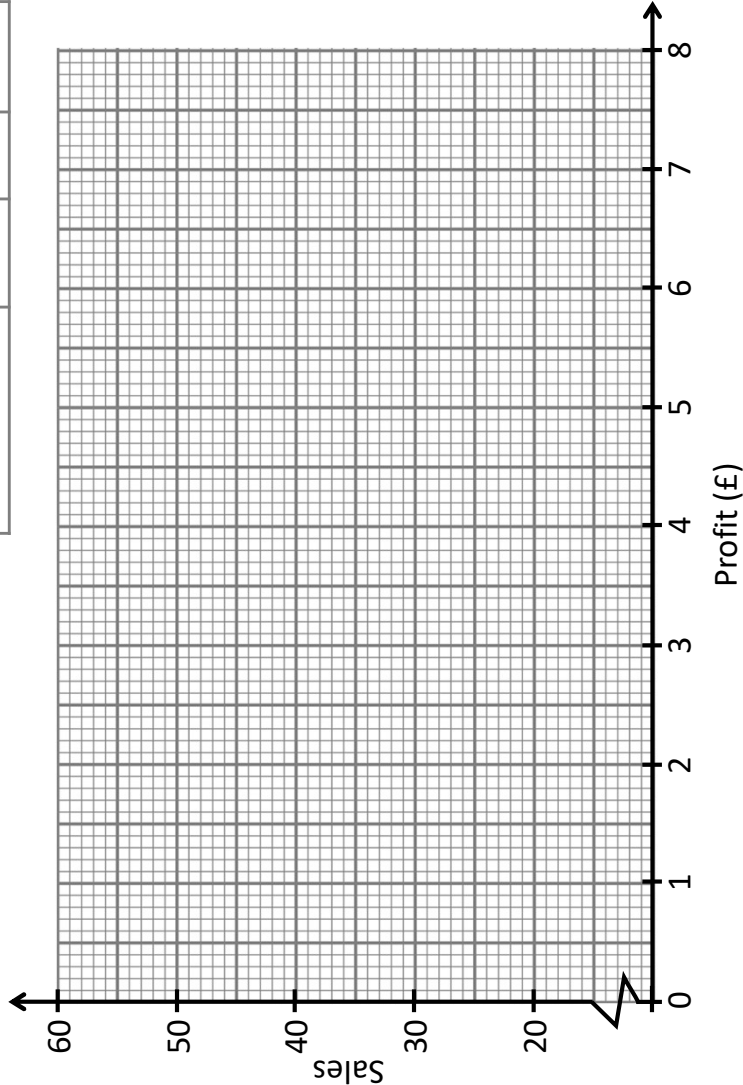
Calculate the mean profit for each shop.

Lucy's Luxury Cakes

Profit p , (£)	Sales	
$2 < p \leq 4$	21	
$4 < p \leq 6$	24	
$6 < p \leq 8$	34	

Derek's Doughnuts

Profit p , (£)	Sales	
$0 < p \leq 1$	54	
$1 < p \leq 2$	56	
$2 < p \leq 3$	51	
$3 < p \leq 4$	39	



Page 176

For each set of data draw a box plot on the grid provided.

2	2	3	4	6	7	8	8	9	13	15	18
---	---	---	---	---	---	---	---	---	----	----	----

[illegible]

11	14	17	18	18	21	22	23	24	27	28
----	----	----	----	----	----	----	----	----	----	----

[illegible]

3 4 6 10 13 14 16 18 19

[illegible]

13	14	15	19	22	24	26	27	29	30
----	----	----	----	----	----	----	----	----	----

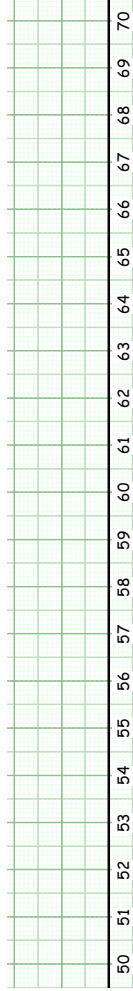
[illegible]

Fluency Practice

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53	53	54	59	62	63	63	64	68	70
----	----	----	----	----	----	----	----	----	----

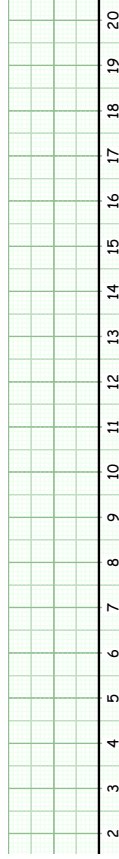
Min =	L.Q. =	Median =	U.Q. =	Max =



6.

3	3	5	9	11	13	14	16	17	19	19	20
---	---	---	---	----	----	----	----	----	----	----	----

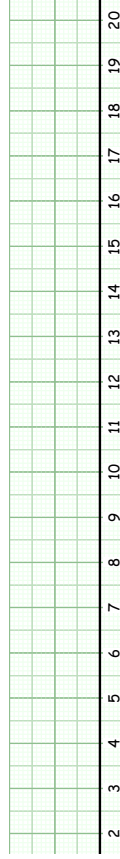
Min =	L.Q. =	Median =	U.Q. =	Max =



7.

2	2	5	8	11	13	14	15	16	16	19	20
---	---	---	---	----	----	----	----	----	----	----	----

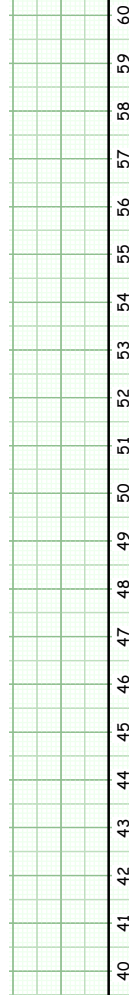
Min =	L.Q. =	Median =	U.Q. =	Max =
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40 40 42 45 46 47 50 50 53 56 57 58

Min =	L.Q. =	Median =	U.Q. =	Max =



Fluency Practice

Drawing box plots

1.	Lightest weight	3 g	Minimum weight	5 g
	Lower quartile	5 g	Lower quartile	12 g
	Median	11 g	Median	15 g
	Upper quartile	18 g	Upper quartile	26 g
	Heaviest weight	23 g	Maximum weight	32 g

Show this data on a box plot.



Show this data on a box plot.



3.	Minimum length	15.4 cm	Minimum weight	42 kg
	Lower quartile	16.8 cm	Lower quartile	48 kg
	Median	17.6 cm	Median	54 kg
	Upper quartile	18.2 cm	Upper quartile	65 kg
	Maximum length	19.8 cm	Maximum weight	71 kg

Show this data on a box plot.



Show this data on a box plot.



5.	Minimum	3	Minimum weight	5 g
	Lower quartile	8	Lower quartile	22 g
	Median	15	Median	34 g
	Interquartile range	12	Interquartile range	18 g
	Range	19	Maximum weight	48 g
	Upper quartile		Upper Quartile	
	Maximum			

Show this data on a box plot.



Show this data on a box plot.



7.	Minimum	4	Maximum	58
	Lower quartile	10	Upper quartile	50
	Median	22	Median	38
	Interquartile range	16	Interquartile range	26
	Range	29	Range	52
	Upper quartile		Lower quartile	
	Maximum		Minimum	

Show this data on a box plot.

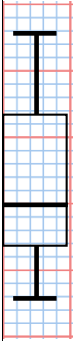
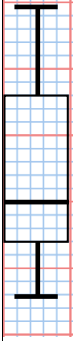
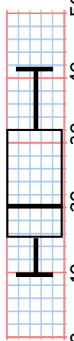
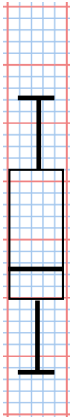
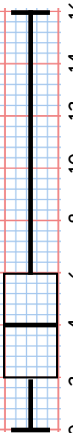
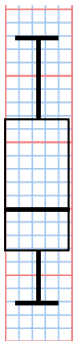
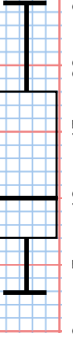
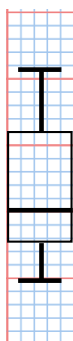
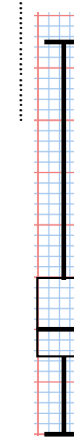


Show this data on a box plot.



Fluency Practice

Interpreting Box plots

1.  10 20 30 cm What is the range? What is the inter-quartile range?	2.  0 5 10 15 20 25 cm What is the range? What is the inter-quartile range?
3.  0 10 20 30 40 50 What is the range? What is the inter-quartile range?	4.  0 10 20 30 40 50 60 min What is the range? What is the inter-quartile range?
5.  0 1 2 3 hours What is the range? What is the inter-quartile range?	6.  0 2 4 6 8 10 12 14 16 What is the range? What is the inter-quartile range?
7.  11 12 13 cm How many plants were taller than 12.7 cm?	8.  0 5 10 15 20 25 cm How many plants were shorter than 18cm?
9.  0 10 20 30 40 50 marks How many pupils scored more than 42?	10.  0 10 20 30 40 50 60 min How many pupils spent less than 18 minutes on their homework?
11.  0 1 2 3 4 5 6 hours If 21 pupils spent more than 2 hours revising, how many pupils were in the class?	12.  0 2 4 6 8 10 12 14 16 If 18 pupils had less than 6 behaviour points, how many pupils are in the form?

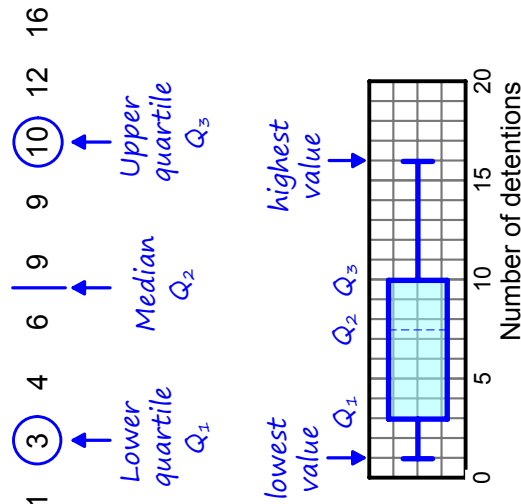
example

The **quartiles** of a data set are the values one quarter and three quarters of the way through the data.

The **interquartile range (IQR)** is the difference between the upper and lower quartiles.

$$\text{IQR} = Q_3 - Q_1$$

The quartiles can be used to draw a **boxplot** for the data. This shows how the data is spread across the range of values.



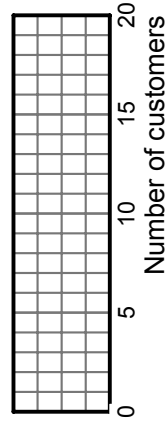
exercise

1. For each data set, work out Q_1 , Q_2 , Q_3 and the interquartile range.

- a) 7 9 11 14 16 16 18
- b) 2 2 4 5 7 8 8 9 10 10 11 11 12
- c) 0.4 0.7 1.0 1.5 1.7 1.7 2.2 2.6
- d) 29 31 34 34 35 39 47 51 52 58 65 82

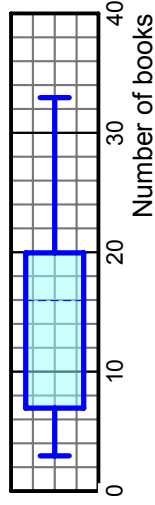
2. Work out the median and quartiles and construct a box plot:

- a) Number of customers each hour
 - b) Maximum day temperature ($^{\circ}\text{C}$)
- | | | | | | | | | | | | | |
|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 2 | 3 | 5 | 7 | 7 | 9 | 12 | 12 | 14 | 16 | 19 | 20 | 21 |
| 9 | 9 | 9 | 13 | 14 | 19 | 23 | 24 | 28 | 28 | 29 | 31 | 31 |



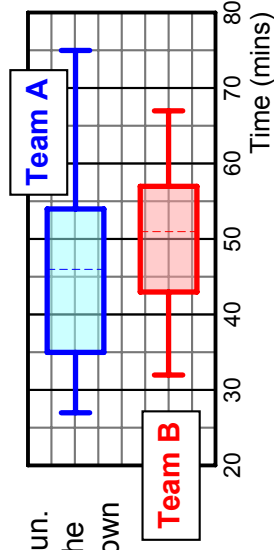
Fluency Practice

3. A group of people were asked how many books they read in a year. The boxplot shows the results.



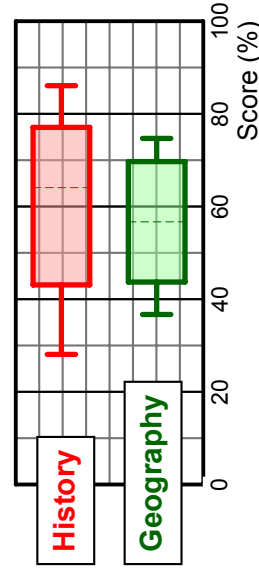
- State the median.
- Work out the range.
- Work out the interquartile range.
- What percentage of the people had read more than 20 books?
- What percentage of the people had read more than 7 books?

4. Two teams took part in a charity run. The times taken for members of the teams to complete the run are shown in the box plots.



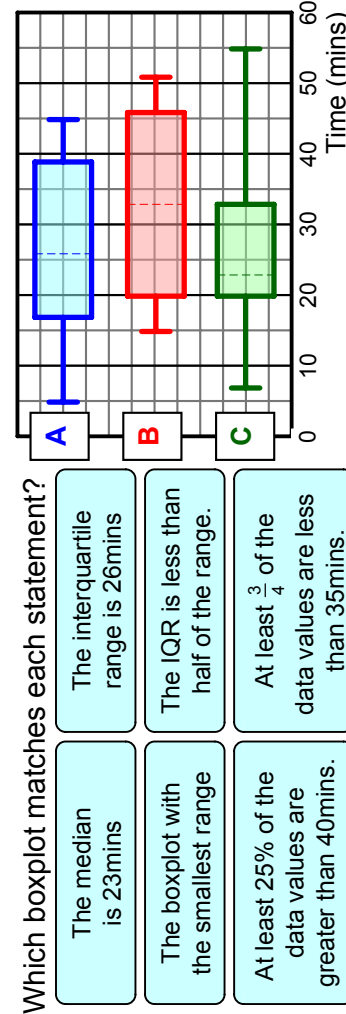
- On average, which team's members were quicker? Justify your answer.
- Which team had a smaller overall spread of times? Justify your answer.
- Which team had a smaller spread of the middle 50% of times.

