KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2025 Mathematics 2026

Unit 16 Booklet – Part 1

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

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Unit 16 Booklet – Part 2

HGS Maths

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Contents

1 Recurring Decimals

Worked Example	Your Turn
Express as a decimal: a) $\frac{2}{9}$ b) $\frac{2}{11}$ c) $\frac{2}{15}$	Express as a decimal: a) $\frac{8}{9}$ b) $\frac{8}{11}$ c) $\frac{4}{15}$

Activity

For each of the following fractions, use your calculator to convert it to a decimal, then decide whether it is terminating or recurring. Now find the denominator as a product of its prime factors. Can you spot any patterns?

Fraction	Decimal using Calculator	Terminating or Recurring	Denominator as Product of Prime Factors
$\frac{1}{2}$			
$\frac{1}{3}$			
$\frac{1}{4}$			
$\frac{1}{5}$			
$\frac{1}{6}$			
$\frac{1}{7}$			
$\frac{1}{8}$			
$\frac{1}{9}$			
$\frac{1}{10}$			
$\frac{1}{11}$			

Fraction	Decimal using Calculator	Terminating or Recurring	Denominator as Product of Prime Factors
$\frac{1}{12}$			
$\frac{1}{13}$			
$\frac{1}{14}$			
$\frac{1}{15}$			
$\frac{1}{16}$			
$\frac{1}{17}$			
$\frac{1}{18}$	$0.0\dot{5}$	Recurring	$2 \times 3 \times 3$
$\frac{1}{19}$			
$\frac{1}{20}$			
$\frac{1}{21}$			

Activity

	Fraction	Factorised	The Law of Cancellation	Simplest Form	Factors of Denominator	Kind of Decimal
(i)	$\frac{8}{12}$					
(ii)	$\frac{3}{16}$					
(iii)	$\frac{9}{27}$					
(iv)	$\frac{12}{30}$					
(v)	$\frac{7}{32}$					
(vi)	$\frac{15}{21}$					
(vii)	$\frac{3}{10}$					
(viii)	$\frac{3}{18}$					
(ix)	$\frac{6}{33}$					
(x)	$\frac{3}{75}$					

Worked Example	Your Turn
Express as a simplified fraction: $0.\dot{4}$	Express as a simplified fraction: $0.\dot{7}$

Dr Frost 277b

Worked Example	Your Turn
Express as a simplified fraction: $0.\dot{5}\dot{4}$	Express as a simplified fraction: $0.\dot{2}\dot{7}$

Worked Example	Your Turn
Express as a simplified fraction: $0.\dot{2}7\dot{9}$	Express as a simplified fraction: $0.\dot{8}3\dot{7}$

Dr Frost 277c

Worked Example	Your Turn
Express as a simplified fraction: $0.7\dot{8}\dot{9}$	Express as a simplified fraction: $0.5\dot{7}\dot{9}$

Worked Example	Your Turn
Express as a simplified fraction: $3.7\dot{6}5\dot{4}$	Express as a simplified fraction: $7.5\dot{3}0\dot{9}$

Fill in the Gaps

x as recurring decimal	Write out multiples of x	Subtract	x as a fraction
$x = 0.\dot{7}$	$10x = 7.\dot{7} = 7.77777\ldots$ $x = 0.\dot{7} = 0.77777\ldots$	$9x = 7$	$x = \frac{7}{9}$
$x = 0.\dot{2}$	$10x =$ $x =$		
$x = 0.\dot{3}\dot{5}$	$100x = 35.\dot{3}\dot{5} = 35.3535\ldots$ $x = 0.\dot{3}\dot{5} = 0.3535\ldots$	$99x = 35$	
$x = 0.\dot{4}\dot{1}$	$100x =$ $x =$		
$x = 0.\dot{2}\dot{7}$			
$x = 0.\dot{6}1\dot{3}$	$1000x =$		
$x = 0.0\dot{2}$	$100x = 2.\dot{2} = 2.22222\ldots$ $10x =$		
$x = 0.1\dot{4}\dot{3}$			
$x = 0.93\dot{2}$			
$x = 0.9\dot{3}\dot{2}$			
$x = 0.0\dot{0}\dot{5}$			

Worked Example	Your Turn
Calculate the value of $0.\dot{7} - 0.0\dot{5}$ Give your answer as a fraction in its simplest form.	Calculate the value of $0.\dot{6} + 0.\dot{8}\dot{1}$ Give your answer as a fraction in its simplest form.

Worked Example	Your Turn
Write the fraction $0.1\dot{3}\dot{6} \times 0.\dot{5}$ as a fraction in its simplest form	Write the fraction $0.6\dot{8}\dot{1} \times 0.\dot{1}$ as a fraction in its simplest form

Dr Frost 277f

Worked Example	Your Turn
$0.7\dot{a}$ is a recurring decimal, where the digit a is a whole number such that $1 \leq a \leq 9$ Find, in terms of a, the recurring decimal $0.7\dot{a}$ as a fraction. Give your fraction in its simplest form.	$0.19\dot{a}$ is a recurring decimal, where the digit a is a whole number such that $1 \leq a \leq 9$ Find, in terms of a, the recurring decimal $0.19\dot{a}$ as a fraction. Give your fraction in its simplest form.

Extra Notes

2 Parallel and Perpendicular Lines

Parallel Lines

Worked Example	Your Turn
Determine if the following equations are parallel. $y = -\frac{5}{2}x + 1$ $5x + 2y - 4 = 0$	Determine if the following equations are parallel. $2x - 3y + 1 = 0$ $y = \frac{2}{3}x - 4$

Dr Frost 274a, 274b and 274c

Worked Example	Your Turn
a) Write down the equation of a line parallel to $y = 2x - 3$ b) Write down the equation of the line that is parallel to $y = 6x + 1$ and passes through $(0, 8)$	a) Write down the equation of a line parallel to $y = -2x + 3$ b) Write down the equation of the line that is parallel to $y = -6x - 1$ and passes through $(0, -8)$

Worked Example	Your Turn
Write down the equation parallel to $y = 4x + 1$ which passes through $(2, 17)$	Write down the equation parallel to $y = 8x + 5$ which passes through $(2, 26)$

Worked Example	Your Turn
Find the equation of the line parallel to $y = -\frac{1}{3}x - 4$ that passes through $(-2, 5)$	Find the equation of the line parallel to $y = -\frac{1}{2}x - 3$ that passes through $(-2, 5)$

Worked Example	Your Turn
Find the equation of the straight line parallel to $4x - 5y = 20$ and that passes through the point $(-4, -2)$ Give your answer in the form $ax + by = c$ where a, b and c are integers in their lowest terms.	Find the equation of the straight line parallel to $2x - 3y = 6$ and that passes through the point $(-2,4)$ Give your answer in the form $ax + by = c$ where a, b and c are integers in their lowest terms.

Worked Example	Your Turn
The straight line l_1 passes through the points $(1, -5)$ and $(5, -9)$ Find an equation of the line l_2 which is parallel to l_1 and passes through the point $(6, -8)$ Give your answer in the form $y = mx + c$	The straight line l_1 passes through the points $(-2,9)$ and $(8, -31)$ Find an equation of the line l_2 which is parallel to l_1 and passes through the point $(4, -12)$ Give your answer in the form $y = mx + c$

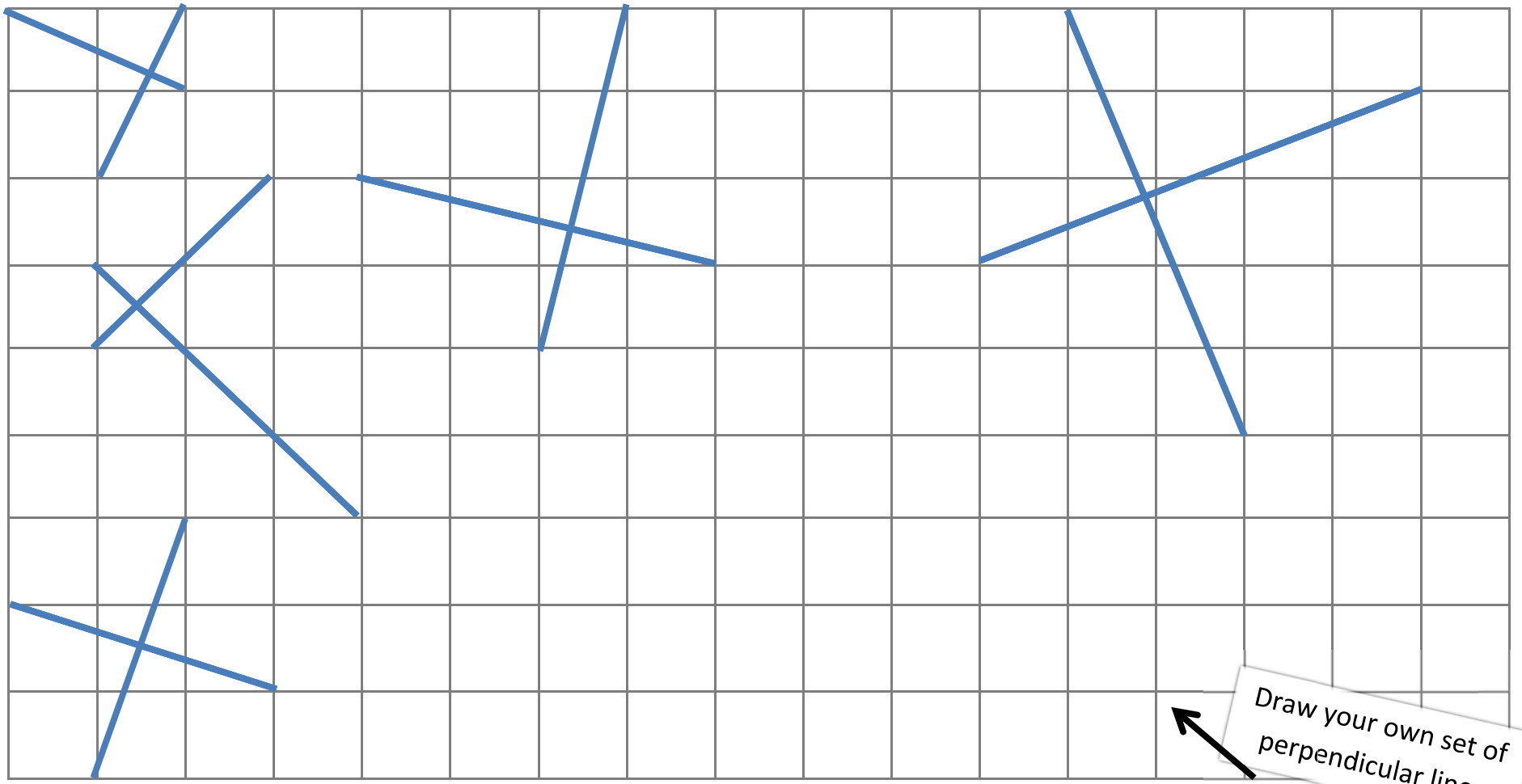
Worked Example	Your Turn
The straight line l_1 passes through the points $(-3, -10)$ and $(5,6)$ Find an equation of the line which is parallel to l_1 and passes through the point $(2, -1)$ Give your answer in the form $ax + by = c$ where a, b and c are integers in their lowest terms.	The straight line l_1 passes through the points $(2, -2)$ and $(6, -4)$ Find an equation of the line which is parallel to l_1 and passes through the point $(-8, 6)$ Give your answer in the form $ax + by = c$ where a, b and c are integers in their lowest terms.

Perpendicular Lines

Worked Example	Your Turn
Write the negative reciprocals of: a) 6 b) $\frac{1}{6}$ c) $-\frac{5}{6}$ d) $1\frac{5}{6}$ e) -1.2	Write the negative reciprocals of: a) -7 b) $\frac{1}{7}$ c) $-\frac{2}{7}$ d) $1\frac{2}{7}$ e) -3.5

Fluency Practice

Calculate the gradient of each of these pairs of perpendicular lines. Simplify your answers. What do you notice?



Worked Example	Your Turn
The line L_1 has equation $-3x + 4y = 8$ The line L_2 has equation $4x + 3y = -5$ Determine whether L_1 and L_2 are perpendicular.	The line L_1 has equation $-2x + y = 6$ The line L_2 has equation $-3x + 2y = 4$ Determine whether L_1 and L_2 are perpendicular.

Dr Frost 445b and 445c

Worked Example	Your Turn
The points P, Q and R have coordinates $(-4, -12)$, $(8, 12)$ and $(16, 6)$ respectively. Determine whether the angle PQR is a right angle.	The points P, Q and R have coordinates $(4, -5)$, $(12, -9)$ and $(8, -17)$ respectively. Determine whether the angle PQR is a right angle.

Worked Example	Your Turn
a) Write down the equation of a line perpendicular to $y = 2x - 3$ b) Write down the equation of the line that is perpendicular to $y = \frac{1}{2}x + 3$ and passes through $(0, -1)$	a) Write down the equation of a line perpendicular to $y = -2x + 3$ b) Write down the equation of the line that is perpendicular to $y = -\frac{1}{2}x + 3$ and passes through $(0, 1)$

Worked Example	Your Turn
Find the equation of the line perpendicular to $y = \frac{1}{2}x - 4$ that passes through $(-2, 5)$	Find the equation of the line perpendicular to $y = -\frac{4}{3}x + 3$ that passes through $(-12, -5)$

Worked Example	Your Turn
Write down the equation perpendicular to $y = 4x + 1$ which passes through $(8, 17)$	Write down the equation perpendicular to $y = 8x + 5$ which passes through $(16, 26)$

Worked Example	Your Turn
Find the equation of the line perpendicular to $3x + 2y = 5$ which passes through the point $(3, 7)$ Give your answer in the form $ax + by = c$ where a, b and c are integers in their lowest terms.	Find the equation of the line perpendicular to $2x + 3y = 5$ which passes through the point $(4, 7)$ Give your answer in the form $ax + by = c$ where a, b and c are integers in their lowest terms.

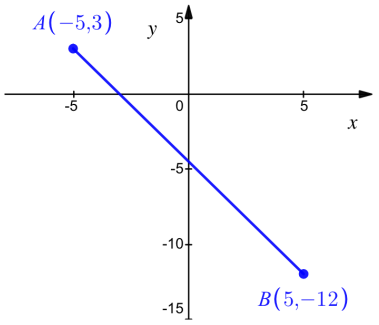
Worked Example	Your Turn
The point A has coordinates $(-8,7)$ and the point B has coordinates $(12,-8)$ The line L_1 passes through A and is perpendicular to AB Find an equation for the line L_1 Give your answer in the form $ax + by = c$ where a, b and c are integers in their lowest terms.	The point A has coordinates $(-8,10)$ and the point B has coordinates $(8,-6)$ The line L_1 passes through A and is perpendicular to AB Find an equation for the line L_1 Give your answer in the form $ax + by = c$ where a, b and c are integers in their lowest terms.

Worked Example	Your Turn
Line L_1 has equation $2x + y = -2$ Line L_2 is perpendicular to L_1 and passes through the points $(2, -6)$ and $(r, -5)$ Find the value of r	Line L_1 has equation $3x + 2y = -2$ Line L_2 is perpendicular to L_1 and passes through the points $(2, -4)$ and $(5, s)$ Find the value of s

Midpoints

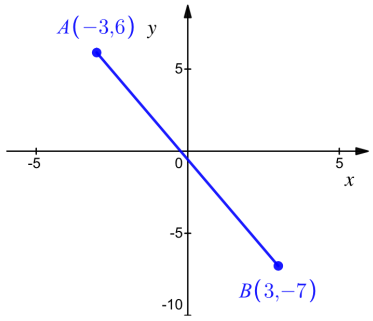
Worked Example

Find the midpoint of the line segment AB where $A(-5, 3)$ and $B(5, -12)$. The line segment is plotted below.



Your Turn

Find the midpoint of the line segment AB where $A(-3, 6)$ and $B(3, -7)$. The line segment is plotted below.



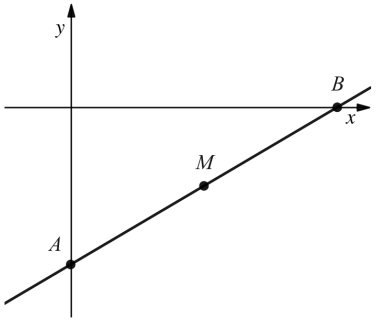
Worked Example	Your Turn
Find the midpoint of the line segment between $(-2,4)$ and $(-9,9)$	Find the midpoint of the line segment between $(2,-4)$ and $(11,8)$

Worked Example	Your Turn
$M(2.5, 0.5)$ is the midpoint of the line segment AB where $A(3, -1)$. Find the coordinates of B	$M(0.5, -2)$ is the midpoint of the line segment AB where $A(2, 1)$. Find the coordinates of B

Worked Example	Your Turn
X is the point $(-10, a)$ Y is the point $(0, -8)$ M is the point $(b, -4)$ M is the midpoint of XY Find the value of a and the value of b	X is the point $(-6, a)$ Y is the point $(2, -2)$ M is the point $(b, 2)$ M is the midpoint of XY Find the value of a and the value of b

Worked Example

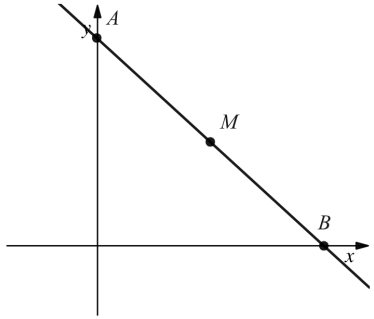
A sketch of the line with equation $3x - 8y = 24$ is shown below. The line passes through the points A, B and M, where M is the midpoint of the line AB.



Calculate the coordinates of M.

Your Turn

A sketch of the line with equation $6x + 5y = 30$ is shown below. The line passes through the points A, B and M, where M is the midpoint of the line AB.



Calculate the coordinates of M.

Fill in the Gaps

Fill in the missing information in the table.

Point A	Point B	**Midpoint** of the line segment AB	**Length** of the line segment AB	**Gradient** of the line segment AB	**Equation** of the line through A and B.
$(1,3)$	$(5,11)$				
$(-3,2)$	$(5,-6)$				
$\left(\frac{-7}{3},\frac{-22}{3}\right)$	$\left(\frac{11}{3},\frac{-4}{3}\right)$				
	$(-7,11)$	$\left(-11,\frac{7}{2}\right)$			
	$(-2,-4)$		$2\sqrt{5}$		$x+2y+10=0$
		$(4,1)$	20	$\frac{3}{4}$	
$(4,1)$			$4\sqrt{13}$		$2x+3y-11=0$

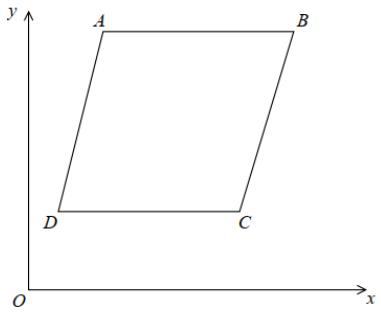
To consider:

- Which of these have multiple possible answers?
- If you were not given either point A or point B, what is the minimum information required to complete the row?

Worked Example	Your Turn
A is the point $(3, 8)$ B is the point $(1, -2)$ C is the midpoint of AB Find the equation of the line perpendicular to AB which passes through C	A is the point $(3, 8)$ B is the point $(1, 4)$ C is the midpoint of AB Find the equation of the line perpendicular to AB which passes through C

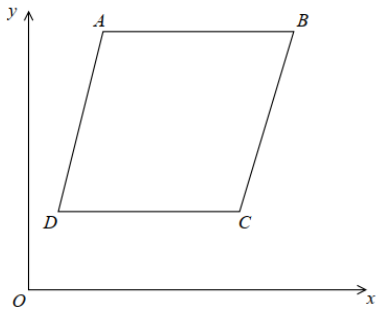
Worked Example

ABCD is a rhombus.
A has coordinates $(5, 10)$
The equation of DB is
$y = \frac{1}{2}x + 5$
Find an equation of diagonal AC



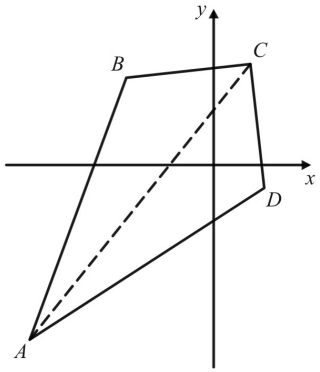
Your Turn

ABCD is a rhombus.
A has coordinates $(5, 11)$
The equation of DB is
$y = \frac{1}{2}x + 6$
Find an equation of diagonal AC



Worked Example

$ABCD$ is a kite with $AB = AD$ and $CB = CD$



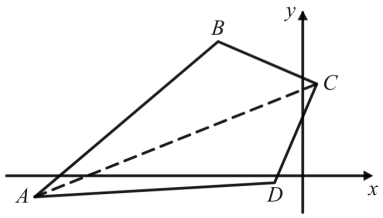
B is the point with coordinates $(-19, 19)$ and D is the point with coordinates $(11, -5)$

Find the equation of the line AC

Give your answer in the form $ax + by + c = 0$
where a, b and c are integers in their lowest terms.

Your Turn

$ABCD$ is a kite with $AB = AD$ and $CB = CD$



B is the point with coordinates $(-12, 19)$ and D is the point with coordinates $(-4, -1)$

Find the equation of the line AC

Give your answer in the form $ax + by + c = 0$
where a, b and c are integers in their lowest terms.

Worked Example	Your Turn
The points $A(5, -7)$, $B(-3, 3)$, $C(7, 11)$ and $D(15, 1)$ are joined to form the quadrilateral $ABCD$ Show that this quadrilateral is a rectangle by finding the gradients of AB, CD, BC and AD	The points $A(-9, 10)$, $B(-13, 18)$, $C(-21, 15)$ and $D(-17, 7)$ are joined to form the quadrilateral $ABCD$ Show that this quadrilateral is a parallelogram by finding the gradients of AB, CD, BC and AD

Fill in the Gaps

Equation	Point on the Line (1)	Point on the Line (2)	Gradient	y intercept	The parallel line that goes through $(2, 5)$	Gradient of all perpendicular lines
$y = 2x + 8$						
$y = 4x - 1$						
	$(1, 5)$	$(3, 11)$				
	$(5, 9)$	$(8, 12)$				
	$(4, 6)$	$(6, 2)$				
	$(4, 3)$		-3			
	$(2, 9)$		6			
	$(-1, 2)$		3			
	$(2, 10)$			$(0, 4)$		
	$(3, 11)$				$y = 5x - 5$	
	$(4, 3)$					-2

Fill in the Gaps

Equation of line	Point on the line (1)	Point on the line (2)	Gradient	y - intercept	x - intercept	Gradient of a perpendicular line
$y = 2x + 1$	$(-2, \square)$	$(2, \square)$				
$y = -1 - x$	$(\square, 1)$	$(\square, -2)$				
	(4, -4)	(4, 3)				
	(-1,5)	(2,-4)				
	(-3, -2)	(-8, -2)				
	(-3, 5)	$(3, \square)$	$\frac{-4}{3}$			
	(4, 0)	$(\square, -6)$	$\frac{3}{4}$			
$2y = 3x - 5$	$(1, \square)$	$(3, \square)$				
$3y = 4x - 7$	$(\square, -1)$	$(\square, 3)$				

Fill in the Gaps

(x_1, y_1)	(x_2, y_2)	Gradient m	Perpendicular Gradient	Midpoint of Line	Length of Line	Equation of Line
$(0, 3)$	$(2, 7)$	$\frac{7-3}{2-0} = 2$	$-\frac{1}{2}$	$(1, 5)$	$\sqrt{2^2 + 4^2} = 4.47$	$y = 2x + 3$
$(0, 2)$	$(4, 14)$		$-\frac{1}{3}$			
$(0, 5)$	$(3, 8)$				$\sqrt{3^2 + 3^2} = 4.24$	
$(2, 1)$	$(0, 9)$					
$(3, 6)$	$(1, 10)$					
$(3, 3)$	$(2, -1)$					
$(3, 7)$	$(6, 8)$					
$(5, 11)$				$(4,9)$		
	$(2, 9)$		1		$\sqrt{8}$	

Ratios

Worked Example	Your Turn
The point M lies on the line segment AB where $A(-1, -3)$ and $B(8, 3)$. Given that $AM : MB = 2 : 1$, find the coordinates of M	The point M lies on the line segment AB where $A(-5, -1)$ and $B(7, 3)$. Given that $AM : MB = 3 : 1$, find the coordinates of M

Worked Example	Your Turn
The points A, B and C lie on a straight line. The coordinates of A are $(5, -2)$ The coordinates of B are $(-1, 3)$ Given that $AB : BC = 1 : 4$, find the coordinates of C	The points P, Q and R lie on a straight line. The coordinates of P are $(-4, 3)$ The coordinates of Q are $(-13, 12)$ Given that $PQ : QR = 3 : 2$, find the coordinates of R

Worked Example	Your Turn
The points X, Y and Z lie on a straight line. The coordinates of X are $(-2, -3)$ The coordinates of Y are $(38, -23)$ $XZ = \frac{3}{7}YZ$ Find the coordinates of Z	The points X, Y and Z lie on a straight line. The coordinates of X are $(8, -1)$ The coordinates of Y are $(20\ -13)$ YZ is 25% of XY Find the coordinates of Z

Worked Example	Your Turn
The points D, E and F lie on a straight line. The coordinates of D are $(0, 8)$ The coordinates of E are $(17.5, -4.5)$ $DF : DE = 2 : 5$ Find the coordinates of F	The points X, Y and Z lie on a straight line. The coordinates of X are $(6, -4)$ The coordinates of Y are $(38, -44)$ $XZ : XY = 3 : 8$ Find the coordinates of Z

Fill in the Gaps

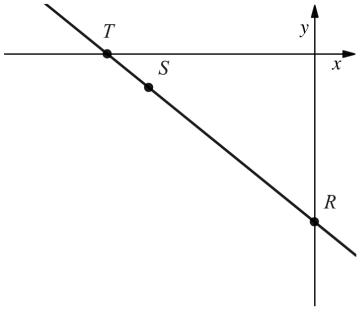
Ratio $AX:XB$	Point A	Point X	Point B
2 : 1	(2,4)		(8,16)
4 : 2	(2,4)		(8,16)
1 : 2	(2,4)		(8,16)
1 : 2	(2,4)	(8,16)	
1 : 2		(2,4)	(8,16)
	(2,4)	(8,16)	(26,52)
	(26,52)	(8,16)	(2,4)
	(13,26)	(4,8)	(1,2)
3 : 1		(4,7)	(0,1)
4 : 1		(4,7)	(0,1)
5 : 1		(4,7)	(0,1)
1 : 1		(4,7)	(0,1)

Point C divides the line segment AB in the given ratio.

Point A	Point B	AC : CB	Point C	Midpoint of AB
$(0, 0)$	$(3, 6)$	$2 : 1$		$(1.5, 3)$
$(1, 1)$	$(7, 4)$	$1 : 2$		
$(10, 5)$	$(0, 0)$	$4 : 1$		
$(0, 0)$	$(10, 5)$	$2 : 3$		
$(-1, 0)$	$(11, 8)$	$3 : 1$		
$(4, 7)$	$(8, -5)$	$1 : 3$		
$(2.3, -5.1)$	$(4.8, 2.4)$	$3 : 2$		
$(9, 0)$	$(-5, -7)$	$5 : 2$		
$(0, 0)$		$3 : 1$		$(2, 4)$
	$(5, 2)$	$1 : 2$		$(3.5, 0.5)$
$(-1, 6)$		$3 : 2$	$(5, 3)$	
$(11, -5)$	$(-3, 2)$		$(5, -2)$	
$(0, 2a)$		$2 : 1$	$(-4a, 4a)$	
$\left(-\frac{9}{10}, \frac{2}{3}\right)$	$\left(\frac{1}{2}, 5\frac{1}{3}\right)$		$\left(-\frac{1}{10}, \frac{10}{3}\right)$	
		$2 : 1$	$(6, -3)$	$(4, -2)$
		$2 : 5$	$\left(\frac{11b}{14}, -2b\right)$	(b, b)

Worked Example

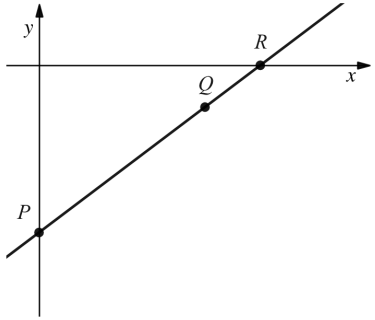
A sketch of $2x + 3y = -30$ is shown below.
The line passes though the points R, S and T, where $RS : ST$ is in the ratio $4 : 1$.



Work out the coordinates of S.

Your Turn

A sketch of $3x - 5y = 60$ is shown below.
The line passes though the points P, Q and R, where $PQ : QR$ is in the ratio $3 : 1$.



Work out the coordinates of Q.

Worked Example	Your Turn
PQR is a straight line. P has coordinates $(a, 1)$ Q has coordinates $(5a, 4)$ R has coordinates $(45, b)$ $PQ : QR$ is $2 : 5$ Find a and b	PQR is a straight line. P has coordinates $(3, b)$ Q has coordinates $(6, 9b)$ R has coordinates $(a, 7)$ $PQ : QR$ is $2 : 3$ Find a and b

Fill in the Gaps

Q	a	b	m	a to m : m to b	a to m : a to b	m is $\frac{?}{?}$ along the line segment ab
1	$(1,5)$	$(13,11)$		$1:1$		
2		$(12,10)$	$(7,8)$		$1:2$	
3	$(2,6)$		$(8,9)$			$\frac{1}{2}$
4	$(2,6)$	$(14,12)$		$1:2$		
5	$(2,6)$	$(14,12)$	$(10,10)$			
6		$(14,12)$	$(9,9)$			$\frac{2}{3}$
7		$(14,12)$	$(9,9)$		$3:4$	
8	$(-6,0)$		$(9,9)$			$\frac{3}{5}$
9	$(-6,0)$	$(14,10)$		$3:2$		
10	$(-6,0)$	$(26,16)$	$(6,6)$			
11	$(-6,0)$		$(-6,-6)$			$\frac{3}{8}$
12	$(-12,0)$	$(-12,-32)$	$(-12,-12)$			

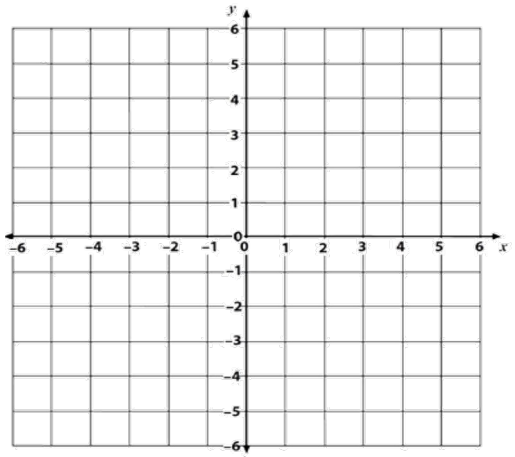
Extra Notes

3 Graphical Inequalities

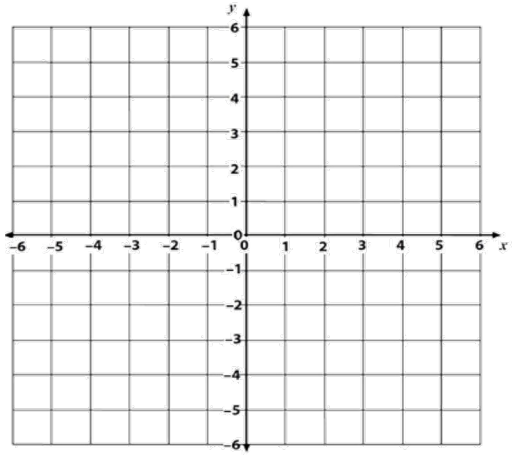
Worked Example

Shade the region which satisfies the inequality:

a) $x > 3$



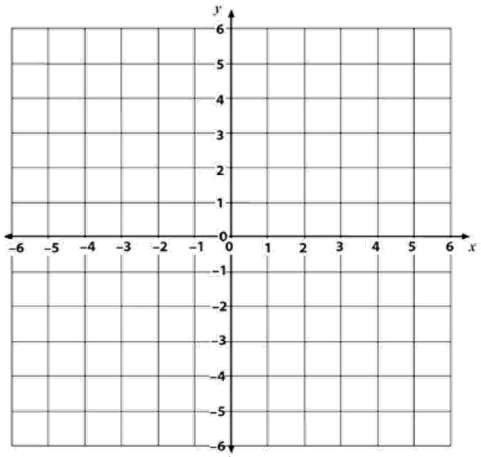
b) $y \leq -2$



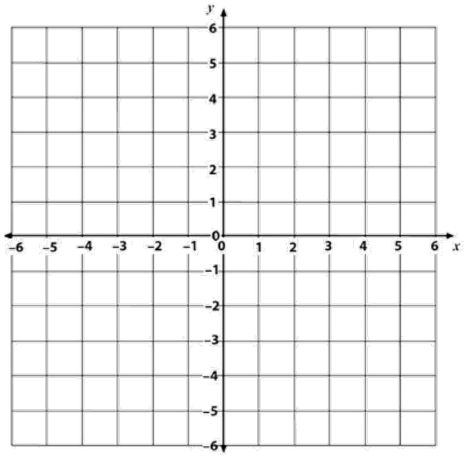
Your Turn

Shade the region which satisfies the inequality:

a) $x < 5$



b) $y \geq -4$



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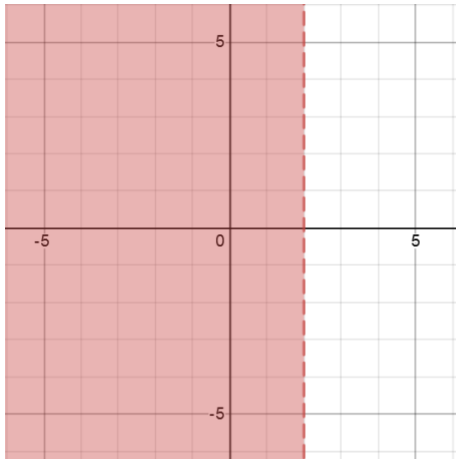
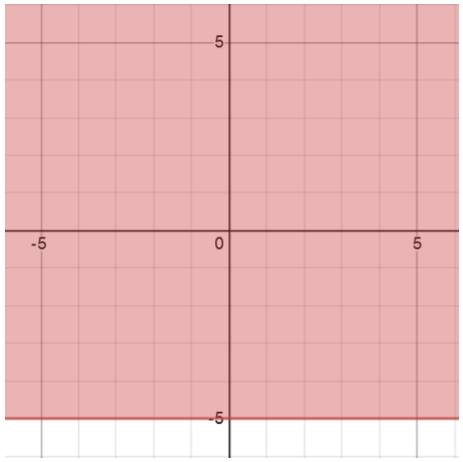
Worked Example

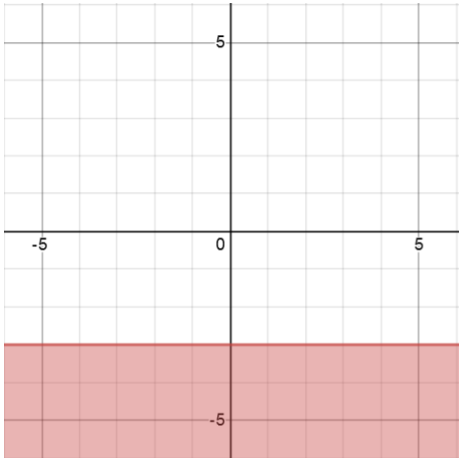
Write the inequality that defines the red region:

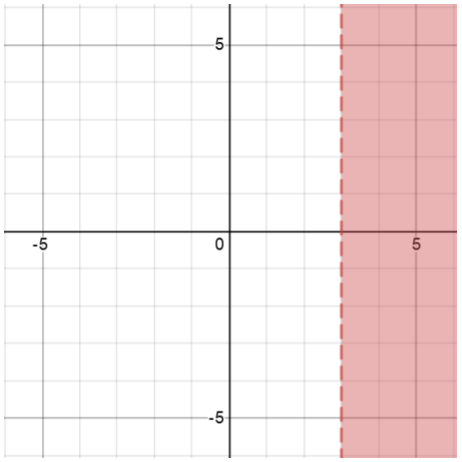




Your Turn

Write the inequality that defines the red region:

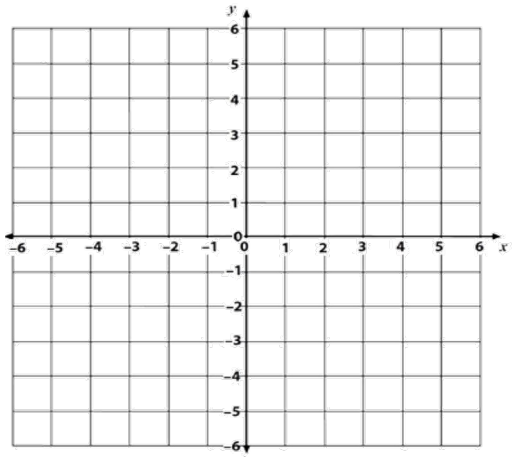




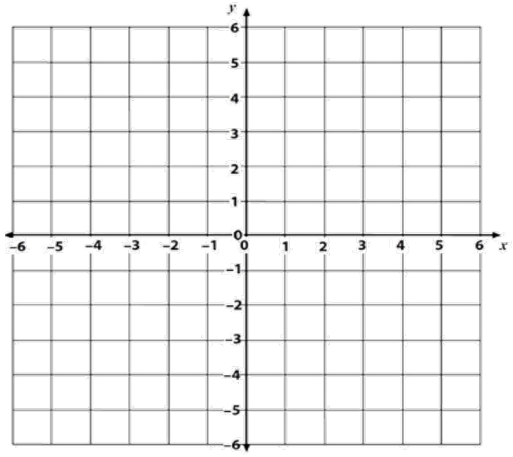
Worked Example

Shade the region which satisfies the inequality:

a) $-2 \leq x < 5$



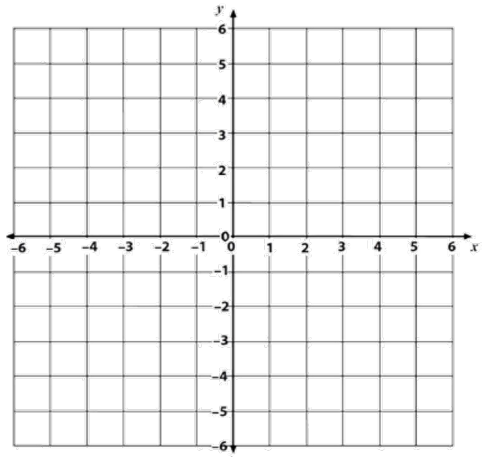
b) $-2 \leq y < 5$



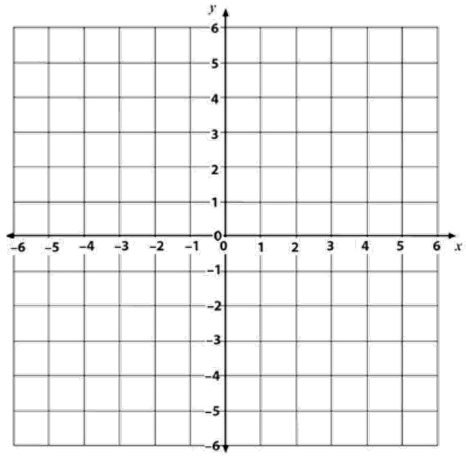
Your Turn

Shade the region which satisfies the inequality:

a) $-4 < x \leq 3$

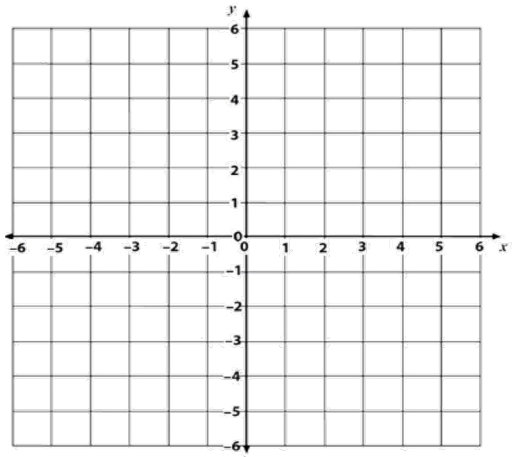


b) $-4 < y \leq 3$



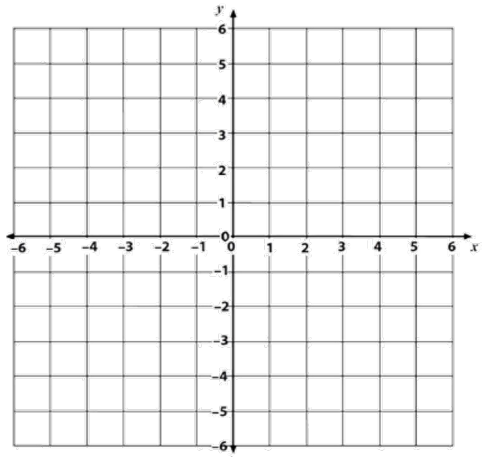
Worked Example

Shade the region which satisfies the inequality:
$x \geq 2$ and $y < 1$



Your Turn

Shade the region which satisfies the inequality:
$x \leq 3$ and $y > 4$



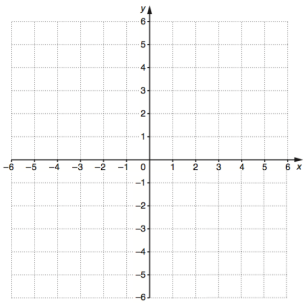
*There are templates for questions 1, 3, and 5 at the end of this exercise

Question 1: On copies of the grid below, clearly indicate the region that satisfies each inequality.

(a) $x > 2$ (b) $x < 4$ (c) $x \leq -1$ (d) $x > 0$

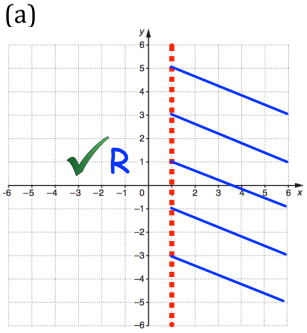
(e) $x \geq -3$ (f) $y < 1$ (g) $y \geq -2$ (h) $y \leq 4$

(i) $y > 2$ (j) $x \geq 3$ (k) $y < 0$ (k) $x < -5$

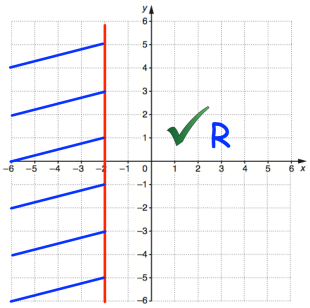


Question 2: Write down the inequality represented in each diagram below.

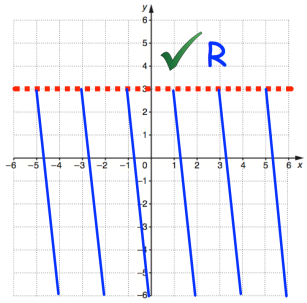
(a)



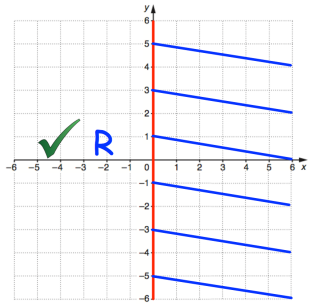
(b)



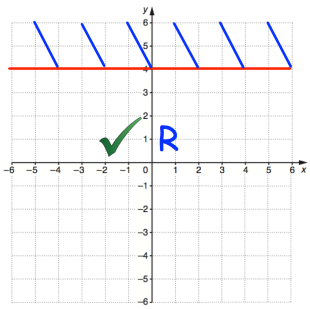
(c)



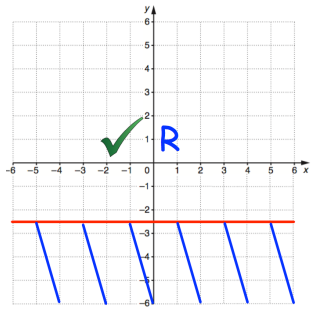
(d)



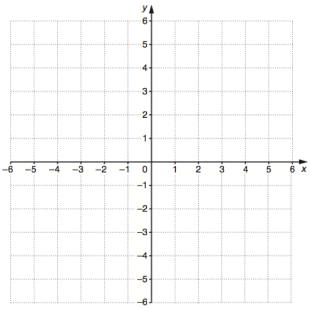
(e)



(f)



Question 3: On copies of the grid below, clearly indicate the region that satisfies each inequality.



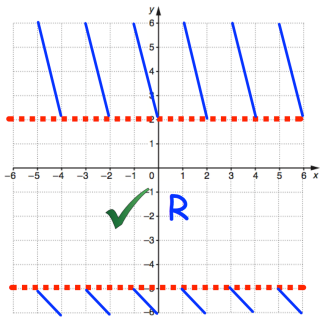
(a) $-4 < x < 1$ (b) $0 \le x \le 5$ (c) $-3 \le x < 3$

(d) $-5 \le y \le -2$ (e) $-1 < y < 4$ (f) $-1 < y \le 2.5$

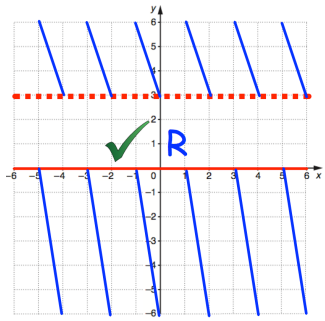
(g) $-2 < x \le 3$ (h) $-4 \le y \le 2$ (i) $-2 \le y < 2$

Question 4: Write down the inequality represented in each diagram below.

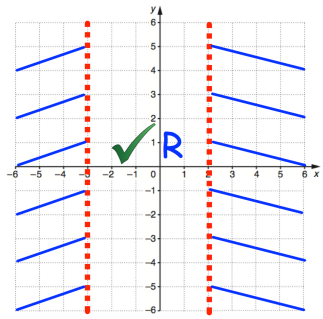
(a)



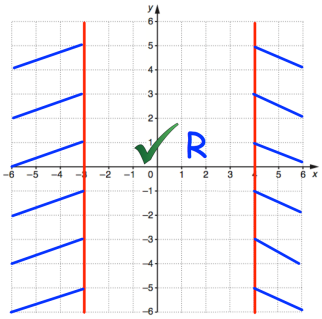
(b)



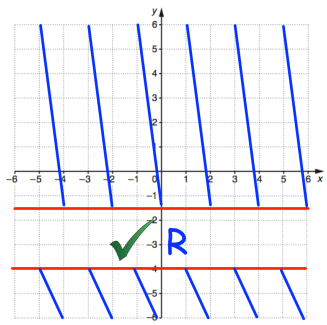
(c)



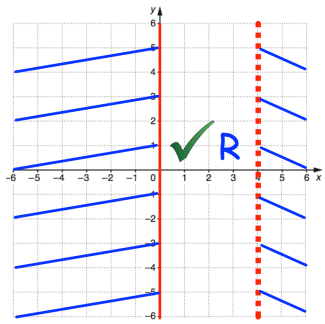
(d)



(e)



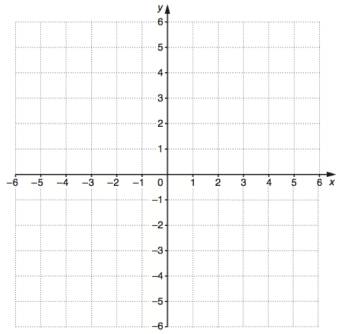
(f)



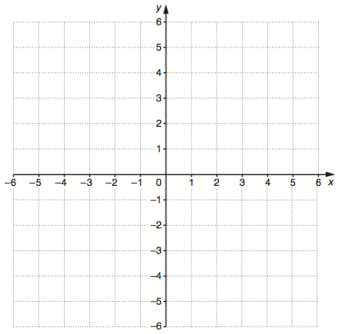
Question 5: On a grid, clearly indicate the region that satisfies the following inequalities.