5. The results of a class of pupils in exams for two subjects are shown in the box plots.



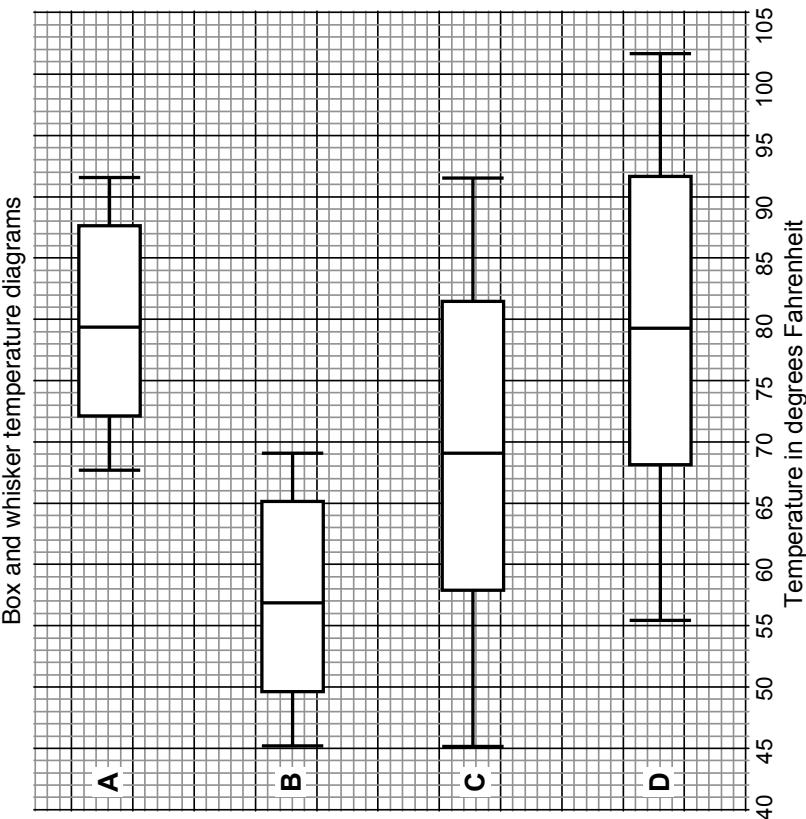
Compare the distributions of exam results in the two subjects.

6. Which boxplot matches each statement?



- | | |
|--|---|
| The median is 23mins | The interquartile range is 26mins |
| The boxplot with the smallest range | The IQR is less than half of the range. |
| At least 25% of the data values are greater than 40mins. | At least $\frac{3}{4}$ of the data values are less than 35mins. |

Fluency Practice



2. Which of the four box diagrams shows the Washington temperatures?

Explain how you decided.

3. Which of the four box diagrams shows the California temperatures?

For which months of the year is the maximum monthly temperature for California between the upper and the lower quartiles?

Explain how you figured it out.

Problem Solving

Sketch it!

Class 9Y2 are taking a maths test. What would the **box plot of their results** look like if...

a) ...the test was far too easy.



b) ...there was a high median & a small interquartile range.



c) ...the distribution was symmetrical.



d) ...the class did well, except George who was late!



e) ...the results were very consistent.



f) ...the results were perfectly distributed.



g) ...the results were low scoring & inconsistent.



h) ...50% of the students got 100%.



i) ...the test was too difficult for most, but Mary got 100%!



j) ...the interquartile range was 20% of the range.



k) ...the lowest result was shared by 25% of the students.



l) ...only the top 50% of students had consistent results.



What interesting results & distributions can you sketch?

Problem Solving

Results Day

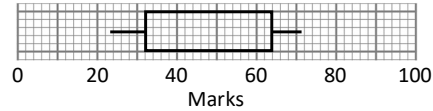
Match each classes Science Test Results with a box plot & complete both. Every class has 32 students.

10X1

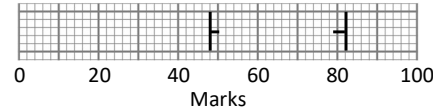
Lowest Score	28
Greatest Score	
Lower Quartile	50
Upper Quartile	
Median	60

a

50% of the class scored more than 52.



75% of the class scored more than 58.



10Y1

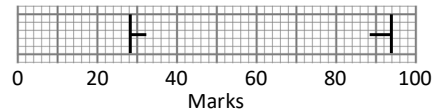
Lowest Score	10
Greatest Score	78
Lower Quartile	32
Upper Quartile	
Median	

10X2

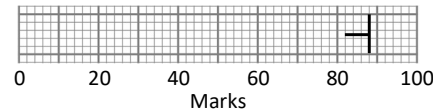
Lowest Score	14
Greatest Score	98
Lower Quartile	
Upper Quartile	76
Median	

b

25% of the class scored more than 80.



The interquartile range was 22.



10Y2

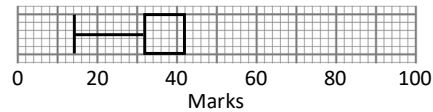
Range	72
Greatest Score	74
Lower Quartile	
Upper Quartile	
Median	54

10X3

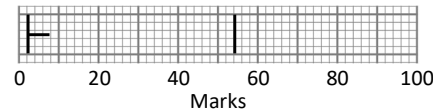
Lowest Score	48
Range	
Lower Quartile	
Upper Quartile	72
Median	68

c

50% of the class scored less than 36.



The middle 50% of students scored between 38 and 68 marks.



10Y3

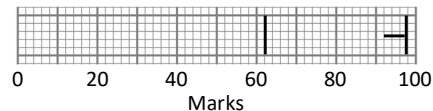
Lowest Score	14
Range	48
Lower Quartile	
Upper Quartile	
Median	

10X4

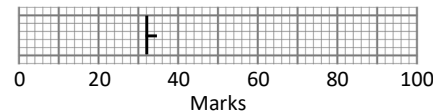
Lowest Score	
Range	64
Lower Quartile	
Upper Quartile	66
Median	58

d

8 students scored less than 28.



24 students scored more than 34.



10Y4

Lowest Score	18
Greatest Score	88
Lower Quartile	
Upper Quartile	48
Median	34

What comments can you make about the results? (Median, Maximums, Minimums, Consistency, Range)

Problem Solving



Comparing Classes 1

2 classes took a science test. We want to compare the scores for the boys in each class (9Y4 & 9X4).
The scores are in the table.



Place the scores in ascending order to identify each quartile (Lower, Median, Upper).
Plot this information as a box plot.

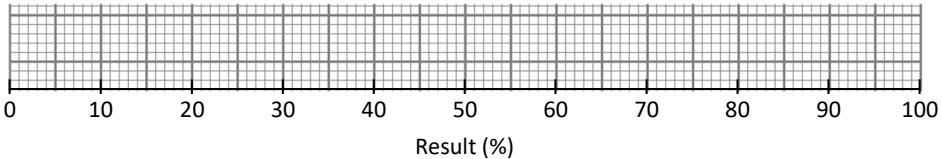
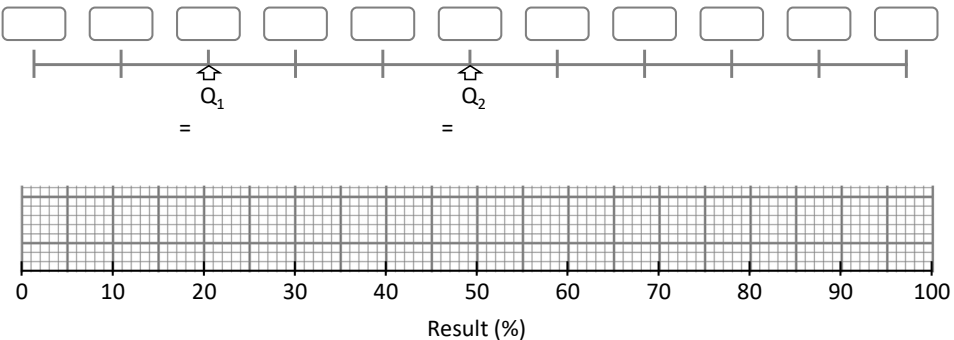
What does the shape of each box plot show?
What comparisons can we make between the classes?

**Class 9Y4
(boys)**

28	44	40
52	56	60
62	26	45
40	54	

**Class 9X4
(boys)**

31	43	58
54	58	31
91	18	54
42	56	40
70	32	90



Problem Solving



Comparing Classes 2

2 classes took a science test. We want to compare the scores for the girls in each class (9Y4 & 9X4).
The scores are in the table.



Place the scores in ascending order to identify each quartile (Lower, Median, Upper).
Plot this information as a box plot.

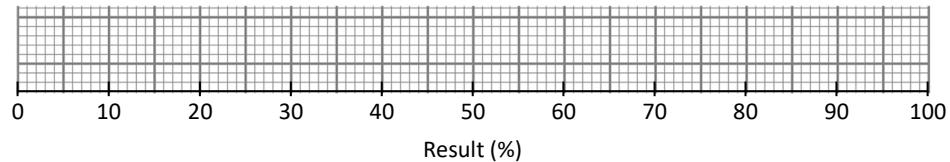
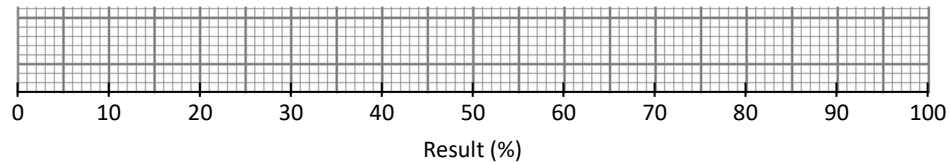
What does the shape of each box plot show?
What comparisons can we make between the classes?

**Class 9Y4
(girls)**

80	72	36
35	15	78
42	79	12
74	71	74
14		

**Class 9X4
(girls)**

42	58	42
52	50	56
46	42	98
62	51	46

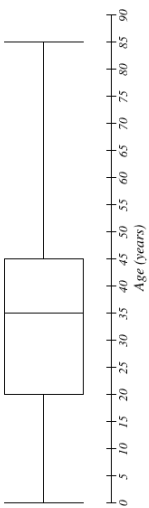
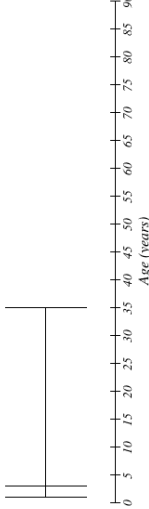
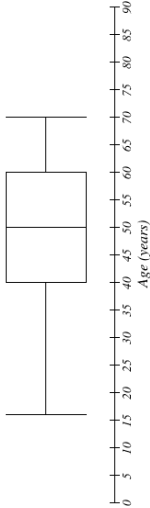
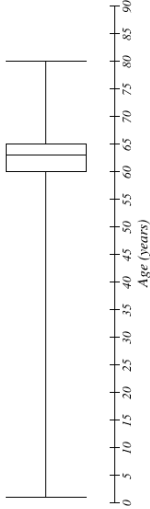
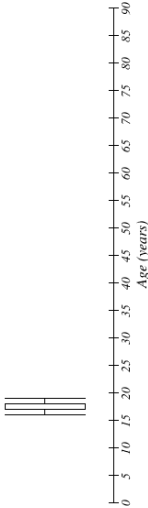
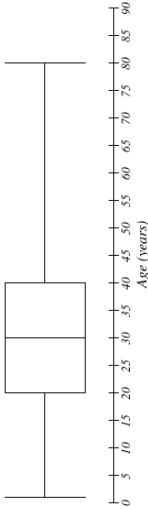


Problem Solving

6 parties were held in the Village Hall last month. They were:

- | | | |
|--|-------------------------|----------------------------------|
| An 18 th birthday party | A retirement party | A wedding reception |
| A 20 th wedding anniversary party | A christening reception | A 3 rd birthday party |

These box plots show the ages of the party guests. Underneath each box plot, write down which party it represents and explain how you know.



Problem Solving

A company has 72 employees, all of whom are at least 20 years of age.

The number of employees aged from 20 to 30 is equal to the number of employees aged over 50.

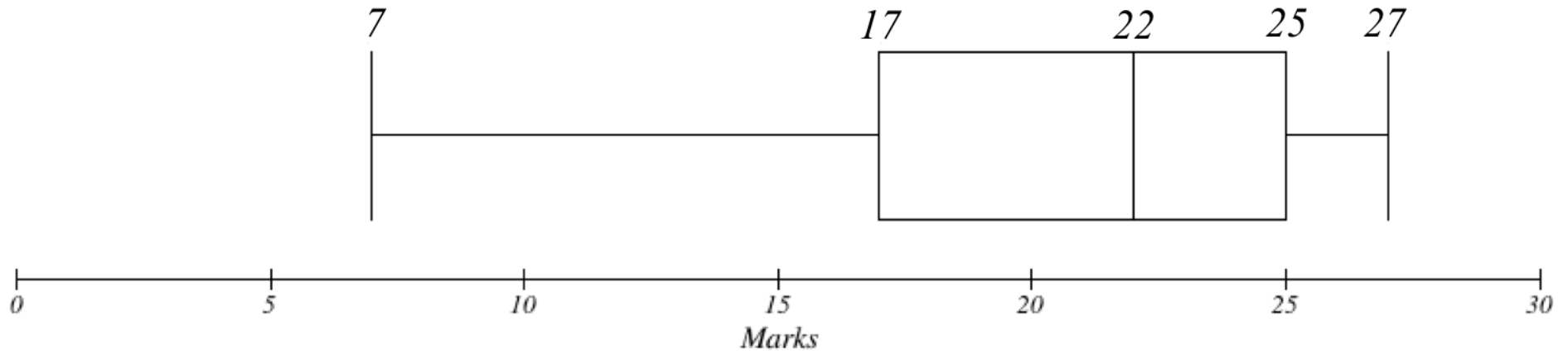
The number of employees aged over 30 but no more than 40 is twice the number of employees aged over 50.