(a) $-2 < x < 3$ and $y \ge -1$ (b) $-5 \le y \le 1$ and $x < 3$ (c) $1 < x \le 3$ and $-2 \le y < 0$

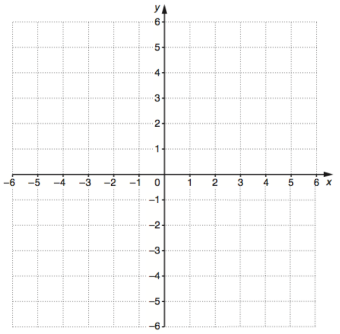
Question 1(a)



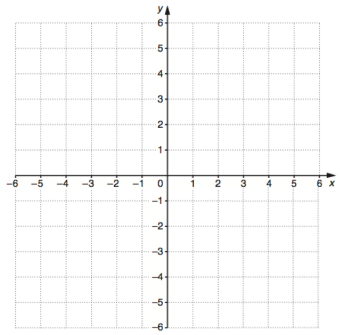
1(b)



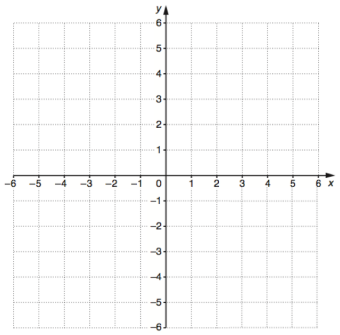
1(c)



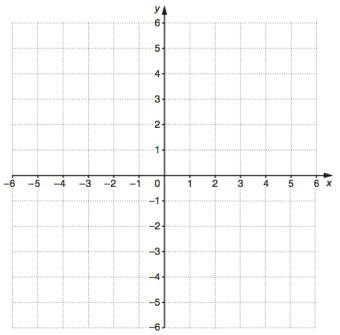
Question 1(d)



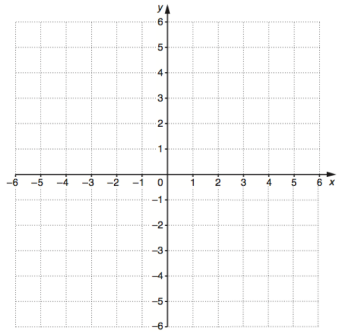
1(e)



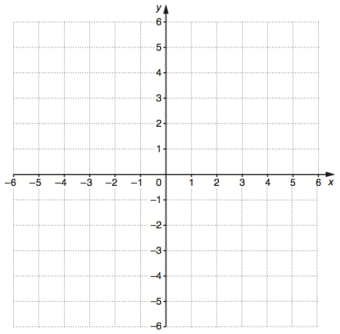
1(f)



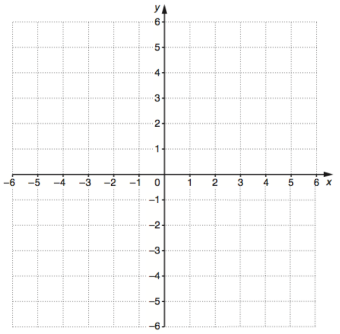
Question 1(g)



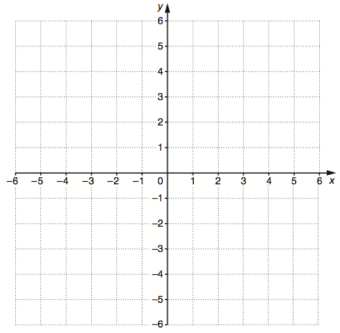
1(h)



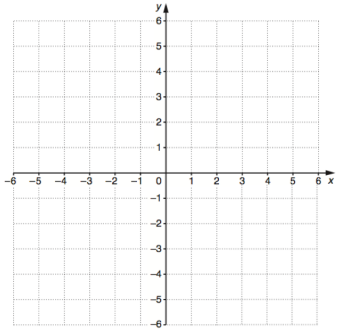
1(i)



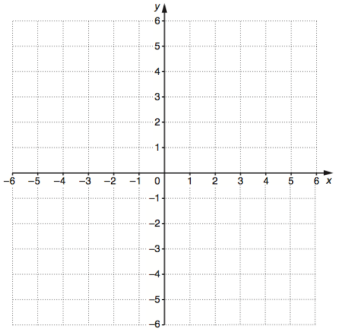
Question 1(j)



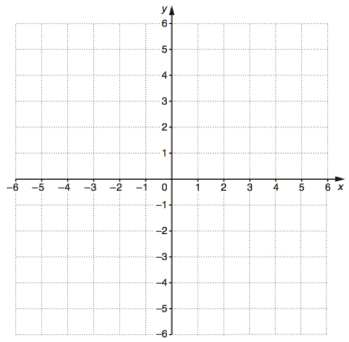
1(k)



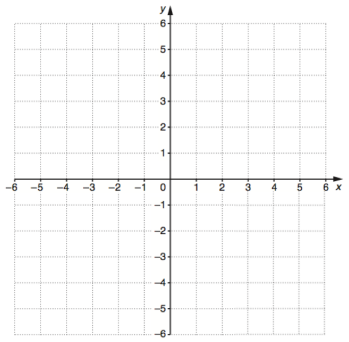
1(l)



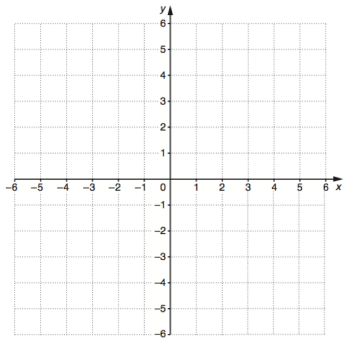
Question 3(a)



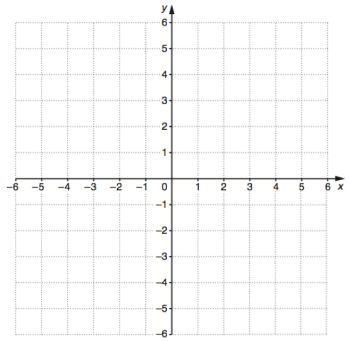
3(b)



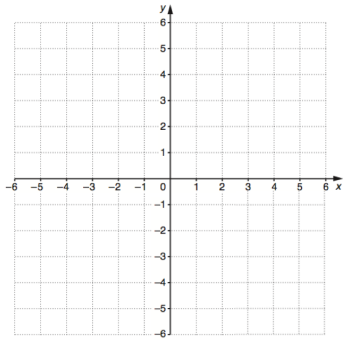
3(c)



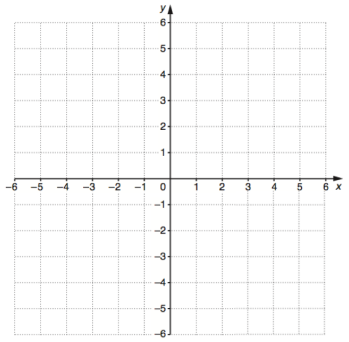
3(d)



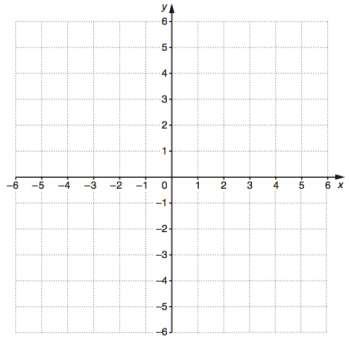
3(e)



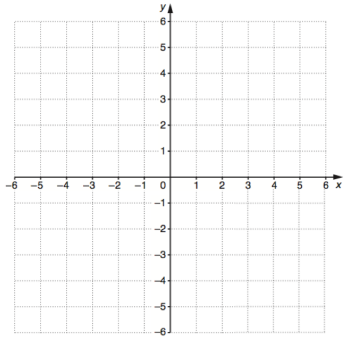
3(f)



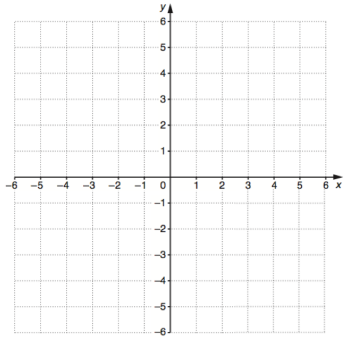
3(g)



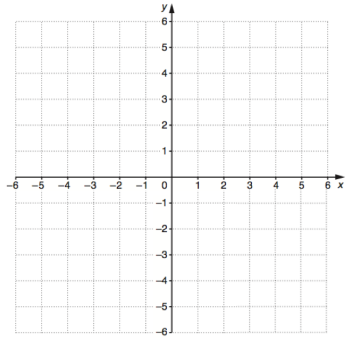
3(h)



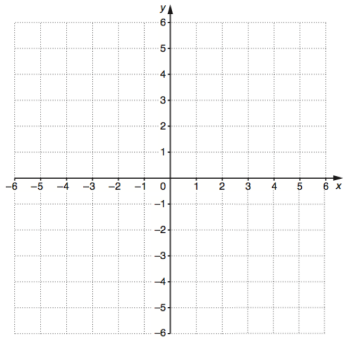
3(i)



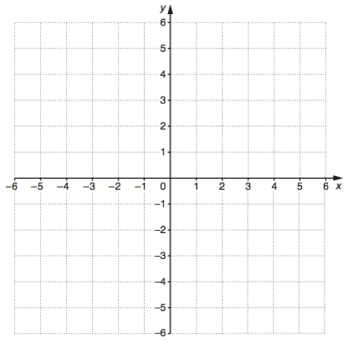
5(a)



5(b)



5(c)



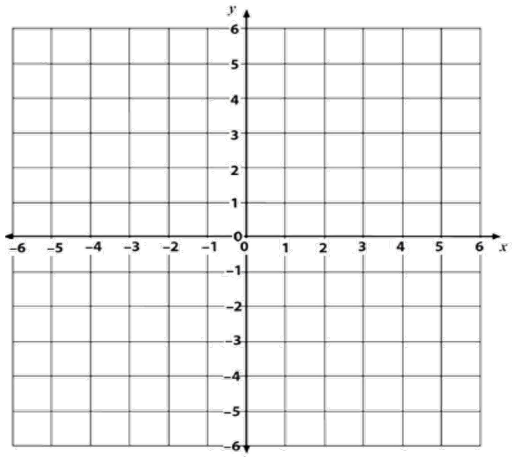
Worked Example	Your Turn
Select the point(s) that satisfy the inequality: $y \leq 2x - 1$ where x and y are both integers. • $(3, -2)$ • $(-1, -5)$ • $(-2, 0)$ • $(-4, -5)$ • $(0, -2)$	Select the point(s) that satisfy the inequality: $y > -2x + 1$ where x and y are both integers. • $(0, -4)$ • $(-2, 5)$ • $(3, 4)$ • $(-1, 4)$ • $(-4, -1)$

Worked Example	Your Turn
Select the point(s) that satisfy the inequality: $-3 < 2x + y < -1$ where x and y are both integers. • $(0, 1)$ • $(-6, -2)$ • $(-2, 5)$ • $(3, 6)$ • $(1, -4)$	Select the point(s) that satisfy the inequality: $-3 \leq -2x + y < 0$ where x and y are both integers. • $(0, -1)$ • $(-2, 4)$ • $(-1, -3)$ • $(3, 0)$ • $(-6, -2)$

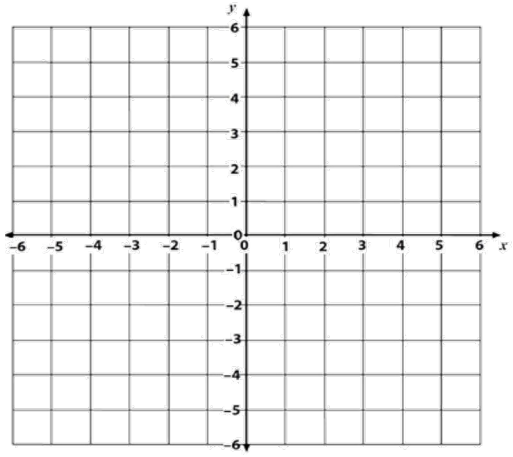
Worked Example

Shade the region which satisfies the inequality:

$y > 2x + 3$



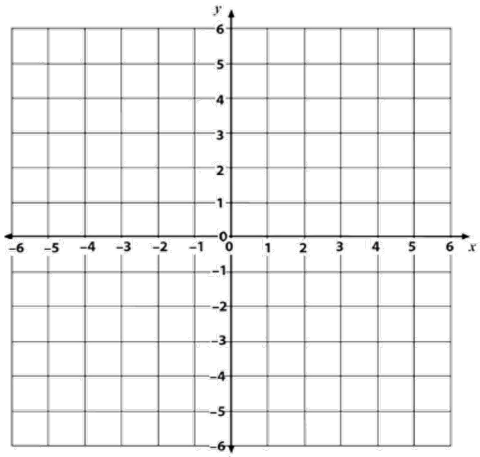
$y < -2x + 3$



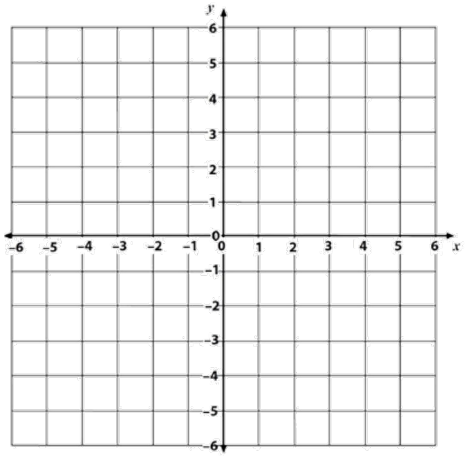
Your Turn

Shade the region which satisfies the inequality:

$y < 4x - 1$

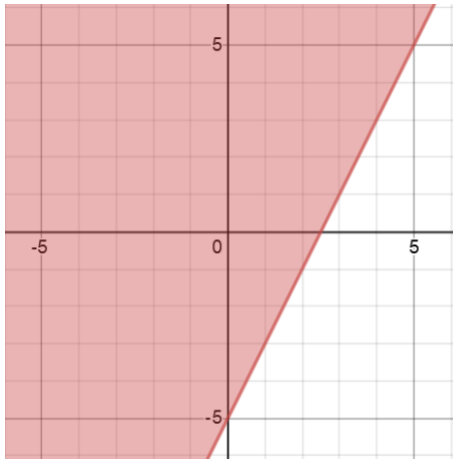


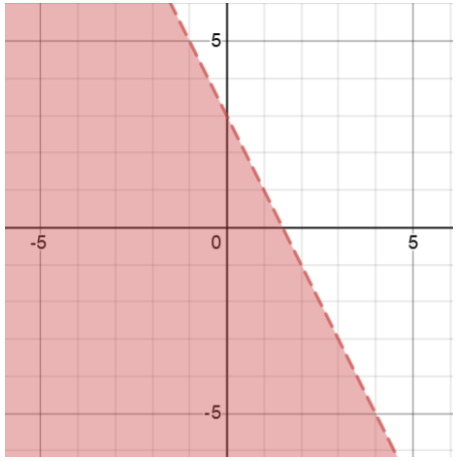
$y > -4x - 1$



Worked Example

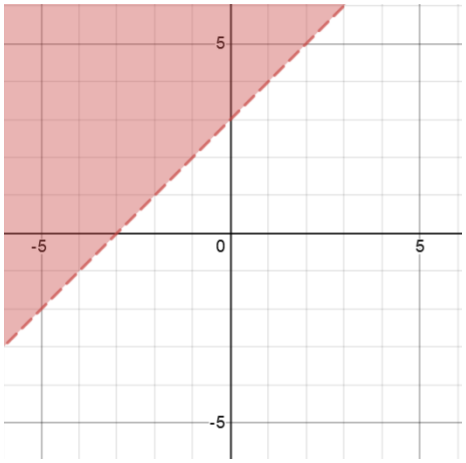
Write the inequality that defines the red region:

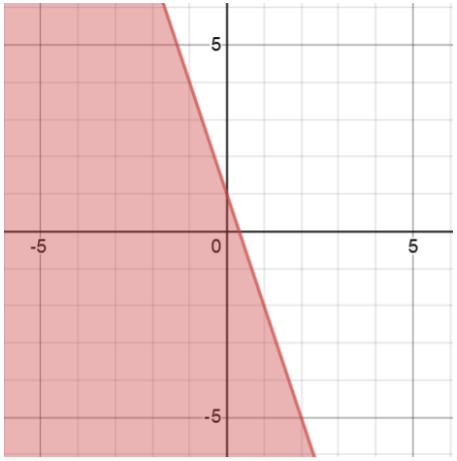




Your Turn

Write the inequality that defines the red region:

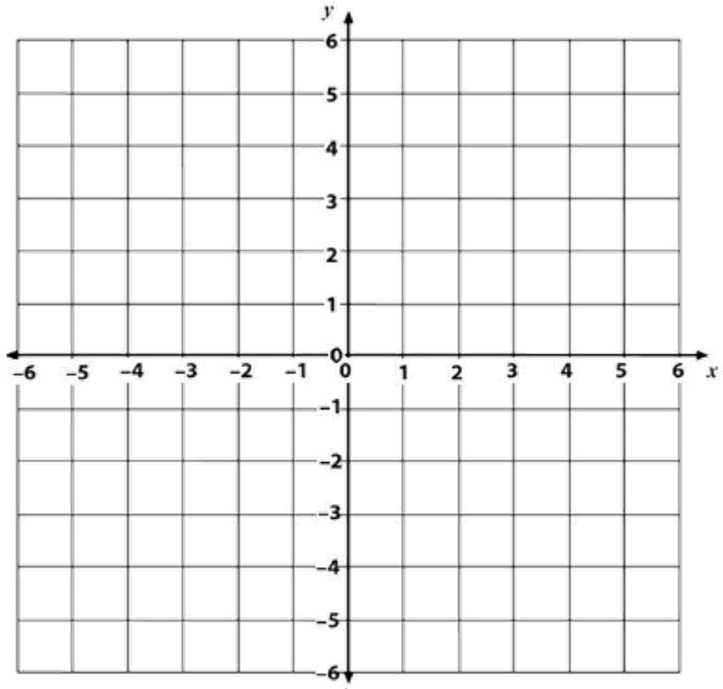




Worked Example

Shade the region which satisfies the inequalities. Label it R.

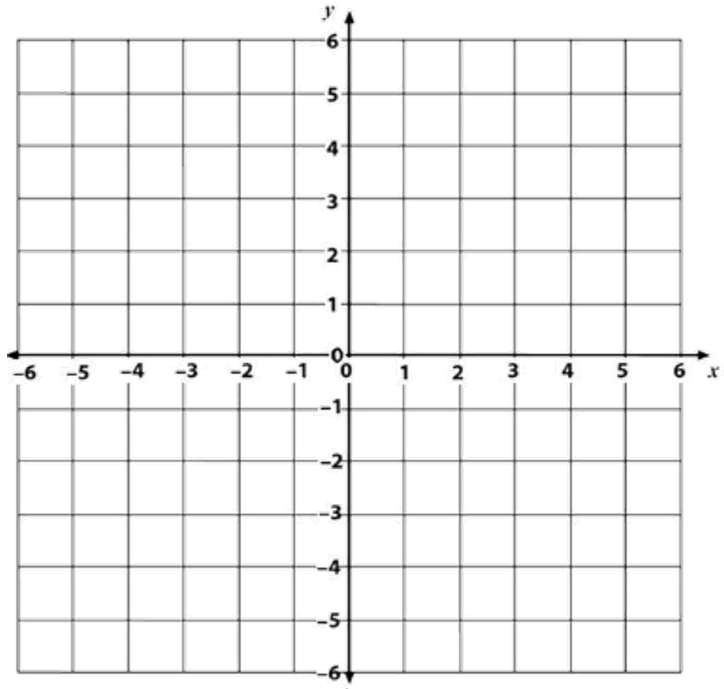
$x \leq 3, y > 1$ and $y \geq x + 3$



Your Turn

Shade the region which satisfies the inequalities. Label it R.

$x < 4, y \geq 3, y \geq x + 2$

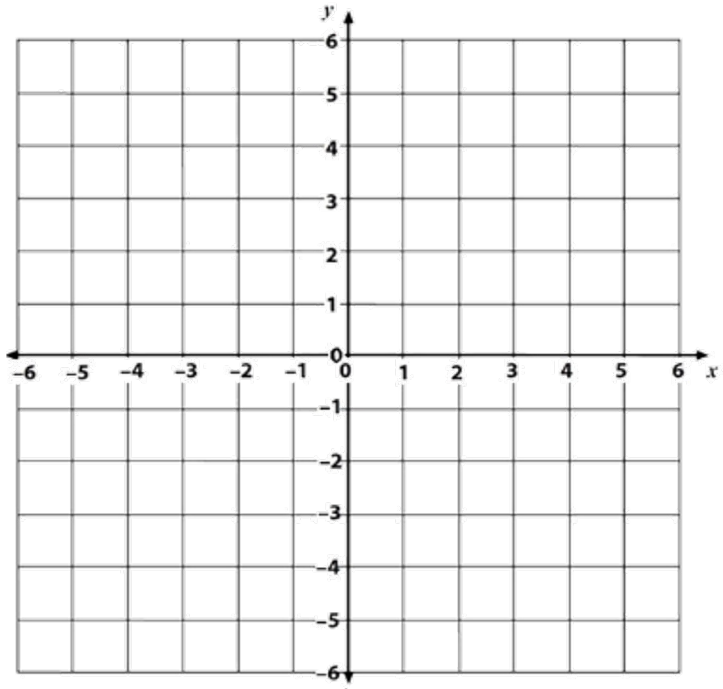


Dr Frost 456e and 456d

Worked Example

Shade the region which satisfies the inequalities. Label it R.

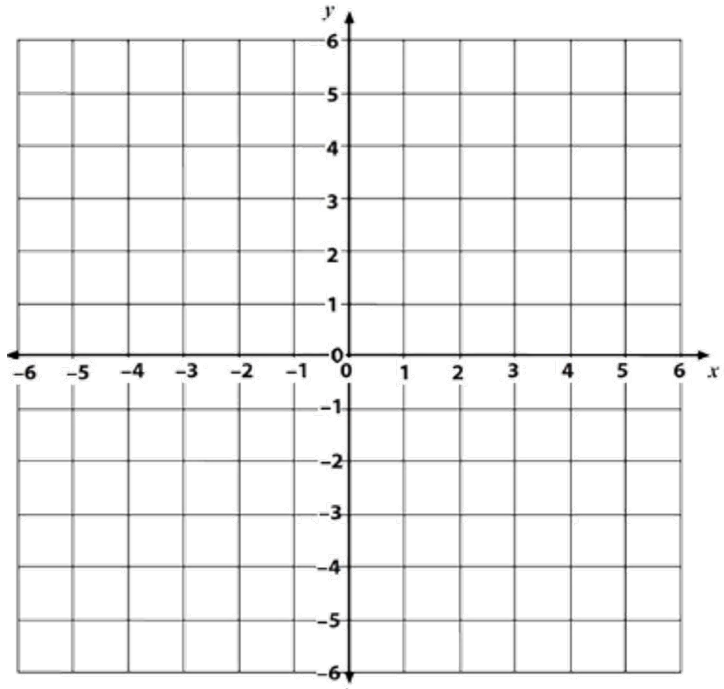
$x \geq 2, y > -1$ and $x + y \leq 5$



Your Turn

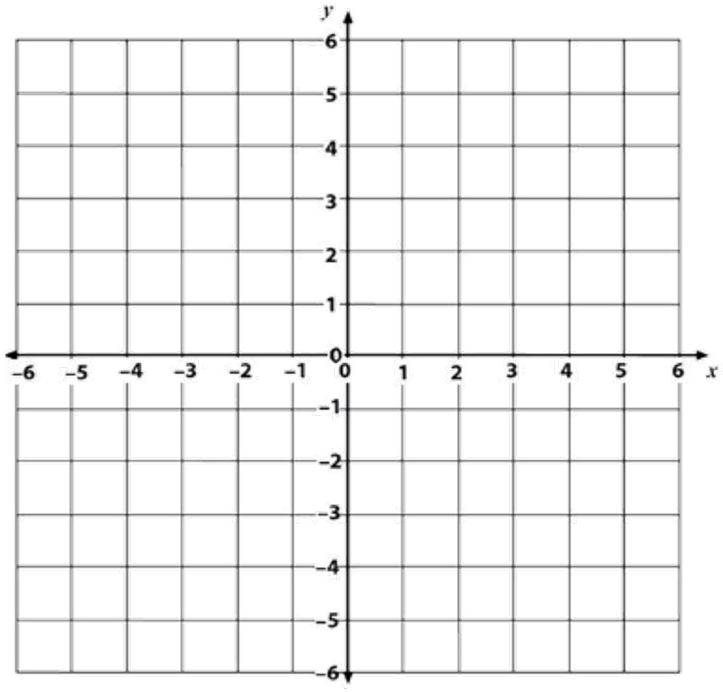
Shade the region which satisfies the inequalities. Label it R.

$x \geq 2, y > 1$ and $x + y \leq 6$



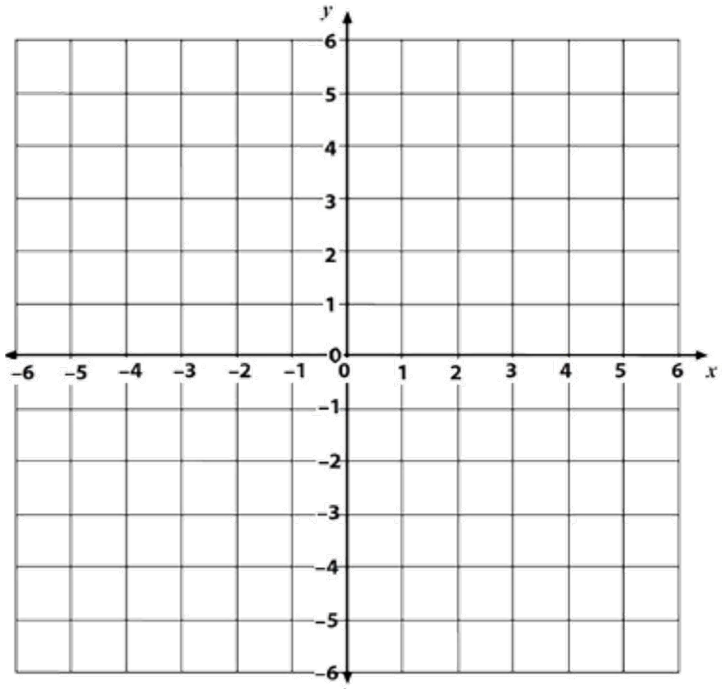
Worked Example

Plot the 4 points that satisfy the inequalities:
$x \leq 1, y \geq -2, y \leq 2x - 2$
where x and y are both integers.



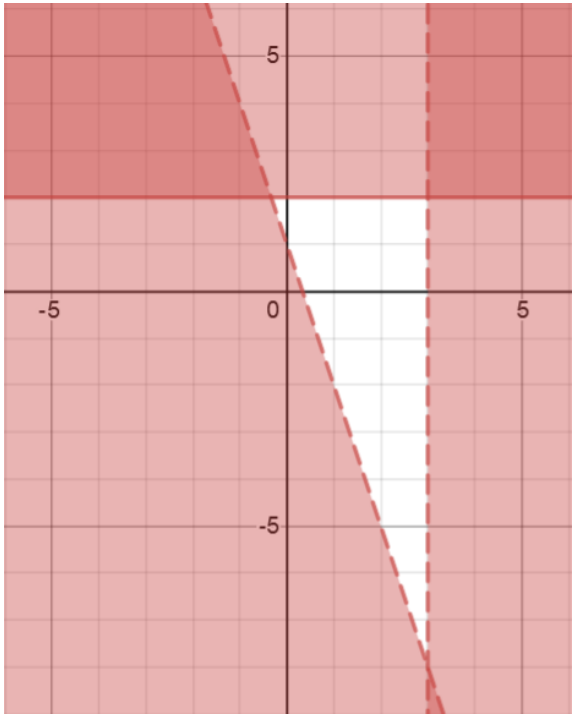
Your Turn

Plot the 4 points that satisfy the inequalities:
$x \leq 1, y \leq 3, y \geq -2x + 3$
where x and y are both integers.



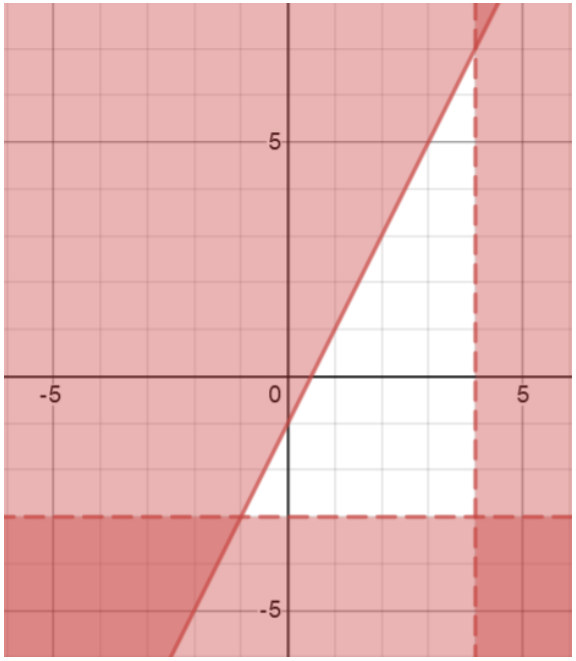
Worked Example

Write the inequalities that define the unshaded region:



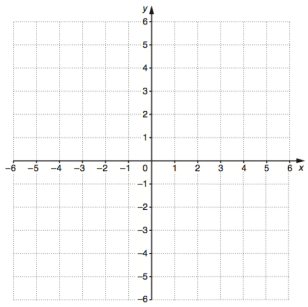
Your Turn

Write the inequalities that define the unshaded region:



*There are templates for the questions workout 1, 2, 4 and apply 4, 5 at the end of exercise

Question 1: On copies of the grid below, clearly indicate the region that satisfies each inequality.

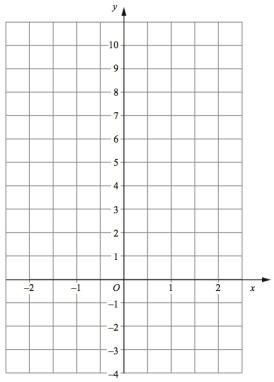


(a) $y < x + 1$ (b) $y \leq 2x + 2$ (c) $y > 3x - 1$

(d) $y \geq x + 3$ (e) $y > 2x$ (f) $y \leq 4x$

(g) $y < -2x + 1$ (h) $y \geq \frac{1}{2}x + 2$ (i) $x + y < 4$

Question 2: On copies of the grid below, clearly indicate the region that satisfies each inequality.



(a) $y > 3x + 4$ (b) $y \geq 5x - 1$

(c) $y \leq 4x + 1$ (d) $y < -2x + 5$

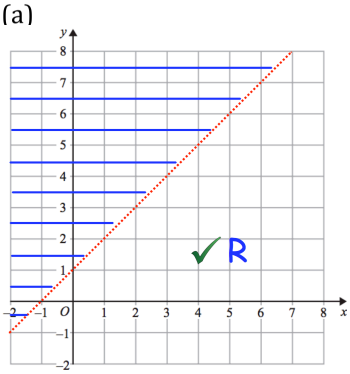
(e) $x + y < 2$ (f) $y > -x - 2$

(g) $y \geq 5 - 2x$ (h) $x + y \geq 7$

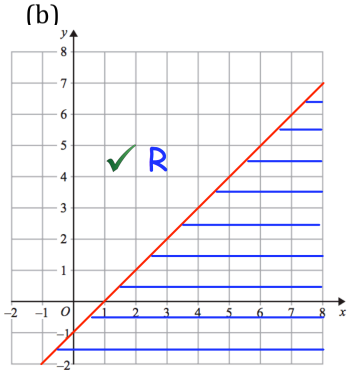
(i) $3x + y > 3$ (j) $5x + 2y > 4$

Question 3: Write down the inequality represented in each diagram below.

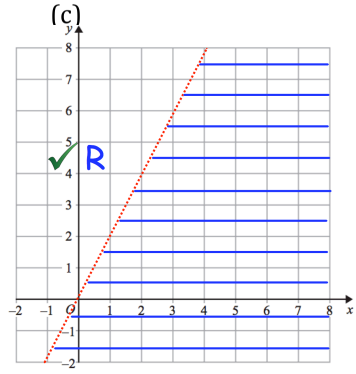
(a)



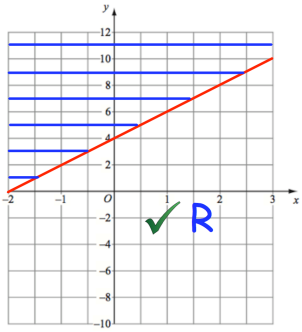
(b)



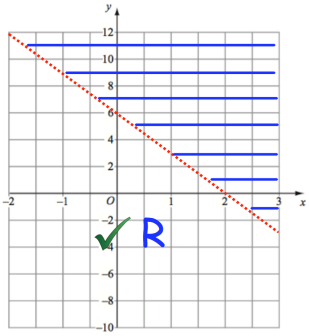
(c)



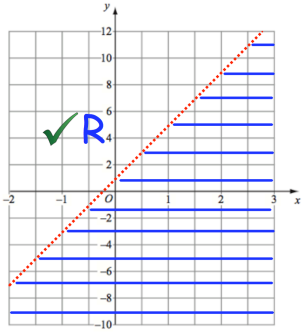
(d)



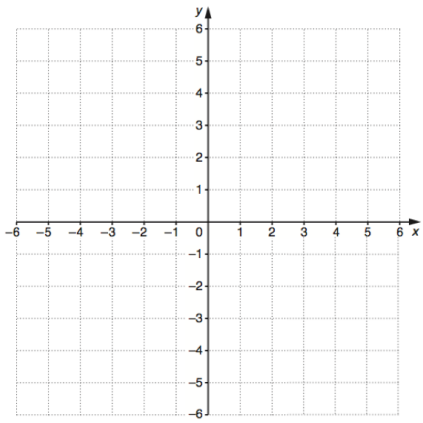
(e)



(f)



Question 4: On copies of the grid below, clearly indicate the region that satisfies the following inequalities.



(a) $y > x - 1$, $x \geq -2$ and $y < 2$

(b) $y \leq 2x$, $x \leq 2$ and $y > -4$

(c) $y \leq -2x + 2$, $x \geq 0$ and $y > x - 4$

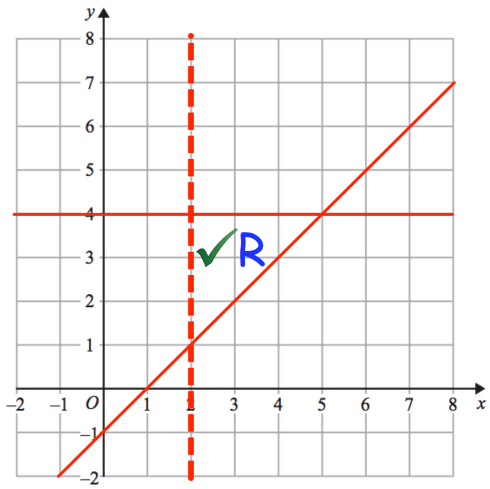
(d) $x + y < 3$, $-2 \leq x < 3$ and $y \geq 0$

(e) $y \leq 5x - 4$, $y > x - 4$ and $y \leq -\frac{1}{2}x + 2$

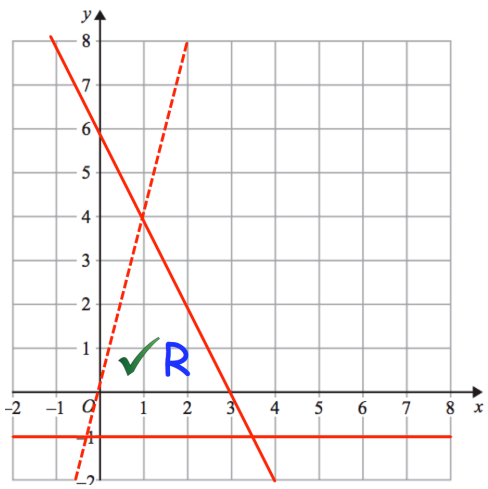
(f) $y \leq -2x + 4$, $y < 2x - 6$ and $-4 < y < -3$

Question 5: State the inequalities that the region labelled R satisfies.

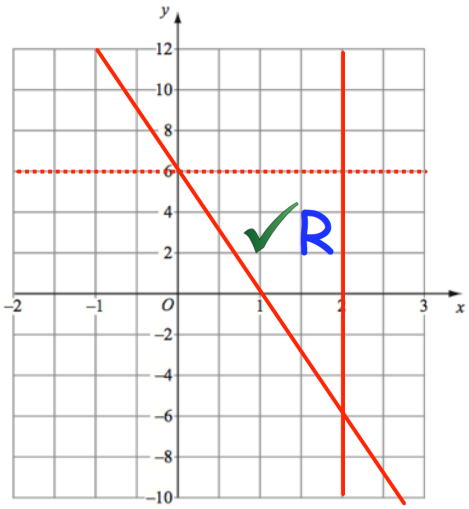
(a)



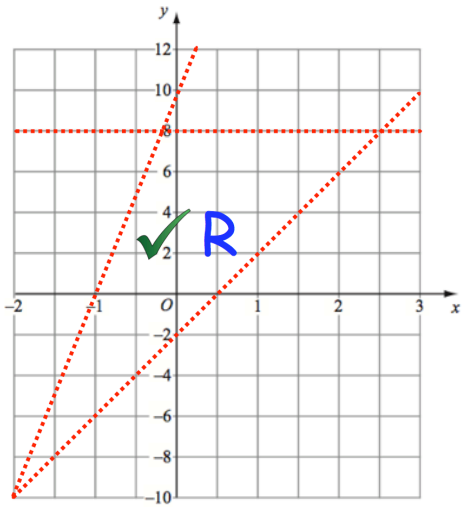
(b)



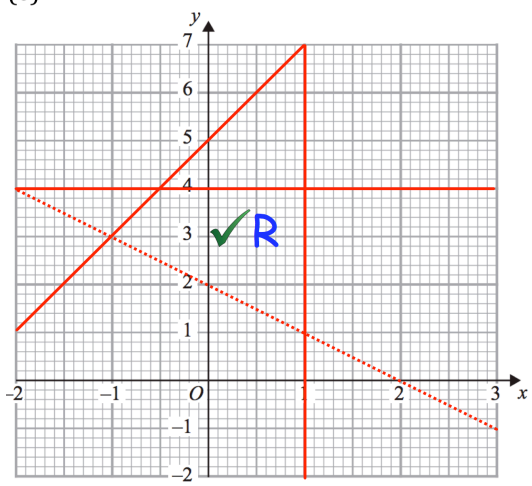
(c)



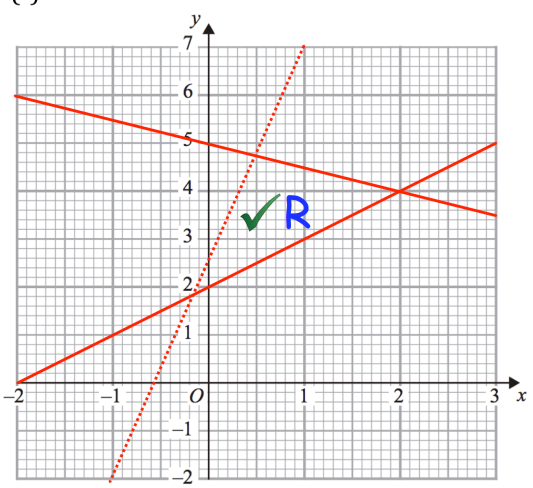
(d)



(e)

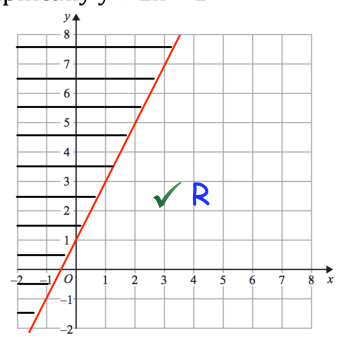


(f)

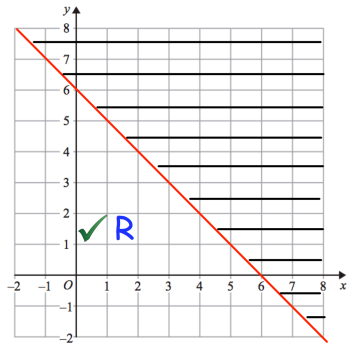


Apply

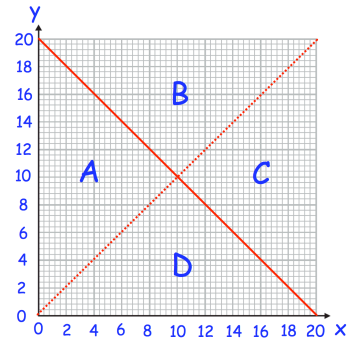
Question 1: Taylor has been asked to represent graphically $y \geq 2x + 1$
Can you spot her mistake?



Question 2: Conor has been asked to represent graphically $x + y < 6$
Can you spot his mistake?



Question 3: At a fitness class, the maximum number of people who can attend is 20.
There are more men than women that attend the fitness class.
y = number of men that attend the fitness class.
x = number of women that attend the fitness class.
Which region A, B, C or D represents the information above?

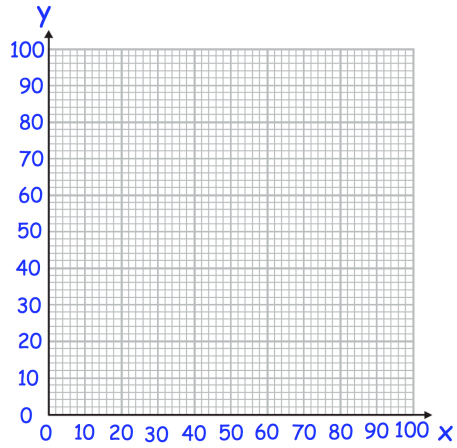


Question 4: A greengrocer sells apples and oranges.

One morning day he sells
up to 50 apples
up to 60 oranges
no more than a total of 90 pieces of fruit.

Let x be the number of apples sold.
Let y be the number of orange sold.

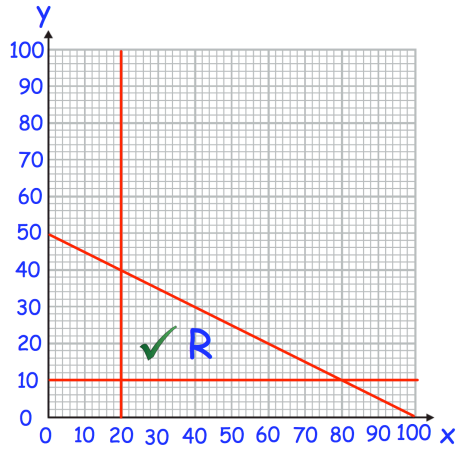
Show the region that satisfies these inequalities



Fluency Practice

Question 5: The region below shows information about the number first class passengers and the number of economy passengers on a flight.

x = number of economy passengers and y = number of first class passengers



(a) Can 15 first class and 60 economy passengers be on the flight?

(b) Can 30 economy and 40 first class passenger be on the flight?

The profit made by the airline for each economy passenger is £90 and for each first class passenger is £200.

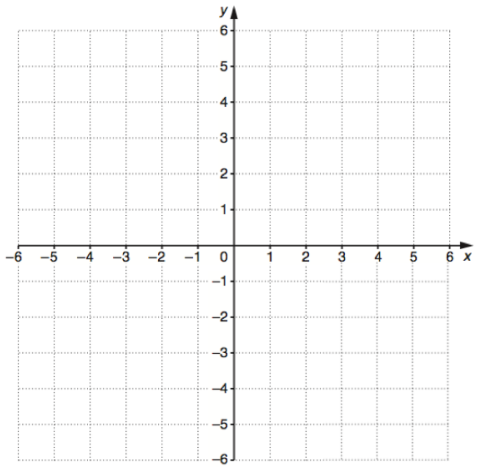
(c) What is the maximum profit the airline can make on one flight?

Question 6: A football stadium holds a maximum of 1000 fans.
Adult tickets cost £5 each and child tickets cost £2 each.
The football club needs to raise at least £3000 to cover costs.
The football club aims to sell at least one child ticket for two adult tickets sold
Let x = number of child tickets sold and y = number of adult tickets sold

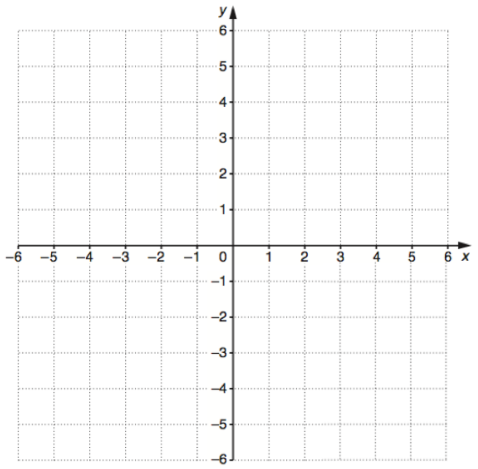
Explain why: (a) $x + y \leq 1000$ (b) $2x + 5y \geq 3000$ (c) $y \leq 2x$

(d) Represent this information on a graph.

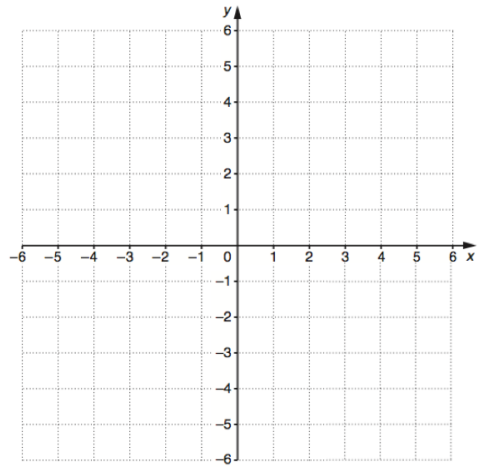
1(a)



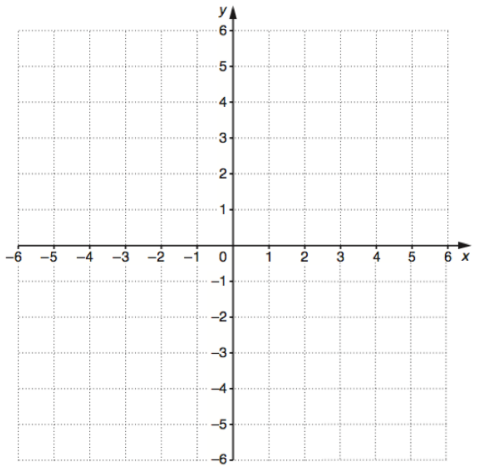
1(b)



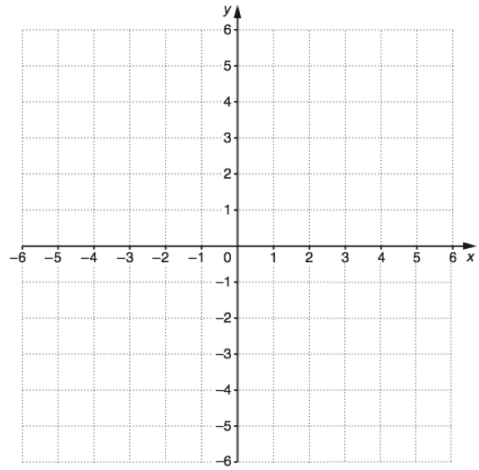
1(c)



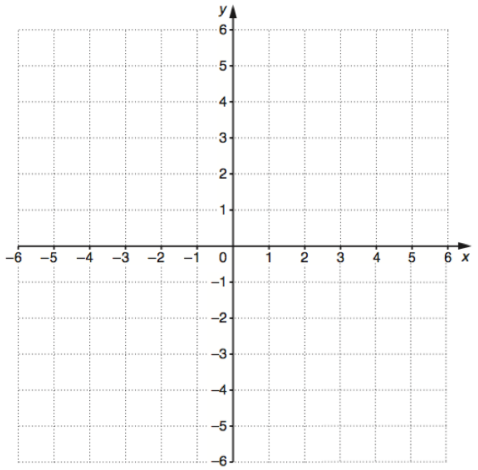
1(d)



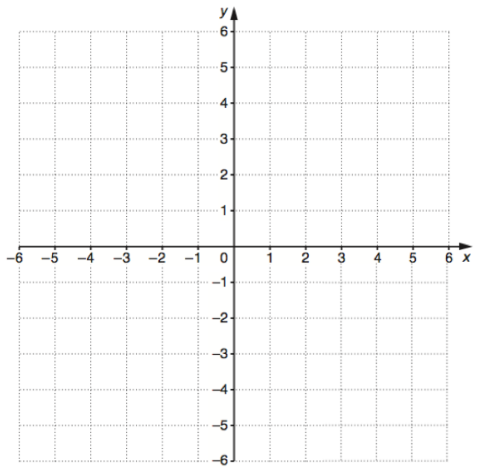
1(e)



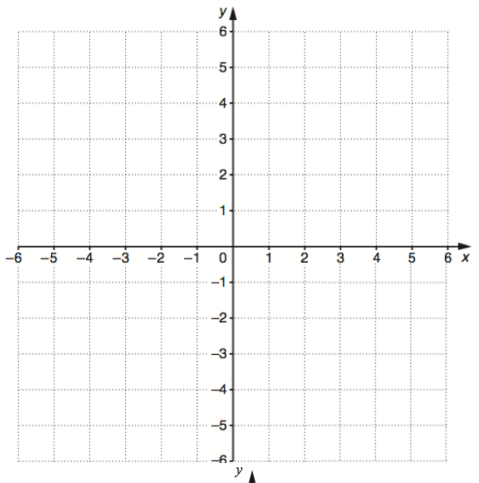
1(f)



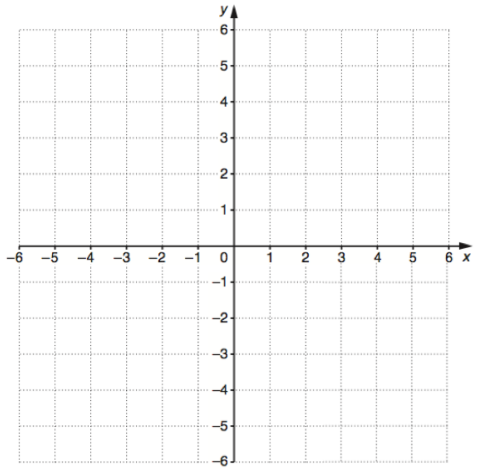
1(g)



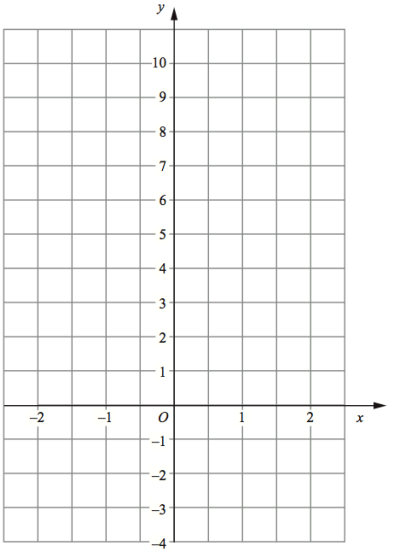
1(h)



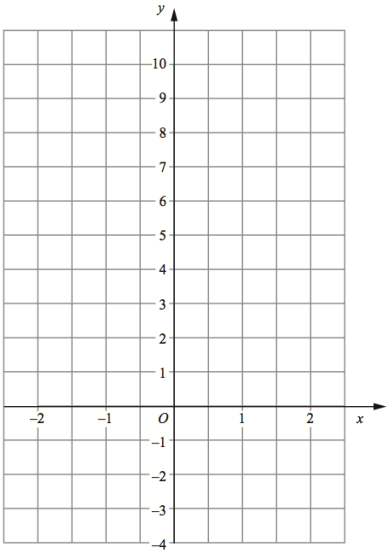
1(i)



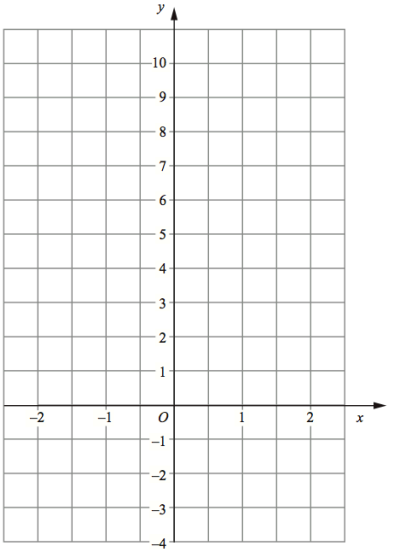
2(a)



2(b)