One third of the company's employees are over 40.

The company's oldest employee is exactly 60.

Construct a cumulative frequency graph for the ages of the employees.

Problem Solving



The boxplot above shows the distribution of marks scored in a test by pupils in a Geography class. There are 31 students in the class.

Find upper and lower bounds for the **mean** score achieved by the 31 students.

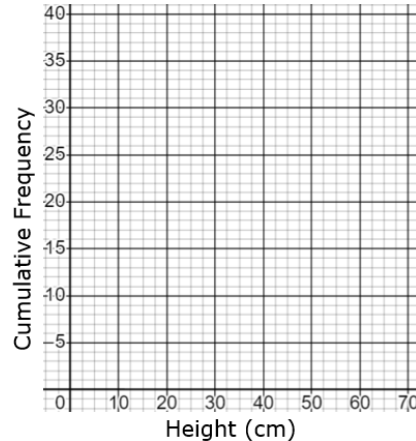
Fluency Practice

Plotting Cumulative Frequency Graphs

(a)

Plot a cumulative frequency graph from the data shown in the table.

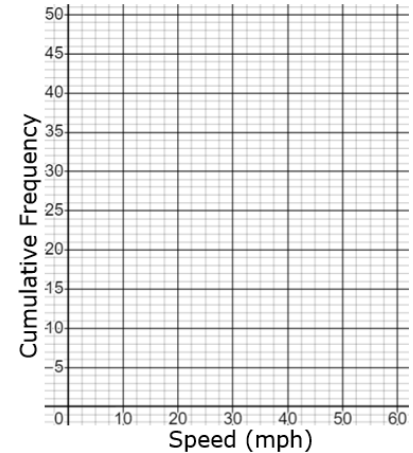
Height (cm)	Cumulative Frequency
$10 < h \leq 20$	3
$10 < h \leq 30$	8
$10 < h \leq 40$	19
$10 < h \leq 50$	29
$10 < h \leq 60$	35
$10 < h \leq 70$	40



(b)

Plot a cumulative frequency graph from the data shown in the table.

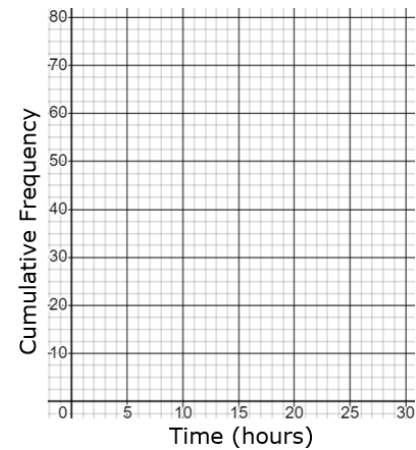
Speed (mph)	Cumulative Frequency
$0 < s \leq 10$	6
$0 < s \leq 20$	10
$0 < s \leq 30$	22
$0 < s \leq 40$	35
$0 < s \leq 50$	44
$0 < s \leq 60$	50



(c)

Plot a cumulative frequency graph from the data shown in the table.

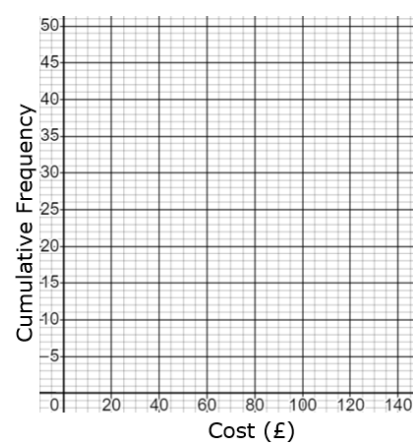
Time (hours)	Cumulative Frequency
$0 < h \leq 5$	9
$0 < h \leq 10$	21
$0 < h \leq 15$	38
$0 < h \leq 20$	60
$0 < h \leq 25$	73
$0 < h \leq 30$	80



(d)

Plot a cumulative frequency graph from the data shown in the table.

Cost (£)	Cumulative Frequency
$20 < c \leq 40$	4
$20 < c \leq 60$	11
$20 < c \leq 80$	19
$20 < c \leq 100$	36
$20 < c \leq 120$	45
$20 < c \leq 140$	48



Finding Frequencies from Cumulative Frequency Graphs

The graph shows the cumulative frequency of time taken. The x-axis is Time (minutes) from 0 to 10. The y-axis is Cumulative Frequency from 0 to 40. The curve starts at (0,0) and increases, passing through points (2, 4), (4, 12), (6, 20), (8, 32), and (10, 40).

Time (minutes)	Cumulative Frequency
$0 < t \leq 2$	6
$0 < t \leq 4$	16
$0 < t \leq 6$	
$0 < t \leq 8$	
$0 < t \leq 10$	40

Time (minutes)	Frequency
$0 < t \leq 2$	6
$2 < t \leq 4$	10
$4 < t \leq 6$	
$6 < t \leq 8$	
$8 < t \leq 10$	

Length (cm)	Cumulative Frequency
$0 < l \leq 10$	0
$0 < l \leq 20$	10
$0 < l \leq 30$	
$0 < l \leq 40$	
$0 < l \leq 50$	
$0 < l \leq 60$	

Length (cm)	Frequency
$0 < l \leq 10$	
$10 < l \leq 20$	
$20 < l \leq 30$	
$30 < l \leq 40$	
$40 < l \leq 50$	
$50 < l \leq 60$	

Weight (kg)	Cumulative Frequency
$0 < w \leq 4$	
$0 < w \leq 8$	
$0 < w \leq 12$	
$0 < w \leq 16$	
$0 < w \leq 20$	

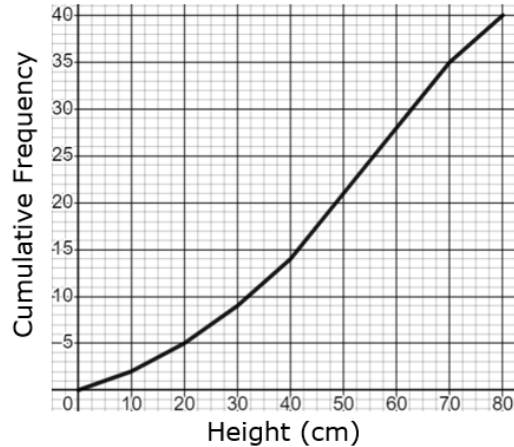
Weight (kg)	Frequency
$0 < w \leq 4$	
$4 < w \leq 8$	10
$8 < w \leq 12$	
$12 < w \leq 16$	
$16 < w \leq 20$	

Fluency Practice

Reading Cumulative Frequency Graphs

(a)

The cumulative frequency graph shows the height of 40 plants in *cm*.



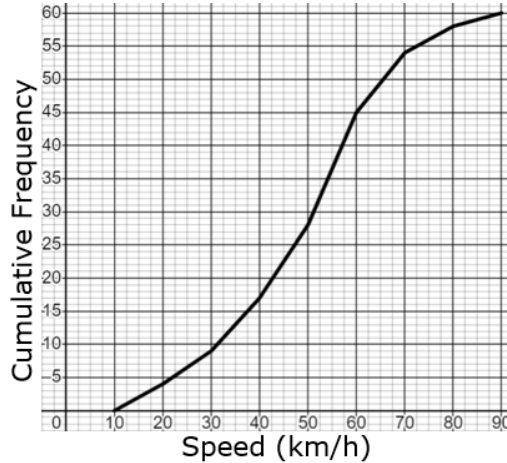
(a) Find the median height of the plants.

(b) Find the interquartile range of the heights of the plants.

(c) Find the number of plants whose height is less than 30 *cm*.

(b)

The cumulative frequency graph shows the speed of 60 cars in *km/h*.



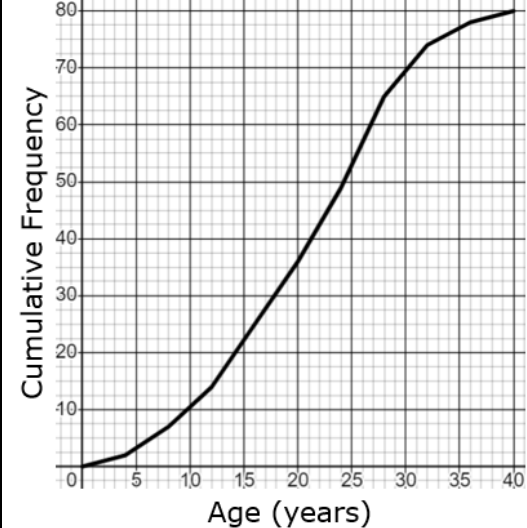
(a) Find the interquartile range of the speeds.

(b) Find the number of cars travelling less than 45 *km/h*.

(c) The speed limit is 70 *km/h*. What fraction of cars are breaking the speed limit?

(c)

The cumulative frequency graph shows the age of 80 apple trees in years.



(a) Find the number of trees that are at least 25 years old.

(b) Oscar says "30% of apple trees are less than 15 years old". Is Oscar correct? You must show working.

Fluency Practice

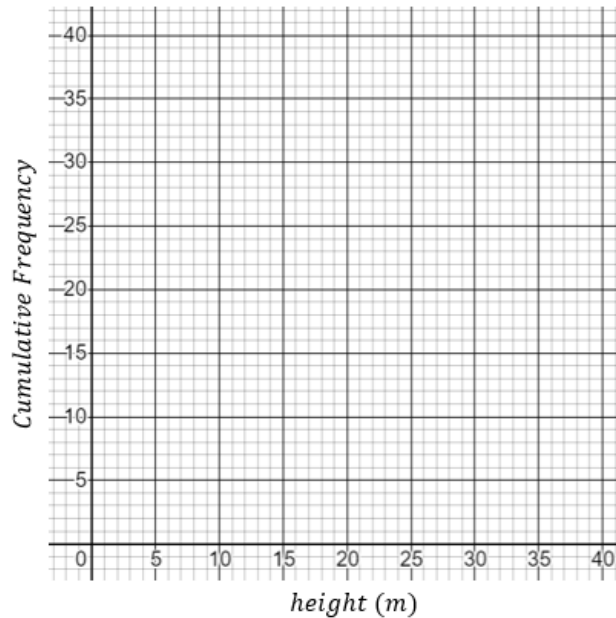
(a)

The heights of 40 oak trees are recorded in a frequency table.

(a) Use the information to calculate cumulative frequencies and plot a cumulative frequency graph.

(b) Use your cumulative frequency graph to find the median and interquartile range of the heights of the oak trees.

Height (m)	Frequency
$10 < h \leq 15$	5
$15 < h \leq 20$	9
$20 < h \leq 25$	13
$25 < h \leq 30$	7
$30 < h \leq 35$	4
$35 < h \leq 40$	2



(b)

The time taken by 100 students to complete a jigsaw is recorded in a grouped frequency table.

(a) Use the information to calculate cumulative frequencies and plot a cumulative frequency graph.

(c) Use your cumulative frequency graph to find the median and interquartile range of the times taken.

Time (minutes)	Frequency
$30 < t \leq 40$	4
$40 < t \leq 50$	11
$50 < t \leq 60$	20
$60 < t \leq 70$	28
$70 < t \leq 80$	19
$80 < t \leq 90$	13
$90 < t \leq 100$	5

(c)

The History test score of 60 students is recorded in a grouped frequency table.

(a) Plot a cumulative frequency graph.

(b) Find the median score in the History test.

(c) Use you graph to estimate the number of students who scored less than 36 marks.

(d) Use your graph to find the minimum score required to come in the top 10% of students in the test.

Test Score	Frequency
$0 < s \leq 10$	1
$10 < s \leq 20$	4
$20 < s \leq 30$	6
$30 < s \leq 40$	12
$40 < s \leq 50$	17
$50 < s \leq 60$	13
$60 < s \leq 70$	5
$70 < s \leq 80$	2

learn by heart

The **cumulative frequency** works like a 'running total'. It tells you the number of pieces of data up to and including a particular value.

example

The length of a number of television adverts is given in the frequency table.
Work out the cumulative frequencies.

Frequency Table

Time t (s)	Frequency
$0 < t \leq 10$	7
$10 < t \leq 20$	12
$20 < t \leq 30$	10
$30 < t \leq 40$	8
$40 < t \leq 50$	3

Cumulative Frequency Table

Time t (s)	Cumulative Frequency
$t \leq 10$	7
$t \leq 20$	19
$t \leq 30$	29
$t \leq 40$	37
$t \leq 50$	40

The second row tells you that 19 adverts lasted less than or equal to 20 seconds.

exercise

1. The widths of some birds are recorded and shown in the frequency table.

Width w (mm)	Frequency
$10 < w \leq 20$	2
$20 < w \leq 30$	6
$30 < w \leq 40$	14
$40 < w \leq 50$	18
$50 < w \leq 60$	7
$60 < w \leq 70$	4

Width w (mm)	Cumulative Frequency
$w \leq 20$	
$w \leq 30$	
$w \leq 40$	
$w \leq 50$	
$w \leq 60$	
$w \leq 70$	

- a) Complete the cumulative frequency table for the data.
- b) How many of the eggs had a width less than or equal to 40mm?
- c) How many eggs had widths up to and including 60mm?
- d) How many eggs were included in the data?

Fluency Practice

2. The hourly wages of the workers at a company are shown in the tables.

Hourly wage W (£)	Frequency
$4 \leq W < 8$	15
$8 \leq W < 12$	11
$12 \leq W < 16$	5
$16 \leq W < 20$	

Hourly wage W (£)	Cumulative Frequency
$W < 8$	
$W < 12$	
$W < 16$	
$W < 20$	33

- Complete the tables.
- How many workers are at the company?
- How many workers earn less than £8 per hour?
- How many workers earn £8 or more per hour?

3. The air temperature at midnight on a number of days was recorded. The data is shown in the cumulative frequency table.

Temperature T (°C)	Cumulative Frequency
$T \leq 0$	3
$T \leq 5$	9
$T \leq 10$	17
$T \leq 15$	21
$T \leq 20$	25

- How many times was the temperature less than or equal to 5°C?
- How many times was the temperature greater than 15°C?
- How many times was the temperature in the range $10^{\circ}\text{C} < T \leq 15^{\circ}\text{C}$?
- How many times was the temperature in the range $5^{\circ}\text{C} < T \leq 15^{\circ}\text{C}$?

4. The test scores of some pupils are shown in the cumulative frequency table.

Score, $x\%$	Cumulative Frequency
$x \leq 20$	2
$x \leq 40$	11
$x \leq 60$	25
$x \leq 80$	34
$x \leq 100$	40

- How many pupils scored more than 80%?
- How many pupils scored more than 40%?
- How many pupils scored between 20% and 60%?
- What fraction of the pupils scored 60% or less?
- What percentage of the pupils scored 80% or less?

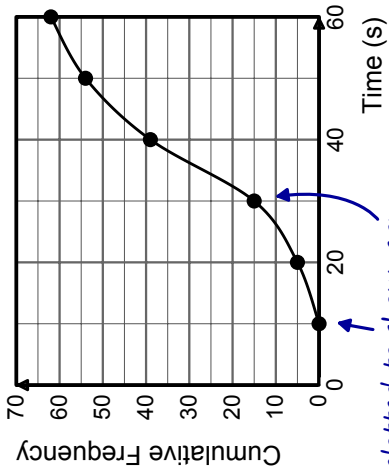
5. At a company, 30% of the workers earn less than £22,000. 88% of the workers earn less than £48,000. What percentage of workers have:

- $\text{salary} \geq \text{£}48,000$
- $\text{£}22,000 \leq \text{salary} < \text{£}48,000$

example

Construct a cumulative frequency graph for this data set:

Time t (s)	Frequency	Cumulative Frequency
$10 \leq t < 20$	5	5
$20 \leq t < 30$	10	15
$30 \leq t < 40$	24	39
$40 \leq t < 50$	15	54
$50 \leq t < 60$	8	62



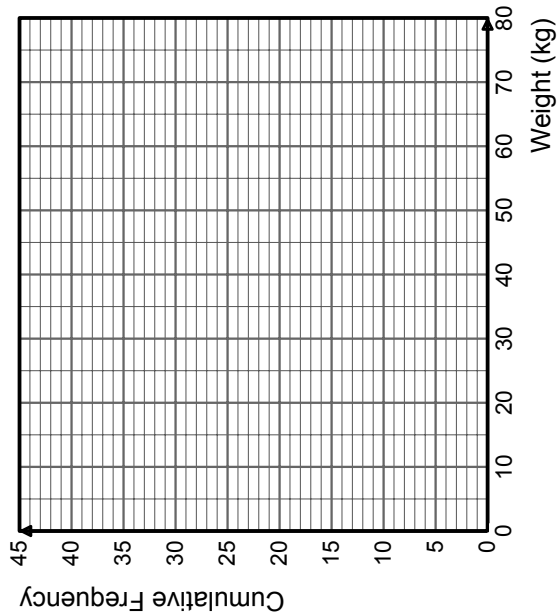
exercise

1. The frequency table shows the weights of 40 dogs.

a) Complete the cumulative frequency table and graph.

Weight w (kg)	Frequency
$0 \leq w < 20$	6
$20 \leq w < 30$	9
$30 \leq w < 40$	14
$40 \leq w < 60$	7
$60 \leq w < 80$	4

- b) Use your graph to estimate the number of dogs that weighed less than 35kg.

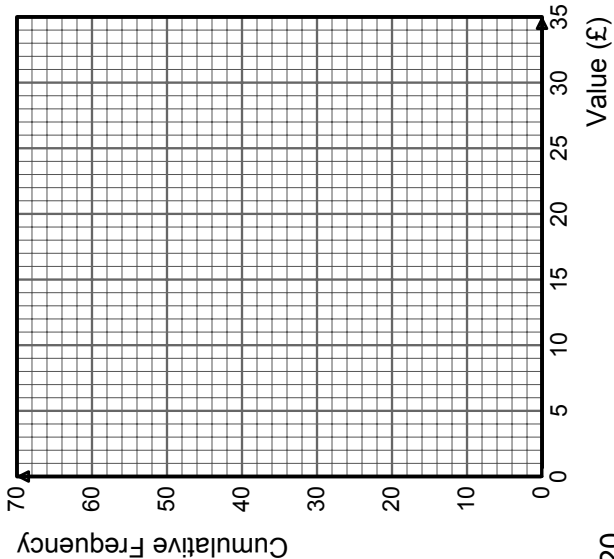


Weight w (kg)	Cumulative Frequency
$w < 20$	6
$w < 30$	15
$w < 40$	
$w < 60$	
$w < 80$	

Fluency Practice

2. The table shows the values of 60 collectable stamps.
a) Complete the table and cumulative frequency graph.

Value v (£)	Freq.	C. Freq.
$0 \leq v < 5$	22	
$5 \leq v < 10$	17	
$10 \leq v < 15$	8	
$15 \leq v < 20$	5	
$20 \leq v < 25$	4	
$25 \leq v < 30$	4	

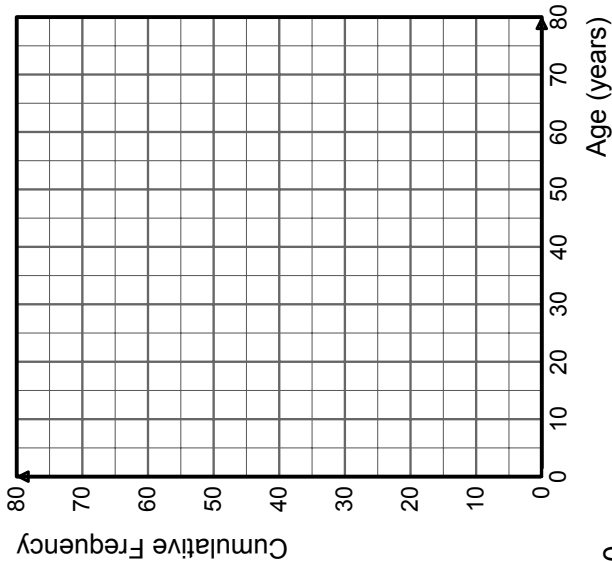


- b) Use your graph to estimate the number of stamps:

- (i) valued at less than £8.
- (ii) valued at £8 or more.
- (iii) valued between £8 and £20.

3. The table shows the ages of people watching a film at the cinema.
a) Complete the table and cumulative frequency graph.

Age a (years)	Freq.	C. Freq.
$0 \leq a \leq 5$	6	
$5 < a \leq 10$	20	
$10 < a \leq 20$	8	
$20 < a \leq 30$	8	
$30 < a \leq 40$	17	
$40 < a \leq 60$	9	
$60 < a \leq 80$	6	



- b) Use your graph to estimate the number of people:

- (i) aged 25 or less.
- (ii) aged 50 or less.
- (iii) aged between 25 and 50.
- (iv) aged more than 35.

Fluency Practice

example

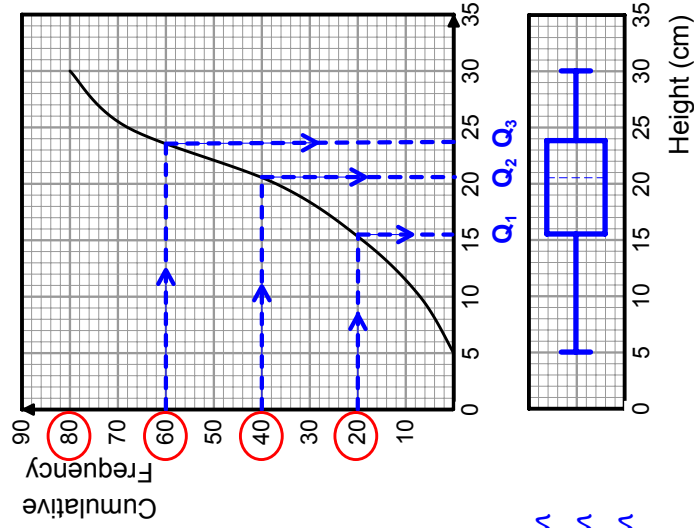
The cumulative frequency graph shows the heights of some plants. Find estimates for Q_1 , Q_2 and Q_3 and draw a box plot for the data.

*There are 80 pieces of data.
The quartiles can be found at:*

$$Q_1 : \frac{1}{4} \times 80 = 20\text{th value}$$

$$Q_2 : \frac{1}{2} \times 80 = 40\text{th value}$$

$$Q_3 : \frac{3}{4} \times 80 = 60\text{th value}$$



$$Q_1 = 16\text{cm}$$

$$Q_2 = 21\text{cm}$$

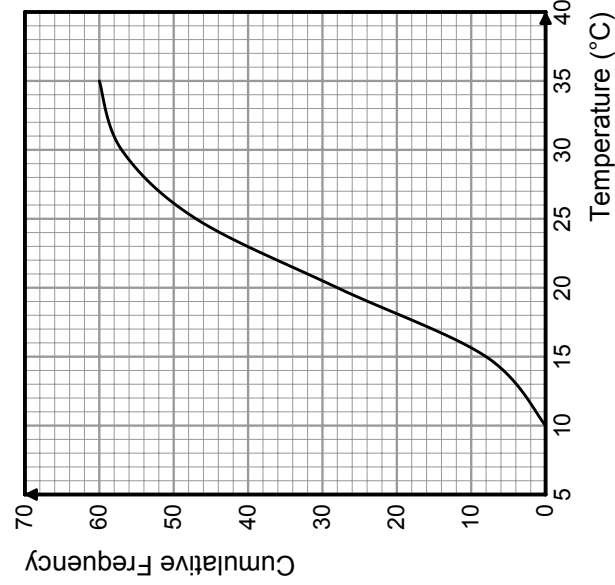
$$Q_3 = 24\text{cm}$$

exercise

1. The cumulative frequency graph shows the maximum temperature recorded on 60 days.

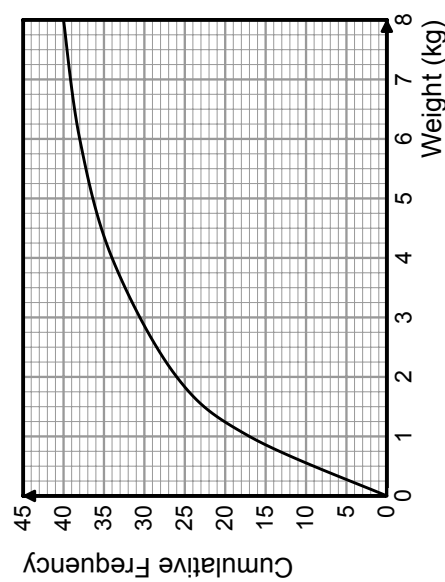
Find estimates for:

- a) the median
- b) Q_1
- c) Q_3
- d) the interquartile range
- e) the range

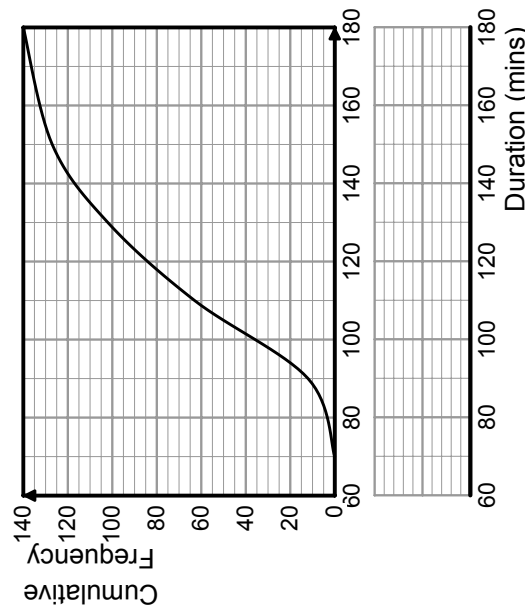


Fluency Practice

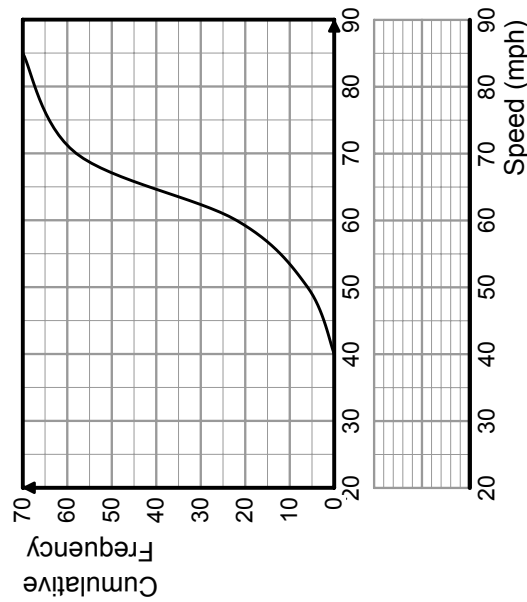
2. The cumulative frequency graph shows the weight of a number of parcels.
- a) Find estimates for:
- i) the lower quartile
 - ii) the median
 - iii) the upper quartile
- b) True or false? 80% of the parcels weighed less than 4kg.



3. The cumulative frequency graph shows the duration of a number of films, in minutes.
- Find estimates for the median and quartiles and construct a boxplot in the space below the graph.

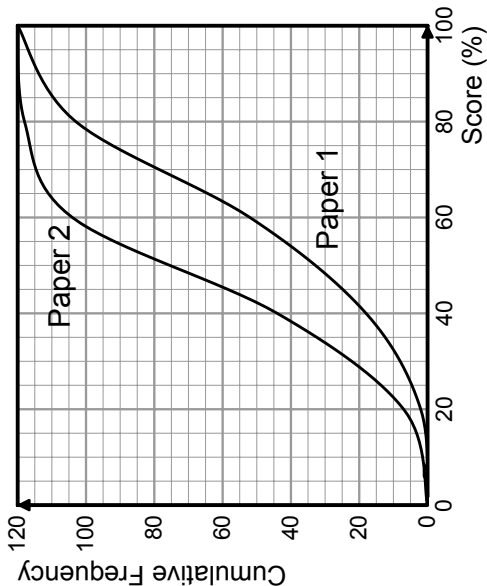


4. The cumulative frequency graph shows the speeds of vehicles on a motorway.
- Find estimates for the median and quartiles and construct a boxplot in the space below the graph.