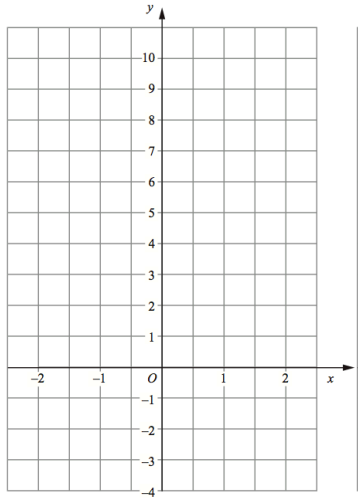


2(c)

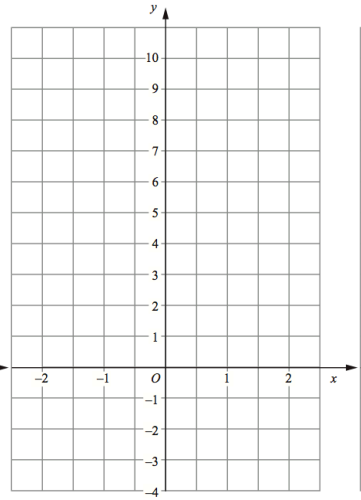


Templates

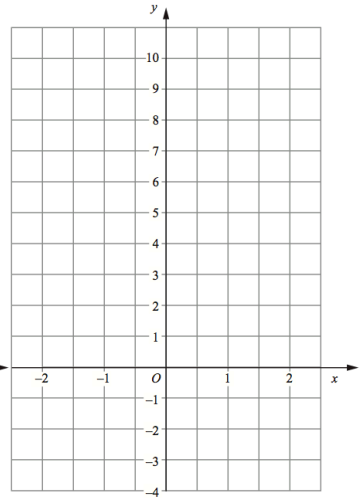
2(d)



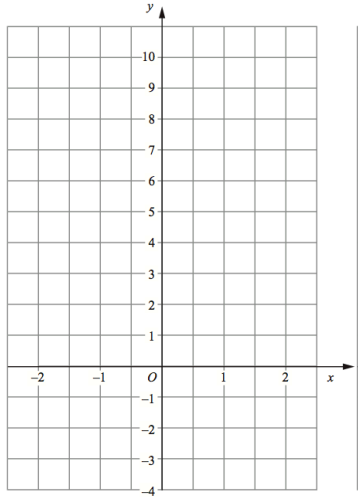
2(e)



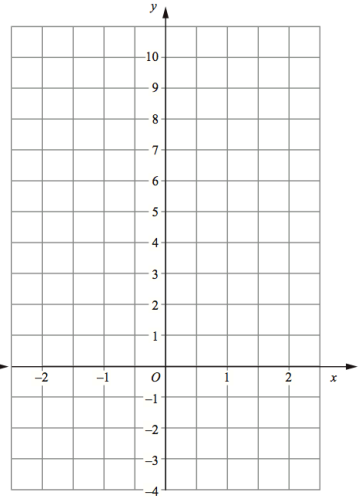
2(f)



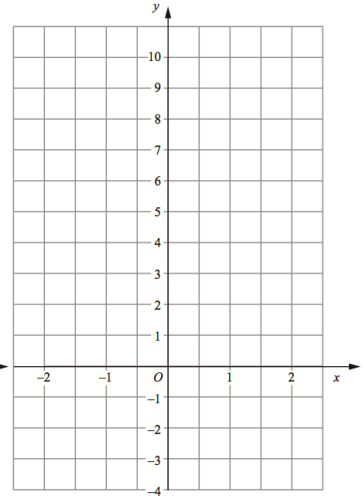
2(g)



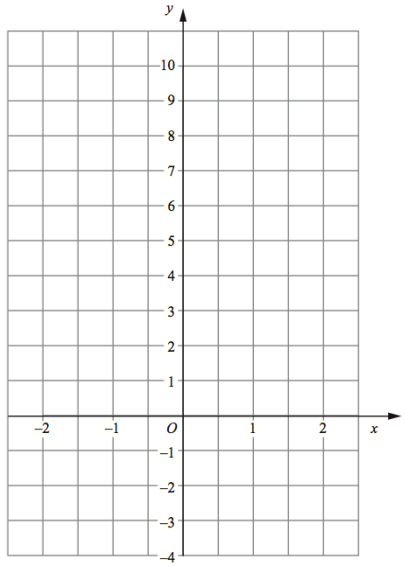
2(h)



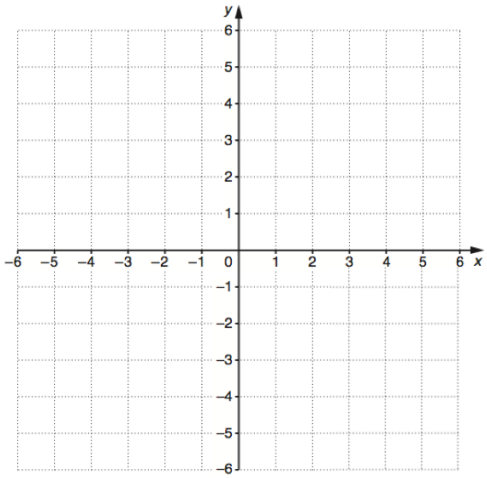
2(i)



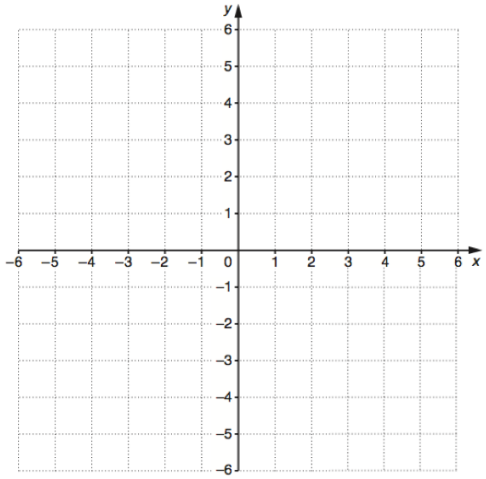
2(j)



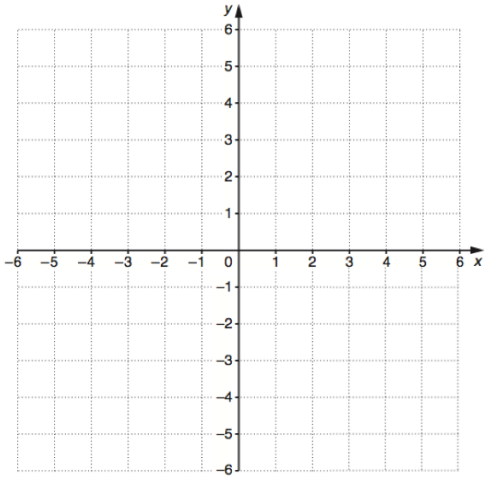
3(a)



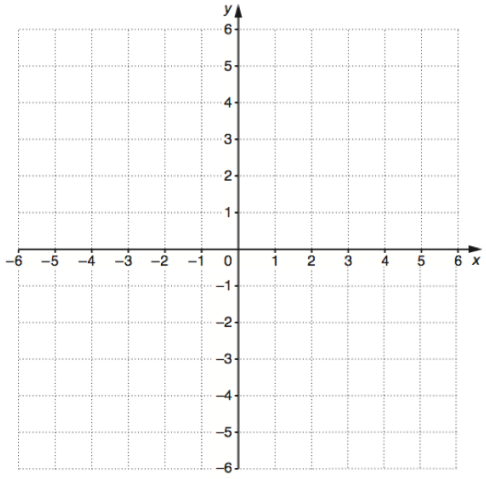
3(b)



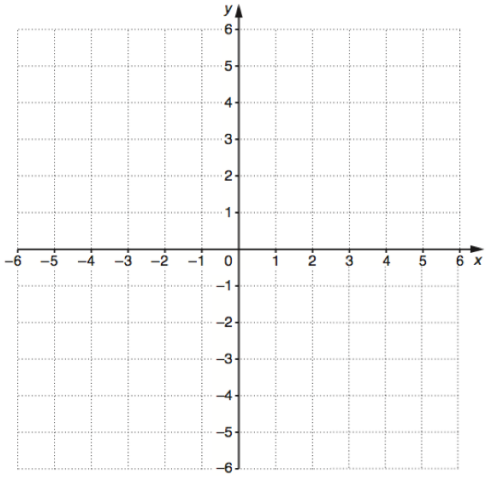
3(c)



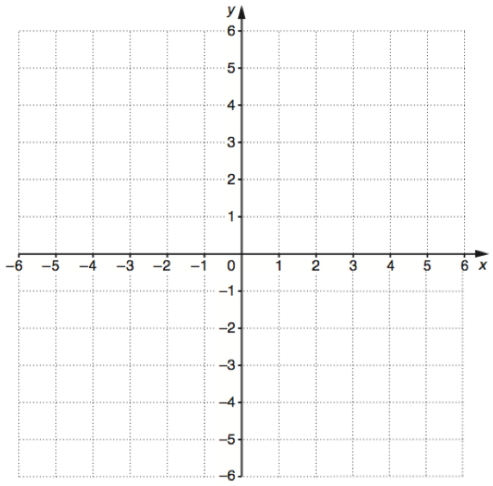
3(d)



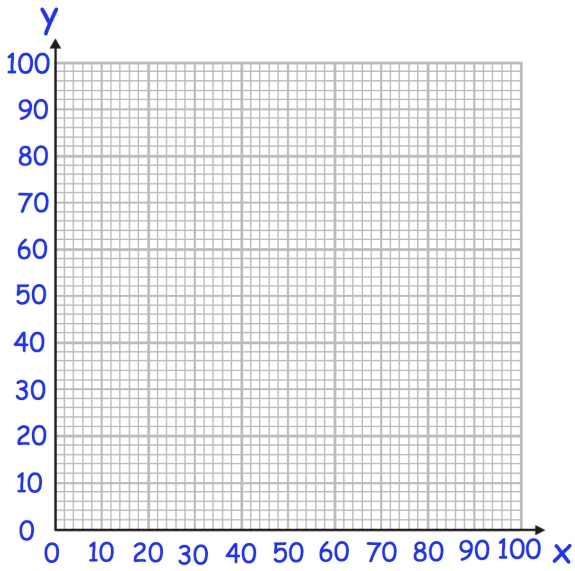
3(e)



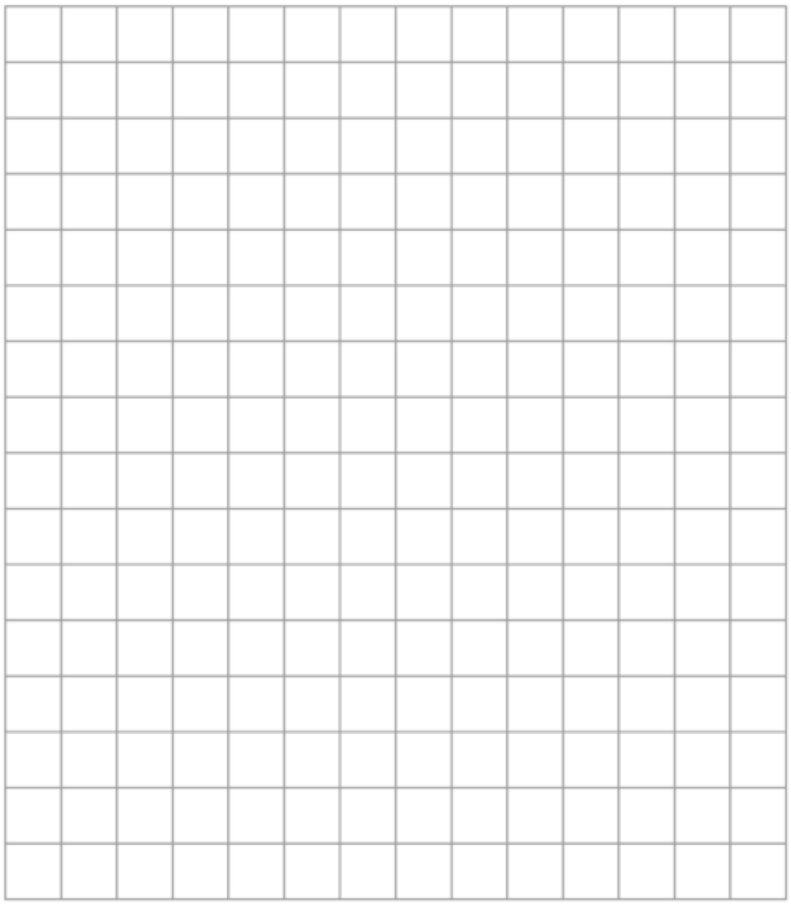
3(f)



Apply 4



Apply 5



Extra Notes

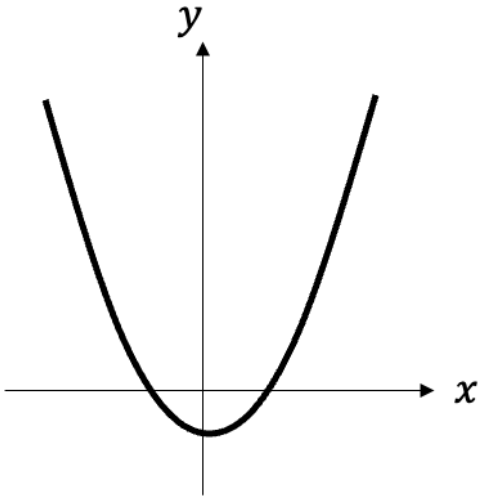
4 Non-Linear Graphs

A **quadratic graph** is produced from an equation of the form

$$y = ax^2 + bx + c$$

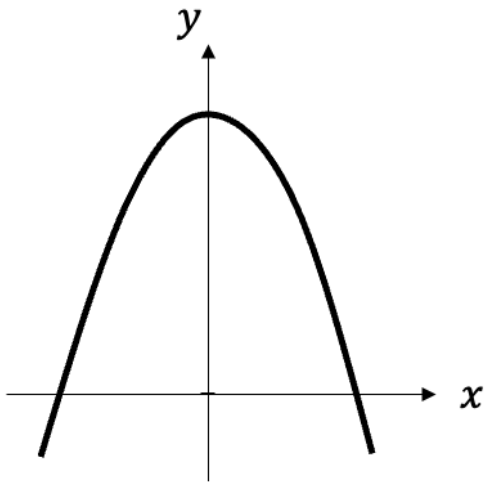
where a, b and c are constants and a is non-zero.

When $a > 0$



For a positive x^2 term, we get a ∪-shaped parabola

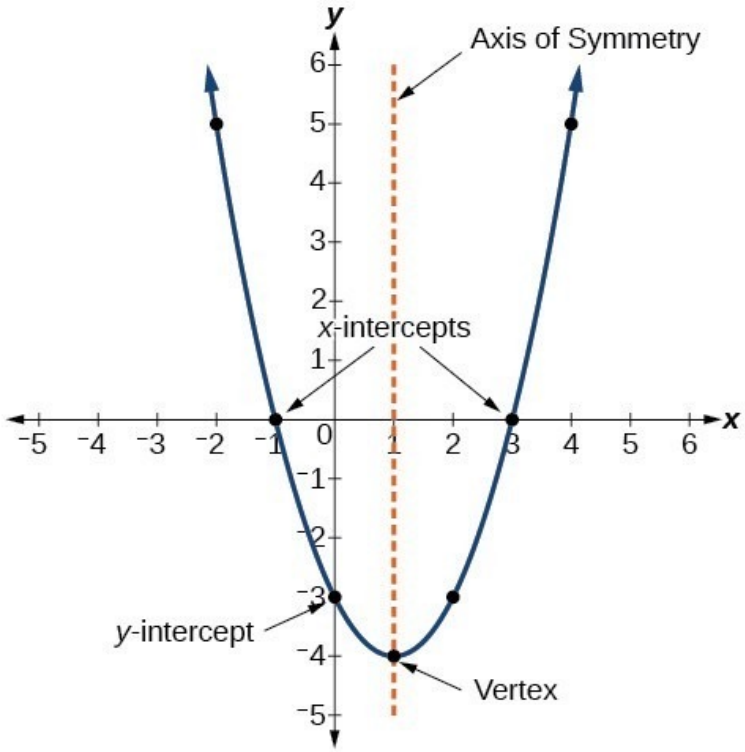
When $a < 0$



For a negative x^2 term, we get a ∩-shaped inverted parabola

Interpreting Quadratic Graphs

- **y-intercept** – where the graph intercepts the y-axis
- **x-intercept** or **root** or **solution** – where the graph intercepts the x-axis
- **Turning point** or **vertex** or **minimum/maximum** – where the graph stops decreasing and starts increasing or vice-versa



Worked Example	Your Turn
Shade the region which satisfies the inequalities. Label it R. $x \leq 3, y > 1$ and $y \geq x + 3$	Shade the region which satisfies the inequalities. Label it R. $x < 4, y \geq 3, y \geq x + 2$

Dr Frost 456e and 456d

Worked Example

a) Complete the table and draw the graph of $y = x^2 + 2x$ for $x = -4$ to $x = 2$

b) Write down the equation of the line of symmetry of your graph

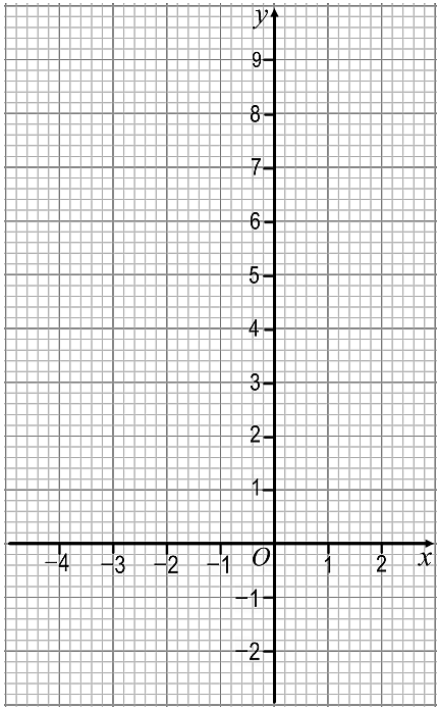
c) Use your graph to find:

i) the value of y when $x = 0.5$

ii) the values of x when $y = 6$

Here is a table of values for $y = x^2 + 2x$.

x	-4	-3	-2	-1	0	1	2
y	8		0	-1			8



Worked Example

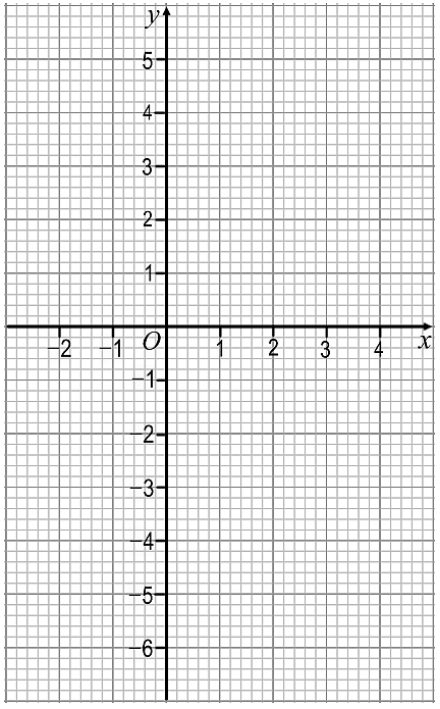
a) Complete the table and draw the graph of $y = x^2 - 2x - 4$ for $x = -2$ to $x = 4$

b) Write down the equation of the line of symmetry of your graph

c) Write down the values of x where the graph crosses the x-axis

Here is a table of values for $y = x^2 - 2x - 4$.

x	-2	-1	0	1	2	3	4
y		-1	-4			-1	



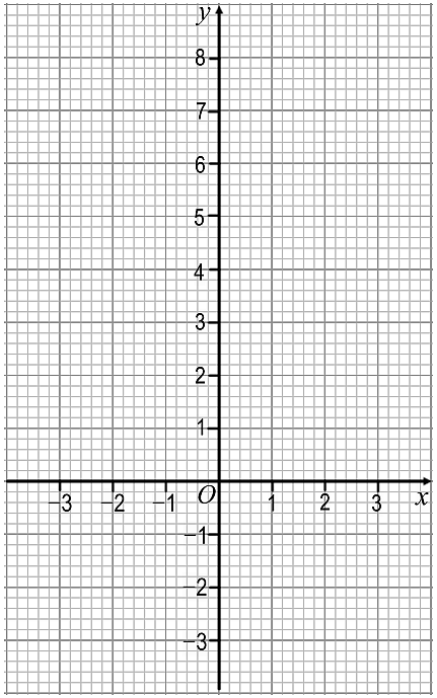
Fluency Practice

1. Here is a table of values for $y = x^2 - 2$.

x	−3	−2	−1	0	1	2	3
y	7		−1	−2			7

a) Complete the table of values.

b) On the grid, draw the graph of $y = x^2 - 2$ for $x = -3$ to $x = 3$.



c) Write down the equation of the line of symmetry of your graph.

d) Write down the coordinates of the minimum point.

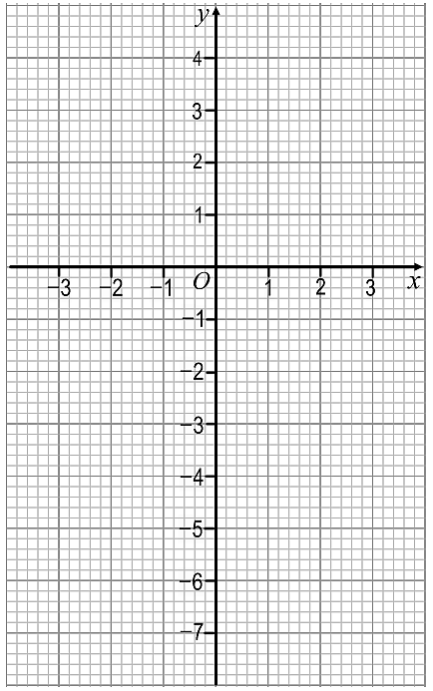
Fluency Practice

2. Here is the table of values for $y = 3 - x^2$.

x	−3	−2	−1	0	1	2	3
y	−6		2	3		−1	

a) Complete the table of values.

b) On the grid, draw the graph of $y = 3 - x^2$ for $x = -3$ to $x = 3$.



c) Write down the coordinates of the maximum point.

d) Write down the values of x where the graph crosses the x-axis.

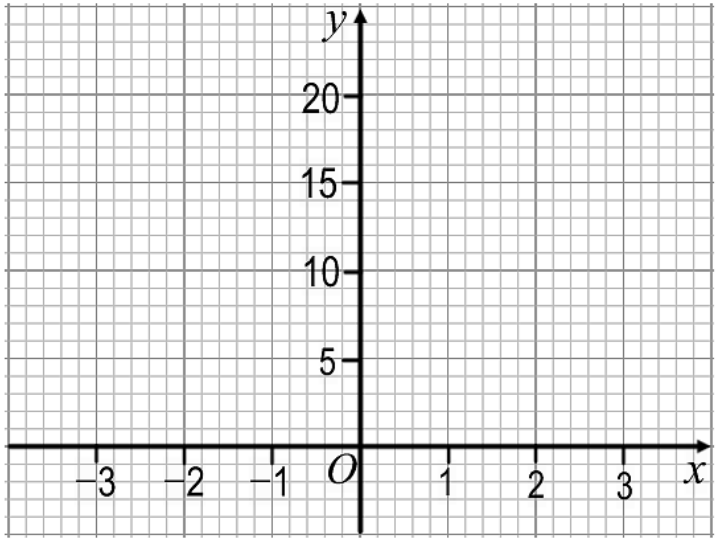
Fluency Practice

3. Here is a table of values for $y = 2x^2 + 1$.

x	-3	-2	-1	0	1	2	3
y		9		1	3	9	

a) Complete the table of values.

b) On the grid, draw the graph of $y = 2x^2 + 1$ for $x = -3$ to $x = 3$.



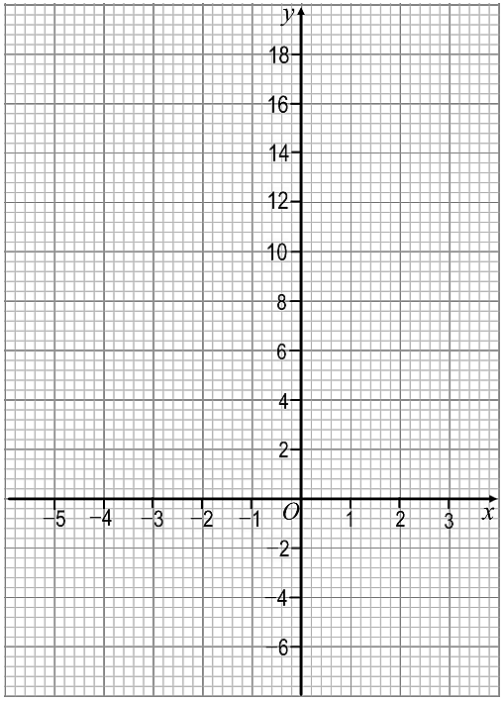
c) Use your graph to find:

i) the value of y when $x = -2.5$

ii) the two values of x when $y = 6$.

Fluency Practice

4. a) On the grid, draw the graph of $y = x^2 + 3x - 2$ for the values of x from -5 to 3.



b) Use your graph to:

i) write down the values of x when the graph crosses the x-axis

ii) draw in and write down the equation of the line of symmetry.