Fluency Practice

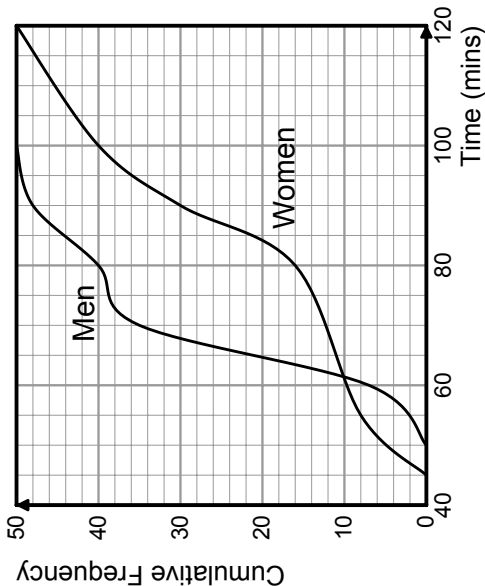
5. The cumulative frequency graph shows the scores of a year group for two exam papers.



a) Which paper did the pupils find easier? Justify your answer.

b) Certificates were given to pupils who achieved over 70% on a paper. Estimate the number of certificates given for each paper.

6. The cumulative frequency graph shows the times taken by 50 men and 50 women to complete a race.

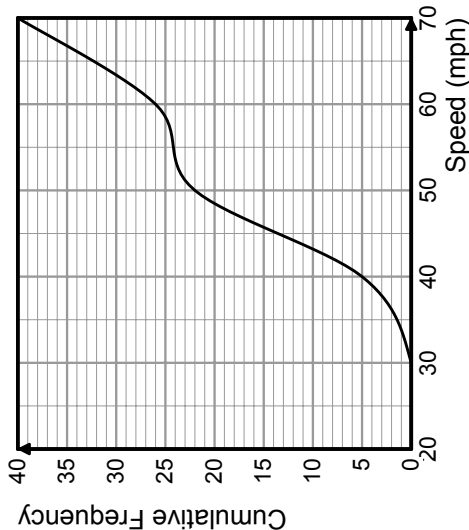


a) Complete the table:

	Men	Women
Median		
Range		
IQR		

b) Compare the two data sets.

7. The cumulative frequency graph shows the speeds of some vehicles on a road.



One of the cars is to be chosen at random. Work out estimates for these probabilities:

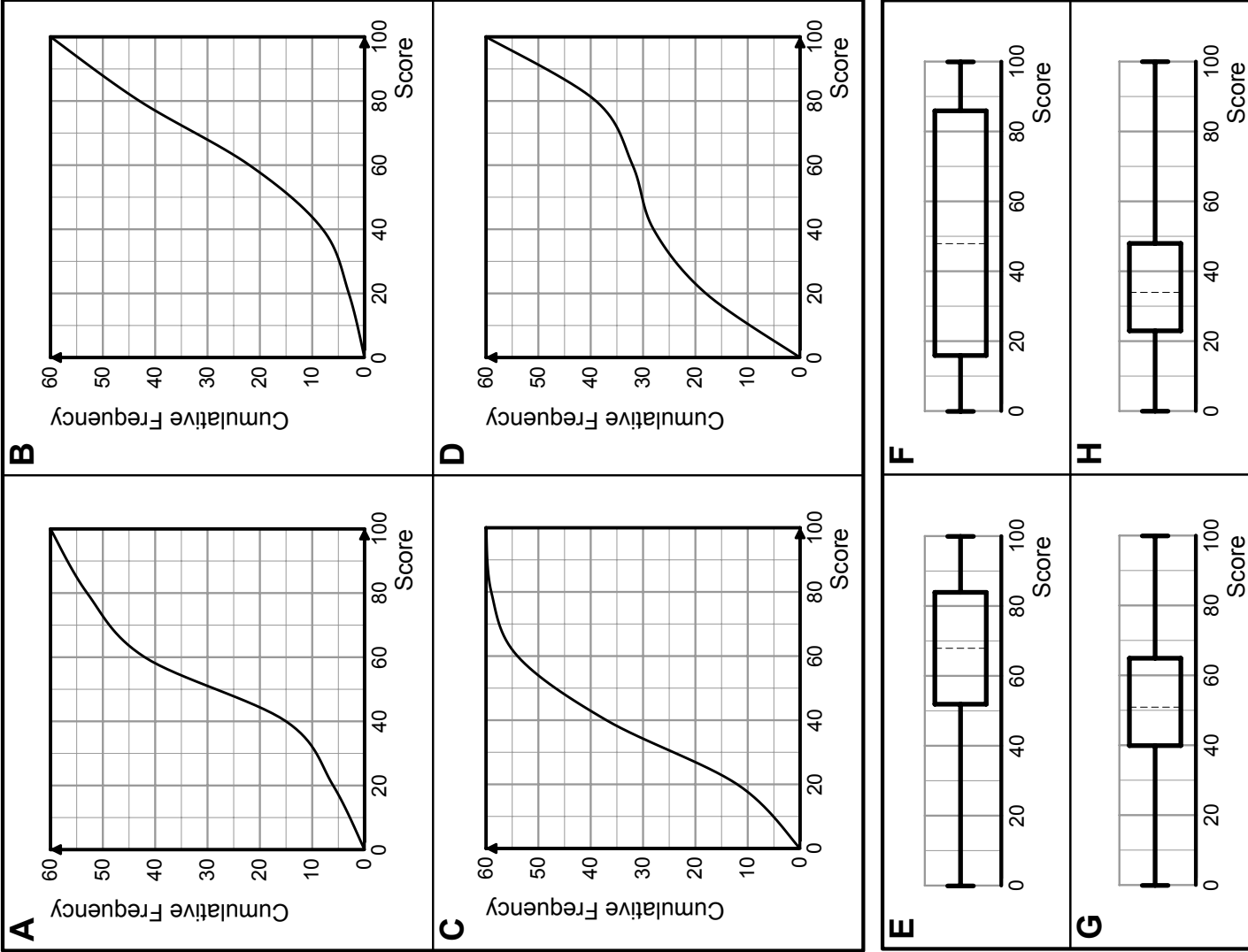
a) $P(\text{speed} \leq 45)$

b) $P(\text{speed} \leq 65)$

c) $P(45 < \text{speed} \leq 65)$

Fluency Practice

Match each cumulative frequency graph with its corresponding boxplot.

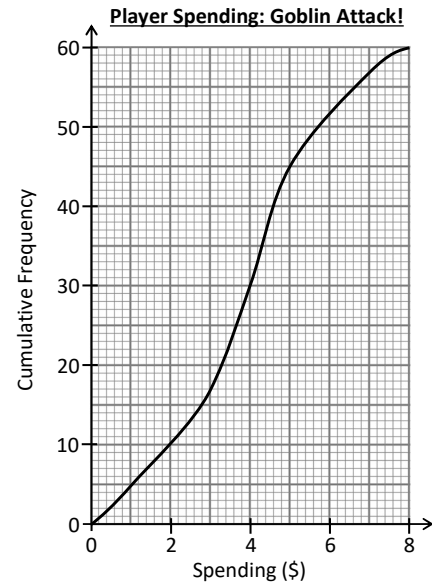


Fluency Practice

①

Calculating Averages From Graphs

For players that spend money in their game, Apex Games records how much they spend in one day.



Use the graph to estimate the **median & interquartile range** for spending.

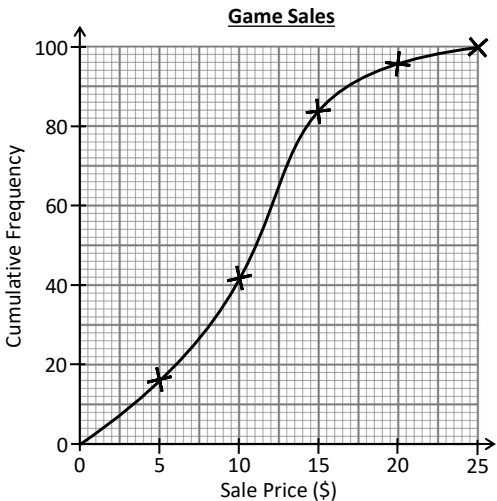
Use the graph to complete the table & calculate the **mean** spending.

Spending, s (\$)	Frequency	Midpoint	$f \times m$
$0 < s \leq 2$			
$2 < s \leq 4$			
$4 < s \leq 6$			
$6 < s \leq 8$			
Totals			

②

Calculating Averages From Graphs

Apex Games has published over 20 games. Over time, they change prices up & down to increase demand or increase profit. They record all the sales made at different price-points.



Use the graph to estimate the **median & interquartile range** for sale prices.

Use the graph to calculate the **mean** sale price.

Fluency Practice

Interpreting the Shapes of Cumulative Frequency Graphs

Each of these cumulative frequency graphs show the test scores for different science classes.

①

How can we describe the results represented by each graph?

How do the results relate to the median?

Nearly all students scored near the median.

Most students scored very high or very low marks.

Most students scored high marks

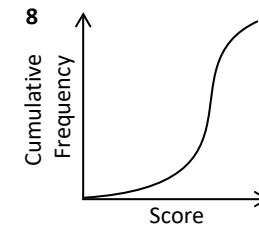
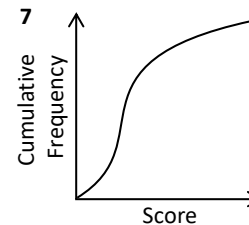
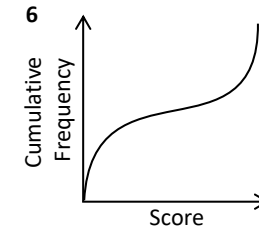
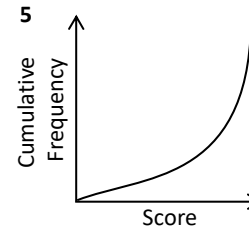
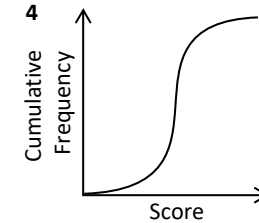
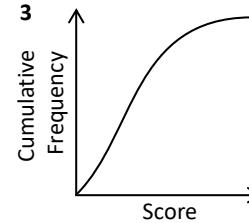
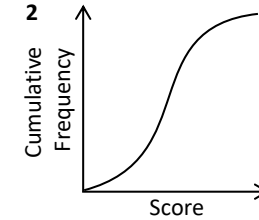
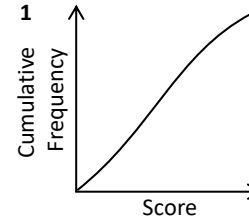
High median score.

Few students scored high marks

Low median score.

Most students scored near the median.
There are equal amounts of high & low scores.

Even distribution of students across the range of scores.



Fluency Practice

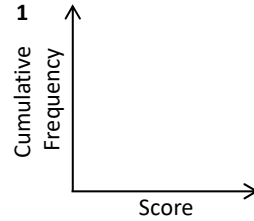
Sketching the Shape of Cumulative Data

8 classes took a science test & their results are described below.

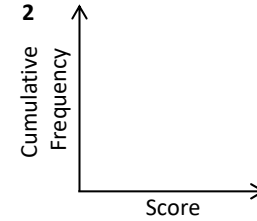
Sketch a cumulative frequency curve for each description.

Think about the rate the cumulative frequency increases above and below the median.

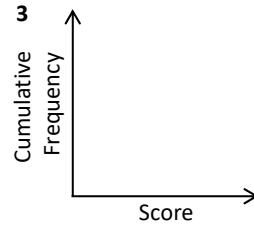
Even distribution of students across the range of scores.



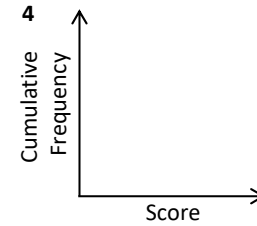
Most students scored near the median.
There are equal amounts of high & low scores.



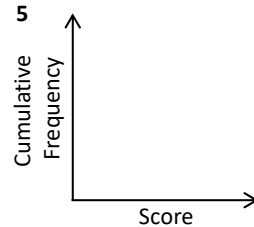
Few students scored high marks



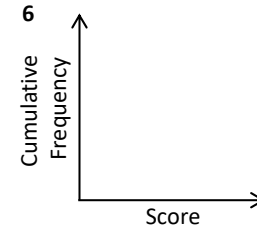
Nearly all students scored near the median.



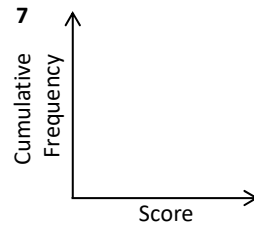
Most students scored high marks



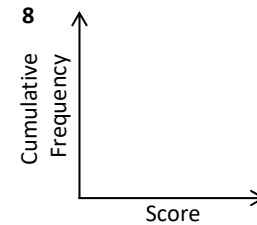
Most students scored very high or very low marks.



Low median score.



High median score.



③

Fluency Practice

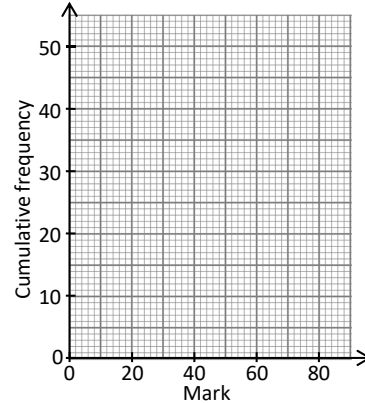
Cumulative Frequency Curves

1) 50 students took a Geography test. Their results were put into 4 groups.

- A) Complete the table to find the cumulative frequency.
B) Plot the table data on the graph.

Remember we plot each group at its **highest** value.