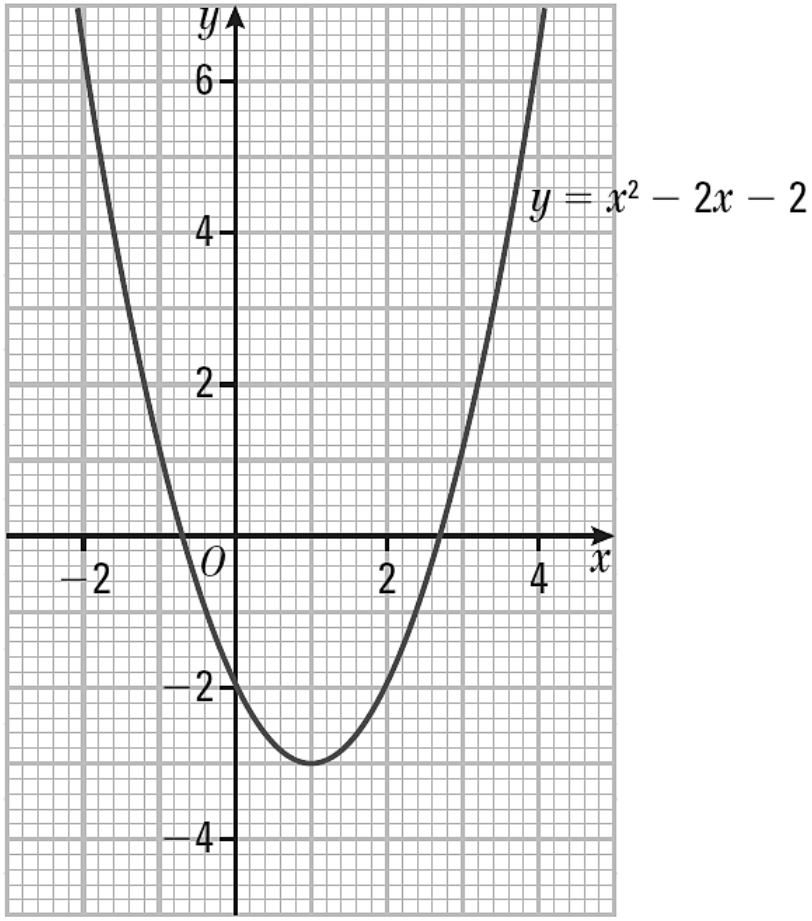
Dr Frost 366b, 366c, 366d, 366e, 366f and 366g

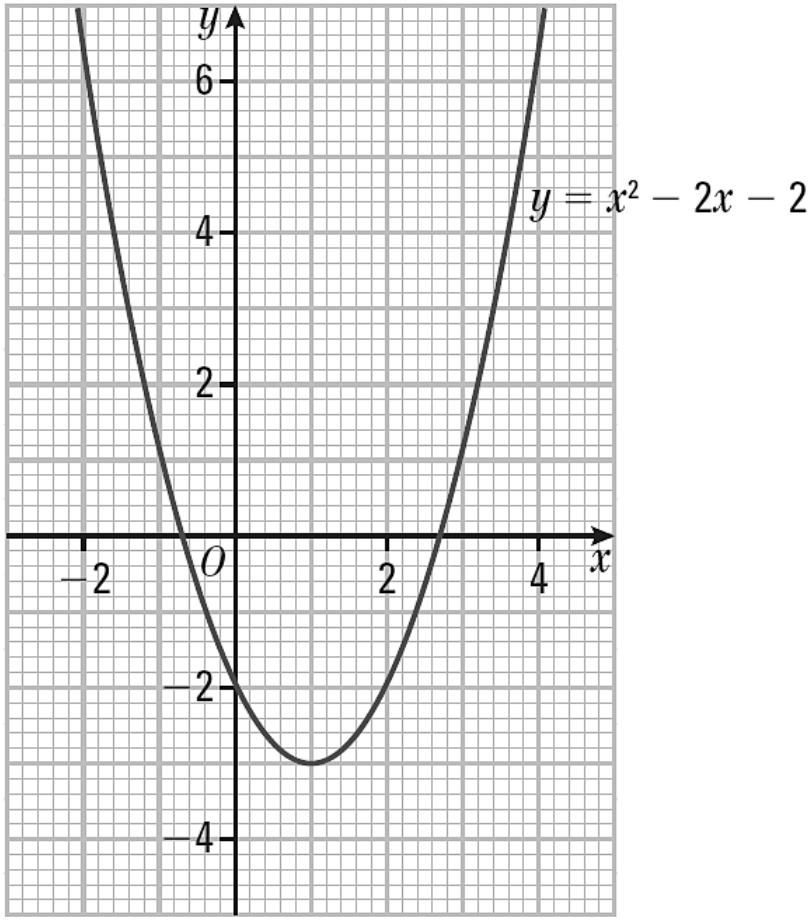
Worked Example

Use this graph to solve these equations:

a) $x^2 - 2x - 2 = 0$

b) $x^2 - 2x - 5 = 0$



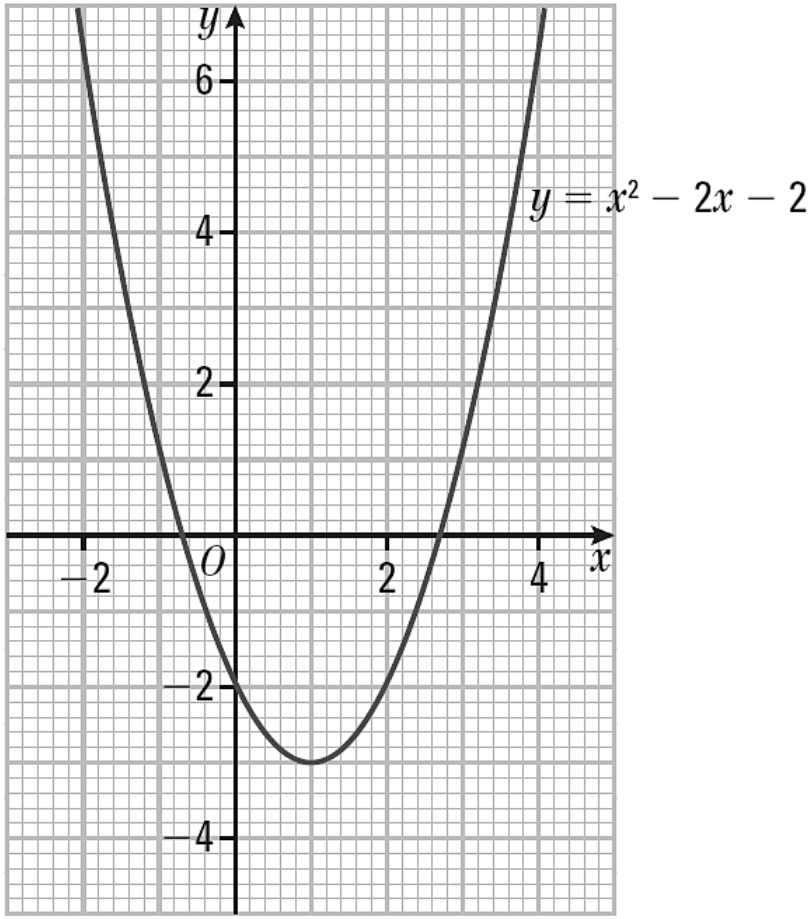


Dr Frost 421d and 421f

Worked Example

Use this graph to solve these equations:

c) $x^2 - 2x - 2 = x$

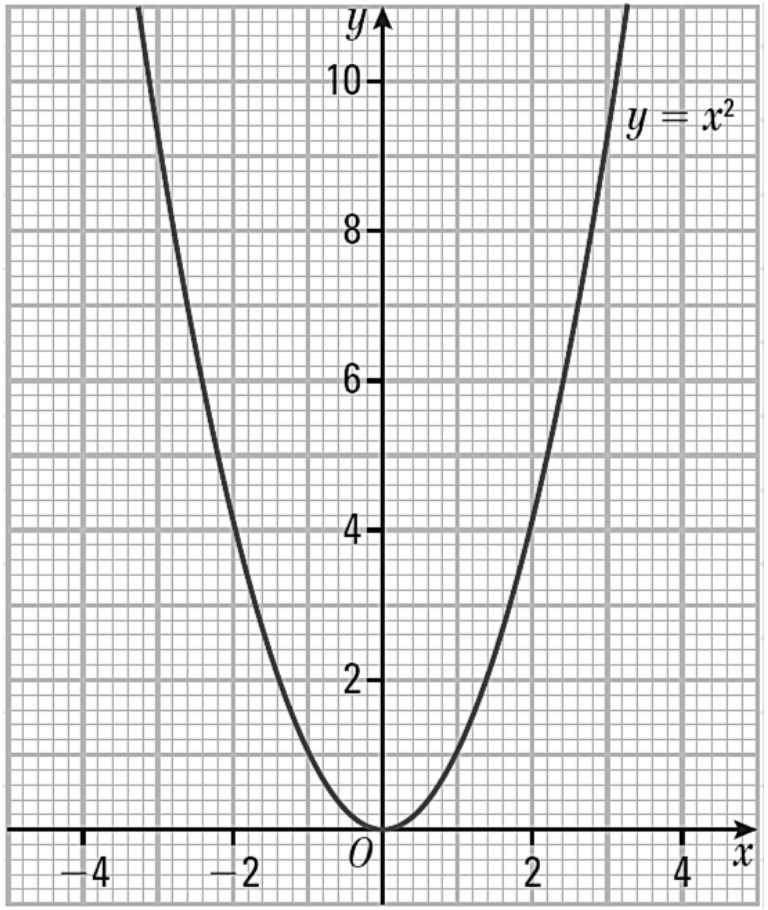


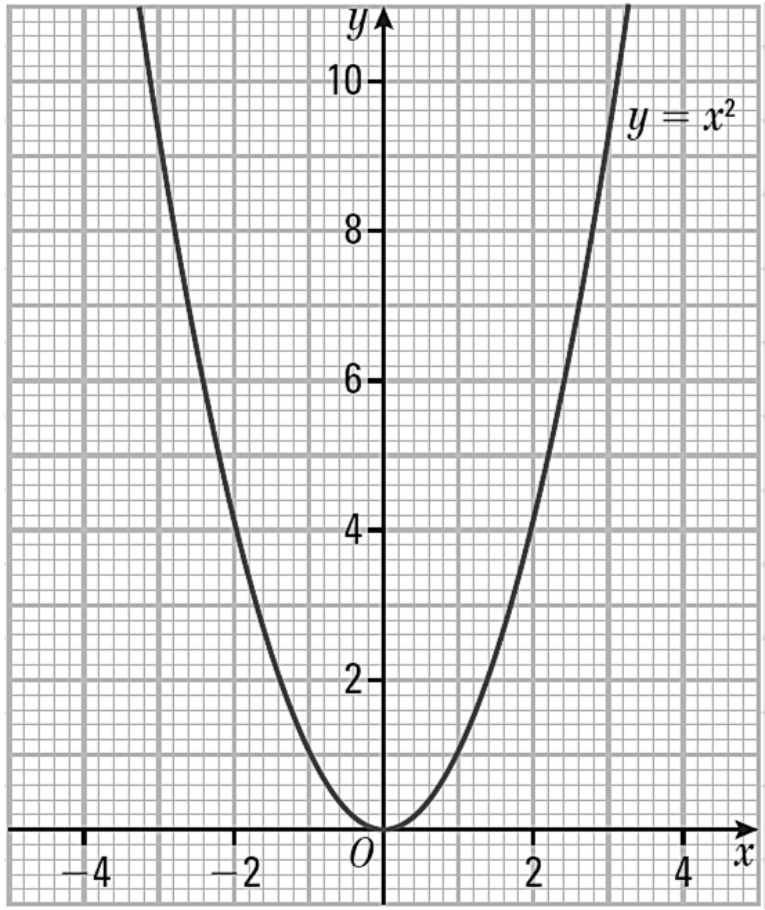
Worked Example

Use this graph to solve these equations:

a) $x^2 = 2x + 3$

b) $x^2 = x + 4$



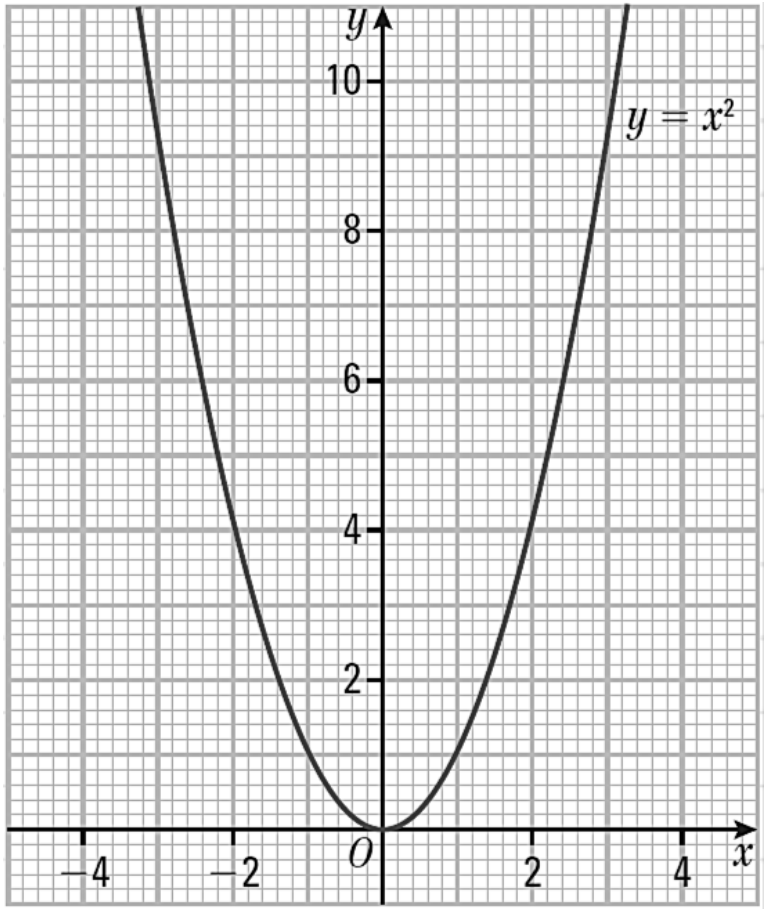


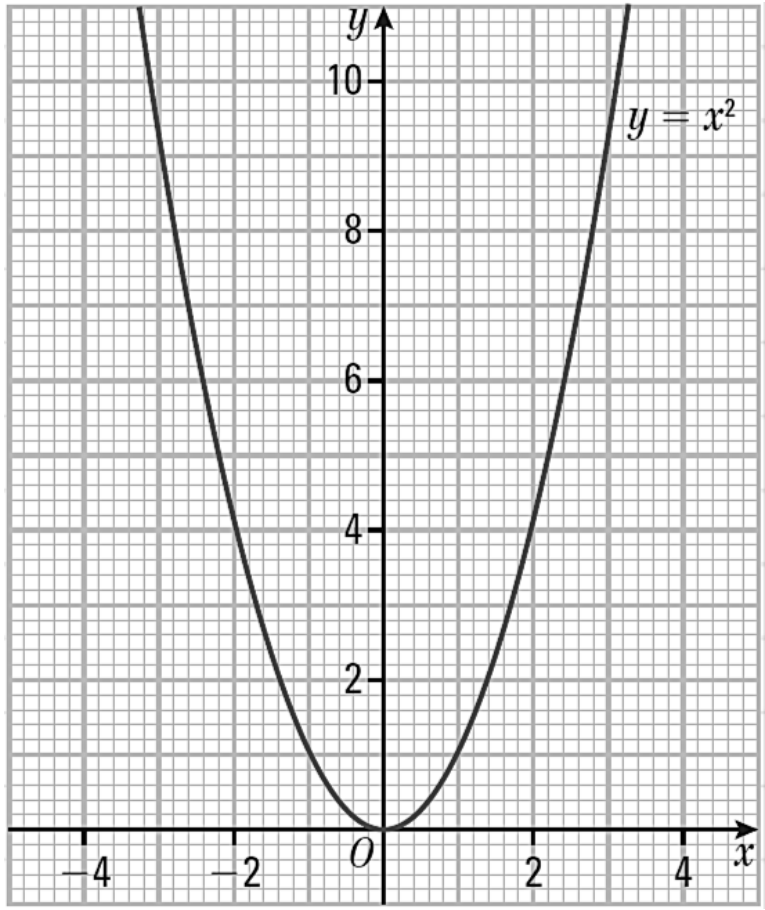
Worked Example

Use this graph to solve these equations:

c) $x^2 + x - 1 = 0$

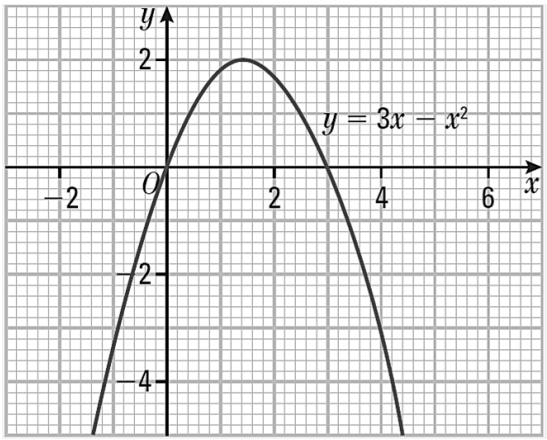
d) $x^2 - 2x - 1 = 0$





Fluency Practice

1. Use this graph to solve the equations.

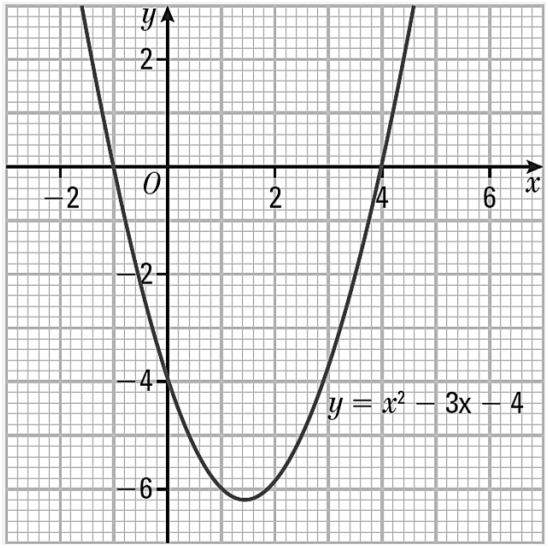


a) $3x - x^2 = 0$

b) $3x - x^2 = 1$

c) $3x - x^2 = -4$

2. Use this graph to solve the equations.



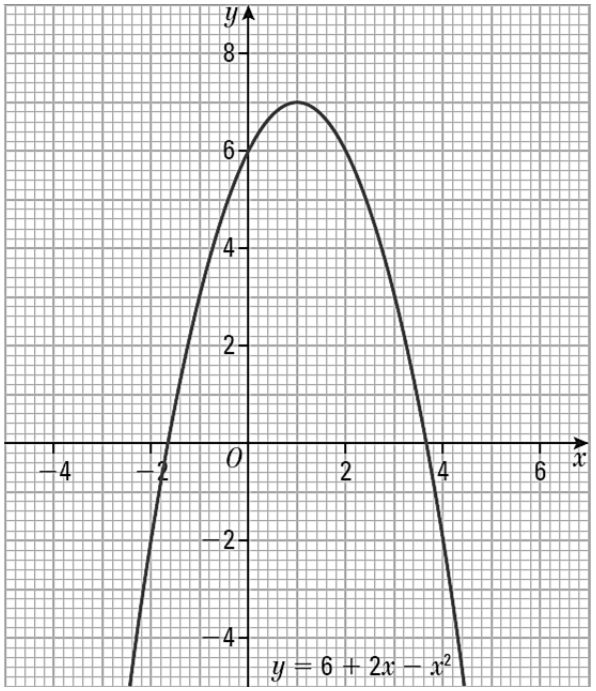
a) $x^2 - 3x - 4 = 0$

b) $x^2 - 3x - 4 = 2$

c) $x^2 - 3x - 4 = -5$

Fluency Practice

3. Use this graph to solve the equations.



a) $6 + 2x - x^2 = 0$

b) $4 + 2x - x^2 = 0$

c) $6 + 2x - x^2 = x$

d) $3 + 3x - x^2 = 0$

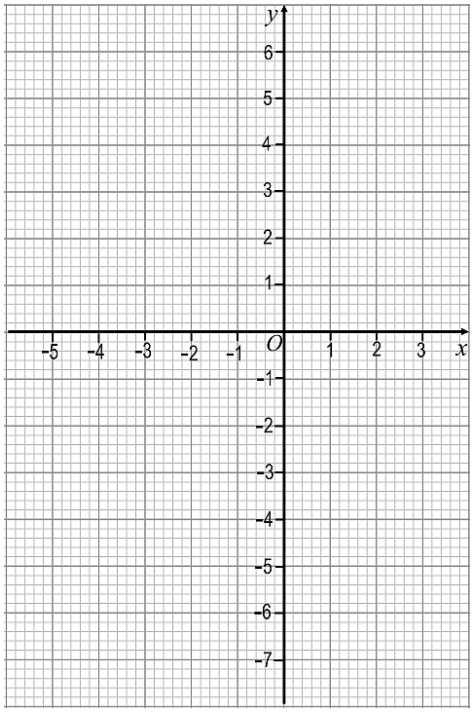
Fluency Practice

4. Here is a table of values for $y = x^2 + 3x - 4$.

x	-5	-4	-3	-2	-1	0	1	2
y	6	0		-6		-4		

a) Complete the table of values.

b) On the grid, draw the graph of $y = x^2 + 3x - 4$.

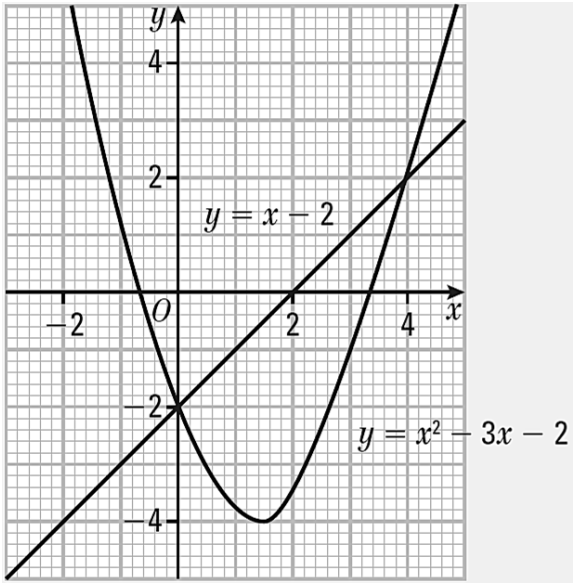


c) Use your graph to solve the equation $x^2 + 3x - 4 = 2$.

d) By drawing a suitable straight line on your graph, solve the equation $x^2 + 3x - 4 = x + 1$.

Fluency Practice

5. The graphs $y = x^2 - 3x - 2$ and $y = x - 2$ are shown below.



a) Show that the equation $x^2 - 3x - 2 = x - 2$ can be rewritten as $x^2 - 4x = 0$.

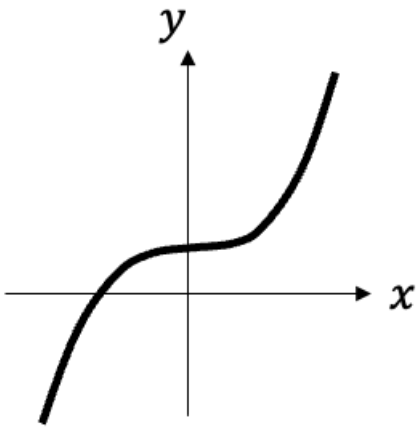
b) Solve the equation $x^2 - 4x = 0$.

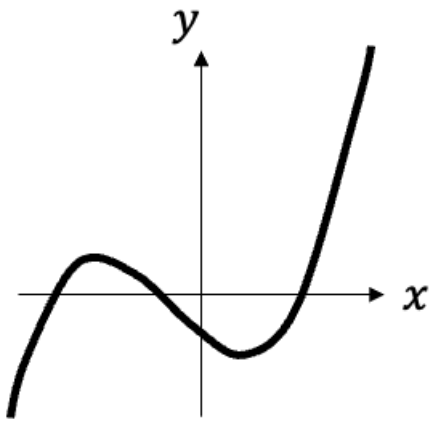
c) The equation $x^2 - 2x - 4 = 0$ can be solved by drawing a suitable straight line on the graph.
Find the equation of this straight line and solve the equation $x^2 - 2x - 4 = 0$.