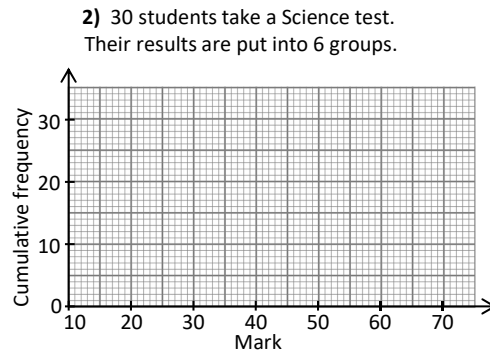
Mark	Frequency	Cumulative Frequency	Coord.
0-20	5		
21-40	22		
41-60	14		
61-80	9		



- C) Join the points with a curve.
Could we join points with straight lines instead?
Should we start the curve at (0, 0)?

- D) We want to find the median. With grouped data we can estimate and use $n \div 2$. Plot a line from $n \div 2$ to the curve. What is the estimate for the median?

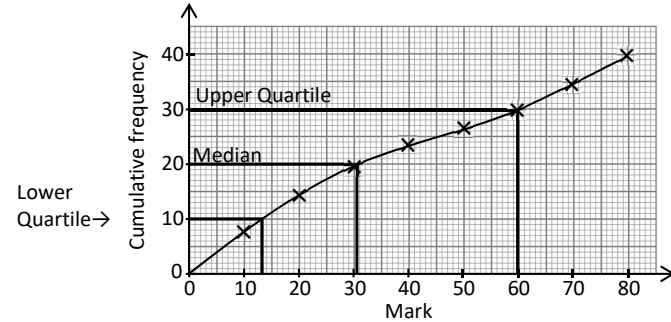
Mark (m)	F.	C.F.
$11 \leq m \leq 20$	2	
$21 \leq m \leq 30$	7	
$31 \leq m \leq 40$	9	
$41 \leq m \leq 50$	8	
$51 \leq m \leq 60$	3	
$61 \leq m \leq 70$	1	



- A) Find an estimate for the median

2) 30 students take a Science test.
Their results are put into 6 groups.

3) Mr Smith has drawn a cumulative frequency graph for the results of his Year 9 students.



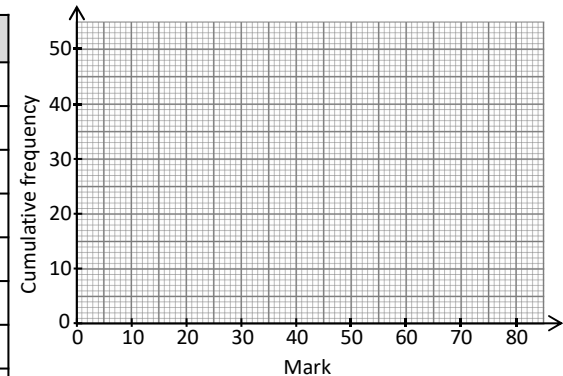
Mr Smith wants to measure the spread of the data, but he wants to eliminate extreme values and focus on the middle 50%. To do this he marks the first quarter (Lower-Quartile) and third quarter (Upper Quartile) of the data.

- A) What is the interquartile range?

4) The data for Mr Smith's Year 10 students is in the table.

- A) Complete the cumulative frequency graph.
B) Find the median and the interquartile range.

Mark (m)	F.	C.F.
$0 \leq m \leq 10$	2	
$11 \leq m \leq 20$	3	
$21 \leq m \leq 30$	9	
$31 \leq m \leq 40$	13	
$41 \leq m \leq 50$	14	
$51 \leq m \leq 60$	6	
$61 \leq m \leq 70$	2	
$71 \leq m \leq 80$	1	



- C) What comparisons can you make between Mr Smith's Year 9 and Year 10 students?

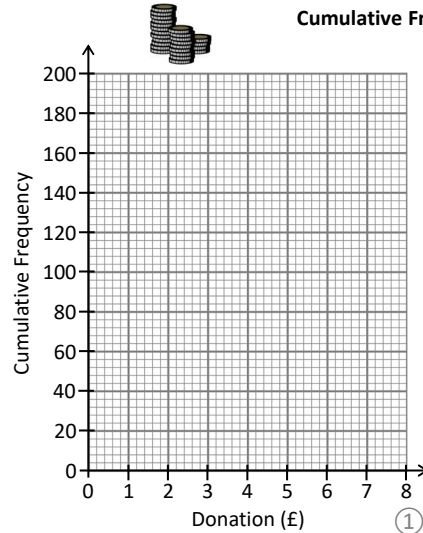
Fluency Practice

A charity recorded the size of some donations it received.

Complete a cumulative frequency graph to show this information.

Donation, d (£)	Frequency	Cumul. Freq.
$0 < d \leq 2$	20	
$2 < d \leq 4$	60	
$4 < d \leq 6$	80	
$6 < d \leq 8$	40	

Estimate the **median** donation size by plotting a horizontal line from 100 on the y-axis.



Cumulative Frequency Graphs

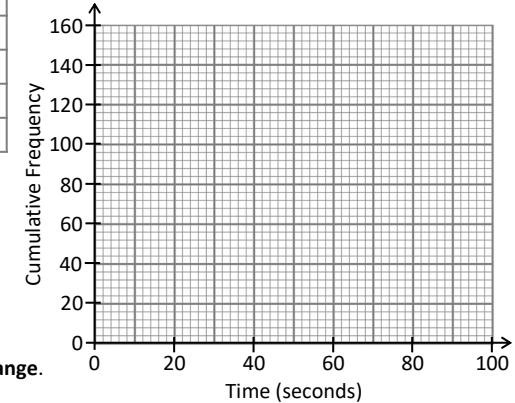
Time, t (seconds)	Freq.	CF
$0 < t \leq 20$	20	
$20 < t \leq 40$	40	
$40 < t \leq 60$	80	
$60 < t \leq 80$	16	
$80 < t \leq 100$	4	

Use the graph to estimate the upper quartile (horizontal line from 120) & lower quartile (horizontal line from 40).

Use these readings to calculate the **interquartile range**.

Customer support recorded how long customers had to wait before their phone call was answered.

Plot a cumulative frequency graph to show this data.

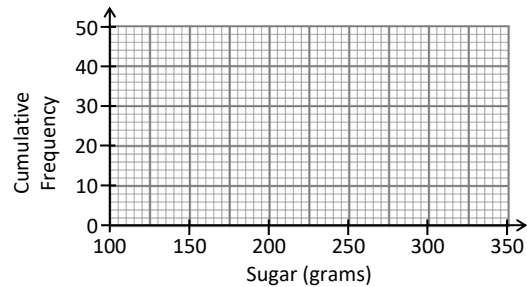


A food scientist records the sugar content in 1 kg of different snacks.

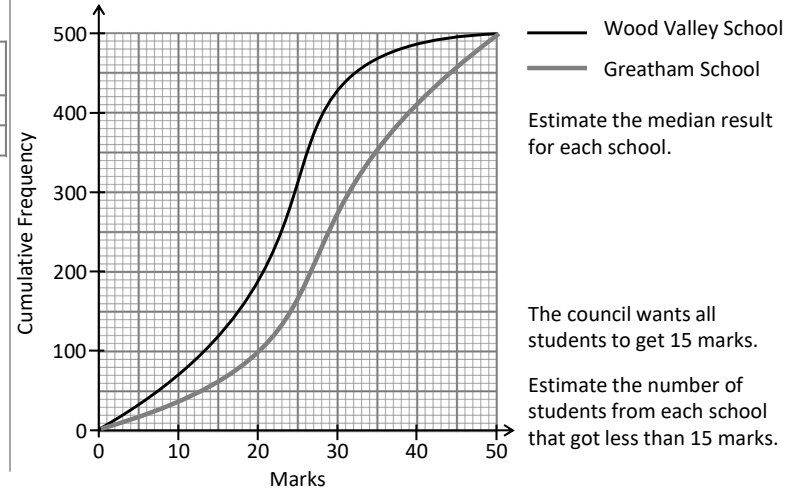
Plot a cumulative frequency graph to show this data.

Sugar, s (grams)	$100 < s \leq 150$	$150 < s \leq 200$	$200 < s \leq 250$	$250 < s \leq 300$	$300 < s \leq 350$
Freq.	4	10	14	6	6

Use the graph to estimate the median & the interquartile range.



A local council records English test results from two schools.



Activity

Wordsearch versus Sudoku

Record the time it takes to find all the words listed in the **wordsearch** below.

S G T E G A R E V A G B D V
 E Y R T R P T E Z W A C T G
 L P G A A S R E P P U Q O R
 I U I D P O C P A M B S D O
 T L D F A H S G U E H I J U
 R S E F L T T L G D P H D P
 A J K O Y L A W F I Z X A E
 U K W L Z T T X R A N G E D
 Q E C Q I V I B N N M T R Q
 R C V V B N S M Q W E A P V
 E R E W J E T R T Y U B S I
 T R T I O A I S G K H L U L
 N K O D X Y C N E U Q E R F
 I P Y U N D S F Z H J X M C

STATISTICS	CUMULATIVE	FREQUENCY	MEDIAN
INTERQUARTILE	RANGE	UPPER	LOWER
TABLE	DATA	AVERAGE	SPREAD
GROUPED	GRAPH		

Time taken to complete wordsearch _____ min _____ sec

Record the time you take to complete the sudoku puzzle below. Each row, column and 3x3 square must contain each of the numbers 1 to 9 once only.

	7			1	5			4
5	8	4	7	3		2	1	9
6				4	9		8	7
7		5				6	4	
	9		4	2	8		3	
	3	8	5	6				2
3	5		9	8				6
9	4		6	5	2	8	7	
	6		3	7	4		5	

Time taken to complete sudoku _____ min _____ sec

Activity

Wordsearch Data

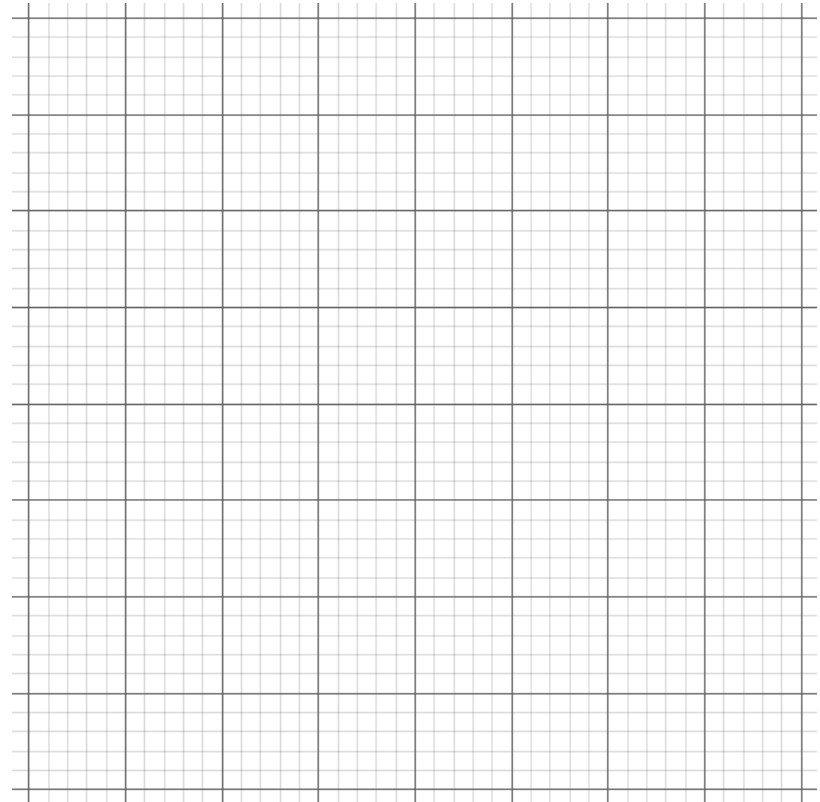
1. Complete a grouped frequency table, then find the cumulative frequencies.

Time (minutes)	Frequency	Cumulative Frequency

3. Find the median, lower quartile, upper quartile and interquartile range (IQR) for the wordsearch data from the cumulative frequency graph.

Median	
Lower Quartile	
Upper Quartile	
Interquartile Range	

2. Plot a cumulative frequency graph.



Activity

Sudoku Data

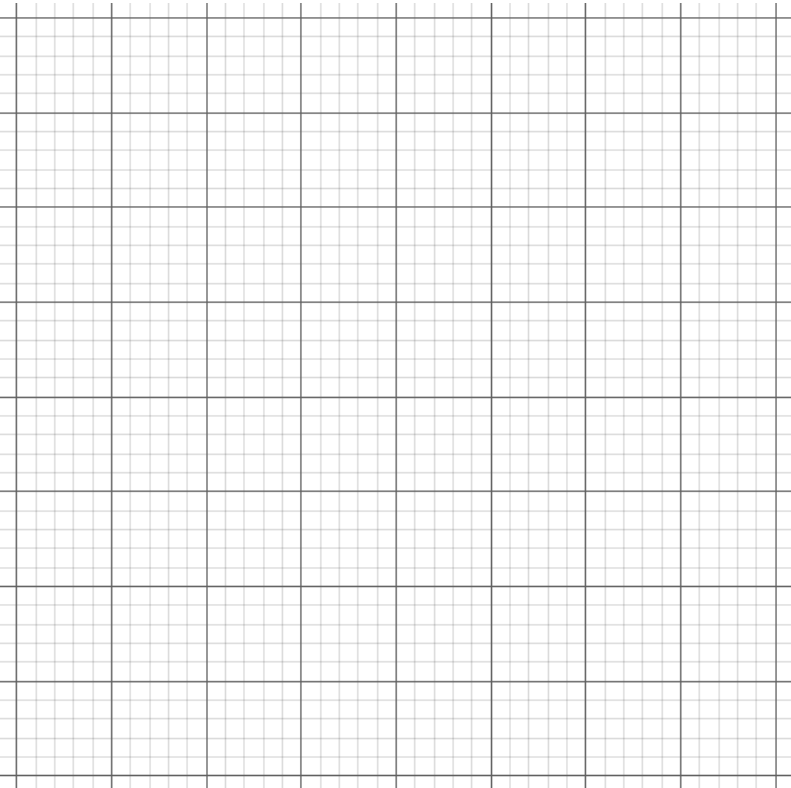
1. Complete a grouped frequency table, then find the cumulative frequencies.

Time (minutes)	Frequency	Cumulative Frequency

3. Find the median, lower quartile, upper quartile and interquartile range (IQR) for the wordsearch data from the cumulative frequency graph.

Median	
Lower Quartile	
Upper Quartile	
Interquartile Range	

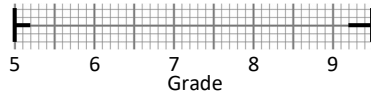
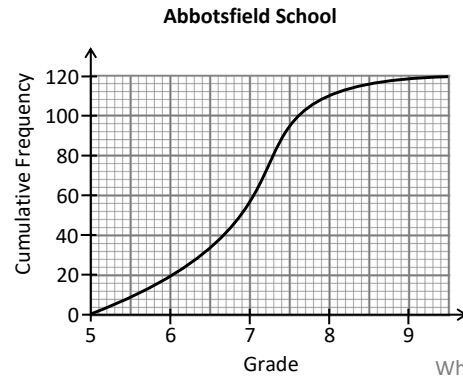
2. Plot a cumulative frequency graph.



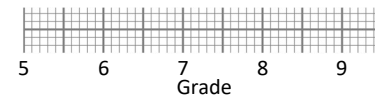
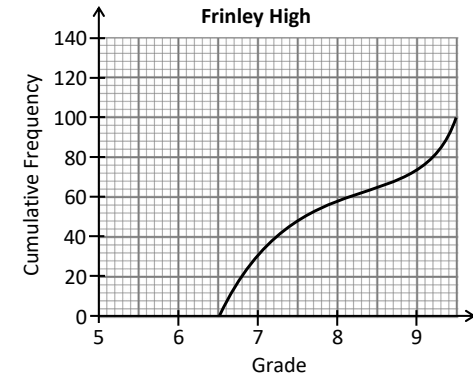
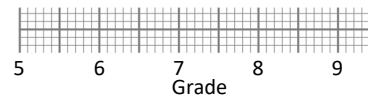
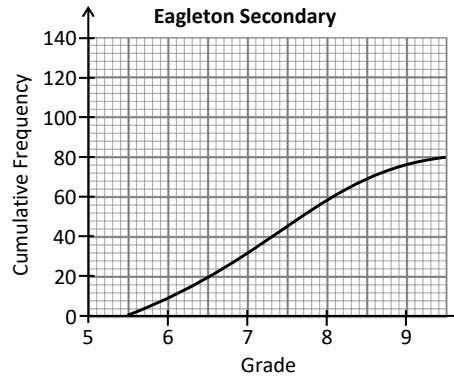
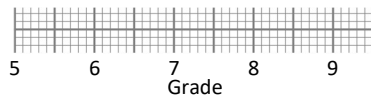
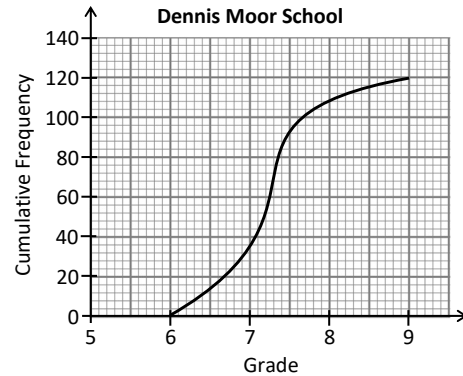
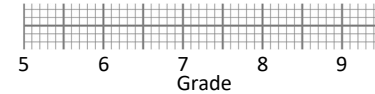
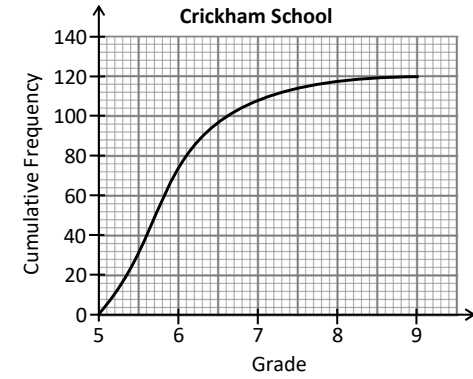
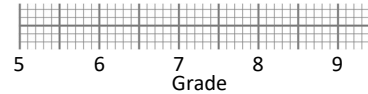
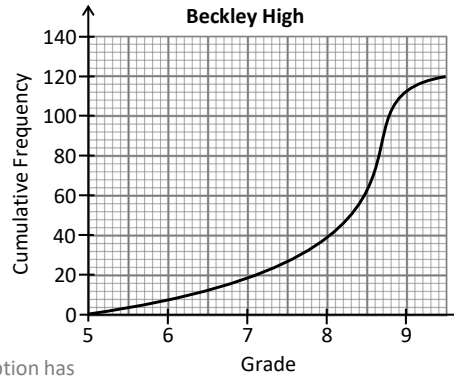
Fluency Practice

Cumulative Frequency Graphs & Box Plots

Students in 6 schools take an English exam (graded 5.0 to 9.5). Complete the box plot for each school.
Which school did best? Which was most consistent? What comments can you make about the maximums & minimums?



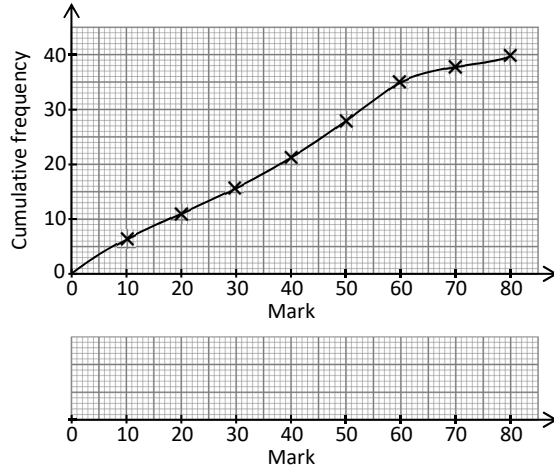
What assumption has been made?



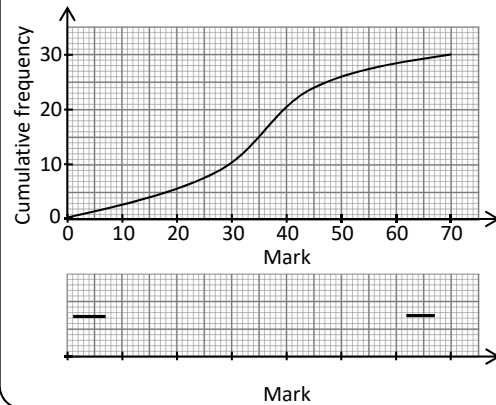
Fluency Practice

Cumulative Frequency Graphs & Box Plots

- 1) The cumulative frequency graph below shows the results of a Year 8 science test. Josh got the highest mark in the class with 76. Samantha got the lowest mark with 7.
- A) Find the Median, Upper Quartile and Lower Quartile from the graph.
- B) Combine all this data to create a box plot.



- 2) This cumulative frequency graph shows the results of a Year 10 science test.



- A) Use the data to complete the box plot.
- B) What comparisons can you make between the Year 8s and the Year 10s?

- 3) These two tables show the results for Year 7 and Year 10 end-of-year English tests.

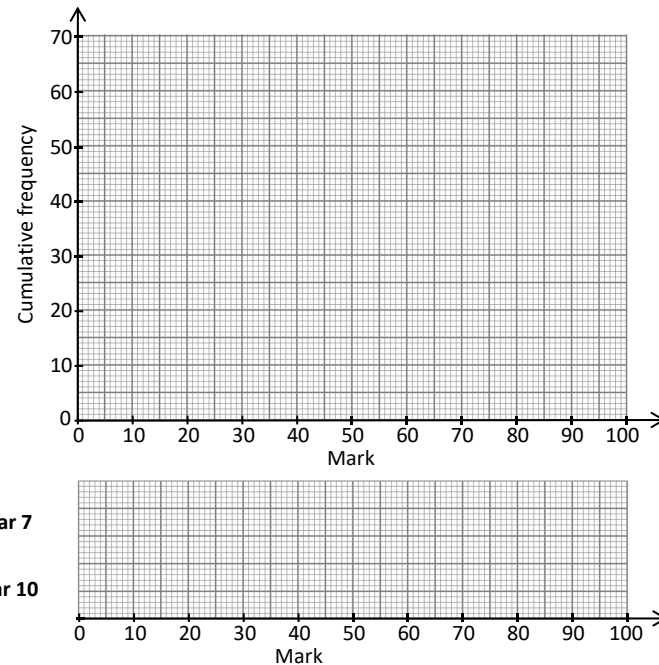
Year 7
Min = 8
Max = 99

Mark (m)	F.	C.F.
$0 \leq m \leq 20$	15	
$21 \leq m \leq 40$	22	
$41 \leq m \leq 60$	18	
$61 \leq m \leq 80$	11	
$81 \leq m \leq 100$	4	

Year 10
Min = 0
Max = 92

Mark (m)	F.	C.F.
$0 \leq m \leq 10$	1	
$11 \leq m \leq 20$	1	
$21 \leq m \leq 30$	7	
$31 \leq m \leq 40$	12	
$41 \leq m \leq 50$	16	
$51 \leq m \leq 60$	13	
$61 \leq m \leq 70$	15	
$71 \leq m \leq 80$	2	
$81 \leq m \leq 90$	2	
$91 \leq m \leq 100$	1	

- A) Plot both sets of data on the single grid.
- B) Use the graphs to plot two box plots on the same grid.
- C) Compare the data. Which class did best?



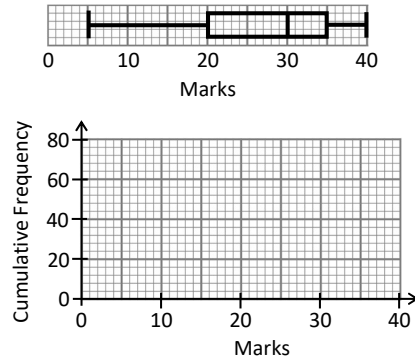
Fluency Practice

①

80 Year 10 students take a science test.

The data was not grouped to create the box plot.

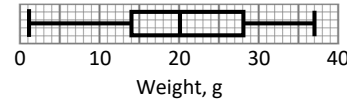
Which points can we plot to create a cumulative frequency curve/polygon?



Converting Box Plots to Cumulative Frequency Graphs

②

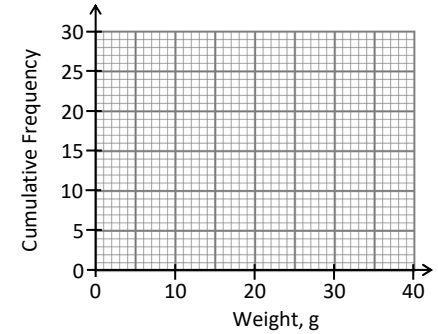
Pirate Penny weighs the 30 diamonds she has plundered!



(She doesn't group the data.)

Plot the data on the grid.

What shape do we expect a cumulative frequency curve to have outside the interquartile range?



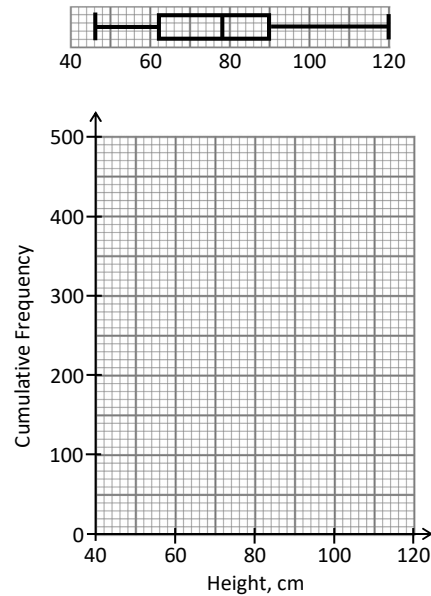
③

A scientist takes a sample of 400 stalks of wheat and measures the heights.

To create the box plot, they group the data and use the minimum & maximum.

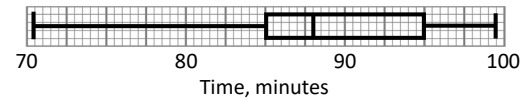
Plot a cumulative frequency curve.

Where do you think it should meet the x -axis?



Race times for 400 charity runners in the Green Park 20K.

④

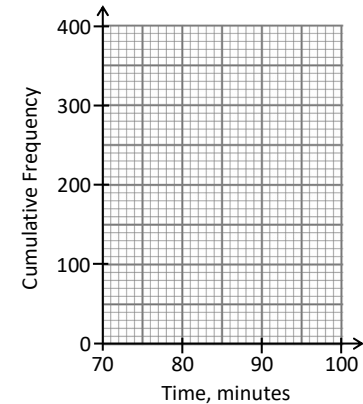


The data was sorted into 3 groups to calculate the median & the quartiles.

Plot a cumulative frequency curve.

How can we use this curve to estimate the mean finishing time for the runners?

Is a curve or a polygon more accurate?

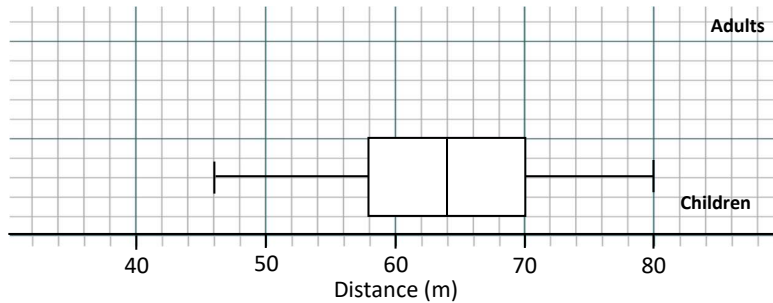


Fluency Practice

1. In an experiment, 50 children and 50 adults were given 10 seconds to run as far as possible. The results for the children are shown below.

The slowest adult went 42 m, the fastest went 88 m. The adults' median was the same as the children. The lower quartile was 54 m and the upper quartile was 74 m.