Cubic Graphs

For a cubic graph $y = ax^3 + bx^2 + cx + d$

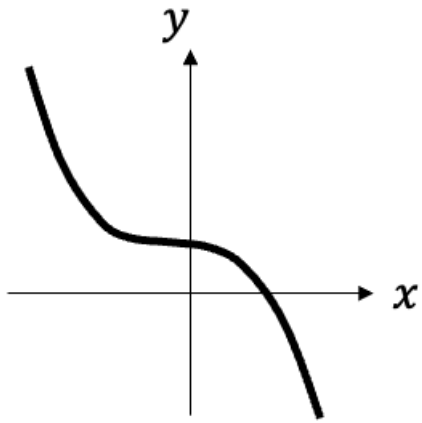
When $a > 0$

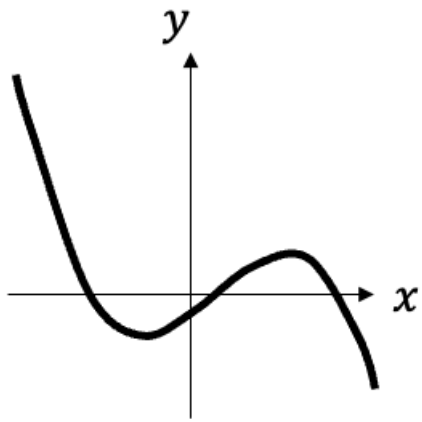




Positive cubics always go from bottom left to top right

When $a < 0$





Negative cubics always go from top left to bottom right

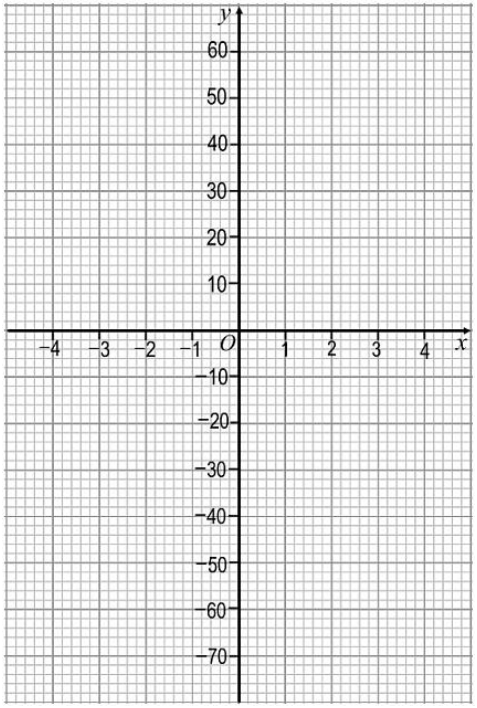
Worked Example

a) Complete the table and draw the graph of $y = x^3 - 4$ for $x = -4$ to $x = 4$

b) Use the graph to find the value of y when $x = 3.4$

Here is a table of values for $y = x^3 - 4$.

x	-4	-3	-2	-1	0	1	2	3	4
y									



Worked Example

a) Complete the table and draw the graph of $y = x^3 - 4x^2 + 5$ for $x = -2$ to $x = 5$

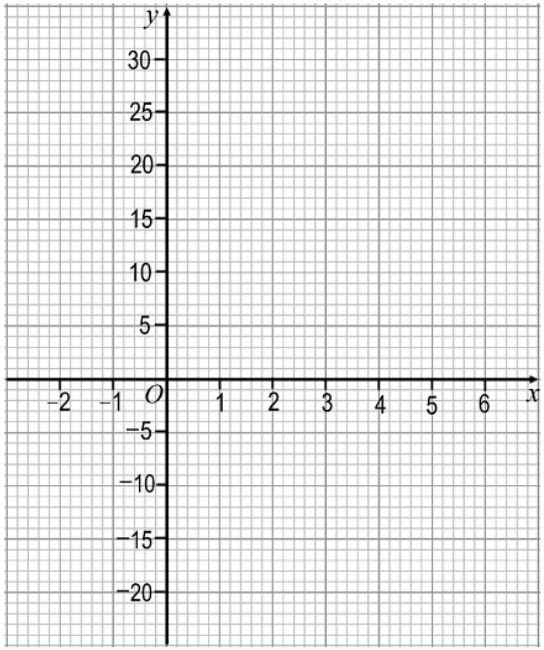
b) Use your graph to find the solutions to:

i) $x^3 - 4x^2 + 5 = 0$

ii) $x^3 - 4x^2 - x + 5 = 0$

Here is a table of values for $y = x^3 - 4x^2 + 5$.

x	-2	-1	0	1	2	3	4	5
y	-19		5			-4	5	



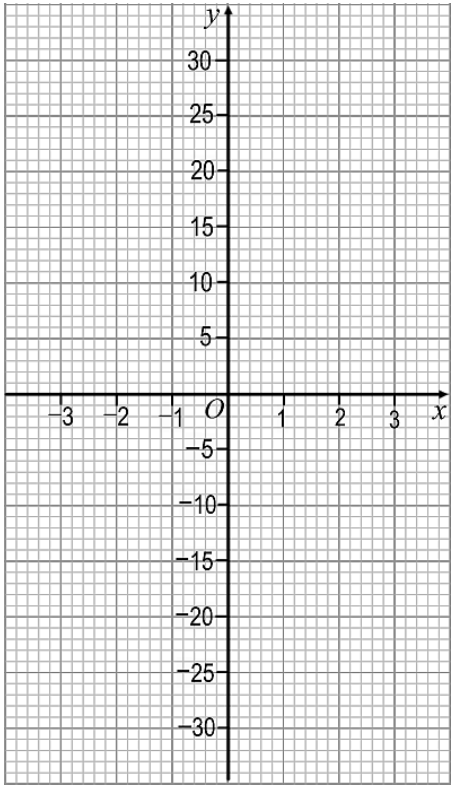
Fluency Practice

1. Here is a table of values for $y = x^3 + 1$.

x	-3	-2	-1	0	1	2	3
y							

a) Complete the table of values.

b) On the grid, draw the graph of $y = x^3 + 1$ for $-3 \leqslant x \leqslant 3$.



c) Use your graph to find the value of y when $x = 1.5$.

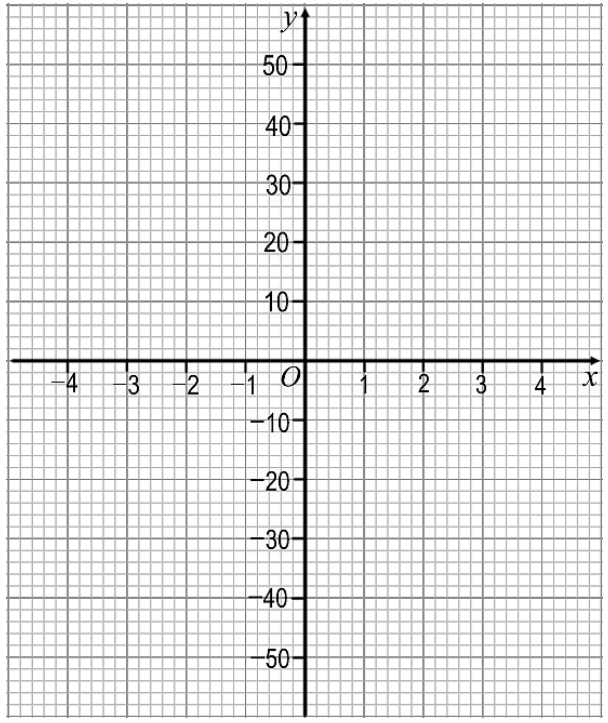
Fluency Practice

2. Here is the table of values for $y = x^3 - 5x$.

x	-4	-3	-2	-1	0	1	2	3	4
y		-12			0	-4		12	44

a) Complete the table of values.

b) On the grid, draw the graph of $y = x^3 - 5x$ for $-4 \leqslant x \leqslant 4$.



c) Use your graph to find the solutions to the equation $x^3 - 5x = 0$.

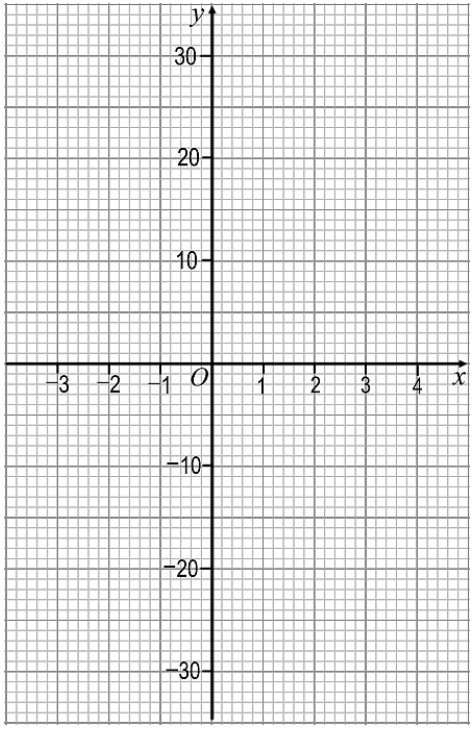
Fluency Practice

3. Here is a table of values for $y = 6x + x^2 - x^3$.

x	-3	-2	-1	0	1	2	3	4
y		0	-4			8	0	

a) Complete the table of values.

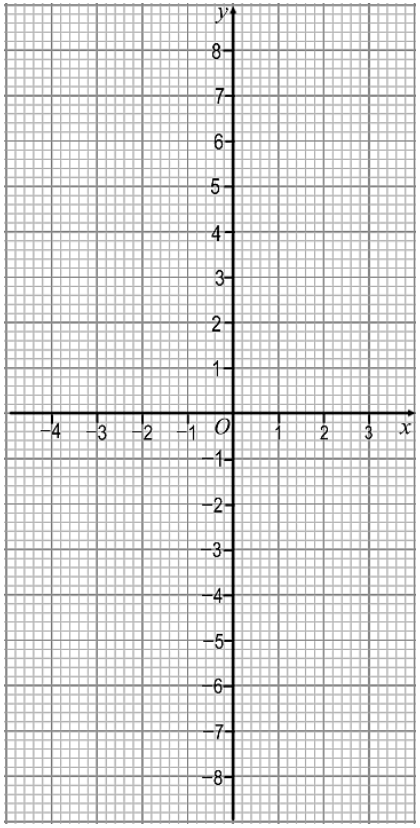
b) On the grid, draw the graph of $y = 6x + x^2 - x^3$ for $-3 \leqslant x \leqslant 4$.



c) By drawing a suitable line on your diagram, solve the equation $6x + x^2 - x^3 = x - 2$.

Fluency Practice

4. a) On the grid, draw the graph of $y = x^3 + x^2 - 4x - 2$ for the values of x from -3 to 2.



b) By drawing a suitable line on your diagram, solve the equation $x^3 + x^2 - 5x - 2 = 0$.

Reciprocal Graphs

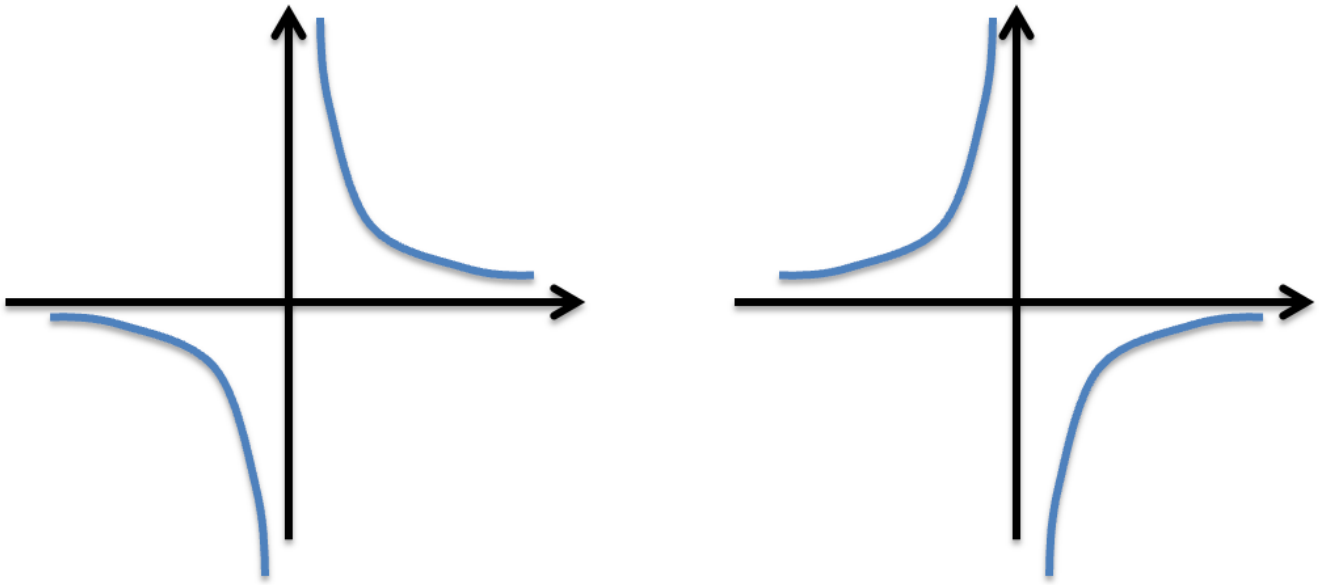
$$y = \frac{k}{x}$$

When $k > 0$

k is a constant while x is a variable, so we might have $y = \frac{3}{x}$

$$y = \frac{k}{x}$$

When $k < 0$



The lines $x = 0$ and $y = 0$ are called asymptotes.

An asymptote is a straight line which the curve approaches at infinity.

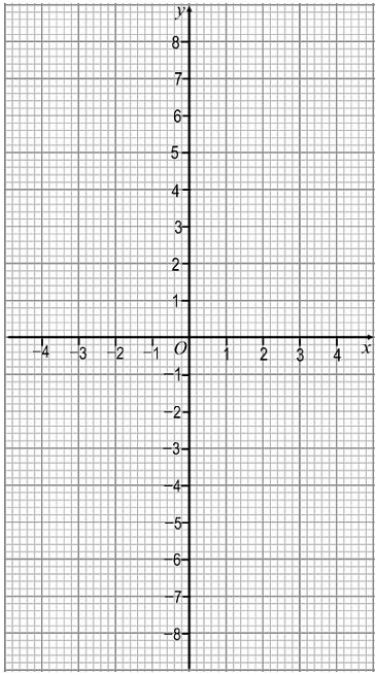
Worked Example

Complete the tables and draw the graph of $y = \frac{2}{x}$ for $x = -5$ to $x = 5$

Here is a table of values for $y = \frac{2}{x}$.

x	0.25	0.4	0.5	0.8	1	2	4	5
y								

x	−0.25	−0.4	−0.5	−0.8	−1	−2	−4	−5
y								



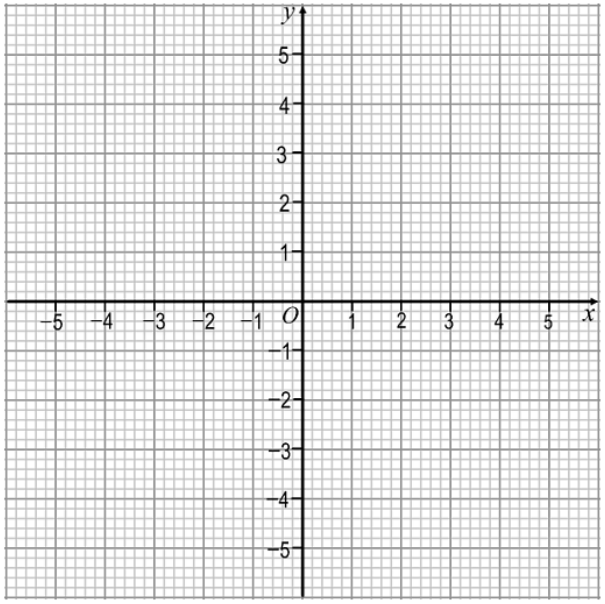
Worked Example

Complete the tables and draw the graph of $y = -\frac{1}{x}$ for $x = -5$ to $x = 5$

Here is a table of values for $y = -\frac{1}{x}$.

x	0.2	0.4	0.5	0.8	1	2	3	4	5
y									

x	−0.2	−0.4	−0.5	−0.8	−1	−2	−3	−4	−5
y									



Dr Frost 422a, 422b and 422d

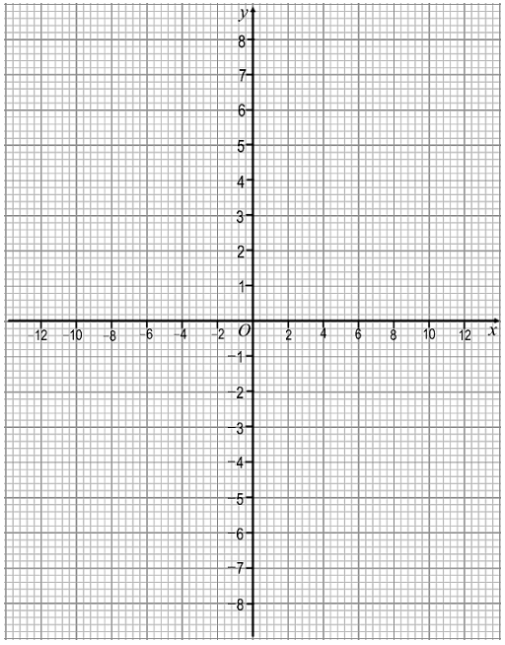
Worked Example

Complete the tables and draw the graph of $y = \frac{4}{x-1}$ for $x = -12$ to $x = 12$

Here is a table of values for $y = \frac{4}{x-1}$.

x	1.5	2	3	5	6	9	11
y							

x	0.5	0	-1	-3	-4	-7	-9
y							



Fluency Practice

1. Here are some table of values for $y = \frac{4}{x}$.

x	0.2	0.4	0.5	1	2	4	5	8	10
y		10		4	2		0.8		

x	-10	-8	-5	-4	-2	-1	-0.5	-0.4	-0.2
y									

a) Complete the table of values.

b) On your additional sheet, draw the graph of $y = \frac{4}{x}$ for $-10 \leqslant x \leqslant 10$.

c) Use your graph to find an estimate for the solutions of $\frac{4}{x} = 4 - x$.

2. On your additional sheet, draw the graph of $y = -\frac{3}{x}$ for $-10 \leqslant x \leqslant 10$.

Fluency Practice

3. a) Here are some table of values for $y = \frac{8}{x+2}$.

x	−12	−10	−7	−6	−4	−3	−1	0	2	3	6	8
y												

b) On your additional sheet, draw the graph of $y = \frac{8}{x+2}$ for $-12 \leqslant x \leqslant 12$.

c) For which values of x is $y = \frac{8}{x+2}$ not defined?

4. a) Complete the table of values for $y = 3 - \frac{2}{x}$, $x \neq 0$.

x	−3	−2	−1	−0.5	−0.1	0.1	0.5	1	2	3
y										

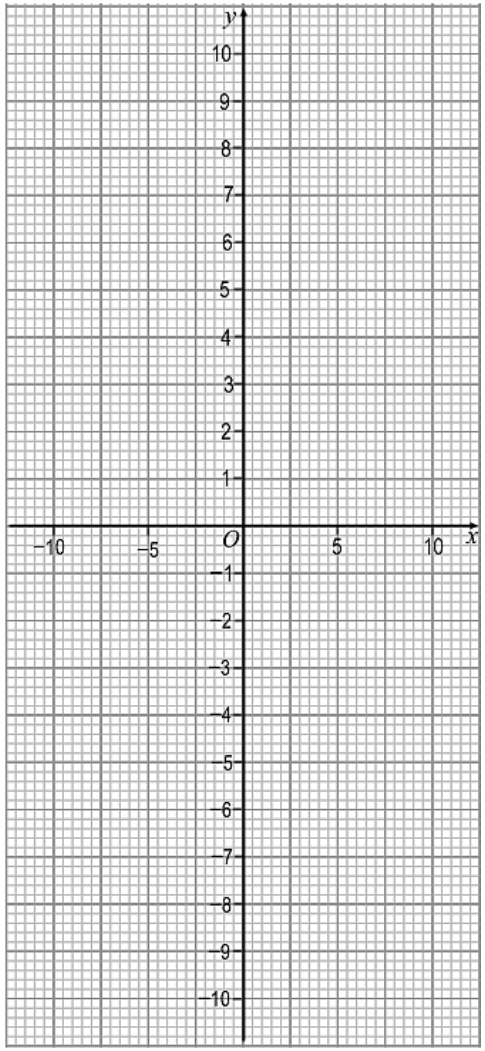
b) On your additional sheet, draw the graph of $y = 3 - \frac{2}{x}$ for $-3 \leqslant x \leqslant 3$.

c) This graph approaches two lines without touching them. These lines are called asymptotes. Write down the equation of each of these two lines.

Fluency Practice

Question 1

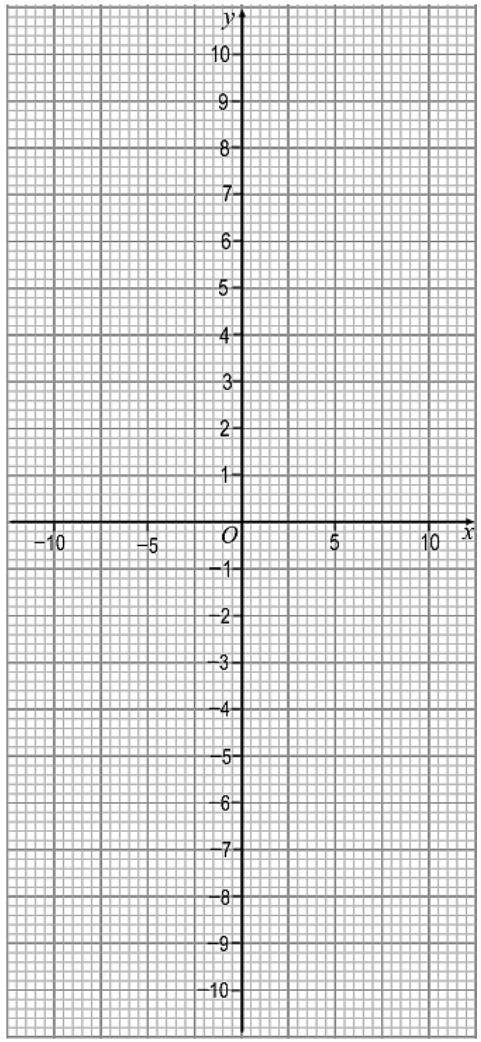
$y = \frac{4}{x}$



Fluency Practice

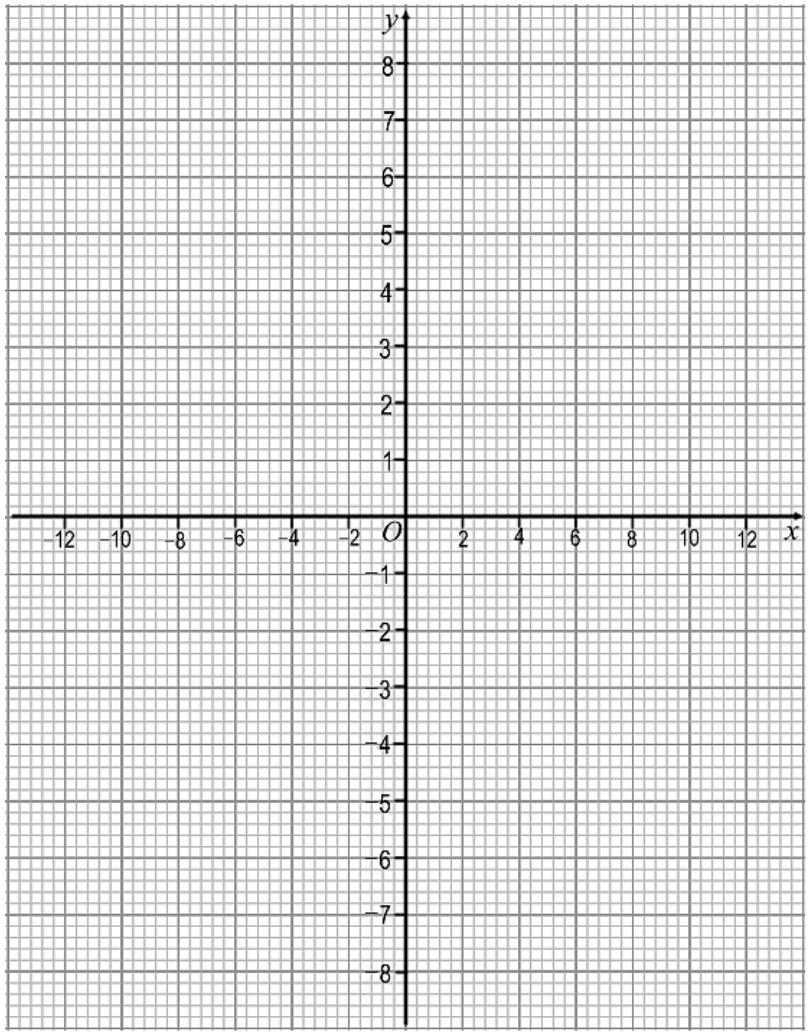
Question 2

$y = -\frac{3}{x}$



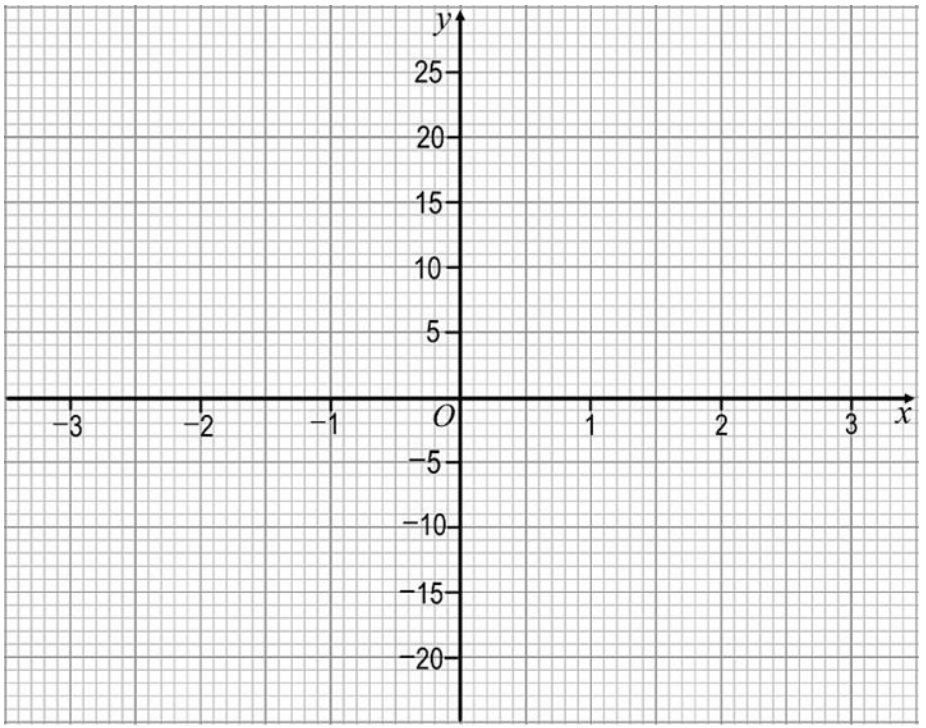
Fluency Practice

Question 3 $y = \frac{8}{x+2}$



Fluency Practice

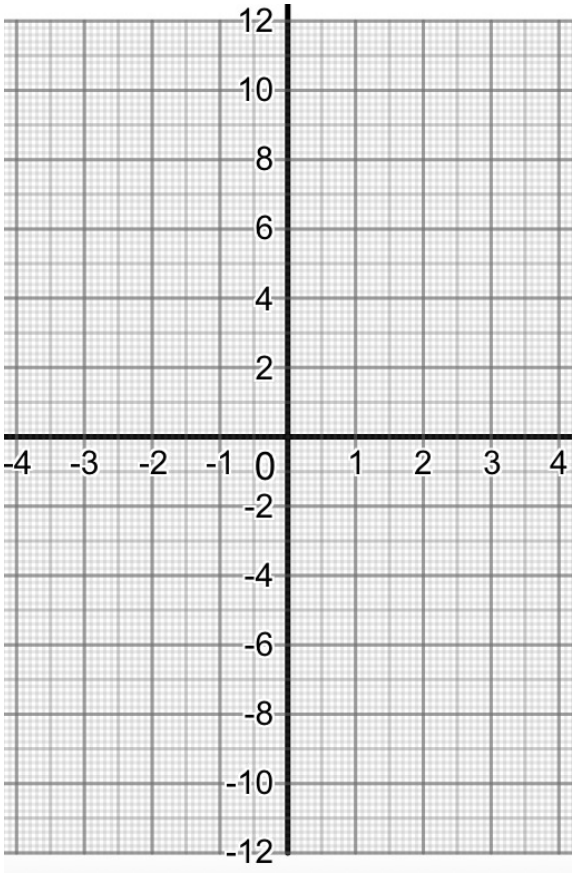
Question 4 $y = 3 - \frac{2}{x}$



Worked Example

By completing the table of values, plot the graph with equation $y = \frac{12}{x^2}$

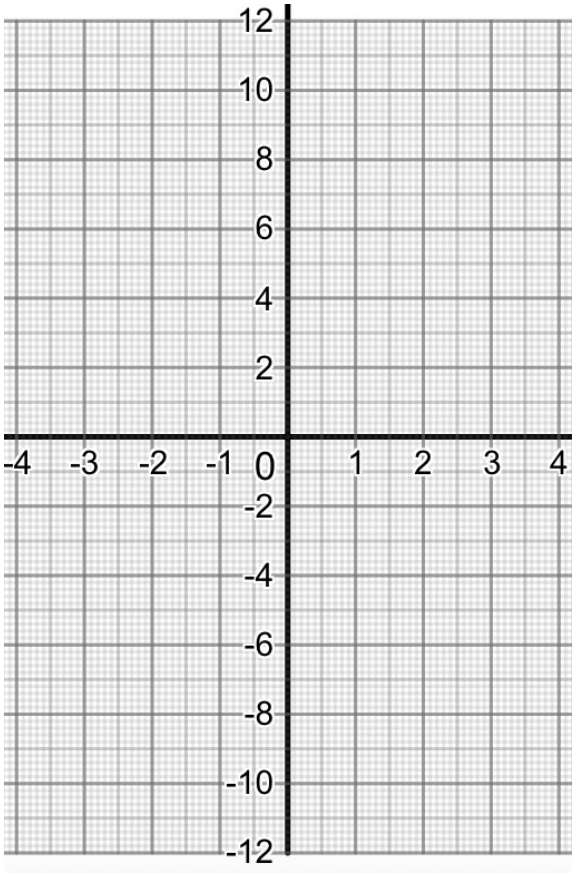
x	-4	-2	-1	1	2	4
y						



Worked Example

By completing the table of values, plot the graph with equation $y = -\frac{12}{x^2}$

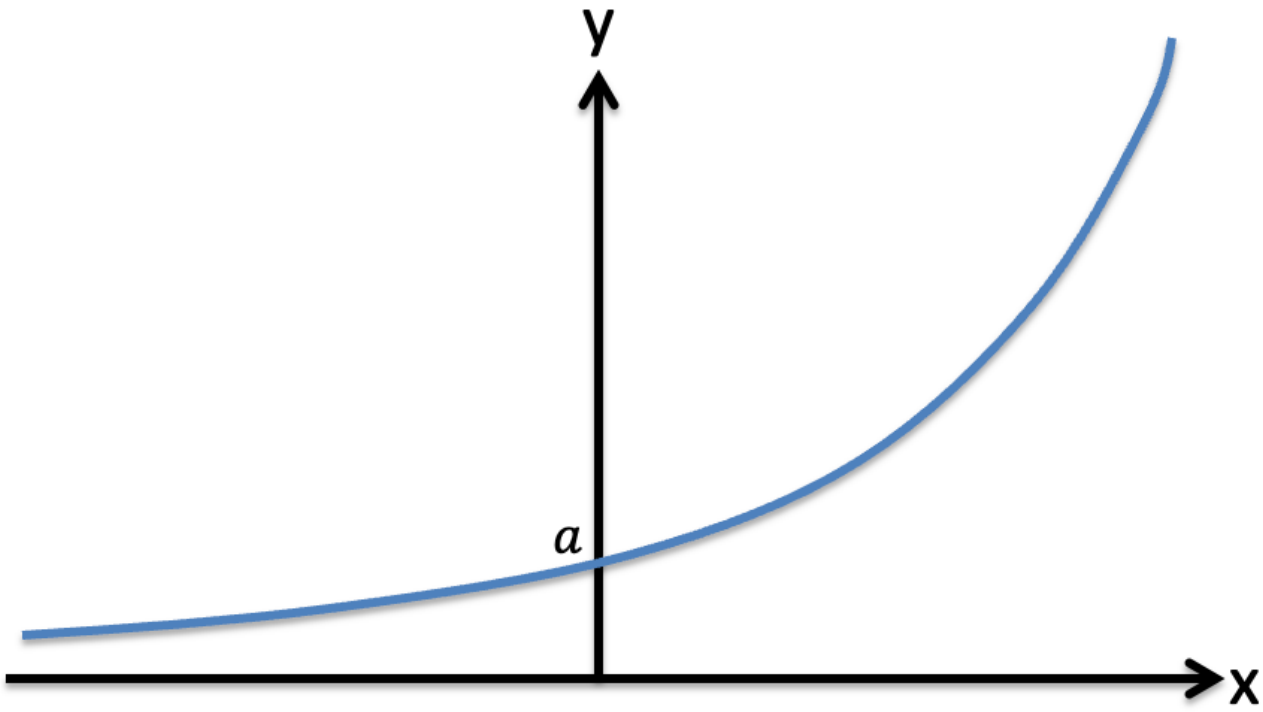
x	-4	-2	-1	1	2	4
y						



Dr Frost 422c, 422e and 422f

Exponential Graphs

$$y = a \times k^x$$



The y-intercept is a because $a \times k^0 = a \times 1 = a$

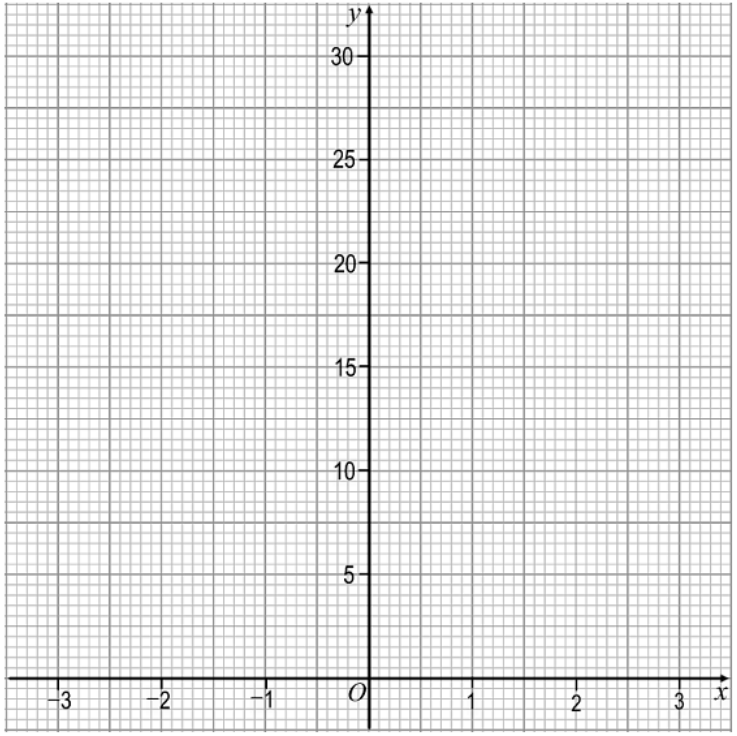
(unless $a = 0$, but let's not go there!)

Worked Example

a) Complete the tables and draw the graph of $y = 3^x$ for $x = -3$ to $x = 3$
b) Use your graph to estimate the solution to $3^x = 20$

Here is a table of values for $y = 3^x$.

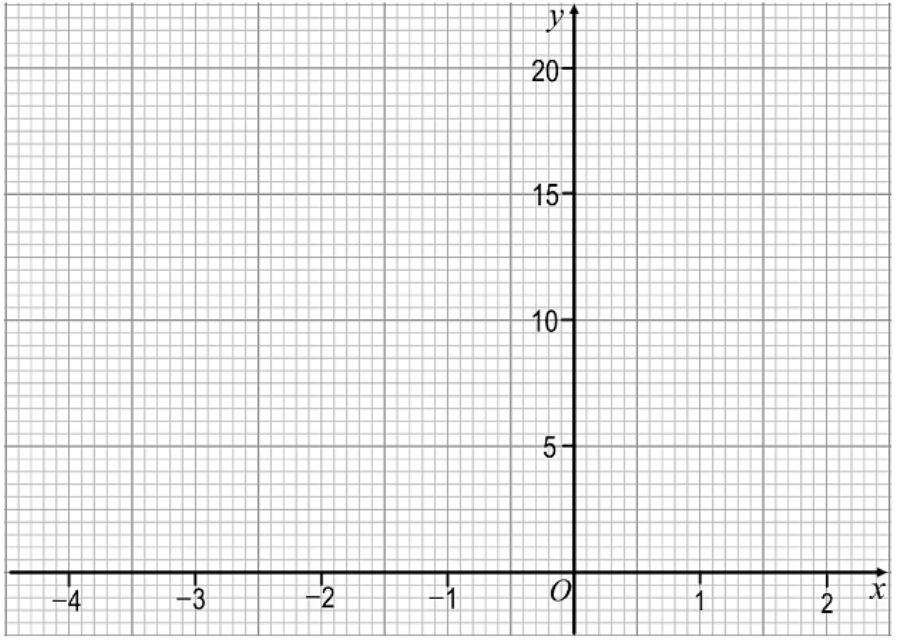
x	-3	-2	-1	0	1	2	3
y							



a) Complete the tables and draw the graph of $y = 2^{-x}$ for $x = -4$ to $x = 2$
b) Use your graph to estimate
i) the value of y when $x = 0.5$
ii) the solution to the equation $2^{-x} = 10$

Here is a table of values for $y = 2^{-x}$.

x	-4	-3	-2	-1	0	1	2
y							



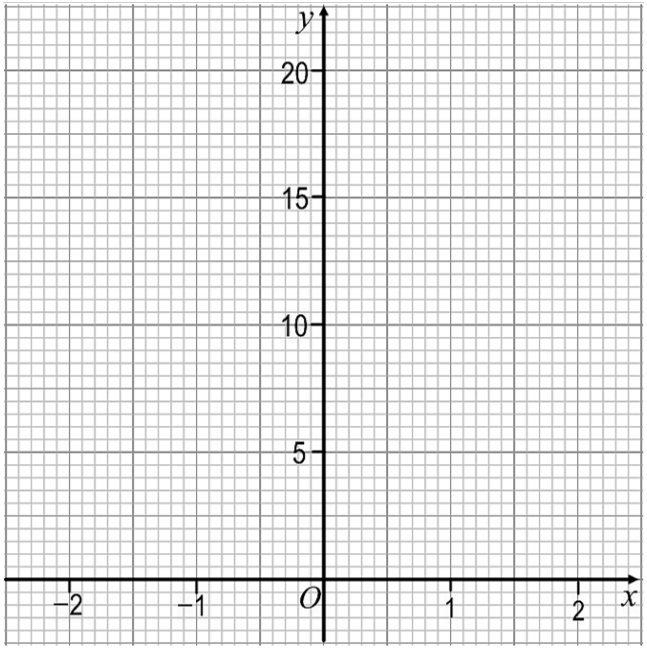
Fluency Practice

1. Here is a table of values for $y = 4^x$.

x	-2	-1	0	1	2
y					

a) Complete the table of values.

b) On the grid, draw the graph of $y = 4^x$ for $-2 \leqslant x \leqslant 2$.



c) Use your graph to find an estimate for:

i) the value of y when $x = 1.5$

ii) the value of x when $y = 11$

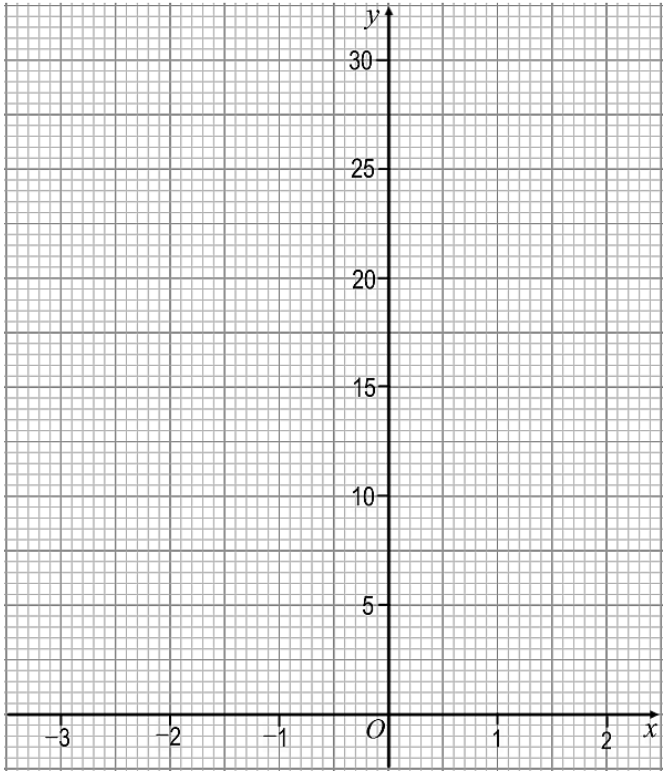
Fluency Practice

2. Here is the table of values for $y = 3^{-x}$.

x	-3	-2	-1	0	1	2
y						

a) Complete the table of values.

b) On the grid, draw the graph of $y = 3^{-x}$ for $-3 \leqslant x \leqslant 2$.

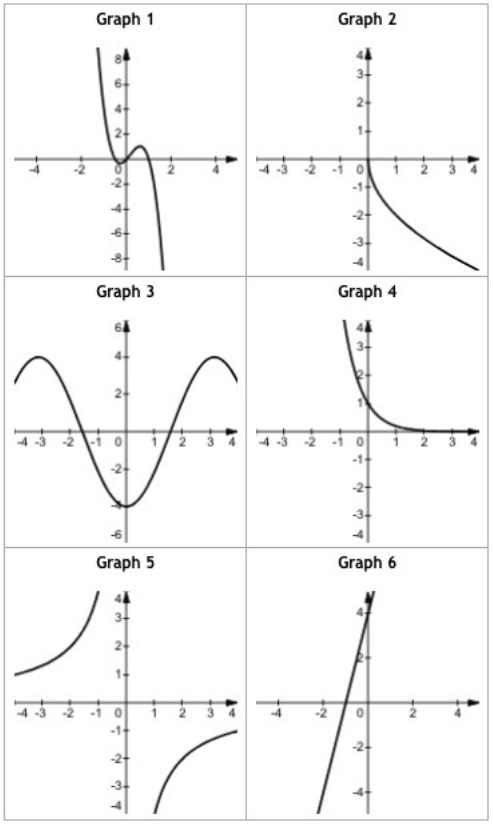


c) Use your graph to find the solution to the equation $3^{-x} = 7$.

Dr Frost 423a, 423b, 423c, 423d and 423e

Worked Example

Here are 6 graphs.

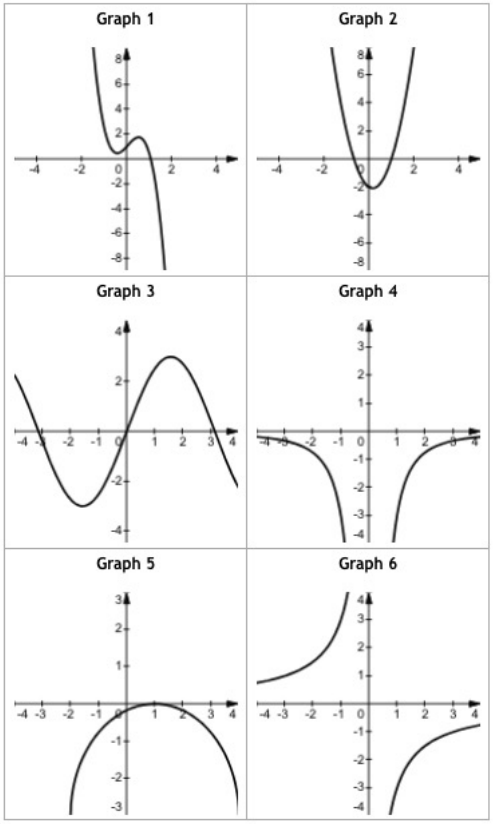


Select the graph that corresponds to $y = \left(\frac{1}{5}\right)^x$

- ◯ Graph 1
- ◯ Graph 2
- ◯ Graph 3
- ◯ Graph 4
- ◯ Graph 5
- ◯ Graph 6

Your Turn

Here are 6 graphs.

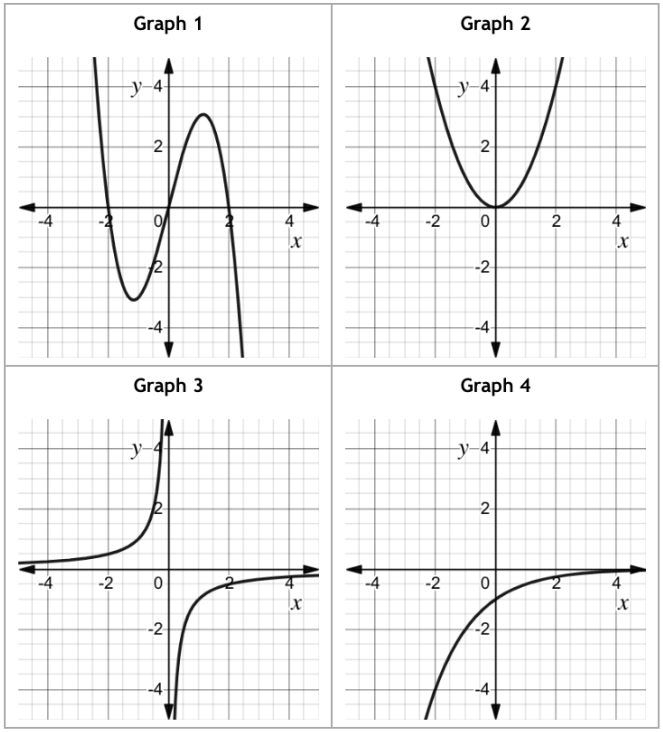


Select the graph that corresponds to $y = -\frac{3}{x}$

- ◯ Graph 1
- ◯ Graph 2
- ◯ Graph 3
- ◯ Graph 4
- ◯ Graph 5
- ◯ Graph 6

Worked Example

Four graphs are sketched below.

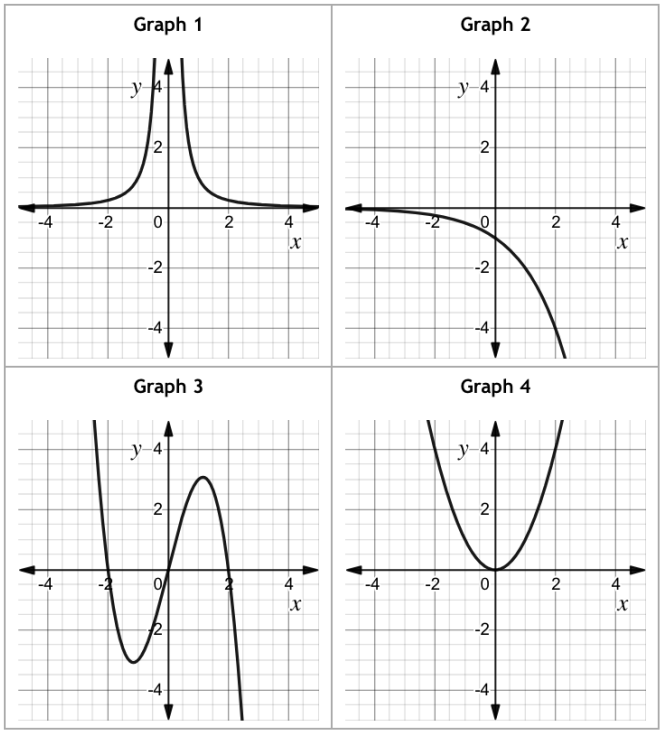


Match each equation in the table with a graph above.

Equation	Graph Number
$y = x^2$	
$y = x(-x+2)(x+2)$	
$y = \dfrac{-1}{x}$	
$y = -0.5^x$	

Your Turn

Four graphs are sketched below.

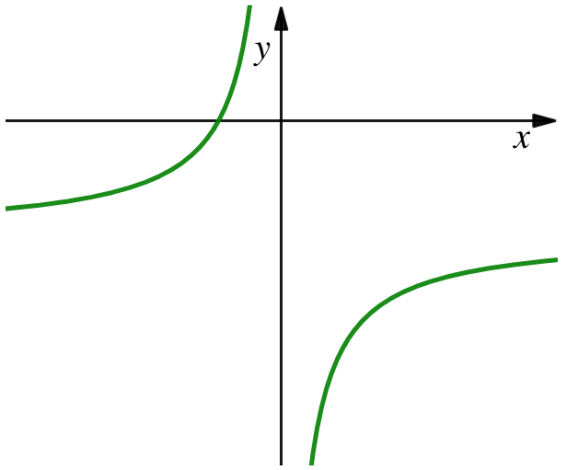


Match each equation in the table with a graph above.

Equation	Graph Number
$y = x^2$	
$y = x(-x+2)(x+2)$	
$y = \dfrac{1}{x^2}$	
$y = -2^x$	

Worked Example

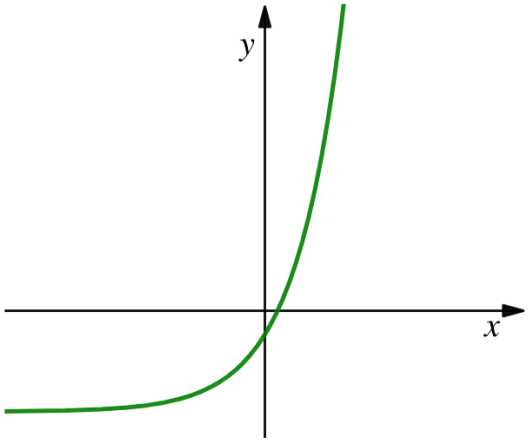
Select the equation of the graph sketched below.



- ○ $y = 2x^3 + 2x^2 + 4x - 4$
- ○ $y = 3x + 3$
- ○ $y = 4^x + 1$
- ○ $y = 4 \times 2^x + 2$
- ○ $y = -\frac{4}{x} - 3$
- ○ $y = 4x + 4$

Your Turn

Select the equation of the graph sketched below.



- ○ $y = -3x^2 - 2x - 3$
- ○ $y = -x^2 - 4x + 2$
- ○ $y = 2x^2 - 3x + 2$
- ○ $y = 3 \times 3^x - 4$
- ○ $y = -\frac{4}{x} - 3$
- ○ $y = 2x + 2$

Extra Notes