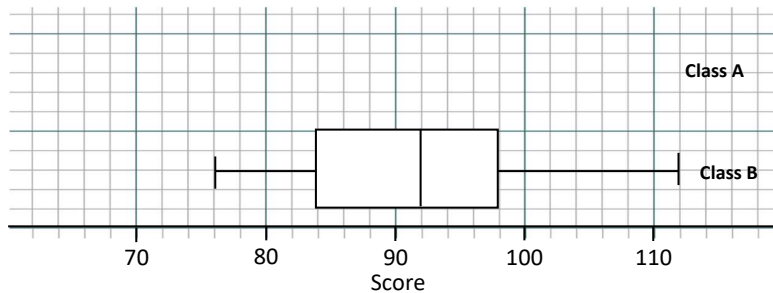
- Complete a box plot to show the adults' results.
- Which group had the furthest and shortest distances?
- Which group had the smallest range?



2. Two classes took the same physics assessment.

Class A scored a median of 90, a lower-quartile of 80 and an interquartile range of 22. The highest score was 108 and the range was 42.

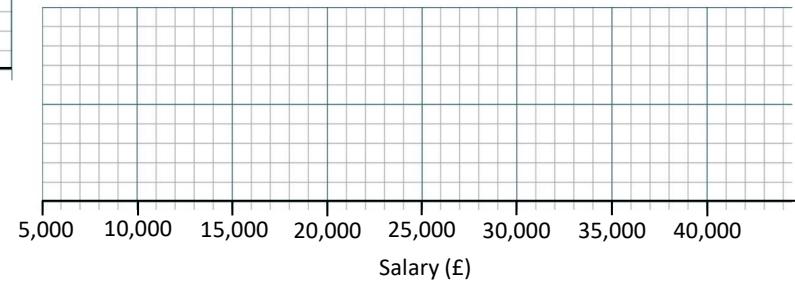
- Complete a box plot for Class A. Which class was more consistent?



Box Plots

- The table below shows data for the salaries of 100 men and 100 women.
 - Complete box plots to show this data.
 - Comment on the difference between men's and women's salaries.

	Lowest Salary	Lower Quartile	Median	Upper Quartile	Highest Salary
Men	£6000	£14,500	£21,000	£23,000	£43,000
Women	£5500	£13,000	£19,000	£22,500	£32,000



- The table below shows the monthly income for 100 individuals.
 - Draw a cumulative frequency graph for this data.
 - Use your graph to estimate the median, as well as Q_1 and Q_2 .
 - The lowest income was £1250, the highest income was 1750.
 - Complete a box plot to show this data.

Income	1201-1300	1301-1400	1401-1500	1501-1600	1601-1700	1701-1800
Frequency	8	22	32	19	14	5

Fluency Practice

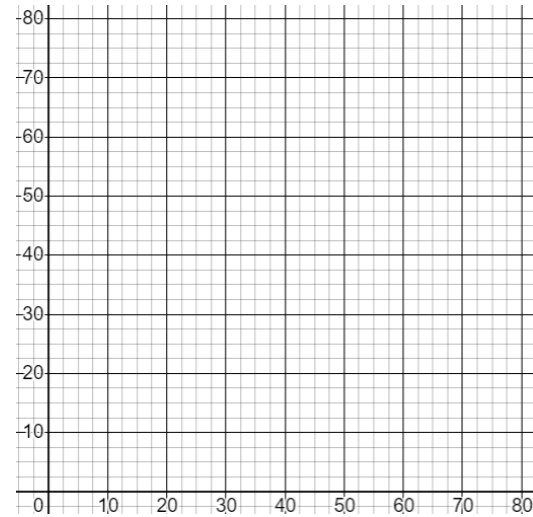
1. The length in mm of 80 leaves is recorded in a grouped frequency table.

Length L (mm)	Frequency
$20 < L \leq 30$	4
$30 < L \leq 40$	7
$40 < L \leq 50$	15
$50 < L \leq 60$	23
$60 < L \leq 70$	22
$70 < L \leq 80$	9

(a) Complete a cumulative frequency table.

Length L (mm)	Cumulative Frequency
$20 < L \leq 30$	
$20 < L \leq 40$	
$20 < L \leq 50$	
$20 < L \leq 60$	
$20 < L \leq 70$	
$20 < L \leq 80$	

(b) Plot a cumulative frequency graph.



(c) Find the median length.

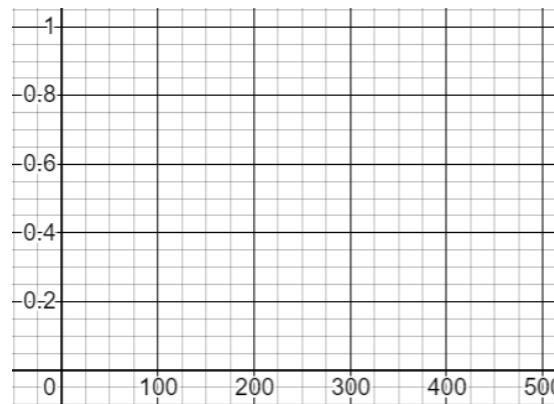
(d) Find the interquartile range of lengths.

(e) Find an estimate for the number of leaves greater than 75 mm in length.

2. The areas in m^2 of 200 gardens are recorded in a grouped frequency table.

Area (m^2)	Frequency		
$0 < A \leq 50$	10		
$50 < A \leq 100$	25		
$100 < A \leq 200$	80		
$200 < A \leq 300$	65		
$300 < A \leq 500$	20		

(a) Plot a histogram.



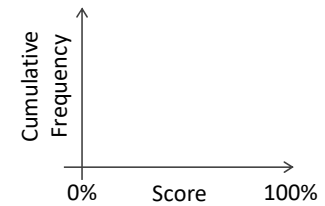
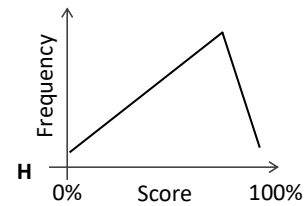
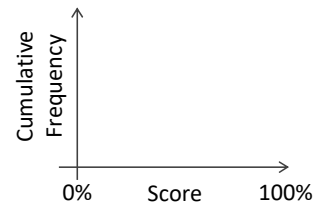
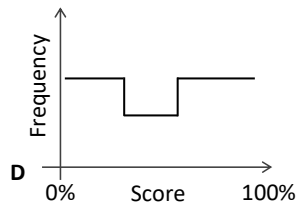
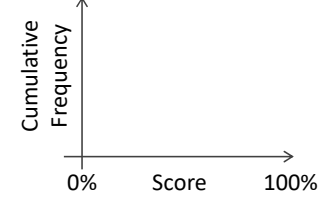
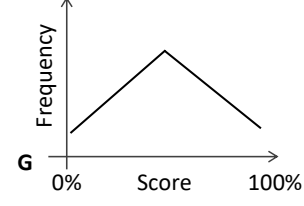
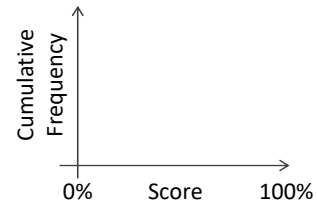
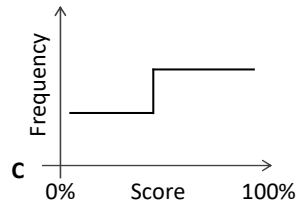
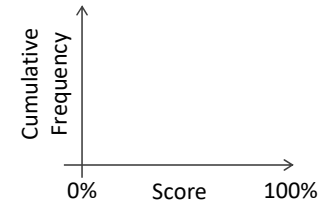
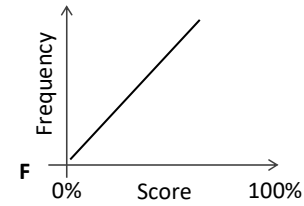
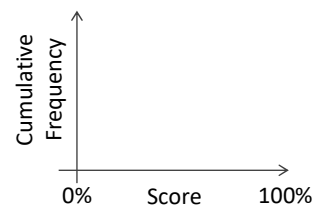
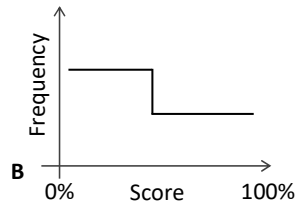
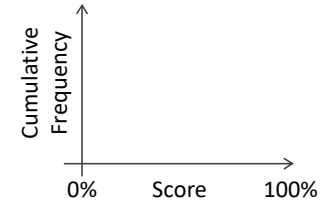
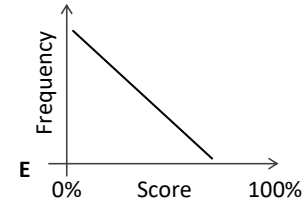
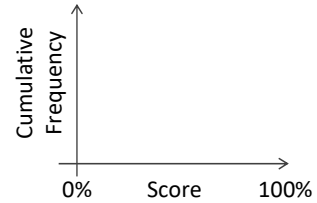
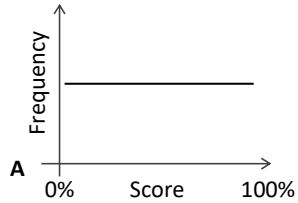
(b) Use your histogram to estimate the number of gardens that are larger than $220 m^2$.

(c) Use your histogram to estimate the median garden size.

Fluency Practice

Frequency Polygons to Cumulative Frequency Graphs

8 different classes took a science test. Can you complete a cumulative frequency graph for each set of results?



Fluency Practice

①

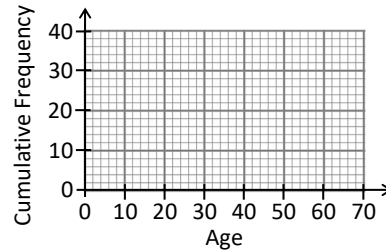
Raw → Grouped

A cinema asked for the ages of people in a screening audience.
Convert the stem & leaf diagram to a cumulative frequency graph.

What groups will you use?

Key 1 | 3 means 13

0	6 6 7 8 9 9
1	2 3 3 3 4
2	1 1 2 5 5 6
3	1 3 3 4 7 7 7 8 9
4	4 4 5 6 6 8 8 9 9
5	1 2 2 2



What is the modal value?

Estimate the median from the graph. Is it accurate?

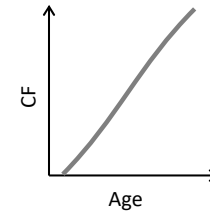
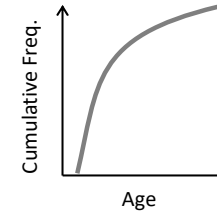
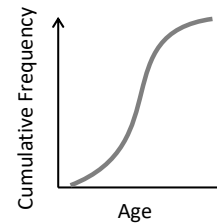
②

Match each stem & leaf diagram with a cumulative frequency graph.

3	2 2 3 4 5 9
4	1 5 5 7 7
5	2 5 5 5
6	3 4 9
7	6

0	2 2 5
1	1 5 5 7
2	2 2 3 4 4 9
3	2 4 4 6 6 6 8 7 9 9
4	1 1 2 3 4 5 5 8
5	2 3 3 7 8
6	7 7 9

2	6 6 7 8 9 9
3	2 3 3 3 4
4	1 1 2 5 5 6
5	1 3 3 4 7 7 7
6	4 4 5 8 8 9 9
7	1 2 2 2 8 9 9
8	1 2 2 2 8



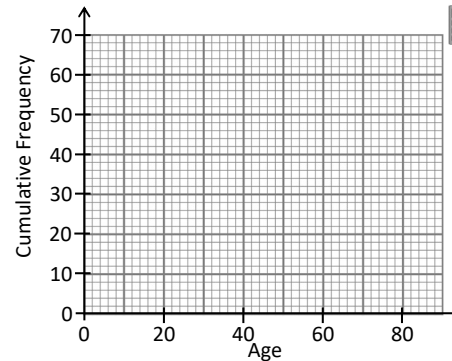
③

To decide who to advertise to, a theatre samples the audiences for both performances on a Saturday.

Plot both sets of data as cumulative frequency curves.

Key 6 | 2 | 3 means 26 for afternoon
23 for evening

afternoon	evening
3 2 2	1 1 7 7 8 9 9
7 6 5 5 5	2 2 2 3 5 5 8 8 8 9
8 8 7 5 4 1 1	3 7 7 8 8 8 8 8 9 9 9 9
8 7 6 5 2 2 1	4 1 2 4 4 5 5 7 8 8
9 8 7 4 3 3 3 2	5 1 1 2 3 6 7 7
8 8 8 5 5 4 4 2 2 1 1 1	6 2 2 2 3 7 8
9 9 9 8 7 7 7 6 6 6	7 1 1 1 2 2 3 4
4 3 3 2 2 2 1	8 2 3 6 6



Use the graph to estimate the **difference** in median age between the audiences.

Are the groups representative of the raw data they contain?



Use this frequency table to estimate a mean age for the evening audience.

Age			
$10 < a \leq 30$			
$30 < a \leq 50$			
$50 < a \leq 70$			
$70 < a \leq 90$			

Would you analyse the data differently if one audience member was actually 20?

Can you think of a quick method to find the mean from the raw data?

Frequency Diagrams with...

30 students have their heights measured to the nearest foot. Draw a **cumulative frequency diagram** for these data.

Height (to nearest foot)	Height (inequality)	Frequency	
4	$\leq h <$	12	
5		14	
6		4	

100 random numbers between 0 and 10 are rounded to the nearest prime. Draw a **histogram** for these data.

Number (to nearest prime)	Number (inequality)	Frequency	
2	$\leq n <$	15	
3		24	
5		24	
7		30	
11		7	

Bounds

The heights of 30 sunflowers are truncated to a whole number of metres. Draw a **histogram** for these data.

Height (truncated)	Height (inequality)	Frequency	
0	$\leq h <$	3	
1		17	
2		10	

The side lengths of 30 cubes are measured to the nearest cm. Draw a **frequency polygon** for the **volumes** of these cubes.

Side Length (to nearest cm)	Volume (inequality)	Frequency	
1	$\leq V <$	11	
2		9	
3		10